NORTH EX PUBLIC SCHOOL
(SENIOR SECONDARY, AFFILIATED TO CBSE)
SCHOOL BLOCK, JAIN NAGAR, SECTOR-38, ROHINI, DELHI-81
HALF-YEARLY EXAMINATION, 2023-24
SUBJECT- MATHEMATICS(041)
CLASS-XII

Time Allowed: 3 Hours Maximum Marks: 80

General Instructions :

1. This Question paper contains - five sections A, B, C, D and E. Each section is compulsory. However, there are internal choices in some questions.
2. Section A has 18 MCQ's and 02 Assertion-Reason based questions of 1 mark each.
3. Section B has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. Section C has 6 Short Answer (SA)-type questions of 3 marks each.
5. Section D has 4 Long Answer (LA)-type questions of 5 marks each.
6. Section E has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(Multiple Choice Questions ,Each question carries 1 mark)

1. The derivative of x^2 w.r.t. x^3 is?
   (a) (3x)/2 (b) 3x (c) 6x (d) 2/(3x)
2. A be a non- singular square matrix of order 3×3. Then $|adj\ A|$ is equal to?
   (a) $|A|$ (b)$|A|^2$ (c)A^2 (d)A
3. The function defined by f(x) = x – [x] is discontinuous at.
   (a) All rational points (b) all irrational points (c) all integers (d) none
4. The value of 'k' so that the function f(x) is continuous at the indicated point: $f(x) = \begin{cases} kx^2, if\ x \leq 2 \\ 3, if\ x > 2 \end{cases}$ at x=2.
   (a) 3/2 (b) 3/4 (c) 4/3 (d) ¼
5. The inverse of the cosine function is defined in
   (a) [-π , 0] (b) [-π/2 , 0] (c) [0, π/2] (d) [π/2 , π]
6. If the matrix A is both symmetric and skew symmetric, then A will be?
   (a) -1 (b) 0 (c) 1 (d) 2
7. The corner points of the feasible region determined by the following system of linear inequalities : $2x + y \leq 10, x + 3y \leq 15,\ x, y \geq 0$ are (0,0), (5,0). Let Z= px+qy, where p,q>0. Condition on p and q so that the maximum of Z occurs at both (3,4) and (0,5) is?
   (a) p=2q (b) p=q (c) p=3q (d) q=3p
8. If the feasible region for a LPP is .....then the optimal value of the objective function z = ax + by may or may not exist.
   (a) Bounded (b) unbounded (c) none
9. If A,B are symmetric matrices of same order, then AB-BA is a
   (a) symmetric matrix (b) zero matrix(c) identity matrix (d) skew symmetric matrix
10. If A is symmetric matric, then A^3 is a _____ matrix.

(a) skew symmetric (b)symmetric (c)null matrix (d) identity

11. The corner points of the feasible region of an LPP are (0, 2), (3, 0) and (6, 0), (6,8) and (0,5). Let F= 4x+6y be the objective function. The minimum value of F occurs at
    (a) (0,2) $only$ (b) (3, 0) only
    (c) the mid point of the line segment joining the points (0,2) and (3, 0) only
    (d) (0,2) and (3,0) both
12. find the value of k if the area of the triangle is 4 sq. Unitsand vertices are: (k,0),(4,0),(0,2)
    (a) 7 (b) 8 (c) 9 (d) 10
13. If A is a square matrix of order 3 and |A| = 5, then $|adjA|$ =
    (a) 5 (b) 25 (c) 125 (d) 35
14. The function $f(x) = e^{2x}$ is ..... on R
    (a) strictly increasing (b) strictly decreasing (c) increasing (d) decreasing
15. Which of the function is decreasing on (0 , π/2)?
    (a) sin2x (b) tanx (c) cosx (d) cos3x
16. The function f:R→R given by f(x) = 2x is
    (a)injective (b)surjective (c) many –one (d) bijective
17. Let A = { 5, 6 } B = {1,2,3,4,5,6} ,then number of possible functions from A to B are
    (a)36 (b) 30 (c) 4 (d) none
18. If A is a non- singular matrix of order 3 and $A^2 = 2A$ then the value of lAl
    (a) 8 (b) 6 (c) 2 (d) 4

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

19. Assertion(A): $\sin^{-1}(\sin(2\pi/3)) = 2\pi/3$
    Reason(R): $\sin^{-1}(\sin \Theta) = \Theta$, if $q \in [(-\pi)/2, \pi/2]$
20. Assertion(A): The point A(a,b+c),B(b,c+a) and C(c,a+b) are collinear.
    Reason(R): Area of a triangle with three collinear points is zero.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each)

21. Find the value of $\tan^{-1}(\tan 7\pi/6)$.
22. Find the second order derivative of the function $\tan^{-1} x$.
23. Differentiate $\sin^2 x$ w. r. to $e^{\cos x}$.
24. On the set of integers Z defined as f:z → z, check for bijectivity.
    F(x) = x/2 , x is even
    0 , x is odd

25 A stone is dropped into a quite lake and waves move in circles at the rate of 5 cm/sec. At the instant when radius of the circular wave is 8 cm, how fast is the enclosed area increasing?

