

**DIRECTORATE OF EDUCATION**  
**Govt. of NCT, Delhi**

**SUPPORT MATERIAL**  
**(2018-2019)**

**Class : XII**

**MATHEMATICS**

Under the Guidance of

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Secretary (Education)

**Mr. Sanjay Goel**  
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### **PREFACE**

It gives me immense pleasure to present the Support Material for various subjects. The material prepared for students of classes IX to XII has been conceived and developed by a team comprising of the Subject Experts, Members of the Academic Core Unit and teachers of the Directorate of Education.

The subject wise Support Material is developed for the betterment and enhancement of the academic performance of the students. It will give them an insight into the subject leading to complete understanding. It is hoped that the teachers and students will make optimum use of this material. This will help us achieve academic excellence.

I commend the efforts of the team who have worked with complete dedication to develop this matter well within time. This is another endeavor of the Directorate to give complete support to the learners all over Delhi.

  
(SANDEEP KUMAR)  
SECRETARY

**Sanjay Goel, IAS**



**Director**

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Date : 8/8/2018

**DIRECTOR'S MESSAGE**

Dear Students,

Through this Support Material, I am getting an opportunity to communicate directly with you and I want to take full advantage of this opportunity.

In Delhi, there are approximately 1020 other government schools like yours, which are run by Directorate of Education. The Head Quarters of Directorate of Education is situated at Old Secretariat, Delhi-54.

All the teachers in your school and officers in the Directorate work day and night so that the standard of our govt. schools may be uplifted and the teachers may adopt new methods and techniques to teach in order to ensure a bright future for the students.

Dear students, the book in your hand is also one such initiative of your Directorate. This material has been prepared specially for you by the subject experts. A huge amount of money and time has been spent to prepare this material. Moreover, every year, this material is reviewed and updated as per the CBSE syllabus so that the students can be updated for the annual examination.

Last, but not the least, this is the perfect time for you to build the foundation of your future. I have full faith in you and the capabilities of your teachers. Please make the fullest and best use of this Support Material.

**DIRECTOR (EDUCATION)**



**Dr. (Mrs.) Saroj Bala Sain**

Addl. Director of Edn. (School)/Exam



सत्यमेव जयते

Govt. of NCT of Delhi  
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D.O. No. PA/ADD/DECSM/80

Date : 16/07/2018

It gives me immense pleasure and a sense of satisfaction to forward the support material for classes IX to XII in all subjects. The support material is continuously revised redesigned and updated by a team of subject experts, members of Core Academic Unit and teachers from various schools of DOE.

Consistent use of support material by the students and teachers will make the year long journey seamless and enjoyable. The purpose of providing support material has always been to make available ready to use material which is matchless and most appropriate.

My commendation for all the team members for their valuable contribution.

Dr. Saroj Bala Sain  
Addl.DE (School)



**DIRECTORATE OF EDUCATION**  
**Govt. of NCT, Delhi**

**SUPPORT MATERIAL**  
**(2018-2019)**

**MATHEMATICS**

**Class : XII**

**NOT FOR SALE**

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## REVIEW TEAM : 2018-19

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### MATHEMATICS : CLASS XII

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3.	VIDYA SAGAR MALIK	LECT. MATHS	CORE ACADEMIC UNIT DIRECTORATE OF EDUCATION

# MATHEMATICS CLASS 12

## SYLLABUS

### Course Structure

Unit	Topic	Marks
I.	Relations and Functions	10
II.	Algebra	13
III.	Calculus	44
IV.	Vectors and 3-D Geometry	17
V.	Linear Programming	06
VI.	Probability	10
Total :		100

### Unit I: RELATIONS AND FUNCTIONS

#### 1. Relations and Functions

Types of Relations: Reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

#### 2. Inverse Trigonometric Functions

Definition, range, domain, principal value branch. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

## **Unit II: ALGEBRA**

### **1. Matrices**

Concept, notation, order, equality, types of matrices, zero and identity matrix, transpose of a matrix, symmetric and skew symmetric matrices.

Operation on Matrices: Addition and multiplication and multiplication with a scalar. Simple properties of addition, multiplication and scalar multiplication. Non commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

### **2. Determinants**

Determinant of a square matrix (up to  $3 \times 3$  matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

## **Unit III : CALCULUS**

### **1. Continuity and Differentiability**

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions.

Derivatives of logarithmic and exponential functions. Logarithmic differentiation, derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's mean Value Theorems (without proof) and their geometric interpretations.

### **2. Applications of Derivatives**

Applications of Derivatives : Rate of change of bodies, increasing/decreasing functions, tangents and normal's, use of derivatives in

approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

### 3. Integrals

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts. Evaluation of simple integrals of the following types and problems based on them.

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}}$$

$$\int \frac{px + q}{ax^2 + bx + c} dx, \int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx, \int \sqrt{x^2 - a^2} dx,$$

$$\int \sqrt{ax^2 + bx + c} dx, \int (px + q)\sqrt{ax^2 + bx + c} dx$$

(Definite integrals as a limit of a sum, Fundamental theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

### 4. Applications of the Integrals

Applications in finding the area under simple curves, especially lines, circles/parabolas/ellipses (in standard form only), area between any of the two above said curves (the region should be clearly identifiable).

### 5. Differential Equations

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, Solution of homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$dy/dx + py = q$ , where  $p$  and  $q$  are functions of  $x$  or constants.

$dx/dy + px = q$ , where  $p$  and  $q$  are functions of  $y$  or constants



## **Unit IV: VECTORS AND THREE-DIMENSIONAL GEOMETRY**

### **1. Vectors**

Vectors and scalars, magnitude and direction of a vector. Direction cosines and direction ratios of a vector. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Definition, Geometrical Interpretation, properties and applications of scalar (dot) product of vectors, vector (cross) product of vectors, scalar triple product of vectors.

### **2. Three-Dimensional Geometry**

Direction cosines and direction ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

## **Unit V: LINEAR PROGRAMMING**

- 1. Linear Programming:** Introduction, related terminology such as constraints, objective function, optimization. Different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions (bounded and unbounded) feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constraints).

## **Unit VI : PROBABILITY**

### **1. Probability**

Conditional probability, Multiplication theorem on probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of a random variable. Repeated independent (Bernoulli) trials and Binomial distribution.

The following will be applicable in the subject Mathematics (041) for class XII for the academic session 2018-19 and Board examination 2019.

### Question Paper Design

S. No.	Typology of Questions	VSA (1) mark	SA (2) marks	LA- I (4) marks	LA -II (6) marks	Marks	Weightage
1	Remembering	2	2	2	1	20	20%
2	Understanding	1	3	4	2	35	35%
3	Application	1	–	3	2	25	25%
4	HOTS	–	3	1	–	10	10%
5	Evaluation	–	–	1	1	10	10%
	<b>Total</b>	1 × 4 = <b>4</b>	2 × 8 = <b>16</b>	4 × 11 = <b>44</b>	6 × 6 = <b>36</b>	100	100%

## QUESTION WISE BREAK UP FOR 2018-19

Type of Questions	Marks per Question	Total Number of Questions	Total Marks
VSA	1	4	04
SA	2	8	16
LA - I	4	11	44
LA - II	6	6	36
<b>Total</b>		<b>29</b>	<b>100</b>

1. No chapter wise weightage. Care to be taken to cover all the chapters.
2. The above template is only a sample. Suitable internal variations may be made for generating similar templates keeping the overall weightage to different form of questions and typology of questions same.

## CHAPTER 1

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### RELATIONS AND FUNCTIONS

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#### IMPORTANT POINTS TO REMEMBER

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- Relation  $R$  from a set  $A$  to a set  $B$  is subset of  $A \times B$  and Relation  $R$  in set  $A$  is a subset of  $A \times A$ .
- If  $n(A) = r, n(B) = s$  from set  $A$  to set  $B$  then  $n(A \times B) = rs$ . and number of relations  $= 2^{rs}$
- $\emptyset$  is also a relation defined on set  $A$ , called the void (empty) relation.
- $R = A \times A$  is called universal relation.
- **Reflexive Relation:** Relation  $R$  defined on set  $A$  is said to be reflexive if  $(a, a) \in R \forall a \in A$ .
- **Symmetric Relation :** Relation  $R$  defined on set  $A$  is said to be symmetric iff  $(a, b) \in R \Rightarrow (b, a) \in R \forall a, b, \in A$
- **Transitive Relation :** Relation  $R$  defined on set  $A$  is said to be transitive if  $(a, b) \in R, (b, c) \in R \Rightarrow (a, c) \in R \forall a, b, c \in A$
- **Equivalence Relation:** A relation defined on set  $A$  is said to be equivalence relation if it is reflexive, symmetric and transitive.
- **Equivalence class of an element:** Let  $R$  be an equivalence relation of set  $A$ , then equivalence class of  $a \in A$  is  $[a] = \{ b \in A : (b, a) \in R \}$ .
- **One-One Function :**  $f : A \rightarrow B$  is said to be one-one if distinct elements in  $A$  have distinct images in  $B$ . i.e.  $\forall x_1, x_2 \in A$  such that  $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ .

OR

$$\forall x_1, x_2 \in A \text{ such that } f(x_1) = f(x_2)$$

$$\Rightarrow x_1 = x_2$$

One-one function is also called injective function.

- **Onto function (surjective):** A function  $f : A \rightarrow B$  is said to be onto iff  $R_f = B$  i.e.  $\forall b \in B$ , there exists  $a \in A$  such that  $f(a) = b$
- **Bijjective Function :** A function which is both injective and surjective is called bijective function.
- **Composition of Two Functions :** If  $f : A \rightarrow B$ ,  $g : B \rightarrow C$  are two functions, then composition of  $f$  and  $g$  denoted by  $g \circ f$  is a function from  $A$  to  $C$  given by,  $(g \circ f)(x) = g(f(x)) \forall x \in A$   
Clearly  $g \circ f$  is defined if Range of  $f$  is a subset of domain of  $g$ . Similarly  $f \circ g$  can be defined.
- **Invertible Function:** A function  $f : X \rightarrow Y$  is invertible iff it is bijective.  
If  $f : X \rightarrow Y$  is bijective function, then function  $g : Y \rightarrow X$  is said to be inverse of  $f$  iff  $f \circ g = I_Y$  and  $g \circ f = I_X$   
when  $I_X, I_Y$  are identity functions.
- Inverse of  $f$  is denoted by  $f^{-1}$ . [  $f^{-1}$  does not mean  $\frac{1}{f}$  ]
- Let  $A$  and  $B$  are two non empty set that  $n(A) = p$  and  $n(B) = q$   
Then
  - a) Number of functions from  $A$  to  $B = q^p$
  - b) Number of one-one functions from  $A$  to  $B = \begin{cases} {}^qP_p & , p \leq q \\ 0 & , p > q \end{cases}$
  - c) Number of onto function from  $A$  to  $B = \begin{cases} \sum_{r=1}^q (-1)^{q-r} {}^qC_r r^p & , p \geq q \\ 0 & , p < q. \end{cases}$
  - d) Number of bijective functions from  $A$  to  $B = \begin{cases} p! & , p = q \\ 0 & , p \neq q. \end{cases}$
- **Binary Operation:** A binary operation '\*' defined on set  $A$  is a function from  $A \times A \rightarrow A$ .  
 $*(a, b)$  is denoted by  $a * b$ .
- Number. of binary operation on set having  $n$  elements  $= n^{(n^2)}$
- Binary operation  $*$  defined on set  $A$  is said to be commutative iff  $a * b = b * a \forall a, b \in A$ .

- Binary operation\*defined on set  $A$  is called associative iff

$$a*(b*c) = (a*b)*c \quad \forall a, b, c \in A$$

- If  $*$  is Binary operation on  $A$ , then an element  $e \in A$  (if exists) is said to be the identity element iff  $a*e = e*a = a \quad \forall a \in A$
- Identity element is unique.
- If  $*$  is Binary operation on set  $A$ , then an element  $b \in A$  (if exists) is said to be inverse of  $a \in A$  iff  $a*b = b*a = e$
- Inverse of an element, if it exists, is unique.

### VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. If  $A$  is the set of students of a school then write, which of following relations are Universal, Empty or neither of the two.

$$R_1 = \{(a, b) : a, b \text{ are ages of students and } |a - b| > 0\}$$

$$R_2 = \{(a, b) : a, b \text{ are weights of students, and } |a - b| < 0\}$$

$$R_3 = \{(a, b) : a, b \text{ are students studying in same class}\}$$

2. Is the relation  $R$  in the set  $A = \{1, 2, 3, 4, 5\}$  defined as

$$R = \{(a, b) : b = a + 1\} \text{ reflexive?}$$

3. If  $R$ , is a relation in set  $N$  given by

$$R = \{(a, b) : a = b - 3, b > 5\},$$

then does element  $(5, 7) \in R$ ?

4. If  $f : \{1, 3\} \rightarrow \{1, 2, 5\}$  and  $g : \{1, 2, 5\} \rightarrow \{1, 2, 3, 4\}$  be given by  $f = \{(1, 2), (3, 5)\}$ ,  $g = \{(1, 3), (2, 3), (5, 1)\}$ , write  $g \circ f$ .

5. Let  $g, f: R \rightarrow R$  be defined by  
 $g(x) = \frac{x+2}{3}$ ,  $f(x) = 3x - 2$ . write  $f \circ g(x)$
6. If  $f: R \rightarrow R$  defined by  
 $f(x) = \frac{2x-1}{5}$   
 be an invertible function, write  $f^{-1}(x)$ .
7. If  $f(x) = \log x$  and  $g(x) = e^x$ . Find  $f \circ g$  and  $g \circ f$ ,  $x > 0$ .
8. Let  $*$  be a Binary operation defined on  $R$ , then if  
 (i)  $a * b = a + b + ab$ , write  $3 * 2$   
 (ii)  $a * b = \frac{(a+b)^2}{3}$ , write  $(2*3)*4$ .
9. If  $n(A) = n(B) = 3$ , then how many bijective functions from  $A$  to  $B$  can be formed?
10. Is  $f: N \rightarrow N$  given by  $f(x) = x^2$ , one-one? Give reason.
11. If  $f: R \rightarrow A$ , given by  
 $f(x) = x^2 - 2x + 2$  is onto function, find set  $A$ .
12. If  $f: A \rightarrow B$  is bijective function such that  $n(A) = 10$ , then  $n(B) = ?$
13. If  $f: R \rightarrow R$  defined by  $f(x) = \frac{x-1}{2}$ , find  $(f \circ f)(x)$
14.  $R = \{(a, b) : a, b \in N, a \neq b \text{ and } a \text{ divides } b\}$ . Is  $R$  reflexive? Give reason
15. Is  $f: R \rightarrow R$ , given by  $f(x) = |x - 1|$  one-one? Give reason
16.  $f: R \rightarrow B$  given by  $f(x) = \sin x$  is onto function, then write set  $B$ .

- 17 If  $f(x) = \log \left( \frac{1+x}{1-x} \right)$ , show that  $f \left( \frac{2x}{1+x^2} \right) = 2f(x)$ .
- 18 If '\*' is a binary operation on set  $Q$  of rational numbers given by  $a * b = \frac{ab}{5}$  then write the identity element in  $Q$ .
- 19 If  $*$  is Binary operation on  $N$  defined by  $a * b = a + ab \forall a, b \in N$ , write the identity element in  $N$  if it exists.

### SHORT ANSWER TYPE QUESTIONS (2 Marks)

20. Consider the binary operation  $*$  on the set  $\{1, 2, 3, 4, 5\}$  defined by  $a * b = \text{HCF of } a \text{ and } b$  write the operation table for the operation  $*$
21. Check the following functions for one-one and onto
- (a)  $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = \frac{2x-3}{7}$
- (b)  $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = |x + 1|$
- (c)  $f : \mathbb{R} - \{2\} \rightarrow \mathbb{R}, f(x) = \frac{3x-1}{x-2}$
- (d)  $f : \mathbb{R} - \{-1, 1\}, f(x) = \sin^2 x$ .
22. If  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be two functions defined by  $f(x) = |x| + x$  and  $g(x) = |x| - x$ , find  $go f$  and  $fog$ .
23. If  $f : [1, \alpha) \rightarrow [2, \alpha)$  is defined by  $f(x) = x + \frac{1}{x}$ , find  $f^{-1}(x)$
24. Let  $A = \{1, 2, 3\}$  and define  $R = \{(a, b) : a - b = 12\}$ . Show that  $R$  is empty relation on Set  $A$ .
25. Let  $A = \{1, 2, 3\}$  and define  $R = \{(a, b) : a + b > 0\}$ . Show that  $R$  is a universal relation on set  $A$ .
26. Let  $A = \{a, b, c\}$ . How many relation can be defined in the set? How many of these are reflexive?
27. Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be defined by  $f(x) = x^2 + 1$ , find the pre image of 17 and -3
28. If  $f : \mathbb{R} \rightarrow \mathbb{R}, g : \mathbb{R} \rightarrow \mathbb{R}$ , given by  $f(x) = [x], g(x) = |x|$ , then find  $fog \left( -\frac{2}{3} \right)$  and  $gof \left( -\frac{2}{3} \right)$ .



**SHORT ANSWER TYPE QUESTIONS ( 4 MARKS)**

29. Let  $f : R - \left\{ \frac{-4}{3} \right\} \rightarrow R - \left\{ \frac{4}{3} \right\}$  be a function given by  $f(x) = \frac{4x}{3x+4}$ .

Show that  $f$  is invertible with  $f^{-1}(x) = \frac{4x}{4-3x}$ .

30. Let  $R$  be the relation on set  $A = \{x : x \in \mathbb{Z}, 0 \leq x \leq 10\}$  given by  $R = \{(a, b) : (a - b) \text{ is divisible by } 4\}$ . Show that  $R$  is an equivalence relation. Also, write all elements related to 4.

31. Show that function  $f : A \rightarrow B$  defined as  $f(x) = \frac{3x+4}{5x-7}$  where  $A = R - \left\{ \frac{7}{5} \right\}$ ,  $B = R - \left\{ \frac{3}{5} \right\}$  is invertible and hence find  $f^{-1}$ .

32. Let  $*$  be a binary operation on  $Q$  such that  $a * b = a + b - ab$ .

(i) Prove that  $*$  is commutative and associative.

(ii) Find identity element of  $*$  in  $Q$  (if it exists).

33. If  $*$  is a binary operation defined on  $R - \{0\}$  defined by  $a * b = \frac{2a}{b^2}$  then check  $*$  for commutativity and associativity.

34. If  $A = N \times N$  and binary operation  $*$  is defined on  $A$  as

$$(a, b) * (c, d) = (ac, bd).$$

(i) Check  $*$  for commutativity and associativity.

(ii) Find the identity element for  $*$  in  $A$  (If it exists).

35. Show that the relation  $R$  defined by  $(a, b) R (c, d) \Leftrightarrow a + d = b + c$  on the set  $N \times N$  is an equivalence relation.

36. Let  $*$  be a binary operation on set  $Q$  defined by  $a * b = \frac{ab}{4}$ , show that

(i) 4 is the identity element in  $Q$ .

(ii) Every non zero element of  $Q$  is invertible with

$$a^{-1} = \frac{16}{a}, \forall a \in Q - \{0\}.$$

37. Show that  $f: R_+ \rightarrow R_+$  defined by  $f(x) = \frac{1}{2x}$  is bijective, where  $R_+$  is the set of all non-zero positive real numbers.
38. Let  $A = \{1, 2, 3, \dots, 12\}$  and  $R$  be a relation in  $A \times A$  defined by  $(a, b) R (c, d)$  if  $ad = bc \forall (a, b), (c, d) \in A \times A$ . Prove that  $R$  is an equivalence relation. Also obtain the equivalence class  $[(3, 4)]$ .
39. If  $*$  is a binary operation on  $R$  defined by  $a * b = a + b + ab$ . Prove that  $*$  is commutative and associative. Find the identity element. Also show that every element of  $R$  is invertible except  $-1$ .

### LONG ANSWER TYPE QUESTIONS (6 MARKS)

41. Let  $N$  denote the set of all natural numbers and  $R$  be the relation on  $N \times N$  defined by  $(a, b) R (c, d)$  if  $ad(b + c) = bc(a + d)$ . Show that  $R$  is an equivalence relation.
42. Let  $f: N \rightarrow R$  be a function defined as  $f(x) = 4x^2 + 12x + 15$ .
- Show that  $f: N \rightarrow S$ , where  $S$  is the range of  $f$ , is invertible. Also find the inverse of  $f$ . Hence find  $f^{-1}(31)$ .
43. If the function  $f: R \rightarrow R$  be defined by  $f(x) = 2x - 3$  and  $g: R \rightarrow R$  by  $g(x) = x^3 + 5$ , then show that  $f \circ g$  is invertible. Also find  $(f \circ g)^{-1}(x)$ , hence find  $(f \circ g)^{-1}(9)$ .
44. Let  $A = Q \times Q$ , where  $Q$  is the set of rational number, and  $*$  be a binary operation on  $A$  defined by  $(a, b) * (c, d) = (ac, b + ad) \forall (a, b), (c, d) \in A$ .
- Is  $*$  commutative?
  - Is  $*$  Associative?
  - Find identity element of  $*$  in  $A$ .
  - Find invertible element of  $A$  and hence write the inverse of  $(1, 2)$  and  $(\frac{1}{3}, -5)$
45. Test whether relation  $R$  defined in  $R$  as  $R = \{(a, b) : a^2 - 4ab + 3b^2 = 0, a, b \in R\}$  is reflexive, summative and transitive

## ANSWERS

1.  $R_1$  : is universal relation.  
 $R_2$  : is empty relation.  
 $R_3$  : is neither universal nor empty.
2. No,  $R$  is not reflexive.
3.  $(5, 7) \notin R$
4.  $g \circ f = \{(1, 3), (3, 1)\}$
5.  $(f \circ g)(x) = x \quad \forall x \in R$
6.  $f^{-1}(x) = \frac{5x+1}{2}$
7.  $g \circ f(x) = x, f \circ g(x) = x$
8. (i)  $3 * 2 = 11$                       (ii)  $\frac{1369}{27}$
9. 6
10. Yes,  $f$  is one-one  $\because \forall x_1, x_2 \in N \Rightarrow x_1^2 = x_2^2$ .
11.  $A = [1, \infty)$  because  $R_f = [1, \infty)$
12.  $n(B) = 10$
13.  $(f \circ f)(x) = \frac{x-3}{4}$
14. No,  $R$  is not reflexive  $\because (a, a) \notin R \quad \forall a \in N$
15.  $f$  is not one-one function  
 $\because f(3) = f(-1) = 2$   
 $3 \neq -1$  i.e. distinct elements have same images.
16.  $B = [-1, 1]$

18.  $e = 5$

19. Identity element does not exist.

20.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

21. (a) Bijective (one-one, onto)  
 (b) Neither one-one nor onto  
 (c) One-one but not onto  
 (d) Neither one-one nor onto

22.  $g \circ f(x) = 0 \forall x \in R$

$$f \circ g(x) = \begin{cases} 0, & x \geq 0 \\ -4x & x < 0 \end{cases}$$

23.  $f^{-1}(x) = \frac{x + \sqrt{x^2 - 4}}{2}$

26. 512, 64

27.  $\pm 4$ , pre image of  $-3$  does not exist.

28.  $f \circ g\left(-\frac{2}{3}\right) = 0, g \circ f\left(-\frac{2}{3}\right) = 1$

30. Elements related to 4 are 0, 4, 8.

31.  $f^{-1}(x) = \frac{7x+4}{5x-3}$

32. 0 is the identity element.

33. Neither commutative nor associative.

34. (i) Commutative and associative.

(ii) (1, 1) is identity in  $N \times N$

38.  $[(3, 4)] = \{(3, 4), (6, 8), (9, 12)\}$

39. 0 is the identity element.

40.  $(fog)(x) = x^2 + x$

$$(gof)(x) = x^2 - x + 1$$

Clearly, they are unequal.

42.  $f^{-1}(y) = \frac{\sqrt{y-6}-3}{2}$ ,  $f^{-1}(31) = 1$

43.  $(f \circ g)^{-1}(x) = \left(\frac{x+7}{2}\right)^{1/3}$ ,  $(f \circ g)^{-1}(9) = 2$

44. I. Not commutative

II. Associative

III. (1,0)

IV. Inverse of (a, b) =  $\left(\frac{1}{a}, \frac{-b}{a}\right)$ , Inverse of (1, 2) = (1, -2) and Inverse of  $\left(\frac{1}{3}, -5\right) = (3, 15)$

45. Reflexive, not symmetric, not transitive

## CHAPTER 2

### INVERSE TRIGONOMETRIC FUNCTIONS

#### IMPORTANT POINTS TO REMEMBER

<i>Function</i>	<i>Domain</i>	<i>Range</i> <i>(Principal Value Branch)</i>
$\sin^{-1}x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$R$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$\cot^{-1}x$	$R$	$(0, \pi)$
$\sec^{-1}x$	$R - (-1, 1)$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$\operatorname{cosec}^{-1}x$	$R - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

- If  $\sin \theta = x$ ,  $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , then  $\theta = \sin^{-1}x$   
 If  $\cos \theta = x$ ,  $\theta \in [0, \pi]$ , then  $\theta = \cos^{-1}x$   
 If  $\tan \theta = x$ ,  $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ , then  $\theta = \tan^{-1}x$   
 If  $\cot \theta = x$ ,  $\theta \in (0, \pi)$ , then  $\theta = \cot^{-1}x$   
 If  $\sec \theta = x$ ,  $\theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$ , then  $\theta = \sec^{-1}x$

If  $\operatorname{cosec} \theta = x$ ,  $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ , then find  $\theta = \operatorname{cosec}^{-1} x$

- $\sin^{-1}(\sin x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$\cos^{-1}(\cos x) = x \quad \forall x \in [0, \pi]$$

$$\tan^{-1}(\tan x) = x \quad \forall x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\cot^{-1}(\cot x) = x \quad \forall x \in (0, \pi)$$

$$\sec^{-1}(\sec x) = x \quad \forall x \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$$

$$\operatorname{cosec}^{-1}(\operatorname{cosec} x) = x \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$$

- $\sin(\sin^{-1} x) = x \quad \forall x \in [-1, 1]$

$$\cos(\cos^{-1} x) = x \quad \forall x \in [-1, 1]$$

$$\tan(\tan^{-1} x) = x \quad \forall x \in \mathbb{R}$$

$$\cot(\cot^{-1} x) = x \quad \forall x \in \mathbb{R}$$

$$\sec(\sec^{-1} x) = x \quad \forall x \in \mathbb{R} - (-1, 1)$$

$$\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x \quad \forall x \in \mathbb{R} - (-1, 1)$$

- $\sin^{-1} x = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right) \quad \forall x \in [-1, 1]$

$$\tan^{-1} x = \cot^{-1}\left(\frac{1}{x}\right) \quad \forall x > 0$$

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right), \quad \forall |x| \geq 1$$

- $\sin^{-1}(-x) = -\sin^{-1} x \quad \forall x \in [-1, 1]$

$$\tan^{-1}(-x) = -\tan^{-1} x \quad \forall x \in \mathbb{R}$$

$$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x \quad \forall |x| \geq 1$$

- $\cos^{-1}(-x) = \pi - \cos^{-1} x \quad \forall x \in [-1, 1]$

$$\cot^{-1}(-x) = \pi - \cot^{-1} x \quad \forall x \in R$$

$$\sec^{-1}(-x) = \pi - \sec^{-1} x \quad \forall |x| \geq 1$$

- $\sin^{-1} X + \cos^{-1} X = \frac{\pi}{2}, X \in [-1, 1]$

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2} \quad \forall x \in R$$

$$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2} \quad \forall |x| \geq 1$$

- $$\tan^{-1} x + \tan^{-1} y = \begin{cases} \tan^{-1} \frac{x+y}{1-xy} & \text{if } xy < 1 \\ \pi + \tan^{-1} \frac{x+y}{1-xy} & \text{if } xy > 1; x > 0 \\ & y > 0 \\ -\pi + \tan^{-1} \frac{x+y}{1-xy} & \text{if } xy > 1; x < 0 \\ & y < 0 \end{cases}$$

- $$\tan^{-1} x - \tan^{-1} y = \begin{cases} \tan^{-1} \frac{x-y}{1+xy} & \text{if } xy > -1 \\ \pi + \tan^{-1} \frac{x-y}{1+xy} & \text{if } xy < -1; x > 0 \\ & y < 0 \\ -\pi + \tan^{-1} \frac{x-y}{1+xy} & \text{if } xy < -1; x < 0 \\ & y > 0 \end{cases}$$

- $2\tan^{-1} x = \tan^{-1} \left( \frac{2x}{1-x^2} \right), |x| < 1$

$$2\tan^{-1} x = \sin^{-1} \left( \frac{2x}{1+x^2} \right), |x| \leq 1,$$

$$2\tan^{-1} x = \cos^{-1} \left( \frac{1-x^2}{1+x^2} \right), x \geq 0.$$



- $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})$

$$\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}(xy \mp \sqrt{1-x^2}\sqrt{1-y^2})$$

### VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Write the principal value of

(i)  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(ii)  $\cos^{-1}(\sqrt{3}/2)$

(iii)  $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(iv)  $\operatorname{cosec}^{-1}(-2)$

(v)  $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(vi)  $\sec^{-1}(-2)$ .

2. What is the value of the following functions (using principal value)

(i)  $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(ii)  $\sin^{-1}\left(-\frac{1}{2}\right) - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(iii)  $\tan^{-1}(1) - \cot^{-1}(-1)$

(iv)  $\operatorname{cosec}^{-1}(\sqrt{2}) + \sec^{-1}(\sqrt{2})$

(v)  $\tan^{-1}(1) + \cot^{-1}(1) + \sin^{-1}(1)$ .

(vi)  $\sin^{-1}\left(\sin \frac{4\pi}{5}\right)$

(vii)  $\tan^{-1}\left(\tan \frac{5\pi}{6}\right)$

(viii)  $\operatorname{cosec}^{-1}\left(\operatorname{cosec} \frac{3\pi}{4}\right)$

3. If  $\tan^{-1}x + \tan^{-1}y = \frac{4\pi}{5}$ , find  $\cot^{-1}x + \cot^{-1}y$ .

### SHORT ANSWER TYPE QUESTIONS (2 MARKS)

4. Find the value of the following

(i)  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(-\frac{1}{2}\right) + \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

(ii)  $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{4\pi}{3}\right)$

(iii)  $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right\}$

(iv)  $\tan^{-1}\left(\tan\frac{7\pi}{6}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

5. Simplify

(i)  $\tan^{-1}\left(\frac{\sin x}{1 + \cos x}\right)$

(ii)  $\cot^{-1}\left(\frac{1}{\sqrt{x^2 - 1}}\right), x < -1$

(iii)  $\cos\left\{\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) + \frac{\pi}{6}\right\}$

(iv)  $\tan\left[\frac{1}{2}\cos^{-1}\left(\frac{3}{\sqrt{11}}\right)\right]$

6. Simplify :  $\sin^{-1}\left\{\frac{\sin x + \cos x}{\sqrt{2}}\right\}, -\frac{\pi}{4} < x < \frac{\pi}{4}$

7. Prove that :  $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$ .

8. Prove that :  $\tan^{-1}\frac{m}{n} - \tan^{-1}\left(\frac{m-n}{m+n}\right) = \frac{\pi}{4}, m, n > 0$

9. Prove that :  $\tan^{-1}\left\{\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right\} = \tan^{-1}\left(\frac{a}{b}\right) - x$

10. Evaluate :  $\tan^{-1}\left[2 \cos\left(2 \sin^{-1}\frac{1}{2}\right)\right]$

### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

11. Show that :  $\tan^{-1} \left[ \frac{\sqrt{1+\cos x} + \sqrt{1-\cos x}}{\sqrt{1+\cos x} - \sqrt{1-\cos x}} \right] = \frac{\pi}{4} + \frac{x}{2}; x \in [0, \pi]$

12. Prove that :

$$\tan^{-1} \left( \frac{\cos x}{1-\sin x} \right) - \cot^{-1} \left( \sqrt{\frac{1+\cos x}{1-\cos x}} \right) = \frac{\pi}{4} \quad x \in (0, \pi/2).$$

13. Prove that  $\tan^{-1} \left( \frac{x}{\sqrt{a^2-x^2}} \right) = \sin^{-1} \frac{x}{a} = \cos^{-1} \left( \frac{\sqrt{a^2-x^2}}{a} \right).$

14. prove that:

$$\cot^{-1} \left[ 2 \tan \left( \cos^{-1} \frac{8}{17} \right) \right] + \tan^{-1} \left[ 2 \tan \left( \sin^{-1} \frac{8}{17} \right) \right] = \tan^{-1} \left( \frac{300}{161} \right)$$

15. Prove that:

$$\tan^{-1} \left( \frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}} \right) = \frac{\pi}{4} + \frac{1}{2} \cos^{-1} x^2$$

16. Solve:

$$\cot^{-1} 2x + \cot^{-1} 3x = \frac{\pi}{4}$$

17. Prove that:

$$\tan \left[ \frac{\pi}{4} + \frac{1}{2} \tan^{-1} \left( \frac{a}{b} \right) \right] + \tan \left[ \frac{\pi}{4} - \frac{1}{2} \tan^{-1} \left( \frac{a}{b} \right) \right] = \frac{2\sqrt{a^2+b^2}}{b}$$

18. Solve for x,  $\cos^{-1} \left( \frac{x^2-1}{x^2+1} \right) + \tan^{-1} \left( \frac{-2x}{1-x^2} \right) = \frac{2\pi}{3}$

19. Prove that:  $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} + \tan^{-1} \frac{1}{7} + \tan^{-1} \frac{1}{8} = \frac{\pi}{4}$

20. Solve for x,  $\tan(\cos^{-1} x) = \sin(\tan^{-1} 2); x > 0$

21. If  $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$ , then prove that  $\sin y = \tan^2\left(\frac{x}{2}\right)$

22. Prove that:

$$\cot\left\{\tan^{-1}x + \tan^{-1}\left(\frac{1}{x}\right)\right\} + \cos^{-1}(1 - 2x^2) + \cos^{-1}(2x^2 - 1) = \pi, x > 0$$

23. Prove that:

$$\tan^{-1}\left(\frac{a-b}{1+ab}\right) + \tan^{-1}\left(\frac{b-c}{1+bc}\right) + \tan^{-1}\left(\frac{c-a}{1+ca}\right) = 0 \text{ where } a, b, c > 0$$

24. If  $\tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \pi$ , then

prove that  $a + b + c = abc$

25. If  $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ , prove that  $x^2 + y^2 + z^2 + 2xyz = 1$

[Hint: Let  $\cos^{-1}x = A$ ,  $\cos^{-1}y = B$ ,  $\cos^{-1}z = C$  then  $A + B + C = \pi$  or  $A + B = \pi - C$  take cos on both the sides].

26. If  $\tan^{-1}\left(\frac{1}{1+1.2}\right) + \tan^{-1}\left(\frac{1}{1+2.3}\right) + \dots + \tan^{-1}\left(\frac{1}{1+n.(n+1)}\right) = \tan^{-1}\emptyset$   
then find the value of  $\emptyset$ .

