DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2018-2019)

Class : XI

MATHEMATICS

Under the Guidance of

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Secretary (Education)

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PREFACE

It gives me immense pleasure to present the Support Material for various subjects. The material prepared for students of classes IX to XII has been conceived and developed by a team comprising of the Subject Experts, Members of the Academic Core Unit and teachers of the Directorate of Education.

The subject wise Support Material is developed for the betterment and enhancement of the academic performance of the students. It will give them an insight into the subject leading to complete understanding. It is hoped that the teachers and students will make optimum use of this material. This will help us achieve academic excellence.

I commend the efforts of the team who have worked with complete dedication to develop this matter well within time. This is another endeavor of the Directorate to give complete support to the learners all over Delhi.

Sandeep K

(SANDEEP KUMAR)
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DIRECTOR'S MESSAGE

Dear Students,

Through this Support Material, I am getting an opportunity to communicate directly with you and I want to take full advantage of this opportunity.

In Delhi, there are approximately 1020 other government schools like yours, which are run by Directorate of Education. The Head Quarters of Directorate of Education is situated at Old Secretariat, Delhi-54.

All the teachers in your school and officers in the Directorate work day and night so that the standard of our govt. schools may be uplifted and the teachers may adopt new methods and techniques to teach in order to ensure a bright future for the students.

Dear students, the book in your hand is also one such initiative of your Directorate. This material has been prepared specially for you by the subject experts. A huge amount of money and time has been spent to prepare this material. Moreover, every year, this material is reviewed and updated as per the CBSE syllabus so that the students can be updated for the annual examination.

Last, but not the least, this is the perfect time for you to build the foundation of your future. I have full faith in you and the capabilities of your teachers. Please make the fullest and best use of this Support Material.

Sanjay Goel

DIRECTOR (EDUCATION)

Dr. (Mrs.) Saroj Bala Sain
Addl. Director of Edn. (School)/Exam

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Date : 16/07/2018

It gives me immense pleasure and a sense of satisfaction to forward the support material for classes IX to XII in all subjects. The support material is continuously revised redesigned and updated by a team of subject experts, members of Core Academic Unit and teachers from various schools of DOE.

Consistent use of support material by the students and teachers will make the year long journey seemless and enjoyable. The purpose of providing support material has always been to make available ready to use material which is matchless and most appropriate.

My commendation for all the team members for their valuable contribution.

S. Sain

Dr. Saroj Bala Sain
Addl.DE (School)

DIRECTORATE OF EDUCATION
Govt. of NCT, Delhi

SUPPORT MATERIAL
(2018-2019)

MATHEMATICS
Class : XI
(English Medium)

NOT FOR SALE

PUBLISHED BY : DELHI BUREAU OF TEXTBOOKS

2018-2019

SUPPORT MATEIRAL

XI

MATHEMATICS

Reviewed by

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COURSE STRUCTURE

CLASS XI (2018-19)

One Paper **Three Hours** **Max. Marks. 100**

Units		**Marks**
I.	Sets and Functions	29
II.	Algebra	37
III.	Coordinate Geometry	13
IV.	Calculus	06
V.	Mathematical Reasoning	03
VI.	Statistics and Probability	12
	Total	**100**

Unit-1 : Sets and Functions

1. Sets:

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set. Properties of Complement Sets. Practice Problems based on sets

2. Relations and Functions:

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the set of reals (upto R × R × R). Definition of relation, pictorial diagrams, domain, codomain and range of a relation. Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co-domain and range of a function. Real valued functions, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum exponential, logarithrnic functions and greatest integer functions, with their graphs. Sum, difference, product and quotients of functions.

YUVA Session

3. Trigonometric Functions:

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions. Domain and range of trigonometric functions and their graphs. Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of $\sin x$, $\sin y$, $\cos x$ and $\cos y$. Deducing the identities like the following:

$$\sin(x \quad y) \quad \frac{\tan x \quad \tan y}{1 \mp \tan x \tan y}, \cot(x \quad y) \quad \frac{\cot x \cot y \mp 1}{\cot y \quad \cot x}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}, \cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2},$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}, \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y,}{2}$$

Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$. General solution of trigonometric equations of the type sin y = sin a, cos y = cos a and tan y = tan a.

Unit-2: Algebra

1. **Principle of Mathematical Induction:**

Process of the proof by induction, motivating the applications of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

2. **Complex Numbers and Quadratic Equations:**

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve some of the quadratic equations. Algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square root of a complex number.

3. **Linear Inequalities:**

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Graphical solution of system of linear inequalities in two variables.

4. **Permutations and Combinations:**

Fundamental principle of counting. Factorial *n* (*n*!) Permutations and combinations, derivation of formulae for ${}^nP_r$ and ${}^nC_r$ and their connections, simple applications.

5. **Binomial Theorem:**

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, General and middle term in binomial expansion, simple applications.

6. **Sequence and Series:**

Arithmetic Progression (A.P.), Arithmetic Mean (A.M), Geometric Progression (G.P.) General term of a G.P., sum of *n* terms of a G.P., Arithmetic and Geometric series, Infinite G.P. and its sum, geometric mean (G.M.), relation between A.M. and G.M. Sum to *n* terms of the special series

$$\sum_{k=1}^{n} k, \sum_{k=1}^{n} k^2 and \sum_{k=1}^{n} k^3$$

Unit-III: Coordinate Geometry

1. Straight Lines:

Brief recall of two-dimensional geometry from earlier classes. Shifting of origin. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two-point form, intercept form and normal form. General equation of a line. Equation of family of lines passing through the point of intersection of two lines. Distance of a point from a line.

2. Conic Sections:

Sections of a cone: circles, ellipse, parabola, hyperbola, a point, a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. Introduction to Three-Dimensional Geometry

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Unit-IV: Calculus

1. Limits and Derivatives:

Derivative introduced as rate of change of distance function and geometrically.

Intuitive idea of limit. Limits of Polynomials and rational function, trigonometric, exponential and logarithmic functions. Definition of derivative, relate it to slope of tangent of a curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit-V: Mathematical Reasoning

1. MathematicalReasoning

Mathematically acceptable statements. Connecting words/phrases consolidating the understanding of “if and only if (necessary and sufficient) condition”, “implies”, “and/or”, “implied by”, “and”, “or”, “there exists” and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words, difference between contradiction, converse and contrapositive.

Unit-VI: Statistics and Probability

1. Statistics

Measure of dispersion, range, mean deviation, variance and standard deviation of ungrouped/grouped data.

Analysis of frequency distributions with equal mean but different variances.

2. Probability

Random experiments, outcomes; Sample spaces (set representation); Event; Occurrence of events, "not', "and" and "or" events. exhaustive events, mutually exclusive events, Axiomatic (set theoretic) probability connections with the theories of earlier classes. Probability of an event, probability of "not", "and" and "or" events.

MATHEMATICS (CODE - 041)

Question paper design class – XI (2018-19)

Time 3 Hours **Max. Marks: 100**

S. No.	Typology of Questions	Very Short Answer (1 M)	Long Answer I (2 marks)	Long Answer I (4 marks)	Long Answer II (6 marks)	Marks	% Weight age
1.	**Remembering** – (Knowledge based Simple recall questions, to know specific facts, terms, concepts, principles, or theories; Identify, define, or recite, information)	2	2	2	1	20	20%
2.	**Understanding** – (Comprehension to be familiar with meaning and to understand conceptually, interpret, compare, contrast, explain, paraphrase information)	1	3	4	2	35	35%
3.	**Application –** (Use abstract information in concrete situation, to apply knowledge to new situations; Use given content to interpret a situation, provide an example, or solve a problem)	1	–	3	2	25	25%
4.	**High Order Thinking Skills** – (Analysis & Synthesis-classify, compare, contrast, or differentiate between different pieces of information; Organise and/or integrate unique pieces of information from a variety of sources)	–	3	1	–	10	10%
5.	**Evaluation** – (Appraise, judge, and/ or justify the value or worth of a decision or outcome, or to predict outcomes based on values)	–	–	1	1	10	10%
	TOTAL	**1×4=16**	**2×8= 16**	**4×11= 44**	**6×6= 36**	**100**	**100**

QUESTION WISE BREAK UP

Type of Questions	Marks per Question	Total Number of Questions	Total Marks
VSA	01	06	04
SA	02	08	16
LA - I	04	13	16
LA - II	06	07	42
Total		**26**	**100**

No chapter wise weightage. Care to be taken to cover all the chapters.

The above template is only a sample. Suitable internal variations may be made for generating similar templates keeping the overall weightage to different form of questions and typology of questions same

CHAPTER - 1

SETS

KEY POINTS

- A set is a well-defined collection of objects.
- There are two methods of representing a set:
  - (a) Roster or Tabular form e.g.natural numbers less than 5 = {1, 2, 3, 4}
  - (b) Set-builder form or Rule method e.g.: Vowels in English alphabet= {*x*: *x* is a vowel in the English alphabet }


Georg Cantor
(1845-1918)

- **Types of sets:**
  - (i) Empty set or Null set or void set
  - (ii) Finite set
  - (iii) Infinite set
  - (iv) Singleton set
- Subset :– A set A is said to be a subset of set B if $a \in A \Rightarrow a \in B, \forall a \in A$. We write it as $A \subseteq B$
- Equal sets :– Two sets A and B are equal if they have exactly the same elements i.e A = B if $A \subset B$ and $B \subset A$
- Power set : The collection of all subsets of a set A is called power set of A, denoted by P(A) i.e. P(A) = { B : $B \subset A$ }
- If A is a set with n(A) = m then n [P(A)] = 2^m.

- Equivalent sets : Two finite sets *A* and *B* are equivalent, if their cardinal numbers are same *i.e., n*(*A*) = *n*(*B*).
- Proper subset and super set : If $A \subset B$ then A is called the proper subset of B and B is called the superset of A.

Types of Intervals

Open Interval (a, b) = $\{ x \in R : a < x < b \}$

Closed Interval [a, b] = $\{ x \in R : a \le x \le b \}$

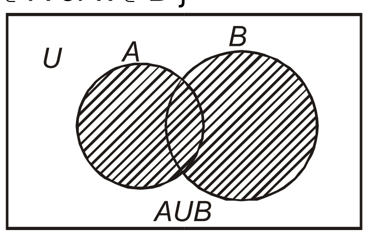
Semi open or Semi closed Interval,

(a,b] = $\{ x \in R : a < x \le b\}$

[a,b) = $\{ x \in R : a \le x < b\}$

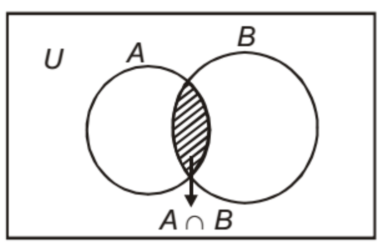
- Union of two sets A and B is,

  $A \cup B = \{ x : x \in A \text{ or } x \in B \}$

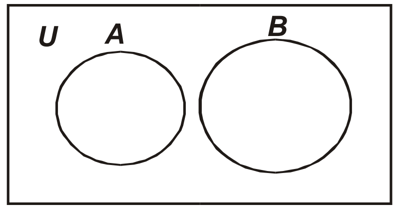


- Intersection of two sets A and B is,

  $A \cap B = \{ x : x \in A \text{ and } x \in B\}$

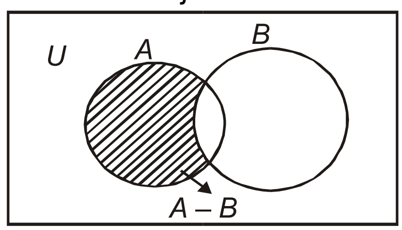


- Disjoint sets: Two sets A and B are said to be disjoint if $A \cap B = \phi$



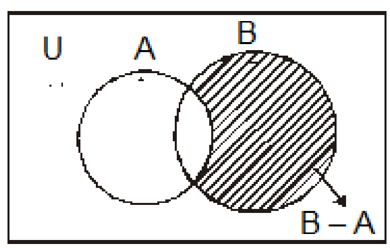
- Difference of sets A and B is,

  $A - B = \{ x : x \in A \text{ and } x \notin B\}$



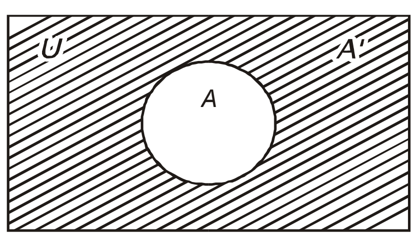
- Difference of sets B and A is,

  $B - A = \{ x : x \in B \text{ and } x \notin A \}$



- Complement of a set A, denoted by A' or A^c is

  $A' = A^c = U - A = \{ x : x \in U \text{ and } x \notin A\}$



- Properties of complement sets :

  1. Complement laws

(i) $A \cup A' = U$ (ii) $A \cap A' = \phi$ (iii) $(A')' = A$

2. De Morgan's Laws

(i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

Note :This law can be extended to any number of sets.

3. $\phi' = \cup$ and $\cup' = \phi$

4. If $A \subset B$ then $B' \subset A'$

Laws of Algebra of sets.

(i) $A \cup \phi = A$

(ii) $A \cap \phi = \phi$

- $A - B = A \cap B' = A - (A \cap B)$
- Commutative Laws :–

  (i) $A \cup B = B \cup A$ (ii) $A \cap B = B \cap A$
- Associative Laws :–

  (i) $(A \cup B) \cup C = A \cup (B \cup C)$

  (ii) $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributive Laws :–

  (i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

  (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- If $A \subset B$, then $A \cap B = A$ and $A \cup B = B$
- When *A* and *B* are disjoint $n(A \cup B) = n(A) + n(B)$
- When A and B are not disjoint $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) -$

$n(A \cap C) + n(A \cap B \cap C)$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Which of the following are sets? Justify your answer.

1. The collection of all the months of a year beginning with letter M
2. The collection of difficult topics in Mathematics.

Let A = {1,3,5,7,9}. Insert the appropriate symbol $\in$ or $\notin$ in blank spaces: – (Question- 3,4)

3. (i) 2 _____ A (ii) {3} _____ A (iii) {3, 5} _____ A
4. Write the set A = { x : x is an integer, $-1 \leq x < 4$} in roster form
5. Write the set B = {3,9,27,81} in set-builder form.

Which of the following are empty sets? Justify. (Question- 6,7)

6. A = { x : x $\in$ N and 3 <x <4}
7. B = { x : x $\in$ N and $x^2 = x$}

Which of the following sets are finite or Infinite? Justify. (Question-8, 9)

8. The set of all the points on the circumference of a circle.
9. B = { x : x $\in$ N and x is an even prime number}
10. Are sets A = { –2,2}, B = { x : x $\in$ Z, $x^2 - 4 = 0$} equal? Why?
11. Write (–5,9] in set-builder form
12. Write {x : x $\in$ R, $-3 \leq x < 7$} as interval.
13. If A = { 1,3,5}, how many elements has P(A)?
14. Write all the possible subsets of A = {5,6}.

If A = Set of letters of the word 'DELHI' and B= the set of letters the words 'DOLL' find (Question- 17,18,19)

15. $A \cup B$

16. $A \cap B$

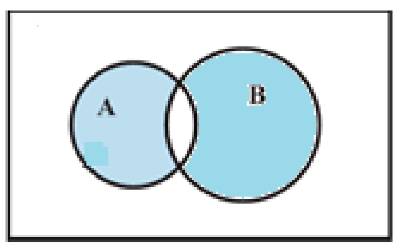
17. $A - B$

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

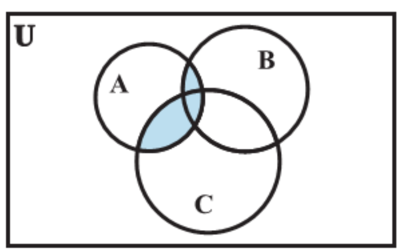
18. Are sets A = {1,2,3,4}, B = { x : x $\in$ N and 5 $\leq$ x $\leq$7} disjoint? Why?

What is Represented by the shaded regions in each of the following Venn-diagrams. (Question 19,20)

19.



20.



SHORT ANSWER TYPE QUESTIONS (4 MARKS)

21. If A = { 1, 3, 5, 7, 11, 13, 15, 17}

B = { 2, 4, 6, 8 ... 18}

and $\cup$ is universal set then find $A' \cup [(A \cup B) \cap B']$

22. Two sets A and B are such that

'n' $(A \cup B) = 21$ 'n' $(A) = 10$ 'n' $(B) = 15$ find 'n'$(A \cap B)$ and 'n' $A - B)$

23. Let $A = \{1, 2, 4, 5\}$ $B = \{2, 3, 5, 6\}$ $C = \{4, 5, 6, 7\}$ Verify the following identity

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

24. If $U = \{x : x \in N \text{ and } x \leq 10\}$

$A = \{x : x \text{ is prime and } x \leq 10\}$

$B = \{x : x \text{ is a factor of } 24\}$

Verify the following result

(i) $A - B = A \cap B'$

(ii) $(A \cup B)' = A' \cap B'$

(iii) $(A \cap B)' = A' \cup B'$

25. Find sets A,B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non-empty sets and $A \cap B \cap C = \phi$

26. For any sets A and B show that

(i) $(A \cap B) \cup (A-B) = A$ (ii) $A \cup (B-A) = A \cup B$

27. On the Real axis, If A=[0,3]and B=[2,6], than find the following

(i) A' (ii) $A \cup B$

(iii) $A \cap B$ (iv) $A-B$

28. In a survey of 450 people, it was found that 110 play cricket, 160 play tennis and 70 play both cricket as well as tennis. How many play neither cricket nor tennis?

29. In a group of students, 225 students know French, 100 know Spanish and 45 know both. Each student knows either French or Spanish. How many students are there in the group?

30. For all set A, B and C is $(A \cap B) \cup C = A \cap (B \cup C)$? Justify your answer.

31. Two sets A and B are such that $n(A \cup B)=21$, $n(A' \cap B')=9$, $n(A \cap B)=7$ find $n(A \cap B)'$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

32. In a town of 10,000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B and 10% families by newspaper C. 5% families buy A and B, 3%, buy B and C and 4% buy A and C. If 2% families buy all the three newspapers, find the no of families which buy(1) A only (2) B only (3) none of A, B and C (4) exactly two newspapers (5) exactly one newspaper (6) A and C but not B (7) at least one of A,B, C. What is the importance of reading newspaper?

33. In a group of 84 persons, each plays at least one game out of three viz. tennis, badminton and cricket. 28 of them play cricket, 40 play tennis and 48 play badminton. If 6 play both cricket and badminton and 4 play tennis and badminton and no one plays all the three games, find the number of persons who play cricket but not tennis. What is the importance of sports in daily life?

34. In a class, 18 students took Physics, 23 students took Chemistry and 24 students took Mathematics of these 13 took both Chemistry and Mathematics, 12 took both Physics and Chemistry and 11 took both Physics and Mathematics. If 6 students offered all the three subjects, find:

    (1) The total number of students.

    (2) How many took Maths but not Chemistry.

(3) How many took exactly one of the three subjects.

35. Using properties of sets and their complements prove that

(1) $(A\cup B)\cap (A\cup B') = A$

(2) $A - (A \cap B) = A - B$

(3) $(A\cup B) - C = (A - C) \cup (B - C)$

(4) $A - (B\cup C) = (A - B) \cap (A - C)$

(5) $A\cap (B - C) = (A\cap B) - (A\cap C)$.

36. If A is the set of all divisors of the number 15. B is the set of prime numbers smaller than 10 and C is the set of even number smaller than 9, then find the value of $(A\cup C) \cap B$.

37. Two finite sets have m and n elements. The total number of subsets of first set is 56 more than the total number of subsets of the second set. Find the value of m and n.

38. If $\begin{aligned} X &= \{4^n - 3n - 1 \quad : \quad n \in N\} \\ Y &= \{9(n-1) \quad : \quad n \in N\} \end{aligned}$

Find the value of $X\cup Y$

39. A survey show that 63% people watch news channel A whereas 76% people watch news channel B. If x% of people watch both news channels, then prove that $39 \le x \le 63$.

40. From 50 students taking examination in Mathematics, Physics and chemistry, each of the student has passed in at least one of the subject, 37 passes Mathematics, 24 Physics and 43 chemistry. At most 19 passed Mathematics and Physics, almost 29 Mathematics and chemistry and at most 20 Physics and chemistry. What is the largest passible number that could have passes in all the three subjects?

ANSWERS

1. Set
2. Not a set
3. (i) $\notin$ (ii) $\notin$ (iii) $\notin$
4. A = {–1, 0, 1, 2, 3}
5. B = { x : x = 3^n , n $\in$ N and 1 $\leq$ n $\leq$ 4}
6. Empty set because no natural number is lying between 3 and 4
7. Non-empty set because B = {1}
8. Infinite set because circle is a collection of infinite points whose distances from the centre is constant called radius.
9. Finite set because B = {2}
10. Yes, because $x^2 - 4 = 0; x = 2, -2$ both are integers
11. {x: x $\in$ R, –5 < x $\leq$ 9}
12. [–3,7)
13. $2^3 = 8$
14. $\phi, \{5\}, \{6\}, \{5,6\}$
15. A $\cup$ B = {D, E, L, H, I, O}
16. A $\cap$ B = {D, L}
17. A – B = {E, H, I}
18. Yes, because A $\cap$ B = ϕ
19. (A –B)$\cup$ (B – A)
20. (A $\cup$ B) $\cap$ C
21. $\cup$
22. n(A $\cap$ B) = 4, n(A – B) = 6
25. A = {1, 2}, B = {1, 3} C = {2, 3}
27. (i) $(-\infty, 0) \cup (3, \infty)$ (ii) [0, 6]

28. **Hint :** $\cup$ = set of people surveyed

A – set of people who play cricket

B = set of people who play tennis

Number of people who play neither cricket nor tennis

$= n [(A \cup B)'] = n(U) - n (A \cup B)$

$= 450 - 200$

$= 250$

29. There are 280 students in the group.

30. No, For example A = {1,2}, B = {2,3},C = {3.4}

31. 23

32. (i) 3300 (ii) 1400 (iii) 4000 (iv) 800 (v) 4800 (vi) 400 (vii) 5800

33. 6

34. (i) 35 (ii) 11 (iii) 11

36. {2,3,5}

37. n = 3 m = 6

38. Y

40. 14

CHAPTER – 2

RELATIONS AND FUNCTIONS

KEY POINTS

- Cartesian Product of two non-empty sets A and B is given by, $A \times B = \{ (a, b) : a \in A, b \in B\}$
- If $(a, b) = (x, y)$, then $a = x$ and $b = y$
- Relation R from a non-empty set A to a non-empty set B is a subset of $A \times B$.
- Domain of $R = \{a : (a, b) \in R\}$
- Range of $R = \{ b : (a, b) \in R\}$
- Co-domain of R = Set B
- Range $\subseteq$ Co-domain
- If $n(A) = p$, $n(B) = q$ then $n(A \times B) = pq$ and number of relations $= 2^{pq}$
- Image : If the element x of A corresponds to $y \in B$ under the function f, then we say that y is image of x under 'f'

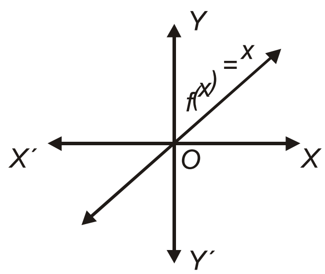
  $\Rightarrow \quad f(x) = y$
- If $f(x) = y$, then x is preimage of y.
- A relation f from a set A to a set B is said to be a function if every element of set A has one and only one image in set B.

- $D_f = \{x : f(x) \text{ is defined}\}$ $R_f = \{f(x) : x \in D_f\}$

- Let A and B be two non-empty finite sets such that n(A) = p and n(B) = q then number of functions from A to B = q^p.

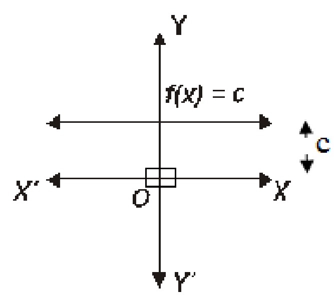
- Identity function, $f : R \to R$; $f(x) = x \ \forall \ x \in R$, where R is the set of realnumbers.

  $D_f = R$ $R_f = R$



- Constant function, $f : R \to R$; $f(x) = c \ \forall \ x \in R$ where c is a constant

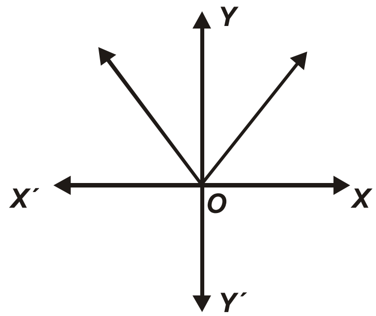
  $D_f = R$ $R_f = \{c\}$



- Modulus function, $f : R \to R$; $f(x) = |x| \ \forall \ x \in R$

  $D_f = R$

  $R_f = R^+ \cup \{0\} = \{ x : x \in R : x \geq 0\}$



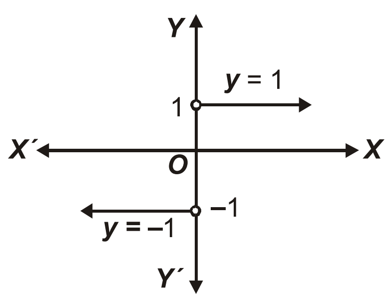
- Signum function

$$f : R \to R \; ; \; f(x) = \begin{cases} 1, \text{if } x > 0 \\ 0, \text{if } x = 0 \\ 1, \text{if } x < 0 \end{cases} \quad \text{and} \quad f(x) = \begin{cases} \dfrac{|x|}{x}, x \neq 0 \\ 0, x = 0 \end{cases}$$

Then

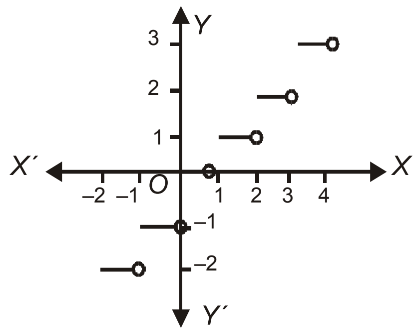
$D_f = R$

and $R_f = \{-1, 0, 1\}$



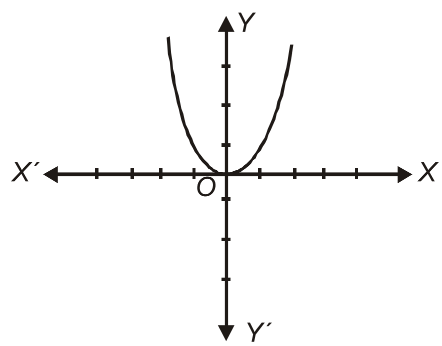
- Greatest Integer functionf : $R \to R$; $f(x) = [x]$, $x \in R$ assumes the value of the greatest integer, less than or equal to x

$D_f = R$ $\quad\quad$ $R_f = Z$



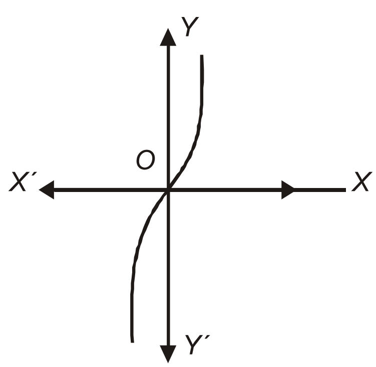
- $f : R \to R$, $f(x) = x^2$

  $D_f = R$ $R_f = [0, \infty)$



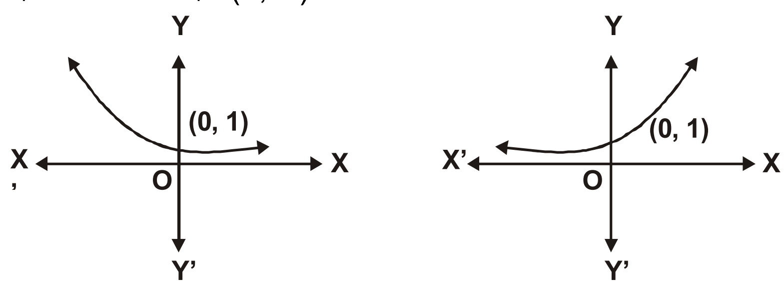
- $f : R \to R$, $f(x) = x^3$

  $D_f = R$ $R_f = R$



- Exponential function, $f : R \to R$; $f(x) = a^x$, $a > 0$, $a \neq 1$

$D_f = R$ $\quad$ $R_f = (0, \infty)$

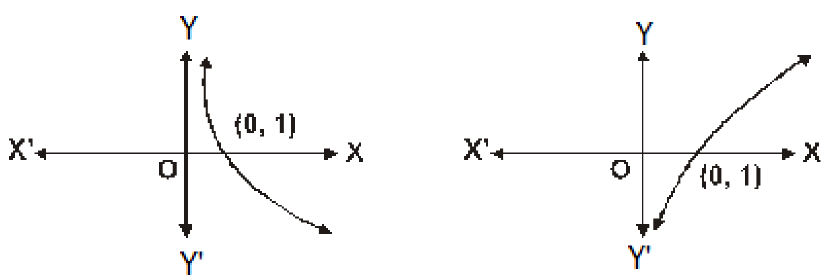


0 < a < 1 $\qquad$ **a > 1**

- Natural exponential function, $f(x) = e^x$

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\infty, 2 < e < 3$$

- Logarithmic functions, $f : (0, \infty) \rightarrow R$; $f(x) \log_a x$, $a > 0$, $a \neq 1$



$D_f = (0, \infty)$ $\quad$ $R_f = R$

- Natural logarithrnic function $f(x) = \log_e x$ or $\log x$.

- Let $f : X \rightarrow R$ and $g : X \rightarrow R$ be any two real functions where $x \subset R$ then

$(f \pm g)(x) = f(x) \pm g(x) \ \forall \ x \in X$

$(fg)(x) = f(x)\, g(x) \ \forall \ x \in X$

$$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)} \quad \forall\ x\in X \ \ provided\ \ g(x)\neq 0$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find a and b if (a – 1, b + 5) = (2, 3)

   If A = {1,3,5}, B = {2,3}, find : (Question-2, 3)

2. A × B

3. B × A

   Let A = {1,2}, B = {2,3,4}, C = {4,5}, find (Question- 4,5)

4. A × (B $\cap$ C)

5. A × (B $\cup$ C)

6. If P = {1,3}, Q = {2,3,5}, find the number of relations from P to Q

7. If R = {(x,y): x,y $\in$ Z, $x^2 + y^2 = 64$}, then,

   Write R in roster form

   Which of the following relations are functions? Give reason. (Questions 8 to 10)

8. R = { (1,1), (2,2), (3,3), (4,4), (4,5)}

9. R = { (2,1), (2,2), (2,3), (2,4)}

10. R = { (1,2), (2,5), (3,8), (4,10), (5,12), (6,12)}

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

11. If A and B are finite sets such that n(A) = 5 and n(B) = 7, then find the number of functions from A to B.

12. If $f(x) = x^2 - 3x + 1$ find $x \in R$ such that $f(2x) = f(x)$

Let f and g be two real valued functions, defined by, $f(x) = x$, $g(x) = |x|$.

Find: (Question 13 to 16)

13. f + g

14. f – g

15. fg

16. $\frac{f}{g}$

17. If $f(x) = x^3$, find the value of,

$$\frac{f(5)-f(1)}{5-1}$$

18. Find the domain of the real function,

$$f(x) = \sqrt{x^2 - 4}$$

19. Find the domain of the function, $f(x) = \dfrac{x^2+2x+3}{x^2-5x+6}$

Find the range of the following functions. (Question- 20,21)

20. $f(x) = \dfrac{1}{4-x^2}$

21. $f(x) = x^2 + 2$

22. Find the domain of the relation,

$R = \{(x, y): x, y \in Z, xy = 4\}$

Find the range of the following relations: (Question-23, 24)

23. $R = \{(a,b) : a, b \in N \text{ and } 2a + b = 10\}$

24. $R = \left\{\left(x, \frac{1}{x}\right) : x \in z, 0 < x < 6\right\}$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

25. Let A = {1,2,3,4}, B = {1,4,9,16,25} and R be a relation defined from A to B as,

$R = \{(x, y): x \in A, y \in B \text{ and } y = x^2\}$

(a) Depict this relation using arrow diagram.

(b) Find domain of R.

(c) Find range of R.

(d) Write co-domain of R.

26. If A = {2,4,6,9} B = {4,6,18,27,54} and a relation R from A to B is defined by R = {(a,b): a$\in$ A, b$\in$ B, a is a factor of b and a < b}, then find in Roster form. Also find its domain and range.

27. Let $f(x) = \begin{cases} x^2, \text{when } 0 \le x \le 2. \\ 2x, \text{when } 2 \le x \le 5. \end{cases}$

$g(x) = \begin{cases} x^2, \text{when } 0 < x \le 3. \\ 2x, \text{when } 3 \le x \le 5. \end{cases}$

Show that f is a function while g is not a function.

