PHYSICS (2 nd YEAR)IIT Material

waves

Introduction

There are essentially two ways of transporting energy from the place where it is produced to the place where it is desired to be utilized. The first involves the actual transport of matter. For example, a bullet fired from a gun carries its kinetic energy with it which can be used at another location. The second method by which energy can be transported is much more useful and important, it involves what we call a wave process.
A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of **matter.** Waves are every where whether we recognize or not, we encounter waves on a daily basis. Sound waves, visible light waves, radio waves, ripples on water surface, earthquake waves and waves on a string are just a few examples of waves.
Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one dimensional, ripples on liquid surface are two dimensional, while sound and light waves are three dimensional.

Types of Waves

Waves can be classified in a number of ways based on the following characteristics

On the basis of necessity of medium:

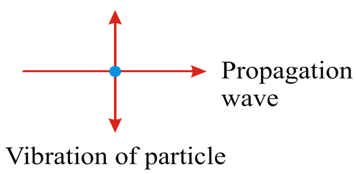
i)Mechanical waves: Require medium for their propagation e.g., Waves on string and spring, waves on water surface, sound waves, seismic waves.

ii) Non-mechanical waves: Do not require medium for their propagation are called e.g., Electromagnetic waves like, light, heat (Infrared), radio waves, γ-rays, x-rays etc.

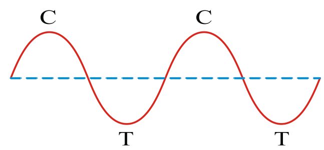
On the basis of vibration of particle:

On the basis of vibration of particle of medium waves can be classified as transverse waves and longitudinal waves.

1) **Transverse waves:** i) Particles of the medium vibrates in a direction perpendicular to the direction propagation of wave



ii) It travels in the form of crests(C) and troughts(T)



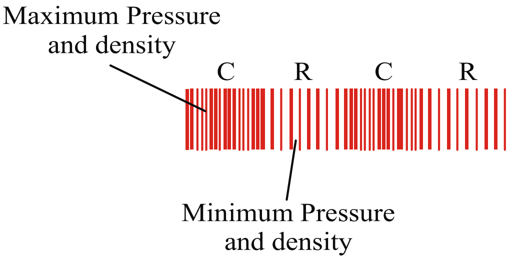
iii) Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they cannot be trnasmitted into liquids and gases.
iv) Medium should posses the property of rigidity

v) Transverse waves can be polarised.
vi) Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope waves setup on the surface of water.

2) Longitudinal waves :

i) Particles of a medium vibrate in the direction of wave motion.

ii) It travels in the form of compression (C) rarefaction (R).



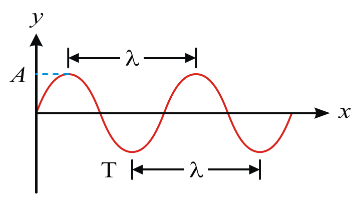
iii) These waves can be transmitted through solids, liquids and gases because for propagation, volume elasticiy is necessary.
iv) Medium should posses the property of elasticity.
v) Longitudinal waves can not be polarized.
vi) Sound waves travel through air, vibration of air column in organ pipes vibration of air column above the surface of water in the tube of resonance apparatus.

On the basis of energy propagation:

i) Progressive wave: These waves advances in a medium with definite velocity. These waves propagate energy in the medium. Eg: Sound wave and light waves.
ii) Stationary wave: These waves remains stationary between two boundaries in medium. Energy is not propagated by these waves but it is confined in segments (or loops) e.g., Wave in a string, waves in organ pipes.

Simple Harmonic wave

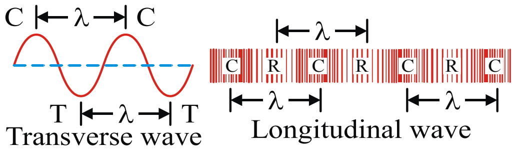
When a wave passes through a medium, if the particles of the medium execute simple harmonic vibrations, then the wave is called a simple harmonic wave. A graph is drawn (fig.) with the displacement of the particles from their mean positions, at any given instant of time, on the y-axis and their location from origin on x-axis.



Characteristics of wave:

1. **Amplitude (A):** Maximum displacement of a vibrating particle of medium from its' mean position is called amplitude.
2. **Wavelength (λ) :** It is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.
Or

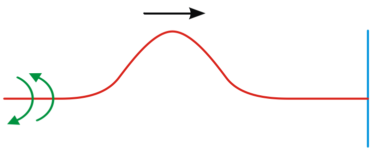
Distance travelled by the wave in one time period is known as wavelength.
Or
It is the distance between the two successive points with same phase.



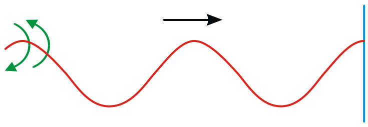
3. Frequency (n): Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.
(Or)it is the number of complete wavelengths traversed by the wave in one second.
Unit of frequency is hertz (Hz) or per second.

4. Time period (T): Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position
Or it is the time taken by the wave to travel a distance equal to one wavelength.
Time period = 1/Frequency $\Rightarrow T = 1/n$

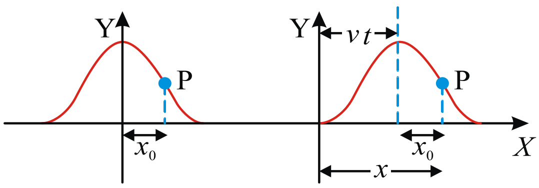
5. Wave pulse: It is a short wave produced in a medium when the disturbance is created for a short time.



6. Wave train: A series of wave pulse is called wave train.



7. Wave function: It is a mathematical description of the disturbance created by a wave. For a string, the wave function is a displacement. For sound waves it is a pressure or density fluctuation where as for light waves it is electric or magnetic field.
Now let us consider a one dimensional wave travelling along x-axis. During wave motion, a particle with equilibrium position x is displaced some distance y in the direction perpendicular to the x-axis. In this case y is a function of position (x) and time (t).
i.e., y = f(x, t). This is called wave function.
Let the wave pulse be travelling with a speed v. After a time t, the pulse reaches a distance vt along the +x-axis as shown. Thus the motion of the particle P^1 at distance 'x' at time 't' is same as the motion of the particle P at time $t = 0$ at position $x_0 = x - vt$. Hence the wave functioon now can be represented as $y = f(x - vt)$.



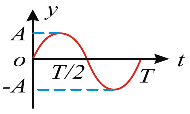
(A) Pulse at time $t=0$ (B) Pulse after time t

In general, then we can represent the transverse position y for all positions and times, measured in stationary frame with the origin at O, as
y(x, t) = f(x – vt) .... (i)
Similarly, if the pulse travels to the left, the transverse position of elements of the string is described by

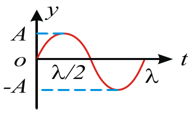
y(x, t) = f(x + vt) .... (ii)

The function y, sometimes called the wave function, depends on the two variables x and t. For this reason, it is often written y(x, t), which is read "y as a function of x and t".

Note-1: The equation y = f(vt – x) represents the displacement of the particle at x = 0 as time passes



Representation of $y = f\left(t - \frac{x}{v}\right)$



Representation of $y = f(x - vt)$

Note-2: If order of a wave function to represent a wave, the three quantities x, v, t must appear in combinations $(x + vt)$ or $(x - vt)$.

Thus $y = (x - vt)^2, \sqrt{(x - vt)}, Ae^{-B(x-vt)^2}$ etc., represents travelling waves while $y = (x^2 - v^2t^2), (\sqrt{x} - \sqrt{vt}),\ A\sin(4x^2 - 9t^2)$ etc. do not represent a wave.

8. Harmonic wave: If a travelling wave is a sin or cos function of $(x \pm vt)$ the wave is said to be harmonic or plane progressive wave.

9. The differential form of wave equation:

All the travelling waves satisfy a differential equation which is called the wave equation. It is given by $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$; where $v = \frac{\omega}{k}$

It is satisfied by any equation of the form $y = f(x \pm vt)$

10. Angular wave number (or) propagation constant (k): Number of wavelengths in the distance 2π is called the wave number or propagation constant i.e., $k = \frac{2\pi}{\lambda}$

It is unit is rad/m.

11. Wave velocity (v): It is the distance travelled by the disturbance in one second. It only depends on the properties of the medium and is independent of time and position.

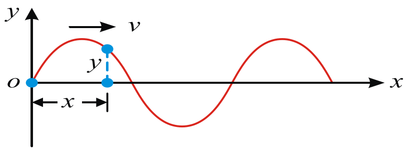
$$v = n\lambda = \frac{\lambda}{T} = \frac{\omega\lambda}{2\pi} = \frac{\omega}{k}$$

12. Phase: Phase gives the state of the vibrating particle at any instant of time as regards to its position and direction of motion.

- Phase is the angular displacement from its mean position. $\theta = (\omega t \pm kx)$
- If phase is constant then the shape of wave remains constant.

Equation of Progressive Wave :

1. If during the propagation of a progressive wave, the particles of the medium perform SHM about their mean position, then the wave is known as a harmonic progressive wave.
2. Suppose a plane simple harmonic wave travels from the origin along the positive direction o f x-axis from left to right as shown in the figure



The displacement y of a particle at O from its mean position at any time t is given by
$y = A\sin\omega t$. --- (1)

The wave reaches the particle P after time $t = \frac{x}{v}$.

So that the motion of the particle 'P' which is at a distance 'x' at a time 't' is same as motion of the particle at $x = 0$, at the earliear time $t - \frac{x}{v}$.

Hence the displacement 'y' of the particle 'P' at 'x' at a time 't' in equation (1) by $\left(t - \frac{x}{v}\right)$.

$$y = A\sin\omega\left(t - \frac{x}{v}\right) = A\sin(\omega t - kx)\left(\because k = \frac{\omega}{v}\right)$$

In general along x-axis, y = A Sin (ωt $\pm$ kx)

\+ sign for a wave travelling along -ve X direction

\- sign for a wave travelling along +ve X direction

where y is displacement of the particle after a time t from mean position, x is displacement of the wave, A is Amplitude.

ω is angular frequency or angular velocity

$\omega = 2\pi / T$ = $2\pi n$

k is propagation constant & k = $2\pi/\lambda$

- For a given time 't', $y - x$ graph gives the shape of pulse on string.

Various forms of progressive wave function:

(i) $y = A\sin(\omega t \pm kx)$ (or) $y = A\sin(kx \pm \omega t)$

(ii) $y = A\cos(\omega t \pm kx)$ (or) $y = A\cos(kx \pm \omega t)$

(iii) $y = A\sin\left(\omega t \pm \frac{2\pi}{\lambda}x\right)$

(iv) $y = A\sin 2\pi\left[\frac{t}{T} \pm \frac{x}{\lambda}\right]$

(v) $y = A\sin\frac{2\pi}{T}\left(t \pm x\frac{T}{\lambda}\right)$

(vi) $y = A\sin\frac{2\pi}{\lambda}(vt \pm x)$

(vii) $y = A\sin\omega\left(t \pm \frac{x}{v}\right)$

(viii) $y = A\sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right)$

General Expression for a Sinusoidal Wave

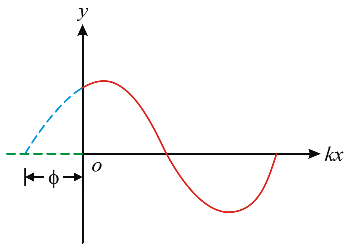
$Y = A\sin(kx - \omega t + \phi)$ (or) $Y = A\sin(\omega t - kx + \phi)$

where ϕ is the phase constant, just as we learned in our study of periodic motion. This constant can be determined from the initial conditions.

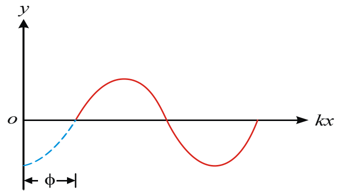
Positive and Negative Initial Phase Constants.

In general, the equation of a harmonic wave travelling along the positive x-axis is expressed as $y = A\sin(kx - \omega t \pm \phi)$. Where ϕ is called the initial phase constant. It determines the initial displacement of the particle at x = 0 when t = 0.

i)Positive initial phase constant $y = A\sin(kx - \omega t + \phi)$ .The sine curve starts from the left of the origin.

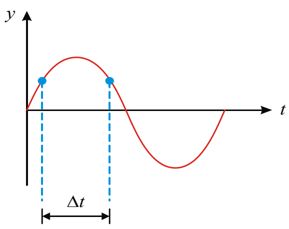


ii) Negative initial phase constant $y = A\sin(kx - \omega t - \phi)$. The sine curve starts from the left of the origin.



Change in Phase with time for a constant x, i.e., at a fixed point in the medium

$$[\phi]_{t_1} = 2\pi\left(\frac{t_1}{T} - \frac{x}{\lambda}\right) + \phi; [\phi]_{t_2} = 2\pi\left(\frac{t_2}{T} - \frac{x}{\lambda}\right) + \phi$$



(For the wave travelling in positive x-direction)

$$\Delta\phi = [\phi]_{t_2} - [\phi]_{t_1} = \frac{2\pi}{T} \times (t_2 - t_1) = \frac{2\pi}{T} \times \Delta t$$

$$\Rightarrow \Delta\phi = \frac{2\pi \times \Delta t}{T}$$

Phase difference $= \frac{2\pi}{T} \times Time\ difference$

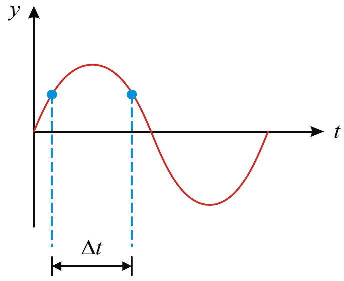
Variation of Phase with Distance

At a given instant of time t = t, phase at $x = x_1$,

$$[\phi]_{x_1} = 2\pi\left(\frac{t}{T} - \frac{x_1}{\lambda}\right) + \phi$$

(For the wave travelling in positive x-direction and phase at $x = x_2$,

$$[\phi]_{x_2} = 2\pi\left(\frac{t}{T} - \frac{x_2}{\lambda}\right) + \phi$$



$$\Rightarrow \Delta\phi = [\phi]_{x_2} - [\phi]_{x_1} = \frac{2\pi}{\lambda}(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta x$$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$$

i.e., Phase difference $= \frac{2\pi}{\lambda} \times Path\ difference$

Particle Velocity: The rate of change of displacement y w.r.t time t is known as particle velocity.

Hence from $y = A\sin(\omega t - kx)$

Particle velocity, $v_p = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$

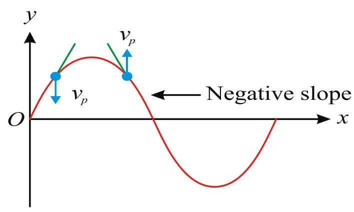
Maximum particle velocity $(v_p)_{max} = A\omega$

Also $\frac{\partial y}{\partial t} = -\frac{\omega}{k} \times \frac{\partial y}{\partial x}$

- Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of wave at that point i.e.

$$v_{particle} = -v_{Wave}\left(\frac{\partial y}{\partial x}\right)$$

Particle velocity = –(wave velocity) × slope of wave curve



ENERGY, POWER AND INTENSITY OF A WAVE:

If a wave given by $y = A\sin(\omega t - kx)$ is propagating through a medium, the particle velocity will be $v_p = \frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$

If ρ is the density of the medium, kinetic energy of the wave per unit volume will be

$$= \frac{1}{2}\rho\left[\frac{\partial y}{\partial t}\right]^2 = \frac{1}{2}\rho\omega^2 A^2\cos^2(\omega t - kx)$$

and its maximum value will be equal to energy per unit volume i.e., energy density U.

$$U = \frac{1}{2}\rho A^2\omega^2$$

The energy associated with a volume $\Delta V = S\Delta x$ will be (where 'S' is the area of cross section).

$$\Delta E = U\Delta V = \frac{1}{2}\rho A^2\omega^2 S\Delta x$$

The power (rate of transmission of energy) will be $P = \frac{\Delta E}{\Delta t} = \frac{1}{2}\rho v\omega^2 A^2 S$

$\left[\text{as } \frac{\Delta x}{\Delta t} = v, \text{(Speed of wave)}\right]$

Intensity is defined as power per unit area.

$$I = \frac{\Delta E}{S\Delta t} = \frac{P}{S} = \frac{1}{2}\rho v\omega^2 A^2 = 2\pi^2 f^2 A^2 \rho v$$

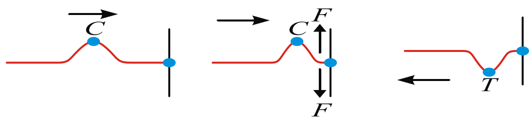
If frequency f is constant then $I \propto A^2$

Reflection and Refraction of Waves : When waves are incident on a boundary between two media a part of incident waves returns back into the initial medium (reflection) while the remaining is partly absorbed and partly transmitted into the second medium (refraction)

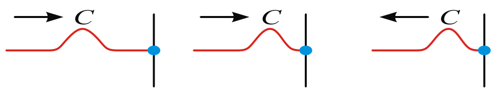
Boundary conditions: Reflection of a wave pulse from some boundary depends on the nature of the boundary.

Rigid end: When the incident wave reaches a fixed end, it exerts an upward pull on the end, according to Newton's third law the fixed end exerts an equal and opposite downward force on the string. It result as inverted pulse or phase change of π.

Crest (C) reflects as trough (T) and vice-versa. Time changes by $\frac{T}{2}$ and Path changes by $\frac{\lambda}{2}$



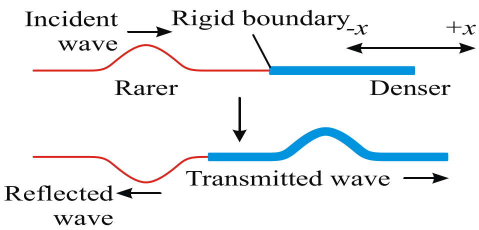
Free end: When a wave or pulse is reflected from a free end, then there is no change of phase (as there is no reaction force).
Crest (C) reflects as crest (C) and trough (T) reflects as trough (T), Time changes by zero and Path changes by zero.



Note: Exception: Longitudinal pressure waves suffer no change in phase from rigid end. i.e., compression pulse reflects as compression pulse. On the other hand if longitudinal pressure wave reflects from free end, it suffer a phase change of π, i.e., compression reflects as rarefaction and vice-versa.

Wave in a combination of string

(i) Wave goes from thin to thick string



Incident wave $y_i = a_i \sin(\omega t - k_1 x)$

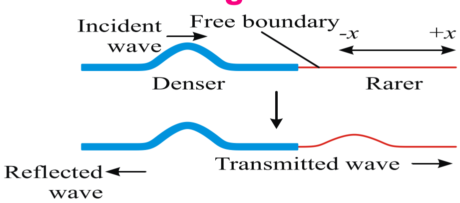
Reflected wave

$$y_r = a_r \sin[\omega t - k_1(-x) + \pi]$$

$$= -a_r \sin(\omega t + k_1 x)$$

Transmitted wave, $y_t = a_t \sin(\omega t - k_2 x)$

(ii) Wave goes from thick to thin string



Incident wave $y_i = a_t \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin[\omega t - k_1(-x) + 0] \quad = a_r \sin(\omega t + k_1 x)$

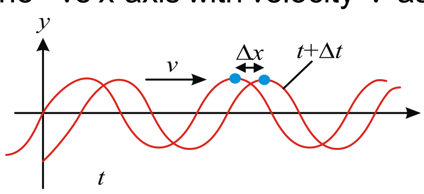
Transmitted wave $y_t = a_t \sin(\omega t - k_2 x)$

Note: Ratio of amplitudes: It is given as follows

$$\frac{a_r}{a_i} = \frac{k_1 - k_2}{k_1 + k_2} = \frac{v_2 - v_1}{v_2 + v_1} \text{ and } \frac{a_t}{a_i} = \frac{2k_1}{k_1 + k_2} = \frac{2v_2}{v_1 + v_2}$$

The Speed of A Travelling Wave

i) Let a wave moves along the +ve x-axis with velocity 'v' as shown in fig.



ii) Let a crest shown by a dot $(\bullet)$ moves a distance Δx in time Δt. The speed of the wave is $v = \Delta x / \Delta t$.

iii) We can put the dot $(\bullet)$ on a point with any other phase. It will move with the same speed *v* (otherwise the wave pattern will not remain fixed).

iv) The motion of a fixed phase point on the wave is given by, $y = \sin(kx - \omega t)$.

v) For the same particle displacement 'y' at two different positions, $kx - \omega t = \text{constant}$ -----(1)

$\Rightarrow k\Delta x - \omega \Delta t = 0$

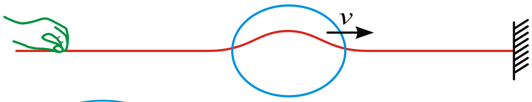
$\Rightarrow \frac{\Delta x}{\Delta t} = \frac{\omega}{k} \Rightarrow v = \frac{\Delta x}{\Delta t} = \frac{\omega}{k}$

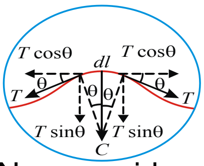
$\Rightarrow v = \frac{2\pi n}{2\pi / \lambda} = n\lambda$

$(\because \omega = 2\pi n$ and $k = 2\pi / \lambda)$

Speed of transverse wave in a string

i) Let a transverse pulse is travelling on a stretched string as shown in fig(a).

a) 

b) 

ii) Now consider a small element of length *dl* on this pulse as shown fig (b). Let this element is forming an arc of radius R and subtending an angle 2θ at center of curvature C.

iii)We can see that two tensions T are acting on the edges of *dl* along tangential directions as shown.

iv)The horizontal components of these tensions cancel each other, but the vertical components add to form a radial restoring force in downward direction, which is given as

$F_R = 2T\sin\theta \approx 2T\theta$ $\qquad$ $(as \sin\theta \approx \theta)$

$= T\frac{dl}{R}$ .....(1) $\qquad$ $\left[2\theta = \frac{dl}{R}\right]$

v) If 'μ' be the mass per unit length (Linear density) of the string, the mass of this element is given as $dm = \mu dl$. In our reference frame if we look at this element, it appears to be moving toward left with speed v then we can say that the acceleration of this element in our reference frame is

$a = \frac{v^2}{R}$ .....(2)

Now from equations (1) and (2) we have

$F_R = \frac{dmv^2}{R}$ or $T\frac{dl}{R} = \frac{(\mu dl)v^2}{R}$

or $v = \sqrt{\frac{T}{\mu}}$ .....(3)

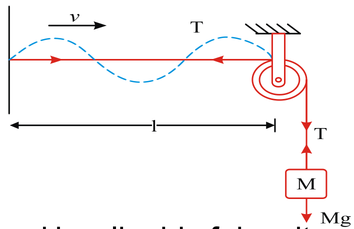
Special cases:

1. If A is the area of cross-section of the wire then linear density $\mu = M / L = \rho AL / L = \rho A$

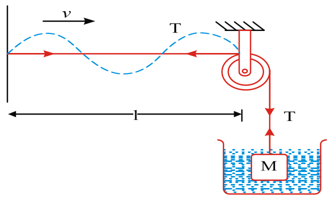
   $\Rightarrow v = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{S}{\rho}}$; where $S = Stress = \frac{T}{A}$

2. If string is stretched by some weight then

   T = Mg $\Rightarrow v = \sqrt{\frac{Mg}{\mu}}$



3. If suspended weight is immersed in a liquid of density σ and ρ = density of material of the suspended load then

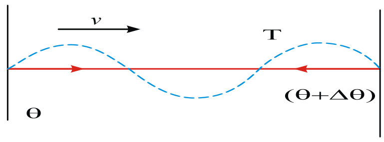


$$T = Mg\left(1-\frac{\sigma}{\rho}\right) \Rightarrow v = \sqrt{\frac{Mg(1-\sigma/\rho)}{\mu}}$$

4. If v_1, v_2 are the velocities of transverse waves while the load is in air medium and in water medium respectively, the relative density of material of load is $d = \frac{v_1^2}{v_1^2 - v_2^2}$

5. If v_1, v_2 and v_3 are the velocities of transverse waves while the load is in air, in water and in a liquid mediums respectively, the relative density of material of load is $d = \frac{v_1^2 - v_3^2}{v_1^2 - v_2^2}$.

6. If the temperature a string varies through $\Delta\theta$ then the thermal force(tension) developed due to elasticity of string is $T = YA\alpha\Delta\theta$

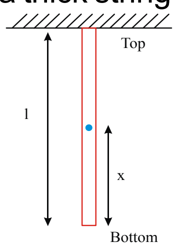


$$\therefore v = \sqrt{\frac{YA\alpha\Delta\theta}{\mu}} = \sqrt{\frac{Y\alpha\Delta\theta}{\rho}}$$

where Y = Young's modulus of elasiticyt of string, A = Area of cross section of string,

α = Temperature coefficient of thermal expansion, ρ = Density of wire $= \frac{\mu}{A}$

7. Velocity of wave in vertical strings. If a thick string is suspended vertically then



Velocity at the bottom $v_B = 0$

($\because$ tension $T_B = 0$)

Velocity at the top $v_T = \sqrt{\frac{T_T}{\mu}} = \sqrt{\frac{mg}{\mu}} = \sqrt{gl}$

($\because$ tension $T_B = mg = \mu lg$)

The average velocity of wave

$v_{avg} = \frac{v_T + v_B}{2} = \frac{\sqrt{gl}}{2}$

$\therefore$ The time taken by the transverse pulse generated at bottom to reach the top is given by

$t = \frac{l}{v_{avg}} = 2\sqrt{\frac{l}{g}}$

Note: Velocity at a distance x from bottom $v = \sqrt{gx}$

The time taken to reach the point p from bottom is $v_x = \frac{x}{v_{avg}} = 2\sqrt{\frac{x}{g}}$

EX-1: ***A longitudinal progressive wave is given by the equation y = 5 × 10^{-2} sinπ (400 t + x)m. Find (i) amplitude (ii) frequency (iii) wave length and (iv) velocity of the wave. (v) velocity and acceleration of particle at $x = \frac{1}{6}m$ at t = 0.01 s (vi) maximum particle velocity and acceleration.***

Sol. Comparing with the general equation of the progressive wave $y = A\sin(\omega t + kx)$ we find, ω = 400π and k = π

We find

(i) A = 5 × 10^{-2} m.

(ii) $n = \frac{\omega}{2\pi} = \frac{400\pi}{2\pi} = 200Hz$

(iii) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi} = 2m$

(iv) $v = \frac{\omega}{k} = \frac{400\pi}{\pi} = 400ms^{-1}$

(v) $v_p = A\omega\cos t(\omega t + kx) = 10\sqrt{3}ms^{-1}$ $\quad$ $a_p = -A\omega^2\sin(\omega t + kx) = -4\times10^4 ms^{-2}$

(vi) $v_{\max} = A\omega = 20\pi\ ms^{-1}$

$a_{\max} = A\omega^2 \Rightarrow 8\times10^4 ms^{-2}$

EX-2: ***The wave function of a pulse is given by*** $\mathbf{y = \frac{3}{(2x+3t)^2}}$ ***where x and y are in metre and t is in second.***

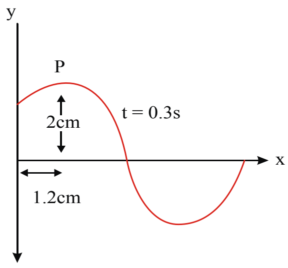
(i) Identify the direction of propagation.

(ii) Determine the wave velocity of the pulse.

Sol. (i) Since the given wave function is of the form y = f(x + vt), therefore, the pulse travels along the negative x-axis.

(ii) Since 2x + 3t = constant for the same particle displacement 'y'. Therefore, by differentiating with respect to time, we get $2\frac{dx}{dt} + 3 = 0 \Rightarrow v = \frac{dx}{dt} = \frac{-3}{2} = -1.5m/s$

EX-3: ***Figure shows a snapshot of a sinusoidal travelling wave taken at t = 0.3s. The wavelength is 7.5 cm and the amplitude is 2 cm. If the crest P was at x = 0 at t = 0, write the equation of travelling wave.***



Sol. The wave has travelled a distance of 1.2 cm in 0.3s. Hence, speed of the wave, $v = 1.2/0.3 = 4cm/s$ and $\lambda = 7.5cm$

$$\Rightarrow k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.5} = 0.84cm^{-1}$$

$\therefore$ Angular frequency $\omega = vk = 4 \times 0.84$

$= 3.36 rad/s$

Since the wave is travelling along positive x-direction and crest (maximum displacement) is at x = 0 at t = 0, we can write the wave equation as, $y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right)$

(or) $y(x,t) = A\cos(kx - \omega t)$

Therefore, the desired equation is,

$$y(x,t) = (2)\cos\left[(0.84)x - (3.36)t\right]cm$$

EX-4: ***A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible tension. Find the speed of transverse waves in this wire at 10°C if Y = 1.3×10¹¹N/m², $\alpha = 1.7 \times 10^{-5} / °C$ and $\rho = 9 \times 10^3 kg/m^3$***

Sol. $v = \sqrt{\frac{Y\alpha\Delta\theta}{\rho}}$

$$= \sqrt{\frac{1.3 \times 10^{11} \times 1.7 \times 10^{-5} \times (30 - 10)}{9 \times 10^3}} = 70m/s$$

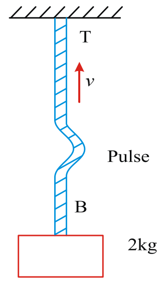
EX-5: ***A 4 kg block is suspended from the ceiling of an elevator through a string having a linear mass density of 19.2 × 10⁻³ kg m⁻¹. Find the speed with which a wave pulse can travel on the string if the elevator accelerates up at 2 ms⁻²? (g = 10 ms⁻²)***

Sol. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M(g+a)}{\mu}}$

$= \sqrt{\frac{4(10+2)}{19.2 \times 10^{-3}}}$ =50ms⁻¹.

EX-6: ***A uniform rope of length 12 m and mass 6 kg hangs vertically from a rigid support. A block of mass 2 kg is attached to the free end of the rope. A transverse pulse of wavelength 0.06 m is produced at the lower end of the rope. What is the wavelength of the pulse when it reaches the top of the rope ?***

Sol. Now as $v = \sqrt{(T/\mu)}$

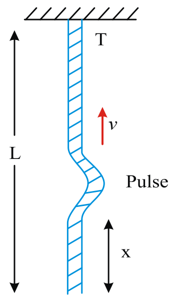


$$\frac{v_T}{v_B} = \sqrt{\frac{T_T}{T_B}} = \sqrt{\frac{(6+2)g}{2g}} = 2$$

So, $\lambda_T = 2\lambda_B = 2 \times 0.06 = 0.12m$

EX-7: ***A uniform rope of mass 0.1 kg and length 2.45m hangs from a ceiling. (a) Find the speed of transverse wave in the rope at a point 0.5m distant from the lower end, b) Calculate the time taken by a transverse wave to travel the full length of the rope (g = 9.8 m/s²)***

Sol. a) If M is the mass of string of length L, the mass of length x of the string will be (M/L)x.



$$\therefore T = \frac{Mx}{L} g$$

So, $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mgx}{L\left(\frac{M}{L}\right)}} = \sqrt{gx}.........(1)$

Hence x = 0.5m

So, $v = \sqrt{0.5 \times 9.8} = 2.21 m/s$

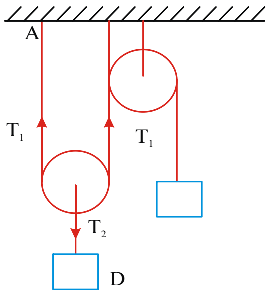
b) $v = \frac{dx}{dt} \Rightarrow \sqrt{gx} = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{\sqrt{gx}}$

$$\Rightarrow \int_0^t dt = \int_0^L \frac{1}{\sqrt{g}} x^{-1/2} dx \Rightarrow t = 2\sqrt{\left(L/g\right)}$$

Here, L = 2.45 m , $\therefore t = 2\sqrt{(2.45/9.8)} = 1\text{s}$

EX-8: ***The strings, shown in figure, are made ofsame material and have same cross-section. The pulleys are light. The wave speed of a transverse wave in the string AB is v_1 and in CD it is v_2. Find v_1/v_2.***

Sol: If T_1 and T_2 are the tensions in strings AB and CD respectively then $T_2 = 2T_1$.



As $v \propto \sqrt{T} \Rightarrow \dfrac{v_1}{v_2} = \sqrt{\dfrac{T_1}{T_2}} \Rightarrow \dfrac{v_1}{v_2} = \dfrac{1}{\sqrt{2}}$

EX-9: ***Two blocks each having a mass of 3.2kg are connected by wire CD and the system is suspended from the ceiling by another wire AB. The linear mass density of the wire AB is 10 g/m and that of CD is 80 g/m. Find the speed of a transverse wave pulse produced in AB and CD and ratio of speeds of transverse pulse in AB to that in CD.***

Sol. Tension in string AB is $T_{AB} = 6.4kg = 64N$

Thus speed of transverse wave in string AB is $v_{AB} = \sqrt{\dfrac{T_{AB}}{\mu_{AB}}} = \sqrt{\dfrac{64}{10\times10^{-3}}} = \sqrt{6400} = 80m/s$

Tension in string CD is $T = 3.2kg = 32N$

Thus speed of transverse waves in string CD is $v_{CD} = \sqrt{\dfrac{T_{CD}}{\mu_{DC}}} = \sqrt{\dfrac{32}{80\times10^{-3}}}$

$= \sqrt{400} = 20m/s \Rightarrow \quad \dfrac{v_{AB}}{v_{CD}} = \dfrac{80}{20} = 4:1$

EX-10 ***A progressive wave travels in a medium M_1 and enters into another medium M_2 in which its speed decreases to 75%. What is the ratio of the amplitude and intensity of the***

a. Reflected and the incident waves, and

b. Transmitted and the incident waves?

Sol. let A_i, A_r and A_t be the amplitudes of the incidents, reflected, and transmitted waves. Given that, velocity in the medium refracted is 75% of that in the initial medium.

$v_2 = \dfrac{3}{4}v_1$

a. $\dfrac{A_r}{A_i} = \dfrac{v_2 - v_1}{v_2 + v_1} = \dfrac{\dfrac{v_2}{v_1} - 1}{\dfrac{v_2}{v_1} + 1} = \dfrac{\dfrac{3}{4} - 1}{\dfrac{3}{4} + 1} = -\dfrac{1}{7}$

i.e., the required ration is $\left|\dfrac{A_r}{A_i}\right| = 1:7$ and $I \propto A^2 \Rightarrow \dfrac{I_r}{I_i} = \dfrac{1}{49}$

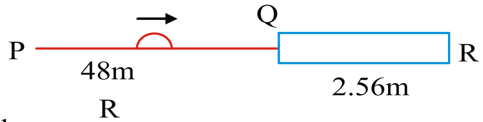
b. $\frac{A_t}{A_i}=\frac{2v_2}{v_2+v_1}=\frac{2v_2/v_1}{\frac{v_2}{v_1}+1}=\frac{2\left(\frac{3}{4}\right)}{\frac{3}{4}+1}=\frac{6}{7}$

i.e., the required ratio is $\left|\frac{A_t}{A_i}\right|=6:7$ and $I\propto A^2\Rightarrow\frac{I_t}{I_i}=\frac{36}{49}$

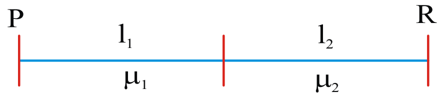
EX-11: ***A long wire PQR is made by joining two wires PQ and QR of equal radii as shown. PQ has length 4.8m and mass 0.06kg. QR has length 2.56 m and mass 0.2kg. The wire PQR is under a tensioon of 80N. A sinusoidal wave pulse of amplitude 3.5cm is sent along the wire PQ from the end P. No power is dissipated during the propagation of the wave pulse.***

a. Find the time taken by the wave pulse to reach the other end R of the wire .

b. The amplitudes of reflected and transmitted wave pulse after incident on the joint Q.



Sol. a.



$$\mu_1=\frac{M_1}{l_1}=\frac{0.06}{4.8}=\frac{1}{80}kg/m$$

$$\mu_2=\frac{M_2}{l_2}=\frac{0.2}{2.56}=\frac{20}{256}kg/m$$

$$v_1=\sqrt{\frac{T}{\mu_1}}=\sqrt{\frac{80}{\frac{1}{80}}}=80m/s$$

$$v_2=\sqrt{\frac{T}{\mu_2}}=\sqrt{\frac{80}{20/256}}=\sqrt{256\times4}=32m/s\left(\because\rho=\frac{\mu}{A}\right)$$

$$t=t_1+t_2=\frac{l_1}{V_1}+\frac{l_2}{V_2}=\frac{4.8}{80}+\frac{2.56}{32}$$

$$=0.06+0.08\ =0.14\text{sec}$$

b. $A_r=\left[\frac{v_2-v_1}{v_2+v_1}\right]A_i=\frac{32-80}{32+80}\times3.5=-1.5cm$

thus $A_r=1.5cm$ and -ve sign represents that the reflected pulse suffers a phase difference of π radian.

$$A_t=\left[\frac{2v_2}{v_1+v_2}\right]A_i=\frac{2\times32}{80+32}\times3.5=2cm.$$

EX-12: ***A wave pulse starts propagating in +ve X-direction along a non-uniform wire of length 'L', with mass per unit length given by $\mu=M_o+\alpha x$ and under a tension of TN. Find the time taken by the pulse to travel from the lighter end (x = 0) to the heavier end.***

Sol. $v = \frac{dx}{dt} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{M_o + \alpha x}}$

$$\int_0^L (M_o + ax)^{1/2} dx = \int_0^L \sqrt{T} dt$$

$$\left[\frac{2}{3}\frac{(M_o + ax)^{3/2}}{\alpha}\right]_0^L = \sqrt{T}[t]_0^t$$

$$t = \frac{2}{3\alpha\sqrt{T}}\left[(M_o + \alpha L)^{3/2} - M_o^{3/2}\right]$$

EX13: ***A stretched string is forced to tranmit transverse waves by means of an oscillator coupled to one end. The string has a diameter of 4 mm. The amplitude of the oscillation is 10^{-4} m and the frequency is 10 Hz. Tension in the string is 100N and mass density of wire $4.2 \times 10^3 kgm^{-3}$. Find***

(a) the equation of the waves along the string

(b) the energy per unit volume of the wave

(c) the average energy flow per unit time across any section of the string

Sol.(a) Speed of transverse wave on the string is $v = \sqrt{\frac{T}{\rho A}}$ $(\because \mu = \rho A)$

$$v = \sqrt{\frac{100}{(4.2 \times 10^3)\left(\frac{\pi}{4}\right)(4.0 \times 10^{-3})^2}}$$

$= 43.53 ms^{-1}$

$\omega = 2\pi n = 20\pi \ rad/s = 62.83$ rad/s

$k = \frac{\omega}{v} = 1.44 m^{-1}$

$\therefore$ Equation of the waves along the string $y(x,t) = A\sin(kx - \omega t)$

$= (10^{-4} m)\sin\left[(1.44 m^{-1})x - (62.83 \text{rad} s^{-1})t\right]$

(b) Energy per unit volume of the string,

$u =$ energy density $= \frac{1}{2}\rho\omega^2 A^2$

$u = \left(\frac{1}{2}\right)(4.2 \times 10^3)(62.83)^2(10^{-4})^2$

$= 8.29 \times 10^{-2} Jm^{-3}$

(c) Average energy flow per unit time P = power

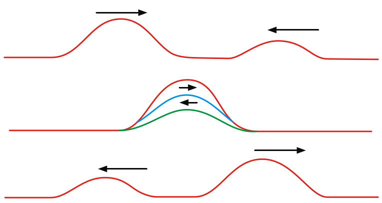
$= \left(\frac{1}{2}\rho\omega^2 A^2\right)(Sv) = (u)(Sv)$

$P = (8.29 \times 10^{-2})\left(\frac{\pi}{4}\right)(4.0 \times 10^{-3})^2(43.53)$

$= 4.53 \times 10^{-5} Js^{-1}$

Principle of Superposition:

1. The displacement at any time due to a number of waves meeting simultaneously at a point in a medium is the vector sum of the individual displacements due to each one of the waves at that point at the same time.
2. If $\vec{y}_1, \vec{y}_2, \vec{y}_3$........... are the displacements at a particular time at a particular position, due to individual waves, then the resultant displacement.
$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

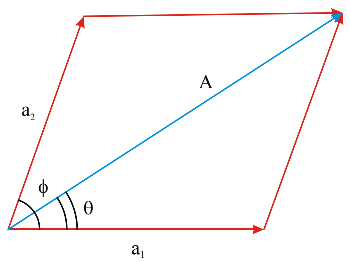


3. Important applications of superposition principle.

i) Interference of waves: Adding waves that differ in phase.
ii) Formation of stationary waves: Adding waves that differ in direction.
iii) Formation of beats: Adding waves that differ in frequency.

INTERFERENCE OF SOUND WAVES

1. When two waves of same frequency, same wavelength, same velocity (nearly equal amplitude) moves in the same direction. Their superimposition results in the interference.
2. Due to interference the resultant intensity of sound at a point is different from the sum of intensities due to each wave separately.
3. Interference is of two type (i) Constructive interference (ii) Destructive interference
4. In interference energy is neither created nor destroyed but is redistributed.
5. For observable interference, the sources (producing interfering waves) must be coherent.
6. Let at a given point two waves arrives with phase difference ϕ and the equation of these waves is given by



$y_1 = a_1 \sin \omega t$, $y_2 = a_2 \sin(\omega t + \phi)$ then by the principle of superposition $\vec{y} = \vec{y}_1 + \vec{y}_2$

$\Rightarrow y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi) = A \sin(\omega t + \phi)$

where $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi}$ and $\tan\theta = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$

Since Intensity (I) $\propto (Amplitude\ \ A)^2$ $\Rightarrow \dfrac{I_1}{I_2} = \left(\dfrac{a_1}{a_2}\right)^2$

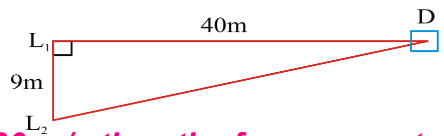
Therefore, the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Table: Constructive and destructive interference	
When the waves meet a point with same phase, constructive interference is obtained at that point (i.e., maximum sound)	When the waves meet a point with opposite phase, destructive interference is obtained at that point (i.e., minimum sound)
Phase difference between the waves at the point of observation $\phi = 0°$(or) $2n\pi$	Phase difference $\phi = 180°$ (or) $(2n-1)\pi$; $n = 1,2,......$
Phase difference between the waves at the point of observation $\Delta = n\lambda$ i.e., even multiple of $\lambda/2$)	Phase difference $\Delta = (2n-1)\frac{\lambda}{2}$ (i.e., odd multiple of $\lambda/2$)
Resultant amplitude at the point of observation will be maximum $A_{max} = a_1 + a_2$ If $a_1 = a_2 = a_0$ $\Rightarrow A_{max} = 2a_0$	Resultant amplitude at the point of observation will be minimum $A_{max} = a_1 - a_2$ If $a_1 = a_2 \Rightarrow A_{max} = 0$
Resultant intensity at the point of observation will be maximum $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $= \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{max} = 4I_0$	Resultant intensity at the point of observation will be minimum $I_{min} = I_1 + I_2 + 2\sqrt{I_1 I_2}$ $= \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ If $I_1 = I_2 = I_0 \Rightarrow I_{min} = 4I_0$

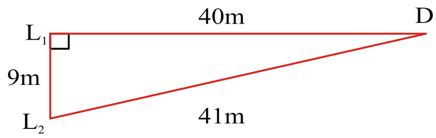
7. $$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \left(\frac{\frac{a_1}{a_2} + 1}{\frac{a_1}{a_2} - 1}\right)^2$$

EX-14: *Two loud speakers L_1 and L_2, driven by a common oscillator and amplifier, are arranged as shown. The frequency of the oscillator is gradually increased from zero and the detector at D records a series of maxima and minima.*



If the speed of sound is 330 m/s then the frequency at which the first maximum is observed is

Sol.

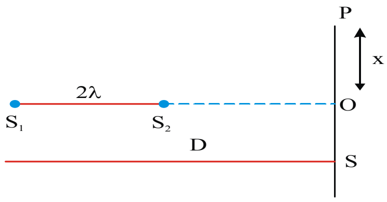


It is clear from figure that the path difference between L_1D and L_2D is $\Delta x = 41 - 40 = 1m$

For maximum $\Delta x = N\lambda$ where N = 1, 2, 3.... for Ist maximum N = 1, $\lambda = \frac{v}{n}$

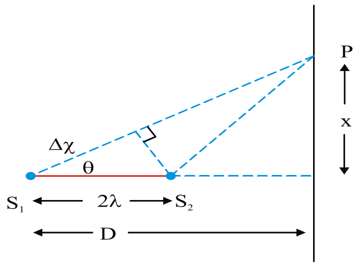
$\Delta x = 1 \times \frac{v}{n} \Rightarrow 1 = 1 \times \frac{330}{n} \Rightarrow n = 330Hz$

EX-15: ***Two coherent narrow slits emitting of wavelength λ in the same phase are placed parallel to each other at a small separation of 2λ. The sound is detected by moving a detector on the screen S at a distance $D(>>\lambda)$ from the slit S_1 as shown in figure.***



Find the distance x such that the intensity at P is equal to the intensity at O.

Sol.



From figure, we get $\cos\theta = \frac{\Delta x}{2\lambda}$

$\Rightarrow \Delta x = 2\lambda\cos\theta$ -------(1)

For maximum intensity path difference

$\Delta x = N\lambda$ --------(2)

From equations (1) and (2) we get

$2\lambda\cos\theta = N\lambda \Rightarrow 2\cos\theta = N$

at least p is Ist maxima $\Rightarrow N = 1$

$\therefore \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$

$\tan\theta = \frac{x}{D} \Rightarrow x = D\tan 60 \Rightarrow x = \sqrt{3}D$

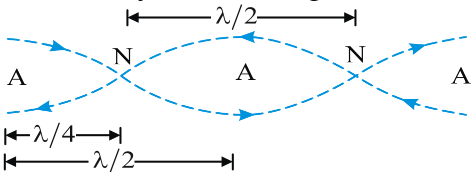
STANDING WAVES OR STATIONARY WAVES:

When two sets of progressive wave trains of same type (both longitudinal or both transverse) having the same amplitude and same time period/frequency/wavelength travelling with same speed along the same straight line in opposite directions superimpose, a new set of waves are formed. These are called stationary waves or standing waves.

These waves are formed only in a bounded medium.

In practice, a stationary wave is formed when a wave train is reflected at a boundary. The incident and reflected waves then interface to produce a stationary wave.

1. Suppose that two super imposing waves are incident wave $y_1 = a\sin(\omega t - kx)$ and reflected wave $y_2 = a\sin(\omega t + kx)$
(As y_2 is the displacement due to reflected wave from a free boundary)
Then by principle of superposition
$y = y_1 + y_2 = a\left[\sin(\omega t - kx) + \sin(\omega t + kx)\right]$
($\because$ sin C + sin D $= 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}$)
$\Rightarrow y = 2a\cos kx \sin\omega t$
(If reflection takes place from rigid end, then equation of stationary wave will be $y = \pm 2a\sin kx\cos\omega t$)
2. As this equation satisfies the wave equation.
$\frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$. It represents a wave
3. As it is not of the form $f(ax \pm bt)$, the wave is not progressive.
4. Amplitude of the wave A_{sw} = 2a cos kx.
5. **Nodes (N):** The points where amplitude is minimum are called nodes.
i) Distance between two successive nodes is $\lambda/2$
ii) Nodes are at permanent rest.
iii) At nodes air pressure and density both are high.



6. **Antinodes (A):** The points of maximum amplitudes are called antinodes.
(i) The distance between two successive antinodes is $\lambda/2$
(ii) At antinodes air pressure and density both are low.
(iii) The distance between a node (N) and adjoining antinode (A) is $\lambda/4$

7. Amplitude of standing waves in two different cases:

Table: : Amplitude in two different cases	
Reflection at open end or free boundary	**Reflection at closed end or rigid boundary**
$A_{sw} = 2a \cos kx$	$A_{sw} = 2a \sin kx$
Amplitude is maximum when cos kx = ≠ 1 $\Rightarrow kx = 0, 2\pi, ...n\pi$ $\Rightarrow x = 0, \frac{\lambda}{2}, \lambda.......\frac{n\lambda}{2}$ Where $k = \frac{2\pi}{\lambda}$ and n = 0, 1, 2, 3.....	Amplitude is maximum when cos kx = ≠ 1 $\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}....\frac{(2n-1)\pi}{2}$ $\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}..............$ Where $k = \frac{2\pi}{\lambda}$ and n = 1, 2, 3,.......
Amplitude is minimum when cos kx = 0 $\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}....\frac{(2n-1)\pi}{2}$ $\Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}.........$	Amplitude is minimum when sin kx = 0 $\Rightarrow kx = \frac{\pi}{2}, \frac{3\pi}{2}....\frac{(2n-1)\pi}{2}$ $\Rightarrow x = 0, \frac{\lambda}{2}, \lambda.......\frac{n\lambda}{2}$

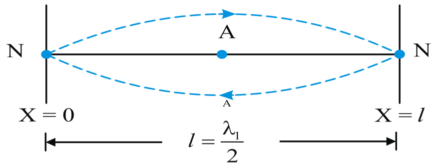
Terms related to the Application of Stationary wave

1. **Harmonics:** The frequency which are the integral multiple of the fundamental frequency are known as harmonics e.g. if n be the fundamental frequency, then the frequencies n, 2n, 3n ...... are termed as first, second, third .... harmonics.
2. **Overtone:** The harmonics other than the first (fundamental note) which are actually produced by the instrument are called overtones. e.g. the tone with frequency immediately higher than the fundamental is defined as first overtone.
3. **Octave:** The tone whose frequency is doubled the fundamental frequency is defined as Octave.
   i) If $n_2 = 2n_1$ it means n_2 is an octave higher than n_1 or n_1 is an octave lower than n_2.
   ii) If $n_2 = 2^3\ n_1$, it means n_2 is 3-octave higher or n_1 is 3-octave lower.
   iii) Similarly, if $n_2 = 2^n n_1$ it means n_2 is n-octave higher or n_1 is n octave lower.
4. **Unison:** If time period is same i.e., two frequencies are equal then vibrating bodies are said to be in unison.

