QUESTION PAPER CODE 65/1/2

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. 5   2. $\left\{\vec{r} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 0$

or

$$\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = a + b + c$$

3. $2x^{3/2} + 2\sqrt{x} + c$ 4. 10   5. $x = 2$

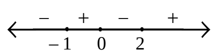
6. $x = \pm 6$ 7. $x = \frac{1}{5}$ 8. $x = 25$

9. $\frac{\pi x}{2} - \frac{x^2}{2} + c$ 10. $\frac{\pi}{6}$ $1 \times 10 = 10$ m

SECTION - B

11. $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x+1)(x-2)$ 1+½ m

$f'(x) > 0, \ \forall x \in (-1, 0) \cup (2, \infty)$ 1 m



$f'(x) < 0, \ \forall x \in (-\infty, -1) \cup (0, 2)$ 1 m

$\therefore$ f(x) is strictly increasing in $(-1, 0) \cup (2, \infty)$

and strictly decreasing in $(-\infty, -1) \cup (0, 2)$ ½ m

OR

Point at $\theta = \pi/4$ is $\left(\frac{a}{2\sqrt{2}}, \frac{a}{2\sqrt{2}}\right)$ ½ m

$\frac{dy}{d\theta} = -3a\cos^2\theta\sin\theta; \ \frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta$ 1 m

$\therefore$ slope of tangent at $\theta = \frac{\pi}{4}$ is $\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = \left.\frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}\right]_{\theta=\frac{\pi}{4}}$

$= -\cot\frac{\pi}{4} = -1$ 1 m

Equation of tangent at the point :

$$y - \frac{a}{2\sqrt{2}} = -1\left(x - \frac{a}{2\sqrt{2}}\right) \Rightarrow x + y - \frac{a}{\sqrt{2}} = 0$$ 1 m

Equation of normal at the point :

$$y - \frac{a}{2\sqrt{2}} = 1\left(x - \frac{a}{2\sqrt{2}}\right) \Rightarrow x - y = 0$$ ½ m

12. $$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x}\, dx = \int \frac{\left(\sin^2 x + \cos^2 x\right)\left[(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x \cos^2 x\right]}{\sin^2 x \cdot \cos^2 x}\, dx$$ 1½ m

$$= \int \left[\frac{1}{\sin^2 x \cdot \cos^2 x} - 3\right] dx$$

$$= \int \left[\frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x} - 3\right] dx$$ ½ m

$$= \int \left(\sec^2 x + \operatorname{cosec}^2 x - 3\right) dx$$ ½ m

$$= \tan x - \cot x - 3x + c$$ 1½ m

(Accept $-2\cot 2x - 3x + c$ also)

OR

$$\int (x-3)\sqrt{x^2 + 3x - 18}\, dx$$

$$= \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\, dx - \frac{9}{2}\int \sqrt{x^2+3x-18}\, dx$$ 1 m

$$= \frac{1}{2}\cdot\frac{2}{3}\left(x^2+3x-18\right)^{3/2} - \frac{9}{2}\int \sqrt{\left(x+3/2\right)^2 - \left(\frac{9}{2}\right)^2}\, dx$$ 1½ m

$$= \frac{1}{3}\left(x^2+3x-18\right)^{3/2} - \frac{9}{2}$$

$$\left\{\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x-18} - \frac{81}{8}\log\left|x+\frac{3}{2}+\sqrt{x^2+3x-18}\right| + c\right.$$ 1½ m

or $= \frac{1}{3}\left(x^2+3x-18\right)^{3/2} - \frac{9}{8}$

$$\left\{ (2x+3)\sqrt{x^2+3x-18} - \frac{81}{2}\log\left| x+\frac{3}{2}+\sqrt{x^2+3x-18}\right| + c\right.$$

13. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{x^2-1}y = \frac{2}{\left(x^2-1\right)^2}$$ 1 m

Integrating factor $= e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2-1$ 1 m

$\therefore$ Solution is $y\cdot(x^2-1) = \int \frac{2}{(x^2-1)^2}\cdot(x^2-1)\ dx + c$ 1 m

$$\Rightarrow \quad y\,(x^2-1) = 2\int \frac{1}{x^2-1}\ dx + c$$

$$\Rightarrow \quad y\,(x^2-1) = \log\left|\frac{x-1}{x+1}\right| + c$$ 1 m

14. $y = x^x \quad \therefore \log y = x\log x$, Taking log of both sides ½ m

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x + 1$, Diff. w r t "x" 1½ m

