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NCERT Solutions Class 11 Physics chapter 14 Oscillations

Oscillations

ExerciseP.362

Q.14.1: Which of the following examples represent periodic motion?
(a) A swimmer completing one (return) trip from one bank of a river to the other and back.
(b) A freely suspended bar magnet displaced from ts N-S direction and released.
(c) A hydrogen molecule rotating about its center of mass.
(d) An arrow released from a bow.

Ans : (b) and (c)

(a) The swimmer's motion is not periodic. The motion of the swimmer between the banks of a river is back and forth. However, it does not have a definite period. his is
because the time taken by the swimmer during his back and forth journey may not be the same.

(b) The motion Of a freely-suspended magnet, if displaced from its N-S direction and released, is periodic. This is because the magnet oscillates about its position with a
definite period of time.

(c) When a hydrogen molecule rotates about its centre of mass, it comes to the same position again and again after an equal interval of time. Such motion is periodic.

(d) An arrow released from a bow moves only in the forward direction. It does not come backward. Hence, this motion is not a periodic.

Q.14.2: Which of the following examples represent (nearly) simple harmonic motion and which represent periodic but not simple harmonic motion?
(a) the rotation of earth about its axis.
(b) motion of an oscillating mercury column in a U-tube.
(c) motion of a ball bearing inside a smooth curved bowl, when released from a point slightly above the lowermost point.
(d) general vibrations of a polyatomic molecule about its equilibrium position.

Ans : (b) and (c) are SHMs
(a) and (d) are periodic, but not SHMs

(a) During its rotation about its axis, earth comes to the same position again and again in equal intervals of time. Hence, it is a periodic motion. However, this motion is not simple harmonic. This is because earth does not have a to and fro motion about its axis.

(b) An oscillating mercury column in a CJ-tube is simple harmonic. This is because the mercury moves to and fro on the same path, about the fixed position, with a certain
period of time.

(c) The ball moves to and fro about the lowermost point of the bowl when released. Also, the ball comes back to ts initial position in the same period of time, again and again. Hence, its motion is periodic as well as simple harmonic.

(d) A polyatomic molecule has many natural frequencies of oscillation. Its vibration is the superposition of individual simple harmonic motions of a number of different

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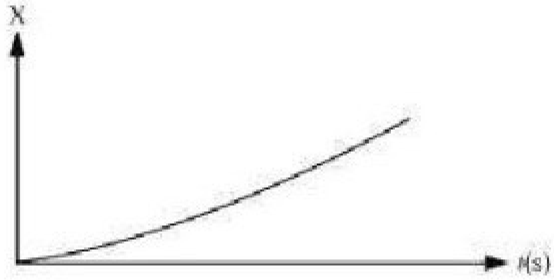
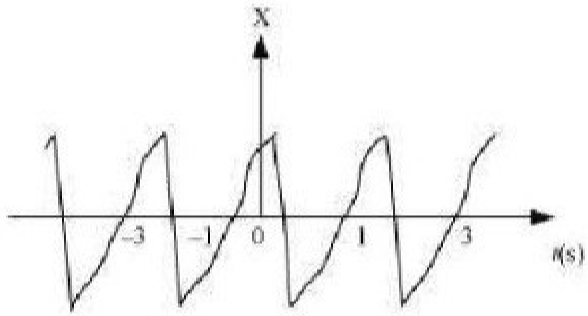
molecules. Hence, it is not simple harmonic, but periodic.

Q.14.3: depicts four x t plots for linear motion of a particle. Which of the plots represent periodic motion? What is the period of motion (in case of periodic motion)?

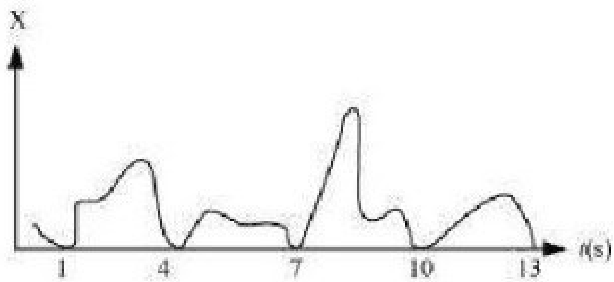
(a)



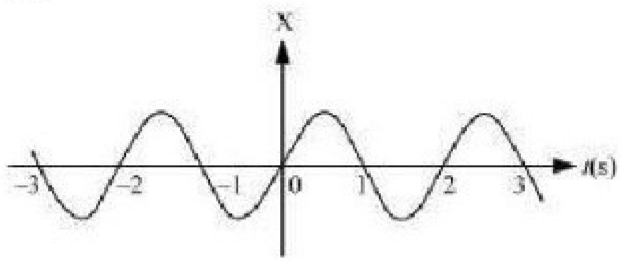
(b)



(c)



(d)



Ans : (b) and (d) are periodic

(a) It is not a periodic motion. his represents a unidirectional, linear uniform motion.

There is no repetition of motion in this case

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(b) In this case, the motion of the particle repeats itself after 2 s. Hence, It is a periodic motion, having a period of 2 s.
(c) It is not a periodic motion. This is because the particle repeats the motion in one position only. For a periodic motion, the entire motion of the particle must be repeated in equal intervals of time.
(d) In this case, the motion of the particle repeats itself after 2 s. Hence, it is a periodic motion, having a period of 2 s .

