

NCERT Solutions Class 11 Physics chapter 15 Waves

Waves

Exercise P.391

**Q.15.1:** A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

**Ans :** Mass Of the string,  $M = 2.50 \text{ kg}$

Tension in the string,  $T = 200 \text{ N}$

Length of the string,  $l = 20.0 \text{ m}$

Mass per unit length

$$\mu = \frac{M}{l} = \frac{2.50}{20} = 0.125 \text{ kg m}^{-1} \quad \mu = \frac{M}{l} = \frac{2.50}{20} = 0.125 \text{ kg m}^{-1}$$

The velocity ( $v$ ) of the transverse wave in the string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/s} \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200}{0.125}} = \sqrt{1600} = 40 \text{ m/s}$$

$\therefore$  Time taken by the disturbance to reach the other

end,  $t = \frac{l}{v} = \frac{20}{40} = 0.50 \text{ s}$ : Time taken by the disturbance to reach the other end,  $t = \frac{l}{v} = \frac{20}{40} = 0.50 \text{ s}$

**Q.15.2:** A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top given that the speed of sound in air is  $340 \text{ ms}^{-1}$ ? ( $g = 9.8 \text{ ms}^{-2}$ )

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**Ans :** Height of the tower,  $s = 300 \text{ m}$  Initial velocity of the stone,  $u = 0$  Acceleration,  $a = g = 9.8 \text{ m/s}^2$  Speed of sound in

air  $= 340 \text{ m/s}$  Height of the tower,  $s = 300 \text{ m}$  Initial velocity of the

stone,  $u = 0$  Acceleration,  $a = g = 9.8 \text{ m/s}^2$  Speed of sound in air  $= 340 \text{ m/s}$

The time ( $t_1$ ) taken by the stone to strike the water in the pond can be calculated using the second equation of motion, as:  $s = ut_1 + \frac{1}{2}gt_1^2$  The

time ( $t_1$ ) taken by the stone to strike the water in the pond can be calculated using the second equation of motion, as:  $s = ut_1 + \frac{1}{2}gt_1^2$

$300 = 0 + \frac{1}{2} \times 9.8 \times t_1^2 \therefore t_1 = \sqrt{\frac{300 \times 2}{9.8}} = 7.82 \text{ s}$  Time taken by the sound to reach the top of the tower,  $t = t_1 + t_2$  Therefore, the time after which the splash is

heard,  $t = t_1 + t_2 = 7.82 + 0.88 = 8.7 \text{ s}$   $300 = 0 + \frac{1}{2} \times 9.8 \times t_1^2 \therefore t_1 = \sqrt{\frac{300 \times 2}{9.8}} = 7.82 \text{ s}$  Time taken by the sound to reach the top of the

tower,  $t = t_1 + t_2$  Therefore, the time after which the splash is

heard,  $t = t_1 + t_2 = 7.82 + 0.88 = 8.7 \text{ s}$

**Q.15.3:** A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at  $20^\circ \text{C}$   $= 343 \text{ ms}^{-1}$   $20^\circ \text{C} = 343 \text{ ms}^{-1}$

**Ans :** Length of the steel wire,  $l = 12 \text{ m}$  Mass of the steel

wire,  $m = 2.10 \text{ kg}$  Velocity of the transverse wave,  $v = 343 \text{ m/s}$  Length of the

steel wire,  $l = 12 \text{ m}$  Mass of the steel wire,  $m = 2.10 \text{ kg}$  Velocity of the transverse wave,  $v = 343 \text{ m/s}$

Mass per unit length

$$\mu = \frac{m}{l} = \frac{2.10}{12} = 0.175 \text{ kg m}^{-1} \quad \mu = \frac{m}{l} = \frac{2.10}{12} = 0.175 \text{ kg m}^{-1}$$

For tension  $T$ , velocity of the transverse wave can be obtained using the relation:

$$v = \sqrt{T\mu} : T = v^2\mu = (343)^2 \times 0.175 = 20588.575 \approx 2.06 \times 10^4 \text{ N}$$

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**Q.15.4:** Use the formula  $v = \sqrt{\gamma P/\rho}$  to explain why the speed of sound in air  
 (a) is independent of pressure,  
 (b) increases With temperature,  
 (c) increases with humidity.

**Ans :**  $v = \sqrt{\gamma P/\rho}$  here, Density,  $\rho = \text{Mass Volume} = M/V$  = Molecular weight of the gas  $V = \text{Volume of the gas}$   
 $v = \sqrt{\gamma P/\rho}$  here, Density,  $\rho = \text{Mass Volume} = M/V$  = Molecular weight of the gas  $V = \text{Volume of the gas}$

Hence, equation ( i ) reduces to:  $v = \sqrt{\gamma PVM}$  Hence, equation ( i ) reduces to:  $v = \sqrt{\gamma PVM}$

Now from the ideal gas equation for  $n=1$ :  $PV = RT$  For constant  $T$ ,  $PV = \text{Constant}$  since both  $M$  and  $\gamma$  are constants,  $v = \text{Constant}$  Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas. Now from the ideal gas equation for  $n=1$ :  $PV = RT$  For constant  $T$ ,  $PV = \text{Constant}$  since both  $M$  and  $\gamma$  are constants,  $v = \text{Constant}$  Hence, at a constant temperature, the speed of sound in a gaseous medium is independent of the change in the pressure of the gas.

(b) Take the relation:  $v = \sqrt{\gamma P/\rho}$  For one mole of an ideal gas, the gas equation can be written as:  $PV = RT$  (b) Take the relation:  $v = \sqrt{\gamma P/\rho}$  For one mole of an ideal gas, the gas equation can be written as:  $PV = RT$   
 $P = RT/V$ ...(ii) Substituting equation (ii) in equation (i), we get:  $v = \sqrt{\gamma RT/V\rho} = \sqrt{\gamma RTM/P} = \sqrt{\gamma RTM/P}$ ...(ii) Substituting equation (ii) in equation (i), we get:  $v = \sqrt{\gamma RTM/P} = \sqrt{\gamma RTM/P}$   
 where,

Mass,  $M = \rho V$  is a constant  $\gamma$  and  $R$  are also constants We conclude from equation (iv) that  $v \propto \sqrt{T}$  . Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa. Mass,  $M = \rho V$  is a constant  $\gamma$  and  $R$  are also constants We conclude from equation (iv) that  $v \propto T$  . Hence, the speed of sound in a gas is directly proportional to the square root of the temperature of the gaseous medium, i.e., the speed of the sound increases with an increase in the temperature of the gaseous medium and vice versa.

