QUESTION PAPER CODE 65/3/A

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

		Marks
1.	Order 2 or degree = 1	½ m
	sum = 3	½ m
2.	Writing $\int \frac{y}{\sqrt{1+y^2}}\,dy = -\int \frac{x\,dx}{\sqrt{1+x^2}}$	½ m
	Getting $\sqrt{1+y^2} + \sqrt{1+x^2} = c$	½ m
3.	getting $\lvert A \rvert = 1$	½ m
	$\lvert A^n \rvert = 1$	½ m
4.	Vector Perpendicular to $\vec{a}$ and $\vec{b}$ $= \frac{\vec{a} \times \vec{b}}{\lvert \vec{a} \times \vec{b} \rvert}$	
	[Finding or using]	½ m
	Required Vector $= \hat{i} - 11\hat{j} - 7\hat{k}$	½ m
5.	Writing standard form	
	$\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ and $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$	½ m
	Finding $\theta = \frac{\pi}{2}$	½ m
6.	$\overrightarrow{OB} = \frac{\overrightarrow{OA} + \overrightarrow{OC}}{2}$	½ m
	$\overrightarrow{OC} = 2\vec{b} - \vec{a}$	½ m

SECTION - B

7. $$I = \int_0^{\frac{\pi}{2}} \frac{dx}{1+4\tan^2 x} = \int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\left(1+\tan^2 x\right)\left(1+4\tan^2 x\right)} dx$$ 1 m

Put $\tan x = t$

$$I = \int_0^{\infty} \frac{dt}{\left(1+t^2\right)\left(1+4t^2\right)} = -\frac{1}{3}\int_0^{\infty}\frac{dt}{1+t^2} + \frac{4}{3}\int_0^{\infty}\frac{dt}{1+(2t)^2}$$ 1 m

$$= -\frac{1}{3}\tan^{-1}t\Big]_0^{\infty} + \frac{4}{3\times 2}\tan^{-1}(2t)\Big]_0^{\infty}$$ 1 m

$$= -\frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{2}{3}\left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$ 1 m

8. $$I = -\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{(\sin x - \cos x)^2 - 2^2} dx$$ 1½ m

Put $\sin x - \cos x = t \Rightarrow t = -1$ to 0 1 m

$(\cos x + \sin x)\, dx = dt$

$$I = -\int_{-1}^{0} \frac{dt}{t^2 - 2^2}$$

$$= -\frac{1}{4}\log\left|\frac{t-2}{t+2}\right|\Bigg]_{-1}^{0}$$ 1 m

$$= -\frac{1}{4}\{0 - \log 3\}$$

½ m

$$= \frac{1}{4}\log 3$$

9. Writing $\vec{d} = \lambda\left(\vec{a} \times \vec{b}\right)$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$ 1 m

$$= \lambda \left(32\hat{i} - \hat{j} - 14\hat{k}\right) \text{ .... (1)}$$ 1 m

$$\vec{c} \cdot \vec{d} = 27$$

$$\left(2\hat{i} - \hat{j} + 4\hat{k}\right) \cdot \lambda \left(32\hat{i} - \hat{j} - 14\hat{k}\right) = 27$$

$$9\lambda = 27$$ 1 m

$$\lambda = 3$$

$$\therefore \vec{d} = 96\hat{i} - 3\hat{j} - 42\hat{k}$$ 1 m

10. Lines are parallel ½ m

$$\therefore \text{S.D} = \left| \frac{\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b}}{\left|\vec{b}\right|} \right|$$ 1 m

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 2\hat{j} + 2\hat{k} \text{ and } \rightarrow b = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 3 & 4 \end{vmatrix}, \quad \left|\vec{b}\right| = \sqrt{29}$$ 1½ + ½ m

$$\therefore \ \text{S.D} = \left| \frac{2\hat{i} - \hat{k}}{\sqrt{29}} \right| = \frac{\sqrt{5}}{\sqrt{29}} \text{ or } \frac{\sqrt{145}}{29}$$ ½ m

