QUESTION PAPER CODE 65/1/3

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. 10   2. $x = 2$ 3. $x = \pm 6$ 3. $2x^{3/2} + 2\sqrt{x} + c$

5. $x = \frac{1}{5}$ 6. $x = 25$ 7. 5

8. $\left\{\vec{r} - \left(a\hat{i} + b\hat{j} + c\hat{k}\right)\right\} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = 0$ 9. 1

or

$$\vec{r} \cdot \left(\hat{i} + \hat{j} + \hat{k}\right) = a + b + c$$

10. $\frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$ 1×10 = 10 m

SECTION - B

11. $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right] = \left(\vec{a}+\vec{b}\right) \cdot \left\{\left(\vec{b}+\vec{c}\right) \times \left(\vec{c}+\vec{a}\right)\right\}$ ½ m

$= \left(\vec{a}+\vec{b}\right)\left\{\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{c} + \vec{c}\times\vec{a}\right\}$ 1 m

$= \vec{a}\cdot\left(\vec{b}\times\vec{c}\right) + \vec{a}\cdot\left(\vec{b}\times\vec{a}\right) + \vec{a}\cdot\left(\vec{c}\times\vec{a}\right) + \vec{b}\cdot\left(\vec{b}\times\vec{c}\right)$ 1½ m

$+ \vec{b}\cdot\left(\vec{b}\times\vec{a}\right) + \vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

$\left\{\vec{a}\cdot\left(\vec{b}\times\vec{a}\right),\ \left\{\vec{a}\cdot\left(\vec{c}\times\vec{a}\right) = \vec{b}\cdot\left(\vec{b}\times\vec{c}\right) = \vec{b}\cdot\left(\vec{b}\times\vec{a}\right) = 0\right\}\right.$

$= 2\left\{\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)\right\} = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$ 1 m

OR

$\vec{a} + \vec{b} + \vec{c} = \vec{0} \quad \therefore \quad \vec{a} + \vec{b} = -\vec{c}$ ½ m

$$\Rightarrow \left(\vec{a} + \vec{b}\right)^2 = \left(-\vec{c}\right)^2 = \left(\vec{c}\right)^2$$ ½ m

$$\Rightarrow \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 2\,\vec{a}\cdot\vec{b} = \left|\vec{c}\right|^2$$ 1 m

$$\Rightarrow 9 + 25 + 2\left|\vec{a}\right|\left|\vec{b}\right|\cos\theta = 49, \quad \theta \text{ being angle between } \vec{a} \,\&\, \vec{b}$$ 1 m

$$\therefore \cos\theta = \frac{15}{2\cdot3\cdot5} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$ 1 m

12. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{2x}{x^2-1}\,y = \frac{2}{\left(x^2-1\right)^2}$$ 1 m

Integrating factor $= e^{\int \frac{2x}{x^2-1}dx} = e^{\log(x^2-1)} = x^2 - 1$ 1 m

$$\therefore \text{ Solution is } \quad y\cdot(x^2-1) = \int \frac{2}{(x^2-1)^2}\cdot(x^2-1)\,dx + c$$ 1 m

$$\Rightarrow y\,(x^2-1) = 2\int \frac{1}{x^2-1}\,dx + c$$

$$\Rightarrow y\,(x^2-1) = \log\left|\frac{x-1}{x+1}\right| + c$$ 1 m

13. $$\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x\cdot\cos^2 x}\,dx = \int \frac{\left(\sin^2 x + \cos^2 x\right)\left[(\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x\right]}{\sin^2 x\cdot\cos^2 x}\,dx$$ 1½ m

$$= \int \left[\frac{1}{\sin^2 x\cdot\cos^2 x} - 3\right] dx$$

$$= \int \left[\frac{\sin^2 x + \cos^2 x}{\sin^2 x\cdot\cos^2 x} - 3\right] dx$$ ½ m

$$= \int \left(\sec^2 x + \text{cosec}^2 x - 3\right) dx$$ ½ m

$$= \tan x - \cot x - 3x + c$$ 1½ m

(Accept – 2 cot 2x – 3x + c also)

