WORK BOOK FOR

INTERMEDIATE

FIRST YEAR

MATHEMATICS PAPER -I(B)

[COORDINATE GEOMETRY AND CALCULUS]

BY

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PREFACE

I hear and I forget; I see and I remember;
I do and I understand; I Think and I learn.

The Board of Intermediate Education, Andhra Pradesh, Vijayawada made an attempt to provide work books for the first time to the Intermediate students with relevant and authentic material with an aim to engage them in academic activity and to motivate them for self learning and self assessment. These work books are tailored based on the concepts of *"learning by doing"* and *"activity oriented approach"* to sharpen the students in four core skills of learning – *Understanding, Interpretation, Analysis and Application.*

The endeavor is to provide ample scope to the students to understand the underlying concepts in each topic. The workbooks enable the students to practice more and acquire the skills to apply the learned concept in any related context with critical and creative thinking. The inner motive is that the students should shift from the existing rote learning mechanism to the conceptual learning mechanism of the core concepts.

I am sure that these compendia are perfect tools in the hands of the students to face not only the Intermediate Public Examinations but also the other competitive Examinations.

My due appreciation to all the course writers who put in all their efforts in bringing out these work books in the desired modus.

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MATHEMATICS IB WORKBOOK

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MATHEMATICS IB WORK BOOK

COORDINATE GEOMETRY
CHAPTERS:
1.LOCUS
2. TRANSFORMATION OF AXES

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LOCUS

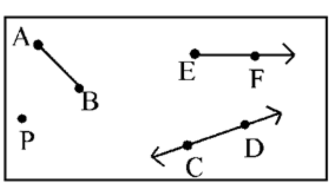
(PRE REQUISITES)

I. State whether the following statements are true or false:

1. A point is dimensionless object i.e. It has no size or shape means neither length nor width or thickness and is shown by dot (.) [ ]
2. The distances from a point to X and Y axes are respectively $|x|, |y|$ [ ]
3. A line contain finite number of points [ ]
4. According to lene Descartes a point in a plane is represented by an ordered pair of real numbers. [ ]
5. The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$AB = \sqrt{(Diff.of\ x-coordinates)^2 + (Diff.of\ y-coordinates)^2}$$ [ ]

II. Fill in the blanks:

6. 

In the adjacent figure P is a _______; $\overline{AB}$ is a _________; $\overline{CD}$ is a _______; $\overline{EF}$ is a _______

7. A line has only _______ and extends _______ in both the directions.
8. The intersection of two lines may be considered as _____________

9. The value of x–coordinate of any point as y–axis is ___________

10. The distance from origin to any point $P(x, y)$ is ___________

III. Math the following:

11. Let $P(x, y)$ be a point on plane. Then

(I) Point on x–axis	a) $(0, 0)$
(II) Point on y–axis	b) $(x, 0)$
(III) Point f intersection of X,Y axes	c) $(0, y)$
	d) (x, y)

12.

List –A	**List – B**
(I) No of points equidistant from two given points	a) 0
(II) No of points equidistant from three collinear points	b) 1
(III) No of points equidistant from three non-collinear points	c) 2
	d) Infinite

13. Let P be a point on $\overline{AB}$ where $A=(x_1, y_1)$, $B=(x_2, y_2)$ and divider in the ratio $m:n$, then

I) P is internal point of $\overline{AB}$	a) $P=\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
II) P is external point of $\overline{AB}$	b) $P=\left(\frac{mx_1+nx_2}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
III) P is midpoint of $\overline{AB}$	c) $P=\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$

14. Let A,B,C be the vertices of a triangle then

I) ΔABC is scalare	a) AB = BC or BC = AC or AC = AB
II) ΔABC is Isoscales	b) AB+BC = AC
III) ΔABC is Equilateral	c) $AB \neq BC \neq CA$
	d) AB = BC = CA

15. Let A,B,C be the vertices of triangle with CA as larger side then

I) ΔABC is acute angled triangle	a) $AB^2+BC^2=CA^2$
II) ΔABC is obtuse angled triangle	b) $AB^2+BC^2>CA^2$
III) ΔABC is right angled triangle	c) $AB^2+BC^2<CA^2$
IV) ΔABC is right angled isosceles triangle	d) $AB=BC$ & $AB^2+BC^2=CA^2$

16. Identify the nature of triangle whose vertices are given

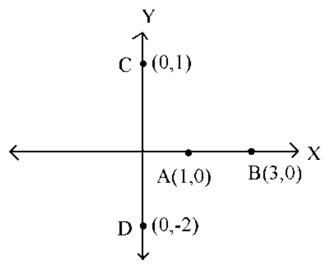
I) $(0, 0)$ $(1, 3)$ $(-1, 3)$	a) Right angled triangle
II) $(3, 4)$ $(3, 5)$ $(6, 5)$	b) Isosceles triangle
III) $(2, -4)$ $(4, -2)$ $(7, 1)$	c) collinear (Triangle cannot be formed)

IV. Answer the following:

17. Define collinear points.

18. In what ratios do the points of trisection divide the line segment.

19. In the adjacent figure find the distance of $\overline{AB}, \overline{CD}$



20. Give the formula for finding area of triangle when its vertices are given

$\left[A=(x_1, y_1), B=(x_2, y_2), C=(x_3, y_3)\right]$

Locus and Equation of Locus

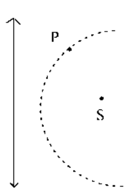
I. Answer the following:

1. Define locus and give an example.

2. What is the locus of point in a plane equidistant from two given parallel lines in the plane?

3. What do you mean by equation of locus?

4. Can you identify the locus of a point 'P' in the adjacent figure and try to name its shape.



5. What is the locus of point equidistant from the two given points A and B?

II. Fill in the blanks:

6. The locus of all points in a plane that are equidistant from given point in the same plane is _____________

7. The equation of locus of point whose distance from x-axis is twice that of distance from y-axis is __________

8. The locus of point which is collinear with the points (3, 4), (–4, 3) is _________

(Hint: find st.line passing through given points)

9. The sum of distances of point ‘P’ from the perpendicular lines in a plane is ‘1’. Then locus of P is _______

10. Locus represented by geometric conditions $x = a + r\cos\theta, y = b + r\sin\theta\left(\theta \in R\right)$

(Hint: Eliminate ‘θ’ from given equations)

III. Choose the correct alternative:

11. The equation of locus of point equidistant from the points A(–2, 3) and B(6, –5) is

12. If A(a, 0), B(–a, 0) then the locus of point such that $PA^2 + PB^2 = 2c^2$

1) $x^2 + y^2 + a^2 - c^2 = 0$ 2) $x^2 + y^2 + a^2 + c^2 = 0$

3) $2x^2 + y^2 + 3a^2 - c^2 = 0$ 4) $x^2 + y^2 + a^2 + 2c^2 = 0$

13. The locus of point such that the sum of its distances from points (0, 2) and (0, –2) is 6 is

1) $9x^2 - 5y^2 = 45$ 2) $5x^2 + 9y^2 = 45$

3) $9x^2 + 5y^2 = 45$ 4) $5x^2 - 9y^2 = 45$

14. The locus of $P\left(x, y\right)$ such that its distance from $A\left(0,0\right)$ is less than 5 units is

1) $x^2 + y^2 < 5$ 2) $x^2 + y^2 < 10$ 3) $x^2 + y^2 < 25$ 4) $x^2 + y^2 < 20$

15. $A\left(-9,0\right), B\left(-1,0\right)$ are two points if P is a point such that $PA : PB = 3:1$ then the locus of ‘P’ is

1) $x^2 + y^2 = 9$ 2) $x^2 + y^2 + 9 = 0$ 3) $x^2 + y^2 = 9$ 4) $x^2 + y^2 - 9 = 0$

16. $A\left(2,3\right), B\left(-2,3\right)$ are two points. The locus of ‘P’ which moves such that $A\left(2,3\right), B\left(-2,3\right)$ is

1) $y + 3 = 0$ 2) $y - 3 = 0$ 3) $y^2 + 3 = 0$ 4) $y^2 - 3 = 0$

17. If $x = \tan\theta + \sin\theta,\ y = \tan\theta - \sin\theta$ then the locus of $\left(x,\ y\right)$ is

1) $\left(x^2 y\right)^{2/3} + \left(xy\right)^{2/3} = 1$ 2) $x^2 - y^2 = 4xy$

3) $x^2 - y^2 = 12xy$ 4) $\left(x^2 - y^2\right)^2 = 16xy$

18. If a point ‘P’ moves such that its distance from the point A(1, 1) and the line $x + y + 2 = 0$ are equal then the locus of ‘P’ is

1) straight line 2) pair of straight lines 3) parabola 4) Ellipse

19. A straight rod of length 9units slides with its ends A,B always on the x and y axes respectively. Then the locus of centroid of ΔOAB is

1) $x^2+y^2=3$ 2) $x^2+y^2=9$ 3) $x^2+y^2=81$

20. Locus of centroid of triangle whose vertices are $(a\cos t, b\sin t)$, $(b\sin t, -b\cos t)$ and (1, 0), where t is parameter is

1) $(3x-1)^2+(3y)^2=a^2-b^2$ 2) $(3x-1)^2+(3y)^2=a^2+b^2$

3) $(3x+1)^2+(3y)^2=a^2+b^2$ 4) $(3x+1)^2+(3y)^2=a^2-b^2$

IV. Remember

(1) Equation of circle is standard from : $x^2+y^2=r^2$ (r=radius)

(2) Equation of circle with centre (h, k) and radius 'r' is $(x-h)^2+(y-k)^2=r^2$

(3) Equation of parabola is standard form is $y^2=4ax$ (vertex = (0, 0)

(4) Equation of ellipse is standard form is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(5) Equation of Hyperbola is standard form is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

(6) Equation of rectangular hyperbola is $xy=c^2$

V. Match the following:

21.

Condition	Locus
I) The sum of the squares of distances from 'p' to the coordinate axes is 25	a) $x^2+y^2=25$
II) The distances to the coordinate axes from 'p' are in the ratio 2 : 3	b) $4x^2-9y^2=0$
III) The square of whose distance from 'p' to the origin is 4-times of its y-coordinate	c) $x^2+y^2=4y$

22. Let A, B be two fixed points. If $PA+PB=k$ then

I) $K=AB$ locus of p is	a) Hyperbola
II) $K>AB$ locus of p is	b) Line segment
III) $K<AB$ locus of p is	c) ellipse
	d) empty set

23. Let A, B be two fixed points. If $|PA-PB|=k$ then

I) $K=AB$ locus of p is	a) hyperbola
II) $K>AB$ locus of p is	b) line through A and B except $\overline{AB}$

III) $K < AB$ locus of p is c) ellipse

d) empty set

24. **List – I** **List – II**

List – I	List – II
I) Locus of point $\left(at^2, 2at\right)$	a) $xy = c^2$
II) Locus of point $\left(ct, c/t\right)$	b) $y^2 + 4x = 4$
III) Locus of point $\left(\cos^2 t,\ 2\sin t\right)$	c) $y^2 + y^2 = 2$
	d) $y^2 = 4ax$

25. **List – I** **List – II**

List – I	List – II
I) Locus of point $\left(a\sec\theta,\ b\tan\theta\right)$	a) $x^2 - y^2 = a^2$
II) Locus of point $\left(2t, \dfrac{2}{t}\right)$	b) $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
III) Locus of point $\left(a\sec\theta,\ a\tan\theta\right)$	c) xy = 4
	d) $x^2 + y^2 - ax + by = 0$

Assertion and Reason:

26. A : The locus of point which is equidistant to the coordinate axes is pair of straight lines.

R : The distance from $P\left(x_1, y_1\right)$ to x–axis is $\left|y_1\right|$ and y-axis is $\left|x_1\right|$

1) A, R are true and R is correct explanation of A
2) A,R are true and R is not correct explanation of A
3) A is true but R is false
4) A is false but R is true

27. A : If $A(4,0), B(-4,0)$ are two points and $PA - PB = 4$ then locus of 'P' is $3x^2 - y^2 = 12$

R : A, B be two points, $PA - PB = K\left(\text{constant}\right) < AB$ the locus of 'P' is hyperbola

1) A, R are true and R is correct explanation of A
2) A,R are true and R is not correct explanation of A
3) A is true but R is false
4) A is false but R is true

28. A : $A(0,2), B(0,-2)$ and $PA + PB = 3$, the locus of P is ellipse

R : The locus of pair sum of whose distances from two fixed pairs is always constant is an ellipse.

1) A, R are true and R is correct explanation of A

2) A,R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

29. A : $A(1,2)$, $B(-1,2)$ then locus of P such that $PA = 3PB$ is x = y

R : A, B are two fixed points. The locus of 'P' such that PA = KPB

$(k \neq 1, a \text{ constatnt})$ is circle.

1) A, R are true and R is correct explanation of A

2) A,R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

30. A : $A(1, 1), B(-2, 3)$ are two points. If a point form a triangle of area 2 sq.units with

A, B then locus of P is $4x^2 + 12xy + 9y^2 - 20x - 36y + 9 = 0$

R : Area of triangle formed by $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ is $\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$

1) A, R are true and R is correct explanation of A

2) A,R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true

KEY

Pre requisites:

I) 1) T 2) F 3) F 4) T 5) T

II) 6) Point, line segment, line, ray 7) length, infinitely 8) point

9) 0 (zero) 10) $\sqrt{x^2 + y^2}$

III) 11) I – b, II-c, III-a 12) I-d, II – a, III- b 13) I –b, II-c, III-a

14) I-c, II-a, III-d 15) I-b, II-c, III-a 16) I-b, II-a, III-c

IV) 17) Three or more points are said to be collinear if they lie on same straight line

18) 1 : 2 or 2 : 1 19) $\overline{AB} = 2$ units, $\overline{CD} = 3$ units

20) $\frac{1}{2}\left|\sum x_1(y_2 - y_3)\right|$ or $\frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix}$ or $\frac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

Locus:

I) 1) The set of all points satisfying a given conditions or property is called locus

2) a line parallel to given lines midway between them

3) The algebraic relation between x and y obtained by applying geometrical condition is called equation of locus

4) path traced by dolted curve, parabola

5) The perpendicular bisector of line segment $\overline{AB}$

II) 6) circle 7) $y^2 = 4x^2$ 8) $x - 7y + 25 = 0$ 9) square

10) $(x-a)^2 + (y-b)^2 = r^2$

III) 11) 2 12) 1 13) 3 14) 3 15) 1 16) 2 17) 4 18) 3

19) 2 20) 2

IV) (21) I-a, II-b, III-c (22) I-b, II-c, III-d (23) I-b, II-d, III-a

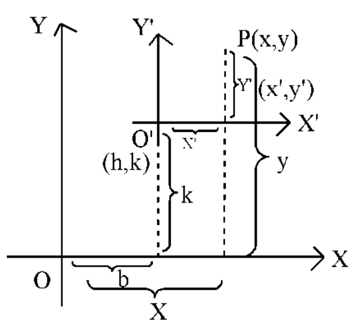
(24) I-d, II-a, III-b (25) I-b, II-c, III-a

V) (26) 1 (27) 1 (28) 4 (29) 4 (30)1

TRANSFORMATION OF AXES

Remember: **Type1: Translation of Axes:**

In this type we shift the origin to some other point say (h, k) without changing the direction of axes. Here we observe the following changes.



Change is coordinates

Original system ⇔ New system

$P = (x, y)$ $P = (x', y')$

$x = x' + h$ $x' = x - h$

$y = y' + k$ $y' = y - k$

Change is equation

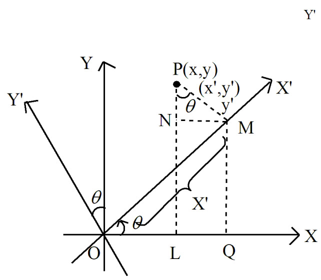
Original equation | Transformed eqn

$f(x, y) = 0$ | $f(x'+h, y'+k) = 0$

$f(x\cos\theta + y\sin gq, -x\sin\theta + y\cos\theta) = 0$ | $f(x', y') = 0$

Remember: **Type2: Rotation of Axes:**

In this type we rotate the coordinate axes through some angle 'θ' without changing the position of origin. Here we observe the following changes.



Change is coordinates

Original system ⇔ New system

$P = (x, y)$ $P = (x', y')$

$x = x'\cos\theta - y'\sin\theta$ $x' = x\cos\theta + y\sin\theta$

$y = x'\sin\theta + y'\cos\theta$ $y' = -x\sin\theta + y\cos\theta$

Change is equation

Original equation ⇔ Transformed eqn

$f(x, y) = 0$ | $f(x'\cos\theta - y\sin\theta, -x'\sin\theta + y'\cos\theta) = 0$

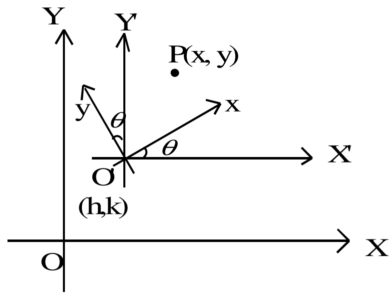
$f(x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta) = 0$ | $f(x', y') = 0$

	x'	y'
x	$\cos\theta$	$-\sin\theta$
y	$\sin\theta$	$\cos\theta$

Easily remembered way

Remember: Type3: General Transformation:

In this type we apply both translation and rotation i.e. say origin is shifted to(h, k) and the axes are rotated about new origin by an angle 'θ' is anticlockwise sense. Here we observe the following changes.



Change is coordinates

Original system ⇔ New system

$P=(x, y)$ $P=(x', y')$

$x=h+x'\cos\theta-y'\sin\theta$ $x'=x\cos\theta+y\sin\theta-h$

$y=k+x'\sin\theta+y'\cos\theta$ $y'=-x\sin\theta+y\sin\theta-k$

$f(x'\cos\theta-y'\sin\theta,\ x'\sin\theta+y'\cos\theta)=0$

$x=x'\cos\theta-y'\sin\theta$ $x'=x\cos\theta+y\sin\theta$

Change is equation

Original equation ⇔ Transformed eqn

$f(x, y)=0$ $f(x'\cos\theta-y'\sin\theta+h,\ x'\sin\theta+y'\cos\theta+k)=0$

$f(x\cos\theta+y\sin\theta-h,\ x\sin\theta+y\cos\theta-k)=0$ $f(x', y')=0$

	x'	y'
x–h	$\cos\theta$	$-\sin\theta$
y–k	$\sin\theta$	$\cos\theta$

Easily remembered way

Note: If the rotation is in clockwise direction then replace 'θ' by ($-\theta$)

LEVEL – I

I. Answer the following:

1. What is the use of transformation?
2. To eliminate 'xy' term from given equation, what type of transformation we apply?
3. What do you mean by rotation of axes?

4. What is the angle of rotation of axes to eliminate 'xy' term from the equation

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

5. Define reflection of a point about line.

II. Fill in the blanks:

6. The point to which the origin has to be shifted to eliminate x, y term in

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ___________

7. If the distance between two given points is 2units and the points are transformed by shifting the origin to (2, 2) then the distance between points in their new position is _________

8. The point to which the origin should be shifted in order to eliminate x and y term in the equation $x^2 + y^2 - 2x + 12y + 1 = 0$ is ____________

9. When the axes are rotated by an angle of 135^0 initial coordinates of (4, –3) are _____

10. The transformed eqn of $x\cos\alpha + y\sin\alpha = p$ when the axes are rotated through an angle 'α' is __________

LEVEL – II

III. Choose the correct alternative:

11. The angle of rotation of axes in order to eliminate 'xy' term in the equation

$x^2 + 2\sqrt{3}xy - y^2 = 2a^2$ is

1) $\pi/6$ 2) $\pi/4$ 3) $\pi/3$ 4) $\pi/2$

12. If the point (5, 7) is transformed to (–1, 2) when the origin is shifted to A, then A =

1) (4, 9) 2) (6, 5) 3) (–6, –5) 4) (2, 4)

13. If the area of triangle is 5sq.units then the area of triangle when the origin is shifted to (1, 2) is

1) 2 sq.unit 2) 3 sq.units 3) 4 sq.units 4) 5 sq.units

14. If (3, –4) is the point to which the origin is shifted and the transformed eqn. Is $X^2 + Y^2 = 4$ then the original equation is

1) $x^2 + y^2 + 6x + 8y + 21 = 0$ 2) $x^2 + y^2 + 6x + 8y - 21 = 0$

3) $x^2 + y^2 - 6x + 8y + 21 = 0$ 4) $x^2 + y^2 - 6x - 8y + 21 = 0$

15. When (0, 0) shifted to (2, –2) the transformed equation of $(x-2)^2 + (y+2)^2 = 9$ is

1) $x^2 + y^2 = 9$ 2) $x^2 + 3y^2 = 9$ 3) $x^2 + y^2 - 2x + 6y = 0$ 4) $4x^2 + 9y^2 = 36$

16. If the axes are rotated through an angle 45^0 in the positive direction then the coordinates of point $\left(\sqrt{2}, 4\right)$ is old system are

1) $\left(1-2\sqrt{2}, 1+2\sqrt{2}\right)$ 2) $\left(1+2\sqrt{2}, 1-2\sqrt{2}\right)$

3) $\left(2\sqrt{2}, \sqrt{2}\right)$ 4) $\left(\sqrt{2}, 2\right)$

17. The transformed equation of $x^2+6xy+8y^2=10$ when the axes are rotated through an angle $\pi/4$ is

1) $15x^2-14xy+3y^2=20$ 2) $15x^2+14xy-3y^2=20$

3) $15x^2+14xy+3y^2=20$ 4) $15x^2-14xy-3y^2=20$

18. If the axes are rotated through an angle 30^0 about the origin then the transformed equation of $x^2+2\sqrt{3}xy-y^2=2a^2$ is

1) $x^2+y^2=a^2$ 2) $x^2-y^2=a^2$

3) $x^2+y=3a^2$ 4) $y^2-x^2=a^2$

19. The line joining the points A(2, 0) and B(3, 1) is rotated through an angle of 45^0, about A is anticlockwise direction. The coordinates of B is the new position

1) $\left(2, \sqrt{2}\right)$ 2) $\left(\sqrt{2}, 2\right)$ 3) $(2, 2)$ 4) $\left(\sqrt{2}, \sqrt{2}\right)$

20. The point (4, 1) undergoes the following transformation successively

i) reflection about the line y = x

ii) transformation through a distance 2unit along +ve directions of x-axis

The final positions of point is

1) (4, 3) 2) (3, 4) 3) (–1, 4) 4) (1, 4)

LEVEL – III

IV. Assertion and Reason type Questions:

21. Assertion (A): If the area of triangle formed by (0, 0), (2, 0), (0, 2) is 2 sq.units. Then the area of triangle an shifting the origin to a point (2, 3) is sq.unit

Reason (R): By the change of axes area does not change.

1) Both A and R are true and R is correct explanation of A

2) Both A and R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true.

22. Assertion (A): By translating the axes the equation $xy - x + 2y = b$ has changed to $xy = c$ and c = 4

Reason (R): If the axes and translated to the point (h, k) then the equation f(x, y) =0 of a curve is transformed to $f(x-h,\ y-k) = 0$

1) Both A and R are true and R is correct explanation of A

2) Both A and R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true.

23. Assertion (A): The angle of rotation to remove xy–term in the equation $2x^2 + \sqrt{3}xy + 3y^2 = 9$ is $\pi/6$

Reason (R): The angle of rotation of axes to eliminate 'xy' term in the equation $ax^2 + 2hxy + by^2 + 2gx + c = 0$ is $\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)$

1) Both A and R are true and R is correct explanation of A

2) Both A and R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true.

24. Assertion (A): The equation of circle is $x^2 + y^2 = 9$. If the axes are rotated through an angle $\tan^{-1}2$ then the transformed equation is $x^2 + y^2 = 9$

Reason (R): In rotation of axes area of circle does not change.

1) Both A and R are true and R is correct explanation of A

2) Both A and R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true.

25. Assertion (A): The angle of rotation of axes so that the equation $\sqrt{3}x - y + 5 = 0$ may be reduced to the form y = constant is $\pi/3$

Reason (R): The angle of rotation of the axes so that the equation $ax + by + c = 0$ may be reduced to the form y = constant is $\tan^{-1}(-a/b)$

1) Both A and R are true and R is correct explanation of A

2) Both A and R are true and R is not correct explanation of A

3) A is true but R is false

4) A is false but R is true.

V. Miscellaneous type:

26. The point (7, 5) undergoes the following transformations successively

i) the origin is translated to (1, 2)

ii) translated through 2 units along the negative direction of new x-axis

iii) rotated through an angle $\pi/4$ about the origin is clockwise direction

The final position of the point (7, 5) is

1) $\left(\frac{9}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ 2) $\left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 3) $\left(\frac{7}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ 4) $\left(\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

27. If the axes are shifted to (–2, –3) and rotated through $\pi/4$ then transformed equation of $2x^2+4xy-5y^2+20x-22y-14=0$ is

1) $x^2-14xy-7y^2=2$ 2) $x^2-14xy-7y^2=4$

3) $x^2-14xy+7y^2=2$ 4) none

28. Statement I: The point to which the origin has to be shifted to eliminate x, y terms is $a(x+\alpha)^2+b(y+\beta)^2=c$ is $(-\alpha, -\beta)$

Statement II: The point to which the origin has to be shifted to eliminate x,y terms in $ax^2+by^2+2gx+2fy+c=0$ is $(-8/a, -8/b)$

The correct statement is

1) only I 2) only II 3) Both I and II 4) Neither I or II

29. To remove the first degree terms in the following equations origin should be shifted to another point then calculate new origins from list – II

List – I	**List – II**
A) $x^2-y^2+2x+4y=0$	1) (5, –1)
B) $4x^2+9y^2-8x+36y+4=0$	2) (1, –2)
C) $x^2+3y^2-2x+12y+1=0$	3) (–1, 2)
D) $2(x-3)^2+3(y+7)^2=10$	4) (–1, –2)
	5) (–5, 7)

The correct rotating is

	A	B	C	D
1)	4	2	2	5
2)	5	3	3	5
3)	3	2	2	1
4)	4	3	3	1

30. Match the following. The angle of rotation of axes to remove 'xy' term.

I) $9x^2+2\sqrt{3}xy+7y^2=0$ a) $\pi/2$

II) $7x^2+2\sqrt{3}xy+9y^2=0$ b) $\pi/4$

III) $3x^2+2xy+3y^2=2$ c) $\pi/3$

IV) $3x^2-2\sqrt{3}xy+9y^2=10$ d) $\pi/6$

1) c,d,a,b 2) d,c,b,a 3) c,a,b,d 4) d,a,b,c

KEY

I. 1. Transformation is used in reducing the general equation of any curve to the desired form

2. Rotation of axes

3. Rotating the system of coordinate axes through an angle 'θ' without changing the position of origin.

4. Angle of rotation, $\theta=\frac{1}{2}\tan^{-1}\left(\frac{2h}{a-b}\right)(a\neq b)$

5. The reflection of a point 'p' in the line AB is the point "p'" such that

(i) $pp'\perp AB$ (ii) AB bisects pp'

II. 6. $\left(\frac{hf-bg}{ab-h^2},\frac{8h-af}{ab-h^2}\right)$ 7. 2 8. (1, –2) 9. $\left(\frac{-1}{2},\frac{7}{\sqrt{2}}\right)$ 10. X = p

III. 11. 1 12. 2 13. 4 14. 3 15. 1 16. 1 17. 3 18. 2 19. 1 20. 2

IV. 21. 1 22. 3 23. 4 25. 1

V. 26. 3 27. 1 28. 3 29. 3 30. 2

STRAIGHT LINES

By R.BHASKER, JL APSWRJC, KONDEPI, PRAKASAM DT

KEY CONCEPTS QUESTIONS

Whether the following statements are true or false.

