QUESTION PAPER CODE 65/3/D

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\cos^2 \pi/2 + \cos^2 \pi/3 + \cos^2\theta = 1 \Rightarrow \theta = \frac{\pi}{6}$ ½ + ½ m

2. $a_{23} = \frac{|2-3|}{2} = \frac{1}{2}$ ½ + ½ m

3. $\frac{dv}{dr} = -\frac{A}{r^2}, \Rightarrow r^2 \frac{d^2v}{dr^2} + 2r\frac{dv}{dr} = 0$ ½ + ½ m

4. $\text{I.F} = e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$ ½ + ½ m

5. $p = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{8}{7}$ ½ + ½ m

6. $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ 0 & \lambda & 3 \end{vmatrix} = 0 \Rightarrow \lambda = 7$ ½ + ½ m

SECTION - B

7. Let E_1 : selecting bag A, and E_2 : selecting bag B.

$\therefore P(E_1) = 1/3,\ P(E_2) = 2/3$ ½ + ½ m

Let A : Getting one Red and one balck ball

$\therefore P(A|E_1) = \frac{{}^4C_1 \cdot {}^6C_1}{{}^{10}C_2} = \frac{8}{15},\ P(A|E_2) = \frac{{}^7C_1 \cdot {}^3C_1}{{}^{10}C_2} = \frac{7}{15}$ 1+1 m

$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2)$

$$= \frac{1}{3}\cdot\frac{8}{15} + \frac{2}{3}\cdot\frac{7}{15} = \frac{22}{45}$$ 1 m

OR

x :	0	1	2	3	4	½ m
P (x) :	${}^4C_0\left(\frac{1}{2}\right)^4$	${}^4C_1\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$	${}^4C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$	${}^4C_3\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3$	${}^4C_4\left(\frac{1}{2}\right)^4$	1½ m
:	$=\frac{1}{16}$	$=\frac{4}{16}$	$=\frac{6}{16}$	$=\frac{4}{16}$	$=\frac{1}{16}$	½ m
x P (x) :	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$	
x^2P (x) :	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$	½ m

$$\text{Mean} = \sum x\,P(x) = \frac{32}{16} = 2$$ ½ m

$$\text{Variance} = \sum x^2\,P(x) - \left(\sum x\,P(x)\right)^2 = \frac{80}{16} - (2)^2 = 1$$ ½ m

8. $$\vec{r}\times\vec{i} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right) x\hat{i} = -y\hat{k} + z\hat{j}$$ 1½ m

$$\vec{r}\times\vec{j} = \left(x\hat{i} + y\hat{j} + z\hat{k}\right)\hat{j} = x\hat{k} - z\hat{i}$$ 1½ m

$$\left(\vec{r}\times\hat{i}\right),\left(\vec{r}\times\vec{j}\right) = \left(o\hat{i} + z\hat{j} - y\hat{k}\right)\cdot\left(-z\hat{i} + o\hat{j} + x\hat{k}\right) = -xy$$ ½ m

$$\left(\vec{r}\times\hat{i}\right)\cdot\left(\vec{r}\times\vec{j}\right) + xy = -xy + xy = 0$$ ½ m

9. Any point on the line $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12}$ is $(3\lambda+2, 4\lambda-1, 12\lambda+2)$ 1 m

If this is the point of intersection with plane $x-y+z=5$

then $3\lambda+2-4\lambda+1+12\lambda+2-5=0 \Rightarrow \lambda=0$ 1 m

$\therefore$ Point of intersection is $(2,-1,2)$ 1 m

Required distance $= \sqrt{(2+1)^2+(-1+5)^2+(2+10)^2} = 13$ 1 m

10. Writing $\cot^{-1}(x+1) = \sin^{-1}\dfrac{1}{\sqrt{1+(x+1)^2}}$ 1½ m

and $\tan^{-1}x = \cos^{-1}\dfrac{1}{\sqrt{1+x^2}}$ 1½ m

$\therefore \sin\left(\sin^{-1}\dfrac{1}{\sqrt{1+(x+1)^2}}\right) = \cos\left(\cos^{-1}\dfrac{1}{\sqrt{1+x^2}}\right)$ ½ m

$1+x^2+2x+1 = 1+x^2 \Rightarrow x = -\dfrac{1}{2}$ ½ m

OR

$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \dfrac{5\pi^2}{8} \Rightarrow (\tan^{-1}x)^2 + \left(\dfrac{\pi}{2} - \tan^{-1}x\right)^2 = \dfrac{5\pi^2}{8}$ 1 m

