Chapter 4

# PRINCIPLE OF MATHEMATICAL INDUCTION

## 4.1 Overview

Mathematical induction is one of the techniques which can be used to prove variety of mathematical statements which are formulated in terms of $n$, where $n$ is a positive integer.

### 4.1.1 *The principle of mathematical induction*

Let P($n$) be a given statement involving the natural number $n$ such that

(i) The statement is true for $n = 1$, i.e., P(1) is true (or true for any fixed natural number) and

(ii) If the statement is true for $n = k$ (where $k$ is a particular but arbitrary natural number), then the statement is also true for $n = k + 1$, i.e, truth of P($k$) implies the truth of P($k$ + 1). Then P($n$) is true for all natural numbers $n$.



## 4.2 Solved Examples

### Short Answer Type

Prove statements in Examples 1 to 5, by using the Principle of Mathematical Induction for all $n \in \mathbf{N}$, that :

**Example 1** $1 + 3 + 5 + ... + (2n - 1) = n^2$

**Solution** Let the given statement P($n$) be defined as P($n$) : $1 + 3 + 5 +...+ (2n - 1) = n^2$, for $n \in \mathbf{N}$. Note that P(1) is true, since

$$P(1) : 1 = 1^2$$

Assume that P($k$) is true for some $k \in \mathbf{N}$, i.e.,

$$P(k) : 1 + 3 + 5 + ... + (2k - 1) = k^2$$

Now, to prove that P($k$ + 1) is true, we have

$$1 + 3 + 5 + ... + (2k - 1) + (2k + 1)$$

$$= k^2 + (2k + 1) \qquad \text{(Why?)}$$

$$= k^2 + 2k + 1 = (k + 1)^2$$

Thus, P($k$ + 1) is true, whenever P($k$) is true.

Hence, by the Principle of Mathematical Induction, P($n$) is true for all $n \in \mathbf{N}$.

**Example 2** $\sum_{t=1}^{n-1} t(t+1) = \frac{n(n-1)(n+1)}{3}$, for all natural numbers $n \geq 2$.

**Solution** Let the given statement P($n$), be given as

$$P(n): \sum_{t=1}^{n-1} t(t+1) = \frac{n(n-1)(n+1)}{3}, \text{ for all natural numbers } n \geq 2.$$

We observe that

$$P(2): \sum_{t=1}^{2-1} t(t+1) = \sum_{t=1}^{1} t(t+1) = 1.2 = \frac{1.2.3}{3}$$

$$= \frac{2.(2-1)(2+1)}{3}$$

Thus, P($n$) in true for $n$ = 2.

Assume that P($n$) is true for $n = k \in \mathbf{N}$.

i.e.,
$$P(k): \sum_{t=1}^{k-1} t(t+1) = \frac{k(k-1)(k+1)}{3}$$

To prove that P($k$ + 1) is true, we have

$$\sum_{t=1}^{(k+1-1)} t(t+1) = \sum_{t=1}^{k} t(t+1)$$

$$= \sum_{t=1}^{k-1} t(t+1) + k(k+1) = \frac{k(k-1)(k+1)}{3} + k(k+1)$$

$$= k(k+1)\left[\frac{k-1+3}{3}\right] = \frac{k(k+1)(k+2)}{3}$$

$$= \frac{(k+1)((k+1)-1))((k+1)+1)}{3}$$

Thus, P($k$ + 1) is true, whenever P($k$) is true.

Hence, by the Principle of Mathematical Induction, P($n$) is true for all natural numbers $n \geq 2$.

**Example 3** $\left(1-\frac{1}{2^2}\right).\left(1-\frac{1}{3^2}\right)...\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}$, for all natural numbers, $n \geq 2$.

