QUESTION PAPER CODE 65/1/P

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $|A| = -19$ ½ m

$A^{-1} = -\frac{1}{19}\begin{pmatrix} -2 & -5 \\ -3 & 2 \end{pmatrix}$ ½ m

2. $\frac{dy}{dx} = c$ ½ m

$y = x\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^2$ ½ m

3. $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1}y}{1+y^2}$ ½ m

I.F. $= e^{\tan^{-1}y}$ ½ m

4. $\vec{a}\cdot\left(\vec{b}\times\vec{a}\right) = \left[\vec{a}\ \vec{b}\ \vec{a}\right] = 0$ 1 m

5. $\vec{a}+\vec{b} = 3\hat{i}+3\hat{j}$ ½ m

$\left(\vec{a}+\vec{b}\right)\cdot\vec{c} = 3$ ½ m

6. $\frac{x+3}{0} = \frac{y-4}{3} = \frac{z-2}{-1}$ ½ m

D.Rs are 0, 3, –1 ½ m

SECTION - B

7. $\begin{matrix} A \\ B \\ C \end{matrix}\begin{pmatrix} 25 & 12 & 34 \\ 22 & 15 & 28 \\ 26 & 18 & 36 \end{pmatrix}\begin{pmatrix} 20 \\ 15 \\ 5 \end{pmatrix}$ 1½ m

$$= \begin{pmatrix} 850 \\ 805 \\ 970 \end{pmatrix}$$ 1½ m

Any relevant value 1 m

8. $$\tan^{-1}\left(\sqrt{\frac{a-b}{a+b}}\tan\frac{x}{2}\right) = \cos^{-1}\left\{\frac{1-\frac{a-b}{a+b}\tan^2\frac{x}{2}}{1+\frac{a-b}{a+b}\tan^2\frac{x}{2}}\right\}$$ 1½ m

$$= \cos^{-1}\left\{\frac{a+b-a\tan^2\frac{x}{2}+b\tan^2\frac{x}{2}}{a+b+a\tan^2\frac{x}{2}-b\tan^2\frac{x}{2}}\right\}$$ 1 m

$$= \cos^{-1}\left\{\frac{a\left(1-\tan^2\frac{x}{2}\right)+b\left(1+\tan^2\frac{x}{2}\right)}{a\left(1+\tan^2\frac{x}{2}\right)+b\left(1-\tan^2\frac{x}{2}\right)}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}+b}{a+b\frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}\right\}$$ ½ m

$$= \cos^{-1}\left\{\frac{a\cos x+b}{a+b\cos x}\right\}$$ ½ m

OR

$$\tan^{-1}\left(\frac{x-2}{x-3}\right) + \tan^{-1}\left(\frac{x+2}{x+3}\right) = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\left(\frac{\frac{x-2}{x-3}+\frac{x+2}{x+3}}{1-\frac{x-2}{x-3}\cdot\frac{x+2}{x+3}}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \tan^{-1}\left(\frac{2x^2-12}{-5}\right) = \frac{\pi}{4}$$ 1½ m

$$\Rightarrow \frac{2x^2-12}{-5} = 1 \Rightarrow x^2 = \frac{7}{2}$$ ½ m

$$\Rightarrow x = \sqrt{\frac{7}{2}}$$

For writing no solution as $|x| < 1$ ½ m

9. $$A^2 = \begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{pmatrix} = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix}$$ 2 m

$$A^2 - 5A + 16I = \begin{pmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{pmatrix} - \begin{pmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{pmatrix} + \begin{pmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{pmatrix}$$ 1 m

$$= \begin{pmatrix} 11 & -1 & -3 \\ -1 & 9 & -10 \\ -5 & 4 & 14 \end{pmatrix}$$ 1 m

10. Taking x from R_2, x (x – 1) from R_3 and (x + 1) from C_3

$$\Delta = x^2 (x-1)(x+1)\begin{vmatrix} 1 & x & 1 \\ 2 & x-1 & 1 \\ -3 & x-2 & 1 \end{vmatrix}$$ 2 m

