

QUESTION PAPER CODE 65/3/B

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{2}{x}$ (Standard form) ½ m

I.F. = $\log x$ ½ m

2. $x = 2, y = 9$ (½ for correct x or y)

$\therefore x + y = 11$ ½ m

3. order 3, or degree 1 ½ m

\therefore Degree + order = 4 ½ m

4. using $\sin \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|}$ ½ m

$\Rightarrow \theta = 0^\circ$ ½ m

5. $\vec{a} \cdot \vec{b} = 0 \Rightarrow x = -6$ ½ m

$y = \pm \sqrt{40}$ or $\pm 2\sqrt{10}$ ½ m

6. $a^2 \sin^2 \alpha + a^2 \sin^2 \beta + a^2 \sin^2 \gamma$ ½ m

$= 2a^2$ ½ m

SECTION - B

7. $\left. \begin{array}{l} \vec{AB} = -2\hat{i} - 5\hat{k} \\ \vec{AC} = \hat{i} - 2\hat{j} - \hat{k} \end{array} \right\}$ 1 m

$\vec{AB} \times \vec{AC} = -10\hat{i} - 7\hat{j} + 4\hat{k}$ 1 m

$|\vec{AB} \times \vec{AC}| = \sqrt{165}$ ½ m

$$\hat{n} = \frac{\vec{AB} \times \vec{AC}}{|\vec{AB} \times \vec{AC}|} \quad 1 \text{ m}$$

$$= \frac{(-10\hat{i} - 7\hat{j} + 4\hat{k})}{\sqrt{165}} \text{ or } \frac{10\hat{i} + 7\hat{j} - 4\hat{k}}{\sqrt{165}} \quad \frac{1}{2} \text{ m}$$

$$8. \quad \left. \begin{aligned} \vec{a}_1 &= -\hat{i}, \quad \vec{b}_1 = \hat{i} + \frac{1}{2}\hat{j} - \frac{1}{12}\hat{k} \\ \vec{a}_2 &= -2\hat{j} + \hat{k}, \quad \vec{b}_2 = \hat{i} + \hat{j} + \frac{1}{6}\hat{k} \end{aligned} \right\} \quad 1 \text{ m}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} - 2\hat{j} + \hat{k} \quad \frac{1}{2} \text{ m}$$

$$\vec{b}_1 \times \vec{b}_2 = \frac{1}{6}\hat{i} - \frac{1}{4}\hat{j} + \frac{1}{2}\hat{k} \quad \frac{1}{2} \text{ m}$$

$$|\vec{b}_1 \times \vec{b}_2| = \frac{7}{12} \quad 1 \text{ m}$$

$$\text{S.D.} = \left| \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} \right| = 2 \quad 1 \text{ m}$$

OR

Foot of perpendicular are $(0, b, c)$ & $(a, 0, c)$ 1 m

Equ. of required plane

$$\begin{vmatrix} x & y & z \\ 0 & b & c \\ a & 0 & c \end{vmatrix} = 0 \quad 2 \text{ m}$$

$$\Rightarrow bcx + acy - abz = 0 \quad 1 \text{ m}$$

$$9. \quad p(x=2) = 9 \cdot P(x=3) \quad 1 \text{ m}$$

$$\Rightarrow {}^3C_2 p^2 q = 9 \cdot {}^3C_3 p^3 \cdot q^0 \quad 1 \text{ m}$$

$$\Rightarrow 3p^2(1-p) = 9p^3 \quad 1 \text{ m}$$

$$\Rightarrow p = \frac{1}{4} \quad 1 \text{ m}$$

OR

Let H_1 be the event that red ball is drawn

H_2 be the event that black ball is drawn

E be the event that both balls are red

$$P(H_1) = \frac{3}{8}, \quad P(H_2) = \frac{5}{8} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{{}^5C_2}{{}^{10}C_2} = \frac{2}{9}, \quad P(E/H_2) = \frac{{}^3C_2}{{}^{10}C_2} = \frac{1}{15} \quad 1 \text{ m}$$

$$P(E) = P(H_1) P(E/H_1) + P(H_2) \cdot P(E/H_2) \quad 1 \text{ m}$$

$$= \frac{3}{8} \cdot \frac{2}{9} + \frac{5}{8} \cdot \frac{1}{15} = \frac{1}{8} \quad 1 \text{ m}$$

$$10. \quad \frac{y}{x} = [\log x - \log(a + bx)] \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow \frac{x \frac{dy}{dx} - y}{x^2} = \frac{1}{x} - \frac{b}{a + bx} \quad 1 \text{ m}$$

$$\Rightarrow x \frac{dy}{dx} - y = \frac{ax}{a + bx} \dots\dots\dots (i) \quad 1 \text{ m}$$

