Government of Tamilnadu

STANDARD EIGHT

TERM II

Volume 2

MATHEMATICS

SCIENCE

SOCIAL SCIENCE

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Untouchability is Inhuman and a Crime

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MATHEMATICS

STANDARD EIGHT

TERM II

1 Algebra

Diophantus

(About 3rd century B.C.)

The Greek Mathematician lived in Alexandria. He is called **"The father of Algebra"**. The equation $x^n + y^n = z^n$ is known as Diophantine equation and for $n > 2$, there are no solutions with positive integral values for x, y and z.

Al-Khwarizmi

(780 - 850 A.D.)

The Arab Mathematician wrote the book *"Kitab al-jabr wa l-mugabala"*. It was the synthesis of Indian Algebra and Greek Geometry which had the most profound effect on the development of mathematics.

1.1 Introduction

The mathematical term *'Algebra'* was derived from the Arabic word *'al-jabr'*. **'Al'** means **'The'** and **'jabr'** means **'the restoration of broken parts'**. It was coined from the title of the book *Kitab al-jabr wa l-mugabala'*. That is **'The Book of Integration and Equation'**. It literally means **'reduction and comparison'** written by the Arab Mathematician *Al - Khwarizmi*.

In ancient India, Algebra was called as 'Bija - Ganitham'. (**'Bija'** means **'the other'** and **'Ganitham'** means **'Mathematics'**) Indian mathematicians like **Aryabhatta, Bhrahmagupta, Mahavir, Bhaskara II, Sridhara** have contributed a lot to this branch of mathematics.

The Greek Mathematician, Diophantus of Alexandria had developed this branch to a great extent. Hence he is called as 'The father of Algebra'.

1.2 Algebraic Expressions - Addition and Subtraction

In class **VII**, we have learnt about variables, constants, coefficient of terms, degree of expressions etc. Let us consider the following examples of expressions :

Illustration

(i) $x + 5$ (ii) $3y - 2$ (iii) $5m^2$ (iv) $2xy + 11$

The expression $x + 5$ is formed with the variable $\boldsymbol{x}$ and the constant 5.

The expression $3y - 2$ is formed with the variable $\boldsymbol{y}$ and the constants 3 and – 2.

The expression $5m^2$ is formed with the variable $\boldsymbol{m}$ and the constant 5.

The expression $2xy + 11$ is formed with the variables $\boldsymbol{x}$ and $\boldsymbol{y}$ and the constants 2 and 11.

1.2.1 Values of the Algebraic Expression

We know that the value of the expression changes with the values chosen for the variables it contains. For example take the expression $x + 5$. The following table shows the value of the expression $x + 5$ when x takes different values :

Value for x	Value of the expression $x + 5$
1	$1 + 5 = 6$
2	$2 + 5 = 7$
3	$3 + 5 = 8$
4	$4 + 5 = 9$
-1	$-1 + 5 = 4$
-2	$-2 + 5 = 3$
-3	$-3 + 5 = 2$
$\frac{1}{2}$	$\frac{1}{2} + 5 = \frac{11}{2}$

Note: The constant 5 remains unchanged, as the values of the expression go on changing for various values of x.

Activity

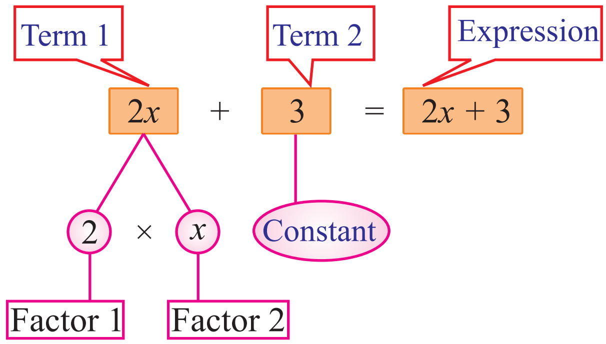
1. Assign different values for the variables given in the remaining illustration and find the values of the expressions.
2. Have you noticed any change in the values of the constants?

1.2.2 Terms, Factors and Coefficients

A single variable or a constant or a combination of these as a product or quotient forms a term.

Examples : $3, -y, ab, \frac{a}{b}, \frac{3x}{5y}, -\frac{21}{3}$.

Terms can be added or subtracted to form an expression. In the expression $2x + 3$ the term $2x$ is made of 2 factors and 2 and x while 3 is a single factor.



Consider the expression $3ab - 5a$. It has two terms $3ab$ and $-5a$. The term $3ab$ is a product of factors 3, a and b. The term $-5a$ is a product of -5 and a. The coefficient of a variable is a factor or factors.

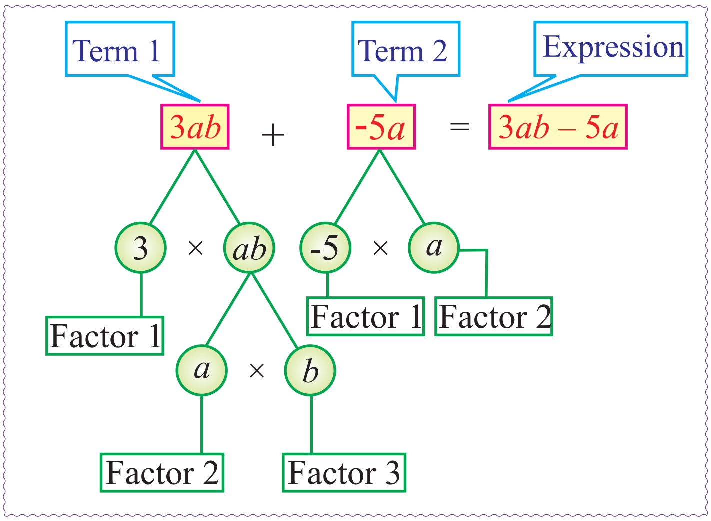
Example : In the term $3ab$;

(i) the coefficient of ab is 3

(ii) the coefficient of a is $3b$

(iii) the coeffcient of b is $3a$.

In the term $-5a$ the coefficient of a is -5



Activity

Identify the number of terms and coefficient of each term in the expression and complete the following table $x^2y^2 - 5x^2y + \frac{3}{5}xy^2 - 11$.

Sl. No.	Term	Coefficient of the term
1	x^2y^2	1
2		
3		
4		

1.2.3 Basic concepts of Polynomial : Monomial, Binomial, Trinomial and Polynomial

Monomial : An Algebraic expression that contains only one term is called a monomial.

Example : $2x^2, 3ab, -7p, \frac{5}{11}a^2b, -8, 81xyz$, etc.

Binomial : An Algebraic expression that contains only two terms is called a binomial.

Example : $x + y, 4a - 3b, 2 - 3x^2y, l^2 - 7m$, etc.

Trinomial : An Algebraic expression that contains only three terms is called a trinomial.

Example : $x + y + z,\ 2a - 3b + 4,\ x^2y + y^2z - z$, etc.

Polynomial : An expression containing a finite number of terms with non-zero coefficient is called a polynomial. In other words, it is an expression containing a finite number of terms with the combination of variables, whole number exponents of variables and constants.

Example :

$a + b + c + d,\ 7xy,\ 3abc - 10,\ 2x + 3y - 5z,\ 3x^5 + 4x^4 - 3x^3 + 72x + 5$ etc.

Degree of the Polynomial : The monomials in the polynomial are called the terms. **The highest power of the terms is the degree of the polynomial**.

The coefficient of the highest power of x in a polynomial is called the **leading coefficient** of the polynomial.

Example :

$2x^5 - x^4 + 7x^3 - 6x^2 + 12x - 4$ is a polynomial in x. Here we have six monomials $2x^5, -x^4, 7x^3, -6x^2, 12x$ and -4 which are called the terms of the polynomial.

Degree of the polynomial is 5.

The leading coefficient of the polynomial is 2.

The teacher asked the students to find the degree of the polynomial $13x^4 - 2x^2y^3 - 4$. Two of the students answered as given below.

Who is correct? Gautham or Ayisha?

Gautham

Polynomial :

$13x^4 - 2x^2y^3 - 4$

The highest power of the terms is 4.

$\therefore$ The degree of the polynomial is 4.

Ayisha

Polynomial :

$13x^4 - 2x^2y^3 - 4$

$= 13x^4 - 2x^2y^3 - 4x^0y^0$

The highest power of the terms is 5.

$\therefore$ The degree of the polynomial is 5.

If your answer is Ayisha, then you are right.

If your answer is Gautham, then where is the mistake?

Let us analyse the given polynomial: $13x^4 - 2x^2y^3 - 4$.

Term 1 : $13x^4 \rightarrow$ coefficient of x^4 is 13, variable x, power of x is 4. Hence the power of the term is 4.

Term 2 : $-2x^2y^3 \rightarrow$ coefficient of x^2y^3 is -2 and the variables are x and y; the power of x is 2 and the power of y is 3.Hence the power of the term x^2y^3 is 2+3 = 5 [Sum of the exponents of variables x and y].

Term 3 : $-4 \rightarrow$ the constant term and it can be written as $-4x^0y^0$. The power of the variables x^0y^0 is zero. Therefore the power of the term –4 is zero.

Why Gautham is wrong?

Gautham thought, the power of the second term $-2x^2y^3$ as **either two or three.** But the right way is explained above. This confusion led him to the wrong conclusion.

Standard form of the polynomial

A polynomial is in standard form when all the terms are written in order of descending powers of the variables.

Example :

Write the polynomial $2 + 9x - 9x^2 + 2x^4 - 6x^3$ in the standard form.

Now we write the polynomial in the standard form as $2x^4 - 6x^3 - 9x^2 + 9x + 2$

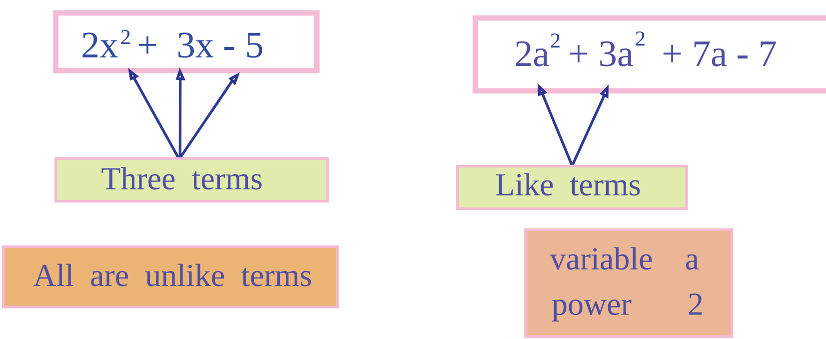
Remember: When no exponent is shown, it is understood to be 1.

Example : $9x = 9x^1$

Like terms and Unlike terms

Like terms contain the same variables having same powers.

Consider the following expressions.



Look at the following expressions :

$$3x,\ 5x^2,\ 2xy,\ -70x,\ -7,\ -3y^2,\ -3x^2,\ -20yx,\ 20,\ 4x,\ -\frac{2}{7},\ 3y.$$

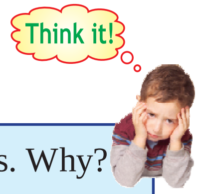
We can list out the like terms as :

(i) $3x, -70x$ and $4x$

(ii) $5x^2$ and $-3x^2$

(iii) $2xy$ and $-20yx$

(iv) $-7, 20$ and $-\frac{2}{7}$



The following are not like terms. Why?
(i) $3x$ and $3y$ and (ii) $5x^2$ and $-3y^2$

1.2.4 Addition and Subtraction of Algebraic Expressions - Revision

In class VII, we have learnt to add and subtract the algebraic expressions. **Only like or similar terms can be added or subtracted**.

Let us have a quick revision now.

Example 1.1

Add : $3x^3 + x^2 - 2$ and $2x^2 + 5x + 5$.

Solution

We first arrange these two as follows and then add:

$$\begin{array}{rrrrr} & 3x^3 & + x^2 & & -2 \\ (+) & & 2x^2 & + 5x & + 5 \\ \hline & 3x^3 & + 3x^2 & + 5x & + 3 \\ \hline \end{array}$$

Or

It can also be written as

$$\begin{array}{rrrrr} & 3x^3 & + x^2 & + 0x & - 2 \\ (+) & 0x^3 & + 2x^2 & + 5x & + 5 \\ \hline & 3x^3 & + 3x^2 & + 5x & + 3 \\ \hline \end{array}$$

Observe, we have written the term $2x^2$ of the second polynomial below the corresponding term x^2 of the first polynomial. Similarly, the constant term +5 is placed below the constant term – 2. Since the term x in the first polynomial and the term x^3 in the second polynomial do not exist, their respective places have been left blank to facilitate the process of addition. Or, for the non existing terms, we annexe the terms with zero coefficients.

Example 1.2

Find out the sum of the polynomials $3x - y$, $2y - 2x$ and $x + y$.

Solution

Column method of addition

$$\begin{array}{rrr} & 3x & - y \\ & -2x & + 2y \\ & x & + y \\ (+) & & \\ \hline & 2x & + 2y \\ \hline \end{array}$$

(Rearranging $2y - 2x$ as $-2x + 2y$)

Row method of addition

$(3x - y) + (2y - 2x) + (x + y)$

$= (3x - 2x + x) + (-y + 2y + y)$

$= (4x - 2x) + (3y - y)$

$= 2x + 2y$.

Therefore, polynomials may be added in a row by combining the like terms.

Example 1.3

(i) Subtract $5xy$ from $8xy$ (ii) Subtract $3c + 7d^2$ from $5c - d^2$

(iii) Subtract $2x^2 + 2y^2 - 6$ from $3x^2 - 7y^2 + 9$

Solution

(i) Subtract $5xy$ from $8xy$.

The first step is to place them as

$$\begin{array}{r} 8xy \\ -\,5xy \\ \hline 3xy \\ \hline \end{array}$$

(The two terms $8xy, -5xy$ are like terms)

$\therefore\ 8xy - 5xy = 3xy$

(ii) Subtract $3c + 7d^2$ from $5c - d^2$

Solution

$$\begin{array}{r} 5c - d^2 \\ -(3c + 7d^2) \\ \hline \\ \hline \end{array} \quad \text{(or)} \quad \begin{array}{r} 5c - d^2 \\ -3c - 7d^2 \\ \hline 2c - 8d^2 \\ \hline \end{array}$$

Often, we do this as →

$$\begin{array}{r} 5c - d^2 \\ 3c + 7d^2 \\ - \quad - \\ \hline 2c - 8d^2 \\ \hline \end{array}$$

Alternatively, this can also be done as :

$$\begin{aligned} (5c - d^2) - (3c + 7d^2) &= 5c - d^2 - 3c - 7d^2 \\ &= (5c - 3c) + (-d^2 - 7d^2) \\ &= 2c + (-8d^2) \\ &= 2c - 8d^2 \end{aligned}$$

(iii) Subtract $2x^2 + 2y^2 - 6$ from $3x^2 - 7y^2 + 9$

Solution

$$\begin{array}{r} 3x^2 - 7y^2 + 9 \\ 2x^2 + 2y^2 - 6 \\ - \quad\ - \quad\ + \\ \hline x^2 - 9y^2 + 15 \\ \hline \end{array}$$

[Change of the sign]

Alternative Method

$(3x^2 - 7y^2 + 9) - (2x^2 + 2y^2 - 6)$

$= 3x^2 - 7y^2 + 9 - 2x^2 - 2y^2 + 6$

$= (3x^2 - 2x^2) + (-7y^2 - 2y^2) + (9 + 6)$

$= x^2 + (-9y^2) + 15$

$= x^2 - 9y^2 + 15$

EXCERCISE 1.1

1. Choose the correct answer for the following:

(i) The coefficient of x^4 in $-5x^7 + \frac{3}{7}x^4 - 3x^3 + 7x^2 - 1$ is ______

(A) – 5 (B) – 3 (C) $\frac{3}{7}$ (D) 7

(ii) The coefficient of xy^2 in $7x^2 - 14x^2y + 14xy^2 - 5$ is ______

(A) 7 (B) 14 (C) – 14 (D) – 5

(iii) The power of the term $x^3y^2z^2$ is ____

(A) 3 (B) 2 (C) 12 (D) 7

(iv) The degree of the polynomial $x^2 - 5x^4 + \frac{3}{4}x^7 - 73x + 5$ is ______

(A) 7 (B) $\frac{3}{4}$ (C) 4 (D) – 73

(v) The degree of the polynomial $x^2 - 5x^2y^3 + 30x^3y^4 - 576xy$ is ______

(A) – 576 (B) 4 (C) 5 (D) 7

(vi) $x^2 + y^2 - 2z^2 + 5x - 7$ is a __________

(A) monomial (B) binomial (C) trinomial (D) polynomial

(vii) The constant term of $0.4x^7 - 75y^2 - 0.75$ is ____

(A) 0.4 (B) 0.75 (C) – 0.75 (D) – 75

2. Identify the terms and their coefficients for the following expressions:

(i) $3abc - 5ca$ (ii) $1 + x + y^2$ (iii) $3x^2y^2 - 3xyz + z^3$

(iv) $-7 + 2pq - \frac{5}{7}qr + rp$ (v) $\frac{x}{2} - \frac{y}{2} - 0.3xy$

3. Classify the following polynomials as monomials, binomials and trinomials:

$3x^2$, $3x + 2$, $x^2 - 4x + 2$, $x^5 - 7$, $x^2 + 3xy + y^2$,

$s^2 + 3st - 2t^2$, $xy + yz + zx$, $a^2b + b^2c$, $2l + 2m$

4. Add the following algebraic expressions:

(i) $2x^2 + 3x + 5, 3x^2 - 4x - 7$ (ii) $x^2 - 2x - 3, x^2 + 3x + 1$

(iii) $2t^2 + t - 4, 1 - 3t - 5t^2$ (iv) $xy - yz, yz - xz, zx - xy$

(v) $a^2 + b^2, b^2 + c^2, c^2 + a^2, 2ab + 2bc + 2ca$.

5. (i) Subtract $2a - b$ from $3a - b$

(ii) Subtract $-3x + 8y$ from $-7x - 10y$

(iii) Subtract $2ab + 5bc - 3ca$ from $7ab - 2bc + 10ca$

(iv) Subtract $x^5 - 2x^2 - 3x$ from $x^3 + 3x^2 + 1$

(v) Subtract $3x^2y - 2xy + 2xy^2 + 5x - 7y - 10$ from
$15 - 2x + 5y - 11xy + 2xy^2 + 8x^2y$

6. Find out the degree of the polynomials and the leading coefficients of the polynomials given below:

(i) $x^2 - 2x^3 + 5x^7 - \frac{8}{7}x^3 - 70x - 8$ (ii) $13x^3 - x^{13} - 113$

(iii) $-77 + 7x^2 - x^7$ (iv) $-181 + 0.8y - 8y^2 + 115y^3 + y^8$

(v) $x^7 - 2x^3y^5 + 3xy^4 - 10xy + 11$

1.3 Multiplication of Algebraic Expressions

1.3.1 Multiplying two Monomials

We shall start with $x + x + x + x + x = 5x$

Similarly, we can write, $5 \times (2x) = (2x) + (2x) + (2x) + (2x) + (2x) = 10x$

Multiplication is repeated **Addition.**

Illustration

(i) $x \times 5y = x \times 5 \times y = 5 \times x \times y = 5xy$

(ii) $2x \times 3y = 2 \times x \times 3 \times y = 2 \times 3 \times x \times y = 6xy$

(iii) $2x \times (-3y) = 2 \times (-3) \times x \times y = -6 \times x \times y = -6xy$

(iv) $2x \times 3x^2 = 2 \times x \times 3 \times x^2 = (2 \times 3) \times (x \times x^2) = 6x^3$

(v) $2x \times (-3xyz) = 2 \times (-3) \times (x \times xyz) = -6x^2yz.$

Note: 1. Product of monomials are also monomials.

2. Coefficient of the product = Coefficient of the first monomial $\times$ Coefficient of the second monomial.

3. Laws of exponents $a^m \times a^n = a^{m+n}$ is useful, in finding the product of the terms.

4. The products of a and b can be represented as: $a \times b$, ab, $a.b$, $a(b)$, $(a)b$, $(a)(b)$, (ab).

(vi) $(3x^2)(4x^3)$

$= (3 \times x \times x)(4 \times x \times x \times x)$

$= (3 \times 4)(x \times x \times x \times x \times x)$

$= 12x^5$

(Or) $(3x^2)(4x^3) = (3 \times 4)(x^2 \times x^3) = 12(x^{2+3})$

$= 12x^5$ (using $a^m \times a^n = a^{m+n}$)

Some more useful examples are as follows:

(vii) $2x \times 3y \times 5z = (2x \times 3y) \times 5z$

$= (6xy) \times 5z$

$= 30\,xyz$

(or) $2x \times 3y \times 5z = (2 \times 3 \times 5) \times (x \times y \times z) = 30xyz$

(viii) $4ab \times 3a^2b^2 \times 2a^3b^3 = (4ab \times 3a^2b^2) \times 2a^3b^3$

$= (12a^3b^3) \times 2a^3b^3$

$= 24a^6b^6$

(or) $4ab \times 3a^2b^2 \times 2a^3b^3 = 4 \times 3 \times 2 \times (ab \times a^2b^2 \times a^3b^3)$

$= 24(a^{1+2+3} \times b^{1+2+3})$

$= 24a^6b^6$

1.3.2 Multiplying a Monomial by a Binomial

Let us learn to multiply a monomial by a binomial through the following examples.

Example 1.4

Simplify: $(2x) \times (3x + 5)$

Solution We can write this as:

$$(2x) \times (3x + 5) = (2x \times 3x) + (2x \times 5) \quad \text{[Using the distributive law]}$$

$$= 6x^2 + 10x$$

Step 1

Step 2

Example 1.5

Simplify: $(-2x) \times (4 - 5y)$

Solution

$$(-2x) \times (4 - 5y) = [(-2x) \times 4] + [(-2x) \times (-5y)]$$

$$= (-8x) + (10xy) \quad \text{[Using the distributive law]}$$

$$= -8x + 10xy$$

Step 1

Step 2

Note: (i) The product of a monomial by a binomial is a binomial.

