

## QUESTION PAPER CODE 65/1

## EXPECTED ANSWER/VALUE POINTS

## SECTION A

$$1. \quad \frac{\pi}{3} - \left( \pi - \frac{\pi}{6} \right) = -\frac{\pi}{2} \qquad \frac{1}{2} + \frac{1}{2}$$

**Note:**  $\frac{1}{2}$  m. for any one of the two correct values and  $\frac{1}{2}$  m. for final answer

$$2. \quad a = -2, b = 3 \qquad \frac{1}{2} + \frac{1}{2}$$

$$3. \quad |\vec{a}| = |\vec{b}| = 3 \qquad \frac{1}{2} + \frac{1}{2}$$

$$4. \quad 5 \circ 10 = (5 * 10) + 3 = 10 + 3 = 13 \qquad \text{For } 5 * 10 = 10 \qquad \frac{1}{2}$$

$$\text{For Final Answer} = 13 \qquad \frac{1}{2}$$

## SECTION B

$$5. \quad \text{In RHS, put } x = \sin \theta \qquad \frac{1}{2}$$

$$\text{RHS} = \sin^{-1} (3 \sin \theta - 4 \sin^3 \theta)$$

$$= \sin^{-1} (\sin 3\theta) \qquad 1$$

$$= 3\theta = 3 \sin^{-1} x = \text{LHS.} \qquad \frac{1}{2}$$

$$6. \quad |A| = 2, \quad \therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \qquad 1$$

$$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \quad \text{RHS} = 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix} \qquad 1$$

$$\therefore \text{LHS} = \text{RHS}$$

$$7. f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right) \quad 1$$

$$= \tan^{-1}\left(\cot x/2\right) = \frac{\pi}{2} - \frac{x}{2} \quad \frac{1}{2}$$

$$\therefore f'(x) = -\frac{1}{2} \quad \frac{1}{2}$$

$$8. \text{ Marginal cost} = C'(x) = 0.015x^2 - 0.04x + 30 \quad 1$$

$$\text{At } x = 3, C'(3) = 30.015 \quad 1$$

$$9. I = \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} dx \quad \frac{1}{2}$$

$$= \int \sec^2 x dx \quad 1$$

$$= \tan x + C \quad \frac{1}{2}$$

$$10. \frac{dy}{dx} = bae^{bx+5} \Rightarrow \frac{dy}{dx} = by \quad 1$$

$$\Rightarrow \frac{d^2y}{dx^2} = b \frac{dy}{dx} \quad \frac{1}{2}$$

$$\therefore \text{ The differential equation is: } y \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2 \quad \frac{1}{2}$$

$$11. \sin \theta = \frac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}| |3\hat{i} - 2\hat{j} + \hat{k}|} \quad \frac{1}{2}$$

$$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6} \quad 1$$

$$\sin \theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7} \quad \frac{1}{2}$$

12. A: Getting a sum of 8, B: Red die resulted in no. < 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad 1$$

$$= \frac{2/36}{18/36} = \frac{1}{9} \quad 1$$

## SECTION C

$$\begin{aligned}
 \mathbf{13.} \quad \text{LHS} &= \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix} \\
 &= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix} \quad (\text{Using } C_2 \rightarrow C_2 - C_1 \text{ \& } C_3 \rightarrow C_3 - C_1) & \quad \begin{array}{l} 1+1 \\ \text{(Any two} \\ \text{relevant} \\ \text{operations)} \end{array} \\
 &= 1 \times (9yz) + 3x(3z + 9yz + 3y) \quad (\text{Expanding along } R_1) & \quad 1 \\
 &= 9(3xyz + xy + yz + zx) = \text{RHS} & \quad 1
 \end{aligned}$$

**14.** Differentiating with respect to 'x'

$$\begin{aligned}
 2(x^2 + y^2) \left( 2x + 2y \frac{dy}{dx} \right) &= x \frac{dy}{dx} + y & \quad 2 \\
 \Rightarrow \frac{dy}{dx} &= \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x} & \quad 2
 \end{aligned}$$