OR

Required equation of plane is

$2x + y - z - 3 + \lambda(5x - 3y + 4z + 9) = 0 \ \rightarrow \ (1)$ 1 m

$x(2 + 5\lambda) + y(1 - 3\lambda) + z(-1 + 4\lambda) + 9\lambda - 3 = 0$ 1 m

(1) is parallel to $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z-5}{5}$

$\therefore \ 2(2 + 5\lambda) + 4(1 - 3\lambda) + 5(-1 + 4\lambda) = 0$

$\Rightarrow \ \lambda = -\frac{1}{6}$ 1 m

(1) $\Rightarrow \ 7x + 9y - 10z - 27 = 0$ 1 m

11. P (step forward) $= \frac{2}{5}$, P (step backword) $= 3/5$ ½ m

He can remain a step away in either of the

ways : 3 steps forward & 2 backwards 1 m

or 2 steps forward & 3 backwards

$\therefore$ required possibility $= {}^5C_3 \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2 + {}^5C_2 \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^3$ 2 m

$= \frac{72}{125}$ ½ m

OR

A die is thrown

Let E_1 be the event of getting 1 or 2
Let E_2 be the event of getting 3, 4, 5 or 6
Let A be the event of getting a tail

$$P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3}$$ 1 m

$$\Rightarrow P\left(\frac{A}{E_1}\right) = \frac{3}{8}, \& \ P\left(\frac{A}{E_2}\right) = \frac{1}{2}$$ 1 m

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\times P\left(\frac{A}{E_2}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right) + P(E_2)\times P\left(\frac{A}{E_2}\right)}$$ 1 m

$$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}}$$

$$= \frac{8}{11}$$ 1 m

12. A = I A

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1 m

Using elementary row trans formations to get

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix} A$$ 2 m

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 & 1 \\ -9 & 6 & -2 \\ 5 & -3 & 1 \end{bmatrix}$$ 1 m

OR

$$AC = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 12 \\ 30 \end{bmatrix}$$ 1 m

$$BC = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \\ -2 \end{bmatrix}$$ 1 m

$$AC + BC = \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ ½ m

$$(A+B)\ C = \begin{bmatrix} 0 & 7 & 8 \\ -5 & 0 & 10 \\ 8 & -6 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$$ ½ m

$$= \begin{bmatrix} 10 \\ 20 \\ 28 \end{bmatrix}$$ 1 m

Yes, $(A + B)\ C = AC + BC$

13. $$f(x) = \begin{cases} -2x+1 & \text{if} \quad x < 0 \\ 1 & \text{if} \quad 0 \le x < 1 \\ 2x-1 & \text{if} \quad x \ge 1 \end{cases}$$ 1½ m

Only possible discontinuties are at $x = 0,\ x = 1$

at $x = 0$: at $x = 1$

L. H. limit = 1 : L. H. limit = 1 1 m

$f(0)$ = R. H. limit = 1 : $f(1)$ = R. H. limit = 1

$\therefore$ $f(x)$ is continuous in the interval $(-1, 2)$ ½ m

At $x = 0$

L. H. D = $-2 \ne$ R. H. D = 1 1 m

$\therefore$ $f(x)$ is not differentiable in the interval $(-1, 2)$

14. $x = a\ (\cos 2t + 2t \sin 2t)$

$y = a\ (\sin 2t\ - 2t \cos 2t)$

$\Rightarrow\ \dfrac{dx}{dt} = 4\text{ at} \cos 2\ t$ 1 m

$\Rightarrow\ \dfrac{dy}{dt} = 4\text{ at} \sin\ 2\ t$ 1 m

$\Rightarrow\ \dfrac{dy}{dx} = \tan 2\ t$ ½ m

$\Rightarrow\ \dfrac{d^2y}{dx^2} = 2 \sec^2 2\ t\ \cdot\ \dfrac{dt}{dx}$ 1 m

$\dfrac{d^2y}{dx^2} = \dfrac{1}{2\text{ at} \cos^3 2\ t}$ ½ m

15. $\dfrac{y}{x} = \log x - \log\ (ax + b)$

differentiating w.r.t. x, 1 m

$= \dfrac{x\dfrac{dy}{dx} - y}{x^2} = \dfrac{1}{x} - \dfrac{a}{ax+b} = \dfrac{b}{x\ (ax+b)}$

