WORK BOOK FOR

INTERMEDIATE

SECOND YEAR

MATHEMATICS PAPER – II(A)

(Algebra and Probability)

BY

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Special commissioner &secretary

Board of intermediate Education

Andhra Pradesh

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CONTENT

PREFACE

I hear and I forget; I see and I remember;
I do and I understand; I Think and I learn.

The Board of Intermediate Education, Andhra Pradesh, Vijayawada made an attempt to provide work books for the first time to the Intermediate students with relevant and authentic material with an aim to engage them in academic activity and to motivate them for self learning and self assessment. These work books are tailored based on the concepts of *"learning by doing"* and *"activity oriented approach"* to sharpen the students in four core skills of learning – *Understanding, Interpretation, Analysis and Application.*

The endeavor is to provide ample scope to the students to understand the underlying concepts in each topic. The workbooks enable the students to practice more and acquire the skills to apply the learned concept in any related context with critical and creative thinking. The inner motive is that the students should shift from the existing rote learning mechanism to the conceptual learning mechanism of the core concepts.

I am sure that these compendia are perfect tools in the hands of the students to face not only the Intermediate Public Examinations but also the other competitive Examinations.

My due appreciation to all the course writers who put in all their efforts in bringing out these work books in the desired modus.

V. RAMAKRISHNA, I.R.S.
SECRETARY
B.I.E., A.P., VIJAYAWADA.

BOARD OF INTERMEDIATE EDUCATION: A.P. VIJAYAWADA

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MATHEMATICS 2A WORK BOOK

Complex Numbers

Level – I

1. If $z_1 = 1 - i$ and $z_2 = 1 + i$ then $z_1 > z_2$ (Yes/No)

2. If $z_1 = 1 - 2i$ and $z_2 = 1 + 2i$, then $|z_1| = |z_2|$ (Yes/No)

3. The conjugate of the complex numbers $(2-3i)(-2+3i)$ (Yes/No)

4. The conjugate of $\overline{i} = -i$ (Yes/No)

Fill up the blanks:

1. The conjugate of $(3+4i)(2-3i)$ is_____

2. The multiplicative inverse of $3-4i$ is ____

3. The locus of $z = x + iy$, where $|z| =$ $|z| = 1$ is acid. Its centre ______ and radius _______

Level – II

1. The value of $i^2 + i^4 + i^6 + ...$ (z_n times) is

   a) 0 b) 1 c) –1 d) +1

2. The amplitude of the complex numbers $-\sqrt{7} + i\sqrt{21}$ is

   a) $\frac{-\pi}{3}$ b) $\frac{\pi}{3}$ c) $\frac{-2\pi}{3}$ d) $\frac{-2\pi}{3}$

3. The amplitude of $-1-i$ is

   a) $\frac{3\pi}{4}$ b) $\frac{-3\pi}{4}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{4}$

Level – III

1. The no of solution is lie in $-\pi \le \theta \le \pi$ for which the complex numbers $\frac{3+2i\sin\theta}{1-2i\sin\theta}$ is a real number

a) 1 b) 2 c) 3 d) 4

2. If the amplitude of $(z-1)$ is $\frac{\pi}{2}$, then the locus of z is

a) a line
b) a line parallel to y-axis
c) line parallel to y-axis and lying is first quadrant
d) line parallel to x-axis

3. If $|z-3+i|=4$, then locus of z is

a) a line b) a circle c) a conic d) none

4. The eccentricity of the ellipse whose complex equation is $|z-3|+|z+3|=10$ is

a) $\frac{1}{3}$ b) $\frac{3}{5}$ c) $\frac{1}{4}$ d) $\frac{3}{4}$

Level – IV

5. The locus of z satisfying $|z-3+1|=4$ respectively a circle in the Argand plane. M Match the following List –I and List –II as given below

List – I	**List – II**
a) centre of circle is	i) (3, 1)
b) Radius of circle is	ii) (3, –1)
c) Length of intercept made by the circle on x-axis is	iii) 4
	iv) 16
d) The point (1, 1) is	v) $2\sqrt{15}$
	vi) $\sqrt{15}$
	vii) $\sqrt{7}$
	viii) $2\sqrt{7}$
	ix) interior point to the ole
	x) external point to the circle

A) (a, ii),(b, iii), (c, v),(d, x) B) (a, i),(b, iii), (c, vi),(d, ix)

C) (a, ii),(b, iii), (c, v),(d, x) D) (a, ii),(b, iii), (c, viii),(d, x)

Note: Student are advised to attempt these problem 4 and 5, after having sufficient knowledge in circle and ellipse.

DE MOIVERE'S THEOREM
Level – I

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1. For any real number θ and any integer $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$
2. If n is a rational number and θ is $\cos n\theta + i\sin n\theta$
3. If 'n' is any integer, then

   i) $(\cos\theta - i\sin\theta)^n = \cos n\theta - i\sin n\theta$

   ii) $(\cos\theta + i\sin\theta)^n = \cos n\theta - i\sin n\theta$

   iii) $\left(\dfrac{1}{\cos\theta + i\sin\theta}\right)^n = \cos n\theta - i\sin n\theta$

4. For any $\alpha, \beta \in R$ (i) $cis(\alpha)\cdot cis(\beta) = cis(\alpha+\beta)$

   (ii) $\dfrac{cis\alpha}{cis\beta} = cis(\alpha - \beta)$

5. For any $\theta \in R, x = \cos\theta + i\sin\theta$, then

   i) $x + \dfrac{1}{x} = -2\cos\theta$ (ii) $x - \dfrac{1}{x} = 2\cos\theta$

6. If n is any integer, then $(\sin\theta + i\cos\theta)^n = \cos n\left(\dfrac{\pi}{2} - \theta\right) + \sin\dfrac{\pi}{2}$
7. Let 'n' be a +ve integer and $z \in d$, if $w \in d$ is such that $w^n = z$, then w is called an nth of z and is denoted by $\sqrt[n]{z}$
8. The nth roots of unity are the solutions of the equation

   a) $x^n - 1 = 0$ b) $x^n + 1 = 0$ c) $x^{n-1} = 0$

9. The nth roots of unity differ by an argument

   a) $\dfrac{2\pi}{n}$ b) $\dfrac{k\pi}{n}$ c) $\dfrac{3\pi}{n}$ d) $\dfrac{\pi}{n}$

10. The sum of the nth roots unity is __0__
11. Product of nth roots of unity is

    a) $(-1)^n$ b) $(-1)^{n-1}$ c) $(-1)^{n+1}$ d) none of these

12. The nth roots of unity $1, \omega, \omega^2, ...\omega^{n-1}$ are in Geometric progression with common ratio $cis\frac{2\pi}{n}$

13. If $x+\frac{1}{x}=2\cos\theta$, then $x=\cos\theta \pm i\sin\theta$

14. The nth roots of unity lies on unit circle.

15. The nth roots of unity are the vertices of n sided regular polygon

16. Amplitudes of all the nth roots of unity are in A.P. with common difference $\frac{2\pi}{n}$

17. The cube roots of unity are the solutions of the equation

a) $x^5-1=0$ b) $x^3=1$ c) $x^3=-1$ d) $x^4+1=0$

18. If $1, \omega, \omega^2$ are cube roots of unity, then

i) $\omega^3=1$ ii) $1+\omega+\omega^2=0$

19. If $1, \omega, \omega^2$ are the cube roots of unity, then $(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)=a^3+b^3$

20. If $1, \omega, \omega^2$ are the cube roots of unity, then $(a+b)(a\omega+b\omega^2)(a\omega^2+b\omega)$ is ____

a) 1 b) 2 c) 3 d) 4

21. One of the cube roots of unity is

a) $\frac{-1+i\sqrt{3}}{2}$ b) $\frac{+1+i\sqrt{3}}{2}$ c) $\frac{1-i\sqrt{3}}{2}$ d) $\frac{\sqrt{3}-i}{2}$

22. The roots of the equation $x^4-1=0$ are

a) $1,1,i,-i$ b) $1,-1,i,-i$ c) $1,-1,\omega,\omega^2$ d) None of these

23. If $\sin\alpha+\sin\beta+\sin\gamma=0=\cos\alpha+\cos\beta+\cos\gamma$, then the value of $\sin^2\alpha+\sin^2\beta+\sin^2\gamma$ is _____

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{1}{2}$ d) 1

24. if n is a positive integer, then $(1+i)^n+(1-i)^n$ is equal to

a) $(\sqrt{2})^{n-2}\cos\frac{n\pi}{4}$ b) $(\sqrt{2})^{n-2}\sin\frac{n\pi}{4}$

c) $(\sqrt{2})^{n+2}\cos\frac{n\pi}{4}$ d) $(\sqrt{2})^{n+2}\sin\frac{n\pi}{4}$

25. If $x=\cos\theta$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ is _____

a) $2i\cos 6\theta$ b) $2\cos 6\theta$ c) $2i\sin 6\theta$ d) $2\cos\theta$

26. The principle amplitude of $(\sin 40+i\cos 40)^5$ is ____

a) 70^0 b) -110^0 c) 110^0 d) -70^0

DE MOIVERE'S THEOREM
Level – 2

1. The cube roots of unity are the vertices of a/an ____________ which are inscribed in a circle of unit radius with its centre at origin [ ]

   a) Right angled triangle b) Equilateral triangle
   c) Scalene triangle d) Isosceles triangle

2. $\left(-i+\sqrt{3}\right)^{300}+\left(-i-\sqrt{3}\right)^{300}=$ ________ [ ]

   a) 2^{300} b) 2^{301} c) 2^{100} d) -2^{100}

3. If ω is a complex cuberoot of unity, then $\sin\left[\left(\omega^{10}+\omega^{23}\right)\pi-\frac{\pi}{4}\right]=$ ___

   a) $\frac{1}{\sqrt{2}}$ b) $\frac{1}{2}$ c) 1 d) $\frac{\sqrt{3}}{2}$

4. If α and β are the roots of the equation $x^2-2x+4=0$, then $\alpha^n+\beta^n=$ _____ $\times\cos\frac{n\pi}{3}$ for any $n\in N$

   a) 2^n b) 2^{n+1} c) 2^{n-1} d) 2^{n-2}

5. If $1,\omega,\omega^2$ are the cube roots of unity, then

   $$\Delta=\begin{vmatrix}1 & \omega^n & \omega^{2n}\\ \omega^n & \omega^{2n} & 1\\ \omega^{2n} & 1 & \omega^n\end{vmatrix}= \_\_\_\_\_\_\_\_$$ [ ]

   a) 0 b) 1 c) ω d) ω^2

6. If $1,\omega,\omega^2$ are the cube roots of unity, then the roots of the equation $(x-1)^3+8=0$ are

   a) $-1,1+2\omega,1+2\omega^2$ b) $-1,1-2\omega,1-2\omega^2$
   c) $-1,-1,-1$ d) None of the these

7. $\frac{(\cos\theta+i\sin\theta)^4}{(\sin\theta+i\sin\theta)^5}$ is ____

   a) $\cos\theta-i\sin\theta$ b) $\cos 9\theta-i\sin 9\theta$

c) $\sin\theta - i\cos\theta$ d) $\sin 9\theta - i\cos 9\theta$

8. $\left(\dfrac{1+\cos 3\theta + i\sin 3\theta}{1+\cos 3\theta - i\sin 3\theta}\right)^{20} =$ ______

a) $\cos 60\theta + i\sin 60\theta$ b) $\cos 60\theta - i\sin 60\theta$

c) $\cos 20\theta + i\sin 20\theta$ d) $\cos 20\theta - i\sin 20\theta$

9. The value of $\left(\dfrac{1+\sqrt{3}i}{1-\sqrt{3}i}\right)^{64} + \left(\dfrac{1-\sqrt{3}i}{1+\sqrt{3}i}\right)^{64}$ is

a) zero b) –1 c) 1 d) i

10. If $\left(\dfrac{1+i}{1-i}\right)^{m} = 1$, the least integral value of m is

a) 2 b) 4 c) 8 d) none of these

11. If ω is a complex root of the equation $z^3 = 1$, then $\omega + \omega^{\left(\frac{1}{2}+\frac{3}{8}+\frac{9}{32}+\frac{27}{128}+....\right)}$ is equal to ___

a) –1 b) 0 c) 9 d) i

12. If $\omega = \dfrac{-1+i\sqrt{3}}{2}$, then $\left(3+\omega+3\omega^2\right)^4 =$ ____

a) 16 b) –16 c) 16ω d) $16\omega^2$

13. The common roots of the equations $x^{12} - 1 = 0$, $x^4 + x^2 + 1 = 0$ are

a) $\pm\omega$ b) $\pm\omega^2$ c) $\pm\omega, \pm\omega^2$ d) none of these

14. If z_1, z_2, z_3, z_4 are the roots of the equation $z^4 = 1$ then the value of $\sum_{i=1}^{4} z_i^3$ is

a) 0 b) 1 c) i d) $1+i$

15. The value of $\dfrac{\cos 30 + i\sin 30}{\cos 60 - i\sin 60}$ is equal to

a) i b) $-i$ c) $\dfrac{1+\sqrt{3}i}{2}$ d) $\dfrac{1-\sqrt{3}i}{2}$

16. $\left(\dfrac{-1+i\sqrt{3}}{2}\right)^{20} + \left(\dfrac{-1-i\sqrt{3}}{2}\right)^{20} =$ ______

a) $20\sqrt{3}i$ b) 1 c) $\dfrac{1}{2^{19}}$ d) 1

17. $\left(\dfrac{-1}{2} + \dfrac{\sqrt{3}i}{2}\right)^{1000} =$ ____

a) $\frac{1}{2}+\frac{\sqrt{3}i}{2}$ b) $\frac{1}{2}-\frac{\sqrt{3}i}{2}$ c) $\frac{-1}{2}+\frac{\sqrt{3}i}{2}$ d) none of these

18. Square of either of the two imaginary cube roots of unity will be
a) Real root of unity b) other imaginary root of unity
c) sum of two imaginary roots of unity d) None of these

19. If n is a +ve integer not a multiple of 3, then $1+\omega+\omega^{2n}=$_______
a) 3 b) 1 c) 0 d) none of these

20. If $a=\sqrt{2i}$ then which of the following is correct
a) $a=1+i$ b) $a=1-i$ c) $a=-(\sqrt{2})i$ d) None of these

21. The least +ve integer n which will reduce $\left(\frac{i-1}{i+1}\right)^n$ to real number is_____
a) 2 b) 3 c) 4 d) 5

22. If $z=\frac{\sqrt{3}+i}{2}$, then the value of z^{69} is
a) $-i$ b) i c) 1 d) –1

23. Which of the following is a fourth root of $\frac{1}{2}+i\frac{\sqrt{3}}{2}$
a) $cis\frac{\pi}{2}$ b) $cis\frac{\pi}{6}$ c) $cis\frac{\pi}{12}$ d) $cis\frac{\pi}{3}$

DE MOIVERE'S THEOREM
Level –3

1. If $\alpha,\beta\in d$ are distinct roots of the equation $x^2-x+1=0$, then $\alpha^{101}+\beta^{107}$ is equal to
a) 2 b) –1 c) 0 d) 1

2. Let 'ω' be a complex number such that $2\omega H=z$ where $z=\sqrt{-3}$

If $\begin{vmatrix}1 & 1 & 1\\ 1 & -\omega^2-1 & \omega^2\\ 1 & \omega^2 & \omega^7\end{vmatrix}=3K$, then K is equal to
a) 1 b) –z c) z d) –1

3. If $\omega(\neq 1)$ is a cube root of unicy and $(1+\omega)^7=A+B\omega$, then (A,B) equals

a) (–1,1) b) (0, 1) c) (1, 1) d) (1, 0)

4. If $x = a + b, y = a\omega + b\omega^2, z = a\omega^2 + b\omega$ then the value of $x^3 + y^3 + z^3$ is equal to (where ω is imaginary cube root of unity)

a) $a^3 + b^3$ b) $3(a^3 + b^3)$ c) $3(a^2 + b^2)$ d) none of these

5. The modulus and amplitude of $(1 + i\sqrt{3})^8$ are respectively

a) 256 and $\frac{8\pi}{3}$ b) 2 and $\frac{2\pi}{3}$

c) 256 and $\frac{2\pi}{3}$ d) 256 and $\frac{\pi}{3}$

6. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$___

a) 2 b) 1 c) –2 d) –1

7. If α and β are imaginary cube roots of unity, then the value of $\alpha^4 + \beta^{28} + \frac{1}{\alpha\beta}$ is

a) i b) $-i$ c) 1 d) –1

8. If $z + \frac{1}{z} = 1$, then $z^{100} + z^{-100}$ is equal to

a) i b) $-i$ c) 1 d) –1

9. If $z = \omega, \omega^2$ where ω is a non real complex cube root of unity, are two vertices of an equilateral triangle in the Argand plane then the third vertex may be represented b

a) $z = 1$ b) $z = 0$ c) $z = -2$ d) $z = -1$

10. If α is the (complex) fifth rod of unity then $|1 + \alpha + \alpha^2| =$_____

a) $2\cos\frac{\pi}{2}$ b) $2\cos\frac{\pi}{10}$ c) $2\cos\frac{\pi}{5}$ d) $2\cos\frac{\pi}{10}$

11. Let α and β be the roots of the equation $x^2 + 2x + 2 = 0$. Then $\alpha^{15} + \beta^{15}$ is equal to

a) 512 b) –512 c) –256 d) 256

12. Let $z = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)^5 + \left(\frac{\sqrt{3}}{2} - \frac{i}{2}\right)^5$. If $R(z)$ and $I(z)$ respectively, denote the real and imaginary parts of z, then

a) $R(z) > 0$ and $I(z) > 0$ b) $R(z) < 0$ and $I(z) > 0$

c) $R(z) = -3$ d) $I(z) = 0$

13. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}$ $(i = \sqrt{-1})$, then $(1 + iz + z^5 - iz^8)^9$ is equal to

a) 0 b) $(-1 + 2i)^9$ c) –1 d) 1

14. The value of $\sum_{k=1}^{6} \sin\frac{2k\pi}{7} - i\cos\frac{2k\pi}{7}$ is

a) -1 b) 0 c) $-i$ d) i

QUADRATIC EXPRESSIONS

SYNOPSIS

Quadratic Expression:

If $a \neq 0$, b, c are real (or) complex number then $ax^2 + bx + c$ is called a quadratic expression in the variable 'x'

A complex number 'α' is said to be zero to a quadratic expression $ax^2 + bx + c = 0$ if $a\alpha^2 + b\alpha + c = 0$

Quadratic Equation:

If $a \neq 0$, b, c are real (or) complex number then $ax^2 + bx + c$ is called a quadratic expression in 'x'

Identity:

An equation is called an identity when it is true for all values of the variables involved.

If a quadratic equation in x has more than two roots then it is an identity in x,

i.e. a=b=c=0

Roots of a quadratic equation:

If $a\alpha^2 + b\alpha + c = 0$ then α is a root or solution of the quadratic equation $ax^2 + bx + c = 0$

The roots of the quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminent:

$b^2 - 4ac$ is called the discriminate of the quadratic expression $ax^2 + bx + c$ as well as the quadratic equation $ax^2 + bx + c = 0$ and it is denoted by the symbol Δ

i) The roots are real or distinct if f $\Delta > 0$

ii)The roots are real and equal iff $\Delta = 0$

iii)The roots are complex with non zero imaginary part, iff $\Delta < 0$

iv)The roots are rational iff a,b,c are rational and D is a perfect square

Properties of roots of the equation:

1. If α, β are the roots of $ax^2 + bx + c = 0$ then the quadratic expression can be written as $ax^2 + bx + c = a(x-\alpha)(x-\beta)$

2. The quadratic equation whose roots are α, β is given by $x^2 - (\alpha + \beta)x + \alpha\beta = 0$

3. If 'α' is a root of the equation $f(x)=0$ then the polynomial $f(x)$ is exactly divisible by $(x-\alpha)$ or $(z-\alpha)$ is a factor of $f(x)$ and conversely
4. Every equation of nth degree $(n\geq 1)$ has exactly n roots and if the equation has more than n roots, then it is an identify
5. If the Coefficients of the equation $f(x)=0$ are all real and $\alpha+i\beta$ is its root then $\alpha-i\beta$ is also a root
   i.e. imaginary roots occur in conjugate pairs.
6. If the coefficients of the equation are all rational and $\alpha+\sqrt{\beta}$ is one of its roots then $\alpha-\sqrt{\beta}$ is also a root, where $\alpha,\beta\in Q$ and β is not a perfect square.
7. If there is any two real numbers a and b such that $f(a)$ and $f(b)$ are of opposite signs, then $f(x)=0$ must have at least one real root between a & b
8. If the signs of a or c in the quadratic equation $ax^2+bx+c=0$ are the same, then the product of the roots $\frac{c}{a}$ is positive, and hence if the roots are real they have the same sign.
9. If the signs of a & c in the quadratic equation $ax^2+bx+c=0$ are opposite, then the product of the roots $\frac{c}{a}$ is negative and hence if the roots are real then they have opposite signs.
10. If $a=c$, then the roots are reciprocal to each other.
11. If both roots are negative then a, b, c will have same sign.
12. If both roots are positive, then a, c will have same sign different from the sign of b
13. If a+ b +c=0 then the roots of quadratic equation $ax^2+bx+c=0$ are 1 and $\frac{c}{a}$
14. If a+c=b, then the roots of quadratic equation $ax^2+bx+c=0$ are -1 & $\frac{-c}{a}$
15. If the roots are in the ratio $m:n$ then $(m+n)^2ac=mnb^2$ (or) $\frac{(m+n)^2}{mn}=\frac{b^2}{ac}$
16. If one root is square of the other, then $a^2c+c^2a=b(3abc-b^2)$
17. If one root is equal to the nth power of the other root then $(ac^n)^{\frac{1}{n+1}}+(a^nc)^{\frac{1}{n+1}}+b=0$
18. If the roots differ by K then $b^2-4ac=a^2K^2$
19. Let α,β be the roots of the equation $ax^2+bx+c=0$ then the equation whose roots are $\alpha+k,\ \beta+k$ is obtained by replacing x by $(x-k)$ in the given equation.
20. Let α,β be the roots of the equation $ax^2+bx+c=0$ then the equation whose roots are $\alpha-k,\ \beta-k$ is obtained by replacing x by $(x+k)$ in the given equation.

21. If α, β are the roots of the equation $ax^2+bx+c=0$ then $k\alpha, k\beta$ are the roots of $f\left(\frac{x}{k}\right)=0$

22. If α, β are the roots of equation $ax^2+bx+c=0$, then $\frac{\alpha}{k}, \frac{\beta}{k}$ are the roots of $f(kx)=0$

23. To obtain an equation whose roots are the reciprocals of the roots of given equation, replace x by $\frac{1}{x}$ in the given equation.

24. If α, β are the roots of $f(x)=0$, then $-\alpha, -\beta$ are the roots of $f(-x)=0$

25. Equation whose roots are the square of the roots of given equation $f(x)=0$ is obtained by replacing x by $\sqrt{x}$ in the given equation.

26. Equation whose roots are the cubes of the roots of the given equation $f(x)=0$ is obtained by replacing x by $x^{1/3}$

Common roots:

Let $a_1x^2+b_1x+c_1=0$ and $a_2x^2+b_2x+c_2=0$ be two quadratic equations such that $a_1, a_2 \neq 0$ and $a_1b_2 \neq a_2b_1$

i) If α is a common root of the given equatins then

$(b_1c_2-b_2c_1)(a_1b_2-a_2b_1)=(c_1a_2-c_2a_1)^2$ which is the condition for roots of two quadratic equations to be common

ii) If two common roots are there for given equations then the required condition is

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Signs of a and $ax^2+bx+c=0$:

1. If the equation $ax^2+bx+c=0$ has non real roots ($\Delta<0$) then a and ax^2+bx+c will have same sign $\forall x \in R$

2. If the equation $ax^2+bx+c=0$ has equal roots then a and ax^2+bx+c will have same sign $\forall x \in R-\left\{\frac{-b}{a}\right\}$

3. If $a,b,c \in R$ and $a \neq 0$ such that the equation $ax^2+bx+c=0$ has real roots α and β with $\alpha<\beta$, then

   i) for $\alpha<x<\beta$, ax^2+bx+c and a have opposite signs

   ii) for $x<\alpha$ or $x>\beta$, ax^2+bx+c and a have same signs

4. Suppose that $a,b,c \in R$. $a \neq 0$ and $f(x)=ax^2+bx+c$

i) If a> 0, then $f(x)$ has absolute minimum at $x=\frac{-b}{2a}$ and the minimum value is $\frac{4ac-b^2}{4a}$

ii) If a < 0, then $f(x)$ has absolute maximum at $x=\frac{-b}{2a}$ and the maximum value is $\frac{4ac-b^2}{4a}$

<u>Quadratic Expression and its Graphs</u>:

Let a, b, c be real numbers, and $a \neq 0$ then $f(x)=ax^2+bx+c$ is known as Quadratic expression (or) Quadratic polynomial.

Consider $y=ax^2+bx+c$

$$y=a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]$$

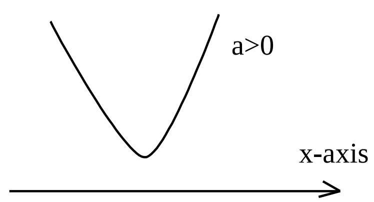
$$y=a\left[\left(x+\frac{b}{2a}\right)^2+\frac{4ac-b^2}{4a^2}\right]$$

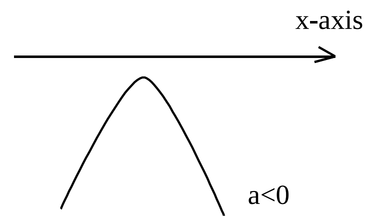
$$y=a\left[\left(x+\frac{b}{a}\right)^2-\frac{\Delta}{4a^2}\right]$$

$$\Rightarrow y+\frac{\Delta}{4a}=a\left(x+\frac{b}{2a}\right)^2$$

This represents a parabola

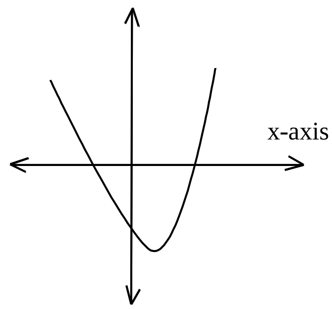
The parabola opens upwards (or) down wards according as a > 0 (or) a < 0





i) <u>The parabola will intersect the x–axis in the distinct points iff $\Delta > 0$</u>

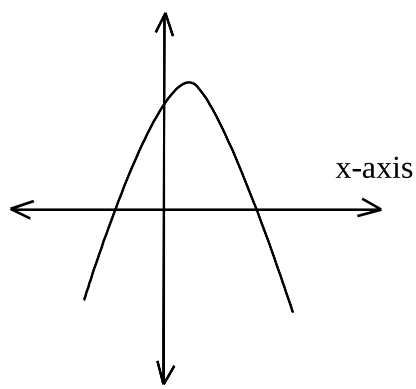
The parabola cuts the x-axis at $(\alpha,0)$ and $(\beta,0)$ where $ga=\frac{-b-\sqrt{\Delta}}{2a}, \beta=\frac{-b+\sqrt{\Delta}}{2a}$



For a > 0, we have

$$y_{\min} = \frac{-\Delta}{4a} \text{ at } x = \frac{-b}{2a} \text{ and } y_{\max} \to \infty \text{ and } y = f(x) \text{ is } \begin{cases} < 0 & if \quad \alpha < x < \beta \\ = 0 & if \quad x = \alpha, \beta \\ > 0 & if \quad x < \alpha \ \ or \ \ x > \beta \end{cases}$$

For a < 0, we have

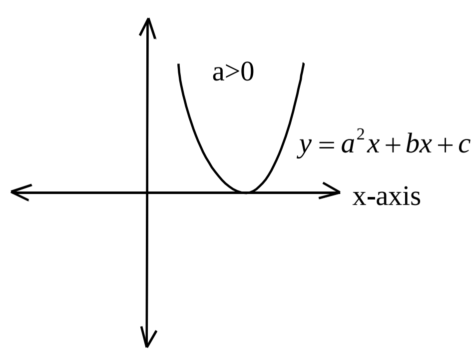


$$y_{\max} = \frac{-\Delta}{4a} \text{ at } x = \frac{-b}{2a} \text{ and } y_{\min} \to \infty \text{ and } y = f(x) \text{ is } \begin{cases} < 0 & if \quad x < \alpha \ \ or \ \ x > \beta \\ = 0 & if \quad x = \alpha, \beta \\ > 0 & if \quad \alpha < x < \beta \end{cases}$$

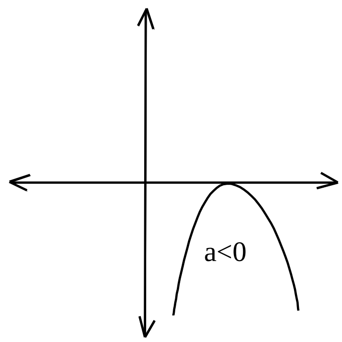
ii) <u>The parabola will touch the x–axis at one point iff $\Delta = 0$</u>

In this case the parabola touches x-axis at $(\alpha, 0)$ when $\alpha = \beta = \frac{-b}{a}$

<u>For a >0</u>, we have $y_{\min} = 0$ at $x = \frac{-b}{2a}$ and $y_{\min} = -\infty$ and $y = f(x)$ is $\begin{cases} < 0 & if \quad \neq \propto \\ = 0 & if \quad x = \propto \end{cases}$



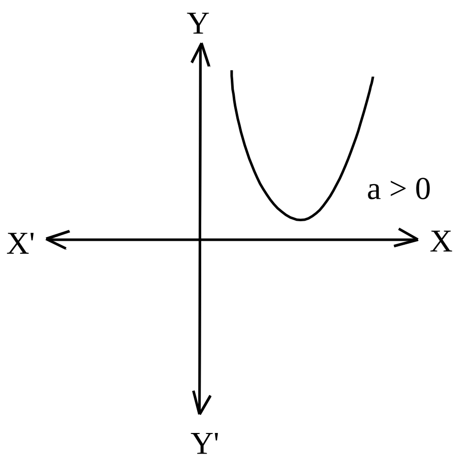
For a < 0, we have $y_{\max} = 0$ at $x = \frac{-b}{2a}$ and $y_{\min} = -\infty$ and $y = f(x)$ is $\begin{cases} < 0 & if \quad \neq \propto \\ = 0 & if \quad x = \propto \end{cases}$



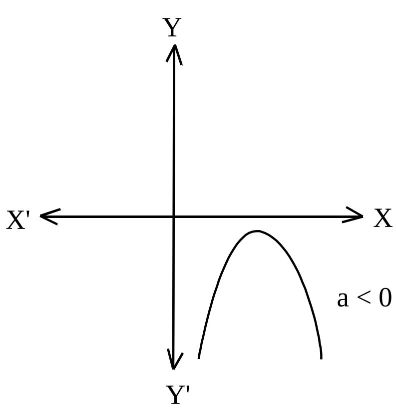
iii) The Parabola will not intersect x-axis iff $\Delta < 0$

In this case, parabola remains either completely above x-axis (or) completely below x-axis according as a >0 (or) a < 0

For a > 0, we have $y_{\min} = -\frac{\Delta}{4a}$ at $x = \frac{-b}{2a}$ and $y_{\max} = \propto$ and $y = f(x) > 0 \forall x$



For a< 0, we have $y_{\max} = -\frac{\Delta}{4a}$ at $x = \frac{-b}{2a}$ and $y_{\min} = -\propto$ and $y = f(x) < 0 \forall x$



Example: What is the minimum height of any point on the curve $y = x^2 - 4x + c$ above the x-axis?

Solution: Here a>0, and $\Delta = (-4)^2 - 4(1)(6) = 16 - 24 = -8 < 0$

$\therefore$ The parabola lies completely above x-axis and minimum value at any point on the curve is $\frac{-\Delta}{4a}$

$\therefore y_{\min} = \frac{-\Delta}{4a} = \frac{8}{4} = 2$

$\therefore$ minimum height = 2

Example: The internal in which a lies when the graph of the function $y = 16x^2 + 8(a+5)x - 7a - 5$ is strictly above the x-axis is

a) (–15, 2) b) (–15, –2) c) (–2, 0) d) (–15, 0)

Solution: x^2 coefficient >0, and graph lies completely above x-axis, then $\Delta < 0$

$\Rightarrow 64(a+5)^5 + 64(7a+5) < 0$

$\Rightarrow a^2 + 10a + 25 + 7a + 5 < 0$

$\Rightarrow a^2 + 17a + 30 < 0$

$\Rightarrow (a+15)(a+2) < 0 \Rightarrow 15 < a < -2$

$\therefore$ option (b) is correct

<u>Maximum and Minimum values of Rational Expression</u>:

1. If $f(x) = \frac{ax^2 + bx + c}{ax^2 - bx + c}$ or $f(x) = \frac{ax^2 - bx + c}{ax^2 + bx + c}$ then the minimum and maximum value of $f(x)$ are given by $f\left(\pm\sqrt{\frac{c}{a}}\right)$

2. If x is real, then the maximum and minimum value of $\frac{(a+x)(b+x)}{(c+x)}(x > -c, a > c, b > c)$ are $\left(\sqrt{a-c} - \sqrt{b-c}\right)^2$ and $\left(\sqrt{a-c} + \sqrt{b-c}\right)^2$

3. The maximum and minimum values of $\frac{a_1x^2 + b_1x + c_1}{a_2x^2 + b_2x + c_2}$ can be obtained as follows.

i) First equate the given rational expression to y, then

ii) Obtain a quadratic equation in x

iii) Then obtain the discriminant of quadratic equation

iv) Then $\Delta \geq 0$ solve for y

$\therefore$ The set of values of y obtained determines the value attained by given rational expression

Example : If x is real, then the value of $\dfrac{x^2-3x+4}{x^2+3x+4}$

a) (3, 4) b) (–3, 4) c) $\left(\dfrac{1}{7},\ 7\right)$ d) $(7, \infty)$

Solution: Minimum & maximum values are $f\left(\pm\sqrt{\dfrac{c}{a}}\right)$

$= f\left(\pm\sqrt{\dfrac{4}{1}}\right) = f(\pm 2)$

$\therefore$ Minimum value = $f(2) = \dfrac{1}{7}$

Maximum value $= f(-2) = 7$

$\therefore$ option (c) is correct

Fill in the blanks: (CUQ):

1. For the equation $6x^2+5x+1=0$ the sum of the roots is ______
2. If 2 & –3 are the roots of the equation $(x-2)\&(x+k)=0$ then k = _______
3. The equation $ax^2+bx+c=0$ can be expressed as $a\alpha^2+b\alpha+c=0$ is greaterthan zero, then the roots are __________
4. If the discriminant of the equation $ax^2+bx+c=0$ is greaterthan zero, then the roots are _____
5. The discriminent of the equation $x^2-7x+2=0$ is _______
6. If α, β are the roots of the equation $x^2-7x+2=0$ then $\alpha^2\beta+\alpha\beta^2=$_____
7. If one of the roots of the equation $x^2-2x+c=0$ is thrice the other, then c = ______
8. The number of real roots of the quadratic equation $3x^2+4=0$ is ______
9. The number of real solution of equation $|x|^2-5|x|+6=0$ is ____
10. Quadratic equation whose roots are reciprocals of the roots of the equation $7x^2-2x+9=0$ is _____
11. Quadratic equation whose roots are 3 and –2 is ______
12. If α, β are the roots of the equation $ax^2+bx+c=0$ then $p\alpha, p\beta$ are the roots of ____
13. Nature of the roots of $3x^2+7x+2=0$ is ______
14. If α, β are the roots of $ax^2+bx+c=0$ then $\alpha^2+\beta^2=$____
15. Quadratic equation whose roots are $2\sqrt{3}-5, -2\sqrt{3}-5$ is _____
16. If α, β are the roots of the equation $ax^2+bx+c=0$ then $\dfrac{1}{\alpha}+\dfrac{1}{\beta}=$_____

17. The values of m for which the equation $x^2+(m+3)x+(m+6)=0$ has equal roots is ______

18. If α,β are the roots of the equation $ax^2+bx+c=0$ then $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=$______

19. Nature of the roots of the equation $2x^2-8x+3=0$ is _____

20. If α,β are the roots of the equation $ax^2+bx+c=0$, then $\alpha^2+\beta^2=$_____

21. Maximum (or) Minimum value of $2x-7-5x^2$ is ______

22. The expression $3x^2+2x+11$ has absolute minimum value at x = _______

23. If α,β are the roots of the equation $x^2+3x-4=0$ then $\frac{1}{\alpha}+\frac{1}{\beta}$ is equal to

a) $\frac{-3}{4}$ b) $\frac{3}{4}$ c) $\frac{-4}{3}$ d) $\frac{4}{3}$

KEY (Fill in the blanks)

1. $\frac{-5}{6}$ 2. K = 3 3. Root (or) solution 4. Real distinct

5. 41 6. 6 7. $\frac{3}{4}$ 8. 0 9. 4

10. $9x^2-2x+7$ 11. $x^2-x-6=0$ 12. $ax^2+pbx+p^2c=0$

13. Distinct rational numbers 14. $\frac{b^2-2ca}{a^2}$ 15. $x^2+10x+13=0$

16. $\frac{-6}{c}$ 17. –5, 3 18. $\frac{b^2-2ca}{c^2}$ 19. Distinct real number

20. $\frac{-b^3+3ca}{a^3}$ 21. $-34/5$ 22. $-1/3$

Quadratic Expression (Objective Type)

Level – 1:

1. If $(x+1)$ is a factor of $x^4+(p-3)x^3-(3p-5)x^2+(2p-9)x+6=0$ then the value of p is

a) 3 b)4 c) 5 d) 8

2. If α,β are the roots of $ax^2+bx+c=0$ then $\frac{1}{\alpha^3}+\frac{1}{\beta^3}=$_____

a) $\frac{3abc-b^3}{a^3}$ b) $\frac{3ab-b^3}{a^2c}$ c) $\frac{3abc-b^3}{c^3}$ d) $\frac{b^2-2ac}{ac}$