STANDING WAVES ON A STRING

1. Consider a string of length *l*, stretched under tension T between two fixed points.
2. If the string is plucked and then released, a transverse harmonic wave propagates along it's length and is reflected at the end.
3. The incident and reflected waves will superimpose to produce transverse stationary waves in a string.

4. Nodes (N) are formed at rigid end and antinodes (A) are formed in between them.
5. Number of antinodes = Number of nodes –1
6. Velocity of wave (incident or reflected wave) is given by $v = \sqrt{\frac{T}{\mu}}$.
7. Frequency of vibration (n) = Frequency of wave $= \frac{v}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{T}{\mu}}$
8. For obtaining p loops (p-segments) in string, it has to be plucked at a distance $\frac{l}{2p}$ from one fixed end.
9. **Fundamental mode of vibration**



i) Number of loops p = 1

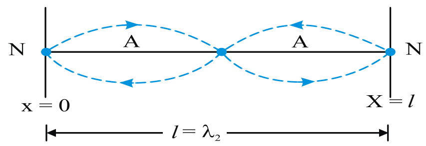
ii) Plucking at $\frac{l}{2}$ (from one fixed end)

iii) $l = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2l$

iv) Fundamental frequency or first harmonic

$$n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

10. **Second mode of vibration:**



i) Number of loops p = 2

ii) Plucking at $\frac{l}{2\times 2} = \frac{l}{4}$ (from one fixed end)

iii) $l = \lambda_2$

iv) Second harmonic or first over tone.

$$n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{\mu}} = \frac{1}{l}\sqrt{\frac{T}{\mu}} = 2n_1$$

11. **Third mode of vibration:**

i) Number of loops p = 3

ii) Plucking at $\frac{l}{2\times 3} = \frac{1}{6}$ (from one fixed one)

iii) $l = \frac{3\lambda_3}{2} \Rightarrow \lambda_3 = \frac{2l}{3}$

iv) Third harmonic or second over tone.

$$n_3 = \frac{1}{\lambda_3}\sqrt{\frac{T}{\mu}} = \frac{3}{2l}\sqrt{\frac{T}{\mu}} = 3n_1$$

More about string vibration

i) In general, if the string is plucked at length $\frac{l}{2p}$, then it vibrates in p segments (loops) and we have the pth harmonic n $n_p = \frac{p}{2l}\sqrt{\frac{T}{\mu}}$

ii) All even and odd harmonics are present. Ratio of harmonic = 1 : 2 : 3......

iii) Ratio of over tones = 2 : 3 : 4 ......

iv) General formula for wavelength $\lambda = \frac{2l}{P}$; where P = 1, 2, 3,..... correspond to 1st, 2nd, 3rd modes of vibratio of the string.

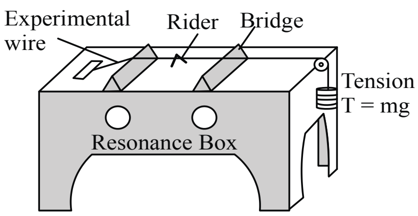
v) General formula for frequency $n = P \times \frac{v}{2l}$

Positions of nodes: $x_N = 0, \frac{l}{P}, \frac{2l}{P}, \frac{3l}{P}.........l$

Positions of antinodes: $x_{AN} = \frac{l}{2P}, \frac{3l}{2P}, \frac{5l}{2P}.....\frac{(2P-1)l}{2P}$

SONOMETER

1. It is an apparatus, used to produce resonance (matching frequency) of tuning fork (or any source of sound) with stretched vibrating string.
2. It consists of a hollow rectangular box of light wood. The experimental set up fitted on the box is shown below.



3. The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire.
4. If the length of the wire between the two bridges is *l*, then the frequency of vibration is $n = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}}$

(r = Radius of the wire, ρ = Density of material of wire) μ = mass per unit length of the wire

Resonance: When a vibrating tuning fork is placed on the box, and if the length between the bridges is property adjusted then if

$(n)_{Fork} = (n)_{string} \rightarrow$ rider is thrown off the wire.

Laws of string

Law of length: If T and μ are constant then $n \propto \frac{1}{l} \Rightarrow nl = \text{constant} \Rightarrow n_1 l_1 = n_2 l_2$

If % change is less than 5% then $\frac{\Delta n}{n}=-\frac{\Delta l}{l}$ or $\frac{\Delta n}{n}\times 100\%=-\frac{\Delta l}{l}\times 100\%$

Law of mass: If T and *l* are constant then $n\propto\frac{1}{\sqrt{\mu}}\Rightarrow\frac{n_1}{n_2}=\sqrt{\frac{\mu_2}{\mu_1}}$

If % change is less than 5% then $\frac{\Delta n}{n}=-\frac{1}{2}\frac{\Delta\mu}{\mu}$

or $\frac{\Delta n}{n}\times 100\%=-\frac{\Delta\mu}{\mu}\times 100\%$

Law of density: If T, *l* and r are constant then $n\propto\frac{1}{\sqrt{\rho}}\Rightarrow n\sqrt{\rho}=\text{const}\Rightarrow\frac{n_1}{n_2}=\sqrt{\frac{\rho_2}{\rho_1}}$

If % change is less than 5% then $\frac{\Delta n}{n}=-\frac{1}{2}\frac{\Delta\rho}{\rho}$ or $\frac{\Delta n}{n}\times 100\%=-\frac{\Delta\rho}{\rho}\times 100\%$

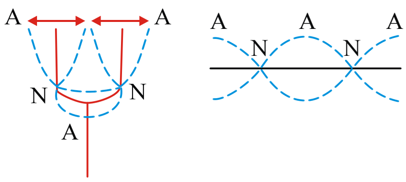
Law of tension: If *l* and μ are constant then $n\propto\sqrt{T}$

$\Rightarrow\frac{n}{\sqrt{T}}=\text{const}\Rightarrow\frac{n_1}{n_2}=\sqrt{\frac{T_1}{T_2}}=\sqrt{\frac{M_1}{M_2}}$

If % change is less than 5% then $\frac{\Delta n}{n}=\frac{1}{2}\frac{\Delta T}{T}$ or $\frac{\Delta n}{n}\times 100\%=-\frac{\Delta T}{T}\times 100\%$

TUNING FORK

i) It is a U shaped metal bar made of steel or an alloy with a handle attached at the bend.
ii) When it is struck against a hard rubber pad, its prongs begin to vibrate as shown in figure(a).



(a) (b)

iii)A tuning fork emits a single frequency note, i.e., a fundamental with no overtones.
iv) A tuning fork may be considered as a vibrating free bar as shown figure(b) that has been bent into U-shape.
v) Two antinodes are formed one at each free end of the bar which are in phase.
vi) The frequency of a tuning fork of arm length '*l*' and thickness 'd' in the direction of vibration is given by

$$n=\frac{d}{l^2}v=\frac{d}{l^2}\sqrt{\frac{Y}{\rho}},\left[Q\,v=\sqrt{\frac{Y}{\rho}}\right]$$

where Y is the Young's modulus and ρ is the density of the material of the turning fork.
vii) Using the tuning fork we can produce transverse waves in solids and longitudinal waves in solids, liquids and gases.
viii) Transverse vibrations are present in the prongs. Longitudinal vibrations are present in the shank.
ix) Loading or waxing a tuning fork increases its inertia and so decreases its frequency, while filing a tuning fork decreases its inertia and so increases its frequency.
x) When tuning fork is heated its frequency decreases due to decrease in elasticity.

EX-16: ***The vibrations of a string of length 60 cm fixed at both ends are represented by the equation.*** $y = 4\sin\left[\frac{\pi x}{15}\right]\cos(96\pi t)$ ***Where x and y are in cm and t in sec.***

a) What is the maximum displacement at x = 5 cm ?

b) What are the nodes located along the string ?

c) What is the velocity of the particle at x = 7.5 cm and t = 0.25 s ?

d) Write down the equations of component waves whose superposition gives the above wave.

Sol. a) For x = 5cm, $y = 4\sin(5\pi/15)\cos(96\pi t)$

(or) $y = 2\sqrt{3}\cos(96\pi t)$

So y will be maximum when cos (96 πt) = 1 i.e., $(y_{max})_{x=5} = 2\sqrt{3}cm$

b) At nodes amplitude of wave is zero.

$4\sin\left[\frac{\pi x}{15}\right] = 0 (or) \frac{\pi x}{15} = 0, \pi, 2\pi, 3\pi......$

So x = 0, 15, 30, 45, 60 cm [as length of string = 60cm]

c) As $y = 4\sin(\pi x/15)\cos(96\pi t)$

$\frac{dy}{dx} = -4\sin\left[\frac{\pi x}{15}\right]\sin(96\pi t)\times(96\pi)$

So the velocity of the particle at x = 7.5cm and t = 0.25s,

v_{pa} = -384π sin(7.5π/15) sin (96π × 0.25)

v_{pa} = -384π × 1 × 0 = 0

d) y = y_1 + y_2 with $y_1 = 2\sin\left[96\pi t + \frac{\pi x}{15}\right]$

$y_2 = -2\sin\left[96\pi t - \frac{\pi x}{15}\right]$

EX-17: ***A guitar string is 90 cm long and has a fundamental frequency of 124 Hz. Where should it be pressed to produce a fundamental frequency of 186 Hz?***

Sol. Since T is constant we have $n \propto \frac{1}{l}$

$l_2 = \frac{n_1}{n_2}l_1 = \frac{124}{186}\times 90 = 60cm$

Thus, the string should be pressed at 60cm from an end.

EX-18: ***A wire having a linear mass density 5.0×10^{-3} kg/m is streched between two rigid supports with a tension of 450 N. The wire resonates at a frequency of 420 Hz. The next higher frequency at which the same wire resonates is 490 Hz. Find the length of the wire.***

Sol. Suppose the wire vibrates at 420 Hz in its nth harmonic and at 490 Hz in its (p + 1)th harmonic.

$$\frac{490}{420}=\frac{p+1}{p}(or)\,p=6$$

$$420=\frac{6}{2l}\sqrt{\frac{450}{5.0\times10^{-3}}}\quad \therefore l=\frac{900}{420}=2.1m$$

EX-19: ***The equation of a standing wave produced on a string fixed at both ends is where 'y' is measured in cm. What could be the smallest length of string?***

Sol. Comparing with $y=2A\sin kx\cos wt$

We have $k=\frac{\pi}{10}$ $\quad\Rightarrow \lambda=20cm$

If the string vibrates in 'p' loops then length of string 'l' is $\frac{p\lambda}{2}$. $\quad\Rightarrow \frac{p\lambda}{2}=l$

'l' is minimum if p = 1 $\Rightarrow l=\frac{\lambda}{2}=10cm$

EX-20: ***The equation for the vibration of a string fixed at both ends, vibrating in its third harmonic is given by*** $y=0.4\sin\frac{\pi x}{10}\cos 600\pi t$ ***where x and y are in cm***

1) What is the frequency of vibration?
2) What are the position of nodes?
3) What is the length of string?
4) What is the wavelength and speed of transverse waves that can interfer to give this vibration?

Sol. Comparing with

$y=2A\sin kx\cos\omega t$ we have

1) $\omega=600\pi$ gives n = 300 Hz

2) To get the position of nodes $\sin\frac{\pi x}{10}=0$

i.e., $\frac{\pi x}{10}=N\pi$ where N = 0, 1, 2....

Hence nodes occur at x = 0, 10, 20 cm...

3) Since the string is in 3rd harmonic

$l=3\frac{\lambda}{2}$ gives l = 30cm; $\left[\because \lambda=\frac{2\pi}{k}=\frac{2\pi}{\pi/10}=20cm\right]$

4)Speed of wave v = $n\lambda$ = 300 × 20=60ms^{-1}.

EX-21 ***A sonometer wire has a length of 114 cm between two fixed ends. Where should two bridges be placed to divide the wire into three segments whose fundamental frequencies are in the ratio 1 : 3 : 4 ?***

Sol. In case of a given wire under constant tension, fundamental frequency of vibration $n\propto(1/l)$

$$\therefore l_1:l_2:l_3=\frac{1}{1}:\frac{1}{3}:\frac{1}{4}=12:4:3$$

$$\therefore l_1=72cm; l_2=24cm; l_3=18cm$$

$\therefore$ First bridge is to be placed at 72 cm from one end.

Second bridge is to be placed at 72 + 24 = 96 cm from one end

EX-22: ***An aluminium wire of cross-sectional area 10^{-6} m^2 is joined to a copper wire of the same cross-section. This compound wire is stretched on a sonometer, pulled by a load of 10 kg. The total length of the compound wire between two bridges is 1.5 m of which the aluminium wire is 0.6 m and the rest is the copper wire. Transverse vibrations are set up in the wire in the lowest frequency of excitation for which standing waves are formed such that the joint in the wire is a node. What is the total number of nodes observed at this frequncy excluding the two at the ends of the wire ? The density of aluminium is 2.6 × 10^4 kg/m^3.***

Sol. As the total length of the wire is 1.5 m and out of which L_A = 0.6 m, so the length of copper wire

L_c = 1.5 - 0.6 = 0.9 m. The tension in the whole wire is same (=Mg = 10g N) and as fundamental frequency of vibration of string is given by

$$n = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2L}\sqrt{\frac{T}{\rho A}} \quad [\because \mu = \rho A]$$

so $n_A = \frac{1}{2L_A}\sqrt{\frac{T}{\rho_A A}}$ and $n_c = \frac{1}{2L_c}\sqrt{\frac{T}{\rho_c A}}$ -------- (1)

Now as in case of composite wire, the whole wire will vibrate with fundamental frequency

$n = p_A n_A = p_C n_C$ --------- (2)

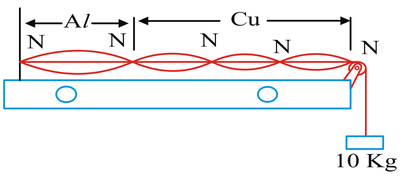
Substituting the values of f_A and f_c from Eqn.(1)in(2)

$$\frac{p_A}{2\times 0.6}\sqrt{\frac{T}{A\times 2.6\times 10^3}}$$

$$= \frac{p_c}{2\times 0.9}\sqrt{\frac{T}{A\times 1.0401\times 10^4}}$$

i.e., $\frac{p_A}{p_c} = \frac{2}{3}\sqrt{\frac{2.6}{10.4}} = \frac{2}{3}\times\frac{1}{2} = \frac{1}{3}$

So that for fundamental frequency of composite string, p_A = 1 and p_c = 3, i.e., aluminium string will vibrate in first harmonic and copper wire at second, overtone as shown in figure.



$\therefore n = n_A = 3n_C$

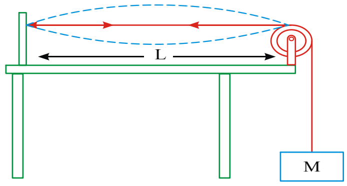
This in turn implies that total number of nodes in the string will be 5 and so number of nodes excluding the nodes at the ends = 5 - 2 = 3

EX-23: ***A wire of uniform cross-section is stretched between two points 1 m apart. The wire is fixed at one end and a weight of 9 kg is hung over a pulley at the other end produces fundamental frequency of 750 Hz.***

(a) What is the velocity of transverse waves propagating in the wire ?

(b) If now the suspended weight is submerged in a liquid of density (5/9) that of the weight, what will be the velocity and frequency of the waves propagating along the wire ?

Sol. a) In case of fundamental vibrations of string $(\lambda/2) = L$, i.e., $\lambda = 2\times 1 = 2m$



Now as $v = n\lambda$ and n = 750 Hz,

$v_T = 2 \times 750 = 1500 \text{m/s}$

b) Now as in case of a wire under tension $v = \sqrt{\frac{T}{\mu}} \Rightarrow v \alpha \sqrt{T} \Rightarrow \frac{v_B}{v_T} = \sqrt{\frac{T_B}{T_T}}$

$\Rightarrow v_B = 1500\sqrt{\frac{T_B}{T_A}} \Rightarrow 1500\sqrt{\frac{mg[1-\rho_l/\rho_b]}{mg}} = 1000 m/s$

From $v = n\lambda \Rightarrow n_B = \frac{v_B}{l_B} = \frac{1000}{2} = 500 Hz$

EX-24: ***A wire of density 9×10³ kg/m³ is stretched between two clamps 1 m apart and is subjected to an extension of 4.9 × 10⁻⁴ m. What will be the lowest frequency of transverse vibrations in the wire ? (Y = 9 × 10¹⁰ N/m²)***

Sol. In case of fundamental vibrations of a string

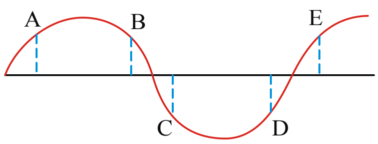
$n = \frac{1}{2L}\sqrt{\frac{Y}{\rho}\frac{\Delta L}{L}}$

$= \frac{1}{2\times 1}\sqrt{\frac{9\times 10^{10} \times 4.9 \times 10^{-4}}{9\times 10^3 \times 1}} = 35 Hz$

EX-25: ***A string 120 cm in length sustains a standing wave, with the points of string at which the displacement amplitude is equal to $\sqrt{2}$ mm being separated by 15.0 cm, Find the maximum displacement amplitude.***

Sol. From figure. points A, B, C, D and E are having equal displacement amplitude.

Further, $x_E - x_A = \lambda = 4 \times 15 = 60 cm$



As $\lambda = \frac{2l}{n} = \frac{2\times 120}{n} = 60$ $\quad \therefore \quad n = \frac{2 \times 120}{60} = 4$

So, it corresponds to 4th harmonic.

Also, distance of node from A is 7.5 cm and no node is between them. Taking node at origin, the amplitude of stationary wave can be written as, $A_{sw} = A_{max} \sin kx$

$A_{sw} = \sqrt{2} mm; k = \frac{2\pi}{\lambda} = \frac{2\pi}{60}$ and x = 7.5 cm

$\therefore \sqrt{2} = A_{max} \sin\left(\frac{2\pi}{60} \times 7.5\right) = A_{max} \sin\frac{\pi}{4}$

Hence, $A_{max} = 2 mm$

SOUND WAVES :

Sound is a form of energy propagated in the form of longitudinal waves. This energy causes the sensation of hearing on reaching the ear. Any vibrating body could be a source of sound. Longitudinal mechanical waves can be transmitted in all the three states of matter namely, solids, liquids and gases. According to their range of frequencies longitudinal mechanical waves are divided into the three categories.

1) Longitudinal waves having frequencies below 20Hz are called infrasonic waves. These are created by earthquakes, elephants and whales. Infrasonic waves can be heard by snakes.

2) Longitudinal waves having range, of frequencies lying between 20Hz and 20kHz are called audible sound waves. The audible wavelength is 16.5 mm to 16.5m at S.T.P when velocity of sound is 330 m/s. These are generated by tuning forks, streched stings and vocal cords. The human ear can detect these waves.

3) Longitudinal waves having frequencies greater than 20 kHz are called ultrasonics. The human ear can't detect these waves. These waves can be produced by high frequency vibrations of a quartz crystal under an alternating electric field. These waves can be detected by mosquito, fish and dog etc.

Application of ultrasonic waves :

i) The fine internal cracks in a metal can be detected by ultrasonic waves.
ii) They are used for determining the depth of the sea and used to detect submarine.
iii) They can be used to clean clothes and fine machinery parts
iv) They can be used to kill animals like rats, fish and frogs etc.

Characteristics of Sound

- Hearing of sound is characterised by following three parameters.

Loudness (Refers to Intensity) :

It is the sensation received by ear due to intensity of sound

Greater the amplitude of vibration, greater will be intensity $(I \alpha A^2)$ and so louder will be sound.

The loudness being the sensation, depends on the sensitivity of listener's ear. Loudness of a sound of a given intensity may be different for different listeners.

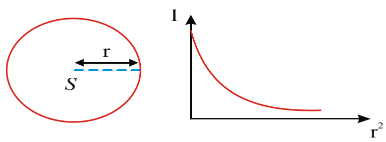
The average energy transmitted by a wave per unit normal area per second is called intensity of a wave. $I = \frac{E}{At}$. ItsSI Unit : W/m^2

- It is the average power transmitted by a wave through the given area.

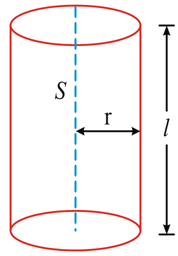
$$I = \frac{P_{\text{avg.}}}{\text{area}}; \quad I = 2\pi^2 n^2 A^2 \rho v$$

where $\rho -$ density of medium, $v -$ velocity of wave, A - Amplitude, n - Frequency

- Human ear responds to sound intensities over a wide range from 10^{-12} W/m^2 to 1 W/m^2.
- In a spherical wave front (i.e. wave starting from a point source) , the amplitude varies inversely with distance from position of source i .e, $A \alpha \frac{1}{r} \Rightarrow I \propto \frac{1}{r^2}$



- In a cylindrical wave front (i.e. wave starting from a linear source), the amplitude varies inversely as the square root of distance from the axis of source i.e. , $A \propto \frac{1}{\sqrt{r}} \Rightarrow I \propto \frac{1}{r}$



Sound level in decibles is given by $\beta = 10 \log\left(\frac{I}{I_0}\right)$

If β_1 and β_2 be the sound levels corresponding to sound intensities I_1 and I_2 respectively. Then,

$\beta_1 = 10 \log \frac{I_1}{I_0}$ and $\beta_2 = 10 \log \frac{I_2}{I_0}$

$\therefore \beta_2 - \beta_1 = 10\left(\log \frac{I_2}{I_0} - \log \frac{I_1}{I_0}\right)$

(or) $\beta_2 - \beta_1 = 10 \log\left(\frac{I_2}{I_1}\right)$

Pitch (Refers to Frequency):

The shrillness or harshness of sound is known as pitch. Pitch depends on frequency. Higher the frequency, higher will be the pitch and shriller will be the sound.

Quality or Timber (Refers to Harmonics):

It is the sensation received by ear due to waveform. Quality of a sound depends on number of overtones. i.e, harmonic present.

Velocity of Sound

- The equation for velocity of sound through a medium is given by $v = \sqrt{\frac{E}{\rho}}$

  where $E =$ modulus of elasticity; ρ=density
- As modulus of elasticity is more for solids and less for gases, so

  $v_{\text{solids}} > v_{\text{liquids}} > v_{\text{gases}}$
- In case of solids $v = \sqrt{\frac{Y}{\rho}}$,

  where Y is Young's modulus,
- In case of fluids (liquids and gases) $v = \sqrt{\frac{B}{\rho}}$

  where B is the Bulk modulus

Velocity of sound in Gases :

- **Newton's formula :**
  Newton assumed that the propagation of sound in a gas takes place under isothermal conditions.
- Isothermal Bulk modulus , $B = P$

  $$\therefore v_s = \sqrt{\frac{P}{\rho}}$$

- At S.T.P. $v = \sqrt{\frac{1.013x10^5}{1.29}} \approx 280 ms^{-1}$

  Which is less than the experimental value ($332\, m/s$)
- **Laplace's correction:** Laplace assumed that the propagation of sound in a gas takes place under adiabatic conditions.
- Adiabatic Bulk modulus, $B = \gamma P$

  $$\therefore v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{m}} = \sqrt{\frac{\gamma nRT}{m}} = \sqrt{\frac{\gamma RT}{M}}$$

  where V = volume, m is mass, M = molecular weight. T is absolute temperature
- For air γ = 1.4. Therefore

  At STP $v_0 = 280\sqrt{1.4} \approx 330 ms^{-1}$, which agrees with the experimentally calculated value.
- Velocity of sound in a gas is directly proportional to the square root of the absolute temperature

  $$\frac{v_t}{v_o} = \sqrt{\frac{T}{T_o}} = \left(\frac{t+273}{273}\right)^{1/2} \left(\because v \propto \sqrt{T}\right)$$

  $$\Rightarrow v_t = v_o\left(1 + \frac{t}{546}\right)$$

  $$\Rightarrow v_t = v_0 + \frac{v_0 t}{546} = v_0 + 0.61 t°C$$

Note:

1) When temperature rises by $1°C$ then velocity of sound increases by 0.61 m/s
2) The velocity of sound increases with increase in humidity. Sound travels faster in moist air than in dry air at the same temperature, because density of humidity air is less than that of dry air.

   $\rho_{\text{moist air}} < \rho_{\text{dry air}} \Rightarrow v_{\text{moist air}} > v_{\text{dry air}}$
3) The velocity of sound at constant temperature in a gas does not depend upon the pressure of air.
4) Amplitude, frequency, phase, loudness, pitch, quality donot effect velocity of sound.

EX-26: ***Find the speed of sound in a mixture of 1 mol of helium and 2 mol of oxygen at 27°C.***

Sol. $\gamma_{mix} = \frac{C_{Pmix}}{C_{Vmix}} = \frac{(19R/6)}{(13R/6)} = \frac{19}{13}$

$$M_{mix} = \frac{n_1 M_1 + n_2 M_2}{n_1 + n_2} = \frac{1 \times 4 + 2 \times 32}{1+2}$$

$$= \frac{68}{3} \times 10^{-3} kg / mol \;;$$

$$v = \sqrt{\frac{\gamma_{mix} RT}{M_{mix}}} = \sqrt{\frac{19}{13} \times \frac{8.314 \times 300}{68 \times 10^{-3}/3}} \approx 401 m/s$$

EX-27: ***A window whose area is $2m^3$ opens on a street where the street noise result in an intensity level at the window of $60dB$. How much 'acoustic power' enters the window via sound waves. Now if an acoustic absorber is fitted at the window, how much energy from street will it collect in five hours ?***

Sol. Sound level $\beta = 10 \log\left(\frac{I}{I_o}\right)$

$$\Rightarrow 60 = 10 \log\left(\frac{I}{I_o}\right) \Rightarrow \frac{I}{I_o} = 10^6 \Rightarrow I = 10^6 I_o$$

$$\Rightarrow I = 10^6 \times 10^{-12} = 10^{-6} W / m^2$$

but intensity $I = \frac{E}{At} \Rightarrow E = IAt$

$$E = 10^{-6} \times 2 \times 5 \times 3600 = 36 \times 10^{-3} J$$

VARIOUS FORMS OF LONGITUDINAL WAVE:

As we know, during a longitudinal wave propagation the particles of the medium oscillate to produce pressure and density variation along the direction of the wave. These variations result in series of high and low pressure (and density) regions called compression and rarefactions respectively. Hence the longitudinal wave can be in terms of displacement of particles called displacement wave y(x, t) or in terms of change in pressure called pressure wave $\Delta P(x,t)$ or change in density called density wave $\Delta d(x,t)$.

1) Pressure Wave:

i)A longitudinal sound wave can be expressed either in terms of the longitudinal displacement of the particles of the medium or in terms of excess pressure produced due to compression or rarefaction. (at compression, the pressure is more than the normal pressure of the medium and at rarefaction the pressure is lesser than the normal).

ii) If the displacement wave is represented by $y = A\sin(\omega t - kx)$ then the corresponding pressure wave will be represented by $\Delta P = -B\frac{dy}{dx}$ (*B* = Bulk modulus of elasticity of medium)

$$\therefore \Delta P = BAk\cos(\omega t - kx) = \Delta P_0 \cos(\omega t - kx)$$

where ΔP_0 = pressure amplitude $= BAk$

iii) Pressure wave is $\pi/2$ out of phase(lags) with displacement wave. *i.e.* pressure is maximum when displacement is minimum and vice-versa.

Note1: At the centre of compression and rarefaction particle velocity is maximum and at the boundary of compression and rarefaction particles are momentarily of rest. This is explained as in a harmonic progressive wave

v_P = -(slope of y-x)$\times v \Rightarrow \frac{v_p}{v} = -\frac{dy}{dx}$

Since the change in pressure of the medium

$$\Delta P = -B\left(\frac{dy}{dx}\right) \qquad \Rightarrow \Delta P = B\left(\frac{v_P}{v}\right)$$

i.e., for a given medium, B and v are constants. Where v_p is maximum, Δp is also maximum, which is true at y = 0

Note 2: As sound sensors (e.g ear or mike) detect pressure changes, description of sound as pressure wave is preferred over displacement wave.

2) Density wave form : Let ρ_o be the normal density of the medium and $\Delta\rho$ be the change in density of the medium during the wave propagation.

Then fraction of change in volume of the element $\frac{\Delta v}{v} = -\frac{\Delta\rho}{\rho_0}$ $\left(\because v = \frac{m}{\rho}\right)$

According to definition of Bulk's modulus

$$B = -\Delta P\left(\frac{v}{\Delta v}\right) = \Delta P\left(\frac{\rho_0}{\Delta\rho}\right)$$

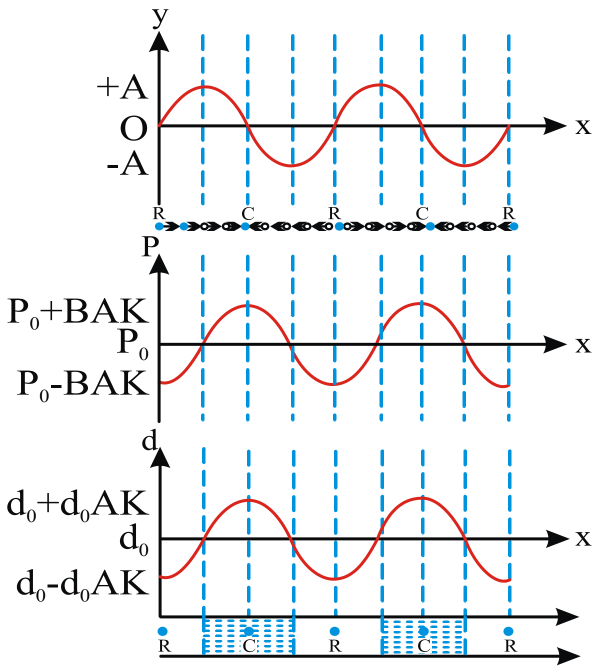
$$\Rightarrow \Delta\rho = \frac{\rho_0}{B}.\Delta p \qquad \Rightarrow \Delta\rho = \frac{\rho_0}{B}(\Delta p)_{\max} Cos(kx - \omega t)$$

$$\Rightarrow \Delta\rho = \rho_0 Ak\, Cos(kx - \omega t)$$

$$\left(\because (\Delta p)_{\max} = BAk\right)$$

$$\Rightarrow \Delta\rho = (\Delta\rho)_{\max} Cos(kx - \omega t),$$

where $(\Delta\rho)_{\max} = \rho_0 Ak$ is called density amplitude. Thus the density wave is in phase with the pressure wave and this is 90^0 out of phase (lags) with the displacement wave as shown in the figure.



Note 1: The relation between density amplitude and pressure amplitude is $(\Delta\rho)_{max} = (\Delta p)_{max}\left(\frac{\rho}{B}\right)$

Note 2: Average Intensity $I = \frac{P}{S} = \frac{1}{2}\rho\omega^2A^2v$

In terms of pressure amplitude, sound intensity

$$I = \frac{1}{2}\rho\omega^2\left(\frac{\Delta p_{max}}{Bk}\right)^2 v = \frac{1}{2}\frac{(\Delta p_{max})^2}{\rho v}$$

$[\because (\Delta P)_{max} = BAk, k = \frac{\omega}{v}\ \ and\ \ B = \rho v^2]$ Thus intensity of wave is proportional to square of pressure amplitude or displacement amplitude or density amplitude and is independent of frequency.

EX-28: ***What is the maximum possible sound level in dB of sound waves in air? Given that density of air*** $= 1.3 kg/m^3, v = 332 m/s$ ***and atmospheric pressure*** $P = 1.01\times10^5\ N/m^2$.

Sol. For maximum possible sound intensity, pressure amplitude of wave will be equal to atmospheric pressure, i.e., $p_0 = P = 1.01\times10^5\ Nm^2$

$$I = \frac{p_0^2}{2\rho v} = \frac{(1.01\times10^5)^2}{2\times1.3\times332} = 1.18\times10^7 W/m^2$$

$$\therefore SL = 10\log\frac{I}{I_0} = 10\log\frac{10^7}{10^{-12}} = 190 dB$$

EX-29: ***The faintest sounds the human ear can detect at a frequency of 1000 Hz correspond to an intensity of about $1.00 \times 10^{-12} W/m^2$, which is called threshold of hearing. The loudest sounds the ear can tolerate at this frequency correspond to an intensity of about $1.00 W/m^2$, the threshold of pain. Determine the pressure amplitude and displacement amplitude associated with these two limits.***

Sol. $\Delta P_{max} = \sqrt{2\rho v I}$ $= \sqrt{2(1.20)(343)(1.00\times 10^{-12})}$

$= 2.87 \times 10^{-5} N/m^2$

$$A = \frac{\Delta P_{max}}{\rho v \omega} = \frac{2.87\times 10^{-5}}{(1.2)(343)(2\pi \times 1000)}$$

$(\because \omega = 2\pi n)$; $= 1.11 \times 10^{-11} m$

EX-30: ***A firework charge is detonated many metres above the ground. At a distance of 400 m from the explosion, the acoustic pressure reaches a maximum of $10.0 N/m^2$. Assume that the speed of sound is constant at 343 m/s throughout the atmosphere over the region considered, the ground absorbs all the sound falling on it, and the air absorbs sound energy at the rate of 7.00 dB/km. What is the sound level (in decibels) at 4.00 km from the explosion?***

Sol. $r = 400m, r^1 = 4000m,$

$\rho = 1.2 kg/m^3, v = 343 m/s$

$$I = \frac{\Delta P_{max}^2}{2\rho v} = \frac{10}{2(1.2)(343)} = 1.21\times 10^{-2} W/m^2$$

as $I \propto \frac{1}{r^2} \Rightarrow \frac{I^1}{I} = \left(\frac{r}{r^1}\right)^2$

$$I^1 = \frac{I(400)}{4000} = 1.21\times 10^{-3} W/m^2 \qquad \beta = 10\log\left(\frac{I}{I^1}\right) = \log\left(\frac{1.21\times 10^{-3}}{1\times 10^{-12}}\right) = 90.8\ dB$$

At a distance of 4 km from the explosion, absorption from the air will decrease the sound level by an additional amount,

$\Delta\beta = (7)(3.60) = 25.2\ dB$

At 4 km, the sound level will be

$\beta_f = \beta - \Delta\beta = 90.8 - 25.2 = 65.6\ dB$

ORGAN PIPES

Organ pipe: An organ pipe is a cylindrical tube of uniform cross section in which a gas is trapped as a column.

Open pipe : If both ends of a pipe are open and a system of air is directed against an edge, standing longitudinal waves can be set up in the tube. The open end is a displacement antinode.

- Due to finite momentum, air molecules undergo certain displacement in the upward direction hence antinode takes place just above the open end but not exactly at the end of the pipe.
- Due to pressure variations, reflection of longitudinal wave takes place at open end and hence longitudinal stationary waves are formed in open tube.

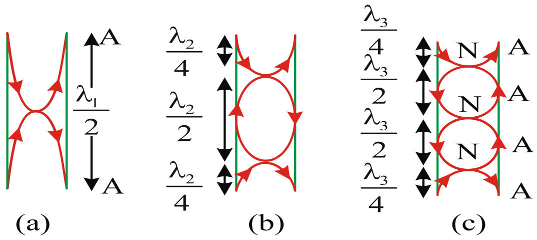
➢ 

fig: a) For fundamental mode of vibrations or I harmonic

$L = \frac{\lambda_1}{2}; \ \therefore \lambda_1 = 2L$

$V = \lambda_1 n_1; \ \therefore V = 2Ln_1 \Rightarrow n_1 = \frac{V}{2L}$ ------ (1)

➢ **fig:b) For the second harmonic or first overtone,**

$L = \lambda_2$

$V = \lambda_2 n_2 \ \therefore V = Ln_2 \ \Rightarrow n_2 = \frac{2V}{2L}$ ------ (2)

➢ **fig:c) For the third harmonic or second overtone,**

$L = 3 \times \frac{\lambda_3}{2} \ \therefore \lambda_3 = \frac{2}{3}L$

$V = \lambda_3 n_3 \ \therefore V = \frac{2}{3}Ln_3 \ \Rightarrow n_3 = \frac{3V}{2L}$ ------- (3)

➢ From (1), (2) and (3) we get,

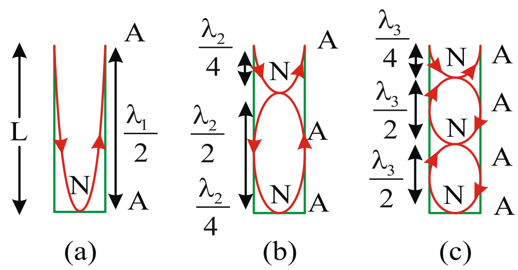
$n_1 : n_2 : n_3 ..... = 1:2:3: ......$

i.e. for a cylindrical tube, open at both ends, the harmonics excitable in the tube are all integral multiples of its fundamental.

➢ $\therefore$ In the general case, $\lambda = \frac{2L}{p}$, where $p = 1, 2, ...$

➢ p[th] harmonic frequency $= \frac{V}{\lambda} = \frac{pV}{2l}$, where $p = 1, 2, ...$

CLOSED PIPE: If one end of a pipe is closed, then reflected wave is $180°$ out of phase with the wave. Thus the displacement of the small volume elements at the closed end must always be zero. Hence the closed end must be a displacement node.



➢ **figure a) for the fundamental mode of vibration or I harmonic :**

$L = \frac{\lambda_1}{4} \ \ \therefore \lambda_1 = 4L$

If n_1 is the fundamental frequency, then the velocity of sound waves is given as

$V = \lambda_1 n_1 \ \therefore V = 4Ln_1 \ \Rightarrow n_1 = \frac{V}{4L}$ ----- (1)

- **figure b) for third harmonic or first overtone.**

$L = 3 \times \frac{\lambda_2}{4}$, $\therefore \lambda_2 = \frac{4}{3}L$

$V = \lambda_2 n_2$, $\therefore V = \frac{4}{3}Ln_2 \Rightarrow n_2 = \frac{3V}{4L}$ ---- (2)

- **figure c) for fifth harmonic or second overtone.**

$L = 5 \times \frac{\lambda_3}{4}$, $\therefore \lambda_3 = \frac{4}{5}L$

$V = \lambda_3 n_3$, $V = \frac{4}{5}Ln_3 \Rightarrow n_3 = \frac{5V}{4L}$ ---- (3)

From (1), (2) and (3) we get,

$n_1 : n_2 : n_3 ..... = 1:3:5: ......$

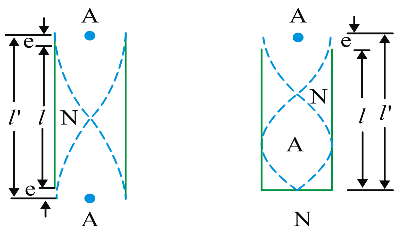
- In the general case, $\lambda = \frac{4L}{(2p+1)}$, where $p = 0,1,2,.....$

- p^{th} harmonic frequency $= \frac{(2p-1)V}{4L}$,

where p = 1, 2....

End Correction

Due to finite momentum of air molecules in organ pipe reflection takes place not exactly at open end but some what above it. Hence antinode is not formed exactly at the open end rather it is formed at a little distance away from open end outside it.
The distance of antinode form the open end is known as end correct (e).
It is given by e = 0.6 r , where r = radius of pipe.



(A) Open pipe (B) Close pipe

Effective length in open organ pipe $l' = (l + 2e)$

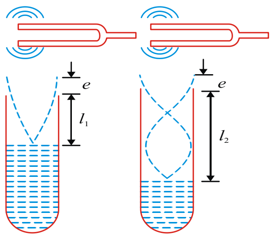
Effective length in closed organ pipe $l' = (l + e)$

Note: When the end correction is considered, then i)the fundamental frequency of open pipe $n = \frac{V}{2(l+2e)} \Rightarrow n = \frac{V}{2(l+1.2r)}$

ii) The fundamental frequency of closed pipe $n = \frac{V}{4(l+e)} \Rightarrow n = \frac{V}{4(l+0.6r)}$

VELOCITY OF SOUND

(Resonance column apparatus) :



- If l_1, l_2 and l_3 are the first, second and third resonating lengths then $l_1 + e = \frac{\lambda}{4}$ ....(1)

$$l_2 + e = \frac{3\lambda}{4} \text{ ...(2)}$$

$$l_3 + e = \frac{5\lambda}{4} \text{ --- (3)}$$

- From equations (1) and (2)

1) $\lambda = 2(l_2 - l_1)$ 2) $V = n\lambda = 2n(l_2 - l_1)$

3) $\frac{(1)}{(2)} \Rightarrow e = \frac{l_2 - 3l_1}{2}$ 4) $l_3 - l_2 = l_2 - l_1 \Rightarrow l_3 = 2l_2 - l_1$

EX31: ***A tube of certain diameter and length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320 m/s. Estimate the diameter of the tube. One end of the tube is now closed. Calculate the frequency of resonance for the tube.***

Sol. $n_0 = \frac{v}{2[L + 2e]} = \frac{v}{2[L + 2 \times 0.6r]} [as\, e = 0.6r]$

So substituting the given data,

$320 = \frac{320 \times 100}{2[48 + 1.2r]} (or)\, r = \frac{10}{6} cm$

So, D = 2r = 2 × (10/6) = 3.33cm.

Now when one end is closed,

$$n_c = \frac{v}{4(L + 0.6r)}$$

$$= \frac{320 \times 100}{4[48 + 0.6 \times (10/6)]} = 163.3 Hz$$

EX-32: ***A tuning fork of frequency 340 Hz is vibrated just above a cylindrical tube of length 120 cm. Water is slowly poured in the tube. If the speed of sound in air is 340 m/s. Find the minimum height of water required for resonance. (v = 340m/s)***

Sol: $n = p\frac{v}{4L}$ with p = 1, 3, 5,.............

So length of air column in the pipe

$L = \frac{pv}{4n} = 25p\,cm$ with p = 1,3,5,.....

i.e., L = 25cm, 75cm, 125cm

Now as the tube is 120 cm , so length of air column must be lesser than 120 cm, i.e., it can be only 25 cm or 75 cm. Further if h is the height of water filled in the tube,

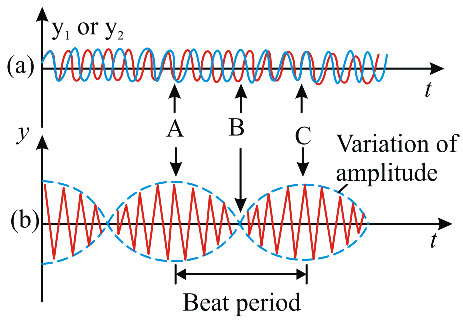
L + h = 120 cm or h = 120 – L

So h will be minimum when L_{max} = 75cm

$\therefore$ $(h)_{min}$ = 120 – 75 = 45cm.

BEATS

- It is the phenomenon of periodic change in the intensity of sound when two waves of slightly different frequencies travelling in same direction superpose with each other.
- Maximum Intensity of sound(Waxing) is produced in the beats when constructive Interference takes place.
- Minimum Intensity of sound(Waning) is produced in the beats when destructive Interference takes place.



Analytical treatment of Beats:

- Equations of waves producing beats are given as $y_1 = a\sin\omega_1 t$ and $y_2 = a\sin\omega_2 t$ let $\omega_1 > \omega_2$
- Resultant wave equation is

$$y = y_1 + y_2 = 2a\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t\sin\left(\frac{\omega_1 + \omega_2}{2}\right)t$$

$$y = A(t)\cos\left(\frac{\omega_1 - \omega_2}{2}\right)t$$

Here $A(t) = 2a\sin\frac{(\omega_1 + \omega_2)}{2}t$

- Amplitude is function of time. Frequency of variation of amplitude $= \frac{n_1 - n_2}{2}$
- Frequency of resultant wave = $\frac{n_1 + n_2}{2}$
- The variation in the intensity of sound between successive maxima or minima is called one beat.
- The number of beats per second is called beat frequency. If n_1 and n_2 are the frequencies of the two sound waves that interfere to produce beats then

  Beat frequency = $n_1 \sim n_2$

- The time period of one beat (or) the time interval between two successive maxima or minima is $\frac{1}{n_1 \sim n_2}$
- The time interval between a minima and the immediate maxima is $\frac{1}{2(n_1 \sim n_2)}$
- As the persistence of human hearing is about 0.1 sec, beats will be detected by the ear only if beat period is $\Delta t \geq 0.1 \sec$ or beat frequency

  $\Delta n = n_1 : n_2 \leq 10 Hz$
- Maximum number of beats that can be heard by a human being is 10 per second.
- If more than 10 beats are produced then no. of beats produced are same but no. of beats heard are zero
- If a_1, a_2 are amplitudes of two sound waves that interfere to produce beats then the ratio of maximum and minimum intensity of sound is, $\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$

Uses of Beats:

i) To determine unknown frequency of a tuning fork with the help of a standard tuning fork.
ii) To tune the stretched string of a musical instrument to a particular frequency.
iii) To detect the presence of dangerous gases in mines.

Note:

1. When wax is added to the arms of one of the tuning forks then its frequency decreases. i.e. $n^1 < n$
2. When arms of one of the tuning forks are filed then its frequency increases. i.e., $n^1 > n$
3. The following table gives the relation for beats produced when sounded together under different conditions.

Fork	Frequency	Relation Δn when	
		$\Delta n^1 > \Delta n$	$\Delta n^1 < \Delta n$
Wax is added to 1st fork	$n_1^1 < n_1$	$\Delta n = n_2 - n_1$	$\Delta n = n_1 - n_2$
Wax is added to 2nd fork	$n_2^2 < n_2$	$\Delta n = n_1 - n_2$	$\Delta n = n_2 - n_1$
1st fork is filled	$n_1^1 < n_1$	$\Delta n = n_1 - n_2$	$\Delta n = n_2 - n_1$
2nd fork is filled	$n_2^1 < n_1$	$\Delta n = n_2 - n_1$	$\Delta n = n_1 - n_2$

EX-33: ***The frequency of tunning fork 'A' is 250 Hz. It produces 6 beats/sec, when sounded together with another tunning fork B. If its arms are loaded with wax then it produces 4 beats/sec. Find the frequency of tuning fork B.***

Sol. $\Delta n = n_A : n_B$ = 6 beats / sec

If wax is added to the tunning fork A then its frequency decreases. i.e., $n_A^1 < n_A$ and given $\Delta n^1 = 4$ beats / sec $< \Delta n$

This is possible when $n_A - n_B = \Delta n$

$\Rightarrow 250 - n_B = 6 \Rightarrow n_B = 244Hz$

EX-34: ***A tunning fork of frequency of 512 Hz when sounded with unknown tunning fork produces 5 beats/sec. If arms of the unknown fork are filed then it produces only 3 beats / sec. Find the frequency of unknown tunning fork.***

Sol. $\Delta n = n_1 : n_2, n_1 = 512Hz$, $\Delta n = 5$ beats/sec

If arms of unknown fork are filed then its frequency increases.

i.e., $n_2^1 > n_2$ and given $\Delta n^1 = 3$ beats / sec.

This is possible when $\Delta n = n_2 - n_1$

$\Rightarrow 5 = n_2 - 512 \Rightarrow n_2 = 517Hz$

EX-35: ***The lengths of two open organ pipes are l and $l + \Delta l\left(\Delta l << l\right)$. If v is the speed of sound, find the frequency of beats between them.***

Sol. Beat frequency $= n_1 - n_2 = \frac{v}{2l} - \frac{v}{2\left(l + \Delta l\right)}$

$$= \frac{v}{2l}\left[1 - \left(1 + \frac{\Delta l}{l}\right)^{-1}\right] \approx \frac{v}{2l}\left[1 - 1 + \frac{\Delta l}{l}\right] = \frac{v\Delta l}{2l^2}$$

EX-36: ***If two sound waves,*** $y_1 = 0.3\sin 596\pi[t - x/330]$ ***and*** $y_2 = 0.5\sin 604\pi[t - x/330]$ ***are superposed, what will be the (a) frequency of resultant wave (b) frequency at which the amplitude of resultant waves varies (c) Frequency at which beats are produced. Find also the ratio of maximum and minimum intensities of beats.***

Sol. Comparing the given wave equation with

$y = A\sin\omega\left[t - \left(x/v\right)\right]\left[as\, k/\omega = 1/v\right]$ we find that here,

$A_1 = 0.3$, $\omega_1 = 2\pi n_1 = 596\pi \Rightarrow n_1 = 298$Hz

and $A_2 = 0.5$, $\omega_2 = 2\pi n_2 = 604\pi \Rightarrow n_2 = 302$ Hz

a) The frequency of the resultant

$$n_{avg} = \frac{n_1 + n_2}{2} = \frac{\left(298 + 302\right)}{2} = 300Hz$$

b) The frequency at which amplitude of resultant wave varies:

$$n_A = \frac{n_1 - n_2}{2} = \frac{\left(298 - 302\right)}{2} = 2Hz$$

c) The frequency at which beats are produced $n_b = 2n_A = n_1 - n_2 = 4$Hz

d) The ratio of maximum to minimum intensities of beat

$$\frac{I_{max}}{I_{min}} = \frac{\left(A_1 + A_2\right)^2}{\left(A_1 - A_2\right)^2} = \frac{\left(0.3 + 0.5\right)^2}{\left(0.3 - 0.5\right)^2} = \frac{64}{4} = 16$$

EX-37: ***The frequency of a tuning fork 'x' is 5% greater than that of a standard fork of frequency 'K'. The frequency of another fork 'y' is 3% less than that of 'K'. When 'x' and 'y' are vibrated together 4 beats are heard per second. Find the frequencies of x and y.***

Sol. Let the frequency of standard fork be K

$n_x = K + \frac{5K}{100} = \frac{105K}{100}$

$n_y = K - \frac{3K}{100} = \frac{97K}{100}$

$\Delta n = n_x - n_y \Rightarrow \quad 4 = \frac{105}{100}K - \frac{97}{100}K$

On solving, K = 50 Hz

The frequency of $x = \frac{105}{100} \times 50 = 52.5Hz$

Similarly frequency of $y = \frac{97}{100} \times 50 = 48.5Hz$

EX-38: ***A string under a tension of 129.6 N produces 10 beats per sec when it is vibrated along with a tuning fork. When the tension in the string is increased to 160 N, it sounds in unison with the same tuning fork. Calculate the fundamental frequency of the tuning fork.***

Sol. Let '*n*' be the frequency of fork.

