

NCERT Solutions Class 11 Physics chapter 13 Kinetic Theory  
Kinetic Theory

Exercise P.338

**Q.13.1:** Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be  $3\text{\AA}$ .

**Ans :** Diameter of an oxygen

molecule,  $d = 3\text{\AA} = 3 \times 10^{-10}\text{m} = 3 \times 10^{-8}\text{cm}$  Diameter of an oxygen molecule,  $d = 3\text{\AA} = 3 \times 10^{-10}\text{m} = 3 \times 10^{-8}\text{cm}$

Actual volume occupied by 1 mole of oxygen gas at STP

$= 22400\text{cm}^3$  Actual volume occupied by 1 mole of oxygen gas at STP  $= 22400\text{cm}^3$

Where,  $N$  is Avogadro's

number  $= 6.023 \times 10^{23}$  molecules/mole  $\therefore V = \frac{4}{3} \pi r^3 \cdot N = \frac{4}{3} \pi (1.5 \times 10^{-8})^3 \times 6.023 \times 10^{23} = 8.51\text{cm}^3$  Where,  $N$  is Avogadro's

number  $= 6.023 \times 10^{23}$  molecules/mole  $\therefore V = \frac{4}{3} \pi r^3 \cdot N = \frac{4}{3} \pi (1.5 \times 10^{-8})^3 \times 6.023 \times 10^{23} = 8.51\text{cm}^3$

Ratio of the molecular volume to the actual volume of

oxygen  $= \frac{8.51}{22400} = 3.8 \times 10^{-4}$  Ratio of the molecular volume to the actual volume of oxygen  $= \frac{8.51}{22400} = 3.8 \times 10^{-4}$

**Q.13.2:** Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP : 1 atmospheric pressure,  $0^\circ\text{C}$ ). Show that it is 22.4 litres.

**Ans :** The ideal gas equation relating pressure ( $P$ ), volume ( $V$ ), and absolute temperature ( $T$ ) is given as:  $PV = nRT$  The ideal gas equation relating pressure ( $P$ ), volume ( $V$ ), and absolute temperature ( $T$ ) is given as:  $PV = nRT$

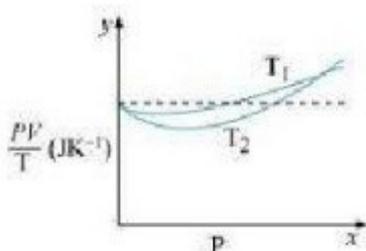
Where,  $R$  is the universal gas constant  $= 8.314\text{Jmol}^{-1}\text{K}^{-1}$   $n =$  Number of moles  $= 1$   $T =$  Standard temperature  $= 273\text{K}$   $P =$  Standard

pressure  $= 1\text{atm} = 1.013 \times 10^5\text{Nm}^{-2}$  Where,  $R$  is the universal gas

constant  $= 8.314\text{Jmol}^{-1}\text{K}^{-1}$   $n =$  Number of moles  $= 1$   $T =$  Standard temperature  $= 273\text{K}$   $P =$  Standard pressure  $= 1\text{atm} = 1.013 \times 10^5\text{Nm}^{-2}$

$\therefore V = \frac{nRT}{P} = \frac{1 \times 8.314 \times 273}{1.013 \times 10^5} = 0.0224\text{m}^3 = 22.4\text{ litres}$   $\therefore V = \frac{nRT}{P} = \frac{1 \times 8.314 \times 273}{1.013 \times 10^5} = 0.0224\text{m}^3 = 22.4\text{ litres}$

**Q.13.3:** Figure 13.8 shows plot of  $PV/T$  versus  $P$  For  $1.00 \times 10^{-3}\text{kg}$  of oxygen gas at two different temperatures. Figure 13.8 shows plot of  $PV/T$  versus  $P$  For  $1.00 \times 10^{-3}\text{kg}$  of oxygen gas at two different temperatures.



(a) does the dotted plot signify?

(b) Which is true.  $T_1 > T_2$  or  $T_1$

**Ans :** (a) The dotted plot in the graph signifies the ideal behaviour of the gas, i.e., the ratio  $PVT$  is equal.  $\mu R$  ( $\mu$  is the number of moles and  $R$  is the

universal gas constant) is a constant quality. It is not dependent on the pressure of the gas. (a) The dotted plot in the graph signifies the ideal behaviour of the gas, i.e., the ratio  $\frac{PV}{T}$  is equal to  $\mu R$  ( $\mu$  is the number of moles and  $R$  is the universal gas constant) is a constant quality. It is not dependent on the pressure of the gas.

(b) The dotted plot in the given graph represents an ideal gas. The curve of the gas at temperature  $T_1$  is closer to the dotted plot than the curve of the gas at temperature  $T_2$ . A real gas approaches the behaviour of an ideal gas when its temperature increases. Therefore,  $T_1 > T_2$  is true for the given plot.