3. The number of real solution of the equation $|x|^2-3|x|+2=0$ is

a) 4 b) 1 c) 3 d) 2

4. If a and b are rational and b is not a perfect square then the quadratic equation with rational coefficients whose one root is $\frac{1}{a+\sqrt{b}}$ is

a) $x^2-2ax+\left(a^2-b\right)=0$ b) $\left(a^2-b\right)x^2-2ax+1=0$

c) $\left(a^2-b\right)x^2-2bx+1=0$ d) None of the above

5. If α,β are the roots of $ax^2+2bx+c=0$ then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=$___

a) $\frac{4b^2-2ac}{ac}$ b) $\frac{4b^2-4ac}{ac}$ c) $\frac{2b^2-2ac}{ac}$ d) $\frac{2b^2-4ac}{ac}$

6. If α,β are the roots of the equation $px^2+qx+r=0$ then the value of $\alpha^2\beta+\beta^2\alpha$ is

a) $\frac{\gamma}{p^3}$ b) $\frac{\gamma\left(q^2-2pr\right)}{p^3}$ c) $\frac{\gamma\left(q^2+2pr\right)}{p^3}$ d) $\frac{q^2-2pr}{p^3}$

7. If α,β are the roots of the equation $2x^2-3x-6=0$ then the equation whose roots are $\alpha^2+2,\ \beta^2+2$ is

a) $4x^2+49x+118=0$ b) $4x^2-49x+118=0$

c) $4x^2-49x-118=0$ d) $x^2-49x+118=0$

8. If $2+i\sqrt{3}$ is a root of $x^2+px+q=0$ where p and q are real then $\left(p,\ q\right)$ is equal to

a) $\left(-4,\ 7\right)$ b) $\left(4,\ -7\right)$ c) $\left(-7,\ 4\right)$ d) $\left(4,\ 7\right)$

9. If a,b,c are real numbers then the real roots of the equation $ax^2+b\left|x\right|+c=0$ are

a) 2 b) 4 c) 0 d) –1

10. If a, b, c are positive numbers in G.P, then the roots of the equation $ax^2+bx+c=0$ are

a) real 2 negative b) have negative real parts

c) are equal d) have negative imaginary parts

11. In a quadratic equation $ax^2+bx+c=0$ if a, c are of opposite signs and b is real then the roots of the equation are ______

a) real & distinct b) real and equal

c) imaginary d) both roots position

12. If both the roots of $ax^2+bx+c=0$ are positive then

a) $\Delta>0,\ ab>0,\ ac>0$ b) $\Delta<0,\ ab<0,\ ac<0$

c) $\Delta>0,\ ab<0,\ ac>0$ d) $\Delta>0,\ ab>0,\ bc>0$

13. If both roots of $ax^2+bx+c=0$ are negative and b < 0 then

a) $a<0,c<0$ b) $a<0,c>0$ b) $a<0,c>0$

c) $a>0,c<0$ d) $a>0,c>0$

14. If a, b, c are positive, then both the roots of the equation $ax^2+bx+c=0$ then ____
a) are real or positive b) real and negative
c) have negative real parts d) have positive real parts

15. The set of values of p for which the roots of the equation $3x^2+2x+p(p-1)=0$ are of opposite sign, is
a) $(-\infty, 0)$ b) $(0, \infty)$ c) $(1, \infty)$ d) $(0, \infty)$

16. If p and q are the roots of the equation $x^2+px+q=0$ then
a) $p=1, q=-2$ b) $p=0, q=1$ c) $p=-2, q=0$ d) $p=-2, q=1$

17. If $|x|^2+|x|-2=0$ then the value of x is equal to
a) 2 b) –2 c) ±1 d) None of the above

18. If $ax^2+bx+c=0$ and $bx^2+cx+a=0$ have a common root and a, b, c are non zero real numbers, then find the value of $\frac{a^3+b^3+c^3}{abc}=$ _______
a) 5 b) 0 c) 3 d) 2

19. If the equation $x^2+5x+8=0$ and $ax^2+bx+c=0$, $a,b,c\in R$ have a common root then a : b : c = _____
a) $1:5:6$ b) $1:5:8$ c) $8:5:1$ d) None of these

20. If $x^2+6x-27>0$ and $x^2-3x-4<0$ then x satisfying these equation is
a) $x>3$ b) $x<4$ c) $3<x<4$ d) $x=\frac{7}{2}$

21. The sum of the real roots of the equation $x^2+5|x|+6=0$ is equal to
a) 5 b) 10 c) –8 d) does not exists

22. The number of real solution of the equation $\left(\frac{9}{10}\right)^x=-3+x-x^2$ is
a) 0 b) 1 c) 2 d) none of these

23. Roots of the equation $ax^3+bx^2+cx+d=0$ remain unchanged by increasing each coefficient by one unit, then
a) $a=b=c=d\neq 0$ b) $a=b\neq c=d\neq 0$
c) $a\neq b=c=d\neq 0$ d) $a=b=c\neq d\neq 0$

24. The number of values of a for which $(a^2-3a+2)(a^2-5a+6)x+(a^2-4)=0$ is an identify in x is :
a) 2 b) 3 c) 0 d) 1

25. If $|x-2|+|x-9|=7$, then the set of values of x is
a) $\{2, 9\}$ b) $\{2, 7\}$ c) $\{2\}$ d) $[2, 9]$

26. The roots of the equation $(x-a)(x-b)=abx^2$ are always

a) real b)imaginary c) cannot be discussed d) None of these

27. If α,β are the roots of the equation $ax^2+bx+c=0$, then the equation whose roots are $\frac{1}{a\alpha+b}$ and $\frac{1}{a\beta+b}$ is

a) $cax^2-bx+1=0$ b) $cax^2+bx+1=0$ c) $cax^2+bx-1=0$ d) None of these

28. The roots of the equation $(q-r)x^2+(r-p)x+(p-q)=0$ are

a) $\frac{r-p}{q-r},1$ b) $\frac{p-q}{p-r},1$ c) $\frac{q-r}{p-q},1$ d) $\frac{r-p}{p-q},1$

29. The sum of the roots of the equation $x^2+px+q=0$ is equal to the sum of the their square then

a) $p^2-q^2=0$ b) $p^2+q^2=2q$ c) $p^2+p=2q$ d) None of these

30. The equation formed by decreasing each root of $ax^2+bx+c=0$ by 1 is $2x^2+8x+2=0$, then

a) $a=-b$ b) $b=-c$ c) $c=-a$ d) $b=a+c$

31. If the equation $ax^2+bx+c=0$ and $x^2+x+1=0$ have a common root then

a) $a+b+c=0$ b) $a=b=c$ c) $a=b\ (or)\ b=c\ (or)\ c=a$

d) $a-b+c=0$

32. If α,β are the roots of the equation $2x^2+6x+b=0\,(b<0)$ then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is less then ___

a) 2 b) –2 c) 18 d) 0

33. If ω is the imaginary cube root of unity, then the equation whose roots are $2\omega+3\omega^2,\ 2\omega^2+3\omega$ is ___

a) $x^2+5x+7=0$ b) $x^2+5x-7=0$

c) $x^2-5x+7=0$ d) $x^2-5x-7=0$

34. If $x^2-(5m-2)x+(4m^2+10m+25)=0$ can be expressed as a perfect square, then m = _____

a) $\frac{8}{3}\ (or)\ 4$ b) $\frac{-8}{3}\ (or)\ 4$ c) $\frac{-8}{3}\ (or)\ 8$ d) $\frac{-4}{3}\ (or)\ 8$

35. If α,β are the roots of the equation $ax^2+bx+c=0$ then the value of $\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}=$ ______

a) $\frac{a}{bc}$ b) $\frac{b}{ac}$ c) $\frac{c}{ab}$ d) $\frac{ab}{c}$

36. if $x^2+4ax+3=0$ and $2x^2+3ax-9=0$ have a common root, then the value of a is ___

a) 3 b) 4 c) 21 d) 23

37. If α, β are the roots of the equation $ax^2+bx+c=0$ then the equation whose roots are $2+\alpha, 2+\beta$ is ____

a) $ax^2+2(b-4a)+(4a-2b+c)=0$ b) $ax^2+4bx+(a+b+c)=0$

c) $ax^2+(4a-b)x+(4a+2b-c)=0$ d) $ax^2+(4a-b)x+(4a+2b+c)=0$

38. If one root of the equation $ax^2+bx+c=0$ is double the other root, then the relation between a,b,c is

a) $9=\frac{2b^2}{a}$ b) $9ac=2b^2$ c) $9ac=2b$ d) $9=2b^2$

39. The roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0\ (a\neq b\neq c\neq 0)$ are ___

a) 1 and $\frac{b(c-a)}{a(b-c)}$ b) 1 and $\frac{c(a-b)}{a(b-c)}$

c) 1 and $\frac{a-b}{b-c}$ d) None of these

40. The nature of the roots of $x^2+x-1=0$ is ______

a) rational b) real c) irrational d) equal

41. Roots of the equation $2x^2-5x+1=0$ and $x^2+5x+2=0$ are

a) Reciprocal and of the same sign b) Reciprocal and of opposite sign

c) Equal in magnitude d) Imaginary

42. If $3^{1+x}+3^{1-x}=10$ then the values of x are

a) 1, –1 b) 1, 0 c) 1, 2 d) –1, –2

43. Max value of $\frac{1}{4x^2+2x+1}$ is ______

a) $\frac{-4}{3}$ b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) $\frac{-3}{4}$

44. If x is real and $5x^2+2x+9>3x^2+10x+7$ then x lies in the internal

a) $(2-\sqrt{3}, 2+\sqrt{3})$ b) $(-\propto, 2-\sqrt{3})\cup(2+\sqrt{3}, \propto)$

c) $(\sqrt{2}-1, \sqrt{2}+1)$ d) $(2+\sqrt{3}, \propto)$

45. If $x\in[2, 4]$ then for the expression x^2-6x+5

a) the least value = –4 b) The greatest value = 4

c) The least value = 3 d) The greatest value = –5

Level -I – Key

1. b	2. c	3. a	4. b	5. a
6. b	7. b	8. a	9. c	10. b
11. a	12. c	13. a	14. c	15. b
16. a	17. c	18. c	19. b	20. c
21. d	22. a	23. a	24. d	25. a
26. a	27. a	28. b	29. c	30. b
31. b	32. b	33. a	34. d	35. b
36. c	37. a	38. b	39. b	40. c
41. b	42. a	43. c	44. b	45. A

Level – 1 (Solutions)

1. $x+1$ is a factor of $x^4+(p-3)x^3-(3p-5)x^2+(2p-9)x+6=0$

$\Rightarrow -1$ is a root of above equation

$\therefore (-1)^4+(p-3)(-1)^3-(3p-5)(-1)^2+(2p-q)(-1)+6=0$

$\Rightarrow 1-(p-3)-(3p-5)-(2p-q)+6=0$

$\Rightarrow 1-p+3-3p+s-2p+9-6=0$

$\Rightarrow -6p+24=0$

$\therefore p=4$

2. $$\frac{1}{\alpha^3}+\frac{1}{\beta^3}=\frac{\alpha^3+\beta^3}{\alpha^3\beta^3}=\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{(\alpha\beta)^3}$$

$$=\frac{(-b/a)^3-3\frac{c}{a}\left(\frac{-b}{a}\right)}{\left(\frac{c}{a}\right)^3}=\frac{3abc-b^3}{c^3}$$

3. $|x|^2-3|x|+2=0$

$(|x|-1)(|x|-2)=0$

$|x|-1=0, |x|-2=0$

$|x|=1, |x|=2$

$\Rightarrow x=\pm1, \Rightarrow x=\pm2$

$\therefore x=1,-,2,-2$

4. Irrational roots always occur in pairs

So roots of the given equation are $\frac{1}{a+\sqrt{b}}, \frac{1}{a-\sqrt{b}}$

$\therefore$ sum of the roots $= \frac{2a}{a^2-b}$

Product of the roots $= \frac{1}{a^2-b}$

$\therefore$ Required quadratic equation is $x^2 - \left(\frac{2a}{a^2-b}\right)x + \frac{1}{a^2-b} = 0$

$\Rightarrow \left(a^2-b\right)x^2 - 2ax + 1 = 0$

5. α, β are the roots of $ax^2 + 2bx + c = 0$

$\alpha+\beta = \frac{-2b}{a}, \alpha\beta = \frac{c}{a}$

Now, $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{(-2b/a)^2 - 2\frac{c}{a}}{\frac{c}{a}}$

$= \frac{4b^2-2ca}{a^2} \times \frac{a}{c} = \frac{4b^2-2ca}{ac}$

6. α, β are the roots of the equation $px^2 + qx + r = 0$

$\Rightarrow \alpha+\beta = \frac{-q}{p}, \alpha\beta = \frac{\gamma}{p}$

Now $\alpha^2\beta + \beta^2\alpha = \gamma\beta\left(\alpha^2+\beta^2\right)$

$= \alpha\beta\left[(\alpha+\beta)^2 - 2\alpha\beta\right]$

$= \frac{\gamma}{p}\left[\frac{q^2}{p^2} - \frac{2\gamma}{p}\right] = \frac{\gamma}{p}\left[\frac{q^2-2\gamma p}{p^2}\right] = \frac{\gamma\left(q^2-2\gamma p\right)}{p^3}$

7. α, β are the roots of $2x^2 - 3x - 6 = 0$

$\alpha+\beta = \frac{3}{2}, \alpha\beta = \frac{-6}{2} = -3$

Then $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2+\beta^2 = \left(\frac{3}{2}\right)^2 - 2(-3) = \frac{33}{4}$

$\therefore$ Equation with roots α^2+2, β^2+2 is

$$x^2-\left(\alpha^2+2+\beta^2+2\right)x+\left(\alpha^2+2\right)\left(\beta^2+2\right)=0$$

$$\Rightarrow x^2-\left(\frac{33}{4}+4\right)x+\left(\alpha^2\beta^2+2\left(\alpha^2+\beta^2\right)+4\right)=0$$

$$\Rightarrow x^2-\left(\frac{49}{4}x\right)+\left(9+2\left(\frac{33}{4}\right)+4\right)=0$$

$$\Rightarrow x^2-\frac{49}{4}x+\left(\frac{59}{2}\right)=0$$

$$\Rightarrow 4x^2-49x+118=0$$

8. Complex roots occur in conjugate pair

So $2+i\sqrt{3},\ 2-i\sqrt{3}$ are the roots

So, Sum of the roots = –p

$\Rightarrow \left(2+i\sqrt{3}\right)+\left(2-i\sqrt{3}\right)=-p$

$\Rightarrow 4=-p$

$\therefore p=-4$

Product of the roots = q

$\Rightarrow \left(2+i\sqrt{3}\right)\left(2-i\sqrt{3}\right)=q$

$\Rightarrow 4-i^2\left(3\right)=q$

$\Rightarrow 4+3=q$

$\Rightarrow q=7$

9. Given equation can be written as $a|x|^2+b|x|+c=0$

$$|x|=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

But $|x|$ can not be $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$, since $|x|\geq 0$

Now since a,b,c are positive, $b^2-4ac<b^2$

$\Rightarrow \sqrt{b^2-4ac}<b^2$

$\Rightarrow -b+\sqrt{b^2-4ac}<0$

$\therefore |x|$ cannot be $-b+\sqrt{b^2-4ac}$ also so it has no real roots

10. If a, b, c are positive numbers and in G.P then $b^2=ac$

$\therefore b^2-4ac<0$ $\quad\therefore$ roots are imaging and they have negative real part

11. In a quadratic equation, if a,c are of opposite signs then ac < 0

$\Rightarrow b^2-4ac<0$ $\therefore$ roots are real and distinct

$\therefore$ option (1) is correct

12. option (c) is correct

13. b < 0 so, a < 0, c < 0

14. If a,b,c are positive, then roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If (i) $b^2 - 4ac < 0,$ then $x = \frac{-b \pm i\sqrt{|b^2 4ac|}}{2a}$

$\therefore$ Roots have negative real parts

Example: $x^2 + 2x + 2 = 0$

(ii) $b^2 - 4ac = 0$ then $x = \frac{-b}{2a}$

$\therefore$ Roots are equal and negative

(iii) $b^2 - 4ac > 0$ then

(A) $\sqrt{b^2 - 4ac} \le b$

Example: $x^2 + 3x + 1 = 0$

$\therefore$ roots are –ve

(B)If $\sqrt{b^2 - 4ac} > b$

$\Rightarrow -b + \sqrt{b^2 - 4ac} > 0, -b - \sqrt{b^2 - 4ac} < 0$

Then we get one positive and one negative roots.

15. Since the roots are of opposite signs product of the roots < 0

$\Rightarrow \frac{p(p-1)}{3} < 0$

$\Rightarrow p(p-1) < 0 \Rightarrow p \in (0, 1)$

16. If p and q are the roots of the equation $x^2 + px + q = 0$

Then sum of the roots $p + q = -p$, product of the roots $p \cdot q = q$

$pq = q \Rightarrow pq - q = 0$

$\Rightarrow q(p-1) = 0 \Rightarrow q = 0 \ (or) \ p = 1$

If q=0 then p =0, and if p=1 then q=–2

$\therefore$ (a) is the correct answer.

17. $|x|^2 + |x| - 2 = 0$

$|x|^2 + 2|x| - |x| - 2 = 0$

$|x|(|x| + 2) - 1(|x| + 2) = 0$

$(|x| + 2)(|x| - 1) = 0$

$|x| = -2$ which is not possible and $|x| = 1 \ \Rightarrow x = \pm 1$

18. $ax^2 + bx + c = 0, bx^2 + cx + a = 0$ have common root, Hence

$\left(bc - a^2\right)^2 = \left(ab - c^2\right)\left(ac - b^2\right)$

$\Rightarrow b^2c^2 + a^4 - 2a^2bc = a^2bc - a^3b^3 - ac^3 + b^2c^2$

$\Rightarrow a^4 + ab^3 + ac^3 = 3a^2bc$

$\Rightarrow \dfrac{a^3 + b^3 + c^3}{abc} = 3$

19. Since the quadratic equation $x^2 + 5x + 8 = 0$ has imaginary roots, the equation $ax^2 + bx + c = 0$ will have both roots same as the equation $x^2 + 5x + 8 = 0$ has

$\therefore \ \dfrac{a}{1} = \dfrac{b}{5} = \dfrac{c}{8}$

$\Rightarrow a = k,\ b = 5k, c = 8k$

$\therefore a : b : c = 1 : 5 : 8$

20. $x^2 + 6x - 27 > 0,\ x^2 - 3x - 4 < 0$

$\Rightarrow \left(x^2 + 9x - 3x - 27\right) > 0,\ x^2 - 4x + x - 4 < 0$

$\Rightarrow x(x+9) - 3(x+9) > 0,\ x(x-4) + 1(x-4) < 0$

$(x+9)(x-3) > 0$ and $(x-4)(x+1) < 0$

$x < -9$ or $x > 3$ and $x \in (-1, 4)$

$\therefore x \in (3, 4)$

21. $|x|^2 + 5|x| + 6 = 0$

$\Rightarrow (|x| + 2)(|x| + 3) = 0$

$\Rightarrow |x| + 2 = 0\ (or)\ |x| + 3 = 0$

$\Rightarrow |x| = -2\ \ (or)\ |x| = -3$ which is not possible

So given equation has no real roots

22. Let $f(x) = -3 + x - x^2$

Since $\Delta < 0$, and coefficient of $x^2 < 0$, so $f(x) < 0 \forall x \in R$

Thus LHS of the given equation is always positive and RHS is always negative

$\therefore$ The given equation has no solution

$\therefore$ option (a) is correct

23. Given equation $ax^3 + bx^2 + cx + d = 0$

Coefficients are increased by one unit

$\Rightarrow (a+1)x^3+(b+1)x^2+(c+1)x+(d+1)=0$

Given that roots remain unchanged, so $\frac{a+1}{a}=\frac{b+1}{b}=\frac{c+1}{c}=\frac{d+1}{d}$

$\Rightarrow a=b=c=d\neq 0$

24. Given equation is an identity

So $a^2-3a+2=0$, $a^2-5a+6=0$, or $a^2-4=0$

$\Rightarrow a=1,2$ $\Rightarrow a=2,3$ $\Rightarrow a=\pm 2$

So ‘2’ is the only solution

$\therefore$ No of the only solution

Equation is identity if a = 2

25. Either $(x-2)+(x-9)=7$ (or) $-(x-2)-(x-9)=7$

$\Rightarrow 2x-11=7$ (or) $-x+2-x+9=7$

$\Rightarrow 2x=18$ (or) $-2x+11=7$

$x=9$ $\Rightarrow -2x=-4$

$\therefore$ solution set = {2, 9} $\Rightarrow x=2$

26. $(x-a)(x-b)=abx^2$

$x^2-x(a+b)+ab-abx^2=0$

$(1-ab)x^2-(a+b)x+ab=0$

$\therefore \Delta=(a+b)^2-4(1-ab)(ab)$

$=a^2+2ab+b^2-4ab+4a^2b^2$

$=(a-b)^2+(2ab)^2$, which is always positive

So roots are real. Option (a) is correct

27. α,β are the roots of the equation

So $a\alpha^2+b\alpha+c=0$ and $a\beta^2+b\beta+c=0$

$\Rightarrow \alpha(a\alpha+b)=-c$, $\beta(a\beta+b)=-c$

$a\alpha+b=\frac{-c}{\alpha}$, $a\beta+b=\frac{-c}{\beta}$

$a=\frac{2}{3}$, $\therefore$

$\therefore$ Equation with roots $\frac{1}{a\alpha+b},\frac{1}{a\beta+b}$ is

$$x^2-\left(\frac{1}{a\alpha+b}+\frac{1}{a\beta+b}\right)x+\left(\frac{1}{a\alpha+b}\right)\left(\frac{1}{a\beta+b}\right)=0$$

$$\Rightarrow x^2-\left(\frac{-\alpha}{c}-\frac{\beta}{c}\right)x+\left(\frac{-\alpha}{c}\right)\left(\frac{-\beta}{c}\right)=0$$

$$\Rightarrow x^2+\left(\frac{\alpha+\beta}{c}\right)x+\frac{\alpha\beta}{c^2}=0$$

$$\Rightarrow x^2+\left(\frac{-b}{ac}\right)x+\left(\frac{c}{ac^2}\right)=0$$

$$\Rightarrow x^2-\frac{b}{ac}x+\frac{1}{ac}=0 \;\Rightarrow ac\,x^2-bx+1=0$$

28. Since sum of the coefficients = 0

The roots are 1 and $\frac{p-q}{q-r}$

29. $ga+\beta=-p, \quad \alpha\beta=q$

Given that $\alpha+\beta=\alpha^2+\beta^2$

$$\Rightarrow \alpha+\beta=(\alpha+\beta)^2-2\alpha\beta$$

$$\Rightarrow -p=(-p)^2-2q$$

$$\Rightarrow p^2-2q=-p$$

$$\Rightarrow p^2+p=2q$$

30. Equation formed by decreasing each root by '1' is f(x+1) = 0

$$\Rightarrow a(x+1)^2+b(x+1)+c=0$$

$$\Rightarrow a\left(x^2+2x+1\right)+bx+b+c=0$$

$\Rightarrow ax^2+(2a+b)x+(a+b+c)=0$ & $2x^2+8x+2=0$ both represent same

$$\therefore a=2,\ 2a+b=8,\ a+b+c=2$$

$$\therefore a=2,\ b=4,\ c=4,\ \therefore b=-c$$

31. $x^2+x1=0 \Rightarrow x=\frac{-1\pm\sqrt{1-4}}{2}$

$=x=\frac{-1\pm i\sqrt{3}}{2}$, so complex roots

$\therefore ax^2+bx+c=0$ also have same roots

$$\therefore \frac{a}{1}=\frac{b}{1}=\frac{c}{1}\Rightarrow a=b=c$$

32. $\alpha+\beta=\frac{-6}{2}=-3,\ \alpha\beta=\frac{6}{2}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{9-2\left(\frac{b}{2}\right)}{\left(\frac{b}{2}\right)}=\frac{9-b}{\left(\frac{b}{2}\right)}$$

$$=(9-b)\times\frac{2}{b}=\frac{18-2b}{b}=\frac{18}{b}-2$$

$\therefore \frac{\alpha}{\beta}+\frac{\beta}{\alpha}<-2 \ \ \therefore$ option (b) is correct

33. Equation of having roots $2\omega+3\omega^2, 2\omega^2+3\omega$ is

$$x^2-\left(2\omega+3\omega^2+2\omega^2+3\omega\right)+\left(2\omega+3\omega^2\right)\left(2\omega^2+3\omega\right)=0$$

$$\Rightarrow x^2-\left(5\omega+5\omega^2\right)x+\left(4\omega^3+6\omega^2+6\omega^4+9\omega^3\right)=0$$

$$\Rightarrow x^2-5\left(\omega+\omega^2\right)x+\left[4+9+6\left(\omega^2+\omega\right)\right]=0$$

$$\Rightarrow x^2-5(-1)x+(13-6)=0$$

$$x^2+5x+7=0$$

34. Perfect square $\Rightarrow$ roots are equal

$$\Rightarrow \Delta=0 \ \Rightarrow 9m^2-60m-96=0$$

$$\Rightarrow 3m^2-20m-32=0$$

$$\therefore m=\frac{20\pm\sqrt{(20)^2-4(3)(-32)}}{2(3)}$$

$$=8\ (or)\ \frac{-4}{3}$$

35. α is the root of the equation $\Rightarrow a\alpha^2+b\alpha+c=0$

$$\Rightarrow \alpha(a\alpha+b)=-c$$

$$\Rightarrow aga+b=\frac{-c}{a}$$

Similarly, β is the root of the equation $\Rightarrow \frac{-\alpha}{c}-\frac{\beta}{c}$

$$=\frac{-(\alpha+\beta)}{c}=\frac{-\left(\frac{b}{a}\right)}{c}=\frac{b}{ac}$$

36. $x^2+4ax+3=0,\ 2x^2+3ax-9=0$ has a common root

$\therefore (c_1a_2-c_2a_1)^2=(b_1c_2-b_2c_1)(a_1b_2-a_2b_1)$

$\Rightarrow (6+9)^2=(3a-8a)(-36a-9a)$

$\Rightarrow 15^2=(-5a)(-45a)$

$\Rightarrow a=\pm 1$

37. $f(x)=ax^2+bx+c$

$f(x-2)=0$

$\Rightarrow a(x-2)^2+b(x-2)+c=0$

$\Rightarrow ax^2+x(b-4a)+(4a-2b+c)=0$

38. If the roots are in the ratio m : n, then the condition is $(m+n)^2 ac=mnb^2$ (or)

$\frac{(m+n)^2}{mn}=\frac{b^2}{ac}$

So here m : n = 1 : 2

$\Rightarrow \frac{(1+2)^2}{1\cdot 2}=\frac{b^2}{ac}\Rightarrow \frac{9}{2}=\frac{b^2}{ac}$

$\Rightarrow 9ac=2b^2$

39. Since the sum of the coefficients = 0

$\therefore$ The roots are 1 and $\frac{c(a-b)}{a(b-c)}$

40. $x^2+a-1=0$

$\Delta=(+1)^2-4(1)(-1)$

$=1+4=5$ which is not a perfect square

$\therefore$ roots are irrational

41. $f(x)=2x^2-5x+1$

$f\left(\frac{-1}{x}\right)=x^2+5x+2$ $\therefore$ Reciprocal and of opposite sign

42. $3\cdot 3^x+3^1 3^{-x}=10$

$\Rightarrow 3t+\frac{3}{t}=10$

$\Rightarrow 3t^2-10t+3=0$

$\Rightarrow (t-3)(3t-1)=0$

43. Max value $=\dfrac{4a}{4ac-b^2}=\dfrac{4}{3}$

44. $\left(5x^2+2x+9-3x^2-10x-7\right)>0$

$\Rightarrow\left(2x^2-8x+2\right)>0$

$\Rightarrow x=\dfrac{8\pm\sqrt{(-8)-4(2)(3)}}{2(2)}$

$=\dfrac{8\pm4\sqrt{3}}{4}=2\pm\sqrt{3}$

$\therefore$ option (b) is correct

45. Minimum value $=\dfrac{4ac-b^2}{4a}$ $=-4$

Level – 2 (Objective Type)

1. The interval in which a lies when the graph of the function

$y=16x^2+8(a+5)x-7a-5=0$ is strictly above the x–axis is ____

a) $(-15,\ 2)$ b) $(-15,\ -2)$ c) $(-2,\ 0)$ d) $(-15,\ 0)$

2. Range of the expression $\dfrac{x^2-x+1}{x^2+x+1}$ is

a) $\left(\dfrac{1}{3},\ 3\right)$ b) $\left(\dfrac{1}{3},\ 0\right)$ c) $(3,\ 0)$ d) $\left(\dfrac{-1}{3},\ 0\right)$

3. If $P(q-r)x^2+q(r-p)x+r(p-q)=0,\ p\neq q\neq r$ has equal roots then the value of $\dfrac{p}{q}$

interms of p & r is _____

a) $\dfrac{2}{q}=\dfrac{1}{pr}$ b) $\dfrac{2}{q}=\dfrac{p+r}{p}$ c) $\dfrac{2}{q}=\dfrac{p+r}{pr}$ d) $\dfrac{2}{q}=\dfrac{p+r}{r}$

4. Sum of the real roots of the equation $x^2+5|x|+6=0$ is equal to

a) 5 b) 10 c) –5 d) does not exist

5. If the absolute value of the difference of roots of the equation $x^2 + px + 1 = 0$ exceeds $\sqrt{3p}$, then

a) p < –1 or p>4 b) p > 4 c) –1 < p < 4 d) $0 \le p < 4$

6. If $\frac{-1}{4-3i}$ is a root of $ax^2 + bx + 1 = 0,$ where a & b are real, then

a) a = 25, b = –8 b) a=25, b = 8

c) a = 5, b = –4 d) None of these

7. The number of positive integral values of k for which $\left(16x^2 + 12x + 39\right) + k\left(9x^2 - 2x + 11\right)$ is a perfect square is _____

a) two b) zero c) one d) None of these

8. For the equation $3x^2 + px + 3 = 0, p > 0$ if one of the roots is the square of the other, then p is equal to

a) $\frac{1}{3}$ b) 1 c) 3 d) $\frac{2}{3}$

9. If a, b, c are positive real numbers, then the number of real roots of the equation $ax^2 + b|x| + c = 0$ is

a) 2 b) 4 c) 0 d) None of these

10. If α, β are the roots of $ax^2 + bx + c = 0$ and $\alpha + k,\ \beta + k$ are the roots of $px^2 + qx + r = 0$ then $\frac{b^2 - 4ac}{q^2 - 4pr} =$ _____

a) $\left(\frac{p}{q}\right)^2$ b) 1 c) $\left(\frac{a}{p}\right)^2$ d) 0

11. If the roots of the equation $ax^2 + bx + c = 0$ are in the ratio m : n, then

a) $mnb^2 = (m+n)^2 ac$ b) $b^2(m+n) = mn$

c) $m + n = b^2 mn$ d) $c^2 mn = ab(m+n)^2$

12. The value of $\sqrt{6 + \sqrt{6 + \sqrt{6 + ...}}} + \infty$ is

a) 3 b) 6 c) –2 d) –4

13. If the roots of the equation $\left(a^2 + b^2\right)x^2 - 2(ac + bd)x + \left(c^2 + d^2\right) = 0$ are equal then

a) ab = cd b) ac = bd c) ad + bc= 0 d) $\frac{a}{b} = \frac{c}{d}$

14. If the roots of the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ are equal then a, b, c are in

a) AP b) GP c) HP d) None of these

15. If α,β are the roots of the equation $8x^2-3x+27=0$ then the value of $\left(\left(\frac{\alpha^2}{\beta^2}\right)+\left(\frac{\beta^2}{\alpha}\right)\right)^{1/3}$ is____

a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{7}{2}$ d) 4

16. If the expression $a^2(b^2-c^2)x^2+b^2(c^2-a^2)x+c^2(a^2-b^2)$ is a perfect square then

a) a,b,c are in AP b) a^2,b^2,c^2 are in AP

c) a^2,b^2,c^2 are in GP d) a^2,b^2,c^2 are in H.P

17. The values of m for which the equation $(1+m)x^2-2(1+3m)x+(1+8m)=0$ has equal roots, are

a) 0, 3 b) 1 c) 2 d) 3

18. If α is a root of $4x^2+2x-1=0$ then the other root is

a) $3\alpha^3-4\alpha$ b) $4\alpha^3-3\alpha$ c) $3\alpha^3+4\alpha$ d) $4\alpha^3+3\alpha$

19. If the roots of the equation $(b-c)x^2+(c-a)x+(a-b)=0$ are equal, then a, b, c are in

a) AP b) GP c) HP d) None of these

20. If the sum of the roots of the equation $ax^2+bx+c=0$ is equal to the sum of the reciprocals of their squares, then bc^2,ca^2 and ab^2 are in _____

a) AP b) GP c) HP d) None of these

21. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2+bx+c=0$, then

a) $(a-c)^2=b^2-c^2$ b) $(a-c)^2=b^2+c^2$

c) $(a+c)^2=b^2-c^2$ d) $(a+c)^2=b^2+c^2$

22. If $a(b-c)x^2+b(c-a)xy+c(a-b)y^2$ is a perfect square then a,b,c are in

a) AP b) GP c) HP d) None of these

23. If $x=2+2^{1/3}+2^{2/3}$ then the value of x^3-6x^2+6x is

a) 3 b) 2 c) 1 d) None of these

24. If α,β are the roots of the quadratic equation $x^2-2\cos\theta\cdot x+1=0$ then the equation whose roots are α^n,β^n is

a) $x^2-(2\cos n\theta)x+1=0$ b) $2x^2+(2\cos n\theta)x+1=0$

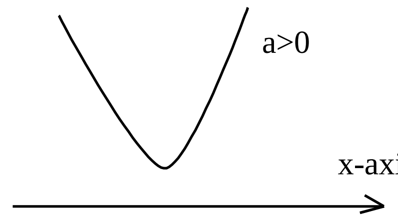
c) $x^2+(2\cos n\theta)x+1=0$ d) $x^2+(2\cos n\theta)x-1=0$

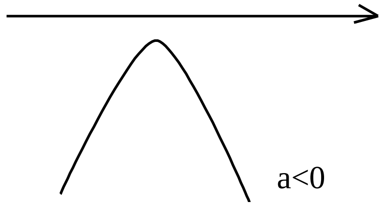
25. The value of $x^2+2bx+c$ is positive if

a) $b^2-4ac>0$ b) $b^2-4ac<0$ c) $c^2<b$ d) $b^2<c$

26. The values of a which make the expression $x^2-ax+(1-2a^2)=0$ always positive for real values of x, are

a) $\frac{-2}{3}<a<\frac{2}{3}$

b) 

c) 

d) $0<a<\frac{2}{3}$

27. **More than one correct option:**

The roots of the equation $(a+\sqrt{b})^{x^2}+(a-\sqrt{b})^{x^2-15}=2a$ where $a^2-b=1$ are

a) ± 3 b) ± 4 c) $\pm\sqrt{14}$ d) $\pm\sqrt{5}$

28. For real x, the function $\frac{(x-a)(x-b)}{x-c}$ will assume all real values provided

a) a > b > c b) $b\le c\le a$ c) $a>c>b$ d) $a\le c\le b$

29. **One option questions:**

If the equation $x^2+2x+3=0,\ ax^2+bx+c=0,\ a,b,c\in R$ have a common root, then a: b : c is

a) 1 : 2 : 3 b) 3 : 2 : 1 c) 1 : 3 : 2 c) 3 : 1 : 2

30. Sacin and Rahul attempted to solve a quadratic equation Sachin made a mistake in writing down the constant term and ended up in roots (4, 3) Rahul made a mistake in writing down coefficient of x to get roots (3, 2) The correct roots of equation are

a) –4, –3 b) 6, 1 c) 4, 3 d) –6, –1

31. If α,β are in the roots of the equation $x^2-x+1=0$, then $\alpha^{2009}+\beta^{2009}$ is equal to ____

a) –2 b) –1 c) 1 d) 2

32. If the difference between the roots of the equation $x^2+ax+1=0$ is less than $\sqrt{5}$, then the set of possible values of a is

a) $(-3, 3)$ b) $(-3, \infty)$ c) $(3, \infty)$ d) $(-\infty, -3)$

33. If the roots of the quadratic equation $x^2 + px + q = 0$ are tan30 and tan15 respectively then the value of 2+q–p is

a) 3 b) 0 c) 1 d) 2

34. If the roots of the equation $x^2 - bx + c = 0$ are two consecutive integers, then $b^2 - 4c =$

a) 1 b) 2 c) 3 d) –2

35. If $(1-p)$ is a root of quadratic equation $x^2 + px + (1-p) = 0$ then its roots are

a)0, 1 b) –1, 1 c) 0, –1 d) –1, 2

36. If one root of the equation $x^2 + px + 12 = 0$ is 4, while the equation $x^2 + px + q = 0$ has equal roots, then the value of q is _____

a) $\frac{49}{4}$ b) 12 c) 3 d) 4

37. Let two numbers have arithmetic mean 9 and geometric mean 4. Then these two numbers are the roots of

a) $x^2 + 18x + 16 = 0$ b) $x^2 - 18x + 16 = 0$

c) $x^2 + 18x - 16 = 0$ d) $x^2 - 18x - 16 = 0$

38. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the square of their reciprocals, then $\frac{a}{c}, \frac{b}{a}$ and $\frac{c}{b}$ are in

a) AP b) GP c) HP d) AGP

39. The values of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice as large as the other is

a) $\frac{2}{3}$ b) $\frac{-2}{3}$ c) $\frac{1}{3}$ d) $\frac{-1}{3}$

40. If the roots of the $x^2 - ax + b = 0$ are two consecutive odd integers, then $a^2 - 4b =$

a) 3 b) 4 c) 5 d) 6

41. If α, β are the roots of $x^2 - ax + b^2 = 0$ then $\alpha^2 + \beta^2 =$

a) $a^2 + 2b^2$ b) $a^2 - 2b^2$ c) $a^2 - 2b$ d) $a^2 + 2b$

42. If the roots of the equation $x^2 + 2bx + c = 0$ are α and β then $b^2 - c$ is equal to

a) $\frac{(\alpha - \beta)^2}{4}$ b) $(\alpha + \beta)^2 - \alpha\beta$ c) $(\alpha + \beta)^2 + \alpha\beta$ d) $\frac{(\alpha - \beta)^2}{2} + \alpha\beta$

