QUESTION PAPER CODE 65/2

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. 3   2. – 2   3. x sin x   4. {1, 2, 3}

5. 1   6. – I   7. $p = -\frac{1}{3}$

8. $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$ 9. log 2   10. $\vec{a} = 5\hat{i} + 5\hat{k}$ 1×10 = 10 m

SECTION - B

11. $y = [x(x-2)]^2 = [x^2 - 2x]^2 \quad \therefore \quad \frac{dy}{dx} = 2(x^2 - 2x)(2x - 2)$ 1 m

$\Rightarrow \quad \frac{dy}{dx} = 4x(x-1)(x-2)$ 1 m

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x = 0, \; x = 1, \; x = 2$ ½ m

$\therefore$ Intervals are $(-\infty, 0), \; (0, 1), \; (1, 2), \; (2, \infty)$ ½ m

since $\frac{dy}{dx} > 0$ in $(0, 1)$ or $(2, \infty)$

$\therefore$ f(x) is increasing in $(0, 1)$ **U** $(2, \infty)$ 1 m

OR

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \Rightarrow \quad \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = \frac{b^2 x}{a^2 y}$ 1 m

slope of tangent at $(\sqrt{2}a, b) = \frac{\sqrt{2}b}{a}$ ½ m

slope of normal at $(\sqrt{2}a, b) = -\frac{a}{\sqrt{2}b}$ ½ m

Equation of tangent is $y - b = \frac{\sqrt{2}\, b}{a}\left(x - \sqrt{2}\, a\right)$ ½ m

i.e. $\sqrt{2}\ bx - ay = ab$ ½ m

and equation of normal is $y - b = -\frac{a}{\sqrt{2}\, b}\left(x - \sqrt{2}\, a\right)$ ½ m

i.e. $ax + \sqrt{2}\ by = \sqrt{2}\ (a^2 + b^2)$ ½ m

12. Let $I = \int_0^{\pi} \frac{4x \sin x}{1 + \cos^2 x}\, dx$

$x \to (\pi - x)$ gives $I = \int_0^{\pi} \frac{4\,(\pi - x) \sin (\pi - x)}{1 + \cos^2 (\pi - x)}\, dx = \int_0^{\pi} \frac{4\,(\pi - x) \sin x}{1 + \cos^2 x}\, dx$ 1 m

$\therefore \quad 2I = 4\pi \int_0^{\pi} \frac{\sin x}{1 + \cos^2 x}\, dx$ ½ m

Put $\cos x = t$

$\therefore \quad \sin x\, dx = - dt$ ½ m

$\therefore \quad I = 2\pi \int_1^{-1} \frac{-dt}{1 + t^2}$ or $2\pi \int_{-1}^{1} \frac{dt}{1 + t^2}$ 1 m

$= 2\pi\ \left[\tan^{-1} t\right]_{-1}^{1} = 2\pi\ \left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \pi^2$ 1 m

OR

$I = \int \frac{x + 2}{\sqrt{x^2 + 5x + 6}}\, dx = \int \frac{\frac{1}{2}(2x + 5) - \frac{1}{2}}{\sqrt{x^2 + 5x + 6}}\, dx$ 1 m

$= \frac{1}{2} \int \frac{2x + 5}{\sqrt{x^2 + 5x + 6}}\, dx - \frac{1}{2} \int \frac{dx}{\sqrt{\left(x + 5/2\right)^2 - \left(\frac{1}{2}\right)^2}}$ ½+½ m

$= \sqrt{x^2 + 5x + 6} - \frac{1}{2} \log \left| \left(x + \frac{5}{2}\right) + \sqrt{x^2 + 5x + 6} \right| + c$ 1+1 m

13. $y = P e^{ax} + Q e^{bx} \Rightarrow \frac{dy}{dx} = a P e^{ax} + b Q e^{bx}$ 1 m

$\frac{d^2y}{dx^2} = a^2 P e^{ax} + b^2 Q e^{bx}$ 1 m

$\therefore$ LHS $= \frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$

$= a^2 P e^{ax} + b^2 Q e^{bx} - (a+b)\{a P e^{ax} + b Q e^{bx}\} + ab\{P e^{ax} + Q e^{bx}\}$ 1 m

$= P e^{ax}\{a^2 - a^2 - ab + ab\} + Q e^{bx}\{b^2 - ab - b^2 + ab\}$ 1 m

$= 0 + 0 = 0. =$ R.H.S.

