

QUESTION PAPER CODE 65/3/RU
EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\frac{dx}{dy} + \frac{2y}{1+y^2} \cdot x = \cot y$ ½ m

Integrating factor = $e^{\log(1+y^2)}$ or $(1+y^2)$ ½ m

2. $\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$ ½ m
 $= 0$ ½ m

3. order 2, degree 1 (any one correct) ½ m
 sum = 3 ½ m

4. $\frac{x - \frac{3}{5}}{\frac{1}{5}} = \frac{y + \frac{7}{15}}{\frac{1}{15}} = \frac{z - \frac{3}{10}}{-\frac{1}{10}}$ ½ m

Direction cosines are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ or $\frac{-6}{7}, \frac{-2}{7}, \frac{3}{7}$ ½ m

5. $|2\hat{a} + \hat{b} + \hat{c}|^2 = (2\hat{a})^2 + (\hat{b})^2 + (\hat{c})^2 + 2(2\hat{a} \cdot \hat{b} + \hat{b} \cdot \hat{c} + \hat{c} \cdot 2\hat{a})$ ½ m

$\therefore |2\hat{a} + \hat{b} + \hat{c}| = \sqrt{6}$ ½ m

6. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j}$ ½ m

unit vector is $-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$ ½ m

SECTION - B

7. $\vec{a} - \vec{b} = -\hat{i} + \hat{j} + \hat{k}$; $\vec{c} - \vec{b} = \hat{i} - 5\hat{j} - 5\hat{k}$ 1½ m

$$(\vec{a} - \vec{b}) \times (\vec{c} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$$
1½ m

\therefore Unit vector perpendicular to both of the vectors $= -\frac{\hat{j}}{\sqrt{2}} + \frac{\hat{k}}{\sqrt{2}}$ 1 m

8. let the equation of line passing through (1, 2, -4) be

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda (\hat{a} + \hat{b} + \hat{c})$$
1 m

Since the line is perpendicular to the two given lines \therefore

$$\begin{aligned} \therefore 3a - 16b + 7c &= 0 \\ 3a + 8b - 5c &= 0 \end{aligned}$$
1½ m

Solving we get, $\frac{a}{24} = \frac{b}{36} = \frac{c}{72}$ or $\frac{a}{2} = \frac{b}{3} = \frac{c}{6}$ 1 m

\therefore Equation of line is : $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k})$ ½ m

OR

Equation of plane is : $\begin{vmatrix} x+1 & y-2 & z \\ 2+1 & 2-2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$ 3 m

Solving we get, $x + 2y + 3z - 3 = 0$ 1 m

9. Let $x =$ No. of spades in three cards drawn

x : 0 1 2 3 1 m

$P(x)$: ${}_{3C_0} \left(\frac{3}{4}\right)^3$ ${}_{3C_1} \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2$ ${}_{3C_2} \left(\frac{1}{4}\right)^2 \frac{3}{4}$ ${}_{3C_3} \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^0$ 2 m

$= \frac{27}{64}$ $= \frac{27}{64}$ $= \frac{9}{64}$ $= \frac{1}{64}$

$x \cdot P(x)$: 0 $\frac{27}{64}$ $\frac{18}{64}$ $\frac{3}{64}$ ½ m

$$\text{Mean} = \sum x \cdot P(x) = \frac{48}{64} = \frac{3}{4} \quad \frac{1}{2} \text{ m}$$

OR

let p = probability of success ; q = Probability of failure

$$\text{then, } 9P(x=4) = P(x=2)$$

$$\Rightarrow 9 \cdot {}^6C_4 p^4 \cdot q^2 = {}^6C_2 \cdot p^2 \cdot q^4 \quad 2 \text{ m}$$

$$\Rightarrow 9p^2 = q^2 \quad \therefore q = 3p \quad 1 \text{ m}$$

$$\text{Also, } p + q = 1 \Rightarrow p + 3p = 1 \quad \therefore p = \frac{1}{4} \quad 1 \text{ m}$$

10. $y = e^{m \sin^{-1} x}$, differentiate w.r.t. "x", we get $\frac{dy}{dx} = \frac{m e^{m \sin^{-1} x}}{\sqrt{1-x^2}}$ 1½ m

$$\Rightarrow \sqrt{1-x^2} \frac{dy}{dx} = my, \text{ Differentiate again w.r.t. "x"}$$

$$\Rightarrow \sqrt{1-x^2} \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}} \frac{dy}{dx} = m \frac{dy}{dx} \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = m \left(\sqrt{1-x^2} \frac{dy}{dx} \right) = m(my) \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2y = 0 \quad \frac{1}{2} \text{ m}$$