1. The equation of x-axis is y = 0 (True)
2. If a straight line makes an angle θ with x-axis in anti clockwise direction then its slope is $-\tan\theta$ (false)
3. The slope of a vertical line is not defined (True)
4. If m_1, m_2 are the slopes of two parallel lines then $m_1=m_2$ (True)
5. If m_1, m_2 are the slopes of two perpendicular lines then $m_1m_2=1$ (false)
6. The equation of the straight line with slope m and making an intercept c on y-axis is y=mx+c (True)

7. The equation of the straight line, which makes an intercepts

8. The equation of a straight line, which makes an intercepts on the coordinate axes respectively is $\frac{x}{a}+\frac{y}{b}=1$ (False)

9. The equation of the line passing through $(x_1, y_1) \& (x_2, y_2)$ is $y-y_1=\left(\frac{y_2-y_1}{x_2-x_1}\right)(x-x_1)(x_1 \neq x_2)$ (True)

10. If θ be the acute angle lines having the slopes $m_1 \& m_2$ then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$ (True)

11. The symmetric form of the line passing through (x_1, y_1) point and making an angle θ with x-axis in anti clock wise direction is $\frac{y-y_1}{\sin\theta}=\frac{x-x_1}{\cos\theta}$ (True)

12. The parametric form of the line equations passing through $A(x_1, y_1)$ and making an angle θ with x-axis and $p(x, y)$ be any point on the line then $x = x_1 + r\sin\theta,\ y = y_1 + r\cos\theta$ where r is the distance of AP. (False)

13. The area of the triangle formed by the line $ax+by+c=0$ with the coordinate axes is $\frac{1}{2}\frac{c^2}{|ab|}$ (False)

14. The image of y = k w.r.t x-axis is y= –k and x = k wr.t to y-axis is x = –k (True)

15. The equation of the line, which is at a distance of p units from the origin and $\alpha \leq \alpha \leq 360$ is the angle made by the normal with +ve x–axis is $x\cos\alpha + y\sin\alpha = p$ (True)

Fill up the blanks in the following:

16. The slope of the line represented by $ax+by+c=0$ is _________

17. If the straight lines $a_1x+b_1y+c_1=0 \& a_2x+b_2y+c_2=0$ is __________

18. If the straight lines $a_1x+b_1y+c_1=0 \& a_2x+b_2y+c_2=0$ represents the same line then $a_1:b_1:c_1=$ _________

19. If the straight lines $a_1x+b_1y+c_1=0 \& a_2x+b_2y+c_2=0$ represents two intersecting lines then their point of intersection is ____________

20. If the line $L=ax+by+c=0$ devides the line segment joining the points $(x_1, y_1) \& (x_2, y_2)$ internally in the ratio m : n then $\frac{m}{n}=$ ______

21. If x–axis devides the line segment joining the points $(x_1, y_1) \& (x_2, y_2)$ internally in the ratio is _________

22. If $a_1x+b_1y+c_1=0, a_2x+b_2y+c_2=0$ & $a_3x+b_3y+c_3=0$ represents the concurrent lines then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} =$ __________

23. If θ be the acute angle between the straight lines $a_1x+b_1y+c_1=0 \& a_2x+b_2y+c_2=0$ then $\cos\theta =$ ___________

24. If θ be the acute angle between the lines $y=m_1x+c_1 \& y=m_2c+c_2$ then $\tan\theta =$ _____

25. The equation of the line parallel to ax+by+c=0 and passing through (x_1, y_1) is ____

26. The equation of the line passing (x_1, y_1) and perpendicular to ax+by+c=0 is ________

27. The perpendicular distance from the point $P(x_1, y_1)$ to the straight line ax+by+c=0 is _______

28. The $\perp er$ distance between two parallel lines $ax+by+c_1=0 \& ax+by+c_2=0$ is ____

29. If $Q(h, k)$ be the foot of $P(x_1, y_1)$ w.r.t the line ax+by+c=0 then $\frac{h-x_1}{a} = \frac{k-y_1}{b} =$ __________

30. If $Q(h,k)$ bet he image of the point $p(x_1, y_1)$ w.r.t the line ax + by + c = 0 then $\frac{h-x_1}{a} = \frac{k-y_1}{b} =$ ________

31. The point of intersection of altitudes in a triangle is called _________

32. The point of intersection of perpendicular bisector in a triangle is called ______

33. The point of intersection of internal angular bisector in a triangle is called ____

34. The point of inter sector of the medium in a triangle is called ______

Answers:

1. True	16. $\frac{-a}{b}$	31. Ortho centre
2. false	17. 0	32. Circum centre
3. True	18. $a_2 : b_2 : c_2$	33. Incentre
4. True	19. $\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1}, \frac{c_1a_2-c_2a_1}{a_1b_2-a_2b_1}$	34. Centroid

5. false 20. $\frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}$

6. True 21. $-y_1 : y_2$

7. True 22. 0

8. false 23. $\frac{|a_1a_2+b_1b_2|}{\sqrt{a_1^2+b_1^2}\sqrt{a_2^2+b_2^2}}$

9. True 24. $\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

10. True 25. $a(x-x_1)+b(y-y_1)=0$

11. True 26. $b(x-x_1)-a(y-y_1)=0$

12. false 27. $\left|\frac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right|$

13. True 28. $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

14. True 29. $\frac{-(ax_1+by_1+c)}{a^2+b^2}$

15. True 30. $\frac{-2(ax_1+by_1+c)}{a^2+b^2}$

LEVE – I (Short Answer Questions)

1. Find the equation of line passing through the points (1, 2) & (1, –2)

SOL: Required equation

$$(y-y_1)(x_2-x_1)=(y_2-y_1)(x-x_1)$$

$$(y-2)(1-1)=(-2-2)(x-1)$$

$$0=-4(x-1)\Rightarrow x=1$$

2. Find the value of x, if the slope of line passing through (2, 5) & (x, 3) is 2.

SOL: $\frac{(5-3)}{2-x}=2\Rightarrow 2-x=1\Rightarrow x=2-1=1$

3. Find the value of y, If the line joining the points (3, y) & (2, 7) is parallel to the line joining the points (–1, 4) & (0, 6)

SOL: Slopes of parallel lines are equal

i.e. $\frac{y-7}{3-2}=\frac{6-4}{0+1}\Rightarrow\frac{y-7}{1}=\frac{2}{1}\Rightarrow y=2+7=9$

4. Find the equation of the line passing through (3, –2) and making an angle 135^0 with +ve x-axis is anticlockwise direction.

SOL: Slope of the line = $\tan 135=\tan(180-45)=\tan 45=-1$

$\therefore$ equation is $y+2=-(x-3)\Rightarrow x+y-1=0$

5. Find the equation of straight line passing through (–4, 5) and cutting of equal non zero intercepts on the coordinate axes.

SOL: Required equations $\frac{x}{a}+\frac{y}{a}=1\Rightarrow x+y=a$. It passes through (–4, 5)

$\therefore -4+5=a=1 \quad i.e. x+y=1$

6. Transform the equation $4x-3y+12=0$ into intercept form & normal form.

SOL: $-4x+3y=12\Rightarrow\frac{x}{3}+\frac{y}{4}=1$

x, y intercepts are –3 & 4

Normal form $\frac{-4x}{5}+\frac{3}{5}y=\frac{12}{5}$

$\cos\alpha=-4/5, \sin\alpha=3/5 \qquad p\frac{12}{5}$

7. Find the sum of the squares of the intercepts of the line $4x-3y=12$ on the coordinate axes.

SOL: Intercept form, $\frac{4x}{312}-\frac{3y}{124}=1$

x intercept=3 & y intercept = –4

$\therefore$ sum of the squares = $3^2+(-4)^2=25$

8. If the area of the triangle formed by the straight line $4x-3y=a$ with coordinate axes is 6. Find the value of a.

SOL: Area of the triangle formed by the line $ax+by+c=0$ is $\frac{1}{2}\left|\frac{c^2}{ab}\right|$

$\therefore\frac{1}{2}\left|\frac{a^2}{3\times4}\right|=6\Rightarrow a^2=6\times6\times4=6^2\times2^2\Rightarrow a=6\times2=12$

9. Find the value of p, if the straight lines x+p=0, y+2=0 & 3x+2y+5=0 are concurrent.

SOL: $x=-p\ \&\ y=-2$

$\therefore 3(-p)+2(-2)+5=0 \quad 3p=1 \Rightarrow p=1/3$

10. Find the value of k, if the lines $2x-3y+k=0,\ 3x-4y-13=0\ \&\ 8x-11y-33=0$ are concurrent

SOL: $\begin{vmatrix} 2 & -3 & k \\ 3 & -4 & -13 \\ 8 & -11 & -33 \end{vmatrix}=0 \Rightarrow 2(132-143)+3(-99+104)+k(-33+32)=0$

$-22+15-k=0 \Rightarrow k=-7$

11. Find the distance between the parallel lines $5x-3y-4=0, 10x-6y-9=0$

SOL: $10x-6y-8=0, 10x-6y-9=0$

Distance = $\left|\dfrac{c_1-c_2}{\sqrt{a^2+b^2}}\right| = \left|\dfrac{-8+9}{\sqrt{10^2+6^2}}\right| = \dfrac{1}{\sqrt{136}} = \dfrac{1}{2\sqrt{34}}$

12. Find the value of p, if the straight lines $3x+7y-1=0\ \&\ 7x-py+3=0$ are mutually perpendicular

SOL: $m_1m_2=-1$

$\Rightarrow \dfrac{+3}{7}\times\dfrac{7}{p}=+1 \Rightarrow p=3$

13. Find the value of k, if the angle between the straight lines $kx+y+g=0$ and $3x-y+4=0$ is $\dfrac{\pi}{4}$

SOL: $\tan 45 = \left|\dfrac{-k-3}{1+(-k)3}\right| = 1$

$\Rightarrow \dfrac{k+3}{3k-1} = \pm 1 \Rightarrow k=2\ \&\ k=\dfrac{-1}{2}$

14. Find the perpendicular distance from (3, 4) to the straight line $3x-4y+10=0$

SOL: $\left|\dfrac{ax_1+by_1+c}{\sqrt{a^2+b^2}}\right| = \left|\dfrac{9-16+10}{\sqrt{9+16}}\right| = \dfrac{3}{5}$

15. Find the equation of the straight line passing through the point (–2, 4) and making intercepts whose sum is zero.

SOL: $a+b=0 \Rightarrow b=-a$

$\therefore$ Required equation is $\frac{x}{a}+\frac{y}{b}=1 \Rightarrow x-y=a$ It passes through (–2, 4)

$\therefore x-y+6=0$

16. State whether the points A(2, –1) & B(1, 1) lie on the same or either side of the line $3x+4y=6$

SOL: $L_{11}=3(2)+4(-1)-6=-4<0$

$L_{22}=3(1)+4(1)-6=1>0$

The given points are on opposite side of the line

17. Find the ratio is which the straight line $3x+3y-20=0$ devides the line segment joining the points (2, 3) & (2, 10)

SOL: $\frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}=\frac{-(4+9-20)}{(4+30-20)}=\frac{7}{14}=\frac{1}{2}$

ratio$\Rightarrow 1:2$

18. Find the value of k if the angle between the straight lines $4x-y+7=0$, $kx-5y-9=0$ us 45

SOL: $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\Rightarrow \left|\frac{4-\frac{k}{5}}{1+\frac{4k}{5}}\right|=1 \Rightarrow 20-k=\pm(5+4k)$

$\therefore$ k = 3or $k=-25/3$

19. Find the equation of the straight line $\perp er$ to the line $5x-3y+1=0$ and passing through the point (4, –3)

SOL: $\perp er$ line equations is the form $3x+5y=k$. It passes through (4, –3)

$\therefore$ required equations is $3x+5y+3=0$

20. Find the equations of vertical line passing through the point of intersection of lines $x-3y+1=0$ & $2x+5y-9=0$ and at a distance of 2 units from the origin.

SOL: Required equation is $(x-3y+1)+\lambda(2x+5y-9)=0$ ———(1)

$(1+2\lambda)x+(5\lambda-3)y+(1-9\lambda)=0$ if it is vertical line then $5\lambda-3=0$

$\Rightarrow \lambda=3/5$ substituting in (1) we get x = 2

21. Find the points on the line $3x-4y-1=0$ which are at a distance of 5 units from the point (3, 2)

SOL: Slope = $\tan\theta = \frac{3}{4}$

$\therefore \sin\theta = \frac{3}{5},\ \cos\theta = \frac{4}{5}$

By parameter form of the line required points $\left(x_1 \pm r\cos\theta,\ y_1 \pm r\sin\theta\right)$

i.e. $\left(3 \pm 5\times\frac{4}{3},\ 2\pm\frac{3}{5}\right) = (7,\ 5)(-1,\ -1)$

22. Find the value of p, if the lines $3x+4y=5, 2x+3y=4, px+4y=6$ Are concurrent.

SOL: point of intersection of the line is (–1, 2) $\therefore\ p(-1)+4(2)=6$

$\Rightarrow p = 8-6 = 2$

23. Find the foot of the perpendicular from (3, 4) to the line $3x-4y=18$

SOL: $\frac{h-3}{3} = \frac{k-4}{-4} = \frac{-(9-16-18)}{9+16} = 1$

$\Rightarrow h = 3+3 = 6,\ k = -4+4 = 0$

$\therefore$ foot (6, 0)

24. Find the image of the point (1, 2) is the straight line $3x-4y-1=0$

SOL: $\frac{h-1}{3} = \frac{k-2}{4} = \frac{-2(3+8-1)}{9+16} = \frac{-20}{25} = \frac{-4}{5}$

$h = \frac{-12}{5}+1 = \frac{-7}{5},\ k = \frac{-16}{5}+2 = \frac{-6}{5}$

$\therefore$ image is $\left(\frac{-7}{5}, \frac{-6}{5}\right)$

25. Find the circumcentre of the triangle whose sides are x = 1, y = 1 & x+y=1

SOL: In a right angle mid point hyperboa is the circum centre

$\therefore \left(\frac{1}{2}, \frac{1}{2}\right)$

26. Find the orthocentre of the triangle whose sides are given by $x+y+10=0$, $x-y-2=0$ & $2x+y-7=0$

SOL: $x+y=-10$

$x-y=2$

$2x=-8 \Rightarrow x=-4$ $\therefore\ y=-6$ $(-4,\ -6)$

LEVE – II (IPE & EAMCET)

(Multiple Choice Questions with solutions)

1. If θ be the inclination of a straight line then the range of θ is

   1) $0 \le \theta < 90$ 2) $0 \le \theta < 190$ 3) $0 \le \theta < 270$ 4) $0 \le \theta < 360$

2. If the points (6, 8), (–2, 2) and (k, –1) are collinear, then the value of k

   1) 5 2) 4 3) 6 4) –6

3. The line $\frac{x}{a} - \frac{y}{b} = 1$ meets x–axis at p. The equation of perpendicular to this line at p is

   1) $\frac{x}{a} + \frac{y}{b} = \frac{a}{b}$ 2) $\frac{x}{a} + \frac{y}{b} = \frac{b}{a}$ 3) $\frac{x}{b} + \frac{y}{a} = \frac{a}{b}$ 4) $\frac{x}{b} + \frac{y}{a} = \frac{b}{a}$

4. P(1, 3) & R(5, 1) are two opposite vertices of a rectangle PQRS. If the slope of the line QS is 2. Then the equation of QS is

   1) $2x - y = 4$ 2) $2x - y = 1$ 3) $4x - 2y = 3$ 4) $2x + y = 1$

5. The equation of the median of the triangle with vertices (4, 3), (–2, 3), (1, –2) passing through (–2, 3)

   1) $5x + 9y + 17 = 0$ 2) $9x - 5y - 11 = 0$

   3) $5x + 9y - 17 = 0$ 4) $5x - 9y + 13 = 0$

6. A straight line meets the coordinate axes at A & B, so that the centroid of the triangle OAB is (1, 2). Then the equation of the line AB is

   1) $x + y = 6$ 2) $2x + y = 6$ 3) $x + 2y = 6$ 4) $3x + y = 0$

7. If the straight line $x + y + 1 = 0$ is transformed into normal form $x\cos\alpha + y\sin\alpha = 0$ then $\alpha =$

   1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{4}$ 3) $\frac{5\pi}{4}$ 4) $\frac{7\pi}{4}$

8. If the area of the triangle formed by the lines x=0, y=0, $3x + 4y = a\left(a > 0\right)$ is 1, then a =

   1) $\sqrt{6}$ 2) $2\sqrt{6}$ 3) $4\sqrt{6}$ 4) $6\sqrt{2}$

9. The area of the triangle formed by the lines $x = 0, y = 0 \,\&\, 3x + 4y = 12$ is

   1) 3 2) 4 3) 6 4) 12

10. A straight line passing through (3, 4) forms a triangle of area 24 sq.units with coordinate axes. Then its equation is

    1) $4x + 3y - 24 = 0$ 2) $2x + 3y + 24 = 0$

    3) $3x + 2y - 24 = 0$ 4) $x + y - 24 = 0$

11. A line passing through (3, 4) meets the coordinate axes at A & B respectively. The maximum area of the triangle OAB is

1) 8.5 2) 10.5 3) 24.5 4) 32.5

12. D(2, 5), E(3, 3) & F(0, 4) are the mid points of the sides of a triangle. Then the area of the triangle ABC is

1) 8 2) 10 3) 12 4) 14

13. If (4, –8), (–9, 7) are two vertices of a triangle whose centroid is (1, 4). Then the area of the triangle is sq.units

1) 165.5 2) 166.5 3) 167.5 4) 168.5

14. The area of the triangle formed by the axes and the line $(\cosh\alpha - \sinh\alpha)x + (\cosh\alpha + \sinh\alpha) = 2$ in sq.units

1) 4 2) 3 3) 2 4) 1

15. The circum centre of the triangle formed by the points (3, 0), (0, 4) & (0, 0) is

1) (3, 4) 2) –3, 4) 3) $\left(2, \frac{3}{2}\right)$ 4) $\left(\frac{3}{2}, 2\right)$

16. The ortho centre of the triangle formed by the points (0, 0), (7, 0), (0, 8) is

1) (7, 8) 2) $\left(\frac{7}{2}, 4\right)$ 3) $\left(\frac{-7}{2}, -4\right)$ 4) (0, 0)

17. The straight line $3x + y = 9$ divides the line joining the points (1, 3) & (2, 7) in the ratio

1) 4 : 2 2) 3 : 4 3) 4 : 5 4) 5 : 6

18. If a, b, c are in A.P., then the st. Line $ax + by + c = 0$ will passes through a fixed point which is

1) (1, –2) 2) (–1, 2) 3) (–2, 1) 4) (1, –2)

19. for all values a, b the line $(a+2b)x + (a-b)y + (a+5b) = 0$ passes through the point

1) (–1, 2) 2) (2, –1) 3) (–2, 1) 4) (1, –2)

20. A straight line passing through Q(2, 3) makes an angle of II with x-axis in +ve direction. If this straight line intersects x + y –7= 0 at p then PQ is

1) $\sqrt{2}$ 2) $3\sqrt{2}$ 3) $5\sqrt{2}$ 4) $7\sqrt{2}$

21. The equation of the st.line passing through (1, 2) & making an angle 60^0 with the line $\sqrt{3}x + y - 2 = 0$ is

1) y =2 2) y = –2 3) x=2 4) x= –2

22. A straight line through $Q\left(\sqrt{3},2\right)$ makes an angle $\frac{\pi}{6}$ with +ve directions of x–axis if this line intersects the line $\sqrt{3}x-4y-8=0$ at p, then the distance PQ is

1) 2 2) 4 3) 6 4) 8

23. A point on the line $2x-3y=5$, which is equidistant from the points (1, 2) & (3, 4) is

1) (2, 3) 2) (4, 6) 3) (1, –1) 4) (4, 1)

24. The point the line $3x+4y=5$, which is equidistant from (1, 2) & (3, 4).

1) (7, –4) 2) (15, –10) 3) $\left(\frac{1}{7},\frac{8}{7}\right)$ 4) $\left(0,\frac{5}{4}\right)$

25. The normal form of the line $x+y+\sqrt{2}=0$

1) $x\cos\frac{\pi}{4}+y\sin\frac{\pi}{4}=1$ 2) $x\cos\frac{3\pi}{4}+y\sin\frac{3\pi}{4}=1$

3) $x\cos\frac{5\pi}{4}+y\sin\frac{5\pi}{4}=1$ 4) $x\cos\frac{7\pi}{4}+y\sin\frac{7\pi}{4}=1$

26. The area of the circle which touch the lines $4x+3y=15\ \&\ 4x+3y=5$

1) 4π 2) 3π 3) 2π 4) π

27. The equation of a line passing through the point of intersection of the lines $x-3y+2=0, 2x+5y-7=0$ and is perpendicular to the line $3x+2y+5=0$ is

1) $2x-3y+1=0$ 2) $6x-9y+11=0$

3) $2x-3y+5=0$ 4) $3x-2y+1=0$

28. The equation of the straight line $\perp er$ to $5x-2y=7$ and passing through the point of intersection of the lines $2x+3y=1\ \&\ 3x+4y=6$ is

1) $2x+5y+17=0$ 2) $2x+5y-17=0$

3) $2x-5y+17=0$ 4) $2x-5y-17=0$

29. The equation of the line passing through the point of intersection of the lines $x+y-5=0$ & $2x-y+4=0$ and having intercepts numerically equal is

1) $x+y-5=0\ \&\ 3x-3y+13=0$ 2) $x-y-5=0\ \&\ 3x-3y+13=0$

3) $x+y-5=0\ \&\ 3x+3y+13=0$ 4) $x+y+5=0\ \&\ 3x-3y-13=0$

30. The equation of the straight line passing through the intersection of $x+2y-19=0$, $x-2y-3=0$ and at a distance of 5 units from (–2, 4) is

1) $5x+12y-7=0$ 2) $5x+12y-103=0$

3) $5x-12y+7=0$ 4) $12x-5y+7=0$

31. A straight line which makes equal intercepts on positive x & y axes and which is at a distance 1 unit from the origin intersect the st.line $y=2x+3+\sqrt{2}$ at $(x_0,\ y_0)$. Then $2x_0+y_0=$

1) $3+\sqrt{2}=$ 2) $\sqrt{2}-1=$ 3) 1 4) 0

32. The angle between the line joining the points (1, –2), (3, 2) and the line $x+2y-7=0$ is

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

33. The value of k such that the lines $2x-3y+k=0, 3x-4y-13=0 \,\&\, 8x-11y-33=0$ are concurrent is

1) 20 2) –7 3) 7 4) –20

34. If the lines $x+ay+a=0,\ bx+y+b=0, cx+cy+1=0$ (a,b,c are distant $\neq 1$) are concurrent then the value of $\frac{a}{a-1}+\frac{b}{b-1}+\frac{c}{c-1}$

1) –1 2) 0 3) 1 4) not defined

35. If the lines $x+2ay+a=0,\ x+3by+b=0, x+4cy+c=0$ are concurrent then a,b,c are in

1)A.P 2) G.P 3) H.P 4) A.G.P

36. The mid points of the sides BC, CA, AB in a triangle ABC are (2,1), (–1, –2) & (3, 3) then the equation of BC

1) $5x+4y+6=0$ 2) $5x-4y-6=0$

3) $5x+4y-6=0$ 4) $5x-4y+6=0$

37. If the equation of one diagonal of a square is $7x-y+8=0$ and one vertex is (–4, 5). Then the equation of the second diagonal

1) $x+7y-7=0$ 2) $x+7y-15=0$

3) $x+7y+8=0$ 4) $7x-y-31=0$

38. A(–1, 1), B(5, 3) are opposite vertices of a square. The equation of the other diagonal (not passing through A, B) of the square is

1) $2x-3y+4=0$ 2) $2x-y+3=0$ 3) $y+3x-8=0$ 4) $x+2y-1=0$

39. If the straight lines $y=4-3x, ay=x+10$ and $2y+bx+9=0$ represent the three conseative sides of a rectangal, then ab =

1) 18 2) –3 3) $\frac{1}{2}$ 4) $\frac{-1}{3}$

40. A line passing through A(1, –2) has slope 1. The points on the line at a distance of $4\sqrt{2}$ units from A are

1) $(3,-6),(5,\ 2)$ 2) $(-3,-6),(5,\ -2)$

3) $(-3,-6),(5,\ 2)$ 4) $(3,\ 6),(-5,\ 2)$

41. If $A(1,\ 2)\ \&\ B(6,\ 5)$ are two points the ratio in which the foot of the $\perp er$ $(4,\ 11)$ to AB devides it is

1) 8:15 2) 5:8 3) –5 :8 4) –8 : 5

42. If the line $3x+4y=8$ denoted by L. Then the points $(2,\ -5),\ (-5,\ 2)$

1) lie on L 2) lie on the same side of L

3) we on the opposite sides 4) equidistant from L

43. Let O be the origin $A(3,-2),B(1,2)\ \&\ C(1,1)$. The pair of points which are on different sides of the line $2x+3y=5$ are

1) A, B 2) A, C 3) B, C 4) None

43(a). The member of wireless that touch all the straight lines $x+y-4=0,\ x-y+2=0$ and y = 2 is

1) 1 2) 2 3) 3 4) 4

44. If the perpendicular bisector of AB is $x-3y-5=0\ \&\ A(-1,\ -3)$ and then B coordinates

1) $\left(\frac{-3}{5},\frac{6}{5}\right)$ 2) $\left(\frac{-8}{5},\frac{-6}{5}\right)$ 3) $\left(\frac{-6}{5},\frac{8}{5}\right)$ 4) $\left(\frac{-6}{5},\frac{-8}{5}\right)$

45. If PM is the $\perp er$ from P(2, 3) on to the line $x+y=3$. Then the coordinates of M are

1) (2, 1) 2) (–1, 4) 3) (1, 2) 4) (4, –1)

46. Suppose A, B are two points on $2x-y+3=0$ and $P(1,\ 2)$ is such that PA = PB, then the midpoint of AB is

1) $\left(\frac{-1}{5},\frac{13}{5}\right)$ 2) $\left(\frac{-7}{5},\frac{9}{5}\right)$ 3) $\left(\frac{7}{5},\frac{-9}{5}\right)$ 4) $\left(\frac{-7}{5},\frac{-9}{5}\right)$

47. The image of the point (4, –13) w.r.t the line $5x+y+6=0$ is

1) (–1, –14) 2)(3, 4) 3) (1, 2) 4) (–4, 13)

48. If (–2, 6) is the image of the point (4, 2) w.r.t the line L = 0 then L =

1) $6x-4y-7$ 2) $2x+3y-5$ 3) $3x-2y+5$ 4) $3x-2y+10$

49. The image of the line $x+y-2=0$ in the y–axis is

1) $x-y+2=0$ 2) $y-x+2=0$ 3) $x+y+2=0$ 4) $x+y-2=0$

50. The image of the line $x+y-2=0$ in the x–axis is

1) $x-y+2=0$ 2) $y-x+2=0$ 3) $x-y-2=0$ 4) $x+y+2=0$

51. The equation of a line, which passes through the point of intersection of the lines $x-3y+1=0, 2x+5y-9=0$ and is it a distance of $\sqrt{5}$ units from the origin is

1) $2x-y=5$ 2) $x+2y=5$ 3) $2x+y=5$ 4) $x-2y=5$

52. The medians $AD \& BE$ of the triangle with vertices $A(0,2b), B(0,0), C(2a,0)$ are mutually perpendicular then

1) $a=\sqrt{2}b$ 2) $b=\sqrt{2}a$ 3) $b=-\sqrt{2}a$ 4) $a=-\sqrt{b}$

53. $\frac{x}{a}+\frac{y}{b}=1$ is variable line where $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$ (c is a constant) locus of the foot of the $\perp er$ drawn from the origin to above variable line is

1) $x^2+y^2=2c^2$ 2) $x^2+y^2=c^2$

3) $2x^2+2y^2=c^2$ 4) $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{c^2}$

54. The lines $x-y-2=0, x+y-4=0 \& x+3y=6$ meet is the common point

1) (1, 2) 2) (2, 2) 3) (3, 1) 4) (1, 1)

55. The area of the parallelogram formed by the line $2x-y+3=0,\ 3x+4y-6=0, 2x-y+9=0 \& 3x+4y+4=0$ is

1) $\frac{60}{11}$ 2) 12 3) $\frac{15}{11}$ 4) $\frac{30}{11}$

56. The point is equidistant from A(1, 3), B(–3, 5) & C(5, –1) then PA

1) 5 2) $5\sqrt{5}$ 3) 25 4) $5\sqrt{10}$

57. The circum centre of the triangle formed by (–2, 3) , (2, –1) , (4, 0) is

1) $\left(\frac{3}{2},\frac{5}{2}\right)$ 2) $\left(\frac{-3}{2},\frac{5}{2}\right)$ 3) $\left(\frac{3}{2},\frac{-5}{2}\right)$ 4) $\left(\frac{-3}{2},\frac{-5}{2}\right)$

58. In a ΔABC, the perpendicular bisector $x-y+5=0$ of the sides AB, AC are $x-y+5=0, x+2y=0$ of $A(1,-2)$ then B vertex

1) $\left(\frac{11}{5},\frac{2}{5}\right)$ 2) $\left(\frac{2}{5},\frac{11}{5}\right)$ 3) $(-7, 6)$ 4) $(-7,-6)$

