QUESTION PAPER CODE 65/2/B

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$ ½ m

$y = \pm \sqrt{40}$ or $\pm 2\sqrt{10}$ ½ m

2. $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$ ½ m

$= 2a^2$ ½ m

3. using $\sin \theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ ½ m

$\Rightarrow \theta = 0^{\circ}$ ½ m

4. $x = 2, \; y = 9$ (½ for correct x or y)

$\therefore \; x + y = 11$ ½ m

5. order 3, or degree 1 ½ m

$\therefore$ Degree + order = 4 ½ m

6. $\dfrac{dy}{dx} + \dfrac{y}{x \log x} = \dfrac{2}{x}$ (Standard form) ½ m

I.F. $= \log x$ ½ m

SECTION - B

7. $\dfrac{y}{x} = [\log x - \log(a + bx)]$ ½ m

$\Rightarrow \dfrac{x\dfrac{dy}{dx} - y}{x^2} = \dfrac{1}{x} - \dfrac{b}{a + bx}$ 1 m

$$\Rightarrow \quad x\frac{dy}{dx} - y = \frac{ax}{a+bx} \text{ .... (i)}$$ 1 m

Differentiating again,

$$x\frac{d^2y}{dx^2} = \frac{a^2}{(a+bx)^2}$$ 1 m

$$x^3 \cdot \frac{d^2y}{dx^2} = \left(\frac{ax}{a+bx}\right)^2 = \left(x\frac{dy}{dx} - y\right)^2 \text{ (using (i))}$$ ½ m

8. $$u = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2\cos^{-1}x \Rightarrow \frac{du}{dx}\ \frac{-2}{\sqrt{1-x^2}}$$ 1½ m

$$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}}$$ 1 m

$$\left.\frac{dv}{dx}\right|_{x=\frac{1}{2}} = \frac{2}{x} = 4$$ 1½ m

9. $$\text{Let } I = \int_0^{\pi/2} \frac{5\sin x + 3\cos x}{\sin x + \cos x}\,dx \text{ .... (i)}$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \frac{5\cos x + 3\sin x}{\cos x + \sin x}\,dx \text{ .... (ii)} \quad \left(\because \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx\right)$$ 1½ m

Adding (i) and (ii) 1+1 m

$$2I = 8\int_0^{\pi/2} 1 \cdot dx = 4\pi$$ ½ m

$$\Rightarrow \quad I = 2\pi$$

OR

put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t\,dt$ 1 m

$$= \int e^t\left(\log t + \frac{1}{t^2}\right)dt$$

$$= \int e^t \left[\left(\log t - \frac{1}{t}\right) + \left(\frac{1}{t} + \frac{1}{t^2}\right)\right] dt$$ 1½ m

$$= e^t \left(\log t - \frac{1}{t}\right) + c$$ 1 m

$$= x\left[\log(\log x) - \frac{1}{\log x}\right] + c$$ ½ m

10. $$[15000 \quad 15000]\begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800]$$ 2 m

$\Rightarrow \quad 300 + 150x = 1800$ 1 m

$\Rightarrow \quad x = 10\%$

yes : compassionate or any other relevant value 1 m

11. $$\cot^{-1}(x+1) = \sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}$$ 1½ m

and $$\tan^{-1}x = \cos^{-1}\frac{1}{\sqrt{1+x^2}}$$ 1½ m

$$\therefore \quad \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$

$$\Rightarrow \quad 1 + x^2 + 2x + 1 = 1 + x^2 \Rightarrow x = -\frac{1}{2}$$ 1 m