The wire frequency would be $(n \pm 10)$

In case of a wire under tension $n \propto \sqrt{T}$

$\therefore \frac{n-10}{n} = \sqrt{\frac{129.6}{160}} \Rightarrow$ n = 100 Hz

EX-39 ***Two open organ pipes 80 cm and 81 cm long found to give 26 beats in 10 sec, when each is sounding its fundamental note. Find the velocity of sound in air.***

Sol. Number of beats per second $\Delta n = \frac{v}{2l_1} - \frac{v}{2l_2}$

$= \frac{26}{10} = \frac{v}{160} \sim \frac{v}{162} \Rightarrow 2.6 = \frac{2v}{160 \times 162}$

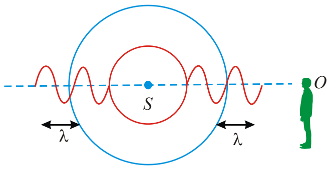
$\Rightarrow v = \frac{2.6 \times 160 \times 162}{2} = 33696 cms^{-1} \cong 337 ms^{-1}$.

DOPPLER'S EFFECT:

Whenever there is a relative motion between a source of sound and the observer (listener), the frequency of sound heard by the observer is different from the actual frequency of sound emitted by the force.

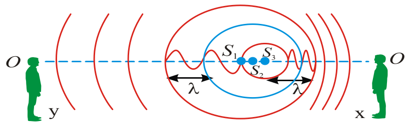
The frequency observed by the observer is called the apparent frequency. It may be less than or greater than the actual frequency emitted by the sound source. The difference depends on the relative motion between the source and observer.

1. When observer and source are stationary



i) Sound waves propagate in the form of spherical wavefronts (shown as circles)
ii) The distance between two successive circles is equal to wavelength λ
iii) Number of waves crossing the observer = Number of waves emitted by the source
iv) Thus apparent frequency (n') = actual frequency (n).

2. When source is moving but observer is at rest



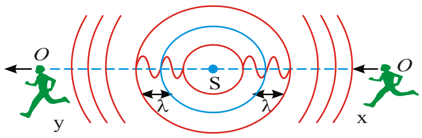
i) S_1, S_2, S_3 are the positions of the source at three different positions.
ii) Waves are represented by non-concentric circles, they appear compressed in the forward direction and spread out in backward direction.
iii) For observer (X)

Apparent wavelength $(\lambda') <$ Actual wavelength (λ)

$\Rightarrow$ Apparent frequency $(n') >$ Actual frequency (n)

For observer (Y) : $\lambda' > \lambda \Rightarrow n' < n$

3. When source is stationary but observer is moving



i) Waves are again represented by concentric circles.
ii) No change in wavelength received by either observer X or Y.
iii) Observer X (moving towards) receives wave fronts at shorter interval thus $n' > n$.
iv) Observer Y receives wavelengths at longer interval thus $n' < n$

4. General expression for apparent frequency:

If v, v_o, v_s are the velocities of sound, observer, source respectively and velocity of medium is v_m then apparent frequency observed by observer when wind blows in the direction of v (from the source to observer)is given by

$n' = \left[\frac{(v+v_m) \pm v_o}{(v+v_m) \pm v_S}\right] n$ and in opposite direction of v

(from observer to source) $n' = \left[\frac{(v-v_m) \pm v_o}{(v-v_m) \pm v_S}\right] n$

If medium is stationary i.e., $v_m = 0$ then $n' = \left(\frac{v \pm v_0}{v \pm v_S}\right) n$

- **Sign convention for different situation**

  i) The direction of v is always taken from source to observer.

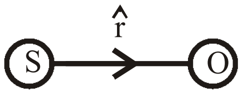
  ii) If the velocities v_o, v_s in the direction of v then positive +ve is taken.

  iii) If the velocities v_o, v_s in the opposite direction of v then positive -ve is taken.

Note:- i) Doppler effect in sound is asymmetric.

ii) Doppler effect in light is symmetric.

iii) Doppler's effect in vector form is written as



$n^1 = \left(\frac{v - \bar{v}_0 . \hat{r}}{v - \bar{v}_s . \hat{r}}\right)$ n

$\hat{r}$ = unit vector along the line joining source and observer $\bar{V}$ = Velocity of sound in the medium. Its direction is always taken from source to observer.

- **limitations of Doppler effect:**

  i) Doppler effect is not observed if

  a) $v_0 = v_s = 0$ (both are in rest)

  b) $v_0 = v_s = 0$ and medium is alone in motion direction.

  d) v_s is perpendicular to the line of sight

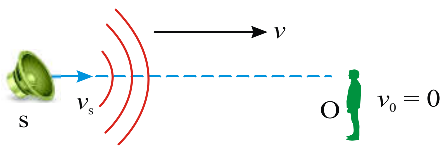
  ii) Doppler effect is applicable only when,

  $v_0 << v$ and $v_s << v$. (v is velocity of sound)

5. Common Cases in Doppler's Effect

- **Source is moving but observer at rest.**

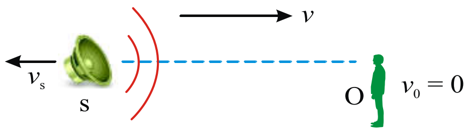
1. Source is moving towards the observer



Apparent frequency $n' = n\left(\frac{v}{v - v_S}\right)$

Apparent wavelength $\lambda' = \lambda\left(\frac{v - v_S}{v}\right)$

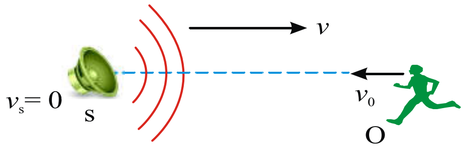
2. **Source is moving away from the observer.**



Apparent frequency $n' = n\left(\frac{v}{v+v_S}\right)$ Apparent wavelength $\lambda' = \lambda\left(\frac{v+v_S}{v}\right)$

➢ Source is at rest but observer is moving.

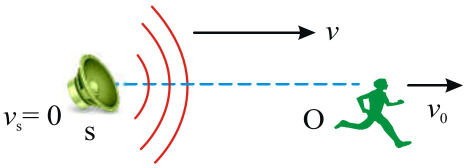
1. Observer is moving towards the source.



Apparent frequency $n' = n\left[\frac{v+v_0}{v}\right]$

Apparent wavelength $\lambda' = \frac{(v+v_0)}{n'} = \frac{(v+v_0)}{n\frac{(v+v_0)}{v}} = \frac{v}{n} = \lambda$

2. Observer is moving away from the source

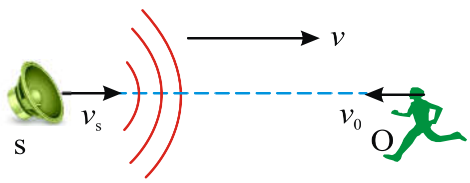


Apparent frequency $n' = n\left[\frac{v-v_O}{v}\right]$

Apparent wavelength $\lambda' = \lambda$

4. When source and observer both are moving

1. When both are moving towards each other

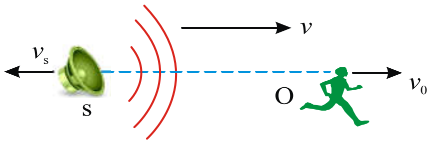


i) Apparent frequency $n' = n\left[\frac{v+v_S}{v-v_S}\right]$

ii) Apparent wavelength $\lambda' = \lambda\left(\frac{v-v_S}{v}\right)$

iii) Velocity of wave with respect to observer = $(v + v_0)$

2. **When both are moving away from each other.**

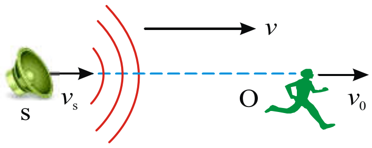


i) Apparent frequency $n' = n\left[\frac{v - v_O}{v + v_S}\right] (n' < n)$

ii) Apparent wavelength $\lambda' = \lambda\left(\frac{v + v_S}{v}\right)(\lambda' > \lambda)$

iii) Velocity of waves with respect to observer $= (v - v_O)$

3. **When source is moving behind observer**



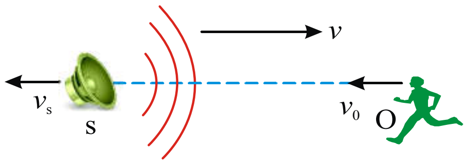
i) Apparent frequency $n' = n\left(\frac{v - v_O}{v - v_S}\right)$

a) If $v_O < v_S$, then $n' > n$
b) If $v_O > v_S$, then $n' < n$
c) If $v_O = v_S$ then $n' = n$

ii) Apparent wavelength $\lambda' = \lambda\left(\frac{v - v_S}{v}\right)$

iii) Velocity of waves with respect to observer $= (v - v_O)$

4. **When observer is moving behind the source**



i) Apparent frequency $n' = n\left(\frac{v + v_0}{v + v_s}\right)$

a) If $v_0 > v_s$, then $n' > n$

b) If $v_0 < v_s$, then $n' < n$

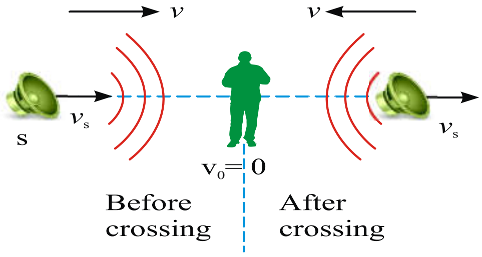
c) If $v_0 = v_s$, then $n' = n$

ii) Apparent wavelength $\lambda' = \lambda\left(\frac{v + v_S}{v}\right)$

iii) The velocity of waves with respect to observer $= (v - v_O)$

CROSSING

1. Moving sound source crosses a stationary observer



Apparent frequency before crossing

$$n'_{Before} = n\left[\frac{v}{v - v_S}\right]$$

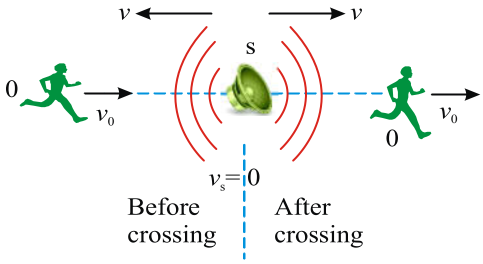
Apparent frequency $n'_{After} = n\left[\frac{v}{v + v_S}\right]$

Ratio of two frequencies $\frac{n'_{Before}}{n'_{After}} = \left[\frac{v + v_S}{v - v_S}\right] > 1$

Change in apparent frequency $n'_{Before} - n'_{After} = n\left(\frac{v}{v - v_s} - \frac{v}{v + v_s}\right) = nv\left(\frac{2v_s}{v^2 - v_s^{\ 2}}\right)$

If $v_s << v$ then $n'_{Before} - n'_{After} = \frac{2nv_S}{v}$

2. Moving observer crosses a stationary source



Apparent frequency before crossing

$$n'_{Before} = n\left(\frac{v + v_O}{v}\right)$$

Apparent frequency after crossing

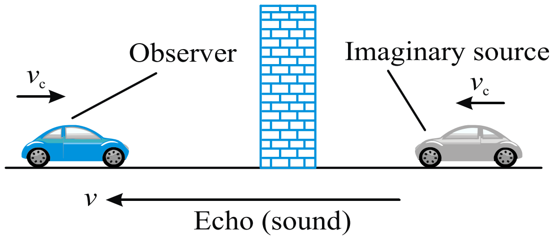
$$n'_{After} = n\left[\frac{v - v_O}{v}\right]$$

Ratio of two frequencies $\frac{n'_{Before}}{n'_{After}} = \frac{v + v_O}{v - v_O}$

Change in apparent frequency $n'_{Before} - n'_{After} = \frac{2nv_O}{v}$

SOME TYPICAL CASES OF DOPPLER' EFFECT

1. **Moving car towards wall:** When a car is moving towards a stationary wall as shown in figure. If the car sounds a horn, wave travels toward the wall and is reflected from the wall. When the reflected wave is heard by the driver, it appears to be of relatively high pitch, if we wish to measure the frequency of reflected sound.



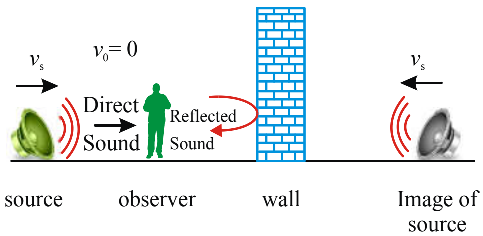
Here we assume that the sound which is reflected by the stationary wall is coming from the image of car which is at the back of it and coming towards it with velocity v_C. Now the frequency of sound heard by car driver be given as

$$n'_{direct} = n;\; n'_{reflected} = n\left(\frac{v+v_c}{v-v_c}\right)$$

No.of beats

$$\Delta n' = n'_{reflected} - n'_{direct} = \frac{2v_c n}{v-v_c}$$

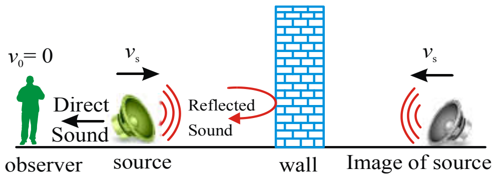
Case (i): If the observer is at rest in between source and wall as shown



$$n^1_{\text{direct}} = \left(\frac{v}{v-v_S}\right)n\;;\quad n^1_{\text{reflected}} = \left(\frac{v}{v-v_S}\right)n$$

No.of beats $\Delta n' = n'_{reflected} - n'_{direct} = 0$

Case (ii): If the source is in between observer and wall



$$n^1_{direct} = \left(\frac{v}{v+v_S}\right)n\;;\; n^1_{reflected} = \left(\frac{v}{v-v_S}\right)n$$

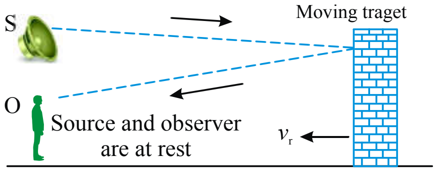
No. of beats $\Delta n' = n'_{reflected} - n'_{direct}$

$$=\left(\frac{v}{v-v_S}\right)n-\left(\frac{v}{v+v_S}\right)n=n\left(\frac{2v_s}{v^2-v_s^{\ 2}}\right)$$

If $v_s << v$ then $\Delta n'=\frac{2nv_S}{v}$

Note: This method of images for solving problems of Doppler effect is very convenient but is used only for velocities of source and observer which are very small compared to the speed of sound and it should not be used frequently when the reflector of sound is moving.

2. Moving target: Let a sound source S and observer O are at rest (stationary). The frequency of sound emitted by the source is n and velocity of waves is v.



A target is moving towards the source and observer, with a velocity v_T. Our aim is to find out the frequency observed by the observer, for the waves reaching it after reflection from the moving target. The formula is derived by applying Doppler equations twice, first with the target as observer and then with the target as source.

The frequency n' of the waves reaching surface of the moving target (treating it as observer) will be $n'=\left(\frac{v+v_T}{v}\right)n$

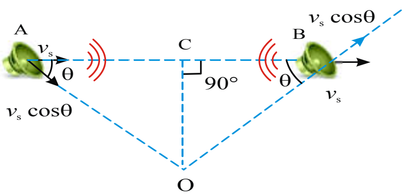
Now these waves are reflected by the moving target (which now acts as a source). Therefore the apparent frequency, for the real observer O will be $n''=\frac{v}{v-v_T}n' \Rightarrow n''=\frac{v+v_T}{v-v_T}n$

i) If the target is moving away from the observer, then $n'=\frac{v-v_T}{v+v_T}n$

ii) If target velocity is much less than the speed of sound, ($v_T << v$), then $n'=\left(1+\frac{2v_T}{v}\right)n$, for approaching target and $n'=\left(1-\frac{2v_T}{v}\right)n$ for receding target

3. Transverse Doppler's effect

i) If a source is moving in a direction making an angle θ w.r.t. the observer.



The apparent frequency herd by observer O at rest

At point A: $n'=\frac{nv}{v-v_S\cos\theta}$

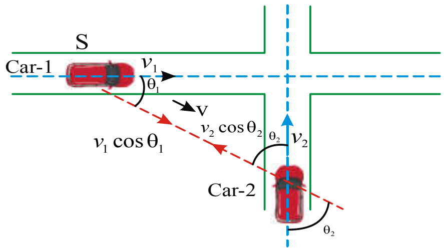
As source moves along AB, value of θ increases, $\cos\theta$ decreases, n' goes on decreasing.

At point C:

$\theta = 90°$, $\cos\theta = \cos 90° = 0, n' = n$

At point B: The apparent frequency of sound becomes $n'' = \frac{nv}{v + v_s \cos\theta}$

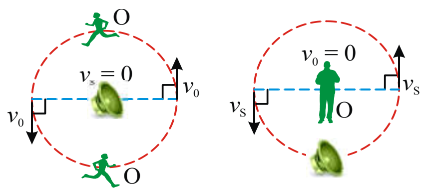
ii) When two cars are moving on perpendicular roads: When car-1 sounds a horn of frequency n, the apparent frequency of sound heard by car-2 can be given as $n' = n\left[\frac{v + v_2 \cos\theta_2}{v - v_1 \cos\theta_1}\right]$



4. Rotating source/observer: Suppose that a source of sound/observer is rotating in a circle of radius r with angular velocity ω (Linear velocity $v_S = r\omega$)

i) When source / observer at rest at centre of circle and observer / source is rotating in a circle

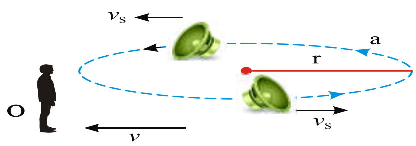
then the line of sight is perpendicular to the direction of motion of observer / source and hence no doppler effect. $\therefore n^1 = n$



ii) When source is rotating

a) Towards the observer heard frequency will be maximum i.e., $n_{max} = \frac{nv}{v - v_S}$

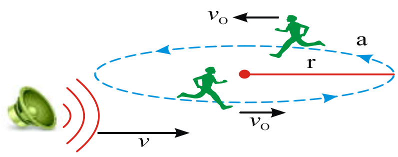
b) Away from the observer heard frequency will be minimum and $n_{min} = \frac{nv}{v + v_S}$



c) Ratio of maximum and minimum frequency

$$\frac{n_{max}}{n_{min}} = \frac{v + v_S}{v - v_S}$$

iii) When observer is rotating



a) Towards the source heard frequency will be maximum i.e., $n_{\max} = n\left(\frac{v+v_0}{v}\right)$

b) Away from the source heard frequency will be minimum and $n_{\min} = n\left(\frac{v-v_0}{v}\right)$

c) Ratio of maximum and minimum frequency

$$\frac{n_{\max}}{n_{\min}} = \frac{v+v_0}{v-v_0}$$

5. Doppler shift in RADAR: A microwave beam is directed towards the aeroplane and is received back after reflection from it. If '*v*' is the speed of the plane and '*n*' is the actual frequency of the microwave beam then the frequency of the microwave beam then the frequency received by moving plane $n^1 = \left(\frac{c+v}{c}\right)n$

Now the plane act as a moving source, the frequency of the wave from it is $n^{11} = \left(\frac{c+v}{c-v}\right)n$

(c is velocity of microwave)

Change in frequency $\Delta n \approx \frac{2nv}{c}$

By measuring Δn, the speed '*v*' can be obtained.

6. Uses of Doppler effect:

It is used in

a) SONAR

b) RADAR (Radio detection and ranging used to determine speed of objects in space)

c) To determine speeds of automobiles by traffic police. The technique is applied in the airports to guide the air crafts.

d) To determine speed of rotation of sun.

e) In Astrophysics, it is applied in the study of the saturn's rings and in the study of binary satrs.

Here the doppler's shift in the frequency of light from the atronomical objects is measured.

f) Accurate navigation and accurate target bombing techniques.

g) Tracking earth's satellite.

h) In medicine, it is applied to study the velocity of blood flow in different parts of the body and the moment of the fetus in the woomb using ultra sound. The conditions of heart beat can be inferred by "echocardiogram" generated from this technique.

EX-40 ***When a train is approaching the observer, the frequency of the whistle is 100 cps while when it has passed the observer, it is 50 cps. Calculate the frequency when the observer moves with the train.***

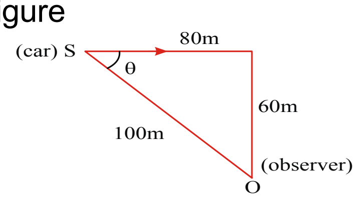
Sol. In case of approaching of source, $100 = \frac{nv}{v - v_S}$

while in case of recession of source, $50 = \frac{nv}{v + v_S}$

Which on simplification gives $n = \frac{200}{3} = 66.67\,Hz$

EX-41: ***A car approaching a crossing at a speed of 20 m/s sounds a horn of frequency 500Hz when at 80m from the crossing. Speed of sound in air is 330 m/s. What frequency is heard by an observer 60 m from the crossing on the straight road which crosses car road at right angles ?***

Sol. The situation is as shown in figure

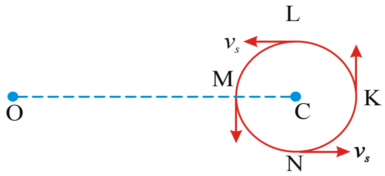


$\cos\theta = \frac{80}{100} = \frac{4}{5}$ $\therefore$ Apparent frequency is

$$n_{app} = \frac{v}{v - v_S\cos\theta}n = \left(\frac{330}{330 - 20\times\frac{4}{5}}\right)(500) = 525.5 \text{ Hz}$$

EX-42: ***A whistle of frequency 540 Hz rotates in a circle of radius 2 m at a linear speed of 30m/s. What is the lowest and highest frequency heard by an observer a long distance away at rest with respect to the centre of circle. Take speed of sound in air as 330 m/s. Can the apparent frequency be ever equal to actual ?***

Sol. Apparent frequency will be minimum when the source is at N and moving away from the observer.



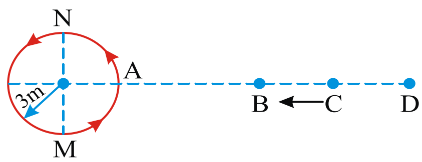
Apparent frequency will be maximum when source is at L and approaching the observer.

$$n'_{max} = \frac{v}{v - v_S}n = \left(\frac{330}{330 - 30}\right)(540) = 594Hz$$

Further when source is at M and K, angle between velocity of source and line joining source and observer is 90° (i.e., line of sight is perpendicular to v_s) or $v_S\cos\theta = v_S\cos 90° = 0$.

So, there will be no Doppler effect.

EX-43: ***A source of sound is moving along a circular orbit of radius 3 m with an angular velocity of 10 rad/s. A sound detector located far away from the source is executing linear simple harmonic motion along the line BD with amplitude BC = CD = 6 m. The frequency of oscillation of the detector is $(5/\pi)$ rev /sec. The source is at the point A when the detector is at the point B. If the source emits a continuous sound wave of frequency 340 Hz, find the maximum and the minimum frequencies recorded by the detector [velocity of sound = 330 m/s].***



Sol. Time period of circular motion $T = (2\pi/\omega) = (2\pi/10)$ is same as that of SHM i.e., T=(1/n) $= (\pi/5)$, so both will complete one periodic motion in same time.

Further more source is moving on a circle, its speed $v_s = r\omega$ = 3 × 10 = 30m/s and as detector is executing SHM, $v_D = \omega\sqrt{A^2 - y^2}$ $= 10\sqrt{6^2 - 0^2} = 60 m/s$ i.e., detector is at C.

So n' will be maximum when both move towards each other. $n'_{max} = n\left(\frac{v+v_D}{v-v_S}\right)$ with v_D = max i.e., the source is at M and detector at C and moving towards B, so

$$n'_{max} = 340\left[\frac{330+60}{330-30}\right] = 442 Hz$$

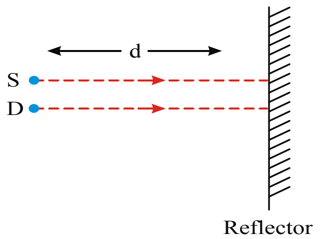
Similarly n' will be minimum when both are moving away from each other i.e.,

$n'_{min} = n\left(\frac{v-v_D}{v+v_S}\right)$ with v_D = max i.e., the source is at N and detector at C but moving towards D, so $n_{min} = 340\left[\frac{330-60}{330+30}\right] = 255 Hz$

ECHO

- The sound reflected by an obstacle which is heard by a listener is called an echo.
- Persistence of hearing is the minimum interval of time between two sound notes to distinguish them.
  Persistence of hearing is 0.1s
- A person is at a distance 'd' from a reflected surface (a wall, mountain etc). The person sounds a horn and hears its echo at the end of a time 't'. If V is the velocity of sound in air then.

$$d = \frac{Vt}{2}$$



To hear a clear echo, the minimum distance of the obstacle,

$$d_{min} = \frac{V \times 0.1}{2} = \frac{V}{20}$$

If V = 330 ms^{-1} then d_{min} = 16.5m
If V = 340 ms^{-1} then d_{min} = 17 m

PREVIOUS MAINS QUESTIONS

MECHANICAL WAVES

1. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound(v), density of air (p) and the frequency (f) . If $\Delta p \sim 10\text{Pa},\ v \sim 300\text{m/s},\ p \sim 1\text{kg/m}^3$ and $f \sim 1000$ Hz, then s will be of the order of (take the multiplicative constant to be 1) [Sep. 05, 2020 (I)]

(a) $\frac{3}{100}$ mm (b) 10 mm (c) $\frac{1}{10}$ mm (d) 1 mm

SOLUTION:(a) As we know, Pressure amplitude, $\Delta P_0 = aKB = S_0 KB = S_0 \times \frac{iJ}{V}(\times \text{p}V^2$

$$\left[\because K = \frac{0\text{J}}{V}, V = \sqrt{\frac{B}{\text{p}}}\right]$$

$\Rightarrow S_0 = \frac{\Delta P_0}{\text{p}V o)} \approx \frac{10}{1 \times 300 \times 1000}\text{m} = \frac{1}{30}\text{mm} \approx \frac{3}{100}$ mm

2. For a transverse wave travelling along a straight line, the distance between two peaks (crests) is 5 m, while the distance between one crest and one trough is 1.5 m. The possible wavelengths (in m) of the waves are: [Sep. 04, 2020 (I)]

(a) 1, 3, 5, ... (b) $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$, (c) 1, 2, 3, (d) $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{6}$,

SOLUTION: (b) Given: Distance between one crest and one trough= $1.5\text{m} = (2n_1 + 1)\frac{\lambda}{2}$

Distance between two crests $= 5\text{m} = n_2 \lambda$

$$\frac{1.5}{5} = \frac{(2n_1 + 1)}{2n_2} \Rightarrow 3n_2 = 10n_1 + 5$$

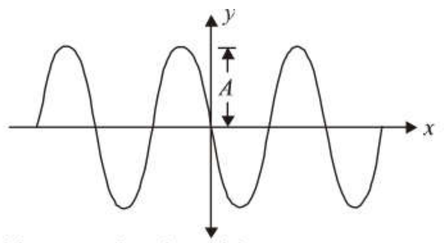
Here n_1 and n_2 are integer. If $n_1 = 1,\ n_2 = 5\ \lambda = 1$

$n_1 = 4, n_2 = 15\ \lambda = 1/3$

$n_1 = 7, n_2 = 25\ \lambda = 1/5$

Hence possible wavelengths $\frac{1}{1}$, $\frac{1}{3}$, $\frac{1}{5}$ metre.

3. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \varphi)$. Its snapshot at $t = 0$ is given in the figure. [12 April 2019 I]



For this wave, the phase ϕ is :

(a) $-\frac{\pi}{2}$ (b) π (c) 0 (d) $\frac{\pi}{2}$

SOLUTION: (b) At $t=0, x=0, y=0 \quad \varphi=\pi$ rad

4. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Given reference intensity of sound as 10 $12 \mathrm{W/m^2}$] [12 April 2019 II]

(a) 40 cm (b) 20 cm (c) 10 cm (d) 30 cm

SOLUTION: (a) Using, $\beta=10$ or $120=10^{\log 10\left(\frac{I}{10^{-12}}\right)}$ (i)

Also $I=\frac{P}{4\pi r^2}=\frac{2}{4\pi r^2}$ (ii) On solving above equations, we get $r=40$ cm.

5. The pressure wave, $\mathrm{P}=0.01 \sin [1000\mathrm{t}-3\mathrm{x}]$ Nm 2, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is $0^{\circ}\mathrm{C}$. On some other day when temperatureis T, the speed of sound produced by the same blade and at the same frequency is found to be 336 m/s Approximate value of T is: [9 April 2019 I]

(a) $4^{\circ}\mathrm{C}$ (b) $11^{\circ}\mathrm{C}$ (c) $12^{\circ}\mathrm{C}$ (d) $15^{\circ}\mathrm{C}$

SOLUTION: (a) On comparing with $\mathrm{P}=\mathrm{P}_0 \sin$ (wt-kx), we have $\mathrm{w}=1000 \mathrm{rad/s},\ \mathrm{K}=3\mathrm{m}^1$

$$v_0=\frac{w}{k}=\frac{1000}{3}=\frac{333.3\mathrm{m}}{\mathrm{s}} \quad \Rightarrow \frac{v_1}{v_2}=\sqrt{\frac{T_1}{T_2}} \text{or } \frac{333.3}{336}=\sqrt{\frac{273+0}{273+t}} \quad \mathrm{t}=4^{\circ}\mathrm{C}$$

6. A travelling harmonic wave is represented by the equation $y(x, \mathrm{t})=10^{-3} \sin (50\mathrm{t}+2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave? [12 Jan. 2019 I]

(a) The wave is propagating along the negative x-axis with speed25 ms^{-1}.

(b) The wave is propagating along the positive x-axis with speed 100 ms^{-1}.

(c) The wave is propagating along the positive x-axis with speed 25 ms^{-1}.

(d) The wave is propagating along the negative x-axis with speed 100 ms^{-1}.

SOLUTION: (a) Comparing the given equation $\mathrm{y}=10^{-3} \sin (50\mathrm{t}+2\mathrm{x})$ with standard equation,

$$\mathrm{y}=\mathrm{a} \sin (\omega \mathrm{t}-\mathrm{kx})$$

$\Rightarrow$ wave is moving along-ve x-axis with speed $\mathrm{v}=\frac{\mathrm{t}0}{\mathrm{K}} \Rightarrow \mathrm{v}=\frac{50}{2}=25 \mathrm{m/sec}.$

7. A transverse wave is represented by $\mathrm{y}=\frac{10}{\pi} \sin \left(\frac{2\pi}{\mathrm{T}} \mathrm{t}-\frac{2\pi}{\lambda} \mathrm{x}\right)$ For what value of the wavelength

the wave velocity is twice the maximum particle velocity? [Online April 9, 2014]

(a) 40 cm (b) 20 cm (c) 10 cm (d) 60 cm

SOLUTION: (a) Given, amplitude $a = 10$ cm wave velocity $= 2 \times$ maximum particle velocity

i.e, $\frac{c0\lambda}{2\pi} = 2\frac{ao)}{\pi}$ or, $\lambda = 4a = 4 \times 10 = 40$ cm

8. In a transverse wave the distance between a crest and neighboring trough at the same instant is 4.0 cm and the distance between a crest and trough at the same place is 1.0 cm. The next crest appears at the same place after a time interval of $0.4s$. The maximum speed of the vibrating particles in the medium is: [Online April 25, 2013]

(a) $\frac{3\pi}{2}$ cm/s (b) $\frac{\frac{5\pi}{2}\text{cm}}{\text{s}}$ (c) $\frac{\pi}{2}$ cm/s (d) 2πcm/s

SOLUTION: (b)

9. When two sound waves travel in the same direction in a medium, the displacements of a particle located at x' at time t' is given by :$y_1 = 0.05\cos(0.50\pi r - 100\pi t)$ $y_2 = 0.05\cos(0.46\tau u\kappa - 92\pi t)$ where y_1, y_2 and x are in meters and t in seconds. The speed of sound in the medium is : [Online April 9, 2013]

(a) 92 m/s (b) 200 m/s (c) 100 m/s (d)332m/s

SOLUTION: (b) Standard equation $y(x,t) =$ Acos $\left(\frac{0)}{V}x - c0t\right)$

From any of the displacement equation Say y_1 $\frac{t0}{V} = 0.50\pi$ and $(j) = 100\pi$

$$\frac{100\pi}{V} = 0.5\pi$$

$$V = \frac{100\pi}{0.5\pi} = 200\text{m/s}$$

10. The disturbance $y(x,t)$ of a wave propagating in the positive x-direction is given by $y = \frac{1}{1+x^2}$ at time $t = 0$ and by $y = \frac{1}{[1+(x-1)^2]}$ at $t = 2s$, where x and y are in meters. The shape of the wave disturbance does not change during the propagation. The velocity of wave in m/s is [Online May 26, 2012]

(a) 2.0 (b) 4.0 (c) 0.5 (d) 1.0

SOLUTION: (c) The equation of wave at any time is obtained by putting $X = x - vt$

$y^{=\frac{1}{1+x^2}} = \frac{1}{1+(x-vt)^2}$ (i)

We know at $t = 2\text{sec}$, $y = \frac{1}{1+(x-1)^2}$ (ii)

On comparing (i) and (ii) we get $vt = 1$ As $t = 2$ sec $V = \frac{1}{2} = 0.5\text{m/s}.$

11. The transverse displacement $y(x,t)$ of a wave is given by $y(x,t) = e^{-(ax^2+bt^2+2\sqrt{ab})xt)}$
This represents a: [2011]

(a) wave moving $\text{in} - x$ direction with speed $\sqrt{\frac{b}{a}}$

(b) standing wave of frequency $\sqrt{b}$

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) wave moving $\text{in} + x$ direction speed $\sqrt{\frac{a}{b}}$

SOLUTION: a) Given $y(x,t) =_e \left(-ax^2 + bt^2 + 2\sqrt{ab}xt\right) - [(\sqrt{ax}\sqrt{b}\sqrt{a}x.\sqrt{b} =_e - \left(\sqrt{a}x + \sqrt{b}t\right)^2$

It is a function of type $y = f(x+vt)$ $\therefore$ $y(x,t)$ represents wave travelling along-ve x direction

$\Rightarrow$Speed ofwave $= \frac{w}{k} = \sqrt{\frac{b}{a}}$

12. A wave travelling along the x-axis is described by the equation $\text{y(x, t)} = 0.005 \cos(\alpha\text{x} - \beta\text{t})$.
Ifthe wavelength and the time period of the wave are $0.08\ \text{m}$ and 2.0s, respectively, then α and β in appropriate units are [2008]

(a) $\alpha = 25.00\pi,\ \beta = \pi$

(b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$

(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$

(d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

SOLUTION: (a) Given, Wavelength, $l = 0.08\text{m}$ Time period, $\text{T} = 2.05$
$\text{y(x, t)} = 0.005 \cos(\alpha\text{x} - \beta\text{t})$ (Given) Comparing it with the standard equation of wave

$\text{y(x, t)} = \text{a} \cos(\text{kx} - \omega\text{t})$ we get $\text{k} = \alpha = \frac{2\pi}{l}$ and $\omega = \beta = \frac{2\pi}{T}$

$\alpha = \frac{2\pi}{0.08} = 25\pi$ and $\beta = \frac{2\pi}{2} = \pi$

13. A sound absorber attenuates the sound level by 20 dB. The intensity decreases by a factor of [2007]

(a) 100 (b) 1000 (c) 10000 (d) 10

SOLUTION: (a) Loudness of sound. $\text{L}_1 = 10 \log\left(\frac{\text{I}_1}{\text{I}_0}\right)$; $\text{L}_2 = 10 \log\left(\frac{\text{I}_2}{\text{I}_0}\right)$

$$\text{L}_\text{I} - \text{L}_2 = 10 \log\left(\frac{\text{I}_1}{\text{I}_0}\right) - 10 \log\left(\frac{\text{I}_2}{\text{I}_0}\right)$$

or, $\Delta\text{L} = 10 \log\left(\frac{\text{I}_1}{\text{I}_0} \times \frac{\text{I}_0}{\text{I}_2}\right) = 10 \log\left(\frac{\text{I}_1}{\text{I}_2}\right)$ The sound level attenuated by20 dB ie $\text{L}_\text{I} - \text{L}_2 = 20\text{dB}$

or, $20 = 10 \log \left(\frac{I_1}{I_2}\right)$ or, $2 = \log \left(\frac{I_1}{I_2}\right)$ or, $\frac{I_1}{I_2} = 10^2$ or, $I_2 = \frac{I_1}{100}$.

⇒ Intensity decreases by a factor 100.

14. The displacement y of a particle in a medium can be expressed as, $y = 10^{-6} \sin \left(100t + 20x + \frac{\pi}{4}\right) m$ where t is in second and x in meter. The speed ofthe wave is [2004]

(a) 20 m/s (b) 5 $\frac{m}{s}$ (c) 2000 m/s (d) $5\pi m/s$

SOLUTION: (b) Given, $y = 10^{-6} \sin \left(100t + 20x + \frac{\pi}{4}\right) m$

Comparing it with standard equation, we get $(\omega) = 100$ and $k = 20$ $v = \frac{\omega}{k} = \frac{100}{20} = 5 m/s$

15. The displacement y ofa wave travelling in the x -direction is given by $y = 10^{-4} \sin \left(600t - 2x + \frac{\pi}{3}\right)$ meters where x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} , is [2003]

(a) 300 (b) 600 (c) 1200 (d) 200

SOLUTION: . (a) $y = 10^{-4} \sin \left(600t - 2x + \frac{\pi}{3}\right)$ On comparing with standard equation

$$y = A \sin(\omega t - kx + \varphi)$$

we get $(\omega) = 600; k = 2$ Velocity of wave $v = \frac{\omega}{k} = \frac{600}{2} = 300 ms^{-1}$

16. When temperature increases, the frequency of a tuning fork [2002]

(a) increases

(b) decreases

(c) remains same

(d) increases or decreases depending on the material

SOLUTION: (b) The frequency of a tuning fork is given by $f = \frac{m^2 k}{4\sqrt{3}\pi \ell^2} \sqrt{\frac{Y}{p}}$ As temperature increases, the length or dimension ofthe prongs increases and also young's modulus increase therefore f decreases.

VIBRATION OF STRINGS AND ORGAN PIPES

17. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom, it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 $\mathrm{m/s}$, the tuning fork frequency is: [Sep. 05, 2020 (I)]

(a) 2200 Hz (b) 550 Hz (c) 1100 Hz (d) 3300 Hz

SOLUTION: (a) Here, $l_1 = 17$ cm and $l_2 = 24.5$ cm, $V = \frac{330\mathrm{m}}{\mathrm{s}}$ $f = ?$

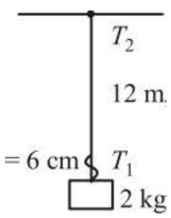
$\lambda = 2(l_2 - l_1) = 2 \times (24.5 - 17) = 15$ cm

Now, from $v = f\lambda \Rightarrow 330 = \lambda \times 15 \times 10^{-2} \Rightarrow \lambda = \frac{330}{15} \times 100 = \frac{1100 \times 100}{5} = 2200$ Hz

18. A uniform thin rope of length 12 m and mass 6 kg hangs vertically fi¡om a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wave train (in cm) when it reaches the top of the rope? [Sep. 03, 2020 (I)]

(a) 3 (b) 6 (c) 12 (d) 9

SOLUTION: (c) Using, $V = f\lambda \Rightarrow \frac{V_1}{\lambda_1} = \frac{V_2}{\lambda_2} \Rightarrow \lambda_2 = \frac{V_2}{V_1}\lambda_1$



Again using,

$n = \frac{V}{\lambda} = \sqrt{\frac{T}{M}} x_2 = \sqrt{\frac{T_2}{T_1}} \lambda_1$ $T_2 = 8\mathrm{g}$ (Top)

$= \sqrt{\frac{8g}{2g}} \lambda_1 = 2\lambda_1 = 12\mathrm{cm} T_1 = 2\mathrm{g}$ (Bottom)

19. Two identical strings X and Z made of same material have tension T_X and T_Z in them. If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio $T\sqrt{\ } T_Z$ is:

[Sep. 02, 2020 (D)]

(a) 2.25 (b) 0.44 (c) 1.25 (d) 1.5

SOLUTION: . (a) Using $f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$,where, $T =$ tension and $\mu = \frac{\text{mass}}{\text{1ength}}$

$f_X = \frac{1}{2\ell}\sqrt{\frac{T_X}{\mu}}$ and $f_Z = \frac{1}{2\ell}\sqrt{\frac{T_Z}{\mu}} \Rightarrow \frac{f_X}{f_Z} = \frac{450}{300} = \sqrt{\frac{T_X}{T_Z}} \Rightarrow \frac{T_X}{T_Z} = \frac{9}{4} = 2.25.$

20. A wire of density 9×10^{-3} kg cm^{-3} is stretched between two clamps 1 m apart. The resulting

strain in the wire is 4.9×10^{-4}. The lowest frequency of the transverse vibrations in the wire is (Young's modulus of wire $Y = 9 \times 10^{10} \mathrm{Nm}^{-2}$), (to the nearest integer), [Sep. 02, 2020 (II)]

SOLUTION: (35.00)Given, Density of wire, $0 = 9 \times 10^{-3}$ kg cm^{-3}

Young's modulus of wire, $Y = 9 \times 10^{10} \mathrm{Nm}^{-2}$ Strain $= 4.9 \times 10^{-4}$

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T/A}{\text{Strain}} \Rightarrow \quad \frac{T}{A} = Y \times \text{ Strain } = 9 \times 10^{9} \times 4.9 \times 10^{-4}$$

Also, mass of wire, $m = Al\rho$

Mass per unit length, $\mu = \frac{m}{l} = A\rho$

Fundamental frequency in the string $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{oA}} = \frac{1}{2\times1}\sqrt{\frac{9\times10^{9}\times49\times10^{-4}}{9\times10^{3}}}$

$$= \frac{1}{2}\sqrt{49 \times 10^{9-4-3}} = \frac{1}{2} \times 70 = 35 \text{ Hz}$$

21. A one meter long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 $\mathrm{m/s}$, the frequency difference between the fundamental and second harmonic of this pipe is Hz. [8 Jan. 2020 (I)]

SOLUTION: (106) Given: $V_{ar} = 300\mathrm{m/s}$, $\mathrm{p_{gas}} = 2\rho$ air Using, $V = \sqrt{\frac{B}{\mathrm{p}}}$

$$\frac{V_{gas}}{V_{air}} = \frac{\sqrt{\frac{B}{2\mathrm{p_{air}}}}}{\sqrt{\frac{B}{\mathrm{p_{air}}}}} \Rightarrow V_{gas} = \frac{V_{air}}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 150\sqrt{2}\mathrm{m/s}$$

And $\mathrm{f_{nth}}$ harmonic $= \frac{nv}{2L}$ (in open organ pipe) $(\mathrm{L} = 1$ metre given)

$$f_{2nd}\text{harmonic} - f_{\text{fundamenta1}} = \frac{2v}{2\times1} - \frac{v}{2\times1} = \frac{v}{2}$$

$$f_{2n}\text{harmonic} - f_{\text{fundamenta1}} = \frac{150\sqrt{2}}{2} = \frac{150}{\sqrt{2}} \approx 106 \ HZ$$

22. A transverse wave travels on a taut steel wire with a velocity of v when tension in it is 2.06×10^{4} N. When the tension is changed to T, the velocity changed to $vl2$.The value of T is close to : [8 Jan. 2020 (II)]

(a) $2.50 \times 10^{4}\mathrm{N}$ (b) $5.15 \times 10^{3}\mathrm{N}$ (c) $30.5 \times 10^{4}\mathrm{N}$ (d) $10.2 \times 10^{2}\mathrm{N}$

SOLUTION: (b) The velocity of a transverse wave in a stretched wire is given by $v = \sqrt{\frac{T}{\mu}}$

Where,$T =$ Tension in the wire,$\mu =$ linear density of wire $\Rightarrow (V \propto T) \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}}$

$$\Rightarrow \frac{v}{v} \times 2 = \sqrt{\frac{206 \times 10^{4}}{T_2}} \Rightarrow T_2 = \frac{2.06 \times 10^{4}}{4} = 0.515 \times 10^{4} N \Rightarrow T_2 = 5.15 \times 10^{3} N$$

23. Speed of a transverse wave on a straight wire (mass $6.0\mathrm{g}$, length 60 cm and area of cross-section $1.0\mathrm{mm}^2$) is $90\mathrm{ms}^{-1}$. Ifthe Young's modulus of wire is $16\times10^{1\prime}\mathrm{Nm}^{-2}$ the extension of wire over its natural length is: [7 Jan. 2020 (I)]

(a) $0.03\mathrm{mm}$ (b) $0.02\mathrm{mm}$ (c) $0.04\mathrm{mm}$ (d) $0.01\mathrm{mm}$

SOLUTION: (a) Given, $l=60$ cm, $m=6\mathrm{g}, A=1\mathrm{mm}^2$, $v=90\mathrm{m/s}$ and $Y=16\times10^{11}\mathrm{Nm}^2$

Using, $v=\sqrt{\frac{T}{m}\times l}\Rightarrow T=\frac{mv^2}{l}$ Again from, $Y=\frac{T}{A}\Delta L l L_0$

$$\Delta L=\frac{Tl}{YA}=\frac{mv^2\times l}{l(\mathrm{YA})}=\frac{6\times10^{-3}\times90^2}{16\times10^{11}\times10^{-6}}=3\times10^{-4}m=0.03\ \text{mm}$$

24. Equation of travelling wave on a stretched string of lineardensity5 $\mathrm{g/m}$ is $\mathrm{y}=0.03\sin(450\mathrm{t}-9\mathrm{x})$ where distance and time are measured in SI units. The tension in the string is: [11 Jan 2019 (I)]

(a) 10 N (b) $7.5\mathrm{N}$ (c) $12.5\mathrm{N}$ (d) $5\mathrm{N}$

SOLUTION: (c) We have given, $\mathrm{y}=0.03\sin(450\mathrm{t}-9\mathrm{x})$

Comparing it with standard equation of wave, we get $(\omega)=450$ $\mathrm{k}=9$

$v=\frac{\omega}{k}=\frac{450}{9}=50m/s$ Velocity of travelling wave on a stretched string is given by

$\mathrm{v}=\sqrt{\frac{\mathrm{T}}{\mu}}\Rightarrow\frac{\mathrm{T}}{\mu}=2500$ $\mu=$ linear mass density$\Rightarrow \mathrm{T}=2500\times5\times10^{-3}=12.5\mathrm{N}$

25. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $\mathrm{m}(\mathrm{m}<<\mathrm{M})$. When the car is at rest, the speed of transverse waves in the string is 60 m/s . when the car has acceleration a, the wave-speed increases to 60.5 m/s . The value of a, in terms of gravitational acceleration g, is closest to: [9 Jan. 2019 (I)]

(a) $\frac{\mathrm{g}}{30}$ (b) $\frac{\mathrm{g}}{5}$ (c) $\frac{\mathrm{g}}{10}$ (d) $\frac{\mathrm{g}}{20}$

SOLUTION: (b) Wave speed $\mathrm{V}=\sqrt{\frac{\mathrm{T}}{\mu}}$when car is at rest $\mathrm{a}=0$ ..$60=\sqrt{\frac{\mathrm{Mg}}{\mu}}$

Similarly when the car is moving with acceleration a,

$$60.5=\sqrt{\frac{\mathrm{M}\left(\mathrm{g}^2+\mathrm{a}^2\right)^{1/2}}{\mu}}$$

$$\frac{60.5}{60}=\sqrt{\sqrt{\frac{\mathrm{g}^2+\mathrm{a}^2}{\mathrm{g}^2}}}$$

$$\left(1+\frac{0.5}{60}\right)^4=\frac{\mathrm{g}^2+\mathrm{a}^2}{\mathrm{g}^2}=1+\frac{2}{60}\quad\Rightarrow\mathrm{g}^2+\mathrm{a}^2=\mathrm{g}^2+\mathrm{g}^2\times\frac{2}{60}$$

$\mathrm{a}=\mathrm{g}\sqrt{\frac{2}{60}}=\frac{\mathrm{g}}{\sqrt{30}}$ [which is closest to $\mathrm{g}/5$]