27. If  $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$  then find  $x$ .

28. If  $\sin[\cot^{-1}(x + 1)] = \cos(\tan^{-1}x)$ , then find  $x$ .

29. Solve the following for  $x$

(i)  $\sin^{-1}(6x) + \sin^{-1}(6\sqrt{3}x) = -\frac{\pi}{2}$

(ii)  $\sin^{-1}x + \sin^{-1}(1 - x) = \cos^{-1}x$

(iii)  $\sin^{-1}\left(\frac{5}{x}\right) + \sin^{-1}\left(\frac{12}{x}\right) = \frac{\pi}{2}$

(iv)  $\sin^{-1}\left(\frac{x}{2}\right) + \cos^{-1}x = \frac{\pi}{6}$

30. If  $\cos^{-1} \frac{x}{2} + \cos^{-1} \frac{y}{3} = \alpha$ , then prove that  
 $9x^2 - 12xy \cos \alpha + 4y^2 = 36 \sin^2 \alpha$
31. Prove that:  $\tan^{-1} \left[ \frac{3 \sin 2\phi}{5 + 3 \cos 2\phi} \right] + \tan^{-1} \left[ \frac{1}{4} \tan \phi \right] = \phi$
32. Prove that:  $\cot^{-1} \left[ \cot \left( \sin^{-1} \sqrt{\frac{2-\sqrt{3}}{4}} + \cos^{-1} \frac{\sqrt{3}}{2} + \cos^{-1} \frac{1}{\sqrt{2}} \right) \right] = \frac{\pi}{2}$
33. Prove that:  $2 \tan^{-1} \left( \sqrt{\frac{a-b}{a+b}} \tan \frac{x}{2} \right) = \cos^{-1} \left( \frac{a \cos x + b}{a + b \cos x} \right)$
34. Prove that:  $2 \tan^{-1} [\tan \alpha / 2 \tan \beta / 2] = \cos^{-1} \left[ \frac{\cos \alpha + \cos \beta}{1 + \cos \alpha \cos \beta} \right]$

### ANSWERS

1. (i)  $-\frac{\pi}{3}$  (ii)  $\frac{\pi}{6}$  (iii)  $\frac{-\pi}{6}$  (iv)  $\frac{-\pi}{6}$  (v)  $\frac{\pi}{3}$  (vi)  $\frac{2\pi}{3}$
2. (i) 0 (ii)  $\frac{-\pi}{3}$  (iii)  $-\frac{\pi}{2}$  (iv)  $\frac{\pi}{2}$
- (v)  $\pi$  (vi)  $\frac{\pi}{5}$  (vii)  $\frac{-\pi}{6}$  (viii)  $\frac{\pi}{4}$
3.  $\pi/5$
4. (i)  $\frac{\pi}{6}$  (ii)  $\pi$  (iii) 1 (iv)  $\pi$
5. (i)  $\frac{x}{2}$  (ii)  $\pi - \sec^{-1} x$  (iii) -1 (iv)  $\frac{\sqrt{11}-3}{\sqrt{2}}$
6.  $x + \frac{\pi}{4}$

$$9. \quad 1 \qquad 10. \quad \frac{\pi}{4} \qquad 12. \quad \tan \frac{\pi}{12} = 2 - \sqrt{3}$$

$$14. \quad \frac{\sqrt{5}}{3} \qquad 16. \quad 1 \qquad 18. \quad \tan \frac{\pi}{12} = 2 - \sqrt{3}$$

$$20. \quad \frac{\sqrt{5}}{3} \qquad 24. \quad \text{Hint: Let } \tan^{-1} a = \alpha$$

$$\tan^{-1} b = \beta$$

$$\tan^{-1} c = \gamma$$

Then given,  $\alpha + \beta + \gamma = \pi$

$$\alpha + \beta = \pi - \gamma$$

Take tangent on both sides

$$\tan(\alpha + \beta) = \tan(\pi - \gamma)$$

$$26. \quad \emptyset = \frac{n}{n+2}$$

$$27. \quad X = -1$$

$$28. \quad x = -\frac{1}{2}$$

$$29. \quad (i) \quad x = -\frac{1}{12}$$

$$(ii) \quad x = 0, \frac{1}{2}$$

$$(iii) \quad x = 13$$

$$(iv) \quad x = 1$$

## CHAPTER: 3 and 4

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### MATRICES AND DETERMINANTS

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#### IMPORTANT POINTS TO REMEMBER

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**Matrix:** It is an ordered rectangular arrangement of numbers (or functions). The numbers (or functions) are called the **elements** of the matrix. Horizontal line of elements is **row** of matrix. Vertical line of elements is **column** of matrix.

Numbers written in the horizontal line form a row of the matrix.  
Number written in the vertical line form a column of the matrix.

**Order of Matrix** with ' $m$ ' rows and ' $n$ ' columns is  $m \times n$  (read as  $m$  by  $n$ ).

#### Types of Matrices

- A **row matrix** has only one row (order:  $1 \times n$ )
- A **column matrix** has only one column (order:  $m \times 1$ )
- A **square matrix** has number of rows equal to number of columns (order:  $m \times m$  or  $n \times n$ .)
- A **diagonal matrix** is a square matrix with all non-diagonal elements equal to zero and diagonal elements not all zeroes.
- A **scalar matrix** is a diagonal matrix in which all diagonal elements are equal.
- An **identity matrix** is a scalar matrix in which each diagonal element is 1 (unity).
- A **zero matrix** or **null matrix** is the matrix having all elements zero.

- **Equal matrices:** two matrices  $A = [a_{ij}]_{m \times n}$  and  $B = [b_{ij}]_{m \times n}$  are equal if
  - (a) Both have same order
  - (b)  $a_{ij} = b_{ij} \forall i \text{ and } j$

### Operations on matrices

- Two matrices can be added or subtracted, if both have same order.
- If  $A = [a_{ij}]_{m \times n}$  and  $B = [b_{ij}]_{m \times n}$ , then
 
$$A \pm B = [a_{ij} \pm b_{ij}]_{m \times n}$$
- $\lambda A = [\lambda a_{ij}]_{m \times n}$  where  $\lambda$  is a scalar
- Two matrices A and B can be multiplied if number of columns in A is equal to number of rows in B.

If  $A = [a_{ij}]_{m \times n}$  and  $[b_{jk}]_{n \times p}$

Then  $AB = [c_{ik}]_{m \times p}$  where  $c_{ik} = \sum_{j=1}^n a_{ij} b_{jk}$

### Properties

- If A, B and C are matrices of same order, then
  - (i)  $A+B = B+A$
  - (ii)  $(A+B)+C = A+(B+C)$
  - (iii)  $A+O = O+A=A$
  - (iv)  $A+(-A) = O$



- If A, B and C are matrices and  $\lambda$  is any scalar, then

- $AB \neq BA$
- $(AB)C = A(BC)$
- $A(B+C) = AB+AC$
- $AB=O$  need not necessarily imply  $A=O$  or  $B=O$
- $\lambda (AB) = (\lambda A)B = A(\lambda B)$

**Transpose of a Matrix:** Let A be any matrix. Interchange rows and columns of A. The new matrix so obtained is transpose of A denoted by  $A' / A^T$ .

[order of A =  $m \times n \Rightarrow$  order of  $A' = n \times m$ ]

Properties of transpose matrices A and B are:

- $(A')' = A$
- $(kA)' = kA'$  ( $k = \text{constant}$ )
- $(A + B)' = A' + B'$
- $(AB)' = B'A'$

#### Symmetric Matrix and Skew-Symmetric matrix

- A square matrix  $A = [a_{ij}]$  is symmetric if  $A' = A$  i.e.  $a_{ij} = a_{ji} \forall i$  and  $j$
- A square matrix  $A = [a_{ij}]$  is skew-symmetric if  $A' = -A$  i.e.  $a_{ij} = -a_{ji} \forall i$  and  $j$   
(All diagonal elements are zero in skew-symmetric matrix)

**Determinant:** to every square matrix  $A = [a_{ij}]$  of order  $n \times n$ , we can associate a number (real or complex). This is called determinant of A ( $\det A$  or  $|A|$ ).

### Properties of Determinants

- I)  $|AB| = |A| |B|$
- II)  $|A'| = |A|$
- III) If we interchange any two rows (or columns), sign of  $|A|$  changes.
- IV) Value of  $|A|$  is zero, if any two rows or columns in  $A$  are identical (or proportional).
- V) 
$$\begin{vmatrix} a+b & x \\ c+d & y \end{vmatrix} = \begin{vmatrix} a & x \\ c & y \end{vmatrix} + \begin{vmatrix} b & x \\ d & y \end{vmatrix}$$
- VI)  $R_i \rightarrow R_i \pm aR_j$  or  $C_i \rightarrow C_i \pm bC_j$  does not alter the value of  $|A|$ .
- VII)  $|k A|_{n \times n} = k^n |A|_{n \times n}$  ( $k$  = scalar)
- VIII)  $K |A|$  means multiplying only one row (or column) by  $k$ .
- IX) Area of triangle with vertices  $(x_1, y_1)$ ,  $(x_2, y_2)$  and  $(x_3, y_3)$  is:

$$\Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

The points  $(x_1, y_1)$ ,  $(x_2, y_2)$ ,  $(x_3, y_3)$  are collinear if area of triangle is zero

### Minors and Cofactors

- Minor ( $M_{ij}$ ) of  $a_{ij}$  in Matrix  $A$  is the determinant of order  $(n-1)$  obtained by leaving  $i^{th}$  row and  $j^{th}$  column of  $A$ .
- Cofactor of  $a_{ij}$ ,  $A_{ij} = (-1)^{i+j} M_{ij}$

### Adjoint of a Square Matrix

adj  $A$  = transpose of the square matrix  $A$  whose elements have been replaced by their cofactors.

**Properties of adj A:** For any square matrix A of order n:

$$(i) \quad A(\text{adj } A) = (\text{adj } A) A = |A| I$$

$$(ii) \quad |\text{adj } A| = |A|^{n-1}$$

$$(iii) \quad \text{adj}(AB) = (\text{adj } B) (\text{adj } A).$$

$$(iv) \quad |k \text{ adj } A| = k^n |A|^{n-1}.$$

**Singular Matrix:** A square matrix A is singular if  $|A| = 0$ .

**Inverse of a Matrix:** An inverse of a square matrix exists if and only if it is non-singular.

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

**Properties of Inverse matrix**

$$(i) \quad AA^{-1} = A^{-1}A = I$$

$$(ii) \quad (A^{-1})^{-1} = A$$

$$(iii) \quad (AB)^{-1} = B^{-1}A^{-1}$$

$$(iv) \quad (A')^{-1} = (A^{-1})'$$

$$(v) \quad |A'| = \frac{1}{|A|}, |A| \neq 0$$

**Solution of system of equations using matrices:**

If  $AX = B$  is a matrix equation, then

$$AX=B \Rightarrow A^{-1}AX = A^{-1}B \Rightarrow IX = A^{-1}B \Rightarrow X = A^{-1}B \text{ gives the solution.}$$

**Criterion of consistency of system of linear equations**

$$(i) \quad \text{If } |A| \neq 0, \text{ system is consistent and has a unique solution.}$$

- (ii) If  $|A| = 0$  and  $(adj A) B \neq 0$ , then the system  $AX=B$  is inconsistent and has no solution.
- (iii) If  $|A| = 0$  and  $(adj A) B = 0$  then system is consistent and has infinitely many solutions.

### VERY SHORT ANSWER TYPE QUESTIONS (1 Mark)

1. If  $[1 \ x \ 1] \begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 3 & 2 \end{bmatrix} \begin{bmatrix} x \\ 1 \\ -2 \end{bmatrix} = 0$ , then What is the value of  $x$ ?
2. For what value of  $\lambda$ , the matrix  $A$  is a singular matrix where

$$A = \begin{bmatrix} 1 & 3 & \lambda + 2 \\ 2 & 4 & 8 \\ 3 & 5 & 10 \end{bmatrix}$$

3. Find the value of  $A^2$ , if

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

4. If  $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$  and  $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$ , then find the value of  $\alpha$  and  $\beta$ .
5. If  $A$  is a square matrix such that  $A^2 = I$ , then write the value of  $(A - I)^3 + (A + I)^3 - 7A$  in simplest form.
6. Write the value of  $\Delta$ , if

$$\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$$

7. If  $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$ , find the value of  $x+y$ .

8. If  $A$  is a  $3 \times 3$  matrix,  $|A| \neq 0$  and  $|3A| = K|A|$ , then write the value of  $K$ .

9. If  $A = \begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$  is a symmetric matrix, then write the value of  $x$ .

10. Matrix  $A = \begin{bmatrix} 0 & 2a & -2 \\ 3 & 1 & 3 \\ 3b & 3 & -1 \end{bmatrix}$  is given to be symmetric, find the value of  $a$  and  $b$ .

11. For any  $2 \times 2$  matrix  $A$ , if  $A(\text{adjoint } A) = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ , then find  $|A|$ .

12. Find  $X$ , if  $A + X = I$ , where

$$A = \begin{bmatrix} 1 & 4 & -1 \\ 3 & 4 & 7 \\ 5 & 1 & 6 \end{bmatrix}$$

13. If  $U = [2 \quad -3 \quad 4]$ ,  $V = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ ,  $X = [0 \quad 2 \quad 3]$  and  $Y = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ , then find  $UV + XY$ .

14. If  $\begin{bmatrix} 2 & -3 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -9 \\ 16 & 15 \end{bmatrix}$

write the equation after applying elementary column transformation  
 $C_2 \rightarrow C_2 + 2C_1$

15. If  $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ , then find the value of  $A^3$ .

16. Find the value of  $a_{23} + a_{32}$  in the matrix

$$A = [a_{ij}]_{3 \times 3} \text{ where } a_{ij} = \begin{cases} |2i - j| & \text{if } i > j \\ -i + 2j + 3 & \text{if } i < j \end{cases}$$

17. If  $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ , then find  $|A^2|$ .

18. For what value of  $x$ , is the matrix

$$A = \begin{bmatrix} 0 & 1 & -2 \\ -1 & x & -3 \\ 2 & 3 & 0 \end{bmatrix} \text{ a skew-symmetric matrix}$$

19. If  $A = \begin{bmatrix} \sin 15^\circ & \cos 15^\circ \\ -\sin 75^\circ & \cos 75^\circ \end{bmatrix}$ , then evaluate  $|A|$ .
20. If  $A$  is a square matrix, expressed as  $A = X + Y$  where  $X$  is symmetric and  $Y$  is skew-symmetric, then write the values of  $X$  and  $Y$ .
21. Write a matrix of order  $3 \times 3$  which is both symmetric and skew-symmetric matrix.
22. What positive value of  $x$  makes the following pair of determinants equal?

$$\begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}, \quad \begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix}$$

23.  $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$ , find the value of  $5A_{31} + 3A_{32} + 8A_{33}$ .

24. If  $A = \begin{bmatrix} 2 & 1 \\ 7 & 5 \end{bmatrix}$ , find  $|A (adj A)|$

25. Find the minimum value of.  $2 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 + \sin \theta & 1 \\ 1 & 1 & 1 + \cos \theta \end{vmatrix}$

26. If  $A$  and  $B$  are square matrices of order 3 and  $|A| = 5$  and  $|B| = 3$ , then find the value of  $|3AB|$ .

27. Evaluate  $\begin{vmatrix} 3 + 2i & -6i \\ 2i & 3 - 2i \end{vmatrix}$ ,  $i = \sqrt{-1}$

28. Without expanding, find the value of  $\begin{vmatrix} \operatorname{cosec}^2 \theta & \cot^2 \theta & 1 \\ \cot^2 \theta & \operatorname{cosec}^2 \theta & -1 \\ 42 & 40 & 2 \end{vmatrix}$

29. Using determinants, find the equation of line passing through  $(0, 3)$  and  $(1, 1)$ .

30. If  $A$  be any square matrix of order  $3 \times 3$  and  $|A| = 5$ , then find the value of  $|\text{adj}(\text{adj}A)|$
31. What is the number of all possible matrices of order  $2 \times 3$  with each entry 0, 1 or 2.
32. Given a square matrix  $A$  of order  $3 \times 3$  such that  $|A|=12$ , find the value of  $|A \text{adj} A|$
33. If  $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$  find  $|(A^{-1})^{-1}|$
34. If  $A = \begin{bmatrix} -1 & 2 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$  find  $|AB|$
35. Find  $|A(\text{adjoint } A)|$  and  $|\text{adjoint } A|$ , if  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

#### SHORT ANSWER TYPE QUESTIONS (2 MARKS)

36. Construct a matrix of order  $2 \times 3$ , whose elements are given by
- (a)  $a_{ij} = \frac{(i-2j)^2}{2}$       (b)  $a_{ij} = \frac{|-2i+j|}{3}$
37. If  $A(x_1, y_1)$ ,  $B(x_2, y_2)$  and  $C(x_3, y_3)$  are vertices of an equilateral triangle with each side equal to  $a$  units, then prove that

$$\begin{bmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{bmatrix}^2 = 3a^4$$

38. Prove that the diagonal elements of a skew-symmetric matrix are all zero.

39. If  $2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$

Find the value of  $x - y$

40. If A and B are skew symmetric matrices of the same order prove that  $AB + BA$  is symmetric matrix.

41. Without expanding prove that  $\begin{bmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{bmatrix} = 0$

42. Evaluate  $\begin{bmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{bmatrix}$

#### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

43. If  $\begin{vmatrix} a & y & z \\ x & b & z \\ x & y & c \end{vmatrix} = 0$ , then prove that  $\frac{a}{a-x} + \frac{b}{b-y} + \frac{c}{c-z} = 2$

44. If  $a \neq b \neq c$ , find the value of  $x$  which satisfies the equation

$$\begin{vmatrix} 0 & x-a & x-b \\ x+a & 0 & x-c \\ x+b & x+c & 0 \end{vmatrix} = 0$$

45. Using properties of determinants, show that

$$\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 0$$

46. Find the value of  $\begin{vmatrix} \sqrt{13} + \sqrt{3} & 2\sqrt{5} & \sqrt{5} \\ \sqrt{15} + \sqrt{26} & 5 & \sqrt{10} \\ 3 + \sqrt{65} & \sqrt{15} & 5 \end{vmatrix}$

47. If  $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ , show that  $A^2 - 12A - I = 0$ . Hence find  $A^{-1}$ .



48. Find the matrix  $X$  so that  $X \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 2 & 0 \end{bmatrix}$
49. If  $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , verify that  $A^2 - 4A - 5I = 0$ .
50. Using elementary transformations find the inverse of the matrix
- $$A = \begin{bmatrix} 2 & 1 \\ 4 & 7 \end{bmatrix}$$
51. If  $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$  and  $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ -\frac{3}{34} & \frac{2}{17} \end{bmatrix}$ , then find the value of  $x$ .
52. If  $A = \begin{bmatrix} 2 & -3 \\ 0 & 1 \end{bmatrix}$ , find  $B$ , such that  $4A^{-1} + B = A^2$
53. If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ ,  $10B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$  and  $B = A^{-1}$ , then find the value of  $\alpha$ .
54. Find the value of  $X$ , such that  $A^2 - 5A + 4I + X = 0$ , if  $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$
55. If  $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & -1 & 4 \\ -2 & 2 & 1 \end{bmatrix}$ , find  $(A')^{-1}$
56. The monthly incomes of Mohan and Sohan are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves ₹ 15000/- per month, find their monthly incomes and expenditures using matrices.

57. If  $A = \begin{bmatrix} 0 & -1 & 2 \\ 4 & 3 & -4 \end{bmatrix}$  and  $B = \begin{bmatrix} 4 & 0 \\ 1 & 3 \\ 2 & 6 \end{bmatrix}$ , then verify that  $(AB)' = B'A'$

58. If  $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  and  $x^2 = -1$

Then show that  $(A + B)^2 = A^2 + B^2$

59. Prove that  $aI + bA + cA^2 = A^3$ , if  $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & b & c \end{bmatrix}$

60. If  $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$ , then find  $A^3$ .

61. If  $A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$  and  $(A + B)^2 = A^2 + B^2 + 2AB$ , find  $a$  and  $b$ .

62. If  $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ , then find the value of  $a$ ,  $b$  and  $c$ . Such that  $A^T A = I$

63. If  $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$ , then prove that  $A^n = \begin{bmatrix} a^n & b(\frac{a^n-1}{a-1}) \\ 0 & 1 \end{bmatrix}$ , for all  $n \in N$ .

64. If  $A = \begin{bmatrix} 1! & 2! & 3! \\ 2! & 3! & 4! \\ 3! & 4! & 5! \end{bmatrix}$ , then find  $A^{-1}$  and hence prove that  $A^2 - 4A - 5I = 0$ .

65. Find the value of  $k$ , if:  $\begin{vmatrix} a+b & b+c & c+a \\ b+c & c+a & a+b \\ c+a & a+b & b+c \end{vmatrix} = k \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$

66. If  $x, y$  and  $z \in \mathbb{R}$ , and

$$\Delta = \begin{vmatrix} x & x+y & x+y+z \\ 2x & 5x+2y & 7x+5y+2z \\ 3x & 7x+3y & 9x+7y+3z \end{vmatrix} = -16, \text{ then find value of } x.$$

67. Find the value of 'k' if  $\begin{vmatrix} 1 & a^2 & a^4 \\ 1 & b^2 & b^4 \\ 1 & c^2 & c^4 \end{vmatrix} = k \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

Using properties of determinants, prove the following (Q. 68 to 74)

68.  $\begin{vmatrix} 1 & a & a^2 - bc \\ 1 & b & b^2 - ac \\ 1 & c & c^2 - ab \end{vmatrix} = 0$

69.  $\begin{vmatrix} 1 & a^2 + bc & a^3 \\ 1 & b^2 + ac & b^3 \\ 1 & c^2 + ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2 + b^2 + c^2)$

70.  $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix} = 3(a+b+c)(ab+bc+ca)$

71.  $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$

72.  $\begin{vmatrix} a^2 & bc & c^2 + ac \\ a^2 + ab & b^2 & ac \\ ab & b^2 + bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

73.  $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 2(3abc - a^3 - b^3 - c^3)$

74. 
$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

75. Given  $A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -2 & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}$ . Find the product AB and also find  $(AB)^{-1}$

78. Using properties of determinants, solve for x:

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

79. If  $\begin{vmatrix} x+a & a^2 & a^3 \\ x+b & b^2 & b^3 \\ x+c & c^2 & c^3 \end{vmatrix} = 0$  and  $a \neq b \neq c$  then find the value of x.

80. Express the following matrix as the sum of symmetric and skew-symmetric matrices and verify your result.

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

81. If  $x = -4$  is a root of a  $\Delta = \begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$ , then find the other two roots.

### LONG ANSWER TYPE QUESTIONS (6 MARKS)

82. Prove that  $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ zx-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$  is divisible by  $(x+y+z)$  and hence find the quotient.

83. Using elementary transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

84. Using matrix method, solve the system of linear equations  
 $x - 2y = 10, 2x - y - z = 8$  and  $-2y + z = 7$
85. Find  $A^{-1}$  if  $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$  and show that  $A^{-1} = \frac{A^2 - 3I}{2}$
86. Find the matrix  $x$  for which  $\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} \times \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$
87. Let  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$  and  $f(x) = x^2 - 4x + 7$ , then show that  $f(A) = 0$ ,  
 using this result find  $A^5$ .
88. If  $a + b + c = 0$  and  $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix} = 0$ , then show that either  
 $x = 0$  or  $x = \pm \sqrt{\frac{3}{2}(a^2 + b^2 + c^2)}$
89. If  $A + B + C = \pi$ , then find the value of  
 $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$
90. If  $\Delta = \begin{vmatrix} (x-2)^2 & (x-1)^2 & x^2 \\ (x-1)^2 & x^2 & (x+1)^2 \\ 2 & (x+1)^2 & (x+2)^2 \end{vmatrix}$  prove that  $\Delta$  is negative.
91. Using properties of determinants prove that:  
 $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$
92. Prove that:  $\begin{vmatrix} a & a+c & a-b \\ b-c & b & b+a \\ c+b & c-a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

93. If  $a, b, c$  are  $p^{\text{th}}, q^{\text{th}}$  and  $r^{\text{th}}$  terms respectively of a G.P. Prove that

$$\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0$$

94. Prove that  $(x-2)(x-1)$  is factor of  $\begin{vmatrix} 1 & 1 & x \\ \beta + 1 & \beta + 1 & \beta + x \\ 3 & x + 1 & x + 2 \end{vmatrix}$  and hence find the quotient.

95. Prove that:

$$\begin{vmatrix} -a(b^2 + c^2 - a^2) & 2b^3 & 2c^3 \\ 2a^3 & -b(c^2 + a^2 - b^2) & 2c^3 \\ 2a^3 & 2b^3 & -c(a^2 + b^2 - c^2) \end{vmatrix} = abc(a^2 + b^2 + c^2)^3$$

96. Determine the product  $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$

and use it to solve the system of equations:

$$x - y + z = 4, \quad x - 2y - 2z = 9, \quad 2x + y + 3z = 1$$

97. If  $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ , find  $A^{-1}$  and use it to solve the system of linear equations:  $x + 2y + z = 4$ ,  $-x + y + z =$ ,  $x - 3y + z = 2$

98. Solve given system of equations by matrix method:

$$\frac{2}{a} + \frac{3}{b} + \frac{4}{c} = -3, \quad \frac{5}{a} + \frac{4}{b} - \frac{6}{c} = 4, \quad \frac{3}{a} - \frac{2}{b} - \frac{2}{c} = 6$$

99. To raise money for an orphanage, students of three schools A, B and C organized an exhibition in their locality, where they sold paper bags,

scrap books and pastel sheets made by them using recycled paper, at the rate of ₹ 20, ₹ 15 and ₹ 5 per unit respectively. School A sold 25 paper bags, 12 scrap books and 34 pastel sheets. School B sold 22 paper bags, 15 scrap books and 28 pastel sheets. While school C sold 26 paper bags, 18 scrap books and 36 pastel sheets. Using matrices, find the total amount raised by each school.

100. Two cricket teams honored their players for three values, excellent batting, to the point bowling and unparalleled fielding by giving ₹ x, ₹ y and ₹ z per player respectively. The first team paid respectively 2, 2 and 1 players for the above values with a total prize money of 11 lakhs, while the second team paid respectively 1, 2 and 2 players for these values with a total prize money of ₹ 9 lakhs. If the total award money for one person each for these values amount to ₹ 6 lakhs, then express the above situation as a matrix equation and find award money per person for each value.

### ANSWERS

- |  |   |
|--|---|
| <p>1. <math>\frac{1}{2}</math></p> <p>2. <math>\lambda = 4</math></p> <p>3. <math>A^2 = I_3</math></p> <p>4. <math>\alpha = a^2 + b^2, \beta = 2ab</math></p> <p>5. A</p> <p>6. 0</p> <p>7. 3</p> <p>8. K=27</p> | <p>9. <math>X = 5</math></p> <p>10. <math>a = \frac{3}{2}, b = \frac{-2}{3}</math></p> <p>11. <math> A  = 10</math></p> <p>12. <math>X = \begin{bmatrix} 0 &amp; -4 &amp; 1 \\ -3 &amp; -3 &amp; -7 \\ -5 &amp; -1 &amp; -5 \end{bmatrix}</math></p> <p>13. [20]</p> <p>14. <math>\begin{bmatrix} 2 &amp; -3 \\ 6 &amp; 5 \end{bmatrix} \begin{bmatrix} 1 &amp; 2 \\ 2 &amp; 7 \end{bmatrix} = \begin{bmatrix} -4 &amp; -17 \\ 16 &amp; 47 \end{bmatrix}</math></p> |
|--|---|

$$15. \quad A^3 = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$16. \quad 11$$

$$17. \quad 0$$

$$18. \quad x = 0$$

$$19. \quad |A| = 1$$

$$20. \quad x = \frac{1}{2}(A + A'), \quad y = \frac{1}{2}(A - A')$$

$$21. \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$22. \quad x = \pm 4$$

$$23. \quad 0$$

$$24. \quad 9$$

$$25. \quad -1$$

$$26. \quad 405$$

$$27. \quad 1$$

$$28. \quad 0$$

$$29. \quad 3 - 2x$$

$$30. \quad 625$$

$$31. \quad 729$$

$$32. \quad 1728$$

$$33. \quad 11$$

$$34. \quad -11$$

$$35. \quad a^9, a^6$$

$$36. \quad (a) \begin{bmatrix} 1 & 9 & 25 \\ 2 & 2 & 2 \\ 0 & 2 & 8 \end{bmatrix}$$

$$(b) \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} \\ 1 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

$$39. \quad x - y = -7$$

$$41. \dots\dots\dots$$

$$42. \quad 4!$$

$$44. \quad x = 0$$

$$46. \quad -5\sqrt{3}(5 - \sqrt{6})$$

$$47. \quad A^{-1} = \begin{bmatrix} -7 & 3 \\ 12 & -5 \end{bmatrix}$$

$$48. \quad \begin{bmatrix} 5 & 0 \\ -\frac{6}{7} & \frac{4}{7} \end{bmatrix}$$



$$50. \quad \frac{1}{10} \begin{bmatrix} 7 & -1 \\ -4 & 2 \end{bmatrix}$$

$$51. \quad x = 4$$

$$52. \quad B = \begin{bmatrix} 2 & -15 \\ 0 & -3 \end{bmatrix}$$

$$53. \quad \alpha = 5$$

$$54. \quad X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & -4 & -2 \end{bmatrix}$$

$$55. \quad \begin{bmatrix} -9 & -8 & -2 \\ 8 & 7 & 2 \\ -5 & -4 & -1 \end{bmatrix}$$

$$56. \quad \text{Incomes: Rs 90,000/- and Rs 1,20,000/-}$$

$$\text{Expenditures: Rs 75,000/- and Rs 10,5000/-}$$

$$79. \quad x = \frac{-abc}{ab+bc+ca}$$

$$80. \quad A = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & -5 & -3 \\ 5 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix}$$

$$81. \quad x = 1, 3$$

$$82. \quad (x + y + z)(xy + yz + zx - x^2 - y^2 - z^2)^2$$

$$60. \quad \begin{bmatrix} \cos 8\theta & \sin 8\theta \\ -\sin 8\theta & \cos 8\theta \end{bmatrix}$$

$$61. \quad a = -1, b = -2$$

$$62. \quad a = \pm \frac{1}{\sqrt{2}}; \quad b = \pm \frac{1}{\sqrt{6}};$$

$$c = \pm \frac{1}{\sqrt{3}}$$

$$63. \quad ..$$

$$64. \quad ..$$

$$65. \quad K = 2$$

$$66. \quad x = 2$$

$$67. \quad K = (a + b)(b + c)(c + a)$$

$$75. \quad AB = \begin{bmatrix} 1 & 2 \\ -2 & 2 \end{bmatrix}$$

$$(AB)^{-1} = \frac{1}{6} \begin{bmatrix} 2 & -2 \\ 2 & -1 \end{bmatrix}$$

$$78. \quad x = 4$$

$$83. \quad A^{-1} = \begin{bmatrix} 0 & 2/3 & -1/3 \\ 1 & -13/3 & 2/3 \\ -1 & 4 & 0 \end{bmatrix}$$

$$84. \quad x = 0; y = -5; z = -3$$

$$86. \quad x = \begin{bmatrix} -16 & 3 \\ 24 & -5 \end{bmatrix}$$

$$89. \quad 0$$

$$94. \quad \beta$$

$$85. \quad A^{-1} = \frac{1}{2} \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$$87. \quad \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$

$$96. \quad \text{Product} = 8I$$

$$x = 3, y = -2, z = -1$$

$$97. \quad A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \quad x = \frac{9}{5}, y = \frac{2}{5}, z = \frac{7}{5}$$

$$98. \quad a = 1, b = -1, c = -2$$

$$99. \quad \text{School A} = ₹ 850$$

$$\text{School B} = ₹ 805$$

$$\text{School C} = ₹ 970$$

$$100. \quad \text{Excellent batting: 3 lakhs}$$

$$\text{point bowling: 2 lakhs}$$

$$\text{fielding: 1 lakh}$$

## CHAPTER 5

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### CONTINUITY AND DIFFERENTIABILITY

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#### POINTS TO REMEMBER

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- A function  $f(x)$  is said to be continuous at  $x = c$  if  $\lim_{x \rightarrow c} f(x) = f(c)$   
*i.e.*,  $\lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^+} f(x) = f(c)$
- $f(x)$  is continuous in  $(a, b)$  iff it is continuous at  $x = c \forall c \in (a, b)$ .
- $f(x)$  is continuous in  $[a, b]$  iff
  - (i)  $f(x)$  is continuous in  $(a, b)$
  - (ii)  $\lim_{x \rightarrow a^+} f(x) = f(a)$
  - (iii)  $\lim_{x \rightarrow b^-} f(x) = f(b)$
- Modulus functions are Continuous on  $\mathbb{R}$
- Trigonometric functions are continuous in their respective domains.
- Exponential function is continuous on  $\mathbb{R}$
- Every polynomial function is continuous on  $\mathbb{R}$ .
- Greatest integer function is continuous on all non-integral real numbers
- If  $f(x)$  and  $g(x)$  are two continuous functions at  $x = a$  and if  $c \in \mathbb{R}$  then
  - (i)  $f(x) \pm g(x)$  are also continuous functions at  $x = a$ .
  - (ii)  $g(x) \cdot f(x), f(x) + c, cf(x), |f(x)|$  are also continuous at  $x = a$ .
  - (iii)  $\frac{f(x)}{g(x)}$  is continuous at  $x = a$  provided  $g(a) \neq 0$ .
- A function  $f(x)$  is derivable or differentiable at  $x = c$  in its domain iff

$$\lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c}, \text{ and is finite}$$

The value of above limit is denoted by  $f'(c)$  and is called the derivative of  $f(x)$  at  $x = c$ .

$$\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$$

- $\frac{d}{dx}(u \cdot v) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$  (Product Rule)

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \cdot \frac{du}{dx} - u \cdot \frac{dv}{dx}}{v^2}$  (Quotient Rule)

- If  $y = f(u)$  and  $u = g(t)$  then  $\frac{dy}{dt} = \frac{dy}{du} \times \frac{du}{dt} = f'(u)g'(t)$  (Chain Rule)

- If  $y = f(u)$ ,  $x = g(u)$  then,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \frac{f'(u)}{g'(u)}$$

- **Rolle's theorem:** If  $f(x)$  is continuous in  $[a, b]$  derivable in  $(a, b)$  and  $f(a) = f(b)$  then there exists at least one real number  $c \in (a, b)$  such that  $f'(c) = 0$ .

- **Mean Value Theorem :** If  $f(x)$  is continuous in  $[a, b]$  and derivable in  $(a, b)$  then there exists at least one real number  $c \in (a, b)$  such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Every differentiable function is continuous but its converse is not true.

### VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

1. Let  $f(x) = \sin x \cos x$ . Write down the set of points of discontinuity of  $f(x)$ .
2. Given  $f(x) = \frac{1}{x+2}$ , write down the set of points of discontinuity of  $f(x)$ .

3. For what value(s) of  $n$ , the function  $f(x) = \begin{cases} x^n \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  is continuous at  $x = 0$ .
4. Write the set of points of continuity of  $f(x) = |x - 1| + |x + 1|$
5. Write the number of points of discontinuity of  $f(x) = [x]$  in  $[3, 7]$ .
6. If  $y = e^{\log(x^5)}$ , find  $\frac{dy}{dx}$ .
7. If  $f(x) = x^2 g(x)$  and  $g(1) = 6$ ,  $g'(x) = 3$ , find the value of  $f'(1)$ .
8. If  $y = a \sin t$ ,  $x = a \cos t$  then find  $\frac{dy}{dx}$

### VERY SHORT ANSWER TYPE QUESTIONS [2 Marks]

9. Differentiate  $\sin(x^2)$  w. r. t.  $e^{\sin x}$
10.  $y = x^y$  then find  $\frac{dy}{dx}$
11. If  $y = x^x + x^3 + 3^x + 3^3$ , find  $\frac{dy}{dx}$
12. If  $y = 2\sin^{-1}(\cos x) + 5 \operatorname{cosec}^{-1}(\sec x)$ . Find  $\frac{dy}{dx}$
13. If  $y = e^{[\log(x+1) - \log x]}$  find  $\frac{dy}{dx}$
14. Differentiate  $\sin^{-1}[x \sqrt{x}]$  w. r. t.  $x$ .

### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

15. Examine the continuity of the following functions at the indicated points.

$$(I) \quad f(x) = \begin{cases} x^2 \cos\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases} \quad \text{at } x = 0$$

$$(II) \quad f(x) = \begin{cases} x - [x] & x \neq 1 \\ 0 & x = 1 \end{cases} \quad \text{at } x = 1$$

$$(III) \quad f(x) = \begin{cases} \frac{e^x - 1}{e^x + 1} & , x \neq 0 \\ 0 & , x = 0 \end{cases} \quad \text{at } x = 0$$

$$(IV) \quad f(x) = \begin{cases} \frac{x - \cos(\sin^{-1}x)}{1 - \tan(\sin^{-1}x)} & x \neq \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & x = \frac{1}{\sqrt{2}} \end{cases} \quad \text{at } x = \frac{1}{\sqrt{2}}$$

16. For what values of constant K, the following functions are continuous at the indicated points.

$$(I) \quad f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} & x < 0 \\ \frac{2x+1}{x-1} & x > 0 \end{cases} \quad \text{at } x = 0$$

$$(II) \quad f(x) = \begin{cases} \frac{e^x - 1}{\log(1+2x)} & x \neq 0 \\ K & x = 0 \end{cases} \quad \text{at } x = 0$$

$$(III) \quad f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & x < 0 \\ K & x = 0 \\ \frac{\sqrt{x}}{\sqrt{16 + \sqrt{x}} - 4} & x > 0 \end{cases} \quad \text{at } x = 0$$

17. For what values a and b

$$f(x) = \begin{cases} \frac{x+2}{|x+2|} + a & \text{if } x < -2 \\ a + b & \text{if } x = -2 \\ \frac{x+2}{|x+2|} + 2b & \text{if } x > -2 \end{cases}$$

Is continuous at  $x = -2$

18. Find the values of a, b and c for which the function

$$f(x) = \begin{cases} \frac{\sin[(a+1)x] + \sin x}{x} & x < 0 \\ c & x = 0 \\ \frac{\sqrt{x+bx^2} - \sqrt{x}}{bx^{3/2}} & x > 0 \end{cases}$$

Is continuous at  $x = 0$

19.  $f(x) = \begin{cases} [x] + [-x] & x \neq 0 \\ \lambda & x = 0 \end{cases}$

Find the value of  $\lambda$ ,  $f$  is continuous at  $x = 0$  ?

20. Let  $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} ; & x < \frac{\pi}{2} \\ a & x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} ; & x > \frac{\pi}{2} \end{cases}$

If  $f(x)$  is continuous at  $x = \frac{\pi}{2}$ , find  $a$  and  $b$ .

21. If  $f(x) = \begin{cases} x^3 + 3x + a & x \leq 1 \\ bx + 2 & x > 1 \end{cases}$

Is everywhere differentiable, find the value of  $a$  and  $b$ .

22. Find the relationship between  $a$  and  $b$  so that the function defined by

$$f(x) = \begin{cases} ax+1, & x \leq 3 \\ bx+3, & x > 3 \end{cases} \text{ is continuous at } x = 3.$$

23. Differentiate  $\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$  w.r.t  $\cos^{-1}(2x\sqrt{1-x^2})$  where  $x \neq 0$ .

24. If  $y = x^{x^x}$ , then find  $\frac{dy}{dx}$ .

25. Differentiate  $(x \cos x)^x + (x \sin x)^{\frac{1}{x}}$  w.r.t.  $x$ .

26. If  $(x + y)^{m+n} = x^m \cdot y^n$  then prove that  $\frac{dy}{dx} = \frac{y}{x}$
27. If  $(x - y) \cdot e^{\frac{x}{x-y}} = a$ , prove that  $y \left( \frac{dy}{dx} \right) + x = 2y$
28. If  $x = \tan \left( \frac{1}{a} \log y \right)$  then show that
- $$(1 + x^2) \frac{d^2 y}{dx^2} + (2x - a) \frac{dy}{dx} = 0$$
29. If  $y = x \log \left( \frac{x}{a+bx} \right)$  prove that  $x^3 \frac{d^2 y}{dx^2} = \left( x \frac{dy}{dx} - y \right)^2$ .
30. Differentiate  $\sin^{-1} \left[ \frac{2^{x+1} \cdot 3^x}{1+(36)^x} \right]$  w.r.t  $x$ .
31. If  $\sqrt{1-x^6} + \sqrt{1-y^6} = a(x^3 - y^3)$ , prove that
- $$\frac{dy}{dx} = \frac{x^2}{y^2} \sqrt{\frac{1-y^6}{1-x^6}}, \text{ Where } -1 < x < 1 \text{ and } -1 < y < 1 \text{ [HINT: put } x^3 \sin A \text{ and } y^3 \sin B]$$
32. If  $f(x) = \sqrt{x^2 + 1}$ ,  $g(x) = \frac{x+1}{x^2+1}$  and  $h(x) = 2x - 3$  find  $f'[h'(g'(x))]$ .
33. If  $x = \sec \theta - \cos \theta$  and  $y = \sec^n \theta - \cos^n \theta$ , then prove that  $\frac{dy}{dx} = n \sqrt{\frac{y^2+4}{x^2+4}}$
34. If  $x^y + y^x + x^x = m^n$ , then find the value of  $\frac{dy}{dx}$ .
35. If  $x = a \cos^3 \theta$ ,  $y = a \sin^3 \theta$  then find  $\frac{d^2 y}{dx^2}$  at  $x = \frac{\pi}{6}$
36. If  $y = \tan^{-1} \left[ \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{\sqrt{1+\sin x} + \sqrt{1-\sin x}} \right]$  where  $0 < x < \frac{\pi}{2}$  find  $\frac{dy}{dx}$



37. If  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , then show that  $\frac{d^2y}{dx^2} = -\frac{b^4}{a^2y^3}$ .

38. Verify Rolle's theorem for the function

$$f(x) = e^x \sin 2x \left[0, \frac{\pi}{2}\right]$$

39. Verify mean value theorem for the function

$$f(x) = \sqrt{x^2 - 4} [2, 4]$$

40. If the Rolle's theorem holds for the function

$$f(x) = x^3 + bx^2 + ax + 5 \text{ on } [1, 3] \text{ with } c = \left(2 + \frac{1}{\sqrt{3}}\right)$$

Find the value of  $a$  and  $b$ .

41. If  $y = [x + \sqrt{x^2 + 1}]^m$ , show that  $(x^2 + 1)y_2 + xy_1 - m^2y = 0$ .

42. Differentiate  $\sin^{-1} \left[ \frac{3x+4\sqrt{1-x^2}}{5} \right]$  w.r.t.  $x$ .

43. If  $x^y = e^{x-y}$ , prove that  $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$

44. If  $f: [-5, 5] \rightarrow R$  is a differentiable function and  $f'(x)$  does not vanish anywhere, then prove that  $f(-5) \neq f(5)$ .

45. If  $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$  then prove that  $(x^2 - 1)y_2 + xy_1 = m^2y$ .

## ANSWERS

- |  |   |
|--|---|
| 1. $\{ \}$                                       | 9. $\frac{2x \cos(x^2)}{\cos x e^{\sin x}}$ |
| 2. $\{-2, -\frac{5}{2}\}$                        | 10. $\frac{y^2}{x[1-y \log x]}$             |
| 3. $n > 1$                                       | 11. $x^x [1 + \log x] + 3x^2 + 3^x \log 3$  |
| 4. $R$   |   |
| 5. Points of discontinuity of $f(x)$ are 4,5,6,7 |   |

Note- At  $x = 3$ ,  $f(x) = [x]$  is continuous because  $\lim_{x \rightarrow 3^+} f(x) = 3 = f(3)$

- |              |  |
|--------------|--|
| 6. $5x^4$    | 12. $-7$                                 |
| 7. $15$      | 13. $-\frac{1}{x^2}$                     |
| 8. $-\cot t$ | 14. $\frac{3}{2} \sqrt{\frac{x}{1-x^3}}$ |

- 15 (I) Continuous  
 (II) Discontinuous  
 (III) Not Continuous at  $x = 0$   
 (IV) Continuous
- 16 (I)  $K = -1$   
 (II)  $K = \frac{1}{2}$   
 (III)  $K = 8$

$$17 \quad a = 0, \quad b = -1$$

$$18 \quad a = \frac{-3}{2}, \quad b = R - \{0\}, \quad c = \frac{1}{2}$$

$$19 \quad \lambda = -1$$

$$20 \quad a = \frac{1}{2}, \quad b = 4$$

$$21 \quad a = 3, \quad b = 5$$

$$22 \quad 3a - 3b = 2$$

$$23 \quad -\frac{1}{2}$$

$$24 \quad x^x x^{x^x} \left\{ (1 + \log x) \log x + \frac{1}{x} \right\}$$

$$25 \quad (x \cos x)^x [1 - x \tan x + (\log x \cos x)] + (x \sin x)^{1/x} \left[ \frac{1+x \cot x - \log(x \sin)^x}{x^2} \right]$$

$$30. \quad \left[ \frac{2^{x+1} 3^x}{1+(36)^x} \right] \log 6 \quad [\text{Hint: } \tan \theta = 6^x]$$

$$32. \quad \frac{2}{\sqrt{5}}$$

$$34. \quad \frac{dy}{dx} = \frac{x^x(1+\log x) + yx^{y-1} - y^x \log y}{x^y \log x + xy^{x-1}}$$

$$35. \quad \frac{32}{27a}$$

$$36. \quad -1/2$$

$$40. \quad a = 11, \quad b = -6$$

$$42. \quad \frac{1}{\sqrt{1-x^2}}$$

## CHAPTER 6

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### APPLICATION OF DERIVATIVES

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#### IMPORTANT POINTS TO REMEMBER

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- **Rate of change:** Let  $y = f(x)$  be a function then the rate of change of  $y$  with respect to  $x$  is given by  $\frac{dy}{dx} = f'(x)$  where a quantity  $y$  varies with another quantity  $x$ .

$$\left\{ \frac{dy}{dx} \right\}_{x=x_1} \text{ or } f'(x_1) \text{ represents the rate of change of } y \text{ w.r.t. } x \text{ at } x = x_1.$$

- **Increasing and Decreasing Function**

Let  $f$  be a real-valued function and let  $I$  be any interval in the domain of  $f$ . Then  $f$  is said to be

- a) Strictly increasing on  $I$ , if for all  $x_1, x_2 \in I$ , we have

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

- b) Increasing on  $I$ , if for all  $x_1, x_2 \in I$ , we have

$$x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2)$$

- c) Strictly decreasing in  $I$ , if for all  $x_1, x_2 \in I$ , we have

$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$$

- d) Decreasing on  $I$ , if for all  $x_1, x_2 \in I$ , we have

$$x_1 < x_2 \Rightarrow f(x_1) \geq f(x_2)$$

- **Derivative Test:** Let  $f$  be a continuous function on  $[a, b]$  and differentiable on  $(a, b)$ . Then

- a)  $f$  is strictly increasing on  $[a, b]$  if  $f'(x) > 0$  for each  $x \in (a, b)$ .
- b)  $f$  is increasing on  $[a, b]$  if  $f'(x) \geq 0$  for each  $x \in (a, b)$ .
- c)  $f$  is strictly decreasing on  $[a, b]$  if  $f'(x) < 0$  for each  $x \in (a, b)$ .
- d)  $f$  is decreasing on  $[a, b]$  if  $f'(x) \leq 0$  for each  $x \in (a, b)$ .
- e)  $f$  is constant function on  $[a, b]$  if  $f'(x) = 0$  for each  $x \in (a, b)$ .

- **Tangents and Normals**

- a) Equation of the tangent to the curve  $y = f(x)$  at  $(x_1, y_1)$  is

$$y - y_1 = \left[ \frac{dy}{dx} \right]_{(x_1, y_1)} (x - x_1)$$

- b) Equation of the normal to the curve  $y = f(x)$  at  $(x_1, y_1)$  is

$$y - y_1 = \frac{-1}{\left[ \frac{dy}{dx} \right]_{(x_1, y_1)}} (x - x_1)$$

- **Maxima and Minima**

- a) Let  $f$  be a function and  $c$  be a point in the domain of  $f$  such that either  $f'(c) = 0$  or  $f'(c)$  does not exist are called critical points.

- b) **First Derivative Test:** Let  $f$  be a function defined on an open interval

I. Let  $f$  be continuous at a critical point  $c$  in I. Then

- i.  $f'(x)$  changes sign from positive to negative as  $x$  increases through  $c$ , then  $c$  is called the point of the local maxima.
- ii.  $f'(x)$  changes sign from negative to positive as  $x$  increases through  $c$ , then  $c$  is a point of *local minima*.

- iii.  $f'(x)$  does not change sign as  $x$  increases through  $c$ , then  $c$  is neither a point of *local maxima* nor a point of *local minima*. Such a point is called a point of *inflexion*.

**c) Second Derivative Test :** Let  $f$  be a function defined on an interval  $I$  and let  $c \in I$ . Let  $f$  be twice differentiable at  $c$ . Then

- i.  $x = c$  is a point of local maxima if  $f'(c) = 0$  and  $f''(c) < 0$ . The value  $f(c)$  is local maximum value of  $f$ .
- ii.  $x = c$  is a point of local minima if  $f'(c) = 0$  and  $f''(c) > 0$ . The value  $f(c)$  is local minimum value of  $f$ .
- iii. The test fails if  $f'(c) = 0$  and  $f''(c) = 0$ .

### Very Short Answer Type Questions (1 Mark)

- Find the angle  $\theta$ , where  $0 < \theta < \frac{\pi}{2}$ , which increases twice as fast as its sine.
- Find the slope of the normal to the curve  $x = a \cos^3 \theta$  and  $y = a \sin^3 \theta$  at  $\theta = \frac{\pi}{4}$
- A balloon which always remains spherical has a variable radius. Find the rate at which its volume is increasing with respect to its radius when the radius is 7cm.
- Write the interval for which the function  $f(x) = \cos x, 0 \leq x \leq 2\pi$  is decreasing

5. For what values of  $x$  is the rate of increasing of  $x^3 - 5x^2 + 5x + 8$  is twice the rate of increase of  $x$  ?
6. Find the point on the curve  $y = x^2 - 2x + 3$  where the tangent is parallel to x-axis.
7. Write the maximum value of  $f(x) = \frac{\log x}{x}$ , if it exists.
8. Find the least value of  $f(x) = ax + \frac{b}{x}$ , where  $a > 0$ ,  $b > 0$  and  $x > 0$ .
9. Find the interval in which the function  $f(x) = x - e^x + \tan(\frac{2\pi}{7})$  increases.
10. For the curve  $y = (2x + 1)^3$  find the rate of change of slope of the tangent.
11. Find the value of  $a$  for which the function  $f(x) = x^2 - 2ax + 6$ ,  $x > 0$  is strictly increasing.

**VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS)**

12. Find the co-ordinates of the point on the curve  $y^2 = 3 - 4x$ , where tangent is parallel to the line  $2x + y - 2 = 0$
13. The sum of the two numbers is 8, what will be the maximum value of the sum of their reciprocals.
14. Find the maximum value of  $f(x) = 2x^3 - 24x + 107$  in the interval  $[1, 3]$
15. If the rate of change of Area of a circle is equal to the rate of change its diameter. Find the radius of the circle.
16. The sides of an equilateral triangle are increasing at the rate of 2 cm/s. Find the rate at which the area increases, when side is 10 cm.

### Rate of Change (4 Mark Questions)

17. In a competition, a brave child tries to inflate a huge spherical balloon bearing slogans against child labour at the rate of  $900 \text{ cm}^3$  of gas per second. Find the rate at which the radius of the balloon is increasing, when its radius is 15 cm. Why is child labour not good for society?
18. An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of  $\frac{3}{2}$  c.c. per minute. Find the rate at which the level of water in the cone is rising when the depth is 4 cm.
19. The volume of a cube is increasing at a constant rate. Prove that the increase in its surface area varies inversely as the length of an edge of the cube.
20. A kite is moving horizontally at a height of 151.5 meters. If the speed of the kite is 10m/sec, how fast is the string being let out when the kite is 250 m away from the boy who is flying the kite ? The height of the boy is 1.5 m.
21. A swimming pool is to be drained for cleaning. If  $L$  represents the number of litres of water in the pool  $t$  seconds after the pool has been plugged off to drain and  $L = 200(10 - t)^2$ . How fast is the water running out at the end of 5 sec. and what is the average rate at which the water flows out during the first 5 seconds?
22. A man 2m tall, walk at a uniform speed of 6km/h away from a lamp post 6m high. Find the rate at which the length of his shadow increases.
23. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi- vertical angle is  $\tan^{-1}(0.5)$ . water is poured into it at a constant rate of  $5\text{m}^3/\text{h}$ . Find the



rate at which the level of the water is rising at the instant, when the depth of Water in the tank is 4m.

24. A spherical ball of salt is dissolving in water in such a manner that the rate of decrease of the volume at any instant is proportional to the surface area. Prove that the radius is decreasing at a constant rate.
25. A conical vessel whose height is 10 meters and the radius of whose base is half that of the height is being filled with a liquid at a uniform rate of  $1.5m^3/min$ . find the rate at which the level of the water in the vessel is rising when it is 3m below the top of the vessel.
26. Let  $x$  and  $y$  be the sides of two squares such that  $y = x - x^2$ . Find the rate of change of area of the second square w.r.t. the area of the first square.
27. The length of a rectangle is increasing at the rate of 3.5 cm/sec. and its breadth is decreasing at the rate of 3 cm/sec. Find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm.
28. If the areas of a circle increases at a uniform rate, then prove that the perimeter varies inversely as the radius.

### Increasing and Decreasing [4 MARK QUESTIONS]

29. Show that  $f(x) = x^3 - 6x^2 + 18x + 5$  is an increasing function for all  $x \in R$ . Find its value when the rate of increase of  $f(x)$  is least.  
[Hint: Rate of increase is least when  $f'(x)$  is least.]
30. Determine whether the following function is increasing or decreasing in the given interval:  $f(x) = \cos\left(2x + \frac{\pi}{4}\right)$ ,  $\frac{3\pi}{8} \leq x \leq \frac{5\pi}{8}$ .
31. Determine for which values of  $x$ , the function  $y = x^4 - \frac{4x^3}{3}$  is increasing and for which it is decreasing.

32. Find the interval of increasing and decreasing of the function  
 $f(x) = \frac{\log x}{x}$

33. Find the interval of increasing and decreasing of the function  $f(x) = \sin x - \cos x$ ,  $0 < x < 2\pi$ .

34. Show that  $f(x) = x^2 e^{-x}$ ,  $0 \leq x \leq 2$  is increasing in the indicated interval.

35. Prove that the function  $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$  is an increasing function of  $\theta$  in  $\left[0, \frac{\pi}{2}\right]$ .

36. Find the intervals in which the following function is decreasing.

$$f(x) = x^4 - 8x^3 + 22x^2 - 24x + 21$$

37. Find the interval in which the function  $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ ,  $x > 0$  is strictly decreasing.

38. Show that the function  $f(x) = \tan^{-1}(\sin x + \cos x)$ , is strictly increasing the interval  $\left(0, \frac{\pi}{4}\right)$ .

39. Find the interval in which the function  $f(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$  is increasing or decreasing.

40. Find the interval in which the function given by

$$f(x) = \frac{3x^4}{10} - \frac{4x^3}{5} - 3x^2 + \frac{36x}{5} + 11$$

(1) strictly increasing

(2) strictly decreasing

### Tangent and Normal [4 MARK QUESTIONS]

41. Find the equation of the tangent to the curve  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$  at the point  $(\sqrt{2}a, b)$ .
42. Find the equation of the tangent to the curve  $y = x^2 - 2x + 7$  which is
- (1) Parallel to the line  $2x - y + 9 = 0$
  - (2) Perpendicular to the line  $5y - 15x = 13$
43. Find the co-ordinates of the point on the curve  $\sqrt{x} + \sqrt{y} = 4$  at which tangent is equally inclined to the axes.
44. Find a point on the parabola  $f(x) = (x - 3)^2$  where the tangent is parallel to the chord joining the points (3,0) and (4,1).
45. Find the equation of the normal to the curve  $y = e^{2x} + x^2$  at  $x = 0$ , also find the distance from origin to the line.
46. Show that the line  $\frac{x}{a} + \frac{y}{b} = 1$  touches the curve  $y = be^{-x/a}$  at the point, where the curve intersects the axis of y.
47. At what point on the circle  $x^2 + y^2 - 2x - 4y + 1 = 0$  the tangent is parallel to
- (1) X – axis
  - (2) Y – axis
48. Show that the equation of the normal at any point ' $\theta$ ' on the curve  $x = 3 \cos \theta - \cos^3 \theta$ ,  $y = 3 \sin \theta - \sin^3 \theta$  is  $4(y \cos^3 \theta - x \sin^3 \theta) = 3 \sin 4 \theta$ .
49. Show that the curves  $xy = a^2$  and  $x^2 + y^2 = 2a^2$  touch each other.
50. For the curve  $y = 5x - 2x^3$ , if x increases at the rate of 2 Units/sec. then how fast is the slope of the curve changing when  $x=3$ ?

- ### APPROXIMATION [4 MARK QUESTIONS]

64. If the radius of a circle increases from 5 cm to 5.1 cm, find the increase in area.

65. If the side of a cube be increased by 0.1%, find the corresponding increase in the volume of the cube.
66. Find the approximate value of  $f(2.01)$  where  $f(x) = x^3 - 4x + 7$ .
67. Find the approximate value of  $\frac{1}{\sqrt{25.1}}$ , using differentials.
68. The radius of a sphere shrinks from 10 cm. to 9.8 cm. Find the approximately decrease in its volume.

### Maxima and Minima (4 Mark Questions)

69. Find the maximum and minimum values of  $f(x) = \sin x + \frac{1}{2}\cos 2x$  in  $\left[0, \frac{\pi}{2}\right]$ .
70. Find the absolute maximum value and absolute minimum value of the following question  $f(x) = \left(\frac{1}{2} - x\right)^2 + x^3$  in  $[-2, 2.5]$
71. Find the maximum and minimum values of  $f(x) = x^{50} - x^{20}$  in the interval  $[0, 1]$
72. Find the absolute maximum and absolute minimum value of  $f(x) = (x - 2)\sqrt{x - 1}$  in  $[1, 9]$
73. Find the difference between the greatest and least values of the function  $f(x) = \sin 2x - x$  on  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

### Maxima and Minima (4/6 Mark Question)

74. Prove that the least perimeter of an isosceles triangle in which a circle of radius  $r$  can be inscribed is  $6\sqrt{3}r$ .
75. If the sum of length of hypotenuse and a side of a right angled triangle is given, show that area of triangle is maximum, when the angle between them is  $\frac{\pi}{3}$ .

76. Show that semi-vertical angle of a cone of maximum volume and given slant height is  $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ .
77. The sum of the surface areas of cuboids with sides  $x$ ,  $2x$  and  $\frac{x}{3}$  and a sphere is given to be constant. Prove that the sum of their volumes is minimum if  $x = 3$  radius of the sphere. Also find the minimum value of the sum of their volumes.
78. Show that the volume of the largest cone that can be inscribed in a sphere of radius  $R$  is  $\frac{8}{27}$  of the volume of the sphere.
79. Show that the cone of the greatest volume which can be inscribed in a given sphere has an altitude equal to  $\frac{2}{3}$  of the diameter of the sphere.
80. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
81. Show that the volume of the greatest cylinder which can be inscribed in a cone of height  $H$  and semi-vertical angle  $\alpha$  is  $\frac{4}{27}\pi h^3 \tan^2 \alpha$ . Also show that height of the cylinder is  $\frac{h}{3}$ .
82. Find the point on the curve  $y^2 = 4x$  which is nearest to the point  $(2,1)$ .
83. Find the shortest distance between the line  $y - x = 1$  and the curve  $x = y^2$ .
84. A wire of length 36 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces, so that the combined area of the square and the circle is minimum?
85. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius  $r$  is  $\frac{2r}{\sqrt{3}}$ .
86. Find the area of greatest rectangle that can be inscribed in an ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .

## Answers

- |     |                               |     |                           |
|-----|-------------------------------|-----|---------------------------|
| 1.  | $\frac{\pi}{3}$               | 13. | $\frac{1}{2}$             |
| 2.  | 1                             | 14. | 89                        |
| 3.  | $196\pi \frac{cm^3}{cm}$      | 15. | $\frac{1}{\pi}$ units     |
| 4.  | $[0, \pi]$                    | 16. | $10\sqrt{3} cm^2 / s$     |
| 5.  | $3, \frac{1}{3}$              | 17. | $\frac{1}{\pi} cm / s$    |
| 6.  | (1,2)                         | 18. | $\frac{3}{8\pi} cm / min$ |
| 7.  | $\frac{1}{e}$                 | 20. | 8 m/sec.                  |
| 8.  | $2\sqrt{ab}$                  | 21. | 3000 L/s                  |
| 9.  | $(-\infty, 0)$                | 22. | 3 km/h                    |
| 10. | 0                             | 23. | $\frac{35}{88} m/h$       |
| 11. | $a \leq 0$                    | 25. | $\frac{6}{49\pi} m/min.$  |
| 12. | $\left(\frac{1}{2}, 1\right)$ | 26. | $1 - 3x + 2x^2$           |
|     |                               | 27. | $8 \frac{cm^2}{sec}$      |

29. 25
30. Increasing
31. Increasing for all  $x \geq 1$   
Decreasing for all  $x \leq 1$
32. Increasing on  $(0, e)$   
Decreasing on  $[e, \infty)$
33. Increasing on  
 $\left(0, \frac{3\pi}{4}\right) \cup \left(\frac{7\pi}{4}, 2\pi\right)$   
Decreasing on  $\left[\frac{3\pi}{4}, \frac{7\pi}{4}\right]$
36.  $(-\infty, 1] \cup [2, 3]$
37.  $[1, \infty)$
39. increasing on  $[0, \infty)$   
Decreasing  $(-\infty, 0]$
40. Strictly increasing  
 $[-2, 1] \cup [3, \infty)$   
Strictly decreasing  
 $(-\infty, -2] \cup [1, 3]$
46.  $\sqrt{2}bx - ay - ab = 0$
47. (1)  $y - 2x - 3 = 0$   
(2)  $36y + 12x - 227 = 0$
43. (4, 4)
44.  $\left(\frac{7}{2}, \frac{1}{4}\right)$
45.  $2y + x - 2 = 0, \frac{2}{\sqrt{5}}$
47. (1) (1, 0) and (1, 4)  
(2) (3, 2) and (-1, 2)
50. decrease 72 units/sec.
51.  $a^2 = b^2$
53.  $2x + 3my - 3a m^4 - 2am^2 = 0$
54.  $x + y = 3, \quad y = x - 1$
55.  $\left(4, \pm \frac{8}{3}\right)$
57. 4.042
58. 20.025
59. 0.1924
60. 5.03



- |     |  |     |   |
|-----|--|-----|---|
| 61. | 7.904  | 71. | max.value=0,  |
| 62. | 2.984  |     | min.value = $\frac{-3}{5} \left[ \frac{2}{5} \right]^{2/3}$ |
| 63. | 1.004343   | 72. | ab. Max = 14 at $x = 9$                                     |
| 64. | $\pi \text{ cm}^2$                                     |     | ab. Min. = $\frac{-3}{4^{4/3}}$ at $x = \frac{5}{4}$        |
| 65. | 0.3%   | 73. | $\pi$   |
| 66. | 7.08   | 77. | $18r^3 + \frac{4}{3} \pi r^3$                               |
| 67. | 0.198  | 82. | (1, 2)  |
| 68. | $80\pi \text{ cm}^3$                                   | 83. | $\frac{3\sqrt{2}}{8}$                                       |
| 69. | max. value = $\frac{3}{4}$ , min value = $\frac{1}{2}$ | 84. | $\frac{144}{\pi+4} m, \frac{36\pi}{\pi+4} m$                |
| 70. | ab. Max. = $\frac{157}{8}$ , ab. Min. = $\frac{-7}{4}$ | 86. | 2ab sq. Units.  |

## CHAPTER 7

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## INTEGRALS

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### POINTS TO REMEMBER

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- Integration or anti derivative is the reverse process of Differentiation.
- Let  $\frac{d}{dx} F(x) = f(x)$  then we write  $\int f(x) dx = F(x) + c$ .
- These integrals are called indefinite integrals and c is called constant of integration.
- From geometrical point of view, an indefinite integral is the collection of family of curves each of which is obtained by translating one of the curves parallel to itself upwards or downwards along y-axis.

### STANDARD FORMULAE

1.  $\int x^n dx = \begin{cases} \frac{x^{n+1}}{n+1} + c & n \neq -1 \\ \log_e|x| + c & n = -1 \end{cases}$
2.  $\int (ax + b)^n dx = \begin{cases} \frac{(ax+b)^{n+1}}{(n+1)a} + c & n \neq -1 \\ \frac{1}{a} \log|ax + b| + c & n = -1 \end{cases}$
3.  $\int \sin x dx = -\cos x + c$ .
4.  $\int \cos x dx = \sin x + c$
5.  $\int \tan x . dx = -\log|\cos x| + c = \log|\sec x| + c$ .
6.  $\int \cot x dx = \log|\sin x| + c$ .
7.  $\int \sec^2 x dx = \tan x + c$

$$8. \quad \int \operatorname{cosec}^2 x \, dx = -\cot x + c$$

$$9. \quad \int \sec x \tan x \, dx = \sec x + c$$

$$10. \quad \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$$

$$11. \quad \int \sec x \, dx = \log|\sec x + \tan x| + c$$

$$= \log \left| \tan \left( \frac{x}{2} + \frac{\pi}{4} \right) \right| + c$$

$$12. \quad \int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x| + c$$

$$= \log \left| \tan \frac{x}{2} \right| + c$$

$$13. \quad \int e^x \, dx = e^x + c$$

$$14. \quad \int a^x \, dx = \frac{a^x}{\log a} + c$$

$$15. \quad \int \frac{1}{\sqrt{1-x^2}} \, dx = \sin^{-1} x + c, |x| < 1$$

$$= -\cos^{-1} x + c$$

$$16. \quad \int \frac{1}{1+x^2} \, dx = \tan^{-1} x + c$$

$$= -\cot^{-1} x + c$$

$$17. \quad \int \frac{1}{|x|\sqrt{x^2-1}} \, dx = \sec^{-1} x + c, |x| > 1$$

$$= -\operatorname{cosec}^{-1} x + c$$

$$18. \quad \int \frac{1}{a^2-x^2} \, dx = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$$

$$19. \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$20. \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$$

$$21. \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + c = -\cos^{-1} \frac{x}{a} + c$$

$$22. \int \frac{1}{\sqrt{a^2 + x^2}} dx = \log |x + \sqrt{a^2 + x^2}| + c$$

$$23. \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log |x + \sqrt{x^2 - a^2}| + c$$

$$24. \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$$

$$25. \int \sqrt{a^2 + x^2} dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \log |x + \sqrt{a^2 + x^2}| + c$$

$$26. \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log |x + \sqrt{x^2 - a^2}| + c$$

### RULES OF INTEGRATION

$$1. \int [(f_1(x) \pm f_2(x) \pm \dots \pm f_x(x))] dx = \int f_1(x) dx \pm \int f_2(x) dx \pm \dots \pm \int f_x(x) dx$$

$$2. \int k.f(x) dx = k \int f(x) dx.$$

$$3. \int e^x \{f(x) + f'(x)\} dx = e^x f(x) + c$$

### INTEGRATION BY SUBSTITUTION

$$1. \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$$

$$2. \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$3. \int \frac{f'(x)}{[f(x)]^n} dx = \frac{(f(x))^{-n+1}}{-n+1} + c$$

## INTEGRATION BY PARTS

$$\int f(x) g(x) dx = f(x) \int g(x) dx - \int \left[ f'(x) \int g(x) dx \right]$$

## DEFINITE INTEGRALS

$$\int_a^b f(x) dx = F(b) - F(a), \text{ where } F(x) = \int f(x) dx$$

## DEFINITE INTEGRAL AS A LIMIT OF SUMS.

$$\int_a^b f(x) dx = \lim_{h \rightarrow 0} h [f(a) + f(a+h) + f(a+2h) + \dots + f(a+n-1h)]$$

$$\text{Where } h = \frac{b-a}{n} \text{ or } \int_a^b f(x) dx = \lim_{n \rightarrow \infty} [h \sum_{r=1}^n f(a+rh)]$$

## PROPERTIES OF DEFINITE INTEGRAL

$$1. \int_a^b f(x) dx = - \int_b^a f(x) dx$$

$$2. \int_a^b f(x) dx = \int_a^b f(t) dt.$$

$$3. \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx.$$

$$4. (i) \int_a^b f(x) dx = \int_a^b f(a+b-x) dx.$$

$$(ii) \int_0^a f(x) dx = \int_0^a f(a-x) dx$$

$$5. \int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx, \quad \text{if } f(x) \text{ is even function}$$

$$6. \int_{-a}^a f(x) dx = 0 \text{ if } f(x) \text{ is an odd function}$$

$$7. \int_0^{2a} f(x) dx = \begin{cases} 2 \int_0^a f(x) dx, & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

### Very Short Answer Type Questions (1 Mark)

Evaluate the following integrals:

$$1. \int (\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}) dx$$

$$2. \int_{-1}^1 e^{|x|} dx$$

$$3. \int \frac{dx}{1-\sin^2 x}$$

$$4. \int_{-1}^1 x^{99} \cos^4 x dx$$

$$5. \int \frac{1}{x \log x \log(\log x)} dx$$

$$6. \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x \log \left( \frac{1+x}{1-x} \right) dx$$

$$7. \int (e^{a \log x} + e^{x \log a}) dx$$

$$8. \int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx$$

9.  $\int_{-\pi/2}^{\pi/2} \sin^7 x \, dx$
10.  $\int \sqrt{10 - 4x + x^2} \, dx$
11.  $\int_{-1}^1 x^3 |x| \, dx$
12.  $\int \frac{1}{\sin^2 x \cos^2 x} \, dx$
13.  $\int_{-2}^2 \frac{dx}{1+|x-1|}$
14.  $\int e^{-\log x} \, dx$
15.  $\int \frac{e^x}{a^x} \, dx$
16.  $\int \frac{x}{\sqrt{x+1}} \, dx$
17.  $\int \frac{x}{(x+1)^2} \, dx$
18.  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$
19.  $\int \cos^2 \alpha \, dx$
20.  $\int \frac{1}{x \cos \alpha + 1} \, dx$
21.  $\int \sec x \log(\sec x + \tan x) \, dx$
22.  $\int \frac{1}{\cos \alpha + x \sin \alpha} \, dx$
23.  $\int \frac{\sec^2(\log x)}{x} \, dx$
24.  $\int \frac{e^x}{\sqrt{4+e^{2x}}} \, dx$
25.  $\int \frac{1}{x(2+3 \log x)} \, dx$
26.  $\int \frac{1-\sin x}{x+\cos x} \, dx$

27.  $\int \frac{1-\cos x}{\cos x} dx$
28.  $\int \frac{x^{e-1}+e^{x-1}}{x^e+e^x} dx$
29.  $\int \frac{(x+1)}{x} (x + \log x) dx$
30.  $\int_0^\pi |\cos x| dx$
31.  $\int_0^2 [x] dx$  where  $[x]$  is greatest integers function.
32.  $\int \frac{1}{\sqrt{9-4x^2}} dx$
33.  $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)} dx$
34.  $\int_{-2}^1 \frac{|x|}{x} dx$
35.  $\int_{-1}^1 x |x| dx$
36.  $\int x \sqrt{x+2} dx$
37.  $\int_a^b f(x) dx + \int_b^a f(x) dx$
38.  $\int \frac{\sin x}{\sin 2x} dx$
39.  $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} |\sin x| dx$
40.  $\int \frac{1}{\sec x + \tan x} dx$
41.  $\int \frac{\sin^2 x}{1+\cos x} dx$
42.  $\int \frac{1-\tan x}{1+\tan x} dx$



**VERY SHORT ANSWER TYPE QUESTIONS [2 MARKS]**

Evaluate :

43.  $\int_e [\log(x+1) - \log x] dx$

44.  $\int \frac{1}{\sqrt{x+1} + \sqrt{x+2}} dx$

45.  $\int \sin x \sin 2x dx$

46.  $\int \left[ \frac{x}{a} + \frac{a}{x} + x^a + a^x \right] dx$

47.  $\int_0^{\pi/2} \left[ \frac{5+3\cos x}{5+3\sin x} \right] dx$

48.  $\int \frac{a^x + b^x}{c^x} dx$

49.  $\int \left( \sqrt{ax} - \frac{1}{\sqrt{ax}} \right)^2 dx$

50.  $\int e^x 2^x dx$

**Short Answer Type Questions (4 Marks)**

Evaluate :

