

NORTH-EX PUBLIC SCHOOL (Session 2020-21)

Class - IX

Subject - MATHS

Unit/Chapter -2 POLYNOMIALS

Topic –REMAINDER THEOREM, FACTOR THEOREM

Worksheet No -5

*Note- Before attempting the question and answers you must check the link given below which will help you understand the chapter thoroughly.

<https://youtu.be/rqN55OL-t08>

You can download the assignment or if you do not have the facility to get printout then you can ask your ward to copy the assignment in a simple notebook and must do question and answers in the notebook.

NOTES

Remainder Theorem: Let $p(x)$ be a polynomial of degree greater than or equal to one and let a be any real number. If $p(x)$ is divided by the linear polynomial $x-a$, then remainder is $p(a)$

Proof: Let $p(x)$ be any polynomial. Suppose that $p(x)$ is divided by $(x-a)$, $q(x)$ is quotient and $r(x)$ is remainder. So by Division algorithm,

$$\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder}$$

Now,
$$p(x) = (x-a)q(x) + r(x)$$

Since the degree of $x-a$ is 1, so degree of $r(x)$ will be 0, it means $r(x)$ is constant. So take $r(x) = r$

If $x=a$, then
$$p(a) = (a-a)q(a) + r$$

$$P(a) = r$$

This proves the theorem

Example: Find the remainder when $x^3 - 3x^2 + 2x - 7$ is divided by $x - 2$

Sol: $p(x) = x^3 - 3x^2 + 2x - 7$, $g(x) = x - 2$

Now to find zero of polynomial $g(x)$, take $g(x) = 0$

So
$$x - 2 = 0$$

$$x = 2 \text{ is zero of } g(x)$$

Now,
$$p(2) = 2^3 - 3 \times 2^2 + 2 \times 2 - 7$$

$$P(2) = 8 - 12 + 4 - 7$$

$$P(2) = -7$$

By remainder theorem, -7 is remainder when $p(x)$ is divided by $g(x)$

Factor Theorem:

If $p(x)$ is any polynomial and a is any real number. By remainder theorem $p(a) = r$

If $r=0$, then $g(x)$ is factor of $p(x)$

Remember that (numerically) if any number divided by other number completely and leaves no remainder, then divisor is factor of dividend. Similarly in solving polynomials also we follow same .

Example:

Check whether $x+2$ is a factor of $x^3 + 3x^2 + 5x + 6$

Sol: The zero of $x+2$ is -2 , then by remainder theorem $p(a) = r$

$$\begin{aligned}\text{So, } p(-2) &= (-2)^3 + 3x(-2)^2 + 5x(-2) + 6 \\ &= -8 + 3 \times 4 - 10 + 6 \\ &= -8 + 12 - 10 + 6 \\ &= 0\end{aligned}$$

By factor theorem, here $r=0$

So $x+2$ is factor of $x^3 + 3x^2 + 5x + 6$

Example: Find the value of k if $x-1$ is the factor of $p(x) = x^2 + 5x - 7k$

Sol: By factor theorem, $x-1$ is factor of $p(x)$, so $r=0$, then $p(a)=0$

Now, $x-1=0$

$x=1$ is zero of $x-1$

$$p(1) = 1^2 + 5 \times 1 - 7k$$

$$p(1) = 1 + 5 - 7k$$

$$p(1) = 6 - 7k \quad (\text{here } p(1) = 0 \text{ by factor theorem})$$

$$\text{So, } 6 - 7k = 0$$

$$6 = 7k$$

$$= k$$

$k=$ satisfies the polynomial when divides by $x-1$

1. Find the remainder when $x^3 + 2x^2 - 4x + 5$ divided by $x - 3$.
2. Find the remainder when $x^4 - x^3 + 4x^2 + 2x - 1$ divided by $x + 3$
3. Check whether $7 + 3x$ is a factor of $3x^3 + 7x$.
4. Find the value of k , if $x - 2$ is a factor of $4x^3 + 3x^2 - 4x + k$.
5. Determine which of the following polynomial has $(x + 2)$ a factor.
 - (a) $x^3 - 2x^2 - 3x + 10$
 - (b) $x^2 + 4x - 3$

ANSWERS

1. Remainder = 38
2. Remainder = 137
3. Remainder = , here remainder 0 so $7 + 3x$ is not the factor of $3x^3 + 7x$.
4. $k = -36$ at $p(x) = 0$
5. (a) remainder = 0, so $(x + 2)$ is the factor of $x^3 - 2x^2 - 3x + 10$
 - (b) remainder = -7, so $(x + 2)$ is not the factor of $x^2 + 4x - 3$