14. Putting $x = \cos\theta$ in LHS, We get

$$\text{LHS} = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{1-\tan\theta/2}{1+\tan\theta/2}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta/2\right)\right]$$ ½+1 m

$$= \frac{\pi}{4} - \frac{1}{2}\theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = \text{R.H.S}$$ ½ m

OR

Given equation can be written as

$$\tan^{-1}\left(\frac{x-2}{x-4}\right) = \tan^{-1} 1 - \tan^{-1}\left(\frac{x+2}{x+4}\right)$$ ½ m

$$= \tan^{-1}\left(\frac{1-\frac{x+2}{x+4}}{1+\frac{x+2}{x+4}}\right) = \tan^{-1}\left(\frac{2}{2x+6}\right)$$ 1+½ m

$$\therefore \quad \frac{x-2}{x-4} = \frac{1}{x+3}$$ ½ m

$$\Rightarrow \quad x^2 + x - 6 = x - 4 \quad \text{or} \quad x^2 = 2 \;\therefore\; x = \pm\sqrt{2}$$ ½+1 m

15. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{1+x^2}\cdot y = \frac{1}{1+x^2}\cdot e^{\tan^{-1}x}$$ 1 m

Integrating factor $= e^{\int\frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$ 1 m

$\therefore$ solution is, $y \cdot e^{\tan^{-1}x} = \int \frac{1}{1+x^2}\, e^{2\tan^{-1}x}\, dx$ 1 m

$$\Rightarrow \quad y\cdot e^{\tan^{-1}x} = \frac{1}{2}\, e^{2\tan^{-1}x} + c$$ 1 m

$$\text{or} \quad y = \frac{1}{2}\, e^{\tan^{-1}x} + c\, e^{-\tan^{-1}x}$$

16. A, B, C, D are coplaner, if $\overrightarrow{AB}\cdot\overrightarrow{AC}\times\overrightarrow{AD} = 0$ 1 m

$\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k},\ \overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k},\ \overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ 1½ m

$$\overrightarrow{AB}\cdot\overrightarrow{AC}\times\overrightarrow{AD} = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$ ½ m

$$= -4(15) + 6(21) - 2(33) = 0$$ 1 m

OR

Given that $\vec{a} \cdot \dfrac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = 1$ ½ m

or $\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = |\vec{b}+\vec{c}|$ ½ m

$(\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+4\hat{j}-5\hat{k}) + (\hat{i}+\hat{j}+\hat{k})\cdot(\lambda\hat{i}+2\hat{j}+3\hat{k}) = |(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}|$ ½ m

$\Rightarrow \quad (2+4-5) + (\lambda+2+3) = \sqrt{(\lambda+2)^2+36+4}$ 1 m

$\therefore \quad (\lambda+6)^2 = (\lambda+2)^2 + 40 \quad \Rightarrow \quad \lambda = 1$ ½ m

Hence $\dfrac{\vec{b}+\vec{c}}{|\vec{b}+\vec{c}|} = \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$ or $\dfrac{3}{7}\hat{i}+\dfrac{6}{7}\hat{j}-\dfrac{2}{7}\hat{k}$ 1 m

17. getting $fog\,(x) = f\left(\dfrac{x}{x-1}\right) = \left(\dfrac{x}{x-1}\right)^2 + 2$ 1½ m

$fog(2) = 6$ ½ m

getting $g\,of\,(x) = g\left(x^2+2\right) = \dfrac{x^2+2}{x^2+1}$ 1½ m

$g\,of\,(-3) = \dfrac{11}{10}$ ½ m

18. Let probability of success be p and that of failure be q

$\therefore \quad p = 3\,q$, and $p + q = 1$

$\therefore \quad p = \dfrac{3}{4}$ and $q = \dfrac{1}{4}$ 1 m

P (atleast 3 successes) = $P(r \geq 3) = P(3) + P(4) + P(5)$ ½ m

$= \;{}^5C_3\left(\dfrac{1}{4}\right)^2\cdot\left(\dfrac{3}{4}\right)^3 + {}^5C_4\left(\dfrac{1}{4}\right)^1\cdot\left(\dfrac{3}{4}\right)^4 + {}^5C_5\left(\dfrac{3}{4}\right)^5$ 1½ m

$$= \frac{10.27}{1024} + \frac{5.81}{1024} + \frac{243}{1024} = \frac{918}{1024} \text{ or } \frac{459}{512}$$ 1 m