51. (I)  $\int \frac{x \operatorname{cosec}(\tan^{-1} x^2)}{1+x^4} dx$

(II)  $\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx$

(III)  $\int \frac{1}{\sin(x-a) \sin(x-b)} dx$

$$(IV) \quad \int \frac{\cos(x+a)}{\cos(x-a)} dx$$

$$(V) \quad \int \cos 2x \cos 4x \cos 6x dx$$

$$(VI) \quad \int \tan 2x \tan 3x \tan 5x dx$$

$$(VII) \quad \int \sin^2 x \cos^4 x dx$$

$$(VIII) \quad \int \cot^3 x \operatorname{cosec}^4 x dx$$

$$(IX) \quad \int \frac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx \quad [\text{Hint: Put } a^2 \sin^2 x + b^2 \cos^2 x = t \text{ or } t^2]$$

$$(X) \quad \int \frac{1}{\sqrt{\cos^3 x \cos(x+a)}} dx$$

$$(XI) \quad \int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} dx$$

$$(XII) \quad \int \frac{\sin x + \cos x}{\sqrt{\sin 2}} dx$$

Evaluate :

$$52. \quad (i) \quad \int \frac{x}{x^4 + x^2 + 1} dx$$

$$(ii) \quad \int \frac{1}{x[6(\log x)^2 + 7 \log x + 2]} dx$$

$$(iii) \quad \int \frac{1}{\sqrt{\sin^3 x \cos^5 x}} dx$$

$$(iv) \quad \int \frac{x^2 + 1}{x^4 + 1} dx$$

$$(v) \quad \int \frac{1}{\sqrt{(x-a)(x-b)}} dx$$

$$(vi) \quad \int \frac{5x-2}{3x^2+2x+1} dx$$

$$(vii) \quad \int \frac{x^2}{x^2+6x+1} dx$$

$$(viii) \quad \int \frac{x+2}{\sqrt{4x-x^2}} dx$$

$$(ix) \quad \int x \sqrt{1+x-x^2} dx$$

$$(x) \quad \int \frac{\sin^4 x}{\cos^8 x} dx$$

$$(xi) \quad \int \sqrt{\sec x - 1} dx \text{ [Hint: Multiply and divided by } \sqrt{\sec x + 1}]$$

Evaluate :

$$53. \quad (I) \quad \int \frac{dx}{x(x^7+1)}$$

$$(II) \quad \int \frac{3x+5}{x^3-x^2-x+1} dx$$

$$(III) \quad \int \frac{\sin \theta \cos \theta}{\cos^2 \theta - \cos \theta - 2} d\theta$$

$$(iv) \quad \int \frac{dx}{(2-x)(x^2+3)}$$

$$(v) \quad \int \frac{x^2+x+2}{(x-2)(x-1)} dx$$

$$(vi) \quad \int \frac{(x^2+1)(x^2+2)}{(x^2+3)(x^2+4)} dx$$

$$(vii) \quad \int \frac{dx}{(2x+1)(x^2+4)}$$

$$(viii) \quad \int \frac{x^2-1}{x^4+x^2+1} dx$$

$$(ix) \quad \int \sqrt{\tan x} dx$$

$$(x) \quad \int \frac{dx}{\sin x - \sin 2x}$$

54. Evaluate:

$$(I) \quad \int x^5 \sin x^3 dx$$

$$(II) \quad \int \sec^3 x dx$$

$$(III) \quad \int e^{ax} \cos(bx + c) dx$$

$$(IV) \quad \int \sin^{-1} \left( \frac{6x}{1+9x^2} \right) dx \quad \text{[Hint: Put } 3x = \tan \theta \text{]}$$

$$(V) \quad \int \cos \sqrt{x} dx$$

$$(VI) \quad \int x^3 \tan^{-1} x dx$$

$$(VII) \quad \int e^{2x} \left( \frac{1+\sin 2x}{1+\cos 2x} \right) dx$$

$$(VIII) \quad \int \left[ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx$$

$$(IX) \quad \int \sqrt{2ax - x^2} dx$$

$$(X) \quad \int e^x \frac{(x^2+1)}{(x+1)^2} dx$$

$$(XI) \quad \int x^3 \sin^{-1} \left( \frac{1}{x} \right) dx$$

$$(XII) \quad \int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$$

[Hint: Put  $\log x = t$   
 $x = e^t$ ]

$$(XIII) \quad \int (6x + 5) \sqrt{6 + x - x^2} dx$$

$$(XIV) \quad \int \frac{1}{x^3 + 1} dx$$

$$(XV) \quad \int \tan^{-1} \left( \frac{x-5}{1+5x} \right) dx$$

$$(XVI) \quad \int \frac{dx}{5+4 \cos x}$$

55. Evaluate the following definite integrals:

$$(i) \quad \int_0^{\pi/4} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$$

$$(ii) \quad \int_0^{\pi/2} \cos 2x \log \sin x dx$$

$$(iii) \quad \int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} dx$$

$$(iv) \quad \int_0^{1/\sqrt{2}} \frac{\sin^{-1} x}{(1-x^2)^{3/2}} dx$$

$$(v) \quad \int_0^{\pi/2} \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$$

$$(vi) \quad \int_0^1 \sin \left( 2 \tan^{-1} \sqrt{\frac{1+x}{1-x}} \right) dx$$

$$(vii) \quad \int_0^{\pi/2} \frac{x + \sin x}{1 + \cos x} dx$$

$$(viii) \int_0^1 x \log \left( 1 + \frac{x}{2} \right) dx$$

$$(ix) \int_{-1}^{1/2} |x \cos \pi x| dx$$

$$(x) \int_{-\pi}^{\pi} (\cos a x - \sin b x)^2 dx$$

56. Evaluate:

$$(i) \int_2^5 [|x - 2| + |x - 3| + |x - 4|] dx$$

$$(ii) \int_0^{\pi} \frac{x}{1 + \sin x} dx$$

$$(iii) \int_{-1}^1 e^{\tan^{-1} x} \left[ \frac{1+x+x^2}{1+x^2} \right] dx$$

$$(iv) \int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$$

$$(v) \int_0^2 [x^2] dx$$

$$(vi) \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$(vii) \int_0^{\pi} \frac{x}{a^2 \cos^2 x + b^2 \sin^2 x} dx \text{ [Hint: use } \int_0^a f(x) dx = \int_0^a f(a-x) dx]$$

57. Evaluate the following integrals:

$$(i) \int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$$

$$(ii) \int_{-\pi/2}^{\pi/2} (\sin |x| + \cos |x|) dx$$

$$(iii) \int_0^{\pi} \frac{e^{\cos x}}{e^{\cos x} + e^{-\cos x}} dx$$

$$(iv) \int_0^{\pi} \frac{x \tan x}{\sec x + \operatorname{cosec} x} dx$$

$$(v) \int_{-a}^a \sqrt{\frac{a-x}{a+x}} dx$$

58. Evaluate

$$(i) \int \frac{\sin^{-1} \sqrt{x} - \cos^{-1} \sqrt{x}}{\sin^{-1} \sqrt{x} + \cos^{-1} \sqrt{x}} dx \quad x \in [0, 1]$$

$$(ii) \int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$$

$$(iii) \int \frac{x^2 e^x}{(x+z)^2} dx$$

$$(iv) \int \frac{x^2}{(x \sin x + \cos x)^2} dx$$

$$(v) \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

$$(vi) \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$$

$$(vii) \int \frac{\sin x}{\sin 4x} dx$$

$$(viii) \int_{-1}^{3/2} |x \sin \pi x| \, dx$$

$$(ix) \int \frac{\sin(x-a)}{\sin(x+a)} \, dx$$

$$(x) \int \frac{x^2}{(x^2+4)(x^2+9)} \, dx$$

$$(xi) \int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} \, dx$$

### Long Answer Type Questions (6 Marks)

59. Evaluate the following integrals:

$$(i) \int \frac{x^5 + 4}{x^5 - x} \, dx$$

$$(ii) \int \frac{2e^t}{e^{3t} - 6e^{2t} + 11e^t - 6} \, dt$$

$$(iii) \int \frac{2x^3}{(x+1)(x-3)^2} \, dx$$

$$(iv) \int \frac{1 + \sin x}{\sin x (1 + \cos x)} \, dx$$

$$(v) \int_0^{\pi/2} (\sqrt{\tan x} + \sqrt{\cot x}) \, dx$$

$$(vi) \int_0^1 x \sqrt{\frac{1-x^2}{1+x^2}} \, dx$$



$$(vii) \int_0^{\pi/2} \frac{\cos x}{1 + \cos x + \sin x} dx$$

60. Evaluate the following integrals as limit of sums:

$$(i) \int_2^4 (2x + 1) dx$$

$$(ii) \int_0^2 (x^2 + 3) dx$$

$$(iii) \int_1^3 (3x^2 - 2x + 4) dx$$

$$(iv) \int_0^4 (3x^2 + e^{2x}) dx$$

$$(v) \int_0^1 e^{2-3x} dx$$

$$(vi) \int_0^1 (3x^2 + 2x + 1) dx$$

61. Evaluate:

$$(i) \int \frac{dx}{(\sin x - 2 \cos x)(2 \sin x + \cos x)}$$

$$(ii) \int_0^1 \frac{\log(1+x)}{1+x^2} dx$$

$$(iii) \int_0^{\pi/2} (2 \log \sin x - \log \sin 2x) dx$$

$$62. \int_0^1 x(\tan^{-1} x)^2 dx$$

$$63. \int_0^{\pi/2} \log \sin x dx$$

$$64. \text{ Prove that } \int_0^1 \tan^{-1} \left( \frac{1}{1-x+x^2} \right) dx = 2 \int_0^1 \tan^{-1} x dx$$

Hence or otherwise evaluate the integral  $\int \tan^{-1}(1-x+x^2) dx$ .

$$65. \text{ Evaluate } \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx.$$

### Answers

$$1. \quad \frac{\pi}{2} x + c$$

$$2. \quad 2e - 2$$

$$3. \quad \tan x + c$$

$$4. \quad 0$$

$$5. \quad \log|\log|\log x|| + c$$

$$6. \quad 0$$

$$7. \quad \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$$

$$8. \quad \tan x + c$$

$$9. \quad 0$$

$$10. \quad \frac{(x-2)\sqrt{x^2-4x+10}}{2} + 3 \log|(x-2) + \sqrt{x^2-4x+10}| + c$$

$$11. \quad 0$$

$$12. \quad \tan x - \cot x + c$$

$$13. \quad 3 \log_e 2$$

$$14. \quad \log|x| + c$$

15.  $\frac{\left(\frac{e}{a}\right)^x}{\log\left(\frac{e}{a}\right)} + c$
16.  $\frac{2}{3}(x+1)^{3/2} - 2(x+1)^{1/2} + c$
17.  $\log|x+1| + \frac{1}{x+1} + c$
18.  $2e^{\sqrt{x}} + c$
19.  $x\cos^2\alpha + c$
20.  $\frac{\log|x\cos\alpha+1|}{\cos\alpha} + c$
21.  $\frac{(\log|\sec x + \tan x|)^2}{2} + c$
22.  $\frac{\log|\cos\alpha + x\sin\alpha|}{\sin\alpha} + c$
23.  $\tan|\log x| + c$
24.  $\log|e^x + \sqrt{4 + e^{2x}}| + c$
25.  $\frac{1}{3}\log|2 + 3\log x| + c$
26.  $\log|x + \cos x| + c$
27.  $2\log\left|\sec\frac{x}{2}\right| + c$
28.  $\frac{1}{e}\log|x^e + e^x| + c$
29.  $\frac{(x+\log)^2}{2} + c$
30.  $2$
31.  $1$
32.  $\frac{1}{2}\sin^{-1}\left(\frac{2x}{3}\right) + c$
33.  $\frac{b-a}{2}$
34.  $-1$
35.  $0$
36.  $\frac{2}{5}(x+2)^{5/2} - \frac{4}{3}(x+2)^{3/2} + c$
37.  $0$
38.  $\frac{1}{2}\log|\sec x + \tan x| + c$
39.  $2-\sqrt{2}$
40.  $\log|1 + \sin x| + c$
41.  $x - \sin x + c$
42.  $\log|\cos x + \sin x| + c$
43.  $x + \log x + c$

$$44. \frac{2}{3} \left[ (x+2)^{3/2} - (x+1)^{3/2} \right] + c$$

$$45. \frac{-1}{2} \left[ \frac{\sin 3x}{3} - \sin x \right] + c$$

$$46. \frac{1}{a} \frac{x^2}{2} + a \log |x| + \frac{x^{a+1}}{a+1} + \frac{a^x}{\log a} + c$$

$$47. 0$$

$$48. \frac{\left(\frac{a}{c}\right)^x}{\log \left|\frac{a}{c}\right|} + \frac{\left(\frac{b}{c}\right)^x}{\log \left|\frac{b}{c}\right|} + c$$

$$49. \frac{ax^2}{2} + \frac{\log |x|}{a} - 2x + c$$

$$50. \frac{2^x e^x}{\log(2e)} + c$$

$$51.$$

$$(I) \quad \frac{1}{2} \log \left[ \operatorname{cosec}(\tan^{-1} x^2) - \frac{1}{x^2} \right] + c$$

$$(II) \quad \frac{1}{2} (x^2 - x\sqrt{x^2 - 1}) + \frac{1}{2} \log |x + \sqrt{x^2 - 1}| + c$$

$$(III) \quad \frac{1}{\sin(a-b)} \log \left| \frac{\sin(x-a)}{\sin(x-b)} \right| + c$$

$$(IV) \quad x \cos 2a - \sin 2a \log |\sec(x-a)| + c$$

$$(V) \quad \frac{3}{8}x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin 4x + c$$

$$(VI) \quad \frac{1}{5} \log |\sec 5x| - \frac{1}{2} \log |\sec 2x| - \frac{1}{3} \log |\sec 3x| + c$$

$$(VII) \quad \frac{1}{32} \left[ 2x + \frac{1}{2} \sin 2x - \frac{1}{2} \sin 4x - \frac{1}{6} \sin 6x \right] + c$$

$$(VIII) \quad - \left( \frac{\cot^6 x}{6} + \frac{\cot^4 x}{4} \right) + c$$

$$(IX) \quad \frac{1}{a^2 - b^2} \sqrt{a^2 \sin^2 x + b^2 \cos^2 x} + c$$

$$(X) \quad -2 \operatorname{cosec} a \sqrt{\cos a - \tan x \sin a} + c$$

$$(XI) \quad \tan x - \cot x - 3x + c$$

$$(XII) \quad \sin^{-1}[\sin x - \cos x] + c$$

$$52. \quad (I) \quad \frac{1}{\sqrt{3}} \tan^{-1} \left( \frac{2x^2 + 1}{\sqrt{3}} \right) + c$$

$$(II) \quad \log \left| \frac{2 \log x}{3 \log x} \right| + c$$

$$(III) \quad \frac{-2}{\sqrt{\tan x}} + \frac{2}{3} \tan^{3/2} x + c$$

$$(IV) \quad \frac{1}{\sqrt{2}} \tan^{-1} \left\{ \frac{1}{\sqrt{2}} \left( x - \frac{1}{x} \right) \right\} + c$$

$$(V) \quad 2 \log |\sqrt{x-a} + \sqrt{x-b}| + c$$

$$(VI) \quad \frac{5}{6} \log |3x^2 + 2x + 1| + \frac{-11}{3\sqrt{2}} \tan^{-1} \left( \frac{3x+1}{\sqrt{2}} \right) + c$$

$$(VII) \quad x - 3 \log |x^2 + 6x + 12| + 2\sqrt{3} \tan^{-1} \left( \frac{x+3}{\sqrt{3}} \right) + c$$

$$(VIII) \quad -\sqrt{4x - x^2} + 4 \sin^{-1} \left( \frac{x-2}{2} \right) + c$$

$$(IX) \quad -\frac{1}{3}(1+x-x^2)^{3/2} + \frac{1}{8}(2x-1)\sqrt{1+x-x^2} + \frac{5}{16} \sin^{-1} \left( \frac{2x-1}{\sqrt{5}} \right) + c$$

$$(X) \quad \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + c$$

$$(XI) \quad -\log \left| \cos x + \frac{1}{2} + \sqrt{\cos^2 x + \cos x} \right| + c$$

$$53. \quad (I) \quad \frac{1}{7} \log \left| \frac{x^7}{x^7+1} \right| + c$$

$$(II) \quad \frac{1}{2} \log \left| \frac{x+1}{x-1} \right| - \frac{4}{x-1} + c$$

$$(III) \quad \frac{-2}{3} \log |\cos \theta - 2| - \frac{1}{3} \log |1 + \cos \theta| + c$$

$$(IV) \quad \frac{1}{14} \log \left| \frac{x^2+3}{(2-x)^2} \right| + \frac{2}{7\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) + c$$

$$(V) \quad x + 4 \log \left| \frac{(x-2)^2}{x-1} \right| + c$$

$$(VI) \quad x + \frac{2}{\sqrt{3}} \tan^{-1} \left( \frac{x}{\sqrt{3}} \right) - 3 \tan^{-1} \left( \frac{x}{2} \right) + c$$

$$(VII) \quad \frac{2}{17} \log |2x+1| - \frac{1}{17} \log |x^2+4| + \frac{1}{34} \tan^{-1} \frac{x}{2} + c$$

$$(VIII) \quad \frac{1}{2} \log \left| \frac{x^2-x+1}{x^2+x+1} \right| + c$$

$$(IX) \quad \frac{1}{\sqrt{2}} \tan^{-1} \left( \frac{\tan x - 1}{\sqrt{2} \tan x} \right) + \frac{1}{2\sqrt{2}} \log \left| \frac{\tan x - \sqrt{2} \tan x + 1}{\tan x + \sqrt{2} \tan x + 1} \right| + c$$

$$(X) -\frac{1}{2}\log|\cos x - 1| - \frac{1}{6}\log|\cos x + 1| + \frac{2}{3}\log|1 - 2\cos x| + c$$

$$54. (i) \quad \frac{1}{3}[-x^3 \cos x^3 + \sin x^3] + c$$

$$(ii) \quad \frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x|] + c$$

$$(iii) \quad \frac{e^{ax}}{a^2+b^2} [a \cos(bx + c) + b \sin(bx + c)] + c$$

$$(iv) \quad 2x \tan^{-1} 3x - \frac{1}{3} \log |1 + 9x^2| + c$$

$$(v) \quad 2[\sqrt{x} \sin \sqrt{x} + \cos \sqrt{x}] + c$$

$$(vi) \quad \left(\frac{x^4-1}{4}\right) \tan^{-1} x - \frac{x^3}{12} + \frac{x}{4} + c$$

$$(vii) \quad \frac{1}{2} e^{2x} \tan x + c$$

$$(viii) \quad \frac{x}{\log x} + c$$

$$(ix) \quad \left(\frac{x-a}{2}\right) \sqrt{2ax - x^2} + \frac{a^2}{2} \sin^{-1} \left(\frac{x-a}{a}\right) + c$$

$$(x) \quad e^x \left(\frac{x-1}{x+1}\right) + c$$

$$(xi) \quad \frac{x^4}{4} \sin^{-1} \left(\frac{1}{x}\right) + \frac{x^2+2}{12} \sqrt{x^2 - 1} + c$$

$$(xii) \quad x \log|\log x| - \frac{x}{\log x} + c$$

$$(xiii) \quad -2(6 + x - x^2)^{\frac{3}{2}} + 8 \left[ \frac{2x-1}{4} \sqrt{6 + x - x^2} + \frac{25}{8} \sin^{-1} \left(\frac{2x-1}{5}\right) \right] + c$$

$$(xiv) \quad \frac{1}{3} \log|x + 1| - \frac{1}{6} \log|x^2 - x + 1| + \frac{1}{\sqrt{3}} \tan^{-1} \left(\frac{2x-1}{\sqrt{3}}\right) + c$$

$$(xv) \quad x \tan^{-1} x - \frac{1}{2} \log|1 + x^2| - x \tan^{-1} 5 + c$$

$$(xvi) \quad \frac{2}{3} \tan^{-1} \left(\frac{1}{3} \tan \frac{x}{2}\right) + c$$

55. (I)  $\frac{1}{20} \log 3$   
 (II)  $-\pi/4$   
 (III)  $\frac{\pi}{4} - \frac{1}{2}$   
 (IV)  $\frac{\pi}{4} - \frac{1}{2} \log 2$   
 (V)  $\frac{\pi}{2}$   
 (VI)  $\pi/4$   
 (VII)  $\pi/2$   
 (VIII)  $\frac{3}{4} + \frac{3}{2} \log \frac{2}{3}$   
 (IX)  $\frac{3}{2\pi} - \frac{1}{\pi^2}$   
 (X)  $2\pi + \frac{1}{2a} \sin 2a\pi - \frac{1}{2b} \sin 2b\pi$
56. (I)  $\frac{1}{2}$   
 (II)  $\pi$   
 (III)  $e^{\pi/4} + e^{-\pi/4}$   
 (IV)  $\frac{1}{4} \pi^2$   
 (V)  $5 - \sqrt{3} - \sqrt{2}$   
 (VI)  $\frac{\pi^2}{16}$   
 (VII)  $\frac{\pi^2}{2a}$



57. (I)  $\frac{\pi}{12}$

(II) 2

(III)  $\frac{\pi}{2}$

(IV)  $\frac{\pi^2}{4}$

(V)  $a\pi$

58. (I)  $\frac{2(2x-1)}{\pi} \sin^{-1} \sqrt{x} + \frac{2\sqrt{x-x^2}}{\pi} - x + c$

(II)  $-2\sqrt{1-x} + \cos^{-1} \sqrt{x} + \sqrt{x-x^2} + c$

(III)  $\frac{x-2}{x+2} e^x + c$

(IV)  $\frac{\sin x - x \cos x}{x \sin x + \cos x} + c$

(V)  $(x+a) \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{ax} + c$

(VI)  $2 \sin^{-1} \frac{\sqrt{3}-1}{2}$

(VII)  $\frac{1}{8} \log \left| \frac{1-\sin x}{1+\sin x} \right| - \frac{1}{4\sqrt{2}} \log \left| \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} \right| + c$

(VIII)  $\frac{3}{\pi} + \frac{1}{\pi^2}$

(IX)  $(\cos 2a)(x+a) - (\sin 2a) \log |\sin(x+a)| + c$

(X)  $-\frac{4}{5} \log |x^2+4| + \frac{9}{5} \log |x^2+9| + c$

(XI)  $-\left(\frac{1}{2} \sin 2x + \sin x\right) + c$

$$59. \quad (I) \quad x - 4 \log|x| + \frac{5}{4} \log|x-1| + \frac{3}{4} \log|x+1| + \log|x^2 + 1| - \frac{1}{2} \tan^{-1} x + c$$

$$(II) \quad \log \left| \frac{(e^t-1)(e^t-3)}{(e^t-2)^2} \right| + c$$

$$(III) \quad 2x - \frac{1}{8} \log|x+1| + \frac{81}{8} \log|x-3| - \frac{27}{2(x-3)} + c$$

$$(IV) \quad \frac{1}{4} \log \left| \frac{1-\cos x}{1+\cos x} \right| + \frac{1}{2(1+\cos x)} + \tan \frac{x}{2} + c$$

$$(V) \quad \frac{\pi}{\sqrt{2}}$$

$$(VI) \quad \frac{\pi-2}{4}$$

$$(I) \quad \frac{\pi}{4} - \frac{1}{2} \log 2$$

$$60. \quad (I) \quad 14$$

$$(II) \quad \frac{26}{3}$$

$$(III) \quad 26$$

$$(IV) \quad \frac{1}{2} (127 + e^8)$$

$$(V) \quad \frac{1}{3} \left( e^2 - \frac{1}{e} \right)$$

$$(VI) \quad 3$$

$$61. \quad (I) \quad \frac{1}{5} \log \left| \frac{\tan x - 2}{2 \tan x + 1} \right| + c$$

$$(II) \quad \frac{\pi}{8} \log 2$$

$$(III) \quad \frac{\pi}{2} \log \frac{1}{2}$$

$$62. \quad \frac{\pi^2}{16} - \frac{\pi}{4} + \frac{1}{2} \log 2$$

$$63. \quad \frac{-\pi}{2} \log 2$$

$$64. \quad \log 2$$

$$65. \quad \frac{1}{\sqrt{2}} \log |\sqrt{2} + 1|$$

## CHAPTER 8

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### APPLICATIONS OF INTEGRALS

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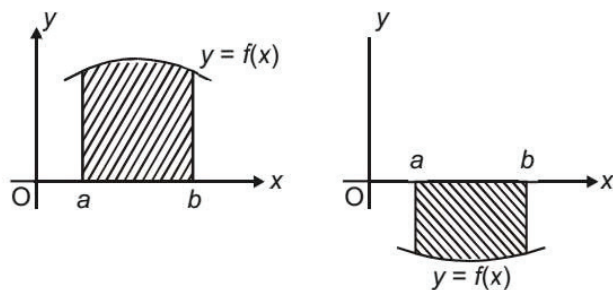
#### POINT TO REMEMBER

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##### AREA OF BOUNDED REGION

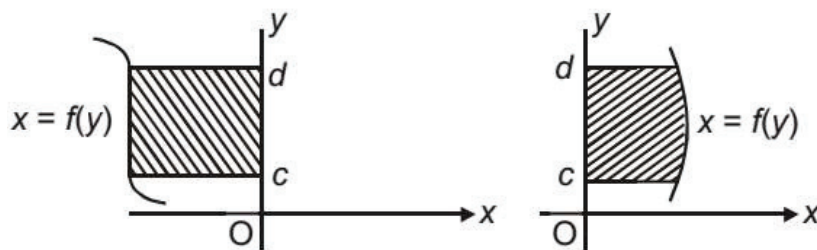
- Area bounded by the curve  $y = f(x)$ , the  $x$  axis and between the ordinates,  $x = a$  and  $x = b$  is given by

$$Area = \left| \int_a^b f(x) dx \right|$$

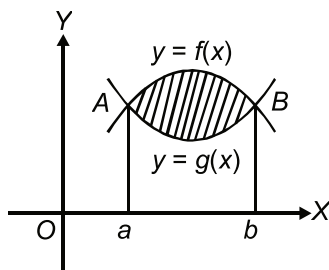


- Area bounded by the curve  $x = f(y)$ , the  $y$ -axis and between the abscissas,  $y = c$  and  $y = d$  is given by

$$Area = \left| \int_c^d f(y) dy \right|$$

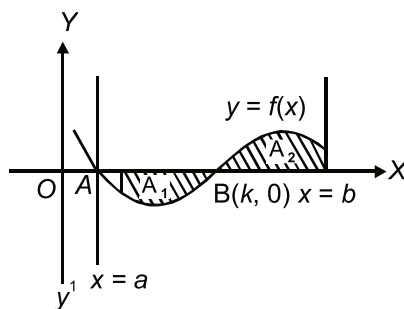


- Area bounded by two curves  $y = f(x)$  and  $y = g(x)$  such that  $0 \leq g(x) \leq f(x)$  for all  $x \in [a, b]$  and between the ordinates  $x = a$  and  $x = b$  is given by



$$\text{Area} = \int_a^b [f(x) - g(x)] dx$$

- Area of the following shaded region =  $\left| \int_a^k f(x) dx \right| + \int_k^b f(x) dx$



### LONG ANSWER TYPE QUESTIONS (6 MARKS)

- Find the area of the parabola  $y^2 = 4ax$  bounded by its Latus rectum.
- Find the area of the region  $\{(x, y): x^2 \leq y \leq |x|\}$ .
- Find the area of region in the first quadrant enclosed by  $x$ -axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

4. Find the area of region  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$
5. Prove that the curve  $y = x^2$  and  $x = y^2$  divide the square bounded by  $x = 0, y = 0, x = 1, y = 1$  into three equal parts.
6. Find the area of the smaller region enclosed between ellipse  $b^2x^2 + a^2y^2 = a^2b^2$  and the line  $bx + ay = ab$ .
7. Find the common area bounded by the circles  $x^2 + y^2 = 4$  and  $(x - 2)^2 + y^2 = 4$ .
8. Using integration, find the area of the triangle whose sides are given by  $2x + y = 4, 3x - 2y = 6$  and  $x - 3y + 5 = 0$ .
9. Using integration, find the area of the triangle whose vertices are  $(-1, 0), (1, 3)$  and  $(3, 2)$ .
10. Find the area of the region  $\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\}$ .
11. Find the area of the region bounded by the curve  $x^2 = 4y$  and the line  $x = 4y - 2$ .
12. Find the area lying above x-axis and included between the circle  $x^2 + y^2 = 8x$  and inside the parabola  $y^2 = 4x$ .
13. Using integration, find the area enclosed by the curve  $y = \cos x, y = \sin x$  and x-axis in the interval  $[0, \pi/2]$ .
14. Using integration, find the area of the following region:  
 $\{(x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2}\}$
15. Using integration, find the area of the triangle formed by positive x-axis and tangent and normal to the circle  $x^2 + y^2 = 4$  at  $(1, \sqrt{3})$ .
16. Using integration, find the area of the region bounded by the line  $x - y + 2 = 0$ , the curve  $x = \sqrt{y}$  and y-axis.
17. Find the area of the region bounded by the curves  $ay^2 = x^3$ , the y-axis and the lines  $y = a$  and  $y = 2a$ .

18. Find the area bounded by x-axis, the curve  $y = 2x^2$  and tangent to the curve at the point whose abscissa is 2.
19. Using integration, find the area of the region bounded by the curve  $y = 1 + |x + 1|$  and lines  $x = -3, x = 3, y = 0$ .
20. Find the area of the region  $\{(x, y): y^2 \geq 6x, x^2 + y^2 \leq 16\}$
21. Find the area of the region enclosed between curves  $y = |x - 1|$  and  $y = 3 - |x|$ .
22. If the area bounded by the parabola  $y^2 = 16ax$  and the line  $y = 4mx$  is  $\frac{a^2}{12}$  sq unit then using integration find the value of m.

### **ANSWERS**

- |  |  |
|--|--|
| 1. $\frac{8}{3}a^2$ sq. units  | 9. 4 sq. units   |
| 2. $\frac{1}{3}$ sq. units   | 10. $\left(\pi - \frac{1}{2}\right)$ sq. units                     |
| 3. $4\pi$ sq. units  | 11. $\frac{9}{8}$ sq. units  |
| 4. $\left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right]$ sq. units | 12. $\frac{4}{3}(8 + 3\pi)$ sq. units                              |
| 5. ....  | 13. $(2 - \sqrt{2})$ sq. units                                     |
| 6. $\left(\frac{\pi-2}{4}\right)ab$ sq. units  | 14. $\left(\frac{5\pi}{4} - \frac{1}{2}\right)$ sq. units          |
| 7. $\left(\frac{8\pi}{3} - 2\sqrt{3}\right)$ sq. units   | 15. $2\sqrt{3}$ sq. units  |
| 8. 3.5 sq. units   | 16. $\frac{10}{3}$ sq. units                                       |
|  | 17. $\frac{3}{5}a^2 \left[(32)^{\frac{1}{3}} - 1\right]$ sq. units |

18.  $\frac{4}{3}$  sq. units

19. 16 sq. units

20.  $\frac{32\pi-4\sqrt{3}}{3}$  sq. units

21. 4 sq. units

22.  $m = 2$



## CHAPTER–9

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### DIFFERENTIAL EQUATIONS

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#### POINTS TO REMEMBER

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- **Differential Equation:** Equation containing derivatives of a dependent variable with respect to an independent variable is called differential equation.
- **Order of a Differential Equation:** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.
- **Degree of a Differential Equation:** Highest power of highest order derivative involved in the equation is called degree of differential equation where equation is a polynomial equation in differential coefficients.
- **Formation of a Differential Equation:** We differentiate the family of curves as many times as the number of arbitrary constant in the given family of curves. Now eliminate the arbitrary constants from these equations.

After elimination, the equation obtained is differential equation.

- **Solution of Differential Equation**

- (i) **Variable Separable Method**

$$\frac{dy}{dx} = f(x, y).$$

We Separate the variables and get

$$f(x)dx = g(y)dy$$

Then  $\int f(x)dx = \int g(y)dy + c$  is the required solutions.

- (ii) **Homogeneous Differential Equation:** A differential equation of the form  $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$  where  $f(x, y)$  and  $g(x, y)$  are both homogeneous functions of the same degree in  $x$  and  $y$  i.e., of the form  $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$  is called a homogeneous differential equation.

For solving this type of equations we substitute  $y = vx$  and then  $\frac{dy}{dx} = v + x \frac{dv}{dx}$ . The equation can be solved by variables separable method.

A homogeneous differential equation can be of the form  $\frac{dx}{dy} = F\left(\frac{x}{y}\right)$

To solve this equation, we substitute  $x = vy$  and then  $\frac{dx}{dy} = v + y \frac{dv}{dy}$  then the equation can be solved by variable separate method.

- (iii) **Linear Differential Equation:** An equation of the form  $\frac{dy}{dx} + Py = Q$

where  $P$  and  $Q$  are constant or functions of  $x$  only is called a linear differential equation. For finding solution of this type of equations, we find integrating factor (I.F.) =  $e^{\int P dx}$

Solution is  $y (I.F.) = \int Q \cdot (I.F.) dx + c$

Similarly, differential equations of the type  $\frac{dx}{dy} + Px = Q$  where  $P$  and  $Q$  are constants or functions of  $y$  only can be solved.

Here, I.F. =  $e^{\int P dy}$  and the solution is  $x (I.F.) = \int Q \times (I.F.) dy + C$

### Very Short Answer Type Questions (1 Mark)

- Write the order and degree of the following differential equations.

$$(i) \quad \frac{dy}{dx} + \cos y = 0$$

$$(ii) \quad \left(\frac{dy}{dx}\right)^2 + 3\frac{d^2y}{dx^2} = 4$$

$$(iii) \quad \frac{d^4y}{dx^4} + \sin x = \left(\frac{d^2y}{dx^2}\right)^5$$

$$(iv) \quad \frac{d^5y}{dx^5} + \log\left(\frac{dy}{dx}\right) = 0$$

$$(v) \quad \sqrt{1 + \frac{dy}{dx}} = \left(\frac{d^2y}{dx^2}\right)^{1/3}$$

$$(vi) \quad \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k\frac{d^2y}{dx^2}$$

$$(vii) \quad \left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 = \sin x$$

$$(viii) \quad \frac{dy}{dx} + \tan\left(\frac{dy}{dx}\right) = 0$$

2. Write integrating factor differential equations :

$$(I) \quad \frac{dy}{dx} + y \cos x = \sin x$$

$$(II) \quad \frac{dy}{dx} + y \sec^2 x = \sec x + \tan x$$

$$(III) \quad x^2 \frac{dy}{dx} + y = x^4$$

$$(IV) \quad x \frac{dy}{dx} + y \log x = x + y$$

$$(V) \quad x \frac{dy}{dx} - 3y = x^3$$

$$(VI) \quad \frac{dy}{dx} + y \tan x = \sec x$$

$$(VII) \quad \frac{dy}{dx} + \frac{1}{1+x^2} y = \sin x$$

3. Write order of the differential equation of the family of following curves.

$$(I) \quad y = Ae^x + Be^{x+c}$$

$$(II) \quad Ay = Bx^2$$

$$(III) \quad (x-a)^2 + (y-b)^2 = 9$$

$$(IV) \quad Ax+By^2 = Bx^2 - Ay$$

$$(V) \quad \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

$$(VI) \quad y = a \cos (ax + b)$$

$$(VII) \quad y=a+be^{x+c}$$

### Very Short Answer Type Questions [2 Marks]

4. Write the general solution of the following differential equations.

$$(i) \quad \frac{dy}{dx} = x^5 + x^2 - \frac{2}{x}$$

$$(e^x + e^{-x}) dy = (e^x - e^{-x}) dx$$

(ii)

$$\frac{dy}{dx} = x^3 + e^x + x^e$$

(iii)

$$\frac{dy}{dx} = 5^{x+y}$$

(iv)

$$\frac{dy}{dx} = \frac{1 - \cos 2x}{1 + \cos 2y}$$

(v)

$$\frac{dy}{dx} = \frac{1 - 2y}{3x + 1}$$

(vi)

### Short Answer Type Questions (4 Marks)

5. (I) Show that  $y = e^{m \sin^{-1} x}$  is a solution of

$$(1 - x^2) \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$$

- (II) Show that  $y = \sin(\sin x)$  is a solution of differential equation

$$\frac{d^2 y}{dx^2} + (\tan x) \frac{dy}{dx} + y \cos^2 x = 0$$

- (III) Show that  $y = Ax + \frac{B}{x}$  is a solution of  $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = 0$

- (IV) Show that  $y = a \cos(\log x) + b \sin(\log x)$  is a solution of

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

- (V) Verify that  $y = \log(x + \sqrt{x^2 + a^2})$  satisfies the differential equation:  $(a^2 + x^2) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = 0$

(VI) Find the differential equation of the family of curves

$$y = e^x(A \cos x + B \sin x), \text{ where } A \text{ and } B \text{ are arbitrary constants.}$$

(VII) Find the differential equation of an ellipse with major and minor axes  $2a$  and  $2b$  respectively.

