QUESTION PAPER CODE 65/2/3

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. $\tan x - \cot x + c$ 2. $\begin{bmatrix} 4 & -6 \\ 3 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ 1 & -1 \end{bmatrix}$ 3. $\cos^{-1}\left(\frac{19}{21}\right)$

4. $k = 27$ 5. $\{8, 27\}$ 6. π 7. 1

8. 0 9. 0 10. $\frac{1}{2}(e-1)$

SECTION - B

11. Here $(\vec{a}+\vec{b}), (\vec{b}+\vec{c}), (\vec{c}+\vec{a})$ are coplanar, 1 m

$(\vec{a}+\vec{b}) \cdot \{(\vec{b}+\vec{c}) \times (\vec{c}+\vec{a})\} = 0$ 1 m

$\Rightarrow (\vec{a}+\vec{b}) \cdot (\vec{b}\times\vec{c} + \vec{b}\times\vec{a} + \vec{c}\times\vec{c} + \vec{c}\times\vec{a}) = 0$ 1 m

$\Rightarrow \vec{a}\cdot(\vec{b}\times\vec{c}) + \vec{a}\cdot(\vec{b}\times\vec{a}) + \vec{a}\cdot(\vec{c}\times\vec{a}) + \vec{b}\cdot(\vec{b}\times\vec{c})$

$+ \vec{b}\cdot(\vec{b}\times\vec{a}) + \vec{b}\cdot(\vec{c}\times\vec{a}) = 0$ ½ m

$\Rightarrow 2\{\vec{a}\cdot(\vec{b}\times\vec{c})\} = 0 \quad \because \quad \vec{b}\cdot(\vec{b}\times\vec{c}) = 0$ ½ m

$\Rightarrow \vec{a}, \vec{b}, \vec{c}$ are coplaner

Similerly converse part can also be proved.

OR

$\vec{a}+\vec{b} = (2\hat{i} + 3\hat{j} + 4\hat{k}),\ \vec{a}-\vec{b} = -\hat{j} - 2\hat{k}$ 1+½ m

Let $\vec{c} = (\vec{a}+\vec{b}) \times (\vec{a}-\vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$ 1½ m

$\Rightarrow \quad \vec{c} = -2\hat{i} + 4\hat{j} - 2\hat{k}$

$\Rightarrow \quad \hat{c} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ 1 m

12. $f^1(x) = 6x + 5$, let $x = 3$, $\Delta x = 0.02$ 1 m

$f^1(x) \cong \frac{f(x+\Delta x) - f(x)}{\Delta x} \Rightarrow f(x+\Delta x) = (\Delta x) f^1(x) + f(x)$ 1½ m

$\therefore \quad f(3.02) = (0.02)\ f^1(3) + f(3)$

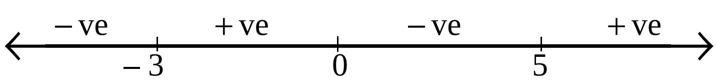
$= (0.02)(23) + 45$

$= 45.46$ 1½ m

OR

$f^1(x) = 6x^3 - 12x^2 - 90x = 6x(x-5)(x+3)$ 1+1 m

$f^1(x) = 0 \Rightarrow x = -3, x = 0, r = 5$ ½ m



$f^1(x) > 0, \ \forall x \in (-3,0) \cup (5,\infty) \Rightarrow$ Strictly increasing

$f^1(x) < 0, \ \forall x \in (-\infty,-3) \cup (0,5) \Rightarrow$ Strictly decreasing 1+½ m

13. $x^2(1-y)\,dy = -y^2(1+x^2)\,dx$ ½ m

$\therefore \quad \int \frac{1-y}{y^2}\,dy = -\int \frac{x^2+1}{x^2}\,dx$ 1 m

$\Rightarrow \quad \int \left(\frac{-1}{y^2} + \frac{1}{y}\right) dy = \int \left(1 + \frac{1}{x^2}\right) dx$

$$\Rightarrow \frac{1}{y} + \log|y| = x - \frac{1}{x} + c$$ 1½ m

Putting x = 1, y = 1 we get, c = 1 ½ m

$$\Rightarrow \frac{1}{y} + \log|y| = x - \frac{1}{x} + 1$$ ½ m

14. $$\cos\left\{\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right\} = \sin\left\{\sin^{-1}\left(\frac{4}{5}\right)\right\}$$ 2 m