$= x\ \cdot\ \dfrac{dy}{dx} - y\ = \dfrac{bx}{(ax+b)}$ .... (1) 1 m

differentiating w.r.t. x again

$x\ \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - \dfrac{dy}{dx} = \dfrac{\left(ax+b\right) b - abx}{(ax+b)^2}$

$$x \frac{d^2y}{dx^2} = \frac{b^2}{(ax+b)^2}$$ 1 m

Writing $\Rightarrow x^3 \frac{d^2y}{dx^2} = \left(\frac{bx}{ax+b}\right)^2$ .... (2) ½ m

From (1) and (2) $\Rightarrow$

$$x^3 \frac{d^2y}{dx^2} = \left(x \cdot \frac{dy}{dx} - y\right)^2$$ ½ m

16. $I = \int \frac{x + \sin x - x\left(1+\cos x\right)}{x\left(x+\sin x\right)} dx$ 1 m

$= \int \frac{1}{x} dx - \int \frac{1+\cos x}{x+\sin x} dx$ put $x + \sin x = t$ 2 m

$\Rightarrow (1+\cos x)\ dx = dt$

$= \log|x| - \log|x+\sin x| + c$ 1 m

OR

$I = \int \frac{\left(x-1\right)\left(x^2+x+1\right)+1}{\left(x-1\right)\left(x^2+1\right)} dx$ ½ m

$= \int \frac{x^2+x+1}{x^2+1} dx + \int \frac{dx}{\left(x-1\right)\left(x^2+1\right)}$ 1 m

$= \int\left(1 + \frac{x}{x^2+1} + \frac{1}{2}\frac{1}{x-1} - \frac{1}{2}\frac{x}{x^2+1} - \frac{1}{2}\frac{1}{x^2+1}\right) dx$ 1½ m

$= x + \frac{1}{4}\log|x^2+1| + \frac{1}{2}\log|x-1| - \frac{1}{2}\tan^{-1}x + c$ 1 m

17. $$\begin{array}{l} \text{Family A} \Rightarrow \\ \text{Family B} \Rightarrow \end{array} \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{array}{cc} C & P \\ \left[\begin{matrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{matrix}\right] & \end{array}$$ 2 m

Writing Matrix Multiplication as $\begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix}$ 1 m

Writing about awareness of balanced diet 1 m

Alt : Method

Taking the given data for all Men, all Women, all Children for each family, the solution must be given marks accordingly

18. $$\tan\left\{\tan^{-1}\left(\frac{1}{5}\right)+\frac{\pi}{4}\right\} = \tan\left\{\tan^{-1}\left(\frac{2/5}{1-\frac{1}{25}}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \tan\left\{\tan^{-1}\left(\frac{5}{12}\right)+\frac{\pi}{4}\right\}$$ 1 m

$$= \frac{\frac{5}{12}+1}{1-\frac{5}{12}} = \frac{17}{7}$$ 1+1 m

19. Writing $C_1 \leftrightarrow C_2$

$$A = -2\begin{vmatrix} 1 & a^3 & a \\ 1 & b^3 & b \\ 1 & c^3 & c \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$A = -\ 2 \begin{vmatrix} 0 & a^3 - b^3 & a - b \\ 0 & b^3 - c^3 & b - c \\ 1 & c^3 & c \end{vmatrix}$$ 1+1 m

$$A = -\ 2\,(a - b)\,(b - c) \begin{vmatrix} 0 & a^2 + ab + b^2 & 1 \\ 0 & b^2 + c^2 + bc & 1 \\ 1 & c^3 & c \end{vmatrix}$$ 1 m