(VIII) Form the differential equation representing the family of curves  $(y - b)^2 = 4(x - a)$ .

6. Solve the following differential equations.

(I)  $(1 - x^2) \frac{dy}{dx} - xy = x^2$ , given that  $x = 0, y = 2$

(II)  $x \frac{dy}{dx} + 2y = x^2 \log x$

(III)  $\frac{dy}{dx} + \frac{1}{x} y = \cos x + \frac{\sin x}{x}$ ,  $x > 0$

(IV)  $dy = \cos x (2 - y \operatorname{cosec} x) dx$ ; given that  $x = \frac{\pi}{2}, y = 2$

(V)  $y dx + (x - y^3) dy = 0$

(VI)  $ye^y dx = (y^3 + 2xe^y) dy$

7. Solve each of the following differential equations:

(I)  $y - x \frac{dy}{dx} = 2 \left( y^2 + \frac{dy}{dx} \right)$

(II)  $\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0$

$$(III) \quad x\sqrt{1-y^2}dx + y\sqrt{1-x^2}dy = 0$$

$$(IV) \quad \sqrt{(1-x^2)(1-y^2)}dy + xy dx = 0$$

$$(V) \quad (xy^2 + x)dx + (yx^2 + y)dy = 0; y(0) = 1$$

$$(VI) \quad \frac{dy}{dx} - y \sin^3 x \cos^3 x + xy e^x = 0$$

$$(VII) \quad \tan x \tan y dx + \sec^2 x \sec^2 y dy = 0$$

$$(VIII) \quad \frac{dy}{dx} = x - 1 + xy - y$$

8. Solve the following differential equations:

$$(I) \quad x^2 y dx - (x^3 + y^3) dy = 0$$

$$(II) \quad x^2 \frac{dy}{dx} = x^2 + xy + y^2$$

$$(III) \quad (x^2 - y^2)dx + 2xy dy = 0, \quad y(1) = 1$$

$$(IV) \quad \left(y \sin \frac{x}{y}\right) dx = \left(x \sin \frac{x}{y} - y\right) dy$$

$$(V) \quad \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

$$(VI) \quad x \frac{dy}{dx} = y (\log y - \log x + 1)$$

$$(VII) \quad \frac{dy}{dx} = e^{x+y} + x^2 e^y$$

$$(VIII) \quad \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

$$(IX) \quad (3xy + y^2)dx + (x^2 + xy)dy = 0$$

9. (I) Form the differential equation of the family of circles touching y-axis at (0, 0).
- (II) Form the differential equation of family of parabolas having vertex at (0,0) and axis along the (i) positive y-axis (ii) positive x-axis.
- (III) Form differential equation of family of circles passing through origin and whose centres lie on x-axis.
- (IV) Form the differential equation of the family of circles in the first quadrant and touching the coordinate axes.
10. Show that the differential equation  $\frac{dy}{dx} = \frac{x+2y}{x-y}$  is homogeneous and solve it.
11. Show that the differential equation:  
 $(x^2 + 2xy - y^2)dx + (y^2 + 2xy - x^2)dy = 0$  is homogeneous and solve it.
12. Solve the following differential equations:
- (I)  $\frac{dy}{dx} - 2y = \cos 3x$
- (II)  $\sin x \frac{dy}{dx} + y \cos x = 2 \sin^2 x \cos x$  if  $y\left(\frac{\pi}{2}\right) = 1$
- (III)  $\log\left(\frac{dy}{dx}\right) = px + qy$
13. Solve the following differential equations:
- (I)  $(x^3 + y^3) dx = (x^2y + xy^2)dy$



$$(II) \quad x \, dy - y \, dx = \sqrt{x^2 + y^2} \, dx$$

$$(III) \quad y \left\{ x \cos \left( \frac{y}{x} \right) + y \sin \left( \frac{y}{x} \right) \right\} dx$$

$$-x \left\{ y \sin \left( \frac{y}{x} \right) - x \cos \left( \frac{y}{x} \right) \right\} dy = 0$$

$$(IV) \quad x^2 dy + y(x + y) \, dx = 0 \text{ given that } y=1 \text{ when } x=1.$$

$$(V) \quad x e^{\frac{y}{x}} - y + x \frac{dy}{dx} = 0 \text{ if } y(e) = 0$$

$$(VI) \quad (x^3 - 3xy^2) \, dx = (y^3 - 3x^2y) \, dy$$

$$(VII) \quad \frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec} \left( \frac{y}{x} \right) = 0 \text{ given that } y = 0 \text{ when } x = 1$$

14. Solve the following differential equations:

$$(I) \quad \cos^2 x \frac{dy}{dx} = \tan x - y$$

$$(II) \quad x \cos x \frac{dy}{dx} + y (x \sin x + \cos x) = 1$$

$$(III) \quad \left( 1 + e^{\frac{x}{y}} \right) dx + e^{\frac{x}{y}} \left( 1 - \frac{x}{y} \right) dy = 0$$

$$(IV) \quad (y - \sin x) \, dx + \tan x \, dy = 0, \, y(0) = 0$$

### Long Answer Type Questions (6 Marks)

15. Solve the following differential equations:

$$(I) \quad (x \, dy - y \, dx)y \sin\left(\frac{y}{x}\right) = (y \, dx + x \, dy)x \cos\left(\frac{y}{x}\right)$$

$$(II) \quad 3e^x \tan y \, dx + (1 - e^x)\sec^2 y \, dy = 0 \text{ given that } y = \frac{\pi}{4}, \text{ when } x = 1$$

$$(III) \quad \frac{dy}{dx} + y \cot x = 2x + x^2 \cot x \text{ given that } y(0) = 0.$$

16. Show that the differential equation

$2y e^{\frac{x}{y}} dx + \left(y - 2x e^{\frac{x}{y}}\right) dy = 0$  is homogenous. Find the particular solution of this differential equation given that  $x = 0$  when  $y = 1$ .

### ANSWERS

1. (i) order = 1, degree is not defined

(ii) order = 2, degree = 1

(iii) order = 4, degree = 1

(iv) order = 5, degree is not defined.

(v) order = 2, degree = 2

(vi) order = 2, degree = 2

(vii) order = 3, degree = 2

(viii) order = 1, degree is not defined

2. (I)  $e^{\sin x}$  (II)  $e^{\tan x}$

(III)  $e^{-1/x}$  (IV)  $e^{\frac{(\log x)^2}{2}}$

(V)  $\frac{1}{x^3}$  (VI)  $\sec x$

(VII)  $e^{\tan^{-1} x}$

3. (I) 1 (II) 1  
(III) 2 (IV) 1  
(V) 1 (VI) 2  
(VII) 2
4. (I)  $y = \frac{x^6}{6} + \frac{x^3}{3} - 2 \log |x| + c$  (II)  $y = \log_e |e^x + e^{-x}| + c$   
(III)  $y = \frac{x^4}{4} + e^x + \frac{x^{e+1}}{e+1} + c$  (IV)  $5^x + 5^{-y} = c$   
(V)  $2(y - x) + \sin 2y + \sin 2x = c$   
(VI)  $2 \log |3x + 1| + 3 \log |1 - 2y| = c$
5. (VI)  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + 2y = 0$   
(VII)  $x \left( \frac{dy}{dx} \right)^2 + xy \frac{d^2y}{dx^2} = y \frac{dy}{dx}$   
(VIII)  $2 \frac{d^2y}{dx^2} + \left( \frac{dy}{dx} \right)^3 = 0$
6. (I)  $y\sqrt{1-x^2} + \frac{x\sqrt{1-x^2}}{2} = \frac{\sin^{-1} x}{2} + 2$   
(II)  $y = \frac{x^2(4\log_e x - 1)}{16} + \frac{c}{x^2}$   
(III)  $y = \sin x + \frac{c}{x}, x > 0$   
(IV)  $2y \sin x = 3 - \cos 2x$   
(V)  $xy = \frac{y^4}{4} + c$   
(VI)  $x = -y^2 e^{-y} + cy^2$

7. (I)  $cy = (x + 2)(1 - 2y)$
- (II)  $(e^x + 2) \sec y = c$
- (III)  $\sqrt{1 - x^2} + \sqrt{1 - y^2} = c$
- (IV)  $\frac{1}{2} \log \left| \frac{\sqrt{1-y^2}-1}{\sqrt{1-y^2}+1} \right| = \sqrt{1-x^2} - \sqrt{1-y^2} + c$
- (V)  $(x^2 + 1)(y^2 + 1) = 2$
- (VI)  $\log y = -\frac{1}{4} \cos^4 x + \frac{1}{6} \cos^6 x - xe^x + e^x + c$
- $$= \frac{1}{16} \left[ \frac{\cos^3 2x}{3} - \cos 2x \right] - (x - 1)e^x + c$$
- (VII)  $\log |\tan y| - \frac{\cos 2x}{4} = c$
- (VIII)  $\log |y + 1| = \frac{x^2}{2} - x + c$
8. (I)  $\frac{-x^3}{y^3} + 3 \log |y| = c$
- (II)  $\tan^{-1} \left( \frac{y}{x} \right) = \log |x| + c$
- (III)  $x^2 + y^2 = 2x$
- (IV)  $y = ce^{\cos(x/y)}$
- (V)  $\sin \left( \frac{y}{x} \right) = cx$
- (VI)  $\log |y/x| = cx$
- (VII)  $-e^{-y} = e^x + \frac{x^3}{3} + c$

$$(VIII) \quad \sin^{-1} y = \sin^{-1} x + c$$

$$(IX) \quad (2xy + y^2) = \frac{c}{x^2}$$

$$9. \quad (I) \quad x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

$$(II) \quad 2y = x \frac{dy}{dx}, \quad y = 2x \frac{dy}{dx}$$

$$(III) \quad x^2 - y^2 + 2xy \frac{dy}{dx} = 0$$

$$(IV) \quad (x - y)^2 (1 + y'^2) = (x + yy')^2$$

$$10. \quad \log|x^2 + xy + y^2| = 2\sqrt{3} \tan^{-1} \left( \frac{x+2y}{\sqrt{3x}} \right) + c$$

$$11. \quad x^2 + y^2 = c(x + y)$$

$$12. \quad (I) \quad y = \frac{3 \sin 3x}{13} - \frac{2 \cos 3}{13} + Ce^{2x}$$

$$(II) \quad y = \frac{2}{3} \sin^2 x + \frac{1}{3} \operatorname{cosec} x$$

$$(III) \quad \frac{e^{px}}{p} + \frac{e^{-qy}}{q} = c$$

$$13. \quad (I) \quad -y = x \log\{c(x - y)\}$$

$$(II) \quad cx^2 = y + \sqrt{x^2 + y^2}$$

$$(III) \quad xy \cos\left(\frac{y}{x}\right) = c$$

$$(IV) \quad 3x^2y = y + 2x$$

$$(V) \quad y = -x \log(\log|x|), \quad x \neq 0$$

$$(VI) \quad c(x^2 + y^2) = \sqrt{x^2 - y^2}$$

$$(VII) \quad \cos \frac{y}{x} = \log|x| + 1$$

$$14. \quad (I) \quad y = \tan x - 1 + ce^{-\tan x}$$

$$(II) \quad y = \frac{\sin x}{x} + c \frac{\cos x}{x}$$

$$(III) \quad x + ye^{\frac{x}{y}} = c$$

$$(IV) \quad 2y = \sin x$$

$$15. \quad (I) \quad cxy = \sec\left(\frac{y}{x}\right)$$

$$(II) \quad (1 - e)^3 \tan y = (1 - e^x)^3$$

$$(III) \quad y = x^2$$

$$16. \quad e^{x/y} = -\frac{1}{2} \log|y| + 1$$

## CHAPTER–10

### VECTORS

#### POINTS TO REMEMBER

- A quantity that has magnitude as well as direction is called a vector. It is denoted by a directed line segment.
- Two or more vectors which are parallel to same line are called collinear vectors.
- Position vector of a point  $P(a, b, c)$  w.r.t. origin  $(0, 0, 0)$  is denoted by  $\overrightarrow{OP}$  where  $\overrightarrow{OP} = a\hat{i} + b\hat{j} + c\hat{k}$  and  $|\overrightarrow{OP}| = \sqrt{a^2 + b^2 + c^2}$ .
- If  $A(x_1, y_1, z_1)$  and  $B(x_2, y_2, z_2)$  be any two points in space, then

$$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- Any vector  $\vec{a}$  is called unit vector if  $|\vec{a}| = 1$ . It is denoted by  $\hat{a}$ .
- If two vectors  $\vec{a}$  and  $\vec{b}$  are represented in magnitude and direction by the two sides of a triangle in order, then their sum  $\vec{a} + \vec{b}$  is represented in magnitude and direction by third side of a triangle taken in opposite order. This is called triangle law of addition of vectors.
- If  $\vec{a}$  is any vector and  $\lambda$  is a scalar, then  $\lambda \vec{a}$  is vector collinear with  $\vec{a}$  and  $|\lambda \vec{a}| = |\lambda| |\vec{a}|$ .
- If  $\vec{a}$  and  $\vec{b}$  are two collinear vectors, then  $\vec{a} = \lambda \vec{b}$  where  $\lambda$  is some scalar.

- Any vector  $\vec{a}$  can be written as  $\vec{a} = |\vec{a}|\hat{a}$  where  $\hat{a}$  is a unit vector in the direction of  $\vec{a}$ .
- If  $\vec{a}$  and  $\vec{b}$  be the position vectors of points A and B, and C is any point which divides  $\overline{AB}$  in ratio m:n internally then position vector  $\vec{c}$  of point C is given as  $\vec{c} = \frac{m\vec{b} + n\vec{a}}{m+n}$ . If C divides  $\overline{AB}$  in ratio m:n externally, then  $\vec{c} = \frac{m\vec{b} - n\vec{a}}{m-n}$ .
- The angles  $\alpha, \beta$  and  $\gamma$  made by  $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$  with positive direction of x, y and z-axis are called angles and cosines of these angles are called direction cosines of  $\vec{r}$  usually denoted as  $l = \cos \alpha$ ,  $m = \cos \beta$ ,  $n = \cos \gamma$

Also  $l = \frac{a}{|\vec{r}|}$ ,  $m = \frac{b}{|\vec{r}|}$ ,  $n = \frac{c}{|\vec{r}|}$  and  $l^2 + m^2 + n^2 = 1$

- The numbers a, b, c proportional to l, m, n are called direction ratios.
- Scalar product or dot product of two vectors  $\vec{a}$  and  $\vec{b}$  is denoted as  $\vec{a} \cdot \vec{b}$  and is defined as  $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos \theta$ ,  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ . ( $0 \leq \theta \leq \pi$ ).
- Dot product of two vectors is commutative i.e.  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} = \vec{0}, \vec{b} = \vec{0}$  or  $\vec{a} \perp \vec{b}$ .
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ , so  $\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$
- If  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$  and  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ , then

$$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3.$$



- Projection of  $\vec{a}$  on  $\vec{b} = \frac{|\vec{a} \cdot \vec{b}|}{|\vec{b}|}$  and

Projection vector of  $\vec{a}$  along  $\vec{b} = \left( \frac{(\vec{a} \cdot \vec{b})}{|\vec{b}|^2} \right) \vec{b}$ .

- Cross product or vector product of two vectors  $\vec{a}$  and  $\vec{b}$  is denoted as  $\vec{a} \times \vec{b}$  and is defined as  $\vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n}$ , where  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$  ( $0 \leq \theta \leq \pi$ ). And  $\hat{n}$  is a unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$  such that  $\vec{a}$ ,  $\vec{b}$  and  $\hat{n}$  form a right handed system.
- Cross product of two vectors is not commutative i.e.,  $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$ , but  $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$ .
- $\vec{a} \times \vec{b} = \vec{0} \Leftrightarrow \vec{a} = \vec{0}, \vec{b} = \vec{0}$  or  $\vec{a} \parallel \vec{b}$ .
- $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = \vec{0}$ .
- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$  and  $\hat{j} \times \hat{i} = -\hat{k}, \hat{k} \times \hat{j} = -\hat{i}, \hat{i} \times \hat{k} = -\hat{j}$
- If  $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$  and  $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$ , then

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

- Unit vector perpendicular to both  $\vec{a}$  and  $\vec{b} = \pm \left( \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} \right)$ .
- $|\vec{a} \times \vec{b}|$  is the area of parallelogram whose adjacent sides are  $\vec{a}$  and  $\vec{b}$
- $\frac{1}{2} |\vec{a} \times \vec{b}|$  is the area of parallelogram where diagonals are  $\vec{a}$  and  $\vec{b}$ .
- If  $\vec{a}, \vec{b}$  and  $\vec{c}$  form a triangle, then area of the triangle

- $= \frac{1}{2} |\vec{a} \times \vec{b}| = \frac{1}{2} |\vec{b} \times \vec{c}| = \frac{1}{2} |\vec{c} \times \vec{a}|.$
- Scalar triple product of three vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  is defined as  $\vec{a} \cdot (\vec{b} \times \vec{c})$  and is denoted as  $[\vec{a} \vec{b} \vec{c}]$
- Geometrically, absolute value of scalar triple product  $[\vec{a} \vec{b} \vec{c}]$  represents volume of a parallelepiped whose coterminal edges are  $\vec{a}, \vec{b}$  and  $\vec{c}$ .
- $\vec{a}, \vec{b}, \vec{c}$  are coplanar  $\Leftrightarrow [\vec{a} \vec{b} \vec{c}] = 0$
- $[\vec{a} \vec{b} \vec{c}] = [\vec{b} \vec{c} \vec{a}] = [\vec{c} \vec{a} \vec{b}]$
- If  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  and  $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$  then
 
$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$
- Then scalar triple product of three vectors is zero if any two of them are same or collinear.

### Very Short Answer Type Questions (1 MARK)

1. If  $\vec{AB} = 3\hat{i} + 2\hat{j} - \hat{k}$  and the coordinate of A are (4,1,1), then find the coordinates of B.
2. Let  $\vec{a} = -2\hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j}$  and  $\vec{c} = 4\hat{i} + 3\hat{j}$ . Find the values of x and y such that  $\vec{c} = x\vec{a} + y\vec{b}$ .
3. Find a unit vector in the direction of the resultant of the vectors  $\hat{i} - \hat{j} + 3\hat{k}, 2\hat{i} + \hat{j} - 2\hat{k}$  and  $\hat{i} + 2\hat{j} - 2\hat{k}$ .

4. Find a vector of magnitude of 5 units parallel to the resultant of vector  $\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$  and  $\vec{b} = (\hat{i} - 2\hat{j} - \hat{k})$
5. For what value of  $\lambda$  are the vector  $\vec{a}$  and  $\vec{b}$  perpendicular to each other?  
Where  $\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$  and  $\vec{b} = 5\hat{i} - 9\hat{j} + 2\hat{k}$
6. Write the value of p for which  $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$  and  $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$  are parallel vectors.
7. For any two vectors  $\vec{a}$  and  $\vec{b}$  write when  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  holds.
8. Find the value of p if  $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = \vec{0}$
9. Evaluate:  $\hat{i} \cdot (\hat{j} \times \hat{k}) + (\hat{i} \times \hat{k}) \cdot \hat{j}$
10. If  $\vec{a} = 2\hat{i} - 3\hat{j}$ ,  $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ ,  $\vec{c} = 3\hat{i} - \hat{k}$ , find  $[\vec{a}\vec{b}\vec{c}]$
11. If  $\vec{a} = 5\hat{i} - 4\hat{j} + \hat{k}$ ,  $\vec{b} = -4\hat{i} + 3\hat{j} - 2\hat{k}$  and  $\vec{c} = \hat{i} - 2\hat{j} - 2\hat{k}$ , then evaluate  $\vec{c} \cdot (\vec{a} \times \vec{b})$
12. Show that vector  $\hat{i} + 3\hat{j} + \hat{k}$ ,  $2\hat{i} - \hat{j} - \hat{k}$ ,  $7\hat{j} + 3\hat{k}$  are parallel to same plane.
13. Find a vector of magnitude 6 which is perpendicular to both the vectors  $2\hat{i} - \hat{j} + 2\hat{k}$  and  $4\hat{i} - \hat{j} + 3\hat{k}$ .
14. If  $\vec{a} \cdot \vec{b} = 0$ , then what can you say about  $\vec{a}$  and  $\vec{b}$ ?
15. If  $\vec{a}$  and  $\vec{b}$  are two vectors such that  $|\vec{a} \times \vec{b}| = \vec{a} \cdot \vec{b}$ , then what is the angle between  $\vec{a}$  and  $\vec{b}$ ?

16. Find the area of a parallelogram having diagonals  $3\hat{i} + \hat{j} - 2\hat{k}$  and  $\hat{i} - 3\hat{j} + 4\hat{k}$ .
17. If  $\hat{i}, \hat{j}$  and  $\hat{k}$  are three mutually perpendicular vectors, then find the value of  $\hat{j} \cdot (\hat{k} \times \hat{i})$ .
18. P and Q are two points with position vectors  $3\vec{a} - 2\vec{b}$  and  $\vec{a} + \vec{b}$  respectively. Write the position vector of a point R which divides the segment PQ in the ratio 2:1 externally.
19. Find  $\lambda$  when scalar projection of  $\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$  on  $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$  is 4 units.
20. Find "a" so that the vectors  $\vec{p} = 3\hat{i} - 2\hat{j}$  and  $\vec{q} = 2\hat{i} + a\hat{j}$  be orthogonal.
21. If  $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{c} = \lambda\hat{i} - \hat{j} + \lambda\hat{k}$  are coplanar, find the value of  $\lambda$ .
22. What is the point of trisection of PQ nearer to P if positions of P and Q are  $3\hat{i} + 3\hat{j} - 4\hat{k}$  and  $9\hat{i} + 8\hat{j} - 10\hat{k}$  respectively?
23. What is the angle between  $\vec{a}$  and  $\vec{b}$ , if  $\vec{a} \cdot \vec{b} = 3$  and  $|\vec{a} \times \vec{b}| = 3\sqrt{3}$ .

### SHORT ANSWER TYPE QUESTIONS (2 MARKS)

- Q.1. A vector  $\vec{r}$  is inclined to x – axis at  $45^\circ$  and y-axis at  $60^\circ$  if  $|\vec{r}| = 8$  units. find  $\vec{r}$ .
- Q.2. if  $|\vec{a} + \vec{b}| = 60$ ,  $|\vec{a} - \vec{b}| = 40$  and  $|\vec{b}| = 46$  find  $|\vec{a}|$

Q.3. Write the projection of  $\vec{b} + \vec{c}$  on  $\vec{a}$  where

$$\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k} \text{ and } \vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$$

Q.4. If the points  $(-1, -1, 2)$ ,  $(2, m, 5)$  and  $(3, 11, 6)$  are collinear, find the value of  $m$ .

Q.5. For any three vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  write value of the following.

$$\vec{a} \times (\vec{b} + \vec{c}) + \vec{b} \times (\vec{c} + \vec{a}) + \vec{c} \times (\vec{a} + \vec{b})$$

Q.6. If  $(\vec{a} + \vec{b})^2 + (\vec{a} - \vec{b})^2 = 144$  and  $|\vec{a}| = 4$ . Find the value of  $|\vec{b}|$ .

Q.7. If for any two vectors  $\vec{a}$  and  $\vec{b}$ ,

$$(\vec{a} + \vec{b})^2 + (\vec{a} - \vec{b})^2 = \lambda [(\vec{a})^2 + (\vec{b})^2] \text{ then write the value of } \lambda.$$

Q.8. if  $\vec{a}, \vec{b}$  are two vectors such that  $|(\vec{a} + \vec{b})| = |\vec{a}|$  then prove that  $2\vec{a} + \vec{b}$  is perpendicular to  $\vec{b}$ .

Q.9. Show that vectors  $\vec{a} = 3\hat{i} - 2\hat{j} + \hat{k}$

$$\vec{b} = \hat{i} - 3\hat{j} + 5\hat{k}, \vec{c} = 2\hat{i} + \hat{j} - 4\hat{k} \text{ form a right angle triangle.}$$

Q.10. If  $\vec{a}, \vec{b}, \vec{c}$  are three vectors such that  $\vec{a} + \vec{b} + \vec{c} = 0$  and  $|\vec{a}| = 5$ ,  $|\vec{b}| = 12$ ,  $|\vec{c}| = 13$ , then find  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

Q.11. The two vectors  $\hat{i} + \hat{j}$  and  $3\hat{i} - \hat{j} + 4\hat{k}$  represents the two sides AB and AC respectively of  $\Delta ABC$ , find the length of median through A.

### Short Answer Type Questions (4 Marks)

- The points A, B and C with position vectors  $3\hat{i} - y\hat{j} + 2\hat{k}$ ,  $5\hat{i} - \hat{j} + \hat{k}$  and  $3x\hat{i} + 3\hat{j} - \hat{k}$  are collinear. Find the values of x and y and also the ratio in which the point B divides AC.
- If sum of two unit vectors is a unit vector, prove that the magnitude of their difference is  $\sqrt{3}$ .
- Let  $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$ ,  $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ . Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{a}$  and  $\vec{b}$  and satisfying  $\vec{d} \cdot \vec{c} = 21$
- If  $\hat{a}$  and  $\hat{b}$  are unit vectors inclined at an angle  $\theta$  then proved that
  - $\cos \frac{\theta}{2} = \frac{1}{2} |\hat{a} + \hat{b}|$
  - $\tan \frac{\theta}{2} = \left| \frac{\hat{a} - \hat{b}}{\hat{a} + \hat{b}} \right|$
- If  $\vec{a}, \vec{b}, \vec{c}$  are three mutually perpendicular vectors of equal magnitude. Prove that  $\vec{a} + \vec{b} + \vec{c}$  is equally inclined with vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$ . Also find angles.
- For any vector  $\vec{a}$  prove that  $|\vec{a} \times \hat{i}|^2 + |\vec{a} \times \hat{j}|^2 + |\vec{a} \times \hat{k}|^2 = 2|\vec{a}|^2$
- Show that  $(\vec{a} \times \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} \\ \vec{a} \cdot \vec{b} & \vec{b} \cdot \vec{b} \end{vmatrix}$
- If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are the position vectors of vertices A, B, C of a  $\Delta ABC$ , show that the area of triangle ABC is  $\frac{1}{2} |\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}|$ . Deduce the condition for points  $\vec{a}, \vec{b}$  and  $\vec{c}$  to be collinear.

9. Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be unit vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$  and the angle between  $\vec{b}$  and  $\vec{c}$  is  $\pi/6$ , prove that  $\vec{a} = \pm 2(\vec{b} \times \vec{c})$ .
10. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are three vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ , then prove that  $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ .
11. If  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{c} = \hat{j} - \hat{k}$  are given vectors, then find a vector  $\vec{b}$  satisfying the equations  $\vec{a} \times \vec{b} = \vec{c}$  and  $\vec{a} \cdot \vec{b} = 3$ .
12. Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be three non zero vectors such that  $\vec{c}$  is a unit vector perpendicular to both  $\vec{a}$  and  $\vec{b}$ . If the angle between  $\vec{a}$  and  $\vec{b}$  is  $\pi/6$ , prove that  $[\vec{a}\vec{b}\vec{c}]^2 = \frac{1}{4}|\vec{a}|^2|\vec{b}|^2$ .
13. If the vectors  $\vec{\alpha} = a\hat{i} + \hat{j} + \hat{k}$ ,  $\vec{\beta} = \hat{i} + b\hat{j} + \hat{k}$  and  $\vec{\gamma} = \hat{i} + \hat{j} + c\hat{k}$  are coplanar, then prove that  $\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$  where  $a \neq 1, b \neq 1$  and  $c \neq 1$ .
14. Find the altitude of a parallelepiped determined by the vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  if the base is taken as parallelogram determined by  $\vec{a}$  and  $\vec{b}$  and if  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$  and  $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$ .
15. Prove that the four points  $(4\hat{i} + 5\hat{j} + \hat{k})$ ,  $-(\hat{j} + \hat{k})$ ,  $(3\hat{i} + 9\hat{j} + 4\hat{k})$  and  $4(-\hat{i} + \hat{j} + \hat{k})$  are coplanar.
16. If  $|\vec{a}| = 3$ ,  $|\vec{b}| = 4$  and  $|\vec{c}| = 5$  such that each is perpendicular to sum of the other two, find  $|\vec{a} + \vec{b} + \vec{c}|$ .
17. Decompose the vector  $6\hat{i} - 3\hat{j} - 6\hat{k}$  into vectors which are parallel and perpendicular to the vector  $\hat{i} + \hat{j} + \hat{k}$ .

18. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ ,  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ ,  $a \neq 0$ , then show that  $\vec{b} = \vec{c}$ .
19. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are three non zero vectors such that  $\vec{a} \times \vec{b} = \vec{c}$  and  $\vec{b} \times \vec{c} = \vec{a}$ . Prove that  $\vec{a}, \vec{b}$  and  $\vec{c}$  are mutually at right angles and  $|\vec{b}| = 1$  and  $|\vec{c}| = |\vec{a}|$ .
20. Simplify  $[\vec{a} - \vec{b}, \vec{b} - \vec{c}, \vec{c} - \vec{a}]$
21. If  $[\vec{a}\vec{b}\vec{c}] = 2$ , find the volume of the parallelepiped whose co-terminus edges are  $2\vec{a} + \vec{b}$ ,  $2\vec{b} + \vec{c}$ ,  $2\vec{c} + \vec{a}$ .
22. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are three vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$  and  $|\vec{a}| = 3$ ,  $|\vec{b}| = 5$ ,  $|\vec{c}| = 7$ , find the angle between  $\vec{a}$  and  $\vec{b}$ .
23. The magnitude of the vector product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of the vector  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to  $\sqrt{2}$ . Find the value of  $\lambda$ .
24. If  $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$  and  $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ , prove that  $(\vec{a} - \vec{d})$  is parallel to  $(\vec{b} - \vec{c})$ , where  $\vec{a} \neq \vec{d}$  and  $\vec{b} \neq \vec{c}$ .
25. Find a vector of magnitude  $\sqrt{171}$  which is perpendicular to both of the vectors  $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$  and  $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$ .
26. If  $a$  is a nonzero real number, prove that the vectors  $\vec{\alpha} = a\hat{i} + 2a\hat{j} - 3a\hat{k}$ ,  $\vec{\beta} = (2a + 1)\hat{i} + (2a + 3)\hat{j} + (a + 1)\hat{k}$  and  $\vec{\gamma} = (3a + 5)\hat{i} + (a + 5)\hat{j} + (a + 2)\hat{k}$  are never coplanar.