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} = \frac{4}{5}$$ 1 m

$$\Rightarrow 1 + x^2 = \frac{25}{16} \Rightarrow x = \frac{3}{4}, \frac{-3}{4}$$ ½ m

$x = \frac{-3}{4}$ does not satisfy so $x = \frac{3}{4}$ ½ m

OR

$$\text{L.H.S.} = \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{18}\right)$$ 1 m

$$= \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{56}}\right) + \tan^{-1}\left(\frac{1}{18}\right)$$ 1 m

$$= \tan^{-1}\frac{3}{11} + \tan^{-1}\frac{1}{18}$$ 1/2 m

$$= \tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{1}{11\times 18}}\right) = \tan^{-1}\left(\frac{1}{3}\right) = \cot^{-1} 3$$ 1½ m

15. Let $x, y \in W$

If x and y both are even, $f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$

If x and y both are odd, $f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$

If x is odd and y is even i.e. $x \neq y$, $(x - 1)$ is even, $(y + 1)$ is odd

$$x \neq y \quad \Rightarrow \quad f(x) \neq f(y)$$

Similerly for x is even and y is odd.

f is one – one 1½ m

Range of $f = \{f(0), f(1), f(2) ....\} = \{1, 0, 3, 2, ...\}$

$= w =$ codomain

f is onto, 1½ m

Hence f is invertible

$$f^{-1} : W \rightarrow W \quad f^{-1}(x) = \begin{cases} x - 1, & x \text{ is odd} \\ x + 1, & x \text{ is even} \end{cases}$$ 1 m

16. $X \rightarrow$ be the number of red cards drawn

$X = 0 \; 1, \; 2, \; 3$ ½ m

$$P(X = 0) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2}{17}$$

$$P(X = 1) = \frac{{}^{26}C_2 \cdot {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$$

$$P(X = 2) = \frac{{}^{26}C_2 \cdot {}^{26}C_1}{{}^{52}C_3} = \frac{13}{34}$$

$$P(X = 3) = \frac{{}^{26}C_3}{{}^{52}C_3} = \frac{2}{17}$$ 2 m

$$\text{Mean} = \sum p_i x_i = 0\times\frac{2}{17} + 1\times\frac{13}{34} + 2\cdot\frac{13}{34} + 3\times\frac{2}{17}$$

$$= \frac{51}{34} = \frac{3}{2} \text{ or } 1.5$$ 1½ m

17. $\frac{dx}{d\theta} = -a\sin\theta + b\cos\theta,\ \frac{dy}{d\theta} = a\cos\theta + b\sin\theta$ 1½ m

$$\frac{dy}{dx} = -\frac{a\cos\theta + b\sin\theta}{a\sin\theta - b\cos\theta} = -\frac{x}{y}$$ 1 m

$$\frac{d^2y}{dx^2} = -\frac{\left(y - x\frac{dy}{dx}\right)}{y^2} \Rightarrow \frac{y^2\,d^2y}{dx^2} - \frac{x\,dy}{dx} + y = 0$$ 1½ m

18. Put $x = \cos\theta \quad dx = -\sin\theta\, d\theta$ 1 m

$$I = \int\frac{\theta\cos\theta}{\sin\theta}(-\sin\theta) = -\int\theta\cos\theta\, d\theta$$ 1 m

$$I = -\{\theta\sin\theta - \int 1\cdot\sin\theta\, d\theta$$

$$\Rightarrow I = -\left\{\theta\sin\theta - \int 1.\sin\theta\, d\theta\right\} = -\theta\sin\theta - \cos\theta + c$$ 1 m

$$\Rightarrow I = -\sqrt{1-x^2}\cdot\cos^{-1}x - x + c$$ 1

OR

$$= \int(3x-2)\sqrt{x^2+x+1}\,dx = \int\left\{\frac{3}{2}(2x+1) - \frac{7}{2}\right\}\sqrt{x^2+x+1}\,dx$$ 1 m

$$= \frac{3}{2}\int(2x+1)\sqrt{x^2+x+1}\,dx - \frac{7}{2}\int\sqrt{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$$ 1 m

$$= (x^2+x+1)^{3/2} - \frac{7}{2}\left(\frac{2x+1}{4}\sqrt{x^2+x+1} + \frac{3}{8}\log\left|x+\frac{1}{2}+\sqrt{x^2+x+1}\right|\right) + c$$ 1 + 1 m

