QUESTION PAPER CODE 65/2/2

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. $\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$ 2. 0 3. π

4. $k = 27$ 5. $\{8, 27\}$ 6. 1 7. $\tan x - \cot x + c$

8. $\cos^{-1}\left(\frac{19}{21}\right)$ 9. $\frac{1}{2}$ 10. $\frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

SECTION - B

11. Here $(\vec{a}+\vec{b}), (\vec{b}+\vec{c}), (\vec{c}+\vec{a})$ are coplanar, 1 m

$(\vec{a}+\vec{b}) \cdot \{(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})\} = 0$ 1 m

$\Rightarrow (\vec{a}+\vec{b}) \cdot (\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{c} + \vec{c}\times\vec{a}) = 0$ 1 m

$\Rightarrow \vec{a} \cdot (\vec{b}\times\vec{c}) + \vec{a} \cdot (\vec{b}\times\vec{a}) + \vec{a} \cdot (\vec{c}\times\vec{a}) + \vec{b} \cdot (\vec{b}\times\vec{c})$

$+ \vec{b} \cdot (\vec{b}\times\vec{a}) + \vec{b} \cdot (\vec{c}\times\vec{a}) = 0$ ½ m

$\Rightarrow 2\{\vec{a} \cdot (\vec{b}\times\vec{c})\} = 0 \quad \because \quad \vec{b} \cdot (\vec{b}\times\vec{c}) = 0$ ½ m

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplaner

Similerly converse part can also be proved.

OR

$\vec{a}+\vec{b} = \left(2\hat{i} + 3\hat{j} + 4\hat{k}\right), \ \vec{a}-\vec{b} = -\hat{j} - 2\hat{k}$ 1+½ m

Let $\vec{c} = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$ 1½ m

$\Rightarrow \vec{c} = -2\hat{i} + 4\hat{j} - 2\hat{k}$

$\Rightarrow \hat{c} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ 1 m

12. $X \rightarrow$ be the number of red cards drawn

$X = 0\ 1,\ 2,\ 3$ ½ m

$P(X=0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2}{17}$

$P(X=1) = \frac{{}^{26}C_2 \cdot {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$

$P(X=2) = \frac{{}^{26}C_2 \cdot {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$

$P(X=3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2}{17}$ 2 m

$\text{Mean} = \sum pi\ xi = 0 \times \frac{2}{17} + 1 \times \frac{13}{34} + 2 \cdot \frac{13}{34} + 3 \times \frac{2}{17}$

$= \frac{51}{34} = \frac{3}{2}$ or 1.5 } 1½ m

13. Let $x, y \in W$

If x and y both are even, $f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$

If x and y both are odd, $f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$

If x is odd and y is even i.e. $x \neq y$, $(x - 1)$ is even, $(y + 1)$ is odd

$$x \neq y \Rightarrow f(x) \neq f(y)$$

Similerly for x is even and y is odd.

f is one – one 1½ m

Range of f = $\{f(0), f(1), f(2) \ldots\ldots\} = \{1, 0, 3, 2, \ldots\ldots\}$

$= w = \text{codomain}$

f is onto, 1½ m

Hence f is invertible

$f^{-1}: W \to W \quad f^{-1}(x) = \begin{cases} x-1, & x \text{ is odd} \\ x+1, & x \text{ is even} \end{cases}$ 1 m

14. $\cos\left\{\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right\} = \sin\left\{\sin^{-1}\left(\frac{4}{5}\right)\right\}$ 2 m

$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$ 1 m

$\Rightarrow 1+x^2 = \frac{25}{16} \Rightarrow x = \frac{3}{4}, \frac{-3}{4}$ ½ m

$x = \frac{-3}{4}$ does not satisfy so $x = \frac{3}{4}$ ½ m

OR

$\text{L.H.S.} = \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{18}\right)$ 1 m

$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{56}}\right) + \tan^{-1}\left(\frac{1}{18}\right)$ 1 m

$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$ 1/2 m

$$= \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{1}{11\times 18}}\right) = \tan^{-1}\left(\frac{1}{3}\right) = \cot^{-1} 3$$ 1½ m