Q.14.4: Which of the following functions of time represent
(a) simple harmonic, (b) periodic but not simple harmonic, and (c) non-periodic motion? Give period for each case of periodic
 motion (ω is any positive
constant): (a) sinωt−cosωt (b) sin3ωt (c) 3cos(π/4−2ωt) (d) cosωt+cos3ωt+cos5ωt (e) exp(−ω2t2) motion (ω is any positive
constant): (a) sinωt−cosωt (b) sin3ωt (c) 3cos(π/4−2ωt) (d) cosωt+cos3ωt+cos5ωt (e) exp(−ω2t2)

Ans : (a) SHM The given function is: sinωt−cosωt=√2[1√2sinωt−1√2cosωt]=√2sin(ωt−π4)=√2sin(ωt−π4) (a) SHM The given function is: sinωt−cosωt=2[12sinωt−12cosωt]=2sin(ωt−π4)=2sin(ωt−π4)
This function represents SHM as it can be written in the form: asin(ωt+ϕ)asin(ωt+ϕ)
Its period is : 2πω2πω
 (b) Periodic, but not SHM The given function is: sin3ωt=12[3sinωt−sin3ωt] (b) Periodic, but not SHM The given function is: sin3ωt=12[3sinωt−sin3ωt]
The terms sinωt and sinωtsinωt and sinωt individually represent simple harmonic motion (SHM). However, the superposition of two SHM IS periodic and not simple harmonic.
 (c) SHM The given function is: 3cos[π4−2ωt]=3cos[2ωt−π4] This function represents simple harmonic motion because it can be written in the form: acos(ωt+ϕ) (c) SHM The given function is: 3cos[π4−2ωt]=3cos[2ωt−π4] This function represents simple harmonic motion because it can be written in the form: acos(ωt+ϕ)
Its period is 2π2ω=πω2π2ω=πω
 (d) Periodic, but not SHM The given function is cosωt+cos3ωt+cos5ωt . Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic. (d) Periodic, but not SHM The given function is cosωt+cos3ωt+cos5ωt . Each individual cosine function represents SHM. However, the superposition of three simple harmonic motions is periodic, but not simple harmonic.
 (e) Non-periodic motion The given function exp(−ω2t2) is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion. (f) The given function 1 +ωt+ω2t2 is non-periodic. (e) Non-periodic motion The given function exp(−ω2t2) is an exponential function. Exponential functions do not repeat themselves. Therefore, it is a non-periodic motion. (f) The given function 1 +ωt+ω2t2 is non-periodic.

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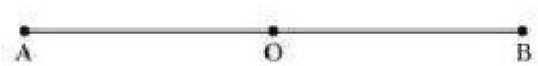
Q.14.5: A particle is in linear simple harmonic motion between two points, A and B, 10 cm apart. Take the direction from A to B as the positive direct on and give the signs of velocity, acceleration and force on the particle when it is
(a) at the end A,
(b) at the end B,
(c) at the mid-point of A3 going towards A,
(d) at 2 cm away from 3 going towards A,
(e) at 3 cm away from A going towards B, and
(f) at 4 cm away from B going towards A.
Ans : (a) Zero, Positive, Positive
(b) Zero, Negative, Negative
(c) Negative, Zero, Zero
(d) Negative, Negative, Negative
(e) Zero, Positive, Positive
(f) Negative, Negative, Negative
The given situation is shown in the following figure. Points A and B are the two end points, with AB = 10 cm. O is the midpoint of the path.



A particle is in linear simple harmonic motion between the endpoints

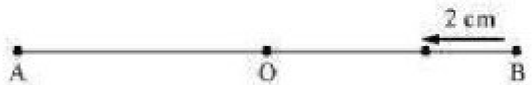
(a) At the extreme point A, the particle is at rest momentarily. Hence, its velocity is zero at this point. Its acceleration is positive as it is directed along AO.
Force is also positive in this case as the particle is directed rightward.

(b) At the extreme point B, the particle is at rest momentarily. Hence, its velocity is zero at this point. Its acceleration is negative as it is directed along 3. Force is also negative in this case as the particle is directed leftward.

(c)

The particle is executing a simple harmonic motion. O is the mean position of the particle. Its velocity at the mean position O is the maximum. The value for velocity is negative as the particle is directed leftward. The acceleration and force of a particle executing SHM is zero at the mean position.

(d)



The particle is moving toward point O from the end B. This direction of motion is opposite to the conventional positive direction, which is from A

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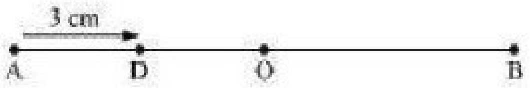
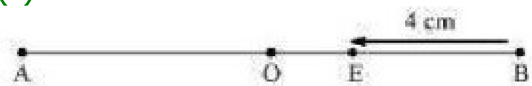
to B, Hence, the particle's velocity and acceleration, and the force on it are all negative.

(e)



The particle is moving toward point O from the end A. This direction of motion is from A to B, which is the conventional positive direction. Hence, the values for velocity, acceleration, and force are all positive.

(f)



This case is similar to the one given in (d).

Q.14.6: Which of the following relationships between the acceleration a and the displacement x of a particle involve simple harmonic motion?
(a) a=0.7x (b) a=−200x2 (c) a=−10x (d) a=100x3 (a) a=0.7x (b) a=−200x2 (c) a=−10x (d) a=100x3

Ans : (c) A motion represents simple harmonic motion if it is governed by the force law
F=−kxma=−k∴a=−kmx Where, F is the force m is the displacement a is the displacement a is the acceleration k is a constant F=−kxma=−k∴a=−kmx Where, F is the force m is the displacement a is the displacement a is the acceleration k is a constant
Among the given equations, only equation a =—10 x is written in the above form with
Hence, this relation represent SHM.

Q.14.7: The motion of a particle executing simple harmonic motion is described by the displacement function. x(t) = A cos (wt+Φ)(wt+Φ).If the initial (t = 0) position of the particle is 1 cm and its initial velocity is w cm/s, what are its amplitude and initial phase angle? The angular frequency of the particle is Tt sl. If instead of the cosine function, we choose the sine function to describe the SHM: Bsin(wt+α)Bsin(wt+α), what are the amplitude and initial phase of the particle with the above initial conditions?

