QUESTION PAPER CODE 65/3

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1-10. 1. $-I$ 2. 3 3. 1 4. -2

5. $x \sin x$ 6. $p = -\frac{1}{3}$ 7. $\{1, 2, 3\}$

8. $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$ 9. $a = 2$ 10. 12 $1 \times 10 = 10$ m

SECTION - B

11. Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{1+x^2} \cdot y = \frac{1}{1+x^2} \cdot e^{\tan^{-1} x}$$ 1 m

Integrating factor $= e^{\int \frac{1}{1+x^2} dx} = e^{\tan^{-1} x}$ 1 m

$\therefore$ solution is, $y \cdot e^{\tan^{-1} x} = \int \frac{1}{1+x^2} e^{2\tan^{-1} x} dx$ 1 m

$\Rightarrow y \cdot e^{\tan^{-1} x} = \frac{1}{2} e^{2\tan^{-1} x} + c$ 1 m

or $y = \frac{1}{2} e^{\tan^{-1} x} + c\, e^{-\tan^{-1} x}$

12. A, B, C, D are coplaner, if $\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = 0$ 1 m

$\overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k}$, $\overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k}$, $\overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k}$ 1½ m

$$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix}$$ ½ m

$= -4(15) + 6(21) - 2(33) = 0$ 1 m

OR

Given that $\vec{a} \cdot \dfrac{\vec{b}+\vec{c}}{\left|\vec{b}+\vec{c}\right|} = 1$ ½ m

or $\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = \left|\vec{b}+\vec{c}\right|$ ½ m

$(\hat{i}+\hat{j}+\hat{k})\cdot(2\hat{i}+4\hat{j}-5\hat{k}) + (\hat{i}+\hat{j}+\hat{k})\cdot(\lambda\hat{i}+2\hat{j}+3\hat{k}) = \left|(\lambda+2)\hat{i}+6\hat{j}-2\hat{k}\right|$ ½ m

$\Rightarrow \quad (2+4-5) + (\lambda+2+3) = \sqrt{(\lambda+2)^2+36+4}$ 1 m

$\therefore \quad (\lambda+6)^2 = (\lambda+2)^2 + 40 \Rightarrow \lambda = 1$ ½ m

Hence $\dfrac{\vec{b}+\vec{c}}{\left|\vec{b}+\vec{c}\right|} = \dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{7}$ or $\dfrac{3}{7}\hat{i} + \dfrac{6}{7}\hat{j} - \dfrac{2}{7}\hat{k}$ 1 m

13. Let $I = \displaystyle\int_0^{\pi} \frac{4x\sin x}{1+\cos^2 x}\,dx$

$x \to (\pi - x)$ gives $I = \displaystyle\int_0^{\pi} \frac{4(\pi-x)\sin(\pi-x)}{1+\cos^2(\pi-x)}\,dx = \int_0^{\pi} \frac{4(\pi-x)\sin x}{1+\cos^2 x}\,dx$ 1 m

$\therefore \quad 2I = 4\pi \displaystyle\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}\,dx$ ½ m

Put $\cos x = t$

$\therefore \quad \sin x\,dx = -\,dt$ ½ m

$\therefore \quad I = 2\pi \displaystyle\int_1^{-1} \frac{-\,dt}{1+t^2}$ or $2\pi \displaystyle\int_{-1}^{1} \frac{dt}{1+t^2}$ 1 m

$= 2\pi \left[\tan^{-1} t\right]_{-1}^{1} = 2\pi\left[\dfrac{\pi}{4} - \left(-\dfrac{\pi}{4}\right)\right] = \pi^2$ 1 m

OR

$$I = \int \frac{x+2}{\sqrt{x^2+5x+6}}\,dx = \int \frac{\frac{1}{2}(2x+5) - \frac{1}{2}}{\sqrt{x^2+5x+6}}\,dx$$ 1 m

$$= \frac{1}{2}\int \frac{2x+5}{\sqrt{x^2+5x+6}}\,dx - \frac{1}{2}\int \frac{dx}{\sqrt{\left(x+\frac{5}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$$ ½+½ m

$$= \sqrt{x^2+5x+6} - \frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right| + c$$ 1+1 m

14. $y = [x\,(x-2)]^2 = [x^2-2x]^2 \;\therefore\; \frac{dy}{dx} = 2\left(x^2-2x\right)(2x-2)$ 1 m