(ii) We use the commutative and distributive laws to solve multiplication sums. $a \times b = b \times a$ **(Commutative Law)**

$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$ **(Distributive laws)**

1.3.3. Multiplying a Monomial by a Polynomial

A Polynomial with more than two terms is multiplied by a monomial as follows:

Example 1.6

Simplify: (i) $3(5y^2 - 3y + 2)$

(ii) $2x^2 \times (3x^2 - 5x + 8)$

Solution

(i) $3(5y^2 - 3y + 2) = (3 \times 5y^2) + (3 \times -3y) + (3 \times 2)$

$= 15y^2 - 9y + 6$

[or] →

$$\begin{array}{rr} & 5y^2 - 3y + 2 \\ \times & 3 \\ \hline & 15y^2 - 9y + 6 \\ \hline \end{array}$$

(ii) $2x^2 \times (3x^2 - 5x + 8)$

$= (2x^2 \times 3x^2) + (2x^2 \times (-5x)) + (2x^2 \times 8)$

$= 6x^4 - 10x^3 + 16x^2$

[or] →

$$\begin{array}{rr} & 3x^2 - 5x + 8 \\ \times & 2x^2 \\ \hline & 6x^4 - 10x^3 + 16x^2 \\ \hline \end{array}$$

1.3.4 Multiplying a Binomial by a Binomial

We shall now proceed to multiply a binomial by another binomial, using the distributive and commutative laws. Let us consider the following example.

Example 1.7

Simplify : $(2a+3b)(5a+4b)$

Solution

Every term in one binomial multiplies every term in the other binomial.

$$
\begin{aligned}
(2a+3b)\ (5a+4b) &= (2a\times 5a)+(2a\times 4b)+(3b\times 5a)+(3b\times 4b)\\
&= 10a^2+8ab+15ba+12b^2\\
&= 10a^2+8ab+15ab+12b^2 \qquad [\because ab = ba]\\
&= 10a^2+23ab+12b^2
\end{aligned}
$$

[Adding like terms $8ab$ and $15ab$]

$$(2a+3b)(5a+4b) = 10a^2+23ab+12b^2$$

Note : In the above example, while multiplying two binomials we get only 3 terms instead of $2\times 2 = 4$ terms. Because we have combined the like terms $8ab$ and $15ab$.

1.3.5 Multiplying a Binomial by a Trinomial

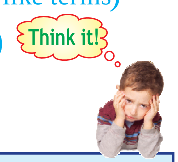
In this multiplication, we have to multiply each of the three terms of the trinomial by each of the two terms in the binomial.

Example 1.8

Simplify: $(x+3)(x^2-5x+7)$

Solution

$$
\begin{aligned}
(x+3)\ (x^2-5x+7) &= x\ (x^2-5x+7) + 3\ (x^2-5x+7) \quad \text{(Using the distributive law)}\\
&= x^3-5x^2+7x+3x^2-15x+21\\
&= x^3-5x^2+3x^2+7x-15x+21 \quad \text{(Grouping the like terms)}\\
&= x^3-2x^2-8x+21 \quad \text{(Combining the like terms)}
\end{aligned}
$$



Alternative Method :

$$
\begin{array}{lcr}
 & & (x+3)\\
 & & \times\ (x^2-5x+7)\\
x(x^2-5x+7) & : & x^3-5x^2+7x\\
3(x^2-5x+7) & : & 3x^2-15x+21\\
\hline
 & & = x^3-2x^2-8x+21\\
\hline
\end{array}
$$

In this example, while multipltying, instead of expecting $2\times 3 = 6$ terms, we are getting only 4 terms in the product. **Could you find out the reason?**

Quit the confusion.

1. Is $2xx = 2x$?

   No. $2xx$ can be written as $2\,(x)\,(x) = 2x^2$.It is the product of the terms 2, x and x. But $2x$ means $x + x$ or $2\,(x)$.

2. Is $7xxy = 7xy$?

   No. $7xxy$ is the product of 7, x, x and y and not the product of 7, x and y. Hence the correct answer for $7xxy = 7\,(x)\,(x)\,(y) = 7x^2y$.

EXERCISE 1.2

1. Find the product of the following pairs of monomials:

   (i) $3, 7x$ (ii) $-7x, 3y$ (iii) $-3a, 5ab$ (iv) $5a^2, -4a$ (v) $\frac{3}{7}x^5, \frac{14}{9}x^2$

   (vi) xy^2, x^2y (vii) x^3y^5, xy^2 (viii) abc, abc (ix) $xyz.\ x^2yz$ (x) $a^2b^2c^3, abc^2$

2. Complete the following table of products:

First monomial → Second Monomial ↓	$2x$	$-3y$	$4x^2$	$-5xy$	$7x^2y$	$-6x^2y^2$
$2x$	$4x^2$	....	....			
$-3y$						
$4x^2$						
$-5xy$				$25x^2y^2$		
$7x^2y$						
$-6x^2y^2$		$18x^2y^3$				

3. Find out the product :

   (i) $2a, 3a^2, 5a^4$ (ii) $2x, 4y, 9z$ (iii) ab, bc, ca

   (iv) $m, 4m, 3m^2, -6m^3$ (v) xyz, y^2z, yx^2 (vi) lm^2, mn^2, ln^2

   (vii) $-2p, -3q, -5p^2$

4. Find the product :

   (i) $(a^3) \times (2a^5) \times (4a^{15})$ (ii) $(5-2x)(4+x)$ (iii) $(x+3y)(3x-y)$

   (iv) $(3x+2)(4x-3)$ (v) $(\frac{2}{3}ab)\ (\frac{-15}{8}a^2b^2)$

5. Find the product of the following :

   (i) $(a+b)(2a^2-5ab+3b^2)$ (ii) $(2x+3y)(x^2-xy+y^2)$

   (iii) $(x+y+z)(x+y-z)$ (iv) $(a+b)(a^2+2ab+b^2)$ (v) $(m-n)(m^2+mn+n^2)$

6. (i) Add $2x\,(x-y-z)$ and $2y\,(z-y-x)$

   (ii) Subtract $3a\,(a-2b+3c)$ from $4a\,(5a+2b-3c)$

1.4 Identities

Let us consider the equality $(x+2)(x+3) = x^2 + 5x + 6$.

We evaluate both the sides of this equality for some value of **x,** say $x = 5$.

For $x = 5$, LHS $= (x+2)(x+3) = (5+2)(5+3) = 7 \times 8 = 56$

RHS $= x^2 + 5x + 6 = 5^2 + 5(5) + 6 = 25 + 25 + 6 = 56$

Thus the values of the two sides of the equality are equal when $x = 5$.

Let us now take, $x = -5$.

LHS $= (x+2)(x+3) = (-5+2)(-5+3) = (-3)(-2) = 6$

RHS $= x^2 + 5x + 6 = (-5)^2 + 5(-5) + 6 = 25 - 25 + 6 = 6$

Thus the values of the two sides of the equality are equal when $x = -5$,

If LHS = RHS is true for every value of the variable in it, then it is called as an Identity.

Thus, $(x+2)(x+3) = x^2 + 5x + 6$ is an identity.

Identity: An equation which is true for all possible values of the variable is called an Identity.

Activity

Check whether the following are Identities:

(i) $5(x+4) = 5x + 20$ and (ii) $6x + 10 = 4x + 20$.

1.4.1 Algebraic Identities

We proceed now to study the three important Algebraic Identities which are very useful in solving many problems. We obtain these Identities by multiplying a binomial by another binomial.

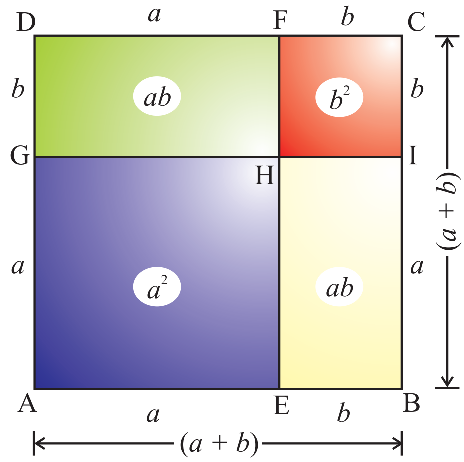
Identity 1

Let us consider $(a+b)^2$.

$$\begin{aligned}(a+b)^2 &= (a+b)(a+b)\\ &= a^2 + ab + ba + b^2\\ &= a^2 + ab + ab + b^2 \qquad [\because ab = ba]\\ &= a^2 + 2ab + b^2\end{aligned}$$

Thus, $\boxed{(a+b)^2 = a^2 + 2ab + b^2}$

Geometrical Proof of $(a+b)^2$



In this diagram,

side of the square ABCD is $(a+b)$.

Area of the square ABCD

$= (a+b)(a+b)$

= Area of the square AEHG +

Area of the rectangle EBIH +

Area of the rectangle GHFD +

Area of the square HICF

$$= (a\times a)+(b\times a)+(a\times b)+(b\times b)$$

$$= a^2+ba+ab+b^2$$

$$(a+b)^2 = a^2+2ab+b^2$$

$$\therefore \quad (a+b)^2 = a^2+2ab+b^2$$

Identity 2

Let us consider $(a-b)^2$

$$(a-b)^2 = (a-b)\ (a-b)$$

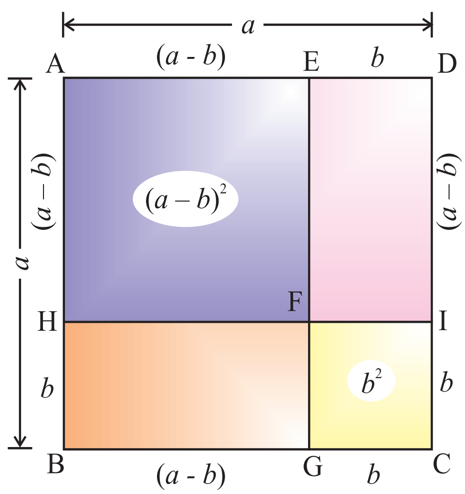
$$= a^2-ab-ba+b^2$$

$$= a^2-ab-ab+b^2$$

$$= a^2-2ab+b^2$$

Thus, $\boxed{(a-b)^2 = a^2-2ab+b^2}$

Geometrical Proof of $(a-b)^2$



The Area of the square ABCD is a^2 sq. units.

The Area of the square AHFE with side $(a-b)$is $(a-b)^2$ sq. units.

This is the area of the blue coloured square portion.

The Area of the rectangles,

BCIH $= a\times b$ sq. units.

EGCD $= a\times b$ sq. units.

The area of the square FGCI $= b^2$ square units.

Area of BGFH = Area of rectangle BCIH –Area of square FGCI

$$= ab - b^2$$

We can see that the area of a square AHFE

= Area of the square ABCD –
[Area of the rectangle EGCD +
Area of the rectangle BGFH]

$$(a-b)^2 = a^2 - (ab + ab - b^2)$$
$$= a^2 - ab - ab + b^2$$
$$= a^2 - 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Identity 3

Let us consider $(a+b)(a-b)$.

$$(a+b)(a-b) = a^2 - ab + ba - b^2$$
$$= a^2 - \cancel{ab} + \cancel{ab} - b^2 \qquad [\because ab = ba]$$
$$= a^2 - b^2$$

Thus, $\boxed{(a+b)(a-b) = a^2 - b^2}$

Geometrical Proof of $(a+b)(a-b)$

The area of the rectangle AHGE = $(a+b)\times(a-b)$

= Area of the square ABCD –

Area of the rectangle EFCD +

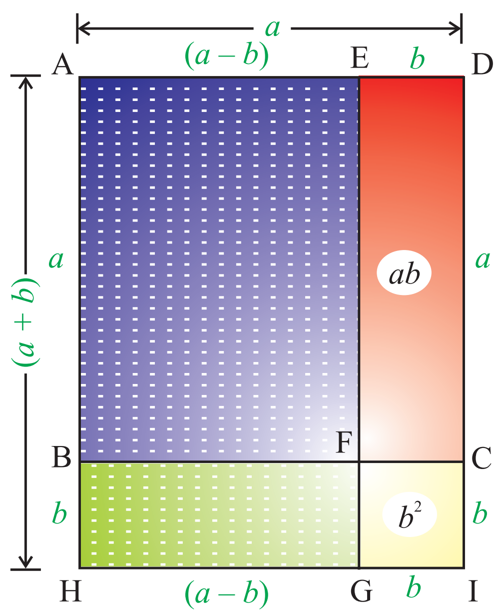
Area of the rectangle BHIC –

Area of the square FGIC.

$$= a^2 - \cancel{ab} + \cancel{ab} - b^2$$

$$= a^2 - b^2$$

$$(a+b)\times(a-b) = a^2 - b^2$$



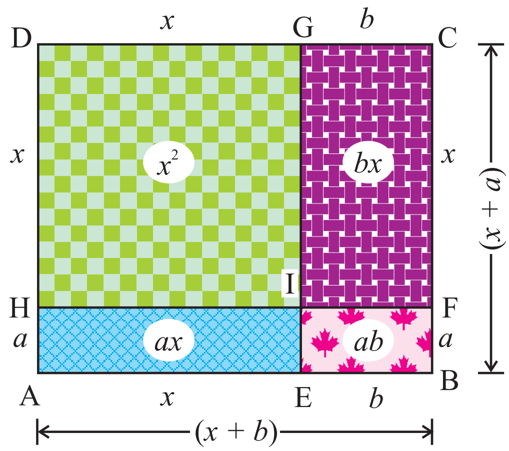
General Identity

Let us consider $(x + a)(x + b)$

$$(x + a)(x + b) = x^2 + bx + ax + ab$$
$$= x^2 + ax + bx + ab$$

Thus, $\boxed{(x + a)(x + b) = x^2 + (a + b)x + ab}$

Geometrical Proof of $(x + a)(x + b)$



The area of the rectangle

ABCD $= (x + a)(x + b)$

$=$ Area of the square DHIG +

Area of the rectangle AEIH +

Area of the rectangle IFCG +

Area of the rectangle EBFI

$= x^2 + ax + bx + ab$

$= x^2 + (a + b)x + ab$

$\therefore \quad (x + a)(x + b) = x^2 + (a + b)x + ab$

Algebraic Identities

- $(a + b)^2 \equiv a^2 + 2ab + b^2$
- $(a - b)^2 \equiv a^2 - 2ab + b^2$
- $(a + b)(a - b) \equiv a^2 - b^2$
- $(x + a)(x + b) \equiv x^2 + (a + b)x + ab$

(Usually, in identities the symbol '$\equiv$' is used. Here we use '$=$' for simplicity)

1.4.2 Applying the Identities

Example 1.9

Expand (i) $(x + 5)^2$ (ii) $(x + 2y)^2$ (iii) $(2x + 3y)^2$ (iv) 105^2.

Solution

(i) $(x + 5)^2 = x^2 + 2(x)(5) + 5^2$

$= x^2 + 10x + 25$

Using the identity:
$(a + b)^2 = a^2 + 2ab + b^2$
Here, $a = x$, $b = 5$.

Aliter: $(x + 5)^2 = (x + 5)(x + 5)$

$= x(x + 5) + 5(x + 5)$

$= x^2 + 5x + 5x + 25$

$= x^2 + 10x + 25$

(ii) $(x+2y)^2 = x^2 + 2(x)(2y) + (2y)^2$

$= x^2 + 4xy + 4y^2$

Using the identity:
$(a+b)^2 = a^2 + 2ab + b^2$
Here, $a = x$, $b = 2y$.

Aliter: $(x+2y)^2 = (x+2y)(x+2y)$

$= x(x+2y) + 2y(x+2y)$

$= x^2 + 2xy + 2yx + 4y^2$ $[\because xy = yx]$

$= x^2 + 4xy + 4y^2$

(iii) $(2x+3y)^2 = (2x)^2 + 2(2x)(3y) + (3y)^2$

$= 4x^2 + 12xy + 9y^2$

Using the identity:
$(a+b)^2 = a^2 + 2ab + b^2$
Here, $a = 2x$, $b = 3y$.

Aliter: $(2x+3y)^2 = (2x+3y)(2x+3y)$

$= 2x(2x+3y) + 3y(2x+3y)$

$= (2x)(2x) + (2x)(3y) + (3y)(2x) + (3y)(3y)$

$= 4x^2 + 6xy + 6yx + 9y^2$ $[\because xy = yx]$

$(2x+3y)^2 = 4x^2 + 12xy + 9y^2$

(iv) $105^2 = (100+5)^2$

$= 100^2 + 2(100)(5) + 5^2$

$= (100 \times 100) + 1000 + 25$

$= 10000 + 1000 + 25$

$105^2 = 11025$

Using the identity:
$(a+b)^2 = a^2 + 2ab + b^2$
Here, $a = 100$, $b = 5$.

Example 1.10

Find the values of (i) $(x-y)^2$ (ii) $(3p-2q)^2$ (iii) 97^2 (iv) $(4.9)^2$

Solution

(i) $(x-y)^2 = x^2 - 2(x)(y) + y^2$

$= x^2 - 2xy + y^2$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = x$, $b = y$.

(ii) $(3p-2q)^2 = (3p)^2 - 2(3p)(2q) + (2q)^2$

$= 9p^2 - 12pq + 4q^2$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 3p$, $b = 2q$.

(iii) $97^2 = (100-3)^2$

$= (100)^2 - 2(100)(3) + 3^2$

$= 10000 - 600 + 9$

$= 9400 + 9$

$= 9409$

Using the identity:
$(a-b)^2 = a^2 - 2ab + b^2$
Here, $a = 100$, $b = 3$.

(iv) $(4.9)^2 = (5.0-0.1)^2$
$= (5.0)^2 - 2(5.0)(0.1) + (0.1)^2$
$= 25.00 - 1.00 + 0.01$
$= 24.01$

> ***Using the identity:***
> $(a-b)^2 = a^2 - 2ab + b^2$
> Here, $a = 5.0$, $b = 0.1$.

Example 1.11

Evaluate the following using the identity $(a+b)(a-b) = a^2 - b^2$

(i) $(x+3)(x-3)$ (ii) $(5a+3b)(5a-3b)$ (iii) 52×48 (iv) $997^2 - 3^2$.

Solution

(i) $(x+3)(x-3) = x^2 - 3^2$
$= x^2 - 9$

> ***Using the identity:***
> $(a+b)(a-b) = a^2 - b^2$
> Here, $a = x$, $b = 3$.

(ii) $(5a+3b)(5a-3b) = (5a)^2 - (3b)^2$
$= 25a^2 - 9b^2$

> ***Using the identity:***
> $(a+b)(a-b) = $ `$a^2 - b^2$
> Here, $a = 5a$, $b = 3b$.

(iii) $52 \times 48 = (50+2)(50-2)$
$= 50^2 - 2^2$
$= 2500 - 4$
$= 2496$

> ***Using the identity:***
> $(a+b)(a-b) = a^2 - b^2$
> Here, $a = 50$, $b = 2$.

(iv) $997^2 - 3^2 = (997+3)(997-3)$
$= (1000)(994)$
$= 994000$

> ***Using the identity:***
> $a^2 - b^2 = (a+b)(a-b)$
> Here, $a = 997$, $b = 3$.

Example 1.12

Using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$, find the values of the following: (i) $(m+3)(m+5)$ (ii) $(p-2)(p-3)$ (iii) $(2x+3y)(2x-4y)$ (iv) 55×56 (v) 95×103 (vi) 501×505

Solution

(i) $(m+3)(m+5) = m^2 + (3+5)m + (3)(5)$
$= m^2 + 8m + 15$

> ***Using the identity:***
> $(x+a)(x+b) = x^2 + (a+b)x + ab$
> Here, $x = m$, $a = 3$, $b = 5$.

(ii) $(p-2)(p-3) = p^2 + (-2-3)p + (-2)(-3)$
$= p^2 + (-5)p + 6$
$= p^2 - 5p + 6$

> ***Using the identity:***
> $(x+a)(x+b) = x^2 + (a+b)x + ab$
> Here, $x = p$, $a = -2$, $b = -3$.

(iii) $(2x+3y)(2x-4y) = (2x)^2 + (3y-4y)(2x) + (3y)(-4y)$

$= 4x^2 + (-y)(2x) - 12y^2$

$= 4x^2 - 2xy - 12y^2$

Using the identity:
$(x + a)(x + b) = x^2 + (a + b)x + ab$
Here : x, a, b are $2x, 3y, -4y$.

(iv) $55 \times 56 = (50+5)(50+6)$

$= 50^2 + (5+6)50 + (5)(6)$

$= (50 \times 50) + (11)50 + 30$

$= 2500 + 550 + 30$

$= 3080$

Using the identity:
$(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 50$, $a = 5$, $b = 6$.

(v) $95 \times 103 = (100-5)(100+3)$

$= (100)^2 + (-5+3)(100) + (-5)(3)$

$= (100 \times 100) + (-2)(100) - 15$

$= 10000 - 200 - 15$

$= 9800 - 15$

$= 9785$

Using the identity:
$(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 100$, $a = -5$, $b = 3$.

(vi) $501 \times 505 = (500+1)(500+5)$

$= (500)^2 + (1+5)(500) + (1)(5)$

$= (500 \times 500) + (6)(500) + (1)(5)$

$= (500 \times 500) + (6)(500) + 5$

$= 250000 + 3000 + 5$

$= 253005$

Using the identity:
$(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 500$, $a = 1$, $b = 5$.

1.4.3 Deducing some useful Identities

Let us consider,

(i) $(a+b)^2 + (a-b)^2 = (a^2 + 2ab + b^2) + (a^2 - 2ab + b^2)$

$= a^2 + \cancel{2ab} + b^2 + a^2 - \cancel{2ab} + b^2$

$= 2a^2 + 2b^2$

$(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$$\boxed{\frac{1}{2}[(a+b)^2 + (a-b)^2] = a^2 + b^2}$$

(ii) $(a+b)^2 - (a-b)^2 = (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$

$= \cancel{a^2} + 2ab + \cancel{b^2} - \cancel{a^2} + 2ab - \cancel{b^2}$

$(a+b)^2 - (a-b)^2 = 4ab$

$$\boxed{\frac{1}{4}[(a+b)^2 - (a-b)^2] = ab}$$

(iii) $(a+b)^2 - 2ab = a^2 + b^2 + \cancel{2}ab - \cancel{2}ab$

$= a^2 + b^2$

$(a+b)^2 - 2ab = a^2 + b^2$

(iv) $(a+b)^2 - 4ab = a^2 + 2ab + b^2 - 4ab$

$= a^2 - 2ab + b^2$

$= (a-b)^2$

$(a+b)^2 - 4ab = (a-b)^2$

(v) $(a-b)^2 + 2ab = a^2 - 2\cancel{a}b + b^2 + 2\cancel{a}b$

$= a^2 + b^2$

$(a-b)^2 + 2ab = a^2 + b^2$

(vi) $(a-b)^2 + 4ab = a^2 - 2ab + b^2 + 4ab$

$= a^2 + 2ab + b^2$

$= (a+b)^2$

$(a-b)^2 + 4ab = (a+b)^2$

Deduced Identities

- $\frac{1}{2}[(a+b)^2 + (a-b)^2] = a^2 + b^2$
- $\frac{1}{4}[(a+b)^2 - (a-b)^2] = ab$
- $(a+b)^2 - 2ab = a^2 + b^2$
- $(a+b)^2 - 4ab = (a-b)^2$
- $(a-b)^2 + 2ab = a^2 + b^2$
- $(a-b)^2 + 4ab = (a+b)^2$

Example 1.13

If the values of $a+b$ and $a-b$ are 7 and 4 respectively, find the values of $a^2 + b^2$ and ab.