(c) Let  $v_m$  and  $v_d$  be the speeds of sound in moist air and dry air respectively. Let  $\rho_m$  and  $\rho_d$  be the densities of moist air and dry air respectively. Take the relation:  $v = \sqrt{\gamma P/\rho}$  Hence, the speed of sound in moist air is: (c) Let  $v_m$  and  $v_d$  be the speeds of sound in moist air and dry air respectively. Let  $\rho_m$  and  $\rho_d$  be the densities of moist air and dry air respectively. Take the relation:  $v = \sqrt{\gamma P/\rho}$  Hence, the speed of sound in moist air is:

$v_m = \sqrt{\gamma P/\rho_m}$  And the speed of sound in dry air is:  $v_d = \sqrt{\gamma P/\rho_d}$  On dividing equations ( i ) and (ii), we get:  $v_m = \sqrt{\gamma P/\rho_m}$  And the speed of sound in dry air is:  $v_d = \sqrt{\gamma P/\rho_d}$  On dividing equations ( i ) and (ii), we get:

$v_{md} = \sqrt{\gamma P_{pm} \times \rho_{dy} P} = \sqrt{\rho_{dpm}}$  However, the presence of water vapour reduces the density of air, i.e.,  $v_{md} = \sqrt{\gamma P_{pm} \times \rho_{dy} P} = \rho_{dpm}$  However, the presence of water vapour reduces the density of air, i.e.,  $\rho_{dvd}$  Hence, the speed of sound in moist air is greater than it is in dry air. Thus, in a gaseous medium, the speed of sound increases with humidity.  $\rho_{dvd}$  Hence, the speed of sound in moist air is greater than it is in dry air. Thus, in a gaseous medium, the speed of sound increases with humidity.

**Q.15.5:** You have learnt that a travelling wave in one dimension is represented by a function  $y = f(x, t)$  where  $x$  and  $t$  must appear in the combination  $x - vt$  or  $x + vt$ , i.e.  $y = f(x \pm vt)$ . Is the converse true? Examine if the following functions for  $V$  can possibly represent a travelling wave:

(a)  $(x - vt)^2 \log[x + ytx0]1(x + vt)$  (a)  $(x - vt)^2 \log[x + ytx0]1(x + vt)$

**Ans :** No;

(a) Does not represent a wave

(b) Represents a wave

(c) Does not represent a wave

The converse of the given statement is not true. The essential requirement for a function to represent a travelling wave is that it should remain finite for all values of  $x$  and  $t$ .

Explanation:

(a) For  $x=0$  and  $t=0$ , the function  $(x - vt)^2$  becomes 0. Hence, for  $x=0$  and  $t=0$ , the function represents a point and not a wave. (a)

For  $x=0$  and  $t=0$ , the function  $(x - vt)^2$  becomes 0. Hence, for  $x=0$  and  $t=0$ , the function represents a point and not a wave.

(b) For  $x=0$  and  $t=0$ , the function  $\log(x + ytx0) = \log 0 = \infty$  (b)

For  $x=0$  and  $t=0$ , the function  $\log(x + ytx0) = \log 0 = \infty$

Since the function does not converge to a finite value for  $x = 0$  and  $t = 0$ , it represents a travelling wave.

(c) For  $x=0$  and  $t=0$ , the function  $1x + vt = 10 = \infty$  since the function does not converge to a finite value for  $x=0$  and  $t=0$ , it does not represent a travelling wave. (c) For  $x=0$  and  $t=0$ , the function  $1x + vt = 10 = \infty$  since the function does not converge to a finite value for  $x=0$  and  $t=0$ , it does not represent a travelling wave.

**Q.15.6:** A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of (a) the reflected sound, (b) the transmitted sound? Speed of sound in air is  $340 \text{ ms}^{-1}$  and in water  $1486 \text{ ms}^{-1}$

**Ans :** (a) Frequency of the ultrasonic sound,  $\nu = 1000 \text{ kHz} = 10^6 \text{ Hz}$  Speed of sound in air,  $\nu_a = 340 \text{ m/s}$  (a) Frequency of the ultrasonic sound,  $\nu = 1000 \text{ kHz} = 10^6 \text{ Hz}$  Speed of sound in air,  $\nu_a = 340 \text{ m/s}$

The wavelength ( $\lambda_r$ ) of the reflected sound is given by the relation:  $\lambda_r = \nu \nu_a = 340 / 10^6 = 3.4 \times 10^{-4} \text{ m}$  The wavelength ( $\lambda_r$ ) of the reflected sound is given by the relation:  $\lambda_r = \nu \nu_a = 340 / 10^6 = 3.4 \times 10^{-4} \text{ m}$

(b) Frequency of the ultrasonic sound,  $\nu = 1000 \text{ kHz} = 10^6 \text{ Hz}$  Speed of sound in water,  $\nu_w = 1486 \text{ m/s}$  The wavelength of the transmitted sound is given as:  $\lambda_1 = 1486 / 10^6 = 1.49 \times 10^{-3} \text{ m}$  (b) Frequency of the ultrasonic sound,  $\nu = 1000 \text{ kHz} = 10^6 \text{ Hz}$  Speed of sound in water,  $\nu_w = 1486 \text{ m/s}$  The

wavelength of the transmitted sound is given

$$\text{as: } \lambda = 1486106 = 1.49 \times 10^{-3} \text{m}$$

**Q.15.7:** A hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is  $1.7 \text{ km s}^{-1}$ ? The operating frequency of the scanner is  $4.2 \text{ MHz}$ .

**Ans :** Speed of sound in the tissue,  $v = 1.7 \text{ km/s} = 1.7 \times 10^3 \text{ m/s}$  Operating frequency of the scanner,  $\nu = 4.2 \text{ MHz} = 4.2 \times 10^6 \text{ Hz}$  The wavelength of sound in the tissue is given

$$\text{as: } \lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4} \text{m}$$

Speed of sound in the tissue,  $v = 1.7 \text{ km/s} = 1.7 \times 10^3 \text{ m/s}$  Operating frequency of the scanner,  $\nu = 4.2 \text{ MHz} = 4.2 \times 10^6 \text{ Hz}$  The wavelength of sound in the tissue is given as:  $\lambda = \frac{v}{\nu} = \frac{1.7 \times 10^3}{4.2 \times 10^6} = 4.1 \times 10^{-4} \text{m}$

**Q.15.8:** A transverse harmonic wave on a string is described by  $y(x,t) = 3.0 \sin(36t + 0.018x + \pi/4)$

Where  $x$  and  $V$  are in  $\text{cm}$  and  $t$  in  $\text{s}$ . The positive direction Of  $x$  is from left to right.

(a) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(b) What are its amplitude and frequency?

(c) What is the initial phase at the origin?

(d) What is the least distance between two successive crests in the wave?

