QUESTION PAPER CODE 65/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|\vec{a}| = |\vec{b}| = 3$ $\frac{1}{2}+\frac{1}{2}$

2. $\frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = -\frac{\pi}{2}$ $\frac{1}{2}+\frac{1}{2}$

Note: $\frac{1}{2}$m. for any one of the two correct values and $\frac{1}{2}$m. for final answer

3. 5o10 = (5 * 10) + 3 = 10 + 3 = 13

For 5 * 10 = 10 $\frac{1}{2}$

For Final Answer = 13 $\frac{1}{2}$

4. $a = -2$, $b = 3$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. A: Getting a sum of 8, B: Red die resulted in no. < 4

$P(A/B) = \frac{P(A \cap B)}{P(B)}$ 1

$= \frac{2/36}{18/36} = \frac{1}{9}$ 1

6. $\sin\theta = \frac{|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})|}{|\hat{i} - 2\hat{j} + 3\hat{k}|\,|3\hat{i} - 2\hat{j} + \hat{k}|}$ $\frac{1}{2}$

$|(\hat{i} - 2\hat{j} + 3\hat{k}) \times (3\hat{i} - 2\hat{j} + \hat{k})| = |4\hat{i} + 8\hat{j} + 4\hat{k}| = 4\sqrt{6}$ 1

$\sin\theta = \frac{4\sqrt{6}}{14} = \frac{2\sqrt{6}}{7}$ $\frac{1}{2}$

7. $\frac{dy}{dx} = bae^{bx+5} \Rightarrow \frac{dy}{dx} = by$ 1

$\Rightarrow \frac{d^2y}{dx^2} = b\frac{dy}{dx}$ $\frac{1}{2}$

$\therefore$ The differential equation is: $y\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ $\frac{1}{2}$

8. $I = \int \frac{1-2\sin^2 x + 2\sin^2 x}{\cos^2 x}\, dx$ $\frac{1}{2}$

$= \int \sec^2 x\, dx$ 1

$= \tan x + C$ $\frac{1}{2}$

9. Marginal cost = $C'(x) = 0.015x^2 - 0.04x + 30$ 1

At $x = 3$, $C'(3) = 30.015$ 1

10. $f(x) = \tan^{-1}\left(\frac{1+\cos x}{\sin x}\right) = \tan^{-1}\left(\frac{2\cos^2 x/2}{2\sin x/2 \cos x/2}\right)$ 1

$= \tan^{-1}\left(\cot x/2\right) = \frac{\pi}{2} - \frac{x}{2}$ $\frac{1}{2}$

$\therefore \quad f'(x) = -\frac{1}{2}$ $\frac{1}{2}$

11. $|A| = 2, \quad \therefore \; A^{-1} = \frac{1}{2}\begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\text{LHS} = 2A^{-1} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}, \; \text{RHS} = 9\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 4 & 2 \end{bmatrix}$ 1

$\therefore$ LHS = RHS

12. In RHS, put $x = \sin\theta$ $\frac{1}{2}$

RHS $= \sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= \sin^{-1}(\sin 3\theta)$ 1

$= 3\theta = 3\sin^{-1} x$ = LHS. $\frac{1}{2}$

SECTION C

13. Let X denote the larger of two numbers

X	2	3	4	5	
$P(X)$	1/10	2/10	3/10	4/10	1
$X \cdot P(X)$	2/10	6/10	12/10	20/10	$\frac{1}{2}$
$X^2 \cdot P(X)$	4/10	18/10	48/10	100/10	$\frac{1}{2}$

$\frac{1}{2}$ (for the X row)

Mean $= \Sigma X \cdot P(X) = \frac{40}{10} = 4$ $\frac{1}{2}$

Variance $= \Sigma X^2 \cdot P(X) - [\Sigma X \cdot P(X)]^2 = \frac{170}{10} - 4^2 = 1$ 1

14. Let side of base = x and depth of tank = y

$V = x^2 y \Rightarrow y = \frac{V}{x^2}$, ($V$ = Quantity of water = constant)

Cost of material is least when area of sheet used is minimum.