(c) The value of the ratio  $\frac{PV}{T}$ , where the two curves meet, is  $\mu R$ . This is because the ideal gas equation is given as:

$$PV = \mu RT \quad \frac{PV}{T} = \mu R$$

Where,  $P$  is the pressure

$T$  is the temperature

$V$  is the volume

$\mu$  is the number of moles

$R$  is the universal constant

Molecular mass of oxygen = 32.0 g

Mass of

$$\text{oxygen} = 1 \times 10^{-3} \text{ kg} = 1 \text{ g} \quad R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1} \quad \frac{PV}{T} = 132 \times 8.314$$

$$\text{Mass of oxygen} = 1 \times 10^{-3} \text{ kg} = 1 \text{ g} \quad R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1} \quad \frac{PV}{T} = 132 \times 8.314$$

$$= 0.26 \text{ J K}^{-1} \quad \text{Therefore, the value of the ratio } \frac{PV}{T}, \text{ where the curves meet}$$

on the y-axis, is  $0.26 \text{ J K}^{-1}$

Therefore, the value of the ratio  $\frac{PV}{T}$ , where the curves meet on the y-axis, is  $0.26 \text{ J K}^{-1}$

(d) If we obtain similar plots for  $1.00 \times 10^{-3}$ – $31.00 \times 10^{-3}$  kg of hydrogen, then we will not get the same value of  $\frac{PV}{T}$  at the point where the curves meet the V axis. This is because the molecular mass of hydrogen (2.02 u) is different from that of oxygen (32.0 u) We have:

$$\frac{PV}{T} = 0.26 \text{ J K}^{-1} \quad R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1} \quad \text{Molecular}$$

mass (M) of  $\text{H}_2 = 2.02 \text{ u} \quad \frac{PV}{T} = \mu R$  at constant

temperature Where,  $\mu = \frac{m}{M} = \frac{\text{Mass of H}_2}{M}$

$$\text{of H}_2 \quad \frac{PV}{T} = 0.26 \text{ J K}^{-1} \quad R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1} \quad \text{Molecular}$$

mass (M) of  $\text{H}_2 = 2.02 \text{ u} \quad \frac{PV}{T} = \mu R$  at constant

temperature Where,  $\mu = \frac{m}{M} = \frac{\text{Mass of H}_2}{M}$

$$m = \frac{PV}{T} \times M \times R = 0.26 \times 2.02 \times 8.31 = 6.3 \times 10^{-2} \text{ g} = 6.3 \times 10^{-5} \text{ kg} \quad \text{Hence, } 6.3 \times 10^{-5} \text{ kg}$$

of  $\text{H}_2$  will yield the same value of  $\frac{PV}{T}$   
Hence,  $6.3 \times 10^{-5} \text{ kg}$  of  $\text{H}_2$  will yield the same value of  $\frac{PV}{T}$

**Q.13.4:** An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder ( $R = 8.31 \text{ J mole}^{-1} \text{ K}^{-1}$ , molecular mass of  $\text{O}_2 = 32 \text{ u}$ ).

**Ans :** Volume of oxygen,  $V_1 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$  Gauge pressure,  $P_1 = 15 \text{ atm} = 15 \times 1.013 \times 10^5 \text{ Pa}$  Volume of oxygen,  $V_1 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$  Gauge pressure,  $P_1 = 15 \text{ atm} = 15 \times 1.013 \times 10^5 \text{ Pa}$  Temperature,  $T_1 = 27^\circ \text{C} = 300 \text{ K}$  Universal gas constant,  $R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1}$  Let the initial number of moles of oxygen gas in the cylinder be  $n_1$  The gas equation is given as:  $P_1 V_1 = n_1 R T_1$  Temperature,  $T_1 = 27^\circ \text{C} = 300 \text{ K}$  Universal gas constant,  $R = 8.314 \text{ J mole}^{-1} \text{ K}^{-1}$  Let the initial number of moles of oxygen gas in the cylinder be  $n_1$  The gas equation is given as:  $P_1 V_1 = n_1 R T_1$   
 $\therefore n_1 = \frac{P_1 V_1}{R T_1} = \frac{15.195 \times 10^5 \times 30 \times 10^{-3}}{(8.314) \times 300} = 18.276$   
 $\therefore n_1 = \frac{P_1 V_1}{R T_1} = \frac{15.195 \times 10^5 \times 30 \times 10^{-3}}{(8.314) \times 300} = 18.276$   
Where,  $m_1 =$  Initial mass of oxygen  $M =$  Molecular mass of oxygen  $= 32 \text{ g}$   
 $\therefore m_1 = n_1 M = 18.276 \times 32 = 584.84 \text{ g}$  Where,  $m_1 =$  Initial mass of oxygen  $M =$  Molecular mass of oxygen  $= 32 \text{ g}$   
 $\therefore m_1 = n_1 M = 18.276 \times 32 = 584.84 \text{ g}$