19. Operating $C_1 \to C_1 - (C_2 + C_3)$, we get

$$\text{LHS} = \begin{vmatrix} -2a & c+a & a+b \\ -2p & r+p & p+q \\ -2x & z+x & x+y \end{vmatrix} = -2 \begin{vmatrix} a & c+a & a+b \\ p & r+p & p+q \\ x & z+x & x+y \end{vmatrix}$$ 1½+½ m

$$\begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix} \Rightarrow \text{LHS} = -2 \begin{vmatrix} a & c & b \\ p & r & q \\ x & z & y \end{vmatrix}$$ 1 m

$$C_2 \leftrightarrow C_3 = +2 \begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = \text{RHS}$$ 1 m

20. $\frac{dx}{dt} = 2a \cos 2t\,(1 + \cos 2t) - 2a \sin^2 2t$ 1 m

$\frac{dy}{dt} = 2b \cos 2t \sin 2t - 2b \sin 2t\,(1 - \cos 2t)$ 1 m

$$\therefore \frac{dy}{dx} = \frac{b}{a}\left(\frac{\sin 2t \cos 2t - \sin 2t\,(1-\cos 2t)}{\cos 2t\,(1+\cos 2t) - \sin^2 2t}\right)$$ 1 m

At $t = \pi/4$, $\sin 2t = 1$ and $\cos 2t = 0$

$$\therefore \frac{dy}{dx}\left(\text{at } t = \pi/4\right) = \frac{b}{a}\left(\frac{0-1}{0-1}\right) = \frac{b}{a}$$ 1 m

21. Given equation can be written as

$$\frac{x}{1+x^2}\,dx - \frac{y}{1+y^2}\,dy = 0$$ 1 m

Integrating to get $\frac{1}{2}\log\left(1+x^2\right)-\frac{1}{2}\log\left(1+y^2\right)=\log c_1$ 1 m

$\Rightarrow \quad \log\left(1+x^2\right)-\log\left(1+y^2\right)=\log c_1^2=\log c$ ½ m

$\therefore \quad \frac{\left(1+x^2\right)}{\left(1+y^2\right)}=c$

$x = 0 \;\; y = 1 \;\; \Rightarrow \;\; c=\frac{1}{2}$ 1 m

$\therefore \quad 1 + y^2 = 2\,(1 + x^2) \quad \text{or} \quad y = \sqrt{2x^2+1}$ ½ m

22. Let the D.R's of the required line be a,b , c

$\therefore \quad a + 2b + 3c = 0$

and $\;-3a + 2b + 5c = 0$ 1 m

$\Rightarrow \quad \frac{a}{4}=\frac{b}{-14}=\frac{c}{8} \quad \therefore$ DRs are 2, – 7, 4 1 m

$\therefore$ Equations of line are $\frac{x-2}{2}=\frac{y-1}{-7}=\frac{z-3}{4}$ 1 m

which, in vector form is, $\vec{r}=\left(2\hat{i}+\hat{j}+3\hat{k}\right)+\lambda\left(2\hat{i}-7\hat{j}+4\hat{k}\right)$ 1 m

SECTION - C

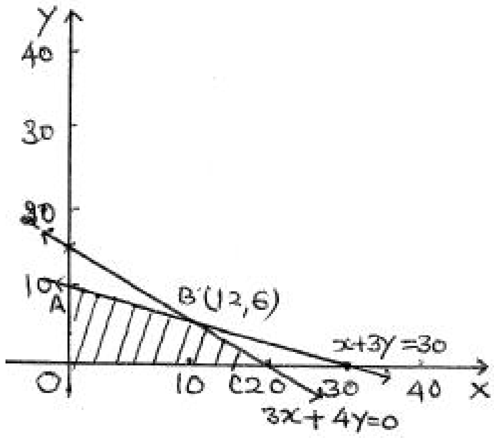
23. Let number of pieces of type A and type B, manufactured per week be x and y respectivily

