# Marking Scheme

## Class-XII

## Mathematics (March 2012)

| Q.No. | Value Pooints/Solution | 65/1//1 | Marks. |
|---|---|---|---|

### SECTION-A

**1-10**

**1.** $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$ **2.** $\lambda = 5$ **3.** $-4\hat{j} - \hat{k}$ **4.** $\log\left(\frac{3}{2}\right)$ **5.** $\frac{3}{2}x^{3/2} - \frac{2}{5}x^{5/2} + c$

**6.** $M_{2,3} = 7$ **7.** 13 **8.** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **9.** $\frac{2\pi}{3}$ **10.** 35. $1\times10 = 10$

### SECTION-B

**11.**

$(\cos x)^y = (\cos y)^x \Rightarrow y \log \cos x = x \log \cos y$ 1/2

$\therefore \quad y.\frac{(-\sin x)}{\cos x} + \log \cos x.\frac{dy}{dx} = x\frac{(-\sin y)}{\cos y}\frac{dy}{dx} + \log \cos y$ 1+1

$(\log \cos x + x \tan y)\frac{dy}{dx} = \log \cos y + y \tan x$ 1

$\therefore \quad \frac{dy}{dx} = \frac{\log \cos y + y \tan x}{\log \cos x + x \tan y}$ 1/2

**OR**

$\sin y = x \sin(a+y) \Rightarrow \cos y \frac{dy}{dx} = x \cos(a+y)\frac{dy}{dx} + \sin(a+y)$ 1

$\therefore \quad \frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - x\cos(a+y)}$ 1

$x = \frac{\sin y}{\sin(a+y)} \Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{\cos y - \frac{\sin y}{\sin(a+y)}.\cos(a+y)}$ 1

$\therefore \quad \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y)\cos y - \cos(a+y)\sin y} = \frac{\sin^2(a+y)}{\sin a}$ 1

**12.** Let the coin be tossed $n$ times

$\therefore$ P(getting at least one heat) > $\dfrac{80}{100}$ 1

$\therefore\ 1 - P(0) > \dfrac{8}{10} \Rightarrow P(o) < 1 - \dfrac{8}{10} = \dfrac{2}{10} = \dfrac{1}{5}$ 1

$\therefore\ {}^nC_0\left(\dfrac{1}{2}\right)^0\left(\dfrac{1}{2}\right)^n < \dfrac{1}{5}$ or $\dfrac{1}{2^n} < \dfrac{1}{5}$ or $2^n > 5$ 1

$\Rightarrow n = 3.$ 1

**13.** Let the vector equation of required line be $\vec{a} = \vec{a} + \lambda\vec{b}$

than $\vec{a} = \hat{i} + 2\hat{j} - 4\hat{k}$

and $\vec{b} = (3\hat{i} - 16\hat{j} + 7\hat{k}) \times (3\hat{i} + 8\hat{j} - 5\hat{k})$ 1

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$ 1

$\therefore$ Vector equation of line is

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(24\hat{i} + 36\hat{j} + 72\hat{k})$

or $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ 1

and cartesian from is $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z+4}{6}$ 1

**14.** $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow (\vec{a} + \vec{b} + \vec{c})^2 = 0$ 1/2

$\Rightarrow \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ 1

or $|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$ 1

$\therefore\ \vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -\dfrac{1}{2}(25 + 144 + 169) = -169.$ 1

**15.** $2x^2\dfrac{dy}{dx} - 2xy + y^2 = 0 \Rightarrow \dfrac{dy}{dx} = \dfrac{2xy - y^2}{2x^2} = \dfrac{2\dfrac{y}{x} - \dfrac{y^2}{x^2}}{2}$ 1/2

Putting $\dfrac{y}{x} = v$ so that $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ 1

$\therefore\ v + x\dfrac{dv}{dx} = v - \dfrac{1}{2}v^2 \ \therefore\ x\dfrac{dv}{dx} = -\dfrac{1}{2}v^2$ 1/2