OR

$$\begin{aligned}
 \frac{dx}{d\theta} &= a(2 - 2\cos 2\theta) = 4a \sin^2 \theta & \quad 1 \\
 \frac{dy}{d\theta} &= 2a \sin 2\theta = 4a \sin \theta \cdot \cos \theta & \quad 1 \\
 \therefore \frac{dy}{dx} &= \frac{4a \sin \theta \cos \theta}{4a \sin^2 \theta} = \cot \theta & \quad 1 \\
 \left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{3}} &= \frac{1}{\sqrt{3}} & \quad 1
 \end{aligned}$$

$$\mathbf{15.} \quad y = \sin(\sin x) \Rightarrow \frac{dy}{dx} = \cos(\sin x) \cdot \cos x \quad 1$$

$$\text{and } \frac{d^2y}{dx^2} = -\sin(\sin x) \cdot \cos^2 x - \sin x \cos(\sin x) \quad 1+1$$

$$\text{LHS} = -\sin(\sin x) \cos^2 x - \sin x \cos(\sin x) + \frac{\sin x}{\cos x} \cos(\sin x) \cos x + \sin(\sin x) \cos^2 x \quad 1$$

$$= 0 = \text{RHS}$$

$$16. \quad x_1 = 2 \Rightarrow y_1 = 3 \quad (\because y_1 > 0)$$

 $\frac{1}{2}$ 

Differentiating the given equation, we get,  $\frac{dy}{dx} = \frac{-16x}{9y}$

 $\frac{1}{2}$ 

$$\text{Slope of tangent at } (2, 3) = \left. \frac{dy}{dx} \right|_{(2,3)} = -\frac{32}{27}$$

 $\frac{1}{2}$ 

$$\text{Slope of Normal at } (2, 3) = \frac{27}{32}$$

 $\frac{1}{2}$ 

$$\text{Equation of tangent: } 32x + 27y = 145$$

1

$$\text{Equation of Normal: } 27x - 32y = -42$$

1

OR

$$f'(x) = x^3 - 3x^2 - 10x + 24$$

 $\frac{1}{2}$ 

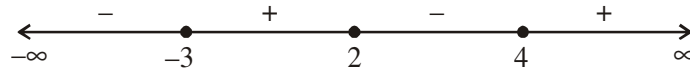
$$= (x - 2)(x - 4)(x + 3)$$

1

$$f'(x) = 0 \Rightarrow x = -3, 2, 4.$$

 $\frac{1}{2}$ 

sign of  $f'(x)$ :



$\therefore f(x)$  is strictly increasing on  $(-3, 2) \cup (4, \infty)$

1

and  $f(x)$  is strictly decreasing on  $(-\infty, -3) \cup (2, 4)$

1

17. Let side of base =  $x$  and depth of tank =  $y$

$$V = x^2y \Rightarrow y = \frac{V}{x^2}, \quad (V = \text{Quantity of water} = \text{constant})$$

Cost of material is least when area of sheet used is minimum.

$$A(\text{Surface area of tank}) = x^2 + 4xy = x^2 + \frac{4V}{x}$$

 $\frac{1}{2} + \frac{1}{2}$ 

$$\frac{dA}{dx} = 2x - \frac{4V}{x^2}, \quad \frac{dA}{dx} = 0 \Rightarrow x^3 = 2V, \quad y = \frac{x^3}{2x^2} = \frac{x}{2}$$

 $\frac{1}{2} + \frac{1}{2}$ 

$$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0, \quad \therefore \text{Area is minimum, thus cost is minimum when } y = \frac{x}{2}$$

 $\frac{1}{2} + \frac{1}{2}$ 

Value: Any relevant value.