OR

$$2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$$

$$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$$ 1 m

$$= \quad \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$$ 1 m

$$= \quad \tan^{-1} 1 = \frac{\pi}{4}$$ 1+1 m

12. $C_1 \to C_1 + C_2 + C_3$,

$$(a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because\ a+b+c \neq 0)$$ 2 m

$$\Rightarrow \quad -a^2 - b^2 - c^2 + ab + bc + ca = 0$$ ½ m

$$\Rightarrow \quad -\frac{1}{2}\,[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$$ ½ m

$$\Rightarrow \quad a = b = c$$

13. $$\begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$ 1 m

$R_2 \to R_2 - 2R_1$,

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_2 \to R_2 - 3R_3$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$R_1 \rightarrow R_1 + R_2\,,\quad R_3 \rightarrow R_3 - 2\,R_2$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A$$ (2 marks for all operations)

$$\therefore \quad A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix}$$ 1 m

14. $f(x) = x - \left| x - x^2 \right| = \left| x - x(1-x) \right| = \begin{cases} 2x - x^2 & , \quad -1 \le x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \quad 0 < x \le 1 \end{cases}$ 1 m

f(x) being a polynomial is continuous on $[-1, 0] \cup [0, 1]$

$\underset{x \to 0^-}{\text{lt}} f(x) = \underset{x \to 0^-}{\text{lt}} (2x - x^2) = 0$ ½ m

$\underset{x \to 0^+}{\text{lt}} f(x) = \underset{x \to 0^+}{\text{lt}} x^2 = 0$ ½ m

Also, $f(0) = 0$

$\because \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$ 1 m

$\Rightarrow$ There is no point of discontinuity on $[-1, 1]$ 1 m

15. $\left.\begin{array}{l} \vec{a}_1 = -\hat{i}\,,\ \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 = -2\hat{j} + \hat{k}\,,\ \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{array}\right\}$ 1 m

$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k}$ ½ m

$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k}$ ½ m

$\left|\vec{b}_1 \times \vec{b}_2\right| = \frac{7}{12}$ 1 m

$$\text{S.D.} = \left|\frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|}\right| = 2$$ 1 m

OR

Foot of perpendicular are (0, b, c) & (a, 0, c) 1 m

Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0$$ 2 m

$\Rightarrow \quad bcx + acy - abz = 0$ 1 m

16. $p(x = 2) = 9 . P(x = 3)$ 1 m

$\Rightarrow \ {}^3C_2\, p^2\, q = 9 \cdot {}^3C_3\, p^3 \cdot q^0$ 1 m

$\Rightarrow \ 3\, p^2\, (1 - p) = 9\, p^3$ 1 m

$\Rightarrow \quad p = \frac{1}{4}$ 1 m

OR

Let H_1 be the event that red ball is drawn

H_2 be the event that black ball is drawn

E be the event that both balls are red

$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8}$ 1 m

$P(E/H_1) = \frac{5_{C_2}}{10_{C_2}} = \frac{2}{9}, \quad P(E/H_2) = \frac{3_{C_2}}{10_{C_2}} = \frac{1}{15}$ 1 m

$P(E) = P(H_1)\ P(E/H_1) + P(H_2) \cdot P(E/H_2)$ 1 m

$$= \frac{3}{8}\cdot\frac{2}{9} + \frac{5}{8}\cdot\frac{1}{15} = \frac{1}{8}$$ 1 m

17. $I = \int \frac{x\cos x}{\cos x + x\sin x}\,dx$ 1 m

put $\cos x + x\sin x = t$

$\Rightarrow \quad x\cos x\,dx = dt$ 1 m

$$= \int \frac{dt}{t}$$ 1 m

$$= \log\left|\cos x + x\sin x\right| + c$$ 1 m

18. $\int \frac{x^4\,dx}{(x-1)(x^2+1)} = \int \left[(x+1) + \frac{1}{(x-1)(x^2+1)}\right]dx$ 1 m

(using partial fractions)

$$= \int (x+1)\,dx + \frac{1}{2}\int \frac{dx}{(x-1)} - \frac{1}{2}\int \frac{x+1}{x^2+1}\,dx$$ 1½ m

$$= \frac{x^2}{2} + x + \frac{1}{2}\log|x-1| - \frac{1}{4}\log(x^2+1) - \frac{1}{2}\tan^{-1}x + c$$ 1½ m

