

QUESTION PAPER CODE 65/3/MT
EXPECTED ANSWERS/VALUE POINTS

SECTION - A

	Marks
1. Order : 2, degree : 2, Product : 4	$\frac{1}{2} + \frac{1}{2}$ m
2. $\frac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$	$\frac{1}{2}$ m
$\frac{d^2y}{dx^2} = -\alpha^2 (A \cos \alpha x + B \sin \alpha x)$	} $\frac{1}{2}$ m
$\frac{d^2y}{dx^2} + \alpha^2 y = 0$	
3. $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$ or any other correct example	$\frac{1}{2} + \frac{1}{2}$ m
4. Value = 3	1 m
5. Writing dr's correctly	$\frac{1}{2}$ m
D.C'S $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$	$\frac{1}{2}$ m
6. Projection of \vec{a} on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{ \vec{b} }$	$\frac{1}{2}$ m
Projection = $\frac{5}{\sqrt{2}}$	$\frac{1}{2}$ m

SECTION - B

7. $\int x \sin^{-1}x \, dx$

$$\frac{x^2}{2} \sin^{-1}x - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} \, dx \quad 1 \text{ m}$$

$$\frac{x^2}{2} \sin^{-1}x + \frac{1}{2} \int \frac{1-x^2-1}{\sqrt{1-x^2}} \, dx \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow \frac{x^2}{2} \sin^{-1}x + \frac{1}{2} \int \sqrt{1-x^2} \, dx - \frac{1}{2} \int \frac{dx}{\sqrt{1-x^2}} \, dx \quad 1 \text{ m}$$

$$\frac{x^2}{2} \sin^{-1}x + \frac{1}{2} \left(\frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1}x \right) - \frac{1}{2} \sin^{-1}x + c \quad 1\frac{1}{2}$$

or $\frac{x^2}{2} \sin^{-1}x + \frac{x}{4} \sqrt{1-x^2} - \frac{1}{4} \sin^{-1}x + c$

8. $\int_0^2 (x^2 + e^{2x+1}) \, dx$

$$h = \frac{2}{n} \quad \frac{1}{2} \text{ m}$$

$$\int_0^2 (x^2 + e^{2x+1}) \, dx = \lim_{h \rightarrow 0} h [f(0) + f(0+h) + f(0+2h) + \dots$$

$$+ \dots \dots \dots f(0+(n-1)h)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [h^2 (1^2 + 2^2 + \dots + (n-1)^2)$$

$$+ e(1 + e^{2h} + e^{4h} + \dots + e^{2(n-1)h})] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} \frac{(nh)(nh-h)(2nh-h)}{6} \quad \frac{1}{2} \text{ m}$$

$$+ \lim_{h \rightarrow 0} e.h. \left(\frac{e^{2nh} - 1}{e^{2h} - 1} \right) \quad \frac{1}{2} \text{ m}$$

$$= \frac{8}{3} + \frac{(e^4 - 1)e}{2} = \frac{8}{3} + \frac{e^5 - e}{2} \quad \frac{1}{2} \text{ m}$$

OR

$$\int_0^{\pi} \frac{x \tan x \, dx}{\sec x \operatorname{cosec} x}$$

$$\int_0^{\pi} x \sin^2 x \, dx \quad 1 \text{ m}$$

$$\text{Let } I = \int_0^{\pi} x \sin^2 x \, dx$$

$$= \int_0^{\pi} (\pi - x) \sin^2 (\pi - x) \, dx \quad \frac{1}{2} \text{ m}$$

$$= \int_0^{\pi} (\pi - x) \sin^2 x \, dx \quad \frac{1}{2} \text{ m}$$

$$2I = \pi \int_0^{\pi} \sin^2 x \, dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2} \, dx \quad \frac{1}{2} \text{ m}$$

$$= \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi} \quad 1 \text{ m}$$

$$= \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4} \quad \frac{1}{2} \text{ m}$$

9. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ ½ m

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ½ m

$x = 3\lambda + 1, y = -\lambda + 1, z = -1$ 1 m

$x = 2\mu + 4, y = 0, z = 3\mu - 1$

At the point of intersection

$\lambda = 1, \mu = 0$ 1 m

so $3\lambda + 1 = 4 = 2\mu + 4$ ½

Hence the lines are intersecting

Point of intersection is $(4, 0, -1)$ ½ m

10. Coordinats of Q are $-3\mu + 1, \mu - 1, 5\mu + 2$ ½ m

D.R's of \vec{PQ} $-3\mu - 2, \mu - 3, 5\mu - 4$ 1 m

as \vec{PQ} is parallel to the plane $x - 4y + 3z = 1$

$1(-3\mu - 2) - 4(\mu - 3) + 3(5\mu - 4) = 0$ 1½ m

$\mu = \frac{1}{4}$ 1 m

OR

The D.R's of the line are $2, -6, 4$ 1 m

mid point of the line $2, 1, -1$ 1 m

The plane passes through $(2, 1, -1)$ and is perpendicular to the plane

eqn. : $2(x - 2) - 6(y - 1) + 4(z + 1) = 0$

$$x - 3y + 2z + 3 = 0 \quad 1 \text{ m}$$

$$\text{Vector from: } \vec{r} \cdot (\hat{i} - 3\hat{j} + 2\hat{k}) + 3 = 0 \quad 1 \text{ m}$$

11. No's divisible by 6 16 1m

No's divisible by 8 12 1m

No's not divisible by 24 20 1m

$$\text{Required probability} = \frac{20}{100} = \frac{1}{5} \quad 1 \text{ m}$$

12. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 m

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix} \quad 1\frac{1}{2} \text{ m}$$

$$a + 4b = -7, \quad c + 4d = 2, \quad 2a + 5b = -8, \quad 2c + 5d = 4$$

$$\text{Solving } a = 1, \quad b = -2, \quad c = 2, \quad d = 0 \quad 1 \text{ m}$$

$$\therefore x = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix} \quad \frac{1}{2} \text{ m}$$

OR

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$|A| = 1 \neq 0$, A^{-1} will exist ½ m

$$\text{adj } A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \quad (\text{Any four correct Cofactors : 1 mark}) \quad \text{2 m}$$

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \quad \text{½ m}$$

$$\begin{aligned} A^{-1} A &= \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{1 m} \end{aligned}$$

13. $f(x) = |x-3| + |x-4|$

$$= \begin{cases} 7-2x, & x < 3 \\ 1, & 3 \leq x < 4 \\ 2x-7, & x \geq 4 \end{cases} \quad \text{1 m}$$

L. H. D at $x = 3 \quad \lim_{x \rightarrow 3^-} \frac{f(x) - f(3)}{x - 3}$

$$\lim_{x \rightarrow 3^-} \frac{6-2x}{x-3} = -2$$

R. H. D at $x = 3 \quad \lim_{x \rightarrow 3^+} \frac{f(x) - f(3)}{x - 3}$

$$= \frac{1-1}{x-3} = 0$$

L. H. D \neq R. H. D $\therefore f(x)$ is not differentiable at $x = 3$

1½ m

$$\text{L. H. D at } x = 4 \lim_{x \rightarrow 4^-} \frac{f(x) - f(4)}{x - 4}$$

$$= \frac{1-1}{x-4} = 0$$

$$\text{R. H. D at } x = 4 \lim_{x \rightarrow 4^+} \frac{f(x) - f(4)}{x - 4}$$

$$\lim_{x \rightarrow 4^+} \frac{2x - 7 - 1}{x - 4} = 2$$

L. H. D at $x = 4 \neq$ R.H.D at $x = 4$

$f(x)$ is not differentiable at $x = 4$

1½ m

14. $y = x^{e^{-x^2}}$

$$\log y = e^{-x^2} \log x$$

1 m

Diff. w. r. t x

$$\frac{1}{y} \frac{dy}{dx} = \frac{e^{-x^2}}{x} + \log x e^{-x^2} (-2x)$$

2 m

$$\frac{dy}{dx} = y \left(\frac{e^{-x^2}}{x} - 2x \log x e^{-x^2} \right)$$

½ m

$$= x^{e^{-x^2}} e^{-x^2} \left(\frac{1}{x} - 2x \log x \right)$$

½ m

OR

$$\log \sqrt{x^2 + y^2} = \tan^{-1} \frac{x}{y}$$

Diff. w. r. t. x

$$\frac{1}{2(x^2 + y^2)} \left(2x + 2y \frac{dy}{dx} \right) = \frac{1}{1 + \frac{x^2}{y^2}} \left(\frac{y - x \frac{dy}{dx}}{y^2} \right) \quad 2 \text{ m}$$

$$\frac{x + y \frac{dy}{dx}}{x^2 + y^2} = \frac{y^2}{x^2 + y^2} \left(\frac{y - x \frac{dy}{dx}}{y^2} \right) \quad 1 \text{ m}$$