1

18. Put  $\sin x = t \Rightarrow \cos x \, dx = dt$

 $\frac{1}{2}$ 

$$\text{Let } I = \int \frac{2 \cos x}{(1 - \sin x)(1 + \sin^2 x)} dx = \int \frac{2}{(1-t)(1+t^2)} dt$$

$$\text{Let } \frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}, \text{ solving we get}$$

$$A = 1, B = 1, C = 1$$

 $1 \frac{1}{2}$ 

$$\therefore I = \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{2t}{1+t^2} + \int \frac{1}{1+t^2} dt$$

$$= -\log |1-t| + \frac{1}{2} \log |1+t^2| + \tan^{-1} t + C$$

 $1 \frac{1}{2}$ 

$$= -\log(1 - \sin x) + \frac{1}{2} \log(1 + \sin^2 x) + \tan^{-1}(\sin x) + C$$

 $\frac{1}{2}$ 

19. Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^x}{e^x - 2} dx$$

 $1 \frac{1}{2}$ 

$$\Rightarrow \log |\tan y| = \log |e^x - 2| + \log C$$

 $1 \frac{1}{2}$ 

$$\Rightarrow \tan y = C(e^x - 2), \text{ for } x = 0, y = \pi/4, C = -1$$

 $\frac{1}{2}$ 

$$\therefore \text{ Particular solution is: } \tan y = 2 - e^x.$$

 $\frac{1}{2}$ 

OR

$$\text{Integrating factor} = e^{\int 2 \tan x dx} = \sec^2 x$$

1

$$\therefore \text{ Solution is: } y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x dx = \int \sec x \cdot \tan x dx$$

1

$$\Rightarrow y \cdot \sec^2 x = \sec x + C, \text{ for } x = \frac{\pi}{3}, y = 0, \therefore C = -2$$

 $1 + \frac{1}{2}$ 

$$\therefore \text{ Particular solution is: } y \cdot \sec^2 x = \sec x - 2$$

 $\frac{1}{2}$ 

$$\text{or } y = \cos x - 2 \cos^2 x$$

$$20. \quad \vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix} \quad 1$$

$$\therefore \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k} \quad 1$$

$$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3} \quad 1$$

$$\therefore \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k} \quad 1$$

$$21. \quad \text{Here } \vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k} \quad 1$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} \quad 1$$

$$\text{Shortest distance} = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \quad 1$$

$$= \left| \frac{-6}{\sqrt{5}} \right| = \frac{6}{\sqrt{5}} \quad \text{or} \quad \frac{6\sqrt{5}}{5} \quad 1$$

$$22. \quad \left. \begin{array}{l} E_1 : \text{She gets 1 or 2 on die.} \\ E_2 : \text{She gets 3, 4, 5 or 6 on die.} \\ A : \text{She obtained exactly 1 tail} \end{array} \right\} \quad 1$$

$$\left. \begin{array}{l} P(E_1) = \frac{1}{3}, P(E_2) = \frac{2}{3} \end{array} \right\} \quad 1$$

$$\left. \begin{array}{l} P(A/E_1) = \frac{3}{8}, P(A/E_2) = \frac{1}{2} \end{array} \right\} \quad 1$$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad 1$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{8}{11} \quad 1$$

23. Let  $X$  denote the larger of two numbers

$X$	2	3	4	5
$P(X)$	1/10	2/10	3/10	4/10
$X \cdot P(X)$	2/10	6/10	12/10	20/10
$X^2 \cdot P(X)$	4/10	18/10	48/10	100/10

$$\text{Mean} = \sum X \cdot P(X) = \frac{40}{10} = 4$$

$$\text{Variance} = \sum X^2 \cdot P(X) - [\sum X \cdot P(X)]^2 = \frac{170}{10} - 4^2 = 1$$

### SECTION D

24. Reflexive:  $|a - a| = 0$ , which is divisible by 4,  $\forall a \in A$

$$\therefore (a, a) \in R, \forall a \in A \quad \therefore R \text{ is reflexive}$$