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduces

Volume,  $V_2 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$  Gauge pressure,  $P_2 = 11 \text{ atm} = 11 \times 1.013 \times 10^5 \text{ Pa}$  Temperature,  $T_2 = 17^\circ \text{C} = 290 \text{ K}$  Volume,  $V_2 = 30 \text{ litres} = 30 \times 10^{-3} \text{ m}^3$  Gauge pressure,  $P_2 = 11 \text{ atm} = 11 \times 1.013 \times 10^5 \text{ Pa}$  Temperature,  $T_2 = 17^\circ \text{C} = 290 \text{ K}$   
 $P_2 V_2 = n_2 R T_2$   
 $\therefore n_2 = \frac{P_2 V_2}{R T_2} = \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290} = 13.86$   
 $P_2 V_2 = n_2 R T_2$   
 $\therefore n_2 = \frac{P_2 V_2}{R T_2} = \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290} = 13.86$   
 $n_2 = \frac{m_2}{M}$  Where,  $m_2$  is the mass of oxygen remaining in the cylinder  
 $n_2 = \frac{m_2}{M}$  Where,  $m_2$  is the mass of oxygen remaining in the cylinder

$\therefore m_2 = n_2 M = 13.86 \times 32 = 453.1 \text{ g}$  The mass of oxygen taken out of the cylinder is given by the relation: Initial mass of oxygen in the cylinder - Final mass of oxygen in the cylinder  $= m_1 - m_2 = 584.84 \text{ g} - 453.1 \text{ g} = 131.74 \text{ g} = 0.131 \text{ kg}$  Therefore, 0.131 kg of oxygen is taken out of the cylinder.  
 $\therefore m_2 = n_2 M = 13.86 \times 32 = 453.1 \text{ g}$  The mass of oxygen taken out of the cylinder is given by the relation: Initial mass of oxygen in the cylinder - Final mass of oxygen in the cylinder  $= m_1 - m_2 = 584.84 \text{ g} - 453.1 \text{ g} = 131.74 \text{ g} = 0.131 \text{ kg}$  Therefore, 0.131 kg of oxygen is taken out of the cylinder.

**Q.13.5:** An air bubble of volume  $1.0 \text{ cm}^3$  rises from the bottom of a lake 40 m deep at a temperature of  $12^\circ \text{C}$ . To what volume does it grow when it reaches the surface, which is at a temperature of  $35^\circ \text{C}$  ?

**Ans :** Volume of the air bubble,  $V_1 = 1.0 \text{ cm}^3 = 1.0 \times 10^{-6} \text{ m}^3$  Bubble rises to height,  $d = 40 \text{ m}$  Temperature at a depth of 40m,  $T_1 = 12^\circ \text{C} = 285 \text{ K}$  Temperature at the surface of the

lake,  $T_2 = 35^\circ\text{C} = 308\text{K}$  Volume of the air bubble,  $V_1 = 1.0\text{cm}^3 = 1.0 \times 10^{-6}\text{m}^3$  Bubble rises to height,  $d = 40\text{m}$  Temperature at a depth of  $40\text{m}$ ,  $T_1 = 12^\circ\text{C} = 285\text{K}$  Temperature at the surface of the lake,  $T_2 = 35^\circ\text{C} = 308\text{K}$

The pressure on the surface of the lake:  $P_2 = 1\text{ atm} = 1 \times 1.013 \times 10^5\text{Pa}$  The pressure at the depth of  $40\text{m}$  :  $P_1 = 1\text{atm} + \rho g h$  Where,  $\rho$  is the density of water  $= 103\text{kg/m}^3$  :  $P_1$  is the acceleration due to gravity  $= 9.8\text{m/s}^2$  :  $P_1 = 1.013 \times 10^5 + 40 \times 103 \times 9.8 = 493300\text{Pa}$  The pressure on the surface of the lake:  $P_2 = 1\text{ atm} = 1 \times 1.013 \times 10^5\text{Pa}$  The pressure at the depth of  $40\text{m}$  :  $P_1 = 1\text{atm} + \rho g h$  Where,  $\rho$  is the density of water  $= 103\text{kg/m}^3$  :  $P_1$  is the acceleration due to gravity  $= 9.8\text{m/s}^2$  :  $P_1 = 1.013 \times 10^5 + 40 \times 103 \times 9.8 = 493300\text{Pa}$   
 $P_1 V_1 T_1 = P_2 V_2 T_2$   
 $P_1 V_1 T_1 = P_2 V_2 T_2$

Where,  $V_2$  is the volume Of the air bubble when it reaches the surface  
 $V_2 = P_1 V_1 T_2 T_1 P_2 = (493300)$   
 $(1.0 \times 10^{-6}) 308 285 \times 1.013 \times 10^5 = 5.263 \times 10^{-6}\text{m}^3$  or  $5.263\text{cm}^3$   
 $V_2 = P_1 V_1 T_2 T_1 P_2 = (493300) (1.0 \times 10^{-6}) 308 285 \times 1.013 \times 10^5 = 5.263 \times 10^{-6}\text{m}^3$  or  $5.263\text{cm}^3$

Therefore, where, the air bubble reaches the surface, its Volume becomes  $5.263\text{ cm}^3$ .