**Solution** Let the given statement be P($n$), i.e.,

$$P(n): \left(1-\frac{1}{2^2}\right).\left(1-\frac{1}{3^2}\right)...\left(1-\frac{1}{n^2}\right)=\frac{n+1}{2n}, \text{ for all natural numbers, } n \geq 2$$

We, observe that P (2) is true, since

$$\left(1-\frac{1}{2^2}\right)=1-\frac{1}{4}=\frac{4-1}{4}=\frac{3}{4}=\frac{2+1}{2\times 2}$$

Assume that P($n$) is true for some $k \in \mathbf{N}$, i.e.,

$$P(k): \quad 1-\frac{1}{2^2} \ . \ 1-\frac{1}{3^2} \ ... \ 1-\frac{1}{k^2} \ = \frac{k+1}{2k}$$

Now, to prove that P ($k$ + 1) is true, we have

$$1-\frac{1}{2^2} \ . \ 1-\frac{1}{3^2} \ ... \ 1-\frac{1}{k^2} \ . \ 1-\frac{1}{(k+1)^2}$$

$$=\frac{k+1}{2k}\left(1-\frac{1}{(k+1)^2}\right)=\frac{k^2+2k}{2k\,(k+1)}=\frac{(k+1)+1}{2\,(k+1)}$$

Thus, P ($k$ + 1) is true, whenever P($k$) is true.

Hence, by the Principle of Mathematical Induction, P($n$) is true for all natural numbers, $n \geq 2$.

**Example 4** $2^{2n} - 1$ is divisible by 3.

**Solution** Let the statement P($n$) given as

P($n$) : $2^{2n} - 1$ is divisible by 3, for every natural number $n$.

We observe that P(1) is true, since

$$2^2 - 1 = 4 - 1 = 3.1 \text{ is divisible by } 3.$$

Assume that P($n$) is true for some natural number $k$, i.e.,

P($k$): $2^{2k} - 1$ is divisible by 3, i.e., $2^{2k} - 1 = 3q$, where $q \in \mathbf{N}$

Now, to prove that P($k$ + 1) is true, we have

$$P(k+1): 2^{2(k+1)} - 1 = 2^{2k+2} - 1 = 2^{2k} . 2^2 - 1$$

$$= 2^{2k} . 4 - 1 = 3.2^{2k} + (2^{2k} - 1)$$

$$= 3.2^{2k} + 3q$$
$$= 3\,(2^{2k} + q) = 3m, \text{ where } m \in \mathbf{N}$$

Thus P($k$ + 1) is true, whenever P($k$) is true.

Hence, by the Principle of Mathematical Induction P($n$) is true for all natural numbers $n$.

**Example 5** $2n + 1 < 2^n$, for all natual numbers $n \geq 3$.

**Solution** Let P($n$) be the given statement, i.e., P($n$) : $(2n + 1) < 2^n$ for all natural numbers, $n \geq 3$. We observe that P(3) is true, since

$$2.3 + 1 = 7 < 8 = 2^3$$

Assume that P($n$) is true for some natural number $k$, i.e., $2k + 1 < 2^k$

To prove P($k$ + 1) is true, we have to show that $2(k + 1) + 1 < 2^{k+1}$. Now, we have

$$2(k + 1) + 1 = 2\,k + 3$$
$$= 2k + 1 + 2 < 2^k + 2 < 2^k\,.\,2 = 2^{k+1}.$$

Thus P($k$ + 1) is true, whenever P($k$) is true.

Hence, by the Principle of Mathematical Induction P($n$) is true for all natural numbers, $n \geq 3$.

## Long Answer Type

**Example 6** Define the sequence $a_1$, $a_2$, $a_3$... as follows :

$a_1 = 2$, $a_n = 5\,a_{n-1}$, for all natural numbers $n \geq 2$.

(i) Write the first four terms of the sequence.

(ii) Use the Principle of Mathematical Induction to show that the terms of the sequence satisfy the formula $a_n = 2.5^{n-1}$ for all natural numbers.

**Solution**

(i) We have $a_1 = 2$

$a_2 = 5a_{2-1} = 5a_1 = 5.2 = 10$

$a_3 = 5a_{3-1} = 5a_2 = 5.10 = 50$

$a_4 = 5a_{4-1} = 5a_3 = 5.50 = 250$

(ii) Let P($n$) be the statement, i.e.,

P($n$) : $a_n = 2.5^{\,n-1}$ for all natural numbers. We observe that P(1) is true

Assume that P($n$) is true for some natural number $k$, i.e., P($k$) : $a_k = 2.5^{k-1}$.