59. The orthocentre of the triangle formed by the points (–2, 3), (2, –1) &(4,0)

1) $\left(\frac{7}{2},\frac{4}{2}\right)$ 2) $\left(\frac{-7}{2},2\right)$ 3) $\left(\frac{-7}{2},\frac{-4}{2}\right)$ 4) $\left(\frac{7}{2},\frac{-4}{2}\right)$

60. The orthocentre of the triangle formed by the lines $x-2y+9=0, x+y-9=0$ is

1) (5, 5) 2) (5, –5) 3) (–5, 5) 4) (–5, –5)

61. The incentre of then triangle formed by the lines x = 1, y=1 & x+y=1 is

1) $\left(1-\frac{1}{\sqrt{2}}, 1-\frac{1}{\sqrt{2}}\right)$ 2) $\left(1-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

3) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ 4) $\left(\frac{1}{\sqrt{2}}, 1-\frac{1}{\sqrt{2}}\right)$

62. The locus of the point of intersection of the lines $x\sin\theta-(1+\cos\theta)\,y=a\sin\theta$ and $x\sin\theta-(1+\cos\theta)\,y+a\sin\theta=0$ is

1) $x^2-y^2=a^2$ 2) $x^2+y^2=a^2$ 3) $y^2=4ax$ 4) $x^2+y^2=4a^2$

63. The equation of the line passing through the point p(1, 2) such that p bisects the part intercepted between the coordinate axes is

1) $x+2y=5$ 2) $x-y+1=0$ 3) $x+y-31=0$ 4) $2x+y-4=0$

64. The line $2x+3y=6,\ 2x+3y=8$ then the x–axis at A,B respectively. A line *l* drawn through the point (2, 2) meets the x-axis at c such that the abscissa of A,B,C are in A.P then the equation of the line *l* is

1) $2x+3y=10$ 2) $3x+2y=10$ 3) $2x-3y=10$ 4) $3x-2y=10$

65. If the points (1, 2), (3, 4) lies on the same side of the straight line $3x-5y+a=0$ Then 'a'

1) $7<a<11$ 2) $a=7$ 3) $a=11$ 4) $a<7$ or $a>11$

1) $[7, 11]$ 2) $[7, \alpha)$ 3) $(-\alpha, 11]$ 4) $R-[7,\ 11]$

66. The image of the point $(3, 8)$ in the line $x+3y=7$

1) $(1, 4)$ 2) $(4, 1)$ 3) $(-1, -4)$ 4) $(-4, -1)$

67. The equation of straight line passing through the point (1, 2) and inclined at 45^0 to the line $y=2x+1$ is

1) $5x+y=7$ 2) $3x+y=5$ 3) $x+y=3$ 4) $x-y+1=0$

68. A point p moves the plane xy such that the sum of its distances from two mutually $\perp er$ lines is always equal to 5. The area enclosed by the locus of the point is

1) $\frac{25}{4}$ 2) 25 3) 50 4) 100

69. If a, b, c from a G.P with a common ratio r, then the sum of ordinates of the points of intersection of the line ax+by+c=0 and the curve x+2y=0 is

1) $\frac{-r^2}{2}$ 2) $\frac{-r}{2}$ 3) $\frac{r}{2}$ 4) r

70. The number of points p(x, y) with natural numbers as coordinates that lie inside the quadrilateral formed by the lines $2x+y=2, x=0, y=0 \& x+y=5$ is

1) 12 2) 10 3) 8 4) 6

71. If p and q are the perpendicular distances from the origin to the straight lines $x\sec\theta - y\cos ec\theta = a$ and $x\cos\theta + y\sin\theta = a\cos 2\theta$

1) $4p^2+q^2=a^2$ 2) $p^2+q^2=a^2$ 3) $p^2+2q^2=a^2$ 4) $4p^2+q^2=2a^2$

72. If $2x+3y=5$ is the perpendicular bisector of the line segment joining the points $A\left(1, \frac{1}{3}\right)$ & B, then B=

1) $\left(\frac{21}{13}, \frac{49}{39}\right)$ 2) $\left(\frac{17}{13}, \frac{31}{39}\right)$ 3) $\left(\frac{7}{13}, \frac{49}{39}\right)$ 4) $\left(\frac{21}{13}, \frac{31}{39}\right)$

73. If a line l passes through $(k,2k),(3k,3k)\&(3,1)$ $k \neq 0$, then the distance

1) $\frac{4}{\sqrt{5}}$ 2) $\frac{3}{\sqrt{5}}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{1}{\sqrt{5}}$

74. If the image of $\left(\frac{-7}{5}, \frac{-6}{5}\right)$ is a line is (1, 2) then the equation of the line is

1) $4x+3y=12$ 2) $4x+3y+24=0$ 3) $3x+4y=12$ 4) $x-2y=6$

75. The equation of the straight line $\perp er$ to $3x-4y=6$ and forming a triangle of area 6 sq.units with the coordinate axes is

1) $4x+3y=12$ 2) $4x+3y+24=0$ 3) $3x+4y=12$ 4) $x-2y=6$

76. If the straight line $2x+3y-1=0,\ x+2y-1=0$ and $ax+by-1=0$ form a triangle with origin as orthocentre, then (a, b) is equal to

1) (6, 4) 2) (–3, 3) 3) (–8, 8) 4) (0, 7)

77. The point on the line 4x-y-2=0 which is equidistant from the points (–5, 6) & (3,2)is

1) (2, 6) 2) (4, 14) 3) (–8, 8) 4) (0, 7)

78. A value of k such that the straight lines $y-3x+4=0(2k-1)x-(8k-1)y-6=0$ are perpendicular is

1) $\frac{1}{6}$ 2) $\frac{-1}{6}$ 3) 1 4)0

79. The length of the segment of the st.line passing through (3, 3) & (7, 6) cut off the coordinate axes is

1) $\frac{4}{5}$ 2) $\frac{5}{4}$ 3) $\frac{7}{4}$ 4) $\frac{4}{7}$

80. If the lines $x+3y-9=0, 4x+by-2=0$ & $2x-y=4$ are concurrent. Then the equation of the line passing through the point (b, 0) and concurrent with given lines is

1) $2x+y+10=0$ 2) $4x-7y+20=0$ 3) $x-y+5=0$ 4) $x-4y+5=0$

81. The midpoint of the line segment joining the centroid and orthocentre of the triangle whose vertices are (a, b), (a, c) & (d, c) is

1) $\left(\frac{5a+d}{6}, \frac{b+5c}{6}\right)$ 2) $\left(\frac{a+5d}{6}, \frac{5b+c}{6}\right)$ 3) (a, c) 4) (0, 0)

82. The distance from the origin to the image of (1, 1) w.r.t the line x+y+5=0 is

1) $7\sqrt{2}$ 2) $3\sqrt{2}$ 3) $6\sqrt{2}$ 4) $4\sqrt{2}$

83. The equation of the straight line passing through the point of contusection of 5x–6y–1=0, 3x+2y+5=0 and $\perp er$ to the line 3x–5y+11=0 is

1) $5x+3y+18=0$ 2) $5x+3y-18=0$ 3) $5x+3y+8=0$ 4) $5x+3y-8=0$

84. The points on the straight line $3x-4y+1=0$ which are at a distance of 5 units from the point (3, 2) are

1) $\left(-2, \frac{-7}{4}\right), \left(-3, \frac{-5}{2}\right)$ 2) $\left(4, \frac{11}{4}\right), (-1, -1)$

3) $\left(1, \frac{1}{2}\right), \left(2, \frac{5}{4}\right)$ 4) $(7, 5), (-1, -1)$

85. The incentre of the triangle formed by the lines $y=\pm\sqrt{3}x$ & $y=3$ is

1) (0,2) 2) (1, 2) 3) (2, 0) 4) (2, 1)

86. The image of the point (2, 4) w.r.t the straight line $2x+3y-6=0$ is

1) $\left(\frac{-14}{13}, \frac{-8}{13}\right)$ 2) $\left(\frac{14}{13}, \frac{8}{13}\right)$ 3) $\left(\frac{-2}{13}, \frac{-4}{13}\right)$ 4) $\left(\frac{-2}{7}, \frac{-8}{7}\right)$

87. The equation of the base of an equilateral triangle is $12x+5y-65=0$ if one of its vertices is (2, 3) Then the length of the side is

1) $\frac{4}{13}$ 2) $\frac{2}{\sqrt{3}}$ 3) $\frac{4}{\sqrt{3}}$ 4) $\frac{2}{13}$

88. A triangle is formed by y–axis, the st line L passing through the points (3, 0), $\left(1,\frac{4}{3}\right)$ and the st line $\perp er$ to the line L passing through the point (8, 1). Then the area of the triangle (in sq.units) is

1) 16 2) 21 3) 36 4) 39

89. For $c \neq 0,1$ if the st lines x+y=1, 2x–y=c and bx+2by=c have one common point then

1) $c<1 \Rightarrow b \in \left(-2, \frac{3}{4}\right)$ 2) $c>1 \Rightarrow b \in \left(\frac{-3}{4}, 3\right)$

3) $c<1 \Rightarrow b \in \left(-3, \frac{3}{2}\right)$ 4) $c>1 \Rightarrow b \in \left(\frac{-3}{4}, \frac{3}{4}\right)$

90. Let $a \neq 0, b \neq 0, c \in R$ and $L(p,q) = \frac{ap+bq+r}{\sqrt{a^2+b^2}}, \forall p,q \in R$. If $L\left(\frac{2}{3}, \frac{1}{3}\right) + L\left(\frac{1}{3}, \frac{2}{3}\right) + L(2, 2) = 0$ Then the line $ax+by+c=0$ always passes through the fixed point

1) $(0, 1)$ 2) $(1, 1)$ 3) $(2, 2)$ 4) $(-1, -1)$

91. The incentre of the triangle formed by the straight line having 3 as x-intercept & 4 as y-intercept, together with coordinate axes is

1) (2, 2) 2) $\left(\frac{3}{2}, \frac{3}{2}\right)$ 3) (1, 2) 4) (1, 1)

92. The equation of the straight line in the normal form, which is parallel to the lines $x+2y+3=0$ & $x+2y+8=0$ and deviding the distance between these two lines is the ratio 1 : 2 internally is

1) $x\cos\alpha + y\sin\alpha = \frac{10}{\sqrt{45}}, \alpha = \tan^{-1}\sqrt{2}$ 2) $x\cos\alpha + y\sin\alpha = \frac{14}{\sqrt{45}}, \alpha = \pi + \tan^{-1} 2$

3) $x\cos\alpha + y\sin\alpha = \frac{14}{\sqrt{45}}, \alpha = \tan^{-1} 2$ 4) $x\cos\alpha + y\sin\alpha = \frac{10}{\sqrt{45}}, \alpha = \pi + \tan^{-1}\sqrt{2}$

93. If the line joining the points $A(b\cos\alpha, b\sin\alpha)$ & $B(a\cos\beta, a\sin\beta)$ is extended to the point N(x, y) such that AN:NB= b : a then

1) $x\cos\frac{\alpha-\beta}{2} + y\sin\frac{(\alpha+\beta)}{2} = 0$ 2) $x\cos\frac{\alpha-\beta}{2} + y\sin\frac{\alpha-\beta}{2} = 0$

3) $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{(\alpha+\beta)}{2} = 0$ 4) $x\cos\frac{(\alpha+\beta)}{2} + y\sin\frac{(\alpha-\beta)}{2} = 0$

94. If α, β are the angles made by the normal drawn from the origin to the lines $x+y+\sqrt{2}=0\ \&\ x-\sqrt{3}y-2=0$ with +ve x-axis in anticlock wise directions, the $\alpha+\beta=$

1) $\frac{-13\pi}{12}$ 2) $\frac{29\pi}{12}$ 3) $\frac{-11\pi}{12}$ 4) $\frac{35\pi}{12}$

95. The straight lines $x+3y-4=0, x+y=4\ \&\ 3x+y=4$

1) forms an isosceles triangle 2) are concurrnent

3) form an equilateral triangle 4) form a right angled isosceles triangle

Answers

1. 2	2. 4	3. 4	4. 1	5. 3
6. 1	7. 3	8. 2	9. 3	10. 1
11. 3	12. 2	13. 2	14. 3	15. 4
16. 4	17. 2	18. 1	19. 3	20. 1
21. 1	22. 3	23. 4	24. 2	25. 3
26. 4	27. 1	28. 1	29. 2	30. 2
31. 2	32. 2	33. 2	34. 3	35. 3
36. 2	37. 1	38. 3	39. 1	40. 3
41. 2	42. 2	43. 1	44. 2	45.3
46. 1	47.1	48.3	49.1	50.3
51.3	52.1	53.2	54.3	55.1
56.4	57.1	58.3	59.1	60.1
61.3	62.2	63.4	64.1	65.4
66.3	67.2	68.3	69.3	70.4
71.1	72.1	73.4	74.3	75.1
76.3	77.2	78.2	79.2	80.4
81.1	82.3	83.3	84.4	85.1
86.1	87.3	88.4	89.1	90.2
91.4	92.2	93.3	94.4	95.1

ASSERTION, REASON & STATEMENT TYPE QUESTIONS

1. Assertion (A): The area of the figure formed by the lines $x \pm y \pm 4 = 0$ is sq.units 32

   Reason (R) : The area of the triangle formed by the x+y+a=0 with coordinate axes in sq.units is a^2

   1) Both A & R are true & A is the correct explanations of A
   2) Both A & R are true & A is not the correct explanations of A
   3) A is false & R is false 4) A is false & R is true

2. Assertion (A): The equations of line passing through (1, 1) and perpendicular to the line $2x+3y-7=0$ and $3x-2y-1=0$

   Reason (R) : The equation of the line passing through (x_1, y_1) and perpendicular to the line $lx+my+n=0$ is $m(x-x_1)-l(y-y_1)=0$

   Which of the following is true

   1) Both A & R are true & A is the correct explanations of A
   2) Both A & R are true & A is not the correct explanations of A
   3) A is false & R is false 4) A is false & R is true

3. Assertion (A): The distance between the lines $2x-y+3=0$ & $3y=6x+4$ is $\frac{\sqrt{5}}{3}$

   Reason (R) : The distance between parallel lines $ax+by+c_1=0$ & $ax+by+c_2=0$ is $\left|\frac{c_1-c_2}{\sqrt{a^2+b^2}}\right|$

   1) Both A & R are true & A is the correct explanations of A
   2) Both A & R are true & A is not the correct explanations of A
   3) A is false & R is false 4) A is false & R is true

4. Assertion (A): The line $2x+3y-20=0$ devides the line segment joining the points (2, 3), (2, 10) in the ratio 1 : 2 internally.

   Reason (R) : The line $L = ax+by+c=0$ devides the line segment joining the points $A(x_1, y_1)$ & $B(x_2, y_2)$ in the ratio $-(ax_1+by_1+c):(ax_2+by_2+c)$

   Which of the following is correct.

   1) Both A & R are true & A is the correct explanations of A
   2) Both A & R are true & A is not the correct explanations of A
   3) A is false & R is false 4) A is false & R is true

5. Assertion (A): The image of (0, 0) with respect to the line $x+y+1=0$ is (–1, –1)

Reason (R) : If (h, k) is the image of (x_1, y_1) with respect to the line $ax+by+c=0$

then $\frac{h-x_1}{a}=\frac{k-y_1}{b}=\frac{-(ax_1+by_1+c)}{a^2+b^2}$

Which of the following statement is correct

1) Both A & R are true & A is the correct explanations of A

2) Both A & R are true & A is not the correct explanations of A

3) A is false & R is false 4) A is false & R is true

6. $L=2x+3y-5=0$ is the line $A(3,-2), B(1,2), C(1,-2))$ are three points

Statement I: The points A, C lies on the either side of the line L = 0

Statement II: The points B, C lies on the same side of the line L = 0

Which of the following is correct

1) only I is true 2) only II is true

3) Both I & II are true 4) Both I & II are false

7. Statement I: P(–2, 2), Q(2, –2), R(1, 1) are the vertices of obtuse angle isosceles triangle

Statement II: Every obtuse angle triangle is a isosceles triangle

Which of the following is correct.

1) only I is true 2) only II is true 3) Both I & II are true 4) Both I & II are false

8. If variable line meets the coordinate areas at P & Q. Let A(a, 0), B(0, B). If BP is always perpendicular to AQ, then the locus of the point of intersection of BP, AQ is $x^2+y^2-ax-by=0$

Statement I: The equation of the circle with centre (a, b) is $x^2+y^2-ax-by=0$

Statement II: The equation of the circle with centre (a, b) is $x^2+y^2-ax-by=0$

Which of the following is correct

1) only I is true 2) only II is true

3) Both I & II are true 4) Both I & II are false

9. Assertion(A): The area of the parallelogram formed by the lines $4x-7y-13=0,$ $8x-y-39=0, 4x-7y+39=0, 8x-y+13=0$ is 52 sq.units

Reason: The are of the parallelogram formed by the lines

$ax+by+p=0, ax+by+q=0, cx+dy+r=0, cx+dy+s=0$ is $\left|\frac{(p-q)(r-s)}{bc-ad}\right|$

1) Both A & R are true & A is the correct explanations of A

2) Both A & R are true & A is not the correct explanations of A

3) A is false & R is false 4) A is false & R is true

Answers

1. 3	2. 1	3. 1	4. 1	5. 3
6. 4	7. 1	8. 2	9. 1	

MATCHING TYPE QUESTIONS

1. A(6, 3), B(–6, 3), C(–6, –2) are the vertices of a triangle If the median through a meets BC at P, AC meets x–axis &PQRS represents orthocentre, centroid of the triangle Match the points of List –I with the coordinates of the List – II

List – I		List – II
i)	P	A(0, 0)
ii)	Q	B(6, 0)
iii)	R	C(–2, 1)
iv)	S	D(–6, 3)

Which is the correct match

	(i)	(ii)	(iii)	(iv)
1.	D	A	D	E
2.	D	B	A	D
3.	D	A	E	C
4.	B	C	C	A

2. The correct match for List – I & List – II

List – I	List – II
i)The equation of the line passing through (5, 4) with slope $\frac{1}{\sqrt{3}}$	A) $\sqrt{3}x-y=0$
ii) A(1, 1); B(–3, 4); C(2, –5) are the vertices of a ΔABC then the altitude through A	B) $9x+5y+4=0$
iii) The $\perp er$ bisector of the line segmental joining the points (1, 2) & (5, 4)	C) $x-\sqrt{\ }y+4\sqrt{3}-5=0$
iv)The equations of the line passing through origin & $\perp er$ to the $x+\sqrt{3}y-5=0$	D) $5x-9y+4=0$
	E) $2x-3y-9=0$

Which is the correct match

	(i)	(ii)	(iii)	(iv)
1.	A	B	D	E
2.	C	D	E	A
3.	A	D	C	B
4.	D	E	A	B

3. Match the straight lines is List – I with areas in List – II formed by the coordinate axes

List – I	**List – II**
i) $y = 2x - 3$	A) 3
ii) $\frac{x}{3} + \frac{y}{4} = 1$	B) 16
iii) $x\cos 135 + y\sin 135 = 4$	C) 6
iv) The line passing through $(0,2),(3,0)$	D) 8
	E) $\frac{9}{4}$

Which is the correct match

	(i)	(ii)	(iii)	(iv)
1.	E	B	D	C
2.	A	B	D	E
3.	E	C	B	A
4.	B	C	D	A

4. Match the family of straight lines is List – I with their point of intersection is List – II

List – I	**List – II**
i) $(3k+1)x - (2k+3)y + 9 - k = 0$	A) (–2, 1)
ii) $(p+2q)x + (p-q)y + (p+5q) = 0$	B) (3, 4)
iii) $(2x+3y+1) + k(3x-2y-5) = 0$	C) (2, 2)
iv) $p(x+y-4) + q(2x-y-2) = 0$	D) (1, –1)
	E) (5, 7)

Which is the correct match

	(i)	(ii)	(iii)	(iv)
1.	A	B	E	C
2.	B	D	A	E
3.	B	A	B	C
4.	C	D	A	B

LEVEL – 3 (AIEEE/JEE PROBLEMS)

1. If the sum of the perpendicular distance from the points (3, 0), (0, 2) & (1, 1) to variable straight line is zero. Then the line passes through a fixed point is

   1) (1, 12) 2) (2, 1) 3) (1, 1) 4) 2, 2)

2. A(–1, –7), B(5, 1), C(1, 4) are the vertices of a triangle then the angular bisector of $\angle ABC$ is

   1) $x+7y-12=0$ 2) $x-7y+2=0$

   3) $x-7y=0$ 4) $x+7y=0$

3. Every line in the family of straight lines $(1+2\lambda)x+(\lambda-1)y+2(1+2\lambda)=0$ passes through a fixed point A. The equation of straight line passing through A and parallel to $3x-y=0$ is

   1) $3x-y+5=0$ 2) $-3x+y+5=0$

   3) $3x-y+6=0$ 4) $3x-y+8=0$

4. If (0, 0), (21, 0), (0, 21) are the vertices of a Δ then the number of points contain integer coordinate in the interior of the triangle is

   1) 231 2) 105 3) 190 4) 133

5. If x_1, x_2, x_3 & y_1, y_2, y_3 are in G.P with same common ratio then the points $(x_1, y_1), (x_2, y_2) \& (x_3, y_3)$

   1) on the line 2) on the ellipse 3) on the circle 4) vertices of a triangle

6. If the x coordinates of the point of intersection of the lines $3x+4y=9,\ y=mx+1$ are integer then the number of value for m is

   1) 2 2) 0 3) 4 4) 1

7. If a line passes through origin intersects the parallel lines $4x+2y=9, 2x+y=-6$ the line segment PQ in the ratio

   1) 1 : 2 2) 3 : 4 3) 2 : 1 4) 4 : 3

8. Let A(2, –3) & B(–2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x+3y=1$, then the locus of the vertex C is the line

   1) $2x+3y=9$ 2) $3x-2y=3$ 3) $3x+2y=5$ 4) $2x-3y=7$

9. The equation of the straight line passing through the point (4, 3) and making an intercepts on the coordinate axes whose sum is –1 is

   1) $\frac{x}{2}+\frac{y}{3}=-1 \& \frac{x}{-2}+\frac{y}{1}=-1$ 2) $\frac{x}{2}-\frac{y}{3}=1 \& \frac{x}{-2}+\frac{y}{1}=1$

3) $\frac{x}{2}+\frac{y}{3}=1 \& \frac{x}{2}+\frac{y}{1}=1$ 4) $\frac{x}{2}-\frac{y}{3}=-1 \& \frac{x}{-2}+\frac{y}{1}=-1$

10. A square of side a lies above the axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0<\alpha<\frac{\pi}{4}\right)$ with the +ve direction of x–axis. The equation of its diagonal not passing through the origin is

1) $y(\cos\alpha-\sin\alpha)-x(\sin\alpha-\cos\alpha)=a$

2) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$

3) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha+\cos\alpha)=a$

4) $y(\cos\alpha+\sin\alpha)+x(\sin\alpha-\cos\alpha)=a$

11. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is

1) $4x+3y=24$ 2) $3x+4y=25$ 3) $x+y=7$ 4) $3x-4y+7=0$

12. Let P(–1, 0), Q(0, 0) and $R\left(3,3\sqrt{3}\right)$ be three points The equation of the bisector of the angle PQR is

1) $\sqrt{3}x+y=0$ 2) $x+\frac{\sqrt{3}}{2}y=0$ 3) $\frac{\sqrt{3}}{2}x+y=0$ 4) $x+\sqrt{3}y=0$

13. If one of the lines $my^2+\left(1-m^2\right)xy-mx^2=0$ is a bisector of the angle between the lines $xy=0$, then m is

1) $\frac{1}{2}$ 2) –2 3) 1 4) 2

14. The perpendicular bisector of the line segment joining P(1, 4) & Q(k, 3) has y–intercept –4. Then a possible value of k is

1) 2 2) –2 3) –4 4) 1

15. The lines $p\left(p^2+1\right)x-y+q=0,\left(p^2+1\right)^2x+\left(p^2+1\right)y+2q=0$ are perpendicular to a commin line. For

1) no value of p 2) exactly are value of p

3) exactly two values of p 4) more than two value of p

16. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1 \& \frac{x}{4}+\frac{y}{4}=1$ meets the coordinate axes at A & B $(A \neq B)$ then the locus of midpoint of AB is

1) $6xy = 7(x+y)$ 2) $7xy = 6(x+y)$

3) $4(x+y)^2 - 28(x+y) + 49 = 0$ 4) $14(x+y)^2 - 97(x+y) + 168 = 0$

17. A straight line through origin meets the lines $3y = 10 - 4x$ & $3x + 6y + 5 = 0$ at the points A & B respectively. Then O devided the segment AB in the ratio

1) 2 : 3 2) 1 ; 2 3) 4 : 1 4) 3 : 4

18. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line k is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L & K is

1) $\frac{23}{\sqrt{15}}$ 2) $\sqrt{17}$ 3) $\frac{17}{\sqrt{15}}$ 4) $\frac{23}{\sqrt{17}}$

19. If the line $2x + y = k$ passes through the point which devides the line segment joining the points (1, 1) & (2, 4) in the ratio 3 : 2, then k =

1) 6 2) $\frac{11}{5}$ 3) $\frac{29}{5}$ 4) 5

20. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

1) $y = \sqrt{3}x - \sqrt{3}$ 2) $\sqrt{3}y = x - 1$

3) $y = x + \sqrt{3}$ 4) $\sqrt{3}y = x - \sqrt{3}$

21. Let a, b, c & d non zero numbers. If the point of intersection of the lines $4ax + 2ay + c = 0$ and $5bx + 2by + d = 0$ lies in the fourth quadrant and is equidistant from the two axes then

1) $3bc - 2ad = 0$ 2) $3bc + 2ad = 0$

3) $2bc - 3ad = 0$ 4) $2bc + 3ad = 0$

22. If PS is the median of the triangle with vertices P(2, 2), Q(6, –1) & R(7, 3). Then the equation of the line passing through (1, –1) and parallel to PS is

1) $4x - 7y - 11 = 0$ 2) $2x + 9y + 7 = 0$

3) $4x + 7y + 3 = 0$ 4) $2x - 9y - 11 = 0$

23. A straight line L passes through (3, –2) is inclined at an angle 60^0 is the line $\sqrt{3}x + y = 1$ and L also intersects x-axis. Equation of L is

1) $y + \sqrt{3}x + 2 - \sqrt{3} = 0$ 2) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

3) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ 4) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

24. The x coordinate of incentre of the triangle that has mid points of its sides as (0, 1), (1, 1) and (1, 0) is

1) $2+\sqrt{2}$ 2) $2-\sqrt{2}$ 3) $1+\sqrt{2}$ 4) $1-\sqrt{2}$

25. Locus of the image of the point (2, 3) in the line $(2x-3y+4)+\lambda(x-2y+3)=0, \lambda \in R$ is a

1) A straight line parallel to y–axis 2) circle of radious $\sqrt{2}$

3) circle of radious $\sqrt{3}$ 4) A straight line parallel to x-axis

26. Two sides of a rhombus are along the lines $x-y+1=0 \,\&\, 7x-y-5=0$ if its diagonals intersect (–1, –2) then which is a vertex of this rhombus

1) (–3, –9) 2) (–3, –8) 3) $\left(\frac{1}{3}, \frac{-8}{3}\right)$ 4) $\left(\frac{1}{3}, \frac{-7}{3}\right)$

27. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) & (–k, 2) has area 28 sq.units. Then the orthocentre of this triangle is

1) $\left(2, \frac{-1}{2}\right)$ 2) $\left(1, \frac{3}{4}\right)$ 3) $\left(1, \frac{-3}{4}\right)$ 4) $\left(2, \frac{1}{2}\right)$

28. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P & Q. If O is the origin and the rectangle OPRQ is completed then the locus of R is

1) $3x+2y=6xy$ 2) $3x+2y=6$

3) $2x+3y=xy$ 4) $3x+2y=xy$

29. Let (0, 0) & A(0, 1) be two fixed points then the locus of a point p such that the perimeter of the triangle AOP is 4 is

1) $8x^2-9y^2+9y=18$ 2) $9x^2-8y^2+8y=16$

3) $9x^2+8y^2-8y=16$ 4) $8x^2+9y^2-9y=18$

30. If the two lines $x+(a-1)y=1$ amd $2x+a^2y=1$ $(a \in R-\{0,1\})$ are perpendicular then the distance of their point of intersection from the origin is

1) $\sqrt{\frac{2}{5}}$ 2) $\frac{2}{5}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{\sqrt{2}}{5}$

31. Suppose that the points (h, k), (1, 2) & (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) & (4, 3) is perpendicular on L_1. Then $\frac{k}{h}=$