$$= -2\,(a - b)\ (b - c)\ \left\{a^2 + ab + b^2 - b^2 - bc - c^2\right\}$$ ½ m

$$= \ 2\ (a - b)\ (b - c)\ (c - a)\ (a + b + c)$$ ½ m

SECTION - C

20. Possible values of x are 0, 1, 2 and x is a random variable 1½ m

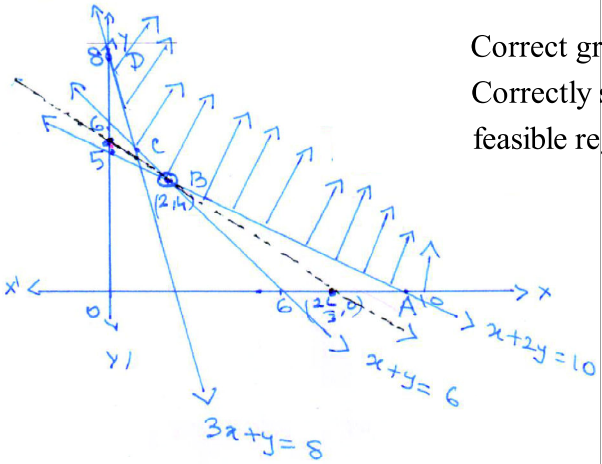
x:	P(x)	x P(x)	x^2 P(x)		
0	$\frac{{}^2C_0 \times {}^5C_2}{{}^7C_2} = \frac{20}{42}$	0	0	For P (x)	1½ m
1	$\frac{{}^2C_1 \times {}^5C_1}{{}^7C_2} = \frac{20}{42}$	$\frac{20}{42}$	$\frac{20}{42}$	For x P (x)	½ m
2	$\frac{{}^2C_2 \times {}^5C_0}{{}^7C_2} = \frac{2}{42}$	$\frac{4}{42}$	$\frac{8}{42}$	For x^2 P (x)	½ m

$$\sum x\ P(x) = \frac{24}{42}\ ;\ \sum x^2\ P(x) = \frac{28}{42}$$ 1 m

$$\text{Mean} = \sum x\ P(x) = \frac{4}{7}\ ;\ \ var\,iance = \sum x^2\ P(x) - \left[\sum x\ P(x)\right]^2$$ 1 m

$$\text{Variance} = \frac{50}{147} \qquad\qquad = \frac{2}{3} - \frac{16}{49} = \frac{50}{147}$$

21.



Correct graphs of 3 lines 3 m

Correctly shading

feasible region ½

Vertices are A (10, 0), B (2, 4), C (1, 5) & D (0, 8) 1 m

$Z = 3x + 5y$ is minimum

at B (2, 4) and the minimum Value is 26. 1 m

on Plotting $(3x + 5y < 26)$

since these it no common point with the feasible

region, Hence, $x = 2$, $y = 4$ gives minimum Z ½ m

22. Here $R = \{(a, b) : a, b \in \mathfrak{R}$ and $a - b + \sqrt{3} \in S$, where

S is the set of all irrational numbers.$\}$

(i) $\forall\ a \in \mathfrak{R}, (a, a) \in R$ as $a - a + \sqrt{3}$ is irrational

$\therefore$ R is reflexive 1½ m

(ii) Let for $a, b \in \mathfrak{R}$, $(a, b) \in R$ i. e. $a - b + \sqrt{3}$ is irrational

$a - b + \sqrt{3}$ is irrational $\Rightarrow$ $b - a + \sqrt{3} \in S$ $\therefore$ $(b, a) \in R$

Hence R is symmetric 2 m

(iii) Let $(a, b) \in R$ and $(b, c) \in R$, for $a, b, c \in \mathfrak{R}$

$\therefore$ $a - b + \sqrt{3} \in S$ and $b - c + \sqrt{3} \in S$

adding to get $a - c + 2\sqrt{3} \in S$ Hence (a, c) ∈ R 2½ m

$\therefore$ R is Transitive

OR

$\forall$ a, b, c, d, e, f $\in \Re$

$$\left(\left(a, b\right) * \left(c, d\right)\right) * \left(e, f\right) = \left(a + c, b + d\right) * \left(e, f\right)$$

1 m

$$= \left(a + c + e, b + d + f\right) \quad \rightarrow \quad (3)$$

$$\left(a, b\right) * \left(\left(c_{,} d\right) * \left(e, f\right)\right) = \left(a, b\right) * \left(c + e, d + f\right)$$