$\Rightarrow \frac{dy}{dx} = 4\,x\,(x-1)(x-2)$ 1 m

$\frac{dy}{dx} = 0 \quad \Rightarrow \quad x = 0,\; x = 1,\; x = 2$ ½ m

$\therefore$ Intervals are $(-\infty, 0),\ (0, 1),\ (1, 2),\ (2, \infty)$ ½ m

since $\frac{dy}{dx} > 0$ in $(0, 1)$ or $(2, \infty)$

$\therefore$ f(x) is increasing in $(0, 1)$ **U** $(2, \infty)$ 1 m

OR

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{b^2x}{a^2y}$ 1 m

slope of tangent at $\left(\sqrt{2}a, b\right) = \frac{\sqrt{2}b}{a}$ ½ m

slope of normal at $\left(\sqrt{2}a, b\right) = -\frac{a}{\sqrt{2}b}$ ½ m

Equation of tangent is $y - b = \frac{\sqrt{2}\,b}{a}\left(x - \sqrt{2}\,a\right)$ ½ m i.e. $\sqrt{2}\ bx - ay = ab$ ½ m

and equation of normal is $y - b = -\frac{a}{\sqrt{2}\ b}\left(x - \sqrt{2}\ a\right)$ ½ m

i.e. $ax + \sqrt{2}\ by = \sqrt{2}\ (a^2 + b^2)$ ½ m

15. getting $fog\,(x) = f\left(\frac{x}{x-1}\right) = \left(\frac{x}{x-1}\right)^2 + 2$ 1½ m

$fog(2) = 6$ ½ m

getting $g\,of\,(x) = g\left(x^2 + 2\right) = \frac{x^2 + 2}{x^2 + 1}$ 1½ m

$g\,of\,(-3) = \frac{11}{10}$ ½ m

16. Putting $x = \cos\theta$ in LHS, We get

$$LHS = \tan^{-1}\left[\frac{\sqrt{1+\cos\theta} - \sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta} + \sqrt{1-\cos\theta}}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{\sqrt{2}\cos\theta/2 - \sqrt{2}\sin\theta/2}{\sqrt{2}\cos\theta/2 + \sqrt{2}\sin\theta/2}\right]$$ 1 m

$$= \tan^{-1}\left[\frac{1-\tan\theta/2}{1+\tan\theta/2}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta/2\right)\right]$$ ½+1 m

$$= \frac{\pi}{4} - \frac{1}{2}\theta = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x = \text{R.H.S}$$ ½ m

OR

Given equation can be written as

$$\tan^{-1}\left(\frac{x-2}{x-4}\right) = \tan^{-1} 1 - \tan^{-1}\left(\frac{x+2}{x+4}\right)$$ ½ m

$$= \tan^{-1}\left(\frac{1-\frac{x+2}{x+4}}{1+\frac{x+2}{x+4}}\right) = \tan^{-1}\left(\frac{2}{2x+6}\right)$$ 1+½ m

$$\therefore \quad \frac{x-2}{x-4} = \frac{1}{x+3}$$ ½ m

$$\Rightarrow \quad x^2 + x - 6 = x - 4 \quad \text{or} \quad x^2 = 2 \;\therefore\; x = \pm\sqrt{2}$$ ½+1 m

17. Let probability of success be p and that of failure be q

$\therefore$ $p = 3q$, and $p + q = 1$

$\therefore$ $p = \frac{3}{4}$ and $q = \frac{1}{4}$ 1 m

P (atleast 3 successes) = $P(r \geq 3) = P(3) + P(4) + P(5)$ ½ m

$$= {}^5C_3\left(\frac{1}{4}\right)^2 \cdot \left(\frac{3}{4}\right)^3 + {}^5C_4\left(\frac{1}{4}\right)^1 \cdot \left(\frac{3}{4}\right)^4 + {}^5C_5\left(\frac{3}{4}\right)^5$$ 1½ m

$$= \frac{10.27}{1024} + \frac{5.81}{1024} + \frac{243}{1024} = \frac{918}{1024} \text{ or } \frac{459}{512}$$ 1 m

18. $y = P e^{ax} + Q e^{bx} \Rightarrow \frac{dy}{dx} = a P e^{ax} + b Q e^{bx}$ 1 m

$$\frac{d^2y}{dx^2} = a^2 P e^{ax} + b^2 Q e^{bx}$$ 1 m

$$\therefore \quad LHS = \frac{d^2y}{dx^2} - (a+b)\frac{dy}{dx} + aby$$

$$= a^2 P e^{ax} + b^2 Q e^{bx} - (a+b)\left\{a P e^{ax} + b Q e^{bx}\right\} + ab\left\{P e^{ax} + Q e^{bx}\right\}$$ 1 m

$$= P e^{ax}\left\{a^2 - a^2 - ab + ab\right\} + Q e^{bx}\left\{b^2 - ab - b^2 + ab\right\}$$ 1 m

$= 0 + 0 \quad = \quad 0. \quad = \quad$ R.H.S.

