

QUESTION PAPER CODE 65/3/C

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1.  $d = \left| \frac{\vec{a} \cdot \vec{n} - p}{|\vec{n}|} \right|$ , distance =  $\frac{13}{7}$  ½ + ½ m

2.  $\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}$ ,  $|\vec{a} \times \vec{b}| = \sqrt{507}$  ½+½ m

3.  $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$ ,  $\theta = \frac{2\pi}{3}$  ½ + ½ m

4.  $\frac{dy}{dx} + \frac{1}{x \log x} y = \frac{2}{x}$ , Integrating factor =  $\log x$  ½ + ½ m

5.  $e^{2x} \sin 2x$  1 m

6.  $y = mx$ ,  $\frac{dy}{dx} = \frac{y}{x}$  ½ + ½ m

SECTION - B

7.  $\frac{dx}{dt} = a [\sin 2t (-2 \sin 2t) + (1 + \cos 2t)(2 \cos 2t)]$  1 m

$\frac{dy}{dt} = b [2 \sin 2t \cos 2t - 2 \sin 2t (1 - \cos 2t)]$  1 m

$\frac{dy}{dx} = \frac{b [2 \sin 2t \cos 2t - 2 \sin 2t (1 - \cos 2t)]}{a [\sin t (-2 \sin 2t) + (1 + \cos 2t)(2 \cos 2t)]}$  1 m

$= \frac{4 b \cos 3t \sin t}{4 a \cos 3t \cos t} = \frac{b}{a} \tan t = \frac{b}{2} \times 1 = \frac{b}{a}$  1 m

8.  $\int \frac{x+3}{(x+5)^3} e^x dx$

$\int \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3} e^x dx$  1 m

$\int \frac{1}{(x+5)^2} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$   $\frac{1}{2}$  m

$= \frac{1}{(x+5)^2} e^x + \int \frac{2}{(x+5)^3} e^x dx - \int \frac{2}{(x+5)^3} e^x dx$  2 m

$= \frac{e^x}{(x+5)^2} + c$   $\frac{1}{2}$  m

9. 
$$\begin{matrix} & \text{F} & \text{M} & \text{T} \\ \begin{matrix} x \\ y \\ z \end{matrix} & \begin{pmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{pmatrix} & \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} & = & \begin{pmatrix} 5450 \\ 5250 \\ 6625 \end{pmatrix} \end{matrix}$$
  $1\frac{1}{2}$  m

Funds collected by school x : ₹ 5450, school y = ₹ 5250

school z = ₹ 6625 1 m

Total collected funds = ₹ 17325  $\frac{1}{2}$  m

For writing any value 1 m

10.  $\overrightarrow{BA} = \hat{i} + (x-1)\hat{j} + 4\hat{k}, \overrightarrow{CA} = \hat{i} - 3\hat{k}, \overrightarrow{DA} = 3\hat{i} + 3\hat{j} - 2\hat{k}$   $1\frac{1}{2}$  m

$[\overrightarrow{BA}, \overrightarrow{CA}, \overrightarrow{DA}] = 0$  1 m

$$\begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$
 1 m

$x = 4$   $\frac{1}{2}$  m

$$11. \quad \vec{r} = (4\hat{i} + 2\hat{j} + 2\hat{k}) + \lambda (2\hat{i} + 3\hat{j} + 6\hat{k}) \vec{a} + \lambda \vec{b} \quad 1 \text{ m}$$

Let L be the foot of perpendicular

$$\text{Position vector of L is } (2\lambda + 4)\hat{i} + (3\lambda + 2)\hat{j} + (6\lambda + 2)\hat{k} \quad \frac{1}{2} \text{ m}$$

$$\vec{PL} = (2\lambda + 3)\hat{i} + 3\lambda\hat{j} + (6\lambda - 1)\hat{k} \quad \frac{1}{2} \text{ m}$$

$$\vec{PL} \cdot \vec{b} = 2(2\lambda + 3) + 3(3\lambda) + 6(6\lambda - 1) = 0$$

$$\Rightarrow \lambda = 0 \quad 1 \text{ m}$$

$$\vec{PL} = 3\hat{i} - \hat{k}$$

$$|\vec{PL}| = \sqrt{10} \text{ units} \quad 1 \text{ m}$$

$$12. \quad \sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$$

$$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right) \quad 1 \text{ m}$$