19. $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}, \quad \vec{b_1} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}, \quad \vec{b_2} = 4\hat{i} + 6\hat{j} + 12\hat{k}$

$\vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k},$ 1 m

$\vec{b_1}$ is parallel to $\vec{b_2}$ $\Rightarrow$ lines are parallel

$$S \cdot D = \frac{\left|\vec{b} \times (\vec{a_2} - \vec{a_1})\right|}{\left|\vec{b}\right|}$$ 1 m

$$= \frac{\left|\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right) \times \left(2\hat{i} + \hat{j} - \hat{k}\right)\right|}{\sqrt{49}}$$ 1 m

$$= \frac{\left|-9\hat{i} + 14\hat{j} - 4\hat{k}\right|}{7} = \frac{\sqrt{293}}{7}$$ 1 m

20. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{x\log x} \cdot y = \frac{2}{x^2}$$ ½ m

Integrating factor $e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$ 1 m

Solution is $y\log x = \int \frac{2}{x^2} \log x \, dx$ 1 m

$$\Rightarrow y\log x = 2\left\{(\log x)\left(\frac{-1}{x}\right) - \int \frac{1}{x}\left(-\frac{1}{x}\right) dx\right.$$

$$\Rightarrow\ y\log x = 2\left(\frac{-\log x}{x} - \frac{1}{x}\right) + c$$ 1½ m

$$\Rightarrow\ y\log x = \frac{-2}{x}(\log x + 1) + c$$

21. $\cos y = x\cos(a+y) \Rightarrow x = \dfrac{\cos y}{\cos(a+y)}$ ½ m

$$\Rightarrow\ 1 = \frac{(-\sin y)\frac{dy}{dx}\cos(a+y) + \cos y\sin(a+y)\frac{dy}{dx}}{\cos^2(a+y)}$$ 1½ m

$$\Rightarrow\ \cos^2(a+y) = \{-\cos(a+y)\sin y + \cos y\sin(a+y)\}\frac{dy}{dx}$$ ½ m

$$\cos^2(a+y) = \sin a\cdot\frac{dy}{dx}$$ 1 m

$$\Rightarrow\ \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$$ ½ m

22. $\text{L.H.S.} = abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$ Takings out a from C_1, b from C_2, and c from C_3 1m

$$= abc\begin{vmatrix} 0 & c & a+e \\ 2b & b & a \\ 2b & b+c & c \end{vmatrix}$$ $C_1 \to C_1 + C_2 - C_3$ 1 m

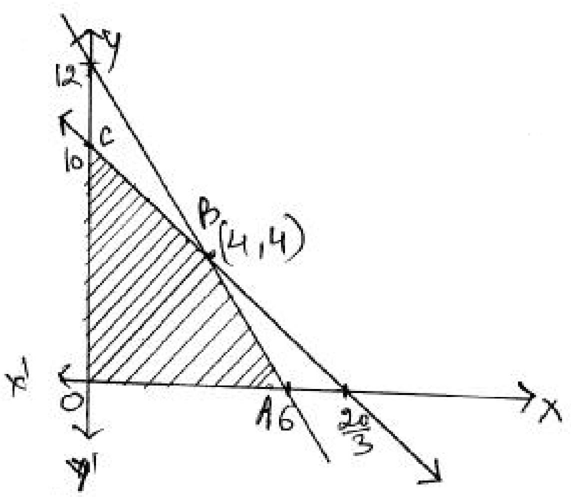
$$= abc\begin{vmatrix} 0 & c & a+c \\ 0 & -c & a-c \\ 2b & b+c & c \end{vmatrix}$$ $R_2 \to R_2 - R_3$ 1 m