19. $R_1 \to \frac{1}{a}R_1,\ R_2 \to \frac{1}{b}R_2,\ R_3 \to \frac{1}{c}R_3$

$$\therefore \quad LHS = abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$ 1 m

$$R_1 \to R_1 + R_2 + R_3 \Rightarrow LHS = abc \begin{vmatrix} 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} & 1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$ 1 m

$$= abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$ ½ m

$$\begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ \frac{1}{b} & 1 & 0 \\ \frac{1}{c} & 0 & 1 \end{vmatrix}$$ 1 m

$$= abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).\ 1 = abc + bc + ca + ab$$ ½ m

$$= \text{RHS}$$

20. $x = 3\cos t - 2\cos^3 t \quad \therefore \quad \frac{dx}{dt} = -3\sin t + 6\cos^2 t \sin t$ 1 m

$y = 3\sin t - 2\sin^3 t \quad \therefore \quad \frac{dy}{dt} = -3\cos t - 6\sin^2 t \cos t$ 1 m

$$\frac{dy}{dx} = \frac{3\cos t\,(1-2\sin^2 t)}{3\sin t\,(-1+2\cos^2 t)} = \cot t$$ 1 m

at $t = \pi/4$, $\frac{dy}{dx} = 1$ 1 m

21. Given differential equation can be written as

$$\frac{dy}{dx} = e^{3x+4y} = e^{3x}\cdot e^{4y}$$ 1 m

$$\therefore \int e^{-4y}\,dy = \int e^{3x}\,dx$$ 1 m

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + c$$ 1 m

$$\therefore \quad 4e^{3x} + 3e^{-4y} + 12c = 0$$

taking x = 0, y = 0 we get $c = -\frac{7}{12}$ ½ m

$\therefore$ The solution is $4e^{3x} + 3e^{-4y} - 7 = 0$ ½ m

22. Given lines can be written as

$$\mathbf{l}_1 : \frac{x-1}{-3} = \frac{y-2}{p/7} = \frac{z-3}{2}; \quad \mathbf{l}_2 : \frac{x-1}{-3p/7} = \frac{y-5}{1} = \frac{z-6}{-5}$$ 1 m

since the lines are perpendicular

$\therefore \quad (-3)\left(-\frac{3p}{7}\right)+\left(\frac{p}{7}\right)(1) + (2)(-5) = 0$ 1 m

$\Rightarrow \quad p = 7$ 1 m

Equation of line passing through (3, 2, – 4) and parallel to $\mathbf{l}_1$ is

$$\frac{x-3}{-3} = \frac{y-2}{1} = \frac{z+4}{2}$$ 1 m

SECTION - C

23. Equation of plane through the intersection of given two planes is :

$x + y + z - 1 + \lambda\ (2x + 3y + 4z - 5) = 0$ 1 m

or $(1+2\lambda)\,x + (1+3\lambda)\,y + (1+4\lambda)\,z - 1 - 5\lambda = 0$ .... (i) ½ m

Plane (i) is perpendicular to the plane x – y + z = 0,

so, $1(1+2\lambda) - 1(1+3\lambda) + 1(1+4\lambda) = 0$ 1½ m

$\Rightarrow \quad 3\lambda = -1 \quad \therefore \quad \lambda = -\frac{1}{3}$ ½ m

$\therefore$ Equation of plane is $\left(1-\frac{2}{3}\right)x + (1-1)\,y + \left(1-\frac{4}{3}\right)z - 1 + \frac{5}{3} = 0$ ½ m

i.e $\quad x - z + 2 = 0$ 1 m

Distance of above plane from origin $= \frac{2}{\sqrt{2}} = \sqrt{2}$ units 1 m

OR

Any point on the line $\vec{r} = 2\hat{i} - 4\hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ is

$(2+3\lambda)\hat{i} + (-4+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$ 1½ m

For the line to intersect the plane, the above point must
satisfy the equation of plane, for some value of λ