$\Rightarrow \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x}$, Diff. w r t "x" 1½ m

$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$ ½ m

15. $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = \left(\vec{a}+\vec{b}\right)\cdot\left\{\left(\vec{b}+\vec{c}\right)\times\left(\vec{c}+\vec{a}\right)\right\}$ ½ m

$= \left(\vec{a}+\vec{b}\right)\left\{\vec{b}\times\vec{c}+\vec{b}\times\vec{a}+\vec{c}\times\vec{c}+\vec{c}\times\vec{a}\right\}$ 1 m

$$= \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)+\vec{a}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{a}\cdot\left(\vec{c}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)$$ 1½ m

$$+\ \vec{b}\cdot\left(\vec{b}\times\vec{a}\right)+\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$$

$$\left\{\vec{a}\cdot\left(\vec{b}\times\vec{a}\right),\ \left\{\vec{a}\cdot\left(\vec{c}\times\vec{a}\right)=\vec{b}\cdot\left(\vec{b}\times\vec{c}\right)=\vec{b}\cdot\left(\vec{b}\times\vec{a}\right)=0\right\}\right.$$

$$=\ 2\ \left\{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right\}=2\left[\vec{a},\vec{b},\vec{c}\right]$$ 1 m

OR

$$\vec{a}+\vec{b}+\vec{c}=\vec{0}\quad\therefore\quad\vec{a}+\vec{b}=-\vec{c}$$ ½ m

$$\Rightarrow\quad\left(\vec{a}+\vec{b}\right)^2=\left(-\vec{c}\right)^2=\left(\vec{c}\right)^2$$ ½ m

$$\Rightarrow\quad\left|\vec{a}\right|^2+\left|\vec{b}\right|^2+2\,\vec{a}\cdot\vec{b}=\left|\vec{c}\right|^2$$ 1 m

$$\Rightarrow\quad 9+25+2\left|\vec{a}\right|\,\left|\vec{b}\right|\cos\theta=49,\quad \theta\text{ being angle between }\vec{a}\ \&\ \vec{b}$$ 1 m

$$\therefore\quad\cos\theta=\frac{15}{2\cdot3\cdot5}=\frac{1}{2}\quad\Rightarrow\quad\theta=\frac{\pi}{3}$$ 1 m

16. $$\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$$

$$=\ \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right\}$$ 2½ m

$$= \cot^{-1}\left\{\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right\} = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$ 1½ m

OR

$$\text{LHS} = 2\left(\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right) + \tan^{-1}\frac{1}{7}$$ 1½+½ m

$$= 2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right) + \tan^{-1}\frac{1}{7}$$ 1 m

$$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{25}{25} = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS}$$ 1 m

17. $\forall\ (a, b) \in A \times A$

$a + b = b + a \quad \therefore \quad (a, b)\ R\ (a, b) \quad \therefore$ R is reflexive 1 m

For $(a, b), (c, d) \in A \times A$

If $(a, b)\ R\ (c, d)$ i.e. $a + d = b + c \Rightarrow c + b = d + a$

then $(c, d)\ R\ (a, b) \quad \therefore$ R is symmetric 1 m

For $(a, b), (c, d), (e, f) \in A \times A$

If $(a, b)\ R\ (c, d)$ & $(c, d)\ R\ (e, f)$ i.e. $a + d = b + c$ & $c + f = d + e$