Solution

(i)
$$a^2 + b^2 = \frac{1}{2}[(a+b)^2 + (a-b)^2]$$
$$= \frac{1}{2}[7^2 + 4^2] \quad \text{[Substituting the values of } a+b=7,\ a-b=4]$$
$$= \frac{1}{2}(49+16)$$
$$= \frac{1}{2}(65)$$
$$= \frac{65}{2}$$
$$a^2 + b^2 = \frac{65}{2}$$

(ii)
$$ab = \frac{1}{4}[(a+b)^2 - (a-b)^2]$$
$$= \frac{1}{4}(7^2 - 4^2) \quad \text{[Substituting the values of } a+b=7,\ a-b=4]$$
$$= \frac{1}{4}(49-16)$$
$$= \frac{1}{4}(33)$$
$$ab = \frac{33}{4}$$

Example 1.14

If $(a+b)=10$ and $ab=20$, find a^2+b^2 and $(a-b)^2$.

Solution

(i)
$$\begin{aligned} a^2+b^2 &= (a+b)^2-2ab \quad \text{[Substituting } a+b=10,\ ab=20] \\ a^2+b^2 &= (10)^2-2(20) \\ &= 100-40=60 \\ a^2+b^2 &= 60 \end{aligned}$$

(ii)
$$\begin{aligned} (a-b)^2 &= (a+b)^2-4ab \quad \text{[Substituting } a+b=10,\ ab=20] \\ &= (10)^2-4(20) \\ &= 100-80 \\ (a-b)^2 &= 20 \end{aligned}$$

Example 1.15

If $(x+l)(x+m)=x^2+4x+2$ find l^2+m^2 and $(l-m)^2$

Solution

By product formula, we know

$$(x+l)(x+m) = x^2+(l+m)x+lm$$

So, by comparing RHS with x^2+4x+2, we have,

$$l+m = 4 \text{ and } lm=2$$

Now,
$$\begin{aligned} l^2+m^2 &= (l+m)^2-2lm \\ &= 4^2-2(2)=16-4 \\ l^2+m^2 &= 12 \\ (l-m)^2 &= (l+m)^2-4lm \\ &= 4^2-4(2)=16-8 \\ (l-m)^2 &= 8 \end{aligned}$$

EXERCISE 1.3

1. Choose the correct answer for the following:

(i) $(a+b)^2=(a+b)\times$ ______

(A) ab (B) $2ab$ (C) $(a+b)$ (D) $(a-b)$

(ii) $(a-b)^2=(a-b)\times$ ______

(A) $(a+b)$ (B) $-2ab$ (C) ab (D) $(a-b)$

(iii) $(a^2-b^2)=(a-b)\times$ ______

(A) $(a-b)$ (B) $(a+b)$ (C) $a^2+2ab+b^2$ (D) $a^2-2ab+b^2$

(iv) $9.6^2 = \_\_\_\_\_$

(A) 9216 (B) 93.6 (C) 9.216 (D) 92.16

(v) $(a+b)^2 - (a-b)^2 = \_\_\_\_\_$

(A) $4ab$ (B) $2ab$ (C) $a^2 + 2ab + b^2$ (D) $2(a^2 + b^2)$

(vi) $m^2 + (c+d)m + cd = \_\_\_\_\_\_$

(A) $(m+c)^2$ (B) $(m+c)(m+d)$ (C) $(m+d)^2$ (D) $(m+c)(m-d)$

2. Using a suitable identity, find each of the following products:

(i) $(x+3)(x+3)$ (ii) $(2m+3)(2m+3)$

(iii) $(2x-5)(2x-5)$ (iv) $\left(a-\frac{1}{a}\right)\left(a-\frac{1}{a}\right)$

(v) $(3x+2)(3x-2)$ (vi) $(5a-3b)(5a-3b)$

(vii) $(2l-3m)(2l+3m)$ (viii) $\left(\frac{3}{4}-x\right)\left(\frac{3}{4}+x\right)$

(ix) $\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)$ (x) $(100+3)(100-3)$

3. Using the identity $(x+a)(x+b) = x^2 + (a+b)x + ab$, find out the following products:

(i) $(x+4)(x+7)$ (ii) $(5x+3)(5x+4)$

(iii) $(7x+3y)(7x-3y)$ (iv) $(8x-5)(8x-2)$

(v) $(2m+3n)(2m+4n)$ (vi) $(xy-3)(xy-2)$

(vii) $\left(a+\frac{1}{x}\right)\left(a+\frac{1}{y}\right)$ (viii) $(2+x)(2-y)$

4. Find out the following squares by using the identities:

(i) $(p-q)^2$ (ii) $(a-5)^2$ (iii) $(3x+5)^2$

(iv) $(5x-4)^2$ (v) $(7x+3y)^2$ (vi) $(10m-9n)^2$

(vii) $(0.4a-0.5b)^2$ (viii) $\left(x-\frac{1}{x}\right)^2$ (ix) $\left(\frac{x}{2}-\frac{y}{3}\right)^2$

(x) $0.54 \times 0.54 - 0.46 \times 0.46$

5. Evaluate the following by using the identities:

(i) 103^2 (ii) 48^2 (iii) 54^2

(iv) 92^2 (v) 998^2 (vi) 53×47

(vii) 96×104 (viii) 28×32 (ix) 81×79

(x) 2.8^2 (xi) $12.1^2 - 7.9^2$ (xii) 9.7×9.8

6. Show that

(i) $(3x+7)^2 - 84x = (3x-7)^2$

(ii) $(a-b)(a+b) + (b-c)(b+c) + (c-a)(c+a) = 0$

7. If $a+b=5$ and $a-b=4$, find a^2+b^2 and ab.

8. (i) If the values of $a+b$ and ab are 12 and 32 respectively, find the values of a^2+b^2 and $(a-b)^2$.

   (ii) If the values of $(a-b)$ and ab are 6 and 40 respectively, find the values of a^2+b^2 and $(a+b)^2$.

9. If $(x+a)(x+b)=x^2-5x-300$, find the values of a^2+b^2.

10. Deduce the Algebraic identity for $(x+a)(x+b)(x+c)$ by using the product formula. [Hint: $(x+a)(x+b)(x+c)=(x+a)\ [(x+b)(x+c)]$]

1.5 Factorization

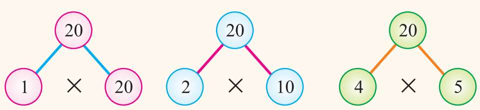
Let us take the natural number 20.

We can write it as the product of following numbers.

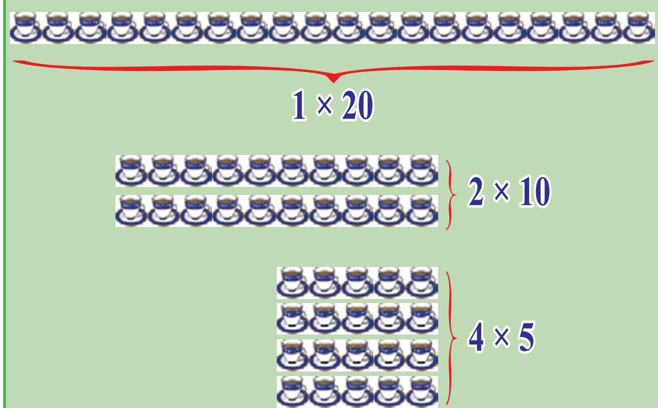
$20 = 1 \times 20$

$20 = 2 \times 10$

$20 = 4 \times 5$



We can arrange the twenty tea cups in different ways as given below:

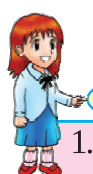


The number 20 has 6 factors : 1, 2, 4, 5, 10 and 20.

Among these factors 2 and 5 are the prime factors of 20.

The prime factor form of 20 = $2 \times 2 \times 5$.

$$\begin{array}{r|l} 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array}$$



Do you know?

1. A whole number greater than 1 for which the only factors are 1 and itself, is called a **prime number**. Example: 2, 3, 5, 7 etc.
2. A whole number greater than 1 which has more than two factors is called a **composite number**. Example: 4, 6, 8, 9, 10 etc.
3. 1 is a factor of any number.
4. Every natural number other than 1 is either prime or composite.
5. 1 is neither prime nor composite.
6. 2 is the only even prime number.

1.5.1. What is factorization?

We can also write any algebraic expression as the product of its factors.

Factorization: The process of expressing any polynomial as a product of its factors is called **factorization**.

We can express the following algebraic expressions as the product of their factors:

(i) $6x^3 = (2x)(3x^2)$

(ii) $3a^2b + 3ab^2 = (3ab)(a+b)$

(iii) $2x^2 + x - 6 = (2x-3)(x+2)$

We can also write the above examples as follows:

Algebraic Expression	Factor 1	Factor 2	Can we factorize further?	
			Factor 1	**Factor 2**
$6x^3$	$2x$	$3x^2$	Yes. $2x = 2 \times x$	Yes. $3x^2 = 3 \times x \times x$
$3a^2b + 3ab^2$	$(3ab)$	$(a+b)$	Yes. $3ab = 3 \times a \times b$	No. $(a+b)$ can't be factorized further
$2x^2 + x - 6$	$(2x-3)$	$(x+2)$	No. $(2x-3)$ cannot be factorized further	No. $(x+2)$ cannot be factorized further

Note: A factor that cannot be factorized further is known as **irreducible factor**. In the above examples $(a+b)$, $(2x-3)$, and $(x+2)$ are irreducible.

1.5.2. Factorization by taking out the common factor

In this method, we rewrite the expression with the common factors outside brackets. Remember that common factors of two or more terms are factors that appear in all the terms.

Example 1.16

Factorize the following expressions:

(i) $2x + 6$ (ii) $4x^2 + 20xy$ (iii) $3x^2 - 12xy$ (iv) $a^2b - ab^2$

(v) $3x^3 - 5x^2 + 6x$ (vi) $7l^3m^2 - 21lm^2n + 28lm$

Solution

(i) $2x + 6 = 2x + (2 \times 3)$

$\therefore 2x + 6 = 2(x+3)$ (Note that '2' is common to both terms.)

Note: (i) Here, the factors of $(2x+6)$ are 2 and $(x+3)$.

(ii) The factors 2 and $(x+3)$ cannot be reduced further. Therefore 2 and $(x+3)$ are irreducible factors.

(ii) $4x^2 + 20xy = (4 \times x \times x) + (4 \times 5 \times x \times y)$

$= 4x\,(x + 5y)$ [Taking out the common factor $4x$]

(iii) $3x^2 - 12xy = (3 \times x \times x) - (3 \times 4 \times x \times y)$

$= 3x\,(x - 4y)$ [Taking out the common factor $3x$]

(iv) $a^2b - ab^2 = (a \times a \times b) - (a \times b \times b)$

$= ab\,(a - b)$ [Taking out the common factor ab]

(v) $3x^3 - 5x^2 + 6x = (3 \times x \times x \times x) - (5 \times x \times x) + (6 \times x)$

$= x\,(3x^2 - 5x + 6)$ [Taking out the common factor x]

(vi) $7l^3m^2 - 21\,lm^2n + 28lm$

$= (7 \times l \times l \times l \times m \times m) - (7 \times 3 \times l \times m \times m \times n) + (7 \times 4 \times l \times m)$

$= 7lm\,(l^2m - 3mn + 4)$ [Taking out the common factor $7lm$]

1.5.3. Factorization by Grouping the terms

In this method, the terms in the given expression can be arranged in groups of two or three so as to get a common factor.

Example 1.17

Factorize the following:

(i) $x^3 - 3x^2 + x - 3$ (ii) $2xy - 3ab + 2bx - 3ay$

(iii) $2m^2 - 10mn - 2m + 10n$ (iv) $ab\,(x^2 + 1) + x\,(a^2 + b^2)$

Solution

(i) $\underbrace{x^3 - 3x^2} + \underbrace{x - 3} = x^2\,(x - 3) + 1\,(x - 3)$ [By grouping the first two and the last two and taking out the common factors]

$= (x^2 + 1)\,(x - 3)$

(ii) $2xy - 3ab + 2bx - 3ay = \underbrace{2xy + 2bx} - \underbrace{3ab - 3ay}$ [Rearranging the factors]

$= 2x\,(y + b) - 3a\,(y + b)$ [Taking out the common factors]

$= (2x - 3a)\,(y + b)$

(iii) $\underbrace{2m^2 - 10mn} - \underbrace{2m + 10n} = 2m\,(m - 5n) - 2\,(m - 5n)$

$= (2m - 2)\,(m - 5n)$ [Taking out the common factors]

(iv) $ab\,(x^2 + 1) + x\,(a^2 + b^2) = abx^2 + ab + xa^2 + xb^2$

$= \underbrace{abx^2 + a^2x} + \underbrace{b^2x + ab}$ [Rearranging the factors]

$= ax\,(bx + a) + b\,(bx + a)$ [Taking out the common factors]

$= (ax + b)\,(bx + a)$

1.5.4. Factorization by using Identities

Recall:

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $(a+b)(a-b) = a^2 - b^2$

Sometimes, the given polynomial or expression can be written in the form of above mentioned Identities. The expressions on the LHS are the factors of the expressions of RHS.

Expression	Factors
$a^2 + 2ab + b^2$	$(a+b)$ and $(a+b)$
$a^2 - 2ab + b^2$	$(a-b)$ and $(a-b)$
$a^2 - b^2$	$(a+b)$ and $(a-b)$

In this method, we consider the following illustrations and learn how to use the identities for factorization.

Example 1.18

Factorize the following using the Identities:

(i) x^2+6x+9 (ii) $x^2-10x+25$ (iii) $49m^2-56m+16$

(iv) x^2-64 (v) $9x^2y-4y^3$ (vi) m^8-n^8

Solution

(i) x^2+6x+9

Comparing x^2+6x+9 with $a^2+2ab+b^2$, we see that $a = x$, $b = 3$.

Now, $x^2+6x+9 = x^2+2(x)(3)+3^2$

Using, $a^2+2ab+b^2 = (a+b)^2$, $a = x$ and $b = 3$,

we get $x^2+6x+9 = (x+3)^2$.

$\therefore$ The factors of x^2+6x+9 are $(x+3)$ and $(x+3)$.

(ii) $x^2-10x+25$

Comparing $x^2-10x+25$ with $a^2-2ab+b^2$, we see that $a = x$, $b = 5$.

Now, $x^2-10x+25 = x^2-2(x)(5)+5^2$

Using, $a^2-2ab+b^2 = (a-b)^2$, $a = x$ and $b = 5$,

we get $x^2-10x+25 = (x-5)^2$.

$\therefore$The factors of $x^2-10x+25$ are $(x-5)$ and $(x-5)$

(iii) $49m^2-56m+16$

In this expression, we can express $49m^2 = (7m)^2$ and $16 = 4^2$

Using the Identity $a^2 - 2ab + b^2 = (a-b)^2$ with $a = 7m$ and $b = 4$,

$$49m^2 - 56m + 16 = (7m)^2 - 2(7m)(4) + 4^2$$
$$= (7m-4)^2$$

$\therefore$The factors of $49m^2 - 56m + 16$ are $(7m-4)$ and $(7m-4)$

(iv) Comparing $x^2 - 64$ with $a^2 - b^2$, we see that $a = x$ and $b = 8$

$$\text{Using, } a^2 - b^2 = (a+b)(a-b),$$
$$x^2 - 64 = x^2 - 8^2$$
$$= (x+8)(x-8)$$

$\therefore$The factors of $x^2 - 64$ are $(x+8)$ and $(x-8)$

(v) $$9x^2y - 4y^3 = y\,[9x^2 - 4y^2]$$
$$= y[(3x)^2 - (2y)^2]$$

Comparing $(3x)^2 - (2y)^2$ with $a^2 - b^2$, we see that $a = 3x$ and $b = 2y$

Using $a^2 - b^2 = (a+b)(a-b)$, where, $a = 3x$, $b = 2y$,

$$9x^2y - 4y^3 = y[(3x+2y)(3x-2y)]$$

(vi) $m^8 - n^8 = (m^4)^2 - (n^4)^2$

$$= (m^4 + n^4)(m^4 - n^4) \qquad [\text{Using the Identity } a^2 - b^2 = (a+b)(a-b)]$$
$$= (m^4 + n^4)[(m^2)^2 - (n^2)^2]$$
$$= (m^4 + n^4)[(m^2 + n^2)(m^2 - n^2)] \qquad [\because m^2 - n^2 = (m+n)(m-n)]$$
$$= (m^4 + n^4)(m^2 + n^2)[(m+n)(m-n)]$$
$$m^8 - n^8 = (m^4 + n^4)(m^2 + n^2)(m+n)(m-n)$$

1.5.5 Factorization by using the Identity $(x+a)(x+b) = x^2 + (a+b)x + ab$

Let us now discuss how we can use the form $(x+a)(x+b) = x^2 + (a+b)x + ab$ to factorize the expressions.

Example 1.19

Factorize $x^2 + 5x + 6$

Solution

Comparing $x^2 + 5x + 6$ with $x^2 + (a+b)x + ab$

We have, $ab = 6$, $a + b = 5$ and $x = x$.

If $ab = 6$, it means a and b are factors of 6.

Let us try with $a = 2$ and $b = 3$. These values satisfy $ab = 6$ and $a + b = 5$.

Therefore the pair of values $a = 2$ and $b = 3$ is the right choice.

> For this $ab = 2 \times 3 = 6$ and $a + b = 2 + 3 = 5$.

Using, $x^2 + (a+b)x + ab = (x+a)(x+b)$

$$x^2 + 5x + 6 = x^2 + (2+3)x + (2 \times 3)$$
$$= (x+2)(x+3)$$

$\therefore (x+2)$ *and* $(x+3)$ are the factors of $x^2 + 5x + 6$.

Example 1.20

Factorize: $x^2 + x - 6$

Solution

Comparing $x^2 + x - 6$ with $x^2 + (a+b)x + ab = (x+a)(x+b)$,

we get $ab = -6$ and $a + b = 1$

To find the two numbers a and b such that $ab = -6$ and $a + b = 1$. The values of a and b may be tabulated as follows:

a	b	ab	$a + b$	Choice
1	6	6	7	✗
1	– 6	– 6	– 5	✗
2	3	6	5	✗
2	– 3	– 6	– 1	✗
– 2	3	– 6	1	✓

Here, we have to select the pair of factors $a = -2$ and $b = 3$, because they alone satisfy the conditions $ab = -6$ and $a + b = 1$

Using $(x+a)(x+b) = x^2 + (a+b)x + ab$, we get

$$x^2 + x - 6 = (x-2)(x+3).$$

Example 1.21

Factorize: $x^2 + 6x + 8$

Solution

Comparing $x^2 + 6x + 8$ with $x^2 + (a+b)x + ab = (x+a)(x+b)$,

we get $ab = 8$ and $a + b = 6$.

$$\therefore\ x^2 + 6x + 8 = x^2 + (2+4)x + (2 \times 4)$$
$$= (x+2)(x+4)$$

Factors of 8	Sum of factors
1, 8	9
2, 4	6

Hence the correct factors are 2, 4

The factors of $x^2 + 6x + 8$ are $(x+2)$ and $(x+4)$.

