

QUESTION PAPER CODE 65/2

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $5 \circ 10 = (5 * 10) + 3 = 10 + 3 = 13$ For $5 * 10 = 10$ $\frac{1}{2}$
 For Final Answer = 13 $\frac{1}{2}$
2. $|\vec{a}| = |\vec{b}| = 3$ $\frac{1}{2} + \frac{1}{2}$
3. $a = -2, b = 3$ $\frac{1}{2} + \frac{1}{2}$
4. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ $\frac{1}{2} + \frac{1}{2}$

Note: $\frac{1}{2}$ m. for any one of the two correct values and $\frac{1}{2}$ m. for final answer

SECTION B

5. Marginal cost = $C'(x) = 0.015x^2 - 0.04x + 30$ 1
 At $x = 3$, $C'(3) = 30.015$ 1
6. $f(x) = \tan^{-1}\left(\frac{1 + \cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2 \cos^2 \frac{x}{2}}{2 \sin \frac{x}{2} \cos \frac{x}{2}}\right)$ 1
 $= \tan^{-1}\left(\cot \frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$ $\frac{1}{2}$
 $\therefore f'(x) = -\frac{1}{2}$ $\frac{1}{2}$
7. $|A| = 2, \therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1
 $\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \text{RHS} = 9 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1
 $\therefore \text{LHS} = \text{RHS}$

8. In RHS, put $x = \sin \theta$

 $\frac{1}{2}$

$$\text{RHS} = \sin^{-1} (3 \sin \theta - 4 \sin^3 \theta)$$

$$= \sin^{-1} (\sin 3\theta)$$

1

$$= 3\theta = 3 \sin^{-1} x = \text{LHS.}$$

 $\frac{1}{2}$

9. A: Getting a sum of 8, B: Red die resulted in no. < 4

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

1

$$= \frac{2/36}{18/36} = \frac{1}{9}$$

1

$$10. \sin \theta = \frac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}| |3\hat{i} - 2\hat{j} + \hat{k}|}$$

 $\frac{1}{2}$

$$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6}$$

1

$$\sin \theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$$

 $\frac{1}{2}$

$$11. \frac{dy}{dx} = ba e^{bx+5} \Rightarrow \frac{dy}{dx} = by$$

1

$$\Rightarrow \frac{d^2 y}{dx^2} = b \frac{dy}{dx}$$

 $\frac{1}{2}$

$$\therefore \text{The differential equation is: } y \frac{d^2 y}{dx^2} = \left(\frac{dy}{dx} \right)^2$$

 $\frac{1}{2}$

$$12. I = \int \frac{1 - 2\sin^2 x + 2\sin^2 x}{\cos^2 x} dx$$

 $\frac{1}{2}$

$$= \int \sec^2 x dx$$

1

$$= \tan x + C$$

 $\frac{1}{2}$

SECTION C

$$13. \quad y = \sin(\sin x) \Rightarrow \frac{dy}{dx} = \cos(\sin x) \cdot \cos x \quad 1$$

$$\text{and } \frac{d^2y}{dx^2} = -\sin(\sin x) \cdot \cos^2 x - \sin x \cos(\sin x) \quad 1+1$$

$$\text{LHS} = -\sin(\sin x) \cos^2 x - \sin x \cos(\sin x) + \frac{\sin x}{\cos x} \cos(\sin x) \cos x + \sin(\sin x) \cos^2 x \quad 1$$

$$= 0 = \text{RHS}$$

14. Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^x}{e^x - 2} dx \quad 1 \frac{1}{2}$$

$$\Rightarrow \log |\tan y| = \log |e^x - 2| + \log C \quad 1 \frac{1}{2}$$

$$\Rightarrow \tan y = C(e^x - 2), \text{ for } x = 0, y = \pi/4, C = -1 \quad \frac{1}{2}$$

$$\therefore \text{ Particular solution is: } \tan y = 2 - e^x. \quad \frac{1}{2}$$

OR

$$\text{Integrating factor} = e^{\int 2 \tan x dx} = \sec^2 x \quad 1$$

$$\therefore \text{ Solution is: } y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x dx = \int \sec x \cdot \tan x dx \quad 1$$

$$\Rightarrow y \cdot \sec^2 x = \sec x + C, \text{ for } x = \frac{\pi}{3}, y = 0, \therefore C = -2 \quad 1 + \frac{1}{2}$$

$$\therefore \text{ Particular solution is: } y \cdot \sec^2 x = \sec x - 2 \quad \frac{1}{2}$$

$$\text{or } y = \cos x - 2 \cos^2 x$$

15. Here $\vec{a}_1 = 4\hat{i} - \hat{j}$, $\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$ 1

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j}$$
 1

Shortest distance = $\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$ 1

$$= \left| \frac{-6}{\sqrt{5}} \right| = \frac{6}{\sqrt{5}} \text{ or } \frac{6\sqrt{5}}{5}$$
 1