Adding, $a + d + c + f = b + c + d + e \Rightarrow a + f = b + e$

then (a, b) R (e, f) $\therefore$ R is transitive 1 m

$\therefore$ R is reflexive, symmetric and transitive

hence R is an equivalance relation ½ m

[(2, 5)] = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)} ½ m

18. let b_2, g_2 be younger boy and girl

and b_1, g_1 be elder, then, sample space of two children is

$S = \{(b_1, b_2), (g_1, g_2), (b_1, g_2), (g_1, b_2)\}$ 1 m

A = Event that younger is a girl = $\{(g_1, g_2), (b_1, g_2)\}$

B = Event that at least one is a girl = $\{(g_1, g_2), (b_1, g_2), (g_1, b_2)\}$

E = Event that both are girls = $\{(g_1, g_2)\}$

(i) $P(E/A) = \frac{P(E \cap A)}{P(A)} = \frac{1}{2}$ 1½ m

(ii) $P(E/B) = \frac{P(E \cap B)}{P(B)} = \frac{1}{3}$ 1½ m

19. $\text{LHS} = \begin{vmatrix} 2(a+b+c) & a & b \\ 2(a+b+c) & b+c+2a & b \\ 2(a+b+c) & a & c+a+2b \end{vmatrix}$ Using, $C_1 \to C_1 + C_2 + C_3$ 1 m

$= \begin{vmatrix} 2(a+b+c) & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix}$ Using, $R_2 \to R_2 - R_1;\ R_3 \to R_3 - R_1$ 2 m

$= 2(a+b+c)\ \{(a+b+c)^2 - 0\}$ Expanding along C_1

$= 2\ (a+b+c)^3 = \text{RHS}$ 1 m

20. let $u = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$; $v = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$; $x = \sin\theta \;\therefore\; \theta = \sin^{-1}x$

$\therefore\quad u = \tan^{-1}\left(\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}\right) = \tan^{-1}(\tan\theta) = \theta = \sin^{-1}x$ 1 m

& $\quad v = \sin^{-1}\left(2x\sqrt{1-x^2}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$ 1 m

$\dfrac{du}{dx} = \dfrac{1}{\sqrt{1-x^2}},\ \dfrac{dv}{dx} = \dfrac{2}{\sqrt{1-x^2}}$ 1 m

$\therefore\quad \dfrac{du}{dv} = \dfrac{1}{\sqrt{1-x^2}} \times \dfrac{\sqrt{1-x^2}}{2} = \dfrac{1}{2}$ 1 m

(In case, if $x = \cos\theta$ then answer is $-\dfrac{1}{2}$)

21. $\operatorname{cosec} x \cdot \log y \dfrac{dy}{dx} = -x^2 y^2 \Rightarrow \dfrac{\log y}{y^2}\, dy = -x^2 \sin x\, dx$ 1 m

Integrating both sides we get

$-\dfrac{\log y}{y} - \dfrac{1}{y} = -\left[-x^2\cos x + 2\int x\cos x\, dx\right]$ 1+1 m

$= -\left[-x^2\cos x + 2\left(x\sin x - \int 1\cdot\sin x\, dx\right)\right]$ ½ m

$\therefore\quad \dfrac{\log y}{y} - \dfrac{1}{y} = --x^2\cos x + 2x\sin x + 2\cos x + c$ ½ m

22. Equations of lines are :

$\dfrac{x-5}{4} = \dfrac{y-7}{4} = \dfrac{z+3}{-5};\ \dfrac{x-8}{7} = \dfrac{y-4}{1} = \dfrac{z-5}{3}$ 1 m

Here, $x_1 = 5,\ y_1 = 7,\ z_1 = -3;\quad x_2 = 8,\ y_2 = 4,\ z_2 = 5$
$a_1 = 4,\ b_1 = 4,\ c_1 = -5;\quad a_2 = 7,\ b_2 = 1,\ c_2 = 3$

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 3 & -3 & 8 \\ 4 & 4 & -5 \\ 7 & 1 & 3 \end{vmatrix} = 3(17) + 3(47) + 8(-24) = 0$$ 1½+1 m

$\therefore$ lines are co-planar ½ m

SECTION - C

23. Let x and y be electronic and manually operated sewing machines purchased respectively

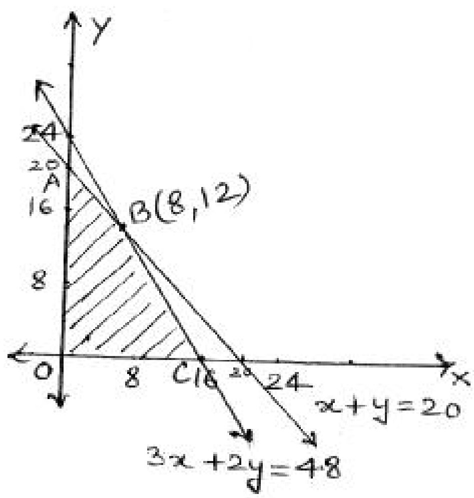
$\therefore$ L.P.P. is Maximize $P = 22x + 18y$ ½ m

subject to $360x + 240y \leq 5760$

or $3x + 2y \leq 48$

$x + y \leq 20$

$x \geq 0, y \geq 0$ 2 m



For correct graph 2 m

Vertices of feasible region are

A (0, 20), B(8, 12), C(16, 0) & O(0, 0)