1) $\frac{1}{3}$ 2) 0 3) 3 4) $\frac{-1}{7}$

32. A point on the straight line $3x+5y=15$ which is equidistant from the coordinate axes will lie only in

1) 4^{th} quadrant 2) 1^{st} quadrant 3) 1^{st} & 2^{nd} quadrants 4) 1,2 &4^{th} quadrants

33. Line are drawn parallel to the line $4x-3y+2=0$ at a distance $\frac{3}{5}$ units from the origin. Then which one of the following points lies on any of these lines

1) $\left(\frac{-1}{4},\frac{2}{3}\right)$ 2) $\left(\frac{1}{4},\frac{-1}{3}\right)$ 3) $\left(\frac{1}{4},\frac{1}{3}\right)$ 4) $\left(\frac{-1}{4},\frac{-2}{3}\right)$

34. The equation $y=\sin x\sin(x+2)-\sin^2(x+1)$ represents a straight line lining is

1) Q_2 & Q_3 (quadrants) only 2) Q_1, Q_2 & Q_4 quadrants only

3) Q_1, Q_3 & Q_4 quadrants 4) Q_3 & Q_4 only

35. A triangle has a vertex at (1, 2) and the mid points of two sides through it are (–1, 1) & (2 3).Then the centroid of this triangle is

1) $\left(1,\frac{7}{3}\right)$ 2) $\left(\frac{1}{3},2\right)$ 3) $\left(\frac{1}{3},1\right)$ 4) $\left(\frac{1}{3},\frac{5}{3}\right)$

36. Consider the set of all lines $px+qy+r=0$ such that $3p+2q+4r=0$ which one of the following statement is true?

1) The lines are concurrent at $\left(\frac{3}{4},\frac{1}{2}\right)$

2) The lines are parallel

3) Each line passes through the origin

4) The line are not concurrent

37. If the line $3x+4y-24=0$ intersects the x-axis at the point A and y-axis at B, then the incentre of the triangle OAB, where O is the

1) (3, 4) 2) (2, 2) 3) (4, 3) 4) (4, 4)

38. A point P moves on the line $2x-3y+4=0$. If $Q(1,4)$ & $R(3,-2)$ are fixed points, then the locus of the centroid of ΔPQR is a line with

1) with slope $2/3$ 2) parallel to x-axis

3) with slope $3/2$ 4) parallel to y-axis

39. Two sides of a parallelogram are along the lines $x+y=3$ & $x-y+3=0$ if its diagonals intersect of (2, 4) then its one of vertex is

1) (3, 5) 2) (2, 1) 3) (2, 6) 4) (3, 6)

40. From any point p on the line x = 2y perpendicular is drawn on y = x. Let foot of perpendicular is Q. Find the locus of mid point of PQ

1) 2x=3y 2) 5x=7y 3) 3x=2y 4) 7x = 5y

41. Let ABC is a triangle whose vertices are A(1, –1), B(0, 2), $C(x', y')$ and area of the triangle is 5 and $C(x', y')$ lies on $3x + y - 4\lambda = 0$ then $\lambda =$

1) 3 2) –3 3) 4 4) 2

42. A(3, –1), B(1, 3), C(2, 4) are vertices of the triangle ABC. If D is the centroid and p is point of intersection of line x+3y–1=0 & 3x–y+1=0 then which of the following points lies on the line joining D & P

1) (–9, –7) 2) (–9, –6) 3) 9, 6) 4) 9, –6)

Answers

1. 3	2. 2	3. 3	4. 3	5. 1
6. 1	7. 2	8. 1	9. 2	10. 4
11. 1	12. 1	13. 3	14. 3	15. 2
16. 2	17. 3	18. 4	19. 1	20. 4
21. 1	22. 2	23. 2	24. 2	25. 2
26. 3	27. 4	28. 4	29. 3	30. 1
31. 1	32. 3	33. 1	34. 4	35. 2
36. 1	37. 2	38. 1	39. 4	40. 2
41. 1	42. 2			

LEVEL – 3 (AIEEE/JEE PROBLEMS)

1. If the sum of the perpendicular distances from the points (3, 0), (0, 2) & (1, 1) to veriable straight line is zero then the line passes through a fixed point is

1) (1, 2) 2) (2, 1) 3) (1, 1) 4) (1, 1)

2. A(–1, –7) B(5, 1) C(1, 4) are the vertices of a triangle then the angular bisector of $\angle ABC$

1) $x + 7y - 12 = 0$ 2) $x - 7y + 2 = 0$

3) $x - 7y = 0$ 4) $x + 7y = 0$

3. every line in the family of straight lines $(1+2\lambda)x + (\lambda - 1)y + 2(1+2\lambda) = 0$ passes through a fixed point A. The equation of straight line passing through A and parallel to $3x - y = 0$ is

1) $3x - y + 5 = 0$ 2) $-3x + y + 5 = 0$

3) $3x - y + 6 = 0$ 4) $3x - y + 8 = 0$

4. If (0, 0),(21, 0), (0, 21) are the vertieces of a Δ then the number of points contain integer coordinate in the interior of the triangle is

1) 231 2) 105 3) 190 4) 133

If x_1, x_2, x_3 & y_1, y_2, y_3 are in G.P with same common ratio then the points $(x_1, y_1), (x_2, y_2) \& (x_3, y_3)$

1) on the line 2) on the ellipse 3) on the circle 4) vertices of a triangle

6. If the x coordinates of the point of intersection of the lines $3x + 4y = 9, y = mx + 1$ are integer then the number of value for m is

1) 2 2) 0 3) 4 4) 1

7. If a line passes through origin intersects the parallel lines $4x + 2y = 9, 2x + y = -6$ the line segment PQ in the ratio

1) 1 : 2 2) 3 : 4 3) 2 : 1 4) 4 : 3

8. Let A(2, –3) & B(–2, 1) be vertices of a triangle ABC. If the centroid of this triangle moves on the line $2x + 3y = 1$, then the locus of the vertex C is the line

1) $2x + 3y = 9$ 2) $3x - 2y = 3$ 3) $3x + 2y = 5$ 4) $2x - 3y = 7$

9. The equation of the straight line passing through the point (4, 3) and making an intercepts on the coordinate axes whose sum is –1 is

1) $\frac{x}{2} + \frac{y}{3} = -1 \& \frac{x}{-2} + \frac{y}{1} = -1$ 2) $\frac{x}{2} - \frac{y}{3} = 1 \& \frac{x}{-2} + \frac{y}{1} = 1$

3) $\frac{x}{2} + \frac{y}{3} = 1 \& \frac{x}{2} + \frac{y}{1} = 1$ 4) $\frac{x}{2} - \frac{y}{3} = -1 \& \frac{x}{-2} + \frac{y}{1} = -1$

10. A square of side a lies above the axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha\left(0 < \alpha < \frac{\pi}{4}\right)$ with the +ve direction of x–axis. The equation of its diagonal not passing through the origin is

1) $y(\cos\alpha - \sin\alpha) - x(\sin\alpha - \cos\alpha) = a$

2) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

3) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha + \cos\alpha) = a$

4) $y(\cos\alpha + \sin\alpha) + x(\sin\alpha - \cos\alpha) = a$

11. A straight line through the point A(3, 4) is such that its intercept between the axes is bisected at A. Its equation is

1) $4x+3y=24$ 2) $3x+4y=25$ 3) $x+y=7$ 4) $3x-4y+7=0$

12. Let P(–1, 0), Q(0, 0) and $R\left(3,3\sqrt{3}\right)$ be three points The equation of the bisector of the angle PQR is

1) $\sqrt{3}x+y=0$ 2) $x+\frac{\sqrt{3}}{2}y=0$ 3) $\frac{\sqrt{3}}{2}x+y=0$ 4) $x+\sqrt{3}y=0$

13. If one of the lines $my^2+\left(1-m^2\right)xy-mx^2=0$ is a bisector of the angle between the lines $xy=0$, then m is

1) $\frac{1}{2}$ 2) –2 3) 1 4) 2

14. The perpendicular bisector of the line segment joining P(1, 4) & Q(k, 3) has y–intercept –4. Then a possible value of k is

1) 2 2) –2 3) –4 4) 1

15. The lines $p\left(p^2+1\right)x-y+q=0,\left(p^2+1\right)^2x+\left(p^2+1\right)y+2q=0$ are perpendicular to a commin line. For

1) no value of p 2) exactly are value of p

3) exactly two values of p 4) more than two value of p

16. If a variable line drawn through the intersection of the lines $\frac{x}{3}+\frac{y}{4}=1\&\frac{x}{4}+\frac{y}{4}=1$ meets the coordinate axes at A & B $\left(A\neq B\right)$ then the locus of midpoint of AB is

1) $6xy=7\left(x+y\right)$ 2) $7xy=6\left(x+y\right)$

3) $4\left(x+y\right)^2-28\left(x+y\right)+49=0$ 4) $14\left(x+y\right)^2-97\left(x+y\right)+168=0$

17. A straight line through origin meets the lines $3y=10-4x\,\&\,3x+6y+5=0$ at the points A & B respectively. Then O devided the segment AB in the ratio

1) 2 : 3 2) 1 ; 2 3) 4 : 1 4) 3 : 4

18. The line L given by $\frac{x}{a}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{a}+\frac{y}{3}=1$. Then the distance between L & K

1) $\frac{23}{\sqrt{15}}$ 2) $\sqrt{17}$ 3) $\frac{17}{\sqrt{15}}$ 4) $\frac{23}{\sqrt{17}}$

19. If the line $2x+y=k$ passes through the point which devides the line segment joining the points (1, 1) & (2, 4) in the ratio 3 : 2, then k =

1) 6 2) $\frac{11}{5}$ 3) $\frac{29}{5}$ 4) 5

20. A ray of light along $x+\sqrt{3}y=\sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is

1) $y=\sqrt{3}x-\sqrt{3}$ 2) $\sqrt{3}y=x-1$

3) $y=x+\sqrt{3}$ 4) $\sqrt{3}y=x-\sqrt{3}$

21. Let a, b, c & d non zero numbers. If the point of intersection of the lines $4ax+2ay+c=0$ and $5bx+2by+d=0$ lies in the fourth quadrant and is equidistant from the two axes then

1) $3bx-2ad=0$ 2) $3bc+2ad=0$

3) $2bc-3ad=0$ 4) $2bc+3ad=0$

22. If PS is the median of the triangle with vertices P(2, 2), Q(6, –1) & R(7, 3). Then the equation of the line passing through (1, –1) and parallel to PS is

1) $4x-7y-11=0$ 2) $2x+9y+7=0$

3) $4x+7y+3=0$ 4) $2x-9y-11=0$

23. A straight line L passes through (3, –2) is inclined at an angle 60^0 is the line $\sqrt{3}x+y=1$ and L also intersects x-axis. Equation of L is

1) $y+\sqrt{3}x+2-\sqrt{3}=0$ 2) $y-\sqrt{3}x+2+3\sqrt{3}=0$

3) $\sqrt{3}y-x+3+2\sqrt{3}=0$ 4) $\sqrt{3}y+x-3+2\sqrt{3}=0$

24. The x coordinate of incentre of the triangle that has mid points of its sides as (0, 1), (1, 1) and (1, 0) is

1) $2+\sqrt{2}$ 2) $2-\sqrt{2}$ 3) $1+\sqrt{2}$ 4) $1-\sqrt{2}$

25. Locus of the image of the point (2, 3) in the line $(2x-3y+4)+\lambda(x-2y+3)=0, \lambda \in R$ is a

1) A straight line parallel to y–axis

2) circle of radious $\sqrt{2}$

3) circle of radious $\sqrt{3}$

4) A straight line parallel to x-axis

26. Two sides of a rhombus are along the lines $x-y+1=0$ & $7x-y-5=0$ if its diagonals intersect (–1, –2) then which is a vertex of this rhombus

1) (–3, –9) 2) (–3, –8) 3) $\left(\frac{1}{3},\frac{-8}{3}\right)$ 4) $\left(\frac{1}{3},\frac{-7}{3}\right)$

27. Let k be an integer such that the triangle with vertices (k, –3k), (5, k) & (–k, 2) has area 28 sq.units. Then the orthocentre of this triangle is

1) $\left(2,\frac{-1}{2}\right)$ 2) $\left(1,\frac{3}{4}\right)$ 3) $\left(1,\frac{-3}{4}\right)$ 4) $\left(2,\frac{1}{2}\right)$

28. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P & Q. If O is the origin and the rectangle OPRQ is completed then the locus of R is

1) $3x+2y=6xy$ 2) $3x+2y=6$

3) $2x+3y=xy$ 4) $3x+2y=xy$

29. Let (0, 0) & A(0, 1) be two fixed points then the locus of a point p such that the perimeter of the triangle AOP is 4 is

1) $8x^2-9y^2+9y=18$ 2) $9x^2-8y^2+8y=16$

3) $9x^2+8y^2-8y=16$ 4) $8x^2+9y^2-9y=18$

30. If the two lines $x+(a-1)y=1$ amd $2x+a^2y=1$ $\left(a\in R-\{0,1\}\right)$ are perpendicular then the distance of their point of intersection from the origin is

1) $\sqrt{\frac{2}{5}}$ 2) $\frac{2}{5}$ 3) $\frac{2}{\sqrt{5}}$ 4) $\frac{\sqrt{2}}{5}$

31. Suppose that the points (h, k), (1, 2) & (–3, 4) lie on the line L_1. If a line L_2 passing through the points (h, k) & (4, 3) is perpendicular on L_1. Then $\frac{k}{h}=$

1) $\frac{1}{3}$ 2) 0 3) 3 4) $\frac{-1}{7}$

32. A point on the straight line $3x+5y=15$ which is equidistant from the coordinate axes will lie only in

1) 4^{th} quadrant 2) 1^{st} quadrant 3) 1^{st} & 2^{nd} quadrants 4) 1,2 &4^{th} quadrants

33. Line are drawn parallel to the line $4x-3y+2=0$ at a distance $\frac{3}{5}$ units from the origin. Then which one of the following points lies on any of these lines

1) $\left(\frac{-1}{4},\frac{2}{3}\right)$ 2) $\left(\frac{1}{4},\frac{-1}{3}\right)$ 3) $\left(\frac{1}{4},\frac{1}{3}\right)$ 4) $\left(\frac{-1}{4},\frac{-2}{3}\right)$

34. The equation $y = \sin x \sin(x+2) - \sin^2(x+1)$ represents a straight line lining is

1) Q_2 & Q_3 (quadrants) only 2) Q_1, Q_2 & Q_4 quadrants only

3) Q_1, Q_3 & Q_4 quadrants 4) Q_3 & Q_4 only

35. A triangle has a vertex at (1, 2) and the mid points of two sides through it are (–1, 1) & (2 3).Then the centroid of this triangle is

1) $\left(1, \frac{7}{3}\right)$ 2) $\left(\frac{1}{3}, 2\right)$ 3) $\left(\frac{1}{3}, 1\right)$ 4) $\left(\frac{1}{3}, \frac{5}{3}\right)$

36. Consider the set of all lines $px + qy + r = 0$ such that $3p + 2q + 4r = 0$ which one of the following statement is true?

1) The lines are concurrent at $\left(\frac{3}{4}, \frac{1}{2}\right)$

2) The lines are parallel

3) Each line passes through the origin

4) The line are not concurrent

37. If the line $3x + 4y - 24 = 0$ intersects the x-axis at the point A and y-axis at B, then the incentre of the triangle OAB, where O is the

1) (3, 4) 2) (2, 2) 3) (4, 3) 4) (4, 4)

Answers

1. 3	2. 2	3. 3	4. 3	5. 1
6. 1	7. 2	8. 1	9. 2	10. 4
11. 1	12. 1	13. 3	14. 3	15. 2
16. 2	17. 3	18.	19.	20.
21.	22.	23. 2	24. 2	25. 2
26. 3	27. 4	28. 4	29. 3	30. 1
31. 1	32. 3	33. 3	34. 4	35. 2
36. 1	37. 2			

PAIR OF STRAIGHT LINES

OBJECTIVE QUESTIONS

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I. Equations of a pair of lines passing through origin Angle between a pair of lines

1. Addtion of equation of two straight lines gives us combined equation of two lines (True/false)

2. Each second degree equation in x and y represents the pair of straight lines. (True/false)

3. If the locus of a second degree equation in x and y contains a straight line, then the equation represents a pair of straight lines (True/false)

4. If a_1h and h are not all zero, then the equation $H \equiv ax^2 + 2hx + by^2 = 0$ represents a pair of straight line if and only if

   a) $h^2 \neq ab$ b) $h^2 < ab$ c) $h^2 > ab$ d) $h^2 \geq ab$

5. If a = 0, then one of the straight line represented by $H \equiv ax^2 + 2hxy + by^2 = 0$ must be x-axis (True/false)

6. If the slopes of the two lines represented by $ax^2 + 2hxy + by^2 = 0$ are m_1 and m_2 then $m_1 + m_2 =$

7. If the slopes of two lines represented by $ax^2 + 2hxy + by^2 = 0$ are m_1 and m_2 then $\frac{(m_1 + m_2)^2}{m_1 m_2} =$_______

8. Let the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of straight lines. If 'θ be the angle between the lines then $\cos\theta =$

9. If $H \equiv ax^2 + 2hxy + by^2 \equiv (l_1x + m_1y)(l_2x + m_2y)$ then $l_1m_2 + l_2m_1 =$_____

10. If $H \equiv ax^2 + 2hxy + my^2 = 0$ represents a pair of concident lines then $h^2 =$_____

11. Let the equation $ax^2 + 2hxy + by^2 = 0$ represent a pair of straight lines. If 'θ' be the angle between the lines then [ ]

    a) $\cos\theta \frac{|a-b|}{\sqrt{(a+b)^2 + ah^2}}$ b) $\sin\theta \frac{\sqrt{h^2 - ab}}{\sqrt{(a+h)^2 + 4h^2}}$

    c) $\tan\theta \frac{2\sqrt{h^2 - ab}}{|a-b|}$ d) None

12. If the lines given by $H \equiv ax^2 + 2hxy + by^2 = 0$ are perpendicular then the sum of co-efficients of x^2 and y^2 is _______

13. $a^2x^2 + 2xy + 9y^2 = 0$ represent a pair of distinct lines then 'a' lies in [ ]

a) $\left[\frac{-1}{3}, \frac{1}{3}\right]$ b) $\left(\frac{-1}{3}, \frac{1}{3}\right)$ c) $\left[\frac{-1}{9}, \frac{1}{9}\right]$ d) $\left(\frac{-1}{9}, \frac{1}{9}\right)$

14. The equation $4x^2 - 12xy + 9y^2 = 0$ represents [ ]

a) real and distinct lines b) real and concident lines

c) imaginary lines d) none

15. If a : b : c = 1 : 2 : 3 Then the lines represented by $ax^2 + bxy + cy^2 = 0$ are [ ]

a) real b) imaginary c) coincident d) perpendicular

16. The difference of the slopes of the lines $3x^2 - 4xy + y^2 = 0$ is

a) 1 b) 2 c) 3 d) 4

17. Which of the given equation doesn't represent a pair of linear

a) $x^2 + xy - y^2 = 0$ b) $6x^2 + 11xy - 10y^2$

c) $2x^2 - 3xy - 6y^2 = 0$ d) None

18. The value 'h' if the slopes of the lines represented by $6x^2 + 2hxy + y^2 = 0$ are in the ratio is 1 : 2 is

19. If $ax^2 + 2hxy + hy^2 = 0$ represents two straight lines such that the slope of one line is twice the slope of the other, then $8h^2 =$______

20. The difference of slopes of lines represented by

$y^2 - 2xy\sec^2\alpha + (3 + \tan^2\alpha)(\tan^2\alpha - 1)x^2 = 0$ is

a) $\frac{1}{4}$ b) 4 c) 0 d) 2

21. The angle between the pair of lines $y^2 - 2xy\operatorname{cosec}\theta + x^2 = 0, 0 \le \alpha \le \frac{\pi}{2}$ is

a) $\frac{\pi}{2} - \theta$ b) $\frac{\pi}{2}$ c) θ d) $\frac{\pi}{4} - \theta$

22. If 'θ' is the acute angle between the pair of line $x^2 + 3xy - 4y^2 = 0$ then $\sin\theta =$

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{5}{\sqrt{34}}$ d) $\frac{3}{\sqrt{34}}$

23. If the pair of lines $\left(x^2+y^2\right)\tan^2\alpha=\left(x-y\tan\alpha\right)^2$ are perpendicular to each other, then r = ____

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{8}$ d) $\frac{\pi}{4}$

24. If the slope of one of the line represented by $2x^2+3xy+ky^2=0$ is '2' then angle between pair of lines is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{4}$

25. The triangle formed by the equations $x^2-4xy+y^2=0$ and $x+y=3$ is an

a) Isosceless b) Scale c) right angle d) equilatered

26. The acute angle between the pair of lines represented by the equation $x^2-7xy+12y^2=0$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $Tan^{-1}\left(\frac{1}{13}\right)$ d) None

27. The acute angle between the pair of lines represented by the equation $y^2-xy-6x^2=0$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) None

28. The acute angle between the pair of lines represented by the equation $\left(x\cos\alpha-y\sin\alpha\right)^2=\left(x^2+y^2\right)\sin^2\alpha$ is

a) α b) 2α c) 4α d) None

29. The nature of the triangle formed by the lines $x^2-3y^2=0$ and x = 2

a) Isosceles b) scalene c) equilateral d) Right angled

30. The acute angle between the pair of lines represented by the equation $x^2+2xy\cot\alpha-y^2=0$ is

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

31. The equation of the pair of st. Line passing through the origin and making an angle of 30^0 with the line $3x-y-1=0$ is

a) $13x^2-12xy-3y^2=0$ b) $13x^2-12xy+3y^2=0$

c) $13x^2+12xy+3y^2=0$ d) none

32. Find the equation to the pair of straight lines passing through the origin and making an acute angle 'α' with the straight line $x+y+5=0$ is

33. The Area of the triangle formed by the following lines

$2y^2 - xy - 6x^2 = 0,\ x + y + 4 = 0$ is _______

34. Centroid of the triangle formed by the lines $12x^2 - 20xy + 7y^2 = 0$ and $2x - 3y + 4 = 0$ is ____

a) $\left(\frac{4}{3}, \frac{4}{3}\right)$ b) $\left(\frac{1}{2}, \frac{1}{2}\right)$ c) $\left(\frac{8}{3}, \frac{8}{3}\right)$ d) none

35. The centroid of the triangle formed by the following locus

$2y^2 - xy - 6x^2 = 0,\ x + y + 4 = 0$ is _____

a) $\left(\frac{20}{9}, \frac{-44}{9}\right)$ b) $\left(\frac{-20}{9}, \frac{44}{9}\right)$ c) $\left(\frac{20}{9}, \frac{44}{9}\right)$ d) none

36. The centroid of the triangle formed by the following lines

$3x^2 - 4xy + y^2 = 0,\ 2x - y = 6$ is ____

a) (0, 4) b) (4, 0) c) (0, –4) d) (–4, 0)

37. One of the lines of $3x^2 + 4xy + y^2 = 0$ is perpendicular to $lx + y + 4 = 0$ then l =

a) (0, 4) b) (4, 0) c) (0, –4) d) –4, 0)

Pair of St. Lines (Objective)

II. Bisectors of Angles.

38. The locus of the points equidistant from two intersecting lines $L_1 = 0$ and $L_2 = 0$ is the pair of lines ___________

39. The internal bisectors of the triangle are _________

40. If the equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of intersecting lines. Then the combined equation of the pair of bisectors of the angle between the lines is ________

41. The equation of pair of angular bisectors of $(a-b)x^2 + 4hxy - (a-b)y^2 = 0$ is

a) $h^2(x^2 - y^2) + xy(a-b) = 0$ b) $h^2(x^2 - y^2) - xy(a-b) = 0$

c) $h^2(x^2 - y^2) - abxy = 0$ d) $ax^2 + 2hxy + by^2 = 0$

42. The equation of the angular bisectors of $2x^2 + 2hxy + y^2 = 0$, $4x^2 + 18xy + y^2 = 0$

a) same b) different c) doesn't exists d) none

43. Equation of the bisector of acute angle between the lines $3x - 4y + 7 = 0$ and $12x + 5y - 2 = 0$ is

a) $11x - 3y + 9 = 0$ b) $21x + 77y - 101 = 0$

c) $11x + 3y - 9 = 0$ d) $21x - 77y + 101 = 0$

44. Equation of the bisector of the obtuse angle between the lines $x+y-5=0$ and $x-7y+7=0$ is

a) $x+3y-8=0$ b) $3x-y-9=0$

c) $x-3y+8=0$ d) $3x+y+9=0$

45. Equation of the straight lines bisects the angles between the lines $7x+y+3=0$ and $7x+y+3=0$ are

a) $\begin{matrix} x+3y-1=0 \\ 3x-y+2=0 \end{matrix}$ b) $\begin{matrix} x-3y+1=0 \\ 3x+y+2=0 \end{matrix}$

c) $\begin{matrix} x+3y+1=0 \\ 3x-y+2=0 \end{matrix}$ d) $\begin{matrix} x-3y-1=0 \\ 3x-y+2=0 \end{matrix}$

46. Equation of the bisector of the acute angle between the lines $7x+y+3=0,\ x-y+1=0$ is

a) $3x-y-2=0$ b) $x+3y-1=0$

c) $3x-y+2=0$ d) $x+3y+1=0$

47. Equation of the bisector of the abtuse angle between the lines $7x+y+3=0,\ x-y+1=0$ is

a) $3x-y-2=0$ b) $x+3y-1=0$

c) $3x-y+2=0$ d) $x+3y+1=0$

48. If the pair of straight lines $x^2-2pxy-y^2=0$ and $x^2-2qxy-y^2=0$ be such that each pair of bisects the angle between the other pair then

a) pq = –1 b) p = 9 c) p = –9 d) pq = 1

III. The product of the perpendicular distances, parallel & perpendicular lines to the given pair and Area of the triangle.