15. $\frac{dx}{d\theta} = -\ a\ \sin\theta + b\cos\theta,\ \frac{dy}{d\theta} = a\cos\theta + b\sin\theta$ 1½ m

$$\frac{dy}{dx} = -\ \frac{a\cos\theta + b\sin\theta}{a\sin\theta - b\cos\theta} = -\ \frac{x}{y}$$ 1 m

$$\frac{d^2y}{dx^2} = -\ \frac{\left(y - x\frac{dy}{dx}\right)}{y^2} \Rightarrow \frac{y^2\ d^2y}{dx^2} - \frac{x\ dy}{dx} + y = 0$$ 1½ m

16. Put $x = \cos\theta \quad dx = -\sin\theta\, d\theta$ 1 m

$$I = \int \frac{\theta\cos\theta}{\sin\theta}\ (-\sin\theta) = -\int \theta\cos\theta\, d\theta$$ 1 m

$$I = -\ \{\theta\sin\theta - \int 1\cdot\sin\theta\, d\theta$$

$$\Rightarrow \quad I = -\ \left\{\theta\sin\theta - \int 1.\sin\theta\, d\theta\right\} = -\theta\sin\theta - \cos\theta + c$$ 1 m

$$\Rightarrow \quad I = -\ \sqrt{1-x^2}\cdot\cos^{-1}x - x + c$$ 1

OR

$$= \int (3x-2)\ \sqrt{x^2+x+1}\, dx = \int \left\{\frac{3}{2}(2x+1) - \frac{7}{2}\right\} \sqrt{x^2+x+1}\, dx$$ 1 m

$$= \frac{3}{2}\int (2x+1)\ \sqrt{x^2+x+1}\, dx - \frac{7}{2}\int \sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\, dx$$ 1 m

$$= \left(x^2 + x + 1\right)^{3/2} - \frac{7}{2}\left(\frac{2x+1}{4}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|x + \frac{1}{2} + \sqrt{x^2+x+1}\right|\right) + c$$ 1 + 1 m

17. $$x^2(1-y)\,dy = -y^2\left(1+x^2\right)dx$$ ½ m

$$\therefore \int \frac{1-y}{y^2}\,dy = -\int \frac{x^2+1}{x^2}\,dx$$ 1 m

$$\Rightarrow \int\left(\frac{-1}{y^2} + \frac{1}{y}\right)dy = \int\left(1 + \frac{1}{x^2}\right)dx$$

$$\Rightarrow \frac{1}{y} + \log|y| = x - \frac{1}{x} + c$$ 1½ m

Putting $x = 1, y = 1$ we get, $c = 1$ ½ m

$$\Rightarrow \frac{1}{y} + \log|y| = x - \frac{1}{x} + 1$$ ½ m

18. $f^1(x) = 6x + 5$, let $x = 3$, $\Delta x = 0.02$ 1 m

$$f^1(x) \cong \frac{f(x+\Delta x) - f(x)}{\Delta x} \Rightarrow f(x+\Delta x) = (\Delta x)f^1(x) + f(x)$$ 1½ m

$$\therefore f(3.02) = (0.02)\ f^1(3) + f(3)$$

$$= (0.02)(23) + 45$$

$$= 45.46$$ 1½ m

OR

$f^1(x) = 6x^3 - 12x^2 - 90x = 6x(x-5)(x+3)$ 1+1 m

$f^1(x) = 0 \Rightarrow x = -3, x = 0, r = 5$ ½ m

$$\xleftarrow{\quad -ve \quad} \underset{-3}{|} \xrightarrow{} +ve \ \underset{0}{|} \ -ve \ \underset{5}{|} \ +ve \rightarrow$$

$f^1(x) > 0, \ \forall x \in (-3,0) \cup (5, \infty) \ \Rightarrow$ Strictly increasing

$f^1(x) < 0, \ \forall x \in (-\infty, -3) \cup (0,5) \ \Rightarrow$ Strictly decreasing 1+½ m