28. Find the domain and range of,

$f(x) = |2x - 3| - 3$

29. Draw the graph of the Greatest Integer function

30. Draw the graph of the Constant functionf : R $\rightarrow$ R; f(x) = 2 $\forall$ x $\in$ R. Also find its domain and range.

31. Draw the graph of the function $|x - 2|$

Find the domain and range of the following real functions (Question 32 to 37)

32. $f(x) = \sqrt{x^2 + 4}$

33. $f(x) = \dfrac{x+1}{x-2}$

34. $f(x) = \dfrac{|x+1|}{x-1}$

35. $f(x) = \dfrac{x^2-9}{x-3}$

36. $f(x) = \dfrac{4-x}{x-4}$

37. $f(x) = 1 - |x-3|$

38. Determine a quadratic function (f) is defined by $f(x) = ax^2 + bx + c$. If f(0) = 6; f(2) = 11, f(–3) = 6

39. Draw the graph of the function $f(x) = \begin{cases} 1+2x & x < 0. \\ 3+5x & x \geq 0. \end{cases}$ also find its range.

40. Draw the graph of following function

$$f(x)=\begin{cases}\dfrac{|x|}{x} & x \neq 0. \\ 0 \quad x=0. \end{cases}$$

Also find its range.

Find the domain of the following function.

41. $f(x)=\dfrac{1}{\sqrt{x+|x|}}$

42. $f(x)=\dfrac{1}{\sqrt{x-|x|}}$

43. $f(x) \quad \dfrac{1}{\sqrt{[x]^2 \quad [x] \quad 6}}$

44. $f(x)=\dfrac{1}{\sqrt{9-x^2}}$

45. $f(x)=\sqrt{4-x}+\dfrac{1}{\sqrt{x^2-1}}$

46. Find the domain for which the followings:

    $f(x)=2x^2-1$ and $g(x)=1-3x$ are equal.

47. If $f(x)=x-\dfrac{1}{x}$ prove that $\left[f(x)\right]^3=f(x^3)-3f\left(\dfrac{1}{x}\right)$.

48. If [x] denotes the greatest integer function. Find the solution set of equation.

    $[x]^2-5[x]+6=0$

49. If $f(x)=\dfrac{ax-b}{bx-a}=y$

    Find the value of f(y)

50. Draw the graph of following function and find range $(\mathrm{R_f})$ of

$f(x) = |x-2| + |2-x| \quad \forall \quad -3 \leq x \leq 3$

ANSWERS

1. a = 3, b = –2
2. A × B = {(1,2), (1,3), (3,2), (3,3), (5,2), (5,3)}
3. B × A = { (2,1), (2,3), (2,5), (3,1), (3,3), (3,5)}
4. {(1,4), (2,4)}
5. {(1,2), (1,3), (1,4), (1,5), (2,2), (2,3), (2,4), (2,5)}
6. $2^6 = 64$
7. R = { (0,8), (0,–8), (8,0), (–8,0)}
8. Not a function because 4 has two images.
9. Not a function because 2 does not have a unique image.
10. Function because every element in the domain has its unique image.
11. 7^5
12. 0,1
13. $f+g = \begin{cases} 2x & x \geq 0 \\ 0 & x < 0 \end{cases}$

14. $f-g = \begin{cases} 0 & x \geq 0 \\ 2x & x < 0 \end{cases}$

15. $fg = \begin{cases} x^2 & x \geq 0 \\ -x^2 & x < 0 \end{cases}$

16. $\frac{f}{g} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$ and Note -: $\frac{f}{g}$ is not defined at x = 0

17. 31
18. (–∞, –2] ∪ [2, ∞)

19. $R - \{2,3\}$

20. $(-\infty, 0) \cup [1/4, \infty)$

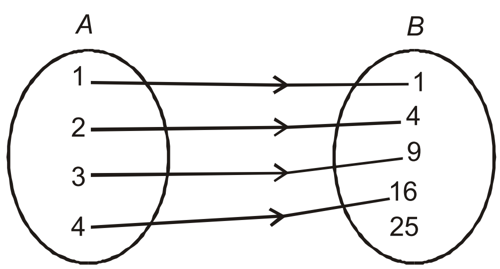
21. $[2,\infty)$

22. $\{-4, -2, -1, 1, 2, 4\}$

23. $\{2,4,6,8\}$

24. $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right\}$

25. (a)



(b) $\{1,2,3,4\}$

(c) $\{1,4,9,16\}$

(d) $\{1,4,9,16,25\}$

26. $R = \{ (2,4)\ (2,6)\ (2,18)\ (2,54)\ (6,18)\ (6,54)\ (9,18)\ (9,27)\ (9,54) \}$

Domain is $R = \{2,6,9\}$

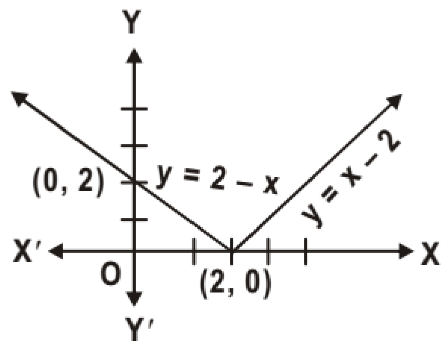
Range of $R = \{ 4, 6, 18, 27, 54\}$

28. Domain is R

Range is $[-3, \infty)$

30. Domain = R, Range = $\{2\}$

31.



32. .Domain = R

Range =[2, ∞)

33. Domain = R – {2}

Range = R – {1}

34. Domain = R

Range = {1, –1}

35. Domain = R – {3}

Range = R – {6}

36. Domain = R – {4}

Range = {–1}

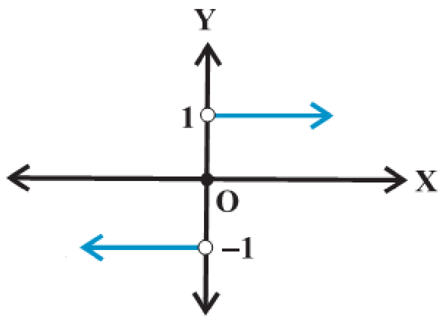
37. Domain = R

Range =(–∞, 1]

38. $\frac{1}{2}x^2+\frac{3}{2}x+6$

39. $(-\infty,1)\cup[3,\infty)$

40. Range of f = {–1,0,1}



41. $(0,\infty)$

42. ϕ (given function is not defined)

43. $(-\infty,-2)\cup[4,\infty)$

44. (–3,3)

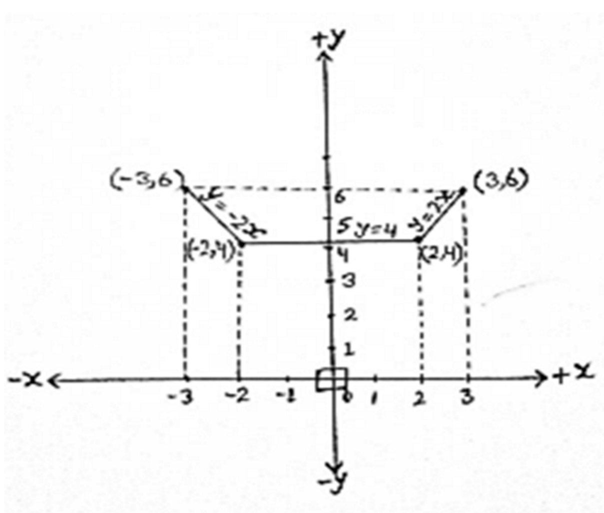
45. $(-\infty,-1)\cup(1,4]$

46. $\left\{-2,\frac{1}{2}\right\}$

48. [2,4)

49. x

50. $R_f = [4,6]$ and graph is



CHAPTER - 3

TRIGONOMETRIC FUNCTIONS

KEY POINTS

- A radian is an angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. We denote 1 radian by 1^c.

- π radian = 180 degree, 1° = 60'

  1 radian = $\frac{180}{\pi}$ degree and 1' = 60''

  1 degree = $\frac{\pi}{180}$ radian

- If an arc of length l makes an angle qradian at the centre of a circle of radius r, we have

  $$\theta = \frac{l}{r}$$

Quadrant →	I	II	III	IV
t- functions which are positive	All	sin x cosec x	tan x cot x	cos x sec x

Function	–x	$\frac{\pi}{2} - x$	$\frac{\pi}{2} + x$	$\pi - x$	$\pi + x$	$2\pi - x$	$2\pi + x$
sin	–sin x	cos x	cos x	sin x	–sin x	–sin x	sin x
cos	cos x	sin x	–sin x	–cos x	–cos x	cos x	cos x
tan	–tan x	cot x	–cot x	–tan x	tan x	–tan x	tan x
cosec	–cosec x	sec x	sec x	cosec x	–cosec x	–cosec x	cosec x
sec	sec x	cosec x	–cosec x	–sec x	–sec x	sec x	sec x
cot	–cot x	tan x	–tan x	–cot x	cot x	–cot x	cot x

Function	Domain	Range
sin x	R	[–1,1]
cos x	R	[–1,1]
tan x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R
Cosec x	$R - \{n\pi; n \in z\}$	R – (–1,1)
Sec x	$R - \left\{(2n+1)\frac{\pi}{2}; n \in z\right\}$	R – (–1,1)
cot x	$R - \{n\pi, n \in z\}$	R

Some Standard Results

- sin (x + y) = sinx cosy + cosx siny

  cos (x + y) = cosx cosy   sinx siny

  $$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x . \tan y}$$

$$\cot(x + y) = \frac{\cot x.\cot y\text{-}1}{\cot y + \cot x}$$

- $\sin (x \quad y) = \sin x \cos y \quad \cos x \sin y$

  $\cos (x \quad y) = \cos x \cos y + \sin x \sin y$

  $$\tan(x \;\, y) = \frac{\tan x - \tan y}{1 + \tan x.\tan y}$$

  $$\cot(x \;\, y) = \frac{\cot x.\cot y + 1}{\cot y - \cot x}$$

- $$\tan(x + y + z) = \frac{\tan x + \tan y + \tan z - \tan x \tan y \tan z}{1 - \tan x \tan y - \tan y.\tan z - \tan z \tan x}$$

- $2\sin x \cos y = \sin(x + y) + \sin(x \quad y)$

  $2\cos x \sin y = \sin(x + y) \quad \sin(x \quad y)$

  $2\cos x \cos y = \cos(x + y) + \cos(x \quad y)$

  $2\sin x \sin y = \cos(x \quad y) \quad \cos(x + y)$

- $\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$

  $\sin x \quad \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$

  $\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$

  $\cos x \quad \cos y = \quad 2 \sin \frac{x+y}{2} \sin \frac{x-y}{2} = 2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{y-x}{2}\right)$

- $\sin 2x = 2 \sin x \cos x = \dfrac{2 \tan x}{1 + \tan^2 x}$

- $\cos 2x = \cos^2 x \quad \sin^2 x = 2 \cos^2 x \quad 1 = 1 \quad 2\sin^2 x = \dfrac{1 - \tan^2 x}{1 + \tan^2 x}$

- $\tan 2x = \dfrac{2 \tan x}{1 - \tan^2 x}$

- $\sin 3x = 3 \sin x \quad 4 \sin^3 x$

- $\cos 3x = 4 \cos^3 x \quad 3 \cos x$

- $\tan 3x = \dfrac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}$

- $\sin(x + y) \sin(x \quad y) = \sin^2 x \quad \sin^2 y = \cos^2 y \quad \cos^2 x$

- $\cos(x + y) \cos(x \quad y) = \cos^2 x \quad \sin^2 y = \cos^2 y \quad \sin^2 x$

- General solution A solution of a trigonometric equation, generalised by means of periodicity, is known as the general solution.

- Principal solutions The solutions of a trigonometric equation which lie in $[0, 2\pi)$ are called its principal solutions.

- $\sin \theta = 0 \Rightarrow \theta = n\pi$, where $n \in z$

- $\cos \theta = 0 \Rightarrow \theta = (2n + 1)\dfrac{\pi}{2}$, where $n \in z$

- $\tan \theta = 0 \Rightarrow \theta = n\pi$, where $n \in z$

- $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (\quad 1)^n \alpha$, where $n \in z$

- $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$, where $n \in z$

- $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$, where $n \in z$

$$\sin 18° = \frac{\sqrt{5}-1}{4}; \qquad \cos 18° = \frac{\sqrt{10+2\sqrt{5}}}{4}$$

$$\sin 36° = \frac{\sqrt{10-2\sqrt{5}}}{4}; \qquad \cos 36° = \frac{\sqrt{5}+1}{4}$$

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the radian measure of 5° 37' 30".
2. Write the value of $\left(\frac{11}{16}\right)^c$.
3. Write the value of $\tan\left(\frac{19\pi}{3}\right)$
4. What is the value of sin (1125°)?
5. What is the radian measure of 37° 30'?
6. Write the general salection of $\sin\left(x+\frac{\pi}{12}\right)=0$
7. Write the value of 2sin 75° sin 15°?
8. What is the maximum value of 3 7 cos 5*x?*
9. Express sin 12θ + sin 4θ as the product of sines and cosines.
10. Express 2 cos 4*x* sin 2*x* as an algebric sum of sines and cosines
11. Write the maximum value of cos (cos *x*).
12. Write the minimum value of cos (cos *x*).
13. Write the radian measure of 22° 30'
14. Write the value of $\tan\frac{\pi}{12}$

15. Write the value of $\tan\frac{\pi}{8}$

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

16. Find the length of an arc of a circle of radius 5cm subtending a central angle measuring 15°.

17. If $\sin A \quad \frac{3}{5}$ and $\frac{\pi}{2} < A < \pi$ Find $\cos A$, $\sin 2A$

18. What is the sign of cos x/2   sin x/2 when

(i) $0 < x < \pi/4$ (ii) $\frac{\pi}{2} < x < \pi$

19. Prove that cos 510° cos 330° + sin 390° cos 120° =   1

20. If $A + B = \pi/4$ from that $(1 + \tan A)(1 + \tan B) = 2$

21. Find the maximum and minimum value of $7 \cos x + 24 \sin x$

22. Evaluate $\sin(\pi + x) \sin(\pi \quad x) \operatorname{cosec}^2 x$

23. Find the angle in radians between the hands sof a clock at 7 : 20 pm.

24. If $\cot\alpha = \frac{1}{2}$ $\sec\beta = \frac{-5}{3}$ where $\pi < \alpha < 3\pi/2$ and $\frac{\quad}{2}$. Find the value of $\tan(\alpha + \beta)$

25. If $\cos x = \frac{-1}{3}$ and $\pi < x < \frac{3\pi}{2}$. Find the value of $\cos x/2$, $\tan x/2$

26. If $\tan A = \frac{a}{a+1}$ and $\tan B = \frac{1}{2a+1}$ then find the value of $A + B$

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

27. Find angle in radians between the hands of a clock at 7:20 pm.

28. A hourse is tied to a post by a rope. If the horse moves along a circular path, always keeping the rope tight and describes 88 metres when it traces 72° at the centre, find the length of the rope.

29. Find the minimum and maximum value of

$\text{Sin}^4 x + \cos^2 x; x \in R.$

30. Solve sec x. cos $5x + 1 = 0$

31. Solve $2 \tan^2 x + \sec^2 x = 2$ for $0 < x^2 \pi$.

32. Solve $\sqrt{3}\cos x \quad \sin x \quad 1.$

33. Solve $\sqrt{2}\sec \quad\quad \tan \quad\quad \sqrt{3}.$

34. Solve $3 \tan x + \cot x = 5 \operatorname{cosec} x$.

35. Find x if $3 \tan (x \quad 15°) = \tan (x + 15°)$

36. Solve $\tan x \quad \tan 2x \quad \sqrt{3} \tan x . \tan 2x \quad \sqrt{3}.$

37. Solve $\tan x \quad \sec x \quad \sqrt{3}.$

38. If $\sec x \quad \sqrt{2}$ and $\frac{3}{2} \quad x \quad 2$, find the value of $\frac{1 \quad \tan x \quad \operatorname{cosec} x}{1 \quad \cot x \quad \operatorname{cosec} x}$.

39. Prove that $\sin 10° \sin 30° \sin 50° \sin 70° = \frac{1}{16}$.

40. If $f(x) = \frac{\cot x}{1 \quad \cot x}$ and $\alpha + \beta = \frac{5\pi}{4}$ then find $f(\alpha). f(\beta)$.

41. Prove that $\tan 70° = \tan 20° + 2 \tan 50°$

42. Prove that $\tan 13x = \tan 4x + \tan 9x + \tan 4x \tan 9x \tan 13x$.

Prove the following Identities

43. $\dfrac{\tan 5\theta + \tan 3\theta}{\tan 5\theta - \tan 3\theta} = 4\cos 2\theta.\cos 4\theta$.

44. $\dfrac{\cos x + \sin x}{\cos x - \sin x} - \dfrac{\cos x - \sin x}{\cos x + \sin x} = 2\tan 2x$.

45. $\dfrac{\cos 4x \sin 3x - \cos 2x \sin x}{\sin 4x.\sin x + \cos 6x.\cos x} = \tan 2x$.

46. $\dfrac{1 + \sin\theta - \cos\theta}{1 + \sin\theta + \cos\theta} = \tan\dfrac{\theta}{2}$.

47. $\tan\alpha.\ \tan(60° - \alpha).\ \tan(60° + \alpha) = \tan 3\alpha$.

48. $\sqrt{2 + \sqrt{2 + 2\cos 4\theta}} = 2\cos\theta$.

49. $\dfrac{\cos x}{1 - \sin x} = \tan\left(\dfrac{\pi}{4} + \dfrac{x}{2}\right)$.

50. $\cos 10° + \cos 110° + \cos 130° = 0$.

51. $\dfrac{\sin(x+y) - 2\sin x + \sin(x-y)}{\cos(x+y) - 2\cos x + \cos(x-y)} = \tan x$.

52. $\sin x + \sin 2x + \sin 4x + \sin 5x = 4\cos\dfrac{x}{2}.\cos\dfrac{3x}{2}.\sin 3x$.

53. $\dfrac{\sec 8\theta - 1}{\sec 4\theta - 1} = \dfrac{\tan 8\theta}{\tan 2\theta}$

54. Find the value of $\sqrt{3}$ cosec 20° sec 20°

55. $\cos\dfrac{\pi}{5}.\cos\dfrac{2\pi}{5}.\cos\dfrac{4\pi}{5}.\cos\dfrac{8\pi}{5} = \dfrac{1}{16}$.

56. $\cos 20°.\cos 40°.\cos 80° = \dfrac{1}{8}$.

Find the general solution of the following equations (Q.No. 57 to Q. No. 59)

57. sin 7x = sin 3x.

58. cos 3x   sin 2x = 0.

59. $\sin x - 3\sin 2x + \sin 3x = \cos x - 3\cos 2x + \cos 3x$.

60. Draw the graph of cosx, sin x and tan x in [0, 2π].

61. Draw sin x, sin 2x and sin 3x on same graph and with same scale.

62. Evaluate:

(ii) Cos 36° (ii) $\tan\left(\frac{13\pi}{12}\right)$

63. Evaluate:

$$\cos^4 \frac{\pi}{8} + \cos^4 \frac{3\pi}{8} + \cos^4\left(\frac{5\pi}{8}\right) + \cos^4\left(\frac{7\pi}{8}\right).$$

64. If tan A tan $B = x$, cot B cot $A = y$ prove that cot (A B) = $\frac{1}{x} + \frac{1}{y}$

65. If $\frac{\sin(x+y)}{\sin(x-y)} = \frac{a+b}{a-b}$ then prove that $\frac{\tan x}{\tan y} = \frac{a}{b}$.

66. If $\cos x = \cos\alpha . \cos\beta$ then prove that

$$\tan\left(\frac{x+\alpha}{2}\right).\tan\left(\frac{x-\alpha}{2}\right) = \tan^2 \frac{\beta}{2}$$

67. If $\tan(\pi \cos\theta) = \cot(\pi \sin\theta)$ then prove that

$$\cos\left(\theta - \frac{\pi}{4}\right) = \pm\frac{1}{2\sqrt{2}}.$$

68. If $\sin(\theta + \alpha) = a$ and $\sin(\theta + \beta) = b$ then prove that

$$\cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = 1 - 2a^2 - 2b^2.$$

69. Find the range of 5 sin x   12 cos x + 7.

70. If α and β are the solution of the equation, $a\tan\theta + b\sec\theta = c$ then show that $\tan(\alpha+\beta) = \dfrac{2ac}{a^2 - c^2}$.

71. Prove that

$$\cos^2 x + \cos^2 y - 2\cos x.\cos y.\cos(x+y) = \sin^2(x+y)$$

72. Prove that

$$2\sin^2\beta + 4\cos(\alpha+\beta)\sin\alpha\sin\beta + \cos 2(\alpha+\beta) = \cos 2\alpha$$

73. Solve

$$81^{\sin^2 x} + 81^{\cos^2 x} = 30 \quad 0 < x < \pi$$

74. Find the minimum value of p for which $\cos(p\sin x) = \sin(p\cos x)$ has a solution in $[0, 2\pi]$.

75. Prove that

$$\cos A \cos 2A \cos 4A \cos 8A = \frac{\sin 16A}{16.\sin A}.$$

76. Solve:

4sinx. sin2x. sin4x = sin3x

77. Solve:

$$\cos\theta \cos 2\theta \cos 3\theta = \frac{1}{4}$$

78. Evaluate:

$$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)$$

79. Prove that

$$4\sin\alpha.\sin\left(\alpha+\frac{\pi}{3}\right).\sin\left(\alpha+\frac{2\pi}{3}\right) = \sin 3\alpha.$$

Answers

1. $\left(\frac{\pi}{32}\right)^C$

2. 39° 22' 30"

3. $\sqrt{3}$

4. $\frac{-1}{\sqrt{2}}$

5. $-\left(\frac{5\pi}{24}\right)^C$

6. $n \quad \frac{}{12}$

7. $\frac{1}{2}$

8. 10

9. 2 sin 8θ cos 4θ

10. sin $6x$ sin $2x$

11. 1

12. cos 1

13. $\frac{\pi}{8}$

14. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

15. $\sqrt{2}-1$

16. 70m

17. $\frac{-4}{5}, \frac{-24}{25}$

18. (i) + ve (ii) ve

21. Max value 25; Min value 25

22. 1

23. $\frac{5\pi}{9}$

24. $\frac{2}{11}$

25. $-1/\sqrt{3}, -2$

26. $\pi/4$

27. $\frac{5\pi}{9}$

28. 70 m

29. $\min = \frac{3}{4}, \quad \max = 1$

30. $x = (2n+1)\frac{\pi}{6}, \quad or \quad x = (2n+1)\frac{\pi}{4}, n \in z$

31. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

32. $2n\pi \pm \frac{\pi}{3} - \frac{\pi}{6}, \quad n \in z$

33. $2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}, \quad n \in z$

34. $2n\pi \pm \frac{\pi}{3}, \quad n \in z$

35. $x = \frac{n\pi}{2} + (-1)^4 \frac{\pi}{4}, \quad n \in z$

36. $\frac{n\pi}{3} + \frac{\pi}{9}$

37, $2n\pi + \frac{\pi}{6}$

38. 1

40. $\frac{1}{2}$

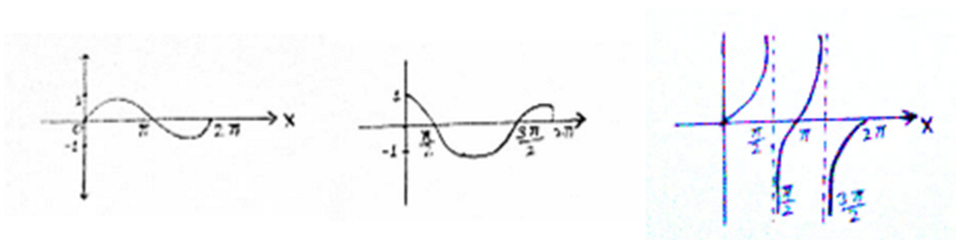
54. 4

57. $(2n+1)\frac{\pi}{10}, \frac{n\pi}{2}, \quad n \in z$

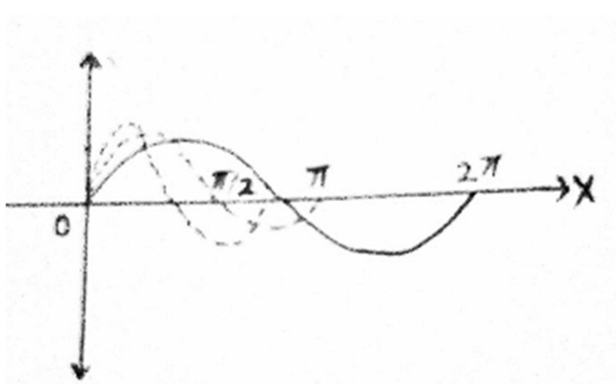
58. $\frac{1}{5}(2n\pi + \frac{\pi}{2}), 2n\pi - \frac{\pi}{2}, \quad n \in z$

59. $x = \frac{n\pi}{2}, \quad n \in z$

60.



61.



62. (i) $\frac{1+\sqrt{5}}{4}$ (ii) $2-\sqrt{3}$

63. $\frac{3}{2}$

69. [6,20]

73. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

74. $\frac{\pi}{\sqrt{8}}, \frac{5\sqrt{2}\pi}{4}$

76. $n\pi,\ n\pi \pm \frac{\pi}{3},\ n \in z$

77. $(2n+1)\frac{\pi}{8},\ n\pi \pm \frac{\pi}{3},\ n \in z$

78. $\frac{1}{8}$

CHAPTER - 4

PRINCIPLE OF MATHEMATICAL INDUCTION

KEY POINTS

- A meaningful sentence which can be judged to be either true or false is called a statement.
- A statement involving mathematical relations is called as mathematical statement.
- Induction and deduction are two basic processes of reasoning.
- Deduction is the application of a general case to a particular case. In contrast to deduction, induction is process of reasoning from particular to general.
- Induction being with observations. From observations we arrive at tentative conclusions called conjectures. The process of induction help in proving the conjectures which may be true.
- Statements like(i) $1 + 2 + 3 + \ldots\ldots\ldots + n = \dfrac{n(n+1)}{2} \;\forall\; \mathrm{n} \in \mathrm{N}.$

  (ii) $2^n \geq 2 \;\forall\; \mathrm{n} \in \mathrm{N}.$

  (iii) If n(A)=n then number of all subsets of $A = 2^n \;\forall\; \mathrm{n} \in \mathrm{N}.$

(iv) $Sn = \frac{2(r^n - 1)}{r-1}$ where Sn is sum of n terms of G.P, a = 1st term and r = common ratio. Are all concerned with n $\in$ N which takes values 1,2, 3,…. Such statements are denoted by P(n). By giving particular values ton, we get particular statement as P(1), P(2),…….. P(k) for some k$\in$N.

Principle of mathematical Induction:

Let P(n) be any statement involving natural number n such that

(i) P(1) is true, and

(ii) If P(k) is true $\Rightarrow$P(k+1) is true for some some k$\in$N. that is P(KH) is true whenever P(K) is true for some k$\in$N then P(n) is true $\forall$ n $\in$ N.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARKS)

1. Let P(n): $n^2 + n$ is even. Is P(1) true?
2. Let P(n): n(n+1) (n+2) is divisible by 3. What is P(3)?
3. Let P(n): $n^2 > 9$. Is P(2) true?

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

4. Give an example of a statement such that P(3) is true but P(4) is not true.
5. If $P(n)$: 1 + 4 + 7+ .... + ($3n$ – 2) = $\frac{1}{2}n$ ($3n$ – 1).Verify $P(n)$ for n = 1, 2,
6. If P(n) is the statement "$n^2 - n + 41$ is Prime" Prove that P(1) and P(2) are the but P(41) is not true.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Prove the following by using the principle of mathematical induction $\forall$ n $\in$N.

Type-1

1. 3.6 + 6.9 + 9.12 + ... + 3n (3n + 3) = 3n(n + 1)(n + 2)

2. $\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)......\left(1-\frac{1}{n+1}\right)=\frac{1}{n+1}$

3. a + (a + d) + (a + 2d) + ... + [a +(n –1) d] = $\frac{n}{2}[2a+(n-1)d]$

HOTS

4. 7 + 77 + 777 + ... + to *n* terms = $\frac{7}{81}(10^{n+1}-9n-10)$

5. $\sin x+\sin 2x+\sin 3x+.........\sin nx=\dfrac{\sin\left(\frac{n+1}{2}x\right)\sin\frac{nx}{2}}{\sin\frac{x}{2}}$.

6. $\sin x+\sin 3x+.........\sin(2n-1)x=\dfrac{\sin^2 nx}{\sin x}$.

7. $\cos\alpha.\cos 2\alpha.\cos 4\alpha...........\cos\left(2^{n-1}\alpha\right)=\dfrac{\sin 2^n\alpha}{2^n\sin\alpha}$.

8. $1^2 \quad 2^2 \quad 3^2 \quad .... \quad n^2 \quad \dfrac{n(n\quad 1)(2n\quad 1)}{6}$.

Type II

9. 2^{3n-1} –1 is divisible by 7.

10. 3^{2n} when divided by 8 leaves the remainder 1.

11. $4^n + 15n - 1$ is divisible by 9.

HOTS

12. $n^3 + (n + 1)^3 + (n + 2)^3$ is a multiple of 9

13. $11^{n+2} + 12^{2n+1}$ is divisible by 133

14. $x^n - y^n$ is divisible by (x–y) if x and y are any two distinct integers.

15. Given that $5^n - 5$ is divisible by 4 $\forall n \in N$. Prove that $2.7^n + 3.5^n - 5$ is a multiple of 24.

16. $7^{2n} + 2^{3n-3}.3^{n-1}$ is divisible by 25.

Type III

17. $2^{n+1} > 2n + 1$

18. $3^n > 2^n$

19. $n < 2^n$

HOTS

20. $1 + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{n^2} < 2 - \frac{1}{n}$.

21. $(1 + x)^n \geq 1 + nx \;\; where \;\; x > -1$.

22. $2^{n+3} \leq (n+3)!$

ANSWER

1. True

2. P(3): 3 (3+1) (3+2) is divisible by 3

3. NO.

4. P(n): $3n^2+n$ is divisible by 3 and soon

5. P(1) and P(2) are true.

CHAPTER - 5

COMPLEX NUMBERS AND QUADRATIC EQUATIONS

KEY POINTS

- The imaginary number $\sqrt{-1} = i$, is called iota
- For any integer k, $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$
- $\sqrt{a} \times \sqrt{b} \neq \sqrt{ab}$ if both a and b are negative real numbers
- A number of the form z = a + ib, where a, b $\in$ R is called a complexnumber.

  a is called the real part of z, denoted by Re(z) and b is called the imaginary part of z, denoted by Im(z)
- a + ib = c + id if a = c, and b = d
- $z_1 = a + ib$, $z_2 = c + id$.