26. A wire of length L and mass per unit length 6.0×10^3 kg/m is put under tension of 540 N. Two

consecutive frequencies that it resonates at are: 420 Hz and 490 Hz. Then L in meters is

: [9 Jan. 2020 (II)]

(a) $2.1\mathrm{m}$ (b) I.I m (c) $8.1\mathrm{m}$ (d) $5.1\mathrm{m}$

SOLUTION: . (a) Fundamental frequency, $f=70$ Hz. The fundamental frequency of wire vibrating under

tension T is given by $f=\frac{1}{2L}\sqrt{\frac{T}{\mu}}$ Here, $\mu=$ mass per unit length of the wire $L=$ length ofwire

$$70=\frac{1}{2L}\sqrt{\frac{540}{6\times10^{-3}}}$$

$$\Rightarrow \mathrm{L}\approx 2.14\mathrm{m}$$

27. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1=30$ cm and $l_2=70$cm. Then, v is equal to: [12 Apri12019 (II)]

(a) 332 m/s (b) 384 m/s (c) 338 m/s (d) 379 m/s

SOLUTION: (b) $V=f\lambda=f\times2(\ell_2-\ell_1)=480\times2(0.70-0.30)=384\mathrm{m/s}$

28. A string $2.0\mathrm{m}$ long and fixed at its ends is driven by a 240 HZ vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is: [9 April 2019 (II)]

(a) 180 $\mathrm{m/s}$, 80 Hz (b) 320 $\mathrm{m/s}$, 80 Hz (c) $320\mathrm{m/s}$, $120\mathrm{Hz}$ (d) $180\mathrm{m/s}$, $120\mathrm{Hz}$

SOLUTION: (b) $\frac{3\lambda}{2}=2$ or $\lambda=\frac{4}{3}m$ Velocity, $v=f_\lambda=240\times\frac{4}{3}=320\mathrm{m/s}$

Also $f_1=\frac{240}{3}=80\mathrm{Hz}$

29. A string is clamped at both the ends and it is vibrating in its 4^{th} harmonic. The equation ofthe stationary wave is $\mathrm{Y}=0.3\sin(0.157\mathrm{x})\cos(200\mathrm{At})$. The length ofthe string is:

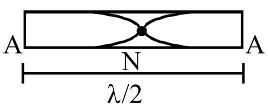
(All quantities are in SI units.) [9 April 2019 (I)]

(a) $20\mathrm{m}$ (b) $80\mathrm{m}$ (c) $40\mathrm{m}$ (d) $60\mathrm{m}$

SOLUTION: (b) Given, $\mathrm{y}=0.3\sin\ (0.157x)\cos(200\pi\mathrm{t})$ So $\mathrm{k}=0.157$ andw $=200\pi$

or $\mathrm{f}=100$ Hz, $v=\frac{w}{k}=\frac{200\pi}{0.157}=\frac{4000\mathrm{m}}{\mathrm{s}}$ Now, using $f=\frac{nv}{2l}=\frac{4v}{2l}=\frac{2v}{l}$

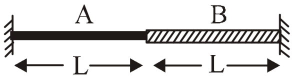
$\frac{\lambda}{2}=\mathrm{L}\ \Rightarrow\ \lambda=2\mathrm{L}$



$l=\frac{2v}{f}=\frac{2\times4000}{100}=80m$

30. A wire of length $2\mathrm{L}$, is made by joining two wires A and B ofsame length but different radii r

and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio $p: q$ is: [8 April 2019 (I)]



(a) 3: 5 (b) 4: 9 (c) 1: 2 (d) 1: 4

SOLUTION: (c) As there must be node at both ends and at the joint of the wire A and B so

$$\frac{V_A}{V_B}=\sqrt{\frac{u_B}{u_A}}=\frac{r_B}{r_A}=2=\frac{\lambda_A}{\lambda_B} \Rightarrow \lambda_A=2 \lambda_B \Rightarrow \frac{P}{q}=\frac{1}{2}$$

31. A closed organ pipe has a fundamental frequency of $1.5 \mathrm{kHz}$. The number of overtones that can be distinctly heard by a person with this organ pipe will be: (Assume that the highest frequency a person can hear is 20,000 Hz) [10 Jan. 2019 (I)]

(a) 6 (b) 4 (c) 7 (d) 5

SOLUTION: .(a) If a closed pipe vibration in N^{th} mode then frequency of vibration

$$\mathrm{n}=\frac{(2 \mathrm{N}-1) \mathrm{v}}{4 l}=(2 \mathrm{N}-1) \mathrm{n}_1$$

(where $\mathrm{n}_1=$ fundamental frequency of vibration) Hence $20,000=(2 \mathrm{N}-1) \times 1500$

$\Rightarrow \mathrm{N}=7.1 \approx 7$ Number ofover tones $=$ (No. of mode of vibration) $-1=7-1=6$

32. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to: [10 Jan. 2019 (I)]

(a) 10.0 cm (b) 33.3 cm (c) 16.6 cm (d) 20.0 cm

SOLUTION: (d) Velocity of wave on string $\mathrm{V}=\sqrt{\frac{\mathrm{T}}{\mu}}=\sqrt{\frac{8}{5} \times 1000}=40 \mathrm{m/s}$

Here, $\mathrm{T}=$ tension and $\mu=\mathrm{mass}/$length Wavelength of wave $\lambda=\frac{\mathrm{v}}{\mathrm{n}}=\frac{40}{100} \mathrm{m}$

Separation $\mathrm{b/w}$ successive nodes, $\frac{\lambda}{2}=\frac{40}{2 \times 100}=\frac{20}{100} \mathrm{m}=20$ cm

33. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^3 \mathrm{kg/m}^3$ and its Young's modulus is 9.27×10^{10} Pa .What will be the fundamental frequency of the longitudinal vibrations? [2018]

(a) 5 ℍb (b) 2.5 ℍb (c) 10 ℍb (d)7.5kHz

SOLUTION: (a) In solids, Velocity of wave $\mathrm{V}=\sqrt{\frac{\mathrm{Y}}{\mathrm{p}}}=\sqrt{\frac{927 \times 10^{10}}{27 \times 10^3}}=5.85 \times 10^3 \mathrm{m/sec}$

Since rod is clamped at middle fundamental wave shape is as follow $\lambda = 1.2\text{m}$

$(\because L = 60\text{cm} = .6\text{m}$(given)   Using $v = f\lambda \Rightarrow f = \frac{v}{\lambda} = \frac{5.85\times10^3}{1.2} = 4.88 \times 10^3 R = 5$ KHz

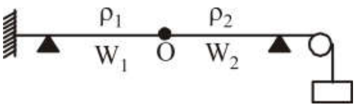
34. The end correction of a resonance column is lcm. If the shortest length resonating with the tuning fork is 10cm, the next resonating length should be [Online Apri116, 2018]

(a) 32cm (b) 4Mn (c) 28cm (d) 36cm

SOLUTION: (a) For first resonance, $\frac{\lambda}{4} = \ell_1 + e = 11$ cm (end correction $e = 1$ cm given)

For second resonance, $\frac{3\lambda}{4} = \ell_2 + e \Rightarrow \ell_2 = 3 \times 11 - 1 = 32$ cm

35. Two wires W_1 and W_2 have the same radius r and respective densities p_1 and p_2 such that $p_2 = 4p_1$. They are joined together at the point O, as shown in the figure. The combination is used as a sonometer wire and kept under tension T. The point O is midway between the two bridges. When a stationary wave is set up in the composite wire, the joint is found to be a node. The ratio of the number of antinodes formed in W_1 to W_2 is:
[Online April 8, 2017]



(a) 1: 1 (b) 1: 2 (c) 1: 3 (d) 4: 1

SOLUTION: (b) $n_1 = n_2$

$T \rightarrow$Same $r \rightarrow$Same$l \rightarrow$Same

Frequency of vibration$n = \frac{p}{2l}\sqrt{\frac{T}{\pi r^2 p}}$

As T, r, and l are same for both the wires $n_1 = n_2$

$$\frac{p_1}{\sqrt{p_1}} = \frac{p_2}{\sqrt{p_2}}$$

$$\Rightarrow \frac{p_1}{p_2} = \frac{1}{2} \quad p_2 = 4p_1$$

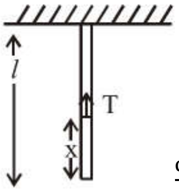
36. A uniform string of length 20 m is suspended from a rigid support. A short-wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the supports is: [2016] (take $g = 10\text{ms}^{-2}$)

(a) $2\sqrt{2}\text{s}$ (b) $\sqrt{2}\text{s}$ (c) $2\pi\sqrt{2}\text{s}$ (d)2s

SOLUTION: (a) We know that velocity in string is given by $v = \sqrt{\frac{T}{\mu}}$ (i)

where $\mu = \frac{m}{1} = \frac{\text{massofstring}}{\text{1engthofstring}}$ The tension $T = \frac{m}{\ell} \times x \times g$ (ii)

From (1) and (2)



$\frac{dx}{dt}=\sqrt{gx}$

$$x^{-1/2}dx=\sqrt{g}dt$$

$$\int_0^p x^{-1/2}\,dx-\sqrt{g}\int_0^p dt$$

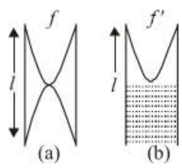
$$\Rightarrow 2\sqrt{l}$$

$$=\sqrt{g}\times t \quad t=2\sqrt{\frac{\ell}{g}}=2\sqrt{\frac{20}{10}}=2\sqrt{2}$$

37. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now: [2016]

(a) $2f$ (b) f (c) $\frac{f}{2}$ (d) $\frac{3f}{4}$

SOLUTION: . (b)The fundamental frequency in case (a) is $f=\frac{v}{2\ell}$



The fundamental frequency in case (b) is $f'=\frac{v}{4(\ell/2)}=\frac{v}{2\ell}=f$

38. A pipe of length 85 cm is closed fi_iom one end. Find the number of possible natural oscillations of air column in the pipe whose frequency lie below 1250 Hz. The velocity of sound in air is 340 m/s. [2014]

(a) 12 (b) 8 (c) 6 (d) 4

SOLUTION: (c) Length of pipe $=85cm=0.85m$ Frequency ofoscillations ofair column in closed organ

pipe is given by, $f=\frac{(2n-1)u}{4L}=\frac{(2n-1)0}{4L}\leq 1250 \quad \Rightarrow \frac{(2n-1)\times 340}{0.85\times 4}\leq 1250 \quad \Rightarrow 2n-1\leq 12.5\approx 6$

39. The total length of a sonometer wire between fixed ends is 110 cm. Two bridges are placed to divide the length of wire in ratio 6: 3 : 2. The tension in the wire is 400 N and the mass per unit length is 0.01 kg/m. What is the minimum common frequency with which three parts can vibrate? [Online Apri119, 2014]

(a) 1100 Hz (b) 1000 Hz (c) 166 Hz (d) 100 Hz

SOLUTION: (b) Total length of sonometer wire, $l = 110$ cm $= 1.1\mathrm{m}$

Length of wire is in ratio, 6: 3: 2 i.e. 60 cm, 30 cm, 20 cm.

Tension in the wire, $\mathrm{T} = 400\mathrm{N}$ Mass per unit length, $\mathrm{m} = 0.01$ kg

Minimum common frequency $=?$ As we know, Frequency, $\mathrm{v} = \frac{1}{21}\sqrt{\frac{\mathrm{T}}{\mathrm{m}}} = \frac{1000}{11}\mathrm{Hz}$

Similarly, $\mathrm{v}_1 = \frac{1000}{6}\mathrm{Hz}$ $\mathrm{v}_2 = \frac{1000}{3}$ Hz $\mathrm{v}_3 = \frac{1000}{2}$ Hz

Hence common frequency $= 1000$ Hz

40. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of I%. What is the fundamental frequency of steel if density and elasticity of steel are $7.7 \times 10^3 \mathrm{kym}^3$ and $2.2 \times 10^{11} \mathrm{N/m^2}$ respectively? [2013]

(a) $188.5\mathrm{R}$ (b) $178.2\mathrm{R}$ (c) 200.5 Hz (d) $770\mathrm{R}$

SOLUTION: (b) Fundamental frequency, $f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}\sqrt{\frac{T}{Ap}}$ [$v = \sqrt{\frac{T}{\mu}}$ and $\mu = \frac{m}{\ell}$]

Also, $Y = \frac{T\ell}{A\Delta\ell} \Rightarrow \frac{T}{A} = \frac{Y\Delta\ell}{\ell}$ $\Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{y\Delta\ell}{\ell p}}$ (i)

$$\ell = 1.5\mathrm{m},\ \frac{\Delta\ell}{\ell} = 0.01,$$

$\mathrm{p} = 7.7 \times 10^3 \mathrm{kg/m^3}$ (given)

$y = 2.2 \times 10^{11} \mathrm{N/m^2}$(given)

Putting the value of ℓ, $\frac{\Delta\ell}{\ell}$, p and y in eq^{n}. (i) we get,

$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3}$ or $f \approx 178.2\mathrm{Hz}$

41. A sonometer wire of length 114 cm is fixed at both the ends. Where should the two bridges be placed so as to divide the wire into three segments whose fundamental frequencies are in the ratio 1: 3: 4? [Online Apri123, 2013]

(a) At 36 cm and 84 cm from one end

(b) At 24 cm and 72 cm from one end

(c) At48 cm and 96 cm from one end

(d) At 72 cm and 96 cm from one end

SOLUTION: (d) Total length of the wire, $\mathrm{L} = 114$ cm $\mathrm{n}_1 : \mathrm{n}_2 : \mathrm{n}_3 = 1:3:4$

Let L_1, L_2 and L_3 be the lengths of the three parts As $\mathrm{n} \propto \frac{1}{\mathrm{L}}$

$$\mathrm{L}_1 : \mathrm{L}_2 : \mathrm{L}_3 = \frac{1}{1} : \frac{1}{3} : \frac{1}{4} = 12:4:3$$

$\mathrm{L}_1 = \left(\frac{12}{12+4+3} \times 114\right) = 72\mathrm{cm}$ $\mathrm{L}_2 = \left(\frac{4}{19} \times 114\right) = 24$ cm and $\mathrm{L}_3 = \left(\frac{3}{19} \times 114\right) = 18$ cm

Hence the bridges should be placed at 72 cm and 72+24 $=96$ cm from one end.

42. A cylindrical tube, open at both ends, has a fundamental frequency f in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now: [2012]

(a) f (b) $f l2$ (c) 3 f /4 (d) 2 f

SOLUTION: (a) Initially for open organ pipe, fundamental frequency $f_0 = v/2l_0$

(i)where l_0 is the length of the tube $v =$ speed of sound

But when it is half dipped in water, it becomes closed organ

pipe of length $\frac{\ell_0}{2}$ Fundamental frequency of closed organ pipe

$v_C = \frac{v}{4l_c}$ ... (ii)

New length, $l_c = \frac{l_0}{2}$

Thus $v_c = \frac{v}{4l_0/2} \Rightarrow v_c = \frac{v}{2l}$ ... (iii)

From equations (i) and(iii) $v_0 = v_c$

Thus, $v_c = f$ ($v_0 = f$ is given)

Therefore, value of l will be $(2n-1)l$ Hence option (b) i.e. $3 \times 31.25 = 93.75$ cm is correct.

43. An air column in a pipe, which is closed at one end, will be in resonance with a vibrating tuning fork of frequency 264 Hz if the length of the column in cm is (velocity of sound $= 330 \mathrm{m/s}$) [Online May 26, 2012]

(a) 125.00 (b) 93.75 (c) 62.50 (d) 187.50

SOLUTION: (b) Given: Frequency of tuning fork, $n = 264$ Hz

Length of column $L = ?$

For closed organ pipe $n = \frac{v}{4l} \Rightarrow l = \frac{v}{4n} = \frac{330}{4 \times 264} = 0.3125\mathrm{m} = 31.25$ cm

In case of closed organ pipe only odd harmonics are possible.

44. A uniform tube of length 60.5 cm is held vertically withits lower end dipped in water. A sound source of frequency 500 Hz sends sound waves into the tube. When the length of tube above water is 16 cm and again when it is 50 cm,the tube resonates with the source of sound. Two lowest frequencies (in Hz), to which tube will resonate when itis taken out of water, are (approximately). [Online May 19, 2012]

(a) 281, 562 (b) 281, 843 (c) 276, 552 (d) 272, 544

SOLUTION: (d) Two lowest frequencies to which tube will resonates are 272 Hz and 544 Hz.

45. The equation of a wave on a string of linear mass density $0.04\ \text{kg m}^{-1}$ is given by

$y = 0.02(\text{m}) \sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$.The tension in the string is [2010]

(a) 4.0N (b) $12.5\ \text{N}$ (c) $0.5\ \text{N}$ (d) $6.25\ \text{N}$

SOLUTION: (d) $y = 0.02(m) \sin\left[2\pi\left(\frac{t}{0.04(s)}\right) - \frac{x}{0.50(m)}\right]$

Comparing it with the standard wave equation $y = a \sin(0)t - kx)$

we get $\omega = \frac{2\pi}{0.04}$ rad s^{-1}

and $k = \frac{2\pi}{0.50}$ Wave velocity, $\text{v} = \frac{w}{k}$ $\Rightarrow v = \frac{2\pi/0.04}{2\pi/0.5} = 12.5mls$

Velocity on a string is given by $v = \sqrt{\frac{T}{\mu}}$ $T = \text{v}^2 \times \mu = (12.5)^2 \times 0.04 = 6.25\text{N}$

46. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then [2008]

(a) $18 > \text{x}$ (b) $\text{x} > 54$ (c) $54 > \text{x} > 36$ (d) $36 > \text{x} > 18$

SOLUTION: (b) Fundamental frequency for first resonant length $\text{v} = \frac{\text{v}}{4\ell_1} = \frac{\text{v}}{4\times18}$ (in winter)

Fundamental frequency for second resonant length $\text{v}' = \frac{3\text{v}'}{4\ell_2} = \frac{3\text{v}'}{4\text{x}}$ (in summer)

According to questions, $\frac{\text{v}}{4\times18} = \frac{3\text{v}'}{4\times\text{x}}$ $\text{x} = 3 \times 18 \times \frac{\text{v}'}{\text{v}}$

$\text{x} = 54 \times \frac{\text{v}'}{\text{v}}$ cm $\text{v}' > \text{v}$ because velocity of light is greater in summer as

compared to winter $(\text{v} \propto \sqrt{\text{T}})$ $\text{x} > 54\text{cm}$

47. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is [2006]

(a) 105 Hz (b) $1.05\ \text{Hz}$ (c) 1050Hz (d) $10.5\ \text{Hz}$

SOLUTION: (a) It is given that 315 Hz and 420 Hz are two resonant frequencies, let these be n^{th} and $(n+1)^{\text{th}}$ harmonies, then we have $\frac{\text{nv}}{2\ell} = 315$ and $(\text{n}+1)\frac{\text{v}}{2\ell} = 420$

$$\Rightarrow \frac{n+1}{n} = \frac{420}{315} \Rightarrow n = 3$$

Hence $3 \times \frac{v}{2\ell} = 315 \Rightarrow \frac{v}{2\ell} = 105$ *HZ*

The lowest resonant frequency is when $n = 1$

Therefore, lowest resonant frequency $= 105$ Hz.

48. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tube A and B is [2002]

(a) 1: 2 (b) 1: 4 (c) 2: 1 (d) 4: 1

SOLUTION: (c) The fundamental frequency for tube B closed at one end is given by $f_B = \frac{v}{4\ell}$

$\left[\because \ell = \frac{\lambda}{4}\right]$ Where $\ell =$ length of the tube and v is the velocity of sound in air.

The fundamental frequency for tube A open with both ends is given by

$f_A = \frac{v}{2\ell} \left[\because \ell = \frac{\lambda}{2}\right]$ $\frac{f_A}{f_B} = \frac{v}{2\ell} \times \frac{4\ell}{v} = \frac{2}{1}$

49. A wave $y = a \sin((j)t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then the equation ofthe unknown wave is [2002]

(a) $y = a \sin(0)t + kx)$

(b) $y = -a \sin(0)t + kx)$

(c) $y = a \sin(0)t - kx)$

(d) $y = -a \sin(0)t - kx)$

SOLUTION: (b) To form a node there should be superposition of this wave with the reflected wave. The reflected wave should travel in opposite direction with a phase change of π. The equation of the reflected wave will be $y = a \sin(\omega t + kx + \pi)$ $\Rightarrow y = -a \sin(\omega t + kx)$

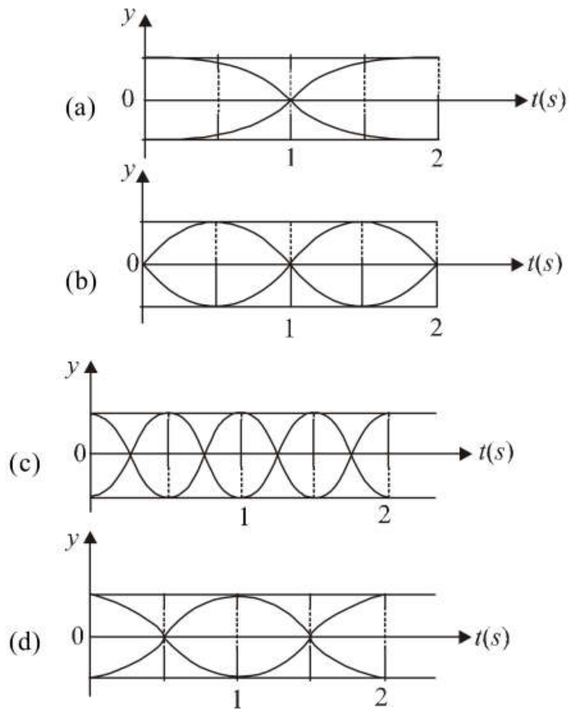
TOPIC 3 Beats, Interference and Superposition of Waves

50. There harmonic waves having equal frequency v and same intensity I_0, have phase angles 0, $\frac{\pi}{4}$ and $-\frac{\pi}{4}$ respectively. When they are superimposed the intensity of the resultant wave is close to: [9 Jan. 2020 I]

(a) $5.8 I_0$ (b) $0.2 I_0$ (c) 3 I_0 (d) I_0

SOLUTION: (a)

51. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is: [10 April 2019 II]



SOLUTION: (c) Beat frequency= difference in frequencies oftwo waves= $11-9=2$ Hz

52. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork offrequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to: [12 Jan. 2019 II]

(a) 322 ms^{-1} (b) 341 ms^{-1} (c) 335 ms^{-1} (d) 328 ms^{-1}

SOLUTION: (d)

53. A tuning fork vibrates with frequency256 Hz and gives one beat per second with the third normal mode of vibration of an open pipe. What is the length of the pipe? (Speed of sound of air is 340 ms^{-1}) [Online April 15, 2018]

(a) 190 cm (b) 180 cm (c) 220 cm (d) 200 cm

SOLUTION: (d) According to question, tuning fork gives 1 beat/ssecond with (N) $3^{r}d$ normal mode. Therefore, organ pipe will have frequency (256 ± 1) Hz. In open organ pipe,

frequency $n=\frac{NV}{2\ell}$ or, $255=\frac{3\times 340}{2\times\ell}\Rightarrow\ell=2m=200$ cm

54. 5 beats/ second are heard when a turning fork is sounded with a sonometer wire under tension, when the length of the sonometer wire is either $0.95m$ or lm. The frequency of the fork will be: [Online Apri115, 2018]

(a) 195Hz (b) 251Hz (c) 150Hz (d)300Hz

SOLUTION: (a) Probable frequencies of tuning fork be $n \pm 5$ Frequency of sonometer wire, $n \propto \frac{1}{l}$

$$\frac{n+5}{n-5} = \frac{100}{95} \Rightarrow 95(n+5) = 100(n-5)$$

or, 95 $n + 475 = 100n - 500$ or, $5n = 975$ or, $n = \frac{975}{5} = 195\text{Hz}$

55. A standing wave is formed by the superposition of two waves travelling in opposite directions. The transverse displacement is given by $y(x,t) = 0.5 \sin\left(\frac{5\pi}{4}x\right) \cos(200\pi t)$ .What is the speed of the travelling wave moving in the positive x direction?(x and t are in meter and second, respectively.)

[Online April 9, 2017]

(a) 160m/s (b) 90m/s (c) 180m/s (d)120m/s

SOLUTION: (a) Given, $\text{y}(\text{x}, \text{t}) = 0.5 \sin\left(\frac{5\pi}{4}\text{x}\right) \cos(200\pi\text{t})$,comparing with equation-

$\text{y} \ (\text{x}, \text{t}) = 2 \ \text{a} \ \sin \ \text{kx} \ \cos \omega \text{t} \Rightarrow (\omega) = 200\pi, \text{k} = \frac{5\pi}{4}$

speed of travelling wave $\text{v} = \frac{0)}{\text{k}} = \frac{200\pi}{5\pi/4} = 160\text{m/s}$

56. A wave represented by the equation $y_1 = a \cos(kx - \omega t)$ is superimposed with another wave to form a stationary wave such that the point $x - 0$ is node. The equation for the other wave is

[Online May 12, 2012]

(a) $a \ \cos(kx - \omega t + \pi)$ (b) $a \ \cos(kx + \omega t + \pi)$

(c) $a \ \cos\left(kx + \omega t + \frac{\pi}{2}\right)$ (d) $a \ \cos\left(kx - \omega t + \frac{\pi}{2}\right)$

SOLUTION (b) Since the point $x = 0$ is a node and reflection is taking place from point $x = 0$. This means that reflection must be taking place from the fixed end and hence the reflected ray must suffer an additional phase change of π or a path change of $\frac{\lambda}{2}$.

So, if $y_{\text{incident}} = a \cos(kx - \omega t) \Rightarrow y_{\text{incident}} = a \cos(-kx - \omega t + \pi) = -a \cos(\omega t + kx)$

Hence equation for the other wave $y = a \cos(kx + \omega t + \pi)$

57. Following are expressions for four plane simple harmonic waves [Online May 7, 2012]

(i) $y_1 = A \cos 2\pi\left(\left(+\frac{x}{\lambda_1}\right)\right)$

(ii) $y_2 = A \cos 2\pi\left(\left(+\frac{x}{\lambda_1} + \pi\right)\right)$

(iii) $y_3 = A \cos 2\pi(^{n_2 t + \frac{x}{\lambda_2}})(\quad)$

(iv) $y_4 = A\cos 2\pi\left(\left(-\frac{x}{\lambda_2}\right)\right)$

The pairs of waves which will produce destructive interference and stationary waves respectively in amedium, are

(a) (iii, iv), (i, ii) (b) (i, iii), (ii, iv) (c) (i, iv), (ii, iii) (d) (i, ii), (iii, iv)

SOLUTION: (d) In case of destructive interference Phase difference $\varphi = 180°$ or π So wave pair (i) and (ii) will produce destructive interference. Stationary or standing waves will produce by equations

(iii) & (iv) as two waves travelling along the same line but in opposite direction. $n' = n + x$

58. A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is

(a) A wave travelling along $+\mathrm{x}$ direction [2011 RS]

(b) A wave travelling along-x direction

(c) A standing wave having nodes at $x = \frac{n\lambda}{2}, n = 0,1,2$ ....

(d) A standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}; n = 0,1,2$ ....

SOLUTION: (d) $y = A\sin(\omega t - kx) + A\sin(\omega t + kx) = 2A\sin\omega t\cos kx$

This is an equation of standing wave. For position of nodes $\therefore$ $\cos kx = 0$

$$\Rightarrow \frac{2\pi}{\lambda}.x = (2n+1)\frac{\pi}{2} \qquad \Rightarrow x = \frac{(2n+1)\lambda}{4},\ n = 0,1,2,3, \dots\dots\dots$$

59. Statement - 1 : Two longitudinal waves given by equations: $y_1(x,t) = 2a\sin\left((j)t - kx\right)$ and $y_2(x,t) = a\sin(20)t - 2kx)$ will have equal intensity. [2011 RS]

Statement-2 : Intensity of waves of given frequency in same medium is proportional to square of amplitude only.

(a) Statement-I is true, statement-2 is false.

(b) Statement-I is true, statement-2 is true, statement-2 is the correct explanation of statement-I

(c) Statement-I is true, statement-2 is true, statement-2 is not the correct explanation of statement-I

(d) Statement-I is false, statement-2 is true.

SOLUTION: (a) Intensity of a wave $I = \frac{1}{2}pw^2A^2v$ Since, $I \propto A^2w^2$ $I_1 \propto (2a)^2w^2$

and $I_2 \propto a^2(2w)^2$ $\therefore I_1 = I_2$

In the same medium, p and v are same. Intensity depends on amplitude and frequency

Note: Had the frequency of unknown fork been 284 cps, then on placing wax its frequency would have decreased thereby increasing the gap between its frequency and the frequency of known fork. This would produce high beat frequency.

65. (a) Let f_1 be the frequency heard by wall,(v)

$$f_1 = f_0 \frac{v}{v - v_c}$$

Here, $v =$ Velocity of sound,

$v =$ Velocity ofCar,

$f_0 =$ actual frequency ofcar horn

Let f_2 be the frequency heard by driver after reflection from wall. $f_2 = \left(\frac{v+v_c}{v}\right) f_1 = \left(\frac{v+v_c}{v-v_c}\right) f_0 \Rightarrow$

$480 = \left[\frac{345+v_c}{345-v_c}\right] 440 \Rightarrow \frac{12}{11} = \frac{345+v_c}{345-v_c} \quad \Rightarrow v_c = 54\mathrm{km/hr}$

66. (a) From the Doppler's effect of sound, frequency appeared at wall $f_w = \frac{330}{330-v} \cdot f$ (i)

Here, $v =$ speed of bus, $f =$ actual frequency of source

Frequency heard after reflection $from$ wall (f') is

$$f' = \frac{330+v}{330} \cdot f_w = \frac{330+v}{330-v} \cdot f$$

$$\Rightarrow 490 = \frac{330+v}{330-v} \cdot 420$$

$$\Rightarrow v = \frac{330 \times 7}{91} \approx 25.38\mathrm{m/s} = 91\mathrm{km/s}$$

67. (d) Permanent magnets(*P*) are made of materials with large retentivity and large coercivity. Transformer cores (*T*) are made of materials with low retentivity and low coercivity.

68. (c) From Doppler's effect, frequency of sound heard (f_1)
when source is approaching $f_1 = f_0(\frac{c}{c-v})$
Here, $c =$ velocity of sound $v =$ velocity of source
Frequency of sound heard (r_2) when source is receding $f_2 = f_0 \frac{c}{c+v}$
Beat frequency $= f_1 - f_2$
$\Rightarrow 2 = f_1 - f_2 = f_0 c \left[\frac{1}{c-v} - \frac{1}{c+v}\right] \quad = f_0 c \frac{2v}{c^2\left[1-\frac{v^2}{c^2}\right]}$

For $c >> v$ $\Rightarrow v = \frac{2c}{2f_0} = \frac{c}{f_0} = \frac{350}{1400} = \frac{1}{4}\mathrm{m/s}$

69. (d) $f_1 = f(\frac{v - v_o}{v - v_s}) = f\left(\frac{1500-5}{1500-7.5}\right)$

No reflected signal,

60. Three sound waves of equal amplitudes have frequencies(v-1), v, $(v+1)$. They superpose to give beats. The number of beats produced per second will be : [2009]

(a) 3 (b) 2 (c) 1 (d) 4

SOLUTION: (b) Maximum number of beats $=$ Maximum frequency- Minimum frequency

$= (v+1) - (v-1) = 2$ Beats per second

61. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2? [2005]

(a) 202Hz (b) 200Hz (c) 204Hz (d) 196Hz

SOLUTION: (d) Frequency of fork 1, $no = 200$ Hz

No. of beats heard when fork2 is sounded with fork $1 = \Delta n = 4$

Now on loading (attaching tape) on unknown fork, the mass of tuning fork increases, So the beat frequency increases (from 4 to 6 in this case) then the frequency of the unknown fork 2 is given by,

$n = n_0 - \Delta n = 200 - 4 = 196$ Hz

62. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string ofa piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was [2003]

(a) (256+2) Hz b) (256-2) Hz (c) (256-5) Hz (d) (256+5) Hz

SOLUTION: (c) It is given that tuning fork offiiequency256 Hz makes 5 beats/second with the vibrating string of a piano. Therefore, possible frequency of the piano are (256 ± 5) Hz. i. e., either261Hz or251 Hz. When the tension in the piano string increases, its frequency will increase. As the original frequency was 261Hz, the beat frequency should decrease, we can conclude that the frequency of

piano string is 251Hz

63. A tuning fork arrangement (pair) produces 4 beats/ sec with one fork offiiequency288 cps. A little wax is placed on the unknown fork and it then produces 2 beats/sec. The frequency of the unknown fork is [2002]

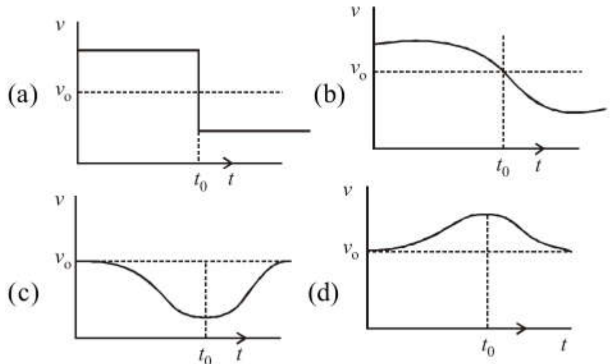
(a) 286 cps (b) 292 cps (c) 294 cps (d) 288 cps

SOLUTION: (b) Frequency of unknown fork $=$ known $frequency \pm$ Beat

frequency $= 288 + 4$ cps or 288-4 cps i.e. 292 cps or 284cps. When a little wax is placed on the unknown fork, it produces 2 beats/sec. When a little wax is placed on the unknown fork, its frequency decreases and simultaneously the beat frequency decreases confirming that the frequency of the unknown fork is 292 cps.

MUSICAL SOUND AND DOPPLER EFFECT

64. A sound source S is moving along a straight track with speed v, and is emitting sound of frequency v_o (see figure). An observer is standing at a finite distance, at the point O, from the track. The time variation of frequency heard by the observer is best represented by:
[Sep. 06, 2020 (I)]
t_0 represents the instant when the distance between the source and observer is minimum)



A driver in a car, approaching a vertical wall notices that

SOLUTION: (b) Frequency heard by the observer $v_{\text{observed}}=\left(\frac{v_{\text{sound}}}{\mathrm{v}_{\text{sound}}-v\cos\theta}\right)v_0$

Observer Initially θ will be less so $\cos\theta$ more. v_{observed} more, then it will decrease.

65.the frequency of his car horn, has changed from 440 Hz to 480 Hz, when it gets reflected from the wall. If the speed of sound in air is 345 $\mathrm{m/s}$, then the speed of the car is:[Sep. 05, 2020 (II)]

(a) 54 $\mathrm{kn}\vee l_{\mathrm{N}}$ (b) 36 km/hr (c) 18 km/hr (d) 24 km/hr

SOLUTION: (a) Let f_1 be the frequency heard by wall,(v)

$$f_1=f_0\frac{v}{v-v_c}$$

Here, $v=$ Velocity of sound,

$v=$ Velocity ofCar,

$f_0=$ actual frequency ofcar horn

Let f_2 be the frequency heard by driver after reflection from wall. $f_2=\left(\frac{v+v_c}{v}\right)f_1=\left(\frac{v+v_c}{v-v_c}\right)f_0\Rightarrow$

$480=\left[\frac{345+v_c}{345-v_c}\right]440\Rightarrow\frac{12}{11}=\frac{345+v_c}{345-v_c}\quad\Rightarrow v_c=54\mathrm{km/hr}$

66. The driver of a bus approaching a big wall notice that the frequency of his bus's horn changes $\mathrm{from}420$ Hz to 490 Hz when he hears it after it gets reflected from the wall. Find the speed of the bus if speed of the sound is 330 ms^{-1}. [Sep. 04, 2020 (II)]

(a) 91 kmh^{-1} (b) 81 kmh^{-1} (c) $61\mathrm{kmh}^{-1}$ (d) $71\mathrm{kmh}^{-1}$

SOLUTION: (a) From the Doppler's effect of sound, frequency appeared at wall $f_w=\frac{330}{330-v}\cdot f$ (i)

Here, $v =$ speed of bus, $f =$ actual frequency of source

Frequency heard after reflection $from$ wall $(f^{'})$ is

$$f' = \frac{330+v}{330} \cdot f_w = \frac{330+v}{330-v} \cdot f$$

$$\Rightarrow 490 = \frac{330+v}{330-v} \cdot 420$$

$$\Rightarrow v = \frac{330 \times 7}{91} \approx 25.38\mathrm{m/s} = 91\mathrm{km/s}$$

67. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required? [Sep. 02, 2020 (I)]

(a) T: Large retentivity, small coercivity

(b) P: Small retentivity, large coercivity

(c) T: Large retentivity, large coercivity

(d) P: Large retentivity, large coercivity

SOLUTION: (d) Permanent magnets(*P*) are made of materials with large retentivity and large coercivity. Transformer cores (*T*) are made of materials with low retentivity and low coercivity.

68. A stationary observer receives sound from two identical tuning forks, one of which approaches and the other one recedes with the same speed (much less than the speed of sound). The observer hears 2 $\mathrm{beats/sec}$. The oscillation frequency of each tuning fork is $\mathrm{v}_0 = 1400$ Hz and the velocity of sound in air is 350 $\mathrm{m/s}$. The speed of each tuning fork is close to: [7 Jan. 2020 I]

(a) $\frac{1}{2}\mathrm{m/s}$ (b) $\mathrm{lm/s}$ (c) $\frac{1}{4}\mathrm{m/s}$ (d)

SOLUTION: (c) From Doppler's effect, frequency of sound heard (f_1)

when source is approaching $f_1 = f_0(\frac{c}{c-v})$

Here, $c =$ velocity of sound $v =$ velocity of source

Frequency of sound heard (r_2) when source is receding $f_2 = f_0\frac{c}{c+v}$

Beat frequency $= f_1 - f_2$

$$\Rightarrow 2 = f_1 - f_2 = f_0 c\left[\frac{1}{c-v} - \frac{1}{c+v}\right] = f_0 c\frac{2v}{c^2\left[1-\frac{v^2}{c^2}\right]}$$

For $c >> v$ $\Rightarrow v = \frac{2c}{2f_0} = \frac{c}{f_0} = \frac{350}{1400} = \frac{1}{4}\mathrm{m/s}$

69. A submarine (A) travelling at 18 $\mathrm{km/hr}$ is being chased along the line of its velocity by another submarine (B) travelling at 27 $\mathrm{km/hr.B}$ sends a sonar signal of500 Hz to detect A and receives a

reflected sound of frequency v. The value of v is close to: [12 April 2019 I]

(Speed of sound in water $= 1500$ ms 1)

(a) 504Hz (b) 507Hz (c) 499 Hz (d) 502 Hz

SOLUTION: 69. (d) $f_1 = f(\frac{v - v_o}{v - v_s}) = f\left(\frac{1500-5}{1500-7.5}\right)$

No reflected signal,

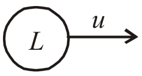
$$f_2 = f_1\left[\frac{v + v_0}{V + V_S}\right] = f_1\left(\frac{1500 + 7.5}{1500 + 5}\right)$$

$$f_2 = 500\left(\frac{1500-5}{1500-7.5}\right)\left(\frac{1500+7.5}{1500+5}\right) = 502 \text{ Hz}$$

70. Two sources of sound S_1 and S_2 produce sound waves of same frequency660 Hz. A listener is moving from sourceS_1 towards S_2 with a constant speed $u \mathrm{m/s}$ and he hears 10 $\mathrm{beats/s}$. The velocity of sound is 330 $\mathrm{m/s}$. Then $u$equals: [12 April 2019 II]

(a) $5.5 \mathrm{~m/s}$ (b) $15.0 \frac{\mathrm{m}}{\mathrm{s}}$ (c) $2.5 \mathrm{m/s}$ (d) $10.0 \mathrm{m/s}$

SOLUTION: . (c) $f_1 = f\frac{v - v_0}{v}$ and $f_2 = f\frac{v + v_0}{v}$



S_1 S_2

But frequency, $f_2 - f_1 = f \times \frac{2v_0}{v}$

or $10 = 660 \times \frac{2u}{330}$ $u = 2.5 \mathrm{m/s}$.

71. A stationary source emits sounds waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 4801 Hz and 530 Hz. Their respective speeds are, inms^{-1},(Given speed of sound $= 300 \mathrm{m/s}$) [10 April 2019 I]

(a) 12, 16 (b) 12, 18 (c) 16, 14 (d) 8, 18

SOLUTION: (b) Frequency of sound source $(\mathrm{f}_0) = 500$ Hz When observer is moving away $\mathrm{fi}_{\mathrm{i}0}\mathrm{m}$ the source

Apparent frequency $\mathrm{f}_1 = 480 = \mathrm{f}_0(\frac{\mathrm{v} - \mathrm{v}_0'}{\mathrm{v}})\ldots.(\mathrm{i})$ And when observer is moving towards the source

$\mathrm{f}_2 = 530 = \mathrm{f}_0(\frac{\mathrm{v} - \mathrm{v}_0''}{\mathrm{v}}$ (ii) From equation (i) $480 = 500\left(\frac{300 - \mathrm{v}_0'}{300}\right)$

$$\mathrm{v}_0 = 12 \mathrm{m/s}$$

From equation (ii)

$$V_0 = 18 m/s$$

72. A source of sound S is moving with a velocity of 50 $\frac{m}{s}$ towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take velocity ofsound in air 350 ffis) [10 April 2019 II]

(a) 750 Hz (b) 857 Hz (c) 1143 Hz (d) 807 Hz

SOLUTION: (a) When source is moving towards a stationary observer, $f_{app} = f_{source}\left(\frac{V-0}{V-50}\right) \Rightarrow 1000 = f_{source}\left(\frac{350}{300}\right)$

When source is moving away from observer $f^{\uparrow} = f_{source}\left(\frac{350}{350+50}\right) = \frac{1000\times300}{350}\times\frac{350}{400} \approx 750$ Hz

73. Two cars A and B are moving away $from$ each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. Ifan observer in car A detects a frequency2000 Hz ofthe sound coming from car B, what is the natural frequency ofthe sound source in car B? (speed of sound in air $= 340ms^{-1}$) [9 April 2019 II]

(a) $2250Hz$ (b) $2060Hz$ (c) $2300Hz$ (d) $2150Hz$

SOLUTION: (a) $f' = f\frac{v-v_0}{v+v_s} \Rightarrow$ or $2000 = f\frac{340-20}{340+20} \Rightarrow f = 2250$ Hz.

74. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1 If the speed of the train is reduced to 17 m/s, the frequency registered is f_2 If speed of sound is 340 m/s, then the ratio $f_1 l f_2$ is: [10 Jan. 2019 I]

(a) 18/17 (b) 19/18 (c) 20/19 (d) 21/20..

SOLUTION: (b) According to Doppler's effect, when source is moving but observer at rest

$$f_{app} = f_0\left[\frac{V}{V-V_s}\right] \Rightarrow f_1 = f_0\left[\frac{340}{340-34}\right] \text{and, } f_2 = f_0\left[\frac{340}{340-17}\right]$$

$$\frac{f_1}{f_2} = \frac{340-17}{340-34} = \frac{323}{306} \text{or,} \frac{f_1}{f_2} = \frac{19}{18}$$

75. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician fiom another end of a hall at a speed of 10 km/h. If the wave speed is 330 m/s, the frequency heard by the running person shall be close to:[9 Jan. 2019 II]

(a) 666 Hz (b) 753 Hz (c) 500 Hz (d) 333 Hz

SOLUTION: (a) Frequency of the sound produced by open flute.

$$f = 2\left(\frac{v}{2\ell}\right) = \frac{2\times330}{2\times0.5} = 660Hz \text{ Velocity of observer, } v_0 = 10\times\frac{5}{18} = \frac{25}{9}mls$$

As the source is moving towards the observer therefore, according to $\text{Doppler}^{\uparrow}\text{s}$ effect.
Frequency detected by observer,

$$f' = \left[\frac{v+v_0}{v}\right] f = \left[\frac{\frac{25}{9}+330}{330}\right] 660$$

$= \frac{2995}{9 \times 330} \times 660$ or, $f' = 665.55 = 666\text{Hz}$

76. Two sitar strings, A and B, playing the note 'Dha' are slightly out of tune and produce beats and frequency 5 Hz. The tension of the string B is slightly increased and the beat frequency is found to decrease by 3 Hz. If the frequency $\text{of} A$ is 425 Hz, the original frequency of B is
[Online Apri116, 2018]

(a) 430Hz (b) 428 Hz (c) 422 Hz (d) 420 Hz

SOLUTION: (d) $n_A = 425$ Hz, $n_B = ?$

Beat frequency $x = 5$ Hz which is decreasing $(5 \rightarrow 3)$ after increasing the tension of the string B.

Also tension of string B increasing so $n_B \uparrow (n \propto \sqrt{T})$

Hence $n_A - n_B \uparrow = x \downarrow \rightarrow$ correct

$n_B \uparrow - n_A = x \downarrow \rightarrow$ incorrect $\quad n_B = n_A - x = 425 - 5 = 420\text{Hz}$

77. A toy-car, blowing its horn, is moving with a steady speed of5 m/s, away fi:om a wall. An observer, towards whom the toy car is moving, is able to hear 5 beats per second. If the velocity of sound in air is 340 m/s, the frequency of the horn of the toy car is close to: [Online Apri110, 2016]

(a) 680Hz (b) 510Hz (c) 340Hz (d)170Hz

SOLUTION: . (d) From Doppler's effect f(direct) $= f\left(\frac{340}{340-5}\right) = f_1$

$$f(\text{bywal1}) = f\left(\frac{340}{340+5}\right) = f_2$$

Beats $= (f_1 - f_2) \Rightarrow 5 = f\left(\frac{340}{340-5} - \frac{340}{340+5}\right) \quad \Rightarrow f = 170$ Hz.

78. Two engines pass each other moving in opposite directions with uniform speed of30 m/s. One of them is blowing a whistle of frequency 540 Hz. Calculate the frequency heard by driver of second engine before they pass each other. Speed of sound is 330 m/sec: [Online April 9, 2016]

(a) 450 Hz (b) 540 Hz (c) 270 Hz (d) 648 Hz

SOLUTION: (d) We know that the apparent frequency $f' = \left(\frac{v-v_0}{v-v_s}\right)$ ffiᵢom Doppler's effect

where $v = v = 30 m/s$, velocity of observer and source Speed or sound is $v = 330 m/s$

$f' = \frac{330+30}{330-30} \times 540 = 648$ Hz.

Frequency of whistle $(f) = 540$ Hz.

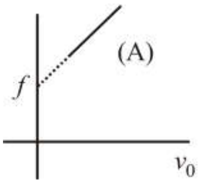
79. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound $= 320 ms^{-1}$) close to: [2015]

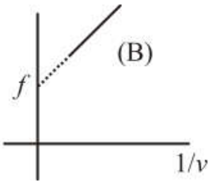
(a) 18% (b) 24% (c) 6% (d) 12%

SOLUTION: (d) $f_1 = f\left[\frac{v}{v-v_s}\right] = f \times \frac{320}{300}$ HZ $f_2 = f\left[\frac{v}{v+v_s}\right] = f \times \frac{320}{340}$ HZ

$$\left(\left(\frac{f_2}{f_1} - 1\right)\right) \times 100 = \left(\frac{300}{340} - 1\right) \times 100 = 12\%$$

80. A source of sound emits sound waves at frequency f_0. It is moving towards an observer with fixed speed v_s ($v_s < v$, where v is the speed of sound in air). If the observer were to move towards the source with speed v_0, one ofthe following two graphs (A and B) will given the correct variation of the frequency f heard by the observer as v_0 is changed.





The variation of f with v_0 is given correctly by :[Online April 11, 2015]

(a) graph A with slope $= \frac{f_0}{(v+v_s)}$

(b) graph B with slope $= \frac{f_0}{(v-v_s)}$

(c) graph A with slope $= \frac{f_0}{(v-v_s)}$

(d) graph B with slope $= \frac{f_0}{(v+v_s)}$

SOLUTION: (c) According to Doppler's effect,

Apparent, frequency $f = \left(\frac{V+V_0}{V-V_S}\right) f_0$

Now, $f = \left(\frac{f_0}{V-V_S}\right) V_0 + \frac{V f_0}{V-V_s} = \underline{f_0}$

So, slope $V-V_S$ Hence, option (c) is the correct answer.