**Q.13.6:** Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity  $25.0\text{ m}^3$  at a temperature of  $27^\circ\text{C}$  and  $1\text{ atm}$  pressure.

**Ans :** Volume of the room,  $V = 25.0\text{m}^3$  Temperature of the room,  $T = 27^\circ\text{C} = 300\text{K}$  Pressure in the room,  $P = 1\text{atm} = 1 \times 1.013 \times 10^5\text{Pa}$  Volume of the room,  $V = 25.0\text{m}^3$  Temperature of the room,  $T = 27^\circ\text{C} = 300\text{K}$  Pressure in the room,  $P = 1\text{atm} = 1 \times 1.013 \times 10^5\text{Pa}$

The ideal gas equation relating pressure ( $p$ ), Volume ( $V$ ), and absolute temperature ( $T$ ) can be written as:

$PV = k_B N T$  Where,  $k_B$  is Boltzmann constant  $= 1.38 \times 10^{-23}\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$   $PV = k_B N T$  Where,  $k_B$  is Boltzmann constant  $= 1.38 \times 10^{-23}\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$   $N$  is the number of air molecules in the room  $k_B$  is Boltzmann constant  $= 1.38 \times 10^{-23}\text{m}^2\text{kgs}^{-2}\text{K}^{-1}$   $N$  is the number of air molecules in the room

$N = PV/k_B T = 1.013 \times 10^5 \times 25 / 1.38 \times 10^{-23} \times 300 = 6.11 \times 10^{26}$  molecules Therefore, the total number of air molecules in the given room is  $6.11 \times 10^{26}$  .  $N = PV/k_B T = 1.013 \times 10^5 \times 25 / 1.38 \times 10^{-23} \times 300 = 6.11 \times 10^{26}$  molecules Therefore, the total number of air molecules in the given room is  $6.11 \times 10^{26}$  .

**Q.13.7:** Estimate the average thermal energy of a helium atom at  
(i) room temperature ( $27^\circ\text{C}$ ),  
(ii) the temperature on the surface of the Sun ( $6000\text{ K}$ ),  
(iii) the temperature of  $10$  million kelvin (the typical core temperature in the case of a star).

**Ans :** (i) At room temperature,  $T=27^\circ\text{C}=300\text{K}$  Average thermal energy  $=3kT$  Where  $k$  is Boltzmann constant  $=1.38\times 10^{-23}\text{m}^2\text{kg}^{-1}\text{K}^{-1}$  (i) At room temperature,  $T=27^\circ\text{C}=300\text{K}$  Average thermal energy  $=3kT$  Where  $k$  is Boltzmann constant  $=1.38\times 10^{-23}\text{m}^2\text{kg}^{-1}\text{K}^{-1}$   
 $\therefore 3kT=3\times 1.38\times 10^{-23}\times 300=6.21\times 10^{-21}\text{J}$  Hence, the average thermal energy of a helium atom at room temperature ( $27^\circ\text{C}$ ) is  $6.21\times 10^{-21}\text{J}$ .  
 $\therefore 3kT=3\times 1.38\times 10^{-23}\times 300=6.21\times 10^{-21}\text{J}$  Hence, the average thermal energy of a helium atom at room temperature ( $27^\circ\text{C}$ ) is  $6.21\times 10^{-21}\text{J}$   
(ii) On the surface of the sun,  $T=6000\text{K}$  Average thermal energy  $=3kT=3\times 1.38\times 10^{-23}\times 6000=1.241\times 10^{-19}\text{J}$  (ii) On the surface of the sun,  $T=6000\text{K}$  Average thermal energy  $=3kT=3\times 1.38\times 10^{-23}\times 6000=1.241\times 10^{-19}\text{J}$   
(iii) At temperature,  $T=10^7\text{K}$  Average thermal energy  $=3kT$  (iii) At temperature,  $T=10^7\text{K}$  Average thermal energy  $=3kT$   
 $=3\times 1.38\times 10^{-23}\times 10^7=2.07\times 10^{-16}\text{J}$  Hence, the average thermal energy of a helium atom at the core of a star is  $2.07\times 10^{-16}\text{J}$ .  
 $=3\times 1.38\times 10^{-23}\times 10^7=2.07\times 10^{-16}\text{J}$  Hence, the average thermal energy of a helium atom at the core of a star is  $2.07\times 10^{-16}\text{J}$ .