$$1-x = \cos(2\sin^{-1}x) \quad 1 \text{ m}$$

$$1-x = 1-2x^2 \quad 1 \text{ m}$$

$$\Rightarrow x = 0, \frac{1}{2} \quad \frac{1}{2} + \frac{1}{2} \text{ m}$$

$$x = \frac{1}{2} \text{ is rejected}$$

OR

$$\text{L.H.S} = 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$$

$$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} \frac{24}{7} - \tan^{-1} \frac{17}{31} \quad 1 \text{ m}$$

$$= \tan^{-1} \left( \frac{\frac{24}{7} - \frac{17}{31}}{1 + \frac{24}{7} \cdot \frac{17}{31}} \right) \quad 1 \text{ m}$$

$$= \tan^{-1} \left( \frac{625}{625} \right) = \frac{\pi}{4} \quad 1 \text{ m}$$

13.  $y = e^{ax} \cos bx$

$$y_1 = ae^{ax} \cos bx - b e^{ax} \sin bx \quad 1 \text{ m}$$

$$y_1 = ay - b e^{ax} \sin bx \quad 1 \text{ m}$$

$$y_2 = ay_1 - b [ae^{ax} \sin bx + b e^{ax} \cos bx] \quad 1 \text{ m}$$

$$y_2 = ay_1 - a b e^{ax} \sin bx - b^2 e^{ax} \cos bx$$

$$y_2 = a y_1 - a (ay - y_1) - b^2 y$$

$$y_2 - 2 a y_1 + (a^2 + b^2) y = 0 \quad 1 \text{ m}$$

14.  $x^x + x^y + y^x = a^b$

$$\text{Let } u = x^x, v = x^y, w = y^x, \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0 \quad \frac{1}{2} \text{ m}$$

$$\frac{du}{dx} = x^x (1 + \log x) \quad 1 \text{ m}$$

$$\frac{dv}{dx} = x^y \left( \frac{y}{x} + \frac{dy}{dx} \log x \right) \quad 1 \text{ m}$$

$$\frac{dw}{dx} = y^x \left( \frac{x}{y} \cdot \frac{dy}{dx} + \log y \right) \quad 1 \text{ m}$$

$$\frac{dy}{dx} = - \left( \frac{x^x (1 + \log x) + y x^{y-1} + y^x \log y}{x^y \log x + x y^{x-1}} \right) \quad \frac{1}{2} \text{ m}$$

15. 
$$A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} \quad 1\frac{1}{2} \text{ m}$$

$$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = O \quad 1 \text{ m}$$

$$A^2 - 4A - 5I = O \Rightarrow A^{-1} = \frac{1}{5} (A - 4I) \quad 1 \text{ m}$$

$$A^{-1} = \frac{1}{5} \left\{ \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right\} = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \quad \frac{1}{2} \text{ m}$$

OR

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad 1 \text{ m}$$

Using elementary row operations to reach at

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A \quad 2 \text{ m}$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} \quad 1 \text{ m}$$

$$16. \quad \begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 3x+7 & x-1 & x+2 \\ 3x+7 & x+2 & x+6 \end{vmatrix} = 0 \quad 1 \text{ m}$$

$$R_2 \rightarrow R_2 - R_1, \quad R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 1 & -7 & 3 \\ 1 & -4 & 7 \end{vmatrix} = 0 \quad 2 \text{ m}$$

$$(3x+7)(-37) = 0 \Rightarrow x = \frac{-7}{3} \quad 1 \text{ m}$$

$$17. \quad I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx \quad 1 \text{ m}$$

$$2I = \int_0^{\pi/2} \frac{\sec^2 \frac{x}{2}}{2 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2}} dx$$

$$I = - \int_0^1 \frac{1}{(t-1)^2 - (\sqrt{2})^2} dt, \quad \text{where } \tan \frac{x}{2} = t \quad 1\frac{1}{2} \text{ m}$$

$$I = \left[ -\frac{1}{2\sqrt{2}} \log \left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| \right]_0^1 \quad 1 \text{ m}$$

$$I = \frac{1}{2\sqrt{2}} \log \left| \frac{1+\sqrt{2}}{1-\sqrt{2}} \right| \quad \frac{1}{2} \text{ m}$$