$C_2 \to C_2 - x\,C_1\,;\quad C_3 \to C_3 - C,$

$$= x^2\,(x^2-1)\begin{vmatrix} 1 & 0 & 0 \\ 2 & -1-x & -1 \\ -3 & 4x-2 & 4 \end{vmatrix}$$ 1 m

$$= x^2\,(x^2-1)\begin{vmatrix} -1\,(1+x) & -1 \\ 4x-2 & 4 \end{vmatrix}$$ ½ m

$$= 6x^2\,(1-x^2)$$ ½ m

11. $$\frac{dx}{dt} = \alpha\left[-2\sin 2t\sin 2t + 2\cos 2t\,(1+\cos 2t)\right]$$ 1 m

$$\frac{dy}{dt} = \beta\left[2\sin 2t\cos 2t - (1-\cos 2t)\cdot 2\sin 2t)\right]$$ 1 m

$$\frac{dy}{dx} = \left(\frac{dy}{dt}\right)\Big/\left(\frac{dx}{dt}\right) = \frac{\beta\left(2\sin 4t - 2\sin 2t\right)}{\alpha\left(2\cos 4t + 2\cos 2t\right)}$$ ½+1 m

$$= \frac{\beta}{\alpha}\cdot\frac{2\cos 3t\sin t}{2\cos 3t\cos t} = \frac{\beta}{\alpha}\tan t$$ ½ m

12. Let y $$= \cos^{-1}\left(\frac{x-x^{-1}}{x-x^{-1}}\right) = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$$ 1 m

$$= \pi - \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$ 1 m

$$= \pi - 2\tan^{-1}x$$ 1 m

$$\therefore\ \frac{dy}{dx} = -\frac{2}{1+x^2}$$ 1 m

13. Let $y = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} + x^x$

Let $u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\}; \quad v = x^x$

$\therefore \quad y = u + v$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ ½ m

$u = \cos^{-1}\left\{\sin\sqrt{\frac{1+x}{2}}\right\} = \cos^{-1}\left[\cos\cdot\left(\frac{\pi}{2} - \sqrt{\frac{1+x}{2}}\right)\right]$ ½ m

$= \frac{\pi}{2} - \sqrt{\frac{1+x}{2}}$

$\therefore \quad \frac{du}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}}$ .... (i) ½ m

$v = x^x$

$\therefore \quad \log v = x \log x$

$\frac{1}{v}\frac{dv}{dx} = x \cdot \frac{1}{x} + 1 \log x = 1 + \log x$

$\frac{dv}{dx} = x^x(1 + \log x)$ ....(ii) 1½ m

$\therefore \quad \frac{dy}{dx} = -\frac{1}{2\sqrt{2}\sqrt{1+x}} + x^x(1 + \log x)$ ½ m

$\left(\frac{dy}{dx}\right)_{\text{at } x=1} = -\frac{1}{4} + 1 = \frac{3}{4}$ ½ m

14. $I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}\, dx$ .... (i)

$= \int_0^{\pi/2} \frac{2^{\sin\left(\frac{\pi}{2} - x\right)}}{2^{\sin\left(\frac{\pi}{2} - x\right)} + 2^{\cos\left(\frac{\pi}{2} - x\right)}}\, dx \left[\text{using} \int_0^a (x)\, dx = \int_0^a f(a - x)\, dx\right]$ 1½ m

$= \int_0^{\pi/2} \frac{2^{\cos x}}{2^{\sin x} + 2^{\cos x}}\, dx$ ....(ii) 1 m

Adding (i) and (ii),

$2I = \int_0^{\pi/2} 1\, dx = [x]_0^{\pi/2} = \frac{\pi}{2}$ 1 m

$\Rightarrow I = \frac{\pi}{4}$ ½ m

OR

$I = \int_0^{3/2} |x \cos(\pi x)|\, dx$

$= \int_0^{1/2} x \cos \pi x\, dx - \int_{1/2}^{3/2} x \cos \pi x\, dx$ 1 m

$= \left[\frac{x \sin \pi x}{\pi}\right]_0^{1/2} - \int_0^{1/2} \frac{\sin \pi x}{\pi}\, dx - \left[\frac{x \sin \pi x}{\pi}\right]_{1/2}^{3/2} + \int_{1/2}^{3/2} \frac{-\sin \pi x}{\pi}\, dx$ 1½ m