OR

$$\int (x-3)\sqrt{x^2+3x-18}\ dx$$

$$= \frac{1}{2}\int (2x+3)\sqrt{x^2+3x-18}\ dx \ - \frac{9}{2}\int \sqrt{x^2+3x-18}\ dx$$ 1 m

$$= \frac{1}{2}\cdot\frac{2}{3}\ \left(x^2+3x-18\right)^{3/2} - \ \frac{9}{2}\int \sqrt{\left(x+3/2\right)^2 - \left(\frac{9}{2}\right)^2}\ dx$$ 1½ m

$$= \frac{1}{3}\left(x^2+3x-18\right)^{3/2} - \ \frac{9}{2}$$

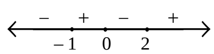
$$\left\{\frac{\left(x+\frac{3}{2}\right)}{2}\sqrt{x^2+3x-18} - \frac{81}{8}\log\left|\ x+\frac{3}{2}+\sqrt{x^2+3x-18}\ \right| + c\right.$$ 1½ m

or $$= \frac{1}{3}\left(x^2+3x-18\right)^{3/2} - \ \frac{9}{8}$$

$$\left\{ (2x+3)\sqrt{x^2+3x-18} - \frac{81}{2}\log\left|\ x+\frac{3}{2}+\sqrt{x^2+3x-18}\ \right| + c\right.$$

14. $f'(x) = 12x^3 - 12x^2 - 24x = 12x(x+1)(x-2)$ 1+½ m

$f'(x) > 0,\ \ \forall\ x \in (-1, 0)\ \mathbf{U}\ (2, \infty)$ 1 m



$f'(x) < 0,\ \ \forall\ x \in (-\infty, -1)\ \mathbf{U}\ (0, 2)$ 1 m

$\therefore$ f(x) is strictly increasing in $(-1, 0)\ \mathbf{U}\ (2, \infty)$

and strictly decreasing in $(-\infty, -1)\ \mathbf{U}\ (0, 2)$ ½ m

OR

Point at $\theta = \pi/4$ is $\left(\frac{a}{2\sqrt{2}},\ \frac{a}{2\sqrt{2}}\right)$ ½ m

$\frac{dy}{d\theta} = -3a\cos^2\theta\sin\theta;\ \frac{dx}{d\theta} = 3a\sin^2\theta\cos\theta$ 1 m

$\therefore$ slope of tangent at $\theta = \frac{\pi}{4}$ is $\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{4}} = \left.\frac{-3a\cos^2\theta\sin\theta}{3a\sin^2\theta\cos\theta}\right]_{\theta=\frac{\pi}{4}}$

$= -\cot\frac{\pi}{4} = -1$ 1 m

Equation of tangent at the point :

$y - \frac{a}{2\sqrt{2}} = -1\left(x - \frac{a}{2\sqrt{2}}\right) \Rightarrow x + y - \frac{a}{\sqrt{2}} = 0$ 1 m

Equation of normal at the point :

$y - \frac{a}{2\sqrt{2}} = 1\left(x - \frac{a}{2\sqrt{2}}\right) \Rightarrow x - y = 0$ ½ m

15. $\forall\ (a, b) \in A \times A$

a + b = b + a $\therefore$ (a, b) R (a, b) $\therefore$ R is reflexive 1 m

For (a, b), (c, d) $\in A \times A$

If (a, b) R (c, d) i.e. a + d = b + c $\Rightarrow$ c + b = d + a

then (c, d) R (a, b) $\therefore$ R is symmetric 1 m

For (a, b), (c, d), (e, f) $\in A \times A$

If (a, b) R (c, d) & (c, d) R (e, f) i.e. a + d = b + c & c + f = d + e

Adding, a + d + c + f = b + c + d + e $\Rightarrow$ a + f = b + e

then (a, b) R (e, f) $\therefore$ R is transitive 1 m

$\therefore$ R is reflexive, symmetric and transitive

hence R is an equivalance relation ½ m

[(2, 5)] = {(1, 4), (2, 5), (3, 6), (4, 7), (5, 8), (6, 9)} ½ m

16. $\cot^{-1}\left\{\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$

$$= \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right\}$$ 2½ m

$$= \cot^{-1}\left\{\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right\} = \cot^{-1}\left(\cot\frac{x}{2}\right) = \frac{x}{2}$$ 1½ m

OR

$$\text{LHS} = 2\left(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8}\right) + \sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)$$

$$= 2\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right) + \tan^{-1}\frac{1}{7}$$ 1½+½ m

$$= 2\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{7} = \tan^{-1}\left(\frac{2\cdot\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2}\right) + \tan^{-1}\frac{1}{7}$$ 1 m

$$= \tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7} = \tan^{-1}\frac{25}{25} = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS}$$ 1 m