**Ans :** (a) Yes; Speed =  $20 \text{ m/s}$  , Direction = Right to

left (b)  $3 \text{ cm}; 5.73 \text{ Hz}$  (c)  $\pi/4$  (d)  $3.49 \text{ m}$  (a) Yes; Speed =  $20 \text{ m/s}$  , Direction = Right to left (b)  $3 \text{ cm}; 5.73 \text{ Hz}$  (c)  $\pi/4$  (d)  $3.49 \text{ m}$

Explanation: (a) The equation of a progressive wave travelling from right to left is given by the displacement function:  $y(x,t) = a \sin(\omega t + kx + \Phi) \dots$

(i) The given equation is:  $y(x,t) = 3.0 \sin(36t + 0.018x + \pi/4)$  Explanation: (a) The equation of a progressive wave travelling from right to left is given by the displacement function:  $y(x,t) = a \sin(\omega t + kx + \Phi) \dots$  (i) The given equation is:  $y(x,t) = 3.0 \sin(36t + 0.018x + \pi/4)$

On comparing both the equations, we find that equation (ii) represents a travelling wave, propagating from right to left. Now, using equations (i) and (ii), we can write: On comparing both the equations, we find that equation (ii) represents a travelling wave, propagating from right to left. Now, using equations (i) and (ii), we can write:

$$\omega = 36 \text{ rad/s and } k = 0.018 \text{ m}^{-1} \text{ We know}$$

that:  $v = \frac{\omega}{2\pi}$  and  $\lambda = \frac{2\pi}{k}$  Also,  $v = \frac{v}{\lambda} \omega = 36 \text{ rad/s and } k = 0.018 \text{ m}^{-1} \text{ We know}$  that:  $v = \frac{\omega}{2\pi}$  and  $\lambda = \frac{2\pi}{k}$  Also,  $v = v \lambda$

$\therefore v = (\omega/2\pi) \times (2\pi/k) = \omega/k = 36/0.018 = 2000 \text{ cm/s} = 20 \text{ m/s}$  Hence, the speed of the given travelling wave

is  $20 \text{ m/s}$ .  $v = (\omega/2\pi) \times (2\pi/k) = \omega/k = 36/0.018 = 2000 \text{ cm/s} = 20 \text{ m/s}$  Hence, the speed of the given travelling wave is  $20 \text{ m/s}$

(b) Amplitude of the given wave,  $a = 3 \text{ cm}$  Frequency of the given

wave:  $\nu = \frac{\omega}{2\pi} = \frac{36}{2 \times 3.14} = 5.73 \text{ Hz}$  (b) Amplitude of the given

wave,  $a = 3 \text{ cm}$  Frequency of the given wave:  $\nu = \frac{\omega}{2\pi} = \frac{36}{2 \times 3.14} = 5.73 \text{ Hz}$

(c) On comparing equations (i) and (ii), we find that the initial phase

angle,  $\phi = \pi/4$  (d) The distance between two successive crests or troughs is

equal to the wavelength of the wave. Wavelength is given by the relation:  $k=2\pi/\lambda \therefore \lambda=2\pi/k=2 \times 3.14/0.018=348.89\text{cm}=3.49\text{m}$  (c) On comparing equations (i) and (ii), we find that the initial phase angle,  $\phi=\pi/4$  (d) The distance between two successive crests or troughs is equal to the wavelength of the wave. Wavelength is given by the relation:  $k=2\pi/\lambda \therefore \lambda=2\pi/k=2 \times 3.14/0.018=348.89\text{cm}=3.49\text{m}$

**Q.15.9:** For the wave described in Exercise 15.8, plot the displacement (V) versus (t) graphs for  $x = 0, 2$  and  $4$  cm. What are the shapes of these graphs? In which aspects does the oscillatory motion in travelling wave differ from one point to another: amplitude, frequency or phase?

**Ans :** All the waves have different phases. The given transverse harmonic wave is:  $y(x,t)=3.0\sin(36t+0.018x+\pi/4)$  All the waves have different phases. The given transverse harmonic wave is:  $y(x,t)=3.0\sin(36t+0.018x+\pi/4)$

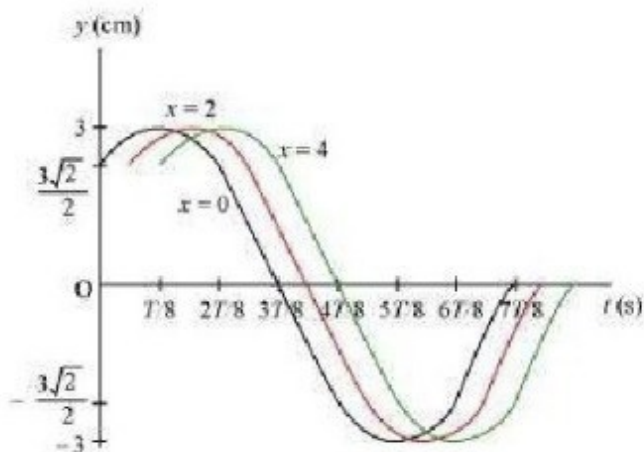
For  $x = 0$ , the equation reduces to:

$y(0,t)=3.0\sin(36t+\pi/4)$  Also,  $\omega=2\pi/T=36\text{rad/s} \therefore T=\pi/8\text{s}$   $y(0,t)=3.0\sin(36t+\pi/4)$  Also,  $\omega=2\pi/T=36\text{rad/s} \therefore T=\pi/8\text{s}$

Now, plotting  $V$  vs.  $t$  graphs using the different values of  $t$ , as listed in the given table.

$t$ (s)	0	$\frac{T}{8}$	$\frac{2T}{8}$	$\frac{3T}{8}$	$\frac{4T}{8}$	$\frac{5T}{8}$	$\frac{6T}{8}$	$\frac{7T}{8}$
$y$ (cm)	$\frac{3\sqrt{2}}{2}$	3	$\frac{3\sqrt{2}}{2}$	0	$-\frac{3\sqrt{2}}{2}$	-3	$-\frac{3\sqrt{2}}{2}$	0

For  $x = 0, x = 2$ , and  $x = 4$ , the phases of the three waves will get changed. This is because amplitude and frequency are invariant for any change in  $x$ . The  $y$ - $t$  plots of the three waves are shown in the given figure.



**Q.15.10:** For the travelling harmonic wave  $y(x,t)=2.0\cos 2\pi(10t-0.0080x+0.35)$  Where  $x$  and  $y$  are in cm and  $t$  in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of For the travelling harmonic wave  $y(x,t)=2.0\cos 2\pi(10t-0.0080x+0.35)$  Where  $x$  and  $y$  are

in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4m, (b) 0.5m t (c)  $\lambda/2$  (d)  $3\lambda/4$  (a) 4m, (b) 0.5m t (c)  $\lambda/2$  (d)  $3\lambda/4$

**Ans :** Equation for a travelling harmonic wave is given

as:  $y(x,t)=2.0\cos 2\pi(10t-0.0080x+0.35)=2.0\cos(20\pi t-0.016\pi x+0.70\pi)$  Equation for a travelling harmonic wave is given

as:  $y(x,t)=2.0\cos 2\pi(10t-0.0080x+0.35)=2.0\cos(20\pi t-0.016\pi x+0.70\pi)$

Where, Propagation constant,  $k=0.0160\pi$  Amplitude,  $a=2\text{cm}$  Angular frequency,  $\omega=20\text{rad/s}$  Phase difference is given by the

relation: Where, Propagation

constant,  $k=0.0160\pi$  Amplitude,  $a=2\text{cm}$  Angular

frequency,  $\omega=20\text{rad/s}$  Phase difference is given by the relation:

$\phi=kx=2\pi\lambda$  (a)

For  $x=4\text{m}=400\text{cm}$   $\Phi=0.016\pi\times 400=6.4\text{rad}$  (b) For  $0.5\text{m}=50\text{cm}$   $\Phi=0.016\pi\times 50=0.8\pi\text{rad}$   $\phi=kx=2\pi\lambda$  (a)

For  $x=4\text{m}=400\text{cm}$   $\Phi=0.016\pi\times 400=6.4\text{rad}$  (b) For  $0.5\text{m}=50\text{cm}$   $\Phi=0.016\pi\times 50=0.8\pi\text{rad}$

(c) For  $x=\lambda/2$   $x=\lambda/2$

$\phi=2\pi\lambda\times\lambda/2=\pi\text{rad}$   $\phi=2\pi\lambda\times\lambda/2=\pi\text{rad}$

(d) For  $\phi=2\pi\lambda\times 3\lambda/4=1.5\pi\text{rad}$  (d) For  $\phi=2\pi\lambda\times 3\lambda/4=1.5\pi\text{rad}$

**Q.15.11:** The transverse displacement of a string (clamped at its both ends) is given by  $y(x,t)=0.06\sin 23x\cos(120\pi t)$  The transverse displacement of a string (clamped at its both ends) is given by  $y(x,t)=0.06\sin 23x\cos(120\pi t)$

Where x and y are in m and t in s. The length of the string is 1.5m and its mass is  $3.0\times 10^{-2}\text{kg}$ . Answer the following: (a) Does the function

represent a travelling wave or a stationary wave? What is the

wavelength, frequency, and speed of each wave? (c) Determine the

tension in the string. Where x and y are in m and t in s. The length of the string

is 1.5m and its mass is  $3.0\times 10^{-2}\text{kg}$ . Answer the following: (a) Does the function

represent a travelling wave or a stationary wave? What is the wavelength,

frequency, and speed of each wave? (c) Determine the tension in the string.

**Ans :** (a) The general equation representing a stationary wave is given by the displacement function:  $y(x,t)=2a\sin kx\cos\omega t$  This equation is similar to

the given equation:  $y(x,t)=0.06\sin(23x)\cos(120\pi t)$  Hence, the given

function represents a stationary wave. (a) The general equation

representing a stationary wave is given by the

displacement function:  $y(x,t)=2a\sin kx\cos\omega t$  This equation is similar to

the given equation:  $y(x,t)=0.06\sin(23x)\cos(120\pi t)$  Hence, the given

function represents a stationary wave.

(b) A wave travelling along the positive x -direction is given

as:  $y_1=a\sin(\omega t-kx)$  The wave travelling along the negative x- direction is

given as:  $y_2=a\sin(\omega t+kx)$  (b) A wave travelling along the positive x -

direction is given as:  $y_1=a\sin(\omega t-kx)$  The wave travelling along the

negative x- direction is given as:  $y_2=a\sin(\omega t+kx)$

The superposition of these two waves yields:  $y=y_1+y_2=a\sin(\omega t-kx)$

$-a\sin(\omega t+kx)$  The superposition of these two waves

yields:  $y=y_1+y_2=a\sin(\omega t-kx)-a\sin(\omega t+kx)$

$= a \sin(\omega t) \cos(kx) - a \sin(kx) \cos(\omega t) - a \sin(\omega t) \cos(kx)$   
 $- a \sin(kx) \cos(\omega t) = -2a \sin(kx) \cos(\omega t) = a \sin(\omega t) \cos(kx) - a \sin(kx) \cos(\omega t)$   
 $- a \sin(\omega t) \cos(kx) - a \sin(kx) \cos(\omega t) = -2a \sin(kx) \cos(\omega t)$   
 $= -2a \sin(2\pi\lambda x) \cos(2\pi\nu t)$  The transverse displacement of the string is given as:  $= -2a \sin(2\pi\lambda x) \cos(2\pi\nu t)$  The transverse displacement of the string is given as:

The transverse displacement of the string is given as:  $y(x,t) = 0.06 \sin(2\pi 3x) \cos(120\pi t)$  Comparing equations (i) and (ii), we have:  $2\pi\lambda = 2\pi 3$  The transverse displacement of the string is given as:  $y(x,t) = 0.06 \sin(2\pi 3x) \cos(120\pi t)$  Comparing equations (i) and (ii), we have:  $2\pi\lambda = 2\pi 3$

$\therefore$  Wavelength,  $\lambda = 3\text{m}$  It is given that:  $120\pi = 2\pi\nu$  Frequency,  $\nu = 60\text{Hz}$  Wave speed,  $v = \nu\lambda = 60 \times 3 = 180\text{m/s}$   $\therefore$  Wavelength,  $\lambda = 3\text{m}$  It is given

that:  $120\pi = 2\pi\nu$  Frequency,  $\nu = 60\text{Hz}$  Wave speed,  $v = \nu\lambda = 60 \times 3 = 180\text{m/s}$

(c) The velocity of a transverse wave travelling in a string is given by the relation:  $v = \sqrt{T/\mu}$ ... (i) (c) The velocity of a transverse wave travelling in a string is given by the relation:  $v = \sqrt{T/\mu}$ ... (i)

Where, Velocity of the transverse wave,  $v = 180\text{m/s}$  Mass of the string,  $m = 3.0 \times 10^{-2}\text{kg}$  Length of the string,  $l = 1.5\text{m}$  Mass per unit length of the string,  $\mu = m/l$  Where, Velocity of the transverse

wave,  $v = 180\text{m/s}$  Mass of the string,  $m = 3.0 \times 10^{-2}\text{kg}$  Length of the string,  $l = 1.5\text{m}$  Mass per unit length of the string,  $\mu = m/l$

$= 3.0 / 1.5 = 2 \times 10^{-2} = 2 \times 10^{-2} \text{kgm}^{-1}$  Tension in the string  $= T$  From equation (i), tension can be obtained

as:  $= (180)^2 \times 2 \times 10^{-2} = 648\text{N} = 3.0 / 1.5 \times 10^{-2} = 2 \times 10^{-2} \text{kgm}^{-1}$  Tension in the string  $= T$  From equation (i), tension can be obtained

as:  $= (180)^2 \times 2 \times 10^{-2} = 648\text{N}$

**Q.15.12:** (i) For the wave on a string described in Exercise 15.11, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude? Explain your answers.

(ii) What is the amplitude of a point 0.375 m away from one end?

**Ans :** (i)(a) Yes, except at the nodes

(b) Yes, except at the nodes

(c) No

(ii) 0.042 m

Explanation: (i) (a) All the points on the string oscillate with the same frequency, except at the nodes which have zero frequency. (b) All the points in any vibrating loop have the same phase, except at the nodes. (c) All the points in any vibrating loop have different amplitudes of vibration.