Now to prove that P ($k$ + 1) is true, we have

$$P(k+1) : a_{k+1} = 5.a_k = 5 \,.\, (2.5^{k-1})$$
$$= 2.5^k = 2.5^{(k+1)-1}$$

Thus P($k$ + 1) is true whenever P ($k$) is true.

Hence, by the Principle of Mathematical Induction, P($n$) is true for all natural numbers.

**Example 7** The distributive law from algebra says that for all real numbers $c$, $a_1$ and $a_2$, we have $c\,(a_1 + a_2) = ca_1 + ca_2$.

Use this law and mathematical induction to prove that, for all natural numbers, $n \geq 2$, if $c, a_1, a_2, ..., a_n$ are any real numbers, then

$$c\,(a_1 + a_2 + ... + a_n) = ca_1 + ca_2 + ... + ca_n$$

**Solution** Let P($n$) be the given statement, i.e.,

P($n$) : $c\,(a_1 + a_2 + ... + a_n) = ca_1 + ca_2 + ... ca_n$ for all natural numbers $n \geq 2$, for $c$, $a_1$, $a_2$, ... $a_n \in$ **R**.

We observe that P(2) is true since

$$c(a_1 + a_2) = ca_1 + ca_2 \qquad \text{(by distributive law)}$$

Assume that P($n$) is true for some natural number $k$, where $k > 2$, i.e.,

$$P(k) : c\,(a_1 + a_2 + ... + a_k) = ca_1 + ca_2 + ... + ca_k$$

Now to prove P($k$ + 1) is true, we have

$$P(k+1) : c\,(a_1 + a_2 + ... + a_k + a_{k+1})$$
$$= c\,((a_1 + a_2 + ... + a_k) + a_{k+1})$$
$$= c\,(a_1 + a_2 + ... + a_k) + ca_{k+1} \qquad \text{(by distributive law)}$$
$$= ca_1 + ca_2 + ... + ca_k + ca_{k+1}$$

Thus P($k$ + 1) is true, whenever P ($k$) is true.

Hence, by the principle of Mathematical Induction, P($n$) is true for all natural numbers $n \geq 2$.

**Example 8** Prove by induction that for all natural number $n$

$\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... + \sin(\alpha + (n-1)\beta)$

$$= \frac{\sin\left(\alpha + \frac{n-1}{2}\beta\right)\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

**Solution** Consider P ($n$) : $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... + \sin(\alpha + (n-1)\beta)$

$$= \frac{\sin\left(\alpha + \frac{n-1}{2}\beta\right)\sin\left(\frac{n\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}, \text{ for all natural number } n.$$

We observe that
P (1) is true, since

$$P(1) : \sin\alpha = \frac{\sin(\alpha + 0)\sin\frac{\beta}{2}}{\sin\frac{\beta}{2}}$$

Assume that P($n$) is true for some natural numbers $k$, i.e.,
P ($k$) : $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... + \sin(\alpha + (k - 1)\beta)$

$$= \frac{\sin\left(\alpha + \frac{k-1}{2}\beta\right)\sin\left(\frac{k\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)}$$

Now, to prove that P ($k$ + 1) is true, we have
P ($k$ + 1) : $\sin\alpha + \sin(\alpha + \beta) + \sin(\alpha + 2\beta) + ... + \sin(\alpha + (k - 1)\beta) + \sin(\alpha + k\beta)$

$$= \frac{\sin\left(\alpha + \frac{k-1}{2}\beta\right)\sin\left(\frac{k\beta}{2}\right)}{\sin\left(\frac{\beta}{2}\right)} + \sin(\alpha + k\beta)$$

$$= \frac{\sin\left(\alpha + \frac{k-1}{2}\beta\right)\sin\frac{k\beta}{2} + \sin(\alpha + k\beta)\sin\frac{\beta}{2}}{\sin\frac{\beta}{2}}$$

$$= \frac{\cos\left(\alpha - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta - \frac{\beta}{2}\right) + \cos\left(\alpha + k\beta - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta + \frac{\beta}{2}\right)}{2\sin\frac{\beta}{2}}$$

$$= \frac{\cos\left(\alpha - \frac{\beta}{2}\right) - \cos\left(\alpha + k\beta + \frac{\beta}{2}\right)}{2\sin\frac{\beta}{2}}$$

$$= \frac{\sin\left(\alpha + \frac{k\beta}{2}\right)\sin\left(\frac{k\beta + \beta}{2}\right)}{\sin\frac{\beta}{2}}$$

$$= \frac{\sin\left(\alpha + \frac{k\beta}{2}\right)\sin\ (k+1)\left(\frac{\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

Thus P $(k + 1)$ is true whenever P $(k)$ is true.