43. If $\alpha \neq \beta$, $\alpha^2 = 5\alpha - 3$, $\beta^2 = 5\beta - 3$ then the equation having $\frac{\alpha}{\beta}$ and $\frac{\beta}{2}$ as its roots is

a) $3x^2 + 19x + 13 = 0$ b) $3x^2 - 19x + 3 = 0$

c) $3x^2 - 19x - 3 = 0$ d) $x^2 - 16x + 1 = 0$

44. The number of real roots of $3^{2x^2-7x+7}=9$ is

a) 0 b) 2 c) 1 d) 4

45. The equation whose roots are the squares of the roots of the equation $2x^2+3x+1=0$ is

a) $4x^2+5x+1=0$ b) $4x^2-x+1=0$

c) $4x^2-5x-1=0$ d) $4x^2-5x+1=0$

46. Roots of the equation $x^2-\sqrt{3}x+1=0$ are

a) $x=\frac{-3\pm 2i}{2}$ b) $x=\frac{-\sqrt{3}\pm i}{2}$ c) $x=-\sqrt{3}\pm i$ d) $\frac{\sqrt{3}\pm i}{2}$

47. If the product of the roots of the equation $x^2-2\sqrt{2}kx+2e^{2\log k}-1=0$ is 31, the the roots of the equation are real for k, in equal to

a) –4 b) 1 c) 4 d) 0

48. If α,β are the roots of the equation $x^2-2x+4=0$ then the value of $\alpha^n+\beta^n=$

a) $2^{n+1}\sin\frac{n\pi}{3}$ b) $2^{n+1}\cos\frac{n\pi}{3}$ c) $2^{n-1}\sin\frac{n\pi}{3}$ d) $2^{n-1}\cos\frac{n\pi}{3}$

49. If the arithmetic mean of the roots of a quadratic equation is 8 and the geometric mean is 5, then the equation is

a) $x^2-16x-25=0$ b) $x^2+16x-25=0$

c) $x^2-16x+25=0$ d) $x^2-8x+5=0$

50. If the equation $2ax^2-3bx+4c=0$ and $3x^2-4x+5=0$ have a common root, then $\frac{a+b}{b+c}$ is equal to

a) $\frac{1}{2}$ b) $\frac{3}{35}$ c) $\frac{34}{31}$ d) $\frac{29}{23}$

51. If a,b,c are in A.P then the roots of the equation $ax^2-2bx+c=0$ are

a) 1 and $\frac{c}{a}$ b) $-\frac{1}{a}$ and –c c) –1 and $-\frac{c}{a}$ d) –2 and $-\frac{c}{ab}$

52. If a,b,c are three real numbers such that a+2b+4c=0 then the equation $ax^2+bx+c=0$

a) has both the roots complex b) has its roots lying with –1<x<0

c) has one of the roots equal to 1/2 d) none of these

53. If α,β are the roots of the equation α,β then the equation whose roots are $\frac{k}{\alpha}$ and $\frac{k}{\beta}$ is ____

a) $cx^2+kbx+k^2a=0$ b) $cx^2+k^2bx+ka=0$

c) $kcx^2+bx+k^2a=0$ d) $k^2cx^2+bx+ka=0$

Key (Level – 2)

1. b	2. a	3. c	4. d	5.b
6. a	7. c	8. c	9.c	10.c
11. a	12. a	13. d	14. c	15. b
16. d	17. a	18. b	19. a	20. a
21. d	22. c	23. b	24. a	25. d
26. a	27. a,c	28. b,d	29.a	30.b
31. c	32. a	33.a	34. a	35. c
36. a	37. b	38. c	39. a	40. b
41. b	42. a	43. b	44. b	45. d
46. d	47. c	48. b	49. c	50. c
51. a	52. c	53.a		

Level –2 (Solutions)

1. a = 16 >0 and graph of the function strictly lies above x–axis then $\Delta < 0$

$\Rightarrow 8(a+5)^2 - 4(16)(-7a-5) < 0$

$\Rightarrow a^2 + 10a + 25 + 7a + 5 < 0$

$\Rightarrow a^2 + 17a + 30 < 0$

$\Rightarrow (a+15)(a+2) < 0 \Rightarrow -15 < a < -2$

$\therefore$ option (b) is correct

2. Here $f(x) = \dfrac{ax^2+bx+c}{ax^2-bx+c}$ (or) $\dfrac{ax^2-bx+c}{ax^2+bx+c}$ then

min & max values of $f(x) = f\left(\pm\sqrt{\dfrac{c}{a}}\right)$ option (1) is correct

3. If a+b+c =0 in $ax^2+bx+c=0$ then the roots are 1 & $\dfrac{c}{a}$ so for the given equation, the roots are 1 and $\dfrac{r(p-q)}{p(q-r)}$ given that roots are equal

$\Rightarrow 1 = \dfrac{r(p-q)}{p(q-r)}$

$\Rightarrow p(q-r) = r(p-q)$

$\Rightarrow pq + qr = 2pr$

$\Rightarrow \dfrac{2}{q} = \dfrac{p+r}{pr}$ $\therefore$ option (c) is correct

4. $|x|^2 + 5|x| + 6 = 0$

$\Rightarrow (|x|+2)(|x|+3)=0$

$|x|+2=0,\ |x|+3=0$

$|x|=-2,\ |x|=-3$ which is not possible

$\therefore$ The given equation does not have real solution

$\therefore$ sum does not exists $\quad\therefore$ option (d) is correct

5. $|\alpha-\beta|>\sqrt{3}p$

$\Rightarrow (\alpha-\beta)^2>3p$

$\Rightarrow \alpha^2+\beta^2-2\alpha\beta>3p$

$\Rightarrow (\alpha+\beta)^2-4\alpha\beta>3p$

$\Rightarrow (-p)^2-4(1)>3p$

$\Rightarrow p^2-3p-4>0,\ p>0$

$\Rightarrow (p-4)(p+1)>0, p>0$

$\therefore\ p>4 \quad \therefore$ option (b) is correct

6. $\frac{1}{4-3i}\ \&\ \frac{1}{4+3i}$ are the roots

$\therefore \alpha\beta=\frac{1}{(4-3i)(4+3i)}=\frac{1}{16+9}=\frac{1}{25}$

But $\alpha\beta=\frac{1}{a} \quad \therefore \frac{1}{a}=\frac{1}{25}\Rightarrow a=25$

Sum of the roots = $\frac{-b}{a}$

$\Rightarrow \frac{1}{4-3i}+\frac{1}{4+3i}=\frac{-b}{a}$

$\Rightarrow \frac{4+3i+4-3i}{(4-3i)(4+3i)}=\frac{-b}{a}$

$\Rightarrow \frac{8}{25}=\frac{-b}{a}$

$\therefore b=-8$

7. $(16x^2+12x+39)+k(9x^2-2x+11)$ is a perfect square

$\Rightarrow \Delta=0 \qquad (16+9k)x^2+(12-2k)x+(39+11k)=0$

$\Rightarrow (12-2k)^2-4(16+9k)(39+11k)=0$

$\Rightarrow 4(6-k)^2 - 4(624+176k+351k+99k^2) = 0$

$\Rightarrow 36+k^2-12k-624-527k-99k^2 = 0$

$\Rightarrow -98k^2-53k-588 = 0$

$\Rightarrow -49\left[2k^2+11k+12\right] = 0$

$\Rightarrow 2k^2+11k+12 = 0$

$\Rightarrow 2k^2+8k+3k+12 = 0$

$\Rightarrow 2k(k+4)+3(k+4) = 0$

$(k+4)(2k+3) = 0$

$\Rightarrow a = \pm 2$ only one integral value for k exists

8. Let $\therefore$ are the roots

Then $\alpha+\alpha^2 = \frac{-p}{3}$, $\alpha \cdot \alpha^2 = 1$

$\Rightarrow \alpha^3 = 1$

$\Rightarrow \alpha = 1, \omega, \omega^2$

If $\alpha = 1$, then p = –6

If $\alpha = \omega$, then p = 3

If $\alpha = \omega^2$, then also p = 3

$\therefore$ but p > 0, so p = 3

9. The given equation can be written as

$a|x|^2 + b|x| + c = 0$ have a,b,c are positive numbers

$\therefore ax^2 + b|x| + c > 0 \forall x$

Hence the equation has real roots

10. $\alpha+\beta = \frac{-b}{a}$, $\alpha\beta = \frac{c}{a}$ and $(\alpha+k+\beta+k) = \frac{-q}{p}$, $(\alpha+k)(\beta+k) = \frac{\gamma}{p}$

Now $|\alpha-\beta| = |(\alpha+k)-(\beta+k)|$

$\Rightarrow \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{(\alpha+k+\beta+k)^2 - 4(\alpha+k)(\beta+k)}$

$\Rightarrow \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}} = \sqrt{\frac{q^2}{p^2} - \frac{4r}{p}}$

$\Rightarrow \frac{b^2-4ca}{a^2} = \frac{q^2-4rp}{p^2}$

$\Rightarrow \frac{b^2-4ca}{q^2-4rp} = \frac{a^2}{p^2}$ $\therefore$ option (c) is correct

11. $\frac{\alpha}{\beta}=\frac{m}{n}$

$$\Rightarrow \frac{\alpha+\beta}{\alpha-\beta}=\frac{m+n}{m-n}\Rightarrow \frac{(\alpha+\beta)^2}{(\alpha-\beta)^2}=\frac{(m+n)^2}{(m-n)^2}$$

$$\Rightarrow \frac{(\alpha+\beta)^2}{(\alpha+\beta)^2-4\alpha\beta}=\frac{(m+n)^2}{(m+n)^2-4mn}$$

$$\Rightarrow \frac{(\alpha+\beta)^2-4\alpha\beta}{(\alpha+\beta)^2}=\frac{(m+n)^2-4mn}{(m+n)^2}$$

$$\Rightarrow 1-\frac{4\alpha\beta}{(\alpha+\beta)^2}=1-\frac{4mn}{(m+n)^2}$$

$$\Rightarrow \frac{4\alpha\beta}{(\alpha+\beta)^2}=\frac{4mn}{(m+n)^2}$$

$$\Rightarrow \frac{4\frac{c}{a}}{\left(\frac{b^2}{a^2}\right)}=\frac{4mn}{(m+n)^2}$$

$$\Rightarrow 4\frac{c}{a}\times\frac{a^2}{b^2}=\frac{4mn}{(m+n)^2}\Rightarrow ca(m+n)^2=mnb^2$$

$\therefore$ option (b)is correct

12. Let $x=\sqrt{6+\sqrt{6+\sqrt{6+........}}}+\infty$

$$\Rightarrow x=\sqrt{6+x}$$

$$\Rightarrow x^2=6+x$$

$$\Rightarrow x^2-x-6=0$$

$$\Rightarrow x^2-3x+2x-6=0$$

$$x(x-3)+2(x-3)=0$$

$(x-3)(x+2)=0$ but x cannot be negative so x = 3

13. roots are equal, so $\Delta=0$

$$\Rightarrow\left[-2(ac+bd)\right]^2-4\left(a^2+b^2\right)\left(c^2+d^2\right)=0$$

$$\Rightarrow 4\left(a^2c^2+2abcd+b^2d^2\right)-4\left(a^2c^2+a^2d^2+b^2c^2+b^2d^2\right)=0$$

$$\Rightarrow a^2c^2+2abcd+b^2d^2-a^2c^2-a^2c^2-a^2d^2-b^2c^2-b^2d^2=0$$

$$\Rightarrow a^2d^2+b^2c^2-2abcd=0$$

$\Rightarrow (ad-bc)^2 = 0$

$\Rightarrow ad = bc \Rightarrow \frac{a}{c} = \frac{b}{\propto}\ (or)\ \frac{a}{b} = \frac{c}{d}$

$\therefore$ option (d) is correct

14. since sum of the coefficients = 0

roots are 1 & $\frac{c(a-b)}{a(b-c)}$

given that roots are equal

so $1 = \frac{c(a-b)}{a(b-c)}$

$\Rightarrow a(b-c) = c(a-b)$

$\Rightarrow ab - ac = ca - bc$

$\Rightarrow ab + bc = 2ca$

$\Rightarrow b(a+c) = 2ac$

$\Rightarrow b = \frac{2ac}{a+c}$ $\quad\therefore$ a,b,c are in H.P

15. $\alpha+\beta = \frac{3}{8},\ \alpha\beta = \frac{27}{8}$

$$\left(\frac{\alpha^2}{\beta}\right)^{1/3} + \left(\frac{\beta^2}{\alpha}\right)^{1/3} = \frac{\alpha^{2/3}}{\beta^{1/3}} + \frac{\beta^{2/3}}{\alpha^{1/3}}$$

$$= \frac{\alpha+\beta}{(\alpha\beta)^{1/3}}$$

$$= \frac{\left(\frac{3}{8}\right)}{\left(\frac{27}{8}\right)^{1/3}} = \frac{3}{8}\times\frac{2}{3} = \frac{1}{4}$$

$\therefore$ option (b) is correct

16. Since sum of the coefficients = 0

The roots are 1 & $\frac{c^2(a^2-b^2)}{a^2(b^2-c^2)}$

Given equation is a perfect square, i.e. roots are equal

So $1 = \frac{c^2(a^2-b^2)}{a^2(b^2-c^2)}$

$\Rightarrow a^2\left(b^2-c^2\right)=c^2\left(a^2-b^2\right)$

$\Rightarrow a^2b^2-a^2c^2=a^2c^2-b^2c^2$

$\Rightarrow a^2b^2+b^2c^2=2a^2c^2$

$b^2=\dfrac{2a^2c^2}{a^2+c^2}$ $\therefore$ a^2,b^2,c^2 are in H.P

$\therefore$ option (d) is correct

17. Roots are equal, so $\Delta=0$

$\Rightarrow m(m-3)=0$

$m=0,3$

18. α is a root of $4x^2+2x-1=0$

$\Rightarrow 4\alpha^2+2\alpha-1=0$

Let the other root be β

Then $\alpha+\beta=\dfrac{-1}{2}$

$\Rightarrow \beta=\dfrac{-1}{2}-\alpha$

Now $4\alpha^2+2\alpha-1=0$

So $4\alpha^3-3\alpha=2\left(4\alpha^2-3\right)=\alpha\left(1-2\alpha-3\right)$

$=\alpha(-2\alpha-2)$

$=-2\alpha^2-2\alpha$

$=\dfrac{-1}{2}\left(1+\alpha^2\right)-2\alpha$

$=\dfrac{-1}{2}(1-2\alpha)-2\alpha$

$=\dfrac{-1}{2}-\alpha$

$=\beta$

$\therefore$ $4\alpha^3-3\alpha$ is the other root

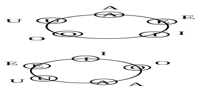
$\therefore$ option (b) is correct

19. sum of the coefficients = 0

$\therefore$ roots are 1& $\dfrac{a-b}{b-c}$

Given that roots are equal

$1=\dfrac{a-b}{b-c}$



$\Rightarrow a, b, c$ are in A.P

20. $\alpha + \beta = \dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$

$$\Rightarrow \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2}$$

$$\Rightarrow \alpha + \beta = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{b^2 - 2ca}{c^2}$$

$$\Rightarrow -bc^2 = ab^2 - 2ca^2$$

$$\Rightarrow ab^2 + bc^2 = 2ca^2$$

$\therefore ab^2, bc^2, ca^2$ are in A.P

21. $\sin\theta + \cos\theta = \dfrac{-b}{a}, \quad \sin\theta \cdot \cos\theta = \dfrac{c}{a}$

Now $(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta$

$$\Rightarrow \left(\frac{-b}{a}\right)^2 = 1 + \frac{2c}{a}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{a + 2c}{a}$$

$$\Rightarrow b^2 = a^2 + 2ca$$

$$\Rightarrow b^2 + c^2 = a^2 + 2ca + c^2$$

$\Rightarrow b^2 + c^2 = (a + c)^2$ $\therefore$ option (d) is correct

22. $a(b-c)x^2 9b(c-a)xy + c(a-b)y^2 = 0$

$$\Rightarrow a(b-c)\left(\frac{x}{y}\right)^2 + b(c-a)\left(\frac{x}{y}\right) + c(a-b) = 0$$

Put $\dfrac{x}{y} = t$, then $a(b-c)t^2 + b(c-a)t + c(a-b) = 0$

Given that it is a perfect square, so $\Delta = 0$

$\Rightarrow$ roots are equal

$$\Rightarrow 1 = \frac{c(a-b)}{a(b-c)}$$

$\Rightarrow ab - ac = ca - bc$

$\Rightarrow ab + bc = 2ca$

$\Rightarrow b(a+c) = 2ca$

$\Rightarrow b = \frac{2ca}{a+c}$ $\therefore$ a,b,c are in H.P

23. $x = 2 + 2^{1/3} + 2^{2/3}$

$x = \frac{-b}{2a}$

$y_{\min} = -\infty$

$\Rightarrow (x-2)^3 = 2 + 4 + 3\cdot 2(x-2)$

$\Rightarrow x^3 - 3x^2(2) + 3x(2^2) - 2^3 = 6 + 6(x-2)$

$\Rightarrow x^3 - 6x^2 + 12x - 8 - 6 - 6x + 12 = 0$

$\Rightarrow x^3 - 6x^2 + 6x = 2$

24. $x^2 - (2\cos\theta)x + 1 = 0$

$\Rightarrow x = \frac{2\cos\theta \pm \sqrt{(2\cos\theta)^2 - 4}}{2}$

$= \frac{2\cos\theta \pm \sqrt{4(\cos^2\theta - 1)}}{2}$

$= \frac{2\cos\theta \pm 2i\sin\theta}{2} = \cos\theta \pm i\sin\theta$

Let $\alpha = \cos\theta + i\sin\theta,\ \beta = \cos\theta - i\sin\theta$

Now $\alpha^n + \beta^n = (\cos\theta + i\sin\theta)^n + (\cos\theta - i\sin\theta)^n$

$= 2\cos n\theta$

$\alpha^n \cdot \beta^n = (\cos\theta + i\sin\theta)^n \cdot (\cos\theta - i\sin\theta)^n$

$= 1$

$\therefore$ Required quadratic equation

$x^2 - (\alpha^n + \beta^n)x + \alpha^n\beta^n = 0 \Rightarrow x^2 - (2\cos n\theta)x + 1 = 0$

25. $x^2 + 2bx + c > 0$

$\Rightarrow \Delta < 0$

$\Rightarrow 4b^2 - 4c < 0$

$b^2 - c < 0 \Rightarrow b^2 < c$

26. x^2 coefficient is positive and given expression is positive for real values of x

So $\Delta < 0$

$\Rightarrow a^2 - 4\left(1-2a^2\right) < 0$

$\Rightarrow 9a^2 - 4 < 0$

$\Rightarrow (3a+2)(3a-2) < 0$

$\Rightarrow \frac{-2}{3} < a < \frac{2}{3}$

27. $a-\sqrt{b} = \frac{\left(a-\sqrt{b}\right)\left(a+\sqrt{b}\right)}{a+\sqrt{b}} = \frac{a^2-b}{a+\sqrt{b}} = \frac{1}{a+\sqrt{b}}$

$\therefore \left(a+\sqrt{b}\right)^{x^2-12} + \frac{1}{\left(a+\sqrt{b}\right)^{x^2-15}} = 2a$

$\Rightarrow y + \frac{1}{y} = 2a$, where $y = \left(a+\sqrt{b}\right)^{x^2-15}$

$\Rightarrow y^2 - 2ay + 1 = 0$

$y = \frac{2a \pm \sqrt{4a^2-4}}{2}$

$= \frac{a \pm \sqrt{a^2-1}}{1} = a \pm \sqrt{b} \quad \left(\because a^2 - b = 1\right)$

$\Rightarrow a^2 - 1 = b$

$\therefore$ (i) If $y = a+\sqrt{b}$

$\Rightarrow \left(a+\sqrt{b}\right)^{x2-15} = 1$

$\Rightarrow x^2 - 15 = 1$

$x^2 = 16 \Rightarrow x = \pm 4$

(ii) If $y = a-\sqrt{b}$

$\Rightarrow \left(a+\sqrt{b}\right)^{x^2-15} = \left(\frac{1}{a+\sqrt{b}}\right)^1$

$\Rightarrow \left(a+\sqrt{b}\right)^{x^2-15} = \left(a+\sqrt{b}\right)^{-1}$

$\therefore x^2 - 15 = -1$

$x^2 = 14 \Rightarrow x = \pm\sqrt{14}$

28. Let $\frac{(x-a)(x-b)}{x-c} = y$

$\Rightarrow x^2 - x(a+b) + ab = xy - cy$

$\Rightarrow x^2 - (a+b+y)x + (ab+cy) = 0$

$\Delta \geq 0 \Rightarrow (a+b+y)^2 - 4(ab+cy) \geq 0$

$\Rightarrow a^2 + b^2 + y^2 + 2ab + 2by + 2ay - 4ab - 4cy \geq 0$

$\Rightarrow y^2 + 2(a+b-2c)y + (a-b)^2 \geq 0$

This is true for all 'y'

$\therefore \Delta \leq 0 \Rightarrow 4(a+b-2c)^2 - 4(a-b)^2 \leq 0$

$\Rightarrow a^2 + b^2 + 4c^2 + 2ab - 4bc - 4ac - (a^2 + b^2 - 2ab) \leq 0$

$\Rightarrow \cancel{a}^2 + \cancel{b}^2 + 4c^2 + 2ab - 4bc - 4ac - \cancel{a}^2 - \cancel{b}^2 \leq 0$

$\Rightarrow 4c^2 + 4ab - 4bc - 4ac \leq 0$

$\Rightarrow c^2 + ab - bc - ac \leq 0$

$\Rightarrow (c-b)(c-a) \leq 0$

$\therefore a \leq c \leq b \ \ (or)\ b \leq c \leq a$

29. Given equation $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$

$x^2 + 2x + 3 = 0$ has complex roots, so $ax^2 + bx + c = 0$ will have both roots same as $x^2 + 2x + 3 = 0$

$\therefore \frac{a}{1} = \frac{b}{2} = \frac{c}{3} \Rightarrow a:b:c = 1:2:3$

30. Sachin made a mistake in writing down the constant term

So sum of the roots $\alpha + \beta = 7$

Rahul made a mistake in writing down the coefficient of x so product of the roots $\alpha\beta = 6$

$\therefore$ Correct quadratic equation $x^2 - (\alpha+\beta)x + \alpha\beta = 0$

$\Rightarrow x^2 - 7x + 6 = 0 \Rightarrow$ roots are 6, 1

31. $x^2 - x + 1 = 0$

$\Rightarrow x = \frac{1 \pm i\sqrt{3}}{2}$

$\therefore \alpha = \frac{1 \pm i\sqrt{3}}{2}, \beta = \frac{1 - i\sqrt{3}}{2}$

$\Rightarrow \alpha = -\omega, \beta = -\omega^2 \ \ (or)\ \alpha = -\omega^2, \beta = -\omega$

$\therefore \alpha^{2009} + \beta^{2009} = (-\omega)^{2009} + (-\omega^2)^{2009}$

$= -\left[(\omega^3)^{669} \cdot \omega^2 + (\omega^3)^{1339} \cdot \omega\right]$

$= -\left[\omega^2 + \omega\right] = 1$

32. Let α, β be the roots $ga+\beta=-a, \quad or \quad \alpha\beta=1$

Now $|\alpha-\beta|<\sqrt{5}$

$\Rightarrow \sqrt{(\alpha+\beta)^2-4\alpha\beta}<\sqrt{5}$

$\Rightarrow \sqrt{a^2-4}<\sqrt{5}$

$\Rightarrow a^2-4<5 \Rightarrow a^2<9$

$a \in (-3, 3)$

33. From the given data $\tan 30^0+\tan 15^0=-p$, $\tan 30\cdot\tan 15=q$

Now $\frac{-b}{a}$

$\Rightarrow \frac{1}{4-3i}+\frac{1}{4+3i}=\frac{-b}{a}$

$\tan 45=\frac{\tan 30+\tan 15}{1-\tan 30\cdot\tan 15} \quad = 3$

$\Rightarrow \tan 30+\tan 15=1-\tan 30\cdot\tan 15$

34. Let K & K+1 are two consecutive roots

Then $k+(k+1)=b,\ k(k+1)=c$

$\Rightarrow 2k+1=b$

$\therefore b^2-4c=(2k+1)^2-4k(k+1)$

$=4\cancel{k}^2+4\cancel{k}+1-4\cancel{k}^2-4\cancel{k}=1$

35. $(1-p)$is a root of given equation

So $(1-p)$ satisfies the given equation

So $(1-p)^2+p(1-p)+1-p=0$

$\Rightarrow 1+\cancel{p}^2-2p+p-\cancel{p}^2+1-\cancel{p}=0$

$\Rightarrow -2p+2=0$

$\Rightarrow p=1$

$\therefore$ given equation becomes $x^2+x=0$

$\Rightarrow x(x+1)=0 \Rightarrow x=0, -1$

36. One root of $x^2+px+12=0$ is 4

$\Rightarrow 16+4p+12=0$

$\Rightarrow 4p=-28 \qquad \therefore p=-7$

$\therefore x^2+px+q=0$ becomes $x^2-7x+q=0$

Let α,α be the roots of this equation (since it has two equal roots)

$\Rightarrow \alpha+\alpha=7 \qquad \alpha\cdot\alpha=q$

$2\alpha=7 \qquad \alpha^2=q$

$\alpha=\frac{7}{2} \qquad \Rightarrow\left(\frac{7}{2}\right)^2=q$

$\therefore q=\frac{49}{4}$

37. Let two numbers be α,β

A.M of two numbers $\frac{\alpha+\beta}{2}=9$

$\Rightarrow \alpha+\beta=18$

G.M of two numbers $\sqrt{\alpha\beta}=4$

$\Rightarrow \alpha\beta=16$

$\therefore$ Quadratic equation having these two numbers α,β as roots is $x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-18x+16=0$

38. $\alpha+\beta=\frac{-b}{a},\ \alpha\beta=\frac{c}{a}$

Given that $\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$

$$\Rightarrow \alpha+\beta=\frac{\alpha^2+\beta^2}{(\alpha\beta)^2}$$

$$\Rightarrow \alpha+\beta=\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}$$

$$\Rightarrow\left(\frac{-b}{a}\right)=\frac{\frac{b^2}{a^2}-\frac{2c}{a}}{\left(\frac{c}{a}\right)^2}$$

$$\Rightarrow \frac{-b}{a}=\frac{b^2}{a^2}\times\frac{a^2}{c^2}-\frac{2c}{a}\times\frac{a^2}{c^2}$$

$$\Rightarrow \frac{-b}{a}=\frac{b^2}{c^2}-\frac{2a}{c}$$

$$\Rightarrow \frac{2a}{c}=\frac{b^2}{c^2}+\frac{b}{a}$$

$$\Rightarrow \frac{2a}{c} = \frac{b}{c}\left(\frac{b}{c} + \frac{c}{a}\right)$$

$$\Rightarrow \frac{2a}{b} = \frac{b}{a} + \frac{c}{a}$$

$\therefore \frac{a}{b}, \frac{b}{c}, \frac{c}{a}$ are in AP

$\Rightarrow \frac{b}{a}, \frac{a}{c}, \frac{c}{b}$ are in HP

39. Let roots are $\alpha, 2\alpha$

$$\alpha + 2\alpha = \frac{-(3a-1)}{a^2-5a+3} \qquad \alpha \cdot 2\alpha = \frac{2}{a^2-5a+3}$$

$$\Rightarrow 3\alpha = \frac{1-3a}{a^2-5a+3} \qquad \Rightarrow 2\alpha^2 = \frac{2}{a^2-5a+3} \text{ ———— } (2)$$

$$\Rightarrow \alpha = \frac{1-3a}{3(a^2-5a+3)} \text{ ———— } (1)$$

Solving (1) & (2)

$$\therefore\ 2\alpha\left(\frac{1-3a}{3(a^2-5a+3)}\right)^2 = \frac{2}{a^2-5a+3}$$

$$\Rightarrow \frac{1+9a^2-6a}{9(a^2-5a+3)} = 1$$

$$\Rightarrow 9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$\Rightarrow -6a + 45a = 26$$

$$39a = 26$$

$$a = \frac{2}{3}$$

40. Let roots are 2k+1, 2k+3

$\therefore$ (2k+1) +(2k+3) = a and (2k+1).(2k+3) = b

$$\Rightarrow 4k + 4 = a \qquad \Rightarrow 4k^2 + 8k + 3 = b$$

$$\therefore a^2 - 4b = (4k+4)^2 - 4(4k^2 + 8k + 3)$$

$$= 16k^2 + 32k + 16 - 16k^2 - 32k - 12$$

$$= 4$$

41. $\alpha + \beta = a,\ \alpha\beta = b^2$

$$\therefore \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$$

$$= a^2 - 2b^2$$

42. $\alpha+\beta=-2b,\ \alpha\beta=c$

$\therefore b^2-c=\left(\frac{\alpha+\beta}{-2}\right)^2-\alpha\beta$

$=\frac{(\alpha+\beta)^2-4\alpha\beta}{4}$

$=\frac{(\alpha-\beta)^2}{4}$

43. clearly α,β are the roots of the equation $x^2+5x+3=0$

$\Rightarrow \alpha+\beta=5,\ \alpha\beta=3$

$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{25-6}{3}=\frac{19}{3}$

$\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha}=1$

$\therefore$ Quadratic equation is $x^2-\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)x+\frac{\alpha}{\beta}\cdot\frac{\beta}{\alpha}=0$

$\Rightarrow x^2-19x+1=0$

44. $3^{2x^2-7x+7}=3^2$

$\Rightarrow 2x^2-7x+7=2$

$\Rightarrow 2x^2-7x+5=0$

$\Delta=(-7)^2-4(2)(5)$

$=49-40$

$=9>0$

$\therefore$ It has two real roots

45. $f(x)=2x^2+3x+1=0$

Required equation is $f\left(\sqrt{x}\right)=0$

$\Rightarrow 2\left(\sqrt{x}\right)^2+3\sqrt{x}+1=0$

$\Rightarrow 2x+1=-3\sqrt{x}$

$\Rightarrow (2x+1)^2=9x$

$\Rightarrow 4x^2+4x+1-9x=0$

$4x^2-5x+1=0$

46. $x=\frac{\sqrt{3}\pm\sqrt{3-4}2}{2}$

$$= \frac{\sqrt{3} \pm i(1)}{2} = \frac{\sqrt{3} \pm i}{2}$$

47. $\therefore$ product of the root = 31

$\Rightarrow 2\ e^{2\log k} - 1 = 31$

$\Rightarrow 2\ e^{\log k^2} = 32$

$\Rightarrow 2\ e^{\log k^2} = 32$

$k^2 = 16$

$k = \pm 4$, for k = –4, logk is not defined

So k = 4

48. $x = \dfrac{2 \pm \sqrt{4-16}}{2}$

$$= \frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm i\ 2\sqrt{3}}{2} = 1 \pm i\sqrt{3}$$

$$\therefore \alpha^n + \beta^n = \left(1 + i\sqrt{3}\right)^n + \left(1 - i\sqrt{3}\right)^n$$

$$= 2^n\left(\frac{1}{2} + i\frac{\sqrt{3}}{1}\right)^n + 2^n\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)^n$$

$$= 2^n\left(\cos 60 + i\sin 60\right)^n + 2^n\left(\cos 60 - i\sin 60\right)^n$$

$$= 2^n\left[\cos\frac{n\pi}{3} + i\sin\frac{n\pi}{3} + \cos\frac{n\pi}{3} - i\sin\frac{n\pi}{3}\right]$$

$$= 2^{n+1}\cos\frac{n\pi}{3}$$

49. A.M of α, β = 8

$\Rightarrow \dfrac{\alpha + \beta}{2} = 8$

$\Rightarrow \alpha + \beta = 16$

G.M of $\alpha, \beta = (\alpha \cdot \beta)^{1/2} = 5$

$\therefore \sqrt{\alpha\beta} = 5$

$\alpha\beta = 25$

$\therefore$ Required quadratic equation $x^2 - 16x + 25 = 0$

50. second equation have imaginary roots

So both the roots of two equations are same

$\dfrac{2a}{3} = \dfrac{-3b}{-4} = \dfrac{4c}{5} = k$ (say)

$\Rightarrow \dfrac{2a}{3} = k \qquad \dfrac{3b}{4} = k \qquad \dfrac{4c}{5} = k$

$\Rightarrow a = \frac{3k}{2} \qquad b = \frac{4k}{3} \qquad \Rightarrow c = \frac{5k}{4}$

$$\therefore \frac{a+b}{b+c} = \frac{\frac{3k}{2}+\frac{4k}{3}}{\frac{4k}{3}+\frac{5k}{4}} = \frac{\left(\frac{9k+8k}{6}\right)}{\left(\frac{16k+15k}{12}\right)} = \frac{17k}{5} \times \frac{12}{31k} = \frac{34}{31}$$

51. a,b,c are in A.P $\Rightarrow 2b = a+c$

$\alpha+\beta = \frac{2b}{a} \qquad \alpha\beta = \frac{c}{a}$

$= \frac{a+c}{a}$

$\Rightarrow \alpha+\beta = 1+\frac{c}{a}$ ———(1)

$\therefore (\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= \left(1+\frac{c}{a}\right)^2 - 4\frac{c}{a}$

$\Rightarrow (\alpha-\beta) = 1-\frac{c}{a}$ ———— (2)

Solving (1) & (2) $2\alpha = 2 \Rightarrow \alpha = 1$

(1) – (2) $\Rightarrow 2\beta = 2\frac{c}{a} \quad \Rightarrow \beta = \frac{c}{a}$

52. $x = \frac{1}{2}$ satisfies given equation

53. $\alpha+\beta = \frac{-b}{a}, \ \alpha\beta = \frac{c}{a}$

$\frac{k}{\alpha}+\frac{k}{\beta} = k\left(\frac{1}{\alpha}+\frac{1}{\beta}\right) = k\left(\frac{\alpha+\beta}{\alpha\beta}\right) = k\left(\frac{-b}{a}\right)$

$\frac{k}{\alpha}\cdot\frac{k}{\beta} = \frac{k^2}{\alpha\beta} = \frac{k^2}{(c/a)} = \frac{k^2 a}{c}$

Required equation is $x^2 - \left(\frac{k}{\alpha}+\frac{k}{\beta}\right)x + \frac{k}{\alpha}\cdot\frac{k}{\beta} = 0$

$\Rightarrow x^2 + \frac{kb}{c}x + \frac{k^2 a}{c} = 0$

$cx^2 + kbx + k^2 a = 0$

PERMUTATIONS & COMBINATONS

Permutation: An arrangement that can be formed by taking some or all of a finite set of things (or object) is called a permutation.

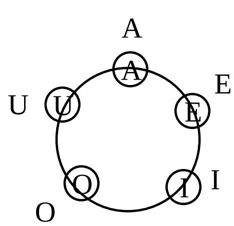
Linear permutation : From a given finite set of elements (similar or not) taking some or all of them arranging them in a line is called linear permutation.

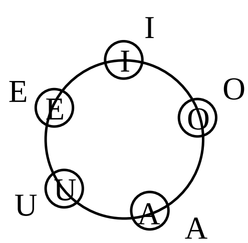
Ex: Amanging vowels of english alphabet

AEIOU, EOUAI, AIUOE, ….. etc

Circular Permutation: A permutation (arrangement) is said to be circular permutation if the object are arranged in the form of a circular (closed curve)

Ex:





.... etc

Fundamental Principal of counting (Product Rule): If a work can be performed in 'm' different ways and a second work can be performed [after ω_1 has been performed in any one of the 'm' ways] in n different ways, then work ω_1 AND ω_2 can be performed in $m \times n$ ways

Ex: If a man has 3 different trousers (T_1, T_2, T_3) and 2 different coloured shirts (S_1, S_2), then he can make a pair (a brouser AND a shirt) below $T_1S_1, T_1S_2, T_2S_1, T_2S_2, T_3S_1, T_3S_2$

Rule of OR (Sum Rule): If event ‘A’ can happen in ‘m’ different ways, event ‘B’ can be happen in n different ways then event A ‘OR’ B can happen in m+n ways.

Ex: If room A has 3 different doors and room B has 2 different doors then a person can enter in room A ‘OR’ B in m+n=3+2=5 ways

Factorial Notation: Note that $\angle D = 1$

The product of first ‘n’ natural numbers is called “Factorial n” and is denoted b $\lfloor n$ or n!