$\Rightarrow 2\int\dfrac{dv}{v^2} = -\int\dfrac{dx}{x} \Rightarrow \dfrac{2}{v} = \log x + c$ 1

$\therefore\ \dfrac{2x}{y} = \log x + c \ \therefore\ y = \dfrac{2x}{\log x + c}$ 1

**16.** $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2 = (1+x^2)(1+y^2)$ ½

$\Rightarrow \int \frac{dy}{1+y^2} = \int (1+x^2)dx$ 1

$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c$ 1

$x = 0,\ y = 1 \Rightarrow c = \pi/4$ 1

$\therefore \tan^{-1} y = x + \frac{x^3}{3} + \frac{\pi}{4}$ or $y = \tan\left(\frac{\pi}{4} + x + \frac{x^3}{3}\right)$ ½

**17.** $I = \int \sin x \sin 2x \sin 3x dx = \frac{1}{2}\int 2\sin 3x \sin x . \sin 2x dx$ ½

$= \frac{1}{2}\int (\cos 2x - \cos 4x)\sin 2x dx = \frac{1}{2}\int (\sin 2x \cos 2x - \cos 4x \sin 2x)dx$ ½

$= \frac{1}{4}\int \sin 4x dx - \frac{1}{4}\int 2\cos 4x \sin 2x dx$ 1

$= -\frac{1}{16}\cos 4x - \frac{1}{4}\int (\sin 6x - \sin 2x)dx$ 1

$= -\frac{1}{16}\cos 4x + \frac{1}{24}\cos 6x - \frac{1}{8}\cos 2x + c$ 1

**OR**

$\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ ½

$2 = A(1+x^2) + (Bx+C)(1-x)$ 1½

$\Rightarrow 0 = A - B,\ B - C = 0\ \ A + C = 2 \Rightarrow A = B = C = 1$

$\therefore \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x+1}{x^2+1}dx$ ½

$= -\log|1-x| + \frac{1}{2}(x^2+1) + \tan^{-1} x + c$ 1½

**18.** Slope of tangent, $y = x - 11$ is 1 ½

$y = x^3 - 11x + 5 \Rightarrow \frac{dy}{dx} = 3x^2 - 11$ ½

If the point is $(x_1, y_1)$ then $3x_1^2 - 11 = 1 \Rightarrow x_1 = \pm 2$ 1

$x_1 = 2$ then $y_1 = 8 - 22 + 5 = -9$ and if $x_1 = -2$ then $y_1 = 19$ 1

Since (–2, 19) do not lie on the tangent $y = x - 11$ ½

$\therefore$ Required point is (2, – 9) ½

**OR**

Let $y = \sqrt{x} \quad \therefore \quad y + \Delta y = \sqrt{x + \Delta x}$ ½

$$\Rightarrow \quad y + \frac{dy}{dx}\Delta x \approx \sqrt{x + \Delta x}$$

$$\Rightarrow \quad \sqrt{x} + \frac{1}{2\sqrt{x}}.\Delta x \approx \sqrt{x + \Delta x}$$ 1

Putting $x = 49$ and $\Delta x = 0.5$ we get 1

$$\sqrt{49} + \frac{1}{2\sqrt{49}}(0.5) \approx \sqrt{49.5}$$ ½

$$\Rightarrow \quad \sqrt{49.5} = 7 + \frac{1}{28} = 7.0357$$ 1

**19.** $y = (\tan^{-1} x)^2 \;\Rightarrow\; \dfrac{dy}{dx} = 2\tan^{-1} x \,\dfrac{1}{1 + x^2}$

$$\Rightarrow \quad (1 + x^2)\frac{dy}{dx} = 2\tan^{-1} x$$

$$\therefore \quad (1 + x^2)\frac{d^2y}{dx^2} + 2x.\frac{dy}{dx} = \frac{2}{1 + x^2}$$

$$\Rightarrow \quad (1 + x^2)^2.\frac{d^2y}{dx^2} + 2x(1 + x^2)\frac{dy}{dx} = 2.$$

