

**NORTH-EX PUBLIC SCHOOL
(SENIOR SECONDARY PUBLIC SCHOOL)
JAIN NAGAR DELHI-110081
PRE-BOARD EXAMINATION
MATHEMATICS-XII**

ALL QUESTIONS ARE COMPULSORY

Q. NO. 1 TO 20 CARRIES 1 MARK EACH

Q. NO. 21 TO 26 CARRIES 2 MARKS EACH

Q. NO. 27 TO 32 CARRIES 4 MARKS EACH

Q. NO. 33 TO 36 CARRIES 6 MARKS EACH

1. Let R be the equivalence relation in the set $A=\{0,1,2,3,4,5\}$ given by $R=\{(a,b) : 2 \text{ divides } (a-b)\}$. Write the equivalence class $[0]$.
2. State the reason for the relation R in the set $\{1,2,3\}$ given by $R=\{(1,2), (2,1)\}$ not to be transitive.
3. Write the principal value of $\tan^{-1}[\sin(\frac{-\pi}{2})]$.
4. Evaluate : $\sin[\frac{\pi}{3}-\sin^{-1}(\frac{-1}{2})]$.
5. If $f(x)=x+1$, find $\frac{d}{dx}(f \circ f)(x)$.
6. The amount of pollution content added in air in a city due to x-diesel vehicles is given by $P(x) = 0.005x^2 + 0.02x^2 + 30x$. Find the marginal increase in pollution content when 3 diesel vehicles are added and write which value is indicated in the above question.
7. Find : $\int \frac{\sin^2 x - \cos^2 x}{\sin^2 x \cos^2 x} dx$.
8. Find the order and the degree of the differential equation $x^2 \frac{d^2 y}{dx^2} = \{1 + (\frac{dy}{dx})^2\}^4$.
9. Write the differential equation representing the family of curves $y = mx$, where m is an arbitrary constant.]
10. If $x \in \mathbb{N}$ and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.
11. For a 2×2 matrix $A=[a_{ij}]$, whose elements are given by $a_{ij} = \frac{(i+2j)^2}{4}$, write the value of a_{21} .
12. Find the magnitude of each of the two vectors \vec{a} and \vec{b} , having the same magnitude such that the angle between them is 60° and their scalar product is $\frac{9}{2}$.
13. Find γ , if the vectors $\vec{a} = \vec{i} + 3\vec{j} + \vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} - \vec{k}$ and $\vec{c} = \gamma\vec{j} + 3\vec{k}$ are coplanar.
14. Find the position vectors of a point which divides the join of points with position vectors \vec{a} , $2\vec{b}$ and $2\vec{a} + \vec{b}$ externally in the ratio 2:1.
15. If a line makes an angle of 90° , 135° , 45° with the x, y and z axes respectively, find its direction cosines.
16. Evaluate : $\int_x^3 3^x dx$.
17. Evaluate : $\int_e^{e^2} \frac{dx}{x \log x}$

18. Write the sum of the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + x^4 = 0$$

19. Write the direction cosines of the line joining the points (1,0,0) and (0,0,1).

$$20. \text{ Write the value of } \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} .$$

21. Prove that : $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$, $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

22. Differentiate $\tan^{-1}\left(\frac{1+\cos x}{\sin x}\right)$ with respect to x.

23. Show that $f:N \rightarrow N$, given by

$$F(x) = \begin{cases} x + 1, & \text{if } x \text{ is odd} \\ x - 1, & \text{if } x \text{ is even} \end{cases}$$

Is both one-one and onto.

24. X and Y are two points with position vectors $3\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively . Write the position vector of a point Z which divides the line segment XY in the ratio 2:1 externally.

25. The x-coordinate of a point on the line joining the points P(2,2,1) and Q(5,1,-2) is 4. Find its z-coordinate.

26. The volume of a sphere is increasing the rate of 3cubic centimeter per second. Find the rate of increase of its surface area, when the radius is 2 cm.

27. A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs. 7 profit and that of B at a profit of Rs. 4. Find the production level per day for maximum profit graphically.

28. Suppose a girl throws a die. If she gets 1 or 2, she tosses a coin three times and notes the number of tails. If she gets 3, 4, 5 or 6, she tosses a coin once and notes whether a 'head' or 'tail' is obtained. If she obtained exactly one 'tail', what is the probability that she threw 3, 4, 5 or 6 with the die .

29. Using elementary row transformations, find the inverse of the matrix

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{vmatrix}$$

30. If $x = a(2\theta - \sin 2\theta)$ and $y = a(1 - \cos 2\theta)$, find $\frac{dy}{dx}$ when $\theta = \frac{\pi}{3}$.

31. A wire of length 28 cm is to be cut into two pieces . One of the two pieces is to be made into a square and the other into a circle. What should be the length of the two pieces so that the combined area of these is maximum.

32. Find $\int \frac{3x+5}{x^2+3x-18} dx$.

33. Using integration , find the area of the region in the first quadrant enclosed by the x-axis , the line $y=x$ and the circle $x^2+y^2=32$.

34. Solve the differential equation $x^2 dy + (xy + y^2) dx = 0$ given $y=1$, when $x=1$.

35. Show that the surface area of a closed cuboid with square base and given volume is minimum , when it is a cube.

36. Two cards are drawn simultaneously (without replacement) from a well shuffled pack of 52 cards. Find the probability distribution of the number of aces. Also find the mean of the distribution.