19. Operating $C_1 \to C_1 + C_2 + C_3$, we get

$$\text{L. H. S.} = \begin{vmatrix} 5x+\lambda & 2x & 2x \\ 5x+\lambda & x+\lambda & 2x \\ 5x+\lambda & 2x & x+\lambda \end{vmatrix} = (5x+\lambda) \begin{vmatrix} 1 & 2x & 2x \\ 1 & x+\lambda & 2x \\ 1 & 2x & x+\lambda \end{vmatrix}$$ 1 m

$$\left.\begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}\right\} \Rightarrow \text{L.H.S} = (5x+\lambda) \begin{vmatrix} 1 & 2x & 2x \\ 0 & \lambda - x & 0 \\ 0 & 0 & \lambda - x \end{vmatrix}$$ 2 m

Expanding along c_1, we get

$$= (5x+\lambda) \left\{(\lambda - x)^2 - 0 + 0\right\}$$

$$= (5x+\lambda) \left((\lambda - x)^2\right) = \text{R. H. S}$$ 1 m

20. $$e^x + e^y \frac{dy}{dx} = e^{x+y} \left(1 + \frac{dy}{dx}\right)$$ 2 m

$$\Rightarrow \frac{dy}{dx} \left(e^y - e^{x+y}\right) = e^{x+y} - e^x$$ 1 m

$$\Rightarrow \frac{dy}{dx} = \frac{e^x + e^y - e^x}{e^y - \left(e^x + e^y\right)} = -\frac{e^y}{e^x}$$

$$\Rightarrow \frac{dy}{dx} + \frac{e^y}{e^x} = 0 \Rightarrow \frac{dy}{dx} + e^{y-x} = 0$$ 1 m

21. Integrating factor is $e^{\int 2\tan x\, dx} = e^{2\log \sec x} = \sec^2 x$ 1 m

$$y\sec^2 x = \int \sin x \cdot \sec^2 x\, dx$$ 1 m

$$\Rightarrow \quad y\sec^2 x = \int \sec x \tan x\, dx$$

$$\Rightarrow \quad y\sec^2 x = \sec x + c$$ 1 m

$x = \frac{\pi}{3},\ y = 0 \qquad 0 = 2 + c \ \Rightarrow\ c = -2$ ½ m

$\therefore \quad y\sec^2 x = \sec x - 2$ or $y = \cos x - 2\cos^2 x$ ½ m

22. The given lines in vector form are

$$\vec{r} = -\hat{i} - \hat{j} - \hat{k} + l\left(7\hat{i} - 6\hat{j} + \hat{k}\right)$$

$$\vec{r} = 3\hat{i} + 5\hat{j} + 7\hat{k} + \mu\left(\hat{i} - 2\hat{j} + \hat{k}\right)$$ ½ m

$$\vec{a_1} = -\hat{i} - \hat{j} - \hat{k},\ \vec{b_1} = 7\hat{i} - 6\hat{j} + \hat{k}$$

$$\vec{a_2} = 3\hat{i} + 5\hat{j} + 7\hat{k},\ \vec{b_2} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{a_2} - \vec{a_1} = 4\hat{i} + 6\hat{j} + 8\hat{k}$$ 1 m

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{j} - 6\hat{j} - 8\hat{k}$$ 1 m

$$\left|\vec{b_1} \times \vec{b_2}\right| = \sqrt{116}$$ ½ m

$$S \cdot D = \left|\frac{\left(\vec{a_2} - \vec{a_1}\right) \cdot \left(\vec{b_1} \times \vec{b_2}\right)}{\left|\vec{b_1} \times \vec{b_2}\right|}\right| = \frac{116}{\sqrt{116}} = \sqrt{116}$$ 1 m

SECTION - C

23. Let equation of plane through (1, –1, 2) with dr's of perpendicular as a, b and c is

$$a(x-1) + b(y+1) + c(z-2) = 0$$ 1 m

The plane is $\perp$ to $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$

$\therefore \quad 2a + 3b - 2c = 0$ and $a + 2b - 3c = 0$ 1½ m

$$\frac{a}{-5} = \frac{b}{4} = \frac{c}{1} = k \Rightarrow a = -5k, \; b = 4k, \; c = k$$ 1½ m