  In general, we cannot compare and say that $z_1 > z_2$ or $z_1 < z_2$ but if b, d = 0 and a > c then $z_1 > z_2$

  i.e. we can compare two complex numbers only if they are purely real.
- $0 + i\,0$ is additive identity of a complex number.
- $-z = -a - ib$ is called the Additive Inverse or negative of $z = a + ib$

- $1 + i\,0$ is multiplicative identity of complex number.
- $\bar{z} = a - ib$ is called the conjugate of z = a + ib
- $i^0 = 1$
- $z^{-1} = \frac{1}{z} = \frac{a - ib}{a^2 + b^2} = \frac{\bar{z}}{|z|^2}$ is called the multiplicative Inverse of

  z = a + ib ($a \neq 0$, $b \neq 0$)
- The coordinate plane that represents the complex numbers is called the complex plane or the Argand plane
- Polar form of z = a + ib is,

  z = r (cosθ + i sinθ) where $r = \sqrt{a^2 + b^2} = |z|$ is called the modulus of z,θ is called the argument or amplitude of z.
- The value of θ such that, $-\pi < \theta < \pi$ is called the principle argument of z.
- Z = x + iy, x > 0 and y > 0 the argument of z is acute angle given by $\tan\alpha = \frac{y}{x}$

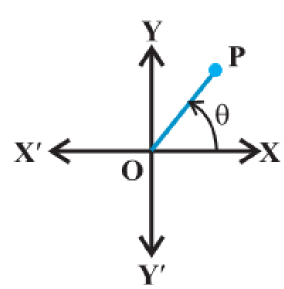
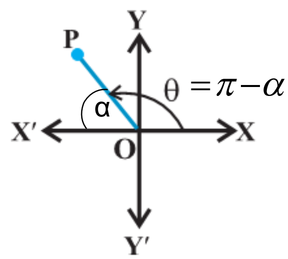


figure (i)

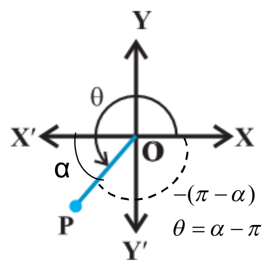
- Z = x + iy, x < 0 and y > 0 the argument of z is $\pi - \alpha$,where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$



$$\theta = \pi - \alpha$$

figure (ii)

- Z = x + iy, x < 0 and y < 0 the argument of z is $\alpha - \pi$,where π is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$



$$\theta = \alpha - \pi$$

figure (iii)

- Z = x + iy, x > 0 and y < 0 the argument of z is $-\alpha$,where α is acute angle given by $\tan\alpha = \left|\frac{y}{x}\right|$

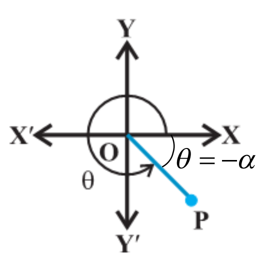


figure (iv)

- $|z_1 + z_2| \le |z_1| + |z_2|$
- $|z_1 z_2| = |z_1| . |z_2|$
- $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}; \left|z^n\right| = |z|^n; |z| = |\bar{z}| = |-z| = |-\bar{z}|; z\bar{z} = |z|^2$
- $|z_1 - z_2| \ge \big||z_1| - |z_2|\big|$
- If $z_1 = r_1 (\cos \theta_1 + i\sin \theta_1)$

  $z_2 = r_2 (\cos \theta_2 + i\sin \theta_2)$

  then $z_1 z_2 = r_1 r_2[\cos (\theta_1 + \theta_2) + i\sin (\theta_1 + \theta_2)]$

  $$\frac{z_1}{z_2} = \frac{r_1}{r_2}\left[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)\right]$$

- For the quadratic equation $ax^2 + bx + c = 0$,

  $a, b, c \in R$, $a \neq 0$, if $b^2 - 4ac < 0$

  then it will have complex roots given by,

  $$x = \frac{-b \pm i\sqrt{4ac - b^2}}{2a}$$

W. R. Hamilton
(1805-1865)

- $\sqrt{a+ib}$ is called square root of $z = a + ib$, $\therefore$ $\sqrt{a+ib} = x + iy$

  squaring both sides we get $a + ib = x^2 - y^2 + 2i(xy)$

$x^2 - y^2 = a$, $2xy = b$. Solving these we get x and y.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the value of $i + i^{10} + i^{20} + i^{30}$

2. Write the additive Inverse of $6i - i\sqrt{-49}$

3. Write the multiplicative Inverse of $2 \quad i\sqrt{3} \;^{2}$

4. Write the conjugate of $\dfrac{2 \quad i}{(1 \quad 2i)^2}$

5. Write the amplitude of $\dfrac{1}{i}$

6. Write the Argument of $(1+\sqrt{3}i)(\cos\theta + i\sin\theta)$

7. Write in the form of a + *ib* $\dfrac{1}{-2+\sqrt{-3}}$

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

8. Evaluate :

   (i) $\sqrt{\;16} \quad 3\sqrt{\;25} \quad \sqrt{\;36} \quad \sqrt{\;625}$

   (ii) $i\sqrt{-16} + i\sqrt{-25} + \sqrt{49} - i\sqrt{-49} + 14$

   (iii) $\left(i^{77} + i^{70} + i^{87} + i^{414}\right)^3$

   (iv) $\dfrac{(3+\sqrt{5}i)(3-\sqrt{5}i)}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-\sqrt{2}i)}$

9. Find x and y if (x + iy) (2 – 3i) = 4 + i.

10. If n is any positi8ve integer, write value of $\dfrac{i^{4n+1} - i^{4n-1}}{2}$

11. If $z_1 = \sqrt{2}(\cos 30° + i \sin 30°)$, $z_2 = \sqrt{3}(\cos 60 + i \sin 30°)$ Find

$R_e\,(z_1 z_2)$

12. If $|z+4| \le 3$ then find the greatest and least values of $|z+1|$.

13. Find the real value of a for which $3i^3 - 2ai^2 + (1-a)i + 5$ is real.

14. If $\arg(z-1) = \arg(z+3i)$ where $z = x + iy$ find $x - 1 : y$.

15. If z = x + iy and the amplitude of $(z - 2 - 3i)$ is $\frac{\pi}{4}$. Find the relation between x and y.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. If x + iy = $\sqrt{\frac{1+i}{1-i}}$ prove that $x^2 + y^2 = 1$

17. Find real value of θ such that, $\frac{1 + i\cos\theta}{1 - 2i\cos\theta}$ is a real number.

18. If $\left|\frac{z-5i}{z+5i}\right| = 1$ show that z is a real number.

19. If $x_n = \cos\frac{\pi}{2^n} + i\sin\frac{\pi}{2^n}$ Prove that $x_1 x_2 \ldots\ldots x_\infty = -1$

20. Find real value of x and y if $\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$.

21. If $(1+i)(1+2i)(1+3i)\ldots\ldots(1+ni) = x + iy$. Show,

    $2.5.10\ldots\ldots(1+n^2) = x^2 + y^2$

22. If z = 2 – 3i show that $z^2 - 4z + 13 = 0$, hence find the value of $4z^3 - 3z^2 + 169$.

23. If $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = a + ib$, find a and b.

24. For complex numbers $z_1 = 6 + 3i$, $z_2 = 3 - i$ find $\frac{z_1}{z_2}$.

25. If $\left(\frac{2+2i}{2-2i}\right)^n = 1$, find the least positive integral value of n

26. If $(x+iy)^{\frac{1}{3}} = a+ib$ prove $\left(\frac{x}{a}+\frac{y}{b}\right) = 4\left(a^2-b^2\right)$.

27. Convert the following in polar form:

(i) $-3\sqrt{2}+3\sqrt{2}i$ (ii) $\frac{\left(\sqrt{3}-1\right)-\left(\sqrt{3}+1\right)i}{2\sqrt{2}}$

(iii) $i(1+i)$ (iv) $\frac{5-i}{2-3i}$

28. Solve

(i) $x^2-\left(3\sqrt{2}-2i\right)x-6\sqrt{2}i=0$

(ii) $x^2-(7-i)x+(18-i)=0$

29. Find the square root of $7-30\sqrt{-2}$.

30. Prove that $x^2 + 4 = (x + 1 + i)(x + 1 - i)(x - 1 + i)(x - 1 - i)$

31. Show that $\left|\frac{z-2}{z-3}\right| = 2$ represent a circle find its centre and radius.

32. Find all non-zero complex number z satisfying $\bar{z} = iz^2$.

33. If $iz^3 + z^2 - z + i = 0$ then show that $|z| = 1$.

34. If z_1, z_2 are complex numbers such that, $\frac{2z_1}{3z_2}$ is purely imaginary number then find $\left|\frac{z_1-z_2}{z_1+z_2}\right|$.

35. If z_1 and z_2 are complex numbers such that,

$\left|1-\overline{z_1}z_2\right|^2-\left|z_1-z_{2^2}\right|^2=k\left(1-\left|z_1\right|^2\right)\left(1-\left|z_2\right|^2\right)$ find value of k.

LONG ANSWER TYPE QUESTIONS (6 MARKS)

36. Find number of solutions of $z^2+\left|z\right|^2=0$.

37. If z_1, z_2 are complex numbers such that $\left|\frac{z_1-3z_2}{3-z_1.\overline{\overline{z_2}}}\right|=1\ and\ \left|z_2\right|\neq 1$ then find $\left|z_1\right|$.

38. Evaluate $x^4 - 4x^3 + 4x^2 + 8x + 44$, When $x = 3 + 2i$

39. If z_1, z_2 are complex numbers, both satisfy $z+\overline{z}=2\left|z-1\right|$

$\arg\left|z_1-z_2\right|=\frac{\pi}{4}$ then find $\text{Im}\left(z_1+z_2\right)$.

40. Solve $2x^2 - (3 + 7i)\,x - (3 - 9i) = 0$

41. What is the locus of z if amplitude of $z - 2 - 3i$ is $\frac{\pi}{4}$

42. If $z = x + iy$ and $w=\frac{1-iz}{z-i}$ show that if $|w| = 1$ then z is purely real.

43. Express the complex number in the form $r(\cos\theta+i\sin\theta)$

(i) $1+i\tan\alpha$

(ii) $1-\sin\alpha+i\cos\alpha$

44. If $\left(\frac{1+i}{1+2^2i}\right)\times\left(\frac{1+3^2i}{1+4^2i}\right)\times\ldots\ldots\ldots\times\left(\frac{1+(2n-1)^2i}{1+(2n)^2i}\right)=\frac{a+ib}{c+id}$ then show that $\frac{2}{17}\times\frac{82}{257}\times\ldots\ldots.\times\frac{1+(2n-1)^4}{1+(2n)^4}=\frac{a^2+b^2}{c^2+d^2}$.

45. Find the values of x and y for which complex numbers $-3 + ix^2y$ and $x^2 + y + 4i$ are conjugate to each other.

46. The complex number z_1, z_2 and z_3 satisfying $\frac{z_1 - z_3}{z_2 - z_3} = \frac{1 - i\sqrt{3}}{2}$ are the vertices of a equilateral triangle.

47. If $f(z) = \frac{7 - z}{1 - z^2}$ where $z = 1 + 2i$ then show that $|f(z)| = \frac{|z|}{2}$.

48. If z_1, z_2, z_3 are complex numbers such that

$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$ then find the value of $|z_1 + z_2 + z_3|$

ANSWERS

1. $-1 + i$ 2. $-7 - 6i$ 3. $\frac{1}{49} \quad \frac{4\sqrt{3}i}{49}$

4. $\frac{-2}{25} - \frac{11i}{25}$ 5. $\frac{-\pi}{2}$ 6. $\frac{}{3}$

7. $\frac{-2}{7} - \frac{i\sqrt{3}}{7}$

8. (i) 0 (ii) 19 (iii) -8

(iv) $\frac{-7}{\sqrt{2}}i$

9. $x = \frac{5}{13}$ $y = \frac{14}{13}$

10. I 11. 0 (zero) 12. 6 and zero

13. $a = -2$ 14. 1:3

15 locus of z is straight line I,e x – y + 1 = 0

17. $\theta = (2n+1)\frac{\pi}{2}$

20. X = 3, y = –1

22. zero

23. a =0, b = –2

24. $\frac{z_1}{z_2} = \frac{3(1+i)}{2}$

25. n = 4

27. (i) $6\left(\cos\frac{3\pi}{4} + i\sin\frac{\pi}{4}\right)$

(ii) $1\left[\cos\left(\frac{-5\pi}{12}\right) + i\sin\left(\frac{-5\pi}{12}\right)\right]$

(iii) $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(iv) $\sqrt{2}\left[\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right]$

28. (i) $3\sqrt{2}$ *and* $-2i$ (ii) $4 - 3i$ *and* $3 + 2i$

29. $\pm\left(5 - 3\sqrt{2}\,i\right)$

31. Centre $\left(\frac{10}{3}, 0\right)$ and radius= $\frac{2}{3}$

32. $z = 0, i, \frac{\sqrt{3}}{2} - \frac{1}{2}i, -\frac{\sqrt{3}}{2} - \frac{1}{2}i$

34. 1

35. K=1

36. Infinitely many solutions of the form $z = 0 \pm iy;\ y \in R$

37. $|z_1| = \sqrt{x^2 + y^2}$

38. 5

39. 2

40. $\frac{3}{2} + \frac{1}{2}i \quad and \quad 3i$

41. x – y + 1 = 0 straight line

43. (i) $\sec\alpha\left(\cos\alpha + i\sin\alpha\right)\ ,\ 0 \le \alpha < \frac{\pi}{2}$

$$-\sec\alpha\left[\cos\left(\alpha - \pi\right) + i\sin\left(\alpha - \pi\right)\right],\ \frac{\pi}{2} < \alpha \le \pi$$

(ii) $\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left[\cos\left(\frac{\pi}{4} + \frac{\alpha}{2}\right) + i\sin\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)\right]\ if\ 0 \le \alpha < \frac{\pi}{2}$

$$-\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right)\right]\ if\ \frac{\pi}{2} < \alpha < \frac{3\pi}{2}$$

$$-\sqrt{2}\left(\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}\right)\left[\cos\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right) + i\sin\left(\frac{\alpha}{2} - \frac{3\pi}{4}\right)\right]\ if\ \frac{3\pi}{2} < \alpha < 2\pi$$

45. When x = 1, y = –4 or x = –1, y = –4

48. 1 (one)

CHAPTER - 6

LINEAR INEQUALITIES

KEY POINTS

- Two real numbers or two algebraic expressions related by the symbol
'<', '>', '≤' or '≥' form an inequality.
- The inequality containing < or > is called strict inequality.
- The inequality containing ≤ or ≥ is called slack inequality.
- 6 < 8 statements 7.2 > –1 are examples of numerical inequalities and 3x + 7 > 8, x + 3 ≤ 7, $\frac{y-3}{2}$ > 2y + 2 are examples of litral inequalities.
- The inequalities of the form ax + b > 0, ax + b < 0, ax + b ≥ 0, ax + b ≤ 0 ; a ≠ 0 are called linear inequalities in one variable x
- The inequalities of the form ax + by + c > 0, ax + by + c < 0, ax + by + c ≥ 0, ax + by + c ≤ 0, a ≠ 0, b ≠ 0 are called linear inequalities
in two variables x and y.
- Rules for solving inequalities :

  (i) a ≥ b then a ± k ≥ b ± k where k is any real number.

  (ii) but if a ≥ b then ka is not always ≥ kb

If k > 0 (i.e. positive) then $a \geq b \Rightarrow ka \geq kb$

If k < 0 (i.e. negative) then $a \geq b \Rightarrow ka \leq kb$

Thus always reverse the sign of inequality while multiplying or dividing both sides of an inequality by a negative number.

- Procedure to solve a linear inequality in one variables.

  (i) Simplify both sides by removing graph symbols and collecting like terms.

  (ii) Remove fractions (or decimals) by multiplying both sides by appropriate factor (L.C.M of denomination or a power of 10 in case of decimals.)

  (iii) Isolate the variable on one side and all constants on the other side. Collect like terms whenever possible.

  (iv) Make the coefficient of the variable.

  (v) Choose the solution set from the replacement set.

- **Replacement Set:** The set from which values of the variable (involved in the inequality) are chosen is called replacement set.

- **Solution Set:** A solution to an inequality is a number which when substituted for the variable, makes the inequality true. The set of all solutions of an inequality is called the solution set. It is obviously a subset of replacement set.

- The graph of the inequality ax + by > c is one of the half planes and is called the solution region.

- When the inequality involves the sign ≤ or ≥ then the points on the line are included in the solution region but if it has the sign <

or > then the points on the line are not included in the solution region and it has to be drawn as a dotted line.

- The common values of the variable form the required solution of the given system of linear inequalities in one variable.
- The common part of coordinate plane is the required solution of the system of linear inequations in two variables when solved by graphical method.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Solve 5x < 24 when $x \in N$
2. Solve 3 – 2x < 9 when $x \in R$. Express the solution in the form of interval.
3. Show the graph of the solution of 2x – 3 > x – 5 on number line.
4. Solve $\frac{1}{x-2} \leq 0$, $x \in R$.
5. Solve $0 < \frac{-x}{3} < 1$, $x \in R$
6. Solve $-3 \leq -3x + 2 < 4$, $x \in R$.
7. Draw the graph of the solution set of $x + y \geq 4$.
8. Draw the graph of the solution set of x < y.

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

9. Solve $\frac{(x-1)(x-2)}{(x-3)(x-4)} \geq 0$, $x \in R$.
10. Solve $\frac{x+3}{x-1} > 0$, $x \in R$.

Solve the inequalities for real x

11. $\frac{2x-3}{4}+9 \geq 3+\frac{4x}{3}$, x $\in$R.

12. $\frac{2x+3}{4}-3 < \frac{x-4}{3}-2$, x $\in$R.

13. $-5 \leq \frac{2-3x}{4} \leq 9$, x $\in$ R.

14. $\frac{x+3}{x-2} > 0$, x $\in$ R

Show the solution set on the number line.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

15. A company manufactures cassettes and its cost and revenue functions are C(x) = 26000 + 30x and R(x) = 43x respectively, where x is number of cassettes produced and sold in a week. How many cassettes must be sold per week to realise some profit.

16. Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23.

17. To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course. What life skills should she acquire in order to get grade 'A' in the course?

18. While drilling a hole in the earth, it was found that the temperature (T°C) at x km below the surface of the earth was given by T = 30 + 25(x – 3), when $3 \leq x \leq 15$.

Between which depths will the temperature be between 200°C and 300°C?

19. The water acidity in a pool is considered normal when the average PH reading of their daily measurements is between 7.2 and 7.8. If the first two PH reading are 7.48 and 7.85. Find the range of PH value for the 3rd reading that will result in acidity level being normal.

Solve the following systems of inequalities for all x $\in$ R

20. $2(2x+3)-10<6(x-2), \quad \frac{2x-3}{4}+6\geq 4+\frac{4x}{3}$

21. $|2x-3|\leq 11, \quad |x-2|\geq 3$

22. $\frac{4x}{3}-\frac{9}{4}<x+\frac{3}{4}, \quad \frac{7x-1}{3}-\frac{7x+2}{6}>x$

23. Solve: $\frac{|x|-1}{|x|-2}\geq 0 \quad x\in R, \quad x\neq \pm 2$

24. Solve for real x, $|x+1|+|x|>3$

LONG ANSWER TYPE QUESTIONS (6 MARKS)

Solve the following system of inequalities graphically:

25. $2x + y \leq 24$, $x + y < 11$, $2x + 5y \leq 40$, $x \geq 0$, $y \geq 0$

26. $3x + 2y \geq 24$, $3x + y \leq 15$, $x \geq 4$

27. $x - 2y \leq 3$

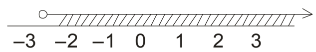
    $3x + 4y > 12$

    $x \geq 0$, $y \geq 1$

ANSWERS

1. $\{1,2,3,4\}$

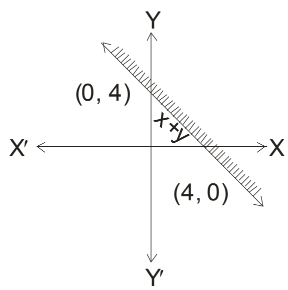
2. $(-3, \infty)$

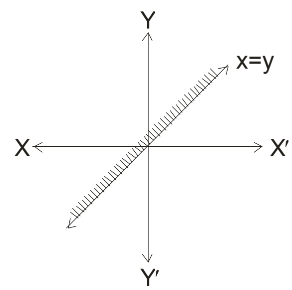
3. 

4. $(-\infty, 2)$ or $x < 2$

5. $-3 < x < 0$

6. $\left(\frac{-2}{3}, \frac{5}{3}\right]$

7. 

8. 

9. $X > 4$

10. $(-\infty, -3) \cup (2, \infty)$

11. $\left(-\infty, \frac{63}{10}\right]$

12. $\left(-\infty, \frac{-13}{2}\right)$

13. $\left[\frac{-34}{3}, \frac{22}{3}\right]$

14. $(-\infty, -3) \cup (2, \infty)$

15. More than 2000 cassettes

16. (6,8), (8,10), (10,12)

17. Minimum of 82 marks. Hard working, self-confidence, diligent and dedicated to her work.

18. Between 9.8 m and 13.8 m

19. Between 6.27 and 8.07

20. Solution set = ϕ

21. $[-4, -1] \cup [5, 7]$

22. $(4,9)$

23. $[-1, 1] \cup (-\infty, -2) \cup (2, \infty)$

24. $(--\infty, -2) \cup (1, \infty)$

CHAPTER - 7

PERMUTATIONS AND COMBINATIONS

KEY POINTS

- **Multiplication Principle (Fundamental Principle of Counting:** If an event can occur in m different ways, following which another event can occur in n different ways, then the total no. of different ways of occurrence of the two events in order is m × n.
- **Fundamental Principle of Addition:**If there are two events such that they can occur independently in m and n different ways respectively, then either of the two events can occur in (m + n) ways.
- **Factorial:**Factorial of a natural number n, denoted by n! or $\underline{|n}$ is the continued product of first n natural numbers.

  $n! = n \times (n - 1) \times (n - 2) \times ... \times 3 \times 2 \times 1$

  $= n \times ((n - 1)!)$

  $= n \times (n - 1) \times ((n - 2)!)$
- **Permutation:**A permutation is an arrangement of a number of objects in a definite order taken some or all at a time.
- The number of permutation of n different objects taken r at a time where$0 \le r \le n$ and the objects do not repeat is denoted by ${}^nP_r$or P(n, r) where,

$$^nP_r = \frac{n!}{(n-r)!}$$

- The number of permutations of n objects, taken r at a time, when repetition of objects is allowed is n^r.
- The number of permutations of n objects of which p_1 are of one kind, p_2 are of second kind, …….. p_k are of k^{th} kind and the rest if any, are of different kinds, is $\frac{n!}{p_1!\,p_2!\,.......p_k!}$
- **Combination:** Each of the different selections made by choosing some or all of a number of objects, without considering their order is called a combination. The number of combination of n objects taken r at a time where

  $0 \le r \le n$, is denoted by ^nC_r or C(n, r) or $\binom{n}{r}$ where $^nC_r = \frac{n!}{r!(n-r)!}$

  Some important result :

  $0! = 1$

  $^nC_0 = {}^nC_n = 1$

  $^nC_r = {}^nC_{n-r}$ where $0 \le r \le n$, and r are positive integers

  $^nP_r = \underline{|n}\ {}^nC_r$ where $0 \le r \le n$, r and n are positive integers.

  $^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$ where $0 \le r \le n$ and r and N are positive integers.

  If $^nC_a = {}^nC_b$ if either a = b or a + b = n

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. How many ways are there to arrange the letters of the word "GARDEN" with the vowels in alphabetical order?

2. In how many ways 7 pictures can be hanged on 9 pegs?

3. Ten buses are plying between two places A and B. In how many ways a person can travel from A to B and come back?

4. There are 10 points on a circle. By joining them how many chords can be drawn?

5. There are 10 non collinear points in a plane. By joining them how many triangles can be made?

6. If ${}^{n}P_4 : {}^{n}P_2 = 12$, find n.

7. How many different words (with or without meaning) can be made using all the vowels at a time?

8. In how many ways 4 boys can be chosen from 7 boys to make a committee?

9. How many different words can be formed by using all the letters of word "SCHOOL"?

10. In an examination there are three multiple choice questions and each question has 4 choices. Find the number of ways in which a student can fail to get all answer correct.

11. A gentleman has 6 friends to invite. In how many ways can he send invitation cards to them if he has three servants to carry the cards?

12. If there are 12 persons in a party, and if each two of them Shake hands with each other, how many handshakes happen in the party?

Section B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

13. From a class of 40 students, in how many ways can five students be chosen

    (i) For an excursion party.

    (ii) As subject monitor (one from each subject)

14. In how many ways can the letters of the word "ABACUS" be arranged such that the vowels always appear together.

15. If $n_{C_{12}} \quad n_{C_{13}}$ then find the value of the 25_{C_n}.

16. In how many ways can the letters of the word "PENCIL" be arranged so that I is always next to L.

Section - C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. In how many ways 12 boys can be seated on 10 chairs in a row so that two particular boys always take seats of their choice.

18. In how many ways 7 positive and 5 negative signs can be arranged in a row so that no two negative signs occur together?

19. From a group of 7 boys and 5 girls, a team consisting of 4 boys and 2 girls is to be made. In how many different ways it can be done?

20. In how many ways can one select a cricket team of eleven players from 17 players in which only 6 players can bowl and exactly 5 bowlers are to be included in the team?

21. A student has to answer 10 questions, choosing at least 4 from each of part A and B. If there are 6 questions in part A and 7 in part B. In how many ways can the student choose 10 questions?

22. Using the digits 0, 1, 2, 2, 3 how many numbers greater than 20000 can be made?

23. If the letters of the word 'PRANAV' are arranged as in dictionary in all possible ways, then what will be 182^{nd} word.

24. From a class of 15 students, 10 are to chosen for a picnic. There are two students who decide that either both will join or none of them will join. In how many ways can the picnic be organized?

25. Using the letters of the word, 'ARRANGEMENT' how many different words (using all letters at a time) can be made such that both A, both E, both R and both N occur together.

26. A polygon has 35 diagonal. Find the number of its sides.

27. How many different products can be obtained by multiplying two or more of the numbers 2, 5, 6, 7, 9?

28. Determine the number of 5 cards combinations out of a pack of 52 cards if at least 3 out of 5 cards are ace cards?

29. How many words can be formed from the letters of the word 'ORDINATE' so that vowels occupy odd places?

30. Find the number of all possible arrangements of the letters of the word "MATHEMATICS" taken four at a time.

31. Prove that 33! in divisible by 2^{15} what is the largest integer n such that 33! is divisible by 2^n?

32. A group consists of 4 girls and 7 boys. In how many ways can a team of 5 members be selected if a team has

(i) no girl (ii) at least 3 girls

(iii) at least one girl and one boy?

33. Find n if

$$16\,{}^{n+2}C_8 = 57\,{}^{n-2}P_4$$

34. In an election, these are ten candidates and four are to be elected. A voter may vote for any number of candidates, not greater than the number to be elected. If a voter vote for at least one candidate, then find the number of ways in which he can vote.

35. Three married couples are to be seated in a row having six seats in a cinema hall. If spouses are to be seated next to each other, in how many ways can they be seated? Find also the number of ways of their seating if all the ladies sit together.

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

36. Using the digits 0, 1, 2, 3, 4, 5, 6 how many 4 digit even numbers can be made, no digit being repeated?

37. There are 15 points in a plane out of which only 6 are in a straight line, then

(a) How many different straight lines can be made?

(b) How many triangles can be made?

38. If there are 7 boys and 5 girls in a class, then in how many ways they can be seated in a row such that (i) No two girls sit together?

    (iii) All the girls never sit together?

39. Using the letters of the word 'EDUCATION' how many words using 6 letters can be made so that every word contains atleast 4 vowels?

40. What is the number of ways of choosing 4 cards from a deck of 52 cards? In how many of these,

    (a) 3 are red and 1 is black.

    (b) All 4 cards are from different suits.

    (c) Atleast 3 are face cards.

    (d) All 4 cards are of the same colour.

41. How many 3 letter words can be formed using the letters of the word INEFFECTIVE?

42. How many different four letter words can be formed (with or without meaning) using the letters of the word "MEDITERRANEAN" such that the first letter is E and the last letter is R.

43. If all letters of word 'MOTHER' are written in all possible orders and the word so formed are arranged in a dictionary order, then find the rank of word 'MOTHER'?

44. In how many ways three girls and nine boys can be seated in two vans, each having numbered seats, 3 in the front and 4 at the back? How many seating arrangements are possible if 3 girls should sit together in a back row on adjacent seats?

45. From 6 different novels and 3 different dictionaries, 4 novels and a dictionary is to be selected and arranged in a row on the shelf so that the dictionary is always in the middle. Then find the number of such arrangements.

46. The set S= {1,2,3,…….12} is to be partitioned into three sets A, B, and C of equal sizes. $A \cup B \cup C = S,\ A \cap B = B \cap C = C \cap A = \phi$. Find the number of ways to partition S.

47. Find the value of ${}^{50}C_4 + \sum_{r=1}^{6} {}^{56-r}C_3$.

48. There are two parallel lines l_1 and l_2 in a plane l_1 contains m different points $A_1, A_2 \ldots\ldots\ldots\ldots\ldots A_m$ and l_2 contains n different points $B_1, B_2 \ldots\ldots\ldots\ldots\ldots B_n$. How many triangles are possible with these vertices?

ANSWERS

1.	$\frac{6!}{2} = 360$	2.	$\frac{9!}{2!}$
3.	100	4.	45
5.	120	6.	n = 6
7.	120	8.	35
9.	360		
10.	63	11.	3^6=729
12.	66	13.	(i) 40_{C_5} (ii) 40_{P_5}
14.	$\frac{3!}{2!}$ $4!$	15.	1

16. 120 17. $90 \times {}^{10}P_8$

18 56,19.350

20. 2772 21. 266

22. 36 23. PAANVR

24. $13C_{10} + 13C_8$ 25. 5040

26. 10 27. ${}^nC_2 - n$

28. 4560 29. 576

30. 2454 31. 31

32. (i) 21;

    (ii) 91;

    (iii) 44133.19

34. ${}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

35. 48,144

36. 420

37. (a) 91 (b) 435

38. (i) $7! \times {}^8P_5$ (ii) $12! - 8! \times 5!$

39. 24480

40. 52C4

    (a) ${}^{26}C_1 \times {}^{26}C_3$ (b) $(13)^4$

(c) 9295 (Hint : Face cards : 4J + 4K + 4Q)

(d) $2 \times {}^{26}C_4$

41. 265 (*Hint* : make 3 cases i.e.

    (i) All 3 letters are different

    (ii) 2 are identical 1 different

    (iv) All are identical, then form the words.)

42. 59

43. 43. 309

44. ${}^{14}P_{12} 2(2 \times 3!) {}^{11}P_9$

45. $4!\, {}^{6}C_4\, {}^{3}C_1$

46. ${}^{12}C_4\, {}^{8}C_4\, {}^{4}C_4$

47. ${}^{56}C_4$

48. ${}^{m+n}C_3 - {}^{m}C_3 - {}^{n}C_3 \;\; or \;\; {}^{m}C_2\, {}^{n}C_1 + {}^{m}C_1\, {}^{n}C_2$

CHAPTER - 8

BINOMIAL THEOREM

KEY POINTS

- If n is a natural number and a, b are any numbers.

  $$(a+b)^n = {}^nc_0 a^n + {}^nc_1 a^{n-1} b + {}^nc_2 a^{n-2} b^2 + \text{...........} + {}^nc_n b^n$$

  $$= \sum_{r=0}^{n} {}^nc_r a^{n-r} b^r, \quad n \in N$$

- T_{r+1} = General term

  $$= {}^nc_r a^{n-r} b^r \quad 0 \le r \le n$$

- Total number of terms in $(a + b)^n$ is (n + 1).

- If n is even, then in the expansion of $(a+b)^n$, middle term is $\left(\frac{n}{2}+1\right)^{th}$ term i,e $\left(\frac{n+2}{2}\right)^{th}$ terms.

- If n is odd, then in the expansion of $(a + b)^n$, middle terms are $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+3}{2}\right)^{th}$

- In $(a + b)^n$, r^{th} term from the end is same as $(n - r + 2)^{th}$ term from the beginning.

- r^{th} term from the end in $(b + a)^n$

  $= r^{th}$ term from the beginning in $(a + b)^n$

- In $(1 + x)^n$, coefficient of x^r is ${}^nC_r$
- Some particular cases:

$$(1+x)^n = \sum_{r=0}^{n} {}^nc_r x^r$$

$$(1-x)^n = \sum_{r=0}^{n} (-1)^r \, {}^nc_r x^r$$

- Some properties of Binomial coefficients:

${}^nC_0 + {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n$

${}^nC_0 - {}^nC_1 + {}^nC_2 - {}^nC_3 + ... + (-1)^{n\,n}C_n = 0$

${}^nC_0 + {}^nC_2 + {}^nC_4 + ... = {}^nC_1 + {}^nC_3 + {}_n{}^{C}{}_5 + ... = 2^{n-1}$

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write number of terms in the expansion of $\left\{\left(2x + y^3\right)^4\right\}^7$.

2. Expand $\left(\sqrt{\frac{x}{a}} - \sqrt{\frac{a}{x}}\right)^6$ using binomial theorem.

3. Write value of

   ${}^{2n-1}c_5 + {}^{2n-1}c_6 + {}^{2n}c_7$ $\qquad$ $use\left[{}^nc_r + {}^nc_{r-1} = {}^{n+1}c_r\right]$

4. Which term is greater $(1.2)^{4000}$ or 800?

5. Find the coefficient of x^{-17},in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$.

6. Find the sum of the coefficients in $(x + y)^8$

[**Hint :** Put x = 1, y = 1]

7. If ${}^nC_{n-3} = 720$, find n.

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

8. How many term are free from radical signs in the expansion of

$$\left(x^{\frac{1}{5}} + y^{\frac{1}{10}}\right)^{55}$$

9. Find the constant term in expansion of $\left(x - \frac{1}{x}\right)^{10}$.

10. Find the value of

$$\frac{{}^8C_0}{6} - {}^8C_1 + {}^8C_2 \cdot 6 - {}^8C_3 \cdot 6^2 + \ldots + {}^8C_8 6^7$$

11. Find 4th term from end in the expansion of find the value of

$$\left(\frac{x^3}{2} - \frac{2}{x^2}\right)^9$$

12. Write the last two digits of the number 3^{400}.

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

Section - C

13. If the first three terms in the expansion of $(a + b)^n$ are 27, 54 and 36 respectively, then find a, b and n.

14. In $\left(3x^2 - \frac{1}{x}\right)^{18}$ which term contains x^{12}?

15. In $\left(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{\sqrt{2}x^2}\right)^{10}$ find the term independent of x.