EXERCISE 1.4

1. Choose the correct answer for the following :

(i) The factors of $3a + 21ab$ are ______

(A) ab , $(3 + 21)$ (B) 3, $(a + 7b)$ (C) $3a$, $(1 + 7b)$ (D) $3ab$, $(a + b)$

(ii) The factors of $x^2 - x - 12$ are ______

(A) $(x + 4)$, $(x - 3)$ (B) $(x - 4)$, $(x - 3)$ (C) $(x + 2)$, $(x - 6)$ (D) $(x + 3)$, $(x - 4)$

(iii) The factors of $6x^2 - x - 15$ are $(2x + 3)$ and _______

(A) $(3x - 5)$ (B) $(3x + 5)$ (C) $(5x - 3)$ (D) $(2x - 3)$

(iv) The factors of $169l^2 - 441m^2$ are _______

(A) $(13l - 21m)$, $(13l - 21m)$ (B) $(13l + 21m)$, $(13l + 21m)$

(C) $(13l - 21m)$, $(13l + 21m)$ (D) $13(l + 21m)$, $13(l - 21m)$

(v) The product of $(x - 1)(2x - 3)$ is ______

(A) $2x^2 - 5x - 3$ (B) $2x^2 - 5x + 3$ (C) $2x^2 + 5x - 3$ (D) $2x^2 + 5x + 3$

2. Factorize the following expressions:

(i) $3x - 45$ (ii) $7x - 14y$ (iii) $5a^2 + 35a$ (iv) $-12y + 20y^3$

(v) $15a^2b + 35ab$ (vi) $pq - pqr$ (vii) $18m^3 - 45mn^2$ (viii) $17l^2 + 85m^2$

(ix) $6x^3y - 12x^2y + 15x^4$ (x) $2a^5b^3 - 14a^2b^2 + 4a^3b$

3. Factorize:

(i) $2ab + 2b + 3a$ (ii) $6xy - 4y + 6 - 9x$ (iii) $2x + 3xy + 2y + 3y^2$

(iv) $15b^2 - 3bx^2 - 5b + x^2$ (v) $a^2x^2 + axy + abx + by$

(vi) $a^2x + abx + ac + aby + b^2y + bc$ (vii) $ax^3 - bx^2 + ax - b$

(viii) $mx - my - nx + ny$ (ix) $2m^3 + 3m - 2m^2 - 3$ (x) $a^2 + 11b + 11ab + a$

4. Factorize:

(i) $a^2 + 14a + 49$ (ii) $x^2 - 12x + 36$ (iii) $4p^2 - 25q^2$

(iv) $25x^2 - 20xy + 4y^2$ (v) $169m^2 - 625n^2$ (vi) $x^2 + \frac{2}{3}x + \frac{1}{9}$

(vii) $121a^2 + 154ab + 49b^2$ (viii) $3x^3 - 75x$

(ix) $36 - 49x^2$ (x) $1 - 6x + 9x^2$

5. Factorize :

(i) $x^2 + 7x + 12$ (ii) $p^2 - 6p + 8$ (iii) $m^2 - 4m - 21$

(iv) $x^2 - 14x + 45$ (v) $x^2 - 24x + 108$ (vi) $a^2 + 13a + 12$

(vii) $x^2 - 5x + 6$ (viii) $x^2 - 14xy + 24y^2$

(ix) $m^2 - 21m - 72$ (x) $x^2 - 28x + 132$

1.6 Division of Algebraic Expressions

1.6.1 Division of a Monomial by another Monomial

Consider $10 \div 2$, we may write this as $\frac{10}{2} = \frac{5 \times \cancel{2}}{\cancel{2}} = 5$

Similarly, (i) $10x \div 2$ may be written as $\frac{10x}{2} = \frac{5 \times \cancel{2} \times x}{\cancel{2}} = 5x$

(ii) $10x^2 \div 2x = \frac{10x^2}{2x} = \frac{5 \times 2 \times x^2}{2x} = \frac{5 \times \cancel{2} \times \cancel{x} \times x}{\cancel{2} \times \cancel{x}} = 5x$

(iii) $10x^3 \div 2x = \frac{10x^3}{2x} = \frac{5 \times \cancel{2} \times \cancel{x} \times x \times x}{\cancel{2} \times \cancel{x}} = 5x^2$

(iv) $10x^5 \div 2x^2 = \frac{10x^5}{2x^2} = \frac{5 \times \cancel{2} \times \cancel{x} \times \cancel{x} \times x \times x \times x}{\cancel{2} \times \cancel{x} \times \cancel{x}} = 5x^3$

Instead, we can also use the law of exponent $\frac{a^m}{a^n} = a^{m-n}$;

Thus in (iv), we can write

$$\frac{10x^5}{2x^2} = \frac{10}{2}x^{5-2} = 5x^3$$

(v) $5a^2b^2c^2 \div 15abc = \frac{5a^2b^2c^2}{15abc} = \frac{\cancel{5} \times \cancel{a} \times a \times \cancel{b} \times b \times \cancel{c} \times c}{\cancel{5} \times 3 \times \cancel{a} \times \cancel{b} \times \cancel{c}} = \frac{abc}{3} = \frac{1}{3}abc$

(or) $5a^2b^2c^2 \div 15abc = \frac{5a^2b^2c^2}{15abc}$

$= \frac{5}{15}a^{2-1}b^{2-1}c^{2-1} = \frac{1}{3}abc$ [using $\frac{a^m}{a^n} = a^{m-n}$]

1.6.2 Division of a Polynomial by a Monomial and Binomial

Let us consider the following example.

Example 1.22

Solve: (i) $(7x^2 - 5x) \div x$ (ii) $(x^6 - 3x^4 + 2x^2) \div 3x^2$ (iii) $(8x^3 - 5x^2 + 6x) \div 2x$

Solution

(i) $(7x^2 - 5x) \div x = \frac{7x^2 - 5x}{x}$

$= \frac{7x^2}{x} - \frac{5x}{x}$

$= \frac{7 \times x \times x}{x} - \frac{5 \times x}{x}$

$= 7x - 5$

Alternative Method:

$\frac{7x^2 - 5x}{x} = 7x^{2-1} - 5x^{1-1}$

$= 7x^1 - 5x^0 = 7x - 5$ (1)

$[\because a^0 = 1]$

$= 7x - 5$

(ii) $(x^6 - 3x^4 + 2x^2) \div 3x^2$

$= \frac{x^6 - 3x^4 + 2x^2}{3x^2}$

$= \frac{x^6}{3x^2} - \frac{3x^4}{3x^2} + \frac{2x^2}{3x^2}$

$= \frac{1}{3}x^4 - x^2 + \frac{2}{3}$

Alternative Method:

we can find the common factor x^2 and simplify it as

$\frac{x^6 - 3x^4 + 2x^2}{3x^2} = \frac{x^2(x^4 - 3x^2 + 2)}{3x^2}$

$= \frac{1}{3}(x^4 - 3x^2 + 2)$

$= \frac{x^4}{3} - x^2 + \frac{2}{3}$

(iii) $(8x^3 - 5x^2 + 6x) \div 2x$

$$= \frac{8x^3 - 5x^2 + 6x}{2x}$$

$$= \frac{8x^3}{2x} - \frac{5x^2}{2x} + \frac{6x}{2x}$$

$$= 4x^2 - \frac{5}{2}x + 3$$

Alternative Method

For $8x^3 - 5x^2 + 6x$, separating $2x$ from each term we get ,

$$8x^3 - 5x^2 + 6x = 2x(4x^2) - 2x\left(\frac{5}{2}x\right) + 2x(3)$$

$$= 2x\left(4x^2 - \frac{5}{2}x + 3\right)$$

$$\frac{8x^3 - 5x^2 + 6x}{2x} = \frac{2x(4x^2 - \frac{5}{2}x + 3)}{2x}$$

$$= 4x^2 - \frac{5}{2}x + 3$$

Example 1.23

Solve: $(5x^2 + 10x) \div (x + 2)$.

Solution

$$(5x^2 + 10x) \div (x + 2) = \frac{5x^2 + 10x}{x + 2}$$

Let us factorize the numerator $(5x^2 + 10x)$.

$$5x^2 + 10x = (5 \times x \times x) + (5 \times 2 \times x)$$

$$= 5x(x + 2) \quad \text{[Taking out the common factor } 5x\text{]}$$

Now, $$(5x^2 + 10x) \div (x + 2) = \frac{5x^2 + 10x}{x + 2}$$

$$= \frac{5x\cancel{(x + 2)}}{\cancel{(x + 2)}} = 5x. \quad \text{[By cancelling } (x + 2)\text{]}$$

EXERCISE 1.5

1. Simplify:

   (i) $16x^4 \div 32x$ (ii) $-42y^3 \div 7y^2$ (iii) $30a^3b^3c^3 \div 45abc$

   (iv) $(7m^2 - 6m) \div m$ (v) $25x^3y^2 \div 15x^2y$ (vi) $(-72l^4m^5n^8) \div (-8l^2m^2n^3)$

2. Work out the following divisions:

   (i) $5y^3 - 4y^2 + 3y \div y$ (ii) $(9x^5 - 15x^4 - 21x^2) \div (3x^2)$

   (iii) $(5x^3 - 4x^2 + 3x) \div (2x)$ (iv) $4x^2y - 28xy + 4xy^2 \div (4xy)$

   (v) $(8x^4yz - 4xy^3z + 3x^2yz^4) \div (xyz)$

3. Simplify the follwing expressions:

   (i) $(x^2 + 7x + 10) \div (x + 2)$ (ii) $(a^2 + 24a + 144) \div (a + 12)$

   (iii) $(m^2 + 5m - 14) \div (m + 7)$ (iv) $(25m^2 - 4n^2) \div (5m + 2n)$

   (v) $(4a^2 - 4ab - 15b^2) \div (2a - 5b)$ (vi) $(a^4 - b^4) \div (a - b)$

1.7 Solving Linear Equations

In class VII, we have learnt about algebraic expressions and linear equations in one variable. Let us recall them now.

Look at the following examples:

(i) $2x = 8$ (ii) $3x^2 = 50$ (iii) $5x^2 - 2 = 102$ (iv) $2x - 3 = 5$

(v) $\frac{2}{5}x + \frac{3}{4}y = 4$ (vi) $3x^3 = 81$ (vii) $2(5x+1) - (2x+1) = 6x + 2$

These are all equations.

A statement in which two expressions are connected by ' = ' sign, is called as an **equation**. In other words, an equation is a statement of equality which contains one or more variables.

In the above equations (i), (iv), (v) and (vii), we see that power of each variable is one. Such equations are called linear equations.

An equation which involves one or more variables whose power is 1 is called a linear equation.

Therefore, the equations (ii), (iii) and (vi) are not linear equations. (Since the highest power of the variable > 1)

Understanding the equations

Consider the equation $2x - 3 = 5$.

(i) An algebraic equation is an equality involving variables and constants.

$$\underbrace{\overset{\text{Variable}}{2x} - 3 \overset{\text{Equality}}{=} 5}_{\text{Equation}}$$

(ii) Every equation has an equality sign. The expression on the left of the equality sign is the Left Hand Side (LHS). The expression on the right of the equality sign is the Right Hand Side (RHS).

$2x - 3 = $ LHS

$5 = $ RHS

(iii) In an equation the values of the expression on the LHS and RHS are equal. It is true only for certain values of the variables. These values are the **solutions or roots** of the equation.

$x = 4$ is the solution of the equation.

$2x - 3 = 5$ when $x = 4$,

LHS $= 2(4) - 3$

$= 8 - 3 = 5 =$ RHS

$x = 5$ is not a solution of this equation.

If $x = 5$, LHS $= 2(5) - 3$

$= 10 - 3 = 7 \neq$ RHS.

Rules for Solving an Equation

We may use any one or two or all the three of the following rules in solving an equation.

1. We can add or subtract the same quantity to both sides of an equation without changing the equality.
2. Both sides of an equation can be multiplied or divided by the same non-zero number without changing the equality.
3. **Transposition Method:** For solving an equation, we need to collect all the terms containing the variables on one side of the equation and constant terms on the other side of the equation. This can be done by transferring some terms from one side to the other. Any term of an equation may be shifted from one side to the other by changing its sign. This process is called as Method of Transposition.

1.7.1 Linear Equation in one variable

We have learnt in class VII to solve linear equations in one variable. Consider the linear equation of the type $ax + b = 0$ where $a \neq 0$.

A Linear equation in one variable has a unique solution.

Example 1.24

Find the solution of $5x - 13 = 42$.

Solution

Step 1 : Add 13 to both sides, $5x - 13 + 13 = 42 + 13$

$$5x = 55$$

Step 2 : Divide both sides by 5, $\frac{5x}{5} = \frac{55}{5}$

$$x = 11$$

Verification:

LHS $= 5 \times 11 - 13$

$= 55 - 13$

$= 42$

$=$ RHS

Transposition method:

$$5x - 13 = 42$$

$$5x = 42 + 13 \quad \text{[Transposing – 13 to RHS]}$$

$$5x = 55$$

$$\frac{5x}{5} = \frac{55}{5} \quad \text{[Dividing both sides by 5]}$$

$$x = 11$$

Example 1.25

Solve: $5y + 9 = 24$

Solution

$$5y + 9 = 24$$

we get, $5y + 9 - 9 = 24 - 9$ (Subtracting 9 from both sides of the equation)

$$5y = 15$$

$$\frac{5y}{5} = \frac{15}{5}$$ [Dividing both the sides by 5]

$$y = 3$$

Verification:

LHS = 5 (3) + 9 = 24 = RHS

Alternative Method

$$5y + 9 = 24$$

$5y = 24 - 9$ [Transposing 9 to RHS]

$5y = 15$ and $y = \frac{15}{5}$. Hence $y = 3$

Example 1.26

Solve: $2x + 5 = 23 - x$

Solution

$$2x + 5 = 23 - x$$

$$2x + 5 - 5 = 23 - x - 5$$

[Adding – 5 both sides]

$$2x = 18 - x$$

$$2x + x = 18 - x + x$$

$$3x = 18$$

$$\frac{3x}{3} = \frac{18}{3}$$ [Dividing both the sides by 3]

$$x = 6$$

Alternative Method

$$2x + 5 = 23 - x$$

$2x + x = 23 - 5$ [By transposition]

$$3x = 18$$

$x = \frac{18}{3}$ [Dividing both sides by 3]

$$x = 6$$

Verification: LHS = $2x + 5 = 2(6) + 5 = 17$,

RHS = $23 - x = 23 - 6 = 17$.

Example 1.27

Solve : $\frac{9}{2}m + m = 22$

Solution

$$\frac{9}{2}m + m = 22$$

$\frac{9m + 2m}{2} = 22$ [Taking LCM on LHS]

$$\frac{11m}{2} = 22$$

$m = \frac{22 \times 2}{11}$ [By cross multiplication]

$$m = 4$$

Verification:

LHS $= \frac{9}{2}m + m = \frac{9}{2}(4) + 4$

$= 18 + 4 = 22 = $ RHS

Example 1.28

Solve: $\frac{2}{x} - \frac{5}{3x} = \frac{1}{9}$

Solution

$$\frac{2}{x} - \frac{5}{3x} = \frac{1}{9}$$

$\frac{6-5}{3x} = \frac{1}{9}$ [Taking LCM on LHS]

$$\frac{1}{3x} = \frac{1}{9}$$

$$3x = 9;\ x = \frac{9}{3};\ x = 3.$$

Verification:

LHS $= \frac{2}{x} - \frac{5}{3x}$

$= \frac{2}{3} - \frac{5}{3(3)} = \frac{2}{3} - \frac{5}{9}$

$= \frac{6-5}{9} = \frac{1}{9} = $ RHS

Example 1.29

Find the two consecutive positive odd integers whose sum is 32.

Solution

Let the two consecutive positive odd integers be x and $(x+2)$.

Then, their sum is 32.

$$\therefore (x)+(x+2) = 32$$
$$2x+2 = 32$$
$$2x = 32-2$$
$$2x = 30$$
$$x = \frac{30}{2} = 15$$

Verification:
$15 + 17 = 32$

Since $x = 15$, then the other integer, $x+2 = 15+2 = 17$

$\therefore$ The two required consecutive positive odd integers are 15 and 17.

Example 1.30

One third of one half of one fifth of a number is 15. Find the number.

Solution

Let the required number be x.

Then, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{5}$ of $x = 15.$

i.e. $\frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times x = 15$

$$x = 15\times3\times2\times5$$
$$x = 45 \times 10 = 450$$

Hence the required number is 450.

Verification:

$$\text{LHS} = \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times x$$
$$= \frac{1}{3}\times\frac{1}{2}\times\frac{1}{5}\times 450$$
$$= 15 = \text{RHS}$$

Example 1.31

A rational number is such that when we multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product we get $\frac{-7}{12}$. What is the number?

Solution

Let the rational number be x.

When we multiply it by $\frac{5}{2}$ and add $\frac{2}{3}$ to the product we get $\frac{-7}{12}$.

i.e., $x\times\frac{5}{2}+\frac{2}{3} = \frac{-7}{12}$

$$\frac{5x}{2} = \frac{-7}{12} - \frac{2}{3}$$
$$= \frac{-7-8}{12}$$
$$= \frac{-15}{12}$$

$$x = \frac{-15}{12} \times \frac{2}{5}$$

$$= \frac{-1}{2}.$$

Verification:

$$\text{LHS} = \frac{-1}{2} \times \frac{5}{2} + \frac{2}{3} = \frac{-5}{4} + \frac{2}{3}$$

$$= \frac{-15+8}{12} = \frac{-7}{12} = \text{RHS}.$$

Hence the required number is $\frac{-1}{2}$.

Example 1.32

Arun is now half as old as his father. Twelve years ago the father's age was three times as old as Arun. Find their present ages.

Solution

Let Arun's age be x years now.

Then his father's age $= 2x$ years

12 years ago, Arun's age was $(x - 12)$ years and

his faher's age was $(2x - 12)$ years.

Given that, $(2x - 12) = 3(x - 12)$

$$2x - 12 = 3x - 36$$

$$36 - 12 = 3x - 2x$$

$$x = 24$$

Verification:

Arun's age	Father's age
Now : 24	48
12 years ago 24 – 12 = 12	48 – 12 = 36 36 = 3 (Arun's age) = 3 (12) = 36

Therefore, Arun's present age = 24 years.

His father's present age = 2 (24) = 48 years.

Example 1.33

By selling a car for ₹ 1,40,000, a man suffered a loss of 20%. What was the cost price of the car?

Solution

Let the cost price of the car be x.

Loss of 20% $= \frac{20}{100}$ of $x = \frac{1}{5} \times x = \frac{x}{5}$

We know that,

Cost price – Loss = Selling price

$$x - \frac{x}{5} = 140000$$

$$\frac{5x - x}{5} = 140000$$

$$\frac{4x}{5} = 140000$$

$$x = 140000 \times \frac{5}{4}$$

$$x = 175000$$

Hence the cost price of the car is ₹ 1,75,000.

Verification:

Loss = 20% of 175000

$= \frac{20}{100} \times 175000$

= ₹ 35,000

S.P = C.P - Loss

= 175000 – 35000

= 140000

EXERCISE 1.6

1. Solve the following equations:

   (i) $3x + 5 = 23$ (ii) $17 = 10 - y$ (iii) $2y - 7 = 1$

   (iv) $6x = 72$ (v) $\frac{y}{11} = -7$ (vi) $3(3x - 7) = 5(2x - 3)$

   (vii) $4(2x - 3) + 5(3x - 4) = 14$ (viii) $\frac{7}{x-5} = \frac{5}{x-7}$

   (ix) $\frac{2x+3}{3x+7} = \frac{3}{5}$ (x) $\frac{m}{3} + \frac{m}{4} = \frac{1}{2}$

2. Frame and solve the equations for the following statements:

   (i) Half of a certain number added to its one third gives 15. Find the number.

   (ii) Sum of three consecutive numbers is 90. Find the numbers.

   (iii) The breadth of a rectangle is 8 cm less than its length. If the perimeter is 60 cm, find its length and breadth.

   (iv) Sum of two numbers is 60. The bigger number is 4 times the smaller one. Find the numbers.

   (v) The sum of the two numbers is 21 and their difference is 3. Find the numbers. (Hint: Let the bigger number be x and smaller number be $x - 3$)

   (vi) Two numbers are in the ratio 5 : 3. If they differ by 18, what are the numbers?

   (vii) A number decreased by 5% of it is 3800. What is the number?

   (viii) The denominator of a fraction is 2 more than its numerator. If one is added to both the numerator and their denominator the fraction becomes $\frac{2}{3}$. Find the fraction.

   (ix) Mary is 3 times older than Nandhini. After 10 years the sum of their ages will be 80. Find their present ages.

   (x) Murali gives half of his savings to his wife, two third of the remainder to his son and the remaining ₹ 50,000 to his daughter. Find the shares of his wife and son.

Concept Summary

- **Monomial:** An Algebraic expression that contains only one term is called a monomial.
- **Binomial:** An expression that contains only two terms is called a binomial.
- **Trinomial**: An expression that contains only three terms is called a trinomial.
- **Polynomial**: An expression containing a finite number of terms with

  non-zero coefficient is called a polynomial.
- **Degree of the Polynomial**: The highest power of the term is called the degree of the polynomial.

  Like terms contain the same variables with same powers.

  - Only like (or) similar terms can be added or subtracted.
  - Products of monomials are also monomials.
  - The product of a monomial by a binomial is a binomal.

Identities
$(a+b)^2 = a^2 + 2ab + b^2$
$(a-b)^2 = a^2 - 2ab + b^2$
$a^2 - b^2 = (a+b)(a-b)$
$(x+a)(x+b) = x^2 + (a+b)x + ab$

- **Factorization:** The process of expressing any polynomial as a product of its factor is called factorization.
- **Linear Equation:** An equation involving one or more variables each with power 1 is called a Linear equation.

  $ax + b = 0$ is the general form of linear equation in one variable, where $a \neq 0, a, b$ are constants and x is the variable.

  A Linear equation in one variable has a unique solution.

Mathematics Club Activity

Algebraic Comedy

Dear students,

Can we prove $2 = 3$? It seems absurd.

Proof : Let us see, how it is proved.

Consider the following equality.

$$4 - 10 = 9 - 15$$

We add $6\frac{1}{4}$ to both sides of the equation

$$4 - 10 + 6\frac{1}{4} = 9 - 15 + 6\frac{1}{4}$$

We can write it as $2^2 - 2(2)(\frac{5}{2}) + (\frac{5}{2})^2 = 3^2 - 2(3)(\frac{5}{2}) + (\frac{5}{2})^2$

$$(2 - \frac{5}{2})^2 = (3 - \frac{5}{2})^2$$

Taking square root on both sides, we get $(2 - \frac{5}{2}) = (3 - \frac{5}{2})$

Adding $\frac{5}{2}$ to both sides,

$$2 - \frac{5}{2} + \frac{5}{2} = 3 - \frac{5}{2} + \frac{5}{2}$$

We arrive at $2 = 3$

Now we have proved that 2 is equal to 3.

Where is the mistake?

Let us discuss in detail.

When we concluded $\left(2 - \frac{5}{2}\right)^2 = \left(3 - \frac{5}{2}\right)^2$ as $2 - \frac{5}{2} = 3 - \frac{5}{2}$ an error slipped in.

From the fact, the squares of number are equal, it does not follow the numbers are equal.

For example, $(-5)^2 = 5^2$ $[\because (-5)(-5) = (5)(5) = 25]$

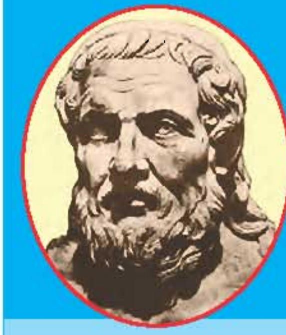
does not imply $-5 = 5$.

In the above problem we arrived $\left(\frac{-1}{2}\right)^2 = \left(\frac{1}{2}\right)^2$, from this we should not conclude that $\frac{-1}{2} = \frac{1}{2}$.

Have you got it now?

Practical Geometry

2



Apolonius
[262 - 200 B.C]

The great mathematician and astronomer Apollonius was born in Perga, Southern Asia Minor. He went to Alexandria and studied under the successors of Euclid. He wrote on variety of mathematical subjects. He discovered the various forms of conic sections Parabola, Hyperbola and Ellipse.

2.1 Introduction

The geometrical figures like square, rectangle and rhombus are often used in our day - to - day life. Our ancient mathematicians found the area with the help of these structures. We can form new structures with the help of these shapes which are more useful and attractive. Many of our machines, materials and all structures are in any one of these forms.

The study of these shapes like square, rectangle and rhombus gives the fundamental knowledge in which one can apply his skill to various fields such as architecture, instrument designing, civil engineering, textile and leather technology.

In Term - 1 we have learnt to draw quadrilateral, trapezium and parallelogram. In this term we are going to learn to draw rhombus, rectangle and square.