$$= abc\ [0 - 0 + 2b\,(ca - c^2 + ca + c^2)]$$

$$= abc\ (4abc) = 4a^2\, b^2\, c^2$$ 1 m

SECTION - C

23. Let the number of lamps and shades manufactured be x and y respectiuely



$\therefore$ L.P.P. is Maximise $Z = 25x + 15y$ ½ m

Subject to $2x + y \leq 12$

$3x + 2y \leq 20$

$x \geq 0,\ y \geq 0$ 2 m

For correct graph 2 m

Vertices of feasible

region are 0(0, 0), A(6, 0), B(4, 4) C(0, 10)

P(A) = 150, P (B) = 160, P(C) = 150 ½ m

For max Prof no. of lamps = 4

No. of shades = 4 1 m

Maximum Profit = Rs. 160

24. Let equation of plane through (1, –1, 2) with dr's of perpendicular as a, b and c is

$a\,(x-1) + b\,(y + 1) + c\,(z - 2) = 0$ 1 m

The plane is $\perp$ to $2x + 3y - 2z = 5$ and $x + 2y - 3z = 8$

$\therefore$ $2a + 3b - 2c = 0$ and $a + 2b - 3c = 0$ 1½ m

$$\frac{a}{-5} = \frac{b}{4} = \frac{c}{1} = k \Rightarrow a = -5k,\ b = 4k,\ c = k$$ 1½ m

Equation of the plane is

$-5k(x-1)+4k(y+1)+k(z-2) = 0 \Rightarrow -5x+4y+z+7=0$ 1 m

Distance of plane from (– 2, 5, 5) is

$$d = \left|\frac{10+20+5+7}{\sqrt{25+16+1}}\right| = \frac{42}{\sqrt{42}} = \sqrt{42}$$ 1 m

OR

Line through A (2, –1, 2) and B (5, 3, 4) is

$$\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2} = \lambda$$ 1½ m

General point on the line is $(3\lambda+2,\ 4\lambda-1,\ 2\lambda+2)$ 1

$\therefore\ 3\lambda+2-(4\lambda-1)+2\lambda+2 = 5 \Rightarrow \lambda=0$ 1½ m

Point of intersection is (2, –1, 2) 1 m

$$d = \sqrt{(3)^2+(4)^2+(12)^2} = \sqrt{169} = 13$$ 1 m

25. Here $3x+2y+z = 2200$

$4x+y+3z = 3100$ 1½ m

$x+y+z = 1200$

$$\therefore \begin{bmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix} \text{ or } AX=B$$

$|A| = 3(-2)-2(1)+1(3) = -5 \neq 0 \therefore X = A^{-1}B$ ½ m cofactors are :

$A_{11} = -2$ $A_{12} = -1$ $A_{31} = 3$

$A_{21} = -1$ $A_{22} = 2$ $A_{32} = -1$

$A_{31} = 5$ $A_{23} = -5$ $A_{33} = -5$ 1½ m

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{bmatrix} \begin{bmatrix} 2200 \\ 3100 \\ 1200 \end{bmatrix}$$

$\therefore$ x = 300 , y = 400 , z = 500 1½ m

One more value like punctuality, honesty etc 1 m

26. Let E_1 : Scooter driver is chosen

E_2 : Car driver is chosen

E_3 : Truck driver is chosen ½ m

A : Person meets with an accident

$P(E_1) = \frac{1}{6}, \; P(E_2) = \frac{1}{3}, \; P(E_3) = \frac{1}{2}$ 1 m

$P\left(A/E_1\right) = 0.01, \; P\left(A/E_2\right) = 0.03, \; P\left(A/E_3\right) = 0.15$ 1 m

$$P\left(E_3/A\right) = \frac{\frac{1}{2}\times(0.15)}{\frac{1}{6}\times(0.01)+\frac{1}{3}\times(0.03)+\frac{1}{2}\times(0.15)} = \frac{45}{52}$$ 1+1 m