$\therefore 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \dfrac{3\pi^2}{8} = 0$ 1½ m

$\tan^{-1}x = \dfrac{\pi \pm \sqrt{\pi^2+3\pi^2}}{4} = 3\pi/4, -\pi/4$ 1 m

$\Rightarrow x = -1$ ½ m

11. Putting $x^2 = \cos\theta$, we get ½ m

$y = \tan^{-1}\left(\dfrac{\sqrt{1+\cos\theta}+\sqrt{1-\cos\theta}}{\sqrt{1+\cos\theta}-\sqrt{1-\cos\theta}}\right)$ ½ m

$= \tan^{-1}\left(\dfrac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right) = \tan^{-1}\left(\dfrac{1+\tan\theta/2}{1-\tan\theta/2}\right)$ 1 + ½ m

$$y = \frac{\pi}{4} + \theta/2 = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x^2$$ ½ m

$$\frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1-x^4}} \cdot 2x = -\frac{x}{\sqrt{1-x^4}}$$ 1 m

12. $$\frac{dx}{d\theta} = -a\sin\theta + b\cos\theta$$ ½ m

$$\frac{dy}{d\theta} = a\cos\theta + b\sin\theta$$ ½ m

$$\therefore \frac{dy}{dx} = \frac{a\cos\theta + b\sin\theta}{a\sin\theta + b\cos\theta} = -\frac{x}{y}$$ 1½ m

or $$y\frac{dy}{dx} + x = 0$$

$$\therefore y\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{dy}{dx} + 1 = 0$$ 1 m

Using (i) we get $y\dfrac{d^2y}{dx^2} - \dfrac{x}{y}\dfrac{dy}{dx} + 1 = 0$ ½ m

$$\therefore y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

13. Let x be the side of an equilateral triangle

$$\therefore \frac{dx}{dt} = 2 \text{ cm/s.}$$ 1 m

Area (A) $= \dfrac{\sqrt{3}x^2}{4}$ 1 m

$$\Rightarrow \quad \frac{dA}{dt} = \frac{\sqrt{3}}{2} \text{ x } \frac{dx}{dt}$$ 1 m

$$\Rightarrow \quad \frac{dA}{dt} = \frac{\sqrt{3}}{2} \cdot (20) \cdot (2) = 20\sqrt{3} \text{ cm}^2|\text{s}$$ 1 m

14. Writing $x + 3 = -\frac{1}{2}(-4 - 2x) + 1$ 1 m

$$\therefore \int (x+3)\sqrt{3-4x-x^2}\, dx = -\frac{1}{2}\int(-4-2x)\sqrt{3-4x-x^2}\, dx + \int\sqrt{7-(x+2)^2}\, dx$$ ½+½ m

$$= -\frac{1}{3}\left(3-4x-x^2\right)^{3/2} + \frac{x+2}{2}\sqrt{3-4x-x^2} + \frac{7}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{7}}\right) + c$$ 1+1 m

15.

$$\begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \text{HF.} & \text{M} & \text{P} \end{array} \\ \begin{pmatrix} 40 & 50 & 20 \\ 25 & 40 & 30 \\ 35 & 50 & 40 \end{pmatrix} \end{array} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 7000 \\ 6125 \\ 7875 \end{pmatrix}$$ 1½ m

Funds collected by school A : Rs. 7000,

School B : Rs. 6125, School C : Rs. 7875 1 m

Total collected : Rs. 21000 ½ m

For writing one value 1 m

16. Getting $A^2 = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$ 1½ m

$$A^2 - 5A + 4I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} + \begin{pmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{pmatrix} + \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$ 1 m

$$= \begin{pmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{pmatrix}$$ 1 m

$$\therefore \quad X = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{pmatrix}$$ ½ m

OR

$$A' = \begin{pmatrix} 1 & 0 & -2 \\ -2 & -1 & 2 \\ 3 & 4 & 1 \end{pmatrix}$$ 1 m

$$|A'| = 1(-9) - 2(-5) = -9 + 10 = 1 \neq 0$$ ½ m

$$\text{Adj } A' = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$ 2 m

$$\therefore \quad (A')^{-1} = \begin{pmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{pmatrix}$$ ½ m