2.2 Rhombus

2.2.1. Introduction

A parallelogram in which the adjacent sides are equal is called a rhombus.

In rhombus ABCD, see Fig. 2.1.

(i) All sides are equal in measure.

i.e., AB = BC = CD = DA

(ii) Opposite angles are equal in measure.

i.e., $\angle A = \angle C$ and $\angle B = \angle D$

(iii) Diagonals bisect each other at right angles.

i.e., AO = OC ; BO = OD,

At 'O', $\overline{AC}$ and $\overline{BD}$ are perpendicular to each other.

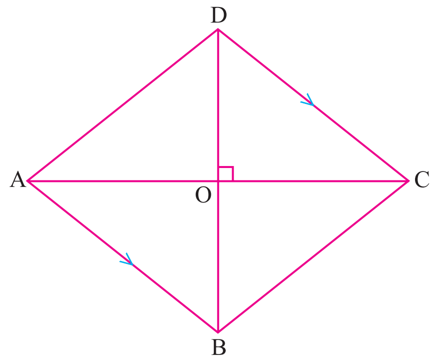


Fig. 2.1

(iv) Sum of any two adjacent angles is equal to 180°.

(v) Each diagonal of a rhombus divides it into two congruent triangles.

(vi) Diagonals are not equal in length.

2.2.2 Area of a rhombus

Let us consider the rectangular sheet of paper JOKE as shown below.

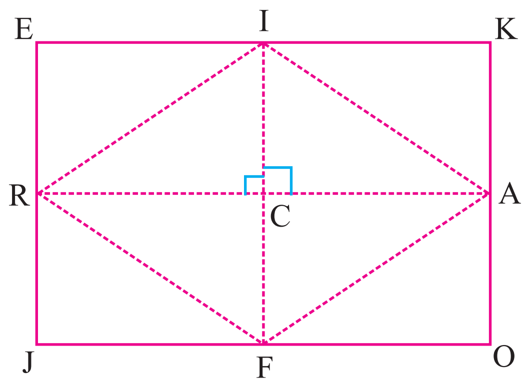


Fig. 2.2

Let us mark the mid-points of the sides. (We use the paper folding technique to find the mid-point), The mid-point of $\overline{JO}$ is F ; the mid-point of $\overline{OK}$ is A ; the mid-point of $\overline{KE}$ is I and the mid-point of $\overline{EJ}$ is R. Let us join $\overline{RA}$ and $\overline{IF}$. They meet at C. FAIR is a rhombus.

Rectangle JOKE has eight congruent right angled triangles. The area of the required rhombus FAIR is the area of four right angled triangles.

In other words, we can say that the area of the rhombus FAIR is half of the rectangle JOKE.

We can clearly see that $\overline{JO}$, the length of rectangle becomes one of the diagonals of the rhombus $(\overline{RA}) = d_1$. The breadth becomes the other diagonal $(\overline{IF}) = d_2$ of the rhombus.

Area of rhombus FAIR $= \frac{1}{2} d_1 \times d_2$

Area of rhombus $A = \frac{1}{2} \times d_1 \times d_2$ sq. units

where d_1 and d_2 are the diagonals of the rhombus.

2.2.3 Construction of a rhombus

Rhombus is constructed by splitting the figure into suitable triangles. First, a triangle is constructed from the given data and then the fourth vertex is found. We need two independent measurements to construct a rhombus.

We can construct a rhombus, when the following measurements are given

(i)	**One side and one diagonal**	**(iii)**	**Two diagonals**
(ii)	**One side and one angle**	**(iv)**	**One diagonal and one angle**

2.2.4 Construction of Rhombus when one side and one diagonal are given

Example 2.1

Construct a rhombus PQRS with PQ = 6 cm and PR = 9 cm and find its area.

Solution

Given: PQ = 6 cm and PR = 9 cm.

To construct a rhombus

Rough Diagram

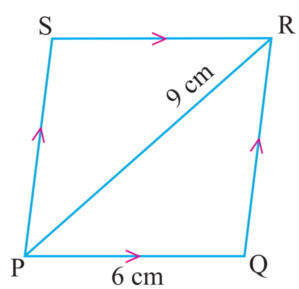


Fig. 2.3

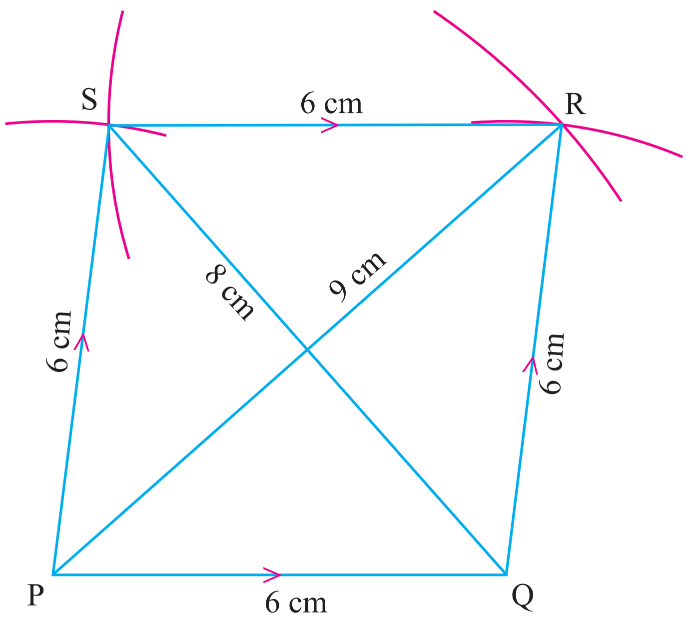


Fig. 2.4

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment PQ = 6 cm.

Step 3 : With P and Q as centres, draw arcs of radii 9 cm and 6 cm respectively and let them cut each other at R.

Step 4 : Join $\overline{PR}$ and $\overline{QR}$.

Step 5 : With P and R as centres draw arcs of radius 6 cm and let them cut each other at S.

Step 6 : Join $\overline{PS}$ and $\overline{RS}$.

PQRS is constructed as required.

Step 7 : Measure the length of QS. QS = d_2 = 8 cm. PR = d_1 = 9 cm.

Calculation of area:

In the rhombus PQRS, d_1 = 9 cm and d_2 = 8 cm.

$$\text{Area of the rhombus PQRS} = \frac{1}{2}\times d_1 \times d_2 = \frac{1}{2}\times 9 \times 8 = 36 \text{ cm}^2.$$

2.2.5 Construction of a rhombus when one side and one angle are given

Example 2.2

Construct a rhombus ABCD with AB = 7 cm and $\angle$A = 60° and find its area.

Solution

Given: AB = 7 cm and $\angle$ A = 60°.

To construct a rhombus

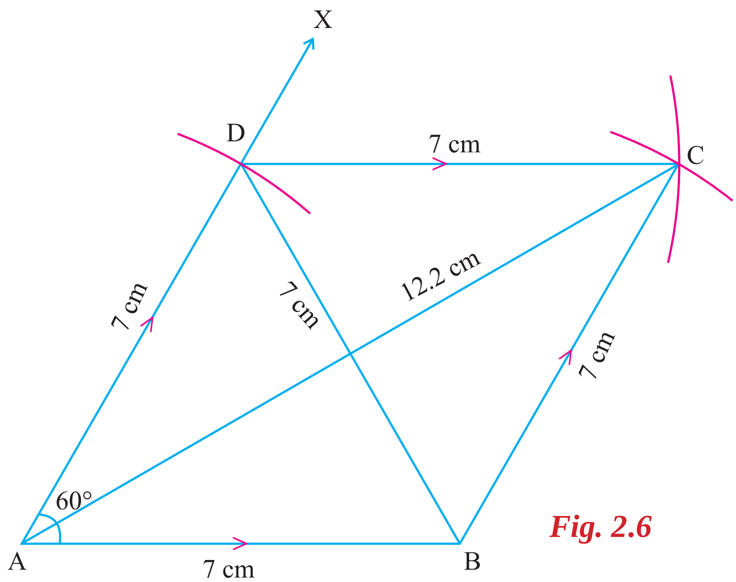


Fig. 2.6

Rough Diagram

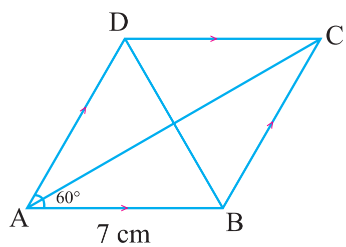


Fig. 2.5

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 7 cm.

Step 3 : At A on $\overline{AB}$ make $\angle BAX$ whose measure is 60°.

Step 4 : With A as centre draw an arc of radius 7 cm. This cuts $\overrightarrow{AX}$ at D.

Step 5 : With B and D as centres draw arcs of radius 7 cm and let them cut each other at C.

Step 6 : Join $\overline{BC}$ and $\overline{DC}$.
ABCD is the required rhombus.

Step 7 : Measure the lengths AC and BD.
AC = d_1 = 12.2 cm and BD = d_2 = 7 cm.

Calculation of area:

In the rhombus ABCD, d_1 = 12.2 cm and d_2 = 7 cm.

Area of the rhombus ABCD $= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 12.2 \times 7 = 42.7 \text{ cm}^2$.

2.2.6 Construction of a rhombus when two diagonals are given

Example 2.3

Cosnstruct a rhombus PQRS with PR = 8 cm and QS = 6 cm and find its area.

Solution

Given: PR = 8 cm and QS = 6 cm.

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment PR = 8 cm

Step 3 : Draw the perpendicular bisector $\overleftrightarrow{XY}$ to $\overline{PR}$. Let it cut $\overline{PR}$ at "O" .

Step 4 : With O as centre and 3 cm (half of QS) as radius draw arcs on either side of 'O' which cuts $\overleftrightarrow{XY}$ at Q and S as shown in Fig. 2.8

Rough Diagram

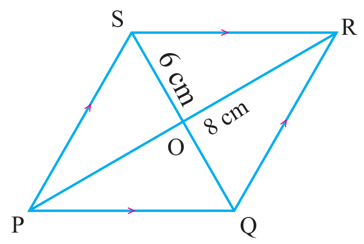


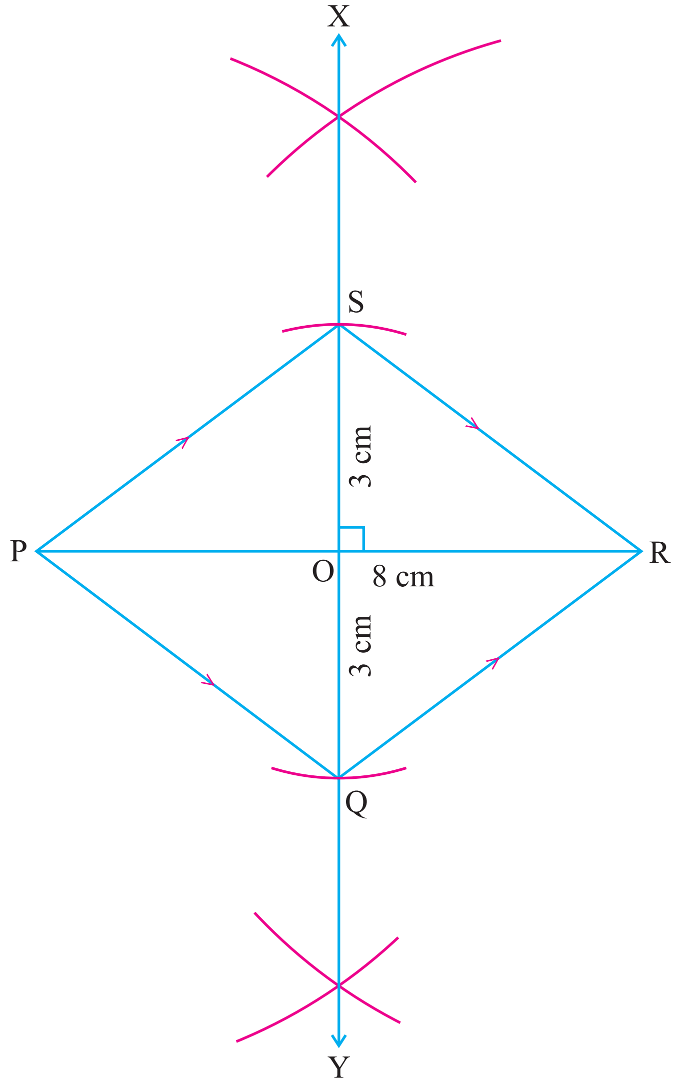
Fig. 2.7

Step 5 : Join $\overline{PQ}$, $\overline{QR}$, $\overline{RS}$ and $\overline{SP}$.

PQRS is the required rhombus.

Step 6 : We know, PR = d_1 = 8 cm and QS = d_2 = 6 cm.

To construct a rhombus



Calculation of area:

In the Rhombus PQRS,

$d_1 = 8$ cm and $d_2 = 6$ cm.

Area of the rhombus PQRS

$$= \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 8 \times 6$$

$$= 24 \text{ cm}^2.$$

Fig. 2.8

2.2.7 Construction of a rhombus when one diagonal and one angle are given

Example 2.4

Construct a rhombus ABCD with AC = 7.5 cm and $\angle$A = 100°. Find its area.

Solution

Given: AC = 7.5 cm and $\angle$A = 100°.

Rough Diagram

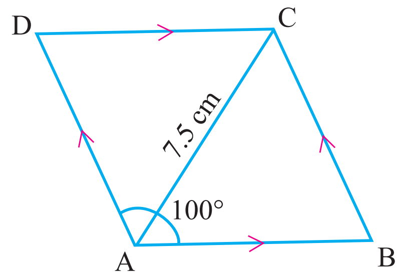


Fig. 2.9

To construct a rhombus

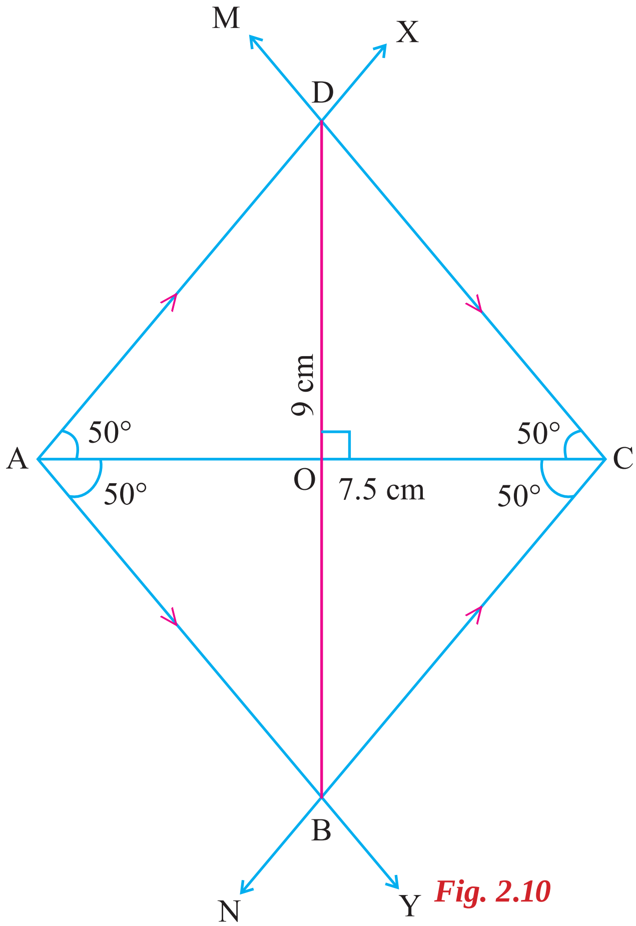


Fig. 2.10

Steps for construction

Step 1 : Draw a rough figure and mark the given measurements.

Step 2 : Draw a line segment AC = 7.5 cm.

Step 3 : At A draw $\overrightarrow{AX}$ and $\overrightarrow{AY}$ on either side of $\overline{AC}$ making an angle 50° with $\overline{AC}$.

Step 4 : At C draw $\overrightarrow{CM}$ and $\overrightarrow{CN}$ on either side of $\overline{CA}$ making an angle 50° with $\overline{CA}$.

Step 5 : Let $\overrightarrow{AX}$ and $\overrightarrow{CM}$ cut at D and $\overrightarrow{AY}$ and $\overrightarrow{CN}$ cut at B.
ABCD is the required rhombus.

Step 6 : Measure the length BD. BD = d_2 = 9 cm. AC = d_1 = 7.5 cm.

Calculation of area:

In the rhombus ABCD, d_1 = 7.5 cm and d_2 = 9 cm.

$$\text{Area of the rhombus ABCD} = \frac{1}{2} \times d_1 \times d_2$$

$$= \frac{1}{2} \times 7.5 \times 9 = 7.5 \times 4.5 = 33.75 \text{ cm}^2.$$

EXERCISE 2.1

Draw rhombus BEST with the following measurements and calculate its area.

1. BE = 5 cm and BS = 8 cm.
2. BE = 6 cm and ET = 8.2 cm.
3. BE = 6 cm and $\angle$ B = 45°.
4. BE = 7.5 cm and $\angle$ E = 65°.
5. BS = 10 cm and ET = 8 cm.
6. BS = 6.8 cm and ET = 8.4 cm.
7. BS = 10 cm and $\angle$ B = 60°.
8. ET = 9 cm and $\angle$ E = 70°.

2.3 Rectangle and Square

2.3.1 Rectangle

A rectangle is a parallelogram in which one of the angles is a right angle.

Its properties are

(i) The opposite sides are equal.

(ii) All angles are equal.

(iii) Each angle is a right angle.

(iv) The diagonals are equal in length.

(v) The diagonals bisect each other.

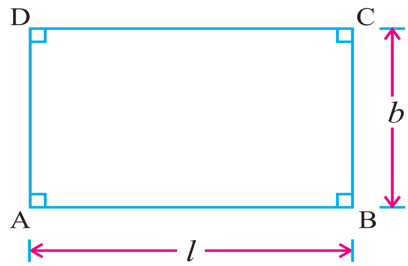


Fig. 2.11

Area of a rectangle:

Area of the rectangle ABCD = length × breadth = $l \times b$ sq. units.

2.3.2 Construction of a rectangle

We can construct a rectangle, when the following measurements are given:

(i) Length and breadth (ii) A side and a diagonal

2.3.3. Construction of a rectangle when length and breadth are given

Example 2.5

Construct a rectangle having adjacent sides of 6 cm and 4 cm and find its area.

Solution

Given:

Adjacent sides are 6 cm and 4 cm.

Rough Diagram

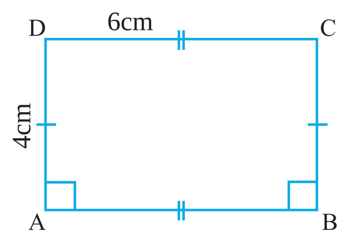


Fig. 2.12

To construct a rectangle

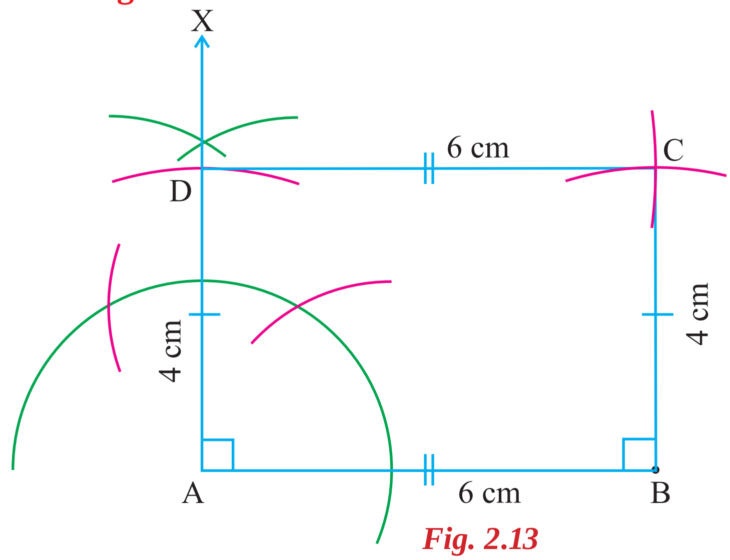


Fig. 2.13

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment AB = 6 cm.

Step 3 : At A, using a compass construct $\overrightarrow{AX} \perp \overline{AB}$.

Step 4 : With A as centre, draw an arc of radius 4 cm and let it cut $\overrightarrow{AX}$ at D.

Step 5 : With D as centre, draw an arc of radius 6 cm above the line segment $\overline{AB}$.

Step 6 : With B as centre, draw an arc of radius 4 cm cutting the previous arc at C. Join $\overline{BC}$ and $\overline{CD}$. ABCD is the required rectangle.

Step 7 : AB = l = 6 cm and BC = b = 4 cm.

Calculation of area:

In the rectangle ABCD, l = 6 cm and b = 4 cm.

Area of the rectangle ABCD $= l \times b = 6 \times 4 = 24$ cm^2.

2.3.4 Construction of a rectangle when one diagonal and one of the sides are given

Example 2.6

Construct a rectangle having diagonal of 7 cm and length of one of its side is 4 cm. Find its area also.

Solution

Rough Diagram

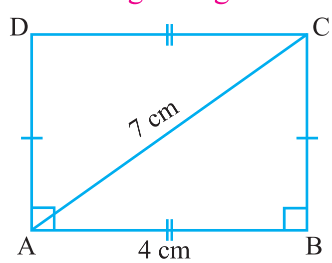


Fig. 2.14

Given:

Diagonal = 7 cm and length of one side = 4 cm.

To construct a rectangle

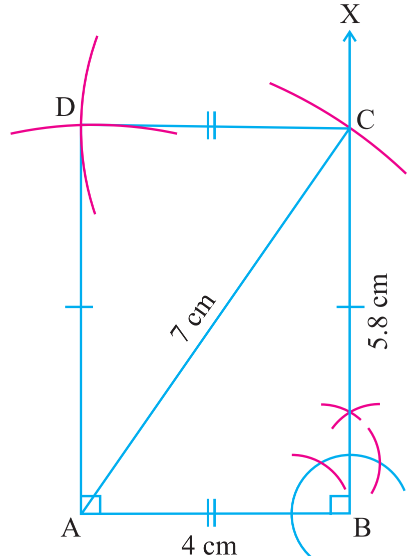


Fig. 2.15

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segmeant AB = 4 cm.

Step 3 : Construct $\overrightarrow{BX} \perp \overline{AB}$.

Step 4 : With A as centre, draw an arc of radius 7 cm which cuts $\overrightarrow{BX}$ at C.