Ans : Initially, at t=0 . Displacement, x=1cm Initial velocity, v=ωcm/sec . Angular frequency, ω=πrad/s−1 It is given that: Initially, at t=0 . Displacement, x=1cm Initial velocity, v=ωcm/sec . Angular frequency, ω=πrad/s−1 It is given that:
x(t)=Acos(ωt+ϕ)1=Acos(ω×0+ϕ)=AcosϕAcosϕ=1x(t)=Acos(ωt+ϕ)1=Acos(ω×0+ϕ)=AcosϕAcosϕ=1
Velocity, v=dxdtω=−Aωsin(ωt+ϕ)1=−Asin(ω×0+ϕ)=−AsinϕAsinϕ=−1 Velocity, v=dxdtω=−Aωsin(ωt+ϕ)1=−Asin(ω×0+ϕ)=−AsinϕAsinϕ=−1
A2(sin2ϕ+cos2ϕ)=1+1A2=2∴A=√2cmA2(sin2ϕ+cos2ϕ)=1+1A2=2∴A=2cm
tanϕ=−1∴ϕ=3π4,7π4,…… SHM is given as: x=Bsin(ωt+α) Putting the given values in this equation, we get: tanϕ=−1∴ϕ=3π4,7π4,…… SHM is given as: x=Bsin(ωt+α) Putting the given values in this equation, we get:

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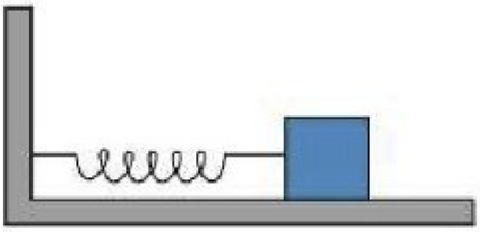
I=Bsin[ω×0+α]Bsinα=1 Velocity, v=ωBcos(ωt+α) Substituting the given values, we get: π=πBsinαBsinα=1I=Bsin[ω×0+ɑ]Bsinɑ=1 Velocity, v=ωBcos(ωt+ɑ) Substituting the given values, we get: п=пBsinɑBsinɑ=1
B2[sin2α+cos2α]=1+1B2=2∴B=√2cmB2[sin2ɑ+cos2ɑ]=1+1B2=2∴B=2cm
BsinαBcosα=11tanα=1=tanπ4∴α=π4,5π4,......
BsinɑBcosɑ=11tanɑ=1=tanп4∴ɑ=п4,5п4,......

Q.14.8: A spring balance has a scale that reads from 0 to 50 kg. The length of the scale is 20 cm. A body suspended from this balance, when displaced and released, oscillates with a period of 0.6 s. What is the weight of the body?

Ans : Maximum mass that the scale can read, M = 50 kg
Maximum displacement of the spring = Length of the scale, l = 20 cm = 0.2 m
Time period, T = 0.6 s
Maximum force exerted on the spring, F = Mg
where,
g= acceleration due to gravity =9.8m/s2F=50×9.8=490 ¿Spring constant, k=Fl=4900.2=2450Nm−1g= acceleration due to gravity =9.8m/s2F=50×9.8=490 ¿Spring constant, k=Fl=4900.2=2450Nm−1
Mass m, is suspended from the balance. T=2π√mk Time period, ∴m=(T2π)2×k=(0.62×3.14)2×2450=22.36kg Mass m, is suspended from the balance. T=2пmk Time period, ∴m=(T2п)2×k=(0.62×3.14)2×2450=22.36kg
∴ Weight of the body =mg=22.36×9.8=219.167N Hence, the weight of the body is about 219N . ∴ Weight of the body =mg=22.36×9.8=219.167N Hence, the weight of the body is about 219N .

Q.14.9: A spring having with a spring constant 1200 Nm−lNm−l is mounted on a horizontal table as shown in Figure A mass of 3 kg is attached to the free end of the spring. The mass is then pulled sideways to a distance of 2.0 cm and released. Determine
(i) the frequency of oscillations,
(ii) maximum acceleration of the mass, and
(iii) the maximum speed of the mass.



Ans : Spring constant, k=1200Nm−1 Mass, m=3kg Displacement, A=2.0cm=0.02cm Spring constant, k=1200Nm−1 Mass, m=3kg Displacement, A=2.0cm=0.02cm

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(i) Frequency of oscillation v, is given by the relation: v=1T=12π√km Where, T is the time period (i) Frequency of oscillation v, is given by the relation: v=1T=12πkm Where, T is the time period
∴v=12×3.14√12003=3.18m/s Hence, the frequency of oscillations is 3.18m/s . (ii) Maximum acceleration (a) is given by the relation: a=ω2A Where, ∴v=12×3.1412003=3.18m/s Hence, the frequency of oscillations is 3.18m/s . (ii) Maximum acceleration (a) is given by the relation: a=ω2A Where,
ω= Angular frequency =√kmA= Maximum displacement ∴a=kmA=1200×0.023=8ms−2 Hence, the maximum acceleration of the mass is 8.0m/s2 . ω= Angular frequency =kmA= Maximum displacement ∴a=kmA=1200×0.023=8ms−2 Hence, the maximum acceleration of the mass is 8.0m/s2 .
(iii) Maximum velocity, vmax=Aω=A√km=0.02×√12003=0.4m/s Hence, the maximum velocity of the mass is 0.4m/s . (iii) Maximum velocity, vmax=Aω=Akm=0.02×12003=0.4m/s Hence, the maximum velocity of the mass is 0.4m/s .