17. $$f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$$

$R_2 \rightarrow R_2 - xR_1$ and $R_3 \rightarrow R_3 - x^2 R_1$

$$f(x) = \begin{vmatrix} a & -1 & 0 \\ 0 & a+x & -1 \\ 0 & ax+x^2 & a \end{vmatrix}$$ (For bringing 2 zeroes in any row/column 1+1 m

$$\therefore \quad f(x) = a\,(a^2 + 2ax + x^2) = a\,(x+a)^2$$ 1 m

$$\therefore \quad f(2x) - f(x) = a\,[2x+a]^2 - a\,(x+a)^2$$

$$= a\,x\,(3x + 2a)$$ 1 m

18. $\int \frac{dx}{\sin x + \sin 2x} = \int \frac{dx}{\sin x\,(1+2\cos x)} = \int \frac{\sin x \cdot dx}{(1-\cos x)\,(1+\cos x)\,(1+2\cos x)}$ 1 m

$= -\int \frac{dt}{(1-t)\,(1+t)\,(1+2t)}$ where $\cos x = t$ ½ m

$= \int \left(\frac{-1/6}{1-t} + \frac{1/2}{1+t} - \frac{4/3}{1+2t} \right) dt$ 1½ m

$= +\frac{1}{6}\log|1-t| + \frac{1}{2}\log|1+t| - \frac{2}{3}\log|1+2t| + c$ ½ m

$= \frac{1}{6}\log|1-\cos x| + \frac{1}{2}\log|1+\cos x| - \frac{2}{3}\log|1+2\cos x| + c$ ½ m

OR

$\int \frac{x^2-3x+1}{\sqrt{1-x^2}}\,dx = \int \frac{2-3x-(1-x^2)}{\sqrt{1-x^2}}\,dx$ ½ m

$= 2\int \frac{1}{\sqrt{1-x^2}}\,dx - 3\int \frac{x}{\sqrt{1-x^2}}\,dx - \int \sqrt{1-x^2}\,dx$ 1 m

$= 2\sin^{-1}x + 3\sqrt{1-x^2} - \frac{x}{2}\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}x + c$ (½+1+1) m

or $= \frac{3}{2}\sin^{-1}x + \frac{1}{2}(6-x)\sqrt{1-x^2} + c$

19. $I = \int_{-\pi}^{\pi} (\cos ax - \sin bx)^2\,dx = \int_{-\pi}^{\pi} (\cos^2 ax + \sin^2 bx)\,dx - \int_{-\pi}^{\pi} 2\cos ax \sin bx\,dx$

$= I_1 - I_2$ ½ m

$I_1 = 2\int_0^{\pi} (\cos^2 ax + \sin^2 bx)\,dx$ (being an even fun.) 1 m

$I_2 = 0$ (being an odd fun.) 1 m

$\therefore\ I = I_1 = \int_0^{\pi} (1+\cos 2ax + 1 - \cos 2bx)\,dx$ ½ m

$= \left[2x + \frac{\sin 2ax}{2a} - \frac{\sin 2bx}{2b} \right]_0^{\pi}$ ½ m

$= \left[2\pi + \frac{1}{2a}\cdot \sin 2a\pi - \frac{\sin 2b\pi}{2b} \right]$ or 2π ½ m

SECTION - C

20. Any point on line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ is $(2\lambda+1, 3\lambda-1, 4\lambda+1)$ 1 m

$\therefore \quad \frac{2\lambda+1-3}{1} = \frac{3\lambda-1-k}{2} = \frac{4\lambda+1}{1} \Rightarrow \lambda = -\frac{3}{2}$, hence $k = \frac{9}{2}$ 2½ m

Eqn. of plane containing three lines is

$$\begin{vmatrix} x-1 & y+1 & z-1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$ 1 m

$\Rightarrow \quad -5(x-1) + 2(y+1) + 1(z-1) = 0$ 1 m

i.e. $\quad 5x - 2y - z - 6 = 0$ ½ m

21. $P(\overline{A} \cap B) = \frac{2}{15} \Rightarrow P(\overline{A}) \cdot P(B) = \frac{2}{15}$ 1 m

$P(A \cap \overline{B}) = \frac{1}{6} \Rightarrow P(A) \cdot P(\overline{B}) = \frac{1}{6}$ 1 m

$\therefore \quad (1 - P(A))P(B) = \frac{2}{15}$ or $P(B) - P(A) \cdot P(B) = \frac{2}{15}$ .... (i) 1 m

$P(A)(1 - P(B)) = \frac{1}{6}$ or $P(A) - P(A) \cdot P(B) = \frac{1}{6}$ ....(ii) 1 m

From (i) and (ii) $P(A) - P(B) = \frac{1}{6} - \frac{2}{15} = \frac{1}{30}$ ½ m

Let $P(A) = x$, $P(B) = y \quad \therefore \quad x = \left(\frac{1}{30} + y\right)$