Hence, by the Principle of Mathematical Induction P$(n)$ is true for all natural number $n$.

**Example 9** Prove by the Principle of Mathematical Induction that

$1 \times 1! + 2 \times 2! + 3 \times 3! + ... + n \times n! = (n + 1)! – 1$ for all natural numbers $n$.

**Solution** Let P$(n)$ be the given statement, that is,

P$(n)$ : $1 \times 1! + 2 \times 2! + 3 \times 3! + ... + n \times n! = (n + 1)! – 1$ for all natural numbers $n$.

Note that P (1) is true, since

$$P(1) : 1 \times 1! = 1 = 2 – 1 = 2! – 1.$$

Assume that P$(n)$ is true for some natural number $k$, i.e.,

P$(k)$ : $1 \times 1! + 2 \times 2! + 3 \times 3! + ... + k \times k! = (k + 1)! – 1$

To prove P $(k + 1)$ is true, we have

P $(k + 1)$ : $1 \times 1! + 2 \times 2! + 3 \times 3! + ... + k \times k! + (k + 1) \times (k + 1)!$

$= (k + 1)! – 1 + (k + 1)! \times (k + 1)$

$= (k + 1 + 1)\ (k + 1)! – 1$

$= (k + 2)\ (k + 1)! – 1 = ((k + 2)! – 1$

Thus P $(k + 1)$ is true, whenever P $(k)$ is true. Therefore, by the Principle of Mathematical Induction, P $(n)$ is true for all natural number $n$.

**Example 10** Show by the Principle of Mathematical Induction that the sum $S_n$ of the $n$ term of the series $1^2 + 2 \times 2^2 + 3^2 + 2 \times 4^2 + 5^2 + 2 \times 6^2$ ... is given by

$$S_n = \begin{cases} \dfrac{n(n+1)^2}{2}, & \text{if } n \text{ is even} \\ \dfrac{n^2(n+1)}{2}, & \text{if } n \text{ is odd} \end{cases}$$

**Solution** Here $P(n) : S_n = \begin{cases} \dfrac{n(n+1)^2}{2}, & \text{when } n \text{ is even} \\ \dfrac{n^2(n+1)}{2}, & \text{when } n \text{ is odd} \end{cases}$

Also, note that any term $T_n$ of the series is given by

$$T_n = \begin{cases} n^2 & \text{if } n \text{ is odd} \\ 2n^2 & \text{if } n \text{ is even} \end{cases}$$

We observe that P(1) is true since

$$P(1) : S_1 = 1^2 = 1 = \frac{1.2}{2} = \frac{1^2.(1+1)}{2}$$

Assume that $P(k)$ is true for some natural number $k$, i.e.

**Case 1** When $k$ is odd, then $k + 1$ is even. We have

$P(k + 1) : S_{k+1} = 1^2 + 2 \times 2^2 + ... + k^2 + 2 \times (k + 1)^2$

$$= \frac{k^2(k+1)}{2} + 2 \times (k+1)^2$$

$$= \frac{(k+1)}{2}[k^2 + 4(k+1)] \text{ (as } k \text{ is odd, } 1^2 + 2 \times 2^2 + ... + k^2 = k^2 \frac{(k+1)}{2}\text{)}$$

$$= \frac{k+1}{2}[k^2 + 4k + 4]$$

$$= \frac{k+1}{2}(k+2)^2 = (k+1)\frac{[(k+1)+1]^2}{2}$$

So $P(k + 1)$ is true, whenever $P(k)$ is true in the case when $k$ is odd.