16. Let X denote the larger of two numbers

X	2	3	4	5	$\frac{1}{2}$
$P(X)$	1/10	2/10	3/10	4/10	1
$X \cdot P(X)$	2/10	6/10	12/10	20/10	$\frac{1}{2}$
$X^2 \cdot P(X)$	4/10	18/10	48/10	100/10	$\frac{1}{2}$

Mean = $\Sigma X \cdot P(X) = \frac{40}{10} = 4$ $\frac{1}{2}$

Variance = $\Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \frac{170}{10} - 4^2 = 1$ 1

17. LHS = $\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix} \quad (\text{Using } C_2 \rightarrow C_2 - C_1 \text{ \& } C_3 \rightarrow C_3 - C_1)$$
 1+1
(Any two relevant operations)

= $1 \times (9yz) + 3x(3z + 9yz + 3y)$ (Expanding along R_1) 1

= $9(3xyz + xy + yz + zx) = \text{RHS}$ 1

$$18. \quad x_1 = 2 \Rightarrow y_1 = 3 \quad (\because y_1 > 0)$$

$$\text{Differentiating the given equation, we get, } \frac{dy}{dx} = \frac{-16x}{9y}$$

$$\text{Slope of tangent at } (2, 3) = \left. \frac{dy}{dx} \right|_{(2,3)} = -\frac{32}{27}$$

$$\text{Slope of Normal at } (2, 3) = \frac{27}{32}$$

$$\text{Equation of tangent: } 32x + 27y = 145$$

$$\text{Equation of Normal: } 27x - 32y = -42$$

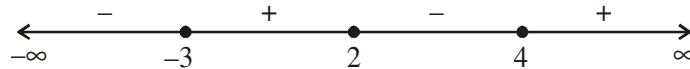
OR

$$f'(x) = x^3 - 3x^2 - 10x + 24$$

$$= (x - 2)(x - 4)(x + 3)$$

$$f'(x) = 0 \Rightarrow x = -3, 2, 4.$$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$

and $f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$

$$19. \quad \text{Put } \sin x = t \Rightarrow \cos x \, dx = dt$$

$$\text{Let } I = \int \frac{2 \cos x}{(1 - \sin x)(1 + \sin^2 x)} dx = \int \frac{2}{(1-t)(1+t^2)} dt$$

$$\text{Let } \frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}, \text{ solving we get}$$

$$A = 1, B = 1, C = 1$$

$$\therefore I = \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{2t}{1+t^2} + \int \frac{1}{1+t^2} dt$$

$$= -\log |1-t| + \frac{1}{2} \log |1+t^2| + \tan^{-1} t + C$$

$$= -\log (1 - \sin x) + \frac{1}{2} \log (1 + \sin^2 x) + \tan^{-1}(\sin x) + C$$

20. E_1 : She gets 1 or 2 on die. }
 E_2 : She gets 3, 4, 5 or 6 on die. } 1
 A: She obtained exactly 1 tail }

$$\left. \begin{aligned} P(E_1) &= \frac{1}{3}, P(E_2) = \frac{2}{3} \\ P(A/E_1) &= \frac{3}{8}, P(A/E_2) = \frac{1}{2} \end{aligned} \right\} 1$$

$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2)} \quad 1$$

$$= \frac{\frac{2}{3} \times \frac{1}{2}}{\frac{1}{3} \times \frac{3}{8} + \frac{2}{3} \times \frac{1}{2}} = \frac{8}{11} \quad 1$$

21. $\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$ } 1

$$\therefore \vec{d} = \lambda \hat{i} - 16\lambda \hat{j} - 13\lambda \hat{k} \quad 1$$

$$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3} \quad 1$$

$$\therefore \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k} \quad 1$$

22. Let side of base = x and depth of tank = y

$$V = x^2y \Rightarrow y = \frac{V}{x^2}, \quad (V = \text{Quantity of water} = \text{constant})$$

Cost of material is least when area of sheet used is minimum.