17. $y = x^x \quad \therefore \log y = x\log x,$ Taking log of both sides ½ m

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \log x + 1,$ Diff. w r t "x" 1½ m

$$\Rightarrow \frac{1}{y}\frac{d^2y}{dx^2} - \frac{1}{y^2}\left(\frac{dy}{dx}\right)^2 = \frac{1}{x},$$ Diff. w r t "x" 1½ m

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ ½ m

18. let b_2, g_2 be younger boy and girl

and b_1, g_1 be elder, then, sample space of two children is

$S = \{(b_1, b_2), (g_1, g_2), (b_1, g_2), (g_1, b_2)\}$ 1 m

A = Event that younger is a girl = $\{(g_1, g_2), (b_1, g_2)\}$

B = Event that at least one is a girl = $\{(g_1, g_2), (b_1, g_2), (g_1, b_2)\}$

E = Event that both are girls = $\{(g_1, g_2)\}$

(i) $P(E/A) = \frac{P(E \cap A)}{P(A)} = \frac{1}{2}$ 1½ m

(ii) $P(E/B) = \frac{P(E \cap B)}{P(B)} = \frac{1}{3}$ 1½ m

19. $$\text{LHS} = \frac{1}{x \cdot y \cdot z}\begin{vmatrix} x^3+x & x^2y & x^2z \\ x y^2 & y^3+y & y^2z \\ x z^2 & y z^2 & z^3+z \end{vmatrix}$$ $R_1 \to x R_1,\ R_2 \to y R_2$, $R_3 \to z R_3$ 1 m

$$= \frac{x y z}{x y z}\begin{vmatrix} x^2+1 & x^2 & x^2 \\ y^2 & y^2+1 & y^2 \\ z^2 & z^2 & z^2+1 \end{vmatrix}$$ ½ m

$$= \begin{vmatrix} 1+x^2+y^2+z^2 & 1+x^2+y^2+z^2 & 1+x^2+y^2+z^2 \\ y^2 & y^2+1 & y^2 \\ z^2 & z^2 & z^2+1 \end{vmatrix}$$ $R_1 \to R_1 + R_2 + R_3$ 1 m

$$= \begin{vmatrix} 1+x^2+y^2+z^2 & 0 & 0 \\ y^2 & 1 & 0 \\ z^2 & 0 & 1 \end{vmatrix}; \quad \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix}$$ 1 m

$= 1 + x^2 + y^2 + z^2 =$ RHS (Expand along C_1) ½ m

20. let $x = \tan\theta \quad \therefore \quad \theta = \tan^{-1} x$

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta} - 1}{\tan\theta}\right) = \tan^{-1}\left(\tan\frac{\theta}{2}\right) = \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$$ 1 m

$$v = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) = \sin^{-1}(\sin 2\theta) = 2\theta = 2\tan^{-1}x$$ 1 m

$$\frac{du}{dx} = \frac{1}{2(1+x^2)}; \quad \frac{dv}{dx} = \frac{2}{1+x^2}$$ 1 m

$$\frac{du}{dv} = \frac{1}{2(1+x^2)} \times \frac{1+x^2}{2} = \frac{1}{4}$$ 1 m

(In case, if $x = \cot\theta$ then answer is $-\frac{1}{4}$)

21. Differential equation can be written as : $(\sin y + y \cdot \cos y)\, dy = x \cdot (2 \cdot \log x + 1)\, dx$ 1 m

Integrating both sides we get

$$-\cos y + y \cdot \sin y + \cos y = 2\left(\frac{x^2}{2}\log x - \frac{x^2}{4}\right) + \frac{x^2}{2} + c$$ 1+1 m

$$\Rightarrow \quad y \sin y = x^2 \log x + c$$

At x = 1 and

$y = \frac{\pi}{2}, \; c = \frac{\pi}{2} \quad \therefore$ solution is : $y \sin y = x^2 \log x + \frac{\pi}{2}$ ½+½ m

22. General points on the lines are

$(1+3\lambda)\hat{i}+(1-\lambda)\hat{j}-\hat{k}$ & $(4+2\mu)\hat{i}+(3\mu-1)\hat{k}$ 1 m

lines intersect if

$1+3\lambda=4+2\mu$ ....(1); $1-\lambda=0$ ....(2); $3\mu-1=-1$ ....(3) for some λ & μ 1 m