Equation of the plane is

$$-5k(x-1) + 4k(y+1) + k(z-2) = 0 \Rightarrow -5x + 4y + z + 7 = 0$$ 1 m

Distance of plane from (– 2, 5, 5) is

$$d = \left| \frac{10+20+5+7}{\sqrt{25+16+1}} \right| = \frac{42}{\sqrt{42}} = \sqrt{42}$$ 1 m

OR

Line through A (2, –1, 2) and B (5, 3, 4) is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$$ 1½ m

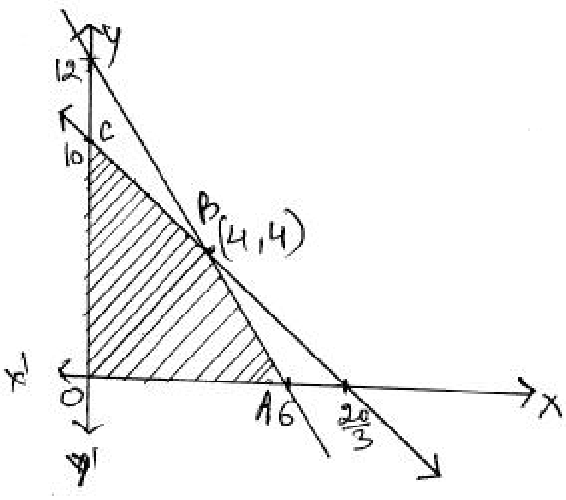
General point on the line is $(3\lambda + 2, 4\lambda - 1, 2\lambda + 2)$ 1

$\therefore \; 3\lambda + 2 - (4\lambda - 1) + 2\lambda + 2 = 5 \Rightarrow \lambda = 0$ 1½ m

Point of intersection is (2, –1, 2) 1 m

$$d = \sqrt{(3)^2 + (4)^2 + (12)^2} = \sqrt{169} = 13$$ 1 m

24. Let the number of lamps and shades manufactured be x and y respectiuely



$\therefore$ L.P.P. is Maximise $Z = 25x + 15y$ ½ m

Subject to $2x + y \leq 12$

$3x + 2y \leq 20$

$x \geq 0, y \geq 0$ 2 m

For correct graph 2 m

Vertices of feasible

region are 0(0, 0), A(6, 0), B(4, 4) C(0, 10)

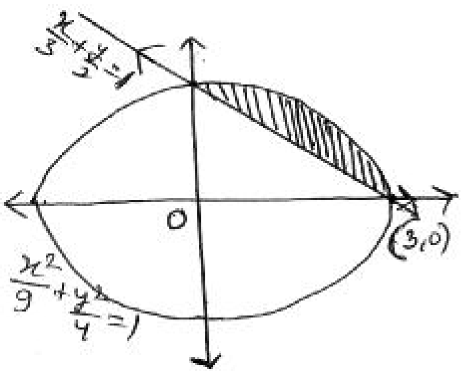
P(A) = 150, P (B) = 160, P(C) = 150 ½ m

For max Prof no. of lamps = 4

No. of shades = 4 1 m

Maximum Profit = Rs. 160

25. Correct figure 1 m



Area of shaded region $= \int_0^3 \left\{ \frac{2}{3}\sqrt{9-x^2} - \frac{2}{3}(3-x) \right\} dx$ 2 m

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + \frac{(3-x)^2}{2}\right]_0^3$$ 2 m

$$= \frac{2}{3}\left[\left(0+\frac{9}{2}\cdot\frac{\pi}{2}+0\right) - \left(0+0+\frac{9}{2}\right)\right]$$

$= \frac{2}{3}\left(9\frac{\pi}{4} - \frac{9}{2}\right) = 3\left(\frac{\pi}{2} - 1\right)$ sq. units 1 m

26. Here $3x + 2y + z = 2200$

$4x + y + 3z = 3100$ 1½ m

$x + y + z = 1200$

$$\therefore \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} \text{ or } AX = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \;\therefore\; X = A^{-1}B$ ½ m

cofactors are :