$= \frac{1}{2\pi} + \frac{1}{\pi^2}[\cos \pi x]_0^{1/2} + \frac{3}{2\pi} + \frac{1}{2\pi} + \frac{1}{\pi^2}[\cos \pi x]_{1/2}^{3/2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} + \frac{3}{2\pi} + \frac{1}{2\pi} + 0$ 1 m

$= \frac{5}{2\pi} - \frac{1}{\pi^2}$ ½ m

15. $I = \int\left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

$= \int \dfrac{\cos x + \sin x}{\sqrt{\sin x \cos x}} dx$ 1 m

$= \sqrt{2} \int \dfrac{(\cos x + \sin x)}{\sqrt{1-(1-2\sin x \cot x)}} dx$ 1 m

$= \sqrt{2} \int \dfrac{\cos x + \sin x}{\sqrt{1-(\sin x - \cos x)^2}} dx$ ½ m

Put $\sin x - \cos x = t \Rightarrow (\cos x + \sin x) dx = dt$ ½ m

$\therefore\ I = \sqrt{2} \int \dfrac{dt}{\sqrt{1-t^2}} = \sqrt{2} \sin^{-1} t + c$ ½ m

$= \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$ ½ m

16. $I = \int \dfrac{x^3 - 1}{x(x^2+1)} dx = \int \left(1 - \dfrac{x+1}{x(x^2+1)}\right) dx$ 1 m

$= x - \int \dfrac{x+1}{x(x^2+1)} dx$ ½ m

$= x - I_1$

Let $\dfrac{x+1}{x(x^2+1)} = \dfrac{A}{x} + \dfrac{Bx + C}{x^2+1} = \dfrac{1}{x} + \dfrac{1-x}{x^2+1}$ 1 m

$\therefore\ I_1 = \int \dfrac{1}{x} + \dfrac{(1-x)}{x^2+1} dx = \log x - \dfrac{1}{2} \log\left| x^2+1 \right| + \tan^{-1} x$ 1 m

$\therefore\ I = x - \log|x| + \dfrac{1}{2} \log\left| x^2+1 \right| - \tan^{-1} x + c$ ½ m

17. Here $\begin{rcases} \overrightarrow{AB} = -4\hat{i} - 6\hat{j} - 2\hat{k} \\ \overrightarrow{AC} = -\hat{i} + 4\hat{j} + 3\hat{k} \\ \overrightarrow{AD} = -8\hat{i} - \hat{j} + 3\hat{k} \end{rcases}$ 1½ m

For them to be coplanar, $\left[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right] = 0$ 1½ m

i.e. $\begin{vmatrix} -4 & -6 & -2 \\ -1 & 4 & 3 \\ -8 & -1 & 3 \end{vmatrix} = -60 + 126 - 66 = 0$ ½ m

$\therefore$ Points A, B, C and D are coplanar ½ m

18. Here $\begin{vmatrix} b-c-(a-d) & b-a & b+c-(a+d) \\ \alpha-\delta & \alpha & \alpha+\delta \\ \beta-\gamma & \beta & \beta+\gamma \end{vmatrix}$ 2½ m

$= 2\begin{vmatrix} b-a & b-a & b+c-a-d \\ \alpha & \alpha & \alpha+\delta \\ \beta & \beta & \beta+\gamma \end{vmatrix}$ $C_1 \to C_1 + C_3$ ½ m