From (2) & (3) $\lambda=1, \quad \mu=0$ ½ m

substituting in equation (1)

Since, 1 + 3(1) = 4 + 2 (0) is true $\therefore$ lines interset 1 m

Point of intersection is : $4\hat{i}-\hat{k}$ or $(4, 0, -1)$ ½ m

SECTION - C

23. Let x and y be electronic and

manually operated sewing machines purchased respectively

$\therefore$ L.P.P. is Maximize P = 22x + 18y ½ m

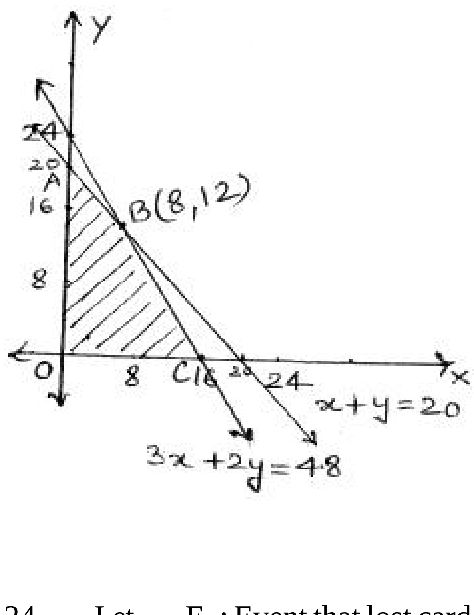
subject to $360x + 240y \leq 5760$

or $3x + 2y \leq 48$

$x + y \leq 20$ 2 m

$x \geq 0, y \geq 0$

For correct graph 2 m



Vertices of feasible region are

A (0, 20), B(8, 12), C(16, 0) & O(0, 0)

P(A) = 360, P(B) = 392, P(C) = 352 ½ m

$\therefore$ For Maximum P, Electronic machines = 8 1 m

Manual machines = 12

24. Let E_1 : Event that lost card is a spade

E_2 : Event that lost card is a non spade ½ m

A : Event that three spades are drawn without replacement from 51 cards

$P(E_1) = \frac{13}{52} = \frac{1}{4}, \quad P(E_2) = 1 - \frac{1}{4} = \frac{3}{4}$ 1 m

$P(A/E_1) = \frac{^{12}C_3}{^{51}C_3}, \quad P(A/E_2) = \frac{^{13}C_3}{^{51}C_3}$ 1½ m

$$P(E_1/A) = \frac{\frac{1}{4} \cdot \frac{^{12}C_3}{^{51}C_3}}{\frac{1}{4} \cdot \frac{^{12}C_3}{^{51}C_3} + \frac{3}{4} \cdot \frac{^{13}C_3}{^{51}C_3}}$$ 1+1 m

$$= \frac{10}{49}$$ 1 m

OR

X = No. of defective bulbs out of 4 drawn = 0, 1, 2, 3, 4 1 m

Probability of defective bulb = $\frac{5}{15} = \frac{1}{3}$ ½ m

Probability of a non defective bulb = $1 - \frac{1}{3} = \frac{2}{3}$ ½ m

Probability distribution is :

x :	0	1	2	3	4
P(x) :	$\frac{16}{81}$	$\frac{32}{81}$	$\frac{24}{81}$	$\frac{8}{81}$	$\frac{1}{81}$
x P(x) :	0	$\frac{32}{81}$	$\frac{48}{81}$	$\frac{24}{81}$	$\frac{4}{81}$

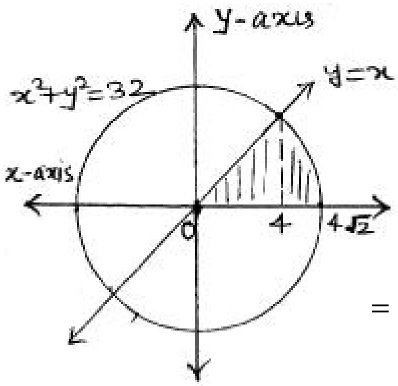
2½ m

½ m

Mean = $\sum x\, P(x) = \frac{108}{81}$ or $\frac{4}{3}$ 1 m

25. Correct Figure 1 m

The line and circle intersect each other at x = $\pm$ 4 1 m



Area of shaded region

$$= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{\left(4\sqrt{2}\right)^2 - x^2}\,dx$$ 1½ m