$\lfloor n$ = n(n–1) (n–2) …..3.2.1

$= n\lfloor n-1$

$= n\lfloor n-1 \lfloor n-2$

Ex: $\lfloor 5 = 5\times4\times3\times2\times1;$ $\lfloor 4 = 4\times3\times2\times1$

=120 =24

The No of permutations (arrangements) of ‘n’ dissimilar things taken ‘r’ at a time is denoted by ${}^nP_r$ or P(n, r) (or)

No of ways of filling ‘r’ blank places which are arranged in a row by ‘n’ dissimilar things is ${}^nP_r$ or P(n, r)

$${}^nP_r = \frac{\lfloor n}{\lfloor n-r}$$

$$= n(n-1)(n-2)(n-3)......{}^nP_1 = n$$

$${}^nP_1 = 1,\ {}^nP_r = \lfloor n,\ {}^nP_1 = n$$

$${}^nP_r = n.\ {}^{(n-1)}P_{(r-1)} = {}^{n(n-1)(n-2)}P_{(r-2)}$$

Ex : ${}^7P_4 = 7.\ {}^6P_3 = 7\cdot6\cdot{}^5P_2$

$= 7\cdot6\cdot5\cdot{}^4P_2$

$= 7\cdot6\cdot5\cdot4 = 840$

$$\frac{{}^nP_r}{{}^{(n-1)}P_{(r-1)}} = n \quad \& \quad \frac{{}^nP_r}{{}^{(n-2)}P_{(r-2)}} = n(n-1)$$

$$\frac{{}^nP_r}{{}^nP_{(r-1)}} = n-r+1$$

The no. of permutaions of ‘n’ disslmilar things taken ‘r’ a time, in which a particular thing always occur is $r\cdot{}^{(n-1)}P_{(r-1)}$

The no. of permutations in which a particular thing will neve occur is ${}^{(n-1)}P_r$

$$^{n}P_r = {}^{(n-1)}P_r + r \cdot {}^{(n-1)}P_{(r-1)}$$

The number of permutaions of n things taken 'r' at a time in which 's' particular things always occur is ${}^{(n-s)}P_{(r-s)} + {}^{n}P_s$

The nomber of permutations of 'n' different things taken not more than 'r' at a time, when each of them may occur any number of times is $= n + n^2 + n^3 + ... + n^r$

$$= \frac{m(n^2-1)}{n-1}, n \neq 1$$

The number of permutaions of n different things taken not more than 'r' at a time is

$$= {}^{n}P_1 + {}^{n}P_2 + {}^{n}P_3 + ... + {}^{n}P_r$$

Sum of the numbers formed by taking all the given n digits $= \lfloor n-1$ (sum of all n digits) (1111….n times)

Sum of the numbers formed by taking all the given n digits (including O) is

= [Sum of all the n digits] [$\lfloor n-1$ (1111….n times) – $\lfloor n-2$ (111..(n+1)times)]

Sum of all the 'r' digit numbers formed by taking the given 'n' digits (without 0) is

$= {}^{(n-1)}P_{(r-1)}$ [sum of all the 'n' digits] (1111 …… r times)

Sum of all the 'r' digit numbers formed by taking the given 'n' digits (including 0) is

=[sum of all the 'r' digits]

$$\left[{}^{(n-1)}P_{(r-1)} \times (1111.....r\ times) - {}^{(n-2)}P_{(r-2)}(1111.....(r-1)\ times) \right]$$

The number of permutations of n dissimilar tings taken r at a time when reptetion of things is allowed any number of times is n^r

$n(A) = r,\ n(B) = n$, then number of functions that can be defined from A into B is

$$n^r = n(B)^{n(A)}$$

$n(A) = n(B) = n$ then the number of bijections can be defined from A onto B is $\lfloor n$

$n(A) = r,\ n(B) = 2$, then the number of surjections (onto) from A to B is $2^r - 2$

PALINDROME: A number or a word which reads same either from left to right or from right to left is called a palindrome

Ex: 1 2 3 2 1, 9 7 7 9, R O T O R, DAD, MALAYALAM, etc

The number of palindromes with 'r' distincict letters that can be formed using the given 'n' distinict letters is (i) $n^{r/2}$, if r is even

(ii) $n^{\frac{r+1}{2}}$, if r is odd

The number of ways in which m (first type of different) things and n (second type of different thengs can be arranged in a row so that no two things of second type come together is

$$\lfloor m \cdot {}^{(m+1)}P_r$$

In the above case number of permutation that all second type come togethe $\lfloor m+1 \cdot \lfloor n$

The number of permutations of 'n' dissimilar things taken r things at a time with atleast one repetation is

$n^r - {}^nP_r$

Number of circular permutations of 'n' different things taken at a time is $\lfloor n-1$

Number of circular permutations of 'n' different things taken all at a time when clockwise and anti clockwise arrangements considered as same is $\frac{1}{2}\lfloor n-1$ [Necklace, Gatland etc]

The number of circular permutations of 'n' things taken 'r' at a time in one direction is $\frac{{}^nP_r}{2r}$

The number of circular permutations of 'n' things taken 'r' all at a time when 'P' of time are all alike and the rest all different is $\frac{\lfloor n}{\lfloor p}$

The number of linear permutations of n things in which they are 'p' like things of one kind, q lke things of second type, r like things of the third kind and the rest are different is $\frac{\lfloor n}{\lfloor p \cdot \lfloor s \cdot \lfloor r}$

No of ways of distributing mn different elements m persons equally is $\frac{\lfloor mn}{(\lfloor r)^m}$

No of ways of dividing mn things into m equal groups is $\frac{\lfloor mn}{(\lfloor n)^m \lfloor m}$

The number of ways in which m (first type of different) things and n (second type of different) things can be arranged in a circle so that

(i) all the second type of things come together is $\lfloor m \cdot \lfloor n$

(ii)no two things of second type come together is $\lfloor m-1 \cdot {}^mP_r$

Combination: A selection that can be made formed by taken some or all of a finite set of things (or objects) is called a combination

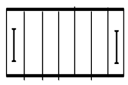
Number of combinations (selections) of ‘n’ dissimilar things taken ‘r’ at a time $=C(n\cdot r)$

$$^{n}C_r=\frac{\lfloor n}{\lfloor r\,\lfloor n-r}$$

$$=\frac{n(n-1)(n-2).....(n-r+1)}{r(r-1)(r-2).....3\cdot 2\cdot 1}$$

Ex: $^{9}C_4=\frac{9\cdot 8\cdot 7\cdot 6}{4\cdot 3\cdot 2\cdot 1}=126$

Where n is positive integer, r is non negative integer & $n\geq r$



$$^{n}C_r=\left(\frac{n}{r}\right)=^{(n-1)}C_{(r-1)}$$

$$=\left(\frac{n}{r}\right)\left(\frac{n-1}{r-1}\right)=^{(n-2)}C_{(r-2)}$$

$^{n}C_r=^{n}C_{n-r}$; $\quad ^{n}C_0=^{n}C_r=1$

$$^{n}C_1=^{n}C_{r-1}=n$$

$^{n}C_r=^{n}C_s\Rightarrow$ either r =s or r+s = n

C V C C V C C

$$^{n}C_{r-1}+^{n}C_r=^{(r+1)}C_r$$

If N is a positive integer such that

$N=p_1^{\alpha_1}\cdot p_2^{\alpha_2}.......p_n^{\alpha_n}$ where $p_1,p_2,p_3......p_n$ are primes and

$\alpha_1,\alpha_2,\alpha_3,.....\alpha_n$ are +ve integers then

(i) Total number of divisors of $n=\phi(n)=(\alpha_1+1)(\alpha_2+2)....(\alpha_n+1)$

(ii) No of proper divisor of $n=(\alpha_1+1)(\alpha_2+2)....(\alpha_n+1)-2$

(iii) Sum of the divisors $=\left(\frac{p_1^{k_1+1}-1}{p_1-1}\right)\left(\frac{p_2^{k_2+1}-1}{p_2-1}\right)........\left(\frac{p_n^{k_n+1}-1}{p_n-1}\right)$

The number of combinations of n things taken ‘r’ at a time in which

(i) ‘s’ particular things always occur = $={}^{(n-s)}C_{(r-s)}$

(ii) ‘s’ particular things always occur and ‘p’ particularthings never occur = $={}^{(n-p-s)}C_{(r-s)}$

The total number of combinations of n different things taken

(i) any number at a time = $={}^nC_0+{}^nC_1+{}^nC_2+........+{}^nC_n+2^n$

(ii)One or more at a tie = 2^n-1

Out of (p+q) things, when p things are alike of one kind and q things are alike of second kind then

(i)Total number of combination of things taken any number at a time =(p+1)(q+1)

(ii)Total number of combination of things taken one or more at a time = (p+1)(q+1)–1

If n is a positive Integer and p is a prime number then the exponent of p in $\lfloor n$ is

$$\left[\frac{n}{p}\right]+\left[\frac{n}{p^2}\right]+\left[\frac{n}{p}\right]+......$$

No of diagonals of a polygon of n sides $={}^nC_2-n$

$$=\frac{n(n-3)}{2}$$

No of paralellograms formed when a set of ‘m’ parallel lines are intersecting another set of ‘n’ parallel lines is ${}^mC_2\times{}^nC_2$

If there are ‘n’ points in a plane out of which ‘p’ points are collinier and no three of the points are collinier unless all the three are from these ‘p’ points then

(i) No of straight lines formed by joining them is is ${}^nC_2-{}^pC_2+1$

(ii) No. of triangles formed by joining them is ${}^nC_3\times{}^pC_3$

There are ‘n’ letters and n addressed envelopes then the number of ways they canbe placed, so that no letter placed correctly is $\lfloor n\left[\frac{1}{\lfloor 2}-\frac{1}{\lfloor 3}+\frac{1}{\lfloor 4}+.......(-1)^n\cdot\frac{1}{\lfloor n}\right]$

1. Find (i) ${}^7p_3$ (ii) ${}^8p_4$ (iii) ${}^8p_2$

Sol: (i) ${}^7p_3$ = $7\times6\times5=210$

(ii) $^{8}p_4 = 8\times7\times6\times5 = 1680$

(iii) $^{8}p_2 = 8\times7 = 56$

Match the following:

i) $^{12}p_3$	( )	(a) 870
ii) $^{30}p_2$	( )	(b) 117600
iii) $^{6}p_4 + {}^{8}p_2$	( )	(c) 1320
iv) $^{50}p_3$	( )	(d) 720
v) $^{6}p_6$	( )	(e) 416

2. $^{n}p_4 = 1680$ then find n

Sol:

$$\begin{array}{r|l} 2 & 1680 \\ \hline 2 & 840 \\ \hline 2 & 420 \\ \hline 2 & 210 \\ \hline 2 & 105 \\ \hline 2 & 35 \\ \hline & 7 \end{array}$$

$$1680 = 2\times2\times2\times2\times3\times5\times7$$
$$= 8\times7\times6\times5$$

(write as product of consecutive natural nos)

Given $^{n}p_4 = 1680$

$$n(n-1)(n-2)(n-3) = 8\times7\times6\times5$$

Comparing $\boxed{n=8}$

Do this: If

(i) $^{n}p_3 = 1320$ find 'n'

(ii) $^{12}p_r = 1320$ find 'r'

(iii) $^{n}p_4 = 1680$ then find n

3. If $^{(r+1)}p_5 : {}^{n}p_5 = 3:2$ then find 'n'

Sol: Given $\dfrac{^{(n+1)}p_5}{^{n}p_5} = \dfrac{3}{2}$

$$\frac{(n+1)n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)(n-4)} = \frac{3}{2}$$

$$\frac{n+1}{1-4} = \frac{3}{2}$$

$$2(n+1) = 3(n-4)$$

$$2n+2 = 3n-12$$

$$3n-2n = 2+12$$

$$\boxed{n = 14}$$

(i) If ${}^{(n+1)}p_5 : {}^{n}p_6 = 2:7$ then n = _____

(ii) If ${}^{n}p_4 : {}^{n}p_3 = 2:1$ then n = ____

(iii)If ${}^{n}p_7 = 42 \cdot {}^{n}p_5$ then n = _____

4. If ${}^{56}p_{r+6} : {}^{54}p_{r+3} = 30800:1$, find 'r'

Sol: Given $\dfrac{{}^{56}p_{r+6}}{{}^{54}p_{r+3}} = \dfrac{30800}{1}$

$$\frac{\dfrac{\lfloor 56}{\lfloor 56-(r+6)}}{\dfrac{\lfloor 54}{\lfloor 54-(r+3)}} = \frac{30800}{1}$$

$$\boxed{{}^{n}p_r = \frac{\lfloor n}{\lfloor n-r}}$$

$$\frac{\lfloor 56}{\lfloor 50-r} \times \frac{\lfloor 51-r}{\lfloor 54} = \frac{30800}{1}$$

$$\boxed{\begin{aligned} \lfloor n &= n\lfloor n-1 \\ &= n(n-1)\ \lfloor n-2 \end{aligned}}$$

$$\frac{56\times 55\times \lfloor 54}{\lfloor 50-r} \times \frac{(51-r)\lfloor 50-r}{154} = \frac{30800}{1}$$

$$56\times 55\times (51-r) = 30800$$

$$51-r = 10$$

$-r = 10 - 51$

$-r = -41$

$\boxed{r = 41}$

(i) If ${}^{18}p_{r-1} : {}^{17}p_{r-1} = 9 : 7$

(ii) If $44 \times {}^{15}p_r : 5 \times {}^{14}p_{r+1}$

5. If ${}^{12}p_5 + 5 \cdot {}^{12}p_4 = {}^{13}p_r$ then find r

Sol: We have $\boxed{{}^{(n-1)}p_r + r \cdot {}^{(n-1)}p_{r-1} = {}^{n}p_r}$

Substitute n = 13 & r = 5 we get ${}^{12}p_5 + 5 \cdot {}^{12}p_4 = {}^{13}p_5$ ————(1)

${}^{12}p_5 + 5 \cdot {}^{12}p_4 = {}^{13}p_r$ ————(2)

Comparing (1) & (2) we have $\boxed{r = 5}$

(i) If ${}^{9}p_5 + 5 \cdot {}^{9}p_4 = {}^{10}p_r$ then r = _____

(ii) If ${}^{14}p_7 + 7 \cdot {}^{14}p_6 = {}^{14}p_7$ then n = ______

(iii) If ${}^{6}p_3 + 3 \cdot {}^{6}p_2 = {}^{n}p_r$ then n = _____, r = _______

6. Find the number of ways of arranging 6 players to throw the cricket ball?

Sol: Here Total number of players = n = 6

No of players to be arranged = r = 6

Number of arrangements $= {}^{n}p_r$

$= {}^{6}p_6$

$= \underline{|6}$

$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

(i)No of ways arranging 5 books in a shelf is _____

(ii)No of arrangements that can be formed out of SCIM are ____

(iii)No of 4 digit numbers can be formed using the digits 1,2,3,4 are _____

(iv)If a number of books can be arranged in linear shelf is 5040 then the number of books is _______

7. How many 4 digited numbers can be formed using the digits 12,5,7,8,9 ?

Sol: Here Total number of digits gives = n = 6

Number of digis to be used = r = 4

No. of permutations of n dissimilar things taken ‘r’ at a time = ${}^{n}p_{r}$

$$= {}^{6}p_{4}$$

$$= 6 \times 5 \times 4 \times 3 = 360$$

(i) No of 3 digit numbers can be formed from1,2,5,7,9 are ______

(ii)Two persons entered a railway compartment in which 5 seats were vacant. The number of ways in which they can be seated is _______

(iii)The number of permutations of 8 things taken ‘r’ at a time is 1680 then r = _____

(iv)A man has 4 sons and then one 5 schools within his reach, then no of ways that he can admit his sons in the school so that no two of thems will be in the same school is _____

(v) There one 25 railway stations on a railway line then the no of single second class tickets must be printed, so as to enable a passenger to travel from one station to other is _____

(vi) No of Numbers formed out of 1,2,3,4 without repetition are ____

(Hint: r not mentioned)

8. Find the number of ways of arraning 6 boys and 5 girls in a row so that (i) all girls sit together (ii) no tow girls sit together (iii) boys and girls sit alternately and (iv) no two boys sit together

Sol: Total persons = $6B + 5G = 11$

Total no of ways of arranging 6 boys & 5 girls in a row is $\underline{|11}$ ways

(i) Treat 5 girls as one unit. Then we have 6 Boys +1 unit = 7 which can be arranged in $\underline{|7}$ ways . The girsl can be arranged among themseleves in $\underline{|5}$ ways

$\therefore$ Required no of permutaion $= \underline{|7} \cdot \underline{|5}$ [AND Rule]

(ii)First arrange 6 boys in a row in $\underline{|6}$ ways

☐ B ☐ B ☐ B ☐ B ☐ B ☐

A girl can be arranged at the beginning or at th end of boys or in between every two bos. Thus there are 7 places to arranged 5 girls.

They can be arranged in ${}^{7}p_{5}$ ways

$\therefore$ Required number of arrangements $= \underline{|6} \times {}^{7}p_{5}$

(iii)To arrange 6 boys and 5 girls alternatively, the odd places will be occuped by 6 boys and even places by 5 girls.

B G B G B G B G B G B

6 Boys can be arranged in 6 odd places in $\underline{|6}$ ways & 5 girls can be arranged in 5 even places in $\underline{|5}$ ways

$\therefore$ Required no of Permutations $= \lfloor 6 \cdot \lfloor 5$

(iv) Since no two boys together, first arrange 5 girls in a row. It can be done in $\lfloor 5$ ways

$\times$ G $\times$ G $\times$ G $\times$ G $\times$ G $\times$

Now a boy can be arranged at the beginning or at the end or in between even two girls. The 6 places should be filled by 6 boys, which can be done in $^6p_6 = \lfloor 6$ ways

Required number of Permutations = $\lfloor 5 \cdot \lfloor 6$

(i) The number of different ways in which 4 boys and 6 girls can be arranged in a row so that no two boys shall be together are ______

(ii)In the above problem, the no of ways in which all girls are together is ______

(iii)The number of ways in which 6 boys and 7 girls can sit in a row such that no two girls are together is _____

(iv)In the above problem, the number of ways in which no two boys sit together is _____

(v) The number of ways in which 5 boys and 5 girls can be arranged in a row so that no two girls are together is _____

(vi) In the above problem, the number of ways in which all the boys together is _____

(vii)In the above problem, the number of ways in which, they set alternatively is ______

9. Find the number of numbers that are greater than 4000 which can be formed using the digits 0,2,4,6,8 without repetition

Sol: Given digits 0,2,4,6,8

Note that Every 5 digit number is greater than 4000

_ _ _ _ _

The first place can be filled in (other than o) can be filled in $4p_1 = 4$ ways and the remaining four places can be filled by 4 digits in $4p_4 = \lfloor 4$ ways

$\therefore$ No of 5 digited numbers $= 4 \cdot \lfloor 4$

4 digit numbers

Starting with '4' $\underline{4}$ _ _ _ $= 4p_3$

Starting with '6' $\underline{6}$ _ _ _ $= 4p_3$

Starting with '8' $\underline{8}$ _ _ _ $= 4p_3$

$\therefore$ No of 4 digited numbers $= 3 \times 4p_3$

$\therefore$ No of numbers greater than 4000 are $(3 \times 4p_3) + (4 \times \lfloor 4)$

$$= (3 \times 24) + (4 \times 24)$$
$$= 72 + 96$$
$$= 168$$

10. Find the number of 5 letters words that can be formed using letters of the word CONSIDER. How many of them begin with C. How many of them and with R and how many of them begin with C and end with R

Sol: Given word contains C, O, N, S, I, D, E, R has 8 letters

No of 5 letter words can be formed $= 8p_5$

$= 6720$

Bigin with C

$\underline{C}\ \_\ \_\ \_\ \_$

The remaining 4 blanks, should be filled by remaining 7 letters which can be done in $7p_4$ ways

$\therefore$ Required No of permutaions = $7p_4$

=840

End with R

$\_\ \_\ \_\ \_\ \underline{R}$

The remaining first 4 blank places should be filled by remaining 7 letters, which can be done in $7p_4$ ways

$\therefore$ Required no of permutaions = $7p_4$

= 840

Begin with C and end with R

$\underline{C}\ \_\ \_\ \_\ \underline{R}$

The remaining 3 blank places in the middle should be filled by remaining 6 letters which can be done in $6p_3$ ways

$\therefore$ Required no of permutations $= 6p_3$

=120

11. Find the number of 4 letter words that can be formed using the letters of the word MIRACLE.

How many of them (i) begin with an vowel

Sol: The word MRACLE has 7 letters. Hence the number of 4 letter words can be formed

$= {}^7p_4$

$= 7 \times 6 \times 5 \times 4 = 840$

(i) **Begin with a vowel**

The first place should be filled with one of the 3 vowels (I, A, E) in $3p_1 = 3$ ways

Now the remaining 3 places can be filled by remaining 6 letters, which can be done in $6p_3 = 120$ ways

Required no of permutaions = $= 3 \times 120 = 360$ (AND Rule)

(ii) **Begin and End with vowels**

Fil the first and last places with 2 **v**owels in $3p_2 = 6$ ways

The remaing 2 places can be filled with remaining 5 letters is $5p_2 = 20$ ways

Required no of permutations = $6 \times 20 = 120$ ways

(iii) **Endwith a conquonant**

We can fill the last place with one of the 4 consonants (M, R, C, L) in $4p_1 = 4$ ways

The remaining 3 places can be filled with remaining 6 letters in $6p_3 = 120$ ways

Required no of permutaions $= 4 \times 120 = 480$

Do this:

(1) Find the number of ways of permuting this letters of the word PICTURE so that (i) all vowels come together (ii) no two vowels come together (iii) the relative positions of vowels and consonants are not disturbed.

12. Find the sum of all 4 digited numbers that can be formed using the digits 1,2,3,4,5,6 without repetation

Given digits are 1,2,4,5,6 $\Rightarrow n = 5$

No of 4 digit numbers can be formed = ${}^{5}p_4$ = 120

We have to find sum of these 120 numbers n = 5, r =4

Sum of all the numbers $= {}^{(n-1)}p_{(r-1)}$ (sum of all digits) (1111…..r times)

$$= {}^{4}p_3(1+2+4+5+6)(1111)$$
$$= 24(18)(1111)$$
$$= 479952$$

Do this:

(i) Find the sum of all 4 digit numbers that can be formed using the digits 1,3,5,7,9

(ii)Find the sum of the all 4 digit numbers that can be formed using the digits 2,3,4,5 (Hint: n=r)

13. Find the sum of all 4 digites numbers that can be formed using the digits 0,2,4,7,8 without repetaition

Here n = 5, r = 4

[Including ‘0’]

Sum of all the numbers =[sum of all the ‘n’ digits]

$$=\left[\,^{(n-1)}p_{(r-1)}\times\left(1111......r\ times\right)-\,^{(n-2)}p_{(r-2)}\times\left(1111......)r-1)\ times\right)\right]$$

$$=\left(0+2+4+7+8\right)\left[\,^{4}p_{3}\left(1111\right)-\,^{3}p_{2}\left(111\right)\right]$$

$$=\left(21\right)\left[24\left(1111\right)-6\left(111\right)\right]$$

$$=\left(21\right)\left[26664-666\right]$$

$$=21\left(25998\right)$$

$$=545958$$

Do this:

(1)Find he sum of 4 digited numbers formed by taking the digits 0,2,4,6

14. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word MASTER

Sol: The alphabetical order of the letters of the given word is A,E,M,R,S,T

The number of words begin with A is $\underline{\lfloor 5}$ = 120

The number of words begin with E is $\underline{\lfloor 5}$ = 120

The number of words begin with MAE is $\underline{\lfloor 3}$ = 6

The number of words begin with MAR is $\underline{\lfloor 3}$ = 6

The number of words begin with MASE is $\underline{\lfloor 2}$ = 2

The number of words begin with MASR is $\underline{\lfloor 2}$ = 2

The nxt word is MASTER = 1

Rank of the work MASTER = 120+120+6+6+2+2+1= 257

(OR)

A , E , M , R , S , T

M					
	A				
		S			
			T		
				E	
					R

$2\times\underline{\lfloor 5}=120$

$0\times\underline{\lfloor 4}=0$

$2\times\underline{\lfloor 3}=12$

$2\times\underline{\lfloor 2}=4$

$0\times\underline{\lfloor 1}=0$

$\underline{1=1}$

$\underline{257}$

Match the Ranks of the following:

(i) PRISON	( )	A) 597
(ii) REMAST	( )	B) 309
(iii) MOTHER	( )	C) 3733
(iv) CARE	( )	D) 391
(v) STREAM	( )	E) 8
(vi) RUBLE	( )	F) 438
(vii) VICTORY	( )	Q) 92

15. Find the number of 4 digited number that can be formed using the digits 1,2,5,6,7. How many of them are divisible by (i) 2 (ii) 3 (iii) 4 (iv) 5 (v) 25

Sol: The number of 4 digited numbers that can be formed using the digits 1,2,5,6,7 is

$^5p_4 = 120$

(i) A number is divisible by '2' when its unit place must be filled with an even digit from among the given integers. This can be done in 2 ways.

			2 & 6

The remaining 3 places can be filled by theremaining 4 digits in $^4p_3 = 24$ ways

The number of 4 digited no divisible by '2' = $2 \times 24 = 48$

(ii)A number is divibible by 3 only when the sum of the digits in that number is a multiple of '3'

Sum of the given 5 digits = 1+2+5+6+7=21

The 4 digits such that their sum is a multiple of 3 from the given digits are 1,2,5,7 (sum is 15)

They can be arranged in $\underline{|4}$ ways and all these 4 digit numbers are divisible by'3'

$\therefore$ The number of 4 digit numbers divisible by 3 = $\underline{|4} = 24$

(iii)A number is divisible by 4 only when the last two places (tens & units) of its is a multiple of 4.

		$\times$	$\times$

$\therefore$ The two places should be filled by one of the following 12,16,52,72,76.

$\therefore$Last 2 places can be filled in 6 ways

From the remaining 3 digits, we can fill the first two digit is $^3p_2 = 6$ ways

$\therefore$No of 4 digit nos divisible by 4 = $6 \times 6 = 36$

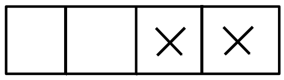
(iv) A number is divisible by 5 when the units place must be filled with 5 from the given integers 1,2,5,6,7 which can be done in 1 way

			5

The remaining first 3 places can be filled by the remaining 4 digits is $^4p_3 = 24$ ways

$\therefore$No of 4 digit nos divisible by 5 = ^np_r

(v)A number is divisible by 25, when the last two places are filled with either 25 or 75 Which can be filled in 2 ways



The remaining first 2 places can be filled by the remaining 3 digits in $^3p_2 = 6$ ways

$\therefore$No of 4 digit nos divisible by 25 = 2x6=12

Do this: (i)Find the number of 4 digit numbers that can be formed using the digits 2,3,5,6,8 (without repetetion). How many of them are divisible by

a) 2 b)3 c) 4 d) 5 e) 25

(ii)The number of 3 digited odd numbers that can be formed with 1,2,3,4,5 repetition being not allowed is _______

(iii)4 digit numbers are formed using the digits 0,1,2,3,4,5,6 (without repetetion) The number of numbers divisible by 5 is ______

(iv)A number of 4 different digits is formed by using the digits 1,2,3,4,5,6,7 in all possible ways Find

a)Total numbers formed

b) howmany of them are > 3400

c)how many of them are divisible by '2'

d)how many of them are divisible by '25'

e)how many of them are divisible by '4'

16. **Permutaions when repetations are allowed:**

If the repetation of things allowed, then the number of permutations of n dissmilar things taken 'n' at a time = n^r

With atleast onerepetition = $n^r - {}^np_r$

1. The number of 4 digit numbers that can be formed using the digit 1,2,4,5,7,8 when repetetaion is allowed is ______

2. No of 5 letter word that can be formed using the letters a the word BRING when repetition is allowed is ____

3. No of 4 letter words that can be formed using the letter of the word PISTON

(a) when repetiton allowed is ________

(b) when repetetion not allowed is _____

(c)with atleast one repetition is ____

4. 9 different letters of an alphabet are given No of 5 letter words that can be formed using these 9 letters when

(i) no letter repeated ( ) (A) $9^5 - {}^9p_5$

(ii) when repetetion allowed ( ) (B) ${}^9p_5$

(iii) atleast one letter repeated ( ) (C) 9^5

5. $n(A) = r,\ n(B) = n$ then No. of

(i) Functioons from $A \to B$ are _( ) (A) ${}^n p_r$

(ii) Injections from A into B ____ ( ) (B) $\lfloor n$

(iii) Bijections (n=r) are ____ ( ) (C) n^r

6. No of injection from a set containing 4 elements into a set B containing 5 elements is _______

7. No of bijections from a set A containing 7 elements onto itself is _____

8. No of 4 digit telephone numbers that can be formed using the digits 1,2,3,4,5,6 when

(i) repetetion allowed is ________

(ii) repetetion not allowed is _____

(iii) with atleast one digit repeated is _______

17. No of palindromes with r letters (or digits) that can be formed using given 'n' digits is

(i) If r is even ( ) (A) n^{r+1}

(ii) If r is odd ( ) (B)4096

(iii)No of 7 letter palidrones can be formed using the letters of the word EQUATION is ___ ( ) (C)500

(iv) No of 6 digital palindromes using digits 1,3,5,7,9 is ____ ( ) (D) $n^{r/2}$

(v)No of 7 digited palindrames using the digits 0,1,2,3,4 is ___ ( ) (E) 125

18. Find the number of 4 digit numbers can be formed using the digits 0,2,5,7,8 that are divisible by (i) 2 (ii) 4 when repetitions are allowed.

Sol: Given digits = 0,2,5,7,8

(i)

Th	H	T	O

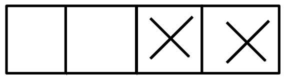
Thousands place can be filled in 4 ways ones place should contain (0 or 2 or 8) which can be filled in ways

Remaining (Tens, Hundreds) 2 places can be filled bygiven 5 digits in = 5^2 (with Repetition)

= 25

By Fundamental principle no of 4 digit Even numbers = $4 \times 3 \times 25 = 300$

(ii) A number is divisible by '4' only when the last two places (tens & units) of its is a multiple of '4'



Last two places can be filled in 8 ways (As Repetition are allowed 00,08,20,28,52,72,80,88)

Hundreds place can be filled n 5 ways

Thousands place can be filled in 4 ways

No. of 4 digit nos divisible by 4 = $8 \times 5 \times 4 = 160$

19. Find the numbe of ways of arranging the letters of the word INDEPENDENCE

Sol: Total number of letters = 12

No of N's = 3

No of D's =2

No of E's =4

and the rest are different total Number of required arrangements $= \frac{\lfloor 12}{\lfloor 3 \lfloor 2 \lfloor 4}$

Number of ways of arranging letters of the word

(i) INTERMEDIATE is _______

(ii) PERMUTATION is _______

(iii) INDEPENDENCE is _____

(iv)COMBINATION is ___________

(v)SINGING is ______

(vi)MATHEMATICS is _______

(vii)MISSISSIPPI is ______

20. If the letters of the word EAMCET are permuted in all possile waysif the words thus formed are arranged in the dictionary order, find the rank of the word EAMCET.

Sol: The dictionary order of the letters of given word is A, C, E, E, M, T

$\underline{A}$ — — — — — $= \frac{\lfloor 5}{\lfloor 2} = 60$

$\underline{C}$ — — — — — $= \frac{\lfloor 5}{\lfloor 2} = 60$

$\underline{E}$ $\underline{A}$ $\underline{C}$ — — — $= \lfloor 3 = 6$

$\underline{E}$ $\underline{A}$ $\underline{E}$ — — — $= \lfloor 3 = 6$

E A M C E T $= 1 = 1$

Rank of EAMCET is 60+60+6+6+1 = 133

A , C , E , E , M , T

E					
	A				
		M			
			C		
				E	
					T

$2 \times \frac{\lfloor 5}{\lfloor 2} = 120$

$0 \times \lfloor 4 = 0$

$2 \times \lfloor 3 = 12$

$0 \times \lfloor 2 = 0$

$0 \times \lfloor 1 = 0$

$1 = 1$

133

(i) Rank of A J A N T A is _____

(ii) Rank of J A N A T A is _____

Circular permutations:

1. The number of circular permutations of n different things taken all at a time is $\lfloor n-1$

2. The number of circular permutaions is $\frac{1}{2}\lfloor n-1$

3. The number of circular permutaions of n things taken r at a time in one direction is $\frac{{}^{n}p_{r}}{2r}$

21.(A)

4.Number of ways of preparing a chain with 6 different coloured beads is ______

5.Number of ways of arranging 7 persons around a circle is ______

6.Number of ways of arranging 5 boys and 5 girls around a circle is ______

7.Number of different chains that can be prepared using 7 different coloured beads is _____

8.Number of chains that can be prepared using 7 different coloured beads is ____

9.Number of ways of preparing chain with '6' different coloured beads is ____

10.The number of ways in which 8 diffreently coloured beads be stung a necklace is ____

21.(B) Find the number of ways of arranging 6 boys and 6 girls around a circular table so that

(i) all the girls set together

(ii) no two girls set together

(iii) boys and girls set alternatively

Sol: No of boys = 6, Girls = 6

(i) Treat all 6 girls as 1 unit, then we have 6 Boys +1 unit = 7, which can be arranged round a circle $= \lfloor(7-1) = \lfloor 6$

All the 6 girls can be arranged among themseleves in $\lfloor 6$ ways.

Total No of permutations $= \lfloor 6 - \lfloor 6$

(ii) Since no two girls sit together First we arrange 6 boys round a circle in $\lfloor 6-1 = \lfloor 5$ ways

In the remaining 6 places

We have to fill by the girls so that, no two girls are together, which can be done in $^6p_6 = \lfloor 6$ ways

No of permutations = $\lfloor 5 \cdot \lfloor 6$

(iii) Since boys and girls sit alternatively. First we have to arrange 6 boys round a circle in $\lfloor 6-1 = \lfloor 5$ ways.

To get allternate arrangement, we have to fill the 6 blank places with 6 girls, which can be done in $\lfloor 6$ ways.

No of permuations = $\lfloor 5 \cdot \lfloor 6$

22. A round table conference is attended by 3 Indeians, 3 chines, 3 canadians and 2 Americans. Find the number of ways of arranging them at the round table so that the delegates belonging to same country sit together.

Sol: Since the delegates belonging to same country sit together, treat each country as 1 unit.

First arrange the 4 units round table in $\lfloor 3$ ways

Now 3 Indians can be arranged among themselves in $\lfloor 3$ ways

3 Chinese can be arranged among themselves in $\lfloor 3$ ways

3 Canadians can be arranged among themseleves in $\lfloor 3$ ways

2 Americans can be arranged among themselves in $\lfloor 2$ ways.

No of required arrangements $= \lfloor 3 \cdot \lfloor 3 \cdot \lfloor 3 \cdot \lfloor 3 \cdot \lfloor 2$

$$= 6 \cdot 6 \cdot 6 \cdot 6 \cdot 2$$

$= 2592$

23. A family consists of a father, mother, 2 daughters and 2 sons. In how many different ways can they sit at a round table if the two daughters wish to sit on eiher side of the father.

Sol: Total number of persons in the family = 6

Treat 2 daughters along with a father as 1 unit

Thus we have 1 mother + 2 sons +1 unit =4 and they can be seated around a table in $\lfloor 4-1 = \lfloor 3$ ways

The two daughters can be arranged on with side of the father in $\lfloor 2$ ways

No of required arrangements $= \lfloor 3 \cdot \lfloor 2$

$= 6 \times 2$

$= 12$

24. Find the number of different ways of preparing a garland rising 7 distinct red roses and 4 distinict yellow rose such that no two yellow roses come together.

Sol: First we arrange 7 red roses in a circuler form (garland form) in $\lfloor 7-1 = 16$ ways Now, there are 7 gaps in between the red roses and we can arrange the 4 yellow roses in these 7 gaps in ${}^7p_4$ ways.

Total number of circular permutations is $\lfloor 6 \times {}^7p_4$

But this being the case of garland, clockwise, and anticlockwise arrangements look alike.

Hence the required number of ways is $\frac{1}{2}\left(\lfloor 6 \times {}^7p_4\right)$

Do this:

1. Find the number of ways of arranging 4 boys and 3 girls around a circle so that all the girls together.

2. Find the number of ways of arranging 7 gents and 4 ladies around a circular table if two ladies wish to sit together.

3. Find the number of ways of arranging 6 boys 7 guests and a host around a circle if 2 particular guesh are wish to sit on either side of the host.

4. Find the number of ways of seating 5 Indians, 4 Americans and 3 Russians at a round table so that (i) all Indians sit together

(ii) no two Russions sit together

(iii) persons of same nationality sit together

Fill in the blanks:

5. Number of ways of arranging 4 boys and 3 girls around a circle so that all the girls sit together is _____

6. Number of ways of arranging 7 gents and 4 ladies around a circular table if no two ladies wish to sit together is______

7. Number of ways of seating 15 Indians, 4 Americals and 3 Russians at a round table so that

(a) all Indians sit together is ______

(b)no two Russians sit together is ______

(c)persons of same nationality sit together is ______

8. A Round table conference is attended by 13 Indians, 3 chinese, 3 Canadians and 2 Americans. The number of arranging them at the round table so that the delegates belonging to same country sit together is ______

9. The No of ways in which 5 boys and 4 girls sit around a circular so that no two girls sit together is ______

10. The No of ways iin which 10 boys and 8 girls can sit around a round table so that all the girls come together is ________

25. Find the number of ways of arranging the letters of the word ASSOCIATIONS In how many of them

(i) all the 3 S's come together

(ii) the two A's do not come together

Sol: The given word has 12 letters in which 2 'A's 3 S's, 2 O's and 2 I's

Hence they can be arranged in $\dfrac{\lfloor 12}{\lfloor 2 \cdot \lfloor 2 \cdot \lfloor 2 \; \lfloor 3}$

(i)3 S's come together

Treat 3 S's as one unit.