$$A(\text{Surface area of tank}) = x^2 + 4xy = x^2 + \frac{4V}{x} \quad \frac{1}{2} + \frac{1}{2}$$

$$\frac{dA}{dx} = 2x - \frac{4V}{x^2}, \frac{dA}{dx} = 0 \Rightarrow x^3 = 2V, y = \frac{x^3}{2x^2} = \frac{x}{2} \quad \frac{1}{2} + \frac{1}{2}$$

$$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0, \quad \therefore \text{Area is minimum, thus cost is minimum when } y = \frac{x}{2} \quad \frac{1}{2} + \frac{1}{2}$$

Value: Any relevant value. } 1

23. Differentiating with respect to 'x'

$$2(x^2 + y^2) \left(2x + 2y \frac{dy}{dx} \right) = x \frac{dy}{dx} + y \quad 2$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x} \quad 2$$

OR

$$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta) = 4a \sin^2 \theta \quad 1$$

$$\frac{dy}{d\theta} = 2a \sin 2\theta = 4a \sin \theta \cdot \cos \theta \quad 1$$

$$\therefore \frac{dy}{dx} = \frac{4a \sin \theta \cos \theta}{4a \sin^2 \theta} = \cot \theta \quad 1$$

$$\left. \frac{dy}{dx} \right|_{\theta = \frac{\pi}{3}} = \frac{1}{\sqrt{3}} \quad 1$$

SECTION D

24. Put $\sin x - \cos x = t$, $(\cos x + \sin x) dx = dt$, $1 - \sin 2x = t^2$ 1

$$\left. \begin{array}{l} \text{when } x = 0, t = -1 \\ \text{and } x = \pi/4, t = 0 \end{array} \right\} \quad \frac{1}{2}$$

$$\therefore I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx = \int_{-1}^0 \frac{1}{16 + 9(1 - t^2)} dt = \int_{-1}^0 \frac{1}{25 - 9t^2} dt \quad 2$$

$$\Rightarrow I = \frac{1}{30} \log \left| \frac{5 + 3t}{5 - 3t} \right| \Bigg|_{-1}^0 \quad 1 \frac{1}{2}$$

$$= \frac{1}{30} \left[0 - \log \frac{1}{4} \right] = -\frac{1}{30} \log \frac{1}{4} \text{ or } \frac{1}{15} \log 2 \quad 1$$

OR

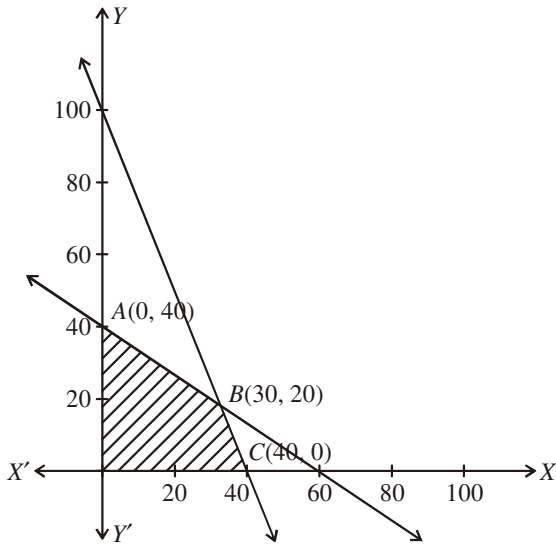
Here $f(x) = x^2 + 3x + e^x$, $a = 1$, $b = 3$, $nh = 2$ 1

$$\therefore \int_1^3 (x^2 + 3x + e^x) dx = \lim_{h \rightarrow 0} [f(1) + f(1+h) + \dots + f(1 + \overline{n-1}h)] \quad 1$$

$$= \lim_{h \rightarrow 0} \left[4(nh) + \frac{(nh-h)(nh)(2nh-h)}{6} + \frac{5(nh-h)(nh)}{2} + \frac{h}{e^h - 1} \times e \times (e^{nh} - 1) \right] \quad 3$$

$$= 8 + \frac{8}{3} + 10 + e(e^2 - 1) = \frac{62}{3} + e^3 - e \quad 1$$

25.

Let number of packets of type A = x and number of packets of type B = y \therefore L.P.P. is: Maximize, $Z = 0.7x + y$

subject to constraints:

$$\left. \begin{aligned} 4x + 6y &\leq 240 & \text{or} & & 2x + 3y &\leq 120 \\ 6x + 3y &\leq 240 & \text{or} & & 2x + y &\leq 80 \end{aligned} \right\}$$

$$x \geq 0, y \geq 0$$

Correct graph

$$Z(0, 0) = 0, Z(0, 40) = 40$$

$$Z(40, 0) = 28, Z(30, 20) = 41 \text{ (Max.)}$$

 \therefore Max. profit is ₹ 41 at $x = 30, y = 20$.26. Reflexive: $|a - a| = 0$, which is divisible by 4, $\forall a \in A$

$$\therefore (a, a) \in R, \forall a \in A \quad \therefore R \text{ is reflexive}$$