$$\frac{dy}{dx} (y + x) = y - x \quad \frac{1}{2} \text{ m}$$

$$\frac{dy}{dx} = \frac{y - x}{y + x} \quad \frac{1}{2} \text{ m}$$

15. $y = \sqrt{x+1} - \sqrt{x-1}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}} \quad 1 \text{ m}$$

$$= \frac{\sqrt{x-1} - \sqrt{x+1}}{2\sqrt{x^2-1}} \quad \frac{1}{2} \text{ m}$$

$$4(x^2 - 1) \left(\frac{dy}{dx} \right)^2 = y^2 \quad \frac{1}{2} \text{ m}$$

$$4(x^2 - 1) 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + 8x \left(\frac{dy}{dx} \right)^2 = 2y \frac{dy}{dx} \quad 1 \text{ m}$$

$$(x^2 - 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = \frac{y}{4} \quad \frac{1}{2} \text{ m}$$

$$(x^2 - 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} - \frac{y}{4} = 0 \quad \frac{1}{2} \text{ m}$$

16. $\int \frac{1 - \cos x}{\cos x (1 + \cos x)} dx$

$= \int \frac{1 + \cos x - 2 \cos x}{\cos x (1 + \cos x)} dx$ 1½ m

$\int \frac{dx}{\cos x} - 2 \int \frac{dx}{1 + \cos x}$ ½ m

$\int \sec x dx - \int \sec^2 \frac{x}{2} dx$ 1 m

$\log |\sec x + \tan x| - 2 \tan \frac{x}{2} + c$ 1 m

17.

	M	W	C	Expenses		Family expenses
Family A	2	3	1	200	=	1050
Family B	2	1	3	150		1150
Family C	4	2	6	200		2300

2 m

Expenses for family A = ₹ 1050

Expenses for family B = ₹ 1150 1 m

Expenses for family C = ₹ 2300

Any relevant impact 1 m

18. $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$ 1 m

$\tan^{-1} \left(\frac{x+y}{1-xy} \right) = \cot^{-1}z$ 1 m

$\tan^{-1} \left(\frac{x+y}{1-xy} \right) = \tan^{-1} \left(\frac{1}{z} \right)$ as $z > 0$ 1 m

$$\frac{x+y}{1-xy} = \frac{1}{z} \quad \frac{1}{2} \text{ m}$$

$$xy + yz + zx = 1 \quad \frac{1}{2} \text{ m}$$

$$19. \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$C_1 \rightarrow C_1 - C_2, \quad C_2 \rightarrow C_2 - C_3$$

$$(a+b+c) \begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} = 0 \quad 2 \text{ m}$$

$$(a+b+c)(ab+bc+ca-a^2-b^2-c^2) = 0$$

$$\text{given } a \neq b \neq c, \text{ so } ab+bc+ca-a^2-b^2-c^2 \neq 0 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow (a+b+c) = 0 \quad \frac{1}{2} \text{ m}$$

SECTION - C

20. Let E_1 be the event of following course of meditation and yoga and E_2 be the event of following course of drugs 1 m

$$P(E_1) = \frac{1}{2}, \quad P(E_2) = \frac{1}{2} \quad 1 \text{ m}$$

$$P(A|E_1) = \frac{70 \times 40}{100 \times 100} \quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100} \quad 1 \text{ m}$$

Formula 1 m

$$P(E_1|A) = \frac{\frac{40}{100} \left(\frac{1}{2} \times \frac{70}{100} \right)}{\frac{40}{100} \left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100} \right)} \quad \left. \vphantom{P(E_1|A)} \right\} 2 \text{ m}$$