OR

$$\int_{-1}^2 (e^{3x} + 7x - 5) dx \quad \text{here } h = \frac{3}{n} \quad \frac{1}{2} \text{ m}$$

$$= \lim_{h \rightarrow 0} h [f(-1) + f(-1+h) + \dots]$$

$$= \lim_{h \rightarrow 0} h [(e^{-3} - 12) + (e^{-3+3h} + 7h - 12) + \dots + (e^{-3+n-1}h + 7(n-1)h - 12)] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h [e^{-3}(1 + e^{3h} + e^{6h} + \dots + e^{3(n-1)h}) + 7h(1 + 2 + 3 + \dots + n-1) - 12nh] \quad 1 \text{ m}$$

$$= \lim_{h \rightarrow 0} h \left[ \frac{e^{-3}(e^{3nh} - 1)h}{e^{3h} - 1} + \frac{7(nh)(nh-h)}{2} - 12nh \right] \quad 1 \text{ m}$$

$$= \frac{e^{-3}(e^9 - 1)}{3} + \frac{63}{2} - 36 = \frac{e^9 - 1}{3e^3} - \frac{9}{2} \quad \frac{1}{2} \text{ m}$$

18.  $\int \frac{x^2}{x^4 + x^2 - 2} dx$

$$\int \frac{x^2}{x^4 + x^2 - 2} = \frac{t}{t^2 + t - 2} = \frac{t}{(t+2)(t-1)} \quad \text{where } x^2 = t \quad 1\frac{1}{2} \text{ m}$$

$$= \frac{2}{3(t+2)} + \frac{1}{3(t-1)} \quad 1\frac{1}{2} \text{ m}$$

$$\int \frac{x^2}{x^4 + x^2 - 2} dx = \int \frac{2}{3(x^2 + 2)} dx + \int \frac{1}{3(x^2 - 1)} dx$$

$$= \frac{2}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + c \quad 1 \text{ m}$$

19. Let  $E_1$  : two headed coin is chosen

$E_2$  : unbiased coin is chosen

A : All 5 tosses are heads

½ m

$$P(E_1) = \frac{1}{5}, P(E_2) = \frac{4}{5}, P\left(\frac{A}{E_1}\right) = 1, P\left(\frac{A}{E_2}\right) = \frac{1}{32}$$

2 m

$$P\left(\frac{E_1}{A}\right) = \frac{P(E_1)P\left(\frac{A}{E_1}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

½ m

$$P\left(\frac{E_1}{A}\right) = \frac{\frac{1}{5} \times 1}{\frac{1}{5} \times 1 + \frac{4}{5} \cdot \frac{1}{32}} = \frac{8}{9}$$

1 m

OR

Let the coin is tossed n times

$$1 - P(0) > \frac{80}{100}$$

1½ m

$$P(0) < \frac{1}{5}$$

½ m

$${}^n C_1 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 < \frac{1}{5}$$

1 m

$$\left(\frac{1}{2}\right)^n < \frac{1}{5} \Rightarrow n \geq 3$$

1 m

### SECTION - C

20.  $x^2 dy = (2xy + y^2) dx$

$$\frac{dy}{dx} = \frac{2xy + y^2}{x^2}$$

½ m



$$y = vx \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \quad 1 \text{ m}$$

$$v + x \frac{dv}{dx} = 2v + v^2 \Rightarrow \int \frac{1}{v^2 + v} dv = \int \frac{1}{x} dx \quad 2 \text{ m}$$

$$\Rightarrow \log \left| \frac{v}{v+1} \right| = \log x + \log c \quad 1 \text{ m}$$

$$\Rightarrow \log \left| \frac{y}{y+x} \right| = \log cx \Rightarrow \frac{y}{y+x} = cx \quad 1 \text{ m}$$

$$x = 1, y = 1 \Rightarrow c = \frac{1}{2}$$

$$x^2 + xy - 2y = 0 \quad \frac{1}{2} \text{ m}$$

OR

Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{1+x^2} y = \frac{e^{m \tan^{-1}x}}{1+x^2} \quad 1 \text{ m}$$