81. A bat moving at 10 ms^{-1} towards a wall sends a sound signal of 8000 Hz towards it. On reflection it hears a sound of frequency f. The value of f in Hz is close to (speed of sound $=$ $320\mathrm{ms}^{-1}$) [Online April 10, 2015]

(a) 8516 (b) 8258 (c) 8424 (d) 8000

SOLUTION: . (a) Reflected frequency of sound reaching bat $=\left[\frac{V-(-V_0)}{V-V_s}\right]f=\left[\frac{V+V_0}{V-V_s}\right]f=\frac{V+10}{V-10}f$

$=\left(\frac{320+10}{320-10}\right)\times 8000=8516$ Hz

82. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground of frequency nearly: (Speed of sound $=343\mathrm{m/s}$) [Online Apri112, 2014]

(a) 2150 Hz (b) 2500 Hz (c) 1800 Hz (d) 2400 Hz

SOLUTION: (b) Given $\mathrm{f_A}=1800$ Hz $\mathrm{v_t}=\mathrm{v}$

$\mathrm{f_B}=2150$ Hz

Reflected wave frequency received by A, $\mathrm{f_{A'}}=?$

Applying doppler's effect of sound, $\mathrm{f}'=\frac{\mathrm{v_s f}}{\mathrm{v_s-v_t}}$

where, $\mathrm{v_t}=\mathrm{v_s}\left(1-\frac{\mathrm{f_A}}{\mathrm{f_B}}\right)$ $=343\left(1-\frac{1800}{2150}\right)=55.8372\mathrm{m/s}$

d_{OW}, for the reflected wave, $\mathrm{f}_{\mathrm{A}}'=\left(\frac{\mathrm{v_s+v_t}}{\mathrm{v_s-v_t}}\right)$ $\mathrm{f_A}=\left(\frac{343+55.83}{343-55.83}\right)\times 1800$ $=2499.44\approx 2500\mathrm{Hz}$

83. Two factories are sounding their sirens at 800 Hz. A man goes from one factory to other at a speed of $2\mathrm{m/s}$. The velocity of sound is 320 $\mathrm{m/s}$. The number of beats heard by the person in one second will be: [Online April 11, 2014]

(a) 2 (b) 4 (c) 8 (d) 10

SOLUTION: (d) Given: Frequency of sound produced by siren, $f=800\mathrm{R}$

Speed of observer, $u=2\mathrm{m/s}$

Velocity of sound, $v=320\mathrm{m/s}$

No. of beats heard per second $=?$

No. of extra waves received by the observer per second $=\underline{+}4\lambda$

$\left(\because\lambda=\frac{V}{f}\right)$ No. of beats/ sec $=\frac{2}{\lambda}-\left(-\frac{2}{\lambda}\right)=\frac{4}{\lambda}=\frac{2\times 2}{\frac{320}{8\ 00}}$ $=\frac{2\times 2\times 800}{320}=10$

84. A and B are two sources generating sound waves. A listener is situated at C. The frequency of the source at A is 500 Hz. A, now, moves towards C with a speed 4 $\frac{m}{s}$.The number of beats heard at C is 6.¥Men A moves away from C with speed 4 m/s, the number ofbeats heard at C is 18. The speed of sound is 340 m/s. The frequency of the source at B is: [Online April 22, 2013]

A C B

(a) 500Hz (b) 506Hz (c) 512Hz (d) 494Hz

SOLUTION: (c) $f = 500$ Hz

A4m/sCB

Listener

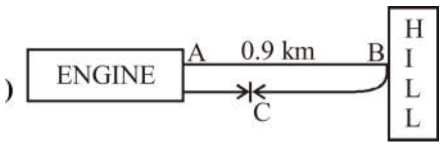
Case 1 : When source is moving towards stationary listener

apparent frequency $\eta' = \eta\left(\frac{v}{v-v_s}\right) = 500\left(\frac{340}{336}\right) = 506\text{Hz}$

Case 2 : When source is moving away from the stationary listener

$\eta'' = \eta\left(\frac{v}{v+v_s}\right) = 500\left(\frac{340}{344}\right) = 494$ Hz

In case 1 number of beats heard is 6 and in case 2 number of beats heard is 18 therefore frequency of the source at $B = 512R$



85. An engine approaches a hill with a constant speed. ¥Menit is at a distance of0.9 km, it blows a whistle whose echo is heard by the driver after 5 seconds. If the speed of sound in air is 330 m/s, then the speed of the engine is: [Online April 9, 2013]

(a) 32 m/s (b) 27.5m/s (c) 60 m/s (d)30m/s

SOLUTION: (d Let after 5 sec engine at point C $t = \frac{AB}{330} + \frac{BC}{330} 5 = \frac{0.9\times1000}{330} + \frac{BC}{330}$

$$BC = 750\text{m}$$

Distance travelled by engine in 5 sec $= 900\text{m} - 750\text{m} = 150\text{m}$

Therefore, velocity of engine $= \frac{150\text{m}}{5\text{ sec}} = 30\text{m/s}$

86. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: Bats emitting ultrasonic waves can detect the location of a prey by hearing the waves reflected from it.

Statement 2: When the source and the detector are moving, the frequency of reflected waves is changed. [Online May 12, 2012]

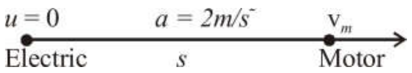
(a) Statement 1 is false, Statement 2 is true.

(b) Statement 1 is true, Statement 2 is false.

(c) Statement 1 is true, Statement 2 is true, Statement2 is not the correct explanation of Statement 1.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of Statement 1.

SOLUTION: (c) Bats catch the prey by hearing reflected ultrasonic waves .When the source and the detector (observer) are moving, frequency of reflected waves change. This is according to Doppler's effect.



87. A motor cycle starts from rest and accelerates along a straight path at $2\mathrm{m/s}^2$. At the starting point of the motor cycle there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound $= 330\mathrm{ms}^{-1}$) [2009]

(a) 98 m (b) 147 m (c) 196 m (d) 49m

SOLUTION: (a) siren cycle Let the motorcycle has travelled a distances, its velocity at that point

$$\mathrm{v}_m^2 - u^2 = 2as \quad \mathrm{v}_m^2 = 2 \times 2 \times s \qquad \mathrm{v}_m = 2\sqrt{s}$$

The observed frequency will be $v' = v\left[\frac{\mathrm{v}-\mathrm{v}_m}{\mathrm{v}}\right]$

$$0.94v = v\left[\frac{330 - 2\sqrt{s}}{330}\right] \Rightarrow s = 98.01m$$

88. A whistle producing sound waves of frequencies 9500 HZ and above is approaching a stationary person with speed vms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies up to a maximum of10,000 HZ, the maximum value of v up to which he can hear whistle is [2006]

(a) $15\sqrt{2}\mathrm{ms}^{-1}$ (b) $\frac{15}{\sqrt{2}}\mathrm{ms}^{-1}$ (c) 15 ms^{-1} (d) 30 ms^{-1}

SOLUTION: (c) Apparent frequency $\mathrm{v}' = \mathrm{v}\left[\frac{\mathrm{v}}{\mathrm{v}-\mathrm{v}_s}\right] \Rightarrow 10000 = 9500\left[\frac{300}{300-\mathrm{v}}\right] \Rightarrow 300 - \mathrm{v} = 300 \times .95$

$$\Rightarrow \mathrm{v} = 300 - 285 = 15\mathrm{ms}^{-1}$$

89. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency? [2005]

(a) 0.5% (b) zero (c) 20% (d) 5%

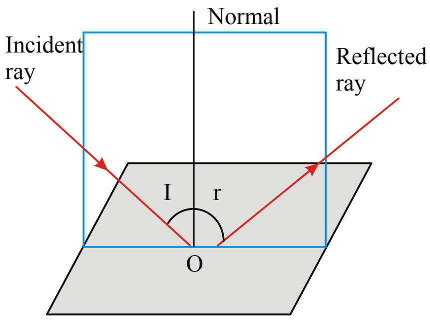
SOLUTION: (c) Apparent frequency $n' = n\left[\frac{v+v_0}{v}\right] = n\left[\frac{v+\frac{v}{5}}{v}\right] = n\left[\frac{6}{5}\right] \frac{n'}{n} = \frac{6}{5}$

The percentage increase in apparent frequency $\frac{n'-n}{n} = \frac{6-5}{5} \times$ 100=20%

Introduction

* Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within a small range of the electromagnetic spectrum.
* Electromagnetic radiation (Wavelength from 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision.
* Light travels along straight line with enormous speed. The speed of light in vacuum is the highest speed attainable in nature. The speed of light in vacuum is $c = 2.99792458 \times 10^8 ms^{-1}$.

  $\approx 3 \times 10^8 ms^{-1}$
* The wavelength of light is very small compared to the size of ordinary objects that we encounter commonly (generally of the order of a few cm or larger). A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a **ray of light**, and a bundle of such rays constitutes a **beam of light**.
* The phenomena of reflection, refraction and dispersion of light are explained using the ray picture of light. We shall study the image formation by plane and spherical reflection and refracting surfaces, using the basic laws of reflection and refraction. The construction and working of some important optical instruments, including the human eye are also explained.
* **Reflection of Light :** When a light ray strikes the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called reflection of light.



In case of reflection at the point of incidence 'O', the angle between incident ray and normal to the reflecting surface is called the angle of incidence (i). The angle between reflected ray and normal to the reflecting surface is called angle of reflection (r).

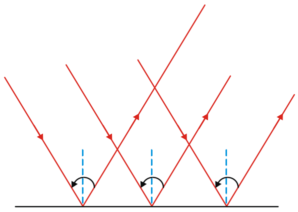
The palne containing incident ray and normal is called plane of incidence.

* **Laws of reflection :** The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.
* The angle of incidence is equal to the angle of reflection $\angle i = \angle r$

Types of reflections

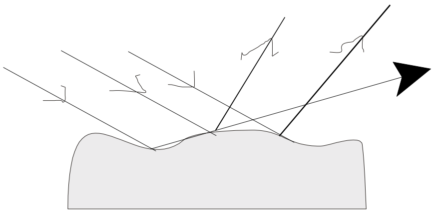
* **Regular reflection:** When the reflection takes place from a perfect smooth plane surface, then the reflection is called regular reflection (or) specular reflection.

  In this case, a parallel beam of light incident will remain parallel even after reflection as shown in the figure.



In case of regular reflection, the reflected light ray has large intensity in one direction and negligibly small intensity in other direction. Regular reflection of light is useful in determining the property of mirror.

* **Diffused reflection:** If the reflecting surface is rough (or uneven), parallel beam of light is reflected in random directions. This kind of refletion is called diffused reflection.

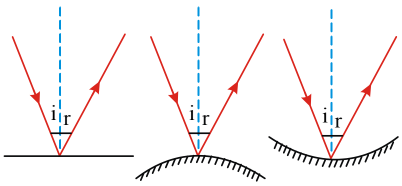


As shown in the above figure if the reflecting surface is rough, the normal at different points will be in different directions, so the rays that are parallel before reflection will be reflected in random directions.

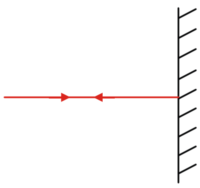
We see non-luminous objects by diffused reflection.

Important points regarding reflection

* Laws of reflection are valid for all reflecting surfaces either plane or curved.



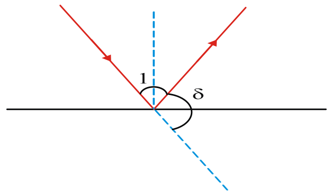
* If a light ray is incident normally on a reflecting surface, after reflection it retraces its path i.e., if $\angle i = 0$ then $\angle r = 0$



* In case of reflection of light frequency, wavelength and speed does not change. But the intensity of light on reflection will decreases.
* If the reflection of light takes place from a denser medium, there is a phase change of π rad.
* If $\hat{I}, \hat{N}$ and $\hat{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively then

  $\hat{R}.(\hat{I}\times\hat{N}) = \hat{N}.(\hat{I}\times\hat{R}) = \hat{I}.(\hat{N}\times\hat{R}) = 0$ This is because incident ray, reflected ray and the normal at the point of incidence lie in the same plane.

* **Deviation of a ray due to reflection:** The angle between the direction of incident ray and reflected light ray is called the angle of deviation (δ).

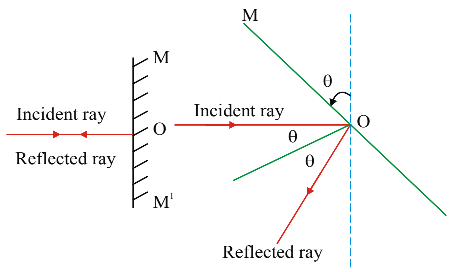


From the above figure $\delta = \pi - (i + r)$

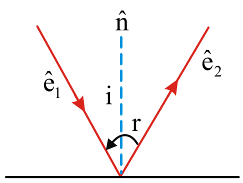
But $i = r$

Hence angle of deviation in the case of reflection is $\delta = \pi - 2i$

* By keeping the incident ray fixed, the mirror is rotated by an angle 'θ', about an axis in the plane of mirror, the reflected ray is rotated hrough an angle '2θ'.



* Vector form of law of reflection:

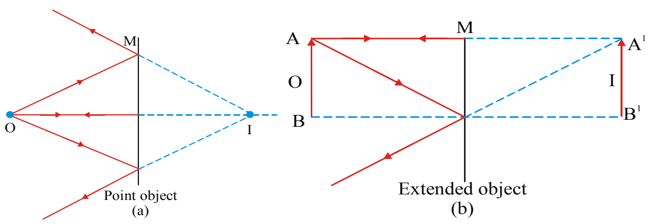


If $\hat{e}_1$ is unit vector along the incidnet ray $\hat{e}_2$ is the unit vector along the reflected ray $\hat{n}$ is the unit vector along the normal then,

$$\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{n})\hat{n}$$

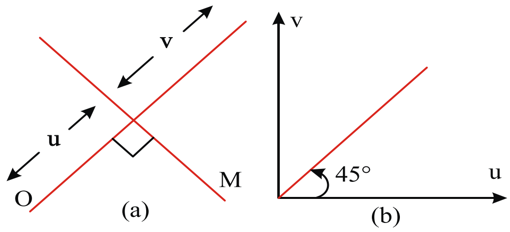
Reflection from Plane Surface

* When you look into a plane mirror, you see an image of yourself that has three properties. The image is up right.
* The image is the same size as you are
* The image is located as far behind the mirror as you are infront of it. This is shown in the figure(b).

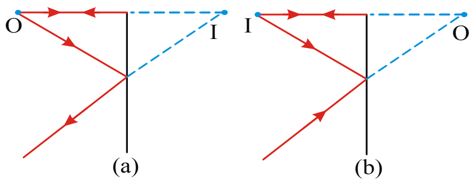


Point object
(a)
Extended object
(b)

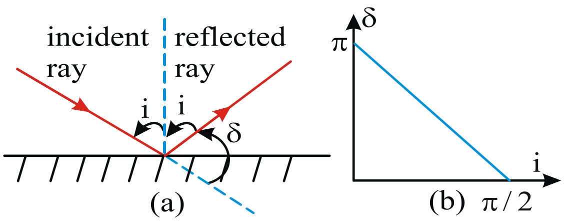
* A plane mirror always form virtual image to a real object and vice versa and the line joining object and image is perpendicular plane mirror as shown in figure (a).



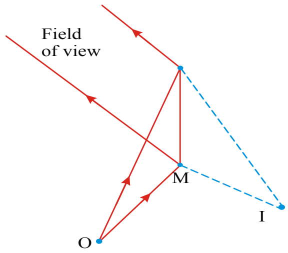
The graph between image distance (v) and object distance (u) for a plane mirror is a straight line as shown in figure (b).
The ratio of image height to the object height is called lateral magnification (m). Thus in case of plane mirror 'm' is equal to one.
* The principle of reversibility states that rays retrace their path when their direction is reversed. In accordance with the principle of reversibility object and image positions are interchangable. The points corresponding to object and image are called conjugate points.
This is illustrated in figure.



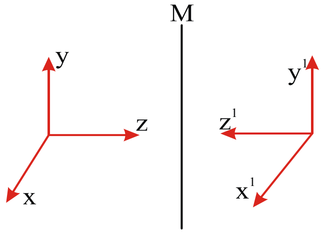
* A mirror whatever may be the size, it forms the complete image of the object lying infront of it. Large mirror gives more bright image than a smaller one. It is seen that the size of reflector must be much larger than the wavelength of the incident light otherwise the light will be scattered in all directions.
* The angle between directions of incident ray and reflected or refracted ray is called deviation (δ).
A plane mirror deviates the incident light through angle $\delta = 180 - 2i$ where 'i' is the angle of incidence. The deviation is maximum for normal incidence, hence $\delta_{max} = 180^0$.



It is noted that, generally anti - clock wise deviation is taken as positive and clock wise deviation as negative.
* Every object has its own field of view for the given mirror. The field of view is the region between the extreme reflected rays and depends on the location of the object infornt of the mirror. If our eye lies in the filed of view then only we can see the image of the object other wise not. This is illustrated in figure.

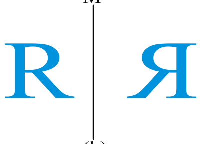


* A plane mirror produces front - back reversal rather than left - right reversal. It must be kept in mind that the mirror produces the reversal effect in the direction perpendicular to plane of the mirror. The figure (a) shows that the right handed co-ordinate system is converted into left handed co-ordinate system.



(a)

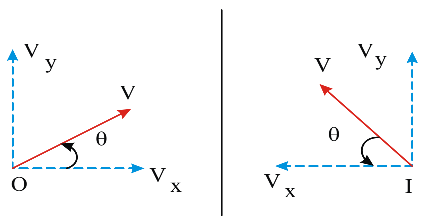
i.e., the image formed by a plane mirror left is turned into right and vice versa with respect to object as shown in figure (b).



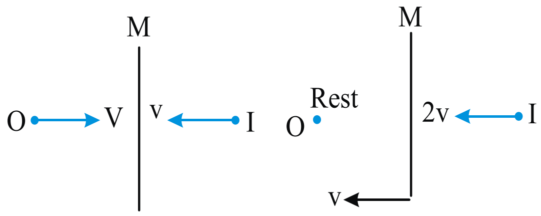
(b)

* When the object moves infront of stationary mirror, the relative speed between object and its image along the plane of the mirror is zero and in perpendicular to plane of mirror relative speed is twice that of the object speed.

$(V_{IO})_y = 0$ and $(V_{IO})_x = 2v_x$



* If an object moves towards (or away from) a plane mirror at speed v, the image will also approach (or recede) at the same speed v, and the relative velocity of image with respect to object will be 2v as shown in figure (a). If the mirror moved towards (or away from) the stationary object with speed v, the image will also move towards (or away from) the object with a speed 2v, as shown figure (b).

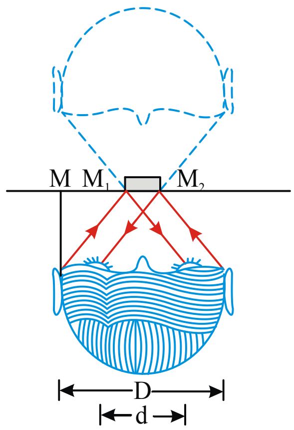


* a) A person of height 'h' can see his full image in a mirror of minimum length $l = \frac{h}{2}$

b) A person standing at the centre of room looking towards a plane mirror hung on a wall, can see the whole height of the wall behind him if the length of the mirror is equal to one-third the height of the wall.

* The minimum width of a plane mirror required for a person to see the complete width of his face is $(D-d)/2$, where, D is the width of his face and d is the distance between his two eyes.

*



* $MM_1 = \frac{1}{2}\left[D - \frac{1}{2}(D-d)\right]$

* $MM_1 = \frac{(D+d)}{4}$ .... (i)

* and $MM_2 = D - \frac{(D+d)}{4}$

* $MM_2 = \frac{(3D-d)}{4}$ ... (ii)

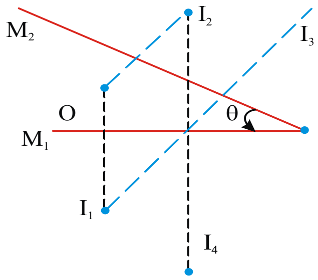
* $\therefore$ Width of the mirror = M_1M_2

$= MM_2 - MM_1$

$= \frac{2D-2d}{4}$ {From (i) and (ii)]

$$=\frac{2(D-d)}{4}=\frac{D-d}{2}$$

* If two plane mirrors inclined to each other at an angle θ, the number of images of a point object formed are determined as follows



* If $\frac{360}{\theta}$ is even number (say m) Number of images formed $n=(m-1)$, for all positions of objectes in between the mirrors.

* If $\frac{360}{\theta}$ is odd integer (say m) number of images formed $n=m$, if the object is not on the bisector of mirrors. $n=(m-1)$, if the object is on the bisector of mirrors.

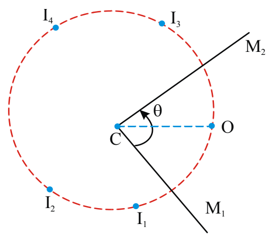
* If $\frac{360}{\theta}$ is a fraction (say m). The number of images formed will be equal to its integer part i.e., $n=[m]$.

* Ex: If m=4.3, the total number of images $n=[4.3]=4$

*

$m=\frac{360}{\theta}$	Position of the object	Number of images (n)
Even	Any where	$m-1$
Odd	Symmetric	$m-1$
	Asymmetric	M
Fraction	Any where	[m]

* **All the images lie on a circle whose radius is equal to the distance between the object 'O' and the point of intersection of mirrors C. If θ is less more number of images on circle with large radius.**



* If the objects is placed in between two parallel

* mirrors $\theta = 0^0$, the number of images formed is infinite but of decreasing intensity in according with $I \alpha r^{-2}$.

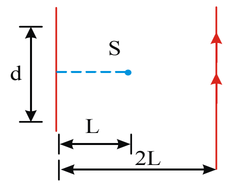
If 'θ' is given n is unique but if 'n' is given θ is not unique. Since same number of images can be formed for different θ.

* The number of images seen may be different from number of images formed and depends on the position of the observer relative to object and mirror.

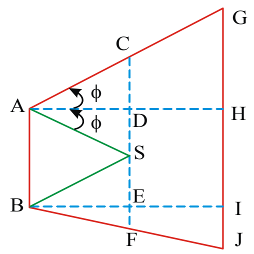
* When a light ray vector incident on a mirror, only the component vector which is parallel to normal of the mirror changes its sign without change of its magnitude on reflection. It is noted that a mirror can reflects entire energy incident on it, hence the magnitude of reflected vector is same as that of incident vector. Incident vector corresponding to an object and reflected vector corresponds to an image. This vector may be position, velocity or acceleration.

* Example: If a plane mirror lies on x-z plane, a light vector $2\hat{i} + 3\hat{j} - 4\hat{k}$ on reflection becomes $2\hat{i} - 3\hat{j} - 4\hat{k}$.

W.E-1: ***A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown in figure. Find the greatest distance over which he can see the image of the light source in the mirror.***



* **Sol**: The ray diagram will be as shown in figure.



HI = AB =d, DS = CD = d/2

Since, AH = 2AD, ∴ GH = 2CD = $2\dfrac{d}{2} = d$

Similarly IJ = d

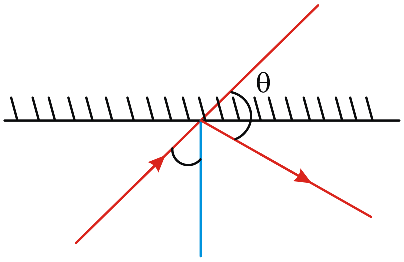
GJ = GH + HI +IJ =d +d + d =3d

W.E-2: ***A ray of light travelling in the direction $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$. The angle of incidence is***

Sol: Let angle between the directions of incident ray and reflected ray be θ,

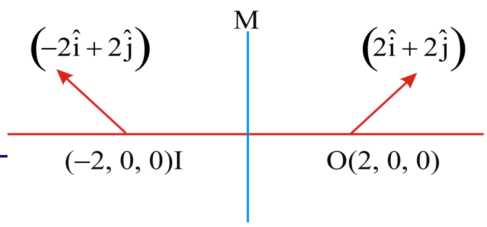
$$\cos\theta = \frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right).\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$$

$$\cos\theta = -\frac{1}{2} \qquad \theta = 120^0$$



W.E-3: ***A plane mirror is placed at origin parallel to y-axis, facing the positive x-axis. An object starts from (2m, 0, 0) with a velocity of $\left(2\hat{i}+2\hat{j}\right)$ m/s. Find the relative velocity of image with respect to object.***

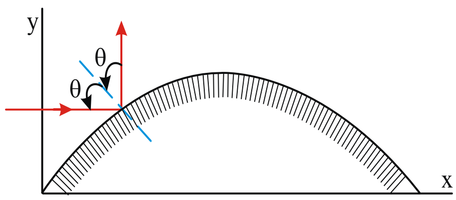
Sol:



The relative velocity of image with respect to object along normal $= 4\hat{i}$ The relative velocity image with respect to object along plane of mirror = 0. Hence the relative velocity of image with respect to object $= -4\hat{i}$

W.E-4: ***A reflecting surface is represented by the equation $Y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right), 0 \le x \le L$. A ray travelling horizontally becomes vertical after reflection. The coordinates of the point(s) where this ray is incident is***

Sol: A horizontal ray becomes vertical after reflection.



$$\tan\theta = \frac{dy}{dx} = 2\cos\frac{\pi x}{L}$$

$$2\theta = 90^0 \Rightarrow \theta = 45^0$$

$$1 = 2\cos(\pi x/2)$$

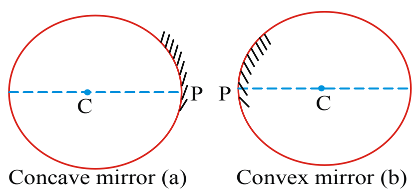
$$\Rightarrow x = L/3$$

$$\therefore y = \frac{2L}{\pi}\sin(\pi/3) = \frac{\sqrt{3}L}{\pi}$$

$$\left(\frac{L}{3}, \frac{\sqrt{3}L}{\pi}\right) \& \left(\frac{2L}{3}, \frac{\sqrt{3}L}{\pi}\right)$$

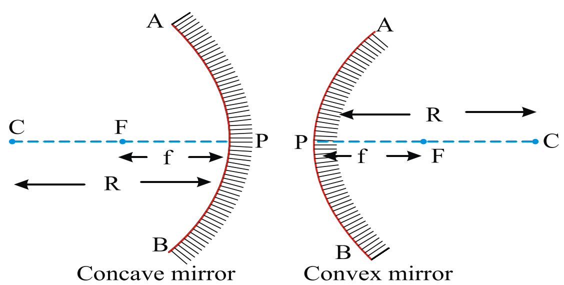
Reflection from Curved Surface

A curved mirror is a smooth reflecting part (in any shape) of a symmetrical curved surface such as spherical, cylindrical or ellipsoidal. In this chapter we consider a piece of spherical surface only.



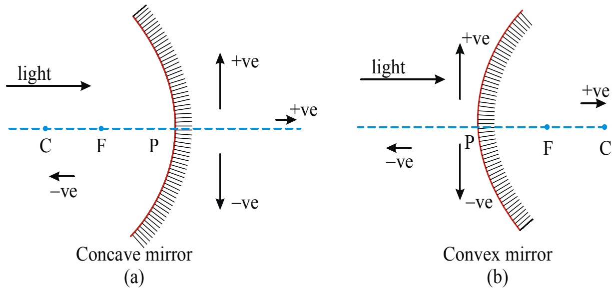
Concave mirror (a) Convex mirror (b)

If the reflection take place from the inner surface, the mirror is called concave and if its outer surface it is convex as shown in the figure. In case of thin spherical mirror, the centre 'C' of the sphere of which the mirror part is called the centre of curvature of the mirror. P is the centre of the mirror surface, is called the pole. The line CP produced is the principal axis, AB is the aperture means the effective diameter of the light reflecting area of the mirror. The distance CP is radius of curvature (R). The point F is the focus and the distance between PF is called focal length (f) and it is related to R as $f = R/2$.



Concave mirror (a) Convex mirror (b)

* **Sign Convertion :** To derive the relevant formula for reflection by spherical mirrors and refraction by spherical surfaces, we must adopt a sign convection for measuring distance. In this book, we shall follow the Cartesian sign convention. According to this convention all distances measured from the pole of the mirror.
* The distance measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of light are taken as negative.
* The heights measured one side with respect to principal axis of the mirror are taken as positive and the heights measured other side are taken as negative.
* Acute angles measured from the normal (principal axis) in the anti-clock wise sense are positive, while that in the clock wise sense are negative.

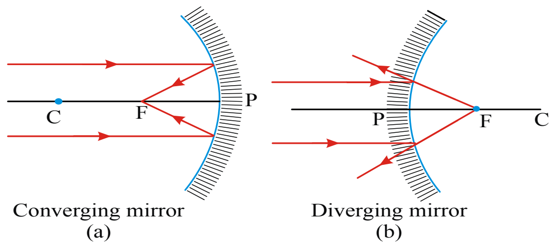


Concave mirror
(a)
Convex mirror
(b)

Paraxial Approximation : Rays which are close to the principal axis or make small angle $\left(\theta < 10^0\right)$ with it i.e. they are nearly parallel to the axis, are called paraxial rays. Accordingly we set $\cos\theta \approx 1, \sin\theta \approx \theta$ and $\tan\theta \approx \theta$. This is known as paraxial approximation or first order theory or "Gaussion" optics. In spherical mirrors we restrict to mirror with small aperture and to paraxial rays.

Focal Length of Spherical Mirrors

* We assume that the light rays are paraxial and they make small angles with the principal axis.
* A beam of parallel paraxial rays is reflected from a concave mirror so that all rays converge to a point F on the principal axis is called principal focus of the mirror and it is real focus.
* A narrow beam of paraxial rays falling on a convex mirror is reflected to form a divergent beam which appears to come from a point 'F' behind the mirror. Thus a convex mirror has a virtual focus 'F'.
* The distance between focus (F) and pole (P) is called the focal length 'f'. Concave mirror is also called as converging mirror. They are used in car head lights, search lights and telescopes. Convex mirror is also called as diverging mirror. Convex mirror gives a wider field of view than a plane mirror and concave mirror, convex mirrors are used as rear view mirrors in vehicles.



Converging mirror
(a)
Diverging mirror
(b)

$(b)\,Diverging\ mirror$

* According to Cartesian sign convention with real object the focal length of concave mirror is negative, because the distance PF (P to F) is measured in opposite direction of light. Similarly with the same reason focal length of convex mirror is positive. The same sign convention is also applicable to virtual object by treating that imaginary light rays from that object.

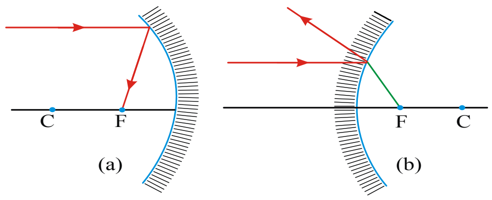
Relation between F and R

a) Concave mirror b) Convex mirror

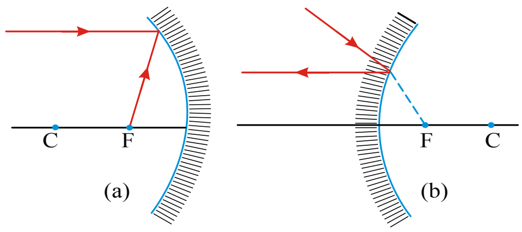
* $f = \dfrac{R}{2}$
* The focal length of mirror is independent on medium in which it placed and wavelength of incident light. To a plane mirror focal length 'f' is infinite (as $R = \infty$)

* **Rules for Image formation:** In general, position of image and its nature [i.e., whether it is real or virtual, erect or inverted, magnified or diminished] to an object depend on the distance of the object from the mirror. Nature of the image can be obtained by drawing a ray diagram. In case of image formation unless stated object is taken to be real, it may be point object or extended.

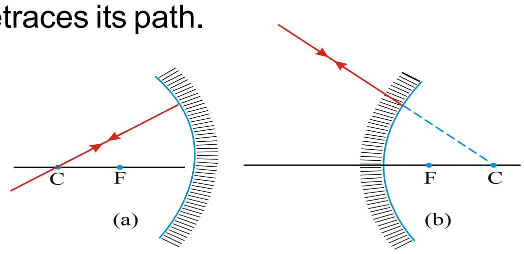
* A ray parallel to principal axis after reflection from the mirror passes or appear to pass through its focus F.



* A ray passing through or directed towards focus, after reflection from the mirror becomes parallel to the principal axis (by principle of reversibility)

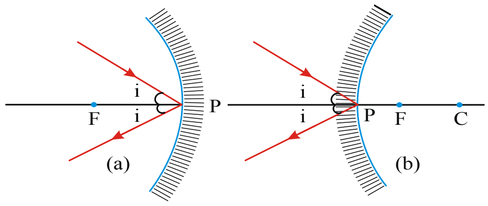


* A ray through or directed towards the centre of curvature C, after reflection from the mirror, retraces its path.

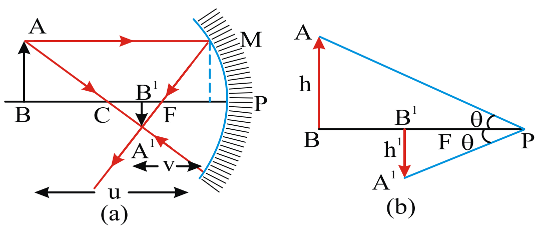


* **A ray striking at pole P is reflected symmetrically back in the opposite side.**

*



* **The Mirror Equation:** Figure (a) shows the ray diagram considering two rays and the image A^1B^1 (in this case real image) of an object AB formed by a concave mirror.



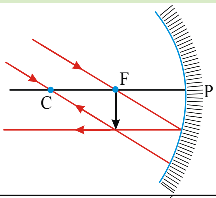
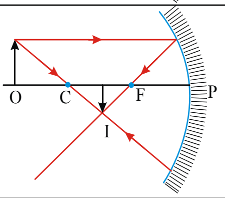
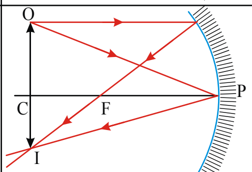
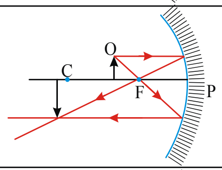
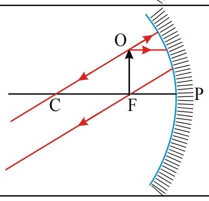
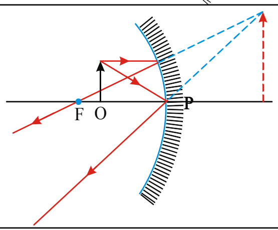
$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad \left(\because f=\frac{R}{2}\right)$$

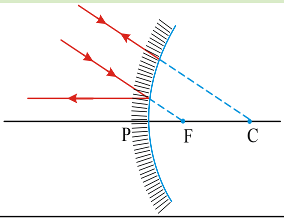
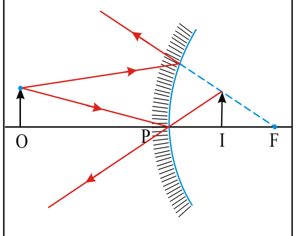
* This relation is known as Gauss's formula for a spherical mirror. It is valid in all other situations with a spherical mirror and also for a convex mirror. In this formula to calculate unknown, known quantities are substituted with proper sign.

* **Image Formation by Spherical Mirrors**

* From the ray diagrams we understand that

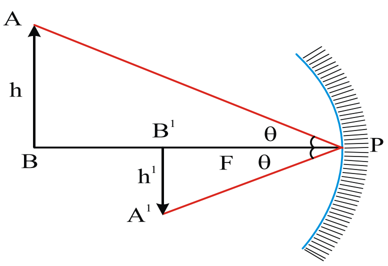
* To a real object in case of concave mirror the image is erect, virtual and magnified when the object is placed between F and P. In all other positions of object the image is real and inverted.
* To a real object the image formed by convex mirror is always virtual, erect and diminished no matter where the object is.
* A concave mirror with virtual object behaviour is similar to convex mirror with real object and convex mirror with virtual object behaviour similar to concave mirror with real object.
* By principle of reversibility a convex mirror can form real and magnified image to a virtual object which is with in the focus and virtual images when virtual object beyond the focus. i.e., the convex mirror can form real and virtual images to virtual object. A concave mirror with virtual object always forms real images.
* If the given mirror breaks in to pieces, each piece of that mirror has own principal axis, but behaviour is similar to that of main mirror with less intensity of image.
* **a) Concave mirror b) Convex Mirror**

Position of the object	Ray diagram	Image details
At Infinity		Real, inverted, very Small, at F
Between ∞ and c		Real, inverted, diminished between F and C
At C		Real, inverted, equal, at C
Between F and C		Real, inverted, enlarged, beyond C
At F		Real, inverted, very large at infinity
Between F and P		Virtual, erect, enlarged behind the mirror

Position of the object	Ray diagram	Image details
At Infinity		Virtual, erect, very small at F
Infront of mirror		Virtual, erect, diminished between P and F

Magnification : The size of the image relative to the size of the object is another important quantity to consider. Hence we define magnification. It is noted that magnification does not mean that the image is enlarged. The image formed by optical system may be larger than, smaller than or of the same size of the object.

Lateral magnification: The ratio of the transverse dimension of the final image formed by an optical system to the corresponding dimension of the object is defined as transverse or lateral or linear magnification (m). Hence it is the ratio of the height of image (h^1) to the height of the object (h). From the figure.



* Lateral magnification $m = \dfrac{A^1B^1}{AB} = \dfrac{h^1}{h}$

here h and h^1 will be taken positive or negative in accordance with the accepted sign convention.

In triangles A^1B^1P and ABP, we have $\dfrac{B^1A^1}{BA} = \dfrac{B^1P}{BP}$, with sign convention this becomes

$\dfrac{-h^1}{h} = \left(\dfrac{-v}{-u}\right)$, so that, $m = \dfrac{h^1}{h} = -\dfrac{v}{u}$

* Here negative magnification implies that image is inverted with respect to object, while positive magnification means that image is erect with respect to object. i.e., m is negetive means for real object, real image formed and for virtual object virtual image is formed. m positive means for real object virtual image formed and for virtual object real image is formed.

* Ex: If $m = -2$, means, if the object is real, image is real, inverted, magnified and mirror used is concave.

Longitudianl magnification: However, if the one dimensional object is placed with its length along the principal axis. The ratio of length of image to length of object is called longitudinal magnification (m_L). Longitudinal magnification can be expressed as

$$m_L = \frac{(v_2 - v_1)}{(u_2 - u_1)}$$

Where v_1 and v_2 are image positions corresponding to u_1 and u_2 positions.

* For small objects $m_L = -\frac{dv}{du}$

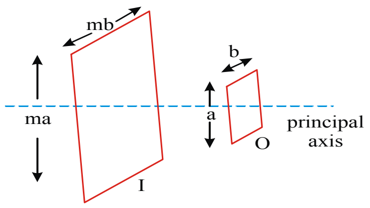
We have $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

In case of small linear objects $-\frac{dv}{v^2} - \frac{du}{u^2} = 0$

* $\therefore m_L = -\frac{dv}{du} = \left[\frac{v}{u}\right]^2 = m^2$

Areal magnification: If a two dimensional object is placed with its plane perpendicular to principal axis, its magnification is called a real or superficial magnification. If m is the lateral magnification and m_A is the areal magnification.

* $m_A = \frac{\text{area of image}}{\text{area of object}} = \frac{(ma)(mb)}{ab} = m^2$



Overall magnification: In case of more than one optical component, the image formed by first component will act as an object for the second and image of second acts as an object for third and so on, the product of all individual magnifications is called over all magnifications.

* $m_0 = \frac{I}{O} = \frac{I_1}{O_1} \times \frac{I_2}{O_2} \times - - \frac{I_n}{O_n}$

* $= m_1 \times m_2 \times - - \times m_n$

Newton's Formula : In case of spherical mirror if the object distance (x_1) and image distance (x_2) are measured from focus instead of the pole of the mirror. Then mirror formula reduces to a simple form called the Newton's formula.

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ reduces to

$$\frac{1}{f+x_2}+\frac{1}{f+x_1}=\frac{1}{f}$$

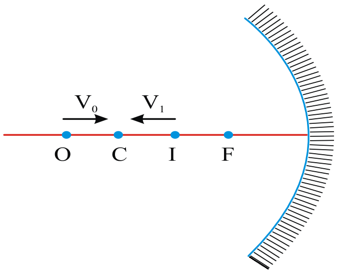
Which on simplification gives $x_1x_2=f^2$

* (Newton's Formula) ($f=\sqrt{x_1x_2}$)

Motion of Object in front of Mirror Along the Principal Axis

* When the position of the object changes with time on the principal axis relative to the mirror, the image position also changes with time relative to the mirror. Hence to know the relation between object and image speed we use the mirror equation.

*



$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Differntiatae with respect to time, we get

$$-\frac{1}{v^2}.\frac{dv}{dt}-\frac{1}{u^2}.\frac{du}{dt}=0 \text{ (or)}$$

$$\frac{dv}{dt}=-\left(\frac{v}{u}\right)^2.\frac{du}{dt} \text{ (or) } V_1=-\left(\frac{v}{u}\right)^2.V_0$$

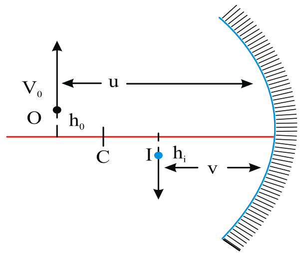
* Where v_1 velocity of image with respect to mirror and v_0 is the velocity of object with respect to mirror along the principal axis. Here negative sign indicates the object and image are always moving opposite to each other. In concave mirror depending on the position of the object image speed may be greater or lesser or equal to the object speed.

* a) $R<u<\infty$ $|m|<1$ $V_1<V_0$
* b) u = R $|m|=1$ $V_1=V_0$
* c) $f<u<R$ $|m|>1$ $V_1>V_0$
* d) $u<f$ $|m|>1$ $V_1>V_0$
* e) $u\approx 0$ $|m|\approx 1$ $V_1\approx V_0$

* Relation between object and image velocity given above is also valid for convex mirror. In convex mirror speed of image slower than the object whatever the position of the object may be. Above relation is not true in terms of acceleration of object and image.

Motion of the object Transverse to the Principal Axis

* If the object moves transverse to principal axis then the image also moves transverse to principal axis.

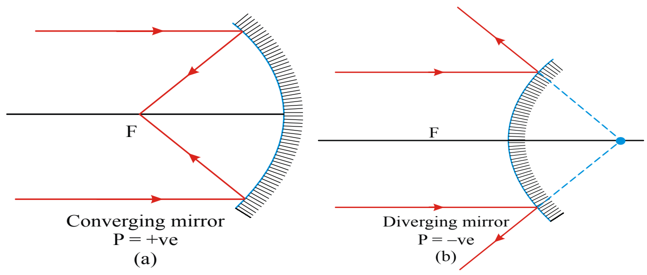
* 

* Consider the diagram. In a mirror

* $$\frac{h_i}{h_0}=\frac{v}{u}=\text{constant}\quad(-m)$$

* $$\therefore\frac{\frac{dh_i}{dt}}{\frac{dh_0}{dt}}=-m\,(\text{or})\,V_I=-mV_0$$

Power of Curved Mirror : Every optical instrument have power, it is the ability of optical instrument to deviate the path rays incident on it. If the instrument converges the rays parallel to principal axis its power is said to be positive and if it diverges its power is said to be negetive.

* 

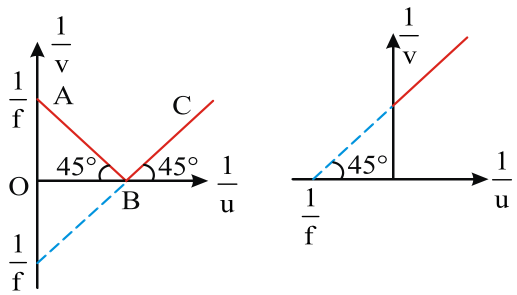
* For a mirror Power 'P'

$$P=-\frac{1}{f(\text{metre})}\,(\text{or})\,P=-\frac{100}{f(\text{cm})}$$

S.I unit of power is dioptre $(D)=m^{-1}$

* For concave mirror (converging mirror) P is positive and for convex mirror (diverging mirror) power is negative.

* $\frac{1}{V}-\frac{1}{U}$ **Graph to Mirrors: The graph between** $\frac{1}{v}$ **and** $\frac{1}{u}$ **to a concave mirror is shown in figure (a)**

* 

* Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

For all real image $-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\therefore \frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$

This is a straight line equation with slope -1.
This is represented by the line AB.

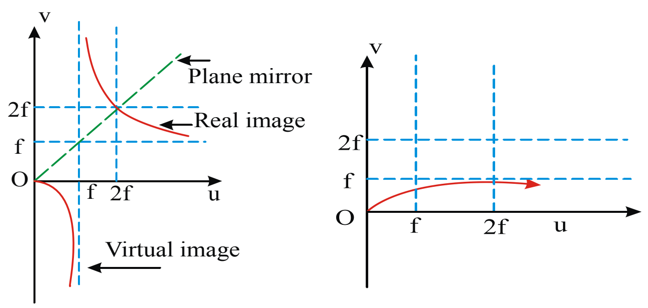
For virtual image, $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\therefore \frac{1}{v}=\frac{1}{u}-\frac{1}{f}$

This is a straight line equation with slope +1.
This represents line BC.

* The graph between 1/v and 1/u to a convex mirror as shown in figure (b).
* Since convex mirror always form virtual image to a real object.
* $\frac{1}{v}+\frac{1}{-u}=\frac{1}{f} \therefore \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$
* This is a straight line equation with slope +1.
* **U-V Graph in Curved Mirror** :In case of concave mirror, the graph between u and v is hyperbola as shown in figure.

*



For real image $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (or) $\frac{1}{v}=\frac{u-f}{uf}$

$v=\frac{f}{1-\frac{f}{u}}$

* For virtual image $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\frac{1}{v}=\frac{f-u}{fu}(\text{or})\, v=\frac{f}{\frac{f}{u}-1}$

* In case of convex mirror, the graph between u and v is hyperbola as shown in figure (b)
* Since convex mirror form only virtual image.

* $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}(or)\, v=\frac{f}{1+\frac{f}{u}}$

Graph in Spherical Mirror : In a spherical mirror: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

* $\therefore 1+\frac{v}{u}=\frac{v}{f}(or)\frac{v}{u}=\frac{v}{f}-1$

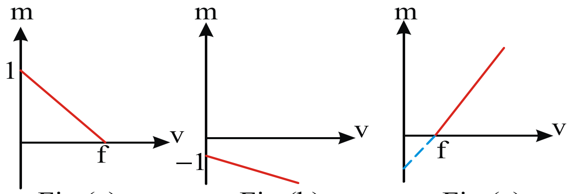


Fig.(a) Fig.(b) Fig.(c)

Concave mirror: If the objects is real,

For real image, u=-ve, v=-ve, f=-ve,

* $\therefore -m=\frac{v}{f}-1(or)\, m=-\frac{v}{f}+1$

Graph as shown in figure (a)

For virtual image, u=-ve, v=+ve, f=-ve

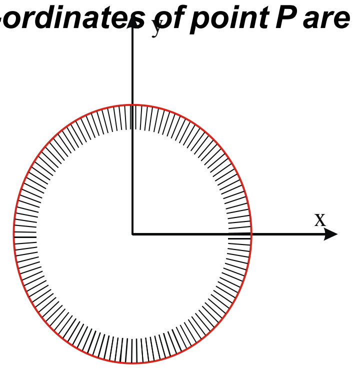
$\therefore m=-\frac{v}{f}-1,$ Graph as shown in figure (b)

Convex mirror: Since convex mirror always form virtual image to a real object, u=-ve, v=+ve, f=+ve,

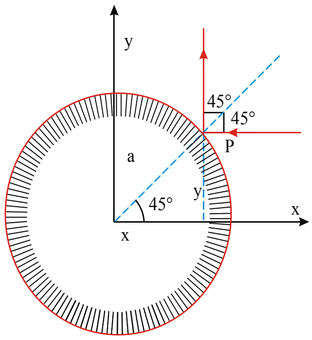
* $\therefore m=\frac{v}{f}-1,$ graph as shown in figure (c).

* From the above graph it is observed that for $v \approx 0, m=1.$ i.e., when an object is very near to pole of the mirror $(u \approx 0)$, then the curved mirror behaves like a plane mirror.

* **W.E-5:** ***A reflecting surface is represented by the equation*** $x^2+y^2=a^2$***. A ray travelling in negative x-direction is directed towards positive y-direction after reflection from the surface at point P. Then co-ordinates of point P are***

* 

* **Sol:** The ray diagram is as shown.

* 

* $x=\frac{a}{\sqrt{2}}$ and $y=\frac{a}{\sqrt{2}}$

* $\therefore P=\left(\frac{a}{\sqrt{2}},\frac{a}{\sqrt{2}}\right)$

* **W.E-6:** ***A point light source lies on the principal axis of concave spherical mirror with radius of curvature 160 cm. Its image appears to be back of the mirror at a distance of 70 cm from mirror. Determine the location of the light source.***

* **Sol:** $\frac{1}{u}+\frac{1}{v}=\frac{2}{R}$, Here $v=70\,cm$,

* $R=-160cm\frac{1}{u}=\frac{2}{R}-\frac{1}{v}$

* $\therefore\frac{1}{u}=\frac{2}{-160\,cm}-\frac{1}{70\,cm}=-\frac{15}{560\,cm}$

* $\therefore u=-\frac{560}{15}cm=-37\,cm$

* The image is at a distance of 37 cm in front of the mirror.

* **W.E-7:** ***A point source of light is located 20 cm in front of a convex mirror with f=15 cm. Determine the position and nature of the image point.***

* **Sol:** $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

* Here $u=-20\,cm, f=15\,cm$

* $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{15cm}-\frac{1}{20cm}=\frac{35}{300cm}$

* $\frac{1}{v}=\frac{7}{60\,cm}$

* $v=8.6\,cm$

* Also v is positive, the image is located behind the mirror.

* **W.E-8:** ***An object is 30.0 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification is 1/2. The image produced is inverted. What is the focal length of the mirror?***

* **Sol:** Image inverted, so it is real u and v both are negative. Magnification is 1/2, therefore,

* $v=\frac{u}{2}$, given, $u=-30\,cm, v=-15cm$

* Using the mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

* We have, $\frac{1}{f}=\frac{1}{-15}-\frac{1}{30}=\frac{-1}{10}$

* $\therefore f=-10\,cm$

* Since focal length is negative the given mirror is concave.