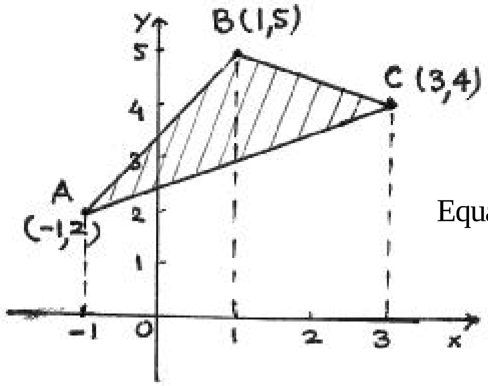
$$\therefore \quad \{(2+3\lambda)\hat{i} + (-4+4\lambda)\hat{j} + (2+2\lambda)\hat{k}\} \cdot (\hat{i} - 2\hat{j} + \hat{k}) = 0$$ 1 m

$$\Rightarrow \quad 2 + 3\lambda + 8 - 8\lambda + 2 + 2\lambda = 0 \quad \Rightarrow \quad \lambda = 4$$ 1½ m

$\therefore$ The point of intersection is $14\hat{i} + 12\hat{j} + 10\hat{k}$ 1 m

Required distance = $\sqrt{12^2 + 0^2 + 5^2} = 13$ units 1 m

24.



Correct fiqure 1 m

Equation of
- AB is : $y = \frac{1}{2}(3x + 7)$ ½ m
- BC is : $y = \frac{1}{2}(11 - x)$ ½ m
- AC is : $y = \frac{1}{2}(x + 5)$ ½ m

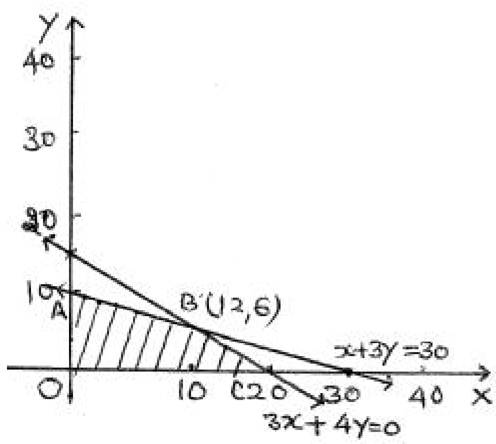
Required area = $\frac{1}{2}\int_{-1}^{1}(3x+7)\,dx + \frac{1}{2}\int_{1}^{3}(11-x)\,dx - \frac{1}{2}\int_{-1}^{3}(x+5)\,dx$ 1 m

$$= \left[\frac{1}{12}(3x+7)^2\right]_{-1}^{1} - \frac{1}{4}\left[(11-x)^2\right]_1^3 - \frac{1}{4}\left[(x+5)^2\right]_{-1}^{3}$$ 1½

$$= 7 + 9 - 12 = 4 \text{ sq. units}$$ 1 m

25. Let number of pieces of type A and type B, manufactured per week be x and y respectivily

$\therefore$ L.P.P. is Maximise $P = 80x + 120y$ ½ m



subject to $9x + 12y \le 180$ or $3x + 4y \le 60$

$x + 3y \le 30$

$x \ge 0 \quad y \ge 0$ 2 m

For correct graph : 2 m

Vertices of feasible region are

A (0, 10), B (12, 6), C (20, 0)

P(A) = 1200, P(B) = 1680, P(C) = 1600

$\therefore$ For Max. P, No. of type A = 12

No. of type B = 6 — 1 m

Maximum Profit = Rs. 1680 — ½ m

26. Let event E_1 : choosing first (two headed) coin

E_2 : choosing 2nd (biased) coin — ½ m

E_3 : choosing 3rd (biased) coin

$\therefore \quad P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ — 1 m

A : The coin showing heads.

$\therefore \quad P(A/E_1) = 1,\ P(A/E_2) = \frac{75}{100} = \frac{3}{4},\ P(A/E_3) = \frac{60}{100} = \frac{3}{5}$ — 1½ m

$$P(E_1/A) = \frac{\frac{1}{3}\cdot 1}{\frac{1}{3}\cdot 1 + \frac{1}{3}\cdot\frac{3}{4} + \frac{1}{3}\cdot\frac{3}{5}}$$ — 1 + 1 m

$$= \frac{20}{47}$$ — 1 m

OR

Total number of ways of selecting two numbers = ${}^6C_2 = 15$ — ½ m

Values of x (larger of the two) can be 2, 3, 4, 5, 6 — 1 m

$P(x = 2) = \frac{1}{15}, \quad P(x = 3) = \frac{2}{15}, \quad P(x = 4) = \frac{3}{15}$