27. If  $\vec{\alpha} = 3\hat{i} - \hat{j}$  and  $\vec{\beta} = 2\hat{i} + \hat{j} + 3\hat{k}$  then express  $\vec{\beta}$  in the form of  $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$ , where  $\vec{\beta}_1$  is parallel to  $\vec{\alpha}$  and  $\vec{\beta}_2$  is perpendicular to  $\vec{\alpha}$ .
28. Find a unit vector perpendicular to plane ABC, when position vectors of A,B,C are  $3\hat{i} - \hat{j} + 2\hat{k}$ ,  $\hat{i} - \hat{j} - 3\hat{k}$  and  $4\hat{i} - 3\hat{j} + \hat{k}$  respectively.
29. Find a unit vector in XY plane which makes an angle  $45^\circ$  with the vector  $\hat{i} + \hat{j}$  at angle of  $60^\circ$  with the vector  $3\hat{i} - 4\hat{j}$ .
30. Suppose  $\vec{a} = \lambda\hat{i} - 7\hat{j} + 3\hat{k}$ ,  $\vec{b} = \lambda\hat{i} + \hat{j} + 2\lambda\hat{k}$ . If the angle between  $\vec{a}$  and  $\vec{b}$  is greater than  $90^\circ$ , then prove that  $\lambda$  satisfies the inequality  $-7 < \lambda < 1$ .
31. Let  $\vec{v} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{w} = \hat{i} + 3\hat{k}$ . If  $u$  is a unit vector, then find the maximum value of the scalar triple products  $\vec{u}, \vec{v}, \vec{w}$ .
32. If  $\vec{a} = \hat{i} - \hat{k}$ ,  $\vec{b} = x\hat{i} + \hat{j} + (1-x)\hat{k}$  and  $\vec{c} = y\hat{i} + x\hat{j} + (1+x-y)\hat{k}$  then prove that  $[\vec{a}\vec{b}\vec{c}]$  depends upon neither  $x$  nor  $y$ .
33. A, b and c are distinct non negative numbers, if the vectors  $a\hat{i} + a\hat{j} + c\hat{k}$ ,  $\hat{i} + \hat{k}$  and  $c\hat{i} + c\hat{j} + b\hat{k}$  lie in a plane, then prove that c is the geometric mean of a and b.
34. If  $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$  and vectors  $(1, a, a^2)$ ,  $(1, b, b^2)$  and  $(1, c, c^2)$  are non-coplanar, then find the value of abc. (Ans. = -1)

## Answers

### Very Short Answer

1.  $(7, 3, 0)$
2.  $x = -1, y = 2$
3.  $\frac{1}{\sqrt{21}}(4\hat{i} + 2\hat{j} - \hat{k})$
4.  $\sqrt{\frac{5}{2}}(3\hat{i} + \hat{j})$
5.  $\lambda = \frac{16}{5}$
6.  $\frac{2}{3}$
7.  $\vec{a}$  and  $\vec{b}$  are perpendicular
8.  $\frac{27}{2}$
9.  $0$
10.  $4$
11.  $-5$
12. ....
13.  $-2\hat{i} + 4\hat{j} + 4\hat{k}$

14. Either  $\vec{a} = 0$  or  $\vec{b} = 0$  or  $\vec{a} \perp \vec{b}$
15.  $45^\circ$
16.  $5\sqrt{3}$  sq. Units
17. 1
18.  $-\vec{a} + 4\vec{b}$
19.  $\lambda = 5$
20.  $a = 3$
21.  $\lambda = 1$
22.  $\left(5, \frac{14}{3}, -6\right)$
23.  $\frac{\pi}{3}$

**SHORT ANSWER TYPE [2 MARKS]**

1.  $4(\sqrt{2}\hat{i} + \hat{j} + \hat{k})$
2. 22
3. 2
4.  $m = 8$
5. 0
6. 3
7.  $\lambda = 2$
8. —
9. —

10.  $-169$

11.  $2\sqrt{2}$

### Short Answer Type Answer (4 marks)

1.  $x = 3, y = 3, 1:2$

3.  $\vec{d} = 7\hat{i} - 7\hat{j} - 7\hat{k}$

5.  $\cos^{-1} \frac{1}{\sqrt{3}}$

11.  $\vec{b} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$

14.  $\frac{4}{\sqrt{38}}$  units

16.  $5\sqrt{2}$

17.  $(-\hat{i} - \hat{j} - \hat{k}) + (7\hat{i} - 2\hat{j} - 5\hat{k})$

20.  $0$

21.  $18 \text{ cu. Units}$

22.  $60^\circ$

23.  $\lambda = 1$

25.  $\hat{i} - 11\hat{j} - 7\hat{k}$

27.  $\vec{\beta} = \left(\frac{3}{2}\hat{i} - \frac{1}{2}\hat{j}\right) + \left(\frac{1}{2}\hat{i} + \frac{3}{2}\hat{j} + 3\hat{k}\right)$

28.  $\frac{-1}{\sqrt{165}}(10\hat{i} + 7\hat{j} - 4\hat{k})$

29.  $\frac{13}{\sqrt{170}} \hat{i} + \frac{1}{\sqrt{170}} \hat{j}$

31.  $\sqrt{59}$

34.  $-1$

## CHAPTER–11

### THREE-DIMENSIONAL GEOMETRY

#### POINTS TO REMEMBER

- **Distance Formula:** Distance (d) between two points  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

- **Section Formula:** line segment AB is divided by P (x, y, z) in ratio m:n

(a) Internally	(b) Externally
$\left( \frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}, \frac{mz_2 + nz_1}{m+n} \right)$	$\left( \frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}, \frac{mz_2 - nz_1}{m-n} \right)$

- **Direction ratio** of a line through  $(x_1, y_1, z_1)$  and  $(x_2, y_2, z_2)$  are  $x_2 - x_1, y_2 - y_1, z_2 - z_1$

- **Direction cosines** of a line having direction ratios as a, b, c are:

$$l = \pm \frac{a}{\sqrt{a^2+b^2+c^2}}, \quad m = \pm \frac{b}{\sqrt{a^2+b^2+c^2}}, \quad n = \pm \frac{c}{\sqrt{a^2+b^2+c^2}}$$

- **Equation of line in space:**

Vector form	Cartesian form
(i) Passing through point $\vec{a}$ and parallel to vector $\vec{b}$ ; $\vec{r} = \vec{a} + \lambda \vec{b}$	(i) Passing through point $(x_1, y_1, z_1)$ and having direction ratios a, b, c;

	$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$
(ii) Passing through two points $\vec{a}$ and $\vec{b}$ ; $\vec{r} = \vec{a} + \lambda (\vec{b} - \vec{a})$	(ii) Passing through two points $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ ; $\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$

- **Angle between two lines:**

Vector form	Cartesian form
(i) For lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ $\cos \theta = \frac{ \vec{b}_1 \cdot \vec{b}_2 }{ \vec{b}_1   \vec{b}_2 }$	(ii) For lines $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and $\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ $\cos \theta = \frac{ a_1 a_2 + b_1 b_2 + c_1 c_2 }{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$
(iii) Lines are perpendicular if $\vec{b}_1 \cdot \vec{b}_2 = 0$	(ii) Lines are perpendicular if $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$
(iv) Lines are parallel if $\vec{b}_1 = k \vec{b}_2$ ; $k \neq 0$	(i) Lines are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

- **Equation of plane:**

If p is length of perpendicular from origin to plane and $\hat{n}$ is unit vector normal to plane $\vec{r} \cdot \hat{n} = p$	If p is length of perpendicular from origin to plane and l, m, n are d.c.s of normal to plane $lx + my + nz = p$
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Passing through $\vec{a}$ and $\vec{n}$ is normal to plane : $(\vec{r} - \vec{a}) \cdot \vec{n} = 0$	Passing through $(x_1, y_1, z_1)$ and a, b, c are d.r.s of normal to plane:  $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$
Passing through three non collinear points $\vec{a}, \vec{b}, \vec{c}$ :  $(\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$	Passing through three non collinear points $(x_1, y_1, z_1)(x_2, y_2, z_2)(x_3, y_3, z_3)$ :  $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$
If a, b, c are intercepts on co-ordinate axes $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$	If $x_1, y_1, z_1$ are intercepts on coordinate axes $\frac{x}{x_1} + \frac{y}{y_1} + \frac{z}{z_1} = 1$
Plane passing through line of intersection of planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is  $\vec{r} \cdot (\vec{n}_1 + \lambda \vec{n}_2) = d_1 + \lambda d_2$ ( $\lambda$ =real no.)	Plane passing through the line of intersection of planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ is  $(a_1x + b_1y + c_1z + d_1) + \lambda(a_2x + b_2y + c_2z + d_2) = 0$

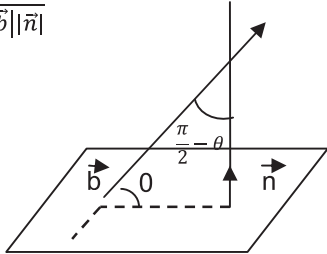
• **Angle between planes:**

Angle $\theta$ between planes $\vec{r} \cdot \vec{n}_1 = d_1$ and $\vec{r} \cdot \vec{n}_2 = d_2$ is  $\cos \theta = \frac{ \vec{n}_1 \cdot \vec{n}_2 }{ \vec{n}_1  \vec{n}_2 }$	Angle $\theta$ between planes $a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is  $\cos \theta = \frac{ a_1a_2 + b_1b_2 + c_1c_2 }{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$
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Planes are perpendicular iff $\vec{n}_1 \cdot \vec{n}_2 = 0$	Planes are perpendicular iff $a_1a_2 + b_1b_2 + c_1c_2 = 0$
Planes are parallel iff $\vec{n}_1 = \lambda \vec{n}_2 ; \lambda \neq 0$	Planes are parallel if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

• **Angle between line and plane:**

<p>Angle <math>\theta</math> between line <math>\vec{r} = \vec{a} + \lambda \vec{b}</math> and plane <math>\vec{r} \cdot \vec{n} = d</math> is <math>\sin \theta = \frac{\vec{b} \cdot \vec{n}}{ \vec{b}   \vec{n} }</math></p> <p><math>\cos(90^\circ - \theta)</math></p> 	<p>Angle <math>\theta</math> between line <math>\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}</math> and plane <math>a_2x + b_2y + c_2z = d</math> is</p> <p><math>\sin \theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}</math></p>
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• **Distance of a point from a plane**

<p>The perpendicular distance <math>p</math> from the point <math>P</math> with position vector <math>\vec{a}</math> to the plane <math>\vec{r} \cdot \vec{n} = d</math> is given by</p> $p = \frac{ \vec{a} \cdot \vec{n} - d }{ \vec{n} }$	<p>The perpendicular distance <math>p</math> from the point <math>P (x_1, y_1, z_1)</math> to the plane <math>Ax + By + Cz + D = 0</math> is given by</p> $p = \frac{ Ax_1 + By_1 + Cz_1 + D }{\sqrt{A^2 + B^2 + C^2}}$
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- Coplanarity**

<p>Two lines <math>\vec{r} = \vec{a}_1 + \lambda \vec{b}_1</math> and <math>\vec{r} = \vec{a}_2 + \mu \vec{b}_2</math> are coplanar iff</p> $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$	<p>Two lines <math>\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}</math> and <math>\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}</math> are coplanar iff</p> $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$
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- Shortest distance between two skew lines**

<p>The shortest distance between lines <math>\vec{r} = \vec{a}_1 + \lambda \vec{b}_1</math> and <math>\vec{r} = \vec{a}_2 + \mu \vec{b}_2</math> is</p> $d = \frac{ (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) }{ \vec{b}_1 \times \vec{b}_2 }$	<p>The shortest distance between <math>\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}</math> and <math>\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}</math> is</p> $d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{D}}$ <p>Where</p> $D = \{(a_1 b_2 - a_2 b_1)^2 + (b_1 c_2 - b_2 c_1)^2 + (c_1 a_2 - c_2 a_1)^2\}$
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### Very Short Answer Type Questions (1 Mark)

1. What is the distance of point (a, b, c) from x-axis?
2. What is the angle between the lines  $2x = 3y = -z$  and  $6x = -y = -4z$ ?
3. Write the equation of a line passing through (2, -3, 5) and parallel to line  $\frac{x-1}{3} = \frac{y-2}{4} = \frac{z+1}{-1}$ .
4. Write the equation of a line through (1, 2, 3) and parallel to  $\vec{r} \cdot (\hat{i} - \hat{j} + 3\hat{k}) = 5$ .
5. What is the value of  $\lambda$  for which the lines  $\frac{x-1}{2} = \frac{y-3}{5} = \frac{z-1}{\lambda}$  and  $\frac{x-2}{3} = \frac{y+1}{-2} = \frac{z}{2}$  are perpendicular to each other?
6. Write line  $\vec{r} = (\hat{i} - \hat{j}) + \lambda (2\hat{j} - \hat{k})$  into Cartesian form.
7. If the direction ratios of a line are 1, -2, 2 then what are the direction cosines of the line?
8. Find the angle between the planes  $2x - 3y + 6z = 9$  and  $xy$  - plane.
9. Write equation of a line passing through (0, 1, 2) and equally inclined to co-ordinate axes.
10. What is the perpendicular distance of plane  $2x - y + 3z = 10$  from origin?

11. What is the y-intercept of the plane  $x - 5y + 7z = 10$ ?
12. What is the distance between the planes  $2x + 2y - z + 2 = 0$  and  $4x + 4y - 2z + 5 = 0$ .
13. What is the equation of the plane which cuts off equal intercepts of unit length on the coordinate axes?
14. Are the planes  $x + y - 2z + 4 = 0$  and  $3x + 3y - 6z + 5 = 0$  intersecting?
15. What is the equation of the plane through the point (1, 4, -2) and parallel to the plane  $-2x + y - 3z = 7$ ?
16. Write the vector equation of the plane which is at a distance of 8 units from the origin and is normal to the vector  $(2\hat{i} + \hat{j} + 2\hat{k})$ .
17. What is equation of the plane if the foot of perpendicular from origin to this plane is (2, 3, 4)?
18. Find the angles between the planes  $\vec{r} \cdot (\hat{i} - 2\hat{j} - 2\hat{k}) = 1$  and  $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = 0$ .
19. If O is origin  $OP = 3$  with direction ratios proportional to -1, 2, -2 then what are the coordinates of P?
20. What is the distance between the line  $\vec{r} = 2\hat{i} - 2\hat{j} + 3\hat{k} + \lambda (\hat{i} + \hat{j} + 4\hat{k})$  from the plane  $\vec{r} \cdot (-\hat{i} + 5\hat{j} - \hat{k}) + 5 = 0$ .
21. Write the line  $2x = 3y = 4z$  in vector form.

22. The line  $\frac{x-4}{1} = \frac{2y-4}{2} = \frac{k-z}{-2}$  lies exactly in the plane  $2x - 4y + z = 7$ .  
Find the value of k.

### SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

- Q.23. What is the angle between the line  $\frac{x+1}{3} = \frac{2y-1}{4} = \frac{2-z}{-4}$  and the plane  $2x + y - 2z + 4 = 0$
- Q.24. Find the equation of a line passing through  $(2, 0, 5)$  and which is parallel to line  $6x - 2 = 3y + 1 = 2z - 2$
- Q.25. Find the equation of the plane passing through the points  $(2, 3, -4)$  and  $(1, -1, 3)$  and parallel to the x – axis.
- Q.26. Find the distance between the planes  $2x + 3y - 4z + 5 = 0$  and  $\hat{r} \cdot (4\hat{i} + 6\hat{j} - 8\hat{k}) = 11$
- Q.27. The equation of a line are  $5x - 3 = 15y + 7 = 3 - 10z$ . Write the direction cosines of the line
- Q.28. If a line makes angle  $\alpha, \beta, \gamma$  with Co-ordinate axis then what is the value of  $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma$
- Q.29. Find the equation of a line passing through the point  $(2, 0, 1)$  and parallel to the line whose equation is  $\vec{r} = (2\lambda + 3)\hat{i} + (7\lambda - 1)\hat{j} + (-3\lambda + 2)\hat{k}$
- Q.30. The plane  $2x - 3y + 6z - 11 = 0$  makes an angle  $\sin^{-1} \alpha$  with x – axis. Find the value of  $\alpha$ .
- Q.31. If  $4x + 4y - cz = 0$  is the equation of the plane passing through the origin that contains the line  $\frac{x+5}{2} = \frac{y}{3} = \frac{z-7}{4}$ , then find the value of c.

- Q.32. Find the equation of the plane passing through the point  $(-2, 1, -3)$  and making equal intercept on the coordinate axes.
- Q.33. Write the sum of intercepts cut off by the plane  $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$  on the three axis.

### Short Answer Type Questions (4 Marks)

34. Find the equation of a plane containing the points  $(0, -1, -1)$ ,  $(-4, 4, 4)$  and  $(4, 5, 1)$ . Also show that  $(3, 9, 4)$  lies on that plane.
35. Find the equation of the plane which is perpendicular to the plane  $\vec{r} \cdot (5\hat{i} + 3\hat{j} + 6\hat{k}) + 8 = 0$  and which is containing the line of intersection of the planes  $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 4$  and  $\vec{r} \cdot (2\hat{i} + \hat{j} - \hat{k}) + 5 = 0$ .
36. Find the distance of the point  $(3, 4, 5)$  from the plane  $x + y + z = 2$  measured parallel to the line  $2x = y = z$ .
37. Find the equation of the plane passing through the intersection of two planes  $x + 2y + 3z - 5 = 0$  and  $3x - 2y - z + 1 = 0$  and cutting equal intercepts on x-axis and z-axis.
38. Find vector and Cartesian equation of a line passing through a point with position vector  $2\hat{i} - \hat{j} + \hat{k}$  and which is parallel to the line joining the points with position vectors  $-\hat{i} + 4\hat{j} + \hat{k}$  and  $\hat{i} + 2\hat{j} + 2\hat{k}$ .
39. Find the equation of the plane passing through the point  $(3, 4, 2)$  and  $(7, 0, 6)$  and is perpendicular to the plane  $2x - 5y = 15$ .

40. Find equation of plane through line of intersection of planes  $\vec{r} \cdot (2\hat{i} + 6\hat{j}) + 12 = 0$  and  $\vec{r} \cdot (3\hat{i} - \hat{j} + 4\hat{k}) = 0$  which is at a unit distance from origin.
41. Find the image of point  $(3, -2, 1)$  in the plane  $3x - y + 4z = 2$ .
42. Find image (reflection) of the point  $(7, 4, -3)$  in the line  $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ .
43. Find equation of a plane passing through the points  $(2, -1, 0)$  and  $(3, -4, 5)$  and parallel to the line  $2x = 3y = 4z$ .
44. Find the distance of the point  $(-1, -5, -10)$  from the point of intersection of line  $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{2}$  and the plane  $x - y + z = 5$ .
45. Find the distance of the point  $(1, -2, 3)$  from the plane  $x - y + z = 5$ , measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ .
46. Find the equation of the plane passing through the intersection of two plane  $3x - 4y + 5z = 10$ ,  $2x + 2y - 3z = 4$  and parallel to the line  $x = 2y = 3z$ .
47. Find the equation of the planes parallel to the plane  $x - 2y + 2z - 3 = 0$  whose perpendicular distance from the point  $(1, 2, 3)$  is 1 unit.
48. Show that the lines  $\text{line } \frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$  and  $\text{line } \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$  intersect each other. Find the point of intersection.

49. Find the shortest distance between the lines:

$$\vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \mu(2\hat{i} + 3\hat{j} + 4\hat{k}) \text{ and}$$

$$\vec{r} = (2\hat{i} + 4\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 5\hat{k}).$$

50. Find the distance of the point  $(-2, 3, -4)$  from the line  $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$  measured parallel to the plane  $4x + 12y - 3z + 1 = 0$ .

51. Find the equation of plane passing through the point  $(-1, -1, 2)$  and perpendicular to each of the plane

$$\vec{r} \cdot (2\hat{i} + 3\hat{j} - 3\hat{k}) = 2 \text{ and } \vec{r} \cdot (5\hat{i} - 4\hat{j} + \hat{k}) = 6$$

52. Find the equation of a plane passing through  $(-1, 3, 2)$  and parallel to each of the line  $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$  and  $\frac{x+2}{-3} = \frac{y-1}{2} = \frac{z+1}{5}$

53. Show that the plane  $\vec{r} \cdot (\hat{i} - 3\hat{j} + 5\hat{k}) = 7$  contains the line

$$\vec{r} = (\hat{i} + 3\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + \hat{j}).$$



### Long Answer Type Questions (6 Marks)

54. Check the co planarity of lines

$$\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k}).$$

$$\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$$

If they are coplanar, find equation of the plane containing the lines.

55. Find shortest distance between the lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{5-y}{2} = \frac{z-7}{1}$$

56. Find the shortest distance between the lines:

$$\vec{r} = (1 - \lambda)\hat{i} + (\lambda - 2)\hat{j} + (3 - 2\lambda)\hat{k}$$

$$\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$$

57. A variable plane is at a constant distance 3 p from the origin and meets the coordinates axes in A, B and C. If the centroid of  $\Delta ABC$  is  $(\alpha, \beta, \gamma)$ , then show that  $\alpha^{-2} + \beta^{-2} + \gamma^{-2} = p^{-2}$

58. A vector  $\vec{n}$  of magnitude 8 units is inclined to x-axis at  $45^\circ$ , y axis at  $60^\circ$  and an acute angle with z-axis. If a plane passes through a point  $(\sqrt{2}, -1, 1)$  and is normal to  $\vec{n}$ , find its equation in vector form.

59. Find the foot of perpendicular from the point  $2\hat{i} - \hat{j} + 5\hat{k}$  on the line  $\vec{r} = (11\hat{i} - 2\hat{j} - 8\hat{k}) + \lambda(10\hat{i} - 4\hat{j} - 11\hat{k})$ . Also find the length of the perpendicular.
60. A line makes angles  $\alpha, \beta, \gamma, \delta$  with the four diagonal of a cube. Prove that  $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$
61. Find the equation of the plane passing through the intersection of planes  $2x + 3y - z = -1$  and  $x + y - 2z + 3 = 0$  and perpendicular to the plane  $3x - y - 2z = 4$ . Also find the inclination of this plane with xy-plane.
62. Find the length and the equations of the line of shortest distance between the lines  $\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$  and  $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ .
63. Show that  $\frac{x-1}{2} = \frac{y+1}{3} = z$  and  $\frac{x+1}{5} = \frac{y-2}{2}, z = 2$ . do not intersect each other.

### ANSWERS

#### 3 DIMENSIONAL GEOMETRY 2 MARKS EACH

- Q.23.  $0^\circ$  (line is parallel to plane )
- Q.24.  $\frac{x-2}{1} = \frac{y}{2} = \frac{z-5}{3}$
- Q.25.  $7y + 4z = 5$
- Q.26.  $\frac{21}{2\sqrt{29}}$  units
- Q.27.  $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$

Q.28. 2

Q.31.  $C = 5$

Q.29.  $\vec{r} = (2\hat{i} + \hat{k}) + \lambda(2\hat{i} + 7\hat{j} - \hat{k})$

Q.32.  $x + y + z = -4$

Q.30.  $\alpha = \frac{2}{7}$

Q.33.  $\frac{5}{2}$

## Answers

1.  $\sqrt{b^2 + c^2}$

11.  $-2$

2.  $90^\circ$

12.  $\frac{1}{6}$

3.  $\frac{x-2}{3} = \frac{y+3}{4} = \frac{z-5}{-1}$

13.  $x + y + z = 1$

4.  $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 3\hat{k})$

14. No

5.  $\lambda = 2$

15.  $-2x + y - 3z = 8$

6.  $\frac{x-1}{0} = \frac{y+1}{2} = \frac{z}{-1}$

16.  $\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) = 24$

7.  $\pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{2}{3}$

17.  $2x + 3y + 4z = 29$

8.  $\cos^{-1}(6/7)$

18.  $\cos^{-1}\left(\frac{11}{21}\right)$

9.  $\frac{x}{a} = \frac{y-1}{a} = \frac{z-2}{a},$

19.  $(-1, 2, -2)$

$a \in R - \{0\}$

20.  $\frac{10}{3\sqrt{3}}$

10.  $\frac{10}{\sqrt{14}}$

21.  $\vec{r} = \vec{0} + \lambda(6\hat{i} + 4\hat{j} + 3\hat{k}).$

22.  $k = 7$
34.  $5x - 7y + 11z + 4 = 0$
35.  $\vec{r} \cdot (-51\hat{i} - 15\hat{j} + 50\hat{k}) = 173$
36. 6 units
37.  $5x + 2y + 5z - 9 = 0$
38.  $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$   
and  $\frac{x-2}{2} = \frac{y+1}{-2} = \frac{z-1}{1}$
39.  $5x + 2y - 3z - 17 = 0$
40.  $\vec{r} \cdot (2\hat{i} + \hat{j} + 2\hat{k}) + 3 = 0$  or  
 $\vec{r} \cdot (-\hat{i} + 2\hat{j} - 2\hat{k}) + 3 = 0$
41.  $(0, -1, -3)$
42.  $\left(-\frac{51}{7}, -\frac{18}{7}, \frac{43}{7}\right)$
43.  $29x - 27y - 22z = 85$
44. 13
45. 1 unit
46.  $x - 20y + 27z = 14$
47.  $x - 2y + 2z = 0$  and  $x - 2y + 2z = 6$
48.  $\left(\frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}\right)$
49.  $\frac{1}{\sqrt{6}}$
50.  $\frac{17}{2}$  units
51.  $\vec{r} \cdot (9\hat{i} + 17\hat{j} + 23\hat{k}) = 20$
52.  $2x - 7y + 4z + 15 = 0$
54.  $x - 2y + z = 0$
55.  $2\sqrt{29}$  units
56.  $\frac{8}{\sqrt{29}}$
58.  $\vec{r} \cdot (\sqrt{2}\hat{i} + \hat{j} + \hat{k}) = 2$
59.  $(1, 2, 3), \sqrt{14}$
61.  $7x + 13y + 4z = 9, \cos^{-1}\left(\frac{4}{\sqrt{234}}\right)$
62.  $SD = 14$  units,  
 $\frac{x-5}{2} = \frac{y-7}{3} = \frac{z-3}{6}$

## CHAPTER 12

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### LINEAR PROGRAMMING

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#### POINTS TO REMEMBER

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- Linear programming is the process used to obtain minimum or maximum value of the linear objectives function under known linear constraints.
- **Objective Functions:** Linear function  $z = ax + by$  where  $a$  and  $b$  are constants, which has to be maximized or minimized is called a linear objective function.
- **Constraints:** the linear inequalities or inequations or restrictions on the variables of a linear programming problem.
- **Feasible Region:** It is defined as a set of points which satisfy all the constraints.
- **To Find Feasible Region:** Draw the graph of all the linear in equations and shade common region determined by all the constraints.
- **Feasible Solutions:** Points within and on the boundary of the feasible region represents feasible solutions of the constraints.
- **Optimal Feasible Solution:** Feasible solution which optimizes the objective function is called optimal feasible solution.

#### Long Answer Type Questions (6 Marks)

1. Solve the following L.P.P. graphically

Minimise and maximise

$$z = 3x + 9y$$

Subject to the constraints

$$x + 3y \leq 60$$

$$x + y \geq 10$$

$$x \leq y$$

$$x \geq 0, y \geq 0$$

2. Determine graphically the minimum value of the objective function  $z = -50x + 20y$ , subject to the constraints.

$$2x - y \geq -5$$

$$3x + y \geq 3$$

$$2x - 3y \leq 12$$

$$x \geq 0, \quad y \geq 0$$

3. Two tailors A and B earn Rs. 150 and Rs. 200 per day respectively. A can stitch 6 shirts and 4 pants per day, while B can stitch 10 shirts and 4 pants per day. How many days shall each work if it is desired to produce at least 60 shirts and 32 pants at a minimum labour cost? Solve the problem graphically.
4. There are two types of fertilisers A and B. A consists of 10% nitrogen and 6% phosphoric acid and B consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that he needs at least 14 kg of nitrogen and 14 kg of phosphoric acid for his crop. If A costs Rs. 6 per kg and B costs Rs. 5 per kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at minimum cost. What is the minimum cost?
5. A man has Rs. 1500 to purchase two types of shares of two different companies  $S_1$  and  $S_2$ . Market price of one share of  $S_1$  is Rs. 180 and  $S_2$  is Rs. 120. He wishes to purchase a maximum of ten shares only. If

one share of type  $S_1$  gives a yield of Rs. 11 and of type  $S_2$  yields Rs. 8 then how much shares of each type must be purchased to get maximum profit? And what will be the maximum profit?

6. A company manufactures two types of lamps say A and B. Both lamps go through a cutter and then a finisher. Lamp A requires 2 hours of the cutter's time and 1 hours of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 100 hours and finisher has 80 hours of time available each month. Profit on one lamp A is Rs. 7.00 and on one lamp B is Rs. 13.00. Assuming that he can sell all that he produces, how many of each type of lamps should be manufactured to obtain maximum profit?
7. A dealer wishes to purchase a number of fans and sewing machines. He has only Rs. 5760 to invest and has space for atmost 20 items. A fan and sewing machine cost Rs. 360 and Rs. 240 respectively. He can sell a fan at a profit of Rs. 22 and sewing machine at a profit of Rs. 18. Assuming that he can sell whatever he buys, how should he invest his money to maximise his profit?
8. If a young man rides his motorcycle at 25 km/h, he has to spend Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/h, the petrol cost increase to Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to cover the maximum distance within one hour. Express this as L.P.P. and then solve it graphically.
9. A producer has 20 and 10 units of labour and capital respectively which he can use to produce two kinds of goods X and Y. To produce one unit of X, 2 units of capital and 1 unit of labour is required. To produce one unit of Y, 3 units of labour and 1 unit of capital is required. If X and

Y are priced at Rs. 80 and Rs. 100 per unit respectively, how should the producer use his resources to maximise the total revenue?

10. A factory owner purchases two types of machines A and B for his factory. The requirements and limitations for the machines are as follows:

Machine	Area Occupied	Labour Force	Daily Output(In units)
<b>A</b>	1000 $m^2$	12 men	50
<b>B</b>	1200 $m^2$	8 men	40

He has maximum area of 7600  $m^2$  available and 72 skilled labourers who can operate both the machines. How many machines of each type should he buy to maximise the daily output?

11. A manufacturer makes two types of cups A and B. Three machines are required to manufacture the cups and the time in minutes required by each in as given below:

Types of Cup	Machines		
	I	II	III
<b>A</b>	12	18	6
<b>B</b>	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise and on B is 50 paise, find how many cups of each type should be manufactures to maximise the profit per day.

12. A company produces two types of belts A and B. Profits on these belts are Rs. 2 and Rs. 1.50 per belt respectively. A belt of type A requires twice as much time as belt of type B. The company can produce at most 1000 belts of type B per day. Material for 800 belts per day is



available. At most 400 buckles for belts of type A and 700 for type B are available per day. How much belts of each type should the company produce so as to maximize the profit?

13. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at a least 20 seats for first class. However at least four times as many passengers prefer to travel by second class than by first class. Determine how many tickets of each type must be sold to maximize profit for the airline.
14. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 units of calories. Two foods A and B are available at a cost of Rs. 5 and Rs. 4 per unit respectively. One unit of food A contains 200 units of vitamins, 1 unit of minerals and 40 units of calories whereas one unit of food B contains 100 units of vitamins, 2 units of minerals and 40 units of calories. Find what combination of the food A and B should be used to have least cost but it must satisfy the requirements of the sick person.
15. Anil wants to invest at most Rs, 12000 in bonds A and B. According to the rules, he has to invest at least Rs. 2000 in Bond A and at least Rs. 4000 in bond B. If the rate of interest on bond A and B are 8% and 10% per annum respectively, how should he invest this money for maximum interest? Formulate the problem as L.P.P. and solve graphically.

## Answers

1. Min  $z = 60$  at  $x = 5, y = 5$   
Max  $z = 180$  at the two corner points  $(0, 20)$  and  $(15, 5)$ .
2. No minimum value
3. Minimum cost = Rs. 1350 at 5 days of A and 3 days of B.
4. 100 kg of fertiliser A and 80 kg of fertilisers B; minimum cost Rs. 1000.
5. Maximum Profit = Rs. 95 with 5 shares of each type.
6. Lamps of type A = 40, Lamps of type B = 20.
7. Fan: 8; Sewing machine: 12, Maximum Profit = Rs. 392.
8. At 25 km/h he should travel  $50/3$  km, at 40 km/h,  $40/3$  km. Maximum distance 30 km in 1 hr.
9. X: 2 units; Y: 6 units; Maximum revenue Rs. 760.
10. Type A: 4; Type B: 3
11. Cup A: 15; Cup B: 30
12. Maximum profit Rs. 1300, No. of belts of type A = 200 No. of belts of type B = 600.
13. No. of first class ticket = 40, No. of second class ticket = 160.
14. Food A: 5 units, Food B: 30 units
15. Maximum interest is Rs. 1160 at  $(2000, 10000)$

## CHAPTER 13

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### PROBABILITY

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#### POINTS TO REMEMBER

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- **Conditional Probability:** If A and B are two events associated with any random experiment, then  $P(A/B)$  represents the probability of occurrence of event A knowing that event B has already occurred.

$$P(A/B) = \frac{P(A \cap B)}{P(B)}, P(B) \neq 0$$

$P(B) \neq 0$ , means that the event should not be impossible.

$$P(A \cap B) = P(A \text{ and } B) = P(B) \times P(A/B)$$

- Similarly  $P(A \cap B \cap C) = P(A) \times P(B/A) \times P(C/AB)$

$$P(A/S) = P(A), P(A/A) = 1, P(S/A) = 1, P(A^1/B) = 1 - P(A/B)$$

- **Multiplication Theorem on Probability:** If the event A and B are associated with any random experiment and the occurrence of one depends on the other, then

$$P(A \cap B) = P(A) \times P(B/A) \text{ where } P(A) \neq 0$$

- When the occurrence of one does not depend on the other then these event are said to be independent events.

Here  $P(A/B) = P(A)$  and  $P(B/A) = P(B)$

$$P(A \cap B) = P(A) \times P(B)$$

- **Theorem on total probability:** If  $E_1, E_2, E_3, \dots, E_n$  be a partition of sample space and  $E_1, E_2, \dots, E_n$  all have non-zero probability. A be any event associated with sample space S, which occurs with  $E_1$ , or  $E_2$ , ..., or  $E_n$ , then

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + \dots + P(E_n) \cdot P(A/E_n)$$

If A & B are independent then (i)  $A \cap B^c$ , (ii)  $A^c \cap B$  & (iii)  $A^c \cap B^c$  are also independent.

- **Bayes' theorem** : Let S be the sample space and  $E_1, E_2, \dots, E_n$  be n mutually exclusive and exhaustive events associated with a random experiment. If A is any event which occurs with  $E_1, \text{ or } E_2 \text{ or } \dots, E_n$ , then

$$P(E_i/A) = \frac{P(E_i)P(A/E_i)}{\sum_{i=1}^n P(E_i)P(A/E_i)}$$

- **Random variable** : It is real valued function whose domain is the sample space of random experiment.
- **Probability distribution** : It is a system of number of random variable (X), such that

X:	$X_1$	$X_2$	$X_3, \dots$	$X_n$
P(X):	$P(X_1)$	$P(X_2)$	$P(X_3), \dots$	$P(X_n)$

Where  $P(x_i) > 0$  and  $\sum_{i=1}^n P(x_i) = 1$

- Mean or expectation of a random variables (X) is denoted by E(X)

$$E(X) = \mu = \sum_{i=1}^n x_i P(x_i)$$

- Variance of X denoted by var(X) or  $\sigma_{x^2}$  and

$$Var(X) = \sigma_{x^2} = \sum_{i=1}^n (x_i - \mu)^2 P(x_i) = \sum_{i=1}^n x_i^2 P(x_i) - \mu^2$$

- The non-negative number  $\sigma_x = \sqrt{var(X)}$  is called standard deviation of random variable X.
- **Bernoulli Trials**: Trials of random experiment are called Bernoulli trials if:

- (xi) Number of trials is finite.
- (xii) Trials are independent.
- (xiii) Each trial has exactly two outcomes-either success or failure.
- (xiv) Probability of success remain same in each trail.

- **Binomial distribution:**

$$P(X = r) = nC_r \cdot q^{n-r} p^r, \text{ where } r = 0, 1, 2, \dots, n$$

P = Probability of Success

q = Probability of Failure

n = total number of trails

r = value of random variables.

### Very Short Answer Type Question (1 Mark)

1. Find  $P(A/B)$  if  $P(A) = 0.4$ ,  $P(B) = 0.8$  and  $P(B/A) = 0.6$
2. Find  $P(A \cap B)$  if A and B are two events such that  $P(A) = 0.5$ ,  $P(B) = 0.6$  and  $P(A \cup B) = 0.8$
3. A soldier fires three bullets on enemy: The probability that the enemy will be killed by one bullet is 0.7. What is the probability that the enemy is still alive?
4. If  $P(A) = \frac{1}{2}$ ,  $P(B) = \frac{7}{12}$  and  $P(\text{not } A \text{ or not } B) = \frac{1}{4}$ . State whether A and B are independent.
5. Three coins are tossed once. Find the probability of getting at least one head.
6. The probability that a student is not a swimmer is  $\frac{1}{5}$ . Find the probability that out of 5 students, 4 are swimmers.
7. Find  $P(A/B)$ , if  $P(B) = 0.5$  and  $P(A \cap B) = 0.32$

### Short Answer Type Questions (2 Marks)

8. If A and B are two events such that  $P(A) \neq 0$ , then find  $P(B/A)$  if (i) A is a subset of B (ii)  $A \cap B = \phi$
9. A random variable X has the following probability distribution find K.

X	0	1	2	3	4	5
P(X)	$\frac{1}{15}$	K	$\frac{15K-2}{15}$	K	$\frac{15K-1}{15}$	$\frac{1}{15}$

10. If  $P(A) = \frac{1}{2}$ ,  $P(A \cup B) = \frac{3}{5}$ , and  $P(B) = q$  find the value of q if A and B are (i) Mutually exclusive (ii) independent events.
11. If  $P(A) = \frac{3}{10}$ ,  $P(B) = \frac{2}{5}$  and  $P(A \cup B) = \frac{3}{5}$ , then find  $P(B/A) + P(A/B)$
12. A die is rolled if the out come is an even number. What is the probability that it is a prime?
13. If A and B are two-events such that  $P(A) = \frac{1}{4}$ ,  $P(B) = \frac{1}{2}$  and  $P(A \cap B) = \frac{1}{8}$  Find  $P(\text{not A and not B})$ .
14. A pair of die is rolled six times. Find the probability that a third sum of 7 is observed in sixth throw.
15. Ten eggs are drawn successively with replacement from a lot containing 10% defective eggs. Find the probability that there is at least one defective egg.

### SHORT ANSWER TYPE QUESTIONS (4 MARKS)

16. A problem in mathematics is given to three students whose chances of solving it are  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$ . What is the probability that the problem is solved?
17. Two aeroplanes X and Y bomb a target in succession. There probabilities to hit correctly are 0.3 and 0.2 respectively. The second plane will bomb only if first miss the target. Find the probability that target is hit by Y plane.
18. A can hit a target 4 times in 5 shots B three times in 4 shots and C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit?
19. Two dice are thrown once. Find the probability of getting an even number on the first die or a total of 8.
20. A and B throw a die alternatively till one of them throws a '6' and wins the game. Find their respective probabilities of winning, if A starts the game.
21. A man takes a step forward with probability 0.4 and backward with probability 0.6. Find the probability that at the end of eleven steps he is one step away from the starting point.
22. Two cards are drawn from a pack of well shuffled 52 cards one by one with replacement. Getting an ace or a spade is considered a success. Find the probability distribution for the number of successes.
23. In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins/loses.
24. Suppose that 10% of men and 5% of women have grey hair. A grey haired person is selected at random. What is the probability that the selected person is male assuming that there are 60% males and 40% females?

25. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn. What is the probability that they both are diamonds?
26. If A and B are two independent events such that  $P(\overline{A} \cap B) = \frac{2}{15}$  and  $P(A \cap \overline{B}) = \frac{1}{6}$  then find P(A) and P(B).