11. $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1}$, $h(x) = 2x-3$

Differentiating w.r.t. "x", we get

$$f'(x) = \frac{x}{\sqrt{x^2+1}}, \quad g'(x) = \frac{1-2x-x^2}{(x^2+1)^2}, \quad h'(x) = 2 \quad 1+1\frac{1}{2}+1 \text{ m}$$

$$\therefore f'(h'(g'(x))) = \frac{2}{\sqrt{5}} \quad \frac{1}{2} \text{ m}$$

$$12. \quad \int (3-2x)\sqrt{2+x-x^2} \, dx = 2 \int \sqrt{\left(\frac{3}{2}\right)^2 - \left(x - \frac{1}{2}\right)^2} \, dx + \int (1-2x)\sqrt{2+x-x^2} \, dx \quad 2 \text{ m}$$

$$= 2 \cdot \left\{ \frac{x - \frac{1}{2}}{2} \sqrt{2+x-x^2} + \frac{9}{8} \sin^{-1} \left(\frac{x - \frac{1}{2}}{\frac{3}{2}} \right) \right\} + \frac{2}{3} (2+x-x^2)^{3/2} + c \quad 2 \text{ m}$$

$$\text{or} \quad \left(\frac{2x-1}{2} \sqrt{2+x-x^2} + \frac{9}{4} \sin^{-1} \left(\frac{2x-1}{3} \right) + \frac{2}{3} (2+x-x^2)^{3/2} + c \right)$$

OR

$$\int \frac{x^2 + x + 1}{(x^2 + 1)(x + 2)} \, dx = \frac{1}{5} \int \frac{2x + 1}{x^2 + 1} \, dx + \frac{3}{5} \int \frac{1}{x + 2} \, dx \quad 2 \text{ m}$$

$$= \frac{1}{5} \int \frac{2x}{x^2 + 1} \, dx + \frac{1}{5} \int \frac{1}{x^2 + 1} \, dx + \frac{3}{5} \int \frac{1}{x + 2} \, dx \quad \frac{1}{2} \text{ m}$$

$$= \frac{1}{5} \log |x^2 + 1| + \frac{1}{5} \tan^{-1} x + \frac{3}{5} \log |x + 2| + c \quad 1\frac{1}{2} \text{ m}$$

$$13. \quad \int_0^{\pi/4} \frac{1}{\cos^3 x \sqrt{2 \sin 2x}} \, dx = \int_0^{\pi/4} \frac{1}{\cos^4 x \cdot 2 \sqrt{\tan x}} \, dx \quad 1 \text{ m}$$

$$= \int_0^{\pi/4} \frac{(1 + \tan^2 x)}{2 \sqrt{\tan x}} \sec^2 x \, dx \quad 1 \text{ m}$$

$$= \frac{1}{2} \int_0^1 \frac{1+t^2}{\sqrt{t}} \, dt \quad \text{Taking, } \tan x = t; \quad 1 \text{ m}$$

$$= \frac{1}{2} \left[2\sqrt{t} + \frac{2}{5} t^{5/2} \right]_0^1 \quad \frac{1}{2} \text{ m}$$

$$= \frac{1}{2} \left[2 + \frac{2}{5} \right] = \frac{6}{5} \quad \frac{1}{2} \text{ m}$$

$$14. \int \log x \cdot \frac{1}{(x+1)^2} dx = \log x \cdot \frac{-1}{x+1} + \int \frac{1}{x} \cdot \frac{1}{x+1} dx \quad 2 \text{ m}$$

$$= \frac{-\log x}{x+1} + \int \frac{1}{x} dx - \int \frac{1}{x+1} dx \quad 1 \text{ m}$$

$$= \frac{-\log x}{x+1} + \log x - \log(x+1) + c \quad 1 \text{ m}$$

$$\text{or } \frac{-\log x}{x+1} + \log \left(\frac{x}{x+1} \right) + c$$

$$15. \begin{pmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{pmatrix} \begin{pmatrix} 50 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 30000 \\ 23000 \\ 39000 \end{pmatrix} \quad 2 \text{ m}$$

cost incurred respectively for three villages is Rs. 30,000, Rs. 23,000, Rs. 39,000 1 m