Then we have remaining 9 letters (2 A's, 2 O's, 2 'I's) + 1unit = 10

These 10 can be arranged in $\lfloor 3$

(ii) 2 'A's do not come together other than 2 'A's remaining 10 letters can be arranged in $\dfrac{\lfloor 10}{\lfloor 3 \; \lfloor 2 \, \lfloor 2}$ ways

No of blanks in eac of the above arrangements = 11

Filling 2 'A's in 11 blank places = $\dfrac{{}^{11}p_2}{\lfloor 2}$

Required number of permutations = $\dfrac{\lfloor 10}{\lfloor 3 \; \lfloor 2 \; \lfloor 2} \times \dfrac{{}^{11}p_2}{\lfloor 2}$

26. Find the number of ways of arranging the letters of the word SINGING so that

(i) They begin and end with I

(ii) the two 'G' is come together

(iii) relative positions of vowels and consonants are not disturbed

Sol: (i) First we fill the first and last places with I's in $\frac{\lfloor 2}{\lfloor 2}=1$ way as shown below

I						I

Now we fill the remaining 5 places with the remaining 5 letters S,N,G,N,G in $\frac{\lfloor 5}{\lfloor 2\ \lfloor 2}=30$ ways

Hence No of required permutations = $1\times 30=30$

(ii)2 'G's come together

Treat 2 'G's as one unit then we have

2 'I's + 2 'N's + 1 'S' +1 unit = 6

Hence they can be arranged in $\frac{\lfloor 6}{\lfloor 2\ \lfloor 2}=180$ ways

Now the 2 'G's among themselves can be arranged in $\frac{\lfloor 2}{\lfloor 2}=1$ ways

Hence No of required permuations = $180\times 1=180$

(iii)In the word SINGING, there are 2 vowels which are alike i.e. I, and there are 5 consonants $\left[2\text{ 'N's, } 2\text{'G's, } 1\text{'S'}\right]$

$\underline{C}\ \ \underline{V}\ \ \underline{C}\ \ \underline{C}\ \ \underline{V}\ \ \underline{C}\ \ \underline{C}$

The 2 vowels can be interchanged among themselves in $\frac{\lfloor 2}{\lfloor 2}=1$ ways

Now the 5 consonants can be arranged in the remaining 5 places in $\frac{\lfloor 5}{\lfloor 2\ \lfloor 2}=30$ ways

Hence No of Required arrangements $=1\times 30=30$

Do this:

1. In how many ways can the letters of the word CHEESE be arranged so that no two E's come together.
2. Find the number of ways of arranging the letters of the word MISSING so that two S's are together and two I's are together.
3. Find the number of ways of arranging the letters of the word SPECIFIC. In how many of them (i) the two 'c's come together (ii) the two 'I's not come together.
4. How many numbers can be formed using all the digit 1,2,3,4,3,2,1 such that even digit always occupy even places.
5. Find the number of 5 digit numbers that can be formed using the digit 1,1,2,2,3. How many of them are even
6. Find the number of 5 digits numbers that can be formed using the digits 0,1,1,2,3

7. No of 7 digit numbers that can be formed using 2,2,2,3,3,4,4 is ______

8. No of ways of arranging the letters of the word $a^4\ b^3\ c^5$ in its Expanded form is ____

9. No of ways of arranging 12 books in which there are 4 copies (alike) each of 3 different books is _____

COMBINATIONS:

1. If ${}^{n}p_r = 5040, {}^{n}c_r = 210$ then find n and r ?

Sol: we have $\frac{{}^{n}p_r}{{}^{n}c_r} = \lfloor r$

$$\frac{5040}{210} = \lfloor r$$

$$24 = \lfloor r$$

$$\lfloor 4 = \lfloor r \Rightarrow \boxed{r = 4}$$

Since ${}^{n}p_r = 5040$

$${}^{n}p_4 = 5040$$

$$n(n-1)(n-2)(n-3) = 10\times 9\times 8\times 7$$

Comparing $\boxed{n = 10}$

(i) If ${}^{10}p_r = 604800, {}^{10}C_r = 120$ then r = _____

(ii) ${}^{8}C_3 = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$

(iiii) ${}^{8}p_3 = 272, {}^{n}C_r = 136$ then then n = ______

2. If ${}^{n}C_4 = 210$ then find n?

Sol: Given ${}^{n}C_4 = 210$

$$\frac{n(n-1)(n-2)(n-3)}{4\cdot 3\cdot 2\cdot 1} = \frac{210}{\lfloor 4}\cdot \lfloor 4$$

$$= \frac{(10\times 7\times 3)(4\times 3\times 2\times 1)}{\lfloor 4}$$

$$= \frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}$$

$${}^{n}C_4 = {}^{10}C_4$$

$$\boxed{n = 10}$$

(1)If ${}^{12}C_r = 495$ then r = ____

(2) If ${}^{10}C_2 = {}^{3\cdot(n+1)}C_3$ then n = ______

(3) If ${}^{(n+2)}C_3 = 120$ then n = ____

3. If ${}^{12}C_{r+1} = {}^{12}C_{3r-5}$ then find r?

Sol: ${}^{n}C_r = {}^{n}C_s \Rightarrow$ either r = s or $r+s=n$

$r=s$	$r+s=n$
$r+1=3r-5$	$(r+1)(3r-s)=12$
$-2r=-6$	$4r-4=12$
$r=3$	$r=4$

$\therefore r=3\ or\ 4$

Match the following:

i) ${}^{15}C_{2r-1} = {}^{15}C_{2r+4}$ then r = ____	( )	A. 8
ii) ${}^{17}C_{2t+1} = {}^{17}C_{3t-5}$ then t = ____	( )	B. 2
iii) ${}^{12}C_{r+1} = {}^{12}C_{r-5}$ then r =____	( )	C. 5
iv) ${}^{12}C_{s+1} = {}^{12}C_{2s-5}$ then $\frac{s}{3} =$ __	( )	D. 6
v) ${}^{15}C_{3r} = {}^{15}C_{r+3}$ then r+2=___	( )	E. 3

4. If ${}^{n}C_{10} = {}^{n}C_{15}$ then find ${}^{27}C_n$?

Sol: Given ${}^{n}C_{10} = {}^{n}C_{15}$

${}^{n}C_r = {}^{n}C_s \Rightarrow$ r = s or r+s = n

${}^{27}C_n = {}^{27}C_{25} = {}^{27}C_2$

$$= \frac{27 \cdot 26}{2} = 351$$

(i) $nc_4 = nc_7$ then n = ______

(ii)If $50c_{48} =$ __________

(iii) If $nc_5 = nc_6$ then $13c_n =$________

(iv) If $nc_{21} = nc_{27}$ then $50c_n =$__________

(v) If $nc_{12} = nc_8$ then nc_{17} ____, $20c_n$________

(vi) If $15c_8 + 15c_9 - 15c_6 - 15c_7 =$___________

5. Find the value of $^{47}C_4 + \sum_{r=1}^{5} {}^{(52-r)}C_3$

Sol: Given that $47c_4 + \sum_{r=1}^{5} {}^{(52-r)}C_3$

$= 47c_4 + 51c_3 + 50c_3 + 49c_3 + 48c_3 + 47c_3$

$= (47c_4 + 47c_3) + 48c_3 + 49c_3 + 50c_3 + 51c_3$

$= \boxed{{}^{n}c_r + {}^{n}c_{r-1} = {}^{(n+1)}c_r}$

$= \left({}^{48}c_4 + {}^{48}c_3\right) + {}^{49}c_3 + {}^{50}c_3 + {}^{51}c_3$

$= \left({}^{49}c_4 + {}^{49}c_3\right) + {}^{50}c_3 + {}^{51}c_3$

$= \left({}^{50}c_4 + {}^{50}c_3\right) + {}^{51}c_3$

$= {}^{51}c_4 + {}^{51}c_3$

$= {}^{52}c_4$

Do this (i) show that $= {}^{n}c_{r-2} + 2{}^{n}c_{r-1} + {}^{n}c_r - {}^{n+2}c_r$

6. If 5 vowels and 6 consonants are given, then how many 6 letter words can be formed with 3 vowels and 3 consonants

Sol: Number of vowels given = 5

Number of consonants given = 6

We have to form a 6 letter word with 3 vowels and 3 consonants from given letters.

3 vowels can be selected from 5 in ${}^{5}c_3$ ways

3 consonants can be selected from 6 in ${}^{6}c_3$ ways

Totalnumber of words $= {}^{5}c_3 + {}^{6}c_3 \times 16$

= 144000

7. Find the number of ways of selecting 11 member cricket team from 7 batsmen, 6 bowlers and 2 wicket keepers so that the team contains 2 wicket keepers and altleast 4 bowlers

Bowlers (6)	Wicket keepers (2)	Batsmens (7)	No of ways of selecting team

4	2	5	${}^{6}C_4 \times {}^{2}C_2 \times {}^{7}C_5$ $15 \times 1 \times 21 = 315$
5	2	4	${}^{6}C_5 \times {}^{2}C_2 \times {}^{7}C_4$ $6 \times 1 \times 35 = 210$
6	2	3	${}^{6}C_6 \times {}^{2}C_2 \times {}^{7}C_3$ $1 \times 1 \times 35 = 35$

No of ways of selecting required cricket team = $315 + 210 + 35 = 560$

Fill in the Blanks:

1) ${}^{34}c_5 + \sum_{r=0}^{8} {}^{38-r}c_4 =$ ________

2) ${}^{25}c_4 + \sum_{r=0}^{4} {}^{29-r}c_3 =$ __________

3) ${}^{10}c_{95} + 2 \cdot {}^{10}c_4 + {}^{10}c_3 =$ ______

4) If $9c_3 + 9c_5 = 10c_r$ then r = ______

5) If $24c_5 + 24c_{18} = Xc_y$ then x + y= _____

6) $\begin{vmatrix} 5c_2 & 5c_3 & 6c_3 \\ 6c_2 & 6c_3 & 7c_3 \\ 8c_4 & 8c_5 & 9c_5 \end{vmatrix} =$

8A. Show that ${}^{(n-3)}C_r + 3 \cdot {}^{(n-3)}C_{(r-1)} + 3 \cdot {}^{(n-3)}C_{(r-2)} + 3 \cdot {}^{(n-3)}C_{(r-3)} = {}^{n}C_r$

L.H.S = ${}^{(n-3)}C_r + 3 \cdot {}^{(n-3)}C_{(r-1)} + 3 \cdot {}^{(n-3)}C_{(r-2)} + {}^{(n-3)}C_{(r-3)}$

$= \left[{}^{(n-3)}C_r + {}^{(n-3)}C_{(r-1)} \right] + 2\left[{}^{(n-3)}C_{(r-1)} + {}^{(n-3)}C_{(r-2)} \right] + \left[{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{(r-3)} \right]$

$\boxed{{}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r}$

$= {}^{(n-2)}C_r + 2 \cdot {}^{(n-2)}C_{(r-1)} + {}^{(n-2)}C_{(r-2)}$

$= \left[{}^{(n-2)}C_r + 2\ {}^{(n-2)}C_{(r-1)} \right] + \left[{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{(r-2)} \right]$

$= {}^{n-1}C_r \quad + \quad {}^{n-1}C_{r-1}$

$= {}^{n}C_{r}$

= R.H.S

Do this:

1. Find the number of ways of selecting a cricket team of is players 7 Batsmen and 6bowlers such that there will be atleast 5 bowlers in the team.
2. Find the number of ways of selecting 3 vowels and 2 consonants from the letter of the word EQUATION
3. Find the number of ways of selecting 3 girls and 3 boys out of 7 girls and 6 boys.
4. Find the number of ways of selecting 4 boys and 3 girls from a group of 8 boys and 5 girls.
5. Find the number of ways of selecting 4 English, 3 telugu and 2 Hindi books out of 7 english, 6 Telugu and 5 Hindi books.
6. Find the number of ways of selecting a committee of 6 members out of 10 members always including a specified member (Hind: 1 members already selected)
7. Find the number of ways of selecting 5 books from 9 different mathematics books such that a particular book is not included

   (Hind: 1 book is excluded)
8. In a class there are 30 students. If each student plays a chess game with each of the other students, then find the total number of chess games played by them

8B. A question paper is divided into 3 sections A, B, C containing 3, 4, 5 questions respectively. Find the number of ways of attempting 6 questions choosing atleast one from each section.

Sol: The selection of a question may be of the following

Section A (3)	Section B (4)	Section C (5)	No. of ways of section
3	2	1	${}^{3}C_{3} \cdot {}^{4}C_{2} \cdot {}^{5}C_{1} = 1 \cdot 6 \cdot 5 = 30$
3	1	2	${}^{3}C_{3} \cdot {}^{4}C_{1} \cdot {}^{5}C_{2} = 1 \cdot 4 \cdot 10 = 40$
2	3	1	${}^{3}C_{2} \cdot {}^{4}C_{3} \cdot {}^{5}C_{1} = 3 \cdot 4 \cdot 5 = 60$
2	2	2	${}^{3}C_{2} \cdot {}^{4}C_{2} \cdot {}^{5}C_{2} = 3 \cdot 6 \cdot 10 = 180$
2	1	3	${}^{3}C_{2} \cdot {}^{4}C_{1} \cdot {}^{5}C_{3} = 3 \cdot 4 \cdot 10 = 120$
1	4	1	${}^{3}C_{1} \cdot {}^{4}C_{4} \cdot {}^{5}C_{1} = 3 \cdot 1 \cdot 10 = 15$
1	3	2	${}^{3}C_{1} \cdot {}^{4}C_{3} \cdot {}^{5}C_{2} = 3 \cdot 4 \cdot 10 = 120$
1	2	3	${}^{3}C_{1} \cdot {}^{4}C_{2} \cdot {}^{5}C_{3} = 3 \cdot 6 \cdot 10 = 180$

1	1	4	${}^{3}C_1 \cdot {}^{4}C_1 \cdot {}^{5}C_4 = 3 \cdot 4 \cdot 15 = 60$

Total number of ways = 30+40+60+180+120+15+120+180+60 = 805

(OR)

$${}^{12}C_6 - {}^{7}C_6 - {}^{9}C_6 - {}^{8}C_6 = 924 - 7 - 84 - 28 = 805$$

Fill in the Blanks:

1. If $n(A) = 8$
   i) number of subsets of A is ________
   ii) No of subsets of A with 6 elements is _________
   iii) No of subsets ofA with 7 elements is______
   iv) No of subsets of A with 8 elements is ______
   v) No of subsets of A with atleast 6 elements is __________
2. $n(A) = 12$ then the number of subsets of A having
   i)4 elements ________
   ii) atleast 34 elements ______
   iii) atmost 3 elements __________

3. Out of 6 boys and 4 girls, number of ways of forming committee having:
   i) If 4 members is ______
   ii) all 4 boy members is ______
   iii) 4 members with atleast one girl in the committee in ______

9. Find the number of positive Integral divisors of 1080

Sol: $1080 = 2^3 \times 3^3 \times 5^1$

$= p_1^{\alpha_1} \times p_2^{\alpha_2} \times p_3^{\alpha_3}$

Divisors of 1080 $= (\alpha_1 + 1)(\alpha_2 + 1)(\alpha_3 + 1)$

$= (3+1)(3+1)(3+1)$

$= (4)(4)(2)$

$= 32$

Fill in the Blanks:

1) No of positive integral divisors of 108 is _____
2) No of positive integral divisors of 2520 is ______
3) No of positive divisors of $2^5 \cdot 3^6 \cdot 7^3$ is_____
4) No of proper divisors of 2520 is ______

10. Find the number of diagonals of a pentagon

Sol: Number of sides = n = 5

No. of diagonals $= \dfrac{n(n-3)}{2}$

$= \dfrac{5(2)}{2}$

$= 5$

1) No of diagonals of a polygon havin sides

i) n is	( )	(A) 27
ii) 10 is	( )	(B) 54
iii) 12	( )	(C) 14
iv) septagon (7)	( )	(D) $\dfrac{n(n-3)}{2}$
v) Nanogon (9)	( )	(E) 35

2) A polygon has 54 diagonals, then the number of its sides are _____

3) A polygon has 275 diagonals, then the number of its sides are ______

11. Find the number of zeroes in $\lfloor 100$

Sol: $\lfloor 100 = 2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma} \cdot 7^{\delta}$........

Where $\alpha = \left[\dfrac{100}{2}\right] + \left[\dfrac{100}{2^2}\right] + \left[\dfrac{100}{2^3}\right] + \left[\dfrac{100}{2^4}\right] + \ldots$

$= 50 + 25 + 12 + 6 + 3 + 1$

= 99

$\gamma = \left[\dfrac{100}{5}\right] + \left[\dfrac{100}{52}\right]$

$= 20 + 4$ = 24

Number of zers in $\lfloor 100 =$ power of 10 in $\lfloor 100$

= 24

= 24

12. If there are 5 black pens, 6 black pencils and 7 white erasers. Find the number of ways of selecting any number of (one of more) things out of thems.

Sol:

13. To pass an examination a student has to pass in each of the 3 papers. In how many ways a student can fail in the examination

Sol:

14. Out of 3 different books on Economics, 4 different books on political science and 5 different books on Geography, how many collections can be made, if each collection consists of (1) exactly one book of each subject (2) atleast one book of each subject

(i) out of 3 books on {conomics exactly one book is chosen in ${}^{3}C_{1}$ ways.

Similarly political science is ${}^{4}C_{1}$ & Geography book can be chosen in ${}^{5}C_{1}$ ways

$\therefore$ Required number of ways = ${}^{3}C_{1}\times{}^{4}C_{1}\times{}^{5}C_{1}$

$=3\times4\times5$

= 60

(ii) The Numberof collections having atleast one book of each subject is

$\left(2^{3}-1\right)\left(2^{4}-1\right)\left(2^{5}-1\right)=7\times15\times31=3255$

Note: In the above problem if the books of each subject are alike then

(i) No of ways of selecting exactly one book of each subject = 1

(ii) Number of collections having atleast one book of eac subject is $3\times4\times5=60$

15. 14 persons are seated at a round table. Find the number of ways of selecting two persons out of them who are not seated adjecent to each other.

Sol: let the seating arrangement of given 14 persons at the table as shown in he fig.

No of ways of selecting 2 persons = ${}^{14}C_{2}=91$

In the above arrangement two persons setting adjacent to each other can be selected in 14 ways (they are

$a_1a_2, a_2a_3, a_3a_4, a_4a_5, a_5a_6, a_6a_7, a_7a_8, a_8a_9, a_9a_{10}, a_{10}a_{11}, a_{11}a_{12}, a_{12}a_{13}, a_{13}a_{14}, a_{14}a_{15}$)

$\therefore$ Required number of ways = 91 – 14 = 77.

Multiple choice questions (Permutaions & Combinations)

Choose the correct Answer:

1. $\frac{1}{\lfloor 4}+\frac{1}{\lfloor 5}+\frac{1}{\lfloor 6}=\frac{x}{26}$ then x = ______ [ ]

a) 35 b) 36 c) 37 d)38

2. A man has 3 jackets, 10 shirts and 5 pair of slacks. If an outfit consists of a jacket, a shirt, and a pair of slacks, the different outfits can the man make is ________ [ ]

a) 35 b) 36 c) 37 d) 38

3. The number of national numbers are rom 1 to 1000 which have none of their digits repeated is ______ [ ]

4. Number of ways in which 7 different colours in a rainbow can be arranged, if green always in the middle is ______ [ ]

a) $\lfloor 5$ b) $\lfloor 7$ c) $\lfloor 6$ d) $\lfloor 8$

5. A number lock has 3 rings and cach ring has 9 digits 1,2,3,….9. The maximum number of unsuccesful attempts that can be made by a person who tries to open the lock without lenouring the key code is ______ [ ]

a) 9^3 b) 3^9 c) 3^9-1 d) 7^3-1

6. The number of words which can be formed using all the letters of the word AKSHI, if each word begin with vowel or terminaters in vowel is ____ [ ]

a) 96 b) 48 c) 84 d) 120

7. Four prizes distributed among 5 students then

(i) If no student get more than one prize, then the number of ways is _ [ ]

a) 5^4 b) 120 c) 4^5 d) 620

(ii) If each student is eligible for all prizes, then the number of ways is ____

a) 625 b) 620 c) 1024 d) 1020

(iii)If no student gets all the prizes, then the number of ways is ____ [ ]

a) 65 b) 620 c) 1027 d) zero

8. If ${}^{(l+m)}p_2=56$ and ${}^{(l\cdot m)}p_2=12$ then $l\times m=$___ [ ]

a) 10 b) 11 c) 12 d) None

9. $1+1\cdot{}^1p_1+2\cdot{}^2p_2+3\cdot{}^3p_3+.....+n\cdot{}^np_n=$____ [ ]

a) $\lfloor n$ b) $\lfloor n-1$ c) $\lfloor n+2$ d) $\lfloor n+1$

10. The number of natural numbers from 1000 to 9999 (both inclusive) that do not have all 4 different digits is ___ [ ]

a) 4048 b) 4464 c) 4518 d) 4536

11. $\begin{vmatrix} {}^nc_r & {}^nc_{r+1} & {}^{n+1}c_{r+1} \\ {}^xc_{r+1} & {}^xc_{r+2} & {}^{x+1}c_{r+2} \\ {}^yc_{r-1} & {}^yc_r & {}^{y+1}c_r \end{vmatrix}=$_____ [ ]

a)1 b) –1 c) ± 1 d) 0

12. No ways in which 5 boys, 4 girls can sit in a row, so that all girls sit together and two particular girls never sit together is ____ [ ]

a) $\lfloor 6\ \lfloor 3$ b) $\lfloor 6\ \lfloor 4$ c) $\lfloor 6\ \lfloor 3\ \lfloor 2$ d) none

13. The number of different arrangemens can be made out of the letters of the word CONCESSION is _______ [ ]

a) $\dfrac{\lfloor 10}{(\lfloor 2)^3}$ b) $\dfrac{\lfloor 10}{(\lfloor 2)^4}$ c) $\dfrac{\lfloor 1}{(\lfloor 12)^3}$ d) $\dfrac{\lfloor 1}{(\lfloor 12)^4}$

14. No of arrangements of the elements of INTRODUCE in which vowels do not come together is______ [ ]

a) $\lfloor 6$ b) $\lfloor 9$ c) $\lfloor 6\,\lfloor 4$ d) $\lfloor 9-\lfloor 6\cdot\lfloor 4$

15. Number of ways that can be arranged from the letters of the expression $p^2q^3 \cdot r^4$ when written in full length is ____ [ ]

a) $\frac{\lfloor 11}{\lfloor 2 \lfloor 3 \lfloor 4}$ b) $\frac{\lfloor 9}{\lfloor 2 \lfloor 3 \lfloor 4}$ c) $\frac{\lfloor 10}{\lfloor 2 \lfloor 3 \lfloor 4}$ d) none

16. Number of ways in which 5 letters can be put in 5 addressed envelopes so that no letter goes into the envelope correctly is ______ [ ]

a) 44 b) 9 c) 2 d) 265

17. No of ways can 3 students go for 4 theatres is _________ [ ]

a) 3^4 b) ${}^4p_3$ c) 4^3 d) none

18. ${}^8c_3 + {}^{n+2}c_4 = {}^9c_4$ then n = _____ [ ]

a) 6 b) 7 c) 8 d) 9

19. The number of ways that the letters of the word ALGEBRA can be arranged without changing the relative positions of vowels are consosnts is ____ [ ]

a) $\lfloor 3 \lfloor 4$ b) $\frac{\lfloor 7}{\lfloor 2}$ c) $\frac{\lfloor 3 \lfloor 4}{\lfloor 2}$ d) none

20. All the numbers that can be formed using the digits 1,2,3,4 are arranged in increasing order of magnitudes then the rank of 3241 is _____ [ ]

a) 56 b) 256 c) 57 d) 257

21. A man has 6 friends. In how many ways can be invite one or more to dinner is __________ [ ]

a) 64 b) 32 c) 31 d) 63

22. No. of combinations of '3n' things taken atleast one at a time is 511 then n = ___[ ]

a) 2 b) 3 c) 4 d) none

23. No. of proper factors (divisols) of 3240 is ____ [ ]

a) 41 b) 40 c) 39 d) 38

24. Sum of the divisors of $2^5 \cdot 3^4 \cdot 5^2$ is ______ [ ]

a) $(2^5-1)(3^4-1)(5^2-1)$ b) $\left(\frac{2^5-1}{2-1}\right)\left(\frac{3^4-1}{3-1}\right)\left(\frac{5^2-1}{5-1}\right)$

c) $\left(\frac{2^6-1}{2-1}\right)\left(\frac{3^5-1}{3-1}\right)\left(\frac{5^3-1}{5-1}\right)$ d) none

25. A polygon has 44 diagonals No of its sides is ______ [ ]

a) 10 b) 11 c) 12 d) 13

26. There are 10 paints in a plane, no three paints are in the straight line excepting 4 points are collinear then noof lines can be formed is _______ [ ]

a) 45 b) 43 c) 40 d) none

27. In the above problem, number of triangles can be formed is _____ [ ]

a) 120 b) 100 c) 80 d) 116

28. No of parallelograms that can be formed from a set of 4 parallel lines intersecting another set of three paralell lines [ ]

a) 18 b) 16 c) 8 d) none

29. Exponent of 5 in $\lfloor 360$ is ______ [ ]

a) 72 b) 14 c) 2 d) 88

30. Number of numbers formed using the digits 4,5,6,7,8 which are greater than 56000 is ______ [ ]

a) 80 b) 90 c) 100 d) none

31. The number of signals that can be generated by using 6 differentl coloured flags, when any no of them may be hoisted at a time is _____ [ ]

a) 2020 b) 1947 c) 1956 d) none

32. No of ways is which 3 letters be posted in 4 letter boxes in a village, if all the 3 letters are not posted in the same letter box is _______ [ ]

a) 64 b)81 c) 77 d) 60

33. Least value of 'n' satisfying ${}^{(n-1)}c_3 + {}^{(n-1)}c_4 > {}^{n}c_3$ is____ [ ]

a) 7 b) 8 c) 6 d) 5

34. No of ways in which 6 maths papers be arranged so that the best and worst may not be together is _____ [ ]

a) $\lfloor 4 \lfloor 5$ b) $4 \lfloor 5$ c) $\lfloor 5 \lfloor 2$ d) $\lfloor 6$

35. 11 animals of a circus have to be placed in 11 cages one in each cage. If 4 of the cages are too small for 6 of the animals the number of ways of caging the animals is __________ [ ]

a) $\lfloor 7 \lfloor 5$ b) $7 \lfloor 6$ c) $6 \lfloor 7$ d) none

36. Total number of 9 digit numbers which have all different digits is ___ [ ]

a) $\lfloor 9$ b) $10 \cdot \lfloor 9$ c) $9 \cdot \lfloor 9$ d) none

37. let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is ______ [ ]

a) 256 b) 220 c) 219 d) 211

38. The large of the function $f(x) = {}^{(7-x)}c_{(x-3)}$ is ____ [ ]

a) $\{1,2,3\}$ b) $\{1,2,3,4\}$ c) $\{1,2,3,4,5\}$ d) none

39. The Number of positive integral solution of abc = 30 is ___ [ ]

a) 30 b) 12 c) 27 d) 8

Passage :

If 52 cards are divided into 4 groups then the number of ways 52 card be

40. Distributed equally among 4 players in order [ ]

a) $\frac{\underline{|52}}{(\underline{|13})^4}$ b) $\frac{\underline{|52}}{(\underline{|13})^4\,\underline{|4}}$ c) $\frac{\underline{|52}}{\underline{|20}\,\underline{|15}\,\underline{|10}\,\underline{|7}}$ d) $\frac{\underline{|52}}{(\underline{|15})^3\,\underline{|3}\,\underline{|7}}$

41. Divided into 4 groups of 13 cards each [ ]

a) $\frac{\underline{|52}}{(\underline{|13})^4}$ b) $\frac{\underline{|52}}{(\underline{|13})^4\,(\underline{|4})}$ c) $\frac{\underline{|52}}{\underline{|20}\,\underline{|15}\,\underline{|10}\,\underline{|7}}$ d) $\frac{\underline{|52}}{(\underline{|15})^3\,\underline{|3}\,\underline{|7}}$

42. Divided into four sets of 20, 15, 10, 7 cards [ ]

a) $\frac{\underline{|52}}{(\underline{|13})^4}$ b) $\frac{\underline{|52}}{(\underline{|13})^4\,(\underline{|4})}$ c) $\frac{\underline{|52}}{\underline{|20}\,\underline{|15}\,\underline{|10}\,\underline{|7}}$ d) $\frac{\underline{|52}}{(\underline{|15})^3\,\underline{|3}\,\underline{|7}}$

43. Divided into four sets,three of them having 15 cards each and fourth having 7 cards [ ]

a) $\frac{\underline{|52}}{(\underline{|113})^4}$ b) $\frac{\underline{|52}}{(\underline{|113})^4\,\underline{|4}}$ c) $\frac{\underline{|52}}{\underline{|20}\,\underline{|15}\,\underline{|10}\,\underline{|7}}$ d) $\frac{\underline{|52}}{(\underline{|15})^3\,\underline{|3}\,\underline{|7}}$

44. The interior angles of a regular polygon measures 150^0 each, the number of diagonals of the polygon is _______ [ ]

a) 35 b) 44 c) 54 d) 36

45. No of squares can be formed in a chess board is ______ [ ]

a) 64 b) 81 c) 420 d) 204

46. No of Rectangles can be formed in a chess board is _____ [ ]

a) 1296 b) 204 c) 1092 d) none

47. $\left({}^7C_0 + {}^7C_1\right) + \left({}^7C_1 + {}^7C_2\right) + \left({}^7C_2 + {}^7C_3\right) + \ldots\left({}^7C_6 + {}^7C_7\right)$ is [ ]

a) $2^8 - 2$ b) $2^8 - 1$ c) $2^8 + 1$ d) 2^8

48. Number of prime numbers among the numbers $\underline{|105} + 2, \underline{|105} + 3, \underline{|105} + 4, \ldots\ldots\ldots, \underline{|105} + 5$ is [ ]

a) 31 b) 32 c) 33 d) none

49. There are 6 roads between A and B and 4 roads between B and C then in howmany ways one can drive the circular trip (to& Fro, up & down) described in without using the same road more than once [ ]

a) 24 b) 576 c) 360 d) none

50. The tamer of wild animals has to bring one by one 5 lions and 4 tigers to the circus area. The nmber of ways this can be done, if no two tigers immediately follow each other is_______ [ ]

a) $\lfloor 5 \cdot {}^{6}p_4$ b) $\lfloor 5 \cdot \lfloor 6$ c) $\lfloor 5 \times \lfloor 4$ d) $\lfloor 6 \times \lfloor 4$

ANSWERS

PERMUTATIONS

1. (i) c (ii)a (iii) e (iv) b (v) d
2. (i) 12 (ii) 3 (iii) 8
3. (i) 11 (ii) 5 (iii) 12
4. (i) 5 (ii)3
5. (i) 5 (ii) 15 (iii) 7, 3
6. (i) 120 (ii) 24 (iii) 24 (iv) 7
7. (i) ${}^{5}p_3 = 60$ ii) ${}^{5}p_2 = 20$ (iii) 4
   (iv) ${}^{5}p_4 = 120$ v) 600 vi) 64
8. (i) $\lfloor 6 \times {}^{7}p_4$ ii) $\lfloor 5 \quad \lfloor 6$ iii) $\lfloor 6 \quad \lfloor 7$
   iv) $\lfloor 7 \times {}^{8}p_6$ v) $\lfloor 5 \times {}^{6}p_5$ vi) $\lfloor 6 \times \lfloor 5$ vii) $2 \times \lfloor 5 \times \lfloor 5$

11. (i) 720 (ii) 1440 (iii) $\lfloor 3 \times \lfloor 4 = 144$
12. (i) 666600 (ii) 93324
13. (i) 77328
14. (i) F (ii) D (iii) B (iv) E v) A
   (vi) G (vii) C
15. (i) ${}^{5}p_4$ (a) 72 ii) 48 c)36 d) 24 e) 6
   (ii) 36 (iii) 220
   iv) a) ${}^{7}p_4$ b) 560 c) 360 d) 40 d) 200
16. (1) $6^4 = 1296$ (2) $6^5 = 3125$ (3) a) 6^4 b) ${}^{6}p_4$ c) $6^4 - {}^{6}p_4$
   (4) i) B ii) C iv) A
   (5) (i) C ii) A iii) B
   (6) ${}^{5}p_4$ (7) $\lfloor 7$ (8) (i) 6^4 (ii) $6^4 - {}^{6}p_4$
17. (i) D (ii) A (iii) B (iv) E (v) C
19. (i) $\dfrac{\lfloor 12}{\lfloor 2 \lfloor 2 \lfloor 3}$ (ii) $\dfrac{\lfloor 1}{\lfloor 2}$ (iii) $\dfrac{\lfloor 12}{\lfloor 4 \lfloor 3 \lfloor 2}$ (iv) $\dfrac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2}$

(v) $\frac{\lfloor 7}{\lfloor 2 \lfloor 2 \lfloor 2}$ (vi) $\frac{\lfloor 11}{\lfloor 2 \lfloor 2 \lfloor 2}$ (vii) $\frac{\lfloor 11}{\lfloor 4 \lfloor 4 \lfloor 2}$

20. (i) 28 (ii) 68

21A. (4) 60 (5) 720 (6) $\lfloor 9$ (7) $\frac{\lfloor 6}{\lfloor 2}$

(8) 360 (9) 60 (10) 2520

24. (1) $\lfloor 4 \times \lfloor 3 = 144$ (2) $\lfloor 6 \times {}^7p_4$ (3) $\lfloor 6 \times \lfloor 2 = 240$

(4) (i) $\lfloor 7 \times \lfloor 5$ (ii) $\lfloor 8 \times {}^9p_3$ (iii) $\lfloor 8 \times \lfloor 5 \times \lfloor 4 \times \lfloor 3$

(5) $\lfloor 6 \times {}^7p_4$ (6) $\lfloor 6 \times {}^7p_4$ (7) (a) $\lfloor 7 \; \lfloor 15$ (b) $\lfloor 18 \cdot {}^{19}p_3$

(c) $\lfloor 2 \; \lfloor 15 \; \lfloor 4 \times \lfloor 3$

(8) $\lfloor 3 \cdot \lfloor 13 \cdot \lfloor 3 \cdot \lfloor 3 \cdot \lfloor 2$ (9) $\lfloor 4 \cdot \lfloor 5$ (10) $\lfloor 10 \cdot \lfloor 8$

26. (1) $\lfloor 3 \times \frac{{}^4p_4}{\lfloor 3} = 24$ (2) $\lfloor 5 = 120$

(3) (i) 2520 (ii) 7560 (4) 18 (5) 30, 12 (6) 48

(7) $\frac{\lfloor 7}{\lfloor 3 \lfloor 2 \lfloor 2}$ (8) $\frac{\lfloor 12}{\lfloor 4 \lfloor 3 \lfloor 5}$ (9) $\frac{\lfloor 12}{\lfloor 4 \lfloor 4 \lfloor 4}$

COMBINATIONS (ANSWERS)

1. (i)7 (ii) 17
2. (1) 4 or 8 (2) 9 (3) 8
3. (i) E (ii) D (iii) A (iv) 1225 v) 1140,231 vi) 0
4. (i) 11 ii) 1225 iii) 78 (iv) 1225 v) 1140,231 vi) 0

7. 1) (1) ${}^{39}C_5$ (2) ${}^{30}C_4$ (3) 792 (4) 4 or 6 (5) 31 or 44 (6) 0

8A. (1) 63 (2) ${}^5C_3 \times {}^7C_2 = 30$ (3) ${}^7C_3 \times {}^6C_2 = 700$

(4) ${}^8C_4 \times {}^5C_3 = 700$ (5) ${}^7C_4 \times {}^6C_3 \times {}^5C_2 = 7000$

8B. (1) i) 63 ii) ${}^5C_3$ iii) ${}^7C_3 \times {}^6C_2 = 700$

(2) (i) ${}^{12}C_4$ ii) $2^{12} - \left({}^{12}C_0 + {}^{12}C_1 + {}^{12}C_2\right) = 4017$

(iii) ${}^{12}C_0 + {}^{12}C_4 + {}^{12}C_2 + {}^{12}C_3 = 299$

(3) (i) ${}^{10}C_4$ ii) ${}^6C_4$ iii) ${}^{10}C_4 - {}^6C_4 = 195$

9. (i) 12 ii) 48 iii) 168 iv) 46
10. (1) i)D ii) E iii) B iv) C v) A

(2) 12 (3) 25

15. Multiple choice question Anaswers.

1. c	2.b	3. a	4. c	5. d
6. c	7. i-b; ii-a; iii-b	8.c	9.d	10.b
11.d	12.c	13. b	14.d	15.b
16.a	17.c	18.a	19.c	20.a
21.d	22.b	23.d	24.c	25.b
26.c	27.d	28.a	29.d	30.b
31.c	32.d	33.b	34.b	35.a
36.c	37.c	38.a	39.c	40.a
41.b	42.c	43.d	44.c	45.d
46.c	47.a	48.d	49.c	50.a

BINOMIAL THEOREM

Level – 1

1. An algebraic expression consisting of two terms with +ve or –ve sign between themis called ____________
2. An example of binomial expression is _____
3. The number of terms in the expression of $(x+y)^n$, where $n \in N$ is ___
4. The number of terms in the expansion of $(x+y+z)^n$ where $n \in N$ is _______
5. The number of terms in the expansion of $(1+x)^n$, where 'n' is a negative integer is ____
6. The general term in the expansion of $(x+a)^n (n \in N)$ is ______
7. The general term in the expansion of $(x-a)^n (n \in N)$ is _____
8. If $n \in N$, then

   i) The coefficient of x^r in $(Hx)^n$ is ______

   ii) The coefficient of x^6 in $(1+2x)^n$ is _____

   iii) The coefficient of x^r in $(1-x)^n$ is ____

   iv) The coefficient of x^7 in $(1-2x)^n$ is _____

v) The coefficient of x^4 in $(3+4x)^6$ is ____

vi) The coefficient of x^3 in $(4-5x)^{11}$ is _____

9. The general term in the expansion of $(3x-5y)^n$ is ________

10. The largest Binomial coefficient in the expansion of $(1+x)^{17}$ is _____

a) ${}^{17}C_8$ b) ${}^{17}C_2$ c) ${}^{17}C_{10}$ d) 1

11. The largest binomial coefficient in the expansion of $(1+x)^{10}$ is______

a) ${}^{10}C_5$ b) ${}^{10}C_4$ c) ${}^{10}C_9$ d) none

12. The middle term in the expansion of $\left(x-\frac{1}{x}\right)^{18}$ is ______

a) ${}^{18}C_9$ b) $-{}^{18}C_9$ c) ${}^{18}C_{10}$ d) $-{}^{18}C_{10}$

13. The middle term in the expansion $\left(\frac{10}{x}+\frac{x}{10}\right)^{10}$ is ______

a) ${}^{8}C_5$ b) ${}^{10}C_5$ c) ${}^{9}C_5$ d) ${}^{7}C_5$

14. When three are two middle terms in the expansion then their binomial coefficients are ____________

15. Which of the binomial coefficients are __________

a) Binomial coefficient of the middle term is the greatest binomial coefficient

b) Binomial coefficient of the middle term is less than the greatest binomial coefficient.