### Long Answer Type Questions (6 Marks)

27. In a hurdle race, a player has to cross 8 hurdles. The probability that he will clear a hurdle is  $\frac{4}{5}$ , what is the probability that he will knock down in fewer than 2 hurdles?
28. Bag A contains 4 red, 3 white and 2 black balls. Bag B contains 3 red, 2 white and 3 black balls. One ball is transferred from bag A to bag B and then a ball is drawn from bag B. The ball so drawn is found to be red. Find the probability that the transferred ball is black.
29. If a fair coin is tossed 10 times, find the probability of getting.
- Exactly six heads,
  - at least six heads,
  - at most six heads.
30. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter by other means of transport are respectively  $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$  and  $\frac{2}{5}$ . The probabilities that he will be late are  $\frac{1}{4}, \frac{1}{3}$  and  $\frac{1}{12}$  if he comes by train, bus and scooter respectively but if comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?



31. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is six. Find the probability that it is actually a six.
32. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?
33. Three cards from a pack of 52 cards are lost. One card is drawn from the remaining cards. If drawn card is heart, find the probability that the lost cards were all hearts.
34. A box X contains 2 white and 3 red balls and a bag Y contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red. Find the probability that it was drawn from bag Y.
35. In answering a question on a multiple choice, a student either knows the answer or guesses. Let  $\frac{3}{4}$  be the probability that he knows the answer and  $\frac{1}{4}$  be the probability that he guesses. Assuming that a student who guesses at the answer will be incorrect with probability  $\frac{1}{4}$ . What is the probability that the student knows the answer, given that he answered correctly?
36. Suppose a girl throws a die. If she gets 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3 or 4 she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head. What is the probability that she throws 1, 2, 3 or 4 with the die?
37. In a bolt factory machines, A, B and C manufacture bolts in the ratio 6:3:1. 2%, 5% and 10% of the bolts produced by them respectively are defective. A bolt is picked up at random from the product and is found to be defective. What is the probability that it has been manufactured by machine A?
38. Two urns A and B contain 6 black and 4 white, 4 black and 6 white balls respectively. Two balls are drawn from one of the urns. If both the balls drawn are white, find the probability that the balls are drawn from urn B.

39. Two cards are drawn from a well shuffled pack of 52 cards. Find the mean and variance for the number of face cards obtained.
40. A letter is known to have come from TATA NAGAR or from CALCUTTA on the envelope first two consecutive letters 'TA' are visible. What is the probability that the letter come from TATA NAGAR?
41. Two groups are competing for the position on the Board of Directors of a corporation. The probabilities that first and the second group will win are 0.6 and 0.4 respectively. Further if the first group wins, the probability of introducing a new product is 0.7 and the corresponding probability is 0.3 if the second group wins. Find the probability that the new product introduced was by the second group.
42. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6, 7. Let X denotes the larger of the two numbers obtained. Find the mean and variance of the probability distribution of X.
43. An urn contains five balls. Two balls are drawn and are found to be white. What is the probability that all the balls are white?
44. Find the probability distribution of the number of doublets in four throws of a pair of dice. Also find the mean and S.D. of his distribution.
45. Three critics review a book. Odds in favour of the book are 5:2, 4:3 and 3:4 respectively for the three critics. Find the probability that the majority are in favour of the book.
46. A box contains 2 Black, 4 White and 3 Red balls. One by one all balls are drawn without replacement and arranged in sequence of drawing. Find the probability that the drawn balls are in sequence of BBWWWR.
47. A bag contains 3 White, 3 Black and 2 Red balls. 3 balls are successively drawn without replacement. Find the probability that third ball is red.

### Answers

1. 0.3

2.  $\frac{3}{10}$

3.  $(0.3)^3$

4. No

5.  $\frac{7}{8}$

6.  $\left(\frac{4}{5}\right)^4$

7.  $\frac{16}{25}$

8. (i) 1 (ii) 0

9.  $K = \frac{4}{15}$

10. (i)  $\frac{1}{10}$  (ii)  $\frac{1}{5}$

11.  $\frac{7}{12}$

12.  $\frac{1}{3}$

13.  $\frac{3}{8}$

14.  $1250 \times \left(\frac{1}{6}\right)^6$

15.  $1 - \left(\frac{9}{10}\right)^{10}$

16.  $\frac{3}{4}$

17.  $\frac{7}{22}$

18.  $\frac{5}{6}$

19.  $\frac{5}{9}$

20.  $\frac{6}{11}, \frac{5}{11}$

21.  $0.3678$  or  $11C_5(0.4)^5(0.6)^5$

22.

X	0	1	2
P(X)	81/169	72/169	16/169

23.  $-\frac{91}{54}$

24.  $\frac{3}{4}$

25.  $\frac{11}{50}$

26.  $P(A) = \frac{1}{5}, P(B) = \frac{1}{6}$  or  $P(A) = 5/6, P(B) = 4/5$

27.  $\frac{12}{5} \left(\frac{4}{5}\right)^7$

28.  $\frac{6}{31}$
29. (i)  $\frac{105}{512}$  (ii)  $\frac{193}{512}$  (iii)  $\frac{53}{64}$
30.  $\frac{1}{2}$
31.  $\frac{3}{8}$
33.  $\frac{10}{49}$
34.  $\frac{25}{52}$
35.  $\frac{12}{13}$
36.  $\frac{8}{11}$
37.  $\frac{12}{37}$
38.  $\frac{5}{7}$
39. Mean =  $\frac{6}{13}$  Variance =  $\frac{60}{169}$
40.  $\frac{7}{11}$
41.  $\frac{2}{9}$
42. Mean =  $\frac{17}{3}$  Variance =  $\frac{14}{9}$
43.  $\frac{1}{2}$

44. Mean =  $\frac{2}{3}$  S.D. =  $\frac{\sqrt{5}}{3}$

45.  $\frac{209}{343}$

46.  $\frac{1}{1260}$

47.  $\frac{1}{4}$

## CBSE Board Paper Delhi-2018

**Time allowed: 3 Hours**

**Maximum Marks: 100**

### **General Instructions:**

- (i) All questions are compulsory.
- (ii) The question paper consists of 29 questions divided into four sections, A, B, C and D. Section A comprises of 4 questions of one mark each, Section B comprises of 8 questions of two marks each, Section C comprises of 11 questions of four marks each and Section D comprises of 6 questions of six marks each.
- (iii) All question in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
- (iv) There is no overall choice. However, internal choice has been provided in 3 questions of four marks each and 3 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
- (v) Use of calculators is not permitted. You may ask for logarithmic tables, if required.

### **SECTION – A**

- 1. Find the value of  $\tan^{-1} \sqrt{3} - \cot^{-1} (-\sqrt{3})$ .
- 2. If the matrix  $A = \begin{pmatrix} 0 & a & -3 \\ 2 & 0 & -1 \\ b & 1 & 0 \end{pmatrix}$  is skew symmetric, find the values of 'a' and 'b'.

3. Find the magnitude of each of the two vectors  $\vec{a}$  and  $\vec{b}$ , having the same magnitude such that the angle between them is  $60^\circ$  and their scalar products is  $\frac{9}{2}$ .
4. If  $a * b$  denotes the larger of 'a' and 'b' and if  $a \circ b = (a * b) + 3$ , then write the value of  $(5) \circ (10)$ , where  $*$  and  $\circ$  are binary operations.

### SECTION – B

5. Prove that :

$$3 \sin^{-1} x = \sin^{-1} (3x - 4x^3), x \in \left[ \frac{1}{2}, \frac{1}{2} \right]$$

6. Given  $A = \begin{pmatrix} 2 & -3 \\ -4 & 7 \end{pmatrix}$ , compute  $A^{-1}$  and show that

$$2A^{-1} = 9I - A.$$

7. Differentiate  $\tan^{-1} \left( \frac{1 + \cos x}{\sin x} \right)$  with respect to  $x$ .

8. The total cost  $C(x)$  associated with the production of  $x$  units of an item is given by  $C(x) = 0.005x^3 - 0.02x^2 + 30x + 5000$ . Find the marginal cost when 3 units are produced, where by marginal cost we mean the instantaneous rate of change of total cost at any level of output.

9. Evaluate :

$$\int \frac{\cos 2x + 2 \sin^2 x}{\cos^2 x} dx$$

10. Find the differential equation representing the family of curves  $y = a e^{bx+5}$ , where  $a$  and  $b$  are arbitrary constants.



11. If  $\theta$  is the angle between two vectors  $\hat{i} - 2\hat{j} + 3\hat{k}$  and  $3\hat{i} - 2\hat{j} + \hat{k}$ , find  $\sin \theta$ .
12. A black and a red die are rolled together. Find the conditional probability of obtaining the sum 8, given that the red die resulted in a number less than 4.

### SECTION C

13. Using properties of determinants, prove that

$$\begin{vmatrix} 1 & 1 & 1+3x \\ 1+3y & 1 & 1 \\ 1 & 1+3x & 1 \end{vmatrix} = 9(3xyz + xy + yz + zx)$$

14. If  $(x^2 + y^2)^2 = xy$ , find  $\frac{dy}{dx}$ .

OR

If  $x = a(2\theta - \sin 2\theta)$  and  $y = a(1 - \cos 2\theta)$ , find  $\frac{dy}{dx}$  when  $\theta = \frac{\pi}{3}$ .

15. If  $y = \sin(\sin x)$ , prove that  $\frac{d^2y}{dx^2} + \tan x \frac{dy}{dx} + y \cos^2 x = 0$ .
16. Find the equation of the tangent and the normal, to the curve  $16x^2 + 9y^2 = 145$  at the point  $(x_1, y_1)$ , where  $x_1 = 2$  and  $y_1 > 0$ .

OR

Find the intervals in which the function

$f(x) = \frac{x^4}{4} - x^3 - 5x^2 + 24x + 12$  is (a) strictly increasing, (b) strictly decreasing.

17. An open tank with a square base and vertical sides is to be constructed from a metal sheet so as to hold a given quantity of water. Show that the cost of material will be least when depth of the tank is half of its width. If the cost is to be borne by nearby settled lower income families, for whom water will be provided, what kind of value is hidden in this question?
18. Find :
- $$\int \frac{2 \cos x}{(1 - \sin x)(1 + \sin^2 x)} dx.$$
19. Find the particular solution of the differential equation  $e^x \tan y \, dx + (2 - e^x) \sec^2 y \, dy = 0$ , given that  $y = \frac{\pi}{4}$  when  $x = 0$ .
- Or
- Find the particular solution of the differential equation  $\frac{dy}{dx} + 2y \tan x = \sin x$ , given that  $y = 0$  when  $x = \frac{\pi}{3}$ .
20. Let  $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$ ,  $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$ .
- Find a vector  $\vec{d}$  which is perpendicular to both  $\vec{c}$  and  $\vec{b}$  and  $\vec{d} \cdot \vec{a} = 21$ .
21. Find the shortest distance between the lines  $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$  and  $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$ .
22. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5, or 6 with the die?

23. Two numbers are selected at random (without replacement) from the first five positive integers. Let  $X$  denote the larger of the two numbers obtained. Find the mean and variance of  $X$ .

#### SECTION D

24. Let  $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ . Show that  $R = \{(a, b) : a, b \in A, |a - b| \text{ is divisible by } 4\}$  is an equivalence relation. Find the set of all elements related to 1. Also write the equivalence class [2].

OR

Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = \frac{x}{x^2 + 1}$ ,  $\forall x \in \mathbb{R}$

is neither one-one nor onto. Also, if  $g : \mathbb{R} \rightarrow \mathbb{R}$  is defined as  $g(x) = 2x - \text{find } \text{fog}(x)$ .

25.  $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ , find  $A^{-1}$ . Use it to solve the system of equations

$$2x - 3y + 5z = 11$$

$$3x + 2y - 4z = -5$$

$$x + y - 2z = -3.$$

OR

Using elementary row transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}.$$

26. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line  $y = x$  and the circle  $x^2 + y^2 = 32$ .

27. Evaluate : 
$$\int_0^{\pi/4} \frac{\sin x + \cos x}{16 + 9 \sin 2x} dx$$

Or

Evaluate: 
$$\int_1^3 (x^2 + 3x + e^x) dx, \text{ as the limit of the sum.}$$

28. Find the distance of the point  $(-1, -5, -10)$  from the point of intersection of the line  $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .
29. A factory manufactures two types of screws A and B, each type requiring the use of two machines, an automatic and a hand-operated. It takes 4 minutes on the automatic and 6 minutes on the hand-operated machines to manufacture a packet of screws 'A' while it takes 6 minutes on the automatic and 3 minutes on the hand-operated machine to manufacture a packet of screws 'B'. Each machine is available for at most 4 hours on any day. The manufacturer can sell a packet of screws 'A' at a profit of 70 paise and screws 'B' at a profit of Rs. 1. Assuming that he can sell all the screws he manufactures, how many packets of each type should the factory owner produce in a day in order to maximize his profit? Formulate the above LPP and solve it graphically and find the maximum profit.

**CBSE Board Paper Delhi-2018**  
**Mathematics**  
**(For Blind Candidates only)**

**Time allowed: 3 hours**

**Maximum Marks: 100**

**General Instructions:**

- (i) All questions are compulsory.
- (ii) This Questions paper contains 29 questions.
- (iii) Questions No. **1-4** in Section-A are very short answer type questions carrying **1** mark each.
- (iv) Questions **5-12** in Section-B are short answer type questions carrying **2** marks each.
- (v) Questions **13-22** in Section-C are long answer I type questions carrying **4** marks each.
- (vi) Questions **23-29** in Section-D are long answer II type questions carrying **6** marks each.

**SECTION – A**

- 1. If A and B are square matrices, each of order 2 such that  $|A| = 3$  and  $|B| = -2$ , then write the value of  $|3AB|$ .
- 2. Write the derivative of  $|x - 5|$  at  $x = 2$ .
- 3. Find :  $\int a^x \cdot e^x dx$

4. Find the cosine of the angle which the vector  $\sqrt{2} \hat{i} + \hat{j} + \hat{k}$  makes with y-axis.

### SECTION – B

5. If  $\sin^{-1}\left(\frac{\sin x + \cos x}{\sqrt{2}}\right) = \frac{\pi}{2}$ , find the value of x.
6. Evaluate :
- $$\tan\left[2 \tan^{-1} \frac{1}{5} - \frac{\pi}{4}\right].$$
7. Find the inverse ( $A^{-1}$ ) of the matrix  $A = \begin{pmatrix} -1 & 4 \\ 7 & 20 \end{pmatrix}$ , using elementary operations.
8. If the curves  $y = 2e^x$  and  $y = ae^{-x}$  intersect orthogonally, find the value of a.
9. Find the points on the curve  $y = 12x - x^3$ , where the tangent drawn is parallel to x-axis.
10. Find :  $\int \sqrt{x^2 - 4x + 13} \, dx$ .
11. Find a vector perpendicular to the plane of ABC, where A, B and C are points (3, -1, 2), (1, -1, -3) and (4, -3, 1) respectively.
12. Eight cards numbered 1 to 8 (one number on one card) are placed in a box, mixed up thoroughly and then a card is drawn randomly. If it is known that the number on the drawn card is more than 2, then find the probability that it is an odd number.

### SECTION – C

13. Using properties of determinants prove that :

$$\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2).$$

Or

If  $A \cdot \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$ , find the matrix A.

14. Find the value of k for which the given function f(x) is continuous at x=0.

$$f(x) = \begin{cases} \frac{2-2\cos 2x}{x^2}; & x < 0 \\ k & ; x = 0 \\ \frac{\sqrt{x}}{\sqrt{4}+\sqrt{x}-2}; & x > 0 \end{cases}$$

OR

If  $x^y = e^{x-y}$ , show that  $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$ .

15. If  $x = a(\cos t + t \sin t)$  and  $y = a(\sin t - t \cos t)$ , find  $\frac{d^2y}{dx^2}$ .

16. Find :  $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$

OR

Find :  $\int \frac{(x-5)e^x}{(x-3)^3} dx$

17. Evaluate:  $\int_0^{\pi} \frac{x \sin x}{4 + \cos^2 x} dx$

18. Find the general solution of the following differential equation :

$$y dx + x \log\left(\frac{y}{x}\right) dy - 2x dy = 0.$$

19. For the differential equation  $xy \frac{dy}{dx} = (x+2)(y+2)$ , find the solution curve passing through the point  $(1, -1)$ .

20. If the scalar product of the vector  $\hat{i} + \hat{j} + \hat{k}$  with a unit vector along the sum of vectors  $2\hat{i} + 4\hat{j} - 5\hat{k}$  and  $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$  is equal to 1, find the value of  $\lambda$ .

21. Find the coordinates of the foot of perpendicular drawn from the point  $P(1, 8, 4)$  to the line joining the points  $A(0, -1, 3)$  and  $B(2, -3, -1)$ . Also find the length of this perpendicular.

22. Find the probability distribution of number of doublets in two throws of a pair of dice. Hence find the mean of the distribution.

23. A and B throw a pair of dice alternately till one of them gets a sum of 5, of the numbers on the two dice and wins the game. Find their respective probabilities of winning, if A starts the game.



### Section – D

24. Let  $R$  be the relation defined in the set  $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$  by

$$R = \{(x, y) : x, y \in A, x \text{ and } y \text{ are either both odd or both even}\}.$$

Show that  $R$  is an equivalence relation, write all the equivalence classes of set  $A$ .

**OR**

Let  $A$  be the set of all real numbers except  $-1$  and let  $*$  be a binary operation on  $A$  defined by  $a * b = a + b + ab$ , for  $\forall a, b \in A$ . Prove that (i)

$*$  is commutative and associative, and (ii) number  $0$  its identity element.

25. Solve the following system of equations, using matrix method :

$$5x - y + z = 4$$

$$3x + 2y - 5z = 2$$

$$X + 3y - 2z = 5.$$

26. Find the intervals in which the function  $f$  given by  $f(x) = \sin 3x - \cos 3x$ ,  $0 < x < \pi$  is (a) strictly increasing, and (b) strictly decreasing.

**OR**

Prove that the height of a solid cylinder of given surface and greatest volume is equal to the diameter of its base.

27. If the area between the curve  $x = y^2$  and the line  $x = 4$  is divided into two equal parts by the line  $x = a$ , find the value of  $a$ .

**OR**

By the method of limit of sum, find the value of the following definite integral :

$$\int_1^3 (3x^2 + 2x + e^x) dx$$

28. Show that the lines  $\frac{x-1}{2} = \frac{y-3}{4} = \frac{z}{-1}$  and

$\frac{x-4}{3} = \frac{1-y}{2} = z-1$  are coplanar. Hence find the equation of the plane containing these lines.

29. If a class XII student aged 17 years, rides his motorcycle at 40 km/hr, the petrol cost is Rs. 2 per km. If he rides at a speed of 70 km/hr, the petrol cost increases to Rs. 7 per km. He has Rs. 100 to spend on petrol and wishes to cover the maximum distance within one hour.

Express the above as an LPP and solve it graphically.

# MATHEMATICS 2017

**Time allowed: 3 hours**

**Maximum Marks: 100**

## **General Instructions:**

- (vi) All questions are compulsory.
- (vii) Questions **1-4** in section A carry **1 mark** each.
- (viii) Question **5-12** in section B carry **2 marks** each.
- (ix) Question **13-22** in section C carry **4 marks** each.
- (x) Question **23-29** in section D carry **6 marks** each.
- (xi) Please write down the serial number of the question before attempting it.

## **SECTION – A**

1. A is a square matrix of order 3 with  $|A| = 4$ . Find the value of  $|A \cdot (\text{adj } A)|$ .
2. The radius of a circle increases at a rate of 3 cm/sec. What is the rate of increase of its area at the instant when radius of circle is 10 cm.
3. Evaluate  $\int_{-1}^1 (x)|x| dx$
4. Write the equation of a line passing through (1, 2, 3) and perpendicular to plane  $x - 3y + z = 9$ .

### SECTION – B

5. If  $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ , show that  $A^2 - 4A + 7I = 0$
6. Examine the continuity of the function  $f(x) = x - [x]$  at  $x = 2$   $[x] = \text{greatest integer } \leq x]$
7. Differentiate  $e^{\sin^2 x}$  w.r.t.  $\cos^2 x$
8. Using differentials find approximate value of  $(8.1)^{\frac{1}{3}}$ .
9. Evaluate  $\int \frac{1}{x^3 + x} dx$
10. Form the differential equation of family of circles  $x^2 + (y - a)^2 = a^2$
11. Find a vector of magnitude 6 units which is perpendicular to both  $\vec{a} = \hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{j} + 5\hat{k}$ .
12. Two balls are drawn at random from a bag containing 6 red and 4 green balls, find the probability that both balls are of same colour.

### SECTION – C

13. A trust invested some money in two type of bonds. The first bond pays 10% interest and second bond pays 12% interest. The trust received Rs 2800 as interest. However, if trust had inter changed money in bonds, they would have got Rs 100 less as interest using matrix method, find the amount invested in each bond by the trust. Interest received on this amount will be given to Helpage India as donation. Which value is reflected in the question.

14. If  $x = a \cos \theta + b \sin \theta$ ,  $y = a \sin \theta - b \cos \theta$ , show that

$$y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$$

15. Find the intervals in which the function  $f(x) = \sin x + \cos x$ ,  $0 < x < 2\pi$  is strictly increasing or strictly decreasing.

16.  $\hat{a}, \hat{b}$  and  $\hat{c}$  are unit vectors such that  $\hat{a} \cdot \hat{b} = \hat{a} \cdot \hat{c} = 0$  and the angle between  $\hat{b}$  and  $\hat{c}$  is  $\frac{\pi}{6}$  prove that  $\hat{a} = \pm 2(\hat{b} \times \hat{c})$

OR

$\vec{a}, \vec{b}$  and  $\vec{c}$  are vectors such that  $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c}$ ,  $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$  and  $\vec{a} \neq \vec{0}$  show that  $\vec{b} = \vec{c}$

17. Evaluate:  $\int_0^{\pi} \frac{x \sin x}{1 + 3 \cos^2 x} dx$

OR

Evaluate:  $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1 + e^x} dx$

18. Find :  $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$

19. Show that  $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$  and  $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$  intersect. Find their point of intersection.

OR

Find the coordinates foot of perpendicular drawn from point (2, 3, 4) on the

plane  $\vec{r} \cdot (2\hat{i} + \hat{j} + 3\hat{k}) = 26$

20. Five bad oranges are accidentally mixed with 20 good ones. If four oranges are drawn one by one successively with replacement, then find the probability distribution of getting bad oranges.
21. Two cards from a pack of 52 cards are lost from the remaining 50 cards, one card is drawn. If the drawn card is a spade, what is the probability that the lost cards were both spades.
22. Solve the differential equation.  $\frac{xdy}{dx} + y - x + xy \cot x = 0, x \neq 0$
23. If  $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$ , then find  $x$

OR

If  $y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$  then prove that  $\sin y = \tan^2\left(\frac{x}{2}\right)$ .

#### Section – D

24. Consider a binary operation  $*$  on  $\mathbb{Q}$ , defined as  $a * b = a + b - ab$ .
- (i) Is  $*$  commutative?
  - (ii) Is  $*$  associative?
  - (iii) Find the identity element of  $*$  in  $\mathbb{Q}$ .
  - (iv) Find the inverse of all  $a \in \mathbb{Q}$ , for which it exists.
25. Using properties of determinants, prove that

$$\begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

26. Show that height of cylinder of greatest volume that can be inscribed in a cone of height  $h$ , is  $\frac{h}{3}$ .

OR

Find the area of greatest rectangle that can be inscribed in an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

27. If a young man rides his motorcycle at a speed of 25 Km/hr, he has to spend ₹ 2 per Km on petrol. If he rides at a faster speed of 40 Km/hr the petrol cost increases to ₹ 5 per Km. He has ₹ 100 to spend on petrol and wished to cover the maximum distance within one hour. Express this as L.P.P. and then solve it graphically.

28. Find the area of the region  $\{(x, y) : y^2 \leq 4x, 4x^2 + 4y^2 \leq 9\}$

or

Find the area of the region  $\{(x-1) : |x-1| \leq y \leq \sqrt{5-x^2}\}$

29. Find the distance of the point  $(1, -2, 3)$  from the plane  $x - y + z = 5$  measured parallel to the line  $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$

## Solution of Mathematics 2017

### SECTION A

$$1. |A \cdot (\text{adj } A)| = \|A\| |I| = |A|^3 |I| = 4^3 \times 1 = 64$$

$$2. \frac{dr}{dt} = 3 \text{ cm/sec}, \quad \frac{dA}{dt} = 2\pi r \frac{dr}{dt} = 60\pi \text{ cm}^2/\text{sec}$$

$$\begin{aligned} 3. \int_{-1}^1 x|x| dx &= \int_{-1}^0 -x^2 dx + \int_0^1 x^2 dx \\ &= \left[ -\frac{1}{3}x^3 \right]_{-1}^0 + \left[ \frac{1}{3}x^3 \right]_0^1 \\ &= -\frac{1}{3} + \frac{1}{3} = 0 \end{aligned}$$

$$4. \frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{1}$$

### SECTION B

$$\begin{aligned} 5. A^2 - 4A + 7I &= \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} (1-8+7) & (12-12+0) \\ (-4+4+0) & (1-8+7) \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$



$$\begin{aligned}
 6. \quad \lim_{x \rightarrow 2^-} f(x) &= \lim_{h \rightarrow 0} f(2-h) \\
 &= \lim_{h \rightarrow 0} (2-h) - [2-h] = 1 \\
 \lim_{x \rightarrow 2^+} f(x) &= \lim_{h \rightarrow 0} f(2+h) = \lim_{h \rightarrow 0} (2+h) - (2+h) = 0 \\
 &\Rightarrow \text{is discontinuous at } x = 2
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \text{Let } y &= e^{\sin^2 x} \text{ and } t = \cos^2 x \\
 \frac{dy}{dx} &= e^{\sin^2 x} \cdot 2 \sin x \cos x, \quad \frac{dt}{dx} = -2 \cos x \sin x
 \end{aligned}$$

$$\therefore \frac{dy}{dt} = e^{\sin^2 x}$$

$$\begin{aligned}
 8. \quad \text{Let } f(x) &= \frac{1}{x^3} \\
 \text{As } f'(x) \Delta x &\approx f(x + \Delta x) - f(x) \\
 \Rightarrow \frac{1}{3x^{2/3}} \Delta x &\approx (x + \Delta x)^{\frac{1}{3}} - x^{\frac{1}{3}} \\
 \text{Put } x = 8, \quad \Delta x = +0.1 &\therefore x + \Delta x = 8.1 \\
 \therefore \frac{1}{3(8)^{\frac{2}{3}}} (+0.1) &\approx (8.1)^{\frac{1}{3}} - 8^{\frac{1}{3}} \\
 \Rightarrow \frac{1}{12} + 2 &\approx (8.1)^{\frac{1}{3}} \\
 \Rightarrow (8.1)^{\frac{1}{3}} &\approx \frac{25}{12} \\
 (8.1)^{\frac{1}{3}} &\approx 2.08
 \end{aligned}$$

$$9. \int \frac{1}{x^3 + x} dx$$

$$= \int \frac{1}{x^3 \left(1 + \frac{1}{x^2}\right)} dx$$

$$\text{Put } 1 + \frac{1}{x^2} = t$$

$$\Rightarrow \frac{-2}{x^3} dx = dt$$

$$= \frac{-1}{2} \int \frac{dt}{t}$$

$$= \frac{-1}{2} \log \left(1 + \frac{1}{x^2}\right) + c$$

$$10. \quad x^2 + (y - a)^2 = a^2$$

$$\Rightarrow x^2 + y^2 - 2ay = 0$$

$$\Rightarrow \frac{x^2 + y^2}{x} = 2a$$

$$\frac{x[2x + 2yy'] - (x^2 + y^2)(1)}{x^2} = 0$$

$$\Rightarrow 2x^2 + 2xy y' - x^2 - y^2 = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} + x^2 - y^2 = 0$$

$$11. \text{ Required vector} = 6 \left( \frac{\vec{a} \times \vec{b}}{(\vec{a} \times \vec{b})} \right)$$

$$|\vec{a} \times \vec{b}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 0 & 1 & 5 \end{vmatrix} = 6\hat{i} - 5\hat{j} + \hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{36 + 25 + 1} = \sqrt{62}$$

Required vector

$$\therefore \frac{6}{\sqrt{62}}[6\hat{i} - 5\hat{j} + \hat{k}]$$

12. P (Balls are of same Colour) = P (both red) + P (both green)

$$\begin{aligned} &= \frac{6}{10} \times \frac{5}{9} + \frac{4}{10} \times \frac{3}{9} \\ &= \frac{30+12}{90} = \frac{42}{90} = \frac{7}{15} \end{aligned}$$

### SECTION C

13. Let money incepted in first type of bond = Rs x

and money incepted in second type of bond = Rs y

$$\frac{10x}{100} + \frac{12y}{100} = 2800$$

$$\frac{12x}{100} + \frac{10y}{100} = 2700$$

$$A = \begin{bmatrix} \frac{10}{100} & \frac{12}{100} \\ \frac{12}{100} & \frac{10}{100} \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 2800 \\ 2700 \end{bmatrix}$$

$$X = A^{-1} B$$

$$= \frac{-10000}{44} \begin{bmatrix} \frac{10}{100} & \frac{-12}{100} \\ \frac{-12}{100} & \frac{10}{100} \end{bmatrix} \begin{bmatrix} 2800 \\ 2700 \end{bmatrix} = \begin{bmatrix} 10,000 \\ 15,000 \end{bmatrix}$$

$$\therefore x = 10000, y = 15000$$

$$\therefore \text{investment in first bond} = ₹ 10000 ;$$

$$\text{And investment in second bond} = ₹ 1500.$$

$$14. X = a \cos \theta + b \sin \theta, \quad y = a \sin \theta - b \cos \theta$$

$$\frac{dx}{d\theta} = -a \sin \theta + b \cos \theta, \quad \frac{dy}{d\theta} = a \cos \theta + b \sin \theta$$

$$\frac{dy}{d\theta} = \frac{a \cos \theta + b \sin \theta}{-a \sin \theta + b \cos \theta} = \frac{-x}{y}$$

$$\frac{dy}{dx} = - \left[ \frac{y \cdot 1 - x \frac{dy}{dx}}{y^2} \right]$$

$$\Rightarrow y^2 \frac{d^2 y}{dx^2} = -y + x \frac{dy}{dx}$$

$$\Rightarrow y^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = 0$$

$$15. \quad f(x) = \sin x + \cos x \quad 0 < x < 2\pi$$

$$f'(x) = \cos x - \sin x$$

$$f'(x) = 0 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$$

These points divide  $(0, 2\pi)$  into three disjoint interval namely

$\left(0, \frac{\pi}{4}\right), \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$  &  $\left(\frac{5\pi}{4}, 2\pi\right)$  where the function is strictly increasing or strictly decreasing.

Intervals	sign of $f'(x)$	Nature of function
$\left(0, \frac{\pi}{4}\right)$	+ ve	Strictly increasing
$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	- ve	Strictly decreasing
$\left(\frac{5\pi}{4}, 2\pi\right)$	+ ve	Strictly increasing

∴ f is strictly increasing on  $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right)$ , f is strictly decreasing on  $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

16.  $\vec{a} \cdot \vec{b} = \hat{a} \cdot \hat{b} = 0 \Rightarrow \hat{a} \perp \hat{b} \text{ and } \hat{a} \perp \hat{c}$

$\Rightarrow \hat{a} \parallel \hat{b} \times \hat{c}$

$\Rightarrow \hat{a} = \lambda (\hat{b} \times \hat{c}), \lambda \in \mathbb{R}$

$\Rightarrow |\hat{a}| = |\lambda| |\hat{b} \times \hat{c}|$

$\Rightarrow 1 = |\lambda| |\hat{b}| |\hat{c}| \sin \frac{\pi}{6}$

$\Rightarrow 1 = |\lambda| (1) (1) \left(\frac{1}{2}\right)$

$\Rightarrow |\lambda| = 2 \Rightarrow \lambda = \pm 2$

$\hat{a} = \pm(\hat{b} \times \hat{c})$

**OR**

$\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$

$\Rightarrow \vec{a} \perp (\vec{b} - \vec{c}) \text{ or } \vec{b} = \vec{c} \dots\dots\dots (1)$

$\vec{a} \times \vec{b} = \vec{a} \times \vec{c} \Rightarrow \vec{a} \times (\vec{b} - \vec{c}) = \vec{0}$

$\Rightarrow \vec{a} \parallel \vec{b} - \vec{c} \text{ or } \vec{b} = \vec{c}$

.....(2)

From (1) & (2) we get  $\vec{b} = \vec{c}$

$$\begin{aligned}
 17. \quad I &= \int_0^{\pi} \frac{x \sin x}{1+3 \cos^2 x} dx \\
 I &= \int_0^{\pi} \frac{(\pi-x) \sin(\pi-x)}{1+3 \cos^2(\pi-x)} dx \\
 I &= \int_0^{\pi} \frac{\pi \sin x}{1+3 \cos^2 x} dx - I \\
 2I &= \int_0^{\pi} \frac{\pi \sin x}{1+3 \cos^2 x} dx
 \end{aligned}$$

Put  $\cos x = t$

$$\Rightarrow \sin x \, dx = dt$$

$$x = 0 \Rightarrow t = 1$$

$$x = \pi \Rightarrow t = -1$$

$$2I = -\pi \int_1^{-1} \frac{dt}{1+3t^2} = -\frac{\pi}{2} \cdot \frac{1}{\sqrt{3}} \left[ \tan^{-1}(\sqrt{3}t) \right]^{-1}$$

$$I = \frac{-\pi}{2\sqrt{3}} \left[ \tan^{-1}(-\sqrt{3}) - \tan^{-1}(\sqrt{3}) \right]$$

$$= \frac{-\pi}{2\sqrt{3}} \left[ \frac{-\pi}{3} - \frac{+\pi}{3} \right]$$

$$I = \frac{-\pi}{2\sqrt{3}} \times \frac{-2\pi}{3}$$

$$\Rightarrow \quad I = \frac{\pi^2}{3\sqrt{3}}$$

OR

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+e^x} dx \quad \dots\dots\dots(1)$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 \left( -\frac{\pi}{2} + \frac{\pi}{2} - x \right)}{1+e^{\left( -\frac{\pi}{2} + \frac{\pi}{2} - x \right)}} dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+e^{-x}} dx$$

$$I = \int_{-\pi/2}^{\pi/2} \frac{e^x \cos^2 x}{e^x + 1} dx \quad \dots\dots\dots(2)$$

$$(1) + (2) \Rightarrow 2 I = \int_{-\pi/2}^{\pi/2} \cos^2 x dx$$

$$= \int_{-\pi/2}^{\pi/2} \frac{1 + \cos 2x}{2} dx$$

$$I = \frac{1}{4} \left[ x + \frac{\sin 2x}{2} \right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{4} \left[ \left( \frac{\pi}{2} + 0 \right) - \left( -\frac{\pi}{2} + 0 \right) \right]$$

$$I = \frac{\pi}{4}$$

$$\begin{aligned}
 18. \quad I &= \int (\sqrt{\tan x} + \sqrt{\cot x}) dx \\
 &= \int \left( \sqrt{\frac{\sin x}{\cos x}} + \sqrt{\frac{\cos x}{\sin x}} \right) dx \\
 I &= \int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx
 \end{aligned}$$

Consider  $(\sin x - \cos x)^2 = 1 - 2 \sin x \cos x$

$$\begin{aligned}
 \Rightarrow \sin x \cos x &= \frac{1 - (\sin x + \cos x)^2}{2} \\
 I &= \sqrt{2} \int \frac{\sin x + \cos x}{\sqrt{1 - (\sin x - \cos x)^2}} dx
 \end{aligned}$$

Put  $\sin x - \cos x = t$

$$\Rightarrow (\cos x + \sin x) dx = dt$$

$$I = \sqrt{2} \int \frac{dt}{\sqrt{1 - t^2}}$$

$$I = \sqrt{2} \sin^{-1}(t) + c$$

$$I = \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$$

$$19. \quad \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda_1(\text{let}) \dots\dots\dots(1)$$

$$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \lambda_2(\text{let}) \dots\dots\dots(2)$$



Coordinates of general point on line (1) & (2) are  $(3\lambda_1 + 1, -\lambda_1 + 1, -1)$  and  $(2\lambda_2 + 4, 0, 3\lambda_2 - 1)$  respectively.