Q.14.10: In Exercise 14.9, let us take the position of mass when the spring is unstreched as x =0, and the direction from eft to right as the positive direction of x axis. Give x as a function of time t for the oscillating mass if at the moment we start the stopwatch (t = 0), the mass is
(a) at the mean position,
(b) at the maximum stretched position, and
(c) at the maximum compressed position.
In what way do these functions for SHM differ from each other, amplitude or the initial phase? in frequency, in

Ans : (a) x=2sin20t (b) x=2cos20t (c) x=−2cos20t The functions have the same frequency and amplitude, but different initial phases. (a) x=2sin20t (b) x=2cos20t (c) x=−2cos20t The functions have the same frequency and amplitude, but different initial phases.
The functions have the same frequency and amplitude, but different initial phases.
Distance traveled by the mass sideways, A = 2.0 cm
Force constant of the spring, k=1200Nm−1k=1200Nm−1
Angular frequency of oscillation:
ω=√km=√12003=√400=20rads−1ω=km=12003=400=20rads−1

(a) When the mass is at the mean position, initial phase is 0.
Displacement, x=Asinωt=2sin20t Displacement, x=Asinωt=2sin20t
=2sin20t=2sin20t

(b) At the maximum stretched position, the mass is toward the extreme right. Hence,
the initial phase is π2π2
=2sin(20t+π2)=2cos20t=2sin(20t+π2)=2cos20t

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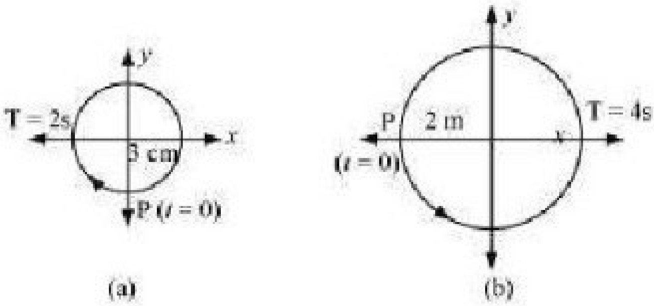
(c) At the maximum compressed position, the mass is toward the extreme left. Hence,
In initial phase 3π23π2
=2sin(20t+3π2)=−2cos20t=2sin(20t+3π2)=−2cos20t

Q.14.11: Following figures correspond to two circular motions. The radius of the circle, the period of revolution, the initial position, and the sense of revolution (i.e., clockwise or anticlockwise) are indicated on each figure.



Obtain the correspond ng simple harmonic motions of the x-projection of the radius vector Of the revolving particle p, in each case.

Ans : (a) Time period, T = 2 s
Amplitude, A = 3 cm
At time, t=0, the radius vector OP makes an angle π2 with the positive x -axis, i.e., phase angle ϕ=+π2 At time, t=0, the radius vector OP makes an angle π2 with the positive x -axis, i.e., phase angle ϕ=+π2
Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given by the displacement equation:
x=Acos[2πtT+ϕ]=3cos(2πt2+π2)=−3sin(2πt2)∴x=−3sinπ/cmx=Acos[2πtT+ϕ]=3cos(2πt2+π2)=−3sin(2πt2)∴x=−3sinπ/cm
(b) Time period, T=4s Amplitude, a=2m At time t=0, OP makes an angle n with the x -axis, in the anticlockwise direction. Hence, phase angle, O=+n (b) Time period, T=4s Amplitude, a=2m At time t=0, OP makes an angle n with the x -axis, in the anticlockwise direction. Hence, phase angle, O=+n
Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given as: x=acos(2πtT+ϕ)=2cos(2πt4+π)∴x=−2cos(π2t)m Therefore, the equation of simple harmonic motion for the x-projection of OP, at time t, is given as: x=acos(2πtT+ϕ)=2cos(2πt4+π)∴x=−2cos(π2t)m

Q.14.12: Plot the corresponding reference circle for each of the following simple harmonic motions. Indicate the initial (t o) position of the particle, the radius of the circle, and the angular speed of the rotating particle. For simplicity, the sense of rotation may be fixed to be anticlockwise in every case: (x is in cm and t is in s).
(a) x =-2 sin (3t + n/3)
(b) x = cos (n/6 - t)
(c) x = 3 sin (2nt + n/4)
(d) x = 2 cos nt

Ans : (a)x=−2sin(3t+π3)=+2cos(3t+π3+π2)=2cos(3t+5π6)
(a)x=−2sin(3t+π3)=+2cos(3t+π3+π2)=2cos(3t+5π6)
If this equation is compared with the standard SHM equation
x=Acos(2πTt+ϕ),x=Acos(2πTt+ϕ),, then we get :

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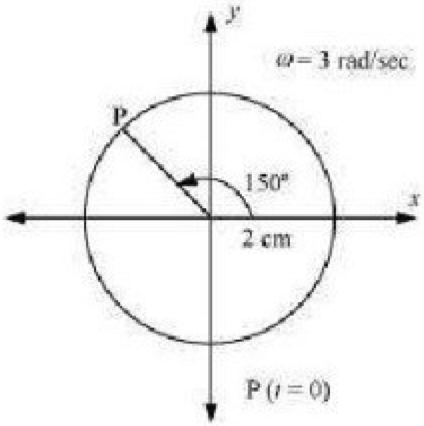
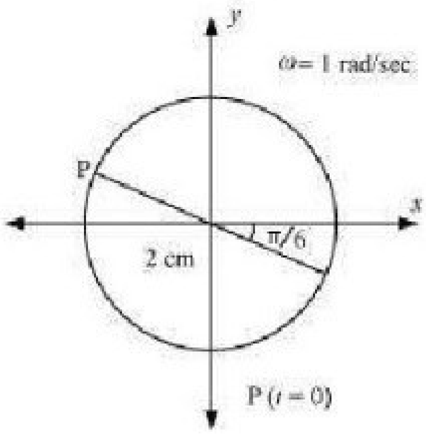
Amplitude, A=2cm Phase angle, ϕ=5π6=150∘ Angular velocity, ω=2πT=3 rad/sec. Amplitude, A=2cm Phase angle, ϕ=5π6=150∘ Angular velocity, ω=2πT=3 rad/sec.
The motion of the particle can be plotted as shown in the following figure.



(b) x=cos(π6−t)=cos(t−π6)x=cos(π6−t)=cos(t−π6)
If this equation is compared with the standard SHM equation x=Acos(2πTt+ϕ)x=Acos(2πTt+ϕ), then we get :
Amplitude, A=1 Phase angle, ϕ=−π6=−30∘ Angular velocity, ω=2πT=1rad/s Amplitude, A=1 Phase angle, ϕ=−π6=−30∘ Angular velocity, ω=2πT=1rad/s
The motion of the particle can be plotted as shown in the following figure.