16. Evaluate $\left(\sqrt{2}+1\right)^5 - \left(\sqrt{2}-1\right)^5$ using binomial theorem.

17. In the expansion of $(1 + x^2)^8$, find the difference between the coefficients of x^6 and x^4.

18. Find the coefficients of x^4 in $(1-x)^2(2+x)^5$ using binomial theorem.

19. show that$3^{2n+2} - 8n - 9$ is divisible by 8.

20. If the term free from x in the expansion of $\left(\sqrt{x} + \frac{k}{x^2}\right)^{10}$ is 405. Find the value of k.

21. Find the number of integral terms in the expansion of $\left(5^{\frac{1}{2}} + 7^{\frac{1}{8}}\right)^{1024}$

22. If for positive integers r > 1, n > 2 the coefficients of the $(3r)^{th}$ term and $(r + 2)^{th}$ powers of x in the expansion of $(1 +)^{2n}$ are equal, then prove that n = 2r + 1.

23. If a, b, c and d in any binomial expansion be the 6^{th} , 7^{th} , 8^{th}, and 9^{th} terms respectively, then prove that $\frac{b^2 - ac}{c^2 - bd} = \frac{4a}{3c}$.

24. If in the expansion of $(1+x)^n$, the coefficients of three consecutive of three consecutive terms are 56, 70 and 56. Then find n and the position of terms of these coefficients.

25. Show that $2^{4n+4} - 15n - 16$ where $n \in N$ is divisible by 225.

26. If the coefficients of three consecutive terms in the expansion of $(1 + x)^n$ are in the ratio 1:3:5, then show that n = 7.

27. Show that the coefficient of middle term in the expansion of $(1 + x)^{20}$ is equal to the sum of the coefficients of two middle terms in the expansion of $(1 + x)^{19}$

Section - D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

28. Show that the coefficient of x^5 in the expansion of product

$(1 + 2x)^6 (1 - x)^7$ is 171.

29. If the 3^{rd}, 4^{th} and 5^{th} terms in the expansion of $(x + a)^n$ are 84, 280 and 560 respectively then find the values of a, x and n

30. If the coefficients of x^7 in $\left[ax^2 + \frac{1}{bx}\right]^{11}$ and x^{-7} in $\left[ax - \frac{1}{bx^2}\right]^{11}$ are equal, then show that ab = 1

31. In the expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}}\right)^n$, the ratio of 7^{th} term from the beginning to the 7^{th} term from the end is 1:6 , find n.

32. If a_1, a_2, a_3 and a_4 are the coefficients of any four consecutive terms in the expansion of $(1+x)^n$ prove that

$$\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}.$$

33. Using binomial theorem, find the remainder when 5^{103} is divided by 13.

HOTS

34. Find the greatest value of the term independent of x in the expansion of $\left(x\sin\alpha+\frac{\cos\alpha}{x}\right)^{10}$; *where* $\alpha\in R$.

35. Find the number of terms in the expansion of $(x + y + z)^n$

36. Find the coefficient of the term independent of x in the expansion of $\left\{\frac{x+1}{x^{213}-x^{\frac{1}{3}}+1}-\frac{x-1}{x-x^{\frac{1}{2}}}\right\}^{10}$

37. For what value of x is the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{-\frac{1}{8}\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ equal to 180.

38. Find the remainder left out when $8^{2n}-(62)^{2n+1}$ is divided by 9.

39. Find the coefficient of x^n in expansions of $(1+x)\,(1-x)^n$.

40. Find the value of $\left(\sqrt{2}+1\right)^6-\left(\sqrt{2}-1\right)^6$ and show that $\left(\sqrt{2}+1\right)^6$ lies between 197 and 198.

41. Find the term independent of x in the expansion of

$(1 + x + 2x^3)\left(\frac{3}{2}x^2 - \frac{1}{3}x\right)^9$

42. If the coefficients of r^{th}, (r+1)th and $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in A.P find the value of r.

43. If ${}^{n-1}c_r = \left(k^2 - 3\right){}^{n}c_{r+1}$ then find the value of k.

44. Find the remainder of 51^{51} when divided by 25.

45. If the coefficient of $x^k\left(0 \le k \le 15\right)$ in the expansion of the expression $1+(1+x)+(1+x)^2+........+(1+x)^{15}$ is the greatest, then find the value of k.

ANSWERS

1. 29

2. $\frac{x^3}{a^3}-\frac{6x^2}{a^2}+15\frac{x}{a}-20+15\frac{a}{x}-\frac{6a^2}{x^2}+\frac{a^3}{x^3}$

3. ${}^{2n+1}C_7$

4. $(1.2)^{4000}$

5. –1365

6. 256

7. n = 10

8. 6 terms (0, 10, 20, 30, 40, 40, 50)

9. $-252 = -{}^{10}C_5$

10. ${}^{31}C_6 - {}^{21}C_6 / \frac{5^8}{6}$

11. $\frac{672}{x^3}$

12. 01

13. a = 3, b = 2, n = 3

14. 9^{th} term

15. $T_3 = \frac{5}{6}$

16. 82

17. 28

18. 10

20. $k = \pm 3$

21. 129 integral terms

22. $x = \frac{1}{\sqrt{10}}$ or 100

24. n = 8, 4^{th}, 5^{th}, and 6^{th}

29. a = 2, x = 1, n = 7

31. 9

33. 8

34. ${}^{10}c_5.2^{-5}$ or $\frac{\underline{|10}}{\left(\underline{|5}\right)^2 2^5}$

35. $\frac{(n+1)(n+2)}{2}$

36. 210

37. $x = \log 5^{15}$ or $x = 1$

38. 2

39. $(-1)^n[1-n]$

40. Zero

41. $\frac{17}{54}$

42. 5

43. [–2, 2]

44. 1 (one)

45. 7

CHAPTER - 9

SEQUENCES AND SERIES

KEY POINTS

- A sequence is a function whose domain is the set N of natural numbers or some subset of it.
- A sequence whose range is a subset of R is called a real sequence.
- A sequence is said to be a progression if the term of the sequence can be expressed by some formula
- A sequence is called an arithmetic progression if the difference of a term and previous term is always same, i.e., $a_{n+1} - a_n$ = constant (=d) for all $n \in N$.
- The term 'series' is associated with the sequence in following way

  Let a_1, a_2, a_3 ..... be a sequence. Then, the expression $a_1 + a_2 + a_3$ + ... is called series associated with given sequence.
- A series is finite or infinite according as the given sequence is finite or infinite.
- General A.P. is,

  a, a + d, a + 2d, ...
- $a_n = a + (n - 1)d = n^{th}$ term of A.P. = l
- S_n = Sum of first n terms of A.P.

$= \frac{n}{2}[a + l]$ where l = last term. N

$= \frac{n}{2}[2a + (n-1)d]$

- If a, b, c are in A.P. then a ± k, b ±k, c ± k are in A.P.

  ak, bk, ck are also in A.P., k ≠ 0, $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ are also in A.P. where k ≠ 0.

- Three numbers in A.P.

  a – d, a, a + d

- Arithmetic mean between a and b is $\frac{a+b}{2}$.

- If A_1, A_2, A_3, ...A_n are n numbers inserted between a and b, such that the resulting sequence is A.P. then,

  $$A_n = a + n.\frac{b-a}{n+1}$$

- $S_k - S_{k-1} = a_k$

- $a_m = n,\ a_n = m \Rightarrow a_r = m + n - r$

- $S_m = S_n \Rightarrow S_{m+n} = 0$

- $S_p = q$ and $S_q = p \Rightarrow S_{p+q} = -p - q$

- In an A.P., the sum of the terms equidistant from the beginning and from the end is always same, and equal to the sum of the first and the last term

- If three terms of A.P. are to be taken then we choose then as a – d, a, a + d.

- If four terms of A.P. are to be taken then we choose then as a – 3d, a – d, a + d, a + 3d.
- If five terms of A.P are to be taken, then we choose then as:

  $a-2d,\ a-d,\ a,\ a+d\ \ a+2d$.
- G.P. (Geometrical Progression)

  a, ar, ar^2, ...(General G.P.)

  a = ar^{n-1}

  $$S_n = \frac{a\left(r^n - 1\right)}{r-1}\ , \qquad r \neq 1$$
- Geometric mean between a and b is $\sqrt{ab}$
- Reciprocals of terms in GP always form a G.P.
- If G_1, G_2, G_3, ...G_n are n numbers inserted between a and b so that the resulting sequence is G.P., then

  $$G_k = a\left(\frac{b}{a}\right)^{\frac{k}{n+1}}\ 1 \le k \le n$$
- If three terms of G.P. are to be taken, then we those then as

  $\frac{a}{r}$, a, ar.
- If four terms of G.P. are to be taken, then we choose then as

  $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3.
- If a, b, c are in G.P. then ak, bk, ck are also in G.P. where $k \neq 0$ and $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}$ also in G.P. wherek $\neq$ o..

- In a G.P., the product of the terms equidistant from the beginning and from the end is always same and equal to the product of the first and the last term.

- If each term of a G.P. be raised to some power then the resulting terms are also in G.P.

- Sum of infinite G.P. is possible if | r | < 1 and sum is given by $\frac{a}{1-r}$

- $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$

- $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

- $\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$

- $S_\infty = a + ar + ar^2 + ar^3 + ...\infty$ term if –1 < r <1 $\Rightarrow S_\infty = \frac{a}{1-r}$.

  Such that –1 < r <1 or $|r| <|$

- If a, b, c are in A.P. then 2b = a+c.

- If a, b, c are in G.P. then $b^2 = ac$

- If A and G be A.M and G.M of two given positive real number 'a' and 'b' respectively then $A = \frac{a+b}{2}$, $G = \sqrt{ab}$ and $A \geq G$.

Section - A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If n^{th} term of an A.P. is 6n – 7 then write its 50^{th} term.

2. If $S_n = 3n^2 + 2n$, then write a_2

3. Which term of the sequence,

   3, 10, 17, ... is 136?

4. If in an A.P. 7^{th} term is 9 and 9^{th} term is 7, then find 16^{th} term.

5. If sum of first n terms of an A.P is $2n^2 + 7n$, write its n^{th} term.

6. Which term of the G.P.

   $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots\ldots\ldots\ldots\ldots$ is $\frac{1}{1024}$?

7. If in a G.P., $a_3 + a_5 = 90$ and if r = 2 find the first term of the G.P.

8. In G.P. $2\sqrt{2}, 4, \ldots\ldots\ldots, 128\sqrt{2}$, find the 4^{th} term from the end.

9. If the product of 3 consecutive terms of G.P. is 27, find the middle term

10. Find the sum of first 8 terms of the G.P. $10, 5, \frac{5}{2}, \ldots\ldots$

11. Find the value of $5^{1/2} \times 5^{1/4} \times 5^{1/8}$ .... upto infinity.

12. Write the value of $0.\overline{3}$

13. The first term of a G.P. is 2 and sum to infinity is 6, find common ratio.

Section - B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

14. Write the n th term of the series, $\frac{3}{7.11^2}+\frac{5}{8.12^2}+\frac{7}{9.13^2}+......$

15. Find the number of terms in the A.P. 7, 10, 13, ..., 31.

16. In an A.P.,

    8, 11, 14, ... find $S_n - S_{n-1}$

17. Find the number of squares that can be formed on chess board?

18. Find the sum of given terms:-

    (a) 81 + 82 + 83 .... + 89 + 90

    (b) 251 + 252 + 253 + ... + 259 + 260

19. (a) If a, b, c are in A.P. then show that 2b = a + c.

    (b) If a, b, c are in G.P. then show that $b^2 = a.c.$

20. If a, b, c are in G.P. then show that $a^2+b^2, ab+bc, b^2+c^2$ are also in G.P.

Section - C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

21. Find the least value of n for which

    $1+3+3^2+...+3^{n-1}>1000$

25. Find the sum of the series

    $(1+x)+(1+x+x^2)+(1+x+x^2+x^3)+...$

23. Write the first negative term of the sequence 20, $19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}$, ...

24. Determine the number of terms in A.P. 3, 7, 11, ... 407. Also, find its 11th term from the end.

25. How many numbers are there between 200 and 500, which leave remainder 7 when divided by 9.

26. Find the sum of all the natural numbers between 1 and 200 which are neither divisible by 2 nor by 5.

27. Find the sum of the sequence,

72 + 70 + 68 + …. + 40

28. If in an A.P

$\frac{a_7}{a_{10}} = \frac{5}{7}$ find $\frac{a_4}{a_7}$.

29. In an A.P. sum of first 4 terms is 56 and the sum of last 4 terms is 112. If the first term is 11 then find the number of terms.

30. Solve: 1 + 6 + 11 + 16 + ... + x = 148

31. The ratio of the sum of n terms of two A.P.'s is (7n – 1): (3n + 11), find the ratio of their 10th terms.

32. If the Ist, 2nd and last terms of an A.P are a, b and c respectively, then find the sum of all terms of the A.P.

33. If $\frac{b+c-2a}{a} \frac{c+a-2a}{b} \frac{a+b-2c}{c}$ are in A.P. then show that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are also in A.P. [**Hint.** : Add 3 to each term] abc

34. The product of first three terms of a G.P. is 1000. If 6 is added to its second term and 7 is added to its third term, the terms become in A.P. Find the G.P.

35. If the continued product of three numbers in G.P. is 216 and the sum of their products in pairs is 156, find the numbers.

36. Find the sum to infinity of the series:

$$1+\frac{3}{2}+\frac{5}{2^2}+\frac{7}{2^3}+.....\infty$$

37. If A = 1 + r^a + r^{2a} + ... up to infinity, then express r in terms of ‘a’ & ‘A’.

38. Find the sum of first terms of the series 0.7 + 0.77 + 0.777 + ...

39. If $x=a+\frac{a}{r}+\frac{a}{r^2}+............+\infty;\ y=b-\frac{b}{r}+\frac{b}{r^2}-...........\infty$ and

$z=c+\frac{c}{r^2}+\frac{c}{r^4}+.....\infty$ prove that $\frac{xy}{z}=\frac{ab}{c}$.

40. The sum of first three terms of a G.P. is 15 and sum of next three terms is 120. Find the sum of first n terms.

41. Prove that $0.03\bar{1}=\frac{7}{225}$

[**Hint:** 0.031 = 0.03 + 0.001 + 0.0001 +..... Now use infinite G.P.]

42. If log2, log(2^n–1) and log (2^n+3) are in A.P. Show that $n=\frac{\log 5}{\log 2}$

43. If a, b, c are in G.P. that the following are also in G.P.

(i) a^2, b^2, c^2

(ii) a^3, b^3, c^3

(iii) $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in G.P.

44. If a, b, c are in A.P. that the following are also in A.P:

(i) $\frac{1}{bc}, \frac{1}{ca}, \frac{1}{ab}$

(ii) b+c, c+a, a+b

(iii) $\frac{1}{a}\left(\frac{1}{b}+\frac{1}{c}\right), \frac{1}{b}\left(\frac{1}{c}+\frac{1}{a}\right), \frac{1}{c}\left(\frac{1}{a}+\frac{1}{b}\right)$ are in A.P.

45. If the numbers a^2, b^2, c^2 are given to be in A.P., show that $\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b}$ are in A.P.

46. Show that: $0.3\overline{56} = \frac{353}{990}$

Section-D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

47. Prove that the sum of n numbers between a and b such that the resultingseries becomes A.P. is $\frac{n(a+b)}{2}$

48. A square is drawn by joining the mid points of the sides of a square. A third square is drawn inside the second square in the same way and the process is continued indefinitely. If the side of

the first square is 15 cm, then find the sum of the areas of all the squares so formed.

49. If a, b, c are in G.P., then prove that

$$\frac{1}{a^2-b^2} - \frac{1}{b^2-c^2} = -\frac{1}{b^2}$$

[**Hint :** Put b = ar, c = ar^2]

50. Find two positive numbers whose difference is 12 and whose arithmetic mean exceeds the geometric mean by 2.

51. If a is A.M. of b and c and c, G_1, G_2, b are in G.P. then prove that

$$G_1^3 + G_2^3 = 2abc$$

52. Find the sum of the series,

1.3.4 + 5.7.8 + 9.11.12 + ... upto n terms.

53. Evaluate

$$\sum_{r=1}^{10} (2r-1)^2.$$

54. The sum of an infinite G.P. is 57 and the sum of the cubes of its term is 9747, find the G.P.

55. If $10^9 + 2(11)^1(10)^8 + 3(11)^2(10)^7 + ... + 10(11)^9 = k.(10)^9$. then find the value of k such that $k \in N$.

56. Find the sum of first n terms of the series $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+.....n$ terms.

57. Three positive numbers form an increasing G.P. If the middle term in the G.P. is doubled, then new numbers are in A.P. then find the common ratio of the G.P.

58. Show that if the positive number a, b, c are in A.P. so are the numbers $\frac{1}{\sqrt{a}+\sqrt{c}}, \frac{1}{\sqrt{c}+\sqrt{a}}, \frac{1}{\sqrt{a}+\sqrt{b}}$ are in A.P.

59. Find the sum of the series:- $1-\frac{3}{2}+\frac{5}{4}-\frac{7}{8}+\frac{9}{16}$........... ∞ .

60. If $a_1, a_2, a_3,$ .... a_n are in A.P. where $a_i > 0 \forall i \in N$. then show that

$$\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\frac{1}{\sqrt{a_3}+\sqrt{a_4}}+......+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}=\frac{n-1}{\sqrt{a_1}+\sqrt{a_n}}$$

61. If the sum of first 'n' terms of an A.P. is $c.n^2$ then prove that the sum of squares of these 'n' terms is $\frac{nc^2\left(4n^2-1\right)}{3}$.

62. Let 'p' and 'q' be the roots of the equation $x^2-2x+A=0$ and let 'r' and 's' be the roots of the equation $x^2-18x+B=0$ if p<q<r<s are in A.P. then prove that A= –3 and B= 77.

63. If $S_1, S_2, S_3 ... S_n$ are the sums of infinite geometric series whose first terms are 1,2,3,…n and whose common ratios are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4} ... \frac{1}{n+1}$ respectively, then show that the value of

$$S_1^2+S_2^2+S_3^2+...S_{2n-1}^2=\frac{1}{6}(2n)(2n+1)(4n+1)-1$$

64. Let $A_n=\left(\frac{3}{4}\right)-\left(\frac{3}{4}\right)^2+\left(\frac{3}{4}\right)^3-\left(\frac{3}{4}\right)^4+...+(-1)^{n-1}.\left(\frac{3}{4}\right)^n$ and $B_n=1-A_n$.

Find at least odd natural number n_o, so that $B_n > A_n \forall n \geq n_o$.

65. If p^{th}, q^{th} and r^{th} terms of an A.P. and G.P. are equal and are x, y and z respectively, then prove that $x^{y-z}.y^{z-x}.z^{x-y} = 1$

ANSWERS

1.	293	2.	11
3.	20th	4.	0
5.	4n + 5	6.	12th
7.	$\frac{9}{2}$	8.	64
9.	3	10.	$20\left(1-\frac{1}{2^8}\right)$
11.	5	12.	$\frac{1}{3}$
13.	$\frac{2}{3}$	14.	$\frac{2n+1}{(n+6)(n+10)^2}$
15.	9	16.	3n + 5
17.	204	18.	(a) 855 (b) 2555
21.	n = 7	22.	$\frac{n}{1-x} - \frac{x^2(1-x^n)}{(1-x)^2}$
23.	$-\frac{1}{4}$	24.	102, 367
25.	33	26.	7999

27.	952	28.	$\frac{3}{5}$
29.	11	30.	36
31.	33:17	32.	$\frac{(b+c-2a)(a+c)}{2(b-a)}$
34.	5, 10, 20, ...; or 20, 10, 5,	35.	18, 6, 2; or 2, 6, 18
36.	6	37.	$\left(\frac{A-1}{A}\right)^{\frac{1}{a}}$
38.	$\frac{7}{81}\left[9n-1+10^{-n}\right]$	40.	$\frac{15}{7}\left(2^{n}-1\right)$
48.	450 cm²	50.	16, 4
52.	$\frac{n(n+1)}{3}(48n^{2}-16n-14)$		
53.	1330	54.	$19, \frac{38}{3}, \frac{76}{9}, ...$
55.	k = 100	56.	$n+2^{-n}-1$
57.	$r=2+\sqrt{3}$	59.	$\frac{2}{9}$
64.	$n_0=7$		

CHAPTER - 10

STRAIGHT LINES

KEY POINTS

- Distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

  $$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

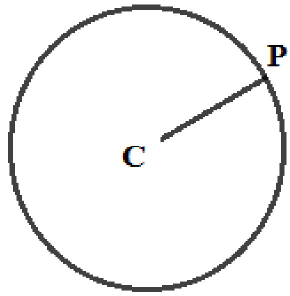
- Let the vertices of a triangle ABC are $A(x_1, y_1)$ B (x_2, y_2) and $C(x_3, y_3)$.Then area of triangle

  $$ABC = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

- NOTE: Area of a Δ is always positive. If the above expression is zero, then a Δ is not possible. Thus the points are collinear.

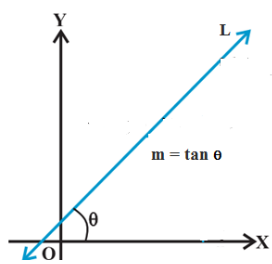
- LOCUS: When a variable point P(x,y) moves under certain condition then the path traced out by the point P is called the locus of the point.

  For example: Locus of a point P, which moves such that its distance from a fixed point C is always constant, is a circle.



CP = constant

- Locus of an equation: In the coordinate plane, locus of an equation is the pictorial representation of the set of all those points which satisfy the given equation.
- Equation of a locus: is the equation in x and y that is satisfied by the coordinates of every point on the locus.
- A line is also defined as the locus of a point satisfying the condition ax + by + c = 0 where a, b, c are constants.
- Slope of a straight line: If θ is the inclination of a line then $\tan\theta$ is defined as slope of the straight line L and denoted by m



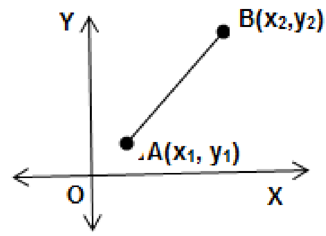
$m = \tan\theta, \ \theta \neq 90°$

If $0° < \theta < 90° \ \ then \ \ m > 0 \ \ and$

$90° < \theta < 180° \ \ then \ m < 0$

- NOTE 1: The slope of a line whose inclination is 90° is not defined. Slope of x-axis is zero and slope of y-axis is not defined
- NOTE 2: Slope of any horizontal line i.e. || to x-axis is zero.Slope of a vertical line i.e. || to y-axis is not zero.

- Three points *A, B* and *C* lying in a plane are collinear, if slope of *AB* = Slope of *BC*.
- Slope of a line through given points (x_1, y_1) and (x_2,y_2) is given by $m = \frac{y_2 - y_1}{x_2 - x_1}$.



- Two lines are parallel to each other if and only if their slopes are equal.

  $i.e\ l_1 \parallel l_2 \Leftrightarrow m_1 = m_2$

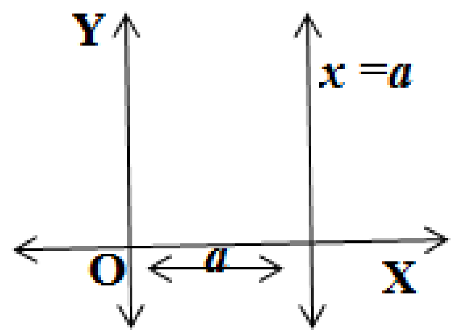
- NOTE: If slopes of lines l_1 and l_2 are not defined then they must be $\perp$ to x-axis, so they are ||. Thus $l_1 \parallel l_2 \Leftrightarrow$ they have same slope or both of them have no slope.
- Two non- vertical lines are perpendicular to each other if and only if their slopes are negative reciprocal of each other.

  $i.e\ l_1 \perp l_2 \Leftrightarrow m_1\ m_2 = -1$

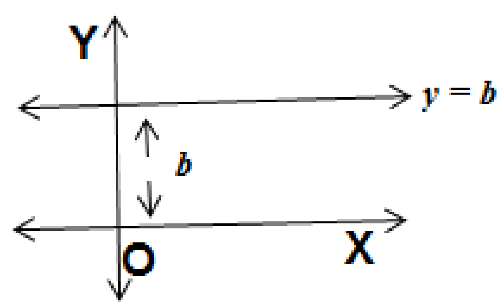
- NOTE: The above condition holds when the lines have non-zero slopes i.e none of them $\perp$ to any axis.
- Acute angle α between two lines, whose slopes are m_1 and m_2 is given by $\tan\alpha = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, 1 + m_1 m_2 \neq 0$

  & obtuse angle is $\phi = 180 - \alpha$

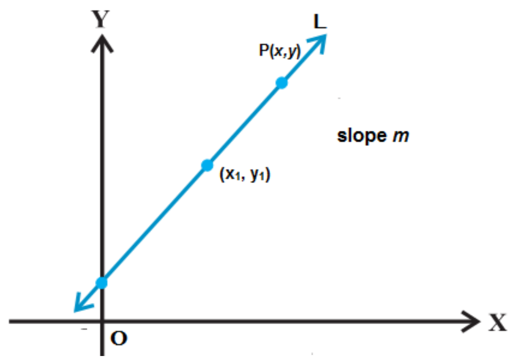
- x = a is a line parallel to y-axis at a distance of a units from y-axis. x = a lies on right or left of y-axis according as a is positive or negative.



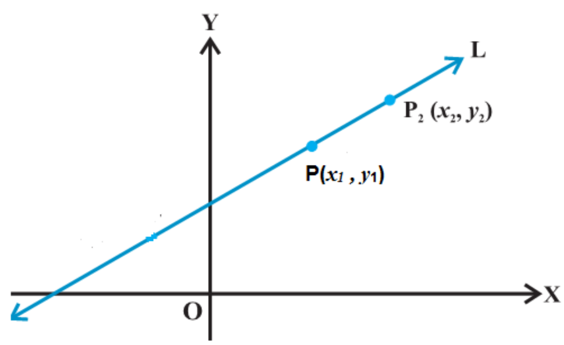
- y = b is a line parallel to x-axis at a distance of ‘b’ units fromx-axis. y=b lies above or below x-axis, according as b is positive or negative.



- Point slope form
- Equation of a line passing through given point (x_1, y_1) and having slope m is given by $y - y_1 = m(x - x_1)$



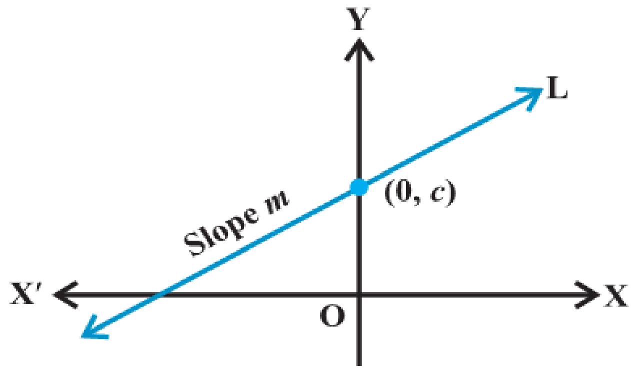
- Two point form



- Equation of a line passing through given points (x_1 , y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

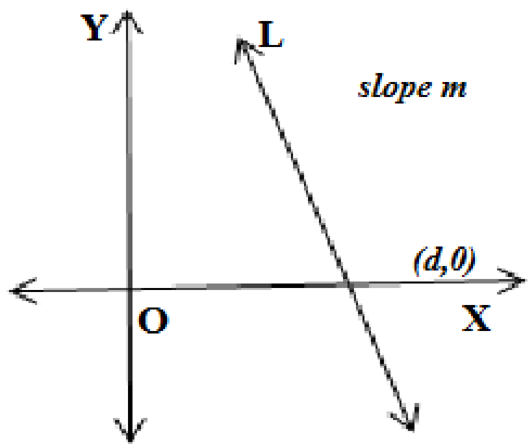
- Slope intercept form(y - intercept)

- Equation of a line having slope m and y-intercept 'c' is given by $y = mx + c$



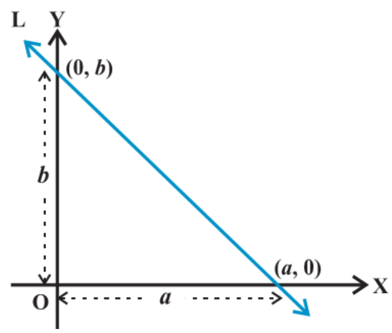
- (x –intercept)
- Equation of a line having slope m and y-intercept c is given by

  y = m (x–d)



- Intercept form.
- Equation of line having intercepts a and b on x-axis and y-axis respectively is given by

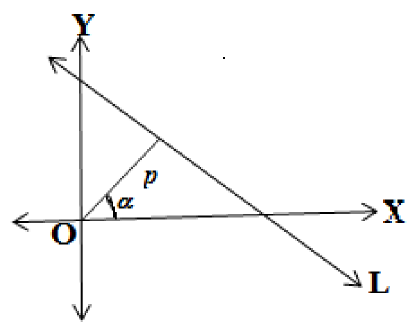
  $\frac{x}{a} + \frac{y}{b} = 1$



- Normal Form
- Equation of line in normal form is given by x cosα + y sinα = p,

  p = Length of perpendicular segment from origin to the line

  α = Angle which the perpendicular segment makes with positive direction of x-axis



- General Equation of a line
- Equation of line in general form is given by Ax + By + C = 0, A, B and C are real numbers and at least one of A or B is non zero.

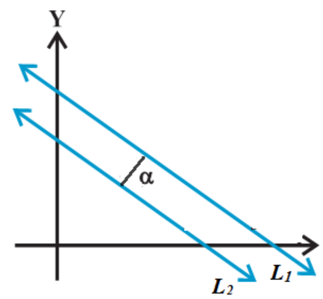
  Slope $= \frac{-A}{B}$ and y-intercept $= \frac{-C}{B}$ x-intercept $= \frac{-C}{A}$

- Distance of a point (x_1, y_1) from line Ax + By + C = 0 is given by

  $$d = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2 + B^2}}$$

- Distance between two parallel lines $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is given by

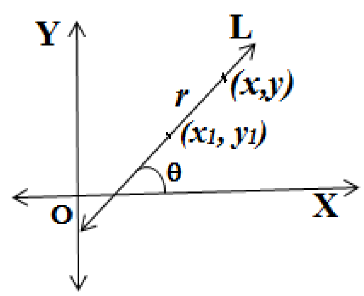
$$d = \frac{|C_1 - C_2|}{\sqrt{A^2 + B^2}}$$



- Symmetrical (or distance) Form

A straight line passing through the point (x_1, y_1) and inclination θ with x-axis is given by

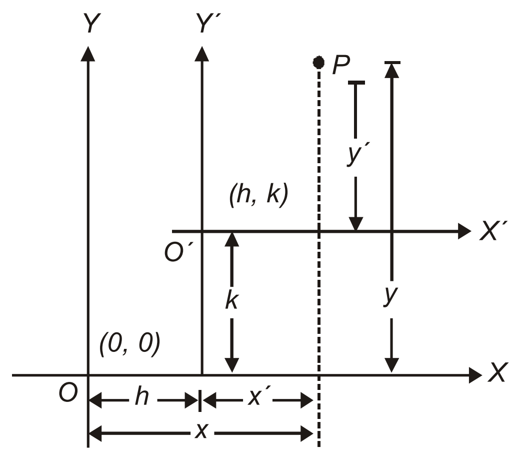
$$\frac{x - x_1}{\cos\theta} = \frac{y - y_1}{\sin\theta} = r$$



Where r is the directed distance of any point (x, y) from the point (x_1, y_1)

- Shifting of Origin
- Shifting of origin to a new point without changing the direction of the axes is known as translation of axes.

Let OX, OY be the original axes and O' be the new origin. Let coordinates of O' referred to original axes be (h, k). Let P(x, y) be point in plane



If the origin is shifted to the point (h,k), then new coordinates (x^{1}, y^{1}) and the original coordinates (x,y) of a point are related to each other by the relation

$x' = x - h$, $y' = y - k$

- Equation of family of lines parallel to $Ax + By + C = 0$ is given by $Ax + By + k = 0$, for different real values of k
- Equation of family of lines perpendicular to $Ax + By + C = 0$ is given by $Bx - Ay + k = 0$, for different real values of k.
- Equation of family of lines through the intersection of lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ is given by $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, for different real values of k.