1 m

$$= \left(a + c + e, b + d + f\right) \quad \rightarrow \quad (4)$$

$\therefore$ * is Associative

Let (x, y) be on identity element in $\Re \times \Re$

$\Rightarrow$ (a, b) * (x, y) = (a, b) = (x, y) * (a, b)

$\Rightarrow$ a + x = a, b + y = b

x = 0 , y = 0 2 m

$\therefore$ (0, 0) is identity element

Let the inverse element of (3, – 5) be $(x_1, y_1,)$

$\Rightarrow$ (3, – 5) * (x_1, y_1) = (0, 0) = (x_1, y_1) * (3, – 5)

$3 + x_1 = 0, \quad -5 + y_1 = 0$

$x_1 = -3 , \quad y_1 = 5$

$\Rightarrow$ (– 3, 5) is an inverse of (3, – 5) 2 m

23.

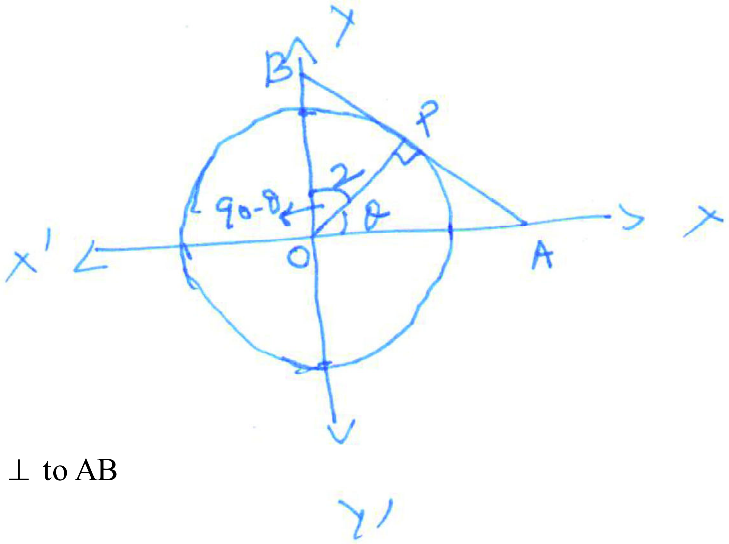


Fig. ½ m

$x^2 + y^2 = 4$. OP is $\perp$ to AB

$\cos\theta = \dfrac{2}{OA}$; $OA = 2\sec\theta$ ½ m

$\cos\left(90^\circ - \theta\right) = \dfrac{2}{OB}$

$OB = 2\,\text{cosec}\,\theta$ ½ m

Let $S = OA + OB = 2\,(\sec\theta + \text{cosec}\,\theta)$ .... (1) 1 m

$\dfrac{dS}{d\theta} = 2\left(\sec\theta\tan\theta - \text{cosec}\,\theta\cdot\cot\theta\right)$ 1m

$= 2\left(\dfrac{\sin^3\theta - \cos^3\theta}{\sin^2\theta\cdot\cos^2\theta}\right)$ .... (2)

for maxima or minima $\dfrac{dS}{d\theta} = 0$

$\Rightarrow \theta = \dfrac{\pi}{4},$ 1 m

(2) $\Rightarrow \dfrac{d^2S}{d\theta^2} > 0$ when $\theta = \dfrac{\pi}{4}$ 1 m

$\therefore$ OA + OB is minimum

$\Rightarrow OA + OB = 4\sqrt{2}$ unit ½ m

24. $(x - y)\ \dfrac{dy}{dx} = x + 2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

$$\frac{dy}{dx} = \frac{1+2\frac{y}{x}}{1-\frac{y}{x}} = f\left(\frac{y}{x}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$\therefore$ differential equation is homogeneous Eqn. 1 m