**20.** Using $R_1 \to R_1 + R_2 + R_3$ we get

$$\text{LHS} = \begin{vmatrix} 2(a+b+c) & 2(p+q+r) & 2(x+y+z) \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$ 1

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$ 1

$$= 2\begin{vmatrix} a+b+c & p+q+r & x+y+z \\ -b & -q & -y \\ -c & -r & -z \end{vmatrix}$$ Using $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$ 1

$$= 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} = \text{RHS}$$ Using $R_1 \to R_1 + R_2 + R_3$, $R_2 \to -R_2$, $R_3 \to -R_3$ 1

**21.** $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \tan^{-1}\left(\frac{\sin\left(\frac{\pi}{2}-x\right)}{1+\cos\left(\frac{\pi}{2}-x\right)}\right)$ 1

$= \tan^{-1}\left(\frac{2\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}{2\cos^2\left(\frac{\pi}{4}-\frac{x}{2}\right)}\right) = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)$ 1+1

$= \frac{\pi}{4}-\frac{x}{2}$ 1

**OR**

Writing $\sin^{-1}\left(\frac{8}{17}\right) = \tan^{-1}\frac{8}{15}$ and $\sin^{-1}\left(\frac{3}{5}\right) = \tan^{-1}\frac{3}{4}$ 1

$\therefore$ LHS $= \tan^{-1}\frac{8}{15}+\tan^{-1}\frac{3}{4} = \tan^{-1}\left(\frac{\frac{8}{15}+\frac{3}{4}}{1-\frac{8}{15},\frac{3}{4}}\right) = \tan^{-1}\left(\frac{77}{36}\right)$ 1+1

Getting $\tan^{-1}\left(\frac{77}{36}\right) = \cos^{-1}\left(\frac{36}{85}\right)$ 1

**22.** Let $x_1, x_2 \in A$ and $f(x_1) = f(x_2)$ ½

$\Rightarrow \frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3} \quad \therefore \quad x_1x_2-2x_2-3x_1 = x_1x_2-2x_1-3x_2$

$\Rightarrow x_1 = x_2$ 1

Hence $f$ is 1 – 1

Let $y \in B$, $\therefore$ $y = f(x) \Rightarrow y = \frac{x-2}{x-3} \Rightarrow xy-3y = x-2$

or $x = \frac{3y-2}{y-1}$ ½

Since $y \neq 1$ and $\frac{3y-2}{y-1} \neq 3 \therefore x \in A$

Hence $f$ is ONTO 1

and $f^{-1}(y) = \frac{3y-2}{y-1}$ 1

## SECTION-C

**23.** Normal to the plane is $\vec{n} = \overrightarrow{AB}\times\overrightarrow{BC}$ ½

$\therefore$ $n = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 12\hat{i}-16\hat{j}+12\hat{k}$ 1½

$\therefore$ Equation of plane is

$$\vec{r}.(12\hat{i} - 16\hat{j} + 12\hat{k}) = (3\hat{i} - \hat{j} + 2\hat{k}).(12\hat{i} - 16\hat{j} + 12\hat{k})$$
$$= 76$$ 2

or $\vec{r}.(3\hat{i} - 4\hat{j} + 3\hat{k}) = 19$ or $3x - 4y + 3z - 19 = 0$

Distance of plane from the point P(6, 5, 9) is

$$d = \frac{|18 - 20 + 27 - 19|}{\sqrt{9 + 16 + 9}} = \frac{6}{\sqrt{34}}$$ 2

**24.** Let $E_1$ : selected student is a hostlier

$E_2$ : selected student is a day scholar 1

A : selected student attain 'A' grade in exam.