c) Binomial coefficient of the middle term is greater than the greatest binomial coefficient

d) None of these

16. The middle term in the expansion of $(x+y)^n$ depends upon the value of _______

a) x b) y c) n d) none of these

17. In the expansion of $(x+a)^n$ $(r+1)^{th}$ term from the end is equal to ______ term from the beginning.

18. The term independent of x in the expansion of $\left(x^2-\frac{1}{x}\right)^9$ is

a) 1 b) –1 c) –48 d) 84

19. The term independent of x in the expansion of $\left(\sqrt{\frac{x}{3}}+\frac{3}{2x^2}\right)^{10}$ will be

a) $\frac{3}{2}$ b) $\frac{5}{4}$ c) $\frac{5}{2}$ d) none of these

20. In $\left(4x^3+\frac{7}{x^2}\right)^{14}$, the term independent of x is _______

a) 28 b) 40 c) does not exist d) 0

21. If 'n' is a +ve integer, then

i) $C_0+C_1+C_2+....+C_n=$_____

ii) $C_0+C_2+C_4+....+C_n=$___________, if n is even

iii) If n is odd, $C_0+C_2+C_4+....+C_{n-1}=$______

iv) If n is even, $C_1+C_3+C_5+....+C_{n-1}=$______

v) If n is odd, $C_1+C_3+C_5+....+C_n=$_____

22. If $x=1/3$, the greatest term in the expansion of $(1+4x)^8$ is _____

23. If n and r are +ve integers and the coefficients of x^r and x^{r+1} are ${}^nC_r$ and ${}^nC_{r+1}$, then

i) $\frac{{}^nC_r}{{}^nC_{r-1}}=$________

ii) $\frac{{}^nC_{r+1}}{{}^nC_r}=$ ______

iii) $\frac{{}^{n+1}C_{r+1}}{{}^{n+1}C_r}=$______

24. The set of values of x for which the binomial expansion of $(2+5x)^{-1/2}$ is valid is ____

25. 6th term in the expansion of $\left(3+\frac{2x}{3}\right)^{\frac{3}{2}}$ is _____

26. 8th term in the expansion of $\left(1-\frac{5x}{2}\right)^{\frac{-3}{5}}$ is ______

27. The general term in the expansion of $(1-4x)^{-3}$ is _________

28. If x^2 and higher power of 'x' can be neglected then $(1+x)^n$ is approximately equal to _______

29. If x^3 and higher power of 'x' can be neglected then $(1+x)^n$ is approximately equal to _______

30. If x^4 and higher power of 'x' can be neglected then $(1+x)^n$ is approximately equal to ______

31. $(1+x)^{-1} =$ __________

32. $(1+x+x^2+.....+x^k+......) =$ _________

33. $(1-x)^{-2} =$

34. $1-2x+3x^1--4x^3......+(-1)^k(k+1)x^k+...... =$

Key (Level – 1)

1.a binomial expression 2. 2x+3y 3. n+1

4. ${}^{n+2}C_2$ or $\frac{(n+2)(n+1)}{2}$ 5. c 6. ${}^nC_r x^{n-r} a^r$

7. $(-1)^r x^{n-r} a^r$

8. i) ${}^nC_r$ ii) ${}^nC_6 2^6$ iii) $(-1)^r \cdot {}^nC_r$ iv) $-{}^nC_7 2^7$ v) ${}^6C_4 2^2 \cdot 4^4$ vi) $-{}^{11}C_3 4^8 5^3$

9. $(-1)^r\ {}^nC_3(3x)^{n-r}(5y)^r$ 10. a 11. a

12. b 13. B 14. 15. a

16.c 17. $(n-r+1)^{th}$ 18. d 19. b

20. c 21. i) 2^n ii) 2^{n-1} iii) 2^{n-1} iv) 2^{n-1} v) 2^{n-1} v) 2^{n-1}

22. 23. i) $\frac{n-r+1}{r}$ ii) $\frac{n-r}{r+1}$ iii) $\frac{n-r+1}{r+1}$ 24. $\left[\frac{-2}{5}, \frac{2}{5}\right]$

25. $\frac{-9\sqrt{3}}{8}\left(\frac{x}{9}\right)^5$ 26. $\frac{(3)(8)(13).....(33)}{7!}\left(\frac{x}{2}\right)^7$ 27. ${}^{r+2}C_r(4x)^r$

28. $1+nx$ 29. $1+nx+\frac{n(n-1)}{2!}x^2$ 30. $\frac{1+nx+n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3$

31. $1-x+x^2-x^3....+(-1)^k x^k$ 32. $(1-x)^{-1}$

33. $1+2x+3x^2+.....+(k+1)x^k+.....$ 34. $(1+x)^{-2}$

Level – 2

1. If the number of terms in the expansion of $(x+y+z)^n$ is 21, then the value of 'n' is ____

2. The number of terms in the expansion of $\left(x_1+x_2+x_3+x_4\right)^n$, where $n \in N$ is ____

3 The number of terms in the expansion of $\left(x_1+x_2+x_3+......+x_r\right)^n$, where $n \in N$ is___

4. The total number of terms in the expansion of $\left(x+a\right)^{100}+\left(x-a\right)^{100}$ after simplication will be

a) 202 b) 51 c) 50 d) none

5. The number of non-zero terms in the expansion of $\left(1+3\sqrt{2}x\right)^9+\left(1-3\sqrt{2}x\right)^9$ is __

a) 9 b) 0 c) 5 d) 10

6. The total number of terms in the expansion of $(1+x)^{2n}-(1-x)^{2n}$ after simplication is ___

a) n+1 b) n –1 c) n d) 4n

7. The number of irrational terms in the binomial expansion of $\left(3^{1/5}+7^{1/3}\right)^{100}$ (or) $\left(\sqrt[5]{3}+\sqrt[3]{7}\right)^{100}$ is ____

a) 94 b) 88 c) 93 d) 95

8. The number of integral terms in the expansion of $\left(5^{1/2}+7^{1/6}\right)^{642}$ is ____

a) 107 b) 108 c) 321 d) none

9. The number of irrational terms in the expansion of $\left(5^{1/6}+2^{1/8}\right)^{100}$ is ___

a) 95 b) 99 c) 97 d) 100

10. If 'n' is a +ve integers, then $\left(\sqrt{3}+1\right)^{2n}-\left(\sqrt{3}-1\right)^{2n}$ is

a) an irrational number
b) an odd +ve integer
c) an even +ve integer
d) a rational number other than +ve integer

11. The value of $\left(\sqrt{5}+1\right)^5-\left(\sqrt{5}-1\right)^5$ is _____

a) 252 b) 352 c) 452 d) 552

12. The sum of the rational terms in the binomial expansion of $\left(2^{1/2}+3^{1/5}\right)^{10}$ is ____

13. The sum of terms with non-zero coefficients in $(4x-74)^{1/9}+(4x+74)^{1/9}$ is ____

14. The sum of the last 20 coefficients in the expansion of $(1+x)^{39}$ is

a) 2^{40} b) 2^{39} c) 2^{38} d) none

15. The sum of the coefficients in the expansion of $(5x-4y)^n$ where n is a +ve integer is ______

a) 0 b) n c) 1 d) –1

16. The sum of all coefficients in the expansion of $(x^2+x-3)^{2021}$ is ___

a) 0 b) 2021 c) 1 d) –1

17. The number of integral terms in the expansion of $(\sqrt{3}+\sqrt[8]{5})^{256}$ is ____

a) 32 b) 33 c) 34 d) 35

18. In the expansion of $(\sqrt[5]{3}+\sqrt[3]{2})^{15}$

a) Number of irrational terms is 3

b) Sum of all irrational terms is 58

c)Sum of all rational terms is greater than the sum of all irrational terms

d) Sum of all irrational terms is greater than the sum of all rational terms

19. The coefficient of x^{10} in the expansion of $(1+x^2-x^3)^8$ is _____

a) 456 b) 476 c) 412 d) 342

20. The coefficient of x^{11} in the expansion of $(1+3x+2x^2)^6$ is ___

a) 144 b) 576 c) 288 d) 216

21. The coefficient of x^4 in the expansion of $(1+x+x^2+x^3)^n$ is ___

a) ${}^nC_4$ b) ${}^nC_4+{}^nC_2$

c) ${}^nC_4+{}^nC_2+{}^nC_4\cdot{}^nC_2$ d) ${}^nC_4+{}^nC_2+{}^nC_1\cdot{}^nC_2$

22. The coefficient of x^{53} in the expansion of $\sum_{m=0}^{100}{}^{100}C_m(x-3)^{100-m}\cdot 2^m$ is _______

a) ${}^{100}C_{47}$ b) ${}^{100}C_{53}$ c) ${}^{-100}C_{53}$ d) ${}^{-100}C_{100}$

23. The coefficient of 4th term in the expansion of $(a+b)^n$ is 56, then ‘n’ is ____

a) 12 b) 10 c) 8 d) 6

24. The coefficient of x^5 in the expansion of $(2-x+3x^2)^6$ is ____

a) –4692 b) 4692 c) 2346 d) –5052

25. If $|x|<1$, then the coefficient of x^n in the expansion of $(1+x+x^2+...)^2$ will be

a) 1 b) n c) n+1 d) none

26. If the coefficients of x^7 and x^8 in $\left(2+\frac{x}{3}\right)^n$ are equal, then n is _____

a) 56 b) 55 c) 45 d) 15

27. The coefficient of a^3b^4c in the expansion of $(1+a-b+c)^9$ is ____

28. The coefficient of x^2y^3 in the expansion of $(1-x+y)^{20}$ is_____

a) $\frac{20!}{2!3!}$ b) $-\frac{20!}{2!3!}$ c) $\frac{20!}{5!2!3!}$ d) $\frac{20!}{15!2!3!}$

29. The coefficient of x^{28} in the expansion of $\left(1+x^3-x^6\right)^{30}$ is _____

a) 1 b) 0 c) ${}^{30}C_6$ d) ${}^{30}C_3$

30. Total number of terms which are dependent on the value of 'x' in the expansion of $\left(x^2-2+\frac{1}{x^2}\right)^n$ is ____

a) $2n+1$ b) $2n$ c) n d) n+1

31. The constant term in the expansion of $\left(\frac{x}{2}+\frac{1}{x}+\sqrt{2}\right)^5$ is $\frac{a\sqrt{2}}{2}$, then a =

a) 67 b) 69 c) 63 d) 65

32. What is the constant term in the expansion of $(1+3x)^n\left(1+\frac{1}{3x}\right)^n$

a) ${}^{2n}C_n$ b) ${}^{2n}C_{n-1}$ c) ${}^{2n}C_{n+1}$ d) no constant term

33. The coefficient of x^5 in the expansion of $\left(1+x+x^2\right)^8$ is _____

a) 405 b) 508 c) 404 d) 504

34. In the expansion of $\left(a+1+\frac{1}{a}\right)^n$ where $n\in N$, there are 2029 terms, then n = _____

a) 1015 b) 1013 c) 1014 d) 1012

35. The coefficient of x^{50} in the expansion of $(1+x)^{101}\left(1-x+x^2\right)^{100}$ is ____

a) 1 b) –1 c) 0 d) 2

36. If the coefficients of x^9 and x^{10} in the binomial expansion of $\left(3+\frac{x}{2}\right)^n$ are equal, then n = ____

a) 69 b) 96 c) 66 d) 99

37. The middle term in the expansion of $\left(x-\frac{1}{x}\right)^{18}$ is ____

a) ${}^{18}C_9$ b) $-{}^{18}C_9$ c) ${}^{18}C_{10}$ d) $-{}^{18}C_{10}$

38. The coefficient of middle term in the expansion of $(1+x)^{10}$ is _____

a) $\frac{10!}{5!6!}$ b) $\frac{10!}{(5!)^2}$ c) $\frac{10!}{5!7!}$ d) none

39. Middle term in the expansion of $(1+3x+3x^2+x^3)^6$ is ______

a) 4^{th} b) 3^{rd} c) 10^{th} d) none

40. The middle term in the expansion of $\left(x^2+\frac{1}{x^2}+2\right)^n$ is ______

41. If the m^{th} term is the middle term in the expansion of $\left(x^2-\frac{1}{2x}\right)^{20}$, find the coefficient of T_{m+3} is

a) ${}^{20}C_{13}2^{-13}$ b) $-{}^{20}C_{13}2^{13}$ c) $-{}^{20}C_{13}2^{-13}$ d) ${}^{20}C_{13}2^{13}$

42. If the term independent of x in the expansion of $\left(\sqrt{x}-\frac{K}{x^2}\right)^{10}$ is 405, then K = ___

a) 3 only b) –3 only c) ± 3 d) 0

43. If $(1+x)^n = \sum C_r x^r$, then $C_1+2C_2+3C_3+....+nC_n = ...$

a) $(n-1)2^n$ b) $n\cdot 2^{n-1}$ c) $n\cdot 2^{n+1}$ d) none

44. Sum of $C_0+3C_1+3^2C_2+....+3^nC_n$ is _____

45. ${}^{10}C_1+{}^{10}C_3+{}^{10}C_5+{}^{10}C_7+{}^{10}C_9 =$ ____

46. ${}^{15}C_0+{}^{15}C_1+{}^{15}C_2+........+{}^{15}C_1 =$ ____

47. If $(1+x)^n = C_0+C_1x+C_2x^2+......+C_nx^n$, then $C_0-C_1+C_2-C_3+....(-1)^n C_n$ is equal to ___

a) 3^n b) 2^n c) 1 d) 0

48. If $(1+x)^n = C_0+C_1x+C_2x^2+....+C_nx^n$, then

i) $C_0^2+C_1^2+C_2^2+.....+C_n^2 =$ ____

ii) $C_0C_1+C_1C_2+...+C_{n-1}\cdot C_n =$

iii) $C_0Cr+C_1C_{r+1}+...+C_{n-r}\cdot C_n =$

49. If 'n' is a +ve integer, then $\sum_{r=1}^{n} r^2 C_r =$ ______ 2^{n-2}

a) $n(n-1)$ b) n c) $n(n+1)$ d) $n+1$

50. If 'n' is a +ve integer, then $\sum_{r=1}^{n} r \cdot C_r =$ ____

a) 2^{n-1} b) $n \cdot 2^{n-1}$ c) $n \cdot 2^{n+1}$ d) 2^{n+1}

51. If $\left(1+x+x^2\right)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n} \cdot x^{2n}$ then

i) $a_0 + a_1 + a_2 + ... + a_2n$ is _____

ii) $a_0 + a_2 + a_4 + ... + a_2n$ is _____

iii) $a_1 + a_3 + a_5 + ... + a_{2n-1}$ is ___

iv) $a_0 + a_3 + a_6 + a_9 + ...$ _____

52. The largest binomial coefficient in the expansion of $(1+x)^{2n+1}$ is ____

a) $\frac{(2n+1)!}{n!(n+1)!}$ b) $\frac{(2n+2)!}{n!(n+1)!}$ c) $\frac{(2n+1)!}{((n+1)!)^2}$ d) $\frac{(2n)!}{(n!)^2}$

53. If $x = \frac{1}{5} + \frac{1\times3}{5\times10} + \frac{1\times3\times5}{5\times10\times15} + ...$, then $3x^2 + 6x =$ ______

a) 1 b) 2 c) –1 d) –2

54. Choose the correct option regarding the following statements

i) $C_0 + C_2 + C_4 + ... + C_n = 2^{n-1}$ if n is even

ii) $C_1 + C_3 + C_5 + ... + C_{n-1} = 2^{n-1}$, it n is even

a) (i) is ture , (ii) is false

b) (i) is false, (ii) is true

c) (i) is false, (ii) is false

d) (i) is true (ii) is true

55. If $(1+x)^n = C_0 + C_1x + C_2x^2 + ... + C_nx^n$ then the value of $C_0 + 2C_1 + 3C_2 + ... + (n+1)C_n$ will be

a) $(n+2)2^{n-1}$ b) $(n+1)2^n$ c) $(n+1)2^{n-1}$ d) none

56. $(1+x)^n - 1$ is always divisible by ____ $\forall n \in N$

a) x b) x^2 c) $2x^2$ d) x^3

57. $(1+x)^n - nx - 1$ is divisible by _____ $\forall n \in N$

a) 2x b) x^2 c) $2x^3$ d) All the above

58. The 11th term in the expansion of $\left(x + \frac{1}{\sqrt{x}}\right)^{14}$ is ____

a) $\frac{999}{x}$ b) $\frac{1001}{x}$ c) 1 d) $\frac{x}{1001}$

59. If the sum of the coefficients in the expansion of $(x-2y+3z)^n$ is 128, then the opeatest coefficient in the expansion of $(1+x)^n$ is

a) 35 b) 20 c) 10 d) none of thse

60. $\sum_{k=1}^{n} K\left(1+\frac{1}{n}\right)^{k-1} =$

a) $n(n-1)$ b) $n(n+1)$ c) n^2 d) $(n+1)^2$

61. The coefficient of x^n, where n is any +ve integer, in the expansion of $(1+2x+3x^2+....+\infty)^{1/2}$ is ____

a) 1 b) $\frac{n+1}{2}$ c) $2n+1$ d) $n+1$

62. The middle term in the expansion of $(1+x)^{2n}$ is _____

a) $\frac{1\times3\times5\times.....\times(2n-1)}{n}\cdot x^n$

b) $\frac{1\times3\times5\times.....\times(2n-1)}{n}2^{n-1}\cdot x^n$

c) $\frac{1\times3\times5\times.....\times(2n-1)}{n!}x^n$

d) $\frac{1\times3\times5\times.....\times(2n-1)}{n!}2^n\cdot x^n$

Level -2 – Key

1. 5	2. ${}^{n+3}C_3$	3. ${}^{n+r-1}C_{r-1}$	4. b	5. c
6. c	7. a	8. b	9. c	10. a
11. b	12. d	13. 25	14. c	15. c
16. d	17. b	18. d	19. b	20. b
21. d	22. c	23. c	24.	25. c
26. b	27. $\frac{9!}{3!4!}$	28. d	29. b	30. b
31. c	32. a	33. d	34. c	35. c
36. a	37. b	38. b	39. c	40. ${}^{2n}C_n$
41. c	42. c	43. b	44. 4^n	45. 2^9
46. 2^{14}	47. d	48. i) ${}^{2n}C_n$ ii) ${}^{2n}C_{n+1}$ iii) ${}^{2n}C_{n+r}$		

49. c 50. B 51. i) 3^n ii) $\frac{3^n+1}{2}$ iii) $\frac{3^n-1}{2}$ iv) 3^{n-1}

52. a 53. b 54. d 55. a

56. a 57. b 58. b 59. a 60. c

61. a 62. D

Level – 3

1. The remainder when 2^{2021} is divided by 17 is ______
   a)15 b) 14 c)20 d)none

2. The remainder when 3^{2003} is divided by 28 is _____
   a) 18 b) 19 c) 28 d) none

3 The remainder when 5^{99} is divided by 13 is ____

4. The last two digits of the number $(23)^{14}$ are _____

5. The last three digits of the number $(27)^{27}$ are _____

6. The coefficient of x^{50} in the expansion of $(1+x)^{1000}+x(1+x)^{999}+x^2(1+x)^{998}+......+x^{1000}$ is ______
   a) ${}^{1000}C_{50}$ b) ${}^{999}C_{50}$ c) ${}^{1000}C_{51}$ d) ${}^{1001}C_{50}$

7. By neglecting x^4 and higher powers of x, find the approximate value of $\sqrt[3]{x^2+64}-\sqrt[3]{x^2+27}$ is ________
   a) $1-\frac{7}{234}x^2$ b) $1-\frac{7}{432}x^2$ c) $1-\frac{7}{32}x^2$ d) $1-\frac{7}{42}x^2$

8. The coefficient of x^n in the expansion of $\left(1-9x+20x^2\right)^{-1}$ is _____

9. The coefficient of x^{11} in the expansion of $\left(1+x^2\right)^4\left(1+x^3\right)^7(1+x)^{12}$ is _____
   a) 1051 b) 1106 c) 1113 d) 1120

10. The coefficient of x^9 in the expansion of $(1+x)\left(1+x^2\right)\left(1+x^3\right).....\left(1+x^{100}\right)$ is ____

11. The sum of binomial coefficients of terms containing power of x more than x^{20} in $(1+x)^{41}$ is divisible by_____
    a) 2^{39} b) 2^{41} c) 2^{42} d) none

12. The sum of binomial coefficients of positive real terms in the expansion of $(1+ix)^{42}$ (x>0) is ___________

a) 2^{40} b) 2^{41} c) 2^{38} d) 2^{39}

13. In the expansion of $\left(7^{1/3}+11^{1/9}\right)^{6561}$ which of the following is incorrect

a)There are exactly 730 rational terms

b)There are exactly 5831 irrational terms

c)The term which involves greatest binomial coefficients is irrational

d)The term which involves greatest binomial coefficients is irrational

14. The middle term in the expansion of $\left(\frac{x}{2}+2\right)^8$ is 1120, then $x \in R$ is equal to

a) ± 2 b) $\pm 2i$ c) ± 3 d) $\pm 3i$

15. **Statement 1:**The sum of coefficients in the expansion $\left(3^{-x/4}+5^{5x/4}\right)^n$ is 2^n

Statement 1: The sum of coefficients in the expansion of $(x+y)^n$ is 2^n when we put $x=1, y=1$ which of the collowing is true.

a) Both the statements are true and statement 2 is the correct explanation of statement 1

b)Both the statements are true but statement 2 is not the correct explanation of statement 1

c) Statement 1 is true and statement 2 is false

d) Statement 1 is False and Statement 2 is true

Key (Level – 3)

1. a	2. b	3. 8	4. 09	5.803
6. d	7. b	8. $5^{n+1}-4^{n+1}$	9.c	10.8
11. a	12. a	13. d	14. a	15. b

PARTIAL FRACTIONS

SYNOPSIS:

1. Polynomial: An expression of the form

$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_n$ is called a polynomial of degree 'n' in the variable 'x', where 'n' is a positive integer and $a_0, a_1, a_2, a_3, ...a_n$ are real numbers and $a_0 \neq 0$

2.Rational fraction: If $f(x)$ & $g(x)$ are two polynomial and $g(x)$ is a non–zero polynomial, then $\frac{f(x)}{g(x)}$ is called a rational fraction.

3.Proper fraction: If the degree of $f(x) <$ degree of $g(x)$, then the rational fraction $\frac{f(x)}{g(x)}$ is called proper fraction.

4. Improper fraction: If the degree of $f(x) \geq$ degree of $g(x)$, then the rational fraction $\frac{f(x)}{g(x)}$ is called improper fraction.

Example: 1. $\frac{5x+1}{x^2+x-2}$ is a proper fraction

2. $\frac{x^4}{x^3-3x+2}$ is an improper fraction

3. $\frac{x^3}{(2x-1)(x+2)(x-3)}$ is an improper fraction

5. Irreducible Polynomial: A polynomial $f(x)$ is said to be irreducible if it cannot be expressed as a product of two polynomials $g(x) \& h(x)$ such that the degree of each polynomial is less than degree of $f(x)$

6. Reducible Polynomial: A polynomial f(x) is said to be reducible polynomial if it can be expressed as a product of two polynomials such that the degree of each polynomial is less than the degree of $f(x)$

Note: If $a \neq 0$, $ax^2 + bx + c$ is irreducible iff $b^2 - 4ac < 0$

7. Division Algorithm for Polynomials: If $f(x)$ and $g(x)$ are polynomials with $g(x) \neq 0$, then there exists a unique polynomials $q(x)$ and $r(x)$ such that $f(x) = g(x) + r(x)$

Where either $r(x) = 0$ (0r) degree of $r(x)$ is less than degree of $g(x)$

Here $f(x)$ is called dividend, $g(x)$ is called divisor. $q(x)$ is called quotient and $r(x)$ is called remainder.

8.Remainder theorem:

If the polynomial $f(x)$ is divided with $(x-a)$, then the remainder is f(a)

9.Factor theorem: If $f(x)$ is a polynomial, and $f(a)=0$ then $(x-a)$ is the factor of $f(x)$

Q: Worked example: If the remainders of the polynomial f(x) when divided by $(x-1),(x-2)$ are 5, 7 then the remainder of f(x) when divided by $(x-1)(x-2)$ is

Sol: The Remainder of f(x) when divided by x–1 is 5 $\Rightarrow f(1)=5$

The Remainder of f(x) when divided by x–2 is 7 $\Rightarrow f(2)=7$

Now, since $(x-1)(x-2)$ is a second degree polynomial in 'x', the remainder of f(x) when divided by $(x-1)(x-2)$ must be of the form ax+b

$f(x)=q(x)\cdot g(x)+r(x)$

$\Rightarrow f(x)=qx\cdot(x-1)(x-2)+(ax+b)$

Put x = 1 $\Rightarrow f(1)=a+b$

$\Rightarrow 5=a+b$ ——(1)

Put x = 2 $\Rightarrow f(2)=2a+b$

$\Rightarrow 7=2a+b$ ——(2)

Solving (1) & (2) a = 2, b = 3

$\therefore$ Remainder = 2x+3

9.Partial Fraction: If a proper fraction is expressed as the sum of two (or) more proper fraction, where in the denominatiors are the powers of irreducible polynomials then each proper fraction in the sum is called a partial fraction of given function.

Resolving Partial Fractions:

Let $\frac{f(x)}{g(x)}$ be a proper fraction

Case(i): Partial fraction of $\frac{f(x)}{g(x)}$ when $g(x)$ contain non–repeated linear factors.

Working rule (i): To each non-repeated linear factor $(ax+b)$ of g(x) there will be a partial fraction of the form $\frac{A}{ax+b}$, where A is a non-zero real number to be determined.

Ex: If $\frac{5x+1}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$ then A = _____

Sol: $\frac{5x+1}{(x+2)(x-1)}=\frac{A}{x+2}+\frac{B}{x-1}$

$\Rightarrow \frac{5x+1}{(x+2)(x-1)} = \frac{A(x-1)+B(x+2)}{(x+2)(x-1)}$

$\Rightarrow 5x+1 = A(x-1)+B(x+2)$

Put x = –2 $\Rightarrow -9 = -3A \therefore A = 3$

Case(ii): Partial fraction of $\frac{f(x)}{g(x)}$ where $g(x)$ contains repeated and non-repeated linear factors

Working rule (2): For each $(ax+b)^n, a \neq 0$ where 'n' is a +ve integer of $g(x)$, there will be a partial fraction of the form $\frac{Ax}{ax+b} + \frac{Az}{(ax+b)^2} + ... + \frac{A_n}{(ax+b)^n}$ where $A_0, A_1, A_2, ...A_n$ are the constants to be determined and for n = 1 Rule 1 can be applied.

Ex: If $\frac{x^2+13x+15}{(2x+3)(x+3)^2} = \frac{A}{2x+3} + \frac{B}{(x+3)} + \frac{C}{(x+3)^2}$ then A+B+C=_____

Sol: $\frac{x^2+13x+15}{(2x+3)(x+3)^2} = \frac{A(x+3)^2 + B(2x+3)(x+3) + C(2x+3)}{(2x+3)(x+3)^2}$

$\Rightarrow x^2+13x+15 = A(x^2+6x+9) + B(2x^2+6x+3x+9) + C(2x+3)$

$x^2+13x+15 = A(x^2+6x+9) + B(2x^2+9x+9) + C(2x+3)$

Comp x^2 coefficients, 1 = A + 2B ——(1)

Comp x coefficients, 13 = 6A+9B+2C ———(2)

Comp constant 15=9A+9B+3C

Solving (2) & (3)

(2) $\times 3 \Rightarrow 39 = 18A + 27B + 6C$

(3) $\times 2 \Rightarrow 30 = 18A + 18B + 6C$

$\therefore$ 9 = 9B

$\therefore$ B = 1

(1) $\Rightarrow$ A=–1

(2) $\Rightarrow$ –6+9+2C=13

3+2C= 13

2C=10 $\Rightarrow$ C= 5

$\therefore A+B+C = -1+1+5 = 5$

Case(iii): Partial fraction of $\frac{f(x)}{g(x)}$ when $g(x)$ contains only repeated linear factors

Working rule (iii): $\frac{f(x)}{(ax+b)^n} = \frac{A_1}{ax+b} + \frac{B}{(ax+b)^2} + ... + \frac{A_n}{(ax+b)^n}$ where $A_1, A_2, A_3, ...A_n$ are to be determined

Ex: If $\frac{2x+3}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$ then C = _____

Sol: $\frac{2x+3}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}$

$$\frac{2x+3}{(x-1)^2}=\frac{A(x-1)^2+B(x-1)+C}{(x-1)^3}$$

$$2x+3=A(x-1)^2+B(x-1)+C$$

Put x = 1 $\Rightarrow C=5 \quad \therefore C=5$

Case(iv): Partial fraction of $\frac{f(x)}{g(x)}$ when $g(x)$ contain non-repeated irreducible factors

Working rule (iv): To each non repeated quadratic factor ax^2+bx+c of $g(x)$ there will be partial fraction of the form $\frac{Ax+B}{ax^2+bx+c}$, where A,B are real numbers to be determined.

Ex: If $\frac{2x^2+1}{x^3-1}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$ then A+B = _____

Sol: $\frac{2x^2+1}{x^3-1}=\frac{A(x^2+x+1)+Bx^2-Bx+Cx-C}{x^3-1}$

$\Rightarrow 2x^2+1=A(x^2+x+1)+Bx^2-Bx+Cx-C$

Companing x^2 coeff, 2=A+B $\quad\quad \therefore A+B=2$

Case(v): Partial fraction of $\frac{f(x)}{g(x)}$ when $g(x)$ contain repeated irreducible factors

Working rule (v): If $n(>1)\in N$ is the largest exponent so that $(ax^2+bx+c)^n, a\neq 0$ is a factor of $g(x)$. Then corresponding to each such factor, there will be partial fraction of the form $\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+.....+\frac{A_{nx}+B_n}{(ax^2+bx+c)^n}$

Where A_i, B_i are to be determined

Ex: If $\frac{x^2+1}{(x^2+x+1)^2}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{(x^2+x+1)^2}$ then $A+B+C+D=$_____

Sol: $\frac{x^2+1}{(x^2+x+1)^2}=\frac{(Ax+B)(x^2+x+1)+(Cx+D)}{(x^2+x+1)^2}$

$x^2+1=Ax^3+Ax^2+Ax+Bx+B+Cx+D$

Comparing coefficients of x^3, A = 0 ———(1)

Comparing coefficients of x^2, A+ B = 1 ———(2)

Comparing coefficients of x, A+B = 1 ———(3)

Comparing constant term, B+D=1 ——(4)

From (1), (2), (3), (4), A=0, B=1, C=–1, D=0

$\therefore A+B+C+D=0+1+(-1)+0 = 0$

Case(vi): Partial fraction of $\frac{f(x)}{g(x)}$ when $g(x)$ is an improper fraction:

If $\frac{f(x)}{g(x)}$ when $g(x)$ is an improper fraction , two cases will arise

Case (α): degree of f(x) = degree of g(x)

Then by division algorithm there exists a unique constant K and the polynomial r(x) such that $f(x)=K\cdot g(x)+r(x)$

Where either $r(x)=0$ (or) the degree of $r(x)$ < degree of g(x) and the constant K is the quotient of the coefficients of highest degree terms of $f(x)\,\&\,g(x)$

So, $\frac{f(x)}{g(x)}$ can be expressed as

$$\frac{f(x)}{g(x)}=K+\frac{r(x)}{g(x)}$$

$\frac{r(x)}{g(x)}$ is a proper fraction and it can be resolved into partial fraction

Working rule (vi):

Ex: $\frac{x^3}{(2x-1)(x+2)(x-3)}=K+\frac{A}{2x-1}+\frac{B}{x+2}+\frac{C}{x-3}$ then K = _______

Sol: $\frac{x^3}{(2x-1)(x+2)(x-3)}$ is an improper fraction with degree of f(x) = degree of g(x)

So K = quotient of coefficient of highest degree term of f(x) & g(x) = $\frac{1}{2}$

Case (6): If $\frac{f(x)}{g(x)}$ is an improper fraction with degree of f(x) > degree of g(x) then using division algorithm $\frac{f(x)}{g(x)} = q(x) + \frac{r(x)}{g(x)}$, where $q(x)$ is a non zero polynomial and $\frac{r(x)}{g(x)}$ is a proper fraction. Further $\frac{r(x)}{g(x)}$ can be resolved into partial fraction.

Ex: If $\frac{x^4}{(x-1)(x-2)} = q(x) + \frac{-1}{x-1} + \frac{16}{x-2}$ then $q(x)$ = _______

Sol: $\frac{x^4}{(x-1)(x-2)}$ is an improper fraction. By dividing x^4 with $(x-1)(x-2)$,

we get quotient $q(x)$

$$\frac{x^4}{(x-1)(x-2)} = \frac{x^4}{x^2-3x+2} = \left(x^2+3x+7\right) + \frac{15x-14}{x^2-3x+2}$$

$$= \left(x^2+3x+7\right) + \frac{15x-14}{(x-1)(x-2)}$$

$$= \left(x^2+3x+7\right) + \frac{A}{x-1} + \frac{B}{x-2}$$

$$= \left(x^2+3x+7\right) + \frac{-1}{x-1} + \frac{16}{x-2} \text{ (By resolving } \frac{15x-14}{(x-1)(x-2)}\text{)}$$

$\therefore g(x) = x^2+3x+7$

Case(vii): Conversion of $\frac{f(x)}{g(x)}$ in power series of x where $|x|<1$,

$\frac{1}{1-x}, \frac{1}{(1-x)^2}, \frac{1}{1+x}, \frac{1}{(1+x)^2}$ have the power series expansion they are given below.

$$\frac{1}{1-x} = 1 + x + x^2 + ... + x^2 + ...$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + ... + (-1)^2 x^2 + ...$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + ... + (k+1)x^{k+1} + ...$$

$$\frac{1}{(1+x)^2} = 1 - 2x + 3x^2 - 4x^3 + ... + (-1)^K x^{K+1} + ...$$

Ex: Coefficient of x^4 is $\frac{3x}{(x-2)(x+1)}$ is _____

Sol: Byresolving the given partial fraction's we get

$$\frac{3x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$$

$$\Rightarrow \frac{3x}{(x-2)(x+1)}=\frac{A(x+1)+B(x-2)}{(x-2)(x+1)}$$

$$\Rightarrow 3x=A(x+1)+B(x-2)$$

Put x = –1, $\Rightarrow -3=-3B \quad \Rightarrow B=1$

Put x = 2 $\Rightarrow 6=3A$

$$\therefore \frac{3x}{(x-2)(x+1)}=\frac{A}{x-2}+\frac{1}{x+1} \quad \Rightarrow A=2$$

$$\therefore \frac{3x}{(x-1)(x+1)}=\frac{A}{x-2}+\frac{B}{x+1}$$

$$=\frac{2}{x-2}+\frac{1}{x+1}$$

$$=\frac{2}{-2\left(1-\frac{x}{2}\right)}+\frac{1}{1+x}$$

$$=-\left(1-\frac{x}{2}\right)^{-1}+(1+x)^{-1}$$

$$=-\left[1+\frac{x}{1}+\left(\frac{x}{2}\right)^2+\left(\frac{x}{2}\right)^3+....+\left(\frac{x}{2}\right)^n+...\right]+\left[1-x+x^2-x^3+x^4+......\right]$$

$\therefore$ coefficient of $x^4=\frac{-1}{2^4}+1$

$$=\frac{-1}{16}+1=\frac{15}{16}$$

Level – I

1. If $\frac{2x+3}{(x+1)(x-3)}=\frac{k}{4(x-3)}-\frac{1}{4(x+1)}$ then k =

a) 9 b) 4 c) 5 d) 2

2. $\frac{13x+43}{2x^2+17x+30}=\frac{A}{2x+5}+\frac{B}{x+6}$ then (A, B)

a) (1,3) b) 4 c) 5 d) 2

3. $\frac{5x+6x+1}{(2+x)(1-x)}=\frac{A}{2+x}+\frac{B}{x-2}$ then A + B =

a) $\frac{4}{3}$ b) $\frac{7}{3}$ c) $\frac{5}{6}$ d) $\frac{7}{8}$

4. If $\frac{x+4}{(x^2-4)(x+1)}=\frac{A}{x-2}+\frac{B}{x+2}+\frac{C}{x+1}$ then B = ____

a) $\frac{1}{5}$ b) $\frac{1}{2}$ c) $\frac{1}{6}$ d) 3

5. $\frac{2x^2+2x+1}{x^3+x^2}=$

a) $\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x+1}$ b) $\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x+1}$

c) $\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x+1}$ d) $\frac{1}{x}-\frac{1}{x^2}-\frac{1}{x+1}$

6. If $\frac{x^2-x+1}{(x+)(x-1)^2}=\frac{A}{x+1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ then C =

a) $\frac{1}{2}$ b) $\frac{1}{4}$ c) $\frac{3}{4}$ d) $\frac{4}{5}$

7. If $\frac{1}{(1-2x)^2(1-3x)}=\frac{1}{x-3}+\frac{11}{(x-3)^2}+\frac{31}{(x-3)^3}$ then A+B+C = ____

a) 9 b) 10 c) 1 d) 0

8. If $\frac{x^2+5x+k}{(x-3)^2}=\frac{1}{x-3}+\frac{11}{(x-3)^2}+\frac{31}{(x-3)^3}$ then K = ____

a) 3 b) 5 c) 7 d) 2

9. If $\frac{x^3}{(x-1)(x+3)}=(x-1)+\frac{A}{x-1}+\frac{B}{x+2}$ then A+B = ___

a) 3 b) 1 c) 2 d) 5

10. If $\frac{1}{x^3(x+a)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x+a}$ then A = ____

a) $\frac{1}{a^2}$ b) $\frac{1}{a^3}$ c) $-\frac{1}{a}$ d $-\frac{1}{a^2}$

11. Number of partial fractional obtained from $\frac{2x+3}{(x-1)^3}=$ ____

a) 3 b_ 2 c) 1 d) 0

12. If $\frac{K}{(x-1)(x+2)^2}=\frac{1}{x-1}-\frac{1}{x+2}-\frac{3}{(x+2)^2}$ then k = ___

a) 3 b) 9 c) 5 d) 8

13. If $\frac{1}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$ then A+B+C =______

a) 0 b) 3 c) 1 d)2

14. The remainder obtained when the polynomial $\left(x^5-5x^4+9x^3-9x^2+5x-1\right)$ is dividedby (x–2) is

a) 3 b) –3 c) 2 d) –2

15. If $\frac{x^3+6x^2+5x}{x^4}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}$ then $A+C-B=$ _____

a) 3 b) 2 c) 0 d) 1

16. Coefficient of x^3 in power series expansion of $\frac{5x+6}{(x+2)(1-x)}$ is

a) $\frac{80}{930}$ b) $\frac{77}{324}$ c) $\frac{23}{67}$ d) $\frac{7}{5}$

17. Degree of the polynomial $f(x)=x^3+3x^2+2x+1=0$ is

a) 2 b) 3 c) 1 d) 0

18. $\frac{x^2+1}{(x^2+4)(x-2)}$ can be resolved as

a) $\frac{Ax+B}{x^2+4}+\frac{C}{x-2}$ b) $\frac{A}{x^2+4}+\frac{Bx+C}{x-2}$

c) $\frac{A}{x^2+4}+\frac{B}{x-2}$ d) $\frac{Ax}{x^2+4}+\frac{C}{x-2}$

19. $\frac{x^4}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}+\left(x^2-x+3\right)$ then $A+B=$ _____

a) 3 b) –5 c) 5 d) –3

20. Number of partial fraction in $\frac{x^2+1}{x^4+x^2+1}=$_____

a) 3 b) –5 c) 5 d) –3

21. If $\frac{x^2+x+1}{x^2+2x+1}=$

a) $\frac{A}{(x+1)}+\frac{B}{(x+1)^2}$ b) $\frac{A}{(x+1)^2}$

c) $A+\frac{B}{x+1}+\frac{C}{(x+1)^2}$ d) $A+\frac{B}{(x+1)^2}$

22. The number of partial fraction in $\frac{3x^2-5x+7}{(x-1)^2}=$___

a) 3 b) 2 c) 4 d) 1

23. The remainder of $(3x^4-x^3+2x^2-2x-4)$ when divided by $(x+2)=$ ____

a) 83 b) 71 c) 64 d)55

24. $\frac{px+q}{(1-x+x^2)(x+2)}=\frac{x}{1-x+x^2}-\frac{1}{x+2}$ then (p, q) =

a) (1, 4) b) (3, –1) c) (2, 3) d) (4, 1)