(i) $\Rightarrow \quad y - \left(\frac{1}{30} + y\right) y = \frac{2}{15} \quad \therefore \quad 30y^2 - 29y + 4 = 0$ ½ m

Solving to get $y = \frac{1}{6}$ or $y = \frac{4}{5}$

$\therefore \quad x = \frac{1}{5}$ or $x = \frac{5}{6}$ ½ m

Hence $P(A) = \frac{1}{5},\ P(B) = \frac{1}{6}$ OR $P(A) = \frac{5}{6},\ P(B) = \frac{4}{5}$ ½ m

22. $f(x) = \sin x - \cos x,\ 0 < x < 2\pi$

$f'(x) = 0 \Rightarrow \cos x + \sin x = 0$ or $\tan x = -1,$ 1 m

$\therefore \quad x = \frac{3\pi}{4},\ \frac{7\pi}{4}$ 1 m

$f''(x) = \cos x - \sin x$ 1 m

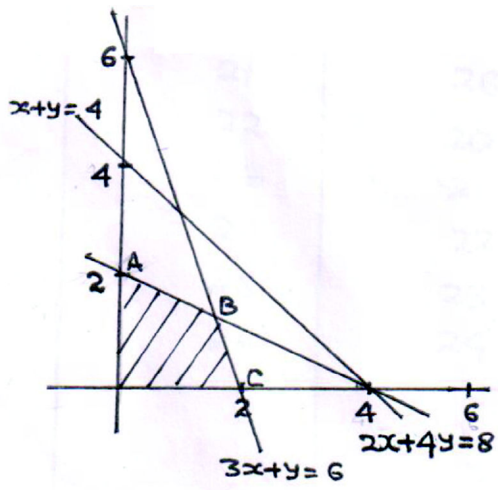
$f''\left(\frac{3\pi}{4}\right) = -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$ i.e – ve so, $x = \frac{3\pi}{4}$ is Local Maxima 1 m

and $f''\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$ i.e + ve so, $x = \frac{7\pi}{4}$ is Local Minima 1 m

Local Maximum value $= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ ½ m

Local Minimum value $= -\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \sqrt{2}$ ½ m

23.



Correct graphs of three lines 1x3 = 3 m

Correctly shading feasible region 1 m

Vertices are

A (0, 2), B (1.6, 1.2), C (2, .0) 1 m

$Z = 2x + 5y$ is maximum

at A (0, 2) and maximum value $= 10$ 1 m

24. $\forall$ a, b $\in$ N, (a, b) R (a, b) as ab (b + a) = ba (a + b)

$\therefore$ R is reflexive .... (i) 2 m

Let (a, b) R (c, d) for (a, b), (c, d) $\in N \times N$

$\therefore$ ad (b + c) = bc (a + d) .... (ii)

Also (c, d) R (a, b) $\because$ cb (d + a) = da (c + b) (using ii)

$\therefore$ R is symmetric .... (iii) 2 m

Let (a, b) R (c, d) and (c, d) R (e, f), for a, b, c, d, e, f, $\in$ N

$\therefore$ ad (b + c) = bc (a + d) and cf (d + e) = de (c + f) 1 m

$$\therefore \frac{b+c}{bc} = \frac{a+d}{ad} \text{ and } \frac{d+e}{de} = \frac{c+f}{cf}$$

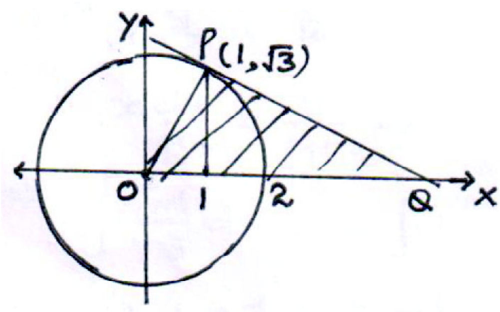
$$\text{i.e } \frac{1}{c}+\frac{1}{b} = \frac{1}{d}+\frac{1}{a} \text{ and } \frac{1}{e}+\frac{1}{d} = \frac{1}{f}+\frac{1}{c}$$

$$\text{adding we get } \frac{1}{c}+\frac{1}{b} + \frac{1}{e}+\frac{1}{d} = \frac{1}{d}+\frac{1}{a} + \frac{1}{f}+\frac{1}{c}$$

$\Rightarrow$ af (b + e) = be (a + f)

Hence (a, b) R (e, f) $\therefore$ R is transitive .... (iv) ½ m

Form (i), (iii) and (iv) R is an equivalence relation ½ m

25. 