P(A) = 360, P(B) = 392, P(C) = 352 ½ m

$\therefore$ For Maximum P, Electronic machines = 8

Manual machines = 12 1 m

24. Let E_1 : Event that lost card is a spade

E_2 : Event that lost card is a non spade ½ m

A : Event that three spades are drawn without replacement from 51 cards

$$P(E_1) = \frac{13}{52} = \frac{1}{4}, \quad P(E_2) = 1 - \frac{1}{4} = \frac{3}{4}$$ 1 m

$$P(A/E_1) = \frac{^{12}C_3}{^{51}C_3}, \quad P(A/E_2) = \frac{^{13}C_3}{^{51}C_3}$$ 1½ m

$$P(E_1/A) = \frac{\frac{1}{4}\cdot\frac{^{12}C_3}{^{51}C_3}}{\frac{1}{4}\cdot\frac{^{12}C_3}{^{51}C_3}+\frac{3}{4}\cdot\frac{^{13}C_3}{^{51}C_3}}$$ 1+1 m

$$= \frac{10}{49}$$ 1 m

OR

X = No. of defective bulbs out of 4 drawn = 0, 1, 2, 3, 4 1 m

Probability of defective bulb = $\frac{5}{15} = \frac{1}{3}$ ½ m

Probability of a non defective bulb = $1 - \frac{1}{3} = \frac{2}{3}$ ½ m

Probability distribution is :

x :	0	1	2	3	4	
P(x) :	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$	2½ m
x P(x) :	0	$\frac{32}{81}$	$\frac{48}{81}$	$\frac{24}{81}$	$\frac{4}{81}$	½ m

Mean = $\sum x\,P(x) = \frac{108}{81}$ or $\frac{4}{3}$ 1 m

25. Here
$$\begin{aligned} 3x + 2y + z &= 1000 \\ 4x + y + 3z &= 1500 \\ x + y + z &= 600 \end{aligned}$$ 1½

$$\therefore \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1000 \\ 1500 \\ 600 \end{pmatrix} \text{ or } A\cdot X = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \;\therefore\; X = A^{-1}B$ ½ m

Co-factors are

$A_{11} = -2,\quad A_{12} = -1,\quad A_{13} = 3$

$A_{21} = -1,\quad A_{22} = 2,\quad A_{23} = -1$ 1½ m

$A_{31} = 5,\quad A_{32} = -5,\quad A_{33} = -5$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5}\begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix}\begin{pmatrix} 1000 \\ 1500 \\ 600 \end{pmatrix}$$

$\therefore \quad x = 100,\ y = 200,\ z = 300$ 1½ m

i.e. Rs. 100 for discipline, Rs 200 for politeness & Rs. 300 for punctuality

One more value like sincerity, truthfulness etc. 1 m

26. Equation of plane through points A, B and C is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0 \Rightarrow 16x + 24y + 32z - 56 = 0$$

i.e. $2x + 3y + 4z - 7 = 0$ 3+1 m

Distance of plane from (7, 2, 4) = $\left| \frac{2(7) + 3(2) + 4(4) - 7}{\sqrt{9+16+4}} \right|$ 1 m

$= \sqrt{29}$ 1 m

OR

General point on the line is $(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$ 1 m

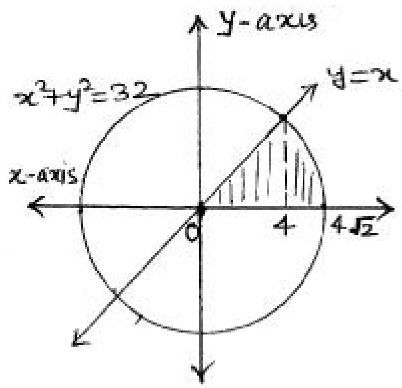
Putting in the equation of plane; we get

$1\cdot(2+3\lambda) - 1\cdot(-1+4\lambda) + 1\cdot(2+2\lambda) = 5$ 1½ m

$\therefore \quad \lambda = 0$ 1 m

Point of intersection is $2\hat{i} - \hat{j} + 2\hat{k}$ or $(2, -1, 2)$ 1½ m

Distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{169} = 13$ 1 m

27. 