Symmetric: let  $(a, b) \in R$

$$\Rightarrow |a - b| \text{ is divisible by 4}$$

$$\Rightarrow |b - a| \text{ is divisible by 4 } (\because |a - b| = |b - a|)$$

$$\Rightarrow (b, a) \in R \quad \therefore R \text{ is symmetric.}$$

**Transitive:** let  $(a, b), (b, c) \in R$

$$\Rightarrow |a - b| \text{ \& } |b - c| \text{ are divisible by 4}$$

$$\Rightarrow a - b = \pm 4m, b - c = \pm 4n, m, n \in Z$$

$$\text{Adding we get, } a - c = 4(\pm m \pm n)$$

$$\Rightarrow (a - c) \text{ is divisible by 4}$$

$$\Rightarrow |a - c| \text{ is divisible by 4 } \quad \therefore (a, c) \in R$$

$\therefore R$  is transitive

Hence  $R$  is an equivalence relation in  $A$

set of elements related to 1 is  $\{1, 5, 9\}$

and  $[2] = \{2, 6, 10\}$ .

OR

Here  $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$  but  $2 \neq \frac{1}{2}$

$\therefore f$  is not 1-1

2

for  $y = \frac{1}{\sqrt{2}}$  let  $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As  $D = (-\sqrt{2})^2 - 4(1)(1) < 0$ ,  $\therefore$  No real solution

$\therefore f(x) \neq \frac{1}{\sqrt{2}}$ , for any  $x \in R(D_f)$   $\therefore f$  is not onto

2

$$\text{fog}(x) = f(2x - 1) = \frac{2x - 1}{(2x - 1)^2 + 1} = \frac{2x - 1}{4x^2 - 4x + 2}$$

2

25.  $|A| = -1 \neq 0 \therefore A^{-1}$  exists

1

Co-factors of A are:

$$\begin{array}{lll} A_{11} = 0 ; & A_{12} = 2 ; & A_{13} = 1 \\ A_{21} = -1 ; & A_{22} = -9 ; & A_{23} = -5 \\ A_{31} = 2 ; & A_{32} = 23 ; & A_{33} = 13 \end{array}$$

1 m for any  
4 correct  
cofactors

2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

 $\frac{1}{2}$ 

For :  $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$  and  $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ , the system of equation is  $A \cdot X = B$

 $\frac{1}{2}$ 

$$\therefore X = A^{-1} \cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

1

$\therefore x = 1, y = 2, z = 3$

1



Using elementary Row operations:

let:  $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

1

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \left\{ \text{Using, } R_2 \rightarrow R_2 - 2R_1; R_3 \rightarrow R_3 + 2R_1 \right.$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \left\{ \text{Using, } R_1 \rightarrow R_1 - 2R_2 \right.$$

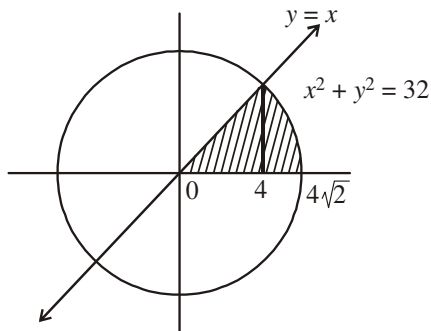
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$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \quad \left\{ \text{Using, } R_1 \rightarrow R_1 - R_3; R_2 \rightarrow R_2 - R_3 \right.$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$

1

26.