19. $$\left.\begin{aligned}\overrightarrow{AB} &= -2\hat{i} - 5\hat{k}\\ \overrightarrow{AC} &= \hat{i} - 2\hat{j} - \hat{k}\end{aligned}\right\}$$ 1 m

$\overrightarrow{AB} \times \overrightarrow{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1 m

$\left|\overrightarrow{AB} \times \overrightarrow{AC}\right| = \sqrt{165}$ ½ m

$$\hat{n} = \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|}$$ 1 m

$$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}} \text{ or } \frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}}$$ ½ m

SECTION - C

20. $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

put $y = v\,x \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1 m

$\Rightarrow \frac{1+v^2}{v^3} = -\frac{dx}{x}$ 1 m

Integrating both sides

$-\frac{1}{2\,v^2} + \log v = -\log x + c$ 1 m

$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$

when $x = 1,\ y = 1 \Rightarrow c = -\frac{1}{2}$ 1 m

$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2}$ ½ m

when $x = x_0,\ y = e \Rightarrow x_0 = \sqrt{3}\,e$ 1½ m

OR

$I\ F = e^{\int \tan x\,dx} = e^{\log \sec x} = \sec x$ 1 m

$\therefore \frac{d}{dx}(y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$ 1 m

$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x\,dx + c$ 2 m

$\Rightarrow y = x^3 + c \cos x$

when $x = \frac{\pi}{3},\ y = 0$; we get $c = \frac{-2\pi^3}{27}$ 1 m

$\therefore \quad y = x^3 - \dfrac{2\pi^3}{27}\cos x$ 1 m

21. Equation of line is $\dfrac{x-3}{2} = \dfrac{y-4}{-3} = \dfrac{z-1}{5}$ 1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0$$ 1 m

$\Rightarrow \quad 2x + y + z - 7 = 0$ .... (i) 1 m

general point on given line $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$ lies on (i) 1 m

$\therefore \quad 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\dfrac{2}{3}$ 1 m

$\therefore$ Point of intersection $\left(\dfrac{5}{3}, 6, -\dfrac{7}{3}\right)$ 1 m

22.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

4 m

$\forall \; a \in \{0, 1, 2, 3, 4, 5, 6\}$

$a * 0 = a = 0 * a \;\Rightarrow\;$ 0 is identity 1 m

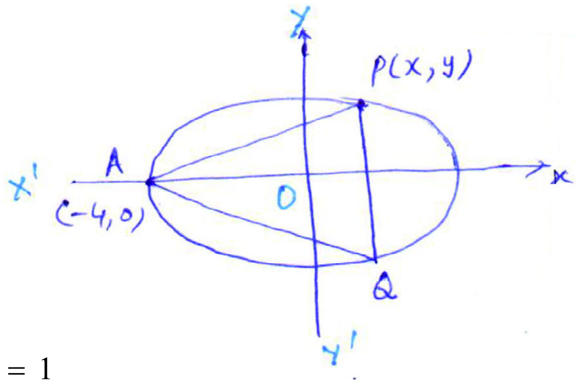
$\forall \; a \in \{1, 2, 3, 4, 5, 6\}$

$a * b = 0 = b * a$

$\Rightarrow \quad a * (7 - a) = 0 = (7 - a) * a$

$\Rightarrow \quad (7 - a)$ is inverse of a 1 m

23. $A = y(x + 4)$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Let $z = A^2 = \frac{9}{16}(16 - x^2)(x + 4)^2 \quad \Rightarrow \quad y^2 = \frac{9}{16}(16 - x^2)$ .... (i) 1 m