**Q.13.8:** Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is the largest?

**Ans :** Yes. All contain the same number of the respective molecules. The root mean square speed of neon is the largest. Since the three vessels have the same capacity, they have the same volume. Hence, each gas has the same pressure, volume, and temperature. According to Avogadro's law, the three vessels will contain an equal number of the respective molecules. This number is equal to Avogadro's number,  $N=6.023\times 10^{23}$   
 $N=6.023\times 10^{23}$  The root mean square speed ( $v_{\text{rms}}$ ) of a gas of mass  $m$ , and temperature  $T$ , is given by the relation:  
 $v_{\text{rms}}=\sqrt{3kT/m}$   
 $v_{\text{rms}}=\sqrt{3kT/m}$

Where,  $k$  is Boltzmann constant  
For the given gases,  $k$  and  $T$  are constants.  
Hence  $v$  depends only on the mass of the atoms, i.e.,  
 $v_{\text{rms}}\propto\sqrt{1/m}$   
 $v_{\text{rms}}\propto\sqrt{1/m}$

Therefore, the root mean square speed of the molecules in the three cases is not the same. Among neon, chlorine, and uranium hexafluoride, the mass of neon is the smallest. Hence, neon has the largest root mean square speed among the given gases.

**Q.13.9:** At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at  $-20^\circ\text{C}$ ? (atomic mass of Ar = 39.9 u, of He = 4.0 u).

**Ans :** Temperature of the helium atom,  $T_{He} = -20^\circ C = 253K$  Atomic mass of argon,  $M_{Ar} = 39.9u$  Atomic mass of helium,  $M_{He} = 4.0u$  Temperature of the helium atom,  $T_{He} = -20^\circ C = 253K$  Atomic mass of argon,  $M_{Ar} = 39.9u$  Atomic mass of helium,  $M_{He} = 4.0u$

Let,  $(v_{rms})_{Ar}$  be the rms speed of argon. Let  $(V_{rms})_{He}$  be the rms speed of helium. The rms speed of argon is given by: Let,  $(v_{rms})_{Ar}$  be the rms speed of argon. Let  $(V_{rms})_{He}$  be the rms speed of helium. The rms speed of argon is given by:

The rms speed of argon is given by:  $(v_{rms})_{Ar} = \sqrt{3RT_{Ar}/M_{Ar}}$  Where,  $T_{Ar}$  is temperature of argon gas The rms speed of helium is given by:  $(v_{rms})_{He} = \sqrt{3RT_{He}/M_{He}}$  Where,  $T_{He}$  is temperature of helium gas

The rms speed of argon is given by:  $(v_{rms})_{Ar} = \sqrt{3RT_{Ar}/M_{Ar}}$

The rms speed of helium is given by:  $(v_{rms})_{He} = \sqrt{3RT_{He}/M_{He}}$

$\sqrt{3RT_{Ar}/M_{Ar}} = \sqrt{3RT_{He}/M_{He}}$   
 $\sqrt{3 \times 8.314 \times 253 / 39.9} = \sqrt{3 \times 8.314 \times 253 / 4}$

$\sqrt{3 \times 8.314 \times 253 / 39.9} = \sqrt{3 \times 8.314 \times 253 / 4}$   
 $\sqrt{3 \times 8.314 \times 253 / 39.9} = \sqrt{3 \times 8.314 \times 253 / 4}$   
 $= 2534 \times 39.9 = 2523.675 = 2.52 \times 10^3 K$  Therefore, the temperature of the argon atom is  $2.52 \times 10^3 K$ .

**Q.13.10:** Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 °C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of  $N_2 = 28.0 u$ ).

**Ans :** Mean free path  $= 1.11 \times 10^{-7} m$  Collision frequency  $= 4.58 \times 10^9 s^{-1}$  Successive collision time  $\approx 500 \times (\text{Collision time})$  Mean free path  $= 1.11 \times 10^{-7} m$  Collision frequency  $= 4.58 \times 10^9 s^{-1}$  Successive collision time  $\approx 500 \times (\text{Collision time})$

Pressure inside the cylinder containing nitrogen,  $P = 2.0 \text{ atm} = 2.026 \times 10^5 \text{ Pa}$  Temperature inside the cylinder,  $T = 17^\circ C = 290K$  Radius of a nitrogen molecule,  $r = 1.0 \text{ \AA} = 1 \times 10^{-10} m$

Pressure inside the cylinder containing nitrogen,  $P = 2.0 \text{ atm} = 2.026 \times 10^5 \text{ Pa}$  Temperature inside the cylinder,  $T = 17^\circ C = 290K$  Radius of a nitrogen molecule,  $r = 1.0 \text{ \AA} = 1 \times 10^{-10} m$