49. Equation of the pair of line passing through $(x_0,\ y_0)$ and parallel to the given lines represented by $ax^2+2hxy+by^2=0$ is

50. Equation of the pair of lines passing through (x_0, y_0) and perpendicular to given pair of lines $ax^2+2hxy+by^2=0$ is

51. The product of the perpendicular distance from a point $(\alpha,\ \beta)$ to the pair of straight lines $ax^2+2hxy+by^2=0$ is

52. The area of the triangle formed by the lines $ax^2+2hxy+by^2=0, lx+my+n=0$ is ___

53. The equation to the pair of lines passing through the point (–2, 3) and parallel to the pair of lines $x^2+4xy+y^2=0$ is

a) $x^2-4xy+y^2-8x+2y-11=0$ b) $x^2+4xy+y^2-8x+2y-11=0$

c) $x^2+4xy+y^2+8x+2y-11=0$ d) $x^2+4xy+y^2-8x-2y-11=0$

54. The equation to the pair of lines passing through the origin and perpendicular to $3x^2-5xy+2y^2=0$ is

a) $2x^2+5xy+3y^2=0$ b) $2x^2-5xy+3y^2=0$

c) $2x^2+5xy-3y^2=0$ d) None

55. Find the equation of the pair of lines intersecting at (2, –1) and perpendicular to the pair of $6x^2-13xy-5y^2=0$ is

a) $5x^2-13xy+6y^2-33x+14y+40=0$ b) $5x^2-13xy-6y^2+33x-14y-40=0$

c) $5x^2-13xy-6y^2-33x+14y+40=0$ d) $5x^2-13xy-6y^2+33x-14y-40=0$

56. Find the equation of the pair of lines inter secting at (2, –1) and parallel to the pair $6x^2-13xy-5y^2=0$

a) $6x^2-13xy-5y^2-37x+16y+45=0$ b) $6x^2-13xy+5y^2-37x+16y+45=0$

c) $6x^2-13xy-5y^2-37x+16y-45=0$ d) $6x^2-13xy-5y^2+37x-16y+45=0$

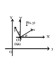
57. The product of the perpendiculars from (p, q) to the pair of lines $x^2-y^2=0$ is

a) $\frac{|p^2-q^2|}{2}$ b) $\frac{p^2+q^2}{2}$ c) $\frac{p^2-q^2}{\sqrt{2}}$ d) $\frac{p^2+q^2}{\sqrt{2}}$

58. If the product of the perpendicular distance from (1, k) to the pair of lines $x^2-4xy+y^2=0$ is $\frac{3}{2}$, then k = _____

a) 4 b) 5 c) 6 d) 8

59. The area of the triangle formed by the lines $x^2-9xy+18y^2=0$ and the line $y-1=0$ is (in sq.units)

a) 3/4 b)  c) 6 d) 3

60. If the area of the triangle formed by the lines $3x^2-2xy-8y^2=0$ and the line $3x+y-k=0$ is 5 sq.units then k = _____

a) 5 b) 6 c) 7 d) 8

61. The area of the triangle formed by the pair of lines $x^2+4xy+y^2=0$ and $x+y-1=0$ is ____

a) $\frac{3}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{3}{4}$ d) None

62. The area of the triangle formed by $x+y+1=0$ and the pair of straight lines $x^2+3xy+2y^2=0$ is _____

a) 7/12 b) 5/12 c) 1/12 d) 1/6

63. The equation of the angular bisectors of $a^2x^2+2h(a+b)xy+b^2y^2=0$ is

a) $h(x^2-y^2)=(a-b)xy$ b) $h(x^2-y^2)+xy(a-b)=0$

c) $h(x^2-y^2)=(a+b)xy$ d) None

64. If the second degree equation $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of lines $l_1x+m_1y+n_1=0, l_2x+m_2y+n_2=0$ Then which of the following is correct

a) $l_1m_2+l_2m_1=2g$, $l_1n_2+l_2n_1=2h$, $m_1n_2+n_1n_2=2f$

b) $l_1m_2+l_2m_1=2h$, $l_1n_2+l_2n_1=2g$, $m_1n_2+m_2n_1=2f$

c) $l_1m_2+l_2m_1=2h$, $l_1n_2+l_2n_1=2f$, $m_1n_2+m_2n_1=2g$

d) none

65. If the equation $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of parallel straight lines then the distance between the parallel lines = _____

66. If the equation $S\equiv ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersecting straight lines then their point of intersection is __________

67. The angle between the lines represented by $2x^2+xy-6y^2+7y-2=0$ is ___

68. The angle between the straight line represented by $2x^2+5xy+2y^2-5x-7y+3=0$ is__

69. The equation of pair of lines passing through the origin and parallel to the lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ is ___

70. The equation of pair of lines passing through the origin and perpendicular to the pair of lines $ax^2+2hxy+by^2+2gx+2fy+c=0$ is _____

71. If $x^2+xy-2y^2+4x-y+k=0$ represents a pair of straight lines then k =___

72. The equation of the pair of lines passing through the origin and parallel to the pair of lines $2x^2+3xy-2y^2-5x+5y-3=0$ is ______

73. The value of λ for which the equation $\lambda x^2-10xy+12y^2+5x-16y-3=0$ represents a pair of straight lines

a) –2 b) 2 c) 4 d) None

74. The angle between the pair of st. Lines represented by $2x^2-13xy-7y^2+x+23y-6=0$ is ____

75. Intersection point of the pair of straight lines represented by $3x^2+7xy+2y^2+5x+5y+2=0$ is ______

76. The value of 'k', if the equation $2x^2+kxy-6y^2+3x+y+1=0$ represents a pair of straight lines then k = _____

77. If represents a pair of straight lines. Then their equation be

a) $x-y-2=0, x+y+1=0$ b) $x+y-2=0, x+y+1=0$

c) $x-y+1=0, x+y-2=0$ d) None

78. If $8x^2-24xy+18y^2-6x+9y-5=0$ represents a pair of st lines then their equations be

a) $2x-3y+12=0,\ 4x-6y-5=0$ b) $2x-3y-5=0,\ 4x-6y+1=0$

c) $2x-3y-1=0,\ 4x-6y+5=0$ d) none

Homogenisins a second degree equation

79. Generally the locus of a second degree equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ whose co-efficients being real numbers determine a second degree curve (True/false)

80. If the graph of the equation $ax^2+2hxy+by^2+2gx+2fy+c=0$ contains more than one point, this second degree curve can be either a pair of straight lines or ____ or ____

81. The equation of pair of lines joining origin to the pair of intersection of the curve $7x^2-4xy+8y^2+2x-4y-8=0$ with the straight line $3x-y=2$ is _____

82. The equation of the line joining the origin to the pair of intersection of $x^2+y^2=1$ and $x+y-1=0$ is ____

83. The angle between the lines joining the origin to the points of intersection of $y^2=x$ and $x+y=1$ is ____

84. The angle between the lines joining the origin to the point of intersection of $x^2-xy+y^2+3x+3y-2=0$ and the straight line ___

85. The angle between the lines joining the origin to the point of intersection of $x^2+2xy+y^2+2x+2y-5=0$ and $3x-y+1=0$ is ______

86. The condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ to subtend at right angle of the origin is _____

87. The condition for the lines joining the origin to the points of intersection of the circle $x^2+y^2=a^2$ and the line $lx+my=1$ to coinside is ___

88. Equation of the pair of straight line joining the origin to the points of intersection of the line 6x–y+8=0 with the pair of straight line $3x^2+4xy-4y^2-11x+2y+6=0$ is ____

Key to Objective Questions

Pair of Strait lines

1. False 2. False 3. Ture 4. [D] 5. True

6. $\left[\frac{-2h}{b}\right]$ 7. $\left(\frac{4h^2}{ab}\right)$ 8. $\frac{|a+b|}{\sqrt{(a-b)^2+4h^2}}$ 9. (2h) 10. (ab)

11.[B] 12. [Zero] 13.[B] 14. [B] 15. [B]

16. [B] 17.[A] 18. $\left[\frac{\pm 3\sqrt{3}}{2}\right]$ 19. [9ab] 20. [B]

21. [A] 22. [C] 23. [D] 24. [A] 25. [D]

26. [C] 27. [A] 28. [B] 29. [C] 30. [D]

31. [A] 32. $y^2 + 2\sec xy + x^2$ 33. $\left[\frac{56}{3}\right]$ 34. [c] 35. [A]

36.[c] 37. [A] 38. [bisecting] 39. [concurrent]

40. $h\left(x^2 - y^2\right) = (a-b)xy$

41. [B] 42.[A] 43. [A] 44. [B] 45.[A]

46. [B] 47.[B] 48. [A]

49. $a\left(x-x_0\right)^2 + 2h\left(x-x_0\right)\left(y-y_0\right) + b\left(y-y_0\right)^2$

50. $b\left(x-x_0\right)^2 - 2h\left(x-x_0\right)\left(y-y_0\right) + a\left(y-y_0\right)^0 = 0$

51. $\dfrac{\left|a\alpha^2+2h\alpha\beta+b\beta^2\right|}{\sqrt{(a-b)^2+4h^2}}$ 52. $\Delta=\left|\dfrac{x^2\sqrt{h^2-ab}}{am^2-2hlm+bl^2}\right|$

53.[B] 54.[A]0 55.[c]

56.[A] 57.[A] 58.[B] 59.[B] 60.[A]

61.[B] 62.[D] 63.[A] 64.[B]

65. $\sqrt[2]{\dfrac{g^2-ac}{a(a+b)}}(or)\sqrt[2]{\dfrac{f^2-bc}{b(a+b)}}$

66. $\left[\dfrac{hd-bg}{ab-h^2},\dfrac{gh-af}{ab-h^2}\right]$ 67. $\theta=\cos^{-1}(4/5) or Tan^{-1}(3/4)$

68. $\theta=\cos^{-1}\left(\dfrac{4}{\sqrt{65}}\right) or Tan^{-1}\left(\dfrac{7}{4}\right)$ 69. $ax^2+2hxy+by^2=0$

70. $bx^2-2hxy+ay^2=0$ 71.[k=3] 72. $2x^2+3xy-2y^2=0$

73.[B] 74. $\theta=Tan^{-1}(3)$ 75.[A]

76.4 (or) –1 77.[c] 78.[A] 79.[True] 80.circle or curve

81. $8x^2-xy-8y^2=0$ 82.xy = 0 83.3 84. $\theta=\dfrac{\pi}{2}(98)$

85. $\theta=Tan^{-1}\left(\dfrac{2\sqrt{6}}{13}\right)$ or $\cos^{-1}\dfrac{13}{\sqrt{193}}$ 86. $a^2\left(l^2+m^2\right)=2$ 87. $a^2\left(l^2+m^2\right)=1$

88. $4x^2-y^2=0$

PREVIOUS COMPETITIVE QUESTIONS

PAIR OF STRAIGHT LINES

(S.V.Satyanarayana, JL in Maths, GJC, Uppugunduru, Prakasam Dt, Cell: 9866624268)

I. Equations of a pair of lines passing through origin Angle between a pair of lines

1. The point of intersection of the straight lines represented by

$6x^2+xy-40y^2-35x-83y+11=0$ is [EAM 1997]

a) (3, 1) b) (3, –1) c) (–3,1) d)(–3, –1)

2. The angle between the pair of lines $2(x+2)^2+3(x+2)(y-2)-2(y-2)^2=0$ is

[EAM 1997]

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

3. If a+b = 2h, then the area of the triangle formed by the lines $ax^2+2hxy+by^2=0$ and the line $x-y+2=0$ in sq. Units is [EAM 1998]

a) $\left|\frac{a+b}{a-b}\right|$ b) $\left|\frac{a^2+b^2}{a-b}\right|$ c) $\left|\frac{a-b}{a+b}\right|$ d) $\left|\frac{a^2+b^2}{a+b}\right|$

4. The equation of the pair of lines through (1, –1) and perpendicular to the pair of lines

$x^2-xy-2y^2=0$ is ____ [EAM 1998]

a) $2x^2-xy+y^2+5x+y+2=0$ b) $2x^2-xy-y^2-5x-y+2=0$

c) $x^2-xy+2y^2-5x-y-2=0$ d) $2x^2-xy-y^2+5x+y-2=0$

5. Equation of the line common to pair of lines

$(p^2-q^2)x^2+(q^2-r^2)xy+(r^2-p^2)y^2=0$ and $(l-m)x^2+(m-n)xy+(n-l)y^2=0$

is ___ [EAM 1998]

a) x –y =0 b) x+y=0 c) x =2y d) 2x–2y

6. If $ax^2+5xy-6y^2-10x+11y+c=0$ represents a pair of perpendicular lines then c =

_____ [EAM 1999]

a) 2 b) –2 c) 4 d) –4

7. If the equation $\lambda x^2-5xy+6y^2+x-3y=0$ represents a pair of straight lines then their point of intersection [EAM 2000]

a) (–3, –1) b) (–1, –3) c) (3, 1) d) (1, 3)

8. The equation of the pair of lines through the point (a, b) parallel to the coordinate axes is [EAM 2000]

a) $(x-b)(y-a)=0$ b) $(x-a)(y+b)=0$

c) $(x-a)(y-b)=0$ d) $(x+a)(y-b)=0$

9. The ortho centre of the triangle formed by the lines $x+3y-10=0$ and $6x^2+xy-y^2=0$ is [EAMCET 2001]

a) (1, 3) b) (3, 1) c) (–1, 3) d) (1, –3)

10. If one of lines is $ax^2+2hxy+by^2=0$ bisects the angle between the coordinate axes then $(a+b)^2=$_____ [EAMCET 2001]

a) $2h^2$ b) h^2 c) $3h^2$ d) $4h^2$

11. The angle between the pair of lines $x^2+4xy+y^2=0$ is [AIEEE 2002]

a) 30^0 b) 45^0 c) 60^0 d) 90^0

12. If the pair of straight lines $xy-x-y+1=0$ and the line $ax+2y-3=0$ is [EAMCET 2002]

a) –1 b) 3 c) 1 d) 0

13. The distance between the pair of parallel lines $9x^2-24xy+16y^2-12x+16y-12=0$ is

a) 5 b) 8 c) 8/5 d) 5/8

14. If the coordinate axes are the bisectors of the angles between the pair of lines $ax^2+2hxy+by^2=0$, when $h^2>ab$ and $a\neq b$ then

a) $a+b=0$ b) h = 0 c) $h\neq 0, a+b=0$ d) $a+b\neq 0$

15. The pair of lines represented by $3ax^2+5xy+(a^2-2)y^2=0$ are perpendicular to each other for

a) two values of a b) $\forall a$

c) for one value of a d) for no values

16. If the pairs of straight lines $x^2-2pxy-y^2=0$ and $x^2-29xy-y^2=0$ be such that each pair bisects the angle between the other pair, then [AIEEE 2003]

a) $p=q$ b) pq = 1 c) pq = –1 d) p = –q

17. If the pair of lines $ax^2+2hxy+by^2=0\left(h^2>ab\right)$ forms an equilateral triangle with the line $lx+my+n=0$ then $(a+3b)(3a+b)=$ _______ [AIEEE 2003]

a) h^2 b) $2h^2$` c) $3h^2$ d) $4h^2$

18. Area of qurdri lateral formed by the pair of lines $a^2x^2-b^2y^2-c(ax+by)=0$ and $a^2x^2-b^2y^2+c(ax-by)=0$ is [EAM 2003]

a) $\frac{c^2}{|ab|}$ b) $\frac{2c^2}{|ab|}$ c) $\frac{c^2}{2|ab|}$ d) $\frac{4c^2}{|ab|}$

19. If the sum of the slopes of the lines given by $x^2 - 20xy - 7y^2 = 0$ is four times their product, then ‘e’ has the value [AIEEE 2004]

a) –2 b) –1 c) 2 d) 1

20. If one of the lines given by $6x^2 - xy + 4cy^2 = 0$ is $3x + 4y = 0$ then ‘c’ equals to [AIEEE 2004]

a) –3 b) –1 c) 3 d) 1

21. Angle between the lines $x^2\left[\cos^2\theta - 1\right] - xy\sin 2\theta + y^2\sin^2\theta = 0$ is [AIEEE 2004]

a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{2}$

22. Area of the triangle formed by the line $3x^2 - 4xy + y^2 = 0$, 2x–y=6 is [EAM 2004]

a) 16 b) 25 c) 36 d) 49

23. The lines represented by the equation $x^2 - y^2 - x + 3y - 2 = 0$ are [EAM 2006]

a) $x + y - 1 = 0$, $x - y + 2 = 0$ b) $x - y - 2 = 0$, $x + y + 1 = 0$

c) $x + y + 2 = 0$, $x - y - 1 = 0$ d) $x - y + 1 = 0$, $x + y - 2 = 0$

24. if one of the lines of $my^2 + (1-m)^2 xy - mx^2 = 0$ is a bisector of angle between the lines xy= 0 then ‘m’ is [EAM 2006]

a) 1 b) 2 c) $-\frac{1}{2}$ d) 2

25. If the lines $x^2 + 2xy - 35y^2 - 4x + 44y - 12 = 0$ and $5x + \lambda y - 8 = 0$ are concurrent then $\lambda =$ [EAM 2007]

a) 0 b) 1 c) –1 d) 2

26. The value of λ such that $\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$ represents a pair of straight lines is [EAM 2008]

a) 1 b) –1 c) 2 d) –2

27. The angle between pair of lines by joining the points of intersection of $x^2 + y^2 = 4$ and y = 3x+c to the origin is a right angle then $c^2 =$ _____ [EAM 2007]

a) 20 b) 13 c) $\frac{1}{5}$ d) 5

28. A pair of perpendicular straight lines passes through the origin and also through the point of intersection of the curve $x^2+y^2=4$ with x+y=a. The set containing the value of 'a' is [EAM 2008]

a) {–2, 2} b) {–3, 3} c) {–4, 4} d) {–5, 5}

29. The area of triangle formed by x+y+1=0 and $x^2-3xy+2y^2=0$ is [EAM 2009]

a) $\frac{7}{12}$ b) $\frac{5}{12}$ c) $\frac{1}{12}$ d) $\frac{1}{6}$

30. The value of λ $(|\lambda|<1)$ such that $2x^2-10xy+12y^2+5x+\lambda y-3=0$ represents a pair of lines is [EAM 2009]

a) –10 b) –9 c) 10 d) 9

31. The figure formed by the pairs of lines $2x^2+3xy-2y^2=0$ and $2x^2+3xy-2y^2-5x+15y-25=0$ is [EAM 2009]

a) parallelogram b) Rhombus

c) Rectangle d) square

32. Two pairs of straight lines with combined equations $xy+4x-3y-12=0$ and $xy-3x+4y-12=0$ form a square then the combined equations of its diagonal is [TSE – 2015]

a) $x^2-3x+4y-12=0$ b) $x^2+2xy+y^2++x+y=0$

c) $x^2-y^2+x-y=0$ d) $x^2-y^2+x+y=0$ 3

33. The angle between the straight lines represented by $\left(x^2+y^2\right)\sin^2\alpha=\left(x\cos\alpha-y\sin\alpha\right)^2$ is [APE – 2015]

a) $\frac{\alpha}{2}$ b) α c) 2α d) $\frac{\pi}{2}$

34. The equation of the pair of straight lines through the point (1, 1) and perpendicular to the pair of straight lines $3x^2-8xy+5y^2=0$ is [TSE–2016]

a) $5x^2+8xy+3y^2-14x-18y+16=0$

b) $5x^2+8xy+3y^2-18x-14y+16=0$

c) $5x^2-8xy+3y^2-18x-14y+32=0$

d) $5x^2-8xy+3y^2-14x-18y+32=0$

35. If each line of a pair of lines passing through origin is at a perpendicular distance of 4 units from the point (3, 4) , then the equation of the pair of lines is [APEAM 2019]

a) $7x^2+24xy=0$ b) $7y^2+24xy=0$

c) $7y^2 - 24xy = 0$ d) $7x^2 - 24xy = 0$

36. The straight line $x + y + 1 = 0$ bisects an angle between a pair of lines, of which one is $2x - 3y + 4 = 0$ Then the equation of the other line in that pair is [APEAM 2019]

a) $2x + 3y + 4 = 0$ b) $x - y + 1 = 0$

c) $5x - 5y + 9 = 0$ d) $3x - 2y + 5 = 0$

37. If the pairs of straight lines represented by $3x^2 + 2hxy - 3y^2 = 0$ and $3x^2 + 2hxy - 3y^2 + 2x - 4y + c = 0$ form a square then (h, c) = [APEAM 2019]

a) (4, –1) b) (–1, 4) c) (–4, 1) d) (1, –4)

38. The equation of the bisectors of the angle between the lines joining the origin to the points of intersection of the curve $x^2 + xy + y^2 + x + 3y + 1 = 0$ and the line $x + y + 2 = 0$ is [APEAM 2019]

a) $x^2 + 4xy - y^2 = 0$ b) $2x^2 + 5xy - y^2 = 0$

c) $x^2 + 6xy - 2y^2 = 0$ d) $2x^2 - 4xy + 2y^2 = 0$

39. The combined equation of two lines L and L_1 is $2x^2 + xy + y^2 + x + 3y + 1 = 0$ and the line $x + y + 2 = 0$ is [APEAM 2019]

a) $x^2 + 3y + 1 = 0$ b) $2x^2 + 5xy - y^2 = 0$

c) $x^2 + 6xy - 2y^2 = 0$ d) $2x^2 - 4xy + 2y^2 = 0$

40. If the pair of lines joining the origin and the points of intersection of the line ax+by=1 and the curve $x^2 + y^2 - x - y - 1 = 0$ are at right angles then the locus of the point (a, b) is a circle of radius is [APEAM 2019]

a) 2 b) $\sqrt{\frac{3}{2}}$ c) $\sqrt{\frac{5}{2}}$ d) $\frac{\sqrt{5}}{2}$

41. The distance of lines joining the origin and the points of intersection of the line ax+by=1 and the curve $x^2 + y^2 - x - y - 1 = 0$ is

a) $4\sqrt{2}$ b) $2\sqrt{2}$ c) 2 d) $2\sqrt{6}$

42. A pair of lines S = 0 together with the lines given by the equation $8x^2 - 14xy + 3y^2 + 10x + 10y - 25 = 0$ from a parallelogram. If its diagonals intersect at the point(3, 2), then the equation S = 0 is [APEAM 2019]

a) $6x^2 - 9xy + y^2 - 25x + 30y + 25 = 0$ b) $8x^2 - 14xy + 3y^2 - 25x + 30y + 50 = 0$

c) $8x^2 - 14xy + 3y^2 - 50x + 50y + 75 = 0$ d) $6x^2 + 14xy - 3y^2 - 30x + 40y - 75 = 0$

43. The line $3x+4y-5=0$ cuts the curve $2x^2+3y^2=5$ at A and B. If ‘O’ is the origin then $\angle AOB=$ [APEAM 2019]

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{8}$

44. The distance from the origin to the orthocentre of the triangle formed by the lines x+y–1=0 and $6x^2-13xy+5y^2=0$ is [APEAM 2019]

a) $\frac{11\sqrt{2}}{2}$ b) 13 c) 11 d) $\frac{11\sqrt{2}}{24}$

45. If A is the orthocentre of the triangle formed by $2x^2-y^2=0, x+y-1=0$ and B is the centroid of the triangle formed by $2x^2-5xy+2y^2=0, 7x-2y-12=0$ then the distance between A and B is [APEAM 2019]

a) $\sqrt{5}$ b) 1 c) 5 d) $\sqrt{2}$

46. If the pair of lines $ax^2+2(a+b)xy+by^2=0$ lie along diameters of a circle and divide the circle into four sectors such that area of one of the sectors in thrice the area of another sector then [AIEEE 2005]

a) $3a^2+10ab+3b^2$ b) $3a^2+2ab+3b^2$

c) $3a^2-10ab+3b^2$ d) $3a^2-2ab+3b^2$

47. The pair of lines $lx^2+2(l+m)xy+my^2=0$ lies along two diameters of a circle and divides the circle into 4 sectors If the area of bigger sector is 5 times the area of smaller sector then $\frac{lm}{(l+m)^2}=$ [APEAM 2019]

a) $\frac{1}{2}$ b) $\frac{2}{\sqrt{3}}$ c) $\frac{11}{12}$ d) $\frac{13}{12}$

Key to Previous Competitive Questions

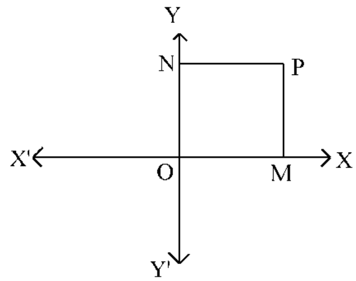
Pair of Strait lines

1.B	2.D	3. C	4. A	5. A
6. D	7. A	8. C	9. A	10. D
11.C	12. C	13.C	14. B	15. A
16. C	17.D	18. C	19. C	20. A
21. D	22. C	23. D	24. A	25. D
26. C	27. A	28. A	29. C	30. B
31. D	32. C	33.C	34. B	35. B
36.D	37. A	38. A	39. A	40. C
41. C	42.C	43. C	44. D	45.A
46. B	47.C			

Chapter: 3-Dimensional Co-ordinates

2–Dimensional System:

We know that in 2–Dimensional system, lines $X'OX, Y'OY$ are the coordinate axes and 'O' is the origin and these lines determine the XY–plane.



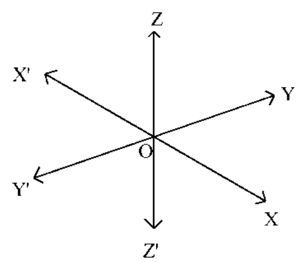
Let P be any point in XY–plane and M, N are the feet of the perpendicular of P to X,Y-axes respectively.

If $OM = |x|, ON = |y|$ then the coordinates of P are (x, y) and conversely P is (x,y)

Then $OM = |x|, ON = |y|$

3–Dimensional System:

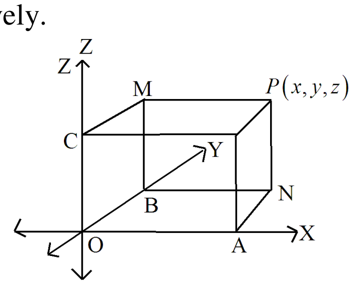
Draw a line $Z'OZ$ which is perpendicular to the XOY–plane and passing through the origin.



Now, these 3-mutually perpendicular lines represent the Rectangular coordinate axes of the 3-Dimensional system

Co-ordinates of a point:

Let $P(x, y, z)$ be any point in the space. Draw the planes which are parallel to the XY, YZ, ZX–planes and passing through P, and let these planes meet the X,Y,Z–axes at A,B,C respectively.



Plane parallel to XY–plane is PLCM
Plane parallel to YZ–plane is PLAN
Plane parallel to ZX–plane is PMBN

Since OA is $\perp er$ to the plane PLAN, so it is $\perp er$ to the every line on that plane and in particular to the line PA

i.e. $OA \perp PA$
$\therefore$ A is the foot of the $\perp er$ of P to x–axis
$\therefore$ OA = |x co ordinate of p| = | x |
And A = (x, 0, 0)
Similarly B, C will be the feet of the $\perp er$ of P to y,z-axes respectively
$\therefore$ $OB = |y|$, $OC = |z|$ and B=(0, y, 0), C(0, 0, z)

Conversely, Let P be a point in the space, A,B,C are the feet of the $\perp r$ s drawn from P to the X,Y,Z –axes and $OA = |x|, OB = |y|, OC = |z|$
Then the co ordinates of P are (x, y, z)

Note:

****Sign of x,y,z be according at A,B,C lie on the '+ve' or '–ve' axes of X,Y,Z

*** $OA = |x|, OB = |y|, OC = |z|$ are the perpendicular distances from the origin to the feet of the $\perp ers$ of P to X,Y,Z–axis

Key concepts and Formulae:

1. P(x, y, z) be a point in space. The $\perp er$ distances of p from yz,zx,xy–planes are $|x|, |y|, |z|$ respectively
   Since, the $\perp r$ distance of p from the
   (i) yz–plane = $PM = OA = |x|$
   (ii)zx–plane = $PN = OB = |y|$
   (iii)xy–plane = $PL = OC = |z|$

2. Every point that lies in xy–plane is of the form (x,y,0)
   Since, if P(x,y,z) lies in xy–plane, then
   The $\perp r$ distance of p from xy-plane = 0
   $\Rightarrow |z| = 0 \Rightarrow z = 0$
   i.e. the z-coordinate of every point in xy-plane is 'o'
   lly every point lies in yz-plane is of the form (x, 0, 0)
   every point lies in zx-plane is of the form (x, 0, z)

3. Every point lies on x-axis is of the form (x, 0, 0)
   Since, if p(x,y,z) lies on x-axis then
   The $\perp r$ distances of p from zx and xy-plane = 0
   $\Rightarrow |y| = 0$ and $|z| = 0 \Rightarrow y = 0$ and z = 0
   The y and z coordinates are 'o'
   lly every point on y-axis is of the form (0, y, 0)
   z-axis is of the form (0, 0, z)

Distance Formula:

1. The distance between any two points $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ in the space is

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

2. The $\perp r$ distance of P(x, y, z) to the x-axis = $\sqrt{y^2+z^2}$
From the diagram,
$\perp r$ distance of p to x-axis = PA, where A = (x, 0, 0)

$$=\sqrt{(x-a)^2+(y-0)^2+(z-0)^2}$$

$$=\sqrt{0+y^2+z^2}=\sqrt{y^2+z^2}$$

lly we can find to the y-axis and z-axis

Section Formula:

3. The point dividing the line segment $\overline{AB}$, where $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ in the ratio $l:m$
(i) internally is $\left(\frac{lx_2+mx_1}{l+m} \quad \frac{ly_2+my_1}{l+m} \quad \frac{lz_2+mz_1}{l+m}\right)$
(ii) Externally is $\left(\frac{lx_2-mx_1}{l-m} \quad \frac{ly_2-my_1}{l-m} \quad \frac{lz_2-mz_1}{l-m}\right)$

4. The point dividing $\overline{AB}$ in $K:1$ ratio is $\left(\frac{kx_2+x_1}{k+1} \quad \frac{ky_2+y_1}{k+1} \quad \frac{kz_2+z_1}{k+1}\right)$

5. Mid point of $\overline{AB}$ is $\left(\frac{x_1+x_2}{2} \quad \frac{y_1+y_2}{2} \quad \frac{z_1+z_2}{2}\right)$

6. If $P(x, y, z)$ lies in the line joining A, B their $\frac{x_1-x}{x-x_2}=\frac{y_1-y}{y-y_2}=\frac{z_1-z}{z-z_2}$ and P divides $\overline{AB}$ in the ratio $(x_1-x):(x-x_2)$ (or) $y_1-y:y-y_2$ (or) $z_1-z:z-z_2$

7. If P divides AB internally in the ratio $l:m$, where
As Q divides externally in the same ratio then P and Q are harmonic conjugate points of A and B and vice-versa

8. centroid of a Δle with vertices $(x_1 \ y_1 \ z_1)$, $i=1,2,3$ is $\left(\frac{\sum x_i}{3} \quad \frac{\sum y_i}{3} \quad \frac{\sum z_i}{3}\right)$

Tetrahedron:

Let ABC be a triangle and D is a point in the space which is not in the plane of the ΔABC, then ABCD is called a tetrahedron. A, B, C, D are the vertices AB, BC, CA, AD, BD, CD are the Edges ABC, ABD,ACD, BCD are the Faces of the tetrahedron

If all the 6-Edges are equal then it is known as a regular tetrahedron.