$= 0 \quad (\because\ C_1 \text{ and } C_2 \text{ are identical})$ ½ m

Hence given lines are coplanar ½ m

OR

D.R's of normal to the plane are $5, -4, 7$ 1 m

D.R's of y – axis : $0, 1, 0$ ½ m

If θ is the angle between the plane and y–axis, then

$$\sin\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\ \sqrt{a_2^2 + b_2^2 + c_2^2}}$$ 1 m

$$= \frac{-4}{3\sqrt{10}}$$ 1 m

$\therefore\ \theta = \sin^{-1}\left(\frac{-4}{3\sqrt{10}}\right)$

$\therefore$ Acute angle is $\sin^{-1}\left(\frac{4}{3\sqrt{10}}\right)$ ½ m

19. Let E be the event of getting number greater than 4

$\therefore\ P(E) = \frac{1}{3}$ and $P(\bar{E}) = \frac{2}{3}$ ½+½ m

Required Probability = P $(\bar{E}\ E \text{ or } \bar{E}\ \bar{E}\ \bar{E}\ E \text{ or } \bar{E}\ \bar{E}\ \bar{E}\ \bar{E}\ \bar{E}\ E \text{ or } ....)$ 1 m

$= \frac{2}{3}\cdot\frac{1}{3} + \left(\frac{2}{3}\right)^3\cdot\frac{1}{3} + \left(\frac{2}{3}\right)^5\cdot\frac{1}{3} + ...\infty$ 1 m

$= \frac{2}{9}\left[1 + \left(\frac{2}{3}\right)^2 + \left(\frac{2}{3}\right)^4 + ...\infty\right]$ ½ m

$= \frac{2}{9}\times\frac{9}{5} = \frac{2}{5}$ ½ m

OR

A = {(5, 6, 1), (5, 6, 2), (5, 6, 3), (5, 6, 4), (5, 6, 5), (5, 6, 6),}

$P(A) = \frac{6}{6\times6\times6} = \frac{1}{36}$, P (B) = P (getting 3 or 4 on the third throw) 1½ m

$A \cap B = \{(5, 6, 3), (5, 6, 4)\} \Rightarrow P(A\cap B) = \frac{2}{6\times6\times6} = \frac{1}{108}$ 1½ m

$P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{1}{3}$ 1 m

SECTION - C

20. Let $y = (fog)(x)$ [say $y = h(x)$]

$= f[g(x)] = f(x^3 + 5)$ 2½ m

$= 2\,(x^3 + 5) - 3$

$= 2\,x^3 + 7$ 2½ m

$\therefore\ x = \sqrt[3]{\dfrac{y-7}{2}} = h^{-1}(y)$ ½ m

$\therefore\ (fog)^{-1} = \sqrt[3]{\dfrac{x-7}{2}}$ ½ m

OR

Let (x, y) be the identity element in Q × Q, then

$(a, b) * (x, y) = (a, b) = (x, y) * (a, b)\ \forall\ (a, b) \in Q \times Q$ 1½ m

$\Rightarrow\ (ax, b + ay) = (a, b)$

$\Rightarrow\ a = ax \text{ and } b = b + ay$

$\Rightarrow\ x = 1 \text{ and } y = 0$ 1 m

$\therefore$ (1, 0) is the identity element in Q × Q ½ m

Let (a, b) be the invertible element in Q × Q, then

there exists $(\alpha, \beta) \in Q \times Q$ such that

$(a, b) * (\alpha, \beta) = (\alpha, \beta) * (a, b) = (1, 0)$ 1½ m

$\Rightarrow\ (a\alpha, b + a\beta) = (1, 0)$ 1 m

$\Rightarrow\ \alpha = \dfrac{1}{a},\ \beta = -\dfrac{b}{a}$

$\therefore$ the invertible element in A is $\left(\dfrac{1}{a}, -\dfrac{b}{a}\right)$ ½ m

21. $f(x) = 2x^3 - 9\,m\,x^2 + 12\,m^2\,x + 1,\ m > 0$

$f'(x) = 6x^2 - 18\,m\,x + 12\,m^2$ 1 m

$f''(x) = 12x - 18\,m$ 1 m

For Max. or minimum, $f'(x) = 0 \Rightarrow 6x^2 - 18\,m\,x + 12\,m^2 = 0$

$$\Rightarrow (x - 2\,m)\,(x - m) = 0$$

$$\Rightarrow \; x = m \text{ or } 2\,m$$ 1 m

At $x = m$, $f''(x) = 12m - 18m = -\text{ve} \Rightarrow x = m$ is a maxima 1 m

At $x = 2\,m$, $f''(x) = 24m - 18m = +\text{ve} \Rightarrow x = 2m$ is manimum 1 m

