# GEO-PHYSICS

## Paper I

| Time Allowed : Three Hours | Maximum Marks : 200 |
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### QUESTION PAPER SPECIFIC INSTRUCTIONS

**Please read each of the following instructions carefully before attempting questions.**

There are **TEN** questions divided under **TWO** Sections.

Candidate has to attempt **SIX** questions in all.

Questions No. **1** and **6** are **compulsory.**

Out of the remaining **EIGHT** questions, **FOUR** questions are to be attempted choosing **TWO** from each Section.

The number of marks carried by a question/part is indicated against it.

Neat sketches may be drawn to illustrate answers, wherever required. These shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

## SECTION 'A'

**1.**(a) Using suitable sketch, draw the elements of geomagnetic fields. Also show the relation between different components of earth's magnetic fields. 10

**1.**(b) Define Bulk modulus $K$ for isotropic solid. Derive the expression of computing Poisson's ratio $\sigma$ using body waves velocity $Vp$ and $Vs$. Give the relationship between $Vp$ and $Vs$ for Poisson solid. What is the dimension of Poisson ratio ? Compute $\frac{Vp}{Vs}$ for a rock with Poisson's ratio of 0·3. 10

**1.**(c) Using a suitable sketch show the relation between electric and magnetic fields. Derive electromagnetic equations for propagation of electric and magnetic field vectors in an isotropic homogeneous medium having conductivity $\sigma$, relative permeability $\mu$, and relative - dielectric permittivity $\epsilon$. 10

**1.**(d) What do you understand by eigenvalues and eigenvectors ? Determine the eigen-vectors of the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$ 10

**2.**(a) What do you mean by reference spheroid and geoid. Explain with the help of a suitable sketch. 10

**2.**(b) Define ray parameter in seismology and its relation to horizontal and vertical slowness. Consider a homogeneous three-layer model with 3 km thickness each and velocities 4, 6 and 8 km/sec for the top, middle and bottom layers, respectively. What is the surface to surface distance and travel time for a ray with $P = 0{\cdot}15$ s/km given below two equations (1) and (2).

$$X(P) = 2P \sum_i \frac{\Delta Z_i}{\left(u_i^2 - P^2\right)^{\frac{1}{2}}} u_i > P \qquad - (1)$$

and

$$T(P) = 2 \sum_i \frac{u_i^2 \, \Delta Z_i}{\left(u_i^2 - P^2\right)^{\frac{1}{2}}}, u_i > P \qquad - (2)$$ 10

**2.**(c) Write an expression for a least square solution of a linear inverse problem $\underline{d} = \underline{G}\,\underline{m}$; where $\underline{m}$ is model parameter, $\underline{d}$ represents the data vector and $\underline{G}$ is data kernel. Use the expression to solve the following for $m_1$ and $m_2$.

$$m_1 = 1$$
$$m_2 = 2$$
$$m_1 + m_2 = 2$$ 10

**3.**(a) Write down Kepler's three laws of planetary motion. Which law is a result of the conservation of energy ? Which law is a result of the conservation of momentum ? Also explain perihelion and aphelion, and perigee and apogee. 10

**3.**(b) What are surface waves in seismology ? What is seismic dispersion ? Derive the dispersion equation relating group velocity and phase velocity. 10

**3.**(c) Show that the gravitational potential satisfies Laplace's equation in free space and Poisson's equation in a region containing a mass. 10

**4.**(a) (i) What is polar wandering ? Give the typical extent in metres and period of wobbling of the earth's axis of rotation.

(ii) Given that for earth the principal moments of inertia : $A = B = 0{\cdot}329591\ Ma^2$, and $C = 0{\cdot}330674\ Ma^2$, calculate the frequency of precession of the earth's axis. Express your answer in both rad $s^{-1}$ and days. 4+6

**4.**(b) (i) Explain Biot-Savart's law with the help of suitable sketch.

(ii) Derive an expression for magnetic field ($H$) for long wire. 5+5

**4.**(c) What do you mean by singular value decomposition (SVD) of a matrix ? Describe a method to determine three matrices of SVD. Do the SVD analysis of the following matrix :

$$A = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$$ 10

**5.**(a) Derive the equation of seismometer in seismology. Also define critical damping and overdamping in seismometer equation. 10

**5.**(b) Derive an expression for gravity effect of the sphere. Draw suitable sketch using depth $z$ from surface to the centre of sphere and radius $a$ in m, and density contrast $\rho$. Also draw the profile over it after deducing expression for gmax. Find out the value of gmax for $z = 500$ m, $a = 50$ m and $\rho = 2{\cdot}6$.
(Given $\gamma = 6{\cdot}67 \times 10^{-11}\ m^3/kg/sec^2$) 10

**5.**(c) Given that $\overline{T}(u, v) = \langle u^2 - v^2,\ 2uv \rangle$.

(i) Find the velocity of $\overline{U}(t) = \langle 1, t^2 \rangle$ when $t = 1$.

(ii) Find the Jacobian and apply it to the vector in (i) as above.