Centroid of the Tetrahedron

The concurrent point of the line segments joining the vertices to the centroids of opposite faces (Δle) is called the centroid of the tetrahedron

This point divides each line segment in the ratio 3 : 1

8. Centroid of the tetrahedron whose vertices are $(x_i \quad y_i \quad z_i)$ i = 1,2,3,4 is

$$\left(\frac{\sum x_i}{4} \quad \frac{\sum y_i}{4} \quad \frac{\sum z_i}{4}\right)$$

Translation of Axes:

9. If the coordinates $(x \quad y \quad z)$ of a point are transformed to $(x \quad y \quad z)$ when the axes are translated by shifting the origin to the point $(h \quad k \quad l)$ then

(i) $\left.\begin{array}{l} X = x-h \\ Y = y-k \\ Z = z-l \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = X+h \\ y = Y+k \\ z = Z+l \end{array}\right\} \Rightarrow \begin{array}{l} h = x-X \\ k = y-Y \\ l = z-Z \end{array}$

(ii) The equation of $f(x_1 \ y_1 \ z_1) = 0$ of a surface is transformed to $f(x+h \quad y+k \quad z+l) = 0$

Note:

1. If (a, b, c) is the midpoint of $\overline{AB}$, where $A(x_1, y_1, z_1)$ then
$B = (2a - x_1 \quad 2b - y_1 \quad 2c - z_1)$

2. If $D(a_1, b_1, c_1), E(a_2, b_2, c_2), F(a_3, b_3, c_3)$ are the midpoint of the sides BC, CA, AB respectively of ΔABC then
$A = (a_2 + a_3 - a_1 \quad b_2 - b_1 + b_3 \quad c_2 + c_3 - c_1)$
$B = (a_3 + a_1 - a_2 \quad b_3 + b_1 - b_2 \quad c_3 + c_1 - c_2)$
$C = (a_1 + a_2 - a_3 \quad b_1 + b_2 - b_3 \quad c_1 + c_2 - c_3)$

3. If G(a, b, c) is the centroid of ΔABC and $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ and then
$C = (3a - x_1 - x_2 \quad 3b - y_1 - y_2 \quad 3c - z_1 - z_2)$

4. $A(x_1, y_1, z_1), B(x_2, y_2, z_2), C(x_3, y_3, z_3)$
(i)A,B,C are 3–conseculative vertices of a parallelogram then the 4th vertex is
$D = (x_1 - x_2 + x_3 \quad y_1 - y_2 + y_3 \quad z_1 - z_2 + z_3)$
($\because$ Midpoint of AC = Midpoint of BD)
(ii) A,B,C are 3–vertices and G(a,b,c) is the centroid of a tetrahedran then the 4th vertex

$$D = (4a - x_1 - x_2 - x_3 \quad 4b - y_1 - y_2 - y_3 \quad 4c - z_1 - z_2 - z_3)$$

5. The line segment joining $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ is divided by the
(i) XY–plane in the ratio $-Z_1 : Z_2$
(ii) YZ–plane in the ratio $-X_1 : X_2$
(iii) ZX–plane in the ratio $-Y_1 : Y_2$
Let YZ-plane divides $\overline{AB}$ at the point P in $m:n$ ratio then

$$P = \left(\frac{mx_2 + nx_1}{m+n} \quad \frac{my_2 + ny_1}{m+n} \quad \frac{mz_2 + nz_1}{m+n}\right)$$

Since P lies in YZ–plane, its x coordinate = 0 i.e. $\frac{mx_2+nx_1}{m+n}=0 \Rightarrow mx_2+nx_1=0$

$\Rightarrow mx_2=-nx_1$

$\Rightarrow \frac{m}{n}=\frac{-x_1}{x_2}$

6. **Incentre of a triangle:**

If a, b, c are the sides of a ΔABC, where $A=(x_1,\ y_1,\ z_1), B=(x_2,\ y_2,\ z_2), C=(x_3,\ y_3,\ z_3)$ are the vertices, then the incentre of the triangle is $I=\left(\frac{ax_1+bx_2+cx_3}{a+b+c}\quad \frac{ay_1+by_2+cy_3}{a+b+c}\quad \frac{az_1+bz_2+cz_3}{a+b+c}\right)$

I. Fill up the Blanks:

1. Distance from the origin to the point P(x, y, z) is __________
2. The locus of P, where distance from y–axis is thrice its distance from (1, 2, –1) is ________
3. If all edges of a Tetrahedron are equal then it is called __________
4. A tetrahedron has how many edges? __________
5. If (2, 4, –1), (3, 6, –1) and (4, 5, 1) are the consecutre vertices of a peralellogram then its 4^{th} vertex is _____
6. The ratio in which XZ–plane divides then line joining A(–2, 3, 4) and B(1, 2, 3) is ____
7. The distance of the point (6, 2, –1) from the z–axis is __________
8. If x–coordinate of a point p on the line joining the points Q(2, 2, 1) and R(5, 1, –2) is 4 then the Z–coordinate of P is ___________

II. Multiple choice Questions:

1. The points $A(-4,\ 9,\ 6), B(-1,\ 6,\ 6), C(0,\ 7,10)$ form a

a) right angle Δle b) right angle isosceles
c) isosceles d) All the above

2. $A(1,\ 2,\ 3)\,B(2,3,\ 1), C(3,\ 1,\ 2)$ form

a) An equilateral b) isosceles Δle
c) scalan Δle d) right angled Δle

3. The point dividing the joing of $(3,\ -2,\ 1)$ and $(-2,\ 3,\ 11)$ in the ratio $2:3$ is

a) $(1\ \ 1\ \ 4)$ b) $(1\ \ 0\ \ 5)$ c) $(2\ \ 3\ \ 5)$ d) $(0\ \ 6\ \ -1)$

4. The point collinear with $(1\ \ -2\ \ -3)$ and $(2\ \ 0\ \ 0)$ among the following is

a) $(0\ \ 4\ \ 6)$ b) $(0\ \ -2\ \ -5)$ c) $(0\ \ -4\ \ -6)$ d) $(0\ \ -4\ \ 6)$

5. If the extremities of a diagonal of a square are $(1\ \ -2\ \ 3)(2\ \ -3\ \ 5)$ then length of its side is

a) $\sqrt{6}$ b) $\sqrt{3}$ c) $\sqrt{5}$ d) $\sqrt{7}$

6. If the line joining A(1 3 4) and B is divided by the point (–2. 3, 5) in the ratio $1:3$ then B is

a) (–11, 3, 8) b) (–8, 12, 20) c) (13, 6, –13) d) (–11, 3, 8)

7. The harmonic conjugate of (2,, 3, 4) w.r.t. the points (3, –2, 2) (6, –17, –4) is

a) $\left(\frac{18}{5},\ -5,\ \frac{4}{5}\right)$ b) $(11,\ -6,\ \ 2)$

c) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\right)$ d) $(0, 0, 0)$

8. If the centroid of a tetrahedron is (2, 3, 4) for which (2, 3, –1) (3, 3, –2), (–1, 4, 3) are three vertices then the fourth vertex is

a) $(4\ 5\ 16)$ b) $(3\ 2\ 4)$ c) $(2\ 3\ 4)$ d) $(2\ 2\ 12)$

III. Matching the following:

List – I	List – II
1. The distance between the points $(\sin\alpha\ \cos\alpha\ 0), (\cos\alpha\ -\sin\alpha\ 0)$ is	a) –2, –1
2. The ratio is which (2, 3, 4) divides the line Segment joining (3 –2 2) (6, –17, –4) is	b) 8
3. XOZ–plane divides the join of (2, 3 1) and (6, 7, 1) in the ratio	c) 2 : 1
4. If $A(1, 2, 3)B(7, 0, 1), C(-2, 3, 4)$ are collinear then the ratio in which A divides $\overline{BC}$ is	d) 1 : 4
5. The line passing through the points (5, 1, a) and (3, b, 1) crosses yz–plane at the point $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$ then a, b value respectively	e) $\frac{1}{2}\sqrt{41}$
6. In ΔABC, the mid-point of the sides AB, BC, CA are respectively. $(l, 0, 0), (0, m, 0), (0, 0, n)$ their $\left(AB^2 + BC^2 + CA^2\right) / \left(l^2 + m^2 + n^2\right) =$	f) 6, 4
7. The circumradius of the triangle formed by the points $(2, -1, 1)(1, -3, -5), (3, -4, -4)$ is	g) $\sqrt{2}$
8. If $(k, 1, 5), (1, 0, 3), (7, -2, 1)$ are collinear then k = , l =	h) –3 : 7

Answers (KEY)

I. Fill up the blanks:

1.
2.
3.
4.
5.
6.
7.
8.

II. Key for Multiple choices:

1.
2.

3.
4.
5.
6.
7.
8.
Solutions:
3.
4.
5.
6.
7.
8.

III. Key for Match the following:

1.g	2. d	3. H	4. c	5.f
6. b	7.	8. A		

Solutions:
1.
2.
3.
4.
5.
6.
7.
8.

Direction Cosines (DCs)

If α, β, γ are the angles made by a directed line segment with the positive direction of the coordinate axes respectively, then $\cos\alpha, \cos\beta, \cos\gamma$ are called the Direction Cosines (DC's) of that directed line segment and they are denoted by *l,m,n* respectively. Thus

$$l = \cos\alpha, m = \cos\beta, n = \cos\gamma$$

If the direction AB are (l, m, n) then the direction consines of line segment BA are $(-l, -m, -n)$. Thus a line can have two sets of DCs according to its directin.

Direction Ratios (DRs)

If *a,b,c* are three numbers proportional to the Direction Cosines *l,m,n* of a straight line, then a,b,c are called its Direction Ratios(DRs). A given line can have infinitely many Direction Ratios. If l,m,n are the DCs and a,b,c are the DRs of line, then $\frac{a}{l} = \frac{b}{m} = \frac{c}{n}$

Key Points:

1. The DCs of line always lie in the interval [–1, 1]
2. If $\cos\alpha = l, \cos\beta = m, \cos\gamma = n$ are the DCs of a line, then

   i) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ or $l^2 + m^2 + n^2 = 1$

   ii) $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
3. Direction Cosines of (i) X-axis are (1, 0, 0) ii) Y-axis are (0, 1, 0)
   iii) Z-axis are (0, 1, 0)
4. If $P(x, y, z)$ be any point in the space and

   $r = OP = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ then the DCs of OP will be $\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$

5. If OP= r and the DCs of OP are (*l, m, n*) then the coordinates of P are (lr, mr, nr).
6. The direction cosines of the line joining the points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$

   where r = OP = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$
7. If (l_1, m_1, n_1) and (l_2, m_2, n_2) be the direction cosines of any two lines and

   i) If θ be the angle between them, then

   $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$ and

   $\sin\theta = \sqrt{(l_1 m_2 - l_2 m_2)^2 + (m_1 n_2 - m_2 n_1)^2 + (n_1 l_2 - l_2 n_1)^2}$

   ii) If the lines are perpendicular , then $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$

   iii) If the lines are parallel, then $\frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$
8. If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points, then the projection of line segment PQ on a line whose direction cosines are (l, m, n) is

   $l(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$
9. If a, b, c are the DRs and *l,m,n* are the DCs of a straight line respectively, then

   $$(l, m, n) = \pm\left(\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}\right)$$
10. If $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are two points, then the

    DRs of $PQ = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$
11. If $(a_1, b_1, c_1), (a_2, b_2, c_2)$ are the DRs of two straight lines and

    i) if θ be the angle between them, then $\cos\theta = \frac{a_1 a_2 + b_1 b_2 + c_1 c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2} + \sqrt{a_2^2 + b_2^2 + c_2^2}}$

    ii) If the lines are perpendicular, then $a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$

    iii) if the lines are parallel then $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Problems:

1. The direction ratios of line joining the points (3, 4, 0) and (4, 4, 4) are _______

2. If the direction ratios of a line are (0, –2, –3) then the direction cosines of the line are ________

3. The DCs of the line passing through two points (–2, –4, –5) and (1, 2, 3) are ______

4. If $\left(\frac{1}{c}, \frac{1}{c}, \frac{1}{c}\right)$ are the directions cosines of a straight line, then the value of c is _____

5. Under what condition do $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, k\right)$ represent DCs of a straight line? Ans ____

6. What are the direction cosies of a line which is equally inclined to the positive direction of the axes

a) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ b) $\left(-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

c) $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ d) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

7. Which of following can be the DCs of a straight line

a) $\left(1, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}\right)$ b) $\left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

c) $\left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$ d) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

8. The angle between the lines with direction ratios (1, –2, 1) and (4, 3, 2) is

a) 0^0 b) 60^0 c) 45^0 d) 90^0

9. If the points A(2, 3, 4), B(–1, –2, 1), C(5, 8, k) are collinear, then the value of k is___

10. A line makes angles α, β, γ with the positive direction of x, y, z axes respectively, then which of the following statements is correct? Ans : ____

1) $\sin^2\alpha + \sin^2\beta = \cos^2\gamma$ 2) $\cos^2\alpha + \cos^2\beta = \sin^2\gamma$ 3) $\sin^2\alpha + \cos^2\beta = \cos^2\gamma$ a)

a) 1 only b) 2 only c) 3 only d) 2 and 3

11. A line makes an angle of 60^0 with each of X–axis and Y–axis. Then what is the acute angle made by the line with Z-axis is _____ Ans: ________

12. If a line makes angles α, β and γ with the coordinate axes, then the value of $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is _________

13. The foot of the perpendicular the point {1, 6, 3} to the line $\frac{x}{1} = \frac{y-1}{5} = \frac{z-2}{3}$ is [ ]

a) (1, 3, 5) b) (–1, –1, –1) c) (2, 5, 8) d) –2, –3, –4)

14. The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$ meets the co-ordinate axes in A, B, C. The centroid of the triangle ABC is

a) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$ b) $\left(\frac{3}{a}, \frac{3}{b}, \frac{3}{c}\right)$ c) $\left(\frac{1}{a}, \frac{1}{b}, \frac{1}{c}\right)$ d) (a, b,, c)

15. The direction cosnes *l, m, n* of two lines are connected by the relations $l+m+n=0$, $lm=0$ Then the angle between them is

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) 0

16. If a line in the space makes angles α, β, γ with the co-ordinate axes then $\cos 2\alpha+\cos 2\beta+\cos 2\gamma+\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ equals

a) –1 b) 0 c) 1 d) 2

17. A line makes 45^0 with positive x-axis and makes equal angles with positive y,z axes respectively. What is the sum of the three angles which the line makes with positive x,y and z axes?

a) 180^0 b) 165^0 c) 150^0 d) 135^0

18. Let L be the line of intersection of the planes $2x+3y+z=1$ and $x+3y+2z=2$. If L makes an angle α with the positive x-axis, then $\cos\alpha$ equals

a) 1 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{2}$

19. What are the direction ratios of the line determined by the planes $x-y+2z=1$ and $x+y-z=3$?

a) $(-1,\ 3,\ 2)$ b) $(-1,\ -3,\ 2)$ c) $(2,\ 1,\ 3)$ d) $(2,\ 3,\ 2)$

20. A line makes the same angle α with each of the x and y axes. If the angle θ which it makes with the z-axis is such that $\sin^2\theta=2\sin^2\alpha$, then what is the value of α?

a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{2}$

PLANE

1. **General equation:** The general equation of a plane is $ax+by+cz+d=0$ (Here a,b,c are the direction ratios of normal to the plane)

   In vector form the general equation of plane is $\bar{r}.\bar{n}=p$ where $\bar{n}$ is a vector perpendicular to the plane

2. The equation of any plane parallel to $ax+by+cz+d=0$ is of the form $ax+by+cz+k=0$

3. The equation of the plane passing through (x_1, y_1, z_1) and perpendicular to a line with DRs (a, b, c) is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

4. Equation of a plane in normal form: If l, m, n be the direction cosines of the normal to a plane and p be the length of the perpendicular from the origin on the plane, then the equation of the plane is $lx+my+nz=p$

   In vector form Normal equation of plane is $\bar{r}.\bar{n}=p$ where $\bar{n}$ is unit vector perpendicular to the plane

5. The perpendicular distance from origin O(0, 0, 0) to the plane $ax+by+cz+d=0$

$$\frac{|d|}{\sqrt{a^2+b^2c^2}}$$

6. The perpendicular distance from $P(x_1, y_1, z_1)$ to the plane $ax+by+cz+d=0$ is

$$\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$

7. The distance between the parallel planes $ax+by+cz+d_1=0$ and $ax+by+cz+d_2=0$ is $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

8. The equation of the plane with x-intercept 'a', y-intercept 'b', z-intercept 'c' is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

9. The equation of the plane passing through three non-collinear points $A(x_1, y_1, z_1)$ is $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$

10. If θ is an angle between the planes $a_1x+b_1y+c_1z+d_1=0$, $a_2x+b_2y+c_2z+d_2=0$, then $\cos\theta=\frac{a_1a_2+b_1b_2+c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$

11. The planes $a_1x+b_1y+c_1z+d_1=0$, $a_2x+b_2y+c_2z+d_2=0$, are

i) parallel $\Leftrightarrow a_1:b_1:c_1=a_2:b_2:c_2$

ii) perpendicualar $\Leftrightarrow a_1a_2+b_1b_2+c_1c_2=0$

12. The equation of i) xy-plane is z = 0 ii) yz-plane is x = 0 iii) zx-plane is y = 0

13. The plane $ax+by+cz+d=0$ is parallel to i) x-axis if a = 0 ii) y-axis if b = 0 iii) z-axis if c = 0

14. The plane $ax+by+cz+d=0$ is parallel to i) x-axis is of the form $by+cz=d$ ii) y-axis is of the form $ax+cz=d$ iii) z-axis is of the form $ax+by=d$

15. **Symmetrical form of a straight line:** Equation of a straight line passing through a fixed point $A(x_1, y_1, z_1)$ and having direction ratios *a, b, c* is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

16. The image of reflection (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax+by+cz+d=0$ is given by $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=-2\left(\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}\right)$

17. The foot (x, y, z) of a point (x_1, y_1, z_1) on the plane $ax+by+cz+d=0$ is given by

$$\frac{x-x_i}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}=-\left(\frac{ax_1+by_1+cz_1+d}{a^2+b^2+c^2}\right)$$

18. Equation of the plane passing through the point (x_1, y_1, z_1) and parallel to the lines whose DRs are $(a_1, b_1, c_1), (a_2, b_2, c_2)$ is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

Problems:

1. The equation of the plane through (1, 2, 3) and parallel to the plane $2x+3y-4z=0$ is ________

2. Distance of the point (2, 3, 4) from the plane $3x-6y+2z+11=0$ is ______

3. Distance between the parallel planes $2x-2y+z+3=0$ and $4x-4y+2z+5=0$ is ______

4. The equation of the plane which is parallel to XY–plane and cuts intercept of length 3 from the Z-axis is _______

5. A point (x, y, z) moves parallel to XY-plane. Which of the three variables x,y,z remains fixed.

   a) z b) y c) x d) x and y

6. If a plane cuts off intercepts 6, 3,4 on the coordinate axes, then the length of the perpendicular from origin to the plane is

   a) $\frac{1}{\sqrt{61}}$ b) $\frac{13}{\sqrt{61}}$ c) $\frac{12}{\sqrt{29}}$ d) $\frac{5}{\sqrt{41}}$

7. In three dimensional space, the equation $3y+4z=0$ represents

   a) A plane containing x–axis

   b) A plane containing y–axis

   c) A plane containing z–axis

   d) A line with DRs 0, 3, 4

8. If a plane cuts off intercepts OA = a, OB = b, OC = c on the co-ordinate axes, then the area of the triangle ABC = _________

   a) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$ b) $\frac{1}{2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}$

   c) $\frac{1}{2}(bc+ca+ab)$ d) $\frac{1}{2}abc$

9. The angle between the planes $2x-y+z=6$ and $x+y+2z=7$ is ________

   a) 30^0 b) 45^0 c) 0^0 d) 60^0

10. The equation of the plane passing through the intersection of the planes $x+y+z=6$ and $2x+3y+4z+5=0$ and $(1, 1, 1)$ is _________

    a) $20x+23y+26z-69=0$ b) $20x+23y+26z+69=0$

    c) $23x+20y+26z-69=0$ d) none of these

11. The equation of the plane passing through the line of intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ and parallel to x-axis is ______

    a) $y-3z-6=0$ b) $y-3z+6=0$

c) $y-z-1=0$ d) $y-z+1=0$

12. The equation of the plane containing the line of intersection of the planes $2x-5y+z=3; x+y+4z=5$ and parallel to the plane $x+3y+6z=1$ is

a) $x+3y+6z=7$ b) $2x+6y+12z=-13$

c) $2x+6y+12z=13$ d) $x+3y+6z=-7$

13. Equation of the plane passing through the line of intersection of the planes $x+y+z=1$ and $2x+3y+z=5$ perpendicular to the plane $x-y-z=0$ is

a) $y-z+3=0$ b) $y-z-3=0$

c) $y+z+2=0$ d) $y+z-2=0$

14. Image of the point (1, 3, 4) in the plane $2x-y+z+3=0$ is _____

a) (3, 5, 2) b) (–3, 5, 2) c) (3, 5, 3) d) None of these

15. If Q is the image of the point P(2, 3, 4) in the plane $x-2y+5z=6$, then the equation of the line PQ is

a) $\frac{x-2}{-1}=\frac{y-3}{2}=\frac{z-4}{5}$ b) $\frac{x-2}{1}=\frac{y-3}{-2}=\frac{z-4}{5}$

c) $\frac{x-2}{-1}=\frac{y-3}{-2}=\frac{z-4}{5}$ d) $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z-4}{5}$

16. Which one of the following is the plane containing the line $\frac{x-2}{2}=\frac{y-3}{3}=\frac{z-4}{5}$ and parallel to Z–axos?

a) $2x-3y=0$ b) $5x-2z=0$ c) $5y-3z=0$ d) $3x-2y=0$

17. What is the distance from origin to the point of intersection of the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-2}{4}$ and the plane $x+2y+3z=15$?

a) $\frac{1}{2}$ b) $\frac{9}{2}$ c) $\frac{5}{2}$ d) 4

18. The lines $\frac{x-2}{1}=\frac{y-3}{1}=\frac{z-4}{-k}$ and $\frac{x-1}{k}=\frac{y-4}{2}=\frac{z-5}{1}$ are coplanar if

a) k = 0 or –1 b) k =1 or –1 c) k =0 or –3 d) k =3 or –3

19. If Q is the image of the point P(0, –1, –3) in the plane 3x–y+4z=2 and R is the point (3, –1, –2) then the area (in sq.units) of ΔPQR is:

a) $2\sqrt{13}$ b) $\frac{\sqrt{91}}{4}$ c) $\frac{\sqrt{91}}{2}$ d) $\frac{\sqrt{65}}{2}$

20. The mirror image of the point (1, 2, 3) in a plane is $\left(\frac{-7}{3}, \frac{-4}{3}, \frac{-1}{3}\right)$ which of the following points lies on this plane?

a) (1, –1, 1) b) (–1, –1, 1) c) (1, 1, 1) d) (–1, –1, –1)

Answers to DCs & DRs Problems

1. (1, 0, 4)	2. $\left(0, \frac{2}{\sqrt{3}}, \frac{-3}{\sqrt{3}}\right)$	3. $\left(\frac{3}{\sqrt{109}}, \frac{6}{\sqrt{109}}, \frac{8}{\sqrt{109}}\right)$		
4. $c = \pm\sqrt{3}$	5. C	6. A	7. C	8. D
9. K = 7	10. B	11. 45^0	12. –1	13.A
14. D	15. A	16. C	17. B	18. C
19. A	20. B			

Answers to Plane problems

1. $2x+3y-4z+4=0$		2. 1	3. $\frac{1}{6}$	4. Z=3
5. A	6. C	7. A	8. A	9. D
10. A	11. B	12. A	13. B	14.A
15. B	16. D	17. B	18.C	19.C
20. A				

Limits and Continuity
(P. Harinatha Achari, J.L.in Maths, SSJC, Tiruchanoor, Cell: 9440820071)

Level – I (I.P.E)

1. For $x \in R$, the modulus of a function x is denoted $|x|$ it is defined as $|x| = x$ if x<0 (Yes/No)

2. For $x \in R$ the step function (or) gratest integer function [x] is defined as [x]=n which is integral part of x such that $n \le x < n+1$ for an integer n. (Yes/No)

3. Let $a \in R$, If $\delta > 0$ be a small positive real number then $(a-\delta, a+\delta)$ is called δ .neighbourhood of a and $(a-\delta, a) \cup (a,\ a+\delta)$ is called deleted neighbour of a

4. let $f(x)$ be a real valued function defined in the deleted neighbourhood of 'a' and $l \in R$. If for any small $\varepsilon > 0$ correspondingly there exists small postline real $\delta > 0$ such that $0 < |x-a| < \delta \Rightarrow |f(x) - l| < \varepsilon$ then we say l is limit of f(x) as x approaches to a and it is denoted by : $\underset{x \to a}{Lt} f(x) = l$

5. Working rule for Left hand limit (L.H.L) Let h>0 is a small positive real number Replace x by a –h in f(x) and make $h \to 0$ i.e. $\underset{x \to a^+}{Lt} f(x) = \underset{h \to 0}{Lt} f(a-h)$ and for right hand limit $\underset{x \to a^+}{Lt} f(x) = \underset{h \to 0}{Lt} f(a+h)$

6. If Left hand limit and Right hand limit both exists and equal to a number K then limit of the function is : K

7. Find $\underset{x \to a^+}{Lt} \frac{|x|}{x} = 1$ and $\underset{x \to 0^-}{Lt} \frac{|x|}{x} = -1$ hence conclude the limit $\underset{x \to 0}{Lt} \frac{|x|}{x} =$ does not exist

8. Find $\underset{x \to 2^+}{Lt} ([x]+x) =$ 4 and $\underset{x \to 2^-}{Lt} ([x]+x) = = 3$ hence conclude the limit $\underset{x \to 2}{Lt} ([x]+x) =$ does not exist

9. **Match the following standard limits:**

List – I **List – I**

a) If $n \in R, a > 0$ then $\underset{x \to a}{Lt} \frac{x^n - a^n}{x-a} =$ i) 1

b) $\underset{x\to 0}{Lt}\,(1+x)^{\frac{1}{x}} =$ ii) e

c) $\underset{x\to 0}{Lt}\,(1+x)^{\frac{1}{x}} =$ iii) $x\cdot a^{n-1}$

d) $\underset{x\to 0}{Lt}\left(\frac{a^x-1}{x}\right) =$ iv) $\log_e a$

	a	b	c	d
1)	(iii)	(ii)	(iv)	(i)
2)	(iii)	(iv)	(ii)	(i)
3)	(iii)	(i)	(ii)	(iv)
4)	(i)	(iv)	(i)	(ii)

10. $\underset{x\to 0}{Lt}\,\frac{e^x-1}{x} = \log_e e = 1$ (Yes/No)

11. $\underset{x\to -\infty}{Lt}\left(\frac{2x+3}{\sqrt{x^2-1}}\right) = 2$ (Yes/No)

12. If f is continuos on the closed integral [a, b] then

i) f is continuous in (a, b)

ii) $\underset{x\to a^+}{Lt}\, f(x) = f(a)$

iii) $\underset{x\to b^-}{Lt}\, f(x) = f(b)$

13. If $\underset{x\to a^+}{Lt}\, f(x)$ and $\underset{x\to a^-}{Lt}\, f(x)$ exist but not equal then the function $f(x)$ at a is discontinueous

14. If $\underset{x\to a^+}{Lt}\, f(x)$ and $\underset{x\to a^-}{Lt}\, f(x)$ exist and are equal but not equal to $f(a)$ then $f(x)$ at x = a is discontinuous

15. If f defined by $f(x) = \begin{cases} \frac{\sin 2x}{x} & if \quad x \neq 0 \\ 1 & if \quad x = 0 \end{cases}$ continuous at 0?