One value : Women welfare or Any other relevant value 1 m

$$16. \tan^{-1} \left(\frac{x+1+x-1}{1-(x+1)(x-1)} \right) = \tan^{-1} \left(\frac{8}{31} \right) \quad 2 \text{ m}$$

$$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31} \quad \therefore 4x^2 + 31x - 8 = 0 \quad 1 \text{ m}$$

$$\therefore x = \frac{1}{4}, -8 \text{ (Rejected)} \quad 1 \text{ m}$$

OR

$$\text{L.H.S.} = \tan^{-1} \left(\frac{x-y}{1+xy} \right) + \tan^{-1} \left(\frac{y-z}{1+yz} \right) + \tan^{-1} \left(\frac{z-x}{1+zx} \right) \quad 2 \text{ m}$$

$$\left. \begin{aligned} &= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \tan^{-1}z - \tan^{-1}x \\ &= 0 = \text{RHS} \end{aligned} \right\} \quad 2 \text{ m}$$

$$17. \begin{vmatrix} a^2 & bc & ac + c^2 \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = abc \begin{vmatrix} a & c & a + c \\ a + b & b & a \\ b & b + c & c \end{vmatrix}$$

Taking a, b & c common from C_1 , C_2 and C_3 1 m

$$= 2 abc \begin{vmatrix} a + c & c & a + c \\ a + b & b & a \\ b + c & b + c & c \end{vmatrix}$$

$C_1 \rightarrow C_1 + C_2 + C_3$ and taking 2 common from C_1 1 m

$$= 2 abc \begin{vmatrix} a + c & c & 0 \\ a + b & b & -b \\ b + c & b + c & -b \end{vmatrix} \quad C_3 \rightarrow C_3 - C_1 \quad 1 m$$

$$= 2 abc \begin{vmatrix} a + c & c & 0 \\ a - c & -c & 0 \\ b + c & b + c & -b \end{vmatrix} \quad R_2 \rightarrow R_2 - R_3 \quad \frac{1}{2} m$$

Expand by C_3 , $= 2 abc (-b) (-ac - c^2 - ac + c^2) = 4a^2 b^2 c^2$ $\frac{1}{2} m$

$$18. \text{Adj } A = \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix}; |A| = 27 \quad 2+1 m$$

$$A. \text{Adj } A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix} = 27 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A| I_3 \quad 1 m$$

$$19. f(x) = |x - 1| + |x + 1|$$

$$L f'(-1) = \lim_{x \rightarrow (-1)^-} \frac{\{-(x-1) - (x+1)\} - 2}{x - (-1)} = \lim_{x \rightarrow (-1)^-} \frac{-2(x+1)}{x+1} = -2 \quad 1 m$$

$$R f'(-1) = \lim_{x \rightarrow (-1)^+} \frac{\{-(x-1)+(x+1)\}-2}{x-(-1)} = \lim_{x \rightarrow (-1)^+} \frac{0}{x+1} = 0 \quad 1 \text{ m}$$

$-2 \neq 0 \therefore f(x)$ is not differentiable at $x = -1$

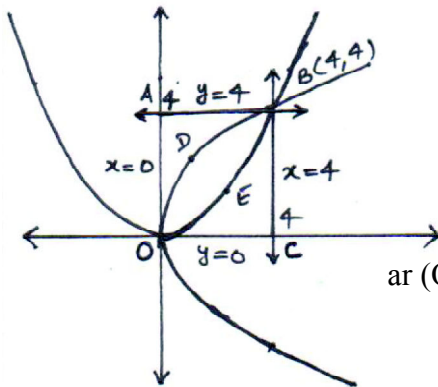
$$L f'(1) = \lim_{x \rightarrow 1^-} \frac{\{-(x-1)+(x+1)\}-2}{x-1} = \lim_{x \rightarrow 1^-} \frac{0}{x-1} = 0 \quad 1 \text{ m}$$

$$R f'(1) = \lim_{x \rightarrow 1^+} \frac{\{x-1+x+1\}-2}{x-1} = \lim_{x \rightarrow 1^+} \frac{2(x-1)}{x-1} = 2 \quad 1 \text{ m}$$

$0 \neq 2 \therefore f(x)$ is not differentiable at $x = 1$

SECTION - C

20.