$$P(E_1) = \frac{60}{100}, \quad P(E_2) = \frac{40}{100}$$ 1

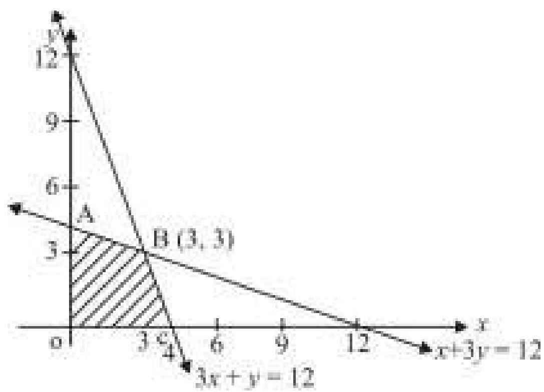
$$P(A/E_1) = \frac{30}{100}, \quad P(A/E_2) = \frac{20}{100}$$ 1

$$P(E_1/A) = \frac{P(E_1) . P(A/E_1)}{P(E_1) . P(A/E_1) + P(E_2) . P(A/E_2)}$$ 1

$$= \frac{60/100 . 30/100}{60/100 . 30/100 + \frac{40}{100}.\frac{20}{100}} = \frac{9}{13}$$ 1+1

**25.** Let $x$ package of nuts and $y$ package of bolts be produced each day

$\therefore$ LPP is maximise $P = 17.5x + 7y$ 1




subject to $x + 3y \le 12$

$3x + y \le 12$ 2

$x \ge 0, y \ge 0$ correct graph

vertices of feasible region are A(0, 4), B (3, 3), C (4, 0)

Profit is maximum at B(3, 3)

i.e. 3 package of nuts and 3 package of bolts 1

**26.** $\text{I} = \int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})dx = \int_0^{\pi/4} \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$ 1

Putting $\sin x - \cos x = t,$ to get $(\cos x + \sin x)dx = dt$ 1

and $\sin x \cos x = \frac{1-t^2}{2}$ 1

$\therefore \quad \text{I} = \sqrt{2}\int_{-1}^{0} \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}.[\sin^{-1} t]_{-1}^{0}$ 1+1

$= \sqrt{2}(\sin^{-1} 0 - \sin^{-1}(-1) = \sqrt{2}.\frac{\pi}{2}$ 1

**OR**

$\text{I} = \int_1^3 (2x^2 + 5x)dx = \lim_{h \to 0} h[f(1) + f(1+h) + f(1+2h) + ... + f(1+\overline{n-1}h)]$

where $f(x) = 2x^2 + 5x$ and $h = \frac{2}{n}$ or $nh = 2.$ 1

$f(1) = 7$

$f(1+h) = 2(1+h)^2 + 5(1+h) = 7 + 9h + 2^2$

$f(1+2h) = 2(1+2h)^2 + 5(1+2h) = 7 + 18h + 22^2 h^2$ 2

$f(1+3h) = 2(1+3h)^2 + 5(1+3h) = 7 + 27h + 2.3^2 h^2$

...........

$f(1+(n-1)h) = 7 + 9(n-1)h + 2.(n-1)^2 h^2$

$\therefore \quad \text{I} = \lim_{h \to 0} h\left[7n + 9h\frac{n(n-1)}{2} + 2h^2.\frac{n(n-1)(2n-1)}{6}\right]$ 1

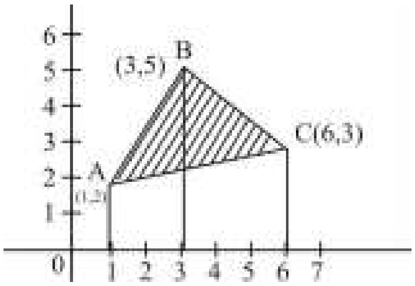
$= \lim_{h \to 0}\left[7nh + \frac{9}{2}nh(nh-h) + \frac{1}{3}nh(nh-h)(2nh-h)\right]$ 1