(c) x=3sin(2πt+π4)x=3sin(2πt+π4)
=−3cos[(2πt+π4)+π2]=−3cos(2πt+3π4)=−3cos[(2πt+π4)+π2]=−3cos(2πt+3π4)
If this equation is compared with the standard SHM equation
We get:
Amplitude, A = 3 crn
ω=2πT=2πrad/sω=2πT=2πrad/s
The motion of the particle can be plotted as shown in the following figure.

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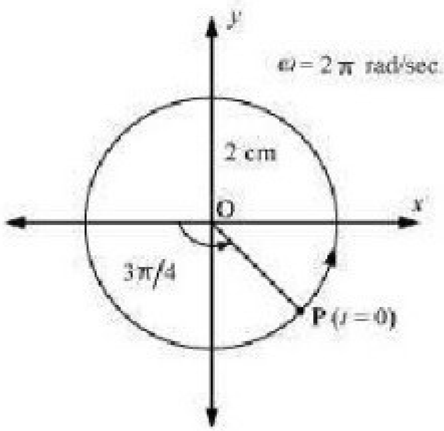
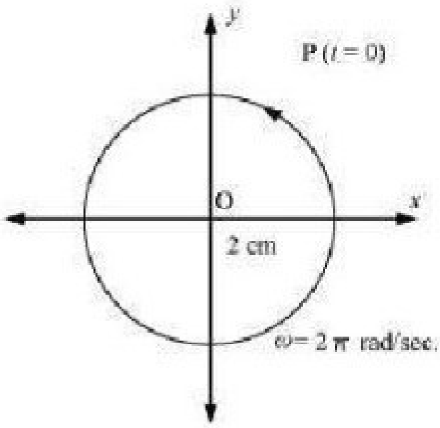
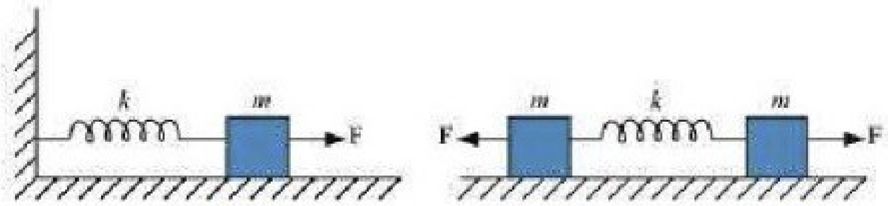


(d) x = 2 cos πt
If this equation is compared with the standard SHM equation
x=Acos(2πTt+ϕ)x=Acos(2πTt+ϕ), then we get
 Amplitude, A=2cm Phase angle, Φ=0 Angular
velocity, ω=πrad/s Amplitude, A=2cm Phase angle, Φ=0 Angular
velocity, ω=πrad/s
The motion of the particle can be plotted as shown in the following figure.



Q.14.13: Figure (a) shows a spring of force constant k clamped rigidly at one end and a mass m attached to its free end. A force F applied at the free end stretches the spring. Figure (b) shows the same spring with both ends free and attached to a mass m at either end. Each end of the spring in Figure (b) is stretched by the same force F.



(a) What is the maximum extension of the spring in the two cases?
(b) If the n Figure (a) and the two masses in Fig. (b) are released, what is the period of oscillation in each case?
Ans : Mien a force F, is applied to the free end of the spring, an extension l, is produced. For the maximum extension, it can be written as:
F = kl

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Where , k is the spring constant
Hence, the maximum extension produced in the spring, l=Fkl=Fk
For the two svstem
The displacement (x) produced in this case is:
x=l2 Net force, F=+2kx∴l=Fkx=l2 Net force, F=+2kx∴l=Fk
(b) For the one block system:
For mass (m) of the block, force is written as :
F=ma=md2xdt2F=ma=md2xdt2
Where, x is the displacement of the block in time t
∴md2xdt2=−kx∴md2xdt2=−kx
It is negative because the direction of elastic force is opposite to the direction of
displacement.
d2xdt2=−(km)x=−ω2x Where, ω2=kmω=√kmd2xdt2=−(km)x=−ω2x W here, ω2=kmω=km
Where, ω2=kmω=√km Where, ω is angular frequency of the oscillation ∴ TIme period of the
oscillation, T=2πω Where, ω2=kmω=km Where, ω is angular frequency of the oscillation ∴ TIme period of the oscillation, T=2πω
for the two system :
F=md2xdt2md2xdt2=−2kxF=md2xdt2md2xdt2=−2kx
It is negative because the direction of elastic force is opposite to the direction of
displacement.
d2xdt2=−[2km]x=−ω2xd2xdt2=−[2km]x=−ω2x
where
ω=√2kmT=2πω=2π√m2kω=2kmT=2πω=2πm2k

Q.14.14: The piston in the cylinder head of a locomotive has a stroke (twice the amplitude) of 1.0 m. If the piston moves with simple harmonic motion with an angular frequency of 200 rad/min, what is its maximum speed?

Ans : Angular frequency of the piston, ω=200rad/min . Stroke =1.0m Angular frequency of the piston, ω=200rad/min . Stroke =1.0m
A Amplitude, A=1.02=0.5mA Amplitude, A=1.02=0.5m
vmax=Aω=200×0.5=100m/minvmax=Aω=200×0.5=100m/min

Q.14.15: The acceleration due to gravity on the surface of moon is 1.7 ms−2ms−2. What is the time period of a simple pendulum on the surface of moon if its time period on the surface of earth is 3.5 s? (g on the surface of earth is 9.8 ms−2ms−2)

Ans : Acceleration due to gravity on the surface of moon, g ′=1.7ms−2 Acceleration due to gravity on the surface of earth, g=9.8ms−2 Time period of a simple pendulum on earth, T=3.5s Acceleration due to gravity on the surface of moon, g ′=1.7ms−2 Acceleration due to gravity on the surface of earth, g=9.8ms−2 Time period of a simple pendulum on earth, T=3.5s
T=2π√lg Where, l is the length of the pendulum ∴l=T2(2π)2×g∴l=(3.5)24×(3.14)2×9.8mT=2πlg Where, l is the length of the pendulum ∴l=T2(2π)2×g∴l=(3.5)24×(3.14)2×9.8m