The line will intersect if  $(3\lambda_1 + 1, -\lambda_1 + 1, -1) = (2\lambda_2 + 4, 0, 3\lambda_2 - 1)$

$$\Rightarrow 3\lambda_1 + 1 = 2\lambda_2 + 4 \quad \Rightarrow \quad 3\lambda_1 - 2\lambda_2 = 3 \dots\dots\dots$$

$$-\lambda_1 + 1 = 0 \quad \Rightarrow \quad \lambda_1 = 1$$

$$-1 = 3\lambda_2 - 1 \quad \Rightarrow \quad \lambda_2 = 0$$

As  $(\lambda_1, \lambda_2) = (1, 0)$  satisfy all three conditions

$\Rightarrow$  lines intersect

Also point of intersection is  $(4, 0, -1)$

Or

Equation of line through  $(2, 3, 4)$  and perpendicular to given plane is

$$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k}) \dots\dots\dots(1)$$

Foot of  $\perp$  is point of intersection of line (1) & given plane

$$\therefore [(2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})] \cdot (2\hat{i} + \hat{j} + 3\hat{k}) = 26$$

$$(4 + 3 + 12) + \lambda(4 + 1 + 9) = 26$$

$$14\lambda = 26 - 19$$

$$\lambda = \frac{7}{14} \Rightarrow \lambda = \frac{1}{2}$$

∴ Position vector of foot of  $\perp$  are

$$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \frac{1}{2}(2\hat{i} + \hat{j} + 3\hat{k})$$

$$\Rightarrow \vec{r} = 3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}$$

and coordinates of foot of  $\perp$  are  $(3, \frac{7}{2}, \frac{11}{2})$

20. Let x denotes number of bad oranges.

∴ Possible values of x are 0, 1, 2, 3, 4.

X	P(X)	
0	${}^4C_0 \left(\frac{1}{5}\right)^0 \left(\frac{4}{5}\right)^4$	= 256/625
1	${}^4C_1 \left(\frac{1}{5}\right)^1 \left(\frac{4}{5}\right)^3$	= 256/625
2	${}^4C_2 \left(\frac{1}{5}\right)^2 \left(\frac{4}{5}\right)^2$	= 96/625
3	${}^4C_3 \left(\frac{1}{5}\right)^3 \left(\frac{4}{5}\right)^1$	= 16/625
4	${}^4C_4 \left(\frac{1}{5}\right)^4 \left(\frac{4}{5}\right)^0$	= 1/625

21 Let event  $A_1$  = Lost Cards are both spades

$A_2$  = Los Cards are both non spades

$A_3$  = Lost cards are one spade and one no spade

$B$  = Cards drawn is a spade

$$P(A_1) = \frac{13}{52} \times \frac{12}{51}, \quad P(A_2) = \frac{39}{52} \times \frac{38}{51}$$

$$P(A_3) = \frac{39}{52} \times \frac{39}{51} \times 2$$

$$P(B/A_1) = \frac{11}{50}, \quad P(B/A_2) = \frac{13}{50}$$

$$P(B/A_3) = \frac{12}{50}$$

$$P(A_1/B) = \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + P(A_3)P(B/A_3)}$$

$$\begin{aligned} &= \frac{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50}}{\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} + \frac{39}{52} \times \frac{38}{51} \times \frac{13}{50} + \frac{13}{52} \times \frac{39}{51} \times 2 \times \frac{12}{50}} \\ &= \frac{12 \times 11}{12 \times 11 + 39 \times 38 + 39 \times 24} = \frac{22}{22 + 13 \times 19 + 39 \times 4} \\ &= \frac{22}{22 + 247 + 156} = \frac{22}{425} \end{aligned}$$

$$22. \quad x \frac{dy}{dx} + y - x + xy \cot x = 0, x \neq 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} - 1 + y \cot x = 0,$$

$$\Rightarrow \quad \frac{dy}{dx} + \left[ \frac{1}{x} + \cot x \right] y = 1$$

$$= e^{\int \left( \frac{1}{x} + \cot x \right) dx} = e^{\log x + \log \sin x} = e^{\log x \sin x} = x \sin x$$

General solution is

$$= y \cdot x \sin x = \int 1 \cdot x \sin x dx + c$$

$$= xy \sin x = -x \cos x + \sin x + c$$

$$23. \quad (\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$(\tan^{-1} x)^2 + \left( \frac{\pi}{2} - \tan^{-1} x \right)^2 = \frac{5\pi^2}{8}$$

$$2(\tan^{-1} x)^2 - \pi \tan^{-1} x + \frac{\pi^2}{4} = \frac{5\pi^2}{8}$$

$$2(\tan^{-1} x) - \pi \tan^{-1} x = \frac{3\pi^2}{8}$$

$$2(\tan^{-1} x)^2 - \pi \tan^{-1} x = \frac{3\pi^2}{8}$$

$$16(\tan^{-1} x)^2 - 8\pi \tan^{-1} x = 3\pi^2$$

$$16(\tan^{-1} x)^2 - 12\pi \tan^{-1} x + 4\pi \tan^{-1} x - 3\pi^2 = 0$$

$$\Rightarrow (4 \tan^{-1} x + \pi)(4 \tan^{-1} x - 3\pi) = 0$$

$$\Rightarrow \tan^{-1} x = \frac{-\pi}{4} \text{ or } \tan^{-1} x = \frac{3\pi}{4}$$

$$\Rightarrow x = \tan\left(\frac{-\pi}{4}\right) \text{ or } x = \tan\frac{3\pi}{4}$$

$$\Rightarrow x = -1$$

OR

$$y = \cot^{-1}\sqrt{\cos x} - \tan^{-1}\sqrt{\cos x}$$

$$\Rightarrow y = \frac{\pi}{2} - 2\tan^{-1}\sqrt{\cos x}$$

$$\Rightarrow y = \frac{\pi}{2} - \cos^{-1}\left(\frac{1 - \cos x}{1 + \cos x}\right)$$

$$\Rightarrow \cos^{-1}\left(\frac{1 - \cos x}{1 + \cos x}\right) = \frac{\pi}{2} - y$$

$$\Rightarrow \tan^2 \frac{x}{2} = \cos\left(\frac{\pi}{2} - y\right)$$

$$\Rightarrow \tan^2 \frac{x}{2} = \sin y$$

#### SECTION D

24. (i)  $a * b = a + b - ab$

$$b * a = b + a - ba = a + b - ab = a * b$$

[ $\therefore$  '+' & 'x' are commutative in Q]

$\Rightarrow$  \* is commutative.

(ii)  $a * (b * c) = a * (b + c - bc)$

$$= a + b + c - ab - ac + abc$$

$$(a * b) * c = (a + b - ab) * c$$

$$= a + b - ab + c - ac - bc + abc$$

$$= a + b + c - ab - bc - ca + abc$$

$$\Rightarrow a * (b * c) = (a * b) * c$$

$$\Rightarrow \text{* is associative}$$

(iii) Let  $e \in Q$  be the identity element

$$\Rightarrow a * e = a = e * a \quad \forall a \in Q$$

Consider  $a * e = a$

$$\Rightarrow a + e - ae = a$$

$$\Rightarrow e = 0, a \neq 1$$

Consider  $a = e * a$

$$\Rightarrow a = e + a - ea$$

$$= e = 0, a \neq 1$$

$\therefore$  0 is the identity element of  $*$  in  $Q$

(iv) Let  $b \in Q$  be the inverse of  $a \in Q$

$$\Rightarrow a * b = 0 \Rightarrow b * a$$

Consider  $a * b = 0$

$$\Rightarrow a + b - ab = 0$$

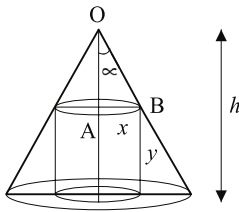
$$\Rightarrow b = \frac{a}{a-1}, a \neq 1$$

$$\therefore a^{-1} = \frac{a}{a-1} \quad \forall a \in Q - \{1\}$$

$$25. \quad \text{L.H.S.} = \begin{vmatrix} -bc & b^2 + bc & c^2 + bc \\ a^2 + ac & -ac & c^2 + ac \\ a^2 + ab & b^2 + ab & ab \end{vmatrix}$$

$$\begin{aligned}
&= \frac{1}{abc} \begin{vmatrix} -abc & ab^2 + bc & ac^2 + abc \\ ba^2 + abc & -abc & bc^2 + abc \\ ca^2 + abc & cb^2 + abc & -abc \end{vmatrix} \quad \begin{array}{l} \text{Applying} \\ R_1 \rightarrow aR_1 \\ R_2 \rightarrow bR_2 \\ R_3 \rightarrow cR_3 \end{array} \\
&\qquad\qquad\qquad R_1 \rightarrow R_1 + R_2 + R_3 \\
&= \begin{vmatrix} ab + bc + ca & ab + bc + ca & ab + bc + ca \\ ab + bc & -ac & bc + ab \\ ac + bc & cb + ac & -ab \end{vmatrix} \\
&= (ab + bc + ca)^3 \begin{vmatrix} 1 & 1 & 1 \\ ab + bc & -ac & bc + ab \\ ac + bc & cb + ac & -ab \end{vmatrix} \quad C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1 \\
&= (ab + bc + ca) \begin{vmatrix} 1 & 0 & 0 \\ ab + bc & -(ac + bc + ca) & 0 \\ ac + bc & 0 & -ab + bc + ca \end{vmatrix} \\
&= (ab + bc + ca)^3 = \text{R.H.S.}
\end{aligned}$$

26. Let radius and height of the cylinder be x units and y units respectively.



In  $\triangle OAB$

$$\tan x = \frac{x}{h - y}$$

Volume of cylinder

$$V = \pi x^2 y$$

$$= \pi \tan^2 \alpha (h-y)^2 y$$

$$\frac{dv}{dy} = \pi \tan^2 \alpha [(h-y)^2 + 2(h-y)(-1)y]$$

$$= \pi \tan^2 \alpha (h-y)(h-3y)$$

$$\frac{dV}{dy} = 0 \Rightarrow y = h \text{ (rejected)}, y = \frac{h}{3}$$

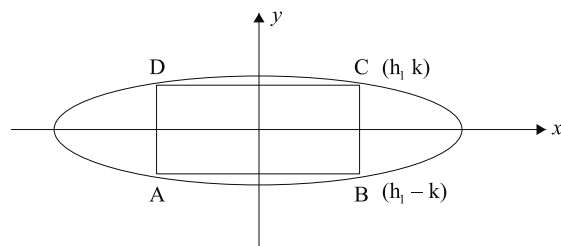
$$\frac{d^2V}{dy^2} = \pi \tan^2 \alpha [(h-y)(-3) + (h-3y)(-1)]$$

$$\text{at } y = \frac{h}{3}, \quad \frac{d^2V}{dy^2} < 0$$

$\therefore$  V is maximum at  $y = \frac{h}{3}$

OR

Let the rectangle ABCD inscribed in the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



$$\text{Let } AB = 2h$$

$$BC = 2k$$

$$\text{Then Area of rectangle } A = 4hk$$

$$\text{Also } \frac{h^2}{a^2} + \frac{k^2}{b^2} = 1 \Rightarrow k = \frac{b}{a} \sqrt{a^2 - h^2}$$



$$\therefore A = \frac{4b}{a} h\sqrt{a^2 - h^2}$$

$$A^2 = \frac{16b^2}{a^2} h^2 (a^2 - h^2)$$

$$\text{Let } f(h) = \frac{16b^2}{a^2} (a^2 h^2 - h^4)$$

$$f'(h) = \frac{16b^2}{a^2} (2a^2 h - 4h^3)$$

$$f'(h) = 0 \Rightarrow h = 0 \quad \text{or} \quad h = \frac{a}{\sqrt{2}}.$$

$$f''(h) = \frac{16b^2}{a^2} (2a^2 - 12h^2)$$

$$\text{at } h = \frac{a}{\sqrt{2}}, \quad f''(h) = -ve$$

$$\therefore A \text{ is maximum at } h = \frac{a}{\sqrt{2}}.$$

$$\therefore \text{Maximum area} = \frac{4b}{a} \times \frac{a}{\sqrt{2}} \sqrt{a^2 - \frac{a^2}{2}} = 2ab \text{ sq. units}$$

27. Let distance covered with speed of 25 km/hr = x km  
distance covered with speed 40 km/hr = y km

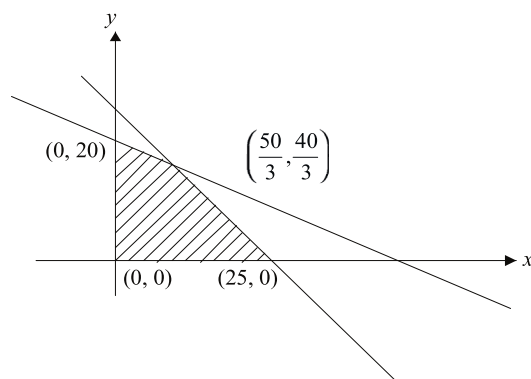
$$\text{Max } (z) = x + y$$

Subject to constraints

$$2x + 5y \leq 100$$

$$\frac{x}{25} + \frac{y}{40} \leq 1$$

$$x \geq 0, y \geq 0.$$



Corner points of bounded feasible region are

$$(0, 0), (25, 0), \left(\frac{50}{3}, \frac{40}{3}\right) \text{ \& } (0, 20)$$

$$Z = 0 \qquad Z = 25 \qquad Z = \frac{50+40}{3} = 30$$

$$(0, 0) \qquad (25, 0) \qquad \left(\frac{50}{3}, \frac{40}{3}\right)$$

$$Z = 0 + 20 = 20$$

$$(0, 20)$$

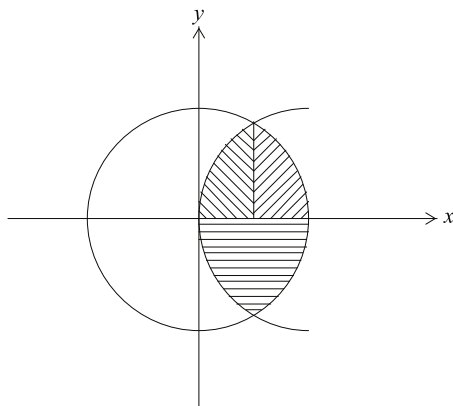
$$\text{Max } (Z) = 30 \text{ when}$$

$$\text{Distance covered with speed } 25 \text{ km / hr} = \frac{50}{3} \text{ km}$$

$$\text{\& Distance covered with speed } 40 \text{ km/hr} = \frac{40}{3} \text{ km}$$

$$28. \quad (x, y) : \quad y^2 \leq 4x, \quad 4x^2 + 4y^2 \leq 9$$

$$\text{Let } y^2 = 4x \quad \& \quad 4x^2 + 4y^2 = 9$$



$$4x^2 + 16x - 9 = 0$$

$$(2x + 9)(2x - 1) = 0$$

$$x = \frac{-9}{2} \text{ or } x = \frac{1}{2}$$

$$\text{Required area} = 2 \left[ \int_0^{1/2} \sqrt{4x} dx + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - x^2} dx \right]$$

$$= 2 \left[ 2 \left[ \frac{x^{3/2}}{3/2} \right]_0^{1/2} + \frac{x}{2} \sqrt{\frac{9}{4} - x^2} + \frac{9}{8} \sin^{-1} \left( \frac{2x}{3} \right) \right]_{1/2}^{3/2}$$

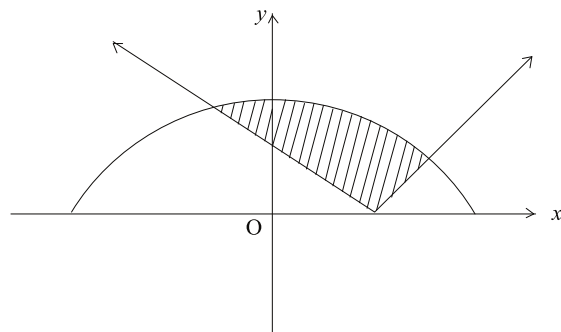
$$= \frac{8}{3} \left[ \frac{1}{2\sqrt{2}} \right] + 2 \left( 0 + \frac{9}{8} \sin^{-1}(1) \right) - \left( \frac{1}{4} \sqrt{2} + \frac{9}{8} \sin^{-1} \left( \frac{1}{3} \right) \right)$$

$$= \frac{2\sqrt{2}}{3} + \frac{9}{4} \times \frac{\pi}{2} - \frac{\sqrt{2} - 9}{2 - 4} \sin^{-1} \left( \frac{1}{3} \right)$$

$$= \frac{2\sqrt{2}}{3} + \frac{9}{4} \times \frac{\pi}{2} - \frac{\sqrt{2}}{2} - \frac{9}{4} \sin^{-1} \left( \frac{1}{3} \right)$$

OR

$$\left\{ (x, y) : |x - 1| \leq y \leq \sqrt{5 - x^2} \right\}$$



$$y = |x - 1| \quad \& \quad y = \sqrt{5 - x^2}$$

$$\Rightarrow |x - 1| = \sqrt{5 - x^2}$$

$$\Rightarrow 2x^2 - 2x - 4 = 0$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x - 2)(x + 1) = 0$$

$$\Rightarrow x = -1, 2$$

Required area

$$= \int_{-1}^2 \sqrt{5 - x^2} dx - \int_{-1}^1 (1 - x) dx - \int_1^2 (x - 1) dx$$

$$= \left[ \left( \frac{x}{2} \sqrt{5 - x^2} + \frac{5}{2} \sin^{-1} \left( \frac{x}{\sqrt{5}} \right) \right) \right] + \left[ \frac{(1 - x)^2}{2} \right] - \left[ \frac{(x - 1)^2}{2} \right]$$

$$= \left[ 1 + \frac{5}{2} \sin^{-1} \left( \frac{2}{\sqrt{5}} \right) \right] - \left( \frac{-1}{2} \times 2 + \frac{5}{2} \sin^{-1} \left( \frac{-1}{\sqrt{5}} \right) \right) + [0 - 2] - \left[ \frac{1}{2} - 0 \right]$$

$$= \cancel{x} + \frac{5}{2} \sin^{-1} \left( \frac{2}{\sqrt{5}} \right) - \cancel{x} + \frac{5}{2} \sin^{-1} \left( \frac{1}{\sqrt{5}} \right) - \cancel{2} - \frac{1}{2}$$

$$\begin{aligned}
&= \frac{5}{2} \left[ \sin^{-1} \left( \frac{2}{\sqrt{5}} \right) + \sin^{-1} \left( \frac{1}{\sqrt{5}} \right) \right] - \frac{1}{2} \\
&= \frac{5}{2} \left[ \cos^{-1} \left( \frac{1}{\sqrt{5}} \right) + \sin^{-1} \left( \frac{1}{\sqrt{5}} \right) \right] - \frac{1}{2} \int_Q \sin^{-1} \frac{2}{\sqrt{5}} = \cos^{-1} \left( \frac{1}{\sqrt{5}} \right) \\
&= \left( \frac{5\pi}{4} - \frac{1}{2} \right) sq. \text{ units}
\end{aligned}$$

29. Equation of the line through (1, -2, 3) & || to given line is

$$\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6} = \lambda [\text{Let}] \dots\dots\dots(1)$$

Co-ordinates of a general point on line are

$$(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$$

For the point of intersection of line (1) and plane

$$(2\lambda + 1) - (3\lambda - 2) + (-6\lambda + 3) = 5$$

$$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5$$

$$-7\lambda + 6 = 5$$

$$\lambda = \frac{1}{7}$$

$$\text{Point of intersection is } \left( \frac{2}{7} + 1, \frac{3}{7} - 2, \frac{-6}{7} + 3 \right)$$

$$= \left( \frac{9}{7}, \frac{-11}{7}, \frac{15}{7} \right)$$

$$\text{Required distance} = \sqrt{\left( \frac{9}{7} - 1 \right)^2 + \left( \frac{-11}{7} + 2 \right)^2 + \left( \frac{15}{7} - 3 \right)^2}$$

$$= \sqrt{\frac{4}{49} + \frac{9}{49} + \frac{36}{49}} = 1 \text{ unit}$$

# PRACTICE PAPER

TIME: ALLOWED: 3 HOURS

M.M. 100

General Instructions:

1. All questions are compulsory.
2. This question paper contains 29 questions.
3. Question 1-4 in section A are very short answer type questions carrying 1 mark each.
4. Question 5-12 in section B are short answer type questions carrying 2 marks each.
5. Question 13-23 in section C are long answer type questions carrying 4 marks each.
6. Questions 24-29 in section D are long answer II type questions carrying 6 marks each.

## SECTION A

- Q.1. Let  $R = \{a, a^3\}$  :  $a$  is a prime number less than 5} be a relation. Find the range of  $R$ .
- Q.2. The elements  $a_{ij}$  of a  $3 \times 3$  matrix are given by  $a_{ij} = \frac{1}{2} | -3i + j |$ . Write the value of elements  $a_{32}$ .
- Q.3. Find a vector in the direction of  $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ . Which has magnitude 6 units.
- Q.4. If  $f(x) = \{4 - (x - 7)^2\}$ , the find  $f^{-1}(x)$ .

## SECTIONS B

- Q.5. Find the value of  $\sin^{-1} \left[ \cos \left( \frac{33\pi}{5} \right) \right]$ .
- Q.6. If the area of a triangle is 35 sq. units with vertices  $(2, -6)$ ,  $(5, 4)$  and  $(K, 4)$ , then find the values of  $K$ .

Q.7. If  $x = t + \frac{1}{t}$ ,  $y = t - \frac{1}{t}$ , then find  $\frac{dy}{dx}$ .

Q.8. Find the interval in which  $y = x^2 e^x$  is increasing.

Q.9.  $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$

Q.10. Find the differential equation of all non-horizontal lines in a plane.

Q.11. Find a vector of magnitude 9, which is perpendicular to both the vectors  $4\hat{i} - \hat{j} + 3\hat{k}$  and  $-2\hat{i} + \hat{j} - 2\hat{k}$ .

Q.12. Three events A, B and C have probabilities  $\frac{2}{5}$ ,  $\frac{1}{3}$  and  $\frac{1}{2}$  respectively. If  $P(A \cap C) = \frac{1}{5}$  and  $P(B \cap C) = \frac{1}{4}$ , then find the values of  $P(C/B)$  and  $P(A^1 \cap C^1)$ .

### SECTION C

Q.13. If  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ , then prove that  $A^2 - 4A - 5I = O$ . Hence find  $A^{-1}$ .

Q.14. Find the value of K for which

$$f(x) = \begin{cases} \frac{\sqrt{1+Kx} - \sqrt{1-Kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1} & \text{if } 0 \leq x \leq 1 \end{cases}$$

is continuous at  $x = 0$

OR

Let  $f(x) = x|x|$ ,  $\forall x \in \mathbb{R}$ . Discuss the differentiability of  $f(x)$  at  $x = 0$

Q.15. If  $x = 2\cos\theta - \cos 2\theta$  and  $y = 2\sin\theta - \sin 2\theta$  then prove that  $\frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$ .

- Q.16. Find the equation of the normal at a point on the curve  $x^2 = 4y$ , which passes through the point (1, 2). Also, find the equation of the corresponding tangent.

OR

The money to be spent for the welfare of the employees of a firm is proportional to the rate of change of its total revenue (in ₹) received from the sale of  $x$  units of a product is given by

$R(x) = 3x^2 + 36x + 5$ , then find the marginal revenue, when  $x = 5$  and write which value does the equation indicate?

- Q.17. If  $e^x + e^y = e^{x+y}$ , then prove that

$$\frac{dy}{dx} + \frac{e^x(e^y - 1)}{e^y(e^x - 1)} = 0$$

- Q.18.  $\int \frac{dx}{\sin x + \sin 2x}$

- Q.19. Solve the following differential equation

$$x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x, x \neq 0$$

OR

Solve the differential equation

$$\frac{dy}{dx} + y \cot x = 4x \operatorname{cosec} x, \text{ given that } y = 0 \text{ when } x = \pi/2$$

- Q.20. If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are three mutually perpendicular vectors of the same magnitude, then prove that  $\vec{a} + \vec{b} + \vec{c}$  is equally inclined with the vectors  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$ .



- Q.21. Find the equation of the perpendicular from point  $(3, -1, 11)$  to line  $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ . Also find the coordinates of foot of perpendicular and length of perpendicular.
- Q.22. There is a group of 100 people who are patriotic out of which 70 believe in non-violence. To persons are selected at random out of them, write the probability distribution for the selected persons who are non-violent. Also find the mean of the distribution.
- Q.23. A letter is known to have come either from LONDON or CLIFTON. On the envelope just two consecutive letters ON are visible. what is the probability that the letter has come from (i) LONDON (ii) CLIFTON.

#### SECTION D

- Q.24. In the set of natural numbers  $N$ , define a relation  $R$  as follows:  $\forall n, m \in N, nRm$ , if on division by 5 each of the integers  $n$  and  $m$  leaves the remainder less than 5. Show that  $R$  is an equivalence relation. Also obtain the pair wise disjoint subset determined by  $R$ .

OR

Consider  $f: R^+ \rightarrow (-9, \infty)$  given by

$$f(x) = 5x^2 + 6x - 9$$

Prove that  $f$  is invertible with

$$f^{-1}(y) = \left[ \frac{\sqrt{54 + 5y} - 3}{5} \right]$$

Where  $R^+$  is the set of all positive real number.

- Q.25. If  $x + y + z = 0$ , then prove that

$$\begin{vmatrix} xa & yb & zc \\ yc & za & xb \\ zb & xc & ya \end{vmatrix} = xyz \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

OR

Find the value of  $\theta$  satisfying

$$\begin{vmatrix} 1 & 1 & \sin 3\theta \\ -4 & 3 & \cos 2\theta \\ 7 & -7 & -2 \end{vmatrix} = 0$$

- Q.26. A farmer has a plot in the shape of a circle  $x^2 + y^2 = 4$ . He divides his property among his son and daughter in such a way that son gets the area interior to the parabola  $y^2 = 3x$  and daughter gets interior to the parabola  $y^2 = -3x$ . How much area his son got? Have both of them get equal share?

- Q.27. Evaluate  $\int_{\frac{1}{3}}^1 \frac{(x - x^3)^{\frac{1}{3}}}{x^4} dx$

OR

Evaluate  $\int_0^3 (2x^2 + e^x) dx$  as limit of sum.

- Q.28. Find the distance of the point  $(1, -2, 3)$  from the plane  $x - y + z = 5$  measured parallel to the line  $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{-6}$ .

- Q.29. If a young man rides his motorcycle at 25 km/h, had to spend of ₹ 2 per km on petrol with very little pollution in the air. If he rides it at a faster speed of 40 km/h, the petrol cost increases to ₹ 5 per km and rate of pollution also increases. He has ₹ 100 to spend on petrol and wishes to find what is the maximum distance he can travel with in one hour? Express this problem as LPP and solve it graphically to find the distance to be covered with different speed.

**ANSWERS**  
**PRACTICE PAPER**

1. Range = {8, 27}
2.  $\frac{7}{2}$
3.  $4\hat{i} - 2\hat{j} + 4\hat{k}$
4.  $7 + (4 - x)^{1/3}$
5.  $\frac{-\pi}{10}$
6. K = 12 and - 2
7.  $\frac{t^2 + 1}{t^2 - 1}$
8. [0, 2]
9. x + c
10.  $\frac{d^2y}{dx^2} = 0$
11.  $-3\hat{i} + 6\hat{j} + 6\hat{k}$
12.  $P(C/B) = \frac{3}{4}, P(A^1 \mid C^1) = \frac{3}{10}$
13.  $A^{-1} = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$
14. K = - 1 OR Differentiable at x = 0
15. \_\_\_\_\_

16. Equation of normal  $x + y = 3$

Equation of tangent  $y = x - 1$

OR

66, The question indicates the value of welfare, which is necessary for each society.

17. \_\_\_\_\_

18.  $\frac{1}{6} \log |1 - \cos x| + \frac{1}{2} \log |1 + \cos x| - \frac{2}{3} \log |1 + 2 \cos x| + c$

19.  $\sin\left(\frac{y}{x}\right) = \log |cx|$  OR  $y = 2x^2 \operatorname{cosec} x - \frac{\pi^2}{2} \operatorname{cosec} x$

20. \_\_\_\_\_

21. Equation of perpendicular  $= \frac{x-3}{-1} = \frac{y+1}{6} = \frac{z-11}{-4}$

Foot of perpendicular is (2, 5, 7)

Length of perpendicular is  $\sqrt{53}$

22. 

x	0	1	2
P(x)	$\frac{29}{330}$	$\frac{140}{330}$	$\frac{161}{330}$

Mean value = 1.4

23. (i)  $\frac{12}{17}$  (ii)  $\frac{5}{17}$

24.  $A_0 = \{5, 10, 15, 20, \dots\}$

$A_1 = \{1, 6, 11, 16, \dots\}$

$A_2 = \{2, 7, 12, 17, \dots\}$

OR \_\_\_\_\_

$A_3 = \{3, 8, 13, 18, \dots\}$

$$A_4 = \{4, 9, 14, 19, \dots\}$$

25. \_\_\_\_\_

26.  $\left[ \frac{4\pi}{3} + \frac{1}{\sqrt{3}} \right]$  sq. units

Both son and daughter get equal share value

27. 6 OR  $17 + e^3$

28. 1 unit

29. Young man covers  $\frac{50}{3}$  km at speed of 25 km/h and  $\frac{40}{3}$  km at the speed of 40 km Distance 30 km.

**CBSE**  
**MATHEMATICS – 2017**  
**(OUTSIDE DELHI)**

Time allowed: 3 hours

Maximum marks: 100

**SET 1**

**SECTION A**

Questions number 1 to 4 carry one mark each.

Q.1. If for any  $2 \times 2$  square matrix A,  $A (\text{adj } A) = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$ , then write the value of  $|A|$ .

Q.2. Determine the value of 'k' for which the following function is continuous at  $x = 3$  :

$$f(x) = \begin{cases} \frac{(x+3)^2 - 36}{x-3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

Q.3. Find:  $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$

Q.4. Find the distance between the planes  $2x - y + 2z = 5$  and  $5x - 2.5y + 5z = 20$ .

**SECTIONS B**

Questions number 5 to 12 carry 2 marks each.

Q.5. If A is a skew-symmetric matrix of order 3, then prove that  $\det A = 0$ .

Q.6. Find the value of c in Rolle's theorem for the function  $f(x) = x^3 - 3x$  in  $[-\sqrt{3}, 0]$ .

- Q.7. The volume of a cube is increasing at the rate of  $9 \text{ cm}^3/\text{s}$ . How fast is its surface area increasing when the length of an edge is  $10 \text{ cm}$ ?
- Q.8. Show that the function  $f(x) = x^3 - 3x^2 + 6x - 100$  is increasing on  $\mathbb{R}$ .
- Q.9. The  $x$ -coordinate of a point on the line joining the points  $P(2, 2, 1)$  and  $Q(5, 1, -2)$  is  $4$ . Find its  $z$ -coordinate.
- Q.10. A die, whose faces are marked  $1, 2, 3$  in red and  $4, 5, 6$  in green, is tossed. Let  $A$  be the event "number obtained is even" and  $B$  be the event "number obtained is red". Find if  $A$  and  $B$  are independent events.
- Q.11. Two tailors,  $A$  and  $B$ , earn ₹  $300$  and ₹  $400$  per day respectively.  $A$  can stitch  $6$  shirts and  $4$  pairs of trousers while  $B$  can stitch  $10$  shirts and  $4$  pairs of trousers per day. To find how many days should each of them work and if it is desired to produce at least  $60$  shirts and  $32$  pairs of trousers at a minimum labour cost, formulate this as an LPP.
- Q.12. Find :  $\int \frac{dx}{5-8x-x^2}$

### SECTION C

Questions number 13 to 23 carry 4 marks each.

- Q.13. If  $\tan^{-1} \frac{x-3}{x-4} + \tan^{-1} \frac{x+3}{x+4} = \frac{\pi}{4}$ , then find the value of  $x$ .

- Q.14. Using properties of determinants, prove that  $\begin{vmatrix} a^2 + 2a & 2a + 1 & 1 \\ 2a + 1 & a + 2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a - 1)^3$

OR

Find matrix  $A$  such that  $\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{pmatrix}$

Q.15. If  $x^y + y^x = a^b$ , then find  $\frac{dy}{dx}$ .

Or

If  $e^y(x+1) = 1$ , then show that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$ .

Q.16. Find :  $\int \frac{\cos \theta}{(4 + \sin^2 \theta)(5 - 4 \cos^2 \theta)} d\theta$ .

Q.17. Evaluate:  $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$ .

Or

Evaluate:  $\int_1^4 \{|x-1| + |x-2| + |x-4|\} dx$ .

Q.18. Solve the differential equation  $(\tan^{-1} x - y) dx = (1 + x^2) dy$ .

Q.19. Show that the points A, B, C with position vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $3\hat{i} - 4\hat{j} - 4\hat{k}$  respectively, are the vertices of a right-angled triangle. Hence find the area of the triangle.

Q.20. Find the value of  $\lambda$ , if four points with position vectors  $3\hat{i} + 6\hat{j} + 9\hat{k}$ ,  $\hat{i} + 2\hat{j} + 3\hat{k}$ ,  $2\hat{i} + 3\hat{j} + \hat{k}$  and  $4\hat{i} + 6\hat{j} + \lambda\hat{k}$  are coplanar.

Q.21. There are 4 cards numbered 1, 3, 5 and 7, one number on one card. Two cards are drawn at random without replacement. Let X denote the sum of the numbers on the two drawn cards. Find the mean and variance of X.

Q.22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report



that 70% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the school and he was found to have an A grade. What is the provability that the student has 100% attendance? Is regularity required only in school? Justify your answer.

Q.23. Maximise  $Z = x + 2y$

Subject to the constraints

$$\begin{aligned} X + 2y &\geq 100, & 2x - y &\leq 0, & 2x + y &\leq 200, \\ x, y, &\geq 0 \end{aligned}$$

Solve the above LPP graphically.

#### SECTION D

Questions number 24 to 29 carry 6 marks each.

Q.24. Determine the product  $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$  and use it to solve the system of equations  $x - y + z = 4$ ,  $x - 2y - 2z = 9$ ,  $2x + y + 3z = 1$ .

Q.25. Consider  $f: \mathbf{R} - \mathbf{R} - \left\{ -\frac{4}{3} \right\} \rightarrow \mathbf{R} - \left\{ \frac{4}{3} \right\}$  given by  $f(x) = \frac{4x+3}{3x+4}$ . Show that  $f$  is bijective. Find the inverse of  $f$  and hence find  $f^{-1}(0)$  and  $x$  such that  $f^{-1}(x) = 2$ .

Or

Let  $A = \mathbf{Q} \times \mathbf{Q}$  and let  $*$  be a binary operation on  $A$  defined by  $(a, b) * (c, d) = (ac, b + ad)$  for  $(a, b), (c, d) \in A$ . Determine, whether  $*$  is commutative and associative. Then, with respect to  $*$  on  $A$ .

- (i) Find the identity element in  $A$ .
- (ii) Find the invertible elements of  $A$ .

Q.26. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

Q.27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(4, 1), B(6, 6) and C(8, 4).

Or

Find the area enclosed between the parabola  $4y = 3x^2$  and the straight line  $3x - 2y + 12 = 0$ .

Q.28. Find the particular solution of the differential equation  $(x - y) \frac{dy}{dx} = (x + 2y)$ , given that  $y = 0$  when  $x = 1$ .

Q.29. Find the coordinates of the point where the line through the points (3, -4, -5) and (2, -3, 1), crosses the plane determined by the points (1, 2, 3), (4, 2, -3) and (0, 4, 3).

Or

A variable plane which remains at a constant distance  $3p$  from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is  $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{p^2}$ .