25. $\frac{x^3+x^2+1}{(x-1)(x^2-1)}=$

a) $\frac{A}{x-1}+\frac{Bx+C}{x^3-1}$ b) $\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{Cx+D}{x^2+x+1}$

c) $\frac{Ax+B}{(x-1)^2}+\frac{D}{x^2+x+1}$ d) $\frac{A}{x-1}+\frac{B}{x+1}+\frac{Cx+D}{x^2-x+1}$

KEY

1. a	2. c	3. b	4. b	5. c
6. a	7. c	8. c	9. a	10. b
11. b	12. c	13. c	14. b	15. c
16. b	17. b	18. a	19. b	20. b
21. c	22. a	23. c	24. b	25. b

Level – 2

1. Number of partial fraction obtained from $\frac{x^2-2x+6}{(x-2)^3}=$

a) 2 b) 3 c) 5 d) 1

2. If $\frac{x-1}{(x+1)(x-2)^2}=\frac{A}{x+3}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$ then A + B =____

a) 2 b) 5 c) 3 d) 0

3. $\frac{Px^2+Q}{(x+2)(x^2+1)}=\frac{1}{5(x+2)}+\frac{1}{5}\left(\frac{4x-2}{x^2+1}\right)$ then $(P, Q)=$___

a) $(1,-3)$ b) $(1,3)$ c) $(-1,3)$ d) $(-1,-2)$

4. If $\frac{1}{x^2-81}=\frac{1}{K}\left[\frac{1}{x-9}+\frac{1}{x+9}\right]$ then K = _____

a) 81 b) 18 c) 9 d) 27

5. $\frac{1}{x^3+1}=$ _____

a) $\frac{Ax+B}{x+1}+\frac{C}{x^2+x+1}$ b) $\frac{A}{x+1}+\frac{Bx+C}{x^2+x+1}$

c) $\frac{A}{x+1}+\frac{Bx+C}{x^2-x+1}$ d) $\frac{Ax+B}{x^3+1}$

6. $\frac{3x-2}{(x-1)(x+2)(x-3)}=\frac{A}{x-1}+\frac{B}{x+2}+\frac{C}{x-3}$ then

a) $\frac{8}{15}$ b) $\frac{-8}{15}$ c) $\frac{7}{10}$ d) $\frac{-9}{5}$

7. If $\frac{1}{(x-a)(x-b)(x-c)}=\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$ then A = ___

a) $\frac{1}{(b-a)(b-c)}$ b) $\frac{1}{(a-b)(a-c)}$

c) $\frac{1}{(c-a)(c-b)}$ d) $\frac{1}{(a-b)(b-c)(c-a)}$

8. Number of partial fraction obtained from $\frac{x^3-2x^2+5}{(x-1)^4}=$ _____

a) 3 b) 2 c) 4 d) 1

9. $\frac{x^3}{(2x-1)(x-1)}=K+\frac{A}{2x-1}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$ then K = ____

a) 3 b) 2 c) $\frac{1}{2}$ d) $\frac{1}{3}$

10. Coefficient of x^n in the power series expansion of $\frac{x-4}{x^2-5x+6}=$ ___

a) $\frac{1}{2^{n+1}}-\frac{1}{2^n}$ b) $\frac{1}{3^{n+1}}-\frac{1}{3^n}$ c) $\frac{1}{3^{n+1}}-\frac{1}{2^n}$ d) $\frac{1}{5^{n+1}}-\frac{1}{6^n}$

11. Coefficient of x^n in the power series expansion of $\frac{3x}{(x-1)(x-2)^2}$ is

a) $3-\frac{3}{2^{n+1}}+\frac{3(n+1)}{2^n}$ b) $3+\frac{1}{2^{n+1}}+\frac{3n}{2^n+1}$

c) $-3+\frac{3}{2^{n+1}}+\frac{3n}{2^{n+1}}$ c) $-3+\frac{3}{2^{n+1}}+\frac{3(n+1)}{2^{n+1}}$

12. $\frac{3x^3-2x^2-1}{x^4+x^2+1}=\frac{Ax+B}{x^2+x+1}+\frac{Cx+D}{x^2-x+1}$ then $A+B+C+D=$

a) 2 b) 0 c) 1 d) 5

13. $\frac{x^3+x^2+1}{(x^2+2)(x^2+3)}=\frac{Ax+B}{x^2+2}+\frac{Cx+D}{x^2+3}$ then $A+C=$ ____

a) 3 b) 5 c) 1 d) 2

14. $\frac{x^4+24x^2+28}{(x^2+1)^3}=\frac{Ax+B}{x^2+1}+\frac{Cx+D}{(x^2+1)^2}+\frac{Ex+F}{(x^2+1)^3}$ then $A+C+E=$___

a) 5 b) 3 c) 1 d)0

15. $\frac{x^2}{(x-1)(x-2)}$ can be reseloved as

a) $\frac{A}{x-1}+\frac{B}{x-2}$ b) $1+\frac{A}{x-1}+\frac{B}{x-2}$ c) $2+\frac{A}{x-1}+\frac{B}{x-2}$ d) $3+\frac{A}{x-1}+\frac{B}{x-2}$

16. $\frac{3x^2-8x^2+10}{(x-1)^2}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{D}{(x-1)^4}$ then $\begin{bmatrix}A & B\\ C & D\end{bmatrix}=$ ____

a) $\begin{bmatrix}3 & -1\\ 7 & 5\end{bmatrix}$ b) $\begin{bmatrix}3 & -1\\ -7 & 5\end{bmatrix}$ c) $\begin{bmatrix}-3 & -1\\ 7 & 5\end{bmatrix}$ d) $\begin{bmatrix}-3 & 1\\ -7 & -5\end{bmatrix}$

17. Number of partial fraction obtained in $\frac{x^4+5x^2+9}{(x^2+1)^5}$ is

a) 2 b) 1 c) 3 d) 5

18. $\frac{2x^2+1}{x^3-1}=$

a) $\frac{2}{x-1}+\frac{x}{x^2+x+1}$ b) $\frac{1}{x-1}-\frac{x}{x^2+x+1}$

c) $\frac{1}{x+1}+\frac{x}{x^2+x+1}$ d) $\frac{1}{x-1}+\frac{x}{x^2+x+1}$

KEY

1. b	2. d	3. a	4. b	5. c
6. a	7. b	8. c	9. c	10. c
11. d	12. a	13. c	14. d	15. b

16. b 17. c 18. d

MEASURE OF DISPERSION

Measures of Dispersion:

Statistics is a branch of mathematics, which consists of the collection of data, classification, analysis. Its application were very important in present days i.e. in economics, business, industry, scientific research and varies other fields.

Measures of dispersion desoniles the spread or scattering value.

Measuring dispersion of a data is significant became it determines the reliability of an average bypointing out as to how far an average is representalie of the entire data.

Data means simply numbers collected for some purpose. The data can be expressed in many forms such as

i) Ungroped data

ii) Grouped data

a)Discrete frequency distribution

b) Continuions frequency distribution

The following measures of dispire and their methods of calculation for ungrouped andgrouped data are

(i) range (ii) Mean deviation

(iii) Variance and standard as the difference

(i)Range: Range is defined as the difference between the maximum value and the minimum value of the series of given observation

(ii)Mean deviation: It is the arithmetic mean of the absolute deviation of the variable from a measure of their central tendency. It is generally denoted by M.D

(a) for ungrouped data $x_1, x_2, ...x_n$

$$M.D = \frac{\sum_{i=1}^{n}|x_i - \overline{x}|}{n}$$ = Mean deviation from mean

Where n = no terms

$\bar{x}$ = Arithmetic mean $= \dfrac{\text{sum of the terms}}{\text{no. of terms}}$

(b) for grouped data

$$M.D = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{\sum f_i} = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$$

Where f_i =frequency of variable x_i

$N = \sum f_i$ =total frequency

$\bar{x} = \sum_{i=1}^{n} f_i x_i / \sum f_i$

Mediation: first we arrange the given data in ascending or descending order.

(i) If the no of terms is odd then the median is $\left(\frac{n+1}{2}\right)^{th}$ term (i.e. middle term)

(ii)If the no. Of terms is even then the median is the average of $\frac{n}{2}^{th}$, $\left(\frac{n}{2}+1\right)^{th}$ term.

Mean deviation frommedian:

(a)for ungrouped data $x_1, x_2, ...x_n$ $M.D = \dfrac{\sum_{i=1}^{n} |x_i - M|}{n}$

Where M = Median of the given data

n = no of the terms

(b)for grouped data:

(i) **Duscrete frequency distribution:**

First the observations are arranged in ascending order. After this the cumulative frequencies are obtained. Find the sum of the frequencies $\sum f_i = N$ Then identify the observation whose cumulative frequency is equal or just greater than N/2. This is the median of the D.F.D

M.D about median $= \dfrac{\sum_{i=1}^{n} |x_i - M|}{N}$

(ii)**Continuous frequency distribution:**

A continuous frequency distribution is a series in which the data is classified into different class – intervals without gaps along with their rsprective frequencies the mid point of the class-intervals are x_i. The cumulative frequencies are obtained and identify the class in which N/2 lies

This class is known as the median class.

Median, $M = L + \left[\dfrac{\frac{N}{2} - P.C.R}{f}\right]h$

Where L = lower limit of the median class

N = sum of the frequencies = $\sum f_i$

f = frequency of the median clas

p.c.f = preceeding cumulative frequency to the median class

h =difference in class interval

Variance and Standard deviation:

The arithmetic mean of the squares of the deviations of the variable from arithemetic mean. This number is called variance and is denoted by $\sigma^2 (sigma\ square)$

Variance, $\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}$ for ungrouped data

Standard deviation:

The postine squre root of variance is called standard deviation and is denoted by σ

$\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n}}$ for ungrouped data

Variance & Standard deviation for grouped data:

Variance, $\sigma^2 = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{\sum f_i} = \dfrac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{N}$

$= \dfrac{1}{N}\sum_{i=1}^{n} f_i x_i^2 - \overline{x}^2$

Standard deviation:

$\sigma = \sqrt{\dfrac{\sum_{i=1}^{n} f_i (x_i - \overline{x})^2}{N}}$ or $= \sqrt{\dfrac{1}{N}\sum_{i=1}^{n} f_i x_i - \overline{x}^2}$

Co-efficient of variation of a distribution:

The measure of variability which is a pure number and is independent of units is called the co-efficient of variation and is denoted by C.V

$C.V = \dfrac{\sigma}{\overline{x}} \times 100,\ \overline{x} \neq 0$

Note: For comparing the variability of two series, we calculate the C.V for each series.
(i)The series having greater C.V is said to have more variable than other.
(ii)The series having lies C.V is said to be more consistent than the other.

Step-deviation Method:

When the class intervals, the mid point of the C.I, x_i as well as their associated frequencies are numerically large then we find a computational tediousues in this method.

We define a new variable, $d_i = \frac{x_i - a}{h}$

Here a= assumed mean which lies in the middle or just close to deviations of mid points x_i

Arithmetic mean, $\overline{x} = a + \left[\frac{\sum_{i=1}^{n} f_i d_i}{N}\right] h$

From this we find the mean deviation about mean by using step-deviation method.

Level – I (Multiple Choice Questions)

1. The simplest Measure of dispersion is
   a) Standard deviation b) Range
   c)Mean deviation d) Quartile deviation
2. The most stable measure of central tendency is
   a) Median b) Mode c) Mean d) Harmonic measure
3. The accurate Measure of dispersion is
   a) Rang b) Mean deviation c) Quartile deviation d) Standard deviation
4. Mean deviation can be calculated from
   a) Mean b) Median c) Mode d) All the three
5. The average of the squares of deviations of the values from arithmetic mean is called
   a) Range b) Variance c) Standard deviation d) Mean deviation
6. If the mean of 3, 4, x, 7, 10 is 6 then the value of 'x' is
   a) 7 b) 6 c) 5 d) 4
7. The mean deviation from the mean of observation 3, 6, 10, 4, 9, 10
   a) 2.67 b) 2.97 c) 2.57 d) 2.75
8. The Mean deviation from the Median of the observations 1, 2, 3, 4
   a) 0 b) 1 c) 2 d) 3
9. The standard deviation of 0, 1, 2, 3, 4 is

a) $\sqrt{\frac{14}{5}}$ b) $\sqrt{10}$ c) $\sqrt{3}$ d) $\sqrt{2}$

10. Variance of 5, 8, 11, 9, 8, 11 is

a) 4.4 b) 10 c) 19.33 d) none

11. The Harmonic Mean of 2, 3, 4 is

a) 3 b) $(24)^{2/3}$ c) $\frac{36}{13}$ d) $\frac{13}{36}$

12. If $\sum_{i=1}^{18}(x_i - 8) = 9$ and $\sum_{i=1}^{18}(x_i - 8)^2 = 45$ then the standard deviation of $x_1, x_2....x_{18}$ is

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) $\frac{5}{2}$ d) $\frac{7}{2}$

13. Mode of the data 3,2,5,2,3,5,6,6,5,3,5,2,5 is

a) 3 b) 4 c) 5 d) 6

14. The Median of $x+4, x-\frac{7}{2}, x-\frac{5}{2}, x-3, x-2, x+\frac{1}{2}, x-\frac{1}{2}, x+5\ (x>0)$ is

a) $x+\frac{1}{2}$ b) $x-\frac{1}{2}$ c) $x+\frac{5}{4}$ d) $x-\frac{5}{4}$

15. The Arithmetic Mean of first 'n' odd natural numbers is

a) n b) n^2 c) n +1 d) n –1

16. Mean deviation when taken from Mediam is

a) Average b) Greatest c) least d) Cannot be defined

17. In any discrete (All the values are not same) the relation between Mean deviation about Mean and Standard deviation is

a) M.D = SD b) M.D > S.D c) M.D < S.D d) $M.D \neq S.D$

18. The variance of first 20 natural numbers is

a) $\frac{379}{12}$ b) $\frac{399}{4}$ c) $\frac{133}{2}$ d) $\frac{133}{4}$

19. If A.M , G.M and H.M in any series are equal then

a) Distribution is symmetric

b) All the values are same

c) Distribution is positively skewed

d)Distribution is negatively skewed

20. The variance of 20 observations is 5. If each of the observation is multiplied by 2 then the variance of the resulting observations

a) 20 b) 100 c) 200 d) 400

Fill in the blanks:

1. The mean of fine observations is 4 and their variance is 5,2 If three of these observations are 1, 2 and c then the other two are _____

2. The standard deviation of a variable 'x' is 'σ' the standard deviation of the variable $\frac{ax+b}{c}$ where a,b,c are constants is _____
3. If the mode of a data is 18 and the mean is 24 then Median is _____
4. The variance of the first 10 positive multiples of '3' is _______
5. The mean deviation from Mean for the data 6, ,7, 10, 12, 13, 4, 16, 12, is ___
6. If $x_1, x_2, \ldots\ldots, x_n$ are 'n' observations and $\bar{x}$ is their mean then $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is called ____
7. The measures of variability which is a pure number and is independent of units is called ______
8. The Median for the data 13, 17, 16, 11, 13, 10, 16, 11, 18, 17, is ____
9. The standard deviation of a data is3, arithematic mean is 20 then the co-eff of variation is ______
10. Median of $\frac{a}{5}, a, \frac{a}{4}, \frac{a}{2}, \frac{a}{3}$ is 8. If a > 0 then the value of 'a' is ____

Matching:

1.

A	B	
a) The most unstable average	1. Harmonic Median	[ ]
b) Affected mostly by extremen observation	2. Median	[ ]
c) Affected least by extreme observations	3. Mode	[ ]
d) which cannot be determined graphically	4. Variance	[ ]
e) Indenpendent of change of origin is	5. Arithmetic Mean	[ ]

2.

A	B	
a) Median	1. $=\frac{(\alpha-\beta)^2}{4}$	[ ]
b) The variance of first 50 even natural numbers	2. α, β	[ ]
c) Mean deviation	3. 833	[ ]
d) Standard deviation of first 'n' natural Nos	4. $x^{\frac{n+1}{2}}$	[ ]
e) The geometric Mean of	5. $l + \left[\frac{\frac{N}{2} - F}{f}\right]C$	[ ]

$x, x^2, x^3, ...x^n$ is

KEY

1. b	2. a	3. b	4. d	5. b
6. b	7. a	8. b	9. d	10. a
11. c	12. b	13. c	14. d	15. a
16. c	17. b	18. d	19. b	20. A

Fill in the blanks:

1. 4, 7 2. $\left(\frac{a}{c}\right)\sigma$ 3. 22 4. 74.25 5. 3.25
6. variance 7. Co-efficient of variation 8. 13 9. 15
10. 24

Matching:

1. $a \to 3, b \to 5, c \to 2, d \to 1, e \to 4$

2. $a \to 5, b \to 3, c \to 1, d \to 2, e \to 4$

Level – 2

1. Which of the following is correct for data –1, 0, 1, 2, 3, 5, 5, 6, 8, 10, 11 ?
   a) Mean =5 b) Mean = Mode c) Mean = median d) Mode = Median
2. If $\bar{x}$ is the mean of distribution then $\sum(x_i - \bar{x})$ is equal to
   a) 0 b) 1 c) Mean deviation d) variances
3. If the standard deviation of a data is '3' arithimetic mean is 20, then te co-efficient of variation is
   a) $\frac{3}{20}$ b) $\frac{20}{3}$ c) $\frac{300}{20}$ d) $\frac{2000}{3}$
4. Mean deviation of 390, 400, 400, 410, 410, 420, 420, 430, 440, 450 through median is
   a) 420 b) 15 c) 210 d) none
5. Mean of 40 terms is 25 and standard deviation is 4. Then the sum of the squares of all terms is
   a) 25640 b) 25000 c) 25645 d) 35645
6. The measure of disparsion which is used to find more consistent data is
   a) Range b) Mean c) Mean deviation d) Standard deviation
7. If $\sum f_i = 100, \sum f_i x_i = 220, \sum f_i\left(x_i - \bar{x}\right) = 104.8$ then the mean deviation is
   a) 1048 b) 104.8 c) 10.48 d) 1.048
8. The A.M. of the series ${}^nC_n$. ${}^nC_1, {}^nC_2, ... {}^nC_n$ is

a) $\frac{2^n}{n}$ b) $\frac{2^n}{n+1}$ c) $\frac{2^n+1}{n+1}$ d) $\frac{2^n-1}{n+1}$

9. The variance of 6,8,10,12,14,16,18,20,22,24

a) 54 b) 45 c) 33 d) 37

10. In 100 members, 80 members are 4's and the rest are 9's then the standard deviation is

a) 2.0 b) 2.1 c) 2.2 d) 2.3

KEY

1. d	2. a	3. c	4. b	5. a
6. a	7.d	8. b	9. c	10. a

WORK BOOK - PROBABILITY

PART – I

Conceptual Questions

1. If carona vaccine is successful, is then a choice of gold rate to decline (Yes/No)
2. Measuring or quantifying such charus of happening or not happening is called Probability (Yes/No)
3. A random experiment can be repeated any no of times under identical conditions (Yes/No)
4. All possible out comes of a random experiment are known in advance. (Yes/No)
5. When a random experiment is conducted any no of times under similar conditions, the actual outcome in any trial can not be prelicted with certainity. (Yes/No)
6. Example of a random experiment

   (1)tossing a fair coin (2) throwing an undbiaked die

   (3) tossing a coin with helds on both sides

   (4) both (1) & (2)
7. Set of all possible out comes of a random experiment is sample space 'S' (Yes/No)
8. When three coins are tossed simultaneously sample spice

   $S=\{(H\ H\ H),(H\ T\ H),(H\ T\ T),(H\ H\ T),(T\ H\ H),(T\ T\ H),(T\ H\ T),(T\ T\ T)\}$ (Yes/No)
9. When a fair coin is tossed and a ball is drawn from a bag containing 3 different red balls simultaneously then sample space (Yes/No)

   $S=\{(H, R_1), (H, R_2),(H, R_3),(T, R_1),(T, R_2),(T, R_3)\}$

10. Any subset of sample space is called an event (Yes/No)
11. When a die is thrown event of getting an odd number = $\{1, 3, 5\}$
12. When two coins are tossed simultaneously, event of getting atmost one head is $\{(T, T), (T, H), (H, T)\}$ (Yes/No)
13. In a trial if the out come of random experiment belongs to event E, then we say that event E has happened (True/False)
14. Not happening of an event E is denoted by E or E
15. An event which never happens is Impossible event
16. Event which surely happens is certains event
17. ϕ is Impossible event
18. Sample space S is certain event.
19. If happening of one event E_1, prevents the happening of another event E_2, then E_1 & E_2 are called Mutually Exclusive
20. $E_1 \cap E_2 = \phi \Rightarrow E_1, E_2$ are Mutually Exclusive
21. If $E_i \cap E_j = \phi$ for $i \neq j$, $1 \leq j,\ j \leq n$; then $E_1, E_2, E_3....E_n$ are called Pair wise mutually Exclusive
22. Events which have equal chaces to occur are called equally likely events.
23. If $\bigcup_{i=1}^{n} E_i = S$, then $E_1, E_2, ..., E_n$ are called Exhaustive events
24. In tossing three coins simultaneously

    Let A = event of getting no head

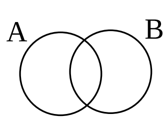
    B = event of getting at least one head

    Which of the following are true.

    1) A &B are mutually are true

    2) A & B are exhaustive events

    3) $A = \phi$

    4) both (1) & (2)
25. 

    From the venn diagram which of the following are true

    1) $A \cap B^C, B$ are mutually exclusive (True/False)

    2) $A^C \cap B, A$ are mutually exclusive (True/False)

    3) $(A^C \cap B) \cup A = A \cup B$ (True/False)

    4) $(A \cap B^C) \cup B = A \cup B$ (True/False)

5) $A \cap B$, A are mutually exclusive (True/False)

6) A – B, $A \cap B$ are mutually exclusive (True/False)

26. A, B are two events of a random experiment

Match the following:

1. Event A or event B to occur. i.e. Atleast one of A, B to occur	(a) $A^C \cap B^C$ (or) $(A \cup B)^C$
2. Neither A, Nor B to occur.	(b) $A \cup B$
3. A occurs but B does not	(c) $A^C \cup B$ or $B - A$
4. Exactly one of A, B to occur	(d) $A \subseteq B$
5. Event B occurs when over A occurs	(e) $(A \cap B^C) \cup (A^C \cap B)$ (or) $(A - B) \cup (B - A)$ (or) $(A \cup B) - A \cap B$
6. Both A & B to occur	(f) $A \cap B$
7. A does not occur but B occurs	(g) $A \cap B^C$ (or) $A - B$

1) b 2) a 3) g 4) e 5) d

6) f 7) c

27. Complement of atleast one of A, B, C solving a given problem = $\bar{A} \cap \bar{B} \cap \bar{C}$

28. Complement of a person losing all the three games A, B, C = $A \cup B \cup C$

29. Complement of India losing at least one of the three matches $= G_1 \cap G_2 \cap G_3$

30. Event of exactly one of A, B, C completing a task

$=(A \cap B^C \cap C^C) \cup (A^C \cap B \cap C^C) \cup (A^C \cap B^C \cap C)$

31. A, B, C are writing an examination. Events “A passing the exam”, “B passing the exam”, “C passing the exam” are

1) mutually exclusive but not equally dikely

2) Equally likely but not mutually exclusive.

3) Not equally likely, not mutually exclsive.

32. In an examination, Events

“A getting first rank”, “B getting first rank”, “C getting first rank”, are (Assuming one rank is given to only one student)

1) mutually exclusive, equally likely.

2) mutually exclusive, equally likely.

3) Neither mutually exclusive, Nor equally likely

33. When the out comes of a random experiment are equally likely, exhaustive, mutually exclusive then probability of occuranu of an event

$$E = \frac{\text{no. of favourable outcomes to E}}{\text{Total number of outcomes}}$$

34. Probability of getting an odd no. When an unbiased lie is thrown = $\frac{3}{6}$

35. If A die is made such that
$P(1)=K\cdot 1^2,\ P(2)=K\cdot 2^2,\ P(3)=K\cdot 3^2,\ P(4)=K\cdot 4^2,\ P(5)=K\cdot 5^2,\ P(6)=K\cdot 6^2$
then P (getting an odd number) $\neq \frac{3}{6}$ as the outcomes are not equally likely. (True/False)

36. For any evernt E, $0 \le P(E) \le 1$ (True/False)

37. P(impossible event) = 0, P (certain event) = 1 (True/False)

38. $P(\phi)=0,\ P(S)=1$ (True/False)

39. $P(E)+P(E^6)=1$

40. If $E_1 \cap E_2 = \phi$ then $P(E_1 \cup E_2) = P(E_1)+P(E_2)$

41. $E_1, E_2, ...E_n$ are pair wise mutually exclusive events , then
$P(E_1 \cup E_2 \cup .... \cup E_n) = P(E_1)+P(E_2)+...+P(E_n)$

42. $P(E_1 \cup (E_2 - E_1)) = P(E_1) + P(E_2 - E_1)$

43. E_1, E_2 are any two events of a random experiment, then
$P(E_1 \cup E_2) = P(E_1)+P(E_2)-P(E_1 \cap E_2)$

44. $P(A \cup B \cup C) = P(A)+P(B)+P(C)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$
Where A,B,C are any three events of a random Experiment

45. If A,B,C are mutually exclusive, then $P(A\cup B\cup C)=P(A)+P(B)+P(C)$ (True/False)

46. $P(A\cup B\cup C)=1-P(\bar{A}\cap\bar{B}\cap\bar{C})$ (True/False)

47. $P(A\cap B)=1-P(A^C\cup B^C)$ (True/False)

48. If $E_1 \le E_2$ then $P(E_2-E_1)=P(E_2)-P(E_1)$ (True/False)

49. $P(E_1)=P(E_1-E_2)+P(E_1\cap E_2)$ (True/False)

50. Event of happening of A after the happening B = A/B

51. $P(A/B)=\frac{P(A\cap B)}{P(B)}$ where A, B are any two events of a random experiment with
$P(B)\neq 0$

52. $P(A\cap B)=P(A)\cdot P(B/A)=P(B)P(A/B)$ (True/False)

53. $P(A\cap B\cap C)=P(A)P(B/A)P(C/A\cap B)$ (True/False)

54. If the happening of one evening does not effect the happening or not happening of another event then the two events are called Independent events

55. If $P(A\cap B)=P(A)\cdot P(B)$ then A and B are called Independent events

56. If A,B,C are independent events then $P(A\cap B\cap C)=P(A)P(B)P(C)$

57. If A and B are Independent, then $P\left[(A-B)\cup(B-A)\right]=$

58. Probability of exactly one of the events A, B to occur is $P\left[(A-B)\cup(B-A)\right]=$

59. $P(A\cup B)-P(A\cap B)=P(A)+P(B)-2P(A\cap B)$ (True/False)

60. For any two events A & B

1) $P(A\cup B)\geq \max\{P(A),\ P(B)\}$ (True/False)

2) $P(A\cap B)\geq \min\{P(A),\ P(B)\}$ (True/False)

61. If A & B are independent events, then

(i) $P(A\cap B)=P(A)\cdot P(B)$ (True/False)

ii) $P(A'\cap B)=P(A')\cdot P(B)$ (True/False)

iii) $P(A\cap B')=P(A)\cdot P(B')$ (True/False)

iv) $P(A'\cap B')=P(A')\cdot P(B')$ (True/False)

62. If $B_1, B_2, B_3, ...B_n$ is a partition of the sample space S, then for any event A,

$P(A)=\sum_{i=1}^{n}P(A\cap B_i)$ (or) $\sum_{i=1}^{n}P(A/B_i)P(B_i)$ (True/False)

63. $E_1, E_2, ...E_n$ are mutually exclusive and exhaustive events of a random experiment with $P(E_i)\neq 0$, then for any event A, with $P(A)\neq 0$

$$P(E_K/A)=\frac{P(E_K)\cdot P(A/E_K)}{\sum_{i=1}^{n}P(E_i)P(A/E_i)}$$ (True/False)

Level – I

1. When a fair coin in thrown the total no of possible outcomes = 2 (True/False)

2. When two fair coins are thrown simultaneously the total no of possible outcomes

$n(S)=2\times 2$

3. When a coin is thrown n times the total no of possible out comes $=2^n$

4. When an unbiased die is rolled total no of possible out comes = 6
5. When two dice are rolled simultaneously the total no of possible out comes = 6×6
6. When an unbiased die is rolled n times successively the total no of possible out comes = 6^n
7. When two cards are drawn at random from a well shiffled pack of 52 cards the total no of possible out comes $= {}^{52}C_2$
8. In a single throw of a normal die, the probability of getting a number between 3 and 6 is $\frac{2}{6}$
9. Probility of getting a sum of 6 or 7 in a single throw of two dice $= \frac{11}{36}$
10. Probability of getting a prime no on one of the dice when two dice are rolled $= 27/36 = \frac{3}{4}$
11. When an unbiased die is rolled, probability of getting even number $= 3/6$

    Probability of getting odd number $= 3/6$
12. When an unbiased die is rolled on two times then the probability of getting the sum of the two outcomes as even number $= 1/2$
13. A biased die is made such that the probability of a number n appearing on it is proportional to n (n=1,2,3,4,5,6). Find the probability of getting an odd number when the die is rolled.

    A) $\frac{1}{21} + \frac{3}{21} + \frac{5}{21} = \frac{3}{7}$ B) $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

    C) $\frac{3}{7}$ D) $\frac{4}{5}$
14. If the biased die in the above problem is rolled two times, then the propability of sum of the two outcomes as even number is _____

    A) $\frac{3}{7} \times \frac{3}{7} + \frac{4}{7} \times \frac{4}{7} = \frac{25}{49}$ B) $\frac{1}{4}$

    C) $\frac{3}{6} \times \frac{3}{6} + \frac{3}{6} \times \frac{3}{6} = \frac{1}{2}$ D) $\frac{3}{5}$
15. When three dice are rolled simultaneously, (the die is unbiased) then the probability getting same no on all the three is ____

    A) $\frac{6}{216}$ B) $\frac{1}{20}$ C) $\frac{1}{36}$ D) $\frac{1}{6}$
16. When a die is rolled 4 times, find the probability of getting a larger no than the previous no each time =

    A) $\frac{{}^{6}C_3}{6^3}$ B) $\frac{{}^{6}C_4 \times 1}{6^4}$ C) $\frac{{}^{6}C_3}{6^4}$ D) $\frac{{}^{6}C_4 \times 4!}{6^4}$

17. When a die is rolled 3 times, find probability of getting each time a number different from its previous out comes

A) $\frac{6\times5\times4}{6^3}$ B) $\frac{6+5+4}{6^3}$ C) $\frac{6\times5\times4}{6^4}$ D) $\frac{1}{2}$

18. When a fair coin is tossed two times successively, probability of getting exactly one tail is _____

A) $\frac{2}{4}$ B) $\frac{1}{4}$ C) $\frac{3}{4}$ D) 1

19. When a fair coin is tossed three times successively, probability of getting exactly two heads is _____

A) $\frac{^3C_2}{2^3}$ B) $\frac{3}{8}$ C) $\frac{1}{2}$ D) $\frac{2}{8}$

20. If a coin is tossed n times the probability that head comes odd number of times is ____

A) $\frac{1}{2}$ B) $\frac{^nc_1+^nc_3+^nc_5+....}{2^n}$ C) $\frac{2^{n-1}}{2^n}$ D) $\frac{1}{2^n}$

21. A coin is tossed 4 times. The probability that at least are head turns up is _____

A) $1-\frac{1}{2^4}$ B) $\frac{15}{16}$ C) $\frac{^4c_1+^4c_2+^4c_3+^4c_1}{2^4}$ D) None of these

22. Two cards are drawn from a well shuffled pack of 52 cards. Probability that one of them is a king of heart is

A) $\frac{2}{^{52}C_2}$ B) $\frac{1\times{}^{51}C_1}{^{52}C_2}$ C) $\frac{1}{26}$ D) $\frac{2}{^{52}C_1}$

23. Two cards are drawn from a well shuftted pack of 52 cards one after the other with replacement (i) probability that both the cards are kings is _____

A) $\frac{4}{52}\times\frac{4}{52}$ B) $\frac{1}{13}\times\frac{1}{13}$ C) $\frac{^4C_2}{^{52}C_2}$ D) None of the above

24. In the above problem if the cards are drawn without replacement then the probability that both the cards are kings is _____

A) $\frac{^4C_1}{^{52}C_1}\times\frac{^3C_1}{^5C_1}$ B) $\frac{1}{221}$ C) $\frac{3}{8}$ D) $\frac{5}{9}$

25. Each of the two boxes A & B contain 10 chits numbered 1 –10. If one unit is drawn at random from A and B each, then the probability that the number drawn from A is smaller than the number drawn from B is

A) $\frac{9}{10}$ B) $\frac{9}{20}$ C) $\frac{19}{20}$ D) $\frac{17}{20}$

26. If A and B throw two dice each 100 times simultaneously. Then probability that both of them will get even number as total at the same time in all the throws is ___

A) 100 B) $\left(\frac{1}{4}\right)^{100}$ C) $\left(\frac{1}{2}\right)^{100}$ D) $\left(\frac{3}{4}\right)^{100}$

27. Find the probability of choosing a multiple of 5 from first 100 natural numbers

A) $\frac{1}{25}$ B) $\frac{1}{20}$ C) $\frac{1}{5}$ D) $\frac{2}{100}$

28. From 30 consecutive +ve integers two no's are chosen at random. Probability that the product of the two numbers is even is ____

A) $1-\frac{{}^{15}C_2}{{}^{30}C_2}$ B) $\frac{{}^{15}C_2+{}^{15}C_1\cdot{}^{15}C_1}{{}^{30}C_2}$ C) $\frac{22}{29}$ D) $\frac{3}{{}^{30}C_2}$

29. Three are 10 letters and 10 envelopes with addresses. Then the probability that all the letters are not kept in the right envelope is

A) $\frac{1}{10!}$ B) $1-\frac{1}{10!}$ C) $\frac{9}{10!}$ D) None

30. A shooter hits a target with probability $\frac{1}{4}$. If he shoots 3 times find the probability that he gets at least one hit is

A) $1-\left(\frac{3}{4}\right)^3$ B) $\frac{37}{64}$ C) $\frac{3}{4}$ D) None

31. A problem in calculus is given to three students A,B,C whose chances of solving the problem are $\frac{1}{3},\frac{1}{4},\frac{1}{5}$. The probability that the question is solved is

A) $1-\frac{2}{3},\frac{3}{4},\frac{4}{5}$ B) $\frac{3}{5}$ C) $\frac{1}{60}$ D) None

32. The probabilities of A,B,C solving a problem are $\frac{2}{5},\frac{1}{4},\frac{1}{5}$ respectively. It all the three try to solve the problem simultaneously, the probability that exactly one of them will solve it is

A) $\frac{2}{5}\times\frac{3}{4}\times\frac{4}{5}+\frac{3}{5}\times\frac{1}{4}\times\frac{4}{5}+\frac{3}{5}\times\frac{3}{4}\times\frac{1}{5}=\frac{9}{20}$

B) $\frac{10}{27}$ C) $\frac{5}{9}$ D) None

33. The probability that A speaks truth is 50% and this probability for B is 60%. Then the probability that they contradict each other about an incident is

A) $\frac{1}{2}$ B) $\frac{1}{2}\times\frac{2}{5}+\frac{1}{2}\times\frac{3}{5}$ C) $\frac{1}{5}$ D) None

34. Three numbers are chosen from 1 to 20 (natural numbers) The probability that they are consecutive is

A) $\frac{2}{190}$ B) $\frac{18}{{}^{20}C_3}$ C) $\frac{3}{190}$ D) None

35. In the above problem probability that the three numbers are not consecutive is

A) $\frac{186}{190}$ B) $\frac{187}{190}$ C) $\frac{188}{190}$ D) None

36. Four digit numbers are formed using the digits 0, 1, 2, 5 then the probability that such a number is divisible by 5 is

A) $\frac{4}{9}$ B) $\frac{5}{9}$ C) $\frac{3!\times 1+2\times 2!\times 1}{3\times 3!}$ D) None

37. Cards are drawn one by one without replacement from a will shuffled pack of 52 cards. The probability that a face card (King, Queen, Jackie) appear for the first time in third draw is

A) $\frac{11}{85}$ B) $\frac{12}{85}$ C) $\frac{40}{52}\times\frac{39}{51}\times\frac{12}{50}$ D) None

38. There are 8 guests and a host. All of them are to sit around a table. Probability that two particular guests sit on either side of last always is

A) $\frac{6!\times 2!}{8!}$ B) $\frac{1}{28}$ C) $\frac{7}{29}$ D) None

39. Find the probability that birthdays of 3 persons A,B,C fall in exactly two calendar months

A) $\frac{7}{48}$ B) $\frac{11}{48}$ C) $\frac{{}^{12}C_2\times\left(2^3-2\right)}{\left(12\right)^3}$ D) $\frac{3}{40}$

40. Probability of drawing a king or a spade from a well shuffled pack of 52 cards is

A) $\frac{4+13-1}{{}^{52}C_1}=\frac{4}{13}$ B) $\frac{9}{13}$ C) $\frac{8}{51}$ D) $\frac{5}{13}$

41. In the above problem probability of drawing a card which is neither a king nor a spade is

A) $1-\frac{4}{13}=\frac{9}{13}$ B) $\frac{39}{52}\times\frac{48}{52}$ C) $\frac{4}{13}$ D) $\frac{2}{13}$

42. If the probability of boy and girl to be born are same; then in a 3 children family the probability of having no girl child is

A) $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$ B) $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{8}$

C) $\frac{3}{8}$ D) None

43. The probability of a leap year has 52 Sundays is_____

A) $\frac{2}{7}$ B) $\frac{5}{7}$ C) $\frac{3}{7}$ D) None

44. Probability of a leap year having 53 Thursdays or 53 Fridays is

A) $\frac{3}{7}$ B) $\frac{4}{7}$ C) $\frac{2}{7}+\frac{2}{7}-\frac{1}{7}$ D) None

45.