Diameter,  $d = 2 \times 1 \times 10^{-10} = 2 \times 10^{-10} m$  Molecular mass of nitrogen,  $M = 28.0g = 28 \times 10^{-3} kg$  The root mean square speed of nitrogen is given by the

relation:  $v_{rms} = \sqrt{3RT/M}$  Diameter,  $d = 2 \times 1 \times 10^{-10} = 2 \times 10^{-10} m$  Molecular mass of nitrogen,  $M = 28.0g = 28 \times 10^{-3} kg$  The root mean square speed of nitrogen is given by the relation:  $v_{rms} = \sqrt{3RT/M}$

Where,  $R$  is the universal gas constant  $= 8.314 \text{ J mole}^{-1} K^{-1}$

$v_{rms} = \sqrt{3 \times 8.314 \times 290 / 28 \times 10^{-3}} = 508.26 m/s$  Where,  $R$  is the universal gas constant  $= 8.314 \text{ J mole}^{-1} K^{-1}$

$v_{rms} = \sqrt{3 \times 8.314 \times 290 / 28 \times 10^{-3}} = 508.26 m/s$

$\therefore$  the mean free path ( $l$ ) is given by the

relation:  $l = kT / \sqrt{2} \times d \times P$  Where,  $k$  is the Boltzmann

constant  $= 1.38 \times 10^{-23} \text{ kg m}^2 \text{ s}^{-2} \text{ K}^{-1}$   $\therefore l = 1.38 \times 10^{-23} \times 290 / \sqrt{2} \times 3.14 \times (2 \times 10^{-10}) \times 2.026 \times 10^5 = 1.11 \times 10^{-7} m$

$0-10)2 \times 2.026 \times 10^5 = 1.11 \times 10^{-7} \text{m}$  ∴ the mean free path ( $l$ ) is given by the relation:  $l = \frac{kT}{\sqrt{2} \times P}$  Where,  $k$  is the Boltzmann constant  $= 1.38 \times 10^{-23} \text{kgm}^2\text{s}^{-2}\text{K}^{-1}$  ∴  $l = \frac{1.38 \times 10^{-23} \times 2902 \times 3.14 \times (2 \times 10^{-10})^2 \times 2.026 \times 10^5}{1.11 \times 10^{-7}} = 1.11 \times 10^{-7} \text{m}$

Collision frequency  $= \frac{v_{rms}}{l} = \frac{508.261}{1.11 \times 10^{-7}} = 4.58 \times 10^9 \text{s}^{-1}$  Collision time is given as: Collision

frequency  $= \frac{v_{rms}}{l} = \frac{508.261}{1.11 \times 10^{-7}} = 4.58 \times 10^9 \text{s}^{-1}$  Collision time is given as:

$T = \frac{d}{v_{min}} = \frac{2 \times 10^{-10}}{508.26} = 3.93 \times 10^{-13} \text{s}$   $T = \frac{d}{v_{min}} = \frac{2 \times 10^{-10}}{508.26} = 3.93 \times 10^{-13} \text{s}$

Time taken between successive collisions:  $T$

$\lambda = \frac{l}{v_{rms}} = \frac{1.11 \times 10^{-7} \text{m}}{508.26 \text{m/s}} = 2.18 \times 10^{-10} \text{s}$  ∴  $T = 2.18 \times 10^{-10}$

$T = 2.18 \times 10^{-10}$   $3.93 \times 10^{-13} = 500$  Time taken between successive

collisions:  $T' = \frac{l}{v_{rms}} = \frac{1.11 \times 10^{-7} \text{m}}{508.26 \text{m/s}} = 2.18 \times 10^{-10} \text{s}$  ∴  $T = 2.18 \times 10^{-10}$

$T = 2.18 \times 10^{-10}$   $3.93 \times 10^{-13} = 500$

Hence, the time taken between successive collisions is 500 times the time taken for a collision.

Additional Exercise P.340

**Q.13.11:** A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom ?

**Ans :** Length of the narrow bore,  $L = 1 \text{m} = 100 \text{cm}$  Length of the mercury thread,  $l = 76 \text{cm}$  Length of the air column between mercury and the closed end,  $l_a = 15 \text{cm}$  Length of the narrow bore,  $L = 1 \text{m} = 100 \text{cm}$  Length of the mercury thread,  $l = 76 \text{cm}$  Length of the air column between mercury and the closed end,  $l_a = 15 \text{cm}$

since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space

is:  $100 - (76 + 15) = 9 \text{cm}$  Hence, the total length of the air

column  $= 15 + 9 = 24 \text{cm}$  Let  $h \text{cm}$  of mercury flow out as a result of

atmospheric pressure. since the bore is held vertically in air with the

open end at the bottom, the mercury length that occupies the air space

is:  $100 - (76 + 15) = 9 \text{cm}$  Hence, the total length of the air

column  $= 15 + 9 = 24 \text{cm}$  Let  $h \text{cm}$  of mercury flow out as a result of atmospheric pressure.