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The length of the pendulum remains constant.
On moon's surface, time period, T′=2π√lg′T′=2πlg′
=3.5√9.81.7=8.4s=3.59.81.7=8.4s

Q.14.16: Answer the following questions:
(a) Time period of a particle in SHM depends on the force constant k and mass m of the particle : T=2π√mkT=2πmkA simple pendulum executes SHM approximately. Why then is the time period of a pendulum independent of the mass of the pendulum?
(b) The motion Of a simple pendulum s approximately simple harmonic for small angle oscillations. For larger angles of oscillation, a more involved analysis shows that T is greater than 2π√lglg2π Think of a qualitative argument to apprec ate this result.
(c) A man with a wristwatch on his hand falls from the top Of a tower. Does the watch give correct time during the free fall?
(d) What is the frequency of oscillation of a simple pendulum mounted in a cabin that is freely falling under gravity?

Ans : (a) The time period of a simple pendulum, T=2π√mkT=2πmk
For a simple pendulum, k is expressed in terms of mass m, as:
kxmmk= Constant kxmmk= Constant .
Hence, the time period T, of a simple pendulum is independent of the mass of the bob.

(b) In the case of a simple pendulum, the restoring force acting on the bob of the pendulum is given as:
F=−mgsinθ Where, F= Restoring force m= Mass of the bob g= Acceleration due to gravity θ= Angle of displacement F=−mgsinθ Where, F= Restoring force m= Mass of the bob g= Acceleration due to gravity θ= Angle of displacement
For small θ,sinθ=θ For large θ,sinθ is greater than θ This decreases the effective value of g . Hence, the time period increases as: T=2π√lg′ Where, l is the length of the simple pendulum For small θ,sinθ=θ For large θ,sinθ is greater than θ This decreases the effective value of g . Hence, the time period increases as: T=2πlg′ Where, I is the length of the simple pendulum

(c) The time shown by the wristwatch of a man falling from the top of a tower is not affected by the fall. Since a wristwatch does not work on the principle of a simple
pendulum, it is not affected by the acceleration due to gravity during free fall. Its working depends on spring action.

(d) When a simple pendu um mounted in a cabin falls freely under gravity, its acceleration is zero. Hence the frequency of oscillation of this simple pendulum is zero.

Q.14.17: A simple pendulum of length and having a bob of mass M is suspended in a car. The car is moving on a circular track of radius R with a uniform speed v. If the pendulum makes small oscillations in a radial direction about its equilibrium position, what will be its time period?

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Ans : The bob Of the simple pendulum will experience the acceleration due to gravity and the centripetal acceleration provided by the circular motion of the car.
Acceleration due to gravity = g
Centripetal acceleration =v2R=v2R
Where,
v is the uniform speed of the car R is the radius of the track Effective acceleration (aeff) is given as: v is the uniform speed of the car R is the radius of the track Effective acceleration (aeff) is given as:
aeff=√g2+(v2R)2T=2π√laeffaeff=g2+(v2R)2T=2πlaeff
 Where, / is the length of the pendulum =2π√lg2+v4R2 Where, / is the length of the pendulum =2πlg2+v4R2

Q.14.18: Cylindrical piece of cork of density of base area A and height h floats in a liquid of density . The cork is depressed slightly and then released. Show that the cork oscillates up and down simple harmonically with a period
T=2π√hρρ,gT=2πhρρ,gwhere p is the density of cork. (Ignore damping due to viscosity of the liquid).

Ans : Base area of the cork =A Height of the cork =h Density of the liquid = ρ1 Density of the cork =ρ Base area of the cork =A Height of the cork =h Density of the liquid = ρ1 Density of the cork =ρ
In equilibrium :
Weight of the cork = Weight of the liquid displaced by the floating cork
Let the cork be depressed slightly by x. As a result, some extra water of a certain volume is displaced. Hence, an extra upthrust acts upward and provides the restoring
force to the cork.
F=−(Volume × Density ×g) Volume = Area × Distance through which the cork is depressed Volume =Ax∴F=−A×Plg...(i) According to the force law: F=kxF=−(Volume × Density ×g) Volume = Area × Distance through which the cork is depressed Volume =Ax∴F=−A×Plg...(i) According to the force law: F=kx
k=Fx Where, k is a constant k=Fx=−Aρ1g The time period of the oscillations of the cork: k=Fx Where, k is a constant k=Fx=−Aρ1g The time period of the oscillations of the cork:
T=2π√mk Where, m= Mass of the cork = volume of the cork × Density T=2πmk Where, m= Mass of the cork = volume of the cork × Density
= Ahp Hence, the expression for the time period becomes: T=2π√AhρAρig=2π√hρρjg= Ahp Hence, the expression for the time period becomes: T=2πAhρAρig=2πhρρjg

Q.14.19: One end of a U-tube containing mercury is connected to a suction pump and the other end to atmosphere. A small pressure difference is maintained between the two columns. Show that, when the suction pump is removed, the column of mercury in the IJ tube executes simple harmonic motion.