Explanation: (i) (a) All the points on the string oscillate with the same frequency, except at the nodes which have zero frequency. (b) All the points in any vibrating loop have the same phase, except at the nodes. (c) All the points in any vibrating loop have different amplitudes of vibration.

(ii) The given equation

is:  $y(x,t) = 0.06 \sin(2\pi 3x) \cos(120\pi t)$  For  $x = 0.375\text{m}$  and  $t = 0$  (ii) The given equation is:  $y(x,t) = 0.06 \sin(2\pi 3x) \cos(120\pi t)$  For  $x = 0.375\text{m}$  and  $t = 0$

Amplitude = Displacement =  $0.06\sin(2\pi 3x)\cos 0 = 0.06\sin(2\pi 3x)\cos 75^\circ \times 1 = 0.06\sin(0.25\pi) = 0.06\sin(\pi/4) = 0.06 \times 1/\sqrt{2} = 0.042\text{m}$   
 Amplitude = Displacement =  $0.06\sin(2\pi 3x)\cos 0 = 0.06\sin(2\pi 3x)\cos 75^\circ \times 1 = 0.06\sin(0.25\pi) = 0.06\sin(\pi/4) = 0.06 \times 1/2 = 0.042\text{m}$

**Q.15.13:** Given below are some functions of  $x$  and  $t$  to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a traveling wave, (ii) a stationary wave or (iii) none at all: (a)  $y = 2\cos(3x)\sin(10t)$  (b)  $y = 2\sqrt{x - vt}$  (c)  $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$  (d)  $y = \cos x \sin t + \cos 2x \sin 2t$   
 Given below are some functions of  $x$  and  $t$  to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a traveling wave, (ii) a stationary wave or (iii) none at all: (a)  $y = 2\cos(3x)\sin(10t)$  (b)  $y = 2x - vt$  (c)  $y = 3\sin(5x - 0.5t) + 4\cos(5x - 0.5t)$  (d)  $y = \cos x \sin t + \cos 2x \sin 2t$

**Ans :** (a) The given equation represents a stationary wave because the harmonic terms  $kx$  and  $\omega t$  appear separately in the equation  
 (b) The given equation does not contain any harmonic term. Therefore, it does not represent either a travelling wave or a stationary wave  
 (c) The given equation represents a travelling wave as the harmonic terms  $kx$  and  $\omega t$  are in the combination  $kx - \omega t$ .  
 (d) The given equation represents a stationary wave because the harmonic terms  $kx$  and  $\omega t$  appear separately in the equation. This equation actually represents the superposition of two stationary waves.

**Q.15.14:** A wire stretched between two rigid supports in its fundamental mode with a frequency of  $45\text{Hz}$ . The mass of the wire is  $3.5 \times 10^{-2}\text{kg}$  and its linear mass density is  $4.0 \times 10^{-2}\text{kgm}^{-1}$ . What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?  
 A wire stretched between two rigid supports in its fundamental mode with a frequency of  $45\text{Hz}$ . The mass of the wire is  $3.5 \times 10^{-2}\text{kg}$  and its linear mass density is  $4.0 \times 10^{-2}\text{kgm}^{-1}$ . What is (a) the speed of a transverse wave on the string, and (b) the tension in the string?

**Ans :** (a) Mass of the wire,  $m = 3.5 \times 10^{-2}\text{kg}$   
 Linear mass density,  $\mu = m/l = 4.0 \times 10^{-2}\text{kgm}^{-1}$   
 Frequency of vibration,  $\nu = 45\text{Hz}$   
 (a) Mass of the wire,  $m = 3.5 \times 10^{-2}\text{kg}$   
 Linear mass density,  $\mu = m/l = 4.0 \times 10^{-2}\text{kgm}^{-1}$   
 Frequency of vibration,  $\nu = 45\text{Hz}$   
 Length of wire,  
 $l = m/\mu = 3.5 \times 10^{-2} / 4.0 \times 10^{-2} = 0.875\text{m}$   
 $l = m/\mu = 3.5 \times 10^{-2} / 4.0 \times 10^{-2} = 0.875\text{m}$   
 The wavelength of the stationary wave ( $\lambda$ ) is related to the length of the wire by the relation:  
 $\lambda = 2l$   
 $\lambda = 2l$

Where,  $n =$  Number of nodes in the wire  
 For fundamental node,  $n = 1: \lambda = 2l = 2 \times 0.875 = 1.75\text{m}$   
 Where,  $n =$  Number of nodes in the wire  
 For fundamental node,  $n = 1: \lambda = 2l = 2 \times 0.875 = 1.75\text{m}$   
 The speed of the transverse wave in the string is given as:  
 $v = \nu \lambda = 45 \times 1.75 = 78.75\text{m/s}$   
 The speed of the transverse wave in the string is given as:  
 $v = \nu \lambda = 45 \times 1.75 = 78.75\text{m/s}$



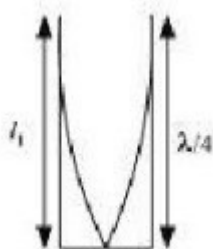
(b) The tension produced in the string is given by the relation:  $T = v^2 \mu = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{ N}$  (b) The tension produced in the string is given by the

relation:  $T = v^2 \mu = (78.75)^2 \times 4.0 \times 10^{-2} = 248.06 \text{ N}$

**Q.15.15:** A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected

**Ans :** Frequency of the tuning fork,  $\nu = 340 \text{ Hz}$

Since the given pipe is attached with a piston at one end, it will behave as a pipe with one end closed and the other end open, as shown in the given figure



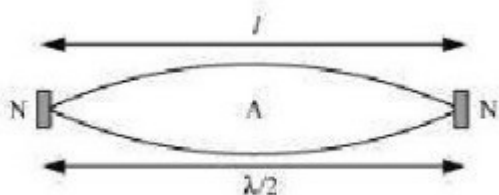
Such a system produces odd harmonics. The fundamental note in a closed pipe is given by the relation.

$l_1 = \lambda/4$  Where, Length of the

pipe,  $l_1 = 25.5 \text{ cm} = 0.255 \text{ m}$ .  $\lambda = 4l_1 = 4 \times 0.255 = 1.02 \text{ m}$  The speed of sound is given by the relation:  $v = \nu \lambda = 340 \times 1.02 = 346.8 \text{ m/s}$

**Q.15.16:** A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel ?

**Ans :** Length of the steel rod,  $l = 100 \text{ cm} = 1 \text{ m}$  Fundamental frequency of vibration,  $\nu = 2.53 \text{ kHz} = 2.53 \times 10^3 \text{ Hz}$  When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) are formed at its two ends, as shown in the given figure. Length of the steel rod,  $l = 100 \text{ cm} = 1 \text{ m}$  Fundamental frequency of vibration,  $\nu = 2.53 \text{ kHz} = 2.53 \times 10^3 \text{ Hz}$  When the rod is plucked at its middle, an antinode (A) is formed at its centre, and nodes (N) are formed at its two ends, as shown in the given figure.