$A_{11} = -2$ $A_{12} = -1$ $A_{31} = 3$

$A_{21} = -1$ $A_{22} = 2$ $A_{32} = -1$

$A_{31} = 5$ $A_{23} = -5$ $A_{33} = -5$ 1½ m

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$\therefore$ $x = 300$, $y = 400$, $z = 500$ 1½ m

One more value like punctuality, honesty etc 1 m

27. Let E_1 : Scooter driver is chosen

E_2 : Car driver is chosen

E_3 : Truck driver is chosen ½ m

A : Person meets with an accident

$$P(E_1) = \frac{1}{6},\ P(E_2) = \frac{1}{3},\ P(E_3) = \frac{1}{2}$$ 1 m

$$P\left(A/E_1\right) = 0.01,\ P\left(A/E_2\right) = 0.03,\ P\left(A/E_3\right) = 0.15$$ 1 m

$$P\left(E_3/A\right) = \frac{\frac{1}{2}\times(0.15)}{\frac{1}{6}\times(0.01)+\frac{1}{3}\times(0.03)+\frac{1}{2}\times(0.15)} = \frac{45}{52}$$ 1+1 m

$$P\left(E_1/A \text{ or } E_2/A\right) = 1-\left(E_3/A\right)$$ 1 m

$$= 1-\frac{45}{52} = \frac{7}{52}$$ ½ m

OR

Let E be the event drawing a diamond card

$$n=5,\ p = \frac{1}{4},\ q = \frac{3}{4}$$ 1½ m

$$P(E) = \frac{13}{52} = \frac{1}{4}$$

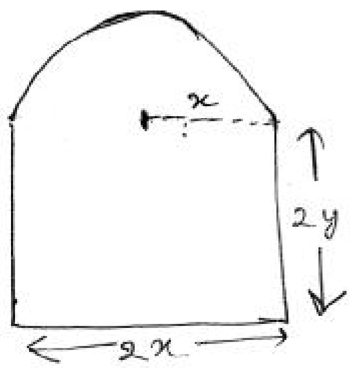
$$P(\overline{E}) = \frac{3}{4}$$

(i) $P(5) = {}^5C_5\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^0 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ 1½ m

(ii) $P(3) = {}^5C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^2 = \frac{45}{512}$ 1½ m

(iii) $P(0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$ 1½ m

28. Let 2x and 2y be the sides of rectangle Correct figure ½ m



$\therefore \quad 2x + 4y + \pi x = 10$ ½ m

Area of window

$A = 4xy + \frac{\pi x^2}{2}$ ½ m

$\Rightarrow \quad A = x(10 - 2x - \pi x) + \frac{\pi x^2}{2}$ 1 m

$\Rightarrow \quad \frac{dA}{dx} = 10 - 4x - 2\pi x + \pi x$ ½ m

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{10}{\pi + 4}$ 1 m

$\Rightarrow \quad \frac{d^2A}{dx^2} = -4 - \pi < 0 \Rightarrow$ Maxima 1 m

Hence A is maximum when $x = \frac{10}{\pi + 4}$ or $2x = \frac{20}{\pi + 4}$ ½ m

$\Rightarrow \quad 2y = \frac{10}{\pi + 4}$ ½ m

29. $I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin^2 x} \Rightarrow I = \int_0^{\pi} \frac{(\pi - x)}{a^2\cos^2 x + b^2 \sin^2 x}\,dx$ 1 m

$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{1}{a^2\cos^2 x + b^2 \sin^2 x}\,dx \Rightarrow I = \pi \int_0^{\pi/2} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x}\,dx$ 2 m

Put $b \tan x = t \Rightarrow \sec^2 x \, dx = \frac{1}{b} dt.$ 1 m

$$\Rightarrow I = \frac{\pi}{b} \int_0^{\infty} \frac{dt}{a^2 + t^2} = \frac{\pi}{b} \left[\frac{1}{a} \tan^{-1} \frac{t}{a} \right]_0^{\infty}$$ 1 m

$$\Rightarrow I = \frac{\pi}{ab} \left(\frac{\pi}{2} - 0 \right) = \frac{\pi^2}{2ab}$$ 1 m