$$= \frac{9}{16}(4 - x)(4 + x)^3$$ 1 m

$$\frac{dz}{dx} = \frac{9}{16}(4 + x)^2 (8 - 4x)$$ 1 m

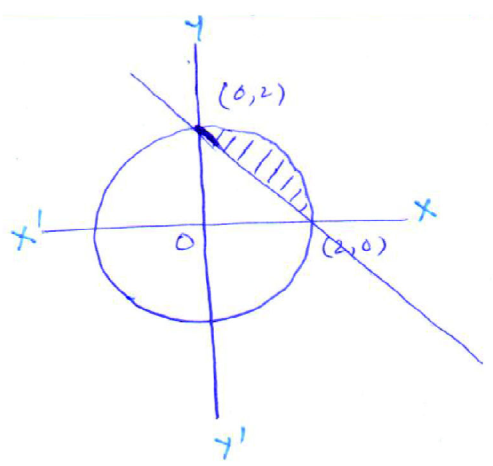
$$\frac{dz}{dx} = 0 \Rightarrow x = 2$$ 1 m

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4 + x)^2 + \frac{9}{8}(4 + x)(8 - 4x)$$

$$\left.\frac{d^2z}{dx^2}\right|_{x=2} < 0$$ 1 m

$\therefore$ Maximum value of A $= 9\sqrt{3}$ sq. units 1 m

24.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2}\,dx - \int_0^2 (2-x)\,dx$$ 2 m

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[2x - \frac{x^2}{2}\right]_0^2$$ 1+1 m

$$= (\pi - 2) \text{ sq. units}$$ 1 m

25. Let us consider the man invested on x electronic and y manually operated machines

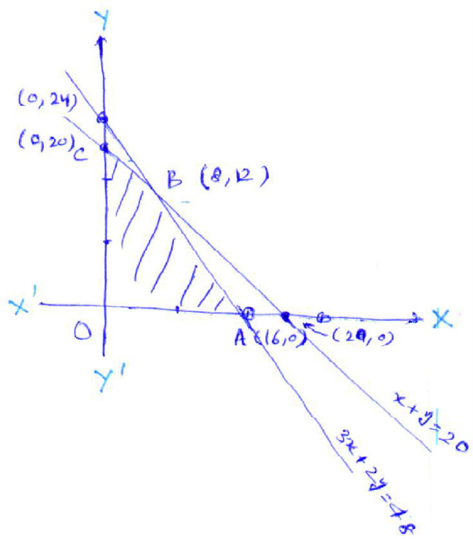
Maximise $P = 220x + 180y$ .... (i) 1 m

subject to

$$x + y \le 20$$

$$3600x + 2400y \le 57600 \quad \Rightarrow \quad 3x + 2y \le 48$$ 1½ m

$$x, y \ge 0$$



(1 mark for plotting each line) = 2 m

(½ to find the vertices of feasible region)

$P\big|_{A(16,0)} = 3520$ Rs.

$P\big|_{B(8,12)} = 3920$ Rs.

$P\big|_{C(0,20)} = 3600$ Rs.

Maximum profit is Rs. 3920 at x = 8, y = 12 — 1 m

26. Let H_1 : be the event 1, 2 appears

H_2 : be the event 3, 4, 5, 6 appears — 1 m

E_3 : be the event that head appears

$P(H_1) = \frac{2}{6} = \frac{1}{3}$, $P(H_2) = \frac{4}{6} = \frac{2}{3}$ — 1 m

$P(E/H_1) = \frac{3}{8}$ $P(E/H_2) = \frac{1}{2}$ — 1 m

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2)\ P(E/H_2)}$$ — 1 m

$$= \frac{8}{11}$$ — 2 m

OR

Let H_1 : be the event that 4 occurs

H_2 : be the event that 4 does not occurs — 1 m

E : be the event that man reports 4 occurs

on a throw of dice

$P(H_1) = \frac{1}{6}$, $P(H_2) = \frac{5}{6}$ — 1 m

$P(E/H_1) = \frac{3}{5}$ $P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5}$ — 1 m

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)}$$ — 1 m

$$= \frac{3}{13}$$ — 2 m