$P(x = 5) = \frac{4}{15}$ and $P(x = 6) = \frac{5}{15}$ — 2½

$\therefore$ Distribution can be written as

x :	2	3	4	5	6
P(x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$
x P(x) :	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$

1 m

$$\text{Mean} = \sum x\, P(x) = \frac{70}{15} = \frac{14}{3}$$ 1 m

27. Here

$$\begin{aligned} 3x + 2y + z &= 1600 \\ 4x + y + 3z &= 2300 \\ x + y + z &= 900 \end{aligned}$$ 1½

$$\therefore \begin{pmatrix} 3 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix} \text{ or } AX = B$$

$|A| = 3(-2) - 2(1) + 1(3) = -5 \neq 0 \therefore X = A^{-1}B$ ½ m

Cofactors are :

$$\begin{aligned} &A_{11} = -2, \quad A_{12} = -1, \quad A_{13} = 3 \\ &A_{21} = -1, \quad A_{22} = 2, \quad A_{23} = -1 \\ &A_{31} = 5, \quad A_{32} = -5, \quad A_{33} = -5 \end{aligned}$$ 1½ m

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{5} \begin{pmatrix} -2 & -1 & 5 \\ -1 & 2 & -5 \\ 3 & -1 & -5 \end{pmatrix} \begin{pmatrix} 1600 \\ 2300 \\ 900 \end{pmatrix}$$

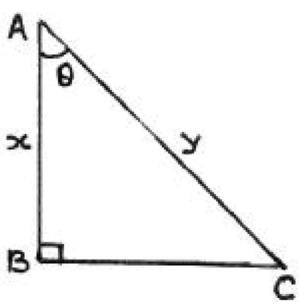
$\therefore \quad x = 200, y = 300, z = 400$ 1½ m

i.e. Rs 200 for sincerity, Rs 300 for truthfulness and

Rs 400 for helpfulness

One more value like, honesty, kindness etc. 1 m

28.



let the length of the side AB of rt. ΔABC be x and that of hypotenuse AC be y, and

$x + y = k$ (given) 1 m

Area of $\Delta ABC = \frac{1}{2}\sqrt{y^2 - x^2} \cdot x$ 1 m

$$\text{let } S = \frac{1}{4}\, x^2 (y^2 - x^2)$$

$$= \frac{1}{4}\, x^2 [(k - x)^2 - x^2]$$

$$= \frac{1}{4}\, [k^2\, x^2 - 2k\, x^3]$$ 1 m

$$\frac{ds}{dx} = 0 \;\Rightarrow\; \frac{1}{4}\left(2k^2 x - 6\,kx^2\right) = 0 \;\Rightarrow\; x = \frac{k}{3}$$ 1 m

$$\text{and } \frac{d^2s}{dx^2} = \frac{1}{4}\left(2k^2 - 12\,kx\right) = \frac{1}{4}\left(2k^2 - 4\,k^2\right) < 0$$ 1 m

$\therefore$ area of Δ is maximum for $x = k/3$ and $y = k - k/3 = 2k/3$

$\therefore$ $\cos\theta = \frac{x}{y} = 1/2$ Hence $\theta = \frac{\pi}{3}$ 1 m

29. $$I = \int \frac{1}{\sin^4 x + \sin^2 x \cos^2 x + \cos^4 x}\, dx$$

$$= \int \frac{\left(\tan^2 x + 1\right)\sec^2 x}{\tan^4 x + \tan^2 x + 1}\, dx,$$ [dividing N & D by $\cos^4 x$] 1 m

$$= \int \frac{t^2 + 1}{t^4 + t^2 + 1}\, dx,$$ where $\tan x = t$ ½ m

$$= \int \frac{1 + \frac{1}{t^2}}{t^2 + \frac{1}{t^2} + 1}\, dx,$$ [dividing N & D by t^2] ½ m

Putting $t - \frac{1}{t} = z$ so that $\left(1 + \frac{1}{t^2}\right) dt = dz$ 1 m

and $t^2 + \frac{1}{t^2} = z^2 + 2$ ½ m

$$\therefore \quad I = \int \frac{dz}{z^2 + \left(\sqrt{3}\right)^2} \qquad 1\text{ m}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{z}{\sqrt{3}} + c = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{t^2 - 1}{\sqrt{3}\, t}\right) + c\ \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{3} \tan x}\right) + c \qquad 1\tfrac{1}{2}\text{ m}$$