* **W.E-9:** ***An object of length 10 cm is placed at right angles to the principal axis of a mirror of radius of curvature 60 cm such that its image is virtual, erect and has a length 6cm. What kind of mirror is it and also determine the position of the object?***

* **Sol:** Since the image is virtual, erect and of a smaller size, the given mirror is 'convex' (concave mirror does not form an image with the said description).

* Given $R=+60\,cm$ $\quad f=\frac{R}{2}=30\,cm$

* Transverse magnification,

* $m=\frac{I}{O}=\frac{6}{10}=+\frac{3}{5}$ Further $m=-\frac{v}{u}=\frac{3}{5}$

* $\therefore v=-\frac{3u}{5}$

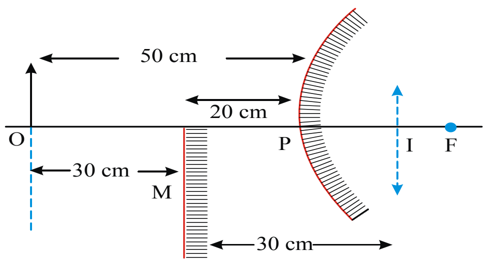
* Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ $\quad \frac{-5}{3v}+\frac{1}{u}=\frac{1}{30}$

* $\frac{-5+3}{3u}=\frac{1}{30}$ $\quad \therefore u=-20\,cm$

* Thus the object is at a distance 20 cm (from the pole) in front of the mirror.

* **W.E-10:** ***An object is placed infront of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, if it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror?***

* **Sol:**



* As shown in figure the plane mirror will form erect and virtual image of same size at a distance of 30 cm behind it. So the distance of image formed by the plane mirror from convex mirror will be PI = MI - MP But as MI = MO, PI= MO-MP = 30-20 = 10 cm.

* Now as this image coincides with the image formed by convex mirror, therefore for convex mirror,

* $u = -50\,cm; v = +10\,cm$

* So $\frac{1}{+10} + \frac{1}{-50} = \frac{1}{f}$, i.e., $f = \frac{50}{4} = 12.5\,cm$

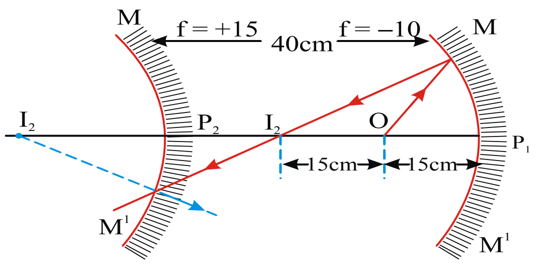
* So $R = 2f = 2 \times 12.5 = 25\,cm$

* **W.E-11:** ***A concave mirror of focal length 10 cm and a convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirrors, on their common axis and 15 cm from the concave mirror. Find the position, nature of the image, and over all magnification produced by the successive reflections, first at concave mirror and then at convex mirror.***

* **Sol:** According to given problem, for concave mirror, u = -15cm and f = -10 cm.

* So $\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}$, i.e., $v = -30\,cm$

* i.e., concave mirror will form real, inverted and enlarged image I_1 of object O at a distance 30 cm from it, i.e., at a distance 40-30=10 cm from convex mirror.

* 

For convex mirror the image I_1 will act as an object and so for it u=-10 cm and f=+15 cm.

* $\frac{1}{v} + \frac{1}{-10} = \frac{1}{15}$, i.e, $v = +6\,cm$

* So final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual as shown in figure.

* Over all magnification

* $= m_1 \times m_2 = -2 \times 6/10 = -6/5$

* negative indicates final image is virtual w.r.t. given object.

* **Ø Refraction of Light :**When a beam of light is travelling from one medium to another medium, a part of light gets reflected back into first medium at the inferface of two media and the remaining part travels through second medium in another direction. The change in the direction of light take place at the interface of two media.

* Deviation or bending of light rays from their original path while passing from one medium to another is called refraction.

* (or)

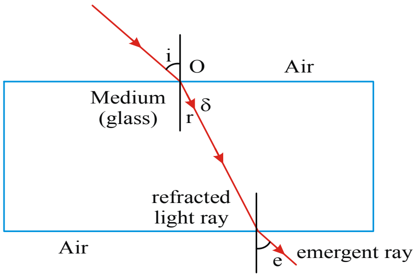
* The phenomenon due to which light deviates from its initial path, while travelling from one optical medium to another optical medium is called refraction.

* Refraction of light is due to change in speed of light passes from one medium to another medium.

* In case of refraction of light frequencey (colour) and phase do not change. But wavelength and velocity will change.

* **Note:** When light passes from one medium to another medium, the colour of light is determined by its frequency not by its wavelength.

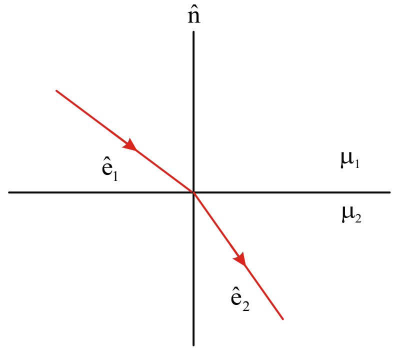
* Ø **Refraction of light at plane surface:**



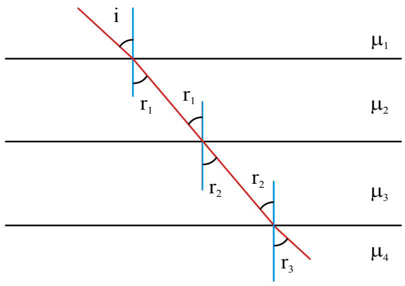
* Ø **Incident ray:** A ray of light, traveling towards another optical medium, is called incident ray.
* Ø **Point of incidence:** The point (O), where an incident ray strikes on another optical medium, is called point of incidence.
* Ø **Normal:** A perpendicular drawn at the surface of seperation of two media on the point of incidence, is called normal.
* Ø **Angle of incidence (i):** The angle which the incident ray makes with normal, is called angle of incidence.
* Ø **Refracted ray:** A ray of light which deviates from its path on entering in another optical medium is called refracted ray.
* Ø **Angle of refraction(r):** The angle which the refracted ray makes with normal, is called the angle of refraction.
* Ø **Angle of deviation due to refraction(δ):** It is the angle between the direction of incident light ray and refracted light ray.
* Ø **Emergent ray:** A ray of light which emerges out from another optical medium as shown in the above figure is called emergent ray.
* Ø **Angle of emergence (e):** The angle which the emergent ray makes with the normal is called the angle of emergence.
* Ø **Laws of Refraction:**
* Ø Incident ray, refracted ray and normal always lie in the same plane.
* Ø The product of refractive index and sine of angle of incidence at a point in a medium is constant,
* $\mu \times \sin$ i = constant
* $\mu_1 \sin i_1 = \mu_2 \sin i_2$
* If $i_1 = i$ and $i_2 = r$ then
* $\mu_1 \sin i = \mu_2 \sin r;$
* This law is called snell's law.
* According to Snell's law,
* $\frac{\sin i}{\sin r}$ = constant $\left(= \frac{\mu_2}{\mu_1}\right)$ for any pair of medium and for light of given wavelength.
* **Note:** The ratio between sine of angle of incidence to sine of angle of refraction is commonly called as refractive index of the material in which angle of refraction is situated with respect to the medium in which angle of incidence is situated.
* When light ray travells from medium 1 to medium 2 then $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2 =$ refractive index of medium (2) with respect to medium (1)
*

* Ø Vector form of Snell's law:

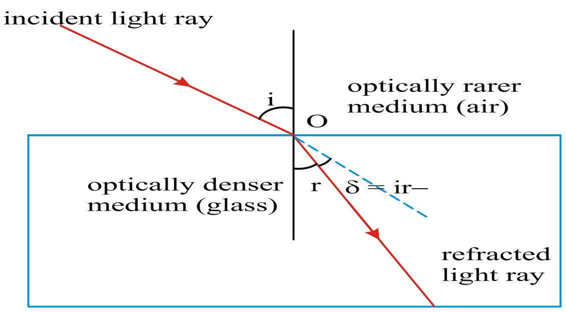
* $\mu_1(\hat{e}_1 \times \hat{n}) = \mu_2(\hat{e}_2 \times \hat{n})$

* 

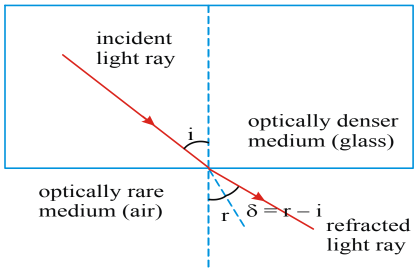
* There $\hat{e}_1$ = unit vector along incident ray
* $\hat{e}_2$ = unit vector along refracted ray
* $\hat{n}$ = unit vector along normal incedence point
* **Note:** Let us consider a ray of light travelling in situation as shown in fig.
* Applying Snell's law at each interface, we get

* 

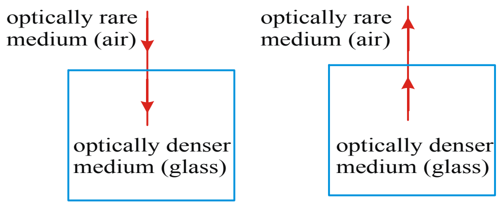
* $\mu_1 \sin i = \mu_2 \sin r_1$; $\mu_2 \sin r_1 = \mu_3 \sin r_2$
* $\mu_3 \sin r_2 = \mu_4 \sin r_3$; It is clear that
* $\mu_1 \sin i = \mu_2 \sin r_1 = \mu_3 \sin r_2 = \mu_4 \sin r_3$
* (or) $\mu \sin i$ = constant
* **Note:** When light ray travells from medium of refractive index μ_1 to another medium of refractive index μ_2 then, $\mu_1 \sin i_1 = \mu_2 \sin i_2$
* $\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = \frac{\sin i_1}{\lambda_1} = \frac{\sin i_2}{\lambda_2}$
* **Ø When a light travels from optically rarer medium to optically denser medium obliquely:**

* 

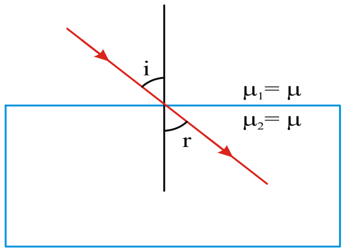
* a) it bends towards normal.
* b) angle of incidence is greater than angle of refraction.
* **Ø When a ray of light travels from opticallly denser medium to optically rarer medium obliquely**

* 

* a) it bends away from the normal at the point of incidence.
* b) angle of refraction is greater than angle of incidence.
* c) angle of deviation $\delta = r - i$.
* **Ø Condition for no refraction :** When an incident ray strikes normally at the point of incidence, it does not deviates from its path.i.e., it suffers no deviation.

* 

* In this case angle of incedence (i) and angle of refraction(r)are equal and $\angle i = \angle r = 0$.
* **Ø** If the refractive indices of two media are equal

* 

* $\mu_1 = \mu_2 = \mu$
* From snell's law,
* $\mu \sin i = \mu \sin r, \sin i = \sin r$
* $\angle i = \angle r$
* Hence, the ray passes without any deviation at the boundary.
* **Note:** Because of the above reason a transperant solid is invisible in a liquid if their refractive indices are same.
* **Ø Refractive Index :**
* **Absolute refractive Index (μ):**
* The absolute refractive index of a medium is the ratio of speed of light in free space (C) to speed of light in a given medium (V).
* $$\mu = \frac{\text{veloctiy of light in free space (C)}}{\text{velocity of light in a given medium (V)}}$$
* It is a scalar.
* It has no units and dimensions.

* From electronmagnetic theory if ε_0 and μ_0 are the permitivity and permeability of free space, ε and μ are the permitivity and refractive index of the given medium

* $$\mu = \frac{C}{V} = \frac{\frac{1}{\sqrt{\varepsilon_0\mu_0}}}{\frac{1}{\sqrt{\varepsilon\mu}}} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

* where ε_r & μ_r are the relative permittivity and permeability of the given medium.
* **Ø** For vaccum of free space, speed of light of all wavelengths is same and is equal to C.
* So,For all wavelengths the refractive index of
* free space is $\mu = \frac{C}{C} = 1$.
* **Ø** For a given medium the speed of light is different for different wavelengths of light, greater will be the the speed and hence lesser will be refractive index.
* $\lambda_R > \lambda_V$, So in medium $\mu_V > \mu_R$
* **Note:** Actually refractive index μ varies with λ according to the equation $\mu = A + \frac{B}{\lambda^2}$.
* (where A & B are constants)
* **Ø** For a given light, denser the medium lesser will be the speed of light and so greater will be the refractive index.
* Example : Glass is denser medium when compared to water, so $\mu_{glass} > \mu_{water}$.
* The refractive index of water $\mu_w = 4/3$
* The refractive index of glass $\mu_g = 3/2$
* **Ø** Foa a given light and given medium, the refractive index is also equal to the ratio of wavelength of light in free space to that in the medium.

* $$\mu = \frac{C}{V} = \left(\frac{f\lambda_{vaccum}}{f\lambda_{medium}}\right) = \frac{\lambda_{vaccum}}{\lambda_{medium}}$$

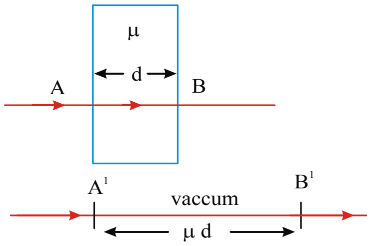
* (when light travells from vaccume to a medium, frequency does not change)
* **Note:** *If C* is velocity of light in free space λ_0 is wavelength of given light in free space then velocity of light in a medium of refractive index (μ) is $V_{medium} = \frac{C}{\mu}$.
* wavelength of given light in a medium of refractive index (μ) is $\lambda_{medium} = \frac{\lambda_0}{\mu}$
* **Ø Relative Refractive Index:** When light passes. from one medium to the other, the refractive index of medium 2 relative to medium 1 is written as ${}_1\lambda_2$ and is given by

* $${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \qquad ...(1)$$

* refractive index of medium 1 relative to medium 2 is ${}_2\mu_1$ and ${}_2\mu_1 = \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{\lambda_2}{\lambda_1}$ ...(2)
* From eq. (1) & (2)
* ${}_1\mu_2 = \frac{1}{{}_2\lambda_1}$ i.e., $({}_1\mu_2).({}_2\mu_1) = 1$
* **W.E-12:** ***The refractive index of glass with respect to water is 9/8. If the velocity and wavelength of light in glass are 2×10^8 m/s and 4000 A^0 respectively, find the velocity and wavelength of light in water.***
* **Sol:** ${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{v_w}{v_g} \Rightarrow \frac{9}{8} = \frac{v_w}{2\times10^8}$;
* $v_w = \frac{9\times2\times10^8}{8} = 2.25\times10^8\, m/s.$
* ${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{\lambda_w}{\lambda_g}\left(\therefore \mu_g = \frac{\lambda}{\lambda_g}, \mu_w = \frac{\lambda}{\lambda_w}\right)$
* $\frac{9}{8} = \frac{\lambda_w}{4000}; \lambda_w = \frac{9\times4000}{8} = 4500A^0.$
* **W.E-13:** ***The wavelength of light in vacuum is λ_0. When it travels normally through glass of thickness 't'. Then find the number of waves of light in 't' thickness of glass (Refractive index of glass is μ)***
* **Sol:** Number of waves in a thickness 't' of a medium of refractive index μ is
* number of waves $= \frac{thickness}{wavelength} = \frac{t}{\lambda_m}$
* But $\lambda_m = \frac{\lambda_0}{\mu}$
* $\therefore$ number of waves = $\frac{t\mu}{\lambda_0}$
* Where λ_0 is the wavelength of light in vacuum.
* **W.E-14:** ***When light of wavelength λ_0 in vacuum travels through same thickness 't' in glass and water, the difference in the number of waves is ______. (Refractive indices of glass and water are μ_g and μ_w respectively.)***
* **Sol:** We know number of waves of a given light in a medium of refractive index μ is $\frac{t\mu}{\lambda_0}$
* $\therefore$ Difference in number of waves $= \frac{t}{\lambda_0}(\mu_g - \mu_w)$
* where μ_g and μ_w are the refractive indicies of glass and water respectively.
* **Ø Optical Path (Δx):** Consider two points A and B in a medium as shown in figure. The shortest distance between any two points A and B is called geometrical path. The length of geometricial path is independent of the medium that surrounds the path AB.When a light ray

travels from the point A to point B it travels with the velocity c if the medium is vacuum and with a lesser velocity v if the medium is other than vaccum. Therefore the light ray takes more time to go from A to B located in a medium.

* The optical path to a given geometrical path in a given medium is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in that medium.

* 

* AB = real path or geometrical path
* A^1B^1 = optical path
* If the light travels a path lenght 'd' in a medium
* at speed v, the time taken by it will be $\left(\frac{d}{v}\right)$
* So optical path length,
* $$\Delta x = c \times t = c \times \left[\frac{d}{v}\right] = \mu d \left(as\ \mu = \frac{c}{v}\right)$$
* Therefore optical path is μ times the geometrical path. As for all media $\mu > 1$, optical path length is always greater than actual path length.
* **Note:** If in a given time t, light has same optical path length in different media, and if light travels a distance d_1 in a medium of refractive index μ_2 in same time t, then $\mu_1 d_2 = \mu_2 d_2$.
* **Note:** The difference in distance travelled by light in vaccum and in a medium in the same interval of time is called optical path difference due to that medium.
* $\Delta x = A'B' - AB = \mu d - d \qquad \Delta x = (\mu - 1)d$
* **Note:** A slab of thickness d and refractive index μ is kept in a medium of refractive index $\mu'(<\mu)$. If the two rays parallel to each other passes through such a system with one ray passing through the slab, then path difference
* Between two rays due to slab will be $\Delta x = \left(\frac{\mu}{\mu'} - 1\right)d$.
* **Note:** The optical phase change $\phi = \frac{2\pi}{\lambda}$ (optical path difference)

* **W.E-15:** ***The optical path of a monochromatic light is the same if it goes through 4.00 m of glass are 4.50 m of a liquid. If the refractive index of glass is 1.5, what is the refractive index of the liquid?***
* **Sol:** When light travells a distance 'x' in a medium of refractive index μ, the optical path is μx
* Given $\mu_1 x_1 = \mu_2 x_2 \Rightarrow 1.5 \times 4.00 = \mu_2 \times 4.50$

* $\mu_2 = \frac{1.5 \times 4.00}{4.50} = 1.333$

* **W.E-16:** ***Find the thickness of a transparent plastic plate which will produce a change in optical path equal to the wavelength λ of the light passing through it normally. The refractive index of the plastic plate is μ.***

* **Sol:** When light travel a distance x in a medium of refractive index μ, its optical path = μx

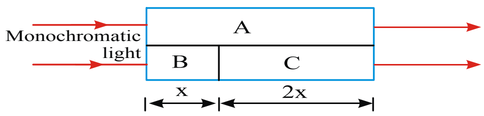
* Change in optical path = $\mu x - x = (\mu - 1)x$.

* This is to be equal to λ

* But $(\mu - 1)x = \lambda$

* The thickness of the plate $x = \frac{\lambda}{\mu - 1}$

* **W.E-17:** ***Consider slabs of three media A, B and C arranged as shown in figure R.I. of A is 1.5 and that of C is 1.4. If the number of waves in A is equal to the number of waves in the combination of B and C then refractive index of B is:***

* 

* **Sol:** $N_A = N_B + N_C$

* $\frac{x_A}{\lambda_A} = \frac{x_B}{\lambda_B} + \frac{x_C}{\lambda_C}$

* $\frac{x_A \mu_A}{\lambda_0} = \frac{x_B \mu_B}{\lambda_0} + \frac{x_C \mu_C}{\lambda_0}$

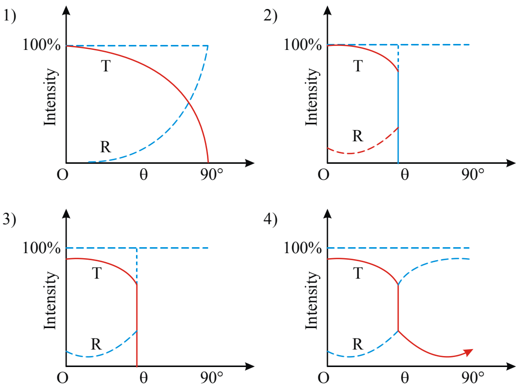
* $3x \times 1.5 = x \times \mu_B + 2x \times 1.4$

* $\therefore \mu_B = 1.7$

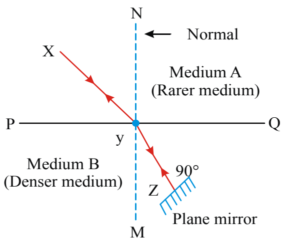
* **W.E-18:** ***Two parallel rays are travelling in a medium of refractive index $\mu_1 = \frac{4}{3}$. One of the rays passes through a parallel glass slab of thickness t and refractive index $\mu_2 = \frac{3}{2}$. The path difference between the two rays due to the glass slab will be***

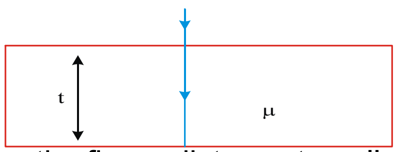
* **Sol:** $\Delta x = \left(\frac{\mu_2}{\mu_1} - 1\right)t = \left(\frac{3/2}{4/3} - 1\right)t = \frac{t}{8}$

* **W.E-19:** ***A light ray travelling in a glass medium is incident on glass - air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is***



* **Sol:** (3) After total internal reflection, there is no refracted ray.
* **Principle of Reversibility of Light**
* **Ø** According to principle of reversibility, if a ray of light travels from X to Z along a certain path, it will follow exactly the same path, while travelling from Z to X. In other words the path of light is reversible.

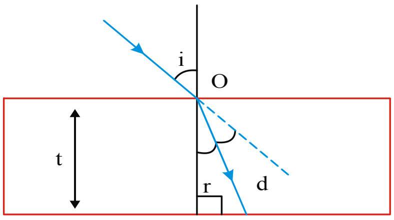


* Figure shows a ray of light XY travelling through medium 'A', such that it travels along YZ, while travelling medium 'B'. NM is the normal at point Y, such $\angle XYN$ is the angle of incidence and $\angle MYZ$ is the angle of refraction.
* $\therefore {}_a\mu_b = \dfrac{\sin\angle XYN}{\sin\angle MYZ}$ ....(1)
* If a plane mirror is placed at right angles to the path of refracted ray 'YZ', it found that light retraces back its path. Now ray ZY acts as incident ray and YX as refracted ray, such that $\angle MYZ$ is angle of incidence and $\angle XYN$
* is the angle of refraction.
* $\therefore {}_b\mu_a = \dfrac{\sin\angle MYZ}{\sin\angle XYN} \therefore \dfrac{1}{{}_b\mu_a} = \dfrac{1}{\dfrac{\sin\angle MYZ}{\sin\angle XYN}} = \dfrac{\sin\angle XYN}{\sin\angle MYZ}$
* Comparing (1) and (2) ${}_a\mu_b = \dfrac{1}{{}_b\mu_a}$
* Thus, the refractive index of medium 'b' with respect to 'a' is equal to the reciprocal of refractive index of medium 'a' with respect to medium 'b'.
* **W.E-20:** ***A light ray is incident normally on a glass slab of thickness 't' and refractive index 'μ' as shown in the figure. Then find time taken by the light ray to travell through the slab.***
* **Sol:** 

* From the figure distance travelled by the light ray through the slab is 't'
* Velocity of light in glass $= \dfrac{\text{distance travelled}}{\text{time}}$

* $\frac{c}{\mu} = \frac{t}{time}, time = \frac{\mu t}{c}$

* **W.E-21:** ***A light ray is incident on a plane glass slab of thickness 't' at an angle of incidence 'i' as shown in the figure. If 'μ'is the refractive index of glass. Then find time taken by the light ray to travel through the slab.***

* **Sol:** 

* As shown in the above figure distance travelled by the light ray through the slab is 'd'. From the figure

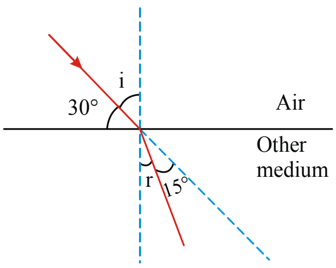
* $\cos r = \frac{t}{d}, d = \frac{t}{\cos r}$

* Velocity of light in glass $= \frac{\text{Distance travelled through the glass}}{\text{time}}$

* $\frac{c}{\mu} = \frac{d}{time}; \; time = \frac{d\mu}{c}$

* $time = \frac{t\mu}{\cos r \times c} = \frac{\mu^2 t}{c\sqrt{\mu^2 - \sin^2 i}}$

* **W.E-22:** ***Light of wavelength 4500 $\overset{0}{A}$ in air is incident on a plane boundary between air and another medium at an angle 30^0 with the plane boundary. As it enters from air into the other medium, it deviates by 15^0 towards the normal. Find refractive index of the medium and also the wavelength of given light in the medium.***

* **Sol:** 

* Angle of incidence i $= 90^0 - 30^0 = 60^0$. As the ray bends towards the normal, it deviates by an angle $i - r = 15^0$ (given)

* $\therefore r = 45^0$ Applying Snell's law

* $\mu_{air} \sin i = \mu_{med} \sin r; \quad \therefore 1 \times \sin 60^0 = \mu \times \sin 45^0$

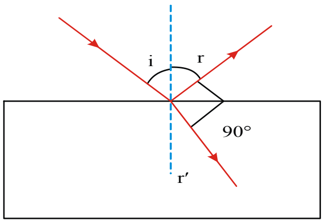
* In terms of wavelengths,

* $\mu = \sqrt{1.5} = \frac{\lambda_{air}}{\lambda_{med}} (or) \lambda_{med} = \frac{\lambda_{air}}{\sqrt{1.5}} = \frac{4500}{\sqrt{1.5}}$

* $\lambda_{med} = 3674 \overset{0}{A}$

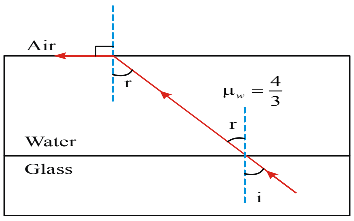
* **W.E-23:** ***Monochromatic light falls at an angle of incidence 'i' on a slab of a transparent material. Refractive index of this material being 'μ'for the given light. What should be the relation between i and μ so that the reflected and the refracted rays are mutually perpendicular?***

* **Sol:** In the given figure let r is the angle of reflection and r' is the angle of refraction. According to the given condition, considering the reflected and the refracted rays to be perpendicular to each other,

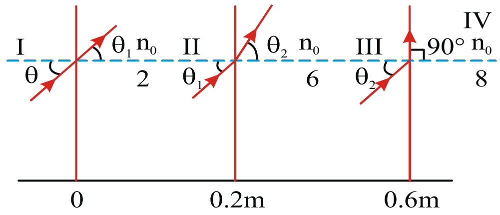
* 

* $\therefore$ From the figure $r + 90^0 + r' = 180^0$
* So, $r' = 90^0 - r$
* $r' = 90^0 - i$ [i = r, by law of reflection]
* According to Snell's law, 1 $\sin i = \mu \sin r'$
* $\sin i = \mu \sin\left(90^0 - i\right)$
* $\sin i = \mu \cos i, \mu = \tan i \Rightarrow i = \tan^{-1}(\mu)$

* **W.E-24:** ***A ray of light is incident at the glass-water interface at an angle i as shown in figure, it emerges finally parallel to the surface of water, then the value of μ_g would be***



* **Sol:** Applying Snell's law ($\mu \sin i =$ constant)
* at first and second interfaces, we have
* $\mu_1 \sin i_1 = \mu_2 \sin i_2$; But, $\mu_1 = \mu_{glass}, i_1 = i$
* $\mu_2 = \mu_{air} = 1$ and $i_2 = 90^0$
* $\therefore \mu_g \sin i = (1)\left(\sin 90^0\right)$ or $\mu_g = \frac{1}{\sin i}$
* **W0.E-25:** ***A light beam is travelling from region I to region IV (Refer figure). The refractive index in regions I, II,III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering region IV is***

* 

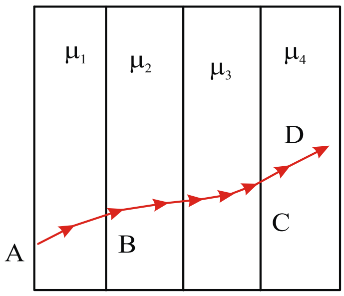
* **Sol:** As the beam just misses entering the region IV, the angle of refraction in the region IV must be 90^0.

* Application of Snell's law successively at different interfaces gives

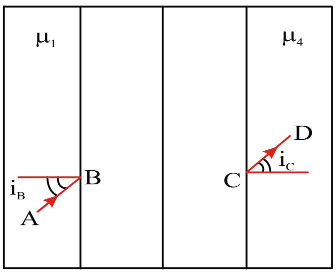
* $$n_0 \sin\theta = \frac{n_0}{2}\sin\theta_1 = \frac{n_0}{6}\sin\theta_2 = \frac{n_0}{8}\sin 90^0$$

* $\Rightarrow \sin\theta = \frac{1}{8}$ or $\theta = \sin^{-1}\frac{1}{8}$

* **W.E-26:** ***A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have***

* 

* **Sol:** Applying Snell's law at B and C,

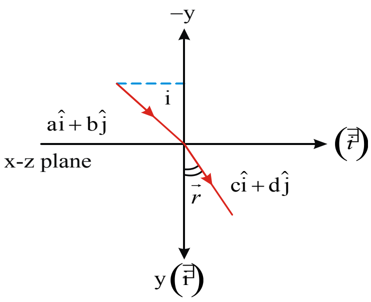
* 

* $\mu \sin i =$ constant or $\mu_1 \sin i_B = \mu_4 \sin i_C$

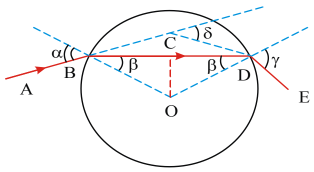
* But $AB \parallel CD$; $\therefore\ i_B = i_C$ or $\mu_1 = \mu_4$

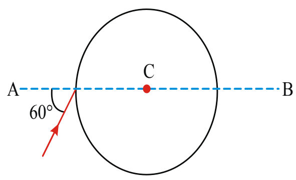
* **W.E-27:** ***The x-z plane separates two media A and B of refractive indices $\mu_1 = 1.5$ and $\mu_2 = 2$. A ray of light travels from A to B. Its directions in the two media are given by unit vectors $\vec{u}_1 = a\hat{i} + b\hat{j}$ and $\vec{u}_2 = c\hat{i} + d\hat{j}$. Then***

* **Sol:**

* 

$\tan i = \frac{a}{b}$ so $\sin i = \frac{a}{\sqrt{a^2+b^2}}$

* and $\tan r = \frac{c}{d}, \sin r = \frac{c}{\sqrt{c^2+d^2}}$
* $\mu_1 \sin i = \mu_2 \sin r$; $\left(\frac{3}{2}\right)\left(\frac{a}{\sqrt{a^2+b^2}}\right) = 2\left(\frac{c}{\sqrt{c^2+d^2}}\right)$
* But as $a\hat{i}+b\hat{j}$ and $c\hat{i}+d\hat{j}$ are unit vectors so
* $\sqrt{a^2+b^2} = \sqrt{c^2+d^2} = 1$;Hence $\frac{3}{2}a = 2c$, so $\frac{a}{c} = \frac{4}{3}$
* **W.E-28:** ***A ray of light is incident on the surface of a spherical glass paper-weight making an angle α with the normal and is refracted in the medium at an angle β. Calculate the deviation.***
* **Sol:** Deviation means the angle through which the incident ray is turned in emerging from the medium. In Figure if AB and DE are the incident and emergent rays respectively, the deviation will be δ.
* 

* Now as at B ; $\angle i = \alpha$ and $\angle r = \beta$
* So from Snell's law, $1 \sin \alpha = \mu \sin \beta$ ....(1)
* Now from geometry of figure at D, $\angle i = \beta$
* So $\mu \sin \beta = 1 \sin \gamma$ .....(2)
* Comparing Eqs. (1) and (2) $\gamma = \alpha$
* Now as in a triangle exterior angle is the sum of remain-ing two interior angles, in ΔBCD,
* $\delta = (\alpha - \beta) + (\alpha - \beta) = 2(\alpha - \beta)$
* **W.E-29:** ***A ray of light falls on a transparent sphere with centre at C as shown in figure. The ray emerges from the sphere parallel to line AB. The refractive index of the sphere is***
* 

* **Sol:** Deviation by a sphere is 2 (i - r)
* Here, deviation $\delta = 60^0 = 2$ (i-r) or i-r = $30^{0;}$
* $\therefore$ r = i$-30^0 = 60^0 - 30^0 = 30^0$

* $\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 60^0}{\sin 30^0} = \sqrt{3}$

* **Apparent Depth**

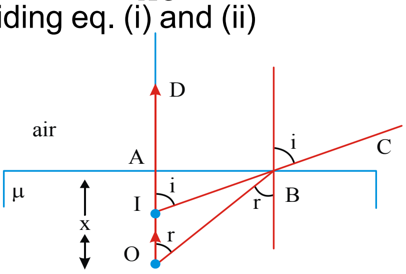
* **Ø Case(1) :** Object in denser medium and observer in rarer medium.

* When object 'O' is placed at a distance 'x' from A in denser medium of refractive index μ as shown in figure. Ray OA, which falls normally on the plane surface, passes undeviated as AD. Ray OB, which 'r'(with normal) on the palne surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at 'I' after extending these two rays backwards. This 'I' is the virtual image of real object 'O' to an observer in rarer medium near to transmitted ray.

* $\sin i \approx \tan i = \frac{AB}{AI}$ ......(i)

* $\sin r \approx \tan r = \frac{AB}{AO}$ ....(ii)

* Dividing eq. (i) and (ii)

* 

* $\frac{\sin i}{\sin} = \frac{AO}{AI}$; According to Snell' law $\mu = \frac{\sin i}{\sin r}$

* $\therefore \mu = \frac{AO}{AI} \therefore AI = \frac{AO}{\mu} = \frac{x}{\mu}$

* The distance of image AI is called apparent depth or apparent distance. The apparent depth x_{app} is given by i.e., $x_{app} = \frac{x_{real}}{\mu}$

* The apparent shift $(OI) = AO - AI = x - \frac{x}{\mu}$

* Hence the apparent shift $(OI) = \left(1 - \frac{1}{\mu}\right)x$

* If the observer is in other than air medium of refractive index $\mu(<\mu)$.

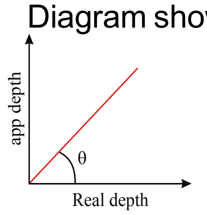
* Then apparent depth

* $= \frac{\text{real depth}}{\mu_{\text{relative}}} = \frac{\text{real depth}}{\left(\frac{\mu}{\mu^1}\right)}$

* $\therefore$ apparent depth $= \frac{\mu^1}{\mu}$ (real depth)

* apparent shift $= \left(1 - \frac{\mu^1}{\mu}\right)x$

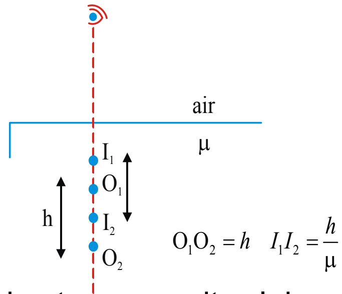
* Diagram shows variation of apparent depth with real depth of the object.

* 

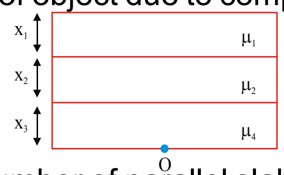
Slope $= \tan\theta = \frac{\mu^1}{\mu}(<1)$

* **Note:** If two objects O_1 and O_2 separated by 'h' on normal line to the boundary in a medium of refractive index μ. These objects are observed from air near to normal line of boundary. The distance between the images I_1 and I_2 of

* O_1 and O_2 is $\frac{h}{\mu}$.

* 

* **Note:** Apparent depth of object due to composite slab

* is $x_a = \frac{x_1}{\mu_1} + \frac{x_2}{\mu_2} + \frac{x_3}{\mu_3}$ 

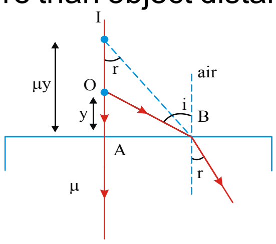
* **Note:** If there are 'n' number of parallel slabs which are may be in contact or may not with different refractive indices are placed between the observer and the object, then the total apparent shift

* $s = \left(1 - \frac{1}{\mu_1}\right)x_1 + \left(1 - \frac{1}{\mu_2}\right)x_2 + ---- + \left(1 - \frac{1}{\mu_n}\right)x_n$

* Where $x_1, x_2 ---- x_n$ are the thickness of the slabs and $\mu_1, \mu_2 .... \mu_n$ are the corresponding refractive indices.

* **Ø Object in rarer medium and observer in denser medium :** When the object in rarer medium (air) at a distance'y' from boundary and an observer near to normal in denser medium of refractive index 'μ'. By ray diagram in figure it is observed that the image is virtual, on same side to boundary and its distance from the boundary is μ times the object distance.

* Since $\mu > 1$ image distance is more than object distance.

* 

* $\sin i \approx \tan i = \frac{AB}{AO}, \sin r \approx \tan r = \frac{AB}{AI}$

* According to Snell's law $1.\sin i = \mu \sin r$

* $\frac{AB}{AO} = \mu \frac{AB}{AI}, AI = \mu.AO$

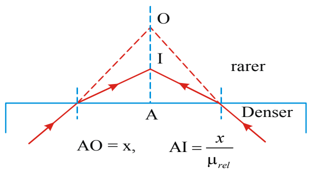
* Therefore apparent height of object (AI) $= \mu$ x real height of object (AO)

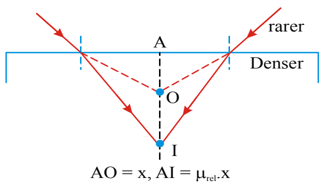
* i.e. $y_{app} = \mu.y_{real}$ Apparent shift $= AI - AO$

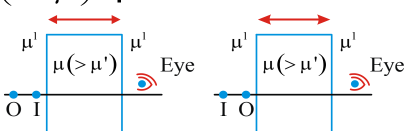
* Apparent shift $= (\mu - 1)y$.

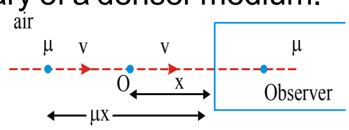
* If the object is in other than air medium of refractive index $\mu^1 (< \mu)$. Then apparent height

$= \mu_{rel}$ (real height) ; i.e., $y_a = \left(\frac{\mu}{\mu^1}\right)y$

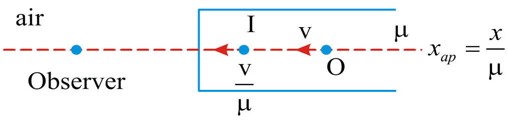
*   Apparent shift $=\left(\frac{\mu}{\mu^{1}}-1\right)y$
*   Diagram shows variation of apparent height with real height of the object.
*   slope $=\tan\phi=\frac{\mu}{\mu'}(>1)$
*   **Note:** When convergent beam of rays passing from denser to rarer medium as shown in the figure. Real image is formed in rarer medium which nearer to boundary than that of virtual object.
*   

*   shift $=x\left(1-\frac{1}{\mu_{real}}\right)$
*   **Note:** When convergent beam of rays passing from rarer to denser medium as shown in the figure. Real image is formmed in denser medium which is far to boundary than that of virtual object.
*   shift $=\left(\mu_{real}-1\right)x$
*   

*   **Ø Application**
*   **Normal shift due to glass slab :** When an object is placed on normal line to the boundary of slab whose thickness is 't' and refractive index 'μ'. On observing this object (real) from other side of the slab, due to refraction, the image of this object shift on the normal line. This shift value is called normal shift. This shift is towards the slab, if the slab is denser relative to the surroundigs and shift is away from the slab, if the slab is rarer relative to the surrounds. Then the Normal shift
* $OI=\left(1-\frac{1}{\mu_{rel}}\right)t=\left(1-\frac{\mu^{1}}{\mu}\right)t$
*   for $\mu'=1,$ normal shift $OI=\left(1-\frac{1}{\mu}\right)t$.
*   

*   **Ø Relation between the velocities of object and image :** The figure shows an object O moving towards the plane boundary of a denser medium.
*   

* $x_{ap}=\mu x$
*   Differentiating the above equation with respective to time, we get
* $V_{ap}=\mu V$
*   To an observer in the denser medium, the object appears to be more distant but moving faster. If the speed of the object is v, then the speed of the image will be μv.
*   (b) Simillarly to an observer in rarer medium and object in denser medium, the image appears to be closer but moving slowly.

* 

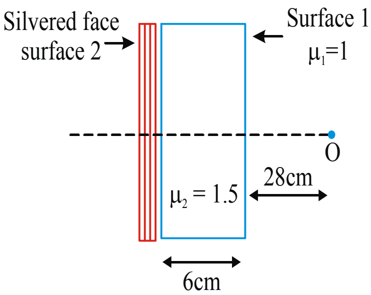
* Differentiating the above equaion with respective to time, we get $V_{ap} = \frac{V}{\mu}$

* If the speed of the object is v. Then the speed of the image will be $\frac{v}{\mu}$.

* **W.E-30:** ***In a tank, a 4cm thick layer of water $\left(\mu = \frac{4}{3}\right)$ floats on a 6 cm thick layer of an organic liquid $(\mu = 1.5)$. Viewing at normal incidence, how far below the water surface does the bottom of tank appear to be?***

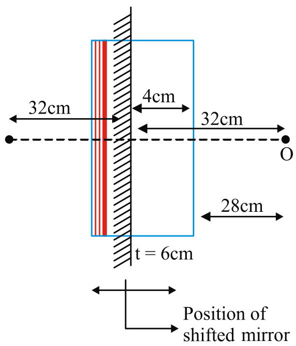
* **Sol:** $d_{AP} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{1.5} + \frac{4}{4/3} = 7$ cm

* **W.E-31:** ***An object is placed in front of a slab $(\mu = 1.5)$ of thickness 6 cm at a distance 28 cm from it. Other face of the slab is silvered. Find the position of final image.***

* **Sol:** 

* By the principle of reversibility of light, we can say if light rays are coming from the mirror and passing through the slab, the mirror will shift

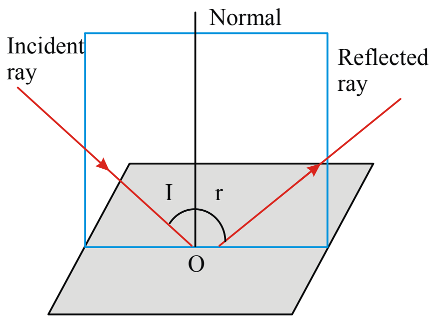
* 2 cm towards right for observer in front of the slab. The position of the object from shifted mirror = 32 cm.

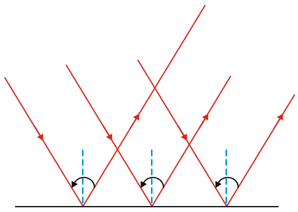
* 

* So, the position of the image formed by shifted mirror will be 32 cm behind it. Hence, position of the image from surface 2 is 30 cm left to it and 36 cm left of surface 1.

* **W.E-32:** ***An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in figure. The beaker height 3h and its radius h. When the beaker is filled with a liquid upto a height 2h, he can see the lower end of the rod. Find the refractiveindex of the liquid.*** **Introduction**

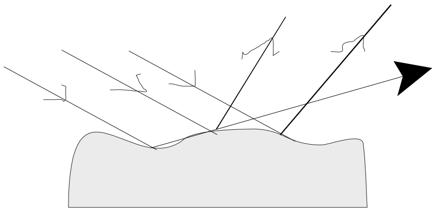
* * Nature has endowed the human eye (retina) with the sensitivity to detect electromagnetic waves within a small range of the electromagnetic spectrum.

* ⁎ Electromagnetic radiation (Wavelength from 400 nm to 750 nm) is called light. It is mainly through light and the sense of vision.
* ⁎ Light travels along straight line with enormous speed. The speed of light in vacuum is the highest speed attainable in nature. The speed of light in vacuum is $c = 2.99792458 \times 10^8 ms^{-1}$.
* $\approx 3 \times 10^8 ms^{-1}$
* ⁎ The wavelength of light is very small compared to the size of ordinary objects that we encounter commonly (generally of the order of a few cm or larger). A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a **ray of light**, and a bundle of such rays constitutes a **beam of light**.
* ⁎ The phenomena of reflection, refraction and dispersion of light are explained using the ray picture of light. We shall study the image formation by plane and spherical reflection and refracting surfaces, using the basic laws of reflection and refraction. The construction and working of some important optical instruments, including the human eye are also explained.
* ⁎ **Reflection of Light :** When a light ray strikes the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called reflection of light.
* 

* In case of reflection at the point of incidence 'O', the angle between incident ray and normal to the reflecting surface is called the angle of incidence (i). The angle between reflected ray and normal to the reflecting surface is called angle of reflection (r).
* The palne containing incident ray and normal is called plane of incidence.
* ⁎ **Laws of reflection :** The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.
* ⁎ The angle of incidence is equal to the angle of reflection $\angle i = \angle r$
* **Types of reflections**
* ⁎ **Regular reflection:** When the reflection takes place from a perfect smooth plane surface, then the reflection is called regular reflection (or) specular reflection.
* In this case, a parallel beam of light incident will remain parallel even after reflection as shown in the figure.
* 

* In case of regular reflection, the reflected light ray has large intensity in one direction and negligibly small intensity in other direction. Regular reflection of light is useful in determining the property of mirror.

* * **Diffused reflection:** If the reflecting surface is rough (or uneven), parallel beam of light is reflected in random directions. This kind of refletion is called diffused reflection.

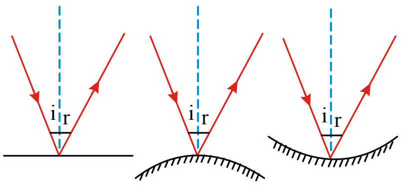
* 

* As shown in the above figure if the reflecting surface is rough, the normal at different points will be in different directions, so the rays that are parallel before reflection will be reflected in random directions.

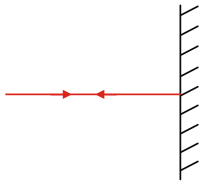
* We see non-luminous objects by diffused reflection.

* **Important points regarding reflection**

* * Laws of reflection are valid for all reflecting surfaces either plane or curved.

* 

* * If a light ray is incident normally on a reflecting surface, after reflection it retraces its path i.e., if $\angle i = 0$ then $\angle r = 0$

* 

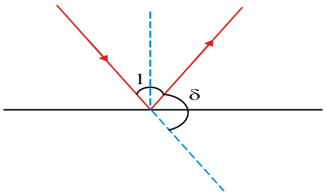
* * In case of reflection of light frequency, wavelength and speed does not change. But the intensity of light on reflection will decreases.

* * If the reflection of light takes place from a denser medium, there is a phase change of π rad.

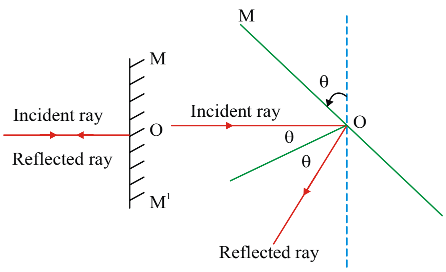
* * If $\hat{I}, \hat{N}$ and $\hat{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively then

* $\hat{R}.\left(\hat{I}\times\hat{N}\right)=\hat{N}.\left(\hat{I}\times\hat{R}\right)=\hat{I}.\left(\hat{N}\times\hat{R}\right)=0$ This is because incident ray, reflected ray and the normal at the point of incidence lie in the same plane.

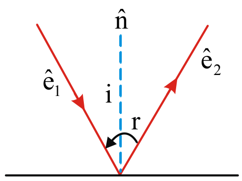
* * **Deviation of a ray due to reflection:** The angle between the direction of incident ray and reflected light ray is called the angle of deviation (δ).

* 

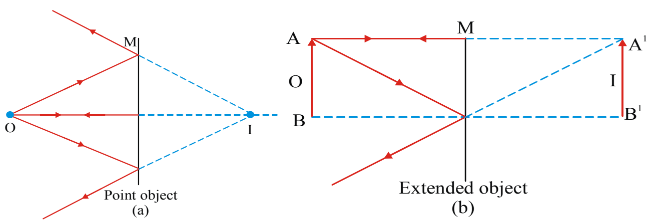
* From the above figure $\delta = \pi - (i + r)$
* But $i = r$
* Hence angle of deviation in the case of reflection is $\delta = \pi - 2i$
* ⁎ By keeping the incident ray fixed, the mirror is rotated by an angle 'θ', about an axis in the plane of mirror, the reflected ray is rotated hrough an angle '2θ'.