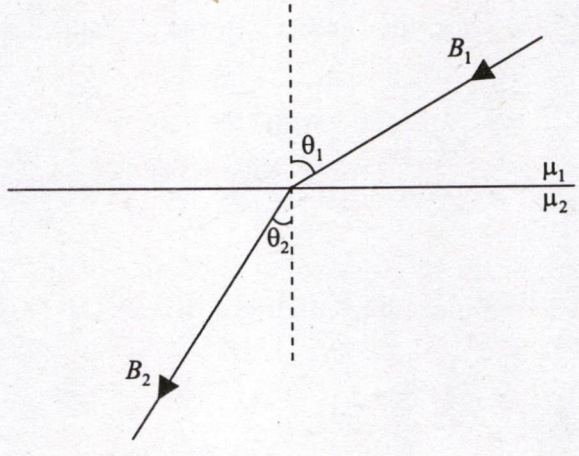
(iii) Also find $x(t) = T(u, t)$ in the $xy$ plane and then find its velocity vector at $t = 1$. 10

## SECTION 'B'

**6.**(a) Solve $\frac{d^2y}{dx^2} - \frac{4dy}{dx} + 3y = 0$, where $y = 0$ at $x = 0$ and $\frac{dy}{dx} = 1$ at $x = 0$. 10

**6.**(b) Six thousand electric lines of force enter a given volume and four thousand lines leave it. Find total charge contained in it. (Given permittivity of free space $\epsilon_0 = 8{\cdot}86 \times 10^{-12}$ Farad/metre) 10

**6.**(c) At the interface between one linear magnetic material and another, the magnetic field lines bend as shown in the figure. Find the relation between $\theta_1$ and $\theta_2$ in terms of $\mu_1$ and $\mu_2$. Assume there is no free current at the boundary. 10




**6.**(d) Given a total pressure $P = 1036$ hPa. Find the vapour pressure $e$ in a mixture of water vapour and air if the water vapour mixing ratio $r = 5{\cdot}6$ gkg$^{-1}$. The ratio of the gas constant of dry air to that of water vapour is 0·622. 10

**7.**(a) Using Simpson's one-third rule, find the value of the integral $\int_0^1 \frac{dx}{1+x}$ correct to third decimal place where the difference between the terms ($h$) is 0·25. 10

**7.**(b) State and prove Poisson's equation regarding electric potential. Show that potential function $\phi(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')}{|r-r'|}\,dv'$ represents the solution of Poisson's equation. 10

**7.**(c) Define the virtual temperature of a moist air parcel. Derive the expression for the virtual temperature in terms of the temperature, partial pressure of water and the total pressure $P$. 10

**8.**(a) Find the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using Fourier's series. 10

**8.**(b) A polarized wave with its $E$ vector of magnitude 20 $kVm^{-1}$ normal to the plane of incidence is reflected at the interface of medium 1 and medium 2 at an angle of 40° from the normal to the interface. Given that the medium 1 has : $\mu_{r1} = 2$ and $n_1 = 1{\cdot}5$, and medium 2 has : $\mu_{r2} = 3$ and $n_2 = 4$, calculate the magnitudes of reflected and transmitted electric fields. 10

**8.**(c) (i) Show that the critical frequency for propagation of electromagnetic waves in plasma is given by $f_c = -9\sqrt{n_0}$ where $n_0$ is number of electrons/$m^3$. 6

(ii) Calculate the plasma frequency and maximum penetration depth for a plasma containing $10^{20}$ electrons/$m^3$. 4

**9.**(a) Consider an air-filled copper waveguide, with dimensions $a = 2{\cdot}286$ cm, $b = 1{\cdot}016$ cm. Find the cutoff frequencies of the first four propagating modes. Also calculate the propagation constant of the $TE_{10}$ mode at 10 GHz. 10

**9.**(b) Apply Runge-Kutta fourth order to find an approximate value of $y$ when $x = 0{\cdot}4$. Given that, $\frac{dy}{dx} = x + y$ and $y = 1$, when $x = 0$. 10

**9.**(c) (i) What are the effects of the coriolis forces on the winds due to pressure gradients in the atmosphere. 6

(ii) Calculate the radius of curvature of wind moving with a speed of 30 $ms^{-1}$. 4

**10.**(a) (i) Explain with a suitable sketch wave propagation along the surface of the earth. 4

(ii) Explain with relevant expressions the wave tilt of surface waves. Sketch the wave tilt for an ideal conductor and for a non-ideal conductor. 6

**10.**(b) A transmission line has the following per unit length parameters : $L = 0{\cdot}2$ $\mu Hm^{-1}$, $C = 300$ $pFm^{-1}$, $R = 5$ $\Omega m^{-1}$, and $G = 0{\cdot}01$ $Sm^{-1}$. Calculate the propagation constant and characteristic impedance of this line at 500 MHz. 10

**10.**(c) A long straight copper tube having an inside radius of 2 cm and outside radius of 4 cm carries a current of 400 amperes. Compute the magnetic field at distance 1 cm, 3 cm and 8 cm from the axis. (Given $\mu_0 = 4\pi \times 10^{-7}$ weber/amp-m). 10

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