∴ Length of the air column in the bore  $= 24 + h \text{cm}$  And, length of the

mercury column  $= 76 - h \text{cm}$  Initial pressure,  $P_1 = 76 \text{cm}$  of mercury Initial

volume,  $V_1 = 15 \text{cm}^3$  ∴ Length of the air column in the bore  $= 24 + h \text{cm}$  And,

length of the mercury column  $= 76 - h \text{cm}$  Initial pressure,  $P_1 = 76 \text{cm}$  of

mercury Initial volume,  $V_1 = 15 \text{cm}^3$

Final pressure,  $P_2 = 76 - (76 - h) = h \text{cm}$  of mercury Final

volume,  $V_2 = (24 + h) \text{cm}^3$  Temperature remains constant throughout the

process. ∴  $P_1 V_1 = P_2 V_2$   $76 \times 15 = h(24 + h)$   $h^2 + 24h - 1140 = 0$  Final

pressure,  $P_2 = 76 - (76 - h) = h \text{cm}$  of mercury Final

volume,  $V_2 = (24 + h) \text{cm}^3$  Temperature remains constant throughout the

process. ∴  $P_1 V_1 = P_2 V_2$   $76 \times 15 = h(24 + h)$   $h^2 + 24h - 1140 = 0$

∴  $h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1 \times 1140}}{2 \times 1} = 23.8 \text{cm}$  or  $-47.8 \text{cm}$  ∴  $h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1 \times 1140}}{2 \times 1} = 23.8 \text{cm}$  or  $-47.8 \text{cm}$

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and

522 cm of mercury will remain in it. The length of the air column Will be  $24 + 23.8 = 47.8$  cm

**Q.13.12:** A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom ?

**Ans :** Rate of diffusion of hydrogen,  $R_1 = 28.7 \text{ cm}^3 \text{ s}^{-1}$  Rate of diffusion of another gas,  $R_2 = 7.2 \text{ cm}^3 \text{ s}^{-1}$  According to Graham's Law of diffusion, we have: Rate of diffusion of hydrogen,  $R_1 = 28.7 \text{ cm}^3 \text{ s}^{-1}$  Rate of diffusion of another gas,  $R_2 = 7.2 \text{ cm}^3 \text{ s}^{-1}$  According to Graham's Law of diffusion, we have:

$R_1 R_2 = \sqrt{M_2 M_1}$  Where,  $M_1$  is the molecular mass of hydrogen  $= 2.020 \text{ g}$   $M_2$  is the molecular mass of the unknown

gas  $R_1 R_2 = \sqrt{M_2 M_1}$  Where,  $M_1$  is the molecular mass of hydrogen  $= 2.020 \text{ g}$   $M_2$  is the molecular mass of the unknown gas

$\therefore M_2 = M_1 (R_1 R_2)^2 = 2.02 (28.77.2)^2 = 32 \text{ g}$  is the molecular mass of oxygen.

Hence, the unknown gas is

oxygen.  $\therefore M_2 = M_1 (R_1 R_2)^2 = 2.02 (28.77.2)^2 = 32 \text{ g}$  is the molecular mass of oxygen. Hence, the unknown gas is oxygen.

**Q.13.13:** A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres

$$n_2 = n_1 \exp[-mg(h_2 - h_1)/kBT] \quad n_2 = n_1 \exp[-mg(h_2 - h_1)/kBT]$$

Where  $n_2, n_1$  refer to number density at heights  $h_2$  and  $h_1$  respectively. use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column:

$$n_2 = n_1 \exp[-mgNA(\rho - \rho')(h_2 - h_1)/(pRT)] \quad n_2 = n_1 \exp[-mgNA(\rho - \rho')(h_2 - h_1)/(pRT)]$$

Where  $p$  is the density of the suspended particle, and  $p'$  that of surrounding medium.  $N_A$  is Avogadro's number, and  $R$  the universal gas constant.] [Hint: use Archimedes principle to find the apparent weight of the suspended particle.]