A(Surface area of tank) $= x^2 + 4xy = x^2 + \frac{4V}{x}$ $\frac{1}{2}+\frac{1}{2}$

$\frac{dA}{dx} = 2x - \frac{4V}{x^2}, \frac{dA}{dx} = 0 \Rightarrow x^3 = 2V, \ y = \frac{x^3}{2x^2} = \frac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

$\frac{d^2A}{dx^2} = 2 + \frac{8V}{x^3} > 0$, $\therefore$ Area is minimum, thus cost is minimum when $y = \frac{x}{2}$ $\frac{1}{2}+\frac{1}{2}$

Value: Any relevant value. 1

15. $x_1 = 2 \Rightarrow y_1 = 3 \quad (\because y_1 > 0)$ $\frac{1}{2}$

Differentiating the given equation, we get, $\frac{dy}{dx} = \frac{-16x}{9y}$ $\frac{1}{2}$

Slope of tangent at $(2, 3) = \left.\frac{dy}{dx}\right]_{(2,3)} = -\frac{32}{27}$ $\frac{1}{2}$

Slope of Normal at $(2, 3) = \frac{27}{32}$ $\frac{1}{2}$

Equation of tangent: $32x + 27y = 145$ 1

Equation of Normal: $27x - 32y = -42$ 1

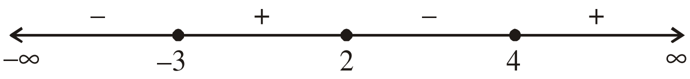
OR

$f'(x) = x^3 - 3x^2 - 10x + 24$ $\frac{1}{2}$

$= (x-2)(x-4)(x+3)$ 1

$f'(x) = 0 \Rightarrow x = -3, 2, 4.$ $\frac{1}{2}$

sign of $f'(x)$:



$\therefore f(x)$ is strictly increasing on $(-3, 2) \cup (4, \infty)$ 1

and $f(x)$ is strictly decreasing on $(-\infty, -3) \cup (2, 4)$ 1

16. Differentiating with respect to 'x'

$2(x^2 + y^2)\left(2x + 2y\frac{dy}{dx}\right) = x\frac{dy}{dx} + y$ 2

$\Rightarrow \frac{dy}{dx} = \frac{y - 4x^3 - 4xy^2}{4x^2y + 4y^3 - x}$ 2

OR

$\frac{dx}{d\theta} = a(2 - 2\cos 2\theta) = 4a\sin^2\theta$ 1

$\frac{dy}{d\theta} = 2a\sin 2\theta = 4a\sin\theta\cdot\cos\theta$ 1

$\therefore \frac{dy}{dx} = \frac{4a\sin\theta\cos\theta}{4a\sin^2\theta} = \cot\theta$ 1

$\left.\frac{dy}{dx}\right]_{\theta=\frac{\pi}{3}} = \frac{1}{\sqrt{3}}$ 1

17. $y = \sin(\sin x) \Rightarrow \frac{dy}{dx} = \cos(\sin x)\cdot\cos x$ 1

and $\frac{d^2y}{dx^2} = -\sin(\sin x)\cdot\cos^2 x - \sin x\cos(\sin x)$ 1+1

LHS $= -\sin(\sin x)\cos^2 x - \sin x\cos(\sin x) + \frac{\sin x}{\cos x}\cos(\sin x)\cos x + \sin(\sin x)\cos^2 x$ 1

$= 0 =$ RHS

18. Separating the variables, we get:

$$\int \frac{\sec^2 y}{\tan y}\,dy = \int \frac{e^x}{e^x - 2}\,dx \qquad 1\frac{1}{2}$$

$\Rightarrow$ $\log|\tan y| = \log|e^x - 2| + \log C$ $\quad 1\frac{1}{2}$

$\Rightarrow$ $\tan y = C(e^x - 2)$, for $x = 0$, $y = \pi/4$, $C = -1$ $\quad \frac{1}{2}$