Step 5 : With BC as radius draw an arc above $\overline{AB}$ with A as centre.

Step 6 : With C as centre, and 4 cm as radius draw an arc to cut the previous arc at D.

Step 7 : Join $\overline{AD}$ and $\overline{CD}$. ABCD is the required rectangle.

Step 8 : Measure the length of BC. BC = l = 5.8 cm

Calculation of area:

In the rectangle ABCD, l = 5.8 cm and b = 4 cm.

Area of the rectangle ABCD = $l \times b$ = 5.8 × 4 = 23.2 cm^2.

2.3.5 Construction of a Square

Square

A square is a rectangle, whose adjacent sides are equal in length.

The properties of a square are :

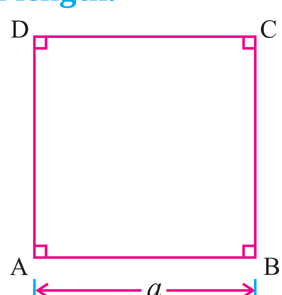


Fig. 2.16

(i) All the angles are equal.

(ii) All the sides are of equal length.

(iii) Each of the angle is a right angle.

(iv) The diagonals are of equal length and

(v) The diagonals bisect each other at right angles.

Area of a square = side × side

A $= a \times a = a^2$ sq. units

If the diagonal is known , then area = $\frac{d^2}{2}$

To construct a square we need only one measurement.

We can construct a square when the following measurements are given:

(i) one side, (ii) a diagonal

MATHEMATICS

2.3.6 Construction of a square when one side is given

Example 2.7

Construct a square of side 5 cm. Find its area also.

Solution

Given: Side = 5 cm.

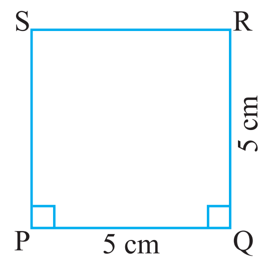


Fig. 2.17

To construct a square

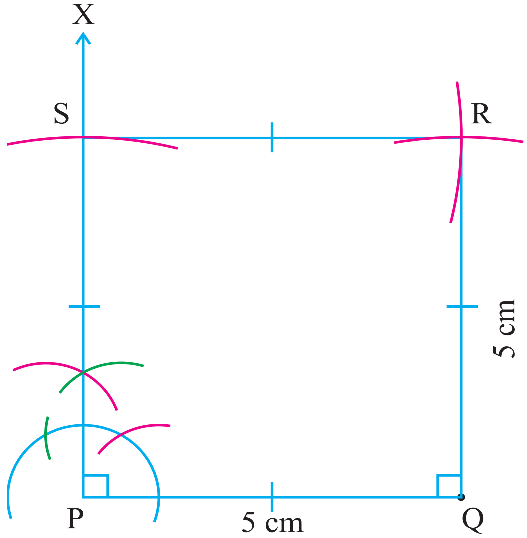


Fig. 2.18

Steps for construction

Step 1 : Draw a rough diagram and mark the given measurements.

Step 2 : Draw a line segment PQ = 5 cm.

Step 3 : At P using a compass construct $\overrightarrow{PX} \perp \overline{PQ}$.

Step 4 : With P as centre, draw an arc of radius 5 cm cutting $\overrightarrow{PX}$ at S.

Step 5 : With S as centre, draw an arc of radius 5 cm above the line segment $\overline{PQ}$.

Step 6 : With Q as centre and same radius, draw an arc, cutting the previous arc at R.

Step 7 : Join $\overline{QR}$ and $\overline{RS}$.

PQRS is the required square.

Calculation of area:

In the square PQRS, side a = 5 cm

Area of the square PQRS = $a \times a$

= $5 \times 5 = 25$ cm^2.

2.3.7 Construction of a square when one diagonal is given

Example 2.8

Construct a square whose diagonal is 6 cm. Measure the side. Find also its area.

Solution

Given: Diagonal = 6 cm.

To construct a square

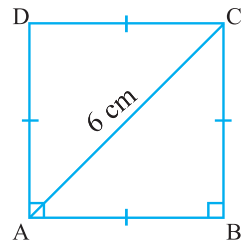


Fig. 2.19

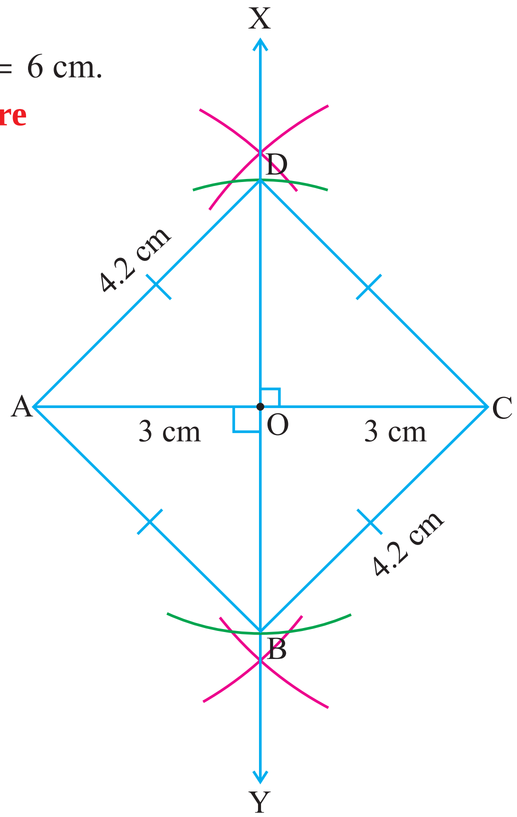


Fig. 2.20

Steps for construction

Step 1 : Draw the rough diagram and mark the given measures.

Step 2 : Draw a line segment AC = 6 cm.

Step 3 : Construct a perpendicular bisector $\overleftrightarrow{XY}$ of $\overline{AC}$.

Step 4 : $\overleftrightarrow{XY}$ intersects $\overline{AC}$ at O. We get OC = AO = 3 cm.

Step 5 : With O as centre draw two arcs of radius 3 cm cutting the line $\overleftrightarrow{XY}$ at points B and D.

Step 6 : Join $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{DA}$.

ABCD is the required square.

Calculation of area:

In the square ABCD, diagonal d = 6 cm

Area of the Square ABCD $= \dfrac{d^2}{2} = \dfrac{6\times 6}{2} = 18\ \text{cm}^2$.

EXERCISE 2.2

1. **Construct rectangle JUMP with the following measurements. Find its area also.**
   - (i) JU = 5.4 cm and UM = 4.7 cm.
   - (ii) JU = 6 cm and JP = 5 cm.
   - (iii) JP = 4.2 cm and MP= 2.8 cm.
   - (iv) UM = 3.6 cm and MP = 4.6 cm.
2. **Construct rectangle MORE with the following measurements. Find its area also.**
   - (i) MO = 5 cm and diagonal MR = 6.5 cm.
   - (ii) MO = 4.6 cm and diagonal OE = 5.4 cm.
   - (iii) OR = 3 cm and diagonal MR = 5 cm.
   - (iv) ME = 4 cm and diagonal OE = 6 cm.
3. **Construct square EASY with the following measurements. Find its area also.**

   (i) Side 5.1 cm. (ii) Side 3.8 cm. (iii) Side 6 cm (iv) Side 4.5 cm.
4. **Construct square GOLD, one of whose diagonal is given below. Find its area also.**

   (i) 4.8 cm. (ii) 3.7 cm. (iii) 5 cm. (iv) 7 cm.

Concept Summary

- A quadrilateral with each pair of opposite sides parallel and with each pair of adjacent sides equal is called a rhombus.
- To contruct a rhombus two independent measurements are necessary.
- The area of a rhombus, A = $\frac{1}{2} d_1 d_2$ sq. units, where d_1 and d_2 are the two diagonals of the rhombus.
- A parallelogram in which each angle is a right angle, is called a rectangle.
- Area of the rectangle = l x b sq. units Where l is the length and b is the breadth.
- Square is a rectangle, whose pair of adjecent sides are equal.
- Area of the square = a x a sq. units Where a is the length of the side.

3 Graphs

Rene Descartes
(1596- 1650 A.D)

The French Mathematician and philosopher who wrote the book "Discourse on Method". His attempts to unify algebra and geometry gave birth to new branches of mathematics, Coordinate Geometry and Graphs. One of his famous statements is

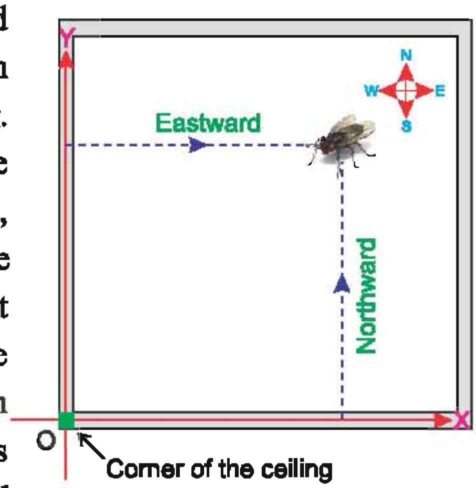
"I think, therefore I am".

3.1 Introduction

The story of a fly and the graph

The mathematician who introduced graph was **Rene Descartes**, a French Mathematician in early 17th century. Here is an interesting anecdote from his life.

Rene Descartes, was a sick child and was therefore, allowed to remain in bed till quiet late in the morning. Later, it became his nature. One day when he was lying on the bed, he saw a small insect(fly) near one corner of the ceiling. It's movement led Rene Descartes to think about the problem of determining its position on the ceiling. He thought that it was sufficient to know the eastward and northward distance of the fly from the corner 'O' of the ceiling (refer figure). This was the beginning of the subject known as **Graphs**.



His system of fixing a point with the help of two measurements one with vertical and another with horizontal is known as 'Cartesian System'. The word 'Cartes' is taken from Rene Descartes and is named as 'Cartesian System', in his honour. The two axes x and y are called as **cartesian axes** .

3.2 Introduction to cartesian plane with axes

3.2.1 Location of a point

Look at the Fig.3.1. Can you tell us where the boy is? Where the church is? Where the temple is? Where the bag is? and Where the mosque is? Is it easy? No. How can we locate the boy, the church, the temple, the bag and the mosque correctly?

Fig. 3.1

Let us first draw parallel horizontal lines with a distance of 1 unit from each other. The bottom line is OX. Now the figure 3.1 will look like the figure 3.2.

Try to express the location of the boy, the church, the temple, the bag and the mosque now. The boy and the church are on the first horizontal line. (i.e.) Both of them are 1 unit away from the bottom line OX. Still we are not able to locate them exactly. There is some confusion for us yet. In the same manner, it is difficult for us to locate the exact positions of the temple, the bag and the mosque because they lie on different parallel lines.

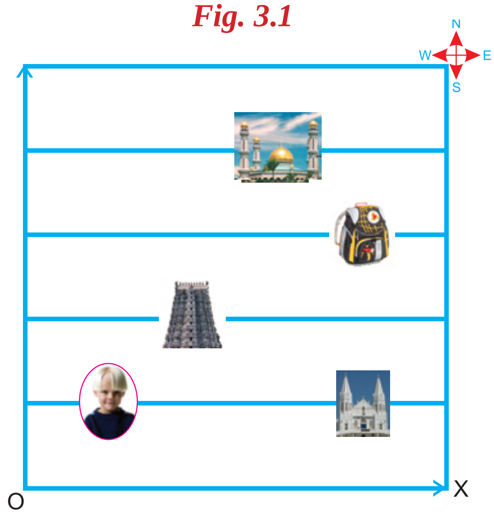


Fig. 3.2

To get rid of this confusion, let us now draw the vertical lines with a distance of 1 unit from each other in the figure 3.2. The left most vertical line is OY. Then it will look like the figure 3.3.

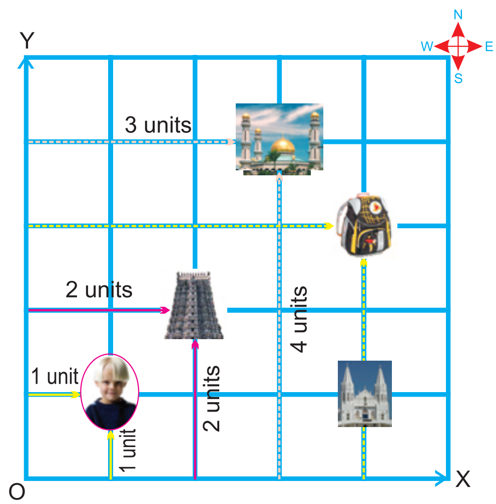


Fig. 3.3

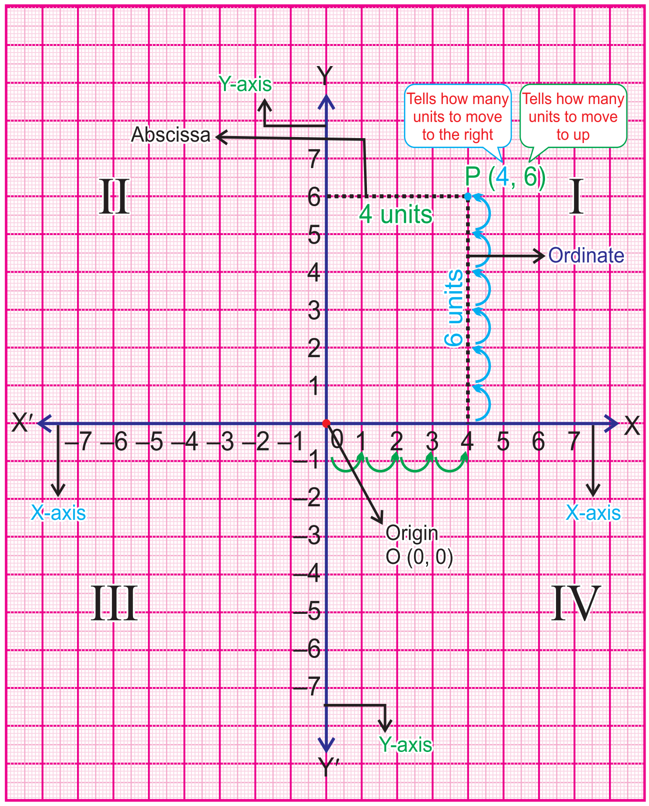
Now with the help of both the horizontal and vertical lines, we can locate the given objects. Let us first locate the boy. He is 1 unit away from the vertical line OY and 1 unit away from the horizontal line OX. Hence his location is represented by a point (1, 1).

Similarly the location of the church is represented by the point (4 , 1), the location of the temple is (2 , 2), the position of the bag is (4 , 3) and the location of the mosque is (3 , 4).

3.2.2. Coordinate system

Now let us define formally what the coordinate system is.

Let X′OX and Y′OY be the two number lines intersecting each other perpendicularly at zero. They will divide the whole plane of the paper into four parts which we call quadrants [I, II, III and IV]. See the figure.



The line X′OX is called the ***x*-axis**.

The line Y′OY is called the ***y*-axis**.

The point 'O' is called the **Origin**.

Thus, Origin is the point of intersection of *x*-axis and *y*-axis.

This is called the Cartesian coordinate system.

Note : To mark a point, we always write the *x*-coordinate (or the number on the horizontal axis) first and then the *y*-coordinate (or the number on the vertical axis). The first number of the pair is called the *x*-coordinate or **abscissa**. The second number of the pair is called the *y*-coordinate or **ordinate**.

Observation : Let us consider the point P (4, 6) in the figure. It is 4 units away from the right side of the *y*-axis and 6 units above the *x*-axis. Then the coordinate of the point P is (4, 6).

3.3 Plotting of Points for different kinds of situations

3.3.1 Plotting a point on a Graph sheet

Example 3.1

Plot the point (4, 5) on a graph sheet. Is it the same as (5, 4) ?

Solution

Draw X′OX and Y′OY and let them cut at the origin at O.

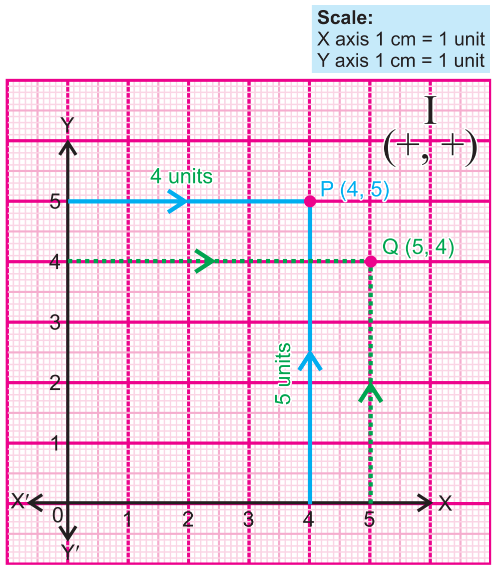
Mark the units along the *x*-axis and *y*-axis with a suitable scale.

The given point is P (4, 5).

Here the *x*-coordinate of P is 4 and the y-coordinate of P is 5.

And both are positive. Hence the point P (4, 5), lies in the first quadrant.

To plot, start at the Origin O(0,0) . Move 4 units to the right along the x-axis. Then turn and move 5 units up parallel to y-axis. You will reach the point P (4, 5). Then mark it. (As shown in the adjoining figure)



Next, let us plot the point Q (5, 4). Here the x-coordinate of Q is 5 and the y-coordinate of Q is 4. And both are positive. Hence, this point Q (5, 4) also lies in the quadrant I. To plot this point Q (5, 4); start at the Origin. Move 5 units to the right along the x-axis . Then turn and move 4 units up parallel to y-axis. You will reach the point Q (5, 4). Then mark it (As shown in the above figure).

Conclusion: From the above figure, it is very clear that the points P (4,5) and Q(5, 4) are two different points.

Example 3.2

Plot the following points on a graph paper and find out in which quadrant do they lie?

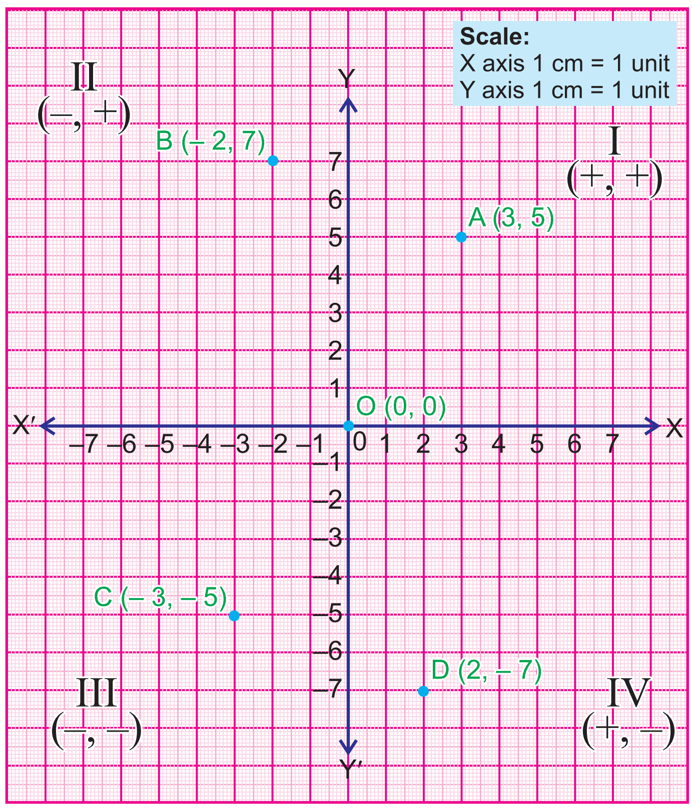
(i) A (3,5) (ii) B (–2 , 7)

(iii) C (–3,–5) (iv) D (2, – 7)

(v) O (0, 0)

Solution

Draw the x and y axes. Mark the units along the x and y axes with a suitable scale.



(i) To plot the point A (3 , 5)

Here, the x-coordinate of A is 3 and the y-coordinate of A is 5. Both are positive. Hence the point A (3 , 5) lies in the quadrant I. Start at the Origin. Move three units to the right along the x-axis.

Then turn and move 5 units up parallel to Y-axis and mark the point A (3 , 5).

(ii) To plot the point B (–2 , 7)

Here, the x-coordinate of B is –2 which is negative and the y-coordinate of B is 7 which is positive. Hence the point B (–2 , 7) lies in the quadrant II. Start at the Origin. Move 2 units to the left along the x-axis. Then turn and move 7 units up parallel to y-axis and mark the point B (–2 , 7).

(iii) To plot the point C (–3 , –5)

Here, the x-coordinate of C is –3 and the y-coordinate of C is –5. Both are negative. Hence the point C (–3 , –5) lies in the quadrant III. Start at the Origin. Move 3 units to the left along the x-axis. Then turn and move 5 units down parallel to y-axis. and mark the point C (–3 , –5).

(iv) To plot the point D (2 , –7)

Here, the x-coordinate of the point D is 2 which is positive and the y-coordinate of D is –7 which is negative. Hence the point D (2 , –7) lies in the quadrant IV. Start at the Origin. Move 2 units to the right along the x-axis. Then turn and move 7 units down parallel to y-axis and mark the point D (2 , –7).

(v) To plot the point O (0, 0)

This is the origin. Both the x and y coordinates are zeros. It is the point of intersection of the axes x and y. Mark the point O (0,0).

Example 3.3

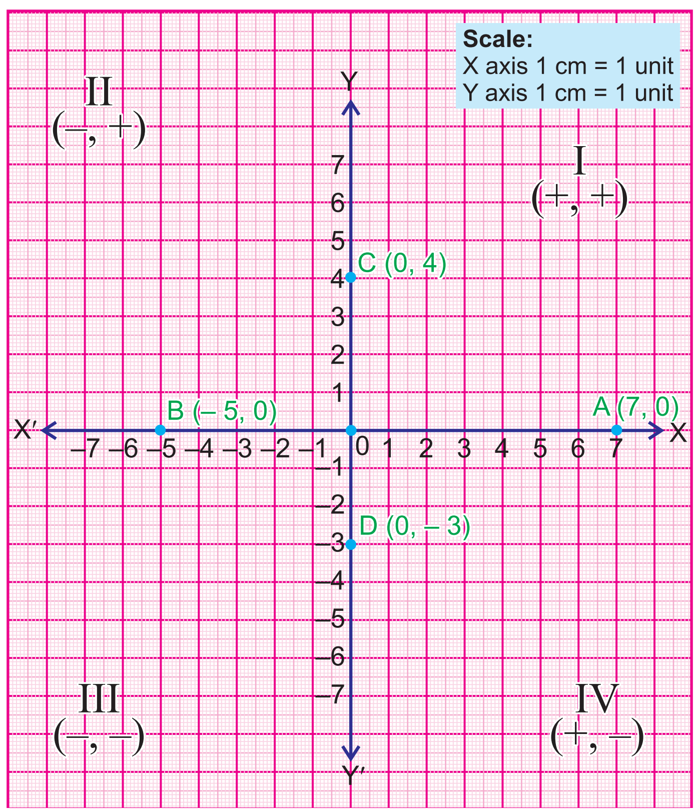
Plot the following points on a graph paper and find out where do they lie?

(i) A (7, 0) (ii) B (– 5, 0)

(iii) C (0 , 4) (iv) D (0, – 3)

Solution

Draw the x and y axes. Mark the units along the x and y axes with a suitable scale.



(i) To plot the point A (7, 0)

Here, the x-coordinate of A is 7 which is positive and the y-coordinate of A is zero. Hence the point A (7, 0) lies on the *x*-axis. Start at the Origin. Move 7 units to the right along the *x*-axis and mark it.

(ii) To plot the point B (–5 , 0)

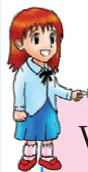
Here, the *x*-coordinate of B is –5 which is negative and the *y*-coordinate is zero. Hence the point B (–5 , 0) lies on the *x*-axis. Start at the Origin. Move 5 units to the left along the *x*-axis and mark it.

(iii) To lot the point C (0 ,4)

Here, the *x*-coordinate of C is zero and the *y*-coordinate of C is 4 which is positive. Hence the point C (0 , 4) lies on the *y*-axis. Start at the Origin. Move 4 units up along the *y*-axis and mark it.

(iv) To plot the point D (0 , –3)

Here the *x*-coordinate of D is zero and the y-coordinate of D is –3 which is negative. Hence the point D (0 , – 3) lies on the *y*-axis. Start at the Origin. Move 3 units down along the *y*-axis and mark it.



Do you know?

Where do the points lie? How can we tell without actually plotting the points on a graph sheet? To know this, observe the following table.

Sl. No.	Examples	*x* coordinate of the point	*y* coordinate of the point	Location of the point
1.	(3,5)	Positive (+)	Positive (+)	Quadrant I
2.	(–4,10)	Negative (–)	Positive (+)	Quadrant II
3.	(–5,–7)	Negative (–)	Negative (–)	Quadrant III
4.	(2,–4)	Positive (+)	Negative (–)	Quadrant IV
5.	(7,0)	Non zero	Zero	On the X axis
6.	(0,–5)	Zero	Non-zero	On the Y axis
7	(0,0)	Zero	Zero	Origin

Try these

Can you tell, where do the following points lie without actually plotting them on the graph paper?

(i) (2 , 7) (ii) (–2 , 7) (iii) (–2 , –7) (iv) (2 , –7)

(v) (2 , 0) (vi) (–2 , 0) (vii) (0 , 7) (viii) (0 , –7)

3.4 Drawing straight lines and parallel lines to the coordinate axes

In this section first we are going to learn to draw straight lines for the given two points and then to draw lines parallel to coordinate axes. Also to find the area of plane figures.

3.4.1 Line joining two given points

Example 3.4

Draw the line joining the following points.

(i) A (2,3) and B (5, 7),

(ii) P (–4,5) and Q (3,–4).

Solution

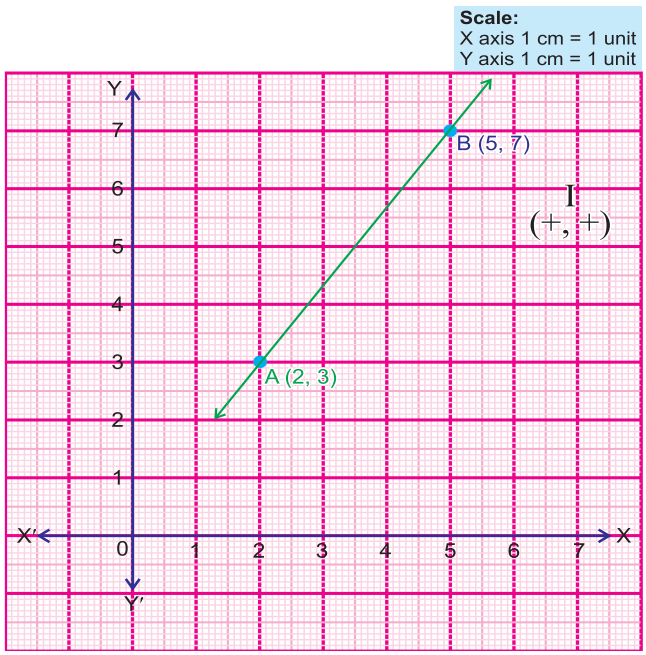
(i) To draw the line joining the points A (2 , 3) and B(5 , 7):

First, plot the point (2 , 3) and denote it by A.

Next, plot the point (5 , 7) and denote it by B.

Then, join the points A and B.

AB is the required line.



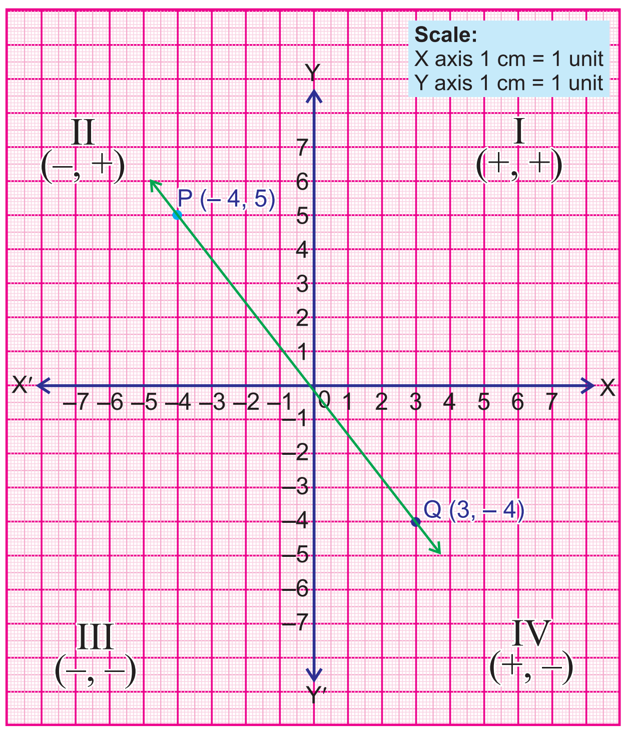
(ii) To draw the line joining the points P (–4 , 5) and Q (3 , –4)

First, plot the point (–4 , 5) and denote it by P.

Next, plot the point (3 , –4) and denote it by Q.

Then, join the points P and Q.

PQ is the required line.



3.4.2 Drawing straight parallel lines to axes

Example 3.5

(i) Draw the graph of $x = 3$.

(ii) Draw the graph of $y = -5$.

(iii) Draw the graph of $x = 0$.

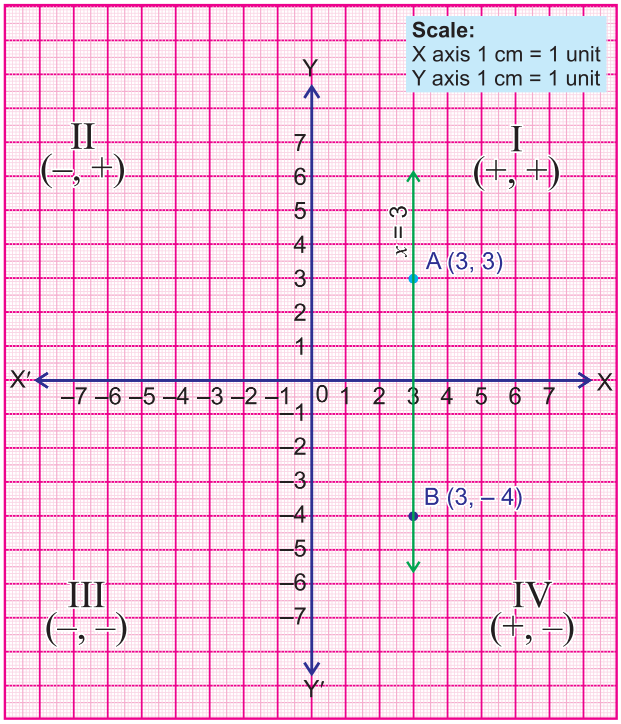
Solution

(i) The equation $x = 3$ means:

Whatever may be y-coordinate, x-coordinate is always 3. Thus, we have

x	3	3
y	3	–4

Plot the points A (3, 3) and B (3 , –4). Join these points and extend this line on both sides to obtain the graph of $x = 3$.



(ii) The equation $y = -5$ means:

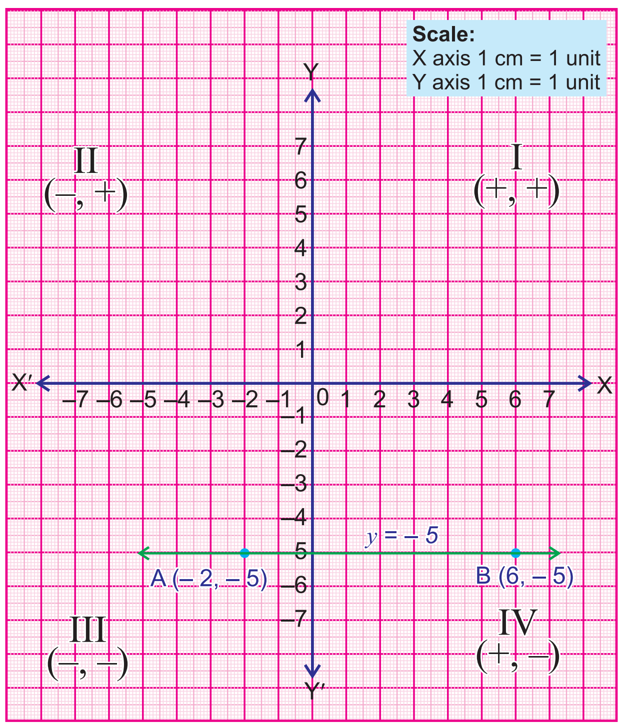
Whatever may be the x-coordinate,

the y-coordinate is always – 5.

Thus we have,

x	–2	6
y	–5	–5

Plot the points A (–2, –5) and B (6 , –5). Join these points A and B and extend this line on both sides to obtain the graph of $y = -5$.



(iii) The equation $x = 0$ means;

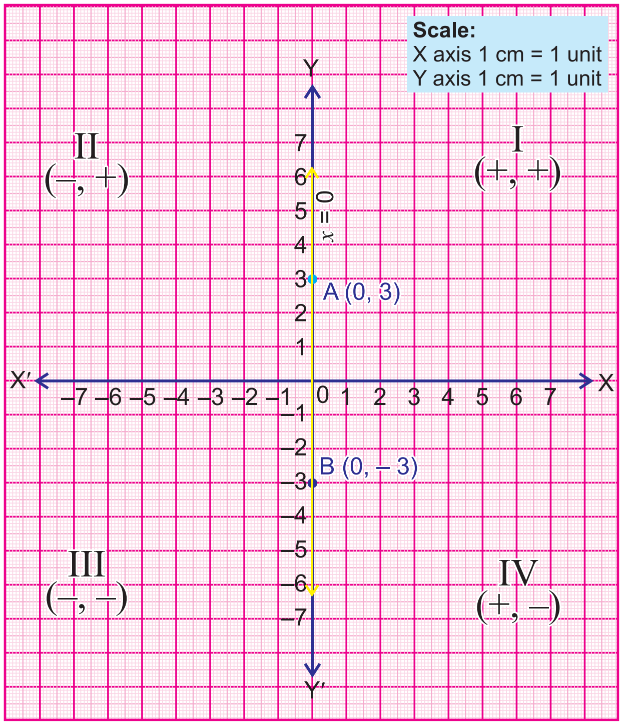
Whatever may be the y-coordinate, x-coordinate is always 0.

Thus, we have

x	0	0
y	3	–3

Plot the points A (0 , 3) and B (0, – 3).

Join the points A and B and extend this line to obtain the graph of $x = 0$.



3.4.3 Area of Plane Figures

Area of regions enclosed by plane figures like square, rectangle, parallelogram, trapezium and triangle drawn in a graph sheet can be determined by actual counting of unit squares in the graph sheet.

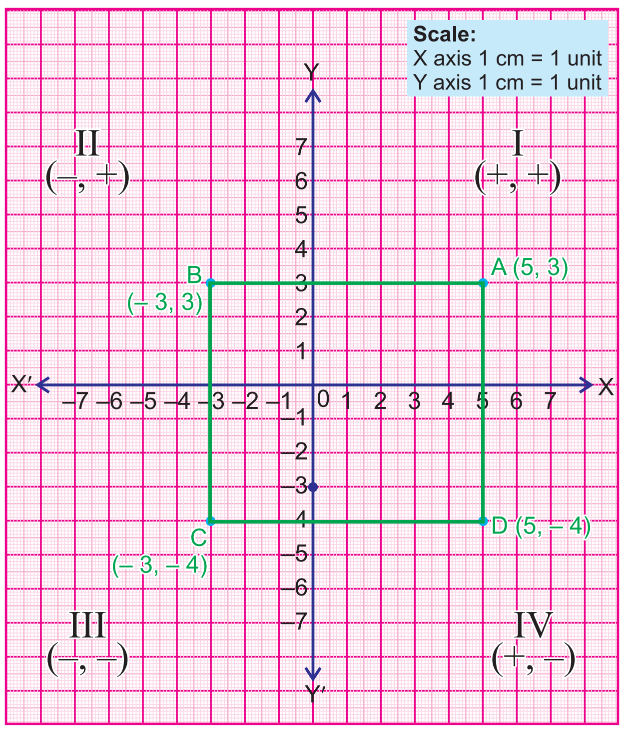
Example: 3.6

Plot the points A (5 , 3), B (–3 , 3), C (–3 , –4), D (5 , –4) and find the area of ABCD enclosed by the figure.

Solution

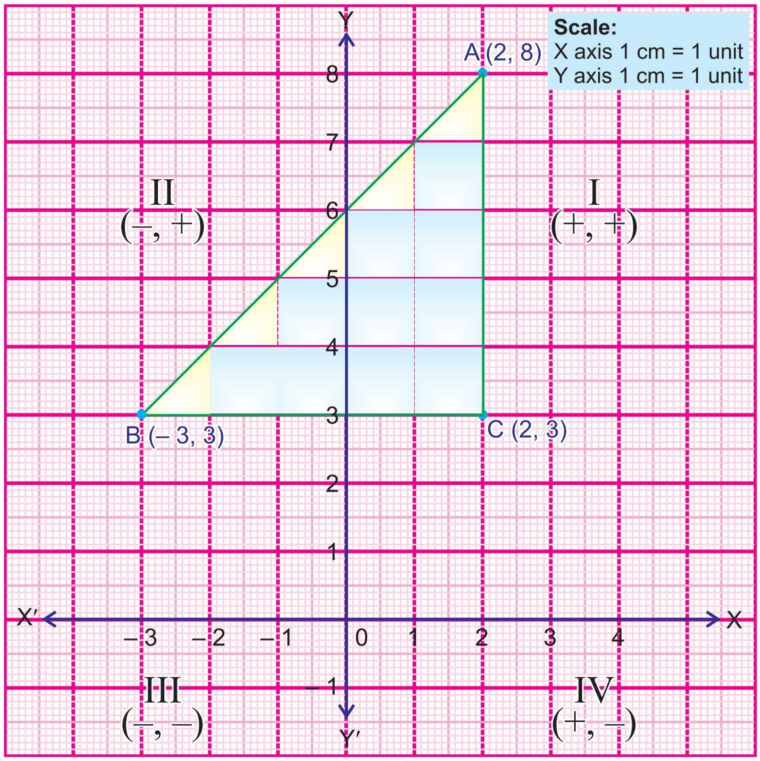
Draw the x-axis and y-axis with a suitable scale.

Plot the points A (5, 3), B (– 3, 3), C (–3 , –4), D (5 , –4). Join the points A and B, B and C, C and D and D and A. We get a closed figure ABCD. Clearly it is a rectangle. Count the number of units squares enclosed between the four sides. There are altogether 56 unit squares. Hence the area of the rectangle ABCD is 56 cm^2.



Example 3.7

Plot the points A (2 , 8), B (–3 , 3) , C (2 , 3) and find the area of the region enclosed by the figure ABC.



Solution

Draw the x-axis and y-axis with a suitable scale.

Plot the points A (2 , 8), B (– 3, 3), C (2 , 3). Join the points A and B, B and C and C and A. We get a closed figure ABC. Clearly it is a triangle. Count the number of full squares. There are 10 full unit squares.

Count the number of half squares. There are 5 half unit squares.

Hence the area of a triangle is $10 + \frac{5}{2} = 10 + 2.5 = 12.5 \text{ cm}^2$.

EXERCISE 3.1

1. Plot the following points in the graph paper and find out where they lie?

(i) A (2 , 3) (ii) B (– 3 , 2) (iii) C (–5 ,–5) (iv) D (5 , –8)

(v) E (6 , 0) (vi) F (–4 , 0) (vii) G (0 , 9) (viii) H (0 , –3)

(ix) J (7 , 8) (x) O (0 , 0)

2. State in which quadrant each of the following points lie without actually plotting the points.

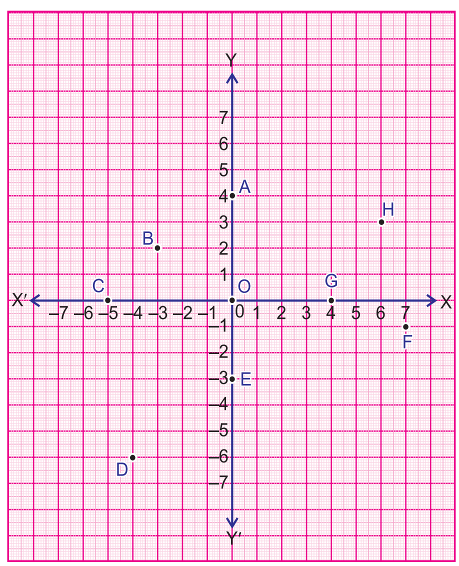
(i) (8 , 15) (ii) (–15 , 2)

(iii) (–20 , –10) (iv) (6 , –9)

(v) (0 , 18) (vi) (–17 , 0)

(vii) (9 , 0) (viii) (–100 , –200)

(ix) (200 , 500) (x) (–50 , 7500).



3. Determine the quadrants and the coordinates of the points A, B, C, D, E, F, G, H and O in the given figure.

4. Plot the following points and draw a line through the points.

   (i) (2 , 7) , (–2 ,– 3) (ii) (5 , 4), (8 , –5)

   (iii) (–3 , 4), (–7 , –2) (iv) (–5 , 3), (5 , –1)

   (v) (2 , 0), (6 , 0) (vi) (0 , 7), (4 , –4)

5. Draw the graph of the following equations:

   (i) $y = 0$ (ii) $x = 5$ (iii) $x = -7$ (iv) $y = 4$ (v) $y = -3$

6. Plot the following points and find out the area of enclosed figures.

   (i) A (3 , 1), B (3 , 6), C (–5 , 6), D (–5 , 1)

   (ii) A (– 2 , – 4), B (5 , – 4), C (5 , 4), D (–2 , 4)

   (iii) A (3 , 3), B (–3 , 3), C (–3 , –3), D (3 , –3)

   (iv) O (0 , 0), A (0 , 7), B (–7 , 7), C (–7 , 0)

   (v) A (0 , – 2), B (–4 , – 6), C (4 ,– 6)

   (vi) A (1 , 2), B (9,2), C (7 , 4), D (3 , 4)

   (vii) A (–4, 1), B (–4, 7), C (–7, 10), D (–7 , 4)

7. Find the perimeter of the rectangle and squares of the previous problems 6 (i), (ii), (iii) and (iv).

3.5 Linear Graphs

We have learnt to draw straight lines and parallel lines in the graph sheet. When we get a straight line by joining any two points, then the graph is called **a linear graph**.

3.5.1 Time and Distance Graph

Let us consider the following example to study the relationship between time and distance.

Example 3.8

Amudha walks at a speed of 3 kilometers per hour. Draw a linear graph to show the relationship between the time and distance.

Solution

Amudha walks at a speed of 3 kilometers per hour. It means she walks 3 Km in 1 hour, 6 Km in 2 hours, 9 km in 3 hours and so on.

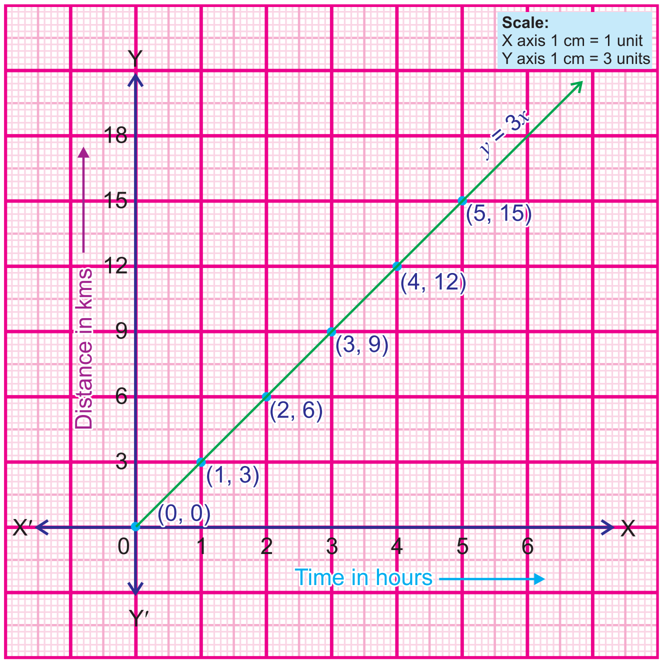
Thus we have the table

Time in hours (x)	0	1	2	3	4	5
Distance in km (y)	0	3	6	9	12	15

Points: (0 , 0), (1 , 3), (2 , 6), (3 , 9), (4 , 12) and (5 , 15).

Plot the points (0 , 0), (1, 3), (2, 6), (3, 9), (4, 12) and (5, 15). Join all these points.

We get a straight line . Hence, it is a linear graph.