Correct Figure 1 m

The line and circle intersect each other at $x = \pm 4$ 1 m

Area of shaded region

$$= \int_0^4 x\, dx + \int_4^{4\sqrt{2}} \sqrt{\left(4\sqrt{2}\right)^2 - x^2}\ dx$$ 1½ m

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\left\{\frac{x\sqrt{32-x^2}}{2} + 16\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right\}\right]_4^{4\sqrt{2}}$$ 1½ m

$$= 8 + 4\pi - 8 = 4\pi \text{ sq.units}$$ 1 m

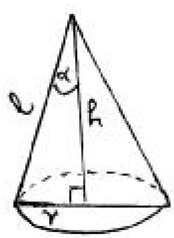
28. Let $I = \int_0^{\pi} \frac{x \tan x}{\sec x \operatorname{cosec} x}\, dx \quad \therefore \quad I = \int_0^{\pi} \frac{(\pi - x)\tan(\pi - x)}{\sec(\pi - x)\operatorname{cosec}(\pi - x)}\, dx$ 1 m

$$\Rightarrow \quad I = \int_0^{\pi} \frac{(\pi - x)\tan x}{\sec x \cdot \operatorname{cosec} x}\, dx$$ ½ m

Adding we get, $2I = \pi \int_0^{\pi} \frac{\tan x}{\sec x \cdot \operatorname{cosec} x}\, dx = \pi \int_0^{\pi} \sin^2 x\ dx$ 1½ m

$$= 2\pi \int_0^{\pi/2} \frac{1 - \cos 2x}{2}\, dx = \pi \left(x - \frac{\sin 2x}{2}\right)\Bigg]_0^{\pi/2}$$ 2 m

$$= \pi \cdot \frac{\pi}{2} = \frac{\pi^2}{2} \quad \therefore \quad I = \frac{\pi^2}{4}$$ 1 m

29. 

For correct fiqure ½ m

let radius, height and slant height of cone

be r, h and **l** respectively $\therefore \quad r^2 + h^2 = l^2$ ½ m

$$V \text{ (volume)} = \frac{\pi}{3} r^2 h, \text{ [V is constant]}$$

$$A = \pi r \mathbf{l}, \quad z = A^2 = \pi^2 r^2 \mathbf{l}^2 = \pi^2 r^2 \left(r^2 + h^2\right) \qquad \text{½ m}$$

$$= \pi^2 r^2 \left(r^2 + \frac{9v^2}{\pi^2 r^4}\right)$$

$$= \pi^2 \left(r^4 + \frac{9v^2}{\pi^2 r^2}\right) \qquad \text{1 m}$$

$$\frac{dz}{dr} = \pi^2 \left(4r^3 - \frac{18v^2}{\pi^2 r^3}\right) \qquad \text{1 m}$$

$$\therefore \frac{dz}{dr} = 0 \Rightarrow r = \sqrt[6]{\frac{9v^2}{2\pi^2}} \qquad \text{½ m}$$

$$\text{At } r = \sqrt[6]{\frac{9v^2}{2\pi^2}};\ \frac{d^2z}{dr^2} = \pi^2 \left(12r^2 + \frac{54v^2}{\pi^2 r^4}\right) > 0 \qquad \text{1 m}$$

$\therefore$ corved surface area is minimum iff $2\pi^2 r^6 = 9v^2$

i.e. $2\pi^2 r^6 = \pi^2 r^4 h^2$

OR

$$h = \sqrt{2}\, r \qquad \text{½ m}$$

$$\therefore \cot\alpha = \frac{h}{r} = \sqrt{2} \Rightarrow \alpha = \cot^{-1}\left(\sqrt{2}\right) \qquad \text{½ m}$$