$\therefore$ Particular solution is: $\tan y = 2 - e^x$. $\quad \frac{1}{2}$

OR

Integrating factor $= e^{\int 2\tan x dx} = \sec^2 x$ $\quad 1$

$\therefore$ Solution is: $y \cdot \sec^2 x = \int \sin x \cdot \sec^2 x\,dx = \int \sec x \cdot \tan x\,dx$ $\quad 1$

$\Rightarrow$ $y \cdot \sec^2 x = \sec x + C$, for $x = \dfrac{\pi}{3}$, $y = 0$, $\therefore C = -2$ $\quad 1+\frac{1}{2}$

$\therefore$ Particular solution is: $y \cdot \sec^2 x = \sec x - 2$ $\quad \frac{1}{2}$

or $y = \cos x - 2\cos^2 x$

19. Here $\vec{a}_1 = 4\hat{i} - \hat{j}, \vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}, \vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$ $\quad 1$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j} \qquad 1$$

$$\text{Shortest distance } = \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|} \qquad 1$$

$$= \left|\frac{-6}{\sqrt{5}}\right| = \frac{6}{\sqrt{5}} \text{ or } \frac{6\sqrt{5}}{5} \qquad 1$$

20. Put $\sin x = t \Rightarrow \cos x \, dx = dt$ $\frac{1}{2}$

Let $I = \int \frac{2\cos x}{(1-\sin x)(1+\sin^2 x)} dx = \int \frac{2}{(1-t)(1+t^2)} dt$

Let $\frac{2}{(1-t)(1+t^2)} = \frac{A}{1-t} + \frac{Bt+C}{1+t^2}$, solving we get

$A = 1, B = 1, C = 1$ $1\frac{1}{2}$

$\therefore I = \int \frac{1}{1-t} dt + \frac{1}{2}\int \frac{2t}{1+t^2} + \int \frac{1}{1+t^2} dt$

$= -\log|1-t| + \frac{1}{2}\log|1+t^2| + \tan^{-1} t + C$ $1\frac{1}{2}$

$= -\log(1-\sin x) + \frac{1}{2}\log(1+\sin^2 x) + \tan^{-1}(\sin x) + C$ $\frac{1}{2}$

21. E_1 : She gets 1 or 2 on die.

E_2 : She gets 3, 4, 5 or 6 on die.

A: She obtained exactly 1 tail 1

$P(E_1) = \frac{1}{3},\ P(E_2) = \frac{2}{3}$

$P(A/E_1) = \frac{3}{8},\ P(A/E_2) = \frac{1}{2}$ 1

$P(E_2/A) = \frac{P(E_2)\cdot P(A/E_2)}{P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)}$ 1

$= \frac{\frac{2}{3}\times\frac{1}{2}}{\frac{1}{3}\times\frac{3}{8}+\frac{2}{3}\times\frac{1}{2}} = \frac{8}{11}$ 1

22. $\vec{d} = \lambda(\vec{c} \times \vec{b}) = \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -1 \\ 1 & -4 & 5 \end{vmatrix}$ 1

$\therefore\ \vec{d} = \lambda\hat{i} - 16\lambda\hat{j} - 13\lambda\hat{k}$ 1

$\vec{d} \cdot \vec{a} = 21 \Rightarrow 4\lambda - 80\lambda + 13\lambda = 21 \Rightarrow \lambda = -\frac{1}{3}$ 1

$\therefore\ \vec{d} = -\frac{1}{3}\hat{i} + \frac{16}{3}\hat{j} + \frac{13}{3}\hat{k}$ 1

23. LHS $= \begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3z & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 0 & 3x \\ 1+3y & -3y & -3y \\ 1 & 3z & 0 \end{vmatrix}$ (Using $C_2 \to C_2 - C_1$ & $C_3 \to C_3 - C_1$) 1+1

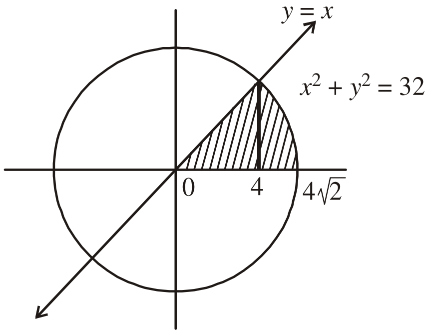
(Any two relevant operations)

$= 1 \times (9yz) + 3x(3z + 9yz + 3y)$ (Expanding along R_1) 1

$= 9(3xyz + xy + yz + zx) =$ RHS 1

SECTION D

24.