$\therefore$ L.P.P. is Maximise $P = 80x + 120y$ ½ m

subject to $9x + 12y \leq 180$ or $3x + 4y \leq 60$

$x + 3y \leq 30$

$x \geq 0 \quad y \geq 0$ 2 m



For correct graph : 2 m

Vertices of feasible region are

A (0, 10), B (12, 6), C (20, 0)

P(A) = 1200, P(B) = 1680, P(C) = 1600

$\therefore$ For Max. P, No. of type A = 12

No. of type B = 6 1 m

Maximum Profit = Rs. 1680 ½ m

24. Let event E_1 : choosing first (two headed) coin

E_2 : choosing 2nd (biased) coin

E_3 : choosing 3rd (biased) coin ½ m

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ 1 m

A : The coin showing heads.

$\therefore \quad P(A/E_1) = 1,\ P(A/E_2) = \frac{75}{100} = \frac{3}{4},\ P(A/E_3) = \frac{60}{100} = \frac{3}{5}$ 1½ m

$$P(E_1/A) = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{3}{5}}$$ 1 + 1 m

$$= \frac{20}{47}$$ 1 m

OR

Total number of ways of selecting two numbers = ${}^6C_2 = 15$ ½ m

Values of x (larger of the two) can be 2, 3, 4, 5, 6 1 m

$P(x = 2) = \frac{1}{15}, \quad P(x = 3) = \frac{2}{15}, \quad P(x = 4) = \frac{3}{15}$

2½

$P(x = 5) = \frac{4}{15} \quad$ and $\quad P(x = 6) = \frac{5}{15}$

$\therefore$ Distribution can be written as

x :	2	3	4	5	6	
P(x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	
x P(x) :	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$	1 m

$$\text{Mean} = \sum x\,P(x) = \frac{70}{15} = \frac{14}{3}$$ 1 m

25. Equation of plane through the intersection of given two planes is :

$$x + y + z - 1 + \lambda\ (2x + 3y + 4z - 5) = 0$$ 1 m

or $(1+2\lambda)\,x + (1+3\lambda)\,y + (1+4\lambda)\,z - 1 - 5\lambda = 0$ .... (i) ½ m

Plane (i) is perpendicular to the plane $x - y + z = 0$,

so, $1(1+2\lambda) - 1(1+3\lambda) + 1(1+4\lambda) = 0$ 1½ m

$\Rightarrow\ 3\lambda = -1 \ \therefore\ \lambda = -\frac{1}{3}$ ½ m

$\therefore$ Equation of plane is $\left(1-\frac{2}{3}\right)x + (1-1)\,y + \left(1-\frac{4}{3}\right)z - 1 + \frac{5}{3} = 0$ ½ m

i.e $x - z + 2 = 0$ 1 m

Distance of above plane from origin $= \frac{2}{\sqrt{2}} = \sqrt{2}$ units 1 m

OR

Any point on the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ is

$$(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}$$ 1½ m

For the line to intersect the plane, the above point must satisfy the equation of plane, for some value of λ

$\therefore\ \left\{(2 + 3\lambda)\hat{i} + (-4 + 4\lambda)\hat{j} + (2 + 2\lambda)\hat{k}\right\}\cdot\left(\hat{i} - 2\hat{j} + \hat{k}\right) = 0$ 1 m

$\Rightarrow \quad 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0 \quad \Rightarrow \quad \lambda = 4$ 1½ m

$\therefore$ The point of intersection is $14\hat{i} + 12\hat{j} + 10\hat{k}$ 1 m

Required distance = $\sqrt{12^2 + 0^2 + 5^2}$ = 13 units 1 m

26. Here

$$\begin{aligned} 3x &+ 2y + z = 1600 \\ 4x &+ y + 3z = 2300 \\ x &+ y + z = 900 \end{aligned}$$