OR

A balloon which always remains spherical on inflation , is being inflated by pumping in 900 cubic centimeters of gas per second. Find the rate at which the radius of the balloon increases when the radius is 15cm.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

26. Find: $\frac{dy}{dx}$, y =$\tan^{-1}\frac{sinx}{1+cosx}$

27. Find the interval in which the function given by f(x)= $4x^3 - 6x^2 - 72x + 30$ is strictly increasing and strictly decreasing.

28. If $y = (\tan^{-1}x)^2$ show that $(x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$

29.Express $= \begin{matrix} 2 & 5 & 6 \\ 1 & 2 & 3 \\ -1 & 0 & 6 \end{matrix}$ as a sum of symmetric and skew symmetric matrix.

30 Find the value of K so that the function f is continuous at the indicated point $f(x) = \frac{k\cos x}{\pi - 2x}$ if $x \neq \frac{\pi}{2}$

3 if $x = \frac{\pi}{2}$

31.Show the relation R in the set A=$\{x \in Z: 0 \leq x \leq 12\}$, given by R=$\{(a,b): |a-b| \text{ is a multiple of } 4\}$ is an equivalence relation.

OR

Show the relation R in the set A=$\{x \in Z: 0 \leq x \leq 12\}$, given by R=$\{(a,b): a = b$ is an equivalence relation.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

32. Solve the following Linear Programming Problem graphically:
Minimize and Maximize Z = 5x + 10y subject to
$x + 2y \leq 120$, $x+y \geq 60$, $x - 2y \geq 0$, $x, y \geq 0$.

33.Show that the semi vertical angle of the cone of the maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$.

OR

Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

34.If $A = \begin{matrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{matrix}$ find A-1 . Using A-1 solve the system of equation

$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

35.Differentiate the function $(\log x)^x + x^{logx}$ w.r.t x.

SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each.)

36.Case-Study 1:Read the following passage and answer the questions given below.

Saloni has a piece of tin rectangular in shape as shown in the diagram given below. She is going to cut squares from each corners and fold up the sides to form an open box.

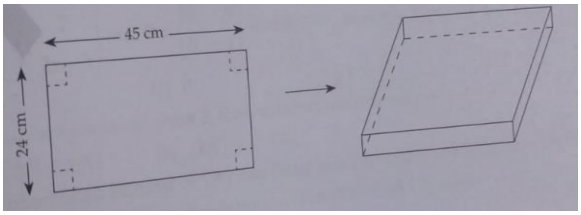
(i) If x is the side of square cut off from each corner of the sheet, then what is the length of open box.

OR

(i) What is the width of open box?

(ii) Find the volume of open box.

(iii) What should be the side of square to be cut off so that the volume of box is maximum?

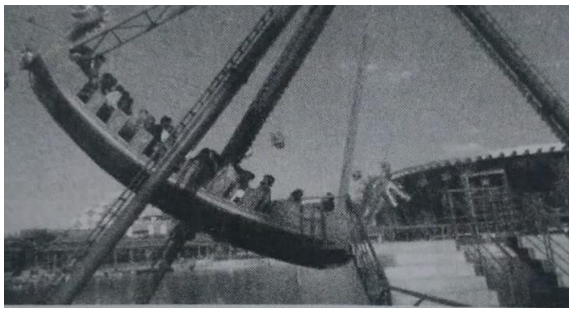


37.Case-Study 2: Read the following passage and answer the questions given below.

Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$.

(i) Let f: R→R be defined by $f(x) = x^2$ is

(a) Neither Surjective nor Injective
(b) Surjective
(c) Injective
(d) Bijective



(ii) Let f : N → N be defined by $f(x) = x^2$ is

(a) Surjective but not Injective
(b) Surjective
(c) Injective
(d) Bijective

(iii) Let f:{ 1, 2, 3 ,..} → { 1, 4, 9 ,...}be defined by $f(x) = x^2$ is

(a) Bijective
(b) Surjective but not Injective
(c) Injective but Surjective
(d) Neither Surjective nor Injective

(iv) The function f : Z → Z defined by $f(x) = x^2$

(a) Neither Injective nor Surjective
(b) Injective
(c) Surjective
(d) Bijective

38.Case-Study 3: Read the following passage and answer the questions given below.

$P(x)= -5x^2+125x+ 37500$ is the total profit function of a company, where x is the production of the company.

i. What will be the production when the profit is maximum?
ii. In which interval the profit is strictly increasing?