Differentiating again,

$$x \frac{d^2y}{dx^2} = \frac{a^2}{(a + bx)^2} \quad 1 \text{ m}$$

$$x^3 \cdot \frac{d^2y}{dx^2} = \left(\frac{ax}{a + bx} \right)^2 = \left(x \frac{dy}{dx} - y \right)^2 \quad (\text{using (i)}) \quad \frac{1}{2} \text{ m}$$

11. $u = \sec^{-1}\left(\frac{1}{2x^2-1}\right) = 2 \cos^{-1}x \Rightarrow \frac{du}{dx} = \frac{-2}{\sqrt{1-x^2}}$ 1½ m

$v = \sqrt{1-x^2} \Rightarrow \frac{dv}{dx} = \frac{-x}{\sqrt{1-x^2}}$ 1 m

$\left. \frac{dv}{dx} \right|_{x=\frac{1}{2}} = \frac{2}{x} = 4$ 1½ m

12. Let $I = \int_0^{\pi/2} \frac{5 \sin x + 3 \cos x}{\sin x + \cos x} dx \dots\dots\dots$ (i)

$\Rightarrow I = \int_0^{\pi/2} \frac{5 \cos x + 3 \sin x}{\cos x + \sin x} dx \dots\dots\dots$ (ii) $\left(\because \int_0^a f(x) dx = \int_0^a f(a-x) dx \right)$ 1½ m

Adding (i) and (ii) 1+1 m

$2I = 8 \int_0^{\pi/2} 1 \cdot dx = 4\pi$ ½ m

$\Rightarrow I = 2\pi$

OR

put $\log x = t \Rightarrow x = e^t \Rightarrow dx = e^t dt$ 1 m

$= \int e^t \left(\log t + \frac{1}{t^2} \right) dt$

$= \int e^t \left[\left(\log t - \frac{1}{t} \right) + \left(\frac{1}{t} + \frac{1}{t^2} \right) \right] dt$ 1½ m

$= e^t \left(\log t - \frac{1}{t} \right) + c$ 1 m

$= x \left[\log(\log x) - \frac{1}{\log x} \right] + c$ ½ m

$$13. \quad I = \int \frac{x \cos x}{\cos x + x \sin x} dx \quad 1 \text{ m}$$

$$\text{put } \cos x + x \sin x = t$$

$$\Rightarrow x \cos x dx = dt \quad 1 \text{ m}$$

$$= \int \frac{dt}{t} \quad 1 \text{ m}$$

$$= \log |\cos x + x \sin x| + c \quad 1 \text{ m}$$

$$14. \quad \int \frac{x^4 dx}{(x-1)(x^2+1)} = \int \left[(x+1) + \frac{1}{(x-1)(x^2+1)} \right] dx \quad 1 \text{ m}$$

(using partial fractions)

$$= \int (x+1) dx + \frac{1}{2} \int \frac{dx}{(x-1)} - \frac{1}{2} \int \frac{x+1}{x^2+1} dx \quad 1\frac{1}{2} \text{ m}$$

$$= \frac{x^2}{2} + x + \frac{1}{2} \log |x-1| - \frac{1}{4} \log (x^2+1) - \frac{1}{2} \tan^{-1} x + c \quad 1\frac{1}{2} \text{ m}$$

$$15. \quad [15000 \quad 15000] \begin{bmatrix} \frac{2}{100} \\ \frac{x}{100} \end{bmatrix} = [1800] \quad 2 \text{ m}$$

$$\Rightarrow 300 + 150x = 1800 \quad 1 \text{ m}$$

$$\Rightarrow x = 10\%$$

yes : compassionate or any other relevant value 1 m

$$16. \quad \cot^{-1}(x+1) = \sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \quad 1\frac{1}{2} \text{ m}$$