**Case 2** When $k$ is even, then $k + 1$ is odd.

Now, $P(k+1) : 1^2 + 2 \times 2^2 + ... + 2.k^2 + (k+1)^2$

$$= \frac{k(k+1)^2}{2} + (k+1)^2 \quad \text{(as } k \text{ is even, } 1^2 + 2 \times 2^2 + ... + 2k^2 = k\ \frac{(k+1)^2}{2}\text{)}$$

$$= \frac{(k+1)^2(k+2)}{2} = \frac{(k+1)^2((k+1)+1)}{2}$$

Therefore, P $(k + 1)$ is true, whenever P $(k)$ is true for the case when $k$ is even. Thus P $(k + 1)$ is true whenever P $(k)$ is true for any natural numbers $k$. Hence, P $(n)$ true for all natural numbers.

## Objective Type Questions

Choose the correct answer in Examples 11 and 12 (M.C.Q.)

**Example 11** Let P$(n)$ : "$2^n < (1 \times 2 \times 3 \times ... \times n)$". Then the smallest positive integer for which P $(n)$ is true is

(A) 1 (B) 2 (C) 3 (D) 4

**Solution** Answer is D, since

P (1) : $2 < 1$ is false

P (2) : $2^2 < 1 \times 2$ is false

P (3) : $2^3 < 1 \times 2 \times 3$ is false

But P (4) : $2^4 < 1 \times 2 \times 3 \times 4$ is true

**Example 12** A student was asked to prove a statement P $(n)$ by induction. He proved that P $(k + 1)$ is true whenever P $(k)$ is true for all $k > 5 \in \mathbf{N}$ and also that P (5) is true. On the basis of this he could conclude that P $(n)$ is true

(A) for all $n \in \mathbf{N}$ (B) for all $n > 5$

(C) for all $n \geq 5$ (D) for all $n < 5$

**Solution** Answer is (C), since P(5) is true and P$(k + 1)$ is true, whenever P $(k)$ is true.

Fill in the blanks in Example 13 and 14.

**Example 13** If P $(n)$ : "$2.4^{2n+1} + 3^{3n+1}$ is divisible by $\lambda$ for all $n \in \mathbf{N}$" is true, then the value of $\lambda$ is _____

**Solution** Now, for $n = 1$,

$$2.4^{2+1} + 3^{3+1} = 2.4^3 + 3^4 = 2.64 + 81 = 128 + 81 = 209,$$

for $n = 2$, $2.4^5 + 3^7 = 8.256 + 2187 = 2048 + 2187 = 4235$

Note that the H.C.F. of 209 and 4235 is 11. So $2.4^{2n+1} + 3^{3n+1}$ is divisible by 11. Hence, $\lambda$ is 11

**Example 14** If P $(n)$ : "$49^n + 16^n + k$ is divisible by 64 for $n \in \mathbf{N}$" is true, then the least negative integral value of $k$ is ______.

**Solution** For $n = 1$, P(1) : $65 + k$ is divisible by 64.

Thus $k$, should be – 1 since, 65 – 1 = 64 is divisible by 64.

**Example 15** State whether the following proof (by mathematical induction) is true or false for the statement.

$$P(n):\ 1^2 + 2^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

**Proof** By the Principle of Mathematical induction, P($n$) is true for $n = 1$,

$1^2 = 1 = \dfrac{1(1+1)(2\cdot1+1)}{6}$. Again for some $k \geq 1$, $k^2 = \dfrac{k(k+1)(2k+1)}{6}$. Now we prove that

$$(k+1)^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

**Solution** False

Since in the inductive step both the inductive hypothesis and what is to be proved are wrong.

## 4.3 EXERCISE

### Short Answer Type

1. Give an example of a statement P($n$) which is true for all $n \geq 4$ but P(1), P(2) and P(3) are not true. Justify your answer.
2. Give an example of a statement P($n$) which is true for all $n$. Justify your answer.

   Prove each of the statements in Exercises 3 - 16 by the Principle of Mathematical Induction :
3. $4^n - 1$ is divisible by 3, for each natural number $n$.
4. $2^{3n} - 1$ is divisible by 7, for all natural numbers $n$.
5. $n^3 - 7n + 3$ is divisible by 3, for all natural numbers $n$.
6. $3^{2n} - 1$ is divisible by 8, for all natural numbers $n$.