Correct figure:

1

Pt. of intersection,  $x = 4$

1

$$\text{Area of shaded region} = \int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} \, dx$$

1

$$= \frac{x^2}{2} \Big|_0^4 + \left\{ \frac{x}{2} \sqrt{32 - x^2} + 16 \sin^{-1} \frac{x}{4\sqrt{2}} \right\} \Big|_4^{4\sqrt{2}}$$

2

$$= 8 + 16 \frac{\pi}{2} - 8 - 4\pi = 4\pi$$

1

27. Put  $\sin x - \cos x = t$ ,  $(\cos x + \sin x) dx = dt$ ,  $1 - \sin 2x = t^2$  1

when  $x = 0, t = -1$  } 1  
 and  $x = \pi/4, t = 0$  } 2

$$\therefore I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx = \int_{-1}^0 \frac{1}{16 + 9(1-t^2)} dt = \int_{-1}^0 \frac{1}{25 - 9t^2} dt$$
 2

$$\Rightarrow I = \frac{1}{30} \log \left| \frac{5+3t}{5-3t} \right| \Big|_{-1}^0$$
 1 \frac{1}{2}

$$= \frac{1}{30} \left[ 0 - \log \frac{1}{4} \right] = -\frac{1}{30} \log \frac{1}{4} \text{ or } \frac{1}{15} \log 2$$
 1

OR

Here  $f(x) = x^2 + 3x + e^x$ ,  $a = 1$ ,  $b = 3$ ,  $nh = 2$  1

$$\therefore \int_1^3 (x^2 + 3x + e^x) dx = \lim_{h \rightarrow 0} [f(1) + f(1+h) + \dots + f(1+n-1h)]$$
 1

$$= \lim_{h \rightarrow 0} \left[ 4(nh) + \frac{(nh-h)(nh)(2nh-h)}{6} + \frac{5(nh-h)(nh)}{2} + \frac{h}{e^h-1} \times e \times (e^{nh}-1) \right]$$
 3

$$= 8 + \frac{8}{3} + 10 + e(e^2-1) = \frac{62}{3} + e^3 - e$$
 1

28. General point on the line is:  $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$  1 \frac{1}{2}

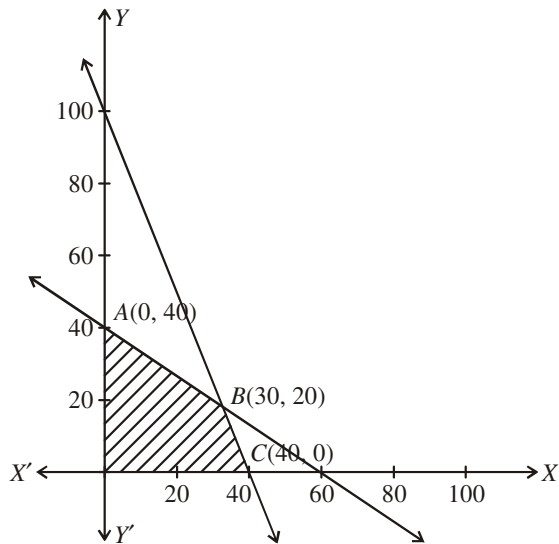
As the point lies on the plane

$$\therefore 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$$
 1 \frac{1}{2}

$$\therefore \text{Point is } (2, -1, 2)$$
 1

$$\text{Distance} = \sqrt{(2-(-1))^2 + (-1-(-5))^2 + (2-(-10))^2} = 13$$
 2

29.

Let number of packets of type  $A = x$ and number of packets of type  $B = y$  $\therefore$  L.P.P. is: Maximize,  $Z = 0.7x + y$ 

1

subject to constraints:

$$\left. \begin{array}{l} 4x + 6y \leq 240 \quad \text{or} \quad 2x + 3y \leq 120 \\ 6x + 3y \leq 240 \quad \text{or} \quad 2x + y \leq 80 \end{array} \right\}$$

2

$$x \geq 0, y \geq 0$$

Correct graph

2

$$Z(0, 0) = 0, Z(0, 40) = 40$$

$$Z(40, 0) = 28, Z(30, 20) = 41 \text{ (Max.)}$$

 $\therefore$  Max. profit is ₹ 41 at  $x = 30, y = 20$ .

1