Integrating factor is  $e^{\tan^{-1}x}$  1 m

$$\text{Solution is } y \cdot e^{\tan^{-1}x} = \int \frac{e^{m \tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x} dx \quad 1\frac{1}{2} \text{ m}$$

$$\Rightarrow y e^{\tan^{-1}x} = \int e^{(m+1)t} dt, \text{ where } \tan^{-1}x = t \quad 1 \text{ m}$$

$$= \frac{e^{(m+1)t}}{m+1} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + c \quad 1 \text{ m}$$

$$y = 1, x = 0 \Rightarrow c = \frac{m}{m+1}$$

$$y e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + \frac{m}{m+1} \quad \frac{1}{2} \text{ m}$$

21.  $f(x) = \sin^2 x - \cos x$
- $f'(x) = \sin x (2 \cos x + 1)$  1 m
- $f'(x) = 0 \Rightarrow \sin x = 0$  and  $2 \cos x + 1 = 0 \Rightarrow x = 0, 2\frac{\pi}{3}, \pi$  2½ m
- $f(0) = -1, f\left(\frac{2\pi}{3}\right) = \frac{5}{4}, f(\pi) = 1$  1½ m
- Absolute maximum value is  $\frac{5}{4}$  ½ m
- Absolute minimum value is  $-1$  ½ m

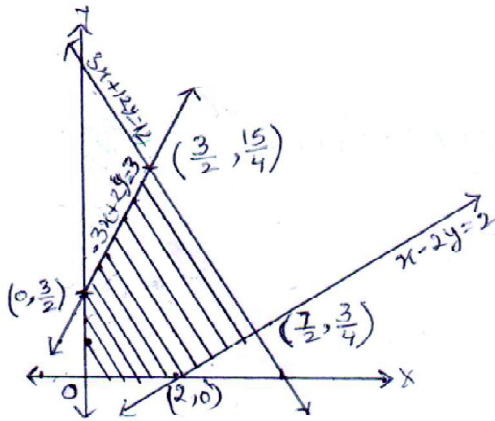
22.  $x:$                     2            3            4            5            6 1 m
- $P(x):$                      $\frac{1}{15}$          $\frac{2}{15}$          $\frac{3}{15}$          $\frac{4}{15}$          $\frac{5}{15}$  2 m
- $x \cdot P(x):$                  $\frac{2}{15}$          $\frac{6}{15}$          $\frac{12}{15}$          $\frac{20}{15}$          $\frac{30}{15}$  ½ m
- $x^2 P(x):$                  $\frac{4}{15}$          $\frac{18}{15}$          $\frac{48}{15}$          $\frac{100}{15}$          $\frac{180}{15}$  ½ m
- Mean =  $\sum x \cdot P(x) = \frac{70}{15} = \frac{14}{3}$  1 m
- Variance =  $\sum x^2 P(x) - (\text{Mean})^2 = \frac{350}{15} - \frac{196}{9} = \frac{14}{9}$  1 m

23. Two lines  $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$  and  $r = \vec{a}_2 + \mu \vec{b}_2$  are coplanar
- if  $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$  1 m
- Here  $(-\hat{i} + 3\hat{j} + \hat{k}) \cdot [(\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} - \hat{j} + 3\hat{k})] = 0$  2 m
- Equation of plane is
- $(\vec{r} - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$  1 m

$$\left[ \vec{r} - (\hat{i} + \hat{j} + \hat{k}) \right] \cdot \left[ (\hat{i} - \hat{j} + \hat{k}) \times (2\hat{i} - \hat{j} + 3\hat{k}) \right] = 0$$

$$\vec{r} \cdot (-2\hat{i} - \hat{j} + \hat{k}) + 2 = 0 \quad 2 \text{ m}$$

24.