Section – A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Three consecutive vertices of a parallelogram are (–2, –1), (1, 0) and (4, 3), find the fourth vertex.
2. For what value of k are the points (8, 1), (k, –4) and (2, –5) collinear?
3. Coordinates of centroid of ΔABC are (1, –1). Vertices of ΔABC are A(–5, 3), B(p, –1) and C(6, q). Find p and q.
4. In what ratio y-axis divides the line segment joining the points (3,4) and (–2, 1)?
5. Show that the points (a, 0), (0, b) and (3a,– 2b) are collinear.
6. Find the equation of straight line cutting off an intercept –1 from *y* axis and being equally inclined to the axes.
7. Write the equation of a line which cuts off equal intercepts on coordinate axes and passes through (2, 5).
8. Find k so that the line 2x + ky – 9 = 0 may be perpendicular to2x + 3y – 1 = 0
9. Find the acute angle between lines x + y = 0 and y = 0
10. Find the angle which $\sqrt{3}x + y + 5 = 0$ makes with positive direction of x-axis.
11. If origin is shifted to (2, 3), then what will be the new coordinates of (–1, 2)?

Section – A

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

12. On shifting the origin to (p, q), the coordinates of point (2, –1) changes to (5, 2). Find p and q.

13. Determine the equation of line through a point (–4, –3) and parallel to *x*-axis.

14. Check whether the points $\left(0, \frac{8}{3}\right)$, (1, 3) and (82, 30) are the vertices a triangle or not?

15. If a vertex of a triangle is (1, 1) and the midpoints of two sides through this vertex are (–1, 2) and (3, 2). Then find the centroid of the triangle.

16. If the medians through A and B of the triangle with vertices A (0, b) , B (0, 0) and C (a, 0) are mutually perpendicular. Then show that $a^2 = 2b^2$.

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. If the image of the point (3, 8) in the line px + 3y – 7 = 0 is the point (–1, –4), then find the value of p.

18. Find the distance of the point (3,2) from the straight line whose slope is 5 and is passing through the point of intersection of lines x + 2y = 5 and x – 3y + 5 = 0

19. The line 2x – 3y = 4 is the perpendicular bisector of the line segment AB. If coordinates of A are (–3, 1) find coordinates of B.

20. The points (1, 3) and (5, 1) are two opposite vertices of a rectangle. The other two vertices lie on line $y = 2x + c$. Find c and remaining two vertices.

21. If two sides of a square are along $5x - 12y + 26 = 0$ and $5x - 12y - 65 = 0$ then find its area.

22. Find the equation of a line with slope –1 and whose perpendicular distance from the origin is equal to 5.

23. If a vertex of a square is at (1, –1) and one of its side lie along the line $3x - 4y - 17 = 0$ then find the area of the square.

24. What is the value of y so that line through (3, y) and (2, 7) is parallel to the line through (–1, 4) and (0, 6)?

25. In what ratio, the line joining (–1, 1) and (5, 7) is divided by the line $x + y = 4$?

26. Find the equation of the lines which cut-off intercepts on the axes whose sum and product are 1 and –6 respectively.

27. Find the area of the triangle formed by the lines $y = x$, $y = 2x$, $y = 3x + 4$.

28. Find the coordinates of the orthocentre of a triangle whose vertices are (–1, 3) (2, –1) and (0, 0). [Orthocentre is the point of concurrency of three altitudes].

29. Find the equation of a straight line which passes through the point of intersection of $3x + 4y - 1 = 0$ and $2x - 5y + 7 = 0$ and which is perpendicular to $4x - 2y + 7 = 0$.

30. If the image of the point (2, 1) in a line is (4, 3) then find the equation of line.

31. The vertices of a Δ are (6,0), (0,6) and (6,6). Find the distance between its circumcentre and centroid.

Section-D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

32. Find the equation of a straight line which makes acute angle with positive direction of x–axis, passes through point(–5, 0) and is at a perpendicular distance of 3 units from origin.

33. One side of a rectangle lies along the line $4x + 7y + 5 = 0$. Two of its vertices are (–3, 1) and (1,1). Find the equation of other three sides.

34. If (1,2) and (3, 8) are a pair of opposite vertices of a square, find the equation of the sides and diagonals of the square.

35. Find the equations of the straight lines which cut off intercepts on x–axis twice that on y–axis and are at a unit distance from origin.

36. Two adjacent sides of a parallelogram are $4x + 5y = 0$ and $7x + 2y = 0$. If the equation of one of the diagonals is $11x + 7y = 4$, find the equation of the other diagonal.

37. A line is such that its segment between the lines $5x - y + 4 = 0$ and$3x + 4y - 4 = 0$ is bisected at the point (1, 5). Obtain its equation.

38. If one diagonal of a square is along the line $8x - 15y = 0$ and one of its vertex is at (1, 2), then find the equation of sides of the square passing through this vertex.

39. If the slope of a line passing through te point A(3, 2) is 3/4 then find points on the line which are 5 units away from the point A.

40. Find the equation of straight line which passes through the intersection of the straight line $3x + 2y + 4 = 0$ and $x - y - 2 = 0$ and forms a triangle with the axis whose area is 8 sq. unit.

41. Find points on the line x + y + 3 = 0 that are at a distance of 5 unitsfrom the line x + 2y + 2 = 0

42. Show that the locus of the midpoint of the distance between the axes of the variable line $x\cos\alpha + y\sin\alpha = p$ is $\frac{1}{x^2}+\frac{1}{y^2}=\frac{4}{p^2}$, where p is a constant.

43. The line $\frac{x}{a}+\frac{y}{b}=1$ moves in such a way that $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ where c is constant. Show that the locus of the foot of perpendicular from the origin to the given line is $x^2+y^2=c^2$.

44. A point p is such that the sum of squares of its distance from the axes of coordinates is equal to the square of its distance from the line x –y =1. Find the locus of P.

45. A straight line L is perpendicular to the line 5x – y =1. The area of the Δ formed by the line L and the coordinate axes is 5. Find the equation of the line L.

46. The vertices of a Δ are $\left[at_1t_2,\ a(t_1+t_3)\right],\left[at_2t_3,\ a(t_2+t_3)\right]$. Find the orthocentre of the Δ.

47. Two equal sides of an isosceles Δ are given by the equation 7x – y + 3 =0 & x + y –3 = 0 & its third side passes through the point (1,–10). Determine the equation of the third side.

48. Let A (2,–3) & B (–2,1) be the vertices of a Δ ABC. If the centroid of this triangle moves on the line 2x + 3y = 1. Then find the locus of the vertex C.

49. ABCD is a rhombus. Its diagonals AC and BD intersect at the point M and satisfy BD = 2 AC. If the coordinates of D and M are (1,1) & (2,–1) respectively. Then find the coordinates of A.

50. Find the area enclosed within the curve $|x|+|y|=1$.

51. If the area of the triangle formed by a line with coordinates axes is $54\sqrt{3}$ square units and the perpendicular drawn from the origin to the line makes an angle 60° with the x-axis, find the equation of the line.

52. Find the coordinators of the circumcentre of the triangle whose vertices are (5,7), (6,6),& (2, -2).

53. Find the equation of a straight line, which passes through the point (a, 0) and whose $\perp$ distance from the point (2a, 2a) is a.

54. Line L has intercepts a and b on the coordinate axis when the axis are rotated through a given angle, keeping the origin fixed, the same line L has intercepts p and q, then prove that $a^{-2}+b^{-2}=p^{-2}+q^{-2}$.

ANSWERS

1.	(1, 2)	2.	k = 3
3.	p = 2, q = –5	4.	3 : 2 (internally)
6.	y = x – 1 and y = – x – 1.	7.	x + y = 7
8.	$\frac{4}{3}$	9.	$\frac{\pi}{4}$
10.	$\frac{2\pi}{3}$	11.	(–3, –1)
12.	p = –3, q = –3	13.	y + 3 = 0
14.	No	15.	$\left(1, \frac{7}{3}\right)$

17. 1

18. $\frac{10}{\sqrt{26}}$

19. (1, –5)

20. c = –4, (2,0), (4, 4)

21. 49 square units

22. $x + y + 5\sqrt{2} = 0,\ x + y \quad 5\sqrt{2} = 0$

23. 4 square units

24. $y = 9$

25. 1 : 2

26. $2x - 3y - 6 = 0$ and $-3x + 2y - 6 = 0$

27. 4 square units

28. (–4, –3)

29. x + 2y = 1

30. x + y – 5 = 0

31. $3\sqrt{2}$

32. 3x – 4y + 15 = 0

33. 4x + 7y – 11 = 0, 7x – 4y + 25 = 0

    7x – 4y – 3 = 0

34. x – 2y + 3 = 0, 2x + y – 14 = 0,

x – 2y + 13 = 0, 2x + y – 4 = 0

3x – y – 1 = 0, x + 3y – 17 = 0

35. $x + 2y + \sqrt{5} = 0, x + 2y \quad \sqrt{5} = 0$

36. x = y

37. 107*x* – 3*y* – 92 = 0

38. 23x – 7y – 9 = 0 and 7x + 23y – 53 = 0

39. (–1, –1) or (7, 5)

40. x – 4y – 8 = 0 or x + 4y + 8 = 0

41. (1,–4), (–9,6)

44. $x^2 + y^2 + 2xy + 2x - 2y - 1 = 0$

45. $x + 5y = \pm 5\sqrt{2}$ 46. $\left(-a, a(t_1 + t_2 + t_3 + t_1t_2t_3\right)$

47. $x - 3y - 31 = 0$
$3x + y + 7 = 0$

48. $\frac{x}{-2} + \frac{y}{1} = 1$ 49. $\left(1, \frac{-3}{2}\right)$ *or* $\left(3, \frac{-1}{2}\right)$

50. $\sqrt{3}$ 51. $x + \sqrt{3}y = 18$

52. $(2,3)$ 53. $3x - 4y - 3a = 0$ *and* $x - a = 0$

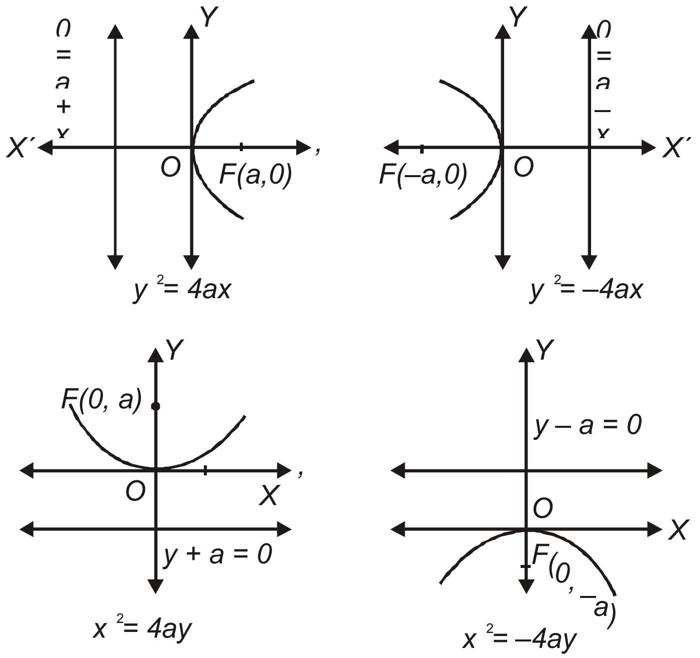
CHAPTER - 11

CONIC SECTIONS

KEY POINTS

- Circle, ellipse, parabola and hyperbola are curves which are obtained by intersection of a plane and cone in different positions
- **Circle:** It is the set of all points in a plane that are equidistant from a fixed point in that plane
- Equation of circle: $(x - h)^2 + (y - k)^2 = r^2$

  Centre (h, k), radius = r
- **Parabola:** It is the set of all points in a plane which are equidistant from a fixed point (focus) and a fixed line (directrix) in the plane. Fixed point does not lie on the line.

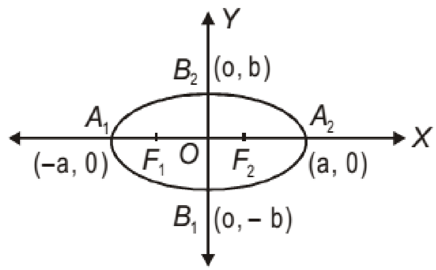


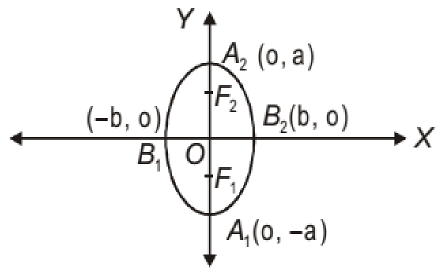
Main facts about the Parabola

Equation	$y^2 = 4ax$ (a > 0) Right hand	$y^2 = -4ax$ a > 0 Left hand	$x^2 = 4ay$ a > 0 Upwards	$x^2 = -4ay$ a > 0 Downwards
Axis	y = 0	y = 0	x = 0	x = 0
Directrix	x + a = 0	x – a = 0	y + a = 0	y – a = 0
Focus	(a, 0)	(–a, 0)	(0, a)	(0, –a)
Length of latus–rectum	4a	4a	4a	4a
Equation of latus–rectum	x – a = 0	x + a = 0	y – a = 0	y + a = 0

- **Latus Rectum:**A chord through focus perpendicular to axis of parabola is called its latus rectum.

- **Ellipse:** It is the set of points in a plane the sum of whose distances from two fixed points in the plane is a constant and is always greater than the distances between the fixed points





$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad\qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$$

$a > b > 0$, $b > a > 0$

$$c = \sqrt{a^2 - b^2}$$

Main facts about the ellipse

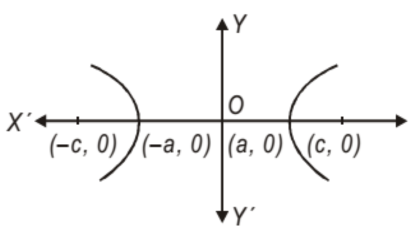
Equation	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$
	$a > 0, b > 0$	$a > 0, b >$
0		
Centre	(0,0)	(0,0)
Major axis lies along	x–axis	y–axis
Length of major axis	2a	2a
Length of minor axis	2b	2b

Foci	(–c, 0), (c, 0)	(0, –c),(0, c)
Vertices	(–a, 0), (a, 0)	(0, –a), (0, a)
Eccentricity (e)	$\frac{c}{a}$	$\frac{c}{a}$
Length of latus–rectum	$\frac{2b^2}{a}$	$\frac{2b^2}{a}$

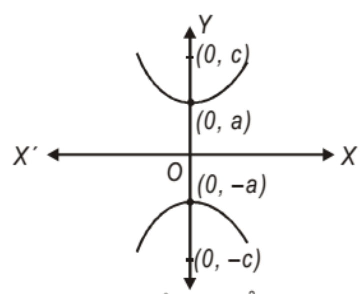
- If e = 0 for ellipse than b = a and equation of ellipse will be converted in equation of the circle its eq. will be $x^2 + y^2 = a^2$. it is called auxiliary circle. For auxiliary circle, diameter is equal to length of major axis and e = 0

- **Latus rectum:**Chord through foci perpendicular to major axis called latus rectum.

- **Hyperbola:**It is the set of all points in a plane, the differences of whose distance from two fixed points in the plane is a constant.



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$$

$$c = \sqrt{a^2 + b^2}$$

Main facts about the Hyperbola

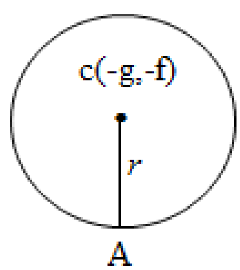
Equation	$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ a > 0, b > 0	$\frac{x^2}{b^2}-\frac{y^2}{a^2}=1$ a > 0, b > 0

Latus Rectum: Chord through foci perpendicular to transverse axis is called latus rectum.

If $e = \sqrt{2}$ for hyperbola, then hyperbola is called rectangular hyperbola. For $e = \sqrt{2}$ then b = a and eq. of its hyperbola will be $x^2 - y^2 = a^2$ or $y^2 - x^2 = a^2$.

(A) **Circle:** If the equation of the circle is in the form of

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

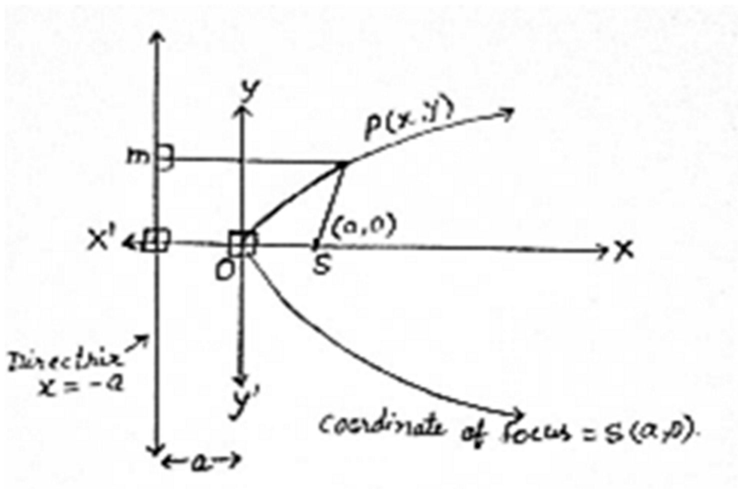


then centre of circle of = c (–g, –f)

and radius of circle = $r = \sqrt{g^2 + f^2 - c}$

(B) **Parabola:** in figure:-

(i) Equation of the parabola $y^2 = 4ax$.



Eccentricity of parabola = e = 1

Equation of directrix will be x = –a

Or x + a = 0

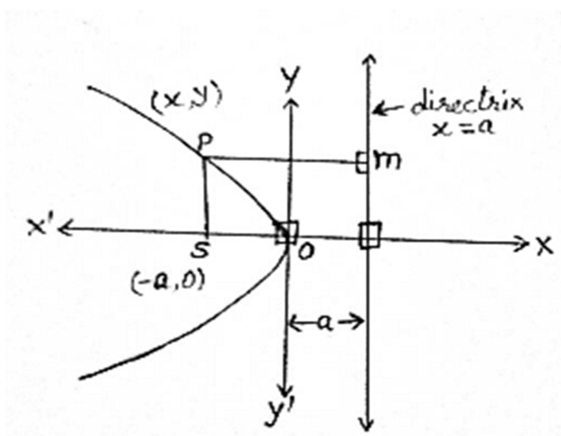
And if a point P lies on the parabola and $PM \perp directrix$ join PS where s is focus of the parabola then

Eccentricity $e = \frac{PS}{PM}.\ ie\ \ 1 = \frac{PS}{PM}\quad i.e = 1 \Rightarrow\ PS = PM.$

(ii) Equation of parabola: $y^2 = -4ax$

Equation of directrix x = a and $e = \frac{PS}{PM} = 1$

Coordinate of focus = S (–a, 0)

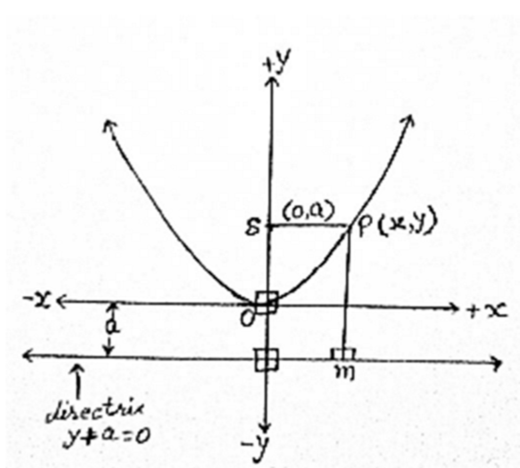


(iii) Equation of parabola: $x^2 = 4ax$
Equation of directrix $y = -a$

$$\Rightarrow y + a = 0$$

And eccentricity $e = \dfrac{PS}{PM} = 1$

Coordinate of focus = S (0, a).

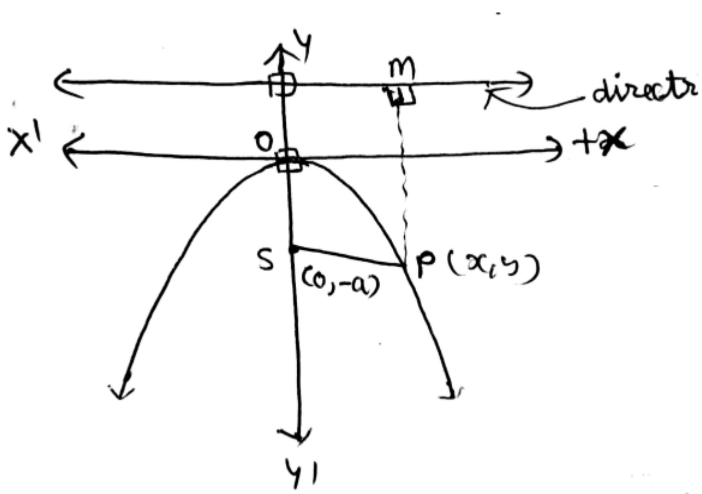


(iv) Equation of parabola: $x^2 = -4ay$

Equation of directrix $y = a$

And eccentricity $e = \frac{PS}{PM} = 1$

$$\Rightarrow PS = PM.$$



Coordinate of focus = S (0, –a)

(C) Ellipse:

(i) Equation of ellipse $\rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

and $b^2 = a^2(1-e^2) \rightarrow a > b > 0$

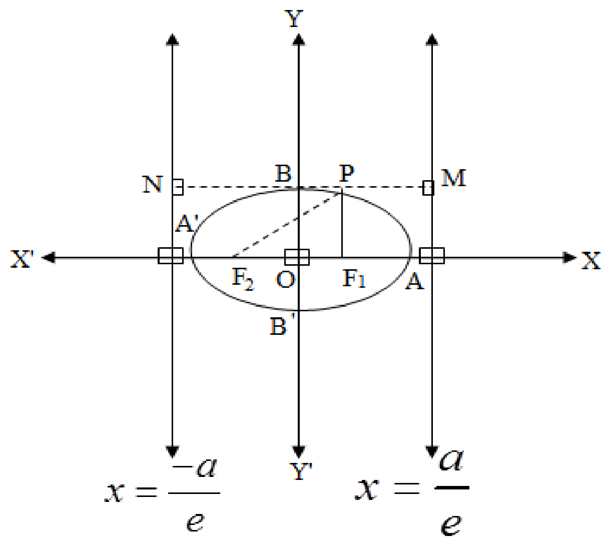
where e = eccentricity and 0 < e < 1.

and $e = \frac{PF_1}{PM}$ or $e = \frac{PF_2}{PN}$

where PM $\perp$ directrix (I)

PN $\perp$ directrix (II)

and equation of directrix: $x = \pm \frac{a}{e}$.

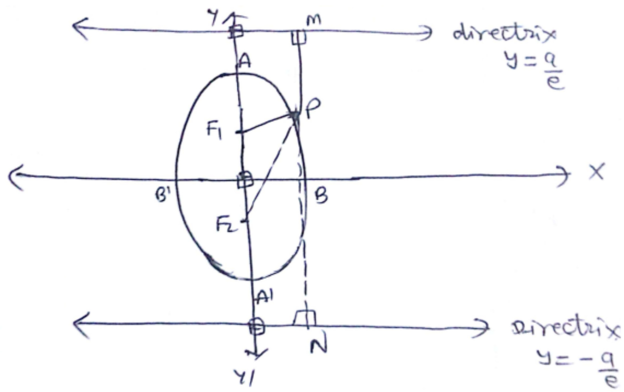


(ii) Equation of ellipse $\rightarrow \frac{y^2}{a^2} + \frac{x^2}{b^2} = 1$ $a > b > 0$

and $b^2 = a^2(1 - e^2)$

eccentricity $e = \frac{PF_1}{PM} = \frac{PF_2}{PN}$

equation of directrix: $y = \pm \frac{a}{e}$., $0 < e < 1$.



(D) (i) for hyperbola: e > 1 and $e = \frac{PF_1}{PM} = \frac{PF_2}{PN}$

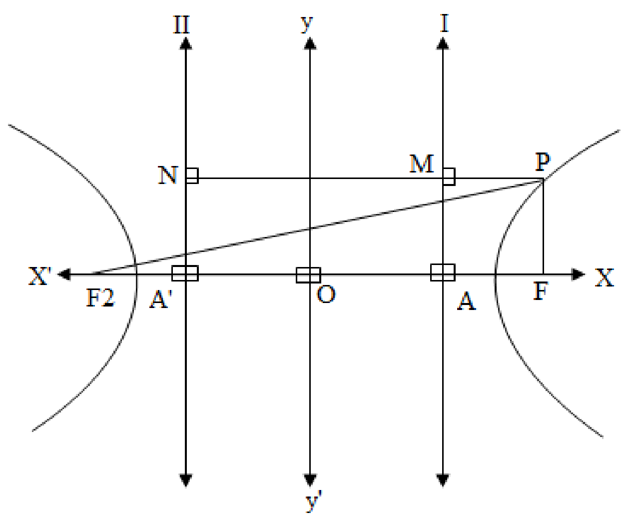
where PM $\perp$ directrix (I)

PN $\perp$ directrix (II)

equation of hyperbola $\rightarrow \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

and $b^2 = a^2(e^2 - 1).,\quad e > 0$

equation of directrix: $x = \pm\frac{a}{e}$



$x = \frac{-a}{e}$ $x = \frac{a}{e}$

(ii) equation of hyperbola $\rightarrow \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

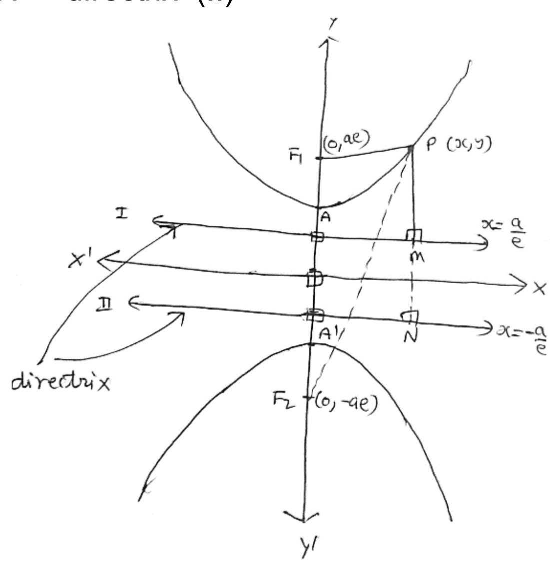
and $b^2 = a^2(e^2 - 1).,\quad e > 1$

equation of directrix: $x = \pm\frac{a}{e}$

eccentricity $e = \frac{PF_1}{PM} = \frac{PF_2}{PN}$

where PM $\perp$ directrix (I)

PN $\perp$ directrix (II)



Section-A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the centre and radius of the circle

$3x^2 + 3y^2 + 6x - 4y - 1 = 0$

2. Does $2x^2 + 2y^2 + 3y + 10 = 0$ represent the equation of a circle? Justify.

3. Find equation of circle whose end points of one of its diameter are (– 2, 3) and (0, –1).

4. Find the value(s) of p so that the equation $x^2 + y^2 – 2px + 4y – 12 = 0$ may represent a circle of radius 5 units.

5. If parabola $y^2 = px$ passes through point (2, –3), find the length of latus rectum.

6. Find the coordinates of focus, and length of latus rectum of parabola

   $3y^2 = 8x$.

7. Find the eccentricity of the ellipse

   $$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

Section – B

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)

8. Find the centre and radius of the circle $x^2 + y^2 – 6x + 4y – 12 = 0$

9. Find the length of major and minor axis of the following ellipse,

   $16x^2 + 25y^2 = 400$

10. Find the eqn. of hyperbola satisfying given conditions foci (±5, 0) and transverse axis is of length 8.

11. Find the coordinates of points on parabola $y^2 = 8x$ whose focal distance is 4.

12. Find the distance between the directrices ellipse $\frac{x^2}{36} + \frac{y^2}{20} = 1$

13. Write the equation of directrix of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ such that a > b.

14. If the eccentricity of the ellipse is zero. Then show that ellipse will be a circle.

15. Write the equation of directrix of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

16. If the eccentricity of the hyperbola is $\sqrt{2}$. Then find the general equation of hyperbola.

Sectinon-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

17. If one end of a diameter of the circle $x^2 + y^2 – 4x – 6y + 11 = 0$ is (3, 4), then find the coordinates of the other end of diameter.

18. Find the equation of the ellipse with foci at (± 5, 0) and $x = \frac{36}{5}$ as one of the directrices.

19. Find equation of an ellipse having vertices (0,± 5) and foci (0, ± 4).

20. If the distance between the foci of a hyperbola is 16 and its eccentricity is 2, then obtain the equation of a hyperbola.

21. Find the equation for the ellipse that satisfies the given condition

    Major axis on the *x*-axis and passes through the points (4, 3) and (6, 2).

22. The foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{25}-\frac{y^2}{9}=1$ find the equation of the hyperbola if its eccentricity is 2.

23. Find the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ which passes through the points (3, 0) and $\left(3\sqrt{2},\ 2\right)$.

24. If the latus rectum of an ellipse is equal to half of minor axis, then find its eccentricity.

25. Find equation of circle concentric with circle $4x^2 + 4y^2 - 12x - 16y - 21 = 0$ and of half its area.

26. Find the equation of a circle whose centre is at (4, –2) and 3x – 4y+ 5 = 0 is tangent to circle.

27. If equation of the circle is in the form of $x^2+y^2+2gx+2fg+c=0$ then prove that its centre and radius will be (–g, –f) and $\sqrt{g^2+f^2-c}$ respectively.

28. If the end points of a diameter of circle are $A\left(x_1, y_1\right)$ and $B\left(x_2, y_2\right)$ then show that equation of circle will be $(x-x_1).(x-x_2)+(y-y_1)(y-y_2)=0$

29. Find the equation of the circle which touches the lines x = 0, y = 0 and x = 2c and c > 0.

30. Find the equation of parabola if its focus at (–1, –2) and equation of directrix is x – 2y + 3 = 0.

31. Find the equation of the set of all points the sum of whose distance from A(3,0) and B(9,0) is 12 unit. Write the name of the curve.

32. Find the equation of the set of all points such that the difference of their distance from (4,0) and (–4,0) is always equal of 2 unit. Write the name of the curve.

Section-D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

33. Prove that the points (1, 2), (3, – 4), (5, – 6) and (11, – 8) ae concyclic.

34. A circle has radius 3 units and its centre lies on the line y = x – 1. If it is passes through the point (7, 3) then find the equations of the circle.

35. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, – 9). Find its centre and radius.

36. Find the equation of circle having centre (1, – 2) and passing through the point of intersection of the lines 3x + y = 14 and 2x + 5y = 18.

37. Prove that the equation $y^2+2Ax+2By+c=0$ is represent aparabola and whose axis is parabola to x axis.

38. Find the coordinates of focus, axis of parabola, the equation of directrix, coordinate of vertex of the parabola $x^2-6x-4y-11=0$.

39. Show that the points A(5,5), B(6,4), C(–2,4) and D(7,1) all lies on the circle. Find the centre, radius and equation of circle.

40. Find the equation of the ellipse in which length of minor axis is equal to distance between focies given length of latus rectum is 10 unit. And major axis is along the x axis.

41. Find the equation of the hyperbola whose axes (transverse and conjugate axis) are parallel to x axis and y axis and centre is origin. Length of latus rectum lengthis 18 unit and distance between focies is 12 unit.

42. Prove that the line 3x + 4y + 7 = 0 touches the circle $x^2 + y^2 - 4x - 6y - 12 = 0$. Also find the point of contact.

43. Find the equation of ellipse whose focus is (1, 0) and the directrix x + y + 1 = 0 and eccentricity is equal to $\frac{1}{\sqrt{2}}$.

44. If y_1, y_2, y_3 be the ordinates of a vertices of the triangle inscribed in a parabola $y^2 = 4ax$, then show that the area of the triangle is $\frac{1}{8a}|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)|$

45. For the hyperbola whose equation is $9x^2 - 16y^2 - 18x + 32y - 151 = 0$, find the length of transverse and conjugate axis, length of latus rectum, co-ordinates of foci and vertices, equation of directrices and eccentricity.

46. Find the equation of parabola whose focus is (1, –1) and whose vertex is (2, 1). Also, find its axis and latus rectum.

47. Find the equation of hyperbola whose focus is (1, 2) and the directrix 2x + y = 1 ad eccentricity is equal to $\sqrt{3}$.