1½

$$\therefore \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix} \text{ or } AX = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \therefore X = A^{-1}B$ ½ m

Cofactors are :

$$\begin{aligned} A_{11} &= -2, & A_{12} &= -1, & A_{13} &= 3 \\ A_{21} &= -1, & A_{22} &= 2, & A_{23} &= -1 \\ A_{31} &= 5, & A_{32} &= -5, & A_{33} &= -5 \end{aligned}$$

1½ m

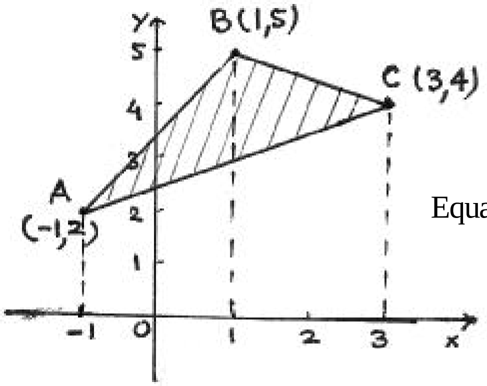
$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

$\therefore \quad x = 200, y = 300, z = 400$ 1½ m

i.e. Rs 200 for sincerity, Rs 300 for truthfulness and

Rs 400 for helpfulness

One more value like, honesty, kindness etc. 1 m

27. 

Correct fiqure 1 m

Equation of
- AB is : $y = \frac{1}{2}(3x+7)$ ½ m
- BC is : $y = \frac{1}{2}(11-x)$ ½ m
- AC is : $y = \frac{1}{2}(x+5)$ ½ m

Required area $= \frac{1}{2}\int_{-1}^{1}(3x+7)\,dx + \frac{1}{2}\int_{1}^{3}(11-x)\,dx - \frac{1}{2}\int_{-1}^{3}(x+5)\,dx$ 1 m

$= \left[\frac{1}{12}(3x+7)^2\right]_{-1}^{1} - \frac{1}{4}\left[(11-x)^2\right]_{1}^{3} - \frac{1}{4}\left[(x+5)^2\right]_{-1}^{3}$ 1½

$= 7 + 9 - 12 = 4$ sq. units 1 m

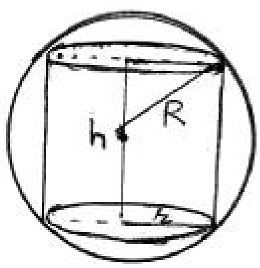
28. $I = \int\left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx = \int \frac{\cos x + \sin x}{\sqrt{\sin x \cos x}}\, dx$ 1 m

Putting $\sin x - \cos x = t$, so that $(\cos x + \sin x)\,dx = dt$ 1 m

and $\sin x \cos x = \frac{1}{2}(1-t^2)$ 1 m

$\therefore\ I = \sqrt{2}\int\frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\,\sin^{-1}t + c$ 1+1 m

$= \sqrt{2}\ \sin^{-1}(\sin x - \cos x) + c$ 1 m

29. 

Correct fiqure ½ m

let the radius and height of cylinder

be r and h respectively

$\therefore \quad V = \pi r^2 h$ .... (i) ½ m

But $r^2 = R^2 - h^2/4$

$\therefore \quad \pi h\left(R^2 - \frac{h^2}{4}\right) = \pi\left(R^2 h - \frac{h^3}{4}\right)$ 1 m

$\frac{dv}{dh} = \pi\left(R^2 - \frac{3h^2}{4}\right)$ ½ m

$\therefore \quad \frac{dv}{dh} = 0 \quad \Rightarrow \quad h^2 = \frac{4R^2}{3} \quad \text{or} \quad h = \frac{2R}{\sqrt{3}}$ ½+1 m

and $\frac{d^2v}{dh^2} = \pi\left(-\frac{6h}{4}\right) < 0 \quad \therefore$ Volume is maximum 1 m

Maximum volume = $\pi \cdot \left[R^2 \cdot \frac{2R}{\sqrt{3}} - \frac{1}{4}\left(\frac{2R}{\sqrt{3}}\right)^3\right] = \frac{4\pi R^3}{3\sqrt{3}}$ cubic units 1 m