Correct graph of three lines 1×3 m

correct shading of feasible region 1 m

vertices are  $\left(0, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{15}{4}\right),$

$\left(\frac{7}{2}, \frac{3}{4}\right), (2, 0)$  1 m

$z = 5x + 2y$  is maximum

at  $\left(\frac{7}{2}, \frac{3}{4}\right) = 19$  and

minimum at  $\left(0, \frac{3}{2}\right) = 3$  1 m

25. (i) Let  $(e, e')$  be the identity element in  $A$

$$(a, b) * (e, e') = (a, b) = (e, e') * (a, b)$$

$$(a e, b + a e') = (a, b)$$

$$\left. \begin{array}{l} a e = a \Rightarrow e = 1 \\ b + a e' = b \Rightarrow e' = 0 \end{array} \right\} \Rightarrow \text{identity} : (1, 0)$$

2 ½ m

(ii) Let  $(x, y)$  is inverse of  $(a, b) \in A$

$$(a, b) * (x, y) = (1, 0) = (x, y) * (a, b)$$

$$(a x, b + a y) = (1, 0)$$

$$\left. \begin{array}{l} ax = 1 \Rightarrow x = \frac{1}{a} \\ b + ay = 0 \Rightarrow y = \frac{-b}{a} \end{array} \right\} \Rightarrow \text{inverse of } (a, b) = \left( \frac{1}{a}, \frac{-b}{a} \right) \quad 2 \frac{1}{2} \text{ m}$$

$$\text{Inverse of } (5, 3) = \left( \frac{1}{5}, \frac{-3}{5} \right) \quad \frac{1}{2} \text{ m}$$

$$\text{Inverse of } \left( \frac{1}{2}, 4 \right) = (2, -8) \quad \frac{1}{2} \text{ m}$$

OR

One – One : - Case I : when x and y are even

$$f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$$

Case II : when x and y are odd

$$f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$$

Case III : one of them is even and one of them is odd

$$f(x) \neq f(y) \Rightarrow x + 1 \neq y - 1 \Rightarrow x \neq y \quad 2 \frac{1}{2} \text{ m}$$

Onto : Let  $y \in W$

$$f(y - 1) = y \text{ if } y \text{ is odd}$$

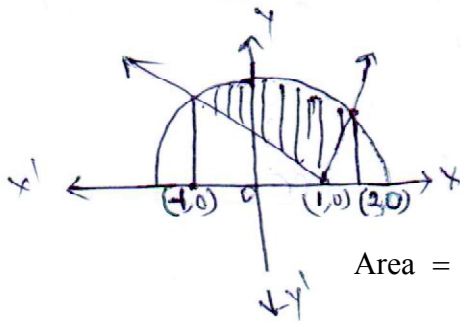
$$f(y + 1) = y \text{ if } y \text{ is even}$$

So  $\forall y \in W$ , there exist some element in domain of f

$\Rightarrow$  f is invertible 2½ m

$$f^{-1}(x) = \begin{cases} x - 1, & x \text{ is odd} \\ x + 1, & x \text{ is even} \end{cases} \quad 1 \text{ m}$$

26.



Figure

1 m

For finding  $(-1, 0)$ ,  $(1, 0)$ ,  $(2, 0)$

1½ m

$$\text{Area} = \int_{-1}^2 \sqrt{5-x^2} \, dx - \int_{-1}^1 -(x-1) \, dx - \int_{-1}^2 (x-1) \, dx$$

1½ m

$$= \left[ \frac{x}{2} \sqrt{5-x^2} + \frac{5}{2} \sin^{-1} \frac{x}{\sqrt{5}} \right]_{-1}^2 + \left[ \frac{(x-1)^2}{2} \right]_{-1}^1 - \left[ \frac{(x-1)^2}{2} \right]_{-1}^2$$

1½ m

$$= \left( 1 + \frac{5}{2} \sin^{-1} \frac{2}{\sqrt{5}} \right) + \left( 1 + \frac{5}{2} \sin^{-1} \frac{1}{\sqrt{5}} \right) - \frac{1}{2} \times 4 - \frac{1}{2} \times 1$$

$$= \frac{5}{2} \left( \sin^{-1} \frac{2}{\sqrt{5}} + \sin^{-1} \frac{1}{\sqrt{5}} \right) - \frac{1}{2} \text{ sq. units}$$

½ m