$$= \left[\frac{x^2}{2}\right]_0^4 + \left[\left\{\frac{x\sqrt{32-x^2}}{2} + 16\sin^{-1}\left(\frac{x}{4\sqrt{2}}\right)\right\}\right]_4^{4\sqrt{2}}$$ 1½ m

$$= 8 + 4\pi - 8 = 4\pi \text{ sq.units}$$ 1 m

26. Equation of plane through points A, B and C is

$$\begin{vmatrix} x-2 & y-5 & z+3 \\ -4 & -8 & 8 \\ 3 & -2 & 0 \end{vmatrix} = 0 \Rightarrow 16x + 24y + 32z - 56 = 0 \quad \text{i.e. } 2x + 3y + 4z - 7 = 0$$ 3+1 m

Distance of plane from (7, 2, 4) = $\left|\frac{2(7)+3(2)+4(4)-7}{\sqrt{9+16+4}}\right|$ 1 m

$= \sqrt{29}$ 1 m

OR

General point on the line is $(2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$ 1 m

Putting in the equation of plane; we get

$$1\cdot(2+3\lambda) - 1\cdot(-1+4\lambda) + 1\cdot(2+2\lambda) = 5$$ 1½ m

$\therefore \quad \lambda = 0$ 1 m

Point of intersection is $2\hat{i} - \hat{j} + 2\hat{k}$ or $(2, -1, 2)$ 1½ m

Distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{169} = 13$ 1 m

27. Here $3x+2y+z = 1000$ 1½

$$4x+y+3z = 1500$$
$$x+y+z = 600$$

$$\therefore \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1000 \\ 1500 \\ 600 \end{pmatrix} \text{ or } A \cdot X = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \quad \therefore \; X = A^{-1} B$ ½ m

Co-factors are

$$A_{11} = -2, \quad A_{12} = -1, \quad A_{13} = 3$$
$$A_{21} = -1, \quad A_{22} = 2, \quad A_{23} = -1$$ 1½ m
$$A_{31} = 5, \quad A_{32} = -5, \quad A_{33} = -5$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} 1000 \\ 1500 \\ 600 \end{pmatrix}$$

$\therefore \quad x = 100, \; y = 200, \; z = 300$ 1½ m

i.e. Rs. 100 for discipline, Rs 200 for politeness & Rs. 300 for punctuality

One more value like sincerity, truthfulness etc. 1 m

28. le $I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} \, dx \; ; \quad \therefore \; I = \int_0^{\pi/2} \frac{\left(\pi/2 - x\right) \cos x \sin x}{\cos^4 x + \sin^4 x} \, dx$ 1 m

Adding we get, $2I = \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} \, dx \; ; \; = \frac{\pi}{4} \int_0^{\pi/2} \frac{2 \tan x \sec^2 x}{1 + \left(\tan^2 x\right)^2} \, dx$ 2 m

$$= \frac{\pi}{4} \tan^{-1} \left(\tan^2 x\right) \Big]_0^{\pi/2} = \frac{\pi^2}{8}$$ 2 m

$$\therefore \; I = \frac{\pi^2}{16}$$ 1 m

29. let r and h be the radius and height of the cylinder then,

Volume of cylinder (V) $\pi r^2 h = 128\pi \quad \therefore \quad h = \frac{128\pi}{\pi r^2} = \frac{128}{r^2}$ 1 m

Surface area of cylinder = $2\pi r^2 + 2\pi rh = 2\pi (r^2 + rh)$ 1 m

$\therefore \quad S = 2\pi\left(r^2 + \frac{128}{r}\right) \quad \therefore \quad \frac{ds}{dr} = 2\pi\left(2r - \frac{128}{r^2}\right)$ 1½ m

$\frac{ds}{dr} = 0 \quad \Rightarrow \quad r^3 = 64 \quad \text{or} \quad r = 4$ ½ m

At $r = 4;\ \frac{d^2s}{dr^2} = 2\pi\left(2 + \frac{256}{r^3}\right) = 2\pi\left(2 + \frac{256}{64}\right) = 12\pi > 0$ 1 m

$\therefore$ surface area is minimum at r = 4 cm ; h = 8 cm 1 m