correct figure 1½ m

$$\text{ar (ABDOA)} = \frac{1}{4} \int_0^4 y^2 dy = \left[\frac{y^3}{12} \right]_0^4 = \frac{16}{3} \dots\dots(i) \quad 1\frac{1}{2} \text{ m}$$

$$\begin{aligned} \text{ar (OEBDO)} &= \int_0^4 2\sqrt{x} dx - \int_0^4 \frac{x^2}{4} dx = \left[\frac{4}{3} x^{3/2} - \frac{x^3}{12} \right]_0^4 \\ &= \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \dots\dots(ii) \quad 1\frac{1}{2} \text{ m} \end{aligned}$$

$$\text{ar (OEBCO)} = \frac{1}{4} \int_0^4 x^2 dx = \left[\frac{x^3}{12} \right]_0^4 = \frac{16}{3} \dots\dots(iii) \quad 1\frac{1}{2} \text{ m}$$

From (i), (ii) and (iii) we get $\text{ar (ABDOA)} = \text{ar (OEBDO)} = \text{ar (OEBCO)}$

21. $\frac{dy}{dx} = \frac{y^2}{xy - x^2} \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2}{\frac{y}{x} - 1}$, Hence the differential equation is homogeneous 1 m

Put $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$, we get $v + x \frac{dv}{dx} = \frac{v^2}{v-1}$ 1+1 m

$$\therefore x \frac{dv}{dx} = \frac{v^2}{v-1} \quad \text{---} \quad v = \frac{v}{v-1} \quad 1 \text{ m}$$

$$\int \frac{v-1}{v} dv = \int \frac{1}{x} dx \Rightarrow v - \log v = \log x + c \quad 1 \text{ m}$$

$$\therefore \frac{y}{x} - \log \frac{y}{x} = \log x + c \quad \left(\text{or, } \frac{y}{x} = \log y + c \right) \quad 1 \text{ m}$$

OR

Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{1+y^2} x = \frac{\tan^{-1}y}{1+y^2}$ 1 m

Integrating factor = $e^{\tan^{-1}y}$ and solution is : $x e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2} dy$ 1+1½ m

$$x e^{\tan^{-1}y} = \int te^t dt = te^t - e^t + c = e^{\tan^{-1}y} (\tan^{-1}y - 1) + c \quad (\text{where } \tan^{-1}y = t) \quad 1\frac{1}{2} \text{ m}$$

$$x = 1, y = 0 \Rightarrow c = 2 \quad \therefore x \cdot e^{\tan^{-1}y} = e^{\tan^{-1}y} (\tan^{-1}y - 1) + 2 \quad 1 \text{ m}$$

$$\text{or } x = \tan^{-1}y - 1 + 2e^{-\tan^{-1}y}$$

22. E_1 : Bolt is manufactured by machine A

E_2 : Bolt is manufactured by machine B

E_3 : Bolt is manufactured by machine C

A : Bolt is defective

$$P(E_1) = \frac{30}{100}; P(E_2) = \frac{50}{100}; P(E_3) = \frac{20}{100};$$

$$P(A/E_1) = \frac{3}{100}; P(A/E_2) = \frac{4}{100}; P(A/E_3) = \frac{1}{100} \quad 3 \text{ m}$$

$$P(E_2/A) = \frac{\frac{50}{100} \times \frac{4}{100}}{\frac{30}{100} \times \frac{3}{100} + \frac{50}{100} \times \frac{4}{100} + \frac{20}{100} \times \frac{1}{100}} = \frac{200}{90 + 200 + 20} = \frac{20}{31} \quad 2 \text{ m}$$

$$P(\bar{E}_2/A) = 1 - P(E_2/A) = \frac{11}{31} \quad 1 \text{ m}$$

23. Equation of line through A and B is $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say) 2 m

General point on the line is $(-\lambda + 3, \lambda - 4, 6\lambda - 5)$ 1 m

If this is the point of intersection with plane $2x + y + z = 7$

then, $2(-\lambda + 3) + \lambda - 4 + 6\lambda - 5 = 7 \Rightarrow \lambda = 2$ 1 m

\therefore Point of intersection is $(1, -2, 7)$ 1 m

Required distance = $\sqrt{(3-1)^2 + (4+2)^2 + (4-7)^2} = 7$ 1 m

24. Let the two factories I and II be in operation for x and y days respectively to produce the order with the minimum cost then, the LPP is :