Ans : Area of cross-section of the U-tube =A Density of the mercury column =ρ Acceleration due to gravity =g Restoring force, F= Weight of the mercury column of a certain height F=−(Volume × Density ×g)F=−(A×2h×ρ×g)=−2Aρgh=−k× Displacement in one of the arms (h) Area of

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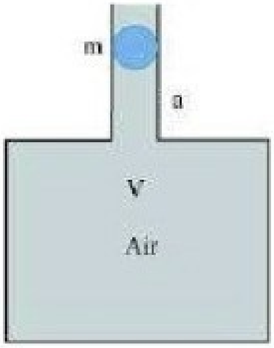
cross-section of the U-tube =A Density of the mercury column =ρ Acceleration due to gravity =g Restoring force, F= Weight of the mercury column of a certain height F=−(Volume × Density ×g)F=−(A×2h×ρ×g)=−2Aρgh=−k× Displacement in one of the arms (h)
2h is the height of the mercury column in the two arms k is a constant, given by k=−Fh=2Aρg 2h is the height of the mercury column in the two arms k is a constant, given by k=−Fh=2Aρg
where, m is the mass of the mercury column
Let 1 be the length of the total mercury in the U-tube.
Mass Of mercury, m = Volume Of mercury x Density of mercury
T=2π√Alρ2Aρg=2π√l2gT=2пAlρ2Aρg=2пl2g
Hence, the mercury column executes simple harmonic motion with time period 2π√12g2п12g

Additional ExerciseP.366

Q.14.20: An air chamber of volume V has a neck area of cross section a into which a ball of mass m just fits and can move up and down without any friction (Fig. 14.33). Show that when the ball is pressed down a little and released, it executes SHM. Obtain an expression for the time period of oscillations assuming pressure-volume variations of air to be isothermal [see Figure].



Ans : Volume of the air chamber =V Area of cross-section of the neck =a Mass of the ball =m Volume of the air chamber =V Area of cross-section of the neck =a Mass of the ball =m
The pressure inside the chamber is equal to the atmospheric pressure. Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber. Decrease in the volume of the air chamber, ΔV=ax The pressure inside the chamber is equal to the atmospheric pressure. Let the ball be depressed by x units. As a result of this depression, there would be a decrease in the volume and an increase in the pressure inside the chamber. Decrease in the volume of the air chamber, ΔV=ax
= Change in volume Original volume ⇒ΔVV=axV= Change in volume Original volume ⇒ΔVV=axV
In this case, stress is the increase in pressure. The negative sign indicates that pressure increases with a decrease in volume.
p=−BaxV The restoring force acting on the ball, F=p×a=−BaxV·a=−Ba2xVp=−BaxV The restoring force acting on the ball, F=p×a=−BaxV·a=−Ba2xV

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In simple harmonic motion, the equation for restoring force is: F=−kx…(ii) Where, k is the spring constant Comparing equations (i) and (iii), we get: In simple harmonic motion, the equation for restoring force is: F=−kx…(ii) Where, k is the spring constant Comparing equations (i) and (iii), we get:

In simple harmonic motion, the equation for restoring force is: F=−kx…(ii) Where, k is the spring constant Comparing equations (i) and (ii), we get: k=Ba2V Time period, T=2π√VmBa2 In simple harmonic motion, the equation for restoring force is: F=−kx…(ii) Where, k is the spring constant Comparing equations (i) and (ii), we get: k=Ba2V Time period, T=2πVmBa2

Q.14.21: You are riding in an automobile of mass 3000 kg. Assuming that you are examining the oscillation characteristics of its suspension system. The suspension sags 15 cm when the entire automobile is placed on it. Also, the amplitude Of oscillation decreases by 50% during one complete oscillation. Estimate the values of (a) the spring constant k and (b) the damping constant b for the spring and shock absorber system Of one wheel, assuming that each wheel supports 750 kg.

Ans : (a) Mass of the automobile, m=3000kg Displacement in the suspension system, x=15cm=0.15m There are 4 springs in parallel to the support of the mass of the automobile. The equation for the restoring force for the system: F=−4kx=mg (a) Mass of the automobile, m=3000kg Displacement in the suspension system, x=15cm=0.15m There are 4 springs in parallel to the support of the mass of the automobile. The equation for the restoring force for the system: F=−4kx=mg

Where, k is the spring constant of the suspension system T=2π√m4k Where, k is the spring constant of the suspension system T=2πm4k

k=mg4x=3000×104×0.15=5000=5×104N/m Spring constant, k=5×104N/mk=mg4x=3000×104×0.15=5000=5×104N/m Spring constant, k=5×104N/m

(b) Each wheel support a mass M=30004=750kgM=30004=750kg

For damping factor b, the equation for displacement is written as:

x=x0e−log/2M The amplitude of oscillation decreases by 50% . x=x0e−log/2M The amplitude of oscillation decreases by 50% .

∴x=x02∴x=x02

loge2=bt2M∴b=2Mloge2tloge2=bt2M∴b=2Mloge2t

Time period, t=2π√m4k=2π√30004×5×104=0.7691s Time period, t=2πm4k=2π30004×5×104=0.7691s

∴b=2×750×0.6930.7691=1351.58kg/s Therefore, the damping constant of the spring is 1351.58kg/s . ∴b=2×750×0.6930.7691=1351.58kg/s Therefore, the damping constant of the spring is 1351.58kg/s .

Q.14.22: Show that for a particle in linear SHM the average kinetic energy over a period of oscillation equals the average potential energy over the same period.

Ans : x=Asinωt Where, A= Amplitude of oscillation x=Asinωt Where, A= Amplitude of oscillation

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ω= Angular frequency =√kMω= Angular frequency =kM
The velocity of the particle is: v=dxdt=Aωcosωt The kinetic energy of the particle is: Ek=12MN2=12MA2ω2cos2ωt The velocity of the particle is: v=dxdt=Aωcosωt The kinetic energy of the particle is: Ek=12MN2=12MA2ω2cos2ωt
Ek=12Mv2=12MA2ω2cos2at The potential energy of the particle is: Ep=12kx2=12Mω2A2sin2ωtEk=12Mv2=12MA2ω2cos2at The potential energy of the particle is: Ep=12kx2=12Mω2A2sin2ωt
For time period T, the average kinetic energy over a single cycle is given as: (Ek) hug =1T∫T0Ekdt=1T∫T012MA2ω2cos2ωtdt For time period T, the average kinetic energy over a single cycle is given as: (Ek) hug =1T∫0TEkdt=1T∫0T12MA2ω2cos2ωtdt =12TMA2ω2∫T0(1+cos2ωt)2dt=14TMA2ω2[t+sin2ωt2ω]T0=14TMA2ω2(T)=14MA2ω2=12TMA2ω2∫0T(1+cos2ωt)2dt=14TMA2ω2[t+sin2ωt2ω]0T=14TMA2ω2(T)=14MA2ω2