$$\text{and } \tan^{-1} x = \cos^{-1} \frac{1}{\sqrt{1+x^2}} \quad 1\frac{1}{2} \text{ m}$$

$$\therefore \sin \left(\sin^{-1} \frac{1}{\sqrt{1+(x+1)^2}} \right) = \cos \left(\cos^{-1} \frac{1}{\sqrt{1+x^2}} \right)$$

$$\Rightarrow 1+x^2+2x+1=1+x^2 \Rightarrow x=-\frac{1}{2} \quad 1 \text{ m}$$

OR

$$2 \sin^{-1} \frac{3}{5} - \tan^{-1} \frac{17}{31}$$

$$= 2 \tan^{-1} \frac{3}{4} - \tan^{-1} \frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} \frac{24}{7} - \tan^{-1} \frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} 1 = \frac{\pi}{4} \quad 1+1 \text{ m}$$

17. $C_1 \rightarrow C_1 + C_2 + C_3,$

$$(a+b+c) \begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$$

$$\Rightarrow \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0 \quad (\because a+b+c \neq 0) \quad 2 \text{ m}$$

$$\Rightarrow -a^2 - b^2 - c^2 + ab + bc + ca = 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow -\frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2] = 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow a = b = c$$

$$18. \begin{pmatrix} 1 & -1 & 0 \\ 2 & 5 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A \quad 1 \text{ m}$$

$$R_2 \rightarrow R_2 - 2R_1,$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 7 & 3 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_2 \rightarrow R_2 - 3R_3$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & -3 \\ 0 & 0 & 1 \end{pmatrix} \cdot A$$

$$R_1 \rightarrow R_1 + R_2, \quad R_3 \rightarrow R_3 - 2R_2$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & +7 \end{pmatrix} \cdot A$$

(2 marks for all operations)

$$\therefore A^{-1} = \begin{pmatrix} -1 & 1 & -3 \\ -2 & 1 & -3 \\ 4 & -2 & 7 \end{pmatrix} \quad 1 \text{ m}$$

$$19. f(x) = x - |x - x^2| = |x - x(1-x)| = \begin{cases} 2x - x^2 & , \quad -1 \leq x < 0 \\ 0 & , \quad x = 0 \\ x^2 & , \quad 0 < x \leq 1 \end{cases} \quad 1 \text{ m}$$

$f(x)$ being a polynomial is continuous on $[-1, 0] \cup [0, 1]$

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2x - x^2) = 0 \quad \frac{1}{2} \text{ m}$$

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x^2 = 0 \quad \frac{1}{2} \text{ m}$$

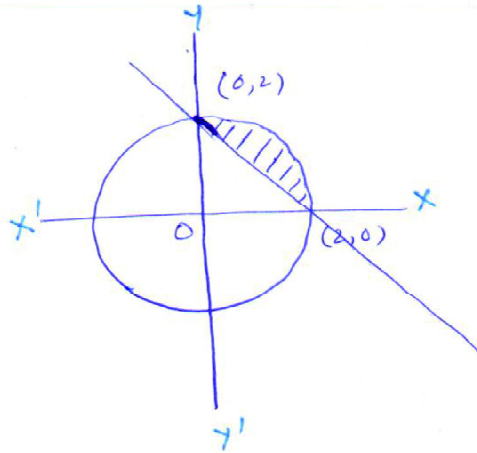
$$\text{Also, } f(0) = 0$$

$$\therefore \lim_{x \rightarrow 0^-} f(x) = f(0) = \lim_{x \rightarrow 0^+} f(x) \quad 1 \text{ m}$$

\Rightarrow There is no point of discontinuity on $[-1, 1]$ 1 m

SECTION - C

20.