$\therefore \; p = m$ and $q = 2\,m$ ½ m

Given $p^2 = q \Rightarrow m^2 = 2\,m \Rightarrow m^2 - 2\,m = 0$

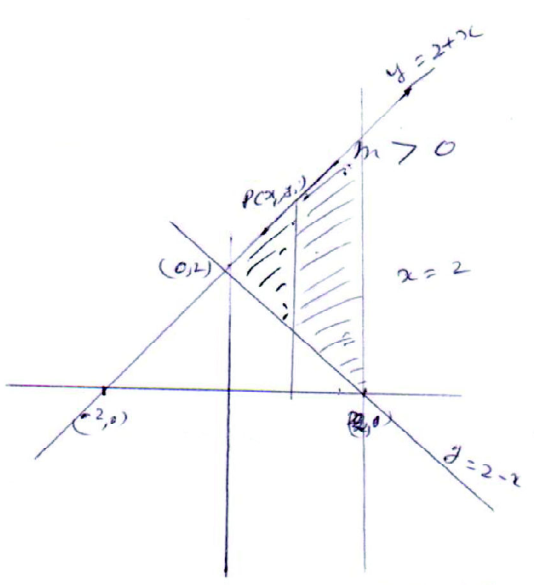
$$\Rightarrow m = 0, 2$$

$$\Rightarrow m = 2 \text{ as } m > 0$$ ½ m

22. $y = 2 + x$ (i)

$y = 2 - x$ (ii)

$x = 2$ (iii),



y_1 is the value of y from (i)

and y_2 is the value of y from (ii)

Required Area $= \int_0^2 (y_1 - y_2)\,dx$ 1 m

correct graph 1+1+1 m

$= \int_0^2 \{(2 + x) - (2 - x)\}\,dx$ correct shading 1 m

$= 2\int_0^2 x\,dx = 2\left[\frac{x^2}{2}\right]_0^2$ ½ m

$= 4$ sq. units ½ m

23. Let the equation of line be $y = mx + c$ 1½ m

the line is at unit distance from the origin

i.e. $\left| \dfrac{0+c}{\sqrt{1+m^2}} \right| = 1 \Rightarrow c = \sqrt{1+m^2}$ 1½ m

$\therefore \; y = mx + \sqrt{1+m^2}$ .... (i) 1 m

$\dfrac{dy}{dx} = m$ 1 m

$\therefore \; y = x\dfrac{dy}{dx} + \sqrt{1+\left(\dfrac{dy}{dx}\right)^2}$ 1 m

OR

$\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2xy} = \dfrac{1+3\left(\dfrac{y}{x}\right)^2}{2\left(\dfrac{y}{x}\right)}$ ....(i) 1 m

Differential equation is homogeneous

Put $y = vx$

$\Rightarrow \dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ 1½ m

$\therefore \; v + x\dfrac{dv}{dx} = \dfrac{1+3v^2}{2v}$ 1 m

$\Rightarrow x\dfrac{dv}{dx} = \dfrac{1+v^2}{2v}$

$\Rightarrow \int \left(\dfrac{2v}{1+v^2}\right) dv = \int \dfrac{dx}{x}$ 1 m

$\Rightarrow \log\left|1+v^2\right| = \log|x| + \log c$ 1 m

$\Rightarrow 1+v^2 = cx$

$\Rightarrow 1+\left(\dfrac{y}{x}\right)^2 = cx \quad \text{or} \quad x^2+y^2 = cx^3$ ½ m

24. Equation of plane passing through (1, 0, 0)

$a(x-1) + b(y-0) + c(z-0) = 0$

or $ax + by + cz - a = 0$ .... (i) 1 m

Plane (i) passes throgh (0, 1, 0)