7. For any natural number $n$, $7^n - 2^n$ is divisible by 5.

8. For any natural number $n$, $x^n - y^n$ is divisible by $x - y$, where $x$ and $y$ are any integers with $x \neq y$.

9. $n^3 - n$ is divisible by 6, for each natural number $n \geq 2$.

10. $n(n^2 + 5)$ is divisible by 6, for each natural number $n$.

11. $n^2 < 2^n$ for all natural numbers $n \geq 5$.

12. $2n < (n + 2)!$ for all natural number $n$.

13. $\sqrt{n} < \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}}$, for all natural numbers $n \geq 2$.

14. $2 + 4 + 6 + \ldots + 2n = n^2 + n$ for all natural numbers $n$.

15. $1 + 2 + 2^2 + \ldots + 2^n = 2^{n+1} - 1$ for all natural numbers $n$.

16. $1 + 5 + 9 + \ldots + (4n - 3) = n(2n - 1)$ for all natural numbers $n$.

## Long Answer Type

Use the Principle of Mathematical Induction in the following Exercises.

17. A sequence $a_1, a_2, a_3 \ldots$ is defined by letting $a_1 = 3$ and $a_k = 7a_{k-1}$ for all natural numbers $k \geq 2$. Show that $a_n = 3.7^{n-1}$ for all natural numbers.

18. A sequence $b_0, b_1, b_2 \ldots$ is defined by letting $b_0 = 5$ and $b_k = 4 + b_{k-1}$ for all natural numbers $k$. Show that $b_n = 5 + 4n$ for all natural number $n$ using mathematical induction.

19. A sequence $d_1, d_2, d_3 \ldots$ is defined by letting $d_1 = 2$ and $d_k = \frac{d_{k-1}}{k}$ for all natural numbers, $k \geq 2$. Show that $d_n = \frac{2}{n!}$ for all $n \in \mathbf{N}$.

20. Prove that for all $n \in \mathbf{N}$
$\cos\alpha + \cos(\alpha + \beta) + \cos(\alpha + 2\beta) + \ldots + \cos(\alpha + (n-1)\beta)$
$$= \frac{\cos\left(\alpha + \left(\frac{n-1}{2}\right)\beta\right)\sin\left(\frac{n\beta}{2}\right)}{\sin\frac{\beta}{2}}$$

21. Prove that, $\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \frac{\sin 2^n\theta}{2^n \sin\theta}$, for all $n \in \mathbf{N}$.

22. Prove that, $\sin\theta + \sin 2\theta + \sin 3\theta + \ldots + \sin n\theta = \frac{\sin\frac{n\theta}{2}\sin\frac{(n+1)}{2}\theta}{\sin\frac{\theta}{2}}$, for all $n \in \mathbf{N}$.

**23.** Show that $\frac{n^5}{5}+\frac{n^3}{3}+\frac{7n}{15}$ is a natural number for all $n \in$ **N.**

**24.** Prove that $\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{2n}>\frac{13}{24}$, for all natural numbers $n > 1$.

**25.** Prove that number of subsets of a set containing $n$ distinct elements is $2^n$, for all $n \in$ **N**.

## Objective Type Questions

Choose the correct answers in Exercises 26 to 30 (M.C.Q.).

**26.** If $10^n + 3.4^{n+2} + k$ is divisible by 9 for all $n \in$ **N**, then the least positive integral value of $k$ is

(A) 5 (B) 3 (C) 7 (D) 1

**27.** For all $n \in$ **N**, $3.5^{2n+1} + 2^{3n+1}$ is divisible by

(A) 19 (B) 17 (C) 23 (D) 25

**28.** If $x^n - 1$ is divisible by $x - k$, then the least positive integral value of $k$ is

(A) 1 (B) 2 (C) 3 (D) 4

Fill in the blanks in the following :

**29.** If P($n$) : $2n < n!$, $n \in$ **N**, then P($n$) is true for all $n \geq$ ________.

State whether the following statement is true or false. Justify.

**30.** Let P($n$) be a statement and let P($k$) $\Rightarrow$ P($k$ + 1), for some natural number $k$, then P($n$) is true for all $n \in$ **N**.