1 m

Required Area

$$= \int_0^2 \sqrt{4-x^2} \, dx - \int_0^2 (2-x) \, dx \quad 2 \text{ m}$$

$$= \left[\frac{x\sqrt{4-x^2}}{2} + \frac{4}{2} \sin^{-1} \frac{x}{2} \right]_0^2 - \left[2x - \frac{x^2}{2} \right]_0^2 \quad 1+1 \text{ m}$$

$$= (\pi - 2) \text{ sq. units} \quad 1 \text{ m}$$

21. $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

put $y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx}$ 1 m

$$\Rightarrow \frac{1+v^2}{v^3} = - \frac{dx}{x} \quad 1 \text{ m}$$

Integrating both sides

$$- \frac{1}{2v^2} + \log v = -\log x + c \quad 1 \text{ m}$$

$$\Rightarrow -\frac{x^2}{2y^2} + \log y = c$$

when $x = 1, y = 1 \Rightarrow c = -\frac{1}{2}$ 1 m

$$\Rightarrow \log y = \frac{x^2 - y^2}{2y^2}$$
 ½ m

when $x = x_0, y = e \Rightarrow x_0 = \sqrt{3} e$ 1½ m

OR

I F = $e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$ 1 m

$$\therefore \frac{d}{dx} (y \cdot \sec x) = 3x^2 \sec x + x^3 \sec x \tan x$$
 1 m

$$\Rightarrow y \sec x = \int 3x^2 \sec x \cdot dx + x^3 \sec x - \int 3x^2 \cdot \sec x \, dx + c$$
 2 m

$$\Rightarrow y = x^3 + c \cos x$$

when $x = \frac{\pi}{3}, y = 0$; we get $c = \frac{-2\pi^3}{27}$ 1 m

$$\therefore y = x^3 - \frac{2\pi^3}{27} \cos x$$
 1 m

22. Let us consider the man invested on x electronic and y manually operated machines

Maximise $P = 220x + 180y$ (i) 1 m

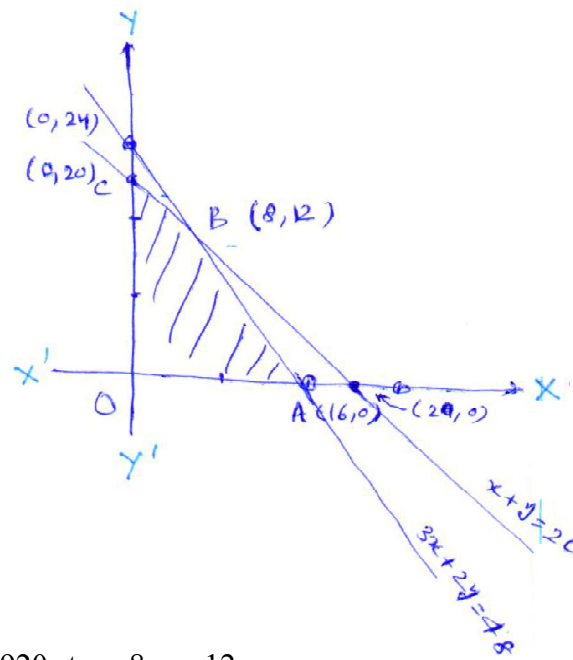
subject to

$$x + y \leq 20$$

$$3600x + 2400y \leq 57600 \Rightarrow 3x + 2y \leq 48$$
 1½ m

$$x, y \geq 0$$

(1 mark for



plotting each
line) = 2 m

(½ to find the vertices
of feasible
region)

$$P \Big|_{A(16, 0)} = 3520 \text{ Rs.}$$

$$P \Big|_{B(8, 12)} = 3920 \text{ Rs.}$$

$$P \Big|_{C(0, 20)} = 3600 \text{ Rs.}$$

Maximum profit is Rs. 3920 at $x = 8, y = 12$

1 m

23. Equation of line is $\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5}$

1 m

Equation of plane is

$$\begin{vmatrix} x-2 & y-1 & z-2 \\ 1 & 0 & -2 \\ 2 & -3 & -1 \end{vmatrix} = 0$$

1 m

$$\Rightarrow 2x + y + z - 7 = 0 \dots\dots\dots (i)$$

1 m

general point on given line $(2\lambda + 3, -3\lambda + 4, 5\lambda + 1)$ lies on (i)

1 m

$$\therefore 2(2\lambda + 3) + (-3\lambda + 4) + (5\lambda + 1) - 7 = 0 \Rightarrow \lambda = -\frac{2}{3}$$