* 

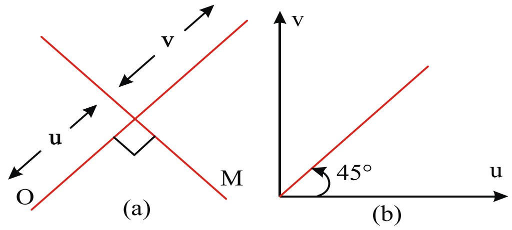
* ⁎ Vector form of law of reflection:

* 

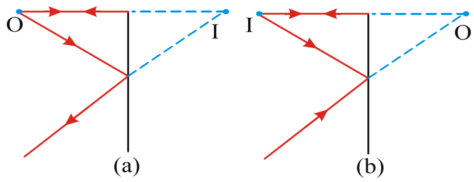
* If $\hat{e}_1$ is unit vector along the incidnet ray $\hat{e}_2$ is the unit vector along the reflected ray $\hat{n}$ is the unit vector along the normal then,
* $\hat{e}_2 = \hat{e}_1 - 2(\hat{e}_1.\hat{n})\hat{n}$
* **Reflection from Plane Surface**
* ⁎ When you look into a plane mirror, you see an image of yourself that has three properties.
* ⁎ The image is up right.
* ⁎ The image is the same size as you are
* ⁎ The image is located as far behind the mirror as you are infront of it. This is shown in the figure(b).

* 

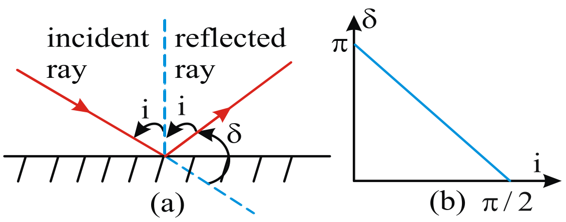
* ⁎ A plane mirror always form virtual image to a real object and vice versa and the line joining object and image is perpendicular plane mirror as shown in figure (a).

* 

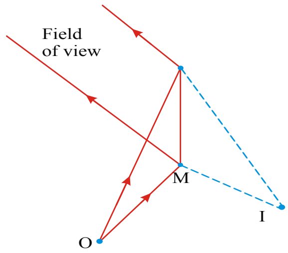
* The graph between image distance (v) and object distance (u) for a plane mirror is a straight line as shown in figure (b).
* The ratio of image height to the object height is called lateral magnification (m). Thus in case of plane mirror 'm' is equal to one.
* * The principle of reversibility states that rays retrace their path when their direction is reversed. In accordance with the principle of reversibility object and image positions are interchangable. The points corresponding to object and image are called conjugate points.
* This is illustrated in figure.

* 

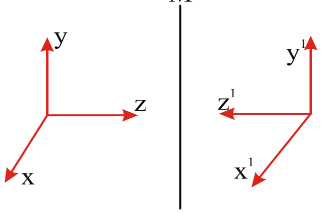
* * A mirror whatever may be the size, it forms the complete image of the object lying infront of it. Large mirror gives more bright image than a smaller one. It is seen that the size of reflector must be much larger than the wavelength of the incident light otherwise the light will be scattered in all directions.
* * The angle between directions of incident ray and reflected or refracted ray is called deviation (δ).
* A plane mirror deviates the incident light through angle $\delta = 180 - 2i$ where 'i' is the angle of incidence. The deviation is maximum for normal incidence, hence $\delta_{max} = 180^0$.

* 

* It is noted that, generally anti - clock wise deviation is taken as positive and clock wise deviation as negative.
* * Every object has its own field of view for the given mirror. The field of view is the region between the extreme reflected rays and depends on the location of the object infornt of the mirror. If our eye lies in the filed of view then only we can see the image of the object other wise not. This is illustrated in figure.

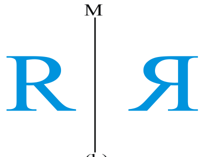
* 

* ⁎ A plane mirror produces front - back reversal rather than left - right reversal. It must be kept in mind that the mirror produces the reversal effect in the direction perpendicular to plane of the mirror. The figure (a) shows that the right handed co-ordinate system is converted into left handed co-ordinate system.

* 

(a)

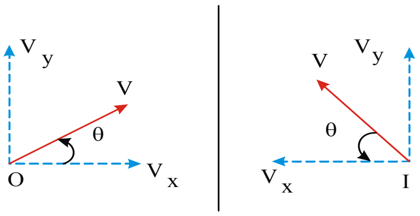
* i.e., the image formed by a plane mirror left is turned into right and vice versa with respect to object as shown in figure (b).

* 

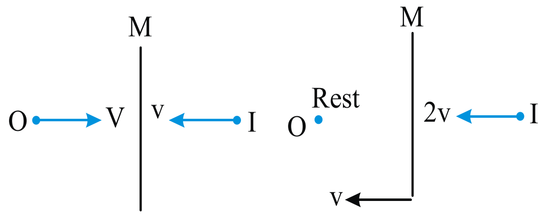
(b)

* ⁎ When the object moves infront of stationary mirror, the relative speed between object and its image along the plane of the mirror is zero and in perpendicular to plane of mirror relative speed is twice that of the object speed.

* $\left(V_{IO}\right)_y = 0$ and $\left(V_{IO}\right)_x = 2v_x$

* 

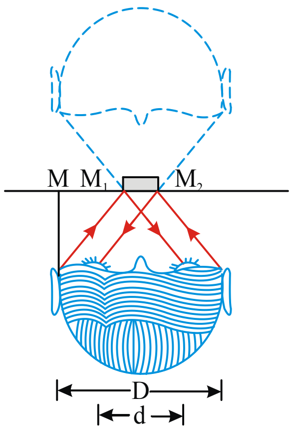
* ⁎ If an object moves towards (or away from) a plane mirror at speed v, the image will also approach (or recede) at the same speed v, and the relative velocity of image with respect to object will be 2v as shown in figure (a). If the mirror moved towards (or away from) the stationary object with speed v, the image will also move towards (or away from) the object with a speed 2v, as shown figure (b).

* 

* * a) A person of height ‘h’ can see his full image in a mirror of minimum length $l = \frac{h}{2}$

* b) A person standing at the centre of room looking towards a plane mirror hung on a wall, can see the whole height of the wall behind him if the length of the mirror is equal to one-third the height of the wall.

* Ø The minimum width of a plane mirror required for a person to see the complete width of his face is $(D-d)/2$, where, D is the width of his face and d is the distance between his two eyes.

* 

* $MM_1 = \frac{1}{2}\left[D - \frac{1}{2}(D-d)\right]$

* $MM_1 = \frac{(D+d)}{4}$ .... (i)

* and $MM_2 = D - \frac{(D+d)}{4}$

* $MM_2 = \frac{(3D-d)}{4}$ ... (ii)

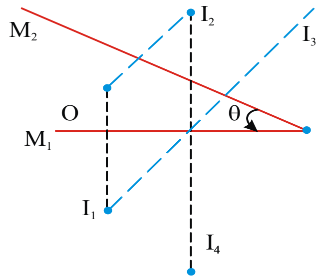
* $\therefore$ Width of the mirror = M_1M_2

* $= MM_2 - MM_1$

* $= \frac{2D-2d}{4}$ {From (i) and (ii)]

* $$=\frac{2(D-d)}{4}=\frac{D-d}{2}$$

* Ø If two plane mirrors inclined to each other at an angle θ, the number of images of a point object formed are determined as follows

* 

* Ø If $\frac{360}{\theta}$ is even number (say m) Number of images formed $n=(m-1)$, for all positions of objectes in between the mirrors.

* Ø If $\frac{360}{\theta}$ is odd integer (say m) number of images formed $n=m$, if the object is not on the bisector of mirrors. $n=(m-1)$, if the object is on the bisector of mirrors.

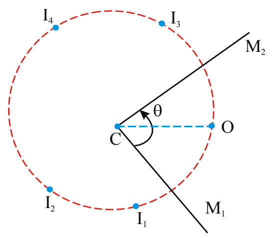
* Ø If $\frac{360}{\theta}$ is a fraction (say m). The number of images formed will be equal to its integer part i.e., $n=[m]$.

* Ex: If m=4.3, the total number of images $n=[4.3]=4$

$m=\frac{360}{\theta}$	Position of the object	Number of images (n)
Even	Any where	$m-1$
Odd	Symmetric	$m-1$
	Asymmetric	M
Fraction	Any where	[m]

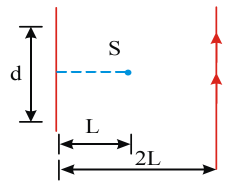
* **Ø All the images lie on a circle whose radius is equal to the distance between the object 'O' and the point of intersection of mirrors C. If θ is less more number of images on circle with large radius.**

*



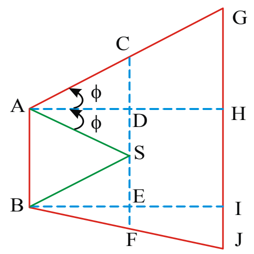
* $*$ If the objects is placed in between two parallel
* mirrors $\theta = 0^0$, the number of images formed is infinite but of decreasing intensity in according with $I \alpha r^{-2}$.
* $*$ If 'θ' is given n is unique but if 'n' is given θ is not unique. Since same number of images can be formed for different θ.
* Ø The number of images seen may be different from number of images formed and depends on the position of the observer relative to object and mirror.
* Ø When a light ray vector incident on a mirror, only the component vector which is parallel to normal of the mirror changes its sign without change of its magnitude on reflection. It is noted that a mirror can reflects entire energy incident on it, hence the magnitude of reflected vector is same as that of incident vector. Incident vector corresponding to an object and reflected vector corresponds to an image. This vector may be position, velocity or acceleration.
* Example: If a plane mirror lies on x-z plane, a light vector $2\hat{i} + 3\hat{j} - 4\hat{k}$ on reflection becomes $2\hat{i} - 3\hat{j} - 4\hat{k}$.
* **W.E-1:** ***A point source of light S, placed at a distance L in front of the centre of a mirror of width d, hangs vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance 2L from it as shown in figure. Find the greatest distance over which he can see the image of the light source in the mirror.***

*



* **Sol:** The ray diagram will be as shown in figure.

*



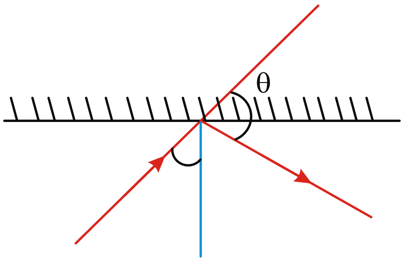
* HI = AB =d, DS = CD = d/2
* Since, AH = 2AD, $\therefore$ GH = 2CD = $2\frac{d}{2} = d$
* Similarly IJ = d
* GJ = GH + HI +IJ =d +d + d =3d

* **W.E-2:** ***A ray of light travelling in the direction*** $\frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right)$ ***is incident on a plane mirror. After reflection, it travels along the direction*** $\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$***. The angle of incidence is***

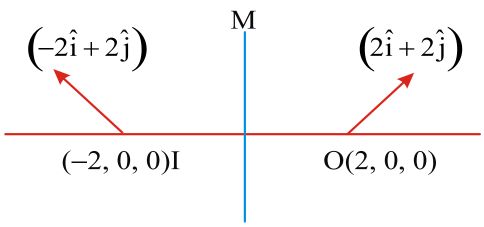
* **Sol:** Let angle between the directions of incident ray and reflected ray be θ,

* $$\cos\theta = \frac{1}{2}\left(\hat{i}+\sqrt{3}\hat{j}\right).\frac{1}{2}\left(\hat{i}-\sqrt{3}\hat{j}\right)$$

* $$\cos\theta = -\frac{1}{2} \qquad \theta = 120^0$$

* 

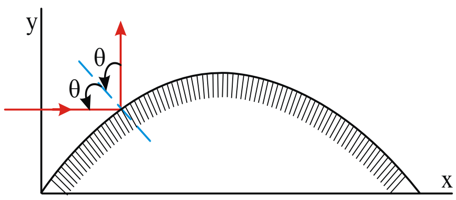
* **W.E-3:** ***A plane mirror is placed at origin parallel to y-axis, facing the positive x-axis. An object starts from (2m, 0, 0) with a velocity of*** $\left(2\hat{i}+2\hat{j}\right)$ ***m/s. Find the relative velocity of image with respect to object.***

* **Sol:** 

* The relative velocity of image with respect to object along normal $= 4\hat{i}$ The relative velocity image with respect to object along plane of mirror = 0. Hence the relative velocity of image with respect to object $= -4\hat{i}$

* **W.E-4:** ***A reflecting surface is represented by the equation*** $Y = \frac{2L}{\pi}\sin\left(\frac{\pi x}{L}\right), 0 \le x \le L$***. A ray travelling horizontally becomes vertical after reflection. The coordinates of the point(s) where this ray is incident is***

* **Sol:** A horizontal ray becomes vertical after reflection.

* 

* $\tan\theta = \frac{dy}{dx} = 2\cos\frac{\pi x}{L}$

* $2\theta = 90^0 \Rightarrow \theta = 45^0$

* $1 = 2\cos(\pi x/2)$

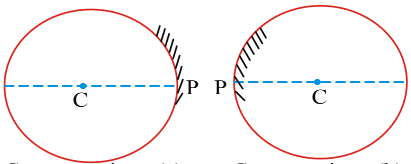
* $\Rightarrow x = L/3$

* $\therefore y = \frac{2L}{\pi}\sin(\pi/3) = \frac{\sqrt{3}L}{\pi}$

* $\left(\frac{L}{3}, \frac{\sqrt{3}L}{\pi}\right) \& \left(\frac{2L}{3}, \frac{\sqrt{3}L}{\pi}\right)$

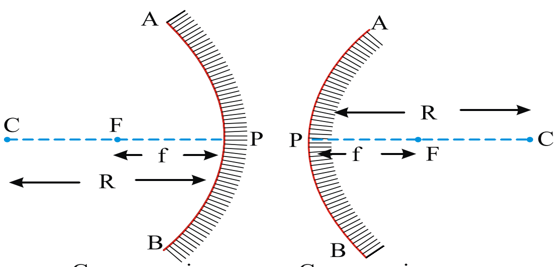
* **Reflection from Curved Surface**

* Ø A curved mirror is a smooth reflecting part (in any shape) of a symmetrical curved surface such as spherical, cylindrical or ellipsoidal. In this chapter we consider a piece of spherical surface only.

* 

Concave mirror (a) Convex mirror (b)

* If the reflection take place from the inner surface, the mirror is called concave and if its outer surface it is convex as shown in the figure. In case of thin spherical mirror, the centre 'C' of the sphere of which the mirror part is called the centre of curvature of the mirror. P is the centre of the mirror surface, is called the pole. The line CP produced is the principal axis, AB is the aperture means the effective diameter of the light reflecting area of the mirror. The distance CP is radius of curvature (R). The point F is the focus and the distance between PF is called focal length (f) and it is related to R as $f = R/2$.

* 

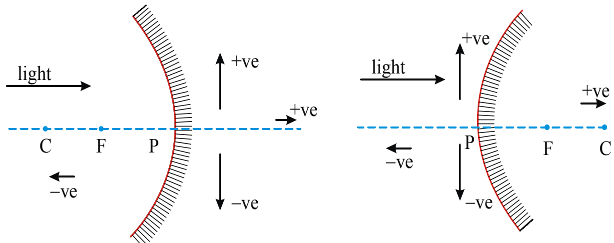
Concave mirror (a) Convex mirror (b)

* **Sign Convertion :** To derive the relevant formula for reflection by spherical mirrors and refraction by spherical surfaces, we must adopt a sign convection for measuring distance. In this book, we shall follow the Cartesian sign convention. According to this convention all distances measured from the pole of the mirror.

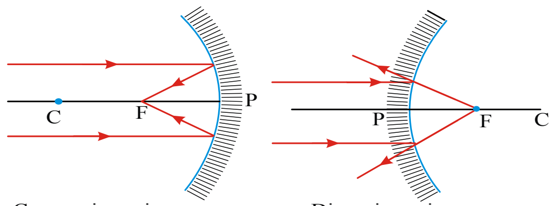
* Ø The distance measured in the same direction as the incident light are taken as positive and those measured in the direction opposite to the direction of light are taken as negative.

* Ø The heights measured one side with respect to principal axis of the mirror are taken as positive and the heights measured other side are taken as negative.

* Ø Acute angles measured from the normal (principal axis) in the anti-clock wise sense are positive, while that in the clock wise sense are negative.

* 

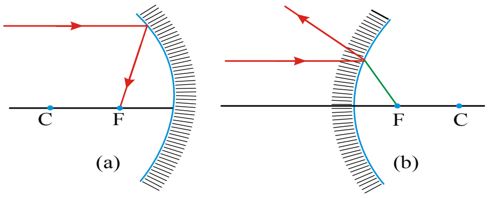
Concave mirror
(a)
Convex mirror
(b)

* Ø **Paraxial Approximation :** Rays which are close to the principal axis or make small angle $\left(\theta < 10^0\right)$ with it i.e. they are nearly parallel to the axis, are called paraxial rays. Accordingly we set $\cos\theta \approx 1, \sin\theta \approx \theta$ and $\tan\theta \approx \theta$. This is known as paraxial approximation or first order theory or "Gaussion" optics. In spherical mirrors we restrict to mirror with small aperture and to paraxial rays.
* **Focal Length of Spherical Mirrors**
* Ø We assume that the light rays are paraxial and they make small angles with the principal axis.
* A beam of parallel paraxial rays is reflected from a concave mirror so that all rays converge to a point F on the principal axis is called principal focus of the mirror and it is real focus.
* A narrow beam of paraxial rays falling on a convex mirror is reflected to form a divergent beam which appears to come from a point 'F' behind the mirror. Thus a convex mirror has a virtual focus 'F'.
* The distance between focus (F) and pole (P) is called the focal length 'f'. Concave mirror is also called as converging mirror. They are used in car head lights, search lights and telescopes. Convex mirror is also called as diverging mirror. Convex mirror gives a wider field of view than a plane mirror and concave mirror, convex mirrors are used as rear view mirrors in vehicles.
* 

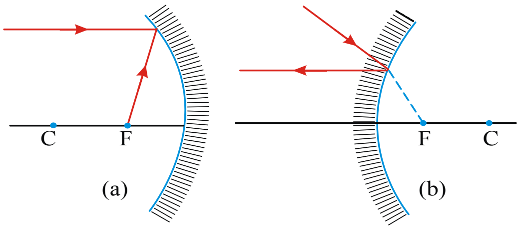
Converging mirror
(a)
Diverging mirror
(b)
* $(b)\, Diverging\ mirror$
* According to Cartesian sign convention with real object the focal length of concave mirror is negative, because the distance PF (P to F) is measured in opposite direction of light. Similarly with the same reason focal length of convex mirror is positive. The same sign convention is also applicable to virtual object by treating that imaginary light rays from that object.
* **Relation between F and R**
* **a) Concave mirror b) Convex mirror**
* $f = \dfrac{R}{2}$
* The focal length of mirror is independent on medium in which it placed and wavelength of incident light. To a plane mirror focal length 'f' is infinite (as $R = \infty$)

* **Ø Rules for Image formation:** In general, position of image and its nature [i.e., whether it is real or virtual, erect or inverted, magnified or diminished] to an object depend on the distance of the object from the mirror. Nature of the image can be obtained by drawing a ray diagram. In case of image formation unless stated object is taken to be real, it may be point object or extended.

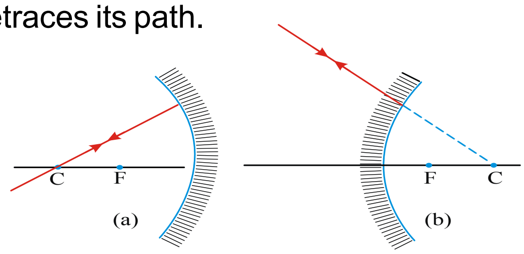
* Ø A ray parallel to principal axis after reflection from the mirror passes or appear to pass through its focus F.

* 

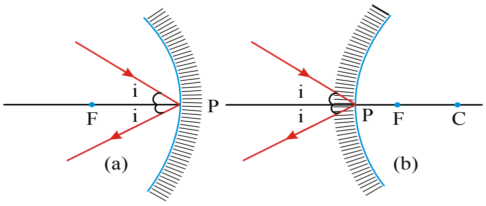
* Ø A ray passing through or directed towards focus, after reflection from the mirror becomes parallel to the principal axis (by principle of reversibility)

* 

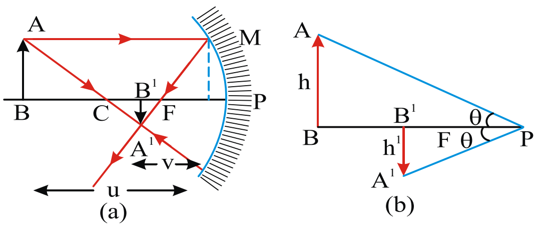
* Ø A ray through or directed towards the centre of curvature C, after reflection from the mirror, retraces its path.

* 

* **Ø A ray striking at pole P is reflected symmetrically back in the opposite side.**

* 

* **The Mirror Equation:** Figure (a) shows the ray diagram considering two rays and the image A^1B^1 (in this case real image) of an object AB formed by a concave mirror.



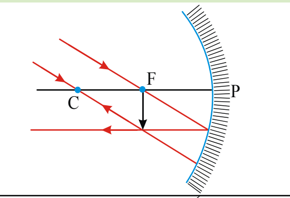
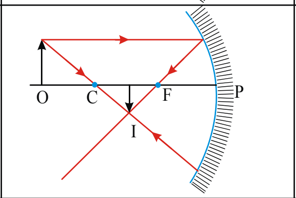
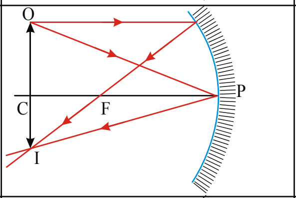
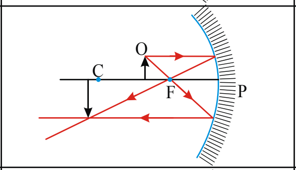
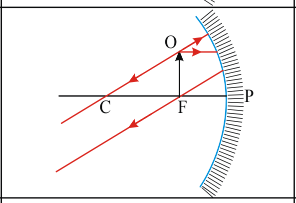
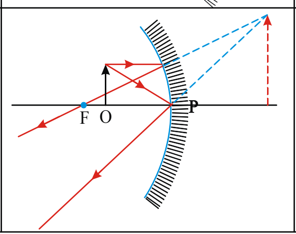
$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \qquad \left(\because f=\frac{R}{2}\right)$$

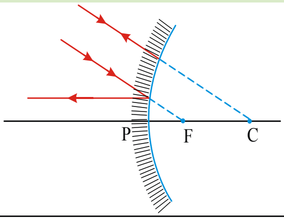
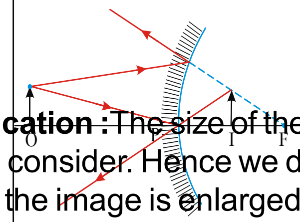
* This relation is known as Gauss's formula for a spherical mirror. It is valid in all other situations with a spherical mirror and also for a convex mirror. In this formula to calculate unknown, known quantities are substituted with proper sign.

* **Image Formation by Spherical Mirrors**

* Ø From the ray diagrams we understand that

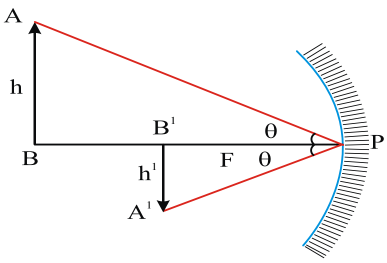
* Ø To a real object in case of concave mirror the image is erect, virtual and magnified when the object is placed between F and P. In all other positions of object the image is real and inverted.
* Ø To a real object the image formed by convex mirror is always virtual, erect and diminished no matter where the object is.
* Ø A concave mirror with virtual object behaviour is similar to convex mirror with real object and convex mirror with virtual object behaviour similar to concave mirror with real object.
* Ø By principle of reversibility a convex mirror can form real and magnified image to a virtual object which is with in the focus and virtual images when virtual object beyond the focus. i.e., the convex mirror can form real and virtual images to virtual object. A concave mirror with virtual object always forms real images.
* Ø If the given mirror breaks in to pieces, each piece of that mirror has own principal axis, but behaviour is similar to that of main mirror with less intensity of image.
* **a) Concave mirror b) Convex Mirror**

Position of the object	Ray diagram	Image details
At Infinity		Real, inverted, very Small, at F
Between ∞ and c		Real, inverted, diminished between F and C
At C		Real, inverted, equal, at C
Between F and C		Real, inverted, enlarged, beyond C
At F		Real, inverted, very large at infinity
Between F and P		Virtual, erect, enlarged behind the mirror

Position of the object	Ray diagram	Image details
At Infinity		Virtual, erect, very small at F
In front of mirror		Virtual, erect, diminished between P and F

* **Magnification :** The size of the image relative to the size of the object is another important quantity to consider. Hence we define magnification. It is noted that magnification does not mean that the image is enlarged. The image formed by optical system may be larger than, smaller than or of the same size of the object.

* **Lateral magnification:** The ratio of the transverse dimension of the final image formed by an optical system to the corresponding dimension of the object is defined as transverse or lateral or linear magnification (m). Hence it is the ratio of the height of image (h^1) to the height of the object (h). From the figure.



* Lateral magnification $m = \frac{A^1B^1}{AB} = \frac{h^1}{h}$

* here h and h^1 will be taken positive or negative in accordance with the accepted sign convention.

* In triangles A^1B^1P and ABP, we have $\frac{B^1A^1}{BA} = \frac{B^1P}{BP}$, with sign convention this becomes

* $\frac{-h^1}{h} = \left(\frac{-v}{-u}\right)$, so that, $m = \frac{h^1}{h} = -\frac{v}{u}$

* Here negative magnification implies that image is inverted with respect to object, while positive magnification means that image is erect with respect to object. i.e., m is negetive means for real object, real image formed and for virtual object virtual image is formed. m positive means for real object virtual image formed and for virtual object real image is formed.

* Ex: If $m=-2$, means, if the object is real, image is real, inverted, magnified and mirror used is concave.

* **Longitudianl magnification:** However, if the one dimensional object is placed with its length along the principal axis. The ratio of length of image to length of object is called longitudinal magnification (m_L). Longitudinal magnification can be expressed as

* $$m_L=\frac{(v_2-v_1)}{(u_2-u_1)}$$

* Where v_1 and v_2 are image positions corresponding to u_1 and u_2 positions.

* For small objects $m_L=-\frac{dv}{du}$

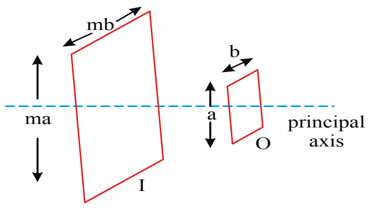
* We have $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

* In case of small linear objects $-\frac{dv}{v^2}-\frac{du}{u^2}=0$

* $$\therefore m_L=-\frac{dv}{du}=\left[\frac{v}{u}\right]^2=m^2$$

* **Areal magnification:** If a two dimensional object is placed with its plane perpendicular to principal axis, its magnification is called a real or superficial magnification. If m is the lateral magnification and m_A is the areal magnification.

* $$m_A=\frac{\text{area of image}}{\text{area of object}}=\frac{(ma)(mb)}{ab}=m^2$$

* 

* **Overall magnification:** In case of more than one optical component, the image formed by first component will act as an object for the second and image of second acts as an object for third and so on, the product of all individual magnifications is called over all magnifications.

* $$m_0=\frac{I}{O}=\frac{I_1}{O_1}\times\frac{I_2}{O_2}\times--\frac{I_n}{O_n}$$

* $$=m_1\times m_2\times--\times m_n$$

* **Newton's Formula :** In case of spherical mirror if the object distance (x_1) and image distance (x_2) are measured from focus instead of the pole of the mirror. Then mirror formula reduces to a simple form called the Newton's formula.

* $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ reduces to

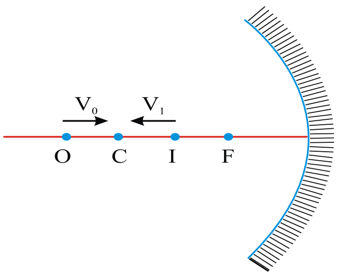
* $$\frac{1}{f+x_2}+\frac{1}{f+x_1}=\frac{1}{f}$$

* Which on simplification gives $x_1x_2=f^2$

* (Newton's Formula) ($f=\sqrt{x_1x_2}$)

* **Motion of Object in front of Mirror Along the Principal Axis**

* Ø When the position of the object changes with time on the principal axis relative to the mirror, the image position also changes with time relative to the mirror. Hence to know the relation between object and image speed we use the mirror equation.

* 

* $$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

* Differntiatae with respect to time, we get

* $$-\frac{1}{v^2}.\frac{dv}{dt}-\frac{1}{u^2}.\frac{du}{dt}=0 \text{ (or)}$$

* $$\frac{dv}{dt}=-\left(\frac{v}{u}\right)^2.\frac{du}{dt} \text{ (or) } V_1=-\left(\frac{v}{u}\right)^2.V_0$$

* Where v_1 velocity of image with respect to mirror and v_0 is the velocity of object with respect to mirror along the principal axis. Here negative sign indicates the object and image are always moving opposite to each other. In concave mirror depending on the position of the object image speed may be greater or lesser or equal to the object speed.

* a) $R<u<\infty$ $|m|<1$ $V_1<V_0$

* b) u = R $|m|=1$ $V_1=V_0$

* c) $f<u<R$ $|m|>1$ $V_1>V_0$

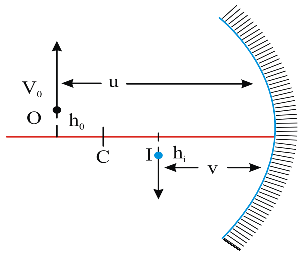
* d) $u<f$ $|m|>1$ $V_1>V_0$

* e) $u\approx 0$ $|m|\approx 1$ $V_1\approx V_0$

* Relation between object and image velocity given above is also valid for convex mirror. In convex mirror speed of image slower than the object whatever the position of the object may be. Above relation is not true in terms of acceleration of object and image.

* **Motion of the object Transverse to the Principal Axis**

* If the object moves transverse to principal axis then the image also moves transverse to principal axis.

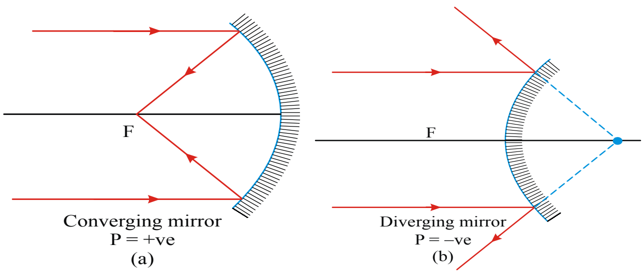
* 

* Consider the diagram. In a mirror

* $\frac{h_i}{h_0} = \frac{v}{u} = \text{cons tan t} \quad (-m)$

* $\therefore \frac{\frac{dh_i}{dt}}{\frac{dh_0}{dt}} = -m \, (\text{or}) \, V_1 = -mV_0$

* **Power of Curved Mirror :** Every optical instrument have power, it is the ability of optical instrument to deviate the path rays incident on it. If the instrument converges the rays parallel to principal axis its power is said to be positive and if it diverges its power is said to be negetive.

* 

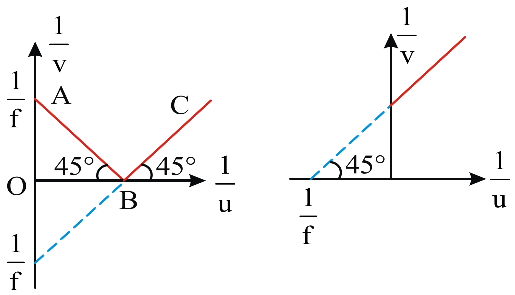
* For a mirror Power ‘P’

* $P = -\frac{1}{f(\text{metre})} (\text{or}) P = -\frac{100}{f(\text{cm})}$

* S.I unit of power is dioptre $(D) = m^{-1}$

* For concave mirror (converging mirror) P is positive and for convex mirror (diverging mirror) power is negative.

* Ø $\frac{1}{V} - \frac{1}{U}$ **Graph to Mirrors: The graph between** $\frac{1}{v}$ **and** $\frac{1}{u}$ **to a concave mirror is shown in figure (a)**

* 

Since $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

For all real image $-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\therefore \frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$

This is a straight line equation with slope -1.
This is represented by the line AB.

For virtual image, $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\therefore \frac{1}{v}=\frac{1}{u}-\frac{1}{f}$

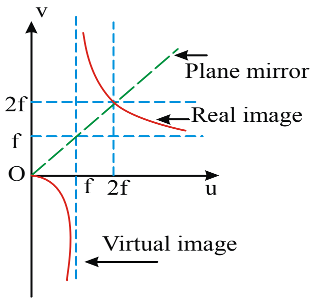
This is a straight line equation with slope +1.
This represents line BC.

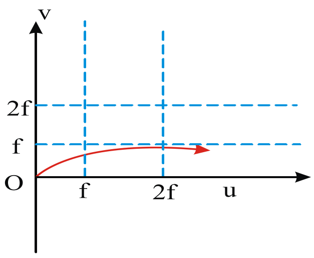
* The graph between 1/v and 1/u to a convex mirror as shown in figure (b).
Since convex mirror always form virtual image to a real object.

* $\frac{1}{v}+\frac{1}{-u}=\frac{1}{f}\therefore \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$

* This is a straight line equation with slope +1.

* **U-V Graph in Curved Mirror :**In case of concave mirror, the graph between u and v is hyperbola as shown in figure.





* For real image $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ (or) $\frac{1}{v}=\frac{u-f}{uf}$

$v=\frac{f}{1-\frac{f}{u}}$

* For virtual image $\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

$\frac{1}{v}=\frac{f-u}{fu}(\text{or})\, v=\frac{f}{\frac{f}{u}-1}$

* In case of convex mirror, the graph between u and v is hyperbola as shown in figure (b)
Since convex mirror form only virtual image.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}(or)\,v=\frac{f}{1+\frac{f}{u}}$$

* **Graph in Spherical Mirror :** In a spherical mirror: $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\therefore 1+\frac{v}{u}=\frac{v}{f}(or)\frac{v}{u}=\frac{v}{f}-1$$

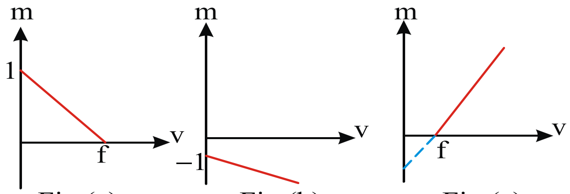
* 

Fig.(a) Fig.(b) Fig.(c)

* **Concave mirror:** If the objects is real,

* For real image, u=-ve, v=-ve, f=-ve,

* $\therefore -m=\frac{v}{f}-1(or)\,m=-\frac{v}{f}+1$

* Graph as shown in figure (a)

* For virtual image, u=-ve, v=+ve, f=-ve

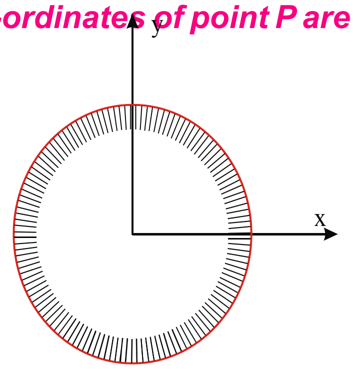
* $\therefore m=-\frac{v}{f}-1,$ Graph as shown in figure (b)

* **Convex mirror:** Since convex mirror always form virtual image to a real object, u=-ve, v=+ve, f=+ve,

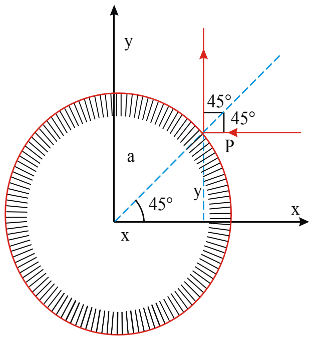
* $\therefore m=\frac{v}{f}-1,$ graph as shown in figure (c).

* From the above graph it is observed that for $v\approx 0, m=1$. i.e., when an object is very near to pole of the mirror $(u\approx 0)$, then the curved mirror behaves like a plane mirror.

W.E-5: ***A reflecting surface is represented by the equation $x^2+y^2=a^2$. A ray travelling in negative x-direction is directed towards positive y-direction after reflection from the surface at point P. Then co-ordinates of point P are***



Sol: The ray diagram is as shown.



* $x = \frac{a}{\sqrt{2}}$ and $y = \frac{a}{\sqrt{2}}$

$$\therefore P = \left(\frac{a}{\sqrt{2}}, \frac{a}{\sqrt{2}}\right)$$

W.E-6: ***A point light source lies on the principal axis of concave spherical mirror with radius of curvature 160 cm. Its image appears to be back of the mirror at a distance of 70 cm from mirror. Determine the location of the light source.***

Sol: $\frac{1}{u} + \frac{1}{v} = \frac{2}{R}$, Here $v = 70\,cm$,

$R = -160cm \frac{1}{u} = \frac{2}{R} - \frac{1}{v}$

$$\therefore \frac{1}{u} = \frac{2}{-160\,cm} - \frac{1}{70\,cm} = -\frac{15}{560\,cm}$$

$$\therefore u = -\frac{560}{15}cm = -37\,cm$$

The image is at a distance of 37 cm in front of the mirror.

W.E-7: ***A point source of light is located 20 cm in front of a convex mirror with f=15 cm. Determine the position and nature of the image point.***

Sol: $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Here $u = -20\,cm, f = 15\,cm$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{15cm} - \frac{1}{20cm} = \frac{35}{300cm}$$

$$\frac{1}{v} = \frac{7}{60\,cm}$$

$v = 8.6\,cm$

Also v is positive, the image is located behind the mirror.

W.E-8: ***An object is 30.0 cm from a spherical mirror, along the central axis. The absolute value of lateral magnification is 1/2. The image produced is inverted. What is the focal length of the mirror?***

Sol: Image inverted, so it is real u and v both are negative. Magnification is 1/2, therefore,

$v = \frac{u}{2}$, given, $u = -30\,cm$, $v = -15cm$

Using the mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

We have, $\frac{1}{f} = \frac{1}{-15} - \frac{1}{30} = \frac{-1}{10}$

$\therefore f = -10\,cm$

Since focal length is negative the given mirror is concave.

W.E-9: ***An object of length 10 cm is placed at right angles to the principal axis of a mirror of radius of curvature 60 cm such that its image is virtual, erect and has a length 6cm. What kind of mirror is it and also determine the position of the object?***

Sol: Since the image is virtual, erect and of a smaller size, the given mirror is 'convex' (concave mirror does not form an image with the said description).

* Given $R = +60\,cm$ $\quad f = \frac{R}{2} = 30\,cm$

Transverse magnification,

$m = \frac{I}{O} = \frac{6}{10} = +\frac{3}{5}$ Further $m = -\frac{v}{u} = \frac{3}{5}$

$\therefore v = -\frac{3u}{5}$

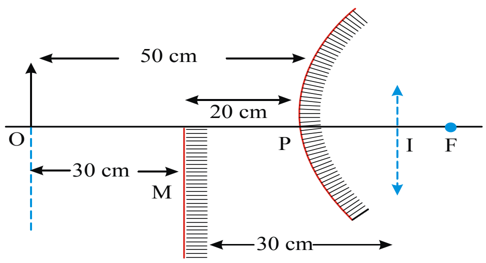
Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ $\quad \frac{-5}{3v} + \frac{1}{u} = \frac{1}{30}$

$\frac{-5+3}{3u} = \frac{1}{30}$ $\quad \therefore u = -20\,cm$

Thus the object is at a distance 20 cm (from the pole) in front of the mirror.

W.E-10: ***An object is placed infront of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, if it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror?***

Sol:



* As shown in figure the plane mirror will form erect and virtual image of same size at a distance of 30 cm behind it. So the distance of image formed by the plane mirror from convex mirror will be PI = MI - MP But as MI = MO, PI= MO-MP = 30-20 = 10 cm.

* Now as this image coincides with the image formed by convex mirror, therefore for convex mirror,

$u = -50\,cm; v = +10\,cm$

So $\frac{1}{+10} + \frac{1}{-50} = \frac{1}{f}$, i.e., $f = \frac{50}{4} = 12.5\,cm$

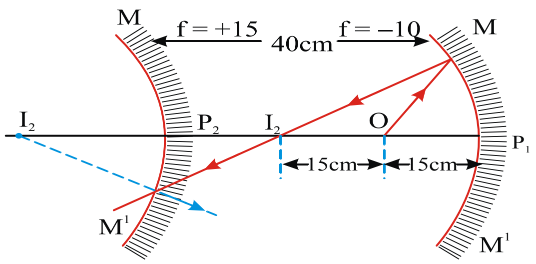
So $R = 2f = 2 \times 12.5 = 25\,cm$

W.E-11: ***A concave mirror of focal length 10 cm and a convex mirror of focal length 15 cm are placed facing each other 40 cm apart. A point object is placed between the mirrors, on their common axis and 15 cm from the concave mirror. Find the position, nature of the image, and over all magnification produced by the successive reflections, first at concave mirror and then at convex mirror.***

Sol: According to given problem, for concave mirror, u = -15cm and f = -10 cm.

So $\frac{1}{v} + \frac{1}{-15} = \frac{1}{-10}$, i.e., $v = -30\,cm$

* i.e., concave mirror will form real, inverted and enlarged image I_1 of object O at a distance 30 cm from it, i.e., at a distance 40-30=10 cm from convex mirror.



For convex mirror the image I_1 will act as an object and so for it u=-10 cm and f=+15 cm.

* $\frac{1}{v} + \frac{1}{-10} = \frac{1}{15}$, i.e, $v = +6\,cm$

So final image I_2 is formed at a distance 6 cm behind the convex mirror and is virtual as shown in figure.

Over all magnification

$= m_1 \times m_2 = -2 \times 6/10 = -6/5$

negative indicates final image is virtual w.r.t. given object.

* **Refraction of Light :**When a beam of light is travelling from one medium to another medium, a part of light gets reflected back into first medium at the inferface of two media and the remaining part travels through second medium in another direction. The change in the direction of light take place at the interface of two media.
* Deviation or bending of light rays from their original path while passing from one medium to another is called refraction.

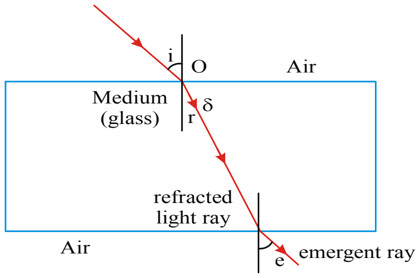
(or)

The phenomenon due to which light deviates from its initial path, while travelling from one optical medium to another optical medium is called refraction.

* Refraction of light is due to change in speed of light passes from one medium to another medium.
* In case of refraction of light frequencey (colour) and phase do not change. But wavelength and velocity will change.

Note: When light passes from one medium to another medium, the colour of light is determined by its frequency not by its wavelength.

* Ø Refraction of light at plane surface:



* **Incident ray:** A ray of light, traveling towards another optical medium, is called incident ray.
* **Point of incidence:** The point (O), where an incident ray strikes on another optical medium, is called point of incidence.
* **Normal:** A perpendicular drawn at the surface of seperation of two media on the point of incidence, is called normal.
* **Angle of incidence (i):** The angle which the incident ray makes with normal, is called angle of incidence.
* **Refracted ray:** A ray of light which deviates from its path on entering in another optical medium is called refracted ray.
* **Angle of refraction(r):** The angle which the refracted ray makes with normal, is called the angle of refraction.
* **Angle of deviation due to refraction(δ):** It is the angle between the direction of incident light ray and refracted light ray.
* **Emergent ray:** A ray of light which emerges out from another optical medium as shown in the above figure is called emergent ray.
* **Angle of emergence (e):** The angle which the emergent ray makes with the normal is alled the angle of emergence.
* **Laws of Refraction:**
* Incident ray, refracted ray and normal always lie in the same plane.
* The product of refractive index and sine of angle of incidence at a point in a medium is constant,

$\mu \times \sin$ i = constant

$\mu_1 \sin i_1 = \mu_2 \sin i_2$

If $i_1 = i$ and $i_2 = r$ then

$\mu_1 \sin i = \mu_2 \sin r;$

This law is called snell's law.

According to Snell's law,

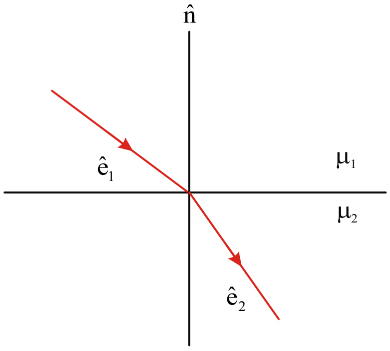
$\frac{\sin i}{\sin r} =$ constant $\left(= \frac{\mu_2}{\mu_1}\right)$ for any pair of medium and for light of given wavelength.

Note: The ratio between sine of angle of incidence to sine of angle of refraction is commonly called as refractive index of the material in which angle of refraction is situated with respect to the medium in which angle of incidence is situated.

When light ray travells from medium 1 to medium 2 then $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = {}_1\mu_2 =$ refractive index of medium (2) with respect to medium (1)

*	Vector form of Snell's law:

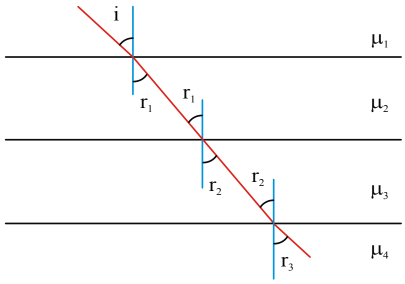
$\mu_1 (\hat{e}_1 \times \hat{n}) = \mu_2 (\hat{e}_2 \times \hat{n})$



There $\hat{e}_1$ = unit vector along incident ray	$\hat{e}_2$ = unit vector along refracted ray

$\hat{n}$ = unit vector along normal incedence point

Note: Let us consider a ray of light travelling in situation as shown in fig.
Applying Snell's law at each interface, we get



$\mu_1 \sin i = \mu_2 \sin r_1$; $\mu_2 \sin r_1 = \mu_3 \sin r_2$

$\mu_3 \sin r_2 = \mu_4 \sin r_3$; It is clear that

$\mu_1 \sin i = \mu_2 \sin r_1 = \mu_3 \sin r_2 = \mu_4 \sin r_3$

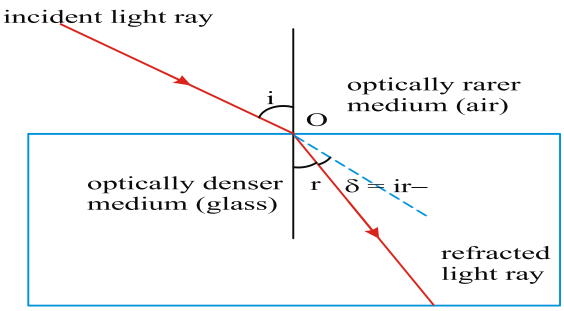
(or) $\mu \sin i =$ constant

Note: When light ray travells from medium of refractive index μ_1 to another medium of refractive index μ_2 then, $\mu_1 \sin i_1 = \mu_2 \sin i_2$

$\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = \frac{\sin i_1}{\lambda_1} = \frac{\sin i_2}{\lambda_2}$

*	**When a light travels from optically rarer medium to optically denser medium obliquely:**

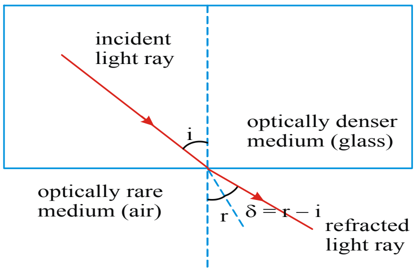
*



a) it bends towards normal.

b) angle of incidence is greater than angle of	refraction.

* **When a ray of light travels from opticallly denser medium to optically rarer medium obliquely**

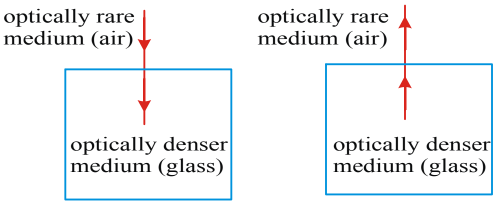


a) it bends away from the normal at the point of incidence.

b) angle of refraction is greater than angle of incidence.

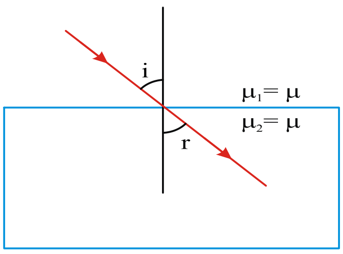
c) angle of deviation $\delta = r - i$.

Condition for no refraction : When an incident ray strikes normally at the point of incidence, it does not deviates from its path.i.e., it suffers no deviation.



In this case angle of incedence (i) and angle of refraction(r)are equal and $\angle i = \angle r = 0$.

* If the refractive indices of two media are equal



$\mu_1 = \mu_2 = \mu$

From snell's law,

$\mu \sin i = \mu \sin r, \sin i = \sin r$

$\angle i = \angle r$

Hence, the ray passes without any deviation at the boundary.

Note: Because of the above reason a transperant solid is invisible in a liquid if their refractive indices are same.

* **Refractive Index :**

Absolute refractive Index (μ):

The absolute refractive index of a medium is the ratio of speed of light in free space (C) to speed of light in a given medium (V).

$$\mu = \frac{\text{veloctiy of light in free space (C)}}{\text{velocity of light in a given medium (V)}}$$

It is a scalar.

It has no units and dimensions.

* From electronmagnetic theory if ε_0 and μ_0 are the permitivity and permeability of free space, ε and μ are the permitivity and refractive index of the given medium

$$\mu = \frac{C}{V} = \frac{\frac{1}{\sqrt{\varepsilon_0\mu_0}}}{\frac{1}{\sqrt{\varepsilon\mu}}} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

where ε_r & μ_r are the relative permittivity and permeability of the given medium.