16. If $f(x) = Tanx$ in continuous on $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ interval

17. While evaluating the limits if $\frac{f(a)}{g(a)}$ is in the indeterminate form $\frac{0}{0}(or)\,\frac{\infty}{\infty}$ then

$\underset{x\to a}{Lt}\,\frac{f(x)}{g(x)} = \underset{x\to a}{Lt}\,\frac{f'(x)}{g'(x)}$, If $\frac{f'(x)}{g'(x)}$ is again of the form $\frac{0}{0}(or)\,\frac{\infty}{\infty}$ then

$\underset{x\to a}{Lt}\,\frac{f(x)}{g(x)} = \underset{x\to a}{Lt}\,\frac{f''(x)}{g''(x)}$ etc, these process is called L–Hospital Rule

18. If $f(x)=\begin{cases}\dfrac{\sin(x)}{[3]} & [3]\neq 0\\ 0 & [x]=0\end{cases}$ where [x] is the greatest integer function then $\underset{x\to 0}{Lt}\, f(x)=$

a) –1 b) 0 c) 1 d) does not exist

19. If $0<x<y$ then $\underset{x\to 0}{Lt}\left(y^n+x^n\right)^{1/n}=$

a) 1 b) x c) y d) e

20. $\underset{x\to 1}{Lt}\dfrac{x^2-1}{|x-1|}=$

a) 0 b) –2 c) 2 d) does not exist

21. If a > 0 and $\underset{x\to a}{lit}\dfrac{a^x-x^a}{x^x-a^a}=-1$ then a = ________

a) 0 b) 1 c) a d) 2e

22. $\underset{x\to\infty}{Lt}\left(\sqrt{x^2+2x-1}-x\right)=$

a) ∞ b) $\dfrac{1}{2}$ c) 4 d) 1

23. The values of a and b so that $\underset{x\to\infty}{Lt}\left(\dfrac{x^2+1}{x+1}-ax-b\right)=0$ are

a) 1, –1 b) 1 c) 2 d) –1

24. If $\Delta(x)=\begin{vmatrix} e^x & -1\\ \sin x-1 & 1\end{vmatrix}$ then $\underset{x\to 0}{Lt}\dfrac{\Delta(x)}{x}=$

a) 0 b) 1 c) 2 d) –1

25. $\underset{x\to\sqrt{5}}{Lt}\dfrac{\sqrt{2x+\sqrt{5}}-\sqrt{3x}}{\sqrt{3\sqrt{5}+x}-2\sqrt{x}}=$

a) $\dfrac{6}{\sqrt{3}}$ b) $\dfrac{2}{3\sqrt{3}}$ c) $\dfrac{2}{3}\sqrt{3}$ d) $\dfrac{\sqrt{3}}{2}$

26. $\underset{n\to\infty}{Lt}\dfrac{2\cdot 5^{n+1}-3\cdot 7^{n+1}}{2\cdot 5^n+3\cdot 7^n}=$

a) 2 b) –3 c) –5 d) –7

27. If $f(x)=\begin{cases}2px+3 & for \quad x<1\\ 1-px^2 & for \quad x>1\end{cases}$ and $\underset{x\to 1}{Lt}\, f(x)$ exist then p =

a) $\dfrac{-3}{2}$ b) $\dfrac{-2}{3}$ c) $\dfrac{2}{3}$ d) $\dfrac{3}{2}$

28. If $f:R\to R$ is defined by $f(x)=\min\left\{1,x^2,x^3\right\}$ then

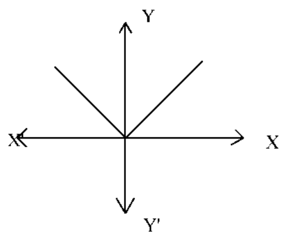
a) f is continuous for all $x \in R$ b) f is continuous for all $x \in R-[1]$

c) f is continuous for all $x \in R-[1]$ d) f is continuous for all $x \in R-\{-1, 0, 1\}$

29. If $f(x)=\frac{|x|}{[x]}$, $x \in (0,1)$ then $\underset{x \to 2^+}{Lt} \frac{f(x)-f(2)}{x-2}$

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) 1 d) does not exist

30. The function whose graph is given below is



a) $f(x)=x$ b) $f(x)=|x|$ c) $f(x)=[x]$ d) $f(x)=-|x|$

Level –III (JEE)

31. Which among the following is deleted neighbourhood of a?

a) $\left(a-\frac{1}{2}, a+1\right)-\{a\}$ b) $\left(a-1, a+\frac{1}{2}\right)-\{a\}$

c) $(a-\frac{1}{2}, a]\cup\left[a, a+\frac{1}{2}\right]$ d) $\left(a-\frac{1}{2}, a\right)\cup\left[a, a+\frac{1}{2}\right]$

32. Assertion (A) : $\underset{x \to 0}{Lt} \frac{1}{x}$ doesnot exist

Reason (R): $\underset{x \to 0}{Lt} f(x)$ exist $\Leftrightarrow \underset{x \to 0^+}{Lt} f(x) = \underset{x \to 0^-}{Lt} f(x)$

a) Both A and R are correct and R is correct explanation of A

b) Both A and R are correct and R is not the correct explanation of A

c) A is true R is false

d) A is false and R is True

33. $\underset{x \to 0}{Lt} \frac{\sin\left(\pi \cos^2 x\right)}{x^2} =$

a) $-\pi$ b) π c) $\frac{\pi}{2}$ d) 1

34. $\underset{n \to \infty}{Lim} \frac{(n!)^{\frac{1}{n}}}{n} =$

a) e b) $\frac{1}{e}$ c) $\frac{2}{e}$ d) 2e

35. If $S_n = \left\{ \frac{1}{1+\sqrt{n}} + \frac{1}{2+\sqrt{2n}} + \frac{1}{3+\sqrt{3n}} + ... + \frac{1}{n+\sqrt{n^2}} \right\}$ then $\underset{n\to\infty}{Lt} S_n =$

a) 2log2 b) log2 c) 3log2 d) $\frac{1}{2}\log 2$

36. $\underset{x\to\infty}{Lim}\left(\frac{x+6}{x+1}\right)^{x+4} =$

a) e b) e^3 c) e^5 d) 1

37. Let $f(x) = \begin{cases} x^2 - 1 & for \quad 0 < 1 < 2 \\ 2x+1 & for \quad x \le x < 3 \end{cases}$

Then the quadratic equation whose roots are $\underset{x\to 2^-}{Lt} f(x)$ and $\underset{x\to 2^+}{Lt} f(x)$ is

a) $x^2 - 21x + 21 = 0$ b) $x^2 - 10x + 21 = 0$

c) $x^2 + 10x - 21 = 0$ d) $x^2 - 10x - 21 = 0$

38. If $f(x) = \left(\frac{x}{2+x}\right)^{2x}$ then

a) $\underset{x\to\infty}{Lt} f(x) = e^{-6}$ b) $\underset{x\to\infty}{Lt} f(x) = 2$

c) $\underset{x\to\infty}{Lt} f(x) = e^{-4}$ d) $\underset{x\to 1}{Lt} f(x) = \frac{1}{9}$

Passage:

If f, g and h are functions having a common domain D and $h(x) \le f(x) \le g(x), \forall x \in D$ and if $\underset{x\to a}{Lt} h(x) = l = \underset{x\to a}{Lt} g(x)$ then $\underset{x\to a}{Lt} f(x) = l$. This is known as sandwich There four using the result, compute the following limits (Qno: 39 to 42)

39. The value of $\underset{x\to 0}{Lt} \frac{|x|}{\sqrt{x^4 + 4x^2 + 7}}$

a) 1 b) 0 c) $\frac{1}{2}$ d) does not exists

40. $\underset{x\to 0}{Lt} x^4 \sin\left(\frac{1}{3\sqrt{2}}\right)$ is

a) 0 b) 1 c) doesnot exist d) $\frac{1}{3}$

41. Let $f(x) = \frac{x^2\left(e^{1/x} - e^{-1/x}\right)}{\left(e^{1/x} + e^{-1/x}\right)}, x \ne 0$ and $f(0) = 1$ then

a) $\underset{x\to 0^+}{Lt} f(x)$ doesnot exist b) $\underset{x\to 0}{Lt} f(x)$ does not exist

c) $\underset{x\to 0}{Lt} f(x)$ exist d) f is continuous function

42. Let $f(x)=x^5\left[\frac{1}{x^3}\right], x\neq 0$ and $f(0)=0$

a) $\underset{x\to 0}{Lt} f(x)$ does not exist b) f is not continuous at x = 0

c) $\underset{x\to 0}{Lt} f(x)=1$ d) $\underset{x\to 0}{Lt} f(x)=0$

Matching:

43. **Column – I** **Column – II**

i) $f(x)=\frac{1}{\sqrt{x-2}}$ a)

ii) $f(x)=\frac{x-\sin x}{x+\sin x}$ b)

iii) $f(xx)=x\cdot\sin\frac{\pi}{x}, f(0)=0$ c)

v) $f(x)=Tan^{-1}\left(\frac{1}{x}\right)$ d)

	a	b	c	d
1)	(ii)	(iii)	(i)	(iv)
2)	(iii)	(iv)	(ii)	(i)
3)	(iv)	(i)	(iii)	(ii)
4)	(i)	(ii)	(iv)	(iii)

Key for Level – I

1. T	2. T	3. F	4. T	5. T
6. F	7. T	8. T	9. F	10. T
11. $\frac{1}{x\log x\log 7}$	12. 5050	13. 0	14. $\sec\sqrt{\tan x}\tan\sqrt{Tanx}$	
15. $2Tanh2x$	16. $x2^x(1+x\log x\log 2+2\log x)$		17. $\frac{1}{\sqrt{22-3}}-\frac{3}{2\sqrt{7-32}}$	
18. $\frac{-e^y}{1+x^2}$	19. $\frac{-1}{2\sqrt{1-x^2}}$	20. –Tant	21. $\frac{ad-bc}{(cz+d)^2}$	22. $\frac{1}{2\sqrt{x-x^2}}(a)$
23. a	24. c	25. B	26. a	27. b
28. b	29. c	30.b	31.a	32. c
33.b	34. a	35.b	36.c	37.c
38. a	39. a	40. b	41. a	42. b
43.a	44. a	45. c	46. D, e, a, b, c	47.c,a,d,e,b
48. b,a,d,e,c	49. d,c,a,b,e	50. c,d,a,e,b		

Key for Level – II

1.d	2.b	3.c	4.c	5.b
6.b	7.b	8.c	9.b	10.b
11.d	12.a	13.b	14.b	15.b
16.b	17. c	18. c	19.b	20.d
21.a	22.a	23.b	24.b	25.b

Key for Level – III

1. a	2. b	3.b	4.a	5.c
6.c	7.c	8.b	9.d	10.c

WORK BOOK FOR INTERMEDIAT STUDENTS

Differentiation (Jr. Inters)

Level – I

I. Write True or False of the following statems:

1. If ‘f’ is differentiable at ‘a’ then $f'(a) = \underset{h\to 0}{Lt} \frac{f(a+h)-f(a)}{h}$
2. If a function f is differentiabe at ‘a’ then ‘f’ is continuous at ‘a’
3. If $f(x)$ is differentiable at x = 0
4. The process of finding the derivative of a function using the definition is called the method of finding the derivative from thefirst priniciple
5. Derivative of constant function is zero.
6. Derivative of the function $f(x)=|x|$ is one
7. Derivative of the function x^x is $x^x(1+\log x)$
8. If $y = ae^{nx}+be^{-nx}$ then $y_2 = n^2 y$
9. Derivative of $\log|\sec x+\tan x|$ is cosec x
10. Derivative of $|x|$ is $\frac{|x|}{x}$

II. Fill the following blanks with suitable answer:

11. If $f(x)=\log_7(\log x)(x>0)$ then $f'(x)=$_____
12. If $f(x)=1+x^2+x^2+.....+x^{100}$ then $f'(1)=$______
13. If $f(x)=2x^2+3x-5$ then $f'(0)+3f'(-1)=$

14. If $y = \sec\sqrt{Tanx}$ then $\frac{dy}{dx} =$ ______

15. If $y = \log(\cosh 2x)$ then $\frac{dy}{dx} =$ ____

16. If $f(x) = x^2 2^7 \log x$ then $f'(x) =$

17. If $f(x) = \sqrt{2x-3} + \sqrt{7-3x}$ then $f'(x) =$ _______

18. $x = \tan(e^{-y})$ then $\frac{dy}{dx} =$ ____

19. If $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ then $\frac{dy}{dx} =$ ____

20. If $x = a\cos^3 t,\ y = a\sin^3 t$ then $\frac{dy}{dx} =$ ____

III. Multiple choice questions:

21. If $f(x) = \frac{ax+b}{cx+d}$ then $f'(x) =$

a) $\frac{bc-ad}{(ax-d)^2}$ b) $\frac{bc+ad}{(ax+d)^2}$ c) $\frac{ad-bc}{(ax+d)^2}$ d) $\frac{ad+bc}{ax+d}$

22. If $y = \sin^{-1}\sqrt{x}$ then $\frac{dy}{dx} =$ ____

a) $\frac{1}{2\sqrt{x-x^2}}$ b) $\frac{1}{\sqrt{x-x^2}}$ c) $\frac{-1}{2\sqrt{x-x^2}}$ d) $\frac{-1}{2\sqrt{x+x^2}}$

23. If $y = \left(\cot^{-1} x^3\right)^2$ then $\frac{dy}{dx} =$ ____

a) $\frac{-6x^2 \cot^{-1} x^3}{1+x^6}$ b) $\frac{6x^2 \cot^{-1} x^3}{1+x^6}$ c) $\frac{6x^3 \cot^{-1} x^3}{1+x^6}$ d) $\frac{-6x^3 \cot^{-1} x^3}{1+x^6}$

24. If $y = e^{\sin^{-1} x}$ then $\frac{dy}{dx} =$ ____

a) $\frac{e^{\sin^{-1} x}}{\sqrt{1+x^2}}$ b) $\frac{-e^{\sin^{-1} x}}{\sqrt{1-x^2}}$ c) $\frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}$ d) $\frac{-e^{\sin^{-1} x}}{\sqrt{1-x^2}}$

25. If $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right),\ |x| < 1$ then $\frac{dy}{dx} =$ ____

a) $\frac{1}{1+x^2}$ b) $\frac{2}{1+x^2}$ c) $\frac{-2}{1+x^2}$ d) $\frac{-1}{1+x^2}$

26. If $y = \sin^{-1}(3x - 4x^3)$ then $\frac{dy}{dx} =$ ____

a) $\frac{3}{\sqrt{1-x^2}}$ b) $\frac{-3}{\sqrt{1-x^2}}$ c) $\frac{2}{\sqrt{1-x^2}}$ d) $\frac{-2}{\sqrt{1-x^2}}$

27. If $y=\tan^{-1}\left(\frac{a-x}{1+ax}\right)$ then $\frac{dy}{dx}=$______

a) $\frac{1}{1+x^2}$ b) $\frac{-1}{1+x^2}$ c) $\frac{-2}{1+x^2}$ d) $\frac{2}{1+x^2}$

28. If $y=\sec^{-1}\left(\frac{1}{2x^2+1}\right)$ $y=\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0<x<\frac{1}{\sqrt{2}}$ then $\frac{dy}{dx}=$______

a) $\frac{2}{\sqrt{1-x^2}}$ b) $\frac{-2}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1-x^2}}$ d) $\frac{-1}{\sqrt{1-x^2}}$

29. If $y=Tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx}=$ _______

a) 1 b) –1 c) $\frac{1}{2}$ d) $\frac{-1}{2}$

30. If $y=e^{a\sin x}$ then $\frac{dy}{dx}=$ _______

a) $\frac{-ay}{\sqrt{1-x^2}}$ b) $\frac{ay}{\sqrt{1-x^2}}$ c) $\frac{-ax}{\sqrt{1-x^2}}$ d) $\frac{ax}{\sqrt{1-x^2}}$

31. If $x=a(\cot t+\log\tan t/2),\ y=a\sin t$ then $\frac{dy}{dx}=$_________

a) Tan t b) –Tan t c) Cot t d) –Cot t

32. If $x=a(\theta-\sin\theta),\ y=a(1-\cos\theta)$ then $\frac{dy}{dx}=$_________

a) $Tan\frac{\theta}{2}$ b) $-Tan\frac{\theta}{2}$

c) $Cot\frac{\theta}{2}$ d) $-Cot\frac{\theta}{2}$

33. If $x^{2/3}+y^{2/3}=a^{2/3}$ then $\frac{dy}{dx}=$ ____________

a) $(y/x)^{\frac{1}{3}}$ b) $-(y/x)^{\frac{1}{3}}$ c) $(x/y)^{\frac{1}{3}}$ d) $-(x/y)^{\frac{1}{3}}$

34. If $x^4+y^4-a^2xy=0$ then $\frac{dy}{dx}=$ ____________

a) $\frac{a^2y-4x^3}{4y^3-a^2x}$ b) $\frac{a^2y+4x^3}{4y^3-a^2x}$

c) $\frac{a^2y-4x^3}{4y^3+a^2x}$ d) none of these

35. If $\sin y = x \sin(a+y)$ then $\frac{dy}{dx} =$ __________

a) $\frac{\sin^2(a+y)}{\sin^2 a}$ b) $\frac{\sin^2(a+y)}{\sin a}$

c) $\frac{\cos^2(a+y)}{\cos^2 a}$ d) $\frac{\cos^2(a+y)}{\cos a}$

36. If $x^y = e^{1-y}$ then $\frac{dy}{dx} =$ __________

a) $\frac{-\log x}{(1+\log x)^2}$ b) $\frac{-\log x}{1+\log x}$

c) $\frac{\log x}{(1+\log x)^2}$ d) $\frac{\log x}{1+\log x}$

37. If $\sqrt{1-x^2}+\sqrt{1-y^2} = a(x-y)$ then $\frac{dy}{dx} =$ _________

a) $\sqrt{\frac{1-x^2}{1-y^2}}$ b) $\sqrt{\frac{1+x^2}{1+y^2}}$ c) $\sqrt{\frac{1-y^2}{1-x^2}}$ d) $\sqrt{\frac{1+y^2}{1+x^2}}$

38. Derivative of e^x with respect to $\sqrt{x}$ is ________

a) $2\sqrt{x}\, e^x$ b) $-2\sqrt{x}\, e^x$ c) $\sqrt{x}\, e^x$ d) $-\sqrt{x}\, e^x$

39. Derivative of $Tan^{-1}\left(\frac{2x}{1-x^2}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is ______

a) 1 b) –1 c) 0 d) None

40. If $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ then $\frac{dy}{dx} =$ _____

a) $\frac{2^x \log 2}{1+4^x}$ b) $\frac{2^{x+1}\log 2}{1+4^x}$ c) $\frac{-2^x \log 2}{1+4^x}$ d) $\frac{-2^{x+1}\log 2}{1+4^x}$

41. If $y = a(1+\cos t), x = a(1-\sin t)$ then $y_2 =$ ______

a) $\frac{1}{4a\sin^4 t/2}$ b) $\frac{1}{2a\sin^4 t/2}$

c) $\frac{-1}{4a\sin^4 t/2}$ d) none

42. If $y = a\, x^{n+1} + b\, x^{-n}$ then $x^2 y^{11} =$ ______

a) $(n+1)y$ b) $n(n+1)y$ c) $n^2(n+1)y$ d) $n^2(n+1)^2 y$

43. If $ay^4 = (x+b)^5$ then $5yy_2 =$ ______

a) y_1^2 b) y_1^3 c) $-y_1^2$ d) $-y_1^3$

44. If $y = a\cos x + (b+2x)\sin x$ then $y^{n} + y =$ _______

a) 4 sinx b) –4 sinx c) 4 cosx d) –4 cosx

45. If $y = \sin(\sin x)$ then $y^{11} + Tanx \cdot y' + y\cos^2 x =$ ______

a) 1 b) –1 c) 0 d) none

IV. Match the following:

46.

List – I	List – II
1) $\frac{d}{dx}\left(x^{-n}\right)$	a) $a^2 \log a$
2) $\frac{d}{dx}\left(\sqrt{x}\right)$	b) $\frac{1}{x}$
3) $\frac{d}{dx}\left(a^x\right)$	c) $\frac{f'(x)}{\left(f(x)\right)^2}$
4) $\frac{d}{dx}(\log x)$	d) $\frac{-n}{x^{n+1}}$
5) $\frac{d}{dx}\left[\frac{1}{f(x)}\right]$	e) $\frac{1}{2\sqrt{x}}$

47.

List – I	List – II
1) $\frac{d}{dx}(Tanx)$	a) $-\cos ec\, x \cot x$
2) $\frac{d}{dx}(\cos ecx)$	b) $g'\left(f(x)\right) \cdot f'(x)$
3) $\frac{d}{dx}\left(\sin^{-1} x\right)$	c) $\sec^2 x$
4) $\frac{d}{dx}\left(Tan^{-1} x\right)$	d) $\frac{1}{\sqrt{1-x^2}}$
5) $\frac{d}{dx}\left[(gof)(x)\right]$	e) $\frac{1}{1+x^2}$

48.

List – I	List – II
1) $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$	a) $\sec h^2 x$
2) $\frac{d}{dx}(Tanhx)$	b) $\frac{g(x)f'(x) - f(x)g'(x)}{\left(g(x)\right)^2}$
3) $\frac{d}{dx}(\cos echx)$	c) $\frac{1}{1-x^2}$

4) $\frac{d}{dx}\left(\sin h^{-1}x\right)$	d) $-\cos ec\, hx\, \cot x$
5) $\frac{d}{dx}\left(Tanh^{-1}x\right)$	e) $\frac{1}{\sqrt{1+x^2}}$

49.

List – I	List – II
1) $\frac{d}{dx}\left(x^3+6x^2+12x-13\right)^{100}$	a) $y^{x^3+3x}\left(3x^2+3\right)$
2) $\frac{d}{dx}\left[\sin\left(\log x\right)\right]$	b) $\frac{-2a}{\left(a+x\right)^2}$
3) $\frac{d}{dx}\left(7^{x^3+3x}\right)$	c) $\frac{\cos\left(\log x\right)}{x}$
4) $\frac{d}{dx}\left(\frac{a-x}{a+x}\right)$	d) $300\left(x+2\right)^2\left(x^3+6x^2+13x-13\right)^{100}$
5) $\frac{d}{dx}\left(x^3e^x\right)$	e) $x^2e^2\left(x+3\right)$

50.

List – I	List – II
1) $\underset{x\to a+}{Lt}\frac{f\left(x\right)-f\left(a\right)}{x-a}$ exists	a) f is differentiable at a
2) $\underset{x\to a-}{Lt}\frac{f\left(x\right)-f\left(a\right)}{x-a}$ exists	b) $f'\left(0\right)=0$
3) $f'\left(a+\right)=f'\left(a-\right)$	c) Right hand derivative of ‘f’ at a
4) $\frac{d}{dx}\left[f\left(1\right)g\left(x\right)\right]$	d) Left hand derivative of ‘f’ at a
5) $f\left(x\right)$ is even function	e) $f\left(x\right)g'\left(x\right)+g\left(x\right)f'\left(x\right)$

Level – II

1. If $x=\sin t\cos 2t,\ y=\cos t\sin 2t$ then $\left(\frac{dy}{dx}\right)_{t=\frac{\pi}{4}}=$_____

a) –2 b) 2 c) $-\frac{1}{2}$ d) $\frac{1}{2}$

2. If $f\left(x\right)=\begin{cases}\frac{x-1}{2x^2-7x+5} & for\quad x\neq 1\\ \frac{-1}{3} & for\quad x=1\end{cases}$ then $f'\left(1\right)=$ _______

a) $\frac{-1}{9}$ b) $\frac{-2}{9}$ c) $\frac{-1}{3}$ d) $\frac{1}{3}$

3. If $y = 2^{2^x}$ then $\frac{dy}{dx} =$______

a) $y(\log_{10} 2)^2$ b) $y(\log_e 2)^2$ c) $y \cdot 2^x (\log_e 2)^2$ d) $y \cdot \log_e 2$

4. If $y = 2^{ax}$ and $\frac{dy}{dx} = \log 256$ at x = 1 then a = _______

a) 0 b) 1 c) 2 d) 3

5. If $f(x) = \frac{1}{1+\frac{1}{x}}$, $g(x) = \frac{1}{1+\frac{1}{f(x)}}$ then $g'(x) =$______

a) $\frac{1}{5}$ b) $\frac{1}{25}$ c) 5 d) $\frac{1}{16}$

6. If $f(x) = \sqrt{ax} + \frac{a^2}{\sqrt{ax}}$ then $f'(a) =$_____

a) a b) 0 c) 1 d) –1

7. $\frac{d}{dx}\left(\cos x^0\right) =$_____

a) $-\sin x^0$ b) $-\frac{\pi}{180}\sin x^0$ c) $\frac{\pi}{180}\sin x^0$ d) $\frac{2\pi}{180}\sin x^0$

8. If $y = \sec\left(Tan^{-1}x\right)$ then $\frac{dy}{dx}$ at x = 1 is equal to ________

a) 1 b) $\sqrt{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{1}{2}$

9. If $f(x) = e^x$, $g(x) = \sin^{-1} x$ and $h(x) = f(g(x))$ then $\frac{h'(x)}{h(x)} =$_______

a) $\sin^{-1} x$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{1-x^2}$ d) $e^{\sin^{-1} x}$

10. If $y = \log\left[\left(\frac{1+x}{1-x}\right)^{\frac{1}{4}}\right] - \frac{1}{2}Tan^{-1}x$ then $\frac{dy}{dx} =$_____

a) $\frac{x}{1-x^2}$ b) $\frac{x^2}{1-x^4}$ c) $\frac{x}{1+x^4}$ d) $\frac{x}{1-x^4}$

11. If $f(x) = \left(x^2 - 1\right)^7$ then $f^{(14)}(x) =$_____

a) 0 b) 2! c) 7! D) 14!