$$\left(E_p\right)_{Avg} = \frac{1}{T}\int_0^T E_p\, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2} M\omega^2 A^2 \sin^2 \omega t\, dt$$

$$= \frac{1}{2T} M\omega^2 A^2 \int_0^T \frac{(1-\cos 2\omega t)}{2}\, dt$$

$$= \frac{1}{4T} M\omega^2 A^2 \left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{1}{4T} M\omega^2 A^2 (T)$$

$$= \frac{M\omega^2 A^2}{4} \qquad \ldots (ii)$$

It can be inferred from equations (i) and (ii) that the average kinetic energy for a given time period is due to the avera e potential energy for the same time period.

Q.14.23: A circular disc of mass 10 kg is suspended by a wire attached to its centre. The wire is twisted by rotating the disc and released. The period of torsional oscillations is found to be 1.5 s. The radius of the disc is 15 cm. Determine the torsional spring constant of the wire. (Torsional spring constant a is defined by the relation J = -a θθ, where J is the restoring couple and e the angle of twist).

Ans : Mass of the circular disc, m = 10 kg
Radius of the disc, r 15 cm 0.15 m

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The torsional oscillations of the disc has a time period, T = 1.5 s
The moment of inertia of the disc is:
I=12mr2=12×(10)×(0.15)2=0.1125kgm2I=12mr2=12×(10)×(0.15)2=0.1125kgm2
T=2π√Iα Time period, a is the torsional constant. T=2πIα Time period, a is the torsional constant.
α=4π2IT2=4×(π)2×0.1125(1.5)2=1.972Nm/radα=4π2IT2=4×(π)2×0.1125(1.5)2=1.972Nm/rad
Hence, the torsional spring constant of the wire is
1.972Nmrad−11.972Nmrad−1

Q.14.24: A body describes simple harmonic motion with amplitude of 5 cm and a period of 0.2 s. Find the acceleration and velocity of the body when the displacement is
(a) 5 cm, (b) 3 cm (c) 0 cm

Ans : Amplitude, A=5cm=0.05m Time period, T=0.2s (a) For disolacement x=5cm=0.05m Acceleration is given by: a=−ω2x=−(2πT)2x=−(2πT)2x=−(2πT)2×=−(2πT)2×0.05 Amplitude, A=5cm=0.05m Time period, T=0.2s (a) For disolacement x=5cm=0.05m Acceleration is given by: a=−ω2x=−(2πT)2x=−(2πT)2x=−(2πT)2×=−(2πT)2×0.05
=−5π2m/s2 Velocity is given by: v=ω√A2−x2=2πT√(0.05)2−(0.05)2=0=−5π2m/s2 Velocity is given by: v=ωA2−x2=2πT(0.05)2−(0.05)2=0
When the displacement of the body is 5cm, its acceleration is −5π2m/s2 and velocity is 0. When the displacement of the body is 5cm, its acceleration is −5π2m/s2 and velocity is 0.
(b) For displacement, x=3cm=0.03m Acceleration is given by: a=−ω2x (b) For displacement, x=3cm=0.03m Acceleration is given by: a=−ω2x
=−(2πT)2x=−(2π0.2)20.03=−3π2m/s2 Velocity is given by: =−(2πT)2x=−(2π0.2)20.03=−3π2m/s2 Velocity is given by: v=ω√A2−x2=2πT√A2−x2=2πT√(0.05)2−(0.03)2v=ωA2−x2=2πTA2−x2=2πT(0.05)2−(0.03)2
=2πT√(0.05)2−(0.03)2=2π0.2×0.04=0.4πm/s=2πT(0.05)2−(0.03)2=2π0.2×0.04=0.4πm/s
(c) For displacement, x=0 Acceleration is given by: a=−ω2x=0 Velocity is given by: v=ω√A2−x2 (c) For displacement, x=0 Acceleration is given by: a=−ω2x=0 Velocity is given by: v=ωA2−x2
=2π0.2√(0.05)2−0=0.5πm/s When the displacement of the body is 0, its acceleration is 0 and velocity is 0.5πm/s . =2π0.2(0.05)2−0=0.5πm/s When the displacement of the body is 0, its acceleration is 0 and velocity is 0.5πm/s .

Q.14.25: A mass attached to a spring is free to oscillate, with angular velocity in a horizontal plane without friction or damping. It is pulled to a distance xo and pushed towards the centre with a velocity vo at time t = O Determine the amplitude of the resulting oscillations in terms Of the parameters ω,x0 and v0ω,x0 and v0. [Hint: Start with the equation x = a cos(ωt+θ)cos(ωt+θ) and note that the initial velocity is negative.]

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Ans : x=Acos(ωt+θ) Where, A is the amplitude x is the displacement θ is the phase
constant velocity, v=dxdt=−Aωsin(at+θ)x=Acos(ωt+θ) Where, A is the amplitude x is the displacement θ is the phase
constant velocity, v=dxdt=−Aωsin(at+θ)
v=dxdt=−Aωsin(at+θ) At t=0,x=x0x0=Acosθ=x0...
(i)v=dxdt=−Aωsin(at+θ) At t=0,x=x0x0=Acosθ=x0...(i)
Squaring and adding equations (i) and (ii), we
get: A2(cos2θ+sin2θ)=x20+(v20ω2)∴A=√x20+(v0ω)2 Squaring and adding equations (i) and (ii), we get: A2(cos2θ+sin2θ)=x02+(v02ω2)∴A=x02+(v0ω)2
∴A=√x20+(v0ct)2∴A=x02+(v0ct)2
Hence, the amplitude of the resulting oscillation is ∴A=√x20+(v0ct)2∴A=x02+(v0ct)2

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