

Assignment Class XII
Mathematics

1. Let $f: \mathbf{N} \rightarrow \mathbf{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: \mathbf{N} \rightarrow \mathbf{S}$, where, \mathbf{S} is the range of f , is invertible. Find the inverse of f .

2. If $f: \mathbf{R} \rightarrow \mathbf{R}$ is defined by $f(x) = x^2 - 3x + 2$, find $f(f(x))$.

3. Show that the function $f: \mathbf{R} \rightarrow \{x \in \mathbf{R} : -1 < x < 1\}$ defined by $f(x) = \frac{x}{1+|x|}$, $x \in \mathbf{R}$ is one one and onto function.

4. Find the value of $\sin^{-1}(\sin \frac{3\pi}{5})$.

5. Find the value of the following expression :

$$\tan(\sin^{-1} \frac{3}{5} + \cot^{-1} \frac{3}{2}).$$

6. Find the value: $\tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$.

7. Express the matrix $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.

8. Obtain the inverse of the following matrix using elementary operations $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

9. If $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$, where n is any positive integer.

10. If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$, then show that $1 + xyz = 0$.

11. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1} .

12. Solve the following system of equations by matrix method.

$$3x - 2y + 3z = 8$$

$$2x + y - z = 1$$

$$4x - 3y + 2z = 4$$

13. If $\cos y = x \cos (a + y)$, with $\cos a \neq \pm 1$, prove that $\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$.

14. If $x = a (\cos t + t \sin t)$ and $y = a (\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

15. If $y = e^{a \cos^{-1} x}$, $-1 \leq x \leq 1$, show that $(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2 y = 0$.

16. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

17. Find the intervals in which the function f given by $f(x) = \sin x + \cos x$, $0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

18. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

19. Show that the semi-vertical angle of the cone of the maximum volume and of given slants height is $\tan^{-1} \sqrt{2}$.

20. Find $\int [\sqrt{\cot x} + \sqrt{\tan x}] dx$.

21. Evaluate $\int_0^{\pi} \frac{x dx}{a^2 \cos^2 x + b^2 \sin^2 x}$.

22. Evaluate $\int_0^{\pi/2} \log \sin x dx$.

23. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

24. Using the method of integration find the area of the region bounded by lines: $2x + y = 4$, $3x - 2y = 6$ and $x - 3y + 5 = 0$

25. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$.

26. Form the differential equation of the family of circles touching the x-axis at origin.

27. Find the general solution of the differential equation $y dx + (x - y^2) dy = 0$.

28. Solve the differential equation $(\tan^{-1} y - x) dy = (1 + y^2) dx$.

29. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

30. If \vec{a} is a unit vector and $(\vec{x} - \vec{a}) \cdot (\vec{x} + \vec{a}) = 8$, then find $|\vec{x}|$.

31. show that the four points A,B,C, and D with position vectors $4\hat{i} + 5\hat{j} + \hat{k}$, $-(\hat{j} + \hat{k})$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $4(-\hat{i} + \hat{j} + \hat{k})$, respectively are coplanar.

32. Find the coordinates of the foot of the perpendicular drawn from the origin to the plane $2x - 3y + 4z - 6 = 0$.

33. Find the coordinates of the point where the line through $(3, -4, -5)$ and $(2, -3, 1)$ crosses the plane $2x + y + z = 7$.

34. Find the equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$.

35. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.

36. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?

37. 4 Determine graphically the minimum value of the objective function $Z = -50x + 20y \dots (1)$

subject to the constraints:

$$2x - y \geq -5 \dots (2)$$

$$3x + y \geq 3 \dots (3)$$

$$2x - 3y \leq 12 \dots (4)$$

$$x \geq 0, y \geq 0 \dots (5)$$