$= 14 + 18 + \frac{16}{3} = \frac{112}{3}$ 1

**27.** Let AB be $3x - 2y + 1 = 0,$ BC be $2x + 3y - 21 = 0$ and AC be $x - 5y + 9 = 0$ correct figure : 1

Solving to get A(1, 2), B (3, 5) and C(6, 3) 1½

area of $(\Delta \text{ABC}) = \frac{1}{2}\int_1^3 (3x+1)dx + \frac{1}{3}\int_3^6 (21-2x)dx - \frac{1}{5}\int_1^6 (x+9)dx$ 1




$$= \frac{1}{12}(3x+1)^2\Big]_1^3 + \frac{(21-2x)^2}{-12}\Big]_3^6 - \frac{(x+9)^2}{10}\Big]_1^6$$ 1½

$$= 7+12-\frac{25}{2}$$ ½

$$= \frac{13}{2} \text{ sq. U.}$$ ½

**28.** Surface area $A = 2\pi rh + 2\pi r^2$ (Given) ½

$\Rightarrow$ $h = \dfrac{A-2\pi r^2}{2\pi r}$ ...(1) 1


$$V = \pi r^2 h = \pi r^2\left(\frac{A-2\pi r^2}{2\pi r}\right)$$ 1

$$= \frac{1}{2}.[Ar-2\pi r^3]$$ ½

$$\frac{dv}{dr} = \frac{1}{2}[A-6\pi r^2]$$ 1

$$\frac{dv}{dr} = 0 = \Rightarrow 6\pi r^2 = A = 2\pi rh + 2\pi r^2$$ 1

$$\Rightarrow 4\pi r^2 = 2\pi rh \Rightarrow h = 2r = \text{diameter}$$ 1

$$\frac{d^2v}{dr^2} = \frac{1}{2}[-12\pi r] < 0 \therefore h = 2r \text{ will give max. volume.}$$ 1

**29.** Given equations can be written as

$$\begin{pmatrix}1 & -1 & 2\\ 3 & 4 & -5\\ 2 & -1 & 3\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \begin{pmatrix}7\\ -5\\ 12\end{pmatrix} \text{ or } AX = B$$ 1

$a_{11} = 7$, $a_{12} = -19$ $a_{13} = -11$

$a_{21} = 1$, $a_{22} = -1$ $a_{23} = -1$

$a_{31} = -3$, $a_{32} = 11$ $a_{33} = 72$

$\Rightarrow$ $$A^{-1} = \frac{1}{4}\begin{pmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{pmatrix}$$ ½

$\therefore$ $$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = \frac{1}{4}\begin{pmatrix}7 & 1 & -3\\ -19 & -1 & 11\\ -11 & -1 & 7\end{pmatrix}\begin{pmatrix}7\\ -5\\ 12\end{pmatrix} = \begin{pmatrix}2\\ 1\\ 3\end{pmatrix}$$

$\Rightarrow$ $x = 2, y = 1, z = 3.$ 1½

**OR**

Let $A = \begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}$ $\therefore$ Writing $\begin{pmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix} = \mathbf{A} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ 1

$c_1 \leftrightarrow c_2 \Rightarrow$ $\begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 1 \end{pmatrix} = A \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ½

$c_2 \to c_2 + c_1$
$c_3 \to c_3 - 2c_1$ $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & -1 \\ 1 & 4 & -1 \end{pmatrix} = A \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$ 1

$c_1 \to c_1 + 2c_3$
$c_2 \to c_2 + 2c_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ -1 & 2 & -1 \end{pmatrix} = A \begin{pmatrix} 0 & 1 & 0 \\ -3 & -3 & -2 \\ 2 & 2 & 1 \end{pmatrix}$ ½

$c_3 \to c_3 + -c_2$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & 1 \end{pmatrix} = A \begin{pmatrix} 0 & 1 & 1 \\ -3 & -3 & -5 \\ 2 & 2 & 3 \end{pmatrix}$ ½

$c_1 \to c_1 + c_3$
$c_2 \to c_2 + 2c_3$ $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = A \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$ 1

$\Rightarrow$ $A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{pmatrix}$ 1