Correct Fig. 1 m

Eqn. of normal (OP) : y = $\sqrt{3}$x ½ + ½ m

Eqn. of tangent (PQ) is

$$y-\sqrt{3} = -\frac{1}{\sqrt{3}}(x-1) \text{ i.e. } y = \frac{1}{\sqrt{3}}(4-x)$$ 1 m

Coordinates of Q (4, 0) ½ m

$$\therefore \quad \text{Req. area} = \int_0^1 \sqrt{3}x \, dx + \int_1^4 \frac{1}{\sqrt{3}}(4-x)\,dx$$ ½+½ m

$$= \sqrt{3}\left.\frac{x^2}{2}\right]_0^1 + \frac{1}{\sqrt{3}}\left[4x - \frac{x^2}{2}\right]_1^4$$ 1 m

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}\left[16 - 8 - 4 + \frac{1}{2}\right] = 2\sqrt{3} \text{ sq. units}$$ ½ m

OR

$$\int_1^3 \left(e^{2-3x} + x^2 + 1\right)dx \qquad \text{here } h = \frac{2}{n}$$ ½ m

$$= \lim_{h\to 0} h\left[f(1) + f(1+h) + f(1+2h) + \text{........} + f\left(1 + (n-1)h\right)\right]$$ 1 m

$$= \lim_{h\to 0} h\left[\left(e^{-1} + 2\right) + \left(e^{-1-3h} + 2 + 2h + h^2\right) + \left(e^{-1-6h} + 2 + 4h + 4h^2\right) + \text{......}\right.$$

$$\left. + \left(e^{-1-3(n\text{-}1)h} + 2 + 2(n-1)h + (n-1)^2 h^2\right)\right]$$ 1 m

$$= \lim_{h\to 0} h\left[e^{-1}\left(1 + e^{-3h} + e^{-6h} + \text{.....} + e^{-3(n-1)h}\right) + 2n + 2h\left(1 + 2 + \text{....} + (n-1)\right) + h^2\left(1^2 + 2^2 + \text{...} + (n-1)^2\right)\right]$$ 1½ m

$$= \lim_{h\to 0} h\left(e^{-1}.\frac{e^{-3nh} - 1}{e^{-3n} - 1}\cdot h + 2nh + 2\,\frac{nh\,(nh-h)}{2} + \frac{nh\,(nh-h)(2nh-h)}{6}\right)$$ 1 m

$$= e^{-1}.\frac{\left(e^{-6} - 1\right)}{-3} + 4 + 4 + \frac{8}{3} = -\ e^{-1}\,\frac{\left(e^{-6} - 1\right)}{3} + \frac{32}{3}$$ 1 m

26. Given differential equation can be written as

$$\frac{dx}{dy} + \frac{1}{1+y^2} \cdot x = \frac{\tan^{-1}y}{1+y^2}$$ 1 m

$\therefore$ Integrating factor is $e^{\tan^{-1} y}$ 1 m

$\therefore$ Solution is : $x \cdot e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \, . \, e^{\tan^{-1}y}}{1+y^2} \, dy$ 1½ m

$\Rightarrow \quad x \cdot e^{\tan^{-1}y} = \int t \, e^t \, dt \;$ where $\tan^{-1}y = t$ 1 m

$= t\,e^t - e^t + c = e^{\tan^{-1}y}\left(\tan^{-1}y - 1\right) + c$ 1½ m

or $\quad x = \tan^{-1}y - 1 + c\,e^{-\tan^{-1}y}$

OR

Given differential equation is $\dfrac{dy}{dx} = \dfrac{y/x}{1+\left(y/x\right)^2}$

Putting $\frac{y}{x} = v$ to get $v + x\frac{dv}{dx} = \frac{v}{1+v^2}$ 1½ m

$\therefore \quad x\frac{dv}{dx} = \frac{v}{1+v^2} - v = \frac{-v^3}{1+v^2}$ 1½ m

$\Rightarrow \int \frac{v^2+1}{v^3} \, dv = -\int \frac{dx}{x}$ ½ m

$\Rightarrow \log|v| - \frac{1}{2v^2} = -\log|x| + c$ 1 m

$\therefore \quad \log y - \frac{x^2}{2y^2} = c$ 1 m

$x = 0, y = 1 \;\Rightarrow\; c = 0 \;\therefore\; \log y - \frac{x^2}{2y^2} = 0$ ½ m