$$= \frac{70}{145} = \frac{14}{29}$$

21. Let the no. of items in the item A = x
 Let the no. of items in the item B = y
 (Maximize) $z = 500x + 150y$ 1 m

$$x + y \leq 60$$

$$2500x + 500y \leq 50,000$$

$$x, y \geq 0$$

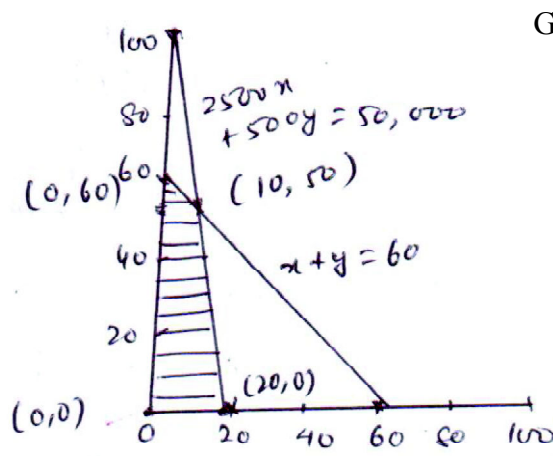
$$z(0,0) = 0$$

$$z(10,50) = 12,500$$

$$z(20,0) = 10,000$$

$$z(0,60) = 9,000$$

$$\text{Max. Profit} = \text{Rs. } 12,500$$



OR 1 m

Let the no. of packets of food X = x

Let the no. of packets of food Y = y

(minimize) $P = (6x + 3y)$ 1 m

subject to

$$12x + 3y \geq 240$$

$$4x + 20y \geq 460$$

$$6x + 4y \leq 300, x, y \geq 0$$

or

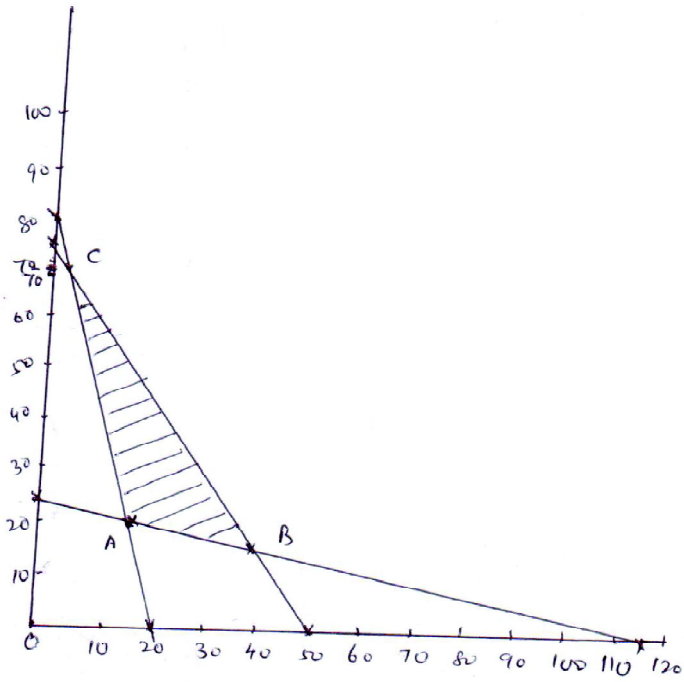
$$4x + y \geq 80$$

$$x + 5y \geq 115$$

$$3x + 2y \leq 150$$

$$x, y \geq 0$$

} 2 m



Correct points
of feasible
region

A (15, 20), B (40, 15),
C (2, 72)

So P (15, 20) = 150
P (40, 15) = 285
P (2, 72) = 228

Graph 2 m

minimum amount of vitamin A = 150 units when 15 packets of food X and 20 packets of food Y are used 1 m

22. For every $a \in A$, $(a, a) \in R$
 $\therefore |a - a| = 0$ is divisible by 2
 $\therefore R$ is reflexive 1 m
- For all $a, b \in A$

$(a, b) \in R \Rightarrow |a - b|$ is divisible by 2

$\Rightarrow |b - a|$ is divisible by 2

$\therefore (b, a) \in R \therefore R$ is symmetric

1 m

For all $a, b, c \in A$

$(a, b) \in R \Rightarrow |a - b|$ is divisible by 2

$(b, c) \in R \Rightarrow |b - c|$ is divisible by 2

So, $a - b = \pm 2k$

1 m

$$\frac{b - c = \pm 2\ell}{a - c = \pm 2m}$$

$\Rightarrow |a - c|$ is divisible by 2

$\Rightarrow (a, c) \in R$

$\Rightarrow R$ is transitive

1 m

Showing elements of $\{1, 3, 5\}$ and

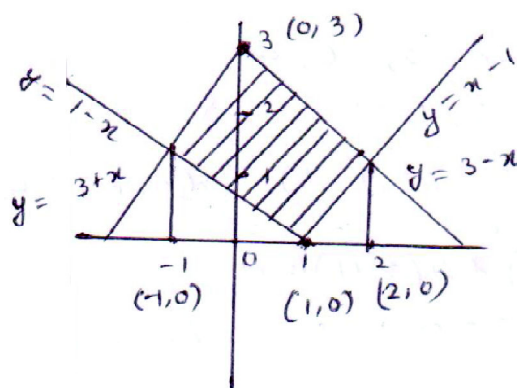
1 m

$\{2, 4\}$ are related to each other

and $\{1, 3, 5\}$ and $\{2, 4\}$ are not related to each other

1 m

23.



Graph

2 + 2 m

Area of shaded region

$$= \int_{-1}^0 (3 + x + x - 1) dx + \int_0^2 (3 - x) dx - 2 \int_1^2 (x - 1) dx$$

1 m

$$= 2 \left[\frac{(x+1)^2}{2} \right]_{-1}^0 - \left[\frac{(3-x)^2}{2} \right]_0^2 - 2 \left[\frac{(x-1)^2}{2} \right]_{-1}^2$$

$$= 1 - \frac{1}{2}(1-9) - 1 = 4 \text{ sq. units}$$

1 m

24. $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

Let $y = vx$, $\frac{dy}{dx} = v + x \frac{dv}{dx}$

½ m

$$v + x \frac{dv}{dx} = \frac{v^2}{v-1}$$

1½ m

$$x \frac{dv}{dx} = \frac{v}{v-1}$$

$$\frac{dx}{x} = \left(\frac{v-1}{v} \right) dv$$

1½ m

$$\int \frac{dx}{x} = \int \left(1 - \frac{1}{v} \right) dv$$

$$\log x = v - \log v + c$$

1 m

$$\log y = \frac{y}{x} + c \text{ or } x \log y - y = c x$$

1½ m

OR

$$\sin 2x \frac{dy}{dx} - y = \tan x$$

$$\frac{dy}{dx} - \frac{y}{\sin 2x} = \frac{\tan x}{\sin 2x}$$

1 m

$$\frac{dy}{dx} - y (\operatorname{cosec} 2x) = \frac{\sec^2 x}{2}$$

$$P = -\operatorname{cosec} 2x, \quad Q = \frac{1}{2} \sec^2 x$$

$$\begin{aligned} \int P \, dx &= - \int \operatorname{cosec} 2x \, dx \\ &= -\frac{1}{2} \log |\tan x| \end{aligned}$$

$$\text{So } e^{\int P \, dx} = \frac{1}{\sqrt{\tan x}} \quad 1\frac{1}{2} \text{ m}$$

Solution is

$$\frac{y}{\sqrt{\tan x}} = \frac{1}{2} \int \frac{\sec^2 x \, dx}{\sqrt{\tan x}} \quad \left(\begin{array}{l} \sqrt{\tan x} = t \\ \Rightarrow \frac{1}{2} \frac{\sec^2 x \, dx}{\sqrt{\tan x}} = dt \end{array} \right) \quad 1\frac{1}{2} \text{ m}$$

$$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c \quad 1 \text{ m}$$

$$\text{Getting } c = 1 \quad \frac{1}{2} \text{ m}$$

$$\Rightarrow y = \tan x - \sqrt{\tan x} \quad \frac{1}{2} \text{ m}$$

25. Eqn. of plane

$$(x + y + z - 6) + \lambda(2x + 3y + 4z + 5) = 0 \quad 2 \text{ m}$$

it passes through (1, 1, 1)

$$-3 + 14\lambda = 0 \Rightarrow \lambda = \frac{3}{14} \quad 2 \text{ m}$$

Eqn. of plane will be

$$20x + 23y + 26z - 69 = 0 \quad 1 \text{ m}$$

$$\text{vector from: } \vec{r} \cdot (20\hat{i} + 23\hat{j} + 26\hat{k}) = 69 \quad 1 \text{ m}$$

26. $y = \frac{x}{1+x^2}$

$$\frac{dy}{dx} = \frac{1-x^2}{(1+x^2)^2} \quad 2 \text{ m}$$

$$\text{Let } f(x) = \frac{1-x^2}{(1+x^2)^2}$$

$$f'(x) = 0 \Rightarrow \frac{-2x(3-x^2)}{(1+x^2)^3} = 0$$

$$\text{For max or min } x(3-x^2) = 0 \Rightarrow x = 0 \text{ or } x = \pm\sqrt{3} \quad 2 \text{ m}$$

$$\text{Calculating } \frac{d^2f(x)}{dx^2} \text{ at } x = 0 < 0$$

1 m

$$\text{at } x = \pm\sqrt{3} > 0$$

$\Rightarrow x=0$ is the point of local maxima

1 m

\Rightarrow the required pt is $(0, 0)$