Minimise cost : $z = 12000x + 15000y$ 1 m

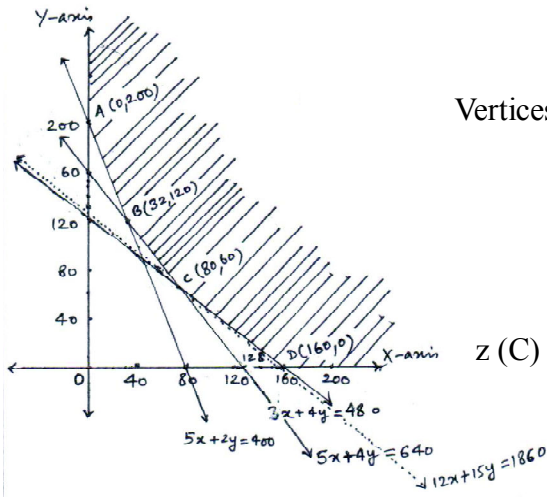
Subject to :

$$50x + 40y \geq 6400 \quad \text{or} \quad 5x + 4y \geq 640$$

$$50x + 20y \geq 4000 \quad \text{or} \quad 5x + 2y \geq 400 \quad 2 \text{ m}$$

$$30x + 40y \geq 4800 \quad \text{or} \quad 3x + 4y \geq 480$$

$$x, y \geq 0$$



correct graph 2 m

Vertices are A (0, 200) ; B (32, 120)

C (80, 60) ; D (160, 0) 1/2 m

$z(A) = \text{Rs. } 30,00,000$; $z(B) = \text{Rs. } 21,84,000$;

$z(C) = \text{Rs. } 18,60,000$ (Min.); $z(D) = \text{Rs. } 19,20,000$;

On plotting $z < 1860000$

or $12x + 15y < 1860$, we get no

point common to the feasible region

\therefore Factory I operates for 80 days

1/2 m

Factory II operates for 60 days

25. $f: \mathbb{R}_+ \rightarrow [-9, \infty)$; $f(x) = 5x^2 + 6x - 9$; $f^{-1}(y) = \frac{\sqrt{54 + 5y} - 3}{5}$

$$f \circ f^{-1}(y) = 5 \left\{ \frac{\sqrt{54 + 5y} - 3}{5} \right\}^2 + 6 \left\{ \frac{\sqrt{54 + 5y} - 3}{5} \right\} - 9 = y \quad 3 \text{ m}$$

$$f^{-1} \circ f(x) = \frac{\sqrt{54 + 5(5x^2 + 6x - 9)} - 3}{5} = x \quad 2\frac{1}{2} \text{ m}$$

Hence 'f' is invertible with $f^{-1}(y) = \frac{\sqrt{54 + 5y} - 3}{5}$ 1/2 m

OR

(i) commutative : let $x, y \in \mathbb{R} - \{-1\}$ then

$$x * y = x + y + xy = y + x + yx = y * x \quad \therefore * \text{ is commutative} \quad 1\frac{1}{2} \text{ m}$$

(ii) Associative : let $x, y, z \in \mathbb{R} - \{-1\}$ then

$$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x(y + z + yz)$$

$$= x + y + z + xy + yz + zx + xyz \quad 1\frac{1}{2} \text{ m}$$

$$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy) \cdot z$$

$$= x + y + z + xy + yz + zx + xyz \quad 1 \text{ m}$$

$$x * (y * z) = (x * y) * z \quad \therefore * \text{ is Associative}$$

(iii) Identity Element : let $e \in \mathbb{R} - \{-1\}$ such that $a * e = e * a = a \quad \forall a \in \mathbb{R} - \{-1\}$ 1/2 m

$$\therefore a + e + ae = a \Rightarrow e = 0 \quad 1/2 \text{ m}$$

(iv) Inverse : let $a * b = b * a = e = 0$; $a, b \in \mathbb{R} - \{-1\}$ 1/2 m

$$\Rightarrow a + b + ab = 0 \quad \therefore b = \frac{-a}{1+a} \quad \text{or} \quad a^{-1} = \frac{-a}{1+a} \quad 1/2 \text{ m}$$

26. Solving the two curves to get the points of intersection $(\pm 3\sqrt{p}, 8)$ 1 1/2 m

$$m_1 = \text{slope of tangent to first curve} = \frac{-2x}{9p} \quad 1\frac{1}{2} \text{ m}$$

$$m_2 = \text{slope of tangent to second curve} = \frac{2x}{p} \quad 1\frac{1}{2} \text{ m}$$

$$\text{curves cut at right angle iff } \frac{-2x}{9p} \times \frac{2x}{p} = -1 \quad 1/2 \text{ m}$$

$$\Leftrightarrow 9p^2 = 4x^2 \quad (\text{Put } x = \pm 3\sqrt{p})$$

$$\Leftrightarrow 9p^2 = 4(9p)$$

$$\therefore p = 0 ; p = 4 \quad 1 \text{ m}$$