* For vaccum of free space, speed of light of all wavelengths is same and is equal to C. So,For all wavelengths the refractive index of

free space is $\mu = \frac{C}{C} = 1$.

* For a given medium the speed of light is different for different wavelengths of light, greater will be the the speed and hence lesser will be refractive index.

$\lambda_R > \lambda_V$, So in medium $\mu_V > \mu_R$

Note: Actually refractive index μ varies with λ according to the equation $\mu = A + \frac{B}{\lambda^2}$.

(where A & B are constants)

* For a given light, denser the medium lesser will be the speed of light and so greater will be the refractive index.

Example : Glass is denser medium when compared to water, so $\mu_{glass} > \mu_{water}$.

The refractive index of water $\mu_w = 4/3$

The refractive index of glass $\mu_g = 3/2$

* Foa a given light and given medium, the refractive index is also equal to the ratio of wavelength of light in free space to that in the medium.

$$\mu = \frac{C}{V} = \left(\frac{f\lambda_{vaccum}}{f\lambda_{medium}}\right) = \frac{\lambda_{vaccum}}{\lambda_{medium}}$$

(when light travells from vaccume to a medium, frequency does not change)

Note: *If C* is velocity of light in free space λ_0 is wavelength of given light in free space then velocity of light in a medium of refractive index (μ) is $V_{medium} = \frac{C}{\mu}$.

wavelength of given light in a medium of refractive index (μ) is $\lambda_{medium} = \frac{\lambda_0}{\mu}$

* **Relative Refractive Index:** When light passes. from one medium to the other, the refractive index of medium 2 relative to medium 1 is written as ${}_1\lambda_2$ and is given by

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} \qquad ...(1)$$

refractive index of medium 1 relative to medium 2 is ${}_2\mu_1$ and ${}_2\mu_1 = \dfrac{\mu_1}{\mu_2} = \dfrac{v_2}{v_1} = \dfrac{\lambda_2}{\lambda_1}$ ...(2)

From eq. (1) & (2)

$${}_1\mu_2 = \frac{1}{{}_2\lambda_1} \text{ i.e.,} \left({}_1\mu_2\right).\left({}_2\mu_1\right) = 1$$

W.E-12: ***The refractive index of glass with respect to water is 9/8. If the velocity and wavelength of light in glass are*** 2×10^8 ***m/s and 4000*** A^0 ***respectively, find the velocity and wavelength of light in water.***

Sol: ${}_w\mu_g = \dfrac{\mu_g}{\mu_w} = \dfrac{v_w}{v_g} \Rightarrow \dfrac{9}{8} = \dfrac{v_w}{2\times10^8};$

$$v_w = \frac{9\times2\times10^8}{8} = 2.25\times10^8\,m/s.$$

$${}_w\mu_g = \frac{\mu_g}{\mu_w} = \frac{\lambda_w}{\lambda_g}\left(\because \mu_g = \frac{\lambda}{\lambda_g}, \mu_w = \frac{\lambda}{\lambda_w}\right)$$

$$\frac{9}{8} = \frac{\lambda_w}{4000}; \lambda_w = \frac{9\times4000}{8} = 4500A^0.$$

W.E-13: ***The wavelength of light in vacuum is*** λ_0 ***. When it travels normally through glass of thickness 't'. Then find the number of waves of light in 't' thickness of glass (Refractive index of glass is*** μ ***)***

Sol: Number of waves in a thickness 't' of a medium of refractive index μ is

number of waves $= \dfrac{thickness}{wavelength} = \dfrac{t}{\lambda_m}$

But $\lambda_m = \dfrac{\lambda_0}{\mu}$

$\therefore$ number of waves = $\dfrac{t\mu}{\lambda_0}$

Where λ_0 is the wavelength of light in vacuum.

W.E-14: ***When light of wavelength*** λ_0 ***in vacuum travels through same thickness 't' in glass and water, the difference in the number of waves is ______. (Refractive indices of glass and water are*** μ_g ***and*** μ_w ***respectively.)***

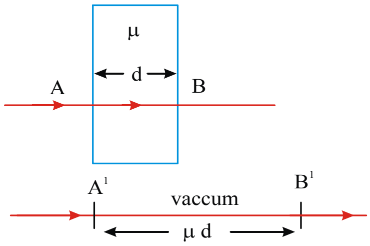
Sol: We know number of waves of a given light in a medium of refractive index μ is $\dfrac{t\mu}{\lambda_0}$

$\therefore$ Difference in number of waves $= \dfrac{t}{\lambda_0}\left(\mu_g - \mu_w\right)$

where μ_g and μ_w are the refractive indicies of glass and water respectively.

Optical Path (Δ x): Consider two points A and B in a medium as shown in figure. The shortest distance between any two points A and B is called geometrical path. The length of geometricial path is independent of the medium that surrounds the path AB.When a light ray travels from the point A to point B it travels with the velocity c if the medium is vacuum and with a lesser velocity v if the medium is other than vaccum. Therefore the light ray takes more time to go from A to B located in a medium.

* The optical path to a given geometrical path in a given medium is defined as distance travelled by light in vacuum in the same time in which it travels a given path length in that medium.



AB = real path or geometrical path

A^1B^1 = optical path

If the light travels a path lenght 'd' in a medium

at speed v, the time taken by it will be $\left(\frac{d}{v}\right)$

So optical path length,

$$\Delta x = c \times t = c \times \left[\frac{d}{v}\right] = \mu d \left(as\ \mu = \frac{c}{v}\right)$$

* Therefore optical path is μ times the geometrical path. As for all media $\mu > 1$, optical path length is always greater than actual path length.

Note: If in a given time t, light has same optical path length in different media, and if light travels a distance d_1 in a medium of refractive index μ_2 in same time t, then $\mu_1 d_2 = \mu_2 d_2$.

Note: The difference in distance travelled by light in vaccum and in a medium in the same interval of time is called optical path difference due to that medium.

$$\Delta x = A'B' - AB = \mu d - d \qquad \Delta x = (\mu - 1)d$$

Note: A slab of thickness d and refractive index μ is kept in a medium of refractive index $\mu'(< \mu)$. If the two rays parallel to each other passes through such a system with one ray passing through the slab, then path difference

Between two rays due to slab will be $\Delta x = \left(\frac{\mu}{\mu'} - 1\right)d$.

Note: The optical phase change $\phi = \frac{2\pi}{\lambda}$ (optical path difference)

W.E-15: ***The optical path of a monochromatic light is the same if it goes through 4.00 m of glass are 4.50 m of a liquid. If the refractive index of glass is 1.5, what is the refractive index of the liquid?***

Sol: When light travells a distance 'x' in a medium of refractive index μ, the optical path is μx

Given $\mu_1 x_1 = \mu_2 x_2 \Rightarrow 1.5 \times 4.00 = \mu_2 \times 4.50$

$$\mu_2 = \frac{1.5 \times 4.00}{4.50} = 1.333$$

W.E-16: ***Find the thickness of a transparent plastic plate which will produce a change in optical path equal to the wavelength λ of the light passing through it normally. The refractive index of the plastic plate is μ.***

Sol: When light travel a distance x in a medium of refractive index μ, its optical path = μx

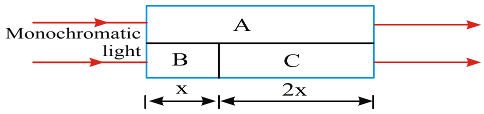
Change in optical path = $\mu x - x = (\mu - 1)x$.

This is to be equal to λ

But $(\mu - 1)x = \lambda$

The thickness of the plate $x = \dfrac{\lambda}{\mu - 1}$

W.E-17: ***Consider slabs of three media A, B and C arranged as shown in figure R.I. of A is 1.5 and that of C is 1.4. If the number of waves in A is equal to the number of waves in the combination of B and C then refractive index of B is:***



Sol: $$N_A = N_B + N_C$$

$$\frac{x_A}{\lambda_A} = \frac{x_B}{\lambda_B} + \frac{x_C}{\lambda_C}$$

$$\frac{x_A \mu_A}{\lambda_0} = \frac{x_B \mu_B}{\lambda_0} + \frac{x_C \mu_C}{\lambda_0}$$

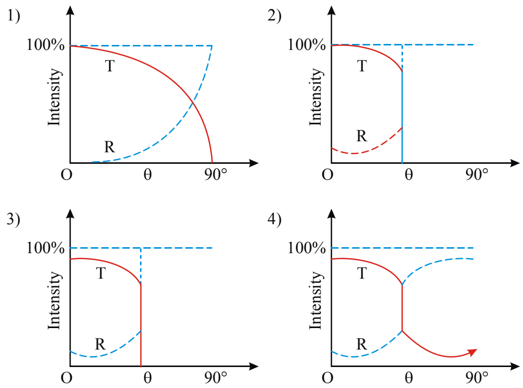
$$3x \times 1.5 = x \times \mu_B + 2x \times 1.4$$

$$\therefore \mu_B = 1.7$$

W.E-18: ***Two parallel rays are travelling in a medium of refractive index $\mu_1 = \frac{4}{3}$. One of the rays passes through a parallel glass slab of thickness t and refractive index $\mu_2 = \frac{3}{2}$. The path difference between the two rays due to the glass slab will be***

Sol: $\Delta x = \left(\dfrac{\mu_2}{\mu_1} - 1\right)t = \left(\dfrac{3/2}{4/3} - 1\right)t = \dfrac{t}{8}$

W.E-19: ***A light ray travelling in a glass medium is incident on glass - air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is***



Sol: (3) After total internal reflection, there is no refracted ray.

Principle of Reversibility of Light

Ø According to principle of reversibility, if a ray of light travels from X to Z along a certain path, it will follow exactly the same path, while travelling from Z to X. In other words the path of light is reversible.

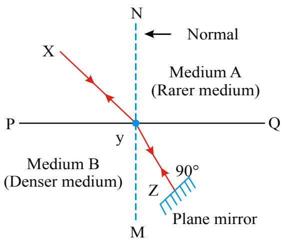


Figure shows a ray of light XY travelling through medium 'A', such that it travels along YZ, while travelling medium 'B'. NM is the normal at point Y, such $\angle XYN$ is the angle of incidence and $\angle MYZ$ is the angle of refraction.

$$\therefore {}_a\mu_b = \frac{\sin\angle XYN}{\sin\angle MYZ} \quad ....(1)$$

If a plane mirror is placed at right angles to the path of refracted ray 'YZ', it found that light retraces back its path. Now ray ZY acts as incident ray and YX as refracted ray, such that $\angle MYZ$ is angle of incidence and $\angle XYN$ is the angle of refraction.

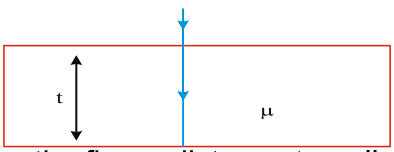
$$\therefore {}_b\mu_a = \frac{\sin\angle MYZ}{\sin\angle XYN} \therefore \frac{1}{{}_b\mu_a} = \frac{1}{\frac{\sin\angle MYZ}{\sin\angle XYN}} = \frac{\sin\angle XYN}{\sin\angle MYZ}$$

Comparing (1) and (2) ${}_a\mu_b = \frac{1}{{}_b\mu_a}$

Thus, the refractive index of medium 'b' with respect to 'a' is equal to the reciprocal of refractive index of medium 'a' with respect to medium 'b'.

W.E-20: ***A light ray is incident normally on a glass slab of thickness 't' and refractive index 'μ' as shown in the figure. Then find time taken by the light ray to travell through the slab.***

Sol:



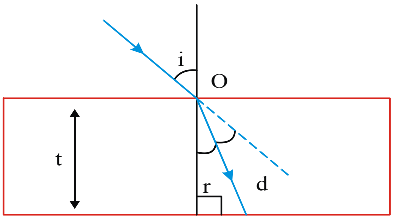
From the figure distance travelled by the light ray through the slab is 't'

Velocity of light in glass $= \frac{\text{distance travelled}}{\text{time}}$

$\frac{c}{\mu} = \frac{t}{time}, time = \frac{\mu t}{c}$

W.E-21: ***A light ray is incident on a plane glass slab of thickness 't' at an angle of incidence 'i' as shown in the figure. If 'μ'is the refractive index of glass. Then find time taken by the light ray to travel through the slab.***

Sol:



As shown in the above figure distance travelled by the light ray through the slab is 'd'. From the figure

$\cos r = \frac{t}{d}, d = \frac{t}{\cos r}$

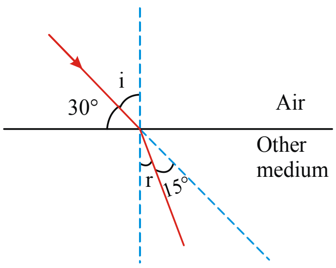
Velocity of light in glass $= \frac{\text{Distance travelled through the glass}}{\text{time}}$

$\frac{c}{\mu} = \frac{d}{time}; \; time = \frac{d\mu}{c}$

$time = \frac{t\mu}{\cos r \times c} = \frac{\mu^2 t}{c\sqrt{\mu^2 - \sin^2 i}}$

W.E-22: ***Light of wavelength 4500 $\overset{0}{A}$ in air is incident on a plane boundary between air and another medium at an angle 30^0 with the plane boundary. As it enters from air into the other medium, it deviates by 15^0 towards the normal. Find refractive index of the medium and also the wavelength of given light in the medium.***

Sol:



Angle of incidence i $= 90^0 - 30^0 = 60^0$. As the ray bends towards the normal, it deviates by an angle $i - r = 15^0$ (given)

$\therefore \; r = 45^0$ Applying Snell's law

$\mu_{air} \sin i = \mu_{med} \sin r; \qquad \therefore \; 1 \times \sin 60^0 = \mu \times \sin 45^0$

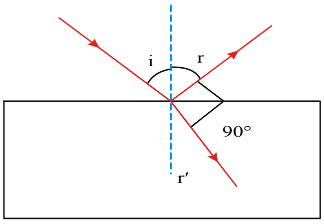
In terms of wavelengths,

$\mu = \sqrt{1.5} = \frac{\lambda_{air}}{\lambda_{med}} (or) \lambda_{med} = \frac{\lambda_{air}}{\sqrt{1.5}} = \frac{4500}{\sqrt{1.5}}$

$\lambda_{med} = 3674\,\overset{0}{A}$

W.E-23: ***Monochromatic light falls at an angle of incidence 'i' on a slab of a transparent material. Refractive index of this material being 'μ'for the given light. What should be the relation between i and μ so that the reflected and the refracted rays are mutually perpendicular?***

Sol: In the given figure let r is the angle of reflection and r' is the angle of refraction. According to the given condition, considering the reflected and the refracted rays to be perpendicular to each other,



$\therefore$ From the figure $r + 90^0 + r' = 180^0$

So, $r' = 90^0 - r$

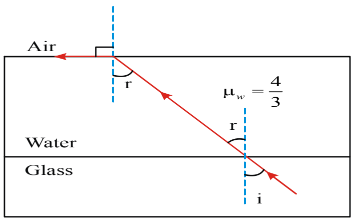
$r' = 90^0 - i$ [i = r, by law of reflection]

According to Snell's law, 1 $\sin i = \mu \sin r'$

$\sin i = \mu \sin\left(90^0 - i\right)$

$\sin i = \mu \cos i, \mu = \tan i \Rightarrow i = \tan^{-1}(\mu)$

W.E-24: ***A ray of light is incident at the glass-water interface at an angle i as shown in figure, it emerges finally parallel to the surface of water, then the value of μ_g would be***



Sol: Applying Snell's law (μ $\sin i =$ constant)

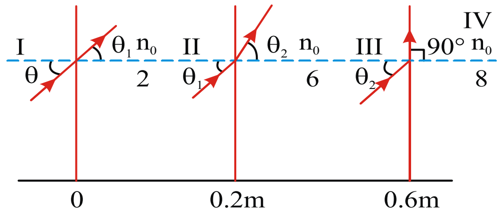
at first and second interfaces, we have

$\mu_1 \sin i_1 = \mu_2 \sin i_2$; But, $\mu_1 = \mu_{glass}, i_1 = i$

$\mu_2 = \mu_{air} = 1$ and $i_2 = 90^0$

$\therefore$ $\mu_g \sin i = (1)\left(\sin 90^0\right)$ or $\mu_g = \dfrac{1}{\sin i}$

W0.E-25: ***A light beam is travelling from region I to region IV (Refer figure). The refractive index in regions I, II,III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$, respectively. The angle of incidence θ for which the beam just misses entering region IV is***



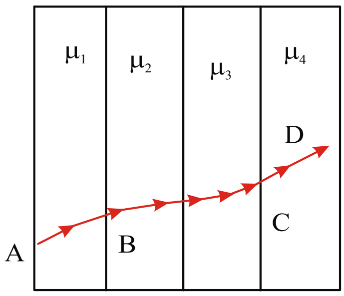
Sol: As the beam just misses entering the region IV, the angle of refraction in the region IV must be 90^0.

Application of Snell's law successively at different interfaces gives

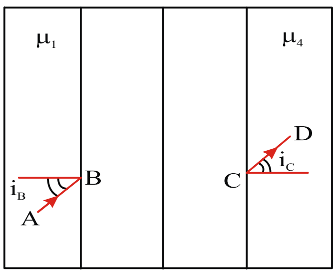
$$n_0 \sin\theta = \frac{n_0}{2}\sin\theta_1 = \frac{n_0}{6}\sin\theta_2 = \frac{n_0}{8}\sin 90^0$$

$\Rightarrow \sin\theta = \frac{1}{8}$ or $\theta = \sin^{-1}\frac{1}{8}$

W.E-26: ***A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have***



Sol: Applying Snell's law at B and C,

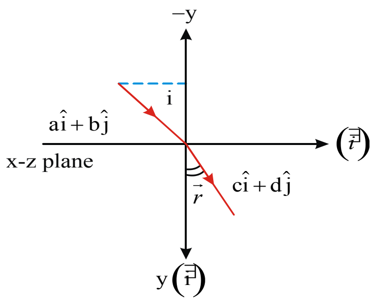


$\mu \sin i =$ constant or $\mu_1 \sin i_B = \mu_4 \sin i_C$

But $AB \parallel CD$; $\therefore i_B = i_C$ or $\mu_1 = \mu_4$

W.E-27: ***The x-z plane separates two media A and B of refractive indices $\mu_1 = 1.5$ and $\mu_2 = 2$. A ray of light travels from A to B. Its directions in the two media are given by unit vectors $\vec{u}_1 = a\hat{i} + b\hat{j}$ and $\vec{u}_2 = c\hat{i} + d\hat{j}$. Then***

Sol:



$\tan i = \frac{a}{b}$ so $\sin i = \frac{a}{\sqrt{a^2+b^2}}$

and $\tan r = \frac{c}{d}, \sin r = \frac{c}{\sqrt{c^2+d^2}}$

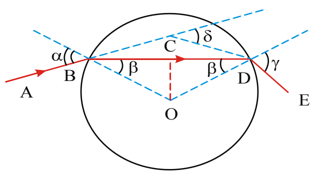
$\mu_1 \sin i = \mu_2 \sin r$; $\left(\frac{3}{2}\right)\left(\frac{a}{\sqrt{a^2+b^2}}\right) = 2\left(\frac{c}{\sqrt{c^2+d^2}}\right)$

But as $a\hat{i}+b\hat{j}$ and $c\hat{i}+d\hat{j}$ are unit vectors so

$\sqrt{a^2+b^2} = \sqrt{c^2+d^2} = 1$;Hence $\frac{3}{2}a = 2c$, so $\frac{a}{c} = \frac{4}{3}$

W.E-28: ***A ray of light is incident on the surface of a spherical glass paper-weight making an angle α with the normal and is refracted in the medium at an angle β. Calculate the deviation.***

Sol: Deviation means the angle through which the incident ray is turned in emerging from the medium. In Figure if AB and DE are the incident and emergent rays respectively, the deviation will be δ.



Now as at B ; $\angle i = \alpha$ and $\angle r = \beta$

So from Snell's law, $1 \sin\alpha = \mu \sin\beta$ ....(1)

Now from geometry of figure at D, $\angle i = \beta$

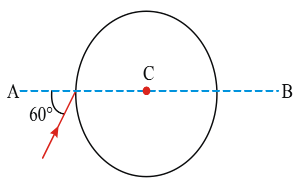
So $\mu \sin\beta = 1 \sin\gamma$ .....(2)

Comparing Eqs. (1) and (2) $\gamma = \alpha$

Now as in a triangle exterior angle is the sum of remain-ing two interior angles, in ΔBCD,

$\delta = (\alpha - \beta) + (\alpha - \beta) = 2(\alpha - \beta)$

W.E-29: ***A ray of light falls on a transparent sphere with centre at C as shown in figure. The ray emerges from the sphere parallel to line AB. The refractive index of the sphere is***



Sol: Deviation by a sphere is 2 (i - r)

Here, deviation $\delta = 60^0 = 2$ (i-r) or i-r = $30^{0;}$

$\therefore$ r = i$-30^0 = 60^0 - 30^0 = 30^0$

$$\therefore \mu = \frac{\sin i}{\sin r} = \frac{\sin 60^0}{\sin 30^0} = \sqrt{3}$$

Apparent Depth

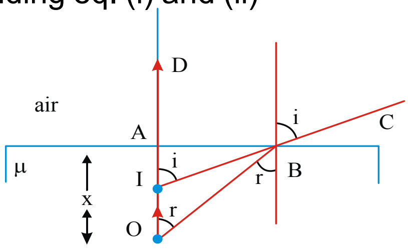
* **Case(1) :** Object in denser medium and observer in rarer medium.

When object 'O' is placed at a distance 'x' from A in denser medium of refractive index μ as shown in figure. Ray OA, which falls normally on the plane surface, passes undeviated as AD. Ray OB, which 'r'(with normal) on the palne surface, bends away from the normal and passes as BC in air. Rays AD and BC meet at 'I' after extending these two rays backwards. This 'I' is the virtual image of real object 'O' to an observer in rarer medium near to transmitted ray.

$$\sin i \approx \tan i = \frac{AB}{AI} \quad ......(i)$$

$$\sin r \approx \tan r = \frac{AB}{AO} \quad ....(ii)$$

Dividing eq. (i) and (ii)



$$\frac{\sin i}{\sin} = \frac{AO}{AI};$$ According to Snell' law $\mu = \frac{\sin i}{\sin r}$

$$\therefore \mu = \frac{AO}{AI} \therefore AI = \frac{AO}{\mu} = \frac{x}{\mu}$$

The distance of image AI is called apparent depth or apparent distance. The apparent depth x_{app} is given by i.e., $x_{app} = \frac{x_{real}}{\mu}$

The apparent shift $(OI) = AO - AI = x - \frac{x}{\mu}$

Hence the apparent shift $(OI) = \left(1 - \frac{1}{\mu}\right)x$

If the observer is in other than air medium of refractive index $\mu(< \mu)$.

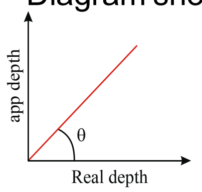
Then apparent depth

$$= \frac{\text{real depth}}{\mu_{\text{relative}}} = \frac{\text{real depth}}{\left(\frac{\mu}{\mu^1}\right)}$$

$\therefore$ apparent depth $= \frac{\mu^1}{\mu}$(real depth)

apparent shift $= \left(1 - \frac{\mu^1}{\mu}\right)x$

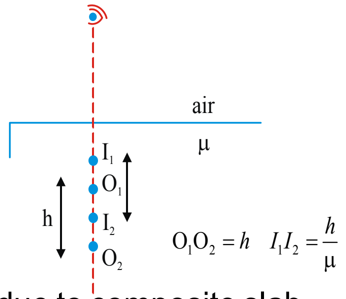
Diagram shows variation of apparent depth with real depth of the object.



Slope $= \tan \theta = \frac{\mu^1}{\mu}(< 1)$

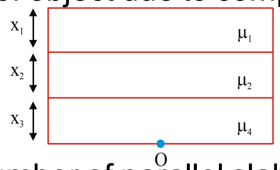
* **Note:** If two objects O_1 and O_2 separated by 'h' on normal line to the boundary in a medium of refractive index μ. These objects are observed from air near to normal line of boundary. The distance between the images I_1 and I_2 of

O_1 and O_2 is $\frac{h}{\mu}$.



Note: Apparent depth of object due to composite slab

is $x_a = \frac{x_1}{\mu_1} + \frac{x_2}{\mu_2} + \frac{x_3}{\mu_3}$

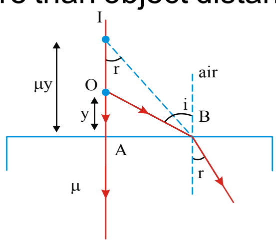


Note: If there are 'n' number of parallel slabs which are may be in contact or may not with different refractive indices are placed between the observer and the object, then the total apparent shift

$$s = \left(1 - \frac{1}{\mu_1}\right)x_1 + \left(1 - \frac{1}{\mu_2}\right)x_2 + ---- + \left(1 - \frac{1}{\mu_n}\right)x_n$$

Where $x_1, x_2 ---- x_n$ are the thickness of the slabs and $\mu_1, \mu_2 .... \mu_n$ are the corresponding refractive indices.

* **Object in rarer medium and observer in denser medium :** When the object in rarer medium (air) at a distance 'y' from boundary and an observer near to normal in denser medium of refractive index 'μ'. By ray diagram in figure it is observed that the image is virtual, on same side to boundary and its distance from the boundary is μ times the object distance. Since $\mu > 1$ image distance is more than object distance.



$$\sin i \approx \tan i = \frac{AB}{AO}, \sin r \approx \tan r = \frac{AB}{AI}$$

According to Snell's law $1.\sin i = \mu \sin r$

$$\frac{AB}{AO} = \mu \frac{AB}{AI}, AI = \mu.AO$$

Therefore apparent height of object (AI) $= \mu$ x real height of object (AO)

i.e. $y_{app} = \mu.y_{real}$ Apparent shift $= AI - AO$

Apparent shift $= (\mu - 1)y$.

If the object is in other than air medium of refractive index $\mu^1 (< \mu)$. Then apparent height

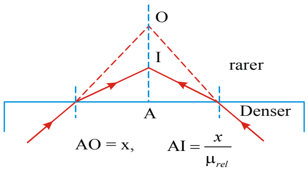
$= \mu_{rel}$ (real height) ; i.e., $y_a = \left(\frac{\mu}{\mu^1}\right)y$

Apparent shift $= \left(\frac{\mu}{\mu^1} - 1\right) y$

Diagram shows variation of apparent height with real height of the object.

slope $= \tan\phi = \frac{\mu}{\mu'} (>1)$

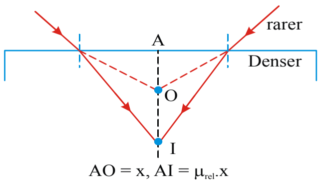
Note: When convergent beam of rays passing from denser to rarer medium as shown in the figure. Real image is formed in rarer medium which nearer to boundary than that of virtual object.



shift $= x\left(1 - \frac{1}{\mu_{real}}\right)$

Note: When convergent beam of rays passing from rarer to denser medium as shown in the figure. Real image is formmed in denser medium which is far to boundary than that of virtual object.

shift $= (\mu_{real} - 1)x$

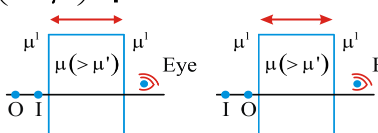


Application

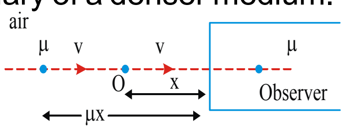
Normal shift due to glass slab : When an object is placed on normal line to the boundary of slab whose thickness is 't' and refractive index 'μ'. On observing this object (real) from other side of the slab, due to refraction, the image of this object shift on the normal line. This shift value is called normal shift. This shift is towards the slab, if the slab is denser relative to the surroundigs and shift is away from the slab, if the slab is rarer relative to the surrounds. Then the Normal shift

$$OI = \left(1 - \frac{1}{\mu_{rel}}\right)t = \left(1 - \frac{\mu^1}{\mu}\right)t$$

for $\mu' = 1$, normal shift $OI = \left(1 - \frac{1}{\mu}\right)t$.



Relation between the velocities of object and image : The figure shows an object O moving towards the plane boundary of a denser medium.



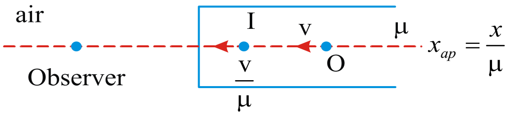
$x_{ap} = \mu x$

Differentiating the above equation with respective to time, we get

$V_{ap} = \mu V$

To an observer in the denser medium, the object appears to be more distant but moving faster. If the speed of the object is v, then the speed of the image will be μv.

(b) Simillarly to an observer in rarer medium and object in denser medium, the image appears to be closer but moving slowly.



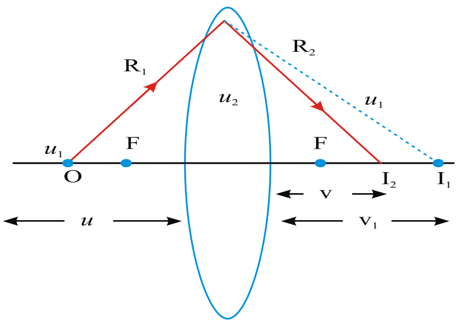
Differentiating the above equaion with respective to time, we get $V_{ap} = \frac{V}{\mu}$

If the speed of the object is v. Then the speed of the image will be $\frac{v}{\mu}$.

Lens Maker's formula and Lens formula :

* In case of image formation by a lens, the incident ray is refracted at first and second surface respectively. The image formed by the first surface acts as object for the second.

  Consider an object O is placed at a distance u from a convex lens as shown in figure. Let its image is I_1 after refraction through first surface. So from the formula for refraction at curved surface.



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

For first surface

$$\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \quad \text{.... (1)}$$

The image I_1 is acts as object to second surface, and form final image I_2

For second surface

$$\frac{\mu_1}{v}-\frac{\mu_2}{v_1}=\frac{\mu_1-\mu_2}{R_2} \quad \text{.... (2)}$$

So adding (1) and (2) equation, we have

$$\mu_1\left[\frac{1}{v}-\frac{1}{u}\right]=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or $\left(\frac{1}{v}-\frac{1}{u}\right)=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{v}-\frac{1}{u}=(\mu_r-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

with $\mu_r=\frac{\mu_2}{\mu_1}(\text{or})\frac{\mu_L}{\mu_M}$

* If object is at infinity, image will be formed at the focus

  i.e. for $u=-\infty, v=f$, so that above equation becomes $\frac{1}{f}=(\mu_r-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

  Which is known as Lens-maker's formula and

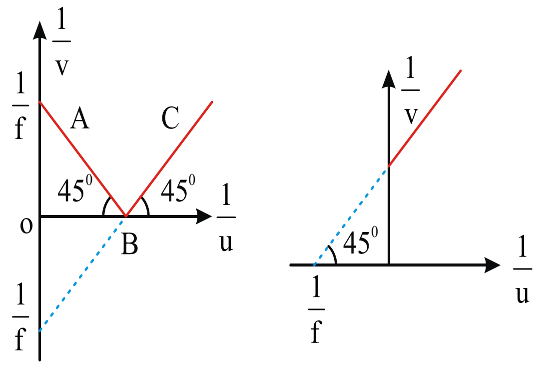
* For a lens it becomes $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ which is known as the "lens - formula" or "Gauss's formula" for a lens.

* Though we derived it for a real image formed by a convex lens, the formula is valid for both convex as well as concave lens and for both real and virtual images.

Note: The lens maker's formula is applicable for thin lenses only and the value of R_1 and R_2 are to be put in accordance with tthe Cartesian sign convention.

* **$\frac{1}{v}$ and $\frac{1}{u}$ Graphs:**

* **Convex lens:** The graph between $\frac{1}{v}$ and $\frac{1}{u}$ in case convex lens is as shown in figure.



For real image:

$$\frac{1}{v}-\frac{1}{(-u)}=\frac{1}{f}; \qquad \frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$$

It is a straight line with slope - 1, for virtual image

$$\frac{1}{(-v)}-\frac{1}{(-u)}=\frac{1}{f}; \qquad \frac{1}{v}=\frac{1}{u}-\frac{1}{f}$$

It is a straight line with slope +1 Hence AB line when the image is real. BC line when the image is virtual.

* **Concave lens:** The graph between $\frac{1}{v}$ and $\frac{1}{u}$ in case of concave lens as shown in figure.

Since concave lens only from virtual image.

$$\frac{1}{-v}-\frac{1}{-u}=-\frac{1}{f}; \qquad \frac{1}{v}=\frac{1}{u}+\frac{1}{f}$$

It is a straight line with slope +1.

U and V Graph

* **Convex lens:** The graph between v and u is hyperbola to convex lens as shown in figure.

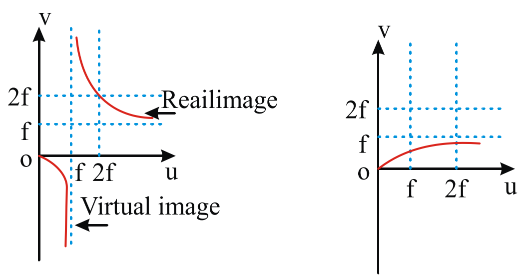
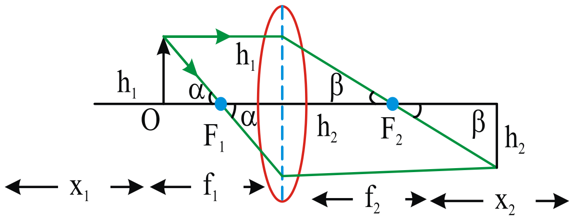


Fig. (a) Fig. (b)

* **Concave lens:** The graph between v and u is hyperbola to concave lens as shown in figure.

In case of thin convex lens if an object is placed at a distance x_1 from first focus and its image is formed at a distance x_2 from the second focus.



From properties of triangles,to the left of the lens

$\frac{h_1}{x_1} = \frac{h_2}{f_1}$ To the right of the lens

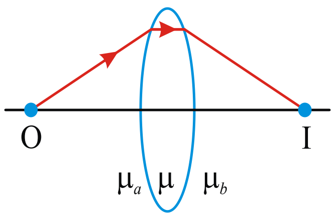
$\frac{h_1}{f_2} = \frac{h_2}{x_2}$ From above two equations $\frac{x_1}{f_1} = \frac{x_2}{f_2}$

$\therefore x_1 x_2 = f_1 f_2$ For $f_1 = f_2$

$x_1 x_2 = f^2$ is called Newton's formula or lens user formula. This relation can also prove by using lens formula.

* **Lenses with Different Media on either side**

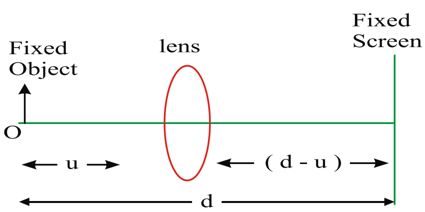
Consider a lens made of a material with refractive index μ with a liquid μ_a on the left and a liquid μ_b.



The governing equation for this system is

$$\frac{\mu_b}{v} - \frac{\mu_a}{u} = \frac{\mu - \mu_a}{R_1} + \frac{\mu_b - \mu}{R_2}$$

* **Determination of the Focal length of a convex lens (or) Size of the object by "LENS DISPLACEMENT METHOD".**



If the distance 'd' between an object and screen is greater than four times the focal length of a convex lens, then there are two positions of lens between the object and screen. This method is called displacement method and is used in the laboratory to determine the focal length of convex lens.

If the object is at a distance u from the lens, the distance of image from the lens $v=(d-u)$,

so from lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\frac{1}{d-u}+\frac{1}{u}=\frac{1}{f}$ i.e., $u^2-du+df=0$

So that $u=\frac{d\pm\sqrt{d(d-4f)}}{2}$

Now there are three possibilities.

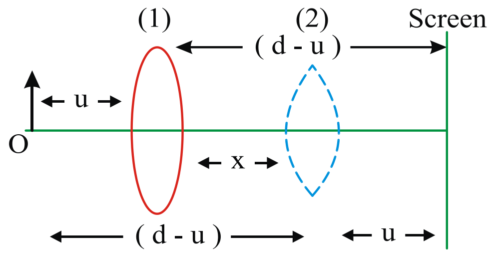
* If $d<4f$, u will be imaginary, so physically no position of lens is possible.
* If $d=4f$, in this $u=\frac{d}{2}=2f$ so only one position is possible and in this $v=2f$. That is why the minimum separation between the real object and real image is 4f.
* If $d>4f, u_1=\frac{d-\sqrt{d(d-4f)}}{2}$ and

$u_2=\frac{d+\sqrt{d(d-4f)}}{2}$ for these two positions of the object real images are formed for

$u=u_1, v=d-u_1=\frac{d+\sqrt{d(d-4f)}}{2}=u_2$

For $u=u_2, v_2=d-u_2=\frac{d-\sqrt{d(d-4f)}}{2}=u_1$

i.e., for two positions of the lens object and image distances are interchangeble as shown in the figure.



So the magnification for the both positions of the object are related as $m_1=\frac{1}{m_2}$

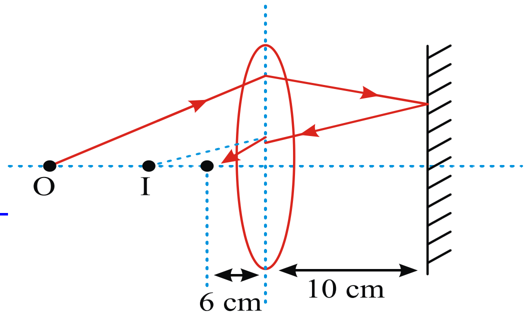
i.e., $m_1.m_2=1$ $\therefore m_1m_2=\frac{I_1}{O}.\frac{I_2}{O}=\frac{I_1I_2}{O^2}=1$

Therefore $O=\sqrt{I_1I_2}$ where I_1 & I_2 are the sizes of images for two positions of the object and O is size of the object.

* It means that the size of object is equal to the geometric mean of the two images. This method of measuring the size of the object is useful when the object inaccessible.
* If 'x' is the distance between the two positions of the lens. Then $f=\frac{x}{m_1-m_2}$

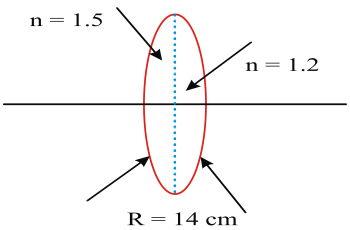
W.E-69: ***A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is***

Sol:



From figure, the image is real and at a distance of 16 cm from the mirror

W.E-70: ***A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index 'n' of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature R=14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be***



Sol: $P_T = (1.5-1)\left(\frac{1}{14}-0\right)+(1.2-1)\left(0-\frac{1}{-14}\right)$

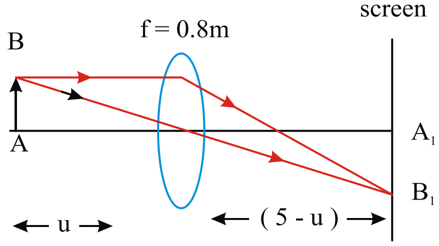
$$P_T = \frac{0.5}{14}+\frac{0.2}{14}=\frac{1}{20}$$

$$f = +20\,\text{cm} \quad ; \quad \frac{1}{v}-\frac{1}{-40}=\frac{1}{20}$$

$$\frac{1}{v}=\frac{1}{20}-\frac{1}{40}=\frac{1}{40};\ \therefore\ v = 40\,\text{cm}$$

W.E-71: ***An object is 5 m to the left of a flat screen. A converging lens for which the focal length is 0.8 m is placed between object and screen. (a) Show that for two positions of lens form images on the screen and determine how far these positions are from the object? (b) How do the two images differ from each other?***

Sol:



(a) Using the lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

We have, $\frac{1}{5-u}+\frac{1}{u}=\frac{1}{0.8}$ or $\frac{1}{5-u}+\frac{1}{u}=1.25$

$\therefore u+5-u=1.25u(5-u)$

or $1.25u^2-6.25u+5=0$; u =4m and 1m

Both the values are real, which means there exist two positions of lens that form images of object on the screen.

(b) $m=\frac{v}{u}$; $\therefore m_1=\frac{(5-4)}{(-4)}=-0.25$ and

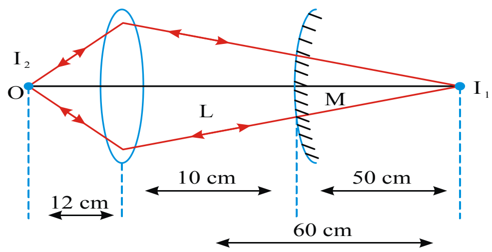
$m_2=\frac{(5-1)}{(-1)}=-4.00$

Hence, both the images are real and inverted, the first has magnification -0.25 and the second -4.00.

W.E-72: ***A point object is placed at a distance of 12 cm on the axis of a convex lens of focal length 10 cm. On the other side of the lens, a convex mirror is placed at a distance of 10 cm from the lens such that the image formed by the combination coincides with the object itself. What is the focal length of convex mirror?***

Sol: For convex lens, $\frac{1}{v}-\frac{1}{-12}=\frac{1}{10}$

i.e., $v=60\,cm$; i.e., in the absence of convex mirror, convex lens will form the image I_1 at a distance of 60 cm behind the lens. Since, the mirror is at a distance of 10 cm from the lens, I_1 will be at a distance of $60-10=50\,cm$ from the mirror, i.e., $MI_1=50\,cm$

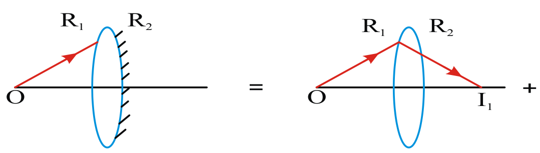


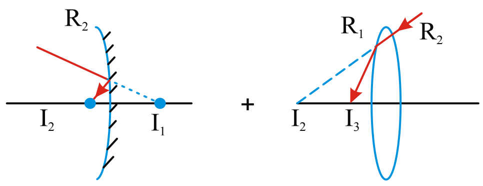
Now as the final image I_2 is formed at the object O itself, the rays after reflection from the mirror retraces its path, i.e., rays on the mirror are incident normally, i.e., I_1 is the centre of the mirror, so that $R=MI_1=50\,cm$ and hence $F=(R/2)=(50/2)=25\,cm$

Lens with one Silvered surface

* **When the back surface of a convex lens is silvered.**

The rays are first refracted by lens, then refracted from the silvered surface and finally refracted by lens, so that we get two refractions and one reflection.





In the diagram if f_l and f_m are respective the focal lengths of lens and mirror. Then

$$\frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_l}=\frac{2}{f_l}+\frac{1}{f_m}$$

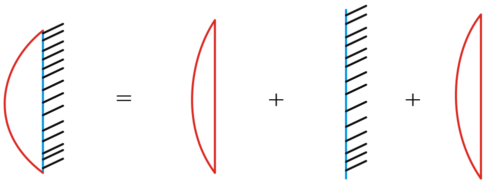
In terms of focal powers of lens and mirror

$$P=P_l+P_m+P_l=2P_l+P_m$$

with $P_l=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ and $P_m=\frac{2}{R_2}$

Here P_l and P_m are substitued with sign.

* The system will behave as
  a concave mirror if 'P' is positive and
* The system will behave as
  a convex mirror if "P" is negative.
  The replacement with the mirror is due to overall reflection of given rays.
* **When the plane surface of plano convex lenss is silvered.**



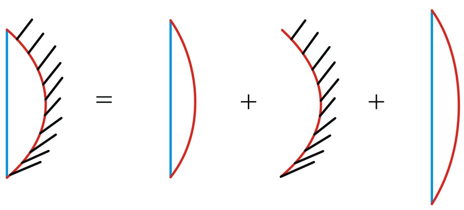
Then, the focal power of the given lens is ($P_m=0$)

$$P=2P_l+P_m$$

$$P=2.\left(\frac{\mu-1}{R}\right)+0=\frac{2(\mu-1)}{R}$$

Since $\mu>1$, 'P' is positive, the system behaves as a concave mirror with focal length $\frac{R}{2(\mu-1)}$

* **When curved surface of a plano convex lens is silvered.**



Then, the focal power of the given lens is

$$P=2P_l+P_m=\frac{2(\mu-1)}{R}+\frac{2}{R}=\frac{2\mu}{R}$$

Since 'P' is positive, the system behaves a concave mirror with focal length $\frac{R}{2\mu}$

W.E-73: *A pin is placed 10 cm in front of a convex lens of focal length 20cm made of material having refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature 22cm. Determine the position of the final image. Is the image real or virtual?*

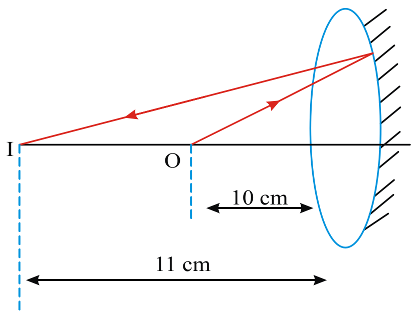
Sol: As radius of curvature of silvered surface is 22 cm, so

$$f_M = \frac{R}{2} = \frac{-22}{2} = -11\,cm$$ and hence,

$$P_M = -\frac{1}{f_M} = -\frac{1}{-0.11} = \frac{1}{0.11}D$$

Further as the focal length of lens is 20cm, i.e., 0.20 m, its power will be given by:

$$P_L = -\frac{1}{f_L} = \frac{1}{0.20}D$$



Now as in image formation, light after passing through the lens will be reflected back by the curved mirror through the lens again

$$P = P_L + P_M + P_L = 2P_L + P_M$$

i.e., $P = \frac{2}{0.20} + \frac{1}{0.11} = \frac{210}{11}D$

SO the focal length of equivalent mirror

$$F = -\frac{1}{P} = -\frac{11}{210}m = -\frac{110}{21}cm$$

i.e., the silvered lens behaves as a concave mirror of focal length (110/21) cm. So for object at a distance 10 cm in front of it,

$$\frac{1}{v} + \frac{1}{-10} = -\frac{21}{110}$$

i.e., $v = -11cm$ i.e., image will be 11 cm in front of the silvered lens and will be real as shown in figure.

W.E-74: *A biconvex thin lens is prepared from glass of refractive index 3/2. The two bounding surfaces have equal radii of 25 cm each. One of the surfaces is silvered from outside to make it reflecting. Where should an object be placed before this lens so that the image coincides with the object.*

Sol: Here, $R_1 = +25cm, R_2 = -25cm \text{ and } \mu = 3/2$

Image coincides with object, hence $u = v = -x$ (say)

$$\frac{1}{F}=\frac{2}{F_L}+\frac{1}{F_C}=2\left(\frac{3}{2}-1\right)\frac{2}{25}+\frac{2}{25}$$

$$\frac{1}{F}=\frac{4}{25}, \text{by using } \frac{1}{v}-\frac{1}{u}=\frac{1}{F}; -\frac{1}{x}-\frac{1}{x}=\frac{4}{25}$$

$x = 12.5cm$

Hence, the object should be placed at a distance 12.5cm in front of the silvered lens.

* **Lens maker's formula-Special Cases**

It relates the focal length of the lens to the re fractive index of material of the lens and the radii of curvature of the two surfaces.

The formula is $\frac{1}{f}=\left(\frac{\mu_{lens}}{\mu_{medium}}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

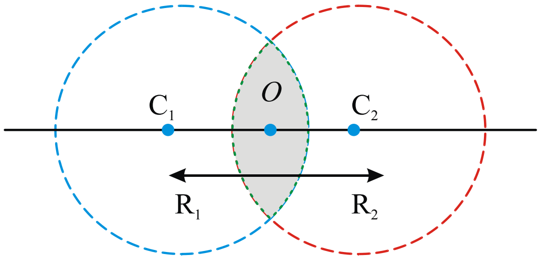
where μ_{lens} is the absolute refractive index of material of the lens, μ_{medium} is the absolute refractive index of the medium in which the lens is placed.

R_1 and R_2 are the radii of curvature of two surfaces of the lens.

If the lens is placed in vacuum then $\frac{1}{f}=(\mu_{lens}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

The lens makers' formula is applicable for thin lenses only and the value of R_1 and R_2 are to be put in accordance with the Cartesian sign convention.

Note: For convex lens

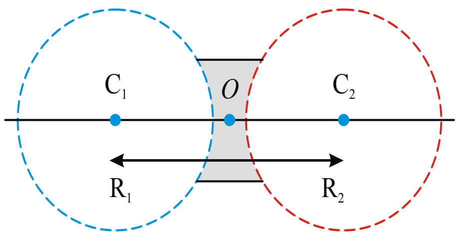


Convex Lens

For convex lens R_1 is +ve and R_2 is -ve so the lens makers formula is $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

For equiconvex lens $\frac{1}{f}=(\mu-1)\left(\frac{2}{R}\right)$

Note: For concave lens



For concave lens R_1 is -ve and R_2 is +ve so the lens makers formula is

$$\frac{1}{f}=(\mu-1)\left(-\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For equiconcave lens $\frac{1}{f}=-(\mu-1)\left(\frac{2}{R}\right)$

Note: For converging meniscus

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right), \text{ if } (R_1<R_2)$$

Note: For diverging meniscus

$$\frac{1}{f}=-(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right), \text{ if } (R_1<R_2)$$

Note: For plano convex lens

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}\right) \quad \because R_2=\infty$$

W.E-75: ***What is the refractive index of material of a plano-convex lens, if the radius of curvature of the convex surface is 10cm and focal length of the lens is 30cm?***

Sol: According to lens-maker's formula

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Here $f=30cm, R_1=10cