

NORTH-EX PUBLIC SCHOOL
(Senior Secondary, Affiliated To CBSE)
School Block, Jain Nagar, Sector-38, Rohini, Delhi – 81
HALF YEARLY EXAMINATION, 2019-20
SUBJECT – MATHEMATICS
CLASS -XII

TIME : 3 hrs

General Instructions:

MM : 80

All questions are compulsory

Questions 1 to 20 carry 1 mark each

Questions 21 to 26 carry 2 marks each

Questions 27 to 32 carry 4 marks each

Questions 33 to 36 carry 6 marks each

1. Find the principal value of $\operatorname{cosec}^{-1}\sqrt{2}$.
2. Evaluate $\tan^{-1}\sqrt{3}$.
3. Differentiate $\cos(x^2+3)$.
4. Find the principal value of $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$.
5. Give an example of equivalence relation. Hence define equivalence relation.
6. Find the inverse of the function $f(x) = \frac{x-2}{x-3}$ where $x \neq 3$.
7. If $A = \begin{pmatrix} 2 & -3 & 0 \\ -1 & 4 & 5 \end{pmatrix}$ find $(3A)^t$.
8. If $\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$ find the integral value of x .
9. Using determinants find the area of a triangle whose vertices are $(3,8)$, $(4,-2)$ and $(5,1)$.
10. Show that the points $(1,0)$, $(6,0)$ and $(0,0)$ are collinear.
11. Is the function $f(x) = \tan x$ continuous at $x = \pi/2$. Show.
12. Differentiate $\cos(x^3)\sin^2(x^5)$.
13. Differentiate $\sec(\tan(\sqrt{x}))$.
14. If $y = \frac{\cos x + \sin x}{\cos x - \sin x}$, prove that $\frac{dy}{dx} = \sec^2\left(\frac{\pi}{4} - x\right)$.
15. Differentiate $\sqrt{3x+2} + \frac{1}{\sqrt{2x+4}}$.
16. Differentiate $\sin^{-1}x + \cos^{-1}x$.
17. What is the slope of the tangent to the curve $y = x^3 - x$ at $x = 2$.
18. What is the slope of the normal to the curve $y = x^3 - 5x^2 - x + 1$ at point $(1, -4)$.
19. Find the antiderivative of $25/(3-5x)^6$.
20. Find $\int \operatorname{cosec} x \, dx$.

21. Given $R = \{(x, y); x, y \in \mathbb{N}, x + y = 10\}$. Check whether it is an equivalence relation or not.

22. Evaluate $\int x^3 / (x^2 + 1)^3 dx$.

23. Evaluate $\int x^2 / \sqrt{1 + x} dx$.

24. Show that the following functions are strictly increasing:

(i) $f(x) = 3x + 17$ (ii) $f(x) = 7x - 3$

25. Show that the greatest integer function is not differentiable at $x = 1$.

26. Show that the relation R on \mathbb{R} defined as $R = \{(a, b): a \leq b\}$ is reflexive and transitive but not symmetric.

27. Let $A = \begin{pmatrix} 2 & -1 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 7 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 2 & 5 \\ 3 & 8 \end{pmatrix}$ then find a matrix such that $CD - AB = O$.

28. Prove that $\cos^{-1}(12/13) + \sin^{-1}(3/5) = \sin^{-1}(56/65)$.

29. Solve for x : $\tan^{-1}4x + \tan^{-1}6x = \pi/4$.

30. Using properties of determinants, prove that
$$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

31. Using differentials find the approximate value of $(3.968)^{3/2}$.

32. Find $\int (\sqrt{\tan x} + \sqrt{\cot x}) dx$

33. Find $\int \frac{1}{\cos^4 x + \sin^4 x} dx$

34. Consider $f: \mathbb{R} - \{-4/3\} \rightarrow \mathbb{R} - \{4/3\}$ given by $f(x) = \frac{4x+3}{3x+4}$. Show that f is bijective. Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$.

35. Show that the surface area of a closed cuboid with square base and given volume is minimum, when it is a cube.

36. Find the equation of the tangent and the normal to the curve $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(\sqrt{2}a, b)$.