Correct figure: 1

Pt. of intersection, $x = 4$ 1

Area of shaded region $= \int_0^4 x\,dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2}\,dx$ 1

$= \left.\frac{x^2}{2}\right]_0^4 + \left.\left\{\frac{x}{2}\sqrt{32 - x^2} + 16\sin^{-1}\frac{x}{4\sqrt{2}}\right\}\right]_4^{4\sqrt{2}}$ 2

$= 8 + 16\frac{\pi}{2} - 8 - 4\pi = 4\pi$ 1

25. Reflexive: $|a-a| = 0$, which is divisible by 4, $\forall\ a \in A$ 1

$\therefore\quad (a, a) \in R, \forall\ a \in A \quad \therefore\ R$ is reflexive

Symmetric: let $(a, b) \in R$

$\Rightarrow\quad |a-b|$ is divisible by 4

$\Rightarrow\quad |b-a|$ is divisible by 4 ($\because\ |a-b| = |b-a|$)

$\Rightarrow\quad (b, a) \in R \quad \therefore\ R$ is symmetric. 1

Transitive: let $(a, b), (b, c) \in R$

$\Rightarrow\quad |a-b|\ \&\ |b-c|$ are divisible by 4

$\Rightarrow\quad a-b = \pm 4m,\ b-c = \pm 4n,\ m, n \in Z$

Adding we get, $a-c = 4(\pm m \pm n)$ 2

$\Rightarrow\quad (a-c)$ is divisible by 4

$\Rightarrow\quad |a-c|$ is divisible by 4 $\quad \therefore\ (a, c) \in R$

$\therefore\ R$ is transitive

Hence R is an equivalence relation in A 1

set of elements related to 1 is $\{1, 5, 9\}$

and $[2] = \{2, 6, 10\}$. 1

OR

Here $f(2) = f\left(\frac{1}{2}\right) = \frac{2}{5}$ but $2 \neq \frac{1}{2}$

$\therefore\quad f$ is not 1-1 2

for $y = \frac{1}{\sqrt{2}}$ let $f(x) = \frac{1}{\sqrt{2}} \Rightarrow x^2 - \sqrt{2}x + 1 = 0$

As $\quad D = (-\sqrt{2})^2 - 4(1)(1) < 0, \quad \therefore$ No real solution

$\therefore\quad f(x) \neq \frac{1}{\sqrt{2}}$, for any $x \in R(D_f) \quad \therefore f$ is not onto 2

$\text{fog}\ (x) = f(2x-1) = \frac{2x-1}{(2x-1)^2+1} = \frac{2x-1}{4x^2-4x+2}$ 2

26. General point on the line is: $(2 + 3\lambda, -1 + 4\lambda, 2 + 2\lambda)$ $1\frac{1}{2}$

As the point lies on the plane

$\therefore\ 2 + 3\lambda + 1 - 4\lambda + 2 + 2\lambda = 5 \Rightarrow \lambda = 0$ $1\frac{1}{2}$