$P\left(E_1/A \text{ or } E_2/A\right) = 1-\left(E_3/A\right)$ 1 m

$= 1-\frac{45}{52} = \frac{7}{52}$ ½ m

OR

Let E be the event drawing a diamond card

$$n = 5,\ p = \frac{1}{4},\ q = \frac{3}{4}$$ 1½ m

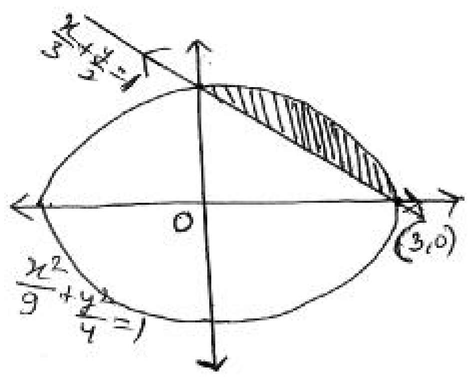
$$P(E) = \frac{13}{52} = \frac{1}{4}$$

$$P(\bar{E}) = \frac{3}{4}$$

(i) $P(5) = {}^5C_5 \left(\frac{1}{4}\right)^5 \left(\frac{3}{4}\right)^0 = \left(\frac{1}{4}\right)^5 = \frac{1}{1024}$ 1½ m

(ii) $P(3) = {}^5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^2 = \frac{45}{512}$ 1½ m

(iii) $P(0) = {}^5C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^5 \left(\frac{3}{4}\right)^5 = \frac{243}{1024}$ 1½ m

27. 

Correct figure 1 m

Area of shaded region $= \int_0^3 \left\{ \frac{2}{3}\sqrt{9-x^2} - \frac{2}{3}(3-x) \right\} dx$ 2 m

$$= \frac{2}{3}\left[\frac{x}{2}\sqrt{9-x^2} + \frac{9}{2}\sin^{-1}\frac{x}{3} + \frac{(3-x)^2}{2}\right]_0^3$$ 2 m

$$= \frac{2}{3}\left[\left(0+\frac{9}{2}\cdot\frac{\pi}{2}+0\right)-\left(0+0+\frac{9}{2}\right)\right]$$

$$= \frac{2}{3}\left(9\frac{\pi}{4} - \frac{9}{2}\right) = 3\left(\frac{\pi}{2} - 1\right) \text{ sq. units}$$ 1 m

28. Let r be the radius and x, the side of the square

$S = \pi r^2 + x^2$, where $2\pi r + 4x = k$ ½ m

$$S = \pi r^2 + \left(\frac{k-2\pi r}{4}\right)^2$$ 1 m

$$\frac{ds}{dr} = 2\pi r + \frac{\pi^2 r}{2} - \frac{k\pi}{4}$$

$$\frac{ds}{dr} = 0 \Rightarrow r = \frac{k}{2(\pi+4)}$$ ½ m

$$\frac{d^2s}{dr^2} = 2\pi + \frac{\pi^2}{2} > 0 \Rightarrow \text{minima}$$ 1 m

Hence s is least when $r = \frac{k}{2(\pi+4)}$ 1 m

$$x = \frac{1}{4}\left(k - 2\pi\frac{k}{2(\pi+4)}\right)$$

$$\Rightarrow x = \frac{k}{\pi+4} = 2r$$ 1 m

29. Let $\sin x - \cos x = t$ then $(\cos x + \sin x)\,dx = dt$ 1 m

and $(\sin x - \cos x)^2 = t^2 \Rightarrow 1 - \sin 2x = t^2$

$\sin 2x = 1 - t^2$ 1½ m

$$I = \int_{-1}^{0} \frac{dt}{9+16\left(1-t^2\right)}, \quad = \frac{1}{16} \int_{-1}^{0} \frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$$ 1 m

$$= \frac{1}{16} \cdot \frac{1}{2\left(5/4\right)} \left[\log \left| \frac{\frac{5}{4}+t}{\frac{5}{4}-t} \right| \right]_{-1}^{0}$$ 1½ m

$$= \frac{1}{40} \left(\log 1 - \log \frac{1}{9} \right)$$

$$= \frac{1}{40} \log 9$$ 1 m