1 m

$$\therefore \text{Point of intersection } \left(\frac{5}{3}, 6, -\frac{7}{3}\right)$$

1 m

24. Let H_1 : be the event 1, 2 appears

H_2 : be the event 3, 4, 5, 6 appears

1 m

E_3 : be the event that head appears

$$P(H_1) = \frac{2}{6} = \frac{1}{3}, \quad P(H_2) = \frac{4}{6} = \frac{2}{3} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{3}{8} \quad P(E/H_2) = \frac{1}{2} \quad 1 \text{ m}$$

$$P(H_2/E) = \frac{P(H_2) \cdot P(E/H_2)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)} \quad 1 \text{ m}$$

$$= \frac{8}{11} \quad 2 \text{ m}$$

OR

Let H_1 : be the event that 4 occurs

H_2 : be the event that 4 does not occurs 1 m

E : be the event that man reports 4 occurs
on a throw of dice

$$P(H_1) = \frac{1}{6}, \quad P(H_2) = \frac{5}{6} \quad 1 \text{ m}$$

$$P(E/H_1) = \frac{3}{5} \quad P(E/H_2) = 1 - \frac{3}{5} = \frac{2}{5} \quad 1 \text{ m}$$

$$P(H_1/E) = \frac{P(H_1) \cdot P(E/H_1)}{P(H_1) \cdot P(E/H_1) + P(H_2) \cdot P(E/H_2)} \quad 1 \text{ m}$$

$$= \frac{3}{13} \quad 2 \text{ m}$$

25.

*	0	1	2	3	4	5	6
0	0	1	2	3	4	5	6
1	1	2	3	4	5	6	0
2	2	3	4	5	6	0	1
3	3	4	5	6	0	1	2
4	4	5	6	0	1	2	3
5	5	6	0	1	2	3	4
6	6	0	1	2	3	4	5

4 m

$$\forall a \in \{0, 1, 2, 3, 4, 5, 6\}$$

$$a * 0 = a = 0 * a \Rightarrow 0 \text{ is identity} \quad 1 \text{ m}$$

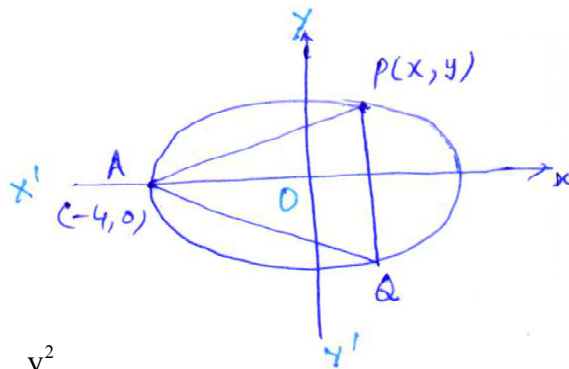
$$\forall a \in \{1, 2, 3, 4, 5, 6\}$$

$$a * b = 0 = b * a$$

$$\Rightarrow a * (7 - a) = 0 = (7 - a) * a$$

$$\Rightarrow (7 - a) \text{ is inverse of } a \quad 1 \text{ m}$$

26. $A = y(x + 4)$



$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\text{Let } z = A^2 = \frac{9}{16}(16 - x^2)(x + 4)^2 \Rightarrow y^2 = \frac{9}{16}(16 - x^2) \dots\dots\dots (i) \quad 1 \text{ m}$$

$$= \frac{9}{16}(4 - x)(4 + x)^3 \quad 1 \text{ m}$$

$$\frac{dz}{dx} = \frac{9}{16}(4 + x)^2(8 - 4x) \quad 1 \text{ m}$$

$$\frac{dz}{dx} = 0 \Rightarrow x = 2 \quad 1 \text{ m}$$

$$\frac{d^2z}{dx^2} = -\frac{9}{4}(4 + x)^2 + \frac{9}{8}(4 + x)(8 - 4x)$$

$$\left. \frac{d^2z}{dx^2} \right|_{x=2} < 0 \quad 1 \text{ m}$$

$$\therefore \text{Maximum value of } A = 9\sqrt{3} \text{ sq. units} \quad 1 \text{ m}$$