The distance between two successive nodes is  $\lambda/2$ .  $l = \lambda/2$  The distance between two successive nodes is  $\lambda/2$ .  $l = \lambda/2$

$$V = v\lambda = 2.53 \times 10^3 \times 2 = 5.06 \times 10^3 \text{ m/s} = 5.06 \text{ km/s}$$

**Q.15.17:** A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe if both ends are open? (Speed of sound in air is 340 m/s)

**Ans :** First (Fundamental); No Length of the pipe,  $l = 20 \text{ cm} = 0.2 \text{ m}$  Source frequency =  $n$ th normal mode of frequency,  $\nu_n = 430 \text{ Hz}$  Speed of sound,  $v = 340 \text{ m/s}$  First (Fundamental); No Length of the pipe,  $l = 20 \text{ cm} = 0.2 \text{ m}$  Source frequency =  $n$ th normal mode of frequency,  $\nu_n = 430 \text{ Hz}$  Speed of sound,  $v = 340 \text{ m/s}$

In a closed pipe, the normal mode of frequency is given by the relation:  
 $\nu_n = (2n-1)v/4l$   
 $430 = (2n-1)340/4 \times 0.2$   
 $430 \times 4 \times 0.2 = (2n-1)340$   
 $1.012n = 2.01$   
 $n \sim 1$

Hence, the first mode of vibration frequency is resonantly excited by the given source.

In a pipe open at both ends, the  $n$ th mode of vibration frequency is given by the relation:

$$\nu_n = nv/2l$$
$$430 = n \times 340 / 2 \times 0.2$$

Since the number of the mode of vibration ( $n$ ) has to be an integer, the given source does not produce a resonant vibration in an open pipe.

**Q.15.18:** Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B?

**Ans :** Frequency of string A,  $f_A = 324 \text{ Hz}$  Frequency of string B =  $f_B$  Beat's frequency,  $n = 6 \text{ Hz}$  Frequency of string A,  $f_A = 324 \text{ Hz}$  Frequency of string B =  $f_B$  Beat's frequency,  $n = 6 \text{ Hz}$

Beat's frequency is given as:  $n = |f_n \pm f_B|$

$$6 = 324 \pm f_B \Rightarrow f_B = 330 \text{ Hz or } 318 \text{ Hz}$$

$$6 = 324 \pm f_B \Rightarrow f_B = 330 \text{ Hz or } 318 \text{ Hz}$$

Frequency decreases with a decrease in the tension in a string. This is because frequency is directly proportional to the square root of tension. It is given as:

$v \propto \sqrt{T}$  Hence, the beat frequency cannot be 330 Hz.  $f_B = 318 \text{ Hz}$

**Q.15.19:** Explain why (or how):

- In a sound wave, a displacement node is a pressure antinode and vice versa,
- Bats can ascertain distances, directions, nature, and sizes of the obstacles without any "eyes",
- A violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,
- Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases, and
- The shape of a pulse gets distorted during propagation in a dispersive medium.

**Ans :** (a) A node is a point where the amplitude of vibration is the minimum and pressure is the maximum. On the other hand, an antinode

is a point where the amplitude of vibration is the maximum and pressure is the minimum. Therefore, a displacement node is nothing but a pressure antinode, and vice versa.

(b) Bats emit very high-frequency ultrasonic sound waves. These waves get reflected back toward them by obstacles. A bat receives a reflected wave (frequency) and estimates the distance, direction, nature, and size of an obstacle with the help of its brain senses.

(c) The overtones produced by a sitar and a violin, and the strengths of these overtones, are different. Hence, one can distinguish between the notes produced by a sitar and a violin even if they have the same frequency of vibration.

(d) Solids have shear modulus. They can sustain shearing stress. Since fluids do not have any definite shape, they yield to shearing stress. The propagation of a transverse wave is such that it produces shearing stress in a medium. The propagation of such a wave is possible only in solids, and not in gases. Both solids and fluids have their respective bulk moduli. They can sustain compressive stress. Hence, longitudinal waves can propagate through solids and fluids.

(e) A pulse is actually a combination of waves having different wavelengths. These waves travel in a dispersive medium with different velocities, depending on the nature of the medium. This results in the distortion of the shape of a wave pulse.

**Q.15.20:** A train, standing at the outer signal of a railway station blows a whistle of frequency 400 Hz in still air. (i) What is the frequency of the whistle for a platform observer when the train (a) approaches the platform with a speed of 10 m s<sup>-1</sup>, (b) recedes from the platform with a speed of 10 m s<sup>-1</sup>? (ii) What is the speed of sound in each case? The speed of sound in still air can be taken as 340 m s<sup>-1</sup>.

**Ans :** (i) (a) Frequency of the whistle,  $\nu = 400 \text{ Hz}$  Speed of the train,  $v_T = 10 \text{ m/s}$  speed of sound,  $v = 340 \text{ m/s}$  (i) (a) Frequency of the whistle,  $\nu = 400 \text{ Hz}$  Speed of the train,  $v_T = 10 \text{ m/s}$  speed of sound,  $v = 340 \text{ m/s}$

The apparent frequency ( $\nu'$ ) of the whistle as the train approaches the platform is given by the relation:  $\nu' = \frac{v + v_T}{v} \nu = \frac{340 + 10}{340} \times 400 = 412.12 \text{ Hz}$  The apparent frequency ( $\nu'$ ) of the whistle as the train approaches the platform is given by the relation:  $\nu' = \frac{v + v_T}{v} \nu = \frac{340 + 10}{340} \times 400 = 412.12 \text{ Hz}$

(b) The apparent frequency ( $\nu'$ ) of the whistle as the train recedes from the platform is given by the relation:  $\nu' = \frac{v - v_T}{v} \nu = \frac{340 - 10}{340} \times 400 = 388.57 \text{ Hz}$  (b) The apparent frequency ( $\nu'$ ) of the whistle as the train recedes from the platform is given by the relation:  $\nu' = \frac{v - v_T}{v} \nu = \frac{340 - 10}{340} \times 400 = 388.57 \text{ Hz}$

(ii) The apparent change in the frequency of sound is caused by the relative motions of the source and the observer. These relative motions produce no effect on the speed of sound. Therefore, the speed of sound in air in both the cases remains the same, i.e., 340 m/s.

**Q.15.21:** A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with a speed of 10 m s<sup>-1</sup>. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 m s<sup>-1</sup>? The speed of sound in still air can be taken as 340 m s<sup>-1</sup>

**Ans :** For the stationary observer: 400Hz;0.875m;350m/s For the running observer: Not exactly identical For the stationary observer: Frequency of the sound produced by the whistle,  $\nu=400\text{Hz}$  Speed of sound = 340m/s velocity of the wind,  $\nu=10\text{m/s}$  For the stationary observer: 400Hz;0.875m;350m/s For the running observer: Not exactly identical For the stationary observer: Frequency of the sound produced by the whistle,  $\nu=400\text{Hz}$  Speed of sound = 340m/s velocity of the wind,  $\nu=10\text{m/s}$

As there is no relative motion between the source and the observer, the frequency of the sound heard by the observer will be the same as that produced by the source, i.e., 400 blowing toward the observer. Hence, the effective speed of the sound is 10 units, Effective speed of the sound  $\nu^\circ=340+10=350\text{m/s}$

Additional Exercise P.393

**Q.15.22:** A travelling harmonic wave on a string is described by  $y(x,t)=7.5\sin(0.0050x+12t+\pi/4)$

(a) What are the displacement and velocity of oscillation of a point at  $x = 1$  cm, and  $t = 1$  s? Is this velocity equal to the velocity of wave propagation?