Symmetric: let $(a, b) \in R$

$$\Rightarrow |a - b| \text{ is divisible by } 4$$

$$\Rightarrow |b - a| \text{ is divisible by } 4 \quad (\because |a - b| = |b - a|)$$

$$\Rightarrow (b, a) \in R \quad \therefore R \text{ is symmetric.}$$

Transitive: let $(a, b), (b, c) \in R$

$$\Rightarrow |a - b| \text{ \& } |b - c| \text{ are divisible by } 4$$

$$\Rightarrow a - b = \pm 4m, b - c = \pm 4n, m, n \in \mathbb{Z}$$

$$\text{Adding we get, } a - c = 4(\pm m \pm n)$$

$$\Rightarrow (a - c) \text{ is divisible by } 4$$

$$\Rightarrow |a - c| \text{ is divisible by } 4 \quad \therefore (a, c) \in R$$

 $\therefore R$ is transitiveHence R is an equivalence relation in A set of elements related to 1 is $\{1, 5, 9\}$ and $[2] = \{2, 6, 10\}$.

OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$

$\therefore f$ is not 1-1

2

for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $D = (-\sqrt{2})^2 - 4(1)(1) < 0$, \therefore No real solution

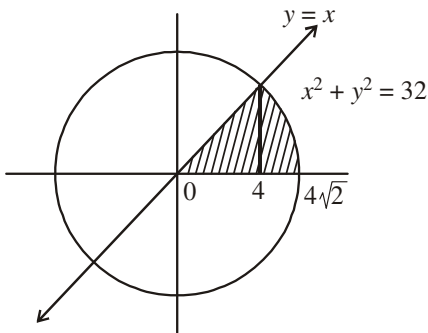
$\therefore f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R(D_f)$ $\therefore f$ is not onto

2

$$\text{fog}(x) = f(2x - 1) = \frac{2x-1}{(2x-1)^2 + 1} = \frac{2x-1}{4x^2 - 4x + 2}$$

2

27.



Correct figure:

1

Pt. of intersection, $x = 4$

1

$$\text{Area of shaded region} = \int_0^4 x \, dx + \int_4^{4\sqrt{2}} \sqrt{32-x^2} \, dx$$

1

$$= \left. \frac{x^2}{2} \right|_0^4 + \left. \left\{ \frac{x}{2} \sqrt{32-x^2} + 16 \sin^{-1} \frac{x}{4\sqrt{2}} \right\} \right|_4^{4\sqrt{2}}$$

2

$$= 8 + 16 \frac{\pi}{2} - 8 - 4\pi = 4\pi$$

1

28. $|A| = -1 \neq 0 \therefore A^{-1}$ exists

1

Co-factors of A are:

$$\begin{array}{lll} A_{11} = 0 ; & A_{12} = 2 ; & A_{13} = 1 \\ A_{21} = -1 ; & A_{22} = -9 ; & A_{23} = -5 \\ A_{31} = 2 ; & A_{32} = 23 ; & A_{33} = 13 \end{array}$$

1 m for any
4 correct
cofactors

2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|} \cdot \text{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$

 $\frac{1}{2}$

$$\text{For : } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}, \text{ the system of equation is } A \cdot X = B \quad \frac{1}{2}$$

$$\therefore X = A^{-1} \cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad 1$$

$$\therefore x = 1, y = 2, z = 3 \quad 1$$

OR

Using elementary Row operations:

let: $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \quad 1$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \left. \begin{array}{l} \text{\{Using, } R_2 \rightarrow R_2 - 2R_1; R_3 \rightarrow R_3 + 2R_1 \} \\ \} \end{array} \right. \quad 4$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \left. \begin{array}{l} \text{\{Using, } R_1 \rightarrow R_1 - 2R_2 \} \\ \} \end{array} \right. \quad 4$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \quad \left. \begin{array}{l} \text{\{Using, } R_1 \rightarrow R_1 - R_3; R_2 \rightarrow R_2 - R_3 \} \\ \} \end{array} \right. \quad 4$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} \quad 1$$

29. General point on the line is: $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ $\frac{1}{2}$

As the point lies on the plane

$$\therefore 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0 \quad \frac{1}{2}$$

$$\therefore \text{Point is } (2, -1, 2) \quad 1$$

$$\text{Distance} = \sqrt{(2 - (-1))^2 + (-1 - (-5))^2 + (2 - (-10))^2} = 13 \quad 2$$