ANSWERS

1. $\left(-1\frac{2}{3}\right), \frac{4}{3}$

2. No

3. $x^2 + y^2 + 2x - 2y - 3 = 0$ or $(x + 1)^2 + (y - 1)^2 = 5$

4. –3 +3

5. $\frac{9}{2}$

6. $\left(\frac{2}{3},0\right), \frac{8}{3}$

7. $\frac{4}{5}$

8. (3,–2), 5

9. 10,8

10. $\frac{x^2}{16}-\frac{y^2}{9}=1$

11. (2, 4) (2, – 4)

12. 18

13. $x=\pm\frac{a}{e}$

15. $x=\pm\frac{a}{e}$

16. $x^2-y^2=a^2 \text{ or } y^2-x^2=a^2$

17. (1,2)

18. $\frac{x^2}{36}+\frac{y^2}{11}=1$

19. $\frac{x^2}{9}+\frac{y^2}{25}=1$

20. $x^2-y^2=32$

21. $\frac{x^2}{52}+\frac{y^2}{13}=1$

22. $\frac{x^2}{4}-\frac{y^2}{12}=1$

23. $e=\frac{\sqrt{13}}{3}$

24. $e=\frac{\sqrt{3}}{2}$

25. $2x^2 + 2y^2 - 6x + 8y + 1 = 0$

26. $x^2 + y^2 - 8x + 4y - 5 = 0$

29. $x^2 + y^2 \quad 2cx \pm 2cy + c^2 = 0$

30. $4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$

31. $3x^2 + 4y^2 = 36, \ ellipse$

32. $15x^2 - y^2 = 15, \ Hyperbola$

34. $\begin{Bmatrix} x^2 + y^2 - 8x - 6y + 16 = 0 \\ x^2 + y^2 = 14x = 12y + 76 = 0 \end{Bmatrix}$

35. $x^2 + y^2 - 14x - 6y - 111 = 0$
$centre\,(7, 3) \ and \ radius = 13 unit$

36. $x^2 + y^2 - 8x + 4y - 5 = 0$

38. $(3, -4), \ x = 3, \ y + 6 = 0 \ and\,(3, -5)$

39. $c(2,1), \ r = 5, \ x^2 + y^2 - 4x - 2y - 20 = 0$

40. $x^2 + 2y^2 = 100$

41. $3x^2 - y^2 = 27$

43. $3x^2 - 2xy + 3y^2 - 10x - 2y + 3 = 0$

46. $4x^2 - 4xy + y^2 + 8x + 46y - 71 = 0$; axis: $2x - y - 3 = 0$, length of L.R. = $4\sqrt{5}$.

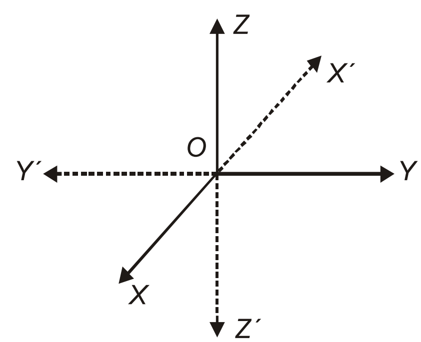
47. $7x^2 - 2y^2 + 12xy - 2x + 14y - 22 = 0$

CHAPTER - 12

INTRODUCTION TO THREE-DIMENSIONAL COORDINATE GEOMETRY

KEY POINTS

- Three mutually perpendicular lines in space define three mutually perpendicular planes, called Coordinate planes, which in turn divide the space into eight parts known as octants and the lines are known as Coordinate axes.



Coordinate axes: XOX', YOY', ZOZ'

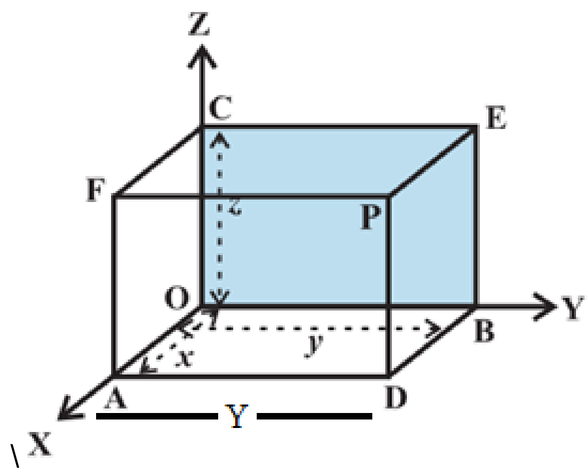
Coordinate planes: XOY, YOZ, ZOX or

XY, YX, ZX planes

Octants: OXYZ, OX'YZ, OXY'Z, OXYZ'

OX' Y'Z, OXY'Z', OX'YZ', OX'Y'Z'

- Coordinates of a points lying on x-axis, y-axis & z-axis are of the from (x,0,0), (0,y,0),(0,0,z) respectively.
- Coordinates of a points lying on xy-plane, yz-plane, & xz- plane are of the from (x,y,0), (0,y,z),(x,0,z) respectively.
- The reflection of the point (x, y, z) in xy-plane yz-plane, &xz-plane is (x,y,–z), (–x,y,z),(x,–y,z) respectively.
- Coordinates of a point P are the perpendicular distances of P from three coordinate planes YZ, ZX and XY respectively.



- The distance between the point A(x1, y1, z1) and B(x2, y2, z2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Let P(x1, y1, z1) and Q (x2, y2, z2) be two points in space and let R be a point on line segment PQ such that it divides PQ in the ratio $m_1 : m_2$

  (I) internally, then the coordinates of R are

  $$\left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}, \frac{m_1z_2 + m_2z_1}{m_1 + m_2}\right)$$

  (II) externally, then coordinates of R are

  $$\left(\frac{m_1x_2 - m_2x_1}{m_1 - m_2}, \frac{m_1y_2 - m_2y_1}{m_1 - m_2}, \frac{m_1z_2 - m_2z_1}{m_1 - m_2}\right)$$

  Coordinates of centroid of a triangle whose vertices are $(x_1, y_1, z_1), (x_2, y_2, z_2)$ and (x_3, y_3, z_3) are

  $$\left(\frac{x_1 + y_1 + z_1}{3}, \frac{x_2 + y_2 + z_2}{3}, \frac{x_3 + y_3 + z_3}{3}\right)$$

Section-A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Find the image of (–5, 4, –3) in xz plane.
2. Name the octant in which (–5, 4, –3) lies.
3. What is the perpendicular distance of the point P(6, 7, 8) from xy plane.
4. Find the distance of point P(3, –2,1) from z–axis.
5. Write coordinates of foot of perpendicular from (3, 7, 9) on x axis y-axis, z-axis.
6. If the distance between the points (a, 2, 1) and (1, –1, 1) is 5, then find the value (s) of a

Section-B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

7. What are the coordinates of the vertices of a cube whose edge is 2 unit, one of whose vertices coincides with the origin and the three edges passing through the origin? Coincides with the positive direction of the axes through the origin?

8. Let A, B, C be the feet ofperpendiculars from point P on the xy, yz and xZ planes respectively. Find the coordinates of A, B, C where the point P is (4, –3, –5).

9. If a parallelopiped is formed by planes drawn through the point (5,8,10) & (3,6,8) parallel to the coordinates planes, then find the length of the diagonal of the parallelopiped.

10. Find the length of the longest piece of a string that can be stretched straight in a rectangular room whose dimensions are 13,10,& 8 unit.

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. Show that points (4, –3, –1), (5, –7, 6) and (3, 1, –8) are collinear.

12. Find the point on y–axis which is equidistant from the point (3,1,2) and (5, 5, 2).

13. Determine the point in yz plane which is equidistant from three pointsA(2, 0, 3), B(0, 3, 2) and C(0, 0, 1).

14. The centroid of ΔABC is at (1,1,1). If coordinates of A and B are (3,–5,7) and (–1, 7, –6) respectively, find coordinates of point C.

15. Find the length of the medians of the triangle with vertices *A*(0,0,6) *B*(0, 4, 0) and *C*(6, 0, 0).

16. If the extremities (end points) of a diagonal of a square are (1,–2,3) and (2,–3,5) then find the length of the side of square.

17. Three consecutive vertices of a parallelogram ABCD are A(6,–2,4) B(2, 4,–8), C(–2, 2, 4). Find the coordinates of the fourth vertex.

18. If the points A(1, 0, –6), B(3, p, q) and C(5, 9, 6) are collinear, find the value of p and q.

19. Show that the point A(1, 3, 0), B(–5, 5,, 2), C(–9, –1, 2) and D(–3, –3, 0) are the vertices of a parallegram ABCD, but it is not a rectangle.

20. The mid points of the sides of a triangle are (5, 7, 11), (0, 8, 5) and (2 3, –1) Find its vertices and hence find centroid.

21. Find the coordinate of the point P which is five-sixth of way from A(–2, 0, 6) to B(10, –6, –12).

22. Prove that the points (0, –1, –7), (2, 1, –9) and (6, 5, –13) are collinear. Find the ratio in which first point divides the join of the other two.

23. Let A(3, 2, 0), B(5, 3, 2) C(–9, 6, –3) be three points forming a triangle. AD, the bisector of $\angle BAC$, meets BC in D. Find the coordinates of the point D.

24. Describe the vertices and edges of the rectangular parallelopiped with one vertex (3,5,6) placed in the first octant with one vertex at origin and edges of parallelopiped lie along x, y & z-axis.

25. Find the coordinates of the point which is equidistant from the point (3,2,2), (–1,2,2), (0,5,6) & (2,1,2).

ANSWERS

1.	(–5, –4, –3)	2.	OX' YZ'
3.	8	4.	$\sqrt{13}\ unit$
5.	(3,0,0)	6.	5, –3

7. (2,0,0) (2,2,0), (0,2,0) (0,2,2), (0,0,2) (2,0,2) (0,0,0), (2,2,2)

8.	(4, –3, 0), (0, –3, –5), (4, 0, –5)	9.	$2\sqrt{3}$
10.	$\sqrt{333}$	12,	(0,5,0)
13.	(0,1,3)	14.	(1,1,2)
15.	$7, \sqrt{34}, 7$	16.	$\sqrt{3}\ unit$

17. (2, –4, 16)

18. p = 6, q = 2

20. Vertices (–3, 4, –7), (7, 2, 5), (3, 21, 17) centroid $\left(\frac{7}{3}, 6, 5\right)$

21. (8,–5,–9)

22. 1:3 externally

23. $\left(\frac{19}{8}, \frac{57}{16}, \frac{17}{16}\right)$

24. (0,0,0) (3,0,0), (3,5,0) (0,5,0), (0,5,6) (0,0,6) (3,0,6), (3,5,6) $\sqrt{61}$, $\sqrt{45}, \sqrt{34}$

25. (1,3,4)

CHAPTER - 13

LIMITS AND DERIVATIVES

KEY POINTS

- $\lim\limits_{x \to c} f(x) = l$ if and only if
- $\lim\limits_{x \to c^-} f(x) = \lim\limits_{x \to c^+} f(x) = l$
- $\lim\limits_{x \to c} \alpha = \alpha$, where α is a fixed real number.
- $\lim\limits_{x \to c} x^n = c^n$, for all $n \in N$
- $\lim\limits_{x \to c} f(x) = f(c)$, where $f(x)$ is a real polynomial in x.

Algebra of limits

Let f, g be two functions such that $\lim\limits_{x \to c} f(x) = l$ and $\lim\limits_{x \to c} g(x) = m$, then

- $\lim\limits_{x \to c} [\alpha\ f(x)] = \alpha \lim\limits_{x \to c} f(x)$

  $= \alpha\ l$ for all $\alpha \in R$

- $\lim_{x\to c} [f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x) = l \pm m$

- $\lim_{x\to c} [f(x).g(x)] = \lim_{x\to c} f(x). \lim_{x\to c} g(x) = lm$

- $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)} = \frac{l}{m}$, $m \neq 0$ $g(x) \neq 0$

- $\lim_{x\to c} \frac{1}{f(x)} = \frac{1}{\lim_{x\to c} f(x)} = \frac{1}{l}$ provided $l \neq 0$ $f(x) \neq 0$

- $\lim_{x\to c} [(f(x)]^n = \left[\left(\lim_{x\to c} f(x)\right)\right]^n = l^n$, for all $n \in N$

Some important theorems on limits

- $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(-x)$

- $\lim_{x\to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

- $\lim_{x\to 0} \frac{\sin x}{x} = 1$ where x is measured in radians.

- $\lim_{x\to 0} \frac{\tan x}{x} = 1 \left[\text{Note that } \lim_{x\to 0} \frac{\cos x}{x} \neq 1\right]$

- $\lim_{x\to 0} \frac{1 - \cos x}{x} = 0$

- $\lim_{x\to 0} \frac{e^x - 1}{x} = 1$

- $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$
- $\lim_{x \to 0} \frac{\log(1 + x)}{x} = 1$
- $\lim_{x \to 0} (1 + x)^{1/x} = e$

$$\frac{d(\sin x)}{dx} = \cos x$$

$$\frac{d(\cos x)}{dx} = -\sin x$$

$$\frac{d(\tan x)}{dx} = \sec^2 x$$

$$\frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$$

$$\frac{d(\sec x)}{dx} = \sec x \tan x$$

$$\frac{d(\operatorname{cosec} x)}{dx} = -\operatorname{cosec} x . \cot x$$

$$\frac{d(x^n)}{dx} = n.x^{n-1}$$

$$\frac{d\left(e^x\right)}{dx} = e^x$$

$$\frac{d\left(a^x\right)}{dx} = a^x.\log a$$

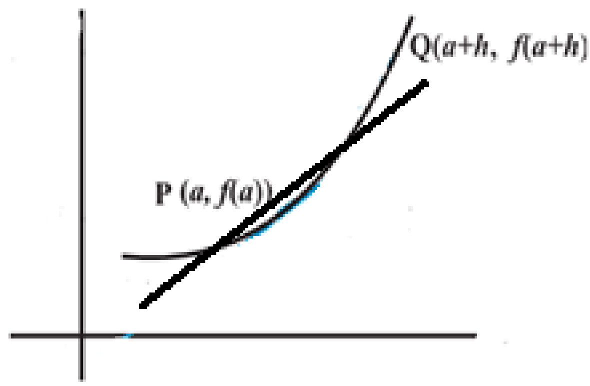
$$\frac{d\left(\log_e x\right)}{dx} = \frac{1}{x}$$

$$\frac{d\left(constant\right)}{dx} = 0$$

<u>Laws of Logarithm</u>

- ❖ $\log_e A + \log_e B = \log_e\left(AB\right)$
- ❖ $\log_e A - \log_e B = \log_e\left(\frac{A}{B}\right)$
- ❖ $\log_e A^m = m\log_e A$
- ❖ $\log_a 1 = 0$
- ❖ If $\log_B A = x$ then $B^x = A$

Let y = f(x) be a function defined in some neighbourhood of the point 'a'. Let P(a, f(a)) and Q(a + h, f (a + h)) are two points on the graph of f(x) where h is very small and $h \neq 0$.



Slope of PQ = $\dfrac{f(a+h)-f(a)}{h}$

If $h \to 0$, point Q approaches to P and the line PQ becomes a tangent to the curve at point P.

$\lim\limits_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$ (if exists) is called derivative of f(x) at the point 'a'. It is denoted by f'(a).

Algebra of derivatives

- $\dfrac{d}{dx}(cf(x)) = c.\dfrac{d}{dx}(f(x))$ where c is a constant
- $\dfrac{d}{dx}(f(x) \pm g(x)) = \dfrac{d}{dx}(f(x)) \pm \dfrac{d}{dx}(g(x))$
- $\dfrac{d}{dx}(f(x).g(x)) = f(x).\dfrac{d}{dx}(g(x)) + g(x)\dfrac{d}{dx}(f(x))$

- $$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)=\frac{g(x)\frac{d}{dx}(f(x))-f(x)\frac{d}{dx}(g(x))}{(g(x))^2}$$

- If y = f(x) is a given curve then slope of the tangent to the curve at the point (h,k) is given by $\left.\frac{dy}{dx}\right]_{(h,k)}$ and is denoted by 'm'.

Section-A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Evaluate the following Limits:

1. $\lim\limits_{x\to 3}\frac{\sqrt{2x+3}}{x+3}$

2. $\lim\limits_{x\to 0}\frac{\sin 3x}{x}$

3. $\lim\limits_{x\to 1}\frac{\sqrt{1+x}-\sqrt{1-x}}{1+x}$

4. $\lim\limits_{x\to \infty}\frac{1+2+3+......+n}{n^2}$

5. Find n, if $\lim\limits_{x\to 2}\frac{x^n-2^n}{x-2}=80.\quad n\in N$

6. $\lim\limits_{x\to 0}\frac{\tan^2 3x}{x^2}$

7. $\lim\limits_{x\to a}\frac{\cos x-\cos a}{x^{\frac{3}{2}}-a^{\frac{3}{2}}}$

Section-B

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Differentiate the following functions with respect to x :

8. $\frac{x}{2}+\frac{2}{n}$

9. $x^2 \tan x$

10. $\frac{x}{\sin x}$

11. 2^x

12. $3^x + x^3 + 4x - 5$

13. $(x^2 - 3x + 2)(x + 2)$

14. $e^x \sin x + x^n . \cos x.$

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

15. Evaluate $\lim\limits_{x \to 0} \frac{\sqrt{2}-\sqrt{1+\cos x}}{\sin^2 x}$

16. Evaluate $\lim\limits_{x \to \frac{\pi}{6}} \frac{\sqrt{3}\sin x - \cos x}{x - \frac{\pi}{6}}$

17. If $f(x) = \begin{cases} \frac{|x-1|}{2(x-1)} & x \neq 1 \\ 3 & x = 1 \end{cases}$ show that $\lim\limits_{x \to 1} f(x)$ does not exists.

18. If $f(x)=\begin{cases}5x-4 & 0\le x\le 1\\ 4x^3-3x, & 1<x<2\end{cases}$ show that $\lim_{x\to 1} f(x)$ exists.

19. Let $f(x)=\begin{cases}\cos x & x\ge 1\\ x+k & , x<0\end{cases}$ find k if $\lim_{x\to 0} f(x)$ exists.

20. Find $\lim_{x\to 5}\left([x+3]-\frac{2|x-5|}{x-5}+4x^2\right)$, if exists.

21. Find the value of 'k' if $\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$.

Evaluate the following Limits:

22. $\lim_{x\to 0}\frac{\sin(x^\circ)}{x}$

23. $\lim_{x\to 1}\frac{\sqrt{x^2-1}+\sqrt{x-1}}{\sqrt{x^2-1}}\quad , \quad x>1$

24. $\lim_{x\to 9}\frac{x^{\frac{3}{2}}-27}{x^2-81}$

25. $\lim_{x\to a}\frac{(x+2)^{\frac{5}{2}}-(a+2)^{\frac{5}{2}}}{x-a}$

26. $\lim_{x\to 0}\frac{\cos ax-\cos bx}{\cos x-1}$

27. $\lim_{x\to 0}\frac{\tan x-\sin x}{x\sin^2 3x}$

28. $\lim_{x\to 0}\frac{x\tan x}{1-\cos x}$

29. $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cot x - \cos x}{(\pi - 2x)^3}$

30. $\lim\limits_{x \to a} \dfrac{\cos x - \cos a}{\cot x - \cot a}$

31. $\lim\limits_{x \to \pi} \dfrac{1 + \sec^3 x}{\tan^2 x}$

32. $\dfrac{e^x - e^{\sin x}}{\sin x - x}$

33. $\lim\limits_{x \to 1} \dfrac{x - 1}{\log_e x}$

34. $\lim\limits_{x \to e} \dfrac{\log_e x - 1}{x - e}$

35. $\lim\limits_{x \to 2} \left[\dfrac{4}{x^3 - 2x^2} + \dfrac{1}{2 - x} \right]$

36. $\lim\limits_{x \to a} \dfrac{\sqrt{a + 2x} - \sqrt{3x}}{\sqrt{3a + x} - 2\sqrt{x}}$

37. $\lim\limits_{x \to 0} \dfrac{\sin(2 + x) - \sin(2 - x)}{x}$

38. $\lim\limits_{x \to 0} \dfrac{1 - \cos x \sqrt{\cos 2x}}{x^2}$

39. $\lim\limits_{x \to 0} \dfrac{10^x - 2^x - 5^x + 1}{x \tan x}$

40. $\lim\limits_{x \to 0} \dfrac{e^{x^2} - \cos x}{\sin^2 x}$

41. $\lim_{x \to 0} \frac{\sin^2(x+a) - \sin^2 a}{x}$

Differentiate the following functions with respect to x by first principle:

42. $f(x) = \frac{x^2+1}{x}$

43. $f(x) = \sqrt{2x+3}$

44. $f(x) = \cos(2x-7)$

45. $f(x) = \tan\sqrt{x}$

46. $f(x) = e^{\tan x}$

47. $f(x) = \cos(2x^2-3)$

Differentiate the following functions with respect to x:

48. $f(x) = \frac{(3x+1)\left(2\sqrt{x}-1\right)}{\sqrt{x}}$

49. $f(x) = \left(x - \frac{1}{x}\right)\left(x^2 - \frac{1}{x^2}\right)$

50. $f(x) = \frac{\sin x - x\cos x}{x\sin x + \cos x}$

51. $f(x) = x^3 e^x \sin x$

52. $f(x) = x^5 e^x + x^3 \log x - 2^x$

53. $f(x) = \frac{2^x . \cot x}{\sqrt{x}}$

54. $f\left(x\right)=e^{2x}\sin x+x^{n}.\cos x$

55. $f\left(x\right)=\dfrac{x^{2}}{\cos x}-2\log(\cos x)+xe^{x}.$

56. $f\left(x\right)=\sqrt{\dfrac{1-\cos 2x}{1+\cos 2x}}$

57. If $f\left(x\right)=x-[x]$ then find $f^{1}\left(\dfrac{1}{2}\right)$.

58. If $y=\sqrt{x}+\dfrac{1}{\sqrt{x}}$, prove that $2x.\dfrac{dy}{dx}+y=2\sqrt{x}$.

59. If $y=\sqrt{\dfrac{x}{a}}+\sqrt{\dfrac{a}{x}}$, prove that $\left(2xy\right)\dfrac{dy}{dx}=\dfrac{x}{a}-\dfrac{a}{x}$.

60. For the curve f(x) = (x^2 + 6x–5)(1–x), find the slope of the tangent at x = 3.

Secstion-D

LONG ANSWER TYPE QUESTIONS (6 MARKS)

Differentiate the following functions with respect to x from first principle:

61. $f\left(x\right)=\sqrt{\cos x}$

62. $f\left(x\right)=xe^{x}$

63. $f\left(x\right)=e^{\sqrt{ax+b}}$

64. $f\left(x\right)=\cos^{2}\left(3x-7\right)$

Evaluate the following Limits:

65. $\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{\sqrt{2} - \sqrt{1 + \sin x}}$

66. $\lim_{x \to 0} \frac{8}{x^8}\left[1 - \cos\left(\frac{x^2}{2}\right) - \cos\left(\frac{x^2}{4}\right) + \cos\left(\frac{x^2}{2}\right).\cos\left(\frac{x^2}{4}\right)\right]$

67. $\lim_{x \to \frac{\pi}{4}} \frac{4\sqrt{2} - (\sin x + \cos x)^5}{1 - \sin 2x}$

68. $\lim_{x \to 0} \frac{9^x - 2.6^x + 4^x}{x^2}$

69. $\lim_{x \to 0} \frac{\sqrt{x+1} - \sqrt{1-x}}{\sqrt[3]{1+x} - \sqrt[3]{1-x}}$

70. $\lim_{x \to 1} \frac{(7+x)^{\frac{1}{3}} - (3+x^2)^{\frac{1}{2}}}{x-1}$

71. Find the value of 'a' and 'b' if $\lim_{x \to 2} f(x)$ and $\lim_{x \to 4} f(x)$ exists where

$$f(x) = \begin{cases} x^2 + ax + b & 0 \le x < 2 \\ 3x + 2 & 2 \le x \le 4 \\ 2ax + 5b & 4 < x \le 8 \end{cases}$$

72. Evaluate $\lim_{x \to 3} f(x)$ where

$$f(x) = \begin{cases} x - [x] & x < 2 \\ 4 & x = 3 \\ 3x - 8 & x > 3 \end{cases}$$

ANSWERS

1. $\frac{1}{2}$

2. 3

3. $\frac{1}{\sqrt{2}}$

4. $\frac{1}{2}$

5. n = 5

6. 9

7. $\frac{-2\sin a}{3\sqrt{a}}$

8. $\frac{1}{2}-\frac{2}{x^2}$

9. $x^2 \sec^2 x + 2x \tan x$

10. $\cos ecx - x\cot x.\cos ecx \quad or \quad \cos ecx(1 - x\cot x)$

11. $2^x \log_e 2$

12. $3^x \log_e 3 + 3x^2 + 4$

13. $3x^2 - 2x - 4$

14. $e^x(\cos x + \sin x) + x^{n-1}[n.\cos x - x\sin x]$

15. $\frac{\sqrt{2}}{8}$

16. $\frac{3+\sqrt{3}}{2}$

19. k = 1

20. LHL = 109, RHL = 106 so, lim does not exist.

21. $k = \frac{8}{3}$

22. $\frac{\pi}{180}$

23. $\frac{\sqrt{2}+2}{2}$

24. $\frac{1}{4}$

25. $\frac{5}{2}(a+2)^{\frac{3}{2}}$

26.	$\dfrac{a^2-b^2}{c}$	27.	$\dfrac{1}{18}$
28.	2	29.	$\dfrac{1}{16}$
30.	$\sin^3 a$	31.	$\dfrac{-3}{2}$
32.	–1	33.	1
34.	$\dfrac{1}{e}$	35.	–1
36.	$\dfrac{2}{3\sqrt{3}}$	37.	2 cos 2
38.	$\dfrac{3}{2}$	39.	log 5. log 2
40.	$\dfrac{3}{2}$	41.	Sin 2a
42.	$1-\dfrac{1}{x^2}$	43.	$\dfrac{1}{\sqrt{2x+3}}$
44.	$-2\sin(2x-7)$	45.	$\dfrac{1}{2\sqrt{x}}\sec^2\sqrt{x}$
46.	$e^{\tan x}.\sec^2 x$	47.	$-4x\sin\left(2x^2-3\right)$
48.	$6-\dfrac{3}{2}x^{\frac{-1}{2}}+\dfrac{1}{2}x^{\frac{-3}{2}}$	49.	$3x^2+\dfrac{1}{x^2}-1-\dfrac{3}{x^4}$
50.	$\dfrac{x^2}{(x\sin x+\cos x)^2}$	51.	$x^2e^x\left[x\sin x+x\cos x+3\sin x\right]$
52.	$x^2\left[1+3\log x+5x^2e^x+x^3e^x\right]-2^x\log^2$		

53. $\frac{2^x}{\sqrt{x}}\left[\log 2.\cot x - \cos ec^2 x - \frac{\cot x}{2.x}\right]$

54. $e^{2x}\left[\cos x + 2\sin x\right] + x^{n-1}\left[n\cos x - x\sin x\right]$

55. $x\sec x\left[2 - x\tan x\right] + 2\tan x + e^x\left(x+1\right)$

56. $\sec^2 x$

57. 1

60. 46.

61. $\frac{-\sin x}{2\sqrt{\cos x}}$

62. $e^x(x+1)$

63. $\frac{a.e^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$

64. $-3.\sin(6x-14)$

65. $4\sqrt{2}$

66. $\frac{1}{32}$

67. $5\sqrt{2}$

68. $\left(\log\frac{3}{2}\right)^2$

69. $\frac{3}{2}$

70. $\frac{-1}{4}$

71. $a = -1,\ b = 6$

72. 1

CHAPTER – 14

MATHEMATICAL REASONING

KEY POINTS

- A sentence is called a statement if it is either true or false but not both simultaneously.
- The denial of a statement p is called its negation and is written as ~p and read as not p.
- Compound statement is made up of two or more simple statements. These simple statements are called component statements.
- 'And', 'or', 'If–then', 'only if' 'If and only if' etc. are connecting words, which are used to form a compound statement.
- Two simple statements p and q connected by the word 'and' namely 'p and q' is called a conjunction of p and q and is written as $p \wedge q$.
- Compound statement with **'And'** is
  - (a) true if all its component statements are true
  - (b) false if any of its component statement is false

p	q	$p \wedge q$
T	T	T

T	F	F
F	T	F
F	F	F

- Two simple statements p and q connected by the word 'or' the resulting compound statement 'p or q' is called disjunction of p and q and is written as $p \vee q$
- Compound statement with **'Or'** is
  (a) true when at least one component statement is true
  (b) false when both the component statements are false

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- The negation of the compound statement 'p or q' is '$\sim p$ *and* $\sim q$'
  $\sim (p \vee q) = \sim p \wedge \sim q$.
- The negation of the compound statement 'p and q' is '$\sim p$ *or* $\sim q$'
  $\sim (p \wedge q) = \sim p \vee \sim q$.
- A statement with **"If p then q"** can be rewritten as
  (a) p implies q
  (b) p is sufficient condition for q
  (c) q is necessary condition for p

(d) p only if q

(e) (~q) implies (~p)

- If in a compound statement containing the connective ‘‘or’’ all the alternatives cannot occur simultaneously, then the connecting word ‘‘or’’ is called as exclusive ‘‘or’’.
- If, in a compound statement containing the connective ‘‘or’’, all the alternative can occur simultaneously, then the connecting word ‘‘or’’ is called as inclusive ‘‘or’’.
- Contrapositive of the statement $p \Rightarrow q$ is the statement $\sim q \Rightarrow \sim p$
- Converse of the statement $p \Rightarrow q$ is the statement $q \Rightarrow p$
- “For all”, “For every” are called universal quantifiers
- A statement is called valid or invalid according as it is true or false.

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Identify which of the following are statements (Q. No 1 to 5)

1. Prime factors of 6 are 2 and 3.
2. $x^2 + 6x + 3 = 0$
3. The earth is a planet.
4. There is no rain without clouds.
5. All complex numbers are real numbers.

Write negation of the following statements (Q. No 6 to 11)

6. π is not a rational number.
7. Everyone in Spain speaks Spanish.
8. Zero is a positive number.

9. There exists a complex number which is not a real number.

10. For every real number x, either x>1 or x<1.

11. For every positive real number x, the number x–1 is also positive

Check whether the compound statement is true or false. Write the component statements. (Q. No. 12 to 15)

12. A square is a quadrilateral and its four sides are equal.

13. All integers are either even or odd.

14. $\sqrt{3}$ is a rational number or an irrational number.

15. "O" is either a positive number or negative number.

Identify the type 'Or' (Inclusive or Exclusive) used in the following statements (Q. No. 16 to 19)

16. Students can take French or Spanish as their third language.

17. To enter in a country, you need a visa or citizenship card.

18. Two lines intersect at a point or are parallel.

19. $\sqrt{2}$ is a rational number or an irrational number?

Write the negation of the following compound statements.

(Question No. 20 to 21)

20. It is daylight and all the people have arisen.

21. Square of an integer is positive or negative.

Identify the quantifiers in the following statements (Q. No.22 to 24)

22. For every integer $p\sqrt{p}$ is a real number.

23. There exists a number which is equal to its square.

24. There exists a capital for every country in the world.

Write the converse of the following statements (Q. No. 25 to 28)

25. If a number x is even, then x^2 is also even.

26. If 3 × 7 = 21 then 3 + 7 = 10

27. If n is a prime number, then n is odd.

28. If x is zero, then x is neither positive nor negative.

Write contrapositive of the following statements (Q. No. 29 to 32)

29. If 5 > 7 then 6 > 7.

30. x is even number implies that x^2 is divisible by 4.

31. If a triangle is equilateral, it is isosceles.

32. Only if he does not tire he will win.

33. Verify by the method of contradiction that $\sqrt{7}$ is irrational.

34. By giving counter example, show that the following statement is false:

    'If n is an odd integer, then n is prime'.

35. Show that the following statement is true by method of contra positive:

    'If x is an integer and x^2 is even, then x is also even'.

36. Prove by direct method that for any integer 'n' ,n^3–n is always even.'

37. Using the word 'necessary and sufficient' rewrite the following statement

    '$A \cup B = \phi$ if and only if $A = B = \phi$'

    Check whether the statement is true.

38. Let p: 'x and y are integers such that x > y' q: ' –x < –y'Describe the following

(i) $p \vee q$ (ii) $p \wedge q$

(iii) $p \rightarrow q$ (iv) $q \rightarrow p$

(v) $q \leftrightarrow p$

ANSWERS

1. Statement
2. Not a statement
3. Statement
4. Statement
5. Statement
6. π is a rational number
7. Everyone in Spain doesn't speak Spanish
8. Zero is not a positive number
9. For all complex number x, x is a real number.
10. There exists a real number x such that $x \leq 1$ and $x \geq 1$.
11. There exists a positive real number x such that x –1 is not positive.
12. True; p: A square is a quadrilateral

    q: All the four sides of a square are equal.
13. False; p: All integers are even.

    q: All integers are odd.
14. True; p: $\sqrt{3}$ is a rational number.

    q: $\sqrt{3}$ is an irrational number.

15. False; p: O is a positive number.

    q: O is a negative number

16. Exclusive
17. Inclusive
18. Exclusive
19. Exclusive
20. It is not daylight or it is false that all the people have arisen.
21. There exists an integer whose square is neither positive nor negative.
22. For every
23. There exists
24. There exists, For every
25. If x^2 is even then x is even
26. If $3 + 7 = 10$ then $3 \times 7 = 21$
27. If n is odd then n a prime number
28. If x is neither positive nor negative then x is zero.
29. If $6 \le 7$ then $5 \le 7$
30. If x^2 is not divisible by 4 then x is not even.
31. If a trignale is not iosceles, then it is not equilateral.
32. If he tires, then he will not win.
37. True
38. (i) x and y are integers $x > y$ or $-x < -y$.