(b) Locate the points of the string which have the same transverse displacements and velocity as the  $x = 1$  cm point at  $t = 2$  s, 5 s and 11 s.

**Ans :** (a) The given harmonic wave

is:  $y(x,t)=7.5\sin(0.0050x+12t+\pi/4)$  For  $x=1\text{cm}$  and  $t=1\text{s}$  (a) The given harmonic wave is:  $y(x,t)=7.5\sin(0.0050x+12t+\pi/4)$  For  $x=1\text{cm}$  and  $t=1\text{s}$   $y=(1,1)=7.5\sin(0.0050+12+\pi/4)=7.5\sin(12.0050+\pi/4)=7.5\sin\theta$

$y=(1,1)=7.5\sin(12.0050+\pi/4)=7.5\sin(12.0050+\pi/4)=7.5\sin\theta$   
 $=12.0050+\pi/4=12.0050+3.144=12.79\text{rad}=1803.14^\circ$   
 $\therefore y=(1,1)=7.5\sin(1803.14^\circ)=7.5\sin(12.81^\circ)=7.5\times 0.2217=1.6629\approx 1.663\text{cm}$

$v=\frac{dy}{dt}(x,t)=\frac{d}{dt}[7.5\sin(0.0050x+12t+\pi/4)]=7.5\times 12\cos(0.0050x+12t+\pi/4)$   
 At  $x=1\text{cm}$  and  $t=1\text{s}$  .  $v=y(1,1)=90\cos(12.005+\pi/4)$   
 $v=\frac{dy}{dt}(x,t)=\frac{d}{dt}[7.5\sin(0.0050x+12t+\pi/4)]=7.5\times 12\cos(0.0050x+12t+\pi/4)$  At  $x=1\text{cm}$  and  $t=1\text{s}$  .  
 $v=y(1,1)=90\cos(12.005+\pi/4)$

$=90\cos(1803.14^\circ)=90\cos(90\times 8+12.81^\circ)=90\cos(12.81^\circ)=90\times 0.975=87.75\text{cm/s}$   
 $=90\cos(12.81^\circ)=90\cos(90\times 8+12.81^\circ)=90\cos(12.81^\circ)=90\times 0.975=87.75\text{cm/s}$

Now, the equation of a propagating wave is given

by:  $y(x,t) = a \sin(kx + \omega t + \phi)$  Now, the equation of a propagating wave is given by:  $y(x,t) = a \sin(kx + \omega t + \phi)$

Where  $k = \frac{2\pi}{\lambda}$ ,  $\lambda = \frac{2\pi}{k}$  Where  $k = \frac{2\pi}{\lambda}$ ,  $\lambda = \frac{2\pi}{k}$

$\therefore v = \frac{\omega}{k}$  Speed,  $v = v\lambda = \omega k$  Where,  $\omega = 12 \text{ rad/s}$ ,  $k = 0.0050 \text{ m}^{-1}$   $\therefore v = \frac{\omega}{k} = \frac{12}{0.0050} = 2400 \text{ cm/s}$

$v = v\lambda = \omega k$  Where,  $\omega = 12 \text{ rad/s}$ ,  $k = 0.0050 \text{ m}^{-1}$

$v = 120.0050 = 2400 \text{ cm/s}$  Hence, the velocity of the wave oscillation

at  $x = 1 \text{ cm}$  and  $t = 1 \text{ s}$  is not equal to the velocity of the wave

propagation. (b) Propagation constant is related to wavelength

as:  $v = 120.0050 = 2400 \text{ cm/s}$  Hence, the velocity of the wave oscillation

at  $x = 1 \text{ cm}$  and  $t = 1 \text{ s}$  is not equal to the velocity of the wave

propagation. (b) Propagation constant is related to wavelength as:

$k = \frac{2\pi}{\lambda}$ ,  $\lambda = \frac{2\pi}{k} = \frac{2 \times 3.140.0050}{1} = 1256 \text{ cm} = 12.56 \text{ m}$

$0.0050 = 1256 \text{ cm} = 12.56 \text{ m}$

**Q.15.23:** A narrow sound pulse (for example, a short pip by a whistle) is sent across a medium. (a) Does the pulse have a definite (i) frequency, (ii) wavelength, (iii) speed of propagation? (b) If the pulse rate is 1 after every 20 s, (that is the whistle is blown for a split of second after every 20 s), is the frequency of the note produced by the whistle equal to 20 or 0.05 Hz?

**Ans :** (a) (i) No

(ii) No

(iii) Yes

(b) No

Explanation:

(a) The narrow sound pulse does not have a fixed wavelength or frequency. However, the speed of the sound pulse remains the same, which is equal to the speed of sound in that medium.

(b) The short pip produced after every 20 s does not mean that the frequency of the

Whistle is  $\frac{1}{20}$  or 0.05 Hz It means that 0.05 Hz is the frequency of the repetition of the pip of the whistle.

Substituting the values from (ii) , (iii) and (iv) in (i) we get the

displacement equation :

$y(x,t) = 0.05 \sin(1.6 \times 10^3 t - 4.84 x) \text{ m}$   $y(x,t) = 0.05 \sin(1.6 \times 10^3 t - 4.84 x) \text{ m}$

**Q.15.24:** One end of a long string of linear mass density  $8.0 \times 10^{-3} \text{ kg m}^{-1}$  is connected to an electrically driven tuning fork of frequency 256 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves at this end have negligible amplitude. At  $t = 0$ , the left end (fork end) of the string  $x = 0$  has zero transverse displacement ( $y = 0$ ) and is moving along positive  $y$ -direction. The amplitude of the wave is 5.0 cm. Write down the transverse displacement  $y$  as function of  $x$  and  $t$  that describes the wave on the string

**Ans :** The equation of a travelling wave propagating along the positive  $y$  - direction is given by the displacement equation:  $y(x,t) = a \sin(\omega t - kx) \dots$

(i) The equation of a travelling wave propagating along the positive  $y$  - direction is given by the displacement equation:  $y(x,t) = a \sin(\omega t - kx) \dots$  (i)