$\therefore$ Point is $(2, -1, 2)$ 1

Distance = $\sqrt{(2-(-1))^2+(-1-(-5))^2+(2-(-10))^2} = 13$ 2

27. Let number of packets of type $A = x$

and number of packets of type $B = y$

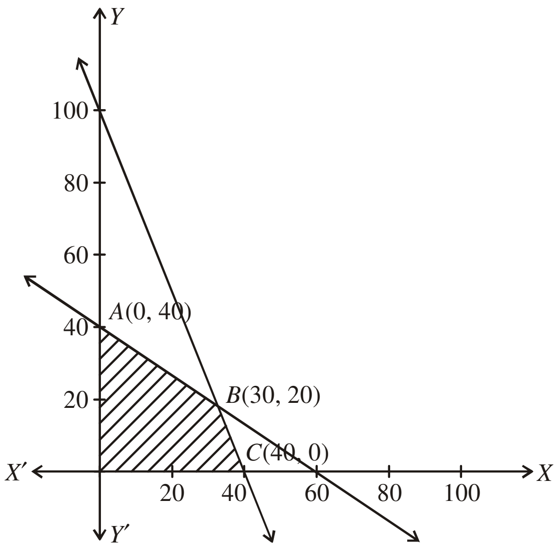
$\therefore$ L.P.P. is: Maximize, $Z = 0.7x + y$ 1

subject to constraints:

$$\left.\begin{array}{l} 4x + 6y \le 240 \quad \text{or} \quad 2x + 3y \le 120 \\ 6x + 3y \le 240 \quad \text{or} \quad 2x + y \le 80 \end{array}\right\}$$ 2

$x \ge 0,\ y \ge 0$

Correct graph 2



$Z(0, 0) = 0,\ Z(0, 40) = 40$

$Z(40, 0) = 28,\ Z(30, 20) = 41$ (Max.)

$\therefore$ Max. profit is ₹ 41 at $x = 30,\ y = 20$. 1

28. Put $\sin x - \cos x = t$, $(\cos x + \sin x)\,dx = dt$, $1 - \sin 2x = t^2$ 1

$$\left.\begin{array}{ll} \text{when} & x = 0, t = -1 \\ \text{and} & x = \pi/4, t = 0 \end{array}\right\}$$ $\frac{1}{2}$

$$\therefore \quad I = \int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9\sin 2x}\,dx = \int_{-1}^{0} \frac{1}{16 + 9(1-t^2)}\,dt = \int_{-1}^{0} \frac{1}{25 - 9t^2}\,dt$$ 2

$$\Rightarrow \quad I = \frac{1}{30}\log\left|\frac{5+3t}{5-3t}\right|\Bigg]_{-1}^{0}$$ $1\frac{1}{2}$

$$= \frac{1}{30}\left[0 - \log\frac{1}{4}\right] = -\frac{1}{30}\log\frac{1}{4} \ \text{ or } \ \frac{1}{15}\log 2$$ 1

OR

Here $f(x) = x^2 + 3x + e^x$, $a = 1$, $b = 3$, $nh = 2$ 1

$$\therefore \quad \int_1^3 (x^2+3x+e^x)\,dx = \lim_{h\to 0}[f(1)+f(1+h)+\ldots+f(1+\overline{n-1}h)]$$ 1

$$= \lim_{h\to 0}\left[4(nh)+\frac{(nh-h)(nh)(2nh-h)}{6}+\frac{5(nh-h)(nh)}{2}+\frac{h}{e^h-1}\times e\times(e^{nh}-1)\right]$$ 3

$$= 8+\frac{8}{3}+10+e(e^2-1)=\frac{62}{3}+e^3-e$$ 1

29. $|A| = -1 \neq 0 \quad \therefore A^{-1}$ exists 1

Co-factors of A are:

$A_{11} = 0$; $A_{12} = 2$; $A_{13} = 1$

$A_{21} = -1$; $A_{22} = -9$; $A_{23} = -5$

$A_{31} = 2$; $A_{32} = 23$; $A_{33} = 13$

1 m for any 4 correct cofactors 2

$$\text{adj}(A) = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{|A|}\cdot\text{adj}(A) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ $\frac{1}{2}$

For : $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$, the system of equation is $A \cdot X = B$ $\frac{1}{2}$

$$\therefore \quad X = A^{-1}\cdot B = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$\therefore \quad x = 1,\ y = 2,\ z = 3$ 1

OR

Using elementary Row operations:

let: $A = IA$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$ 1

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_2 \to R_2 - 2R_1; R_3 \to R_3 + 2R_1$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - 2R_2$$ 4

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix} A \quad \{\text{Using, } R_1 \to R_1 - R_3; R_2 \to R_2 - R_3$$

$$\therefore \quad A^{-1} = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$$ 1