    (ii) x and y are integers $x > y$ and $-x < -y$.

    (iii) If x and y are integers such that $x > y$ then $-x < -y$.

    (iv) If x and y are integers such that $-x < -y$ then $x > y$.

    (v) x and y are integers such that $x > y$ if and only if $-x < -y$.

CHAPTER - 15

STATISTICS

KEY POINTS

- Range of Ungrouped Data and Discrete Frequency

  Distribution = Largest observation – smallest observation.

- Range of Continuous FrequencyDistribution

  = Upper Limit of Highest Class — Lower Limit of Lowest Class

- Mean deviation for ungrouped data or raw data

$$M.D.(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n}$$

$$M.D.(M) = \frac{\sum |x_i - M|}{n}, \ where \qquad M = Median$$

- Mean deviation for grouped data (Discrete frequency distribution and Continuous frequency distribution).

$$M.D.(\bar{x}) = \frac{\sum f_i |x_i - \bar{x}|}{N}$$,where N = $\sum$ f_i

$$M.D.(M) = \frac{\sum f_i |x_i - M|}{N}$$, where N = Σ f_i

- Variance is defined as the mean of the squares of the deviations from mean.
- Standard deviation 'σ' is positive square root of variance.

$$\sigma = \sqrt{\text{variance}}$$

- variance σ^2 and standard deviation (SD) σ for ungrouped data

$$\sigma^2 = \frac{1}{n}\Sigma\left(x_i - \overline{x}\right)^2$$

$$SD = \sigma = \sqrt{\frac{1}{n}\Sigma\left(x_i - \overline{x}\right)^2}$$

- Standard deviation of a discrete frequency distribution

$$\sigma = \sqrt{\frac{1}{N}\sum f_i\left(x_i - \overline{x}\right)^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f x_i\right)^2}$$

- Standard deviation of a continuous frequency distribution

$$\sigma = \sqrt{\frac{1}{N}\sum f_i\left(x_i - \overline{x}\right)^2} = \frac{1}{N}\sqrt{N\sum f_i x_i^2 - \left(\sum f x_i\right)^2}$$

where x_i are the midpoints of the classes.

- Short cut method to find variance and standard deviation

$$\sigma = \frac{h^2}{N^2}\left[N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2\right]$$

$$\sigma = \frac{h}{N}\sqrt{N\sum f_i y_i^2 - \left(\sum f_i y_i\right)^2}$$

Where $y_i = \frac{x_i - A}{h}$

- Coefficient of variation (C.V) $= \frac{\sigma}{\overline{x}} \times 100, \qquad \overline{x} \neq 0$

- If each observation is multiplied by a positive constant k then variance of the resulting observations become k^2 times of the original value and standard deviation becomes k times of the original value.

- If each observation is increased by k, where k is positive or negative, the variance and standard deviation remains same.

- Standard deviation is independent of choice of origin but depends on the scale of measurement.

- The series having higher coefficient of variation is called more variable than the other. While the series having lesser coefficient of variation is called more consistent or more stable. For series with equal means the series with lesser standard deviation is more stable.

- The mean and standard deviation of a set of n_1 observations are $\overline{x_1}$ and s_1 respectively while the mean and standard deviation of another set of n_2 observations are $\overline{x_2}$ and s_2 respectively then the standard deviation of the combined set of $(n_1 + n_2)$ observations is given by

$$S.D = \sqrt{\frac{n_1 (s_1)^2 + n_2 (s_2)^2}{n_1 + n_2} + \frac{n_1 n_2 \left(\overline{x_1} - \overline{x_2}\right)^2}{(n_1 + n_2)^2}}$$

Section-A

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

1. The sum of the squares of deviation for 10 observations taken from their mean 50 is 250. Find the coefficient of variation.

2. If the standard deviation of a variable X is σ, then find the standard deviation of variable $\frac{ax+b}{c}$.

3. If the variance of 14, 18, 22, 26, 30, is 32, then find the variance of 28, 36, 44, 52, 60.

4. Find range of the following data.

Class	10-19	20-29	30-39	40-49	50-59
Frequency	5	4	5	3	2

5. The variance of 10 observations is 16 and their mean is 12. If each observation is multiplied by 4, what are the new mean and the newvariance?

6. The standard deviation of 25 observations is 4 and their mean is 25. If each observation is increased by 10, what are the new mean and the new standard deviation?

Section-B

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

7. In a series of 2n observations, half of them equal ‘a’ and remaining half equal –a. If the standard deviation of the observations is 2, then find the value of |a|.

8. The frequency distribution

x	A	2A	3A	4A	5A	6A
f	2	1	1	1	1	1

Where A is positive integer, has a variance of 160. Determine the value of A.

9. Find the mean deviation from the mean of A.P:

   a, a+d, a+2d,…., a+2nd.

10. Consider the first 10 positive integers. If we multiply each number by –1 and then add 1 to each number, find the variance of the numbers so obtained.

11. Coefficients of variation of two distributions are 60 and 80 and their standard deviations are 21 and 36. What are their means?

12. Life of bulbs produced by two factors A and B given below

Length of Life in (Hours)	*Factory A (Number of bulbs)*	*Factory B (Number of bulbs)*
550-650	10	8
650-750	22	60
750-850	52	24
850-950	20	16
950-1050	16	12

The bulb of which factory are more consistent from point of view of length.

13. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6. Find the other two observations.

14. Calculate the possible values of x if standard deviation of the numbers 2, 3, 2x and 11 is 3.5.

15. Mean and standard deviation of the data having 18 observations were found to be 7 and 4 respectively. Later it was found that 12 was miscopied as 21 in calculation. Find the correct mean and the correct standard deviation.

16. Find the standard deviation of

a, a+d, a+2d,…, a+2nd

17. Suppose a population A has 100 observations 101, 102,…., 200. Another population B has 100 observations 151, 152,…, 250. If V_A and V_B represent the variances of the two populations respectively then find the ratio of V_A and V_B.

Section-C

LONG ANSWER TYPE QUESTIONS (6 MARKS)

18. Calculate the mean deviation about mean for the following data.

Size	2	4	6	8	10	12	14	16
Frequency	2	2	4	5	3	2	1	1

19. Calculate the mean deviation about median for the following data:

20.

x	10	15	20	25	30	35	40	45
f	7	3	8	5	6	8	4	4

There are 60 students in a class. The following is the frequency distribution of the marks obtained by the students in a test :

x_i	0	1	2	3	4	5
f_i	$x-2$	X	x^2	$(x+1)^2$	$2x$	$2x+1$

where x is positive integer. Determine the mean and standard deviation of the marks.

21. Find the mean and variance of the frequency distribution given below

X	$1 \le x < 3$	$3 \le x < 5$	$5 \le x < 7$	$7 \le x < 10$
f	6	4	5	1

22. Calculate the mean deviation about mean (Q. No. 22 & 23)

Classes	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency	2	3	8	14	8	3	2

23.

Marks	0-10	10-20	20-30	30-40	40-50
Number of Students	5	8	15	16	6

24. Find the mean deviation about the median

Weight (in kg.)	30-40	40-50	50-60	60-70	70-80	80-90
Number of Persons	8	10	10	16	4	2

25. The mean and standard deviation of some data taken for the time to complete a test are calculated with following results:

    Number of observations = 25, mean = 18.2

    Standard deviation = 3.25 seconds

    Further another set of 15 observations x_1, x_2, ..., x_{15} also in

    $\sum_{i=1}^{15} x_i^2$ =5524.

    Calculate the standard deviation based on all 40 observations.

26. Find the coefficient of variation of the following data

Classes	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
Frequency	5	12	15	20	18	10	6	4

27. Which group of students is more stable- Group A or Group B?

Classes	5-15	15-25	25-35	35-45	45-55	55-65	65-73
Number in Group A	4	12	22	30	23	5	4
Number in Group B	5	15	20	33	15	10	2

28. Mean and standard deviation of 100 observations were found to be 40 and 10 respectively. If at the time of calculation two observations were wrongly taken as 30 and 70 in place of 3 and 27 respectively. Find correct standard deviation.

29. For a distribution $\sum(x_i - 5) = 3,\ \sum(x_i - 5)^2 = 43$ and total number of items is 18. Find the mean and standard deviation.

30. The following is the record of goals scored by team A in a football session:

Number in Goals scored	0	1	2	3	4
Number of Matches	1	9	7	5	3

For the team B, mean number of goals scored per match was 2 with standard deviation 1.25 goals. Find which of the two teams is more consistent in their performance.

ANSWERS

1.	10	2.	$\left\lvert\frac{a}{c}\right\rvert\sigma$
3.	128	4.	50
5.	48,256	6.	35, 4
7.	2	8.	A = 7
9.	$\frac{n(n+1)}{2n+1}d$	10.	8.25
11.	35, 45	12.	Factory A
13.	4, 9	14.	3, 7/3

15. 6.5, 2.5

16. $\sqrt{\frac{n(n+1)}{3}d}$

17. 1

18. 2.8

19. 10.1

20. Mean = 2.8, Standard deviation = 1.12

21. 4.16, 3.96

22. 10

23. 9.44

24. 11.44

25. 3.87

26. 31.24

27. Group A

28. 10.24

29. 5.17, 1.53

30. Team A

CHAPTER - 16

PROBABILITY

KEY POINTS

- **Random Experiment:** If an experiment has more than one possible outcome and it is not possible to predict the outcome in advance then experiment is called random experiment.
- **Sample Space:** The collection or set of all possible outcomes of a random experiment is called sample space associated with it. Each element of the sample space(set) is called a sample point.
- **Some examples of random experiments and their sample spaces**

  (i) A coin is tossed

  $S = \{H, T\}$, $n(S) = 2$

  Where n(S) is the number of elements in the sample space S.

  (ii) A die is thrown

  $S = \{ 1, 2, 3, 4, 5, 6]$, $n(S) = 6$

  (iii) A card is drawn from a pack of 52 cards n (S) = 52.

  (iv) Two coins are tossed

  $S = \{HH, HT, TH, TT\}$, $n(S) = 4$.

  (v) Two dice are thrown

$$S=\begin{Bmatrix}(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)\\(2,1),(2,2),......................,(2,6)\\\vdots\\\vdots\\(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\end{Bmatrix}\qquad n(s)=36.$$

(vi) Two cards are drawn from a well shuffled pack of 52cards

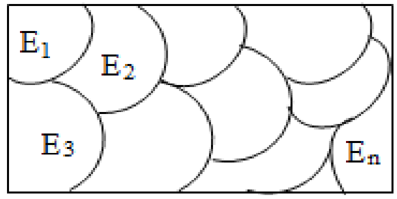
(a) with replacement n(S) = 52 × 52

(b) without replacement $n(S)={}^{52}C_2$

- **Event:** A subset of the sample space associated with a random experiment is called an event.
- **Elementary or Simple Event:** An event which has only one Sample point is called a simple event.
- **Compound Event:**An event which has more than one Sample point is called a Compound event.
- **Sure Event:** If event is same as the sample space of the experiment, then event is called sure event.
- **Impossible Event:** Let S be the sample space of the experiment, $\phi \subset S$, $\phi$is called impossible event.
- **Exhaustive and Mutually Exclusive Events:** Events E_1, E_2, E_3……..E_n are such that

  (i) $E_1 \cup E_2 \cup E_3 \cup \ldots\ldots \cup E_n = S$ then Events E_1, E_2, E_3……..E_n are called exhaustive events.

  (ii) $E_i \cap E_j = \phi$ for every $i \neq j$ then Events E_1, E_2, E_3……..E_n are called mutually exclusive.



Then we say that $E_1, E_2, \ldots\ldots E_n$ partitions the sample space S.

Probability of an Event: For a finite sample space S with equally likely outcomes, probability of an event A is defined as:

$$P(A) = \frac{n(A)}{n(S)}$$

where n(A) is number of elements in A

and n(S) is number of elements in set S and $0 \le P(A) \le 1$.

- (a) If A and B are any two events then

  $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  $= P(A) + P(B) - P(A \text{ and } B)$

  (b) If A and B are mutually exclusive events, then

  $P(A \cup B) = P(A) + P(B)$ (since $P(A \cap B) = 0$ for mutually exclusive events)

  (c) $P(A) + P(\overline{A}) = 1$ or P(A) + P(not A) = 1

  (d) P (Sure event) = P(S) = 1

  (e) P (impossible event) = $P(\phi) = 0$

  (f) $P(A - B) = P(A) - P(A \cap B) = P(A \cap \overline{B})$

  (g) $P(B - A) = P(B) - P(A \cap B) = P(B \cap \overline{A})$

  (h) $P(\overline{A} \cap \overline{B}) = P(\overline{A \cup B}) = 1 - P(A \cup B)$

  (i) $P(\overline{A} \cup \overline{B}) = P(\overline{A \cap B}) = 1 - P(A \cap B)$

- Addition theorem for three events

  Let A, Band Cbe any three events associated with a random experiment, then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C).$$

- Axiomatic Approach to Probability:

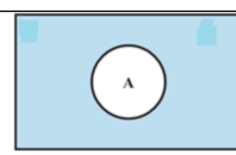
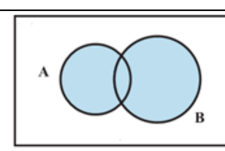
  Let S be a sample space containing elementary outcomes $w_1, w_2, \ldots\ldots\ldots w_n$

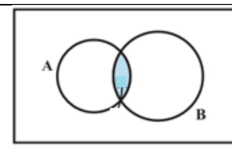
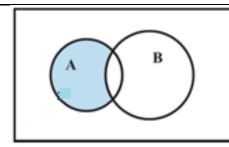
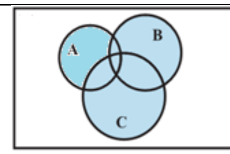
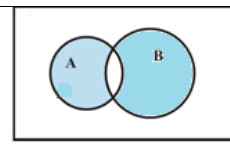
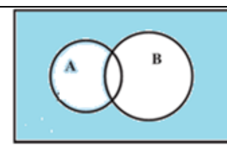
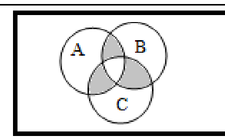
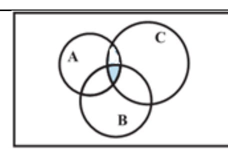
  i.e. $S = \{w_1, w_2, \ldots\ldots\ldots w_n\}$

  (i) $0 \le P(w_i) \le 1$ for each $w_i \in S$

  (ii) $P(w_1) + P(w_2) + \ldots\ldots\ldots + P(w_n) = 1$

  (iii) $P(A) = \sum P(w_i)$ for any event A containing elementary events w_i.

Verbal description of the event	Equivalent Set Theoretic Notation	Venn Diagram
Not A	$\bar{A}$ or A'	
A or B	$A \cup B$	

A and B	$A\cap B$	
A but not B (only A)	$A\cap\overline{B}$	
At least one of A, B or C	$A\cup B\cup C$	
Exactly one of A and B	$(A\cap\overline{B})\cup(\overline{A}\cap B)$	
Neither A nor B	$(A\cap\overline{B})$	
Exactly two of A, B and C	$(A\cap B\cap\overline{C})\cup$ $(A\cap\overline{B}\cap C)$ $\cup(\overline{A}\cap B\cap C)$	
All there of A, B and C	$A\cap B\cap C$	

The cards J, Q and K are called face cards. There are 12 face cards in a deck of 52 cards.

There are 64 squares in a chess board i.e.32 white and 32 Black.

Total n digit numbers = $9\times10^{n-1}$, $n\geq2$

e.g. there are $9\times10^{2}=900$. Three digit numbers.

Section-A

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Describe the Sample Space for the following experiments

(Q. No. 1 to 5)

1. A coin is tossed twice and number of heads is recorded.
2. A card is drawn from a deck of playing cards and its colour is noted.
3. A coin is tossed repeatedly until a tail comes up.
4. A coin is tossed. If it shows head, we draw a ball from a bag consisting of 2 red and 3 black balls. If it shows tail, coin is tossed again.
5. Two balls are drawn at random in succession without replacement from a box containing 1 red and 3 identical white balls.
6. A coin is tossed n times. Find the number of element in its sample space.
7. One number is chosen at random from the numbers 1 to 21. What is the probability that it is prime?
8. What is the probability that a given two-digit number is divisible by 15?
9. If $P(A \cup B) = P(A) + P(B)$, then what can be said about the events A and B?
10. If $P(A \cup B) = P(A \cap B)$, then find relation between P(A) and P(B).
11. Is $P(A \cap B) = 0$? If A and B are exhaustive events.

Section-B

SHORT ANSWER TYPE QUESTIONS (2 MARK)

12. Let A and B be two events such that $P(A) = 0.3$ and $P(A \cup B) = 0.8$. find $P(B)$ if $P(A \cap B) = P(A)\ P(B)$.

13. Three identical dice are rolled. Find the probability that the same number appears on each of them.

14. In an experiment of rolling of a fair die.

    Let A,B and C be three events defined as under:

    A : a number which is a perfect square

    B : a prime number

    C : a number which is greater than 5.

    Is A, B, and C exhaustive events?

15. Punching time of an employee is given below:

Day	Mon	Tue	Wed	Thurs	Fri	Sat
Time(a.m)	10:35	10:20	10:22	10:27	10:25	10:40

    If the reporting time is 10:30 a.m, then find the probability of his coming late.

16. A game has 18 triangular block out of which 8 are blue and rest are red, and 19 square blocks out of which 7 are blue and rest are yellow. On piece is lost. Find the probability that it was a square of blue colour.

17. A card is drawn from a pack of 52 cards. Find the probability of getting:

(i) a jack or a queen

(ii) a king or a diamond

(iii) a heart or a club

(iv) either a red or a face card.

(v) neither a heart nor a king

(vi) neither an ace nor a jack

(vii) a face card

18. In a leap year find the probability of

    (i) 53 Mondays and 53 Tuesdays

    (ii) 53 Mondays and 53 Wednesday

    (iii) 53 Mondays or 53 Tuesdays.

19. In a non-leap year, find the probability of

    (i) 53 Mondays and 53 Tuesdays.

    (ii) 53 Mondays or 53 Tuesdays.

20. Two card are drawn at random from a deck of 52 playing cards. Find the probability of drawing two kings.

Section-C

SHORT ANSWER TYPE QUESTIONS (4 MARKS)

21. Find the probability that in a random arrangement of the letters of the word UNIVERSITY two I's come together.

22. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.

23. Out of 8 points in a plane 5 are collinear. Find the probability that 3 points selected at random form a triangle.

24. Find the probability of at most two tails or at least two heads in a toss of three coins.

25. A, B and C are events associated with a random experiment such that P(A) = 0.3, P(B) = 0.4, P(C) = 0.8, P(A $\cap$ B) = 0.08 P(A $\cap$ C) =0.28and P(A$\cap$B$\cap$C) = 0.09. If P(A$\cup$B$\cup$C) $\geq$ 0.75

then prove that P(B $\cap$ C)lies in the interval [0.23, 0.48]

[***Hint:*** P (A $\cup$ B $\cup$ C) = P(A) + P(B) + P(C) – P(A $\cap$ B) – P(B $\cap$ C) – P(A $\cap$ C) + P (A $\cap$ B $\cap$ C)]

26. If $\frac{1+3p}{3}$, $\frac{1-p}{4}$, and $\frac{1-2p}{2}$ are the probability of three mutually exclusive events. Then find the set of all values of p.

27. An urn A contains 6 red and 4 black balls and urn B contain 4 red and 6 black balls. One ball is drawn at random from urn A and placed in urn B. Then one ball is drawn at random from urn B and placed in urn A. Now if one ball is drawn at random from urn A then find the probability that it is found to be red.

28. If three distinct numbers are chosen randomly from the first 100 natural numbers, then find the probability that all three of them are divisible by both 2 and 3.

29. S = {1, 2, 3, -----, 30}, A = {x : x is multiple of 7}, B = { x : x is multiple of 5}, C = {x : x is a multiple of 3}. If x is a member of S chosen at random find the probability that

    (i) x $\in$ A $\cup$ B

    (ii) x $\in$ B $\cap$ C

    (iii) x $\in$ A $\cap$ C'

30. One number is chosen at random from the number 1 to 100. Find the probability that it is divisible by 4 or 10.

31. The number lock of a suitcase has 4 wheels with 10 digits, i.e. fro 0 to 9. The lock open with a sequence of 4 digits with repeats allowed. What is the probability of a person getting the right sequence to open the suit case?

32. If A and B are any two events having P(A $\cup$ B) = $\frac{1}{2}$ and $P(\overline{A}) = \frac{2}{3}$ then find the $P(\overline{A} \cap B)$.

33. Three of the six vertices of a regular hexagon are chosen at random. What is probability that the triangle with these vertices is equilateral?

34. A typical PIN (Personal identification number) is a sequence of any four symbol chosen from the 26 letters in the alphabet and ten digits. If all PINs are equally likely, what is the probability that a randomly chosen PIN contains a repeated symbol?

35. An urn contains 9 red, 7 white and 4 black balls. If two balls are drawn at random. Find the probability that the balls are of same colour.

36. One mapping /function is selected at random from all the mappings of the set A = {1,2,3,4,5}. Into itself. Find the probability that the mapping selected is one to one.

37. A girl calculates that the probability of her winning the first prize in a lottery is 0.02. If 6000 tickets were sold, how many tickets has she bought?

38. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9?

39. All the face cards are removed from a deck of 52 playing cards. The remaining cards are well shuffled and then one card is drawn at random. Giving ace a value 1 and similar value for other cards. Find the probability of getting a card with value less than 7.

40. If A,B,C are three mutually exclusive and exhaustive events of an experiment such that 3P(A) = 2P(B) = P(C), then find the value of P(A).

ANSWERS

1. {0, 1, 2}
2. {Red, Black}
3. {T, HT, HHT, HHHT.........}
4. {HR1, HR2, HB1, HB2, HB3, TH, TT}
5. {RW, WR, WW}
6. 2^n
7. $\frac{8}{21}$
8. $\frac{1}{15}$
9. A and B are Mutually exclusive events.
10. P(A) = P(B)
11. No.
12. $\frac{5}{7}$
13. $\frac{1}{36}$
14. Yes as $A \cup B \cup C = S$
15. $\frac{1}{3}$
16. $\frac{1}{4}$

17. (i) $\frac{2}{13}$ (ii) $\frac{4}{13}$ (iii) $\frac{1}{2}$ (iv) $\frac{8}{13}$

(v) $\frac{9}{13}$ (vi) $\frac{11}{13}$ (vii) $\frac{3}{13}$

18. (i) $\frac{1}{7}$ (ii) 0 (iii) $\frac{3}{7}$

19. (i) 0 (ii) $\frac{2}{7}$

20. $\frac{1}{221}$

21. $\frac{1}{5}$

22. 3

23. $\frac{23}{28}$

24. $\frac{7}{8}$

25. $0.23 \le P(B) \le 0.48$

26. $\frac{-1}{3} \le p \le \frac{1}{2}$

27. $\frac{32}{55}$

28. $\frac{4}{1155}$

29. (i) $\frac{1}{3}$ (ii) $\frac{1}{15}$ (iii) $\frac{1}{10}$

30. $\frac{3}{10}$

31. $\frac{1}{10000}$

32. $\frac{1}{6}$

33. $\frac{1}{10}$

34. $\frac{265896}{1679616}$

35. $\frac{63}{190}$

36. $\frac{5!}{5^5} = \frac{24}{125}$

37. 120

38. $\frac{5}{12}$

39. $\frac{3}{5}$

40. $\frac{2}{11}$

MODEL TEST PAPER-I (SOLVED)

Time: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) This questions paper contains 29 questions.

(iii) Questions nos. 1-4 in section A are very short answer type questions carrying 1 mark each

(iv) Question nos. 5 -12 in section B are short answer type questions carrying 2 marks each

(v) Questions non. 13 -23 in section C are long answer-I type questions carrying 4 marks each

(vi) Question 24 -29 in section D are long answer –II type questions carrying 6 marks each.

SECTION A

1. Differentiate f(x) = $\dfrac{x^3+x^2+1}{x}$ with respect to x.

2. Find the component statements for the compound statement : Number seven is prime and odd

3. Solve for x : $x^2 + 3x + 9 = 0$.

4. If A = {1, 2, 3, 4, 5, 6}, B = {2, 4, 6, 8}, then find A - B

SECTION B

5. (a) Write the contra positive of the statement : "If a triangle is equilateral then it is isosceles."

(b) Write the negation of the statement "All triangles are not equilateral triangles.."

6. Let A and B be two sets containing 3 and 6 elements respectively. Find the maximum and number of elements in $A \cup B$.

7. Find the coordinate of the point R which divide the joint of the points P(0, 0, 0) and Q(4, –1, –2) in the ratio 1 : 2 externally and verify that P is the mid point of RQ.

8. Find the derivative of $f(x) = \dfrac{\cos x}{1 + \sin x}$ w.r.t. '*x*'

9. If $z_1 = z - i$, $z_2 = -2 + i$ then find the value of $\mathrm{Re}\left(\dfrac{z_1 z_2}{\overline{z}_1}\right)$

10. Find the range of the real function $f(x) = 1 - |x - 2|$.

11. Using binomial theorem prove that $6^n - 5n - 1$ is divisible by 25, $\forall\, n \in N$.

12. If the letters of the word "ALGORITHM" are arranged at random in a row, what is the probability that the letters G, O and R must remain together?

SECTION C

13. Find the general solution of the equation (sin 2x – sin 4x + sin 6x = 0)

14. Find the equation of the circle which passes through the points (2, – 2), (3, 4) and has its centre on the line 2x + 2y = 7

OR

Find the equation of the hyperbola whose foci are $\left(\pm 3\sqrt{5}, 0\right)$ and the length of length of lat us rectum is 8 units

15. Find the sixth term of the expansion $\left(y^{\frac{1}{2}} + x^{\frac{1}{3}}\right)^n$, if the binomial coefficient of the third from the end is 45

16. Three squares of a chess board are selected at random Find the probability of selecting two squares of one colour and the other of a different colour. What is the importance of games in life?

17. In how many of the distinct permutations of the letters in MISSISSIPPI do the four I's not come together?

18. In a plane there are 27 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B and no two are parallel. Find the number of points of intersection of the straight lines.

19. Is g = {(1, 1), (2, 3), (3, 5), (4, 7)} a function? Justify. If this is described by the relation g(x) = ax + b then what value should be assigned to a and b?

20. If A = {2, 3, 4, 5, 6, 7, 8, 9}. Let R be a relation on A defined by {(x, y) : x $\in$ A, y $\in$ A and x divides y}.

    (a) Draw arrow diagram of R

    (b) Find : (i) R in roster form (ii) Domain of R (iii) Range of R

21. Find the square root of $2-2\sqrt{3}i$.

OR

If $a+ib=\dfrac{c+i}{c-i}$.; a, b, c $\in$ R then show that $a^2 + b^2 = 1$ and $\dfrac{b}{a}=\dfrac{2c}{c^2-1}$

22. Solve the following system of linear inequalities graphically :

    X – 2y $\leq$ 3; 3x + 4y $\geq$ 12; x $\geq$ 0; y $\geq$ 1

23. Evaluate : $\lim_{x \to 0} \frac{\sin x - \tan x}{x^3}$

Find the derivative of x sin x with respect to x from first principle of derivative.

SECTION D

24. Find the mean, variance and standard deviation for the following data :

Class-Interval	Frequency
30-40	3
40-50	7
50-60	12
60-70	15
70-80	8
80-90	3
90-100	2

25. Find the direction in which a straight line must be drawn through the point (–1, 2) so that its point of intersection with the line

x + y = 4 may be at a distance of 3 units from this point

OR

The hypotenuse of an isosceles right angled triangle has its ends at the points (1, 3) and (–4, 1) find the equation of the legs (perpendicular sides) of the triangle.

26. Between 1 and 31, m numbers have been inserted in such a way that the resulting sequence is an AP and the ratio of 7^{th} and (m –1) the numbers is 5 : 9 Find the value of m.

OR

Let S be the sum, P the product and R the sum of reciprocals of n terms of a GP Prove that $P^2 R^n = S^n$.

27. In a town of 10000 families it was found that 40% families buy newspaper A, 20% families buy newspaper B, 10% families buy newspaper C. 5% of families buy newspaper A and B, 3% of families buy newspaper B and C and 4% of families buy newspaper A and C. If 12% of families buy all the three newspaper that find.

    (a) the number of families which buy newspaper A only.

    (b) the number of families which buy none of the newspapers A, B and C.

28. Prove that $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

OR

If $x \cos\theta = y \cos\left(\theta + \frac{2\pi}{3}\right) = z \cos\left(\theta + \frac{4\pi}{3}\right)$ Prove that $xy + yz + zx = 0$

29. Using principle of mathematical induction for all n $\in$ N, prove that

$$1.3 + 2.3^2 + 3.3^3 + ... + n.3^n = \frac{(2n \quad 1)\ 3^{n \quad 1} \quad 3}{4}$$

MODEL TEST PAPER-II

Time: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into four sections A, B and C. D Section A comprises of 4 questions of one mark each, section B comprises of 8 questions of 2 marks each and section C comprises of 11 questions of 4 marks each. And Section D comprises of 6 questions of six marks each.

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted.

SECTION A

1. Write the interval [6, 12] in the set builder form

2. Find the 6^{th} term in the expansions of $(2x - y)^{12}$

3. Find the length of lat us rectum ellipse $\frac{x^2}{49}+\frac{y^2}{36}=1$

4. A coin is tossed twice what is the probability that at least one head occurs?

SECTION-B

5. Solve the following trigonometric equation: tan 2θ = $\sqrt{3}$

6. Prove that : cos 4x = 1 – 8 $\sin^2$ x $\cos^2$ x.

7. Reduce the following equation into intercept form and find the intercepts on the axes : 4x – 3y = 6

8. Find the equation of the line passing through the point (3, 0) and perpendicular to the line x – 7y + 5 = 0.

9. Find the equation of the line whose perpendicular distance from the origin is 5 units and angle mode by the perpendicular with positive axis is 30°

10. Find the multiplicative inverse of following complex number :

    4 – 3i

11. Write the contrapositive of the following statements :

    (i) If x is prime number, then x is odd.

    (ii) If the two lines are parallel then they do not intersect in the same plane.

12. Write the negation of the following statements :

    (i) π is not a rational number.

    (ii) Zero is a positive number.

SECTION-C

13. Find the equation of the hyperbola whose foci are $\left(\pm 3\sqrt{5}, 0\right)$ and length of latus rectum is 8.

OR

Find the equation of the circle with radius 5 whose centre lies on x-axis sand passes through the point (2, 3).

14. Find the derivative of cot x with respect to x from first principle.

OR

Evaluate : $\lim_{x \to 0} f(x), \text{when}\, f(x) = \begin{cases} \frac{|x|}{x}, x \neq 0. \\ 0, x = 0. \end{cases}$

15. Find the co-ordinates of a point on y-axis which is at a distance of $5\sqrt{2}$ from the point R(3, –2, 5)

16. Find the square root of –15 – 8i.

OR

Convent the complex number $\frac{1+3i}{1-2i}$ in polar form.

17. In how many of the distinct permutations of the letters in MISSISSIPPI do four I's not come together?

OR

How many words or without meaning can be formed with letters of the word EQUATION at a time so that vowels and consonants occur together ?

18. In an examination, a question paper consists of 12 questions divided into two parts i.e., Part I and Part II containing 5 and 7 questions respectively A student is required to attempt 8 questions all, selecting at least 3 from each part. In how many ways can a student select the questions? Write one importance of examination.

19. In the binomial expansion of $(1 + x)^n$. the coefficient of the 5th, 6th, and 7^{th} terms are in A.P. Find all the value of n for which this can happen.

20. Find the domain and range of the real function $f(x) = \dfrac{1}{1-x^2}$

21. The function f is defined by $f(x) \quad \begin{matrix} 1 & x, x & 0 \\ 1 & , x & 0 \\ x & 1, x & 0 \end{matrix}$ Draw the graph of f(x).

22. If tan A – tan B = x

cot B – cot A = y Prove that : cot(A – B) = $\dfrac{1}{x} + \dfrac{1}{y}$

23. In a class of 60 students, 30 opted of NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that

(i) the student opted for NCC or NSS.

(ii) the student has opted NSS but not NCC.

SECSTION D

24. Prove the following by using the principle of mathematical induction for all n $\in$ N.

$3^{2n+2} - 8n - 9$ is divisible by 8.

25. Solve the following system of inequalities graphically :

$x + y \le 4,\ y \le 3,\ x + 5y \ge 4,\ 6x + 2y \ge 8,\ x \ge 0,\ y \ge 0$

26. The ratio of the A.M and G. M of two positive numbers a and b be m : n(m > n). Show that a : b = $\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right)$

27. Prove that : $\cos^2 A \quad \cos^2 \quad A \quad \frac{2}{3} \quad \cos^2 \quad A \quad \frac{2}{3} \quad 3$

Prove that : $\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+\cos\frac{7\pi}{8}\right)=\frac{1}{8}+.$

28. In a survey it was found that 21 people like product A, 26 people like product B and 29 like product C. If 14 people like product A and B. 15 people like product B and C. 12 people like product C and A and 8 people like all three products, find :

(i) How many people like a least one of the products?