Relationship between x and y:

We know that,

Distance = Speed × Time.

From the above table,

$$
\begin{aligned}
0 &= 3 \times 0 \\
3 &= 3 \times 1 \\
6 &= 3 \times 2 \\
9 &= 3 \times 3 \\
12 &= 3 \times 4 \\
15 &= 3 \times 5 \\
\Longrightarrow y &= 3x
\end{aligned}
$$

[Here, y = Distance, x = Time in hour and 3 is the speed]

The linear equation of this problem is $\mathbf{y = 3}x$.

3.5.2 Perimeter–side graph of a square

Example 3.9

Draw a linear graph to show the perimeter–side relationship of a square.

Solution

We know that the perimeter of a square is four times of its side. (i.e) P = 4a.

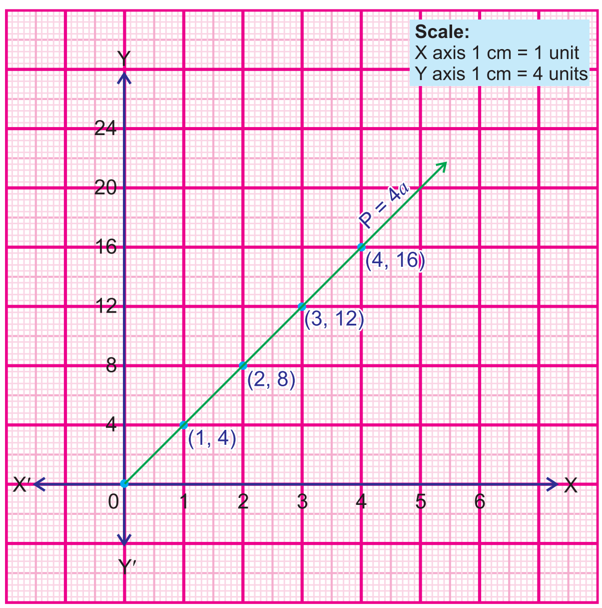
(Here, P = Perimeter and a = side)

For different values of a, the values of P are given in the following table.

a (in cm)	1	2	3	4
P = 4*a* (in cm)	4	8	12	16

Points: (1, 4), (2, 8), (3, 12), (4, 16).

Plot the above points. Join all the points. We get the linear graph of P = 4a.



3.5.3 Area as a function of side of a square

Example 3.10

Draw a graph to show the area-side of a square.

Solution

We know that the area of a square is the square of its side. (i.e) A = a^2.

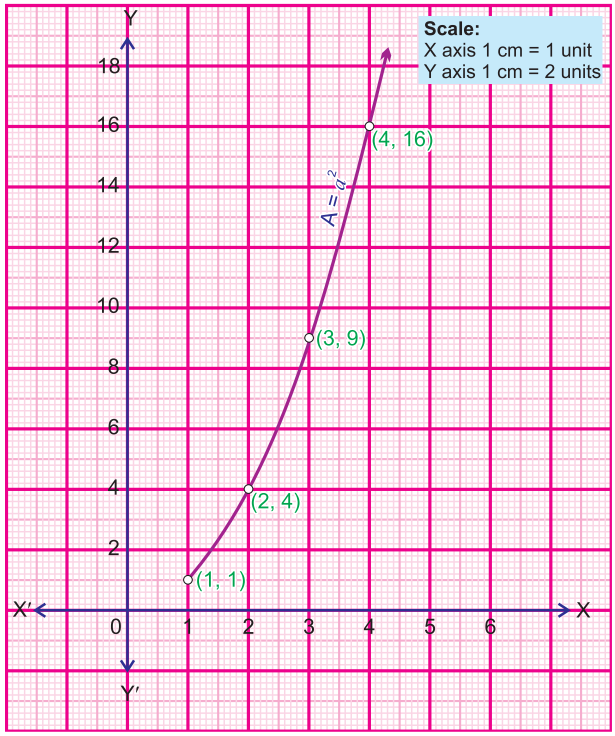
(Here, A = Area, a = side). For different values of a, the values of A are given in the following table.

a (in cm)	1	2	3	4	5
$A = a^2$ (in cm^2)	1	4	9	16	25

Points: (1 , 1), (2 , 4), (3 , 9), (4 ,16), (5 , 25)

Plot the above points.

Join all the points. Observe the graph. Is it a linear graph? **No. It is a curve.**



3.5.4 Plotting a graph of different multiples of numbers

Example 3.11

Draw a graph of multiples of 3.

Solution

Let us write the multiples of 3. Multiples of 3 are 3, 6, 9, 12, 15... etc.

We can also write this as Multiples of $3 = 3 \times n$, where $n = 1, 2, 3, ...$

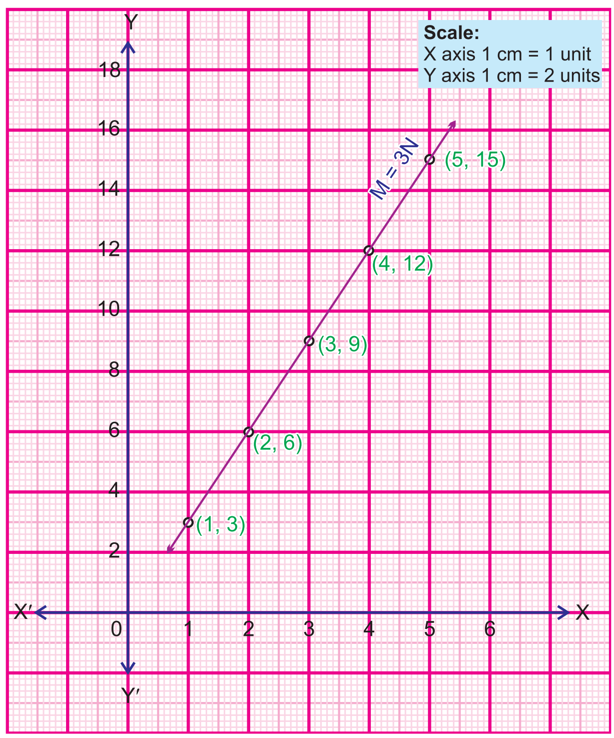
$m = 3\,n$. m is the multiple of 3

Thus, we have the following table.

n	1	2	3	4	5
$m = 3n$	3	6	9	12	15

Points: (1 , 3), (2 , 6), (3 , 9), (4 , 12), (5 , 15).

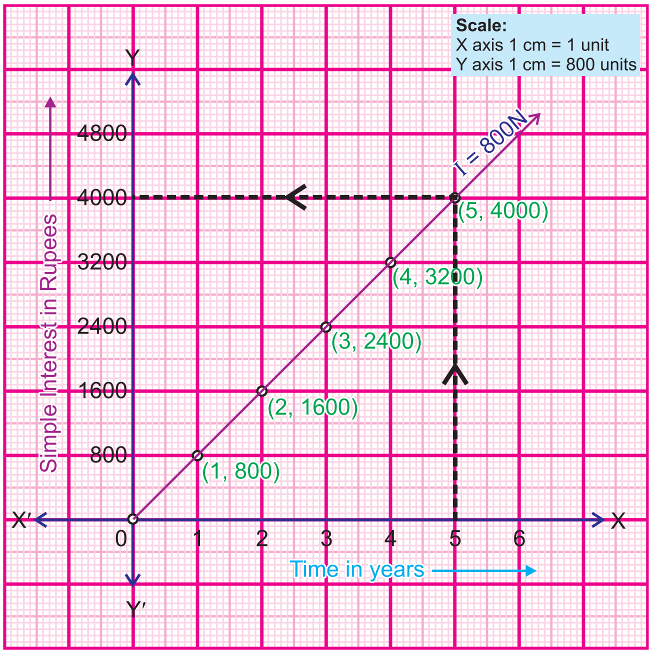
Plot all these points and join them. We get the graph for multiples of 3.



3.5.5. Simple Interest–Time graph

Example 3.12

Ashok deposited ₹ 10,000 in a bank at the rate of 8% per annum. Draw a linear graph to show the simple interest-time relationship. Also, find the simple interest for 5 years.



Solution

We know that,

Simple interest, $I = \frac{Pnr}{100}$

[where P = Principal, n = Time in years,

r = Rate of interest]

Principal, P = 10000

Time, n = ?

Rate, r = 8%

$$I = \frac{P \times n \times r}{100}$$

$$I = \frac{10000 \times n \times 8}{100}$$

$$I = 800\, n.$$

(Here, the simple interest, I depends upon N)

For different values of n, the values of I are given in the following table.

n (Time in Yrs)	1	2	3	4	5
I = 800 *n* (in ₹)	800	1600	2400	3200	4000

Points: (1, 800), (2, 1600), (3, 2400), (4, 3200), (5, 4000)

Plot all the points. Join them all. Draw the linear graph.

So, Ashok will get ₹ 4000 as simple interest after 5 years. (In the graph, the answer is shown by the dotted lines.)

3.6 Reading Linear Graphs

Money Exchange: The world has become very small today. It is inevitable to do business with foreign countries. When we are doing business with other countries, we have to transact our money (Indian currency) in terms of their currencies. Different countries use different currencies under different names. Hence we should know the concepts related to money exchange. Let us consider the following example.

Example 3.13

On a particular day the exchange rate of 1 Euro was ₹ 55. The following linear Graph shows the relationship between the two currencies. Read the graph carefully. and answer the questions given below:

(i) Find the values of 4 Euros in terms of Rupees.

(ii) Find the values of 6 Euros in terms of Rupees.

(iii) Find the value of ₹ 275 in terms of Euros.

(iv) Find the value of ₹ 440 in terms of Euros.

Solution

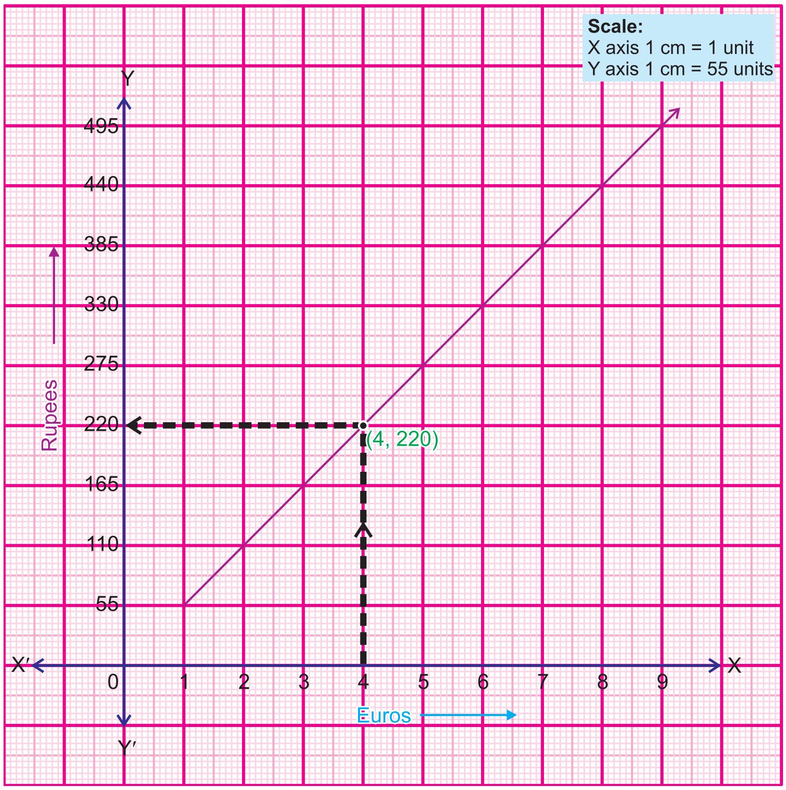
(i) To find the value of 4 Euros. In this graph, draw a dotted line at $x = 4$ parallel to y-axis.

Locate the point of intersection of this line with the given line.

From this point draw a dotted line parallel to x-axis.

It cuts the y-axis at 220. (See figure)

Hence the value of 4 Euros is ₹ 220.



Activity

Try and answer the remaining questions (ii), (iii) and (iv).

EXERCISE 3.2

1. Draw a linear graph for the following data.

(i)

x	5	5	5	5	5	5
y	1	2	3	4	5	6

(ii)

x	1	2	3	4	5
y	1	2	3	4	5

2. Draw the linear graph and find the missing entries.

x	1	2	3	4	–
y	6	12	–	–	30

3. Draw the following graph of side–area relationship of a square.

Side (in m)	2	3	4	5	6
Area (in m^2)	4	9	16	25	36

4. Draw the graph of $y = 7x$.

5. If Akbar is driving a car at a uniform speed of 40 km/hr. Draw the distance time graph. Also find the time taken by Akbar to cover a distance of 200 km.

6. Eliza deposited ₹ 20,000 in a bank at the rate of 10% per annum. Draw a linear graph showing the time and simple interest relationship. Also, find the simple interest for 4 years.

ANSWERS

Chapter 1

Exercise 1.1

1. i) C ii) B iii) D iv) A v) D vi) D vii) C

2.

Sl. No.	Terms	Coeffficients of variables
i)	$3\,abc$ $-5\,ca$	3 -5
ii)	$1, x, y^2$	constant term, 1, 1
iii)	$3x^2 y^2$ $-3xyz$ y^2	3 -3 1
iv)	-7 $2\,pq$ $\frac{-5}{7}qr$ rp	constant term 2 $\frac{-5}{7}$ 1
v)	$\frac{x}{2}$ $\frac{-y}{2}$ $-0.3\,xy$	$\frac{1}{2}$ $\frac{-1}{2}$ -0.3

3. Monomials : $3x^2$

   Binomials : $3x + 2$, $x^5 - 7$, $a^2b + b^2c$, $2l + 2m$.

   Trinomials : $x^2 - 4x + 2$, $x^2 + 3xy + y^2$, $s^2 + 3st - 2t^2$

4. i) $5x^2 - x - 2$ ii) $2x^2 + x - 2$ iii) $-3t^2 - 2t - 3$

   iv) 0 v) $2(a^2 + b^2 + c^2 + ab + bc + ca)$

5. i) a ii) $-4x - 18y$ iii) $5ab - 7bc + 13ca$

   iv) $-x^5 + x^3 + 5x^2 + 3x + 1$ v) $5x^2y - 9xy - 7x + 12y + 25$

6. i) 7, 5 ii) 13, -1 iii) 7, -1 iv) 8, 1 v) 8, -2

Exercise 1.2

1. i) $21\,x$ ii) $-21\,xy$ iii) $-15a^2b$ iv) $-20a^3$ v) $\frac{2}{3}x^7$ vi) x^3y^3

   vii) x^4y^7 viii) $a^2b^2c^2$ ix) $x^3y^2z^2$ x) $a^3b^3c^5$

2.

First Monomial → Second Monomial ↓	$2x$	$-3y$	$4x^2$	$-5xy$	$7x^2y$	$-6x^2y^2$
$2x$	$4x^2$	$-6xy$	$8x^3$	$-10x^2y$	$14x^3y$	$-12x^3y^2$
$-3y$	$-6xy$	$9y^2$	$-12x^2y$	$15xy^2$	$-21x^2y^2$	$18x^2y^3$
$4x^2$	$8x^3$	$-12x^2y$	$16x^4$	$-20x^3y$	$28x^4y$	$-24x^4y^2$
$-5xy$	$-10x^2y$	$15xy^2$	$-20x^3y$	$25x^2y^2$	$-35x^3y^2$	$30x^3y^3$
$7x^2y$	$14x^3y$	$-21x^2y^2$	$28x^4y$	$-35x^3y^2$	$49x^4y^2$	$-42x^4y^3$
$-6x^2y^2$	$-12x^3y^2$	$18x^2y^3$	$-24x^4y^2$	$30x^3y^3$	$-42x^4y^3$	$36x^4y^4$

3. i) $30a^7$ ii) $72\,xyz$ iii) $a^2b^2c^2$ iv) $-72m^7$ v) $x^3y^4z^2$
   vi) $l^2m^3n^4$ vii) $-30\,p^3q$
4. i) $8a^{23}$ ii) $-2x^2-3x+20$ iii) $3x^2+8xy-3y^2$ iv) $12x^2-x-6$
   iv) $\frac{-5}{4}a^3b^3$
5. i) $2a^3-3a^2b-2ab^2+3b^3$ ii) $2x^3+x^2y-xy^2+3y^3$
   iii) $x^2+2xy+y^2-z^2$ iv) $a^3+3a^2b+3ab^2+b^3$ v) m^3-n^3
6. i) $2(x^2-2xy+yz-xz-y^2)$ ii) $17a^2+14ab-21ac$

Exercise 1.3

1. i) C ii) D iii) B iv) D v) A vi) B
2. i) x^2+6x+9 ii) $4m^2+12m+9$ iii) $4x^2-20x+25$
   iv) $a^2-2+\frac{1}{a^2}$ v) $9x^2-4$ vi) $25a^2-30\,ab+9b^2$
   vii) $4l^2-9m^2$ viii) $\frac{9}{16}-x^2$ ix) $\frac{1}{x^2}-\frac{1}{y^2}$ x) 9991
3. i) $x^2+11x+28$ ii) $25x^2+35x+12$ iii) $49x^2-9y^2$
   iv) $64x^2-56x+10$ v) $4m^2+14\,mn+12n^2$ vi) $x^2y^2-5xy+6$
   vii) $a^2+\left(\frac{x+y}{xy}\right)a+\frac{1}{xy}$ viii) $4+2x-2y-xy$
4. i) $p^2-2\,pq+q^2$ ii) $a^2-10a+25$ iii) $9x^2+30x+25$
   iv) $25x^2-40x+16$ v) $49x^2+42xy+9y^2$ vi) $100m^2-180mn+81n^2$
   vii) $0.16a^2-0.4ab+0.25\,b^2$ viii) $x^2-2+\frac{1}{x^2}$
   ix) $\frac{x^2}{4}-\frac{xy}{3}+\frac{y^2}{9}$ x) 0.08
5. i) 10609 ii) 2304 iii) 2916 iv) 8464 v) 996004 vi) 2491
   vii) 9984 viii) 896 ix) 6399 x) 7.84 xi) 84 xii) 95.06
7. $ab=\frac{9}{4}$, $a^2+b^2=\frac{41}{2}$ 8. i) 80, 16, ii) 196, 196 9. 625
10. $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc.$

Exercise 1.4

1. i) C ii) D iii) A iv) C v) B

2. i) $3(x-15)$ ii) $7(x-2y)$ iii) $5a(a+7)$
   iv) $4y(5y^2-3)$ v) $5ab(3a+7)$ vi) $pq(1-r)$
   vii) $9m(2m^2-5n^2)$ viii) $17(l^2+5m^2)$ ix) $3x^2(2xy-4y+5x^2)$
   x) $2a^2b(a^3b^2-7b+2a)$

3. i) $a(2b+3)+2b$ (or) $2b(a+1)+3a$ vi) $(a+b)(ax+by+c)$
   ii) $(3x-2)(2y-3)$ vii) $(ax-b)(x^2+1)$
   iii) $(x+y)(3y+2)$ viii) $(x-y)(m-n)$
   iv) $(5b-x^2)(3b-1)$ ix) $(2m^2+3)(m-1)$
   v) $(ax+y)(ax+b)$ x) $(a+11b)(a+1)$

4. i) $(a+7)^2$ ii) $(x-6)^2$ iii) $(2p+5q)(2p-5q)$ iv) $(5x-2y)^2$
   v) $(13m+25n)(13m-25n)$ vi) $\left(x+\frac{1}{3}\right)^2$
   vii) $(11a+7b)^2$ viii) $3x(x+5)(x-5)$ ix) $(6+7x)(6-7x)$
   x) $(1-3x)^2$

5. i) $(x+3)(x+4)$ ii) $(p-2)(p-4)$ iii) $(m-7)(m+3)$
   iv) $(x-9)(x-5)$ v) $(x-18)(x-6)$ vi) $(a+12)(a+1)$
   vii) $(x-2)(x-3)$ viii) $(x-2y)(x-12y)$
   ix) $(m-24)(m+3)$ x) $(x-22)(x-6)$

Exercise 1.5

1. i) $\frac{x^3}{2}$ ii) $-6y$ iii) $\frac{2}{3}a^2b^2c^2$ iv) $7m-6$
   v) $\frac{5}{3}xy$ vi) $9l^2m^3n^5$

2. i) $5y^2-4y+3$ ii) $3x^3-5x^2-7$ iii) $\frac{5}{2}x^2-2x+\frac{3}{2}$
   iv) $x+y-7$ v) $8x^3-4y^2+3xz^3$.

3. i) $(x+5)$ ii) $(a+12)$ iii) $(m-2)$ iv) $(5m-2n)$
   v) $(2a+3b)$ vi) $(a^2+b^2)(a+b)$

Exercise 1.6

1. i) $x = 6$ ii) $y = -7$ iii) $y = 4$ iv) $x = 12$ v) $y = -77$

vi) $x = -6$ vii) $x = 2$ viii) $x = 12$ ix) $x = 6$ x) $m = \frac{6}{7}$

2. i) 18 ii) 29, 30, 31 iii) l = 19 c.m, b = 11 c.m.,

iv) 12, 48 v) 12, 9 vi) 45, 27 vii) 4000 viii) $\frac{3}{5}$

ix) Nandhini's present age is 15 years and Mary's present age is 45 years.

x) Wife's share = ₹ 1,50,000 ; Son's share = ₹ 1,00,000.

Chapter 3

Exercise 3.1

2. i) Quadrant I ii) Quadrant II iii) Quadrant III

iv) Quadrant IV v) on the y-axis vi) on the x-axis

vii) on the x-axis viii) Quadrant III ix) Quadrant I

x) Quadrant II

3.

Point	Quadrants/Axes	Coordinates
A	On the y - axis	(0,4)
B	Quadrant II	(-3,2)
C	On the x-axis	(-5,0)
D	Quadrant III	(-4,-6)
E	On the y-axis	(0.-3)
F	Quadrant IV	(7,-1)
G	On the x-axis	(4,0)
H	Quadrant I	(6,3)
O	The origin	(0,0)

6. i) 40 cm^2 ii) 56 cm^2 iii) 36 cm^2 iv) 49 cm^2

v) 16 cm^2 vi) 12 cm^2 vii) 18 cm^2

7. i) 26 cm ii) 30 cm iii) 24 cm iv) 28 cm

Exercise 3.2

5. 5 hours 6. ₹ 8,000

‘I can, I did’

Student’s Activity Record

Subject:

Sl. No	Date	Lesson No.	Topic of the Lesson	Activities	Remarks