$b - a = 0$ ....(ii) ½ m

Angle between plane (i) and plane $x + y = 3$ is $\frac{\pi}{4}$ ½ m

$\therefore \quad \cos\frac{\pi}{4} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\,\sqrt{2}}$ 1 m

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a+b}{\sqrt{a^2+b^2+c^2}\,\sqrt{2}}$ 1 m

$\Rightarrow a + b = \sqrt{a^2+b^2+c^2}$

$\Rightarrow 2a = \sqrt{2a^2+c^2}$ (using ii)

$\Rightarrow c = \pm\sqrt{2}\,a$ .... (iii) 1 m

$\therefore$ Equation (i) becomes

$a(x-1) + a(y-0) \pm \sqrt{2}\,a(z-0) = 0$

$\Rightarrow x + y \pm \sqrt{2}\,z - 1 = 0$ ½ m

D.R's of the normal is $1, 1, \pm\sqrt{2}$ ½ m

25. Let E_1, E_2 and E be the events such that

E_1 : students residing in hostel

E_2 : students residing outside hostel 1½ m

E_3 : students getting 'A' grade

$$\therefore \quad P(E_1) = \frac{40}{100}, \quad P(E/E_1) = \frac{50}{100}$$

$$P(E_2) = \frac{60}{100}, \quad P(E/E_2) = \frac{30}{100}$$ 2 m

$$P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E_1) \cdot P(E/E_1) + P(E_2)\ \ P(E/E_2)}$$ 1 m

$$= \frac{\frac{40}{100} \times \frac{50}{100}}{\frac{40}{100} \times \frac{50}{100} + \frac{30}{100} \times \frac{60}{100}}$$ 1 m

$$= \frac{10}{19}$$ ½ m

26. Let x be the man helpers and y be the woman helpers

Pay roll : $Z = 225x + 200y$ 1 m

Subject to constraints :

$x + y \leq 10$

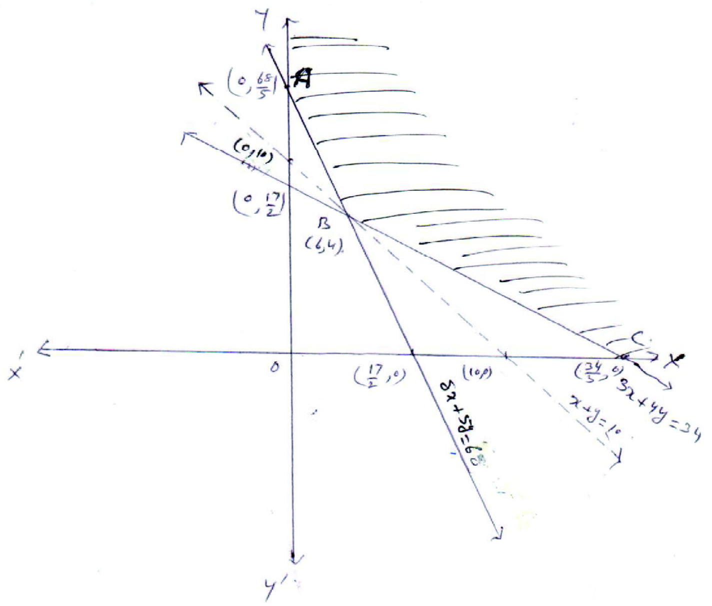
$3x + 4y \geq 34$

$8x + 5y \geq 68$

$x \geq 0, y \geq 0$

½×4 =2 m

correct graph : 2 m



At A $\left(0, \frac{68}{5}\right)$, Z (A) = Rs. 2720

At B $(6, 4)$, Z (B) = Rs. 2150 Minimum ½ m

At C $\left(\frac{34}{5}, 0\right)$, Z (C) = Rs. 2550

Minimum Z = Rs. 2150 at (6, 4) ½ m

[Feasible region is unbounded and to check minimum

of Z, $225x + 200y < 2150$

corresponding line is outside of the shaded region]