**Ans :** According to the law of atmospheres, we have:  $n_2 = n_1 \exp[-mg(h_2 - h_1)/kBT]$ ... (i) Where,  $n_1$  is the number density at height  $h_1$ , and  $n_2$  is the number density at height  $h_2$   $mg$  is the weight of the particle suspended in the gas column According to the law of atmospheres, we have:  $n_2 = n_1 \exp[-mg(h_2 - h_1)/kBT]$ ... (i) Where,  $n_1$  is the number density at height  $h_1$ , and  $n_2$  is the number density at height  $h_2$   $mg$  is the weight of the particle suspended in the gas column

Density of the medium  $= \rho'$  Density of the suspended particle  $= \rho$  Mass of one suspended particle  $= m'$  Mass of the medium displaced  $= m$  Volume of a suspended particle  $= v$  Density of the medium  $= \rho'$  Density of the suspended particle  $= \rho$  Mass of one suspended particle  $= m'$  Mass of the medium displaced  $= m$  Volume of a suspended particle  $= v$

According to Archimedes' principle for a particle suspended in a liquid column, effective weight of the suspended particle is given as: Weight of the medium displaced - Weight of the suspended particle According to

Archimedes' principle for a particle suspended in a liquid column, effective weight of the suspended particle is given as: Weight of the medium displaced- Weight of the suspended particle  
 $=mg - m'g = mg - V\rho'g = mg - (m\rho)\rho'g = mg(1 - \rho'\rho)$  Gas constant,  $R = k_B N_A$   
 $n_2 = n_1 \exp[-mg(h_2 - h_1)/k_B T]$   
 $n_2 = n_1 \exp[-mg(\rho - \rho')(h_2 - h_1)/k_B T]$

**Q.13.14:** Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms:

Substance	Atomic Mass (u)	Density ( $10^3 \text{ Kg m}^{-3}$ )
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

[Hint : Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].

Substance	Radius (Å)
Carbon (diamond)	1.29
Gold	1.59
Nitrogen (liquid)	1.77
Lithium	1.73
Fluorine (liquid)	1.88

**Ans :**

Atomic mass of a substance = M Density of the substance =  $\rho$  Avogadro's number =  $N = 6.023 \times 10^{23}$  Volume of each atom =  $\frac{4}{3}\pi r^3$   
 Atomic mass of a substance = M Density of the substance =  $\rho$  Avogadro's number =  $N = 6.023 \times 10^{23}$  Volume of each atom =  $\frac{4}{3}\pi r^3$   
 Volume of N number of molecules =  $\frac{4}{3}\pi r^3 N$  Volume of one mole of a substance =  $\frac{M}{\rho}$ ... (ii) Volume of N number of molecules =  $\frac{4}{3}\pi r^3 N$  Volume of one mole of a substance =  $\frac{M}{\rho}$ ... (ii)  
 $\frac{4}{3}\pi r^3 N = \frac{M}{\rho}$   $\therefore r = \sqrt[3]{\frac{3M}{4\pi\rho N}}$  For carbon:  $M = 12.01 \times 10^{-3} \text{ kg}$   $\rho = 2.22 \times 10^3 \text{ kg m}^{-3}$   
 For carbon:  $M = 12.01 \times 10^{-3} \text{ kg}$   $\rho = 2.22 \times 10^3 \text{ kg m}^{-3}$   
 $\therefore r = \left(\frac{3 \times 12.01 \times 10^{-3}}{4\pi \times 2.22 \times 10^3 \times 6.023 \times 10^{23}}\right)^{1/3} = 1.29 \text{ \AA}$  Hence, the radius of a carbon atom

is 1.29 Å.  $\therefore r = (3 \times 12.01 \times 10^{-34} \pi \times 2.22 \times 10^3 \times 6.023 \times 10^{23})^{1/3} = 1.29 \text{ Å}$  Hence, the radius of a carbon atom is 1.29 Å.

For Gold

$$M = 197.00 \times 10^{-3} \text{ kg} \rho = 19.32 \times 10^3 \text{ kg m}^{-3} \therefore r = (3 \times 197.00 \times 10^{-3} / (19.32 \times 10^3 \times 6.023 \times 10^{23}))^{1/3} = 1.29 \text{ Å}$$

$$\therefore r = (3 \times 12.01 \times 10^{-34} \pi \times 2.22 \times 10^3 \times 6.023 \times 10^{23})^{1/3} = 1.29 \text{ Å}$$

For liquid nitrogen :

$$M = 14.01 \times 10^{-3} \text{ kg} \rho = 1.00 \times 10^3 \text{ kg m}^{-3} \therefore r = (3 \times 14.01 \times 10^{-34} \pi \times 1.00 \times 10^3 \times 6.23 \times 10^{25})^{1/3} = 1.77 \text{ Å}$$

For lithium :

$$M = 6.94 \times 10^{-3} \text{ kg} \rho = 0.53 \times 10^3 \text{ kg m}^{-3} \therefore r = (3 \times 6.94 \times 10^{-34} \pi \times 0.53 \times 10^3 \times 6.23 \times 10^{23})^{1/3} = 1.73 \text{ Å}$$

For liquid Fluorine :

$$M = 19.00 \times 10^{-3} \text{ kg} \rho = 1.14 \times 10^3 \text{ kg m}^{-3} \therefore r = (3 \times 19.00 \times 10^{-34} \pi \times 1.14 \times 10^3 \times 6.023 \times 10^{23})^{1/3} = 1.88 \text{ Å}$$