$y = vx$ to give

$$v + x \cdot \frac{dv}{dx} = \frac{1+2v}{1-v}$$ ½ m

$$\Rightarrow \int \frac{1-v}{1+v+v^2}\, dv = \int \frac{dx}{x}$$ 1 m

$$\Rightarrow -\frac{1}{2}\int \frac{2v+1}{1+v+v^2}\, dv + \frac{3}{2}\int \frac{dv}{\left(v+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \int \frac{dx}{x}$$ 1½ m

$$-\frac{1}{2}\log\left|1+v+v^2\right| + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log|x| + c$$ 1 m

$$-\frac{1}{2}\log\left|\frac{x^2+xy+y^2}{x^2}\right| + \sqrt{3}\tan^{-1}\left(\frac{2y+x}{x\sqrt{3}}\right) = \log|x| + c$$ 1 m

OR

$$(x-h) + (y-k)\ \frac{dy}{dx} = 0$$ 1 m

and $1 + (y-k)\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0$ 1 m

$$\Rightarrow \ (y-k) = \frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\frac{d^2y}{dx^2}}$$ 1 m

(1) $$\Rightarrow \ (x-h) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} \ \frac{dy}{dx}$$ 1 m

Putting in the given eqn.

$$\frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} \cdot \left(\frac{dy}{dx}\right)^2 + \frac{\left(1+\left(\frac{dy}{dx}\right)^2\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2} = r^2$$ 1 m

or $$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = r^2 \left(\frac{d^2y}{dx^2}\right)^2$$ 1 m

25. Eqn. of a plane through

and Points A (6, 5, 9), B (5, 2, 4) & C (– 1, – 1, 6) is

$$\Rightarrow \begin{vmatrix} x-6 & y-5 & z-9 \\ 2 & 3 & 2 \\ -6 & -3 & 2 \end{vmatrix} = 0$$ 2½ m

$\Rightarrow$ 3x – 4y + 3z – 25 = 0 → (2) 1½ m

distance from (3, – 1, 2) to (2)

$$d = \left| \frac{9+4+6-25}{\sqrt{9+16+9}} \right| = \frac{6}{\sqrt{34}} \text{ units}$$ 2 m

26.

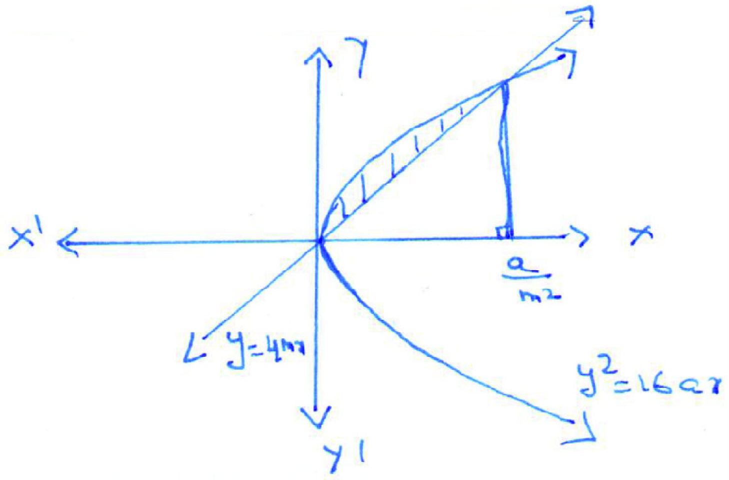


Figure .... ½ m

$y = 4mx \rightarrow (1)$ and $y^2 = 16\,ax \rightarrow (2)$ 1 m

$$\Rightarrow x = \frac{a}{m^2}$$

Required area $= 4\sqrt{a}\int_0^{\frac{a}{m^2}} \sqrt{x}\,dx - 4m\int_0^{\frac{a}{m^2}} x\,dx$ 2 m

$$= \frac{8}{3}\sqrt{a}\,x^{3/2}\Big]_0^{\frac{a}{m^2}} - 2m\;x^2\Big]_0^{\frac{a}{m^2}}$$

$= \frac{8}{3}\,\frac{a^2}{m^3} - \frac{2a^2}{m^3} = \frac{2}{3}\,\frac{a^2}{m^3}$ 2 m

$$\Rightarrow \frac{2}{3}\cdot a^2/_{m^3} = \frac{a^2}{12}\text{ given}$$

$$m^3 = 8$$

$m = 2$ ½ m