46. A matrix is chosen at random from a set of 2×2 matrices with elements 0, 2 only. The probability that the matrix chosen is non-singular is___

A) $\frac{3}{8}$ B) $\frac{5}{8}$ C) $\frac{3}{16}$ D) None

47. In the above problem, probability that the determinant of the selected matrix is positive is_____

A) $\frac{3}{8}$ B) $\frac{5}{8}$ C) $\frac{3}{16}$ D) None

48. A bag contains 4 red, 5 black balls second bag contains 5 red, 7 black balls. A ball is drawn from each bag Probability that (1) one is red another is black is P (2) both are of red colour is r (3) both are of same colour is t. Then

A) $p=\frac{53}{108}, r=\frac{5}{27}, t=\frac{55}{108}$

B) $p=\frac{53}{108}, r=\frac{3}{55}, t=\frac{53}{108}$

C) $p=\frac{1}{2}, r=\frac{3}{4}, t=\frac{1}{6}$

49. A bag contains 4 black, 6 red balls. Two balls are drawn one after the other without replacement. Then

i)probability of second ball is red if first ball drawn is black = $\frac{6}{4}$

ii)probability of first ball is black and second ball is red = $\frac{4}{10}\times\frac{6}{9}$

iii)probability of second ball is red = $\frac{4}{10}\times\frac{6}{4}+\frac{6}{10}\times\frac{5}{4}$

50. A bag contains 5 white, 3 red, 4 black balls. Two balls are drawn one after the other with replacement. Then

A) probability of second ball is black if first ball drawn is black = $4/12$

B) probability of first ball is red, second ball is white = $\frac{3}{12}\times\frac{5}{12}$

C)probability of second ball is red = $\frac{5}{12}\times\frac{3}{12}+\frac{3}{12}\times\frac{3}{12}+\frac{4}{12}\times\frac{3}{12}$

51. A couple has two children. What is the probability that both are girls given that at least one is girl = $\frac{1}{3}$

52. A couple has three children (1) probability of having 1 son and two daughters, if the younger one is son = $\frac{1}{4}$

ii) probability of having one son & two daughters = $\frac{3}{8}$

iii) P (younger one is son) = $\frac{4}{8}$

iv) P(one son & two daughters) $\cap$ (younger one is son) = $\frac{1}{8}$

53. A bag contains 20 tickets numbered 1,2,3....20. Three are drawn at random and arranged in ascending order of magnitude. Then the probability that middle no in that order is 6 = — $\frac{{}^{5}C_1 \times {}^{14}C_1}{{}^{20}C_3}$

54. In a town 50% own a house, 40% own a car, 20% own both. Probability of random only selected family owns a car or a house but not both is 50%

55. A bag contains 5 black, 4 white, 3 red balls. If a ball is drawn the probability that it is a black or red ball is = $\frac{{}^{5}C_1 \times {}^{3}C_1}{{}^{12}C_3}$

56. $P(A\cup B)=\frac{2}{3},\ P(\bar{A})=\frac{3}{4},$ then $P(\bar{A}\cap B)=5/12$

57. Three horses A,B,C are in a race probability of A winning the race is twice the probability of B winning the race and probability of B winning in twice that of C winning. Then probability of B or C winning = $\frac{2}{7}+\frac{1}{7}=\frac{3}{7}$

58. $P(A\cup B)=\frac{4}{5},\ P(A\cap B)=\frac{1}{4},\ P(A^C)=\frac{2}{3}$

Then (i) $P(A\cap B^C)=\frac{1}{12}$

(ii) $P(A^C\cap B)=\frac{7}{15}$

59. $P(A\cap B)=\frac{1}{2},\ P(\bar{A}\cap\bar{B})=\frac{1}{4}.\ P(A)=x, P(B)=3x$ then x = $5/16$

60. Probability that at least one of A & B to occur is 0.8. If A & B occur simultaneously with probability 0.2 then $P(A^C)+P(B^C)=1$

61. In a class of 75 students, 70 passed in maths 45 in physics, 30 in both. Then the probability that a student selected at random passed in only one subject = $\frac{70}{75}+\frac{45}{75}-2\times\frac{30}{75}=\frac{11}{15}$

62. A fair coin is tossed repeatedly. Then problitiy of head appearing on 3rd toss if fail appears in first two tosses = $1/2$

63. A die marked 1,4,5 in red & 2,3,6 in green is rolled. Let A be the event "The no is odd", B be the event "The no is green. Then A & B are independent. (Yes/No)

64. $P(A/B)=P(A/B')$ then A & B are independent (Yes/No)

65. If a die is thrown twice Let A = event of getting 2 in second thrown only

Then (1) $P(A)=\frac{5}{6}\times\frac{1}{6}$ (True/False)

(2) P(A) = P (not getting 2 in first throw). P (getting 2 in second throw) (True/False)

66. Probability of hitting a target by three persons A,B,C are $\frac{1}{2},\frac{1}{3},\frac{1}{4}$. The probability of only one of them hits the target is

A) $P(A)P(B^C)P(C^C)+P(A^C)P(B)P(C^C)+P(A^C)P(B^C)P(C)$

B) $\frac{11}{24}$

C) $\frac{1}{6}$

D) None

67. A bag contains 4 white, 2 pink, 4 yellow roses. Three are drawn one after the other without replacement. Then the probability third rose is pink is ____

A) $\frac{57}{180}$ B) $\frac{4}{180}$ C) $\frac{58}{180}$ D) None

68. $P(A)=0.6,\ P(B)=0.6,\ P(A\cap B\cap C)=0.2,\ P(B\cap C)=\beta,\ P(A\cup B\cup C)=\alpha,$ $0.85\le\alpha\le 0.95$ Then β lies in interval

A) $[0.35, 0.36]$ B) $[0.25, 0.35]$ C) $[0.2, 0.25]$ D) None

69. There are 3 families in which 2 families have 3 members each and third family has 4 members. They are arranged in a line Then probability that members of the same family are together is

A) $\frac{1}{700}$ B) $\frac{3!3!3!4!}{10!}$ C) $\frac{3}{700}$ D) None

70. Two persons A & B play a game of throwing a pair of dice until one of them wins. A will win if sum of numbers on dice appear to be 6 and B will win if sum is 7. Then which of the following are true.

A)In any trial, P(A getting success) $=\frac{5}{36}$

B)In any trial, P(B getting success) $=\frac{6}{36}$

C)If A starts the game, then P(A winning the game) $=P(A)+P(\bar{A})P(\bar{B})P(A)+...$

D) If a starts the game, P(A winning the game) = $\frac{30}{31}$

E) All the above

71. If two unbiased dice are rolled simultaneously until a sum of either 7 or 11 occurs, then probability that 7 comes before 11 is to be found. Which of the following are true.

A)P(getting a sum of 7 or 11) in any trial = $\frac{6}{36}+\frac{2}{36}$

B)P(sum is neither 7 nor 11) in any trial = $\frac{7}{9}$

C) P(getting sum 7 before 11) = $\frac{1}{6}+\frac{1}{6}\times\frac{7}{9}+\left(\frac{7}{9}\right)^2\frac{1}{6}+.....=\frac{3}{4}$

D) All the above

72. An unbiased die is tossed until a number greater than 5 appears. Let E be the event that even number of tosses is required to get success. Then which of the following are correct.

A) In any trial P (success) $=\frac{1}{6}$, P(Failure) $=\frac{5}{6}$

B) $P(E)=P(F\,S)+P(F\,F\,F\,S)+P(F\,F\,F\,F\,F\,S)+....$

C) $P(E)=\frac{5}{11}$

D)All the above

73. In the above problem, the probability that he gets '6' in 4^{th} attempt = _____

A) $\frac{5}{6}\times\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}$ B) $\frac{5^3}{6^3}$ C) $\frac{1}{6}$ D) None

74. One of 10 keys open the door. If we try the keys one after the other, then which of the following are true.

A) P (the door is opened in first attempt) $=\frac{1}{10}$

B) P (the door is opened in 4th attempt) $=\frac{9}{10}\times\frac{8}{9}\times\frac{7}{8}\times\frac{1}{7}$

C) P (the door is opened in the 10th attempt) $=\frac{1}{10}$

D) All the above

75. A & B independent events probability that both A & B occur is $\frac{1}{12}$ and neither of them occur is $\frac{1}{2}$. Then probability of A = ____

A) $\frac{1}{3}$ *or* $\frac{1}{4}$ B) $\frac{1}{2}$ *or* $\frac{1}{3}$ C) $\frac{1}{12}$ D) None

76. When two dice are rolled simultaneously, the probability of sum of numbers on two dice is 11 it 5 appears on the first die is $1/6$

77. $P(A)=\frac{1}{2}, P(B)=\frac{1}{3}, P(B/A)=\frac{1}{6}$ then $P(A\cap B)=\frac{1}{12}$, $P(A/B)=\frac{1}{4}$, $P(A\cup B)=\frac{3}{4}$

78. A and B are two independent events $P(A)=0.8, P(B)=0.5$ then $P(A\cap B)=0.4$, $P(A/B)=0.8$, $P(B/A)=0.5$, $P(A\cup B)=0.9$

79. In tossing 3 coins simultaneously, if two of them shows a head, then the probability that all three shows head = $\underline{1/4}$

80. A die is thrown three times and the sum of the no thrown is 15. Probability for which 5 appears in second throw = $\underline{3/10}$

81. A bag contains 6 eggs out of which 2 are totten. Two eggs are taken out together. It one of them is good, then the probability that other is also good = $\underline{3/7}$

82. $P(A^C)=0.6,\ P(B)=0.3\ P(A\cap B^C)=0.2$ Then $P(B/A\cup B^C)=$ ____

A) $\frac{P(B\cap(A\cup B^C))}{P(A\cup B^C)}$ B) $\frac{2}{9}$

C) $\frac{P(A)-P(A\cap B^C)}{P(A)+P(B^C)-P(A\cap B^C)}$ D) All the above

83. From a group of 6 boys and 4 girls three are selected one after the other. Then probability of the three selected are girls = __ $\frac{4}{10}\times\frac{3}{9}\times\frac{2}{8}$

84. Probability of happening of an event A in one trial $=\frac{3}{5}$, then the probability of happening of the at least once in three trials is

A) $1-P(A^C)P(A^C)P(A^C)$ B) $1-\left(\frac{2}{5}\right)^3$

C) both (1) & (2) D) None

85. A and B are seeking admission in Indian Institute of statistics Probability of A getting admission = 0 – 4, probability that both are selected is atmost 0.3 Then

A) $0\le P(B)\le\frac{3}{4}$ B) $0\le P(B)\le\frac{1}{2}$

C) both (1) & (2) D) None

86. Probability of India winning a test match Assuming independence from match to match, the probability that in a test match series, India's second win occurs at the third test is ______

A) $P(W\ L\ W)+P(L\ W\ W)$ B) $\frac{1}{4}$ C) both (1) & (2) D) None

KEY

1. True	2. True	3. True	4. True	5. True
6. True	7. True	8. $\frac{2}{6}$	9. $\frac{11}{36}$	10. $\frac{27}{36}=\frac{3}{4}$
11. $\frac{3}{6}$	12. $\frac{1}{2}$	13. A	14. A	15. A
16. B	17. A	18. A	19. A	20. A,B,C
21.A,B,C	22. B,C	23. A	24. A,B	25.B
26.B	27.C	28.A,B,C	29.B	30.A,B
31.A,B	32.A	33.A,B	34.B,C	35.B
36.B	37.B,C	38.A,B	39.B,C	40.A
41.A,B	42.A	43.B	44.A,C	45.
46.A	47.C	48.	49.	50.
51.	52.	53.	54.	55.
56.	57.	58.	59.	60.
61.	62.	63.	64.	65.
66.	67.	68.	69.	70.
71.	72.	73.	74.	75.
76.	77.	78.	79.	80.
81.	82.	83.	84.	85.
86.				

Level – II

1. The letters of the word "ASSASSINATION" are arranged at random in a row, then the probability that no two 'A's come together is

1) $\frac{15}{26}$ 2) $\frac{25}{26}$ 3) $\frac{23}{26}$ 4) $\frac{17}{26}$

2. Given that a throw of three unbiased dice shows different faces, then the probability that their total is 8 is

1) $\frac{1}{10}$ 2) $\frac{23}{256}$ 3) $\frac{{}^6P_3}{2\times 3!}$ 4) $\frac{2\times 3!}{{}^6P_3}$

3. 5 different books are distributed to 4 students. The probability that each student gets atleast one book is

1) $\frac{15}{64}$ 2) $\frac{21}{64}$ 3) $\frac{31}{64}$ 4) $\frac{51}{64}$

4. $P(A\cup B)=a\ P(A\cap B)+b\ P(A)+c\ P(B)$ then 3a + 4b + 6c =

5. If the probability for A to fail an exam is 0.2 and for B is 0.3 then the probability that either A or B fails is ≤ _____

1) 0.5 2) 0.4 3) 0.2 4) None

6. A bag contains 5 balls and an unknown number x of red balls. Two balls are drawn at random. If the probability drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, then

1) 3 2) 5 3) 14 4) None

7. A number is selected at random from {1,2,3,4....1000} then the probability of getting a number which is a perfect unbe or a natural number having odd number of devisors is _____

1) $\frac{19}{500}$ 2) $\frac{481}{500}$ 3) $\frac{2}{125}$ 4) None

8. Three squares of a chess board are selected at random. The probability of selecting two squares of one colour and the other of different colour is _____

1) $\frac{16}{21}$ 2) $\frac{2\times {}^{32}C_2\cdot 31}{{}^{64}C_3}$ 3) $\frac{15}{21}$ 4) $\frac{8}{21}$

9. A and B are two candidates seeking admission in a college. The probability that A is selected is 0.7 and the probability that exactly one of them is selected is 0.6. Find the probability that B is selected is

1) 0.25 2) 0.75 3) 0.5 4) None

10. Bill and Gearge go target shooting together. Both shoot at a target at the same time. Suppose Bill hits the target with probability 0.7 where as George independly hits the target with probability 0.4, Given that exactly one shot hit the target, what is the probability that it was George's shot

1) $\frac{1}{9}$ 2) $\frac{2}{9}$ 3) $\frac{3}{9}$ 4) 0

11. A big consists of 3 red, 5 blue, 8 green balls. A ball is selected at random. Then the probability that the ball selected is not a blue ball

1) $\frac{11}{16}$ 2) $\frac{10}{19}$ 3) $\frac{5}{16}$ 4) None

12. A, B be two events P(A) = 0.9, P(B) = 0.8, $P(A\cap B)\ge 0.7$.

Then we can conclude that such a case is ____

1) Always true 2) Alwasys false 3) Not true in some examples

4) True only in same cases

13. A bag contains 4 defective and 6 good machines. Two machines are selected at random without replacement. Find the probability that both the machines are good

1) $\frac{1}{3}$ 2) $\frac{2}{3}$ 3) $\frac{3}{4}$ 4) None

14. The content of 3 boxes are as follows. If one box is selected at random and 3 balls are drawn from it and they are all of different colours, find the probability that they came from box 2.

	B_1	B_2	B_3
Black	1	1	6
White	2	1	4
Red	3	2	1

15. An envelope is known to have come from either LONDON or CLIFTON. On the postal mark only two successive letters 'ON' are legible. The brobability that the envelope comes from "LONDON" is

1) $\frac{12}{17}$ 2) $\frac{5}{17}$ 3) $\frac{3}{17}$ 4) $\frac{2}{5}$

16. If a carton is selected at random and a toy drawn randomly from it is found to be defective, then the probability that it is drawn from B is

1) $\frac{15}{47}$ 2) $\frac{20}{47}$ 3) $\frac{20}{59}$ 4) $\frac{15}{59}$

17. A bag contains 2n coins out of which $n-1$ are unfair with head on both sides and remaining are four. One coin is picked from a bag and tossed. If the probability that head falls in the toss is $\frac{41}{56}$, then the no of unfair coins in the bag is

1) 18 2) 15 3) 13 4) 14

18. Bag A contains 6 green, 8 red balls. B contains 9 green 5 red balls. A card is drawn from a well shuffled pack of 52 cards. If it is a shade, two balls are drawn at random from bag A, otherwise two ball are drawn from bag A, otherwise two ball are drawn from B. If the two balls drawn are found to be of same colour, then the probability that they are drawn from bag A is

1) $\frac{43}{181}$ 2) $\frac{1}{4}$ 3) $\frac{48}{131}$ 4) $\frac{43}{138}$

19. There are 3 bags, B contains 4 white and 2 balck, C contains 3 white, 2 black balls. If a ball is drawn bag, then the probability that the ball drawn is black is

1) $\frac{2}{3}$ 2) $\frac{4}{9}$ 3) $\frac{5}{9}$ 4) $\frac{1}{9}$

20. Probability of occurance of an event is $\frac{2}{5}$ and the probability of non-occurance of an event is $\frac{3}{10}$. If these two events are independent then the probability that only one of the two events occur is

1) $\frac{27}{25}$ 2) $\frac{27}{50}$ 3) $\frac{7}{25}$ 4) $\frac{14}{25}$

21. Let α be a root of $x^2+x+1=0$ suppose that a fair die is thrown 3 times. If a,b,c are no.s shown on die then probability that $\alpha^a+\alpha^b+\alpha^c=0$ is

1) $\frac{2}{36}$ 2) $\frac{1}{27}$ 3) $\frac{1}{72}$ 4) $\frac{2}{9}$

22. E_1, E_2 are two events of a random Experiment. $P(E_1)=\frac{1}{8}$, $P(E_2/E_2)=\frac{1}{4}$, $P(E_1/E_2)=\frac{1}{3}$ then match the following

List – I	**List – II**
a) $P(E_2)$	i) $\frac{3}{16}$
b) $P(E_1 \cup E_2)$	ii) $\frac{3}{29}$
c) $P(\bar{E}_1 \cup \bar{E}_2)$	iii) $\frac{26}{32}$
d) $P(E_1/\bar{E}_2)$	iv) $\frac{26}{29}$
	v) $\frac{13}{16}$

23. Box I contains 30 cards numbered 1 to 30. Box B_2 contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The no on the card is found to be non-prime number. The probability that the card was drawn from box I is

1) $\frac{2}{13}$ 2) $\frac{8}{17}$ 3) $\frac{4}{17}$ 4) $\frac{2}{5}$

24. E_1, E_2, E_3 be pair wise independent events with $P(E_1)>0$ and $P(E_1 \cap E_2 \cap E_3)=0$ Then $P(E_2{}^C \cap E_3{}^C/E_1)=0$ is

1) $P(E_1{}^C \cap E_3{}^C)=0$ 2) $P(E_3{}^C)-P(E_2{}^C)$

3) $P(E_3)-P(E_2{}^C)$ 4) $P(E_3{}^C)-P(E_2)$

25. In a random Experiment a fair die is rolled until two fours are obtained in succession. The probability that the experiment will end at the fifth throw of the die is

A) $\frac{175}{6^5}$ B) $\frac{125}{6^5}$ C) $\frac{150}{6^5}$ D) $\frac{200}{6^5}$

26. Four persons hit a target correctly with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{8}$. If all hit target independently, then the probability that the target would be hit is

1) $\frac{25}{32}$ 2) $\frac{27}{32}$ 3) $\frac{7}{32}$ 4) None

27. A bag contains n coins. It is known that $\frac{1}{4}$ of the coins show tails on both sides. Where as the other coins we fair. One coin is selected at random and tossed. Find the probability that toss results in heads

1) $\frac{3}{8}$ 2) $\frac{5}{8}$ 3) $\frac{5}{8}$ 4) $\frac{1}{2}$

28. Bag I contains 4 white, 2 red balls and Bag II contains 2 white, 3 red balls . A bag is chosen at raondom and a ball is drawn from it. Then

1) Probability of drawing a white ball from bag I is $\frac{1}{2} \times \frac{4}{6} = \frac{1}{3}$ (True/False)

2)Probability of drawing a white ball is $\frac{1}{2} \times \frac{4}{6} + \frac{1}{2} \times \frac{2}{5} = \frac{11}{15}$ (True/False)

3)If the ball drawn is white then the probability that the ball was drawn from bag I is

$$\frac{\frac{1}{3}}{\frac{11}{15}} = \frac{5}{11}$$

29. From a group of 50 students, two sections comprising of 20 & 30 students are formed If Ram & Rahim are two particular students among 50 students, then the

i) Probability that they belong to same section is ___ $\frac{{}^{48}C_{18} + {}^{48}C_{28}}{{}^{50}C_{20}} = \frac{25}{49}$

(True/False)

30. Probability of a person speaks truth is $\frac{4}{5}$. He toss two coins simultaneously and announced the out come as (H, H) Then

1) P[he announcing (H, H) when it is actually (H H)] = $\frac{1}{4} \times \frac{4}{5}$ (True/False)

2) P[he anounsing out come as (H, H)] = $\frac{1}{4} \times \frac{4}{5} + \frac{3}{4} \times \frac{1}{5} = \frac{7}{20}$ (True/False)

3) P[actual out come is (H, H)/annonce the out come as (H, H) $\frac{\frac{1}{4} \times \frac{4}{5}}{7/20} = \frac{4}{7}$ (True/False)

=

Level – 3

1. A bag contains n red , 2 balk balls and another bag contains 2 red and n black balls. One of the two bags is selected at random and two balls are drawn from it at a time when it is known that the two balls drawn are red if the probability that those two balls drawn are from A is $\frac{6}{7}$, then n = ___

   A) 6 B) 4 C) 8 D) 7

2. A box B_1 contains 3 blue balls, 6 red balls Another one B_2 contains 8 blue balls and n red balls. A ball is selected at random from a box is found to be red. If P is the probability that this red ball drawn is from box B_2, then

   A) $\frac{1}{7} \le P < \frac{3}{5}$ B) $\frac{1}{7} \le P < \frac{1}{5}$ C) $P \ge \frac{1}{7}$

3. Probability of 5 digit no.s that are made up of exactly two district digits.

   A) $\frac{135}{10^4}$ B) $\frac{{}^9C_2\left(2^5-2\right)+{}^9C_1\left(2^4-1\right)}{9\times10^4}$ C) $\frac{30}{10^4}$ D) None

4. An urn contains 5 red and 2 green balls. A ball is drawn at random from the urn. If the drawn ball is green, then a red ball is added to the urn and if the drawn ball is red, then a green ball is added. The original ball is green ball is added. The original ball is not returned to the urn. Now a second ball is drawn at random from it. The probability that the second ball is red is

   A) $\frac{32}{49}$ B) $\frac{31}{49}$ C) $\frac{30}{49}$ D) $\frac{29}{49}$

5. An unbiased coin is tossed. If the outcome is a head then a pair of unbiased dice is rolled and sum of the numbers obtained on them is noted. If toss of the coin results in tail then a card from a well shuffled pack of 9 cards numbered 1 to 9 is randanly picked and number on the lard is noted. The probability that the number noted The probability that the number noted is 7 or 8

   A) $\frac{18}{65}$ B) $\frac{19}{72}$ C) $\frac{2}{9}$ D) $\frac{11}{36}$

6. Assume that each born child is equally likely to be a boy or a girl. If two families have two children each, then the conditional probability that all children are girls given that at least two are girls is ______

A) $\frac{1}{11}$ B) $\frac{2}{11}$ C) $\frac{10}{11}$ D) 1

RANDOM VARIABLES AND PROBABILITY DISTRIBUTION

Random variable: Let S be the sample space of a random experiment. A function $X:S\to R$ where R is set of all real number is called a random variable.

Note: If X is a random variable, then $X^{-1}(P(R))=P(s)$ where $P(s)$ stands for the power set of S and $P(R)$ is the set of all subsets of real numbers

Probability Distribution function: If $X:S\to R$ is a random variable and 'P' is a probability function associated with it then $F:R\to R$ defined by $F(x)=P(X\le n)$ for each $x\in R$ is called the probability distribution function (p.d.f) of X.

i) $0\le F(x)\le \forall x\in R$

ii) $x_1\le x_2 \Rightarrow F(x_1)\le F(x_2)$

iii) $\lim_{n\to\infty} F(x)=1$ and $\lim_{n\to-\infty} F(x)=0$

iv) $\lim_{t\to x_+} F(t)=F(x)$

Diserete Random Variable: Let $X:S\to R$ be a random variable. If the range of X is either finite or countably infinite then X is called discrete random variable.

Continuous Random Variable: A random variable which can take all real values in an interval (a, b) is called a continuous random variable.

Mean and Variance: Let $X:S\to R$ be a discrete random variable with range $\{x_1,x_2,......x_i,....\}$. If the sum of the infinite series $\sum x_i P(x=x_i)$ is finite is called the mean of X and is denoted by μ.

$$\boxed{Mean, \mu=\sum x_i\, P(x-x_i)}$$

If $\sum (x_i-\mu)^2 P(x=x_i)$ is finite then it is called variance of X and is denoted by σ^2

$$Variance,\ \sigma^2 = \sum (x_1-\mu)^2 P(x=x_i)$$
$$= \sum x_i^2 P(X=x_i) - \mu^2$$

The positive square root of the variance is called the standard deviation of X and is denoted by σ.

Binomial distribution: It was discovered by James Beruouli

If 'n' is a +ve integer $\Rightarrow 16+4p+12=0$

Here p,q are the mutually exclusme and exhaustine units

$\Rightarrow 4p=-28$

A discrete random variable x is said to binomial distribution with parameters n, p

i.e. $\therefore p=-7$ if $P(x=r)={}^nc_r\ p^r q^{n-r}$

Note:

1.The number of trails must be finite

2. Each trail results only two mutually exclusine out comes

3.The probability 'p' then mean, $\mu=np$ and each trail

If $x \sim B(n,\ p)$ then mean $\mu=np$ and various, $\sigma^2=npq$

Poisson distribution: If we know the number of times an event occurred but not how many times it did not occur then the poisson distribution is applicable.

Let $\lambda>0$ be a real number. A random variable x with range $\{0,1,2,.....\}$ is said to have poisson distribution with parameter λ, if $P(X=r)=\dfrac{e^{-\lambda}\cdot\lambda^r}{r!}$ for r = 0, 1, 2....

Note:

1. Each trail in two mutually exclusine out comes formed as success and failure.
2. The probability of a success 'p' is very very small.

If x is a poisson variate with parameter λ then mean, μ = variance, $\sigma^2=\lambda$

Level – I

1. **Column I** **Column II**

Column I	Column II
A) $\sum P(x=x_i)$	1. variance
B)Mean & Variance are same in	2. $\sigma^2+\mu^2$
C) $\sum (x_i-\mu)^2 P(x=x_i)$	3.1
D) $\sum x_i^2 P(x=x_i)$	4.Poisson distribution

a) $A\to4, B\to3, C\to2, D\to1$

b) $A\to3, B\to4, C\to1, D\to2$

c) $A\to2, B\to1, C\to4, D\to3$

d) $A \to 1, B \to 2, C \to 3, D \to 4$

2. **Column I** / **Column II**

Column I	Column II
A) e^{λ}	1. $^{n}c_r p^r q^{n-r}$
B) In P.D, P(X = r)	2. $\sum_{r=0}^{n} \frac{\lambda^r}{r!}$
C) In B.D, P(X = r)	3. n_P
D) In B.D, mean	4. $\frac{e^{-\lambda} \cdot \lambda^r}{r!}$

a) $A \to 1, B \to 2, C \to 3, D \to 4$

b) $A \to 2, B \to 3, C \to 4, D \to 1$

c) $A \to 2, B \to 4, C \to 1, D \to 3$

d) $A \to 1, B \to 3, C \to 2, D \to 4$

II. State 'true or false'

1. An experiment with two possible outcomes is called a Bernouli traul ( )
2. If 'P' is the probability of success and 'q' is the probability of faiture the P–2=1
3. A random variable which can take all real values in an interval (a, b) is called continuous random variable ( )
4. $\sigma^2 = \sum x_i^2 P(x = x_i) - \mu^2$ ( )
5. The non-negative square root of variance is called mean ( )
6. In Binomial distribution the number 'n' of trails is infinite ( )
7. In poisson distribution the trails are independent on each other ( )
8. The mean and variance in poisson distribution are unequal ( )
9. If a sample space is countable them it is called a disrete sample space ( )
10. $\sigma^2 + \mu^2 = E(x^2)$

III. Multiple choice questions:

1. For a random experiment of fossing two coins symeultanensly. The probability of getting two heads is [ ]

   a) 1 b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{8}$

2. In a random experiment of throwing a die, the probability distribution of $X = X_i$, the number '3' on the face of the die is [ ]

   a) $\frac{1}{6}$ b) $\frac{2}{6}$ c) $\frac{3}{6}$ d) $\frac{4}{6}$

3. The range of random variable X = {1,2,3,....} and $P(X = k) = \frac{C^K}{K^1}$ then the value of 'c' is [ ]

a) $\log_c^1$ b) $\log_c^2$ c) $\log_c^3$ d) $\log_c^4$

4. In Binomial distribution the number of 'n' trails is [ ]

a) 0 b) 1 c) finite d) infinite

5. The mean and variance of binomial distribution are 4 and 3 respectively. Then the value of 'n' is [ ]

a) 3 b) 4 c) 9 d) 16

6. A person variable satisfies P(X=1)=P(X=2) then the value of λ is [ ]

a) –2 b) –1 c) 1 d) 2

7. If the standard deviation of the binomial distribution $(P+2)^{16}$ is 2 then the mean is [ ]

a) 2 b) 3 c) 4 d) 5

8. If the mean of the binomial distribution with 9 trails is 6, then its variance is [ ]

a) 2 b) 3 c) 4 d) 5

9. If the mean of the passion distribution is 2.56 then the standard deviation is [ ]

a) 0.256 b) 25.6 c) 0.16 d) 1.6

10. In a book of 450 pages, there are 400 typographical errors. Assuming that the no of errors per page follow the poisson law, the average number of errors per page in the book is [ ]

a) $\frac{8}{9}$ b) $\frac{9}{8}$ c) $\frac{4}{5}$ d) $\frac{5}{4}$

11. Out of 10,000 families with 4 children each, the probability number of families all of whose children are daughters is [ ]

a) 125 b) 625 c) 1250 d) 2500

12. A random variable x has its range {0, 1, 2} = $3k^3$, $P(x=1)=4k-10k^2$ and $P(x=2)=5k-1$ where k is constant then k = [ ]

a) 0 b) $\frac{1}{3}$ b) $\frac{2}{3}$ d) 3

KEY

I.	1. b	2. c			
II.	1. True	2. False	3. True	4.True	5.False
	6.False	7.True	8.False	9.True	10.True
III.	1. c	2. a	3.b	4.c	5.d
	6.d	7.c	8.a	9.d	10.a
	11.b	12.b			

Level – II

I. Fill in the blanks:

1. The probability of gessing atleast 6 out of 10 answers in time or false examination is ____

2. On an average rain falls 12 days in every 30 days. The probability that rain will fall on joint 3 days of a given week is ___________

3. In a poisson distribution, $P(X=0)=2p(X=1)$ then the standard deviation is _____

4. A telephone exchange receives on are average 180 calls per hour. Then the probability that it will receive only 2 calls in a given minute is _____

5. If the range of a random variable X is {0, 1, 2, 3, 4,.....} with $P(X=k)=\frac{(k+1)c}{3^k}$ then the value of c is _______

6. If the difference between the mean and variance of the binomial distribution for 5 trails is $\frac{5}{9}$ then the distribution is _____

7. Five coins are tossed 3200 times using the poisson distribution. The probability of gelting heads 2 times is ____

8. The probability that a person chose of random is lift hunded (in hand writing) is 0.1 then the probability that is a group of 10 people there is one who is left handed is ____

9. 8 coins are toned simultaneously. Then the probability of getting atleast six heads is ____

10. A poisson variable satisfies P(X=1) = P(X=2) then P(X=5) is ____

11. The probability that a student is not a swimmer is $\frac{1}{5}$. The probability that a out of 5 students exactly 4 one swimmer is ______

12. A cubical die in thrown. The mean of X that the member on the face shows up is ____

13. One in 9 ships is likely to be arecked when they are set on soil when 6 ships set on soil the probability atleast 1(one) will arrive safely is _____

14. $1+\frac{\lambda}{1!}+\frac{\lambda^2}{2!}+\frac{\lambda^3}{3!}+......=$______

Level –2

II. Multiple choice questions:

1. The probability distribution of a radom variable X is given below.

X = x	0	1	2	3
P(X=n)	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$

Then the variance of X is [ ]

a) 0.1 b) 1 c) 0.2 d) 2

2. The probability distribution of a random variable X is given below [ ]

X = x	–2	–1	0	1	2	3
P(X=n)	$\frac{1}{10}$	K	$\frac{1}{5}$	2K	$\frac{3}{10}$	K

a) 0.1 b) 0.2 c) 0.3 d) 0.4

3. X follows binomial distribution with parameters n = 100, $p=\frac{1}{2}$ then P(X=r) is maximum when r = [ ]

a) 32 b) 33 c) 50 d) 67

4. If the standard deviation of the binomial distribution $(2+p)^{16}$ is 2 the mean is [ ]

a) 4 b) 8 c) 12 d) 16

5. In a poisson distribution the variance is in the sum f the terms in odd places in this distribution is [ ]

a) $e^{-m}\sinh m$ b) $e^{-m}\operatorname{cosech} m$ c) $e^{-m}\operatorname{sech} m$ d) $e^{-m}\cosh m$

6. The probability that a bomb dropped from a plane strikes the tangent is 1/5 the probability that out of six bombs dropped at least 2 bombs strike the target is [ ]

a) 0.345 b) 0.246 c) 0.543 d) 0.426

7. The probability that an individual suffers a had reaction from an injection is 0.001. The probability that out of 2000 individuals exactly three will suffer bad reaction is[ ]

a) $\frac{1}{3e^2}$ b) $\frac{2}{3e^2}$ c) $\frac{8}{3e^2}$ d) $\frac{4}{3e^2}$

8. In a poisson variate X, if $P(X=0)=0.2$ find the variance of the distribution is [ ]

a) log2 b) log5 c) log6 d) log 8

9. If the mean of the binomial distribution is 25. Then the standard deviation lies in the interval [ ]

a) $(0, 5)$ b) $[0, 5]$ c) [0, 5) d) (0, 5]

10. for a binomial variate X with n = 6, if $P(X=2)=9(PX=4)$, then the variance is [ ]

a) 6/9 b) 9/6 c) 8/9 d) 9/8

11. If the mean and variance of a binomial variate X and 2 and 1 respectively , then $P(X\geq 1)$ is [ ]

a) $\frac{2}{3}$ b) $\frac{4}{5}$ c) $\frac{7}{8}$ d) $\frac{15}{16}$

12. Let X be a binomially distributed variate with mean 10 and variance 5. Then $P(X>10)$ is

a) $\frac{1}{20}\sum_{r=1}^{20} {}^{20}C_r$ b) $\frac{1}{2^{20}}\sum_{r=1}^{20} {}^{n}C_r$ c) $\frac{1}{2^{20}}\sum_{r=1}^{20} {}^{20}C_r$ d) $\sum_{r=11}^{20} {}^{20}C_r \frac{1}{2^r}\cdot\left(\frac{2}{3}\right)^{20-r}$

13. Consider 5 independent Bernouli‘s trails each with probability of success P. If the probability of at least one failure is greater than or equal to $31/32$ the P lies in the interval [ ]

a) $\left[0, \frac{1}{2}\right]$ b) $\left(0, \frac{1}{2}\right)$ c) $\left(\frac{1}{2}, \frac{3}{4}\right]$ d) $\left[\frac{1}{2}, \frac{3}{4}\right)$

14. If the mean and variance of a binomial variate X and 8 and 4 respectively then $P(X\ C_3)=$ [ ]

a) $\frac{265}{2^{15}}$ b) $\frac{265}{2^{16}}$ c) $\frac{137}{2^{15}}$ d) $\frac{137}{2^{16}}$

15. If X is a proisson variate such that $P(X=2)=9P(X=4)+90P(X=6)$ then mean of X is [ ]

a) $\frac{1}{2}$ b) $\frac{3}{2}$ c) 1 d) 2

Level – II Key

I. 1. $\sum_{k=6}^{10} {}^{10}C_k \left(\frac{1}{2}\right)^{10}$ 2. ${}^{7}C_3\left(\frac{2}{5}\right)^3\left(\frac{3}{5}\right)^4$ 3. $\frac{1}{\sqrt{2}}$ 4. 0.23 5. $\frac{4}{9}$

6. $\left[\frac{2}{3}+\frac{1}{3}\right]^5$ 7. $\frac{e^{-100}\times(100)^2}{2}$ 8. $(0.9)^9$ 9. $\left(\frac{1}{2}\right)^8\left[{}^8C_6+{}^8C_7+{}^8C_8\right]$

10. $\frac{e^{-2}\times 2^5}{5!}$ 11. $\left(\frac{4}{5}\right)^4$ 12. $\frac{7}{2}$ 13. $1-{}^6C_0\left(\frac{8}{9}\right)^0\left(\frac{1}{9}\right)^6=1-\frac{1}{9^6}$

14. $e^{\lambda}\ (or)\sum_{r=0}^{n}\frac{\lambda^n}{r!}$

II.

1.b	2.a	3.c	4.b	5.d
6.a	7.d	8.b	9.c	10.d
11.d	12.b	13.a	14.d	15.c