Linear mass density,  $\mu = 8.0 \times 10^{-3} \text{ kg m}^{-1}$  Frequency of the tuning fork,  $\nu = 256 \text{ Hz}$  Amplitude of the wave,  $a = 5.0 \text{ cm} = 0.05 \text{ m}$ ... (ii) Mass of the pan,  $m = 90 \text{ kg}$  Linear mass density,  $\mu = 8.0 \times 10^{-3} \text{ kg m}^{-1}$  Frequency of the tuning fork,  $\nu = 256 \text{ Hz}$  Amplitude of the wave,  $a = 5.0 \text{ cm} = 0.05 \text{ m}$ ... (ii) Mass of the pan,  $m = 90 \text{ kg}$

Tension in the string,  $T = mg = 90 \times 9.8 = 882 \text{ N}$  The velocity of the transverse wave  $v$ , is given by the relation: Tension in the

string,  $T = mg = 90 \times 9.8 = 882 \text{ N}$  The velocity of the transverse wave  $v$ , is given by the relation:

$$v = \sqrt{T/\mu} = \sqrt{882/8.0 \times 10^{-3}} = 332 \text{ m/s}$$

Angular frequency,  $\omega = 2\pi\nu = 1608.5 = 1.6 \times 10^3 \text{ rad/s}$   $v = T/\mu = 882/8.0 \times 10^{-3} = 332 \text{ m/s}$

Angular frequency,  $\omega = 2\pi\nu = 1608.5 = 1.6 \times 10^3 \text{ rad/s}$

Wavelength,  $\lambda = v/\nu = 332/256 = 1.297 \text{ m}$  ∴ Propagation

constant,  $k = 2\pi/\lambda = 2 \times 3.14/1.297 = 4.84 \text{ m}^{-1}$  Wavelength,  $\lambda = v/\nu = 332/256 = 1.297 \text{ m}$

∴ Propagation constant,  $k = 2\pi/\lambda = 2 \times 3.14/1.297 = 4.84 \text{ m}^{-1}$

**Q.15.25:** A SONAR system fixed in a submarine operates at a frequency  $40.0 \text{ kHz}$ .

An enemy submarine moves towards the SONAR with a speed of  $360 \text{ km h}^{-1}$

What is the frequency of sound reflected by the submarine? Take the speed of sound in water to be  $1450 \text{ m s}^{-1}$

**Ans :** Operating frequency of the SONAR system,  $\nu = 40 \text{ kHz}$  Speed of the

enemy submarine,  $v_e = 360 \text{ km/h} = 100 \text{ m/s}$  Speed of sound in

water,  $v = 1450 \text{ m/s}$  Operating frequency of the SONAR

system,  $\nu = 40 \text{ kHz}$  Speed of the enemy

submarine,  $v_e = 360 \text{ km/h} = 100 \text{ m/s}$  Speed of sound in water,  $v = 1450 \text{ m/s}$

The source is at rest and the observer (enemy submarine) is moving toward it. Hence the apparent frequency ( $\nu'$ ) received and reflected by the submarine is given by the relation: The source is at rest and the observer (enemy submarine) is moving toward it. Hence the apparent frequency ( $\nu'$ ) received and reflected by the submarine is given by the relation:

$$\nu' = \left( \frac{v + v_o}{v} \right) \nu = \left( \frac{1450 + 100}{1450} \right) \times 40 = 42.76 \text{ kHz}$$

$$\nu' = \left( \frac{v + v_o}{v} \right) \nu = \left( \frac{1450 + 100}{1450} \right) \times 40 = 42.76 \text{ kHz}$$

The frequency ( $\nu''$ ) received by the enemy submarine is given by the relation:  $\nu'' = \left( \frac{v}{v + v_s} \right) \nu'$  Where,  $v_s = 100 \text{ m/s}$  ∴  $\nu'' = \left( \frac{1450}{1450 + 100} \right) \times 42.76 = 45.93 \text{ kHz}$

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**Q.15.26:** Earthquakes generate sound waves inside the earth. Unlike a gas, the earth can experience both transverse (S) and longitudinal (P) sound waves. Typically the speed of S wave is about  $4.0 \text{ km s}^{-1}$ , and that of P wave is  $8.0 \text{ km s}^{-1}$ . A seismograph records P and S waves from an earthquake. The first P wave arrives 4 min before the first S wave. Assuming the waves travel in straight line, at what distance does the earthquake occur ?

**Ans :** Let  $v_s$  and  $v_p$  be the velocities of S and P waves

respectively. Let L be the distance between the epicentre and the

seismograph. We have: Let  $v_s$  and  $v_p$  be the velocities of S and P waves

respectively. Let L be the distance between the epicentre and the

seismograph. We have:

$L = v_s t_s$  (i)  $L = v_p t_p$  (ii) Where,  $L = v_s t_s$  (i)  $L = v_p t_p$  (ii) Where,  $t_s$  and  $t_p$  are the respective times taken by the S and P waves to reach the seismograph from the epicentre It is given

that:  $v_p = 8 \text{ km/s}$   $v_s = 4 \text{ km/s}$  and  $t_p$  are the respective times taken by the S and P waves to reach the seismograph from the epicentre It is given that:  $v_p = 8 \text{ km/s}$   $v_s = 4 \text{ km/s}$

$v_s t_s = v_p t_p$   $4 t_s = 8 t_p$   $t_s = 2 t_p$  (iii)  $v_s t_s = v_p t_p$   $4 t_s = 8 t_p$   $t_s = 2 t_p$  (iii)

It is also given that:  $t_s - t_p = 4 \text{ min} = 240 \text{ s}$   $2 t_p - t_p = 240$   $t_p = 240$  It is also given that:  $t_s - t_p = 4 \text{ min} = 240 \text{ s}$   $2 t_p - t_p = 240$   $t_p = 240$

And  $t_s = 2 \times 240 = 480 \text{ s}$  From equation (iii), we

get:  $L = 8 \times 240 = 1920 \text{ km}$  And  $t_s = 2 \times 240 = 480 \text{ s}$  From equation (iii), we

get:  $L = 8 \times 240 = 1920 \text{ km}$

Hence, the earthquake occurs at a distance of 1920 km from the seismograph.

**Q.15.27:** A bat is flitting about in a cave, navigating via ultrasonic beeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall?

**Ans :** Ultrasonic beep frequency emitted by the bat,  $v = 40 \text{ kHz}$  velocity of the bat,  $v_b = 0.03v$  Where,  $v =$  velocity of sound in air The apparent frequency of the sound striking the wall is given as: Ultrasonic beep frequency emitted by the bat,  $v = 40 \text{ kHz}$  velocity of the bat,  $v_b = 0.03v$  Where,  $v =$  velocity of sound in air The apparent frequency of the sound striking the wall is given as:

$$v' = (v - v_b) / v = (v - 0.03v) / 40 = 400.97 \text{ kHz}$$

$$v'' = (v - v_b) / v = (v - 0.03v) / 40 = 400.97 \text{ kHz}$$

This frequency is reflected by the stationary wall ( $v_s = 0$ ) toward the bat. The frequency ( $v''$ ) of the received sound is given by the

$$\text{relation: } v^* = (v + v_b) / v'' = (v + 0.03v) / 400.97 = 1.03 \times 400.97 = 42.47 \text{ kHz}$$