(ii) How many people like product C only?

29. Find the mean variance and standard deviation for the following data

C.I.	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	16	6

ANSWER OF MODEL TEST PAPER-I

1. $f'(x) = 2x + 1 - \frac{1}{x^2}$

2. p : Numbers seven is prime

q : Numbers seven is odd.

3. $x \quad \frac{3 \quad 3\sqrt{3}i}{2}$

4. $A - B = \{1, 3.5\}$

5. (a) If a triangle is not isosceles them it is not equilateral.

   (b) All triangles are equilateral triangles.

6. 6

7. R(–4, 1, 2)

8. $f'(x) = \frac{-1}{1 + \sin x}$

9 $\mathrm{Re}\left(\frac{z_1 z_2}{\overline{z}_1}\right) = \frac{-2}{5}$

10. (– ∞, 1) 12. $\frac{1}{12}$ 13. $x = n\pi \pm \frac{\pi}{6}$

14. $x \quad \frac{5}{2} \quad ^{2} \quad y \quad 1 \;^{2} \quad \frac{37}{4}$ or $\frac{x^2}{25} \quad \frac{y^2}{20} \quad 1$

15. $T_6 \quad 252 y^{\frac{5}{2}} x^{\frac{5}{3}}$

16. $\frac{16}{21}$ Games keep is fit and healthy.

17. 33810 18. 220 19. a = 2, b = –1

20. (b) (i) R = $\begin{array}{l}(2,2),(2,4),(2,6),(2,8)(3,3),(3,6),(3,9),\\(4,4),(4,8),(5,5),(6,6),(7,7),(8,8),(9,9)\end{array}$

   (ii) D(R) = A (iii) Range (R) = A

21. $\pm\left(\sqrt{3}-i\right)$ 23. $x\cos x+\sin x$

24. Mean = 62; variance = 201 s.d. = $\sqrt{201}=14.17$

25. m = 0; required line is parallel to x –axis.

OR

7y + 3x – 24 = 0; 3y – 7x – 2 = 0 and 7x – 3y + 31 = 0; 3x + 7y + 5 = 0

26. m = 14

27. (a) 3300 families (b) 4000 families.

ANSWER OF MODEL TEST PAPER-II

1. $\{x : 6 \leq x \leq 12, \quad x \in R\}$ 2. T_6 = –101376 x^7 y^5

3. $\frac{72}{7}$ units 4. $\frac{3}{4}$

5. $\frac{n}{2} \quad \frac{}{6}$

7. $\frac{x}{\left(\frac{3}{2}\right)}+\frac{y}{(-2)}=1;$ x-intercept is $\frac{3}{2}$ and Y-intercept is –2.

8. 7x + y = 21 9. $\sqrt{3}x+y=+10$

10. $z^{1} \quad \frac{4 \quad 3i}{25}$

11. (i) If x is not odd then x is not prime.

(ii) If two lines intersect in the plane then they are not parallel in the same plane.

12. (i) π is a rational number

(ii) Zero is not a positive number.

13. $\frac{x^2}{25} - \frac{y^2}{20} = 1$

14. – cosec^2 x or LHL = –1; RHL = 1 so, $\lim_{x \to 0} f(x)$ does not exist.

15. (0, 2, 0) or (0, – 6, 0) 16. ±(1 – 4i)

or $\sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

17. 33810 or 1440 18. 36750 19. n = 7, 14

20. D(f) = R – {–1,1}; Range (f) = (– ∞, 0) ∪ [1, ∞]

21. (i) $\frac{19}{30}$ (ii) $\frac{2}{15}$ 28. (i) 43 (ii) 10

MODEL TEST PAPER-III (Solved)

Time: 3 hours **Maximum Marks: 100**

General Instructions:

1. Find the argument of complex number z = $\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}$

   Solution. Z = $\sin\frac{\pi}{6}+i\cos\frac{\pi}{6}$

   $\Rightarrow$ z = $\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}$

   So, arg(z) = $\frac{\pi}{3}$.

2. Evaluate : $\lim_{x\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$

   Solution. $\lim_{x\to 0}\frac{\sin\frac{1}{x}}{\frac{1}{x}}$ let $\frac{1}{x}=y$

   = $\lim_{y\to\infty}\frac{\sin y}{y}=0$

3. Find the number of terms in the expansion of $(3x+y)^8-(3x-y)^8$

   Solution. 4 terms.

4. Write the domain of the function, $f(x)=\frac{x}{x^2-5x+6}$

Solution. f(x) $\dfrac{x}{x^2-5x+6}=\dfrac{x}{(x-3)(x-2)}$

For Domain (f) = R–{3, 2}

5. Two finite set have m and n element. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. Find the values of m and n.

Solution. Let A and B are two sets having m and n elements. A.T.Q

$2^m - 2^n = 56$

$\Rightarrow 2^n(2^{m-n} - 1) = 8 \times 7$

$\Rightarrow 2^n(2^{m-n} - 1) = 2^3 \times (2^3 - 1)$

As comparing, n = 3; m – n = 3

$\Rightarrow$ m = 6

Thus, m = 6; n = 3.

6. Let f : R $\rightarrow$ R be a function given by $f(x) = x^2 + 1$. Find $f^{-1}(-5)$.

Solution. let $f^{-1}(-5) = x \Rightarrow f(x) = -5$

$\Rightarrow x^2 + 1 = -5$

$\Rightarrow x^2 = -6$

$\Rightarrow$ x = no real value.

So, $f^{-1}(-5) = \phi$

7. If $\dfrac{a+ib}{c+id}=x+iy$ prove that $\dfrac{a+ib}{c+id}=x-iy$.

Solution $\because$ $\dfrac{a+ib}{c+id}=x+iy$ [Given]

$$\Rightarrow \overline{\left(\frac{a+ib}{c+id}\right)} = \overline{x+iy} \quad [\text{If } z_1 = z_2 \;\Rightarrow\; \overline{z}_1 = \overline{z}_2]$$

$$\Rightarrow \frac{(\overline{a+ib})}{(\overline{c+id})} = x - iy \quad \left[\because \overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z}_1}{\overline{z}_2}\right]$$

$$\Rightarrow \frac{a-ib}{c-id} = x - iy$$

8. If (n +1) ! = 12 (n – 1) !, find n.

Solution. (n + 1)! = 12 (n – 1) !

$\Rightarrow$ (n + 1) . n . (n – 1)! = 12 (n – 1) !

$\Rightarrow$ (n + 1) n = 12

$\Rightarrow$ (n + 1) n = 4 × 3

$\Rightarrow$ n = 3

9. Find the middle term in the expansion of $\left(\frac{x}{3}+9y\right)^{10}$

Solution. In the expansion of $\left(\frac{x}{3}+9y\right)^{10}$, the middle term is T_6.

$T_6 = 10_{C_5}\left(\frac{x}{3}\right)^5 (9y)^5$

$= \frac{\lfloor 10}{\lfloor 5 \lfloor 5}\frac{x^5}{3^5}\times 9^5 y^5$

$= 252 \times 3^5 x^5 y^5$

$= 61236 x^5 y^5$.

10. Find the sum of first 24 te4rms of the A. P.

a_1, a_2, a_3. .... if it is known that

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$.

Solution. $\because$ $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow (a_1 + a_{24}) + (a_5 + a_{20}) + (a_{10} + a_{20}) = 225$

$\Rightarrow 3(a_1 + a_{24}) = 225$ $\quad \left[\begin{array}{l}\because a_1 + a_n = a_2 + a_{n-1} = a_3 + a_{n-2} = ...\\ \text{in an A.P}\end{array}\right]$

$\Rightarrow a_1 + a_{24} = 75$

Now, $S_{24} = \dfrac{24}{2}(a_1 + a_{24})$

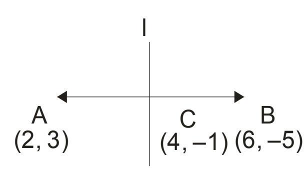
$= 12 \times 75 = 900$.

11. Find the equation of the perpendicular bisector of the line segment joining the points A(2, 3) and B(6, –5).

Solution. Slope of AB = $\dfrac{-5-3}{6-2} = \dfrac{-8}{4} = -2$

$\because$ l $\perp$ AB,

So, slope of line l is m = $\dfrac{1}{2}$



equation of line l is

$y + 1 = \dfrac{1}{2}(x-4)$

$\Rightarrow$ x – 2y – 6 = 0.

12. Find the derivative of sin x. Cos x w.r.t. 'x'

Solution. y = sin x cos x

Diffr w.r.t 'x'......

$$\frac{dy}{dx} = \sin x \frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(\sin x)$$

= sin x(– sin x) + cos x . cos x

= – $\sin^2$ x + $\cos^2$ x

= cos 2x.

13. Show that $\sqrt{3}$ cosec $20° - \sec 20° = 4$

Solution. LHS= $\sqrt{3}$ cosec $20° - \sec 20°$

$$= \frac{\sqrt{3}}{\sin 20°} - \frac{1}{\cos 20°} = \frac{\sqrt{3}\cos 20° - \sin 20°}{\sin 20° \cos 20°}$$

$$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20° - \frac{1}{2}\sin 20°\right)}{\sin 20° \cos 20°} = \frac{2(\sin 60° \cos 20° - \cos 60° \sin 20°)}{\sin 20° \cos 20°}$$

$$= \frac{2\sin(60 - 20°)}{\sin 20° \cos 20°} = \frac{2\sin 40°}{\sin 20 \cos 20°} = \frac{4\sin 40}{2\sin 20° \cos 20°}$$

$$= \frac{4\sin 40°}{\sin 40°} = 4$$

14. Solve (x + iy) (2 – 3i) = 4 + i, where x and y are seal

Solution. (x + iy) (2 – 3i) = 4 + i

$$\Rightarrow x + iy = \frac{4+i}{2-3i} = \frac{(4+i)}{(2-3i)} \times \frac{(2+3i)}{(2+3i)}$$

$= \dfrac{5+14i}{13} = \dfrac{5}{13} + \dfrac{14}{13}i$

So, x = $\dfrac{5}{13}$ and $y = \dfrac{14}{13}$.

15. Let P be the solution set of 3x + 1 > x – 3 and is Q be the solution set of 5x + 2 ≤ 3(x + 2), x ∈ n. Find the set P ∩ Q

Solution ∵ 3x + 1 > x – 3	Also, 5x + 2 ≤ 3(x + 2)
⇒ 3x – x > –3 –1	⇒ 5x – 3x ≤ 6 – 2
⇒ 2x > – 4	⇒ 2x ≤ 4
⇒ x > –2	⇒ x ≤ 2
But x ∈ N. ∴, P = {1, 2, 3 ...}	But x ∈ N, ∴ Q = {1, 2}

∴ P ∩ Q = {1, 2}

16. If there are six periods in each working day of a school, in how many ways can one arrange 5 subjects such that each subject is allowed at least on period?

Solution. Six periods can be arranged for 5 subject in 6/5 ways.

= 720 ways.

One periods is left, which can be arranged for any of the five subject, one left period can be arranged in 5 ways.

Required no, of arrangements = 720 × 5 = 3600.

17. Find the term in dependent of x in $2x^2 \quad \dfrac{1}{3x^3}^{\,10}$.

Solution. General term, T_{r+1} = $10_{C_r}(2x^2)^{10\ \ r} \quad \dfrac{1}{3x^3}^{\,r}$

$= 10_{C_r} 2^{10 \quad r} \quad \frac{1}{3}^{\ r} x^{20 \quad 5r}$

It will be independent of x if 20 – 5r = 0, i.e. if r = 4

so, $T_5 = 10_{C_4} \quad 2^6 \quad \frac{1}{2}^{\ 4} \quad \frac{4480}{27}$.

18. Divide 63 into three parts such that they are in G.P. and the product of the first and the second term is $\frac{3}{4}$ of the third term.

Solution. Let the three numbers be a, ar, ar^2.

Given $a + ar + ar^2 = 63$ ...(1) and a. ar = $\frac{3}{4}ar^2$

$\Rightarrow$ a = $\frac{3}{4}r$ ...(2)

From (1) and (2) are get

$$\frac{3}{4}r + \frac{3}{4}r^2 + \frac{3}{4}r^3 = 63$$

$\Rightarrow r^3 + r^2 + r - 84 = 0$

$\Rightarrow (r - 4)(r^2 + 5r + 21) = 0$

$\Rightarrow$ r = 4, $\frac{-5 \pm \sqrt{25 - 84}}{2}$

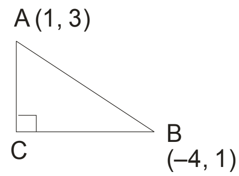
Real value of r is 4. So, a = 3.

$\therefore$, Three numbers are 3, 12, 48,

19. The hypotenuse of a right angled triangle has its ends at the points ((1, 3)) and (–4, 1). Find the equation of the legs of the triangle.

Solution. Let ABC be the right angled triangle such that $\angle c = 90°$

Let m be the slope of the line AC then the slope of BC = $\frac{1}{m}$.



Equation of AC is : y – 3 = m(x – 1) and equation of BC is

y = – 1 $-\frac{1}{m}(x+4)$.

or x – 1 = $\frac{1}{m}(y-3)$

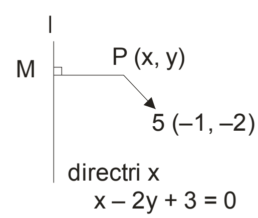
For m = 0, these lines are x + 4 = 0, y – 3 = 0

For m = ∞, the lines are x – 1 = 0, y – 1 = 0.

20. Find the equation of parabola whose focus at (–1, –2) and directrix is x – 2y + 3 = 0

Solution. Let P(x, y) be any point on the parabola is using focus-directrix property of the parabola, SP = PM

$\therefore \sqrt{(x \quad 1)^2 \quad (y \quad 2)^2} = \frac{|x-2y+3|}{\sqrt{1^2+(-2)^2}}$



$\Rightarrow (x + 1)^2 + (y + 2)^2 = \frac{(x-2y+3)^2}{5}$

$\Rightarrow 5x^2 + 5 + 10x + 5y^2 + 20 + 20y = x^2 + 4y^2 + 9 - 4xy - 12y + 6x$

$\Rightarrow 4x^2 + 4xy + y^2 + 4x + 32y + 16 = 0$. This is required equation of parabola

21. Evaluate : $\lim\limits_{x\to\sqrt{3}} \dfrac{x^2-3}{x^2+3\sqrt{3}x-12}$

Solution. $\lim\limits_{x\to\sqrt{3}} \dfrac{x^2-3}{x^2+3\sqrt{3}x-12}$ = $\lim\limits_{x \quad \sqrt{3}} \dfrac{(x \quad \sqrt{3})(x \quad \sqrt{3})}{x^2 \quad 4\sqrt{3}x \quad \sqrt{3}x \quad 12}$

= $\lim\limits_{x\to\sqrt{3}} \dfrac{(x-\sqrt{3})(x+\sqrt{3})}{(x+4\sqrt{3})(x-\sqrt{3})} = \lim\limits_{x\to\sqrt{3}} \dfrac{(x+\sqrt{3})}{(x+4\sqrt{3})}$

= $\dfrac{2\sqrt{2}}{5\sqrt{3}} = \dfrac{2}{5}$

22. In a single throw of three dice, determine the probability of getting total of at most 5.

Solution. Number of exhaustive cases in a single throw of three dice = 6 × 6 × 6 = 216. (favarouble number of cases = 10{i.e. (1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1)}

So, required Probolility = $\dfrac{10}{216} = \dfrac{5}{108}$.

23. Let f be defined by f(x) = x – 4 and g be defined by

g(x) = $\begin{cases} \dfrac{x^2-16}{x+4}, & x \neq -4 \\ k, & x = -4 \end{cases}$

Find k such that f(x) = g(x) for all x.

Solution. we have f(–4) = –4 –4 = –8 and g(–4) = k.

But f(x) = g(x) $\forall$ x.

$\therefore$, –8 = k i.e. k = –8 Ans.

24. Calculate the mean deviation from the median of following data.

Wages per week (in Rs)	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of workers	4	6	10	20	10	6	4

Solution.

Wages per Week in Rs	Mid value x_i	Frequency f_i	Cumulative frequency	Deviation $\|d\|$ $= \|x_i - 45\|$	$f_i\|d_i\|$
10-20	15	4	4	30	120
20-30	25	6	10	20	120
30-40	35	10	20	10	100
40-50	45	20	40	0	0
50-60	55	10	50	10	100
60-70	65	6	56	20	120
70-80	75	4	60	30	120
		N = f_i = 60			$\Sigma f_i\|di\|$ = 680

Here N = 60, so, $\frac{N}{2}$ = 30; Median = $l + \left(\frac{\frac{n}{2} - f_c}{f_m}\right) \times h$

= $40 + \left(\frac{30 - 20}{20}\right) \times 10 = 45$

Mean definition from median = $\frac{f_i|di|}{N} \quad \frac{680}{60} \quad 11.33$ Ans.

25. If p and p' be the perpendiculars from the orign upon the straight lines x sec θ – y cosec θ = a and x cos θ + y sin θ = a cos 2θ prove that $4p^2 + p'^2 = a^2$.

Solution. one line is x sec θ – y cosec θ – a = 0 ...(1)

P = length of perpendicular from the origin (0, 0) on (1)

$$= \left|\frac{-a}{\sqrt{\sec^2\theta+\text{cosec}^2\theta}}\right| = \left|\frac{-a}{\sqrt{\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}}}\right| = \left|\frac{-a}{\frac{1}{\sin\theta\cos\theta}}\right|$$

$\Rightarrow$ p = a sin θ cos θ ...(2)

The other line is x cos θ + y sin θ – a cos 2θ = 0 ...(3)

P' = length of perpendicular from origin (0, 0) on (3) is

$$= \left|\frac{a\cos 2}{\sqrt{\cos^2 \quad \sin^2}}\right| \quad a\cos 2$$

$\therefore$, $4p^2 + p'^2 = 4a^2 \cos^2\theta \sin^2\theta + a^2 \cos^2 2\theta$

$= a^2(2\cos\theta \sin\theta)^2 + a^2 \cos^2 2\theta$

$= a^2 \sin^2 2\theta + a^2 \cos^2 2\theta$

$= a^2(\sin^2 2\theta + \cos^2 2\theta)$

$= a^2$

Hence $4p^2 + p'^2 = a^2$.

26. Sum the series $\frac{1^3}{1}+\frac{1^3+2^3}{2}+\frac{1^3+2^3+3^3}{3}+\ldots$ to n terms.

Solution. Here

$$t_n = \frac{1^3+2^3+3^3+\ldots+n^3}{n} = \frac{\sum_{k=1}^{n} k^3}{n} = \frac{n^2(n+1)^2}{4n}$$

$$= \frac{n}{4}(n^2+2n+1) = \frac{1}{4}n^3+\frac{1}{2}n^2+\frac{1}{4}n$$

$$S_n = \frac{1}{4}\sum_{k=1}^{n}k^3+\frac{1}{2}\sum_{k=1}^{n}k^2+\frac{1}{4}\sum_{k=1}^{n}k$$

$$= \frac{1}{4}\cdot\frac{n^2(n+1)^2}{4}+\frac{1}{2}\cdot\frac{n(n+1)(2n+1)}{6}+\frac{1}{4}\cdot\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{48}[3n(n+1)+4(2n+1)+6]$$

$$= \frac{n(n+1)}{48}(3n^2+1(1n+10) = \frac{n(n+1)(n+2)(3n+5)}{48}$$

27. For any two sets A and B, prove that P(A) = P(B) $\Rightarrow$ A = B

Solution. Let x be an arbitrary element of A. Then, there exists a subset, say X, of set A such that x $\in$ X. Now,

$X \subset A \Rightarrow X \in P(A)$

$\Rightarrow X \in P(B)$ $\quad$ [$\because$ P(A) = P(B)]

$\Rightarrow X \subset (B)$

$\Rightarrow x \in B$ $\quad$ [$\because$ $x \in X$ and $X \subset B$ $\therefore$ $x \in B$]

Thus, $x \in A \Rightarrow x \in B$

$\therefore$ $A \subseteq B$ $\quad$ ...(1)

Now, let y be an arbitrary element of B. Then, there exists a subset, say Y, of set B such that $y \in Y$.

Now, $y \subset B \Rightarrow Y \in P(B)$

$\Rightarrow Y \in P(A)$ $\quad$ [$\because$ P(A) = P(B)]

$\Rightarrow Y \subset A$

$\Rightarrow Y \in A$

Thus, $y \in B \Rightarrow y \in A$

$\therefore$ $B \subseteq A$

...(2)

From (1) and (2), we obtain A = B .

28. Prove that cos 20° cos 40° cos 60° cos 80° = $\frac{1}{16}$

L.H.S = cos 20° cos 40° cos 60° cos 80°

$= \frac{1}{2}\cos 20°\cos 40°\cos 80° \qquad \left[\because \cos 60° = \frac{1}{2}\right]$

$= \frac{1}{4}(2\cos 20°\cos 40°)\cos 80°$

$= \frac{1}{4}[\cos(20°+40°)\cos(20°-40°)]\cos 80°$

[$\because$ 2 cos A cos B = cos (A + B) + cos (A – B)]

$= \frac{1}{4}(\cos 60° + \cos 20°)\cos 80° \qquad [\because \cos(-20°) = \cos 20°]$

$= \frac{1}{4} \; \frac{1}{2}\cos 80 \qquad \cos 20 \;\; \cos 80$

$= \frac{1}{8}(\cos 80° + 2\cos 20°\cos 80°)$

$= \frac{1}{8}[\cos 80° + \cos(20°+80°) + \cos(20°-80°)]$

[$\because$ 2 cos A cos B = cos (A + B) + cos (A – B)]

= $\frac{1}{8}[\cos 80° + \cos 100° + \cos(-60°)] = \frac{1}{8}\left[\cos 80° - \cos 80° + \frac{1}{2}\right]$

= $\frac{1}{8} \quad \frac{1}{2} \quad \frac{1}{16}$ R.H.S

$\left[\because \cos 100° = \cos(180° - 80°) = -\cos 80° \text{ and } \cos(-60°) = \cos 60° = \frac{1}{2}\right]$

29. By the principle of mathematical induction, prove that $(1 + x)^n \geq 1 + nx$ for all $n \in N$ and $x > -1$.

Solution. Let P(n): $(1 + x)^n \geq 1 + nx$, for $x > -1$, $n \in N$ be the given statement. For n = 1, P(1): $(1 + x)^1 \geq 1 + x$, which is true, P(1) is true. Assume that P(k) $(1 + x)^k \geq 1 + kx$ holds. We shall prove that

P(k + 1): $(1 + x)^{k+1} \geq 1 + (k + 1)x$

Since $x > -1 \Rightarrow 1 + x > 0$

Multiplying both sides of (1) by 1 + x, we get

$(1 + x)^{k+1} \geq (1 + kx)(1 + x) = 1 + kx + x + kx^2 \geq 1 + (k + 1)x$

[$\because k \in N, x^2 \geq 0 \Rightarrow kx^2 \geq 0$ for all $x \in R$]

$\therefore (1 + x)^{k+1} \geq 1 + (k + 1)x \Rightarrow$ P(k + 1) is also true. Hence by mathematical induction, P(n) holds for all $n \in N$.

MODEL TEST PAPER-IV

Time: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) This questions paper contains 29 questions.

(iii) Questions nos. 1-4 in section A are very short answer type questions carrying 1 mark each

(iv) Question nos. 5 -12 in section B are short answer type questions carrying 2 marks each

(v) Questions non. 13 -23 in section C are long answer-I type questions carrying 4 marks each

(vi) Question 24 -29 in section D are long answer –II type questions carrying 6 marks each.

SECTION A

1. Write the domain and range of the real function $f(x) = -|x|$.
2. What is the real value of a for which

   $3i^3 - 2ai^2 + (1 - a)i + 5$

   is real
3. Write the intercepts of line $2x - 3y = 7$ on coordinate axes.
4. Write the converse of the following statements

   ‘I go to a beach whenever it is a sunny day’.

SECTION B

5. Draw the Venn diagram to illustrate 'All the students who study Mathematics study English but some students who study English do not study Mathematics'. If E is the set of students studying English in a school, M is the set of students studying Mathematics in the same school, U is the set of all students in that school.
6. Prove that

   $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2\left(\frac{x-y}{2}\right)$.

7. If x + iy = $\frac{(a^2+1)^2}{2a-i}$, what is the value of $x^2 + y^2$?
8. Find the sum of all two digit numbers which when divided by 4, yield 1 as remainder.

9. If $|x| < 1$ and $y = x + x^2 + x^3 + \ldots\ldots \infty$ show that :

   $X = \frac{y}{1+y}$

10. $\lim_{x\to 0} \frac{\sin ax + bx}{ax + \sin bx}$ when a, b, a + b $\neq$ 0.
11. (i) Write the contra positive of the following conditional statements :

    'If my grandmother had wheels, then she would be a bus'.

    (iii) Write the negation of the following statements :

    'Australia is a continent'.'

12. Find the probability of having exactly one girl in a family of three children.
13. Let A ={1, 2, 4, 5}, B = {2, 3, 5, 6}, C = {4, 5, 6, 7} Verify the identity :

$A \cap (B - C) = (A \cap B) - (A \cap C)$

14. If $2f(x) - 3f\left(\frac{1}{x}\right)$ = x^2, $x \neq 0$,, then find f(2).

15. If tan x = $\frac{3}{4}, \pi < x < \frac{3\pi}{2}$, find the value of :

$\sin\frac{x}{2}, \cos\frac{x}{2} \text{and} \tan\frac{x}{2}$

16. Prove the following by using the principle of mathematical induction for all $n \in N$.

$$\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + ... + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

17. Find the square root of 16 – 30i.

18. Solve the following linear inequalities graphically :

$3x + 4y \leq 60$

$X + 3y \leq 30$

$X \geq 0, y \geq 0$.

19. A line is such that its segment between the lines

5x – y + 4 = 0 and 3x + 4y – 4 = 0

is bisected at the point (1, 5). Obtain its equation.

OR

Find the image of the point (3, 8) with respect to the line x + 3y = 7 assuming the line to be a plane mirror.

20. Find the equation of the ellipse having axes along the coordinate axes and passing through the points (4, 3) and (–1, 4).

21. Determine the values of a and b so that the points (a, b, 3), (2, 0, –1) and (1, –1, –3) are collinear.

22. Find the derivative of $\frac{x \sin x}{1+\cos x}$ with respect to x.

OR

Find the derivative of $\frac{e^x - e^{-x}}{e^x + e^{-x}}$ with respect to x.

23. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75. What is the probability of passing Hindi examination?

SECTION C

24. Out of 100 students; 15 passed in English, 12 passed in Mathematics, 8 in Science, 6 in English and Mathematics, 7 in Mathematics and Science, 4 in English and Science, 4 in all the three. Find how many passed :

(i) In English and Mathematics but not in Science?

(ii) in more than one subject only?

25. Prove that :

$\text{Cos}^3 A + \cos^3 (120° + A) + \cos^3 (240° + A) = \frac{3}{4}\cos 3A$.

OR

If $x\cos\theta = y\cos\left(\theta+\frac{2\pi}{3}\right) = z\cos\left(\theta+\frac{4\pi}{3}\right)$ prove that :

xy + yz + zx = 0.

26. A committee of 10 is to be formed from 8 gentlemen and 8 ladies. In how many ways this can be done if at least five ladies have to be included? In how many of these committees :

(i) The ladies are in majority?

(ii) The gentlemen are in majority?

27. The second, third and fourth terms in the binomial expansion $(x + a)^n$ are 240, 720 and 1080 respectively. Find x, a and n.

OR

Show that the middle terms in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^n$ is

$$\frac{1.3.5.......(2n-1)}{n!}2^n$$

28. If a and b are the roots of $x^2 - 3x + p = 0$ and c, d are roots of $x^2 - 12x + q = 0$ where a, b, c, d form a G.P. Prove that :

(q + p) : (q – p) = 17 : 15.

OR

Let S be the sum, P the product and R the sum of reciprocals of n terms is a G.P. Prove that :

$$P = \left(\frac{S}{R}\right)^n$$

29. Calculate mean deviation about median for the following data :

Class	0-10	10-20	20-30	30-40	40-50	50-60
Frequency	6	7	15	16	4	2

MODEL TEST PAPER-V

Time: 3 hours **Maximum Marks: 100**

General Instructions:

(i) All questions are compulsory.

(ii) This questions paper contains 29 questions.

(iii) Questions nos. 1-4 in section A are very short answer type questions carrying 1 mark each

(iv) Question nos. 5 -12 in section B are short answer type questions carrying 2 marks each

(v) Questions non. 13 -23 in section C are long answer-I type questions carrying 4 marks each

(vi) Question 24 -29 in section D are long answer –II type questions carrying 6 marks each.

SECTION A

1. Define Modulus function
2. If $\tan\alpha = \frac{x}{x+1}$ and $\tan\beta = \frac{1}{2x+1}$, then find the value of α + β.
3. Evaluate

$$\lim_{x\to 0} \frac{\sin x^o}{x}$$

4. If the latus rectum of an ellipse is one half of its minor axis, then what will be its eccentricity?

SECTION B

5. For sets A, B and C using properties of sets, prove that

   A – (B – C) = (A – B) $\cup$ (A $\cap$ C)

6. Find the modulus of $\frac{1+i}{1-i}-\frac{1-i}{1+i}$.

7. Find the 4th term from the end in the expansion $\left(\frac{3}{x^2}-\frac{x^3}{6}\right)^7$

8. Solve 1 + 6 + 11 + 16 ….. + x = 148.

9. Find the ratio in which the line joining the points (1, 2, 3) and (–3, 4, –5) is divided by xy plane.

10. If the letters of word " ALGORITHM" are arranged in random in a row. What is the probability that the letters "GOR" must remain together as a unit?

11. Differentiate $\tan\sqrt{x}$ wrt . 'x'.

12. Find that equation of the line passing through (–3, 5) and perpendicular to the line through (2, 5) and (–3, 6)

SECTION C

13. Find the domain and range of

    f(x) = $\frac{1}{\sqrt{x-[x]}}$

14. Prove that $\left|\sqrt{\frac{1-\sin x}{1+\sin x}}+\sqrt{\frac{1+\sin x}{1-\sin x}}\right|=\frac{-2}{\cos x}$ where $\frac{\pi}{2}<x<\pi$.

15. Prove that $2.7^n + 3.5^n - 5$ is divisible by 24 for all n $\in$ N.

16. In the expansion of $(1 + x)^n$ the binomial coefficients of three consecutive terms are respectively 220, 495 and 792. Find the value of n.

17. In a plane, there are 37 straight lines of which 13 pass through the point A and 11 pass through the point B. Besides no three lines pass through one point, no line passes through both points A and B and no two are parallel. Find the number of prints of intersection of the straight lines.

18. Differentiate by first principle, $\sqrt{ax+b}$

OR

Evaluate $\lim\limits_{x \to 0} \frac{\tan 2x - \sin 2x}{x^3}$

19. Calculate the mean deviation from the median

Class	0-6	6-12	12-18	18-24	24-30
Frequency	8	10	12	9	5

20. Find the equation of the circle passing through the points (5, –8), (2, –9), (2, 1)

21. Find the equation of the hyperbola whose directrix is 2x+ y = 1, Focus (1, 2) and eccentricity $\sqrt{3}$.

22. If z_1, z_2 are complex number such that $\frac{2z_1}{3z_2}$ is purely imaginary number, then find $\left|\frac{z_1 - z_2}{z_1 + z_2}\right|$.

23. Insert 4 geometric means between 576 and 18.

SECTION D

24. In a survey of 100 students, the number of students studying the various languages were found to be : English only 18, English but not Hindi 23, English and German 8, English 26, German 48, German and Hindi 8, no language 24. Find the number of students who were studying (i) Hindi (ii) English and Hindi (iii) English, Hindi and German.

25. Exhibit graphically the solution set of the linear in equations $x + y \leq 5, 4x + y \geq 4, x + 5y \geq 5, x \leq 4, y \leq 3$.

26. Find the sum of n terms of the series

$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots\ldots\ldots\ldots$.

27. Calculate the mean and standard deviation for the given distribution

Age :	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No. of person	3	51	122	141	130	51	2

28. Using the words "necessary and sufficient" rewrite the statement.

"The integer n is odd if and only if n^2 is odd." Also check whether the statement is true.

29. Solve for x: $\sin 2x - \sin 4x + \sin 6x = 0$

OR

In a ΔABC, prove that

$(b^2 - c^2) \cot A + (c^2 - a^2) \cot B + (a^2 - b^2) \cot c = 0$