12. If $x = \theta - \frac{1}{\theta}$, $y = \theta + \frac{1}{\theta}$ then $\frac{dy}{dx} =$__________

a) x/y b) y/x c) $-x/y$ d) $-y/x$

13. If $x = 3\cos\theta - 2\cos^3\theta,\ y = 3\sin\theta - 2\sin^3\theta$ then $\frac{dy}{dx} =$ _____

a) $Tan\theta$ b) $Cot\theta$ c) $Cot\,\theta/2$ d) $Tan\,\theta/2$

14. If $x^2 + y^2 = t + \frac{2}{t}$ and $x^4 + y^4 = t^2 + \frac{4}{t^2}$ then $x^3 y \frac{dy}{dx} =$ ______

a) –1 b) –2 c) y/x d) xy

15. If $x = \frac{1-\sqrt{y}}{1+\sqrt{y}}$ then $\frac{dy}{dx} =$ _____

a) $\frac{4}{(x+1)^2}$ b) $\frac{4(x-1)}{(1+x)^3}$ c) $\frac{x-1}{(1+x)^3}$ d) $\frac{4}{(x+1)^3}$

16. If $xy = (x+y)^n$ and $\frac{dy}{dx} = \frac{y}{x}$ then n = ____

a) 1 b) 2 c) 3 d) 4

17. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{4x^3}{27} - x\right)\right] =$ ________

a)m $\frac{3}{\sqrt{9-x^2}}$ b) $\frac{1}{\sqrt{9-x^2}}$ c) $\frac{-3}{\sqrt{9-x^2}}$ d) $\frac{-1}{\sqrt{9-x^2}}$

18. $\frac{d}{dx}\left[\cos^{-1}\left(\frac{a^2 - x^2}{a^2 + x^2}\right) + \sin^{-1}\left(\frac{2ax}{a^2 + x^2}\right)\right] =$ ______

a) $\frac{a}{x^2 + a^2}$ b) $\frac{2a}{x^2 + a^2}$ c) $\frac{4a}{x^2 + a^2}$ d) $\frac{a^2}{x^2 + a^2}$

19. If $y = Tan^{-1}\left[\frac{5\cos x - 12\sin x}{12\cos + 5\sin x}\right]$ then $\frac{dy}{dx} =$ _____

a)1 b) –1 c) –2 d) $\frac{1}{2}$

20. $\frac{d}{dx}\left|Tan^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x}\sqrt{1-\sin x}}\right)\right|$ then $\frac{dy}{dx} =$

a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{-1}{2}$

21. Derivative of $\sin^{-1}\left(\frac{1}{2x^2 - 1}\right)$ w.r.to $\sqrt{1+3x}$ at $x = \frac{-1}{3}$ is _____

a) 0 b) 1 c) $\frac{1}{2}$ d) $\frac{-2}{3}$

22. If $\cos^{-1}\left(\frac{x^2 - y^2}{x^2 + y^2}\right) - K$ (a constant) then $\frac{dy}{dx} =$

a) $\frac{y}{x}$ b) $\frac{x}{y}$ c) $\frac{x^2}{y^2}$ d) $\frac{y^2}{x^2}$

23. If $y=\sqrt{Tanx+\sqrt{Tanx+\sqrt{Tanx+............\infty}}}$ then $\frac{dy}{dx}=$ _______

a) $\frac{\cos^2 x}{2y-1}$ b) $\frac{\sec^2 x}{2y-1}$ c) $\frac{Tanx}{2y-1}$ d) $\frac{\cot x}{2y-1}$

24. If $y=\sin\left(m\sin^{-1}x\right)$ then $\left(1-x^2\right)y_2-xy_1=$____

a) m^2y b) $-m^2y$ c) $2m^2y$ d) $-2m^2y$

25. If $f(x)=\sin x+\cos x$ then $f\left(\frac{\pi}{4}\right)f^{(iv)}\left(\frac{\pi}{4}\right)=$____

a) 1 b) 2 c) 3 d) 4

Level – III

1. If the function ‘f’ is defined by $f(x)=\frac{x}{1+|x|}$ then at what points is ‘f’ differentiable

a) every wheres b) at $x=\pm1$ c) except at x = 0 d) except at x = 0 or ±1

2. If f is defined by $f(x)=\begin{cases} x, & for \quad 0\le x\le 1 \\ 2-x & for \quad x\ge 1\end{cases}$ then at z =1, f is ____

a) continous and differentiable b) continuous but not differentiable

c) Discontinuous but differentiable d) neither continuous not differentiable

3. If $f(x)=\begin{cases}(x-1)\sin\left(\frac{1}{x-1}\right) & if \quad x\neq 1 \\ 0 & if \quad x=1\end{cases}$ then _____

a) f is differentiable at x = 0 and at x = 1

b) f is differentiable at x = 1 but not at x =1

c) f is differentiable at x = 1 but not at x =0

d) f is neither differentiable at x = 0 nor at x = 1

4. If ‘f’ is an even function and $f'(x)$ exists then $f'(0)=$ ____

a) 0 b) 1 c) –1 d) f(0)

5. If $f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$ then $f'(1)=$_____

a) 1 b) –1 c) log2 d) –log2

6. If $f(x)=\begin{vmatrix}2\cos x & 1 & 0 \\ 1 & 2\cos x & 1 \\ 0 & 1 & 2\cos x\end{vmatrix}$ then $f'\left(\frac{\pi}{3}\right)=$_______

a) –5 b) –4 c) $-\sqrt{3}$ d) –2

7. If $f(x)=\frac{x}{1+x}$ and $g(x)=f(f(x))$ then $g'(x)=$ ____

a) $\frac{1}{(x+1)^2}$ b) $\frac{1}{x^2}$ c) $\frac{1}{(2x+1)^2}$ d) $\frac{1}{(2x+3)^2}$

8. Let $f:(-1,1)\to R$ be a differentiable function with $f(0)=-1$ and $f'(0)=1, g(x)=[f(2(x)+2)]^2$ then $g'(0)=$______

a) 4 b) –4 c) 0 d) –2

9. If x = a is a root of multiplicity two of a polynomial equation $f(x)=0$ then _____

a) $f'(a)=f''(a)=0$ b) $f''(a)=f(a)=0$

c) $f'(a)\neq 0=f''(a)$ d) $f(a)=f'(a)=0,\ f''(a)=0$

10. If $y=Tan^{-1}\left[\frac{\sqrt{1+a^2x^2}-1}{ax}\right]$ then $(1+a^2x^2)y''+2a^2xy'=$_____

a) a^2 b) $2a^2$ c) 0 d) $-2a^2$

APPLICATIONS OF DERIVATIVES
(B. Rtamesh Chandra Babu, JL in Maths, PVKN Govt College, Chittor)

Derivative as a rate measurer

Level – I

1. What is the 1^{st} principle of differentiation?
2. How do define derivative of a function $f(x)$ at a point?
3. If $y=f(x)$ and $x=g(x)$ then $\frac{df}{dx}\frac{dx}{dt}=$____
4. Define ‘Average rate of change’.
5. Define ‘Instantaneous rate of change’.
6.

	Triangle		Equilateral triangle	Quadril triangle	Trapezium	Rhombus
Given	Base, Height	b,c and SinA	Side	d, h_1, h_2	Hight, Parallel sides	Diagonals
Area						

7.

	Sector		Ellipse	Sphere	Cone	Cylinder
Given	l, r	θ, r	a, b	R	r, h, l	r, h
Area/ Surfac e area						
Perime ter/Vol ume						

8. A circular wound circumference is reducing 0.2 cm/day. Find the rate of change of healing of the wound when its radius is 4cms.

a) 0.08 sq cm/day b) 0.8 sq cm/day

c) 0.008 sq cm/day d) None

9. In a rice mill, husk of $3\pi/2\ c.\ ft/hr$ is filling as a conical pile from the delivery pipe which is at a height 9ft from the groung. The height of the pile is always twice of base radius. Find the time taken for the pile to touch the delivery pipe, when height of the pile is 3ft from the ground.

a) 13.5 hrs b) 12 hrs c) 9 hrs d) 12.5 hrs

10. The rate of change of oxygen in a cylinder of a covid-19 patient on ventilator is decreasing 3 gms/min. Find the rate at which volume of oxygen is changing per minute when pressure is 500gm-wt/sq cm, if oxygen follos PV = 500000

a) 3 cc/min b) 6 cc/min c) 1.5 cc/min d) None

11. Base curve of water tank is ellipse. If 6 cc/min of water is leaking from the tank then find the rate of change of water level. The major and minor axes lengths are 4mts and 6mts.

a) $4\pi\ cc/\text{sec}$ b) $4\pi^2\ cc/\text{sec}$ c) $1/4\pi\ cc/\text{sec}$ d) $1/4\pi^2\ cc/\text{sec}$

12. Flood water if flowing in to reservoir (whose cross section) water entering face is in the shape of trapezium. Lower and upper width of the reservoir are 20mts, 400mts and length 500 mts. If the water level is increasing at the rate of 0.04 cm/sec. Find the rate at which of the water increasing.

a) 6000 cc/sec b) 16,000 cc/sec c) 12,000 cc/sec d) None

13. Aquaculture water pond measurements are length 60ft, breadth 30ft and depth 3ft. The level of the water is decreasing 1/3 feet per 12hrs due to sun. Find the rate of evaporation of water per hour when pond is full of water.

a) 180 c.ft/hr b) 18 c.ft/hr c) 50 c.ft/hr d) 500c.ft/hr

14. An orange in a tree is increasing 1 c.c/day find the rate at which its surface is increasing per day when radius is 3cm.

a) 2 sq cm/day b) 0.2 sq cm/day c) 1/2 sq m/day d) 0.02 sq cm/day

15. If the radius of a circular hole in the Ozone layer is decreasing 1/11 mts/day due to Lock down then find the rate at which hole is refilling when radius is 14 mts.

(Assume that thickness of the ozone layer is 0cm)

a) 8 sq mts/day b) 6 sq mts/day c) 4 sq mts/day d) None

16. A burning cylindrical candle of radius 1cm is melting 11 cc/min. Find the rate at which height of the candle is decreasing.

a) 11/2 cm/min b) 9/2 cm/min c) 7/2 cm/min d) 5/2 cm/min

17. A(0, 8) and point B moves 5 units/min on X–axis. Find the rate at which $\overline{AB}$ is changing when B is at (6, 0)

a) 1 unit/min b) 3 units/min c) 2 units/min d) 4 units/min

Level – 3

18. Newly born baby is 3kgs weight. If the rate of change of weight of the baby is 3/2 kg/month then find the weight of the baby after 4months.

a) 9kgs b) 6kgs c) 12 kgs d) None

19. The trunk of the tree which is in cylindrical shape is observed that its circumferences is changing 0.1 cm/month, Find the rate of change of wood when height is 200mts. The relation between radius and height is 10r = h.

a) 600 cc/month b) 6000 cc/month

c) 60,000 cc/month d) None

20. The rate of change of the height of the plant is 0.5cm/day. Find the height of the plant after a month. If its initial height is 15cms.

a) 7 1/2cm b) 15cms c) 22 1/2cm d) 30cm

21. The leaf in elliptic shape in a tree increase 3π sq.cm/day. Find the rate of increase in the length of leaf if its breadth is increasing 1/3 cm/day, when length is 3cm and width is 2cm.

a) 4 cm/day b) 3 cm/day c) 1 cm/day d) 2 cm/day

22. The number of fishes in a lake an 31/8/2019 are 250 in a lake. Population of fishes raised to 1000 on 31/8/2020. Find the population of fishes on 31/8/2022.

a) 2250 b) 16250 c) 1600 d) 2500

23. The number of COVID-19 patients in 30 days from the day of 1st case identified is 146 and 78,512 patients were identified in 180 days. How many COVID-19 patients will be there, after 300 days from the day of 1st case identified. (log 573.75=1.2704 and $e^{11.4336} = 92373.92$)

a) 1,34,869 b) 13,48,692 c) 134,86,923 d) 13,48,232

24. Leaf of a tree in the shape of equilateral triangle placed on the diameter of a semicircle. The number of leaves in 1st stem is 1,2nd stem are2------100th stem are 100. Rate of change radius 1/10 Cm/day. If the relation between absorption of CO_2 when the average radius of semicircle is 10 cm.

a) 111 $l$ts/day b) 1111 $l$ts/day c) 11111 $l$ts/day d) None

25. The inner and outer radii of a car tube are 7 cm and 14cm respectively. Radius of cross section of air in the tube is decreasing 0.2 cm/hr due to puncher. Find (approximately) at what rate air is coming out when radius of cross section of air in the tube is 7 cm.

a) 580.8 cc/hr b) 58.08 cc/hr c) 290.4 cc/hr d) 29.04 cc/hr

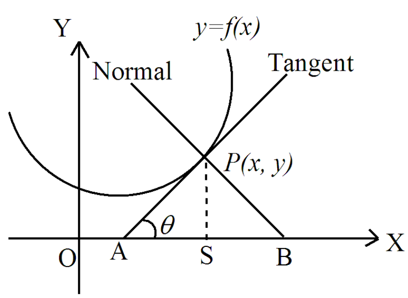
26. When water is pumping into a cylindrical water tank of radius 7 ft, the level of water increases 9 inches/minute and when out let is open $\frac{77}{2}$ c.fit/minute of water flows out. Find the rate at which volume of the water is changing in the tank when in flow

a) 192.5 c.ft/min b) 38.5 c.ft/min c) 77 c.ft/min d) 231 c.ft/min

Applications of Derivatives to Geometry

Level – I

27. Define slope of a line in Geometricaly.
28. Define Slope of a line in Trigonometrically.
29. Define Slope of a line in Calculus.
30. Slope of X-axis and Y-axis.
31. Slope of the line parallel to X-axis and parallel to Y-axis
32. What do you mean by general slope of a curve?
33. What do you mean by slope of a curve at a point?
34. Define Secant line, Tangent line and Normal line
35. How do you define angle between two curves?
36. Formula to find angle between two curves? (In terms of slopes)



Write the name and formula of the following to the curve C at the point $P(x_1, y_1)$

37. AP =
38. PB =
39. AS =
40. SB =

Level – 2

41. If the sub tangent and sub normal of a particular curve at some point "P" are 2 and 8 then match the following.

1) 1.Ordinate	A) $4\sqrt{5}$
2) 2.Length of the tangent	B) 2
3) 3.Length of the normal	C) 4
4) 4.Slope	D) $2\sqrt{5}$
	E) $8\sqrt{5}$

Ans:-----------------------

42. If the length of the normal and tangent of a particular curve at "P" are $4\sqrt{2}$ and $2\sqrt{2}$ then match the following.

1. Slope	A) $4\sqrt{(2/5)}$
2. Ordinate	B) $2\sqrt{(2/5)}$
3. Length of sub tangent	C)
4. Length of Sub normal	D) 2
	E) $8\sqrt{(2/5)}$

Ans:----------------

43. If the tangent at "P" to the curve $3x^2+4y^2=1$ is the normal at "P" to the curve $4x^2+ky^2=1$. Then find "k".

a) $\frac{2}{3}$ b) $\frac{-3}{2}$ c) $\frac{-2}{3}$ d) $\frac{3}{2}$

44. Find the angle between the normals drawn at the points $A\left(\frac{3}{2}, 1\right)$ and $B\left(\frac{5}{2}, 2\right)$ to $B\left(\frac{5}{2}, 2\right)$ to $B\left(\frac{5}{2}, 2\right)$

a) 90^0 b) -90^0 c) 0^0 d) None

45. If $\overline{AB}$ is the chord of $x=2\cos\theta, y=2\sin\theta$ drawn parallel to x-axis then find the angle made by the tangent to the curve at B with y-axis . Where $A\left(\sqrt{3}, 1\right)$.

a) 75^0 b) 60^0 c) 45^0 d) 30^0

46. Angle between the curves $x^2+y^2-2x-4y-20=0$ and $x^2+y^2-18x-16y+120=0$ at the point A(5, 5)

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) 0^0

47. Find the slope of the tangent to the derivative of the curve $y=x^3-x^2+x+3$ at (2, 9).

a) 6 b) 9 c) 10 d) None

48. Find the point at which the tangent at (1, 3) to $Y = x^3 - x + 3$ intersects the same curve.

a) (2, 9) b) (4, 9) c) (2, 5) d) (–2, –3)

49. Find the distance between tangents parallel to x-axis of the curve $y = 2x^3 - 6x + 5$

a) 4units b) 8units c) 12units d) None

50. Find the point at which, the tangent at $\left(5, \sqrt{3}\right)$ to $x^2 + y^2 - 8x + 12 = 0$ is the normal to the curve $y = x^2$

a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{4}\right)$ b) $\left(\frac{-\sqrt{3}}{2}, \frac{4}{3}\right)$ c) $\left(\frac{3}{4}, \frac{\sqrt{3}}{2}\right)$ d) $\left(\frac{4}{3}, \frac{-\sqrt{3}}{2}\right)$

51. Find the equation of the normal at P(–4, 3) to the curve C. Where C is the locus of a point which moves 5 units from the origin.

a) $4x - 3y + 25 = 0$ b) $4x + 3y + 25 = 0$

c) $3x + 4y = 0$ d) $3x + 4y - 25 = 0$

52. If the slopes of f(x) = sinx and g(x)=cosx are m_1 and m_2, then write tanx in terms of slopes of f(x) and g(x).

a) $\frac{m_1}{m_2}$ b) $\frac{m_2}{m_1}$ c) $\frac{-m_2}{m_1}$ d) $\frac{-m_1}{m_2}$

53. Assume that center of the moon is at origin. Let "P" be a point on the earth such that $\overrightarrow{OP}$ is x-axis. An artificial satellite is moving in the orbit $x^2 + y^2 = 8$ around the moon. Find distance from the satellite to the point "P" when satellite is at T(2, 2).

a) $2\sqrt{2}$ units b) $\sqrt{2}$ units c) $2\sqrt{2}$ units d) None

54. Assume that hill is in the shape of parabola $x^2 + 16y - 128 = 0$ and bottom of the hill is x-axis. A soldier is on the edge of the hill (take positive side) at a point "P" whose altitude is 4uints. Find the angle of depression at which soldier at "P" has to shoot his enemy at Q. Where Q is point on the positive side x-axis.

a) 30^0 b) 45^0 c) 60^0 d) None

55. Terrorists suicide bomber is coming the path y=3x to hit the city at origin "O". Army camp at "A" (on negative side of x-axis) projected missile in the path $x^2 + y - 4 = 0$ to hit the terrorist bomber at "P". If the fragments (after hitting) travels in the tangential direction and fall at B, find the distance between P and B. Where A, O, B lines on x-axis.

a) $3\sqrt{5}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{3\sqrt{5}}{2}$ d) None

56. Find the distance between the tangents drawn to the curve x=2cosθ, y=2sinθ at $\left(-\sqrt{2}, \sqrt{2}\right)$ and $y^2 = 4$ at (1, 1)

a) $2\sqrt{2} - 1$ b) $2\sqrt{2} - \frac{1}{2}$ c) $2 - \frac{\sqrt{2}}{2}$ d) None

57. If the length of the sub normal to the curve $\left(\frac{x}{2}\right)^n + \left(\frac{y}{2}\right)^n = 2$ at (2, 3) is one of the diagonals of the rhombus with area 15sq units then find the length of another diagonal.

a) 15 b) 10 c) 5 d) None

58. Find the equation of the tangent at P(3, 4) tot he curve C. Where C is the locus of a point which moves 1unit distance from the curve $x^2 + y^{2`} = 16$.

a) $3x+4y+25=0$ b) $3x-4y=0$

c) $3x+4y=0$ d) $3x+4y-25=0$

59. Find the point at which tangent drawn to $y = 2x^2 + 3x - 4$ is parallel to the secant line through A(0, –4) and B(4, 40).

a) (2, 10) b) (10, 2) c) (–2, 10) d) (10, 2)

60. Equation of the tangent to $y = |x|$ at origin.

a) y=0 b) x=0 c) does not exist d) y = x

Errors and approximations

Level – I

61. Δy is called ____________

62. $\frac{\Delta y}{y}$ is called _________

63. $\frac{\Delta y}{y}$X100 is called _____

64. (i) $\Delta y =$__________ (ii) $dy =$_________ (iii) Δy ______ dy

Level – I

65. Time period of a simple pendulum is directly proportional to the square root of its length. If there is an error of 1% in measuring time period, then the percentage error in length is

a) $\sqrt{2}$ b) 1 c) 2 d) None

66. Find the approximate value of $Tan^{-1}(1.0349)$

a) 46^0 b) 47^0 c) $45^0\ 45^0\ 30'$ d) None

67. Find the approximately $(2.0125)^5$.

a) 33 b) 34 c) 32 d) None

68. If there is an error of 6% in measuring total surface area of semi sphere then find the relative error in volume of semi-sphere.

a) 0.9 b) 0.3 c) 0.09 d) 0.03

Level – 3

69. While measuring a land which is in the shape of rhombus, the short diagonal was measured as 5.01 meters instead of 5 meters. Find the error in measuring its area if both diagonals were measured with same instrument. Length of the long diagonal is 4 times of the short diagonal

a) 0.01 mt^2 b) 0.1 mt^2 c) 0.2 mt^2 d) 0.02 mt^2

70. Pressure "P" and volume "V" follows PV=Constant. The decrease in pressure from 1.5kg-wt/cm^2 to 1.4 $kg - wt/cm^2$ when 12,000 c.c. Then find the increase in volume.

a) 0.8 c.cm b) 80 c.cm c) 800 c.cm d) none

71. An electric current is measured by a tangent galvanometer. The current "c" is directly proportional to '$\tan\theta$' ('θ' is angle of deflection). Find the appropriate relative error in "c" corresponding to an error of 1^0 in measuring 15^0 deflection.

a) 4 units b) $\frac{4}{\sqrt{3}}$ units c) $\sqrt{3}$ units d) None

Applications of Derivatives to Maxima and Minima

Level – I

72. f(x) is a real valued function defined on the interval 1.

If $x_1 \le x_2$ and $f(x_1) \le f(x_2)$ then f(x) is called ___________

73. If $x_1 \ge x_2$ and $f(x_1) \ge f(x_2)$ then f(x) is called __________

74. If $x_1 < x_2$ and $f(x_1) < f(x_2)$ then $f(x)$ is called _________

75. If $x_1 < x_2$ and $f(x_1) > f(x_2)$ then $f(x)$ is called _________

76. Define Critical point.

77. Define Stationary point.

78. Define Turning point.

79. FIRST DERIVATIVE test.

80. State SECOND DERIVSTIVE test.

81. Explain absolute maximum.

82. If the tangent to the curve at any point $c \in [a, b]$ to the curve $y = f(x)$ makes an acute angle with the X-axis then f(x) is

a) Increasing in [a, b] b) decreasing in [a, b]

c) Neither increasing nor decreasing d) None

83. If the tangent to the curve at any point $c \in [a, b]$ to the curve $y = f(x)$ makes an obtuse angle with the x-axis then f(x) is

a) Increasing in [a, b] b) decreasing in [a, b]

c) Neither increasing nor decreasing d) None

84. State Rolle's theorem.

85. State Lagrange mean value theorem.

86. Find the minimum value of $f(t) = t^3 - 3t^2 - 9t + 27$

a) –1 b) 1 c) 0 d) None

87. A merchant wants to fence a empty plane for parking place using an existing wll in one side. He has 64 mts of fencing and wants to know the dimensions of parking plance.

a) 44, 10 b) 48, 8 c) 32, 16 d) 40, 12

88. The $f(x) = \cot^{-1} x$ is strictly decreasing in

a) [–1, 1] b) $(-\infty, \infty)$ c) [0, ∞) d) None

Level – 2

89. The maximum possible area that can be enclosed by a wire of length 100ft, by bending it into the form of sector is

a) 125 sq ft b) 625 sq ft c) 250 sq ft d) 650 sq ft

90. For what value of "a" the sum of the squares of the roots of the equation v will be minimum.

a) 3 b) 1 c) –1 d) 2

91. Find the minimum length of intercept made by the tangent drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

a) 25 b) 7 c) –1 d) 2

92. If $f(x) = 2x^3 - 5x + 5$ has local maximum value as 8 at $x = a \in Z$. Then find $f''(a)$

a) –11 b) –12 c) –17 d) None

93. Find the maximum value of $f(x) = \sin^4 x \cos^2 x$

a) $\frac{4}{27}$ b) $\frac{27}{4}$ c) $\frac{23}{27}$ d) None

94. The strength of a rectangular wooden beam is equal to the product of square of breadth and cube of thickness. Find the relation between breadth and thickness so that beam strength is maximum, which can be cut from the log,

a) $\sqrt{3}$ breadth =thickness b) thickness=$\sqrt{2}$ thickness

c) thickness=$\frac{\sqrt{3}}{2}$breadth d) None

95. Find the maximum area of the rectangle that can be inscribed in the $Y = \sqrt{25 - x^2}$

a) 25 sq units b) 16 sq units c) 9 sq units d) None

96. Find the maximum area of the triangle which can be inscribed in the semi-circle of radius 'r'.

a) $\sqrt{2}r^2$ b) $\sqrt{2}r$ c) $2r^2$ d) r^2

97. Rs 2 is the production cost per unit and 'x' is the selling price per unit. The profit function $P(x) = 1 + 36000x - 600x^2$. Find the maximum profit per unit

a) Rs 3 b) Rs 2 c) Rs 1 d) None

98. If the Production cost function of a company is $C(x)=1300x+3200$ and revenue function is $R(x)=(4000-2x)x$, then find for what value of 'x' profit will be maximum

a) 1000 b) 625 c) 675 d) 500

99. The mileage functions of petrol engines A and B are given by $F(x)=x^3-6x^2+9x+15$ and $G(x)=2x^3-9x^2+12x+6$ respectively. Where x is the number of litres of petrol consumed by an engine in 1 hr when tested both engines at two constant speeds 20 km/hr. Which is engine preferable

a) A and B engines b) B engine

c) A engine d) None

100. The day wise (including holidays) sales function of air conditioner units from 16-04-2019 to 15-05-2019 is $f(x)=5+30x-x^2$. Find the date on which maximum number of air conditioner units were sold.

a) 30-04-2019 b) 01-05-2019 c) 29-04-2019 d) 02-05-2019

101. The day wise (including holidays) sales function of a shopping mall rupees in Lakshs between10th day to 30th day of May month is $2\ S(t)=t^2-40t+440$. Find the minimum sales in Lakhs.

a) 130 b) 140 c) 120 d) None

102. $f(x)=x^3-12x+5$ is

a) monotonically increasing in (–2, 2)

b) monotonically decreasing in (–2, 2)

c) Monotonically decreasing in $(-\infty, 2)$

d) Monotonically increasing in $(2,\infty)$

103. A polynomial of degree 'n' will have at most number of turning points.

a) n b) n+1 c) n –2 d) n –1

104. $f(x)=x^3+3x^2+3$ is decreasing function in

a) $(-\infty, 2)$ b) $(-2, 2)$ c) $(-2, 0)$ d) $(2, \infty)$

Level – 3

105. If $\overline{AB}=2i-xj+3k,\ \overline{BC}=-2j+\overline{k}$ then find for what value of "x" the area of triangle ABC will be minimum.

a) 4 b) 6 c) 3 d) 2

106. Find where to cut the wire of length 8mts such that the sum of the areas of square and equilateral triangle (made from the wire) is minimum.

a)6.88 mts b) 6.75 mts c) 6.25 mts d) None

107. The maximum area of the rectangle that can be inscribed in the ellipse $\frac{x^2}{23}+\frac{y^2}{16}=1$

a) 20π sq units b) 40 sq units c) 10π sq units d) 20 sq units

108. Find the biggest granite stone in the cuboid shape that can be cut from the semi sphere rocky hill of radius $10\sqrt{3}$ ft.

a) 2000 e ft b) 1000 c ft c) 6000 c ft d) 8000 c ft

109. Toys manufacturing company has 3 branches at A, B, C places. Distance between B and C is 160 kms and A is 100 kms equidistant from B and C. Godown is to be built such that the distances from godown ot A,B,C are minimum. Find the distance between godown and branch A.

a) 45 kms b) 60 kms c) 55 kms d) 50 kms

110. If it takes 9 minutes in polar region to raise temperature from -5^0C ti 76^0C then find the average rate of change in temperature per minutes.

a) 7^0C b) 8^0C c) 9^0C d) 10^0C

111. If $f(x)=x^4e^{-x}$ then find the length of the interval in which f(x) in increasing.

a) 2 b) ∞ c) 4 d) $-\infty$

112. Find the number of stationary points of the function $f(x)=\sin^3 x+\cos^3 x$ in $(0, \pi/2)$.

a) 1 b) 2 c) 3 d) 4

113. All critical points of $f(x)=x^3-3x^2+3$ lies in

a) $(-2, 2)$ b) $(-5, 2)$ c) $(1, 5)$ d) $(-2, 4)$

Applications of Derivatives to Motion of a particle

Level – 1

114. If S = f(t) is the distance travelled by the particle then

$\frac{ds}{dt}$ represents-----------------, $\frac{d^2s}{dt^2}$ represents--------------

115. Acceleration in terms of velocity ------------------

116. When does the object projected vertically reaches maximum height?

117. When does the object projected vertically has maximum velocity?

118. When does the object projected vertically has maximum acceleration?

119. When does the object reverse its direction of motion?

Level – 2

120. The distance travelled by the stone projected vertically at time "t" is given by $S = 2t^3 + pt^2 + 2t + 3$. If stone takes 1 minute to reach the maximum height then find "p".

a) 4 b) –6 c) –4 d) 6

121. The relation between velocity and time of a particle moving on a straight line is $V(t) = 12t - 9t^2 + 2t^3$. Find its minimum velocity.

a) 3 units/sec b) 5 units/sec c) 4 units/sec d) None

Level – 3

122. The velocity "V" of a particle changes the cube of its displacements "S" then its acceleration is proportional to

a) $\frac{1}{3^3}$ b) 5^5 c) 3^3 d) $\frac{1}{5^5}$

123. the acceleration of a moving particle which started from rest is $a(t) = 6t - 2$. Its velocity after 1 sec is 4units/sec. Find its displacement after 3sec.

a) 24 units b) 27 units c) 16 units d) None

124. The distance travelled by a particle in "t" sec is given by $S(t) = 3t^2 + 4t - 5$. Find the time $t \in [1, 3]$ when the instantaneous velocity of the particle equals to its average velocity in the given interval.

a) 2/3 b) 3 c) 2 d) –2/3

125. The time and distance relation of particle is given by $S(t) = 8 + 3t^2 - t^3$. Find the distance at which the direction of the particle gets reversed.

a) 12 units b) 8units c) 4units d) 6nits

Answers

1.	2.	3.	4.	5.
6.	7.	8. B	9. C	10. B
11. C	12. A	13. C	14. A	15. A
16. C	17. B	18. A	19. B	20. D
21. C	22. C	23. B	24. B	25. C
26. C	27.	28.	29.	30.
31.	32.	33.	34.	35.
36.	37.	38.	39.	40.
41.	42.	43.1-C,2-D,3-A,4-B		
44. 1-D, 2-A, 3-B, 4-E		45.D		
46. D	47.C	48.D	49.B	50.A
51.D	52.C	53.A	54.B	55.C
56.C	57.A	58.D	59.A	60.C
61.	62.	63.	64.	65.C
66.A	67.A	68.C	69.C	70.C
71.A	72.	73.	74.	75.
76.	77.	78.	79.	80.
81.	82.	83.	84.	85.
86.C	87.C	88.B	89.B	90.D
91.B	92.C	93.A	94.C	95.A
96.D	97.C	98.C	99.C	100.C
101.	102.	103.	104.	105.
106.	107.	108.	109.	110.
111.C	112.A	113.D	114.	115.
116.	117.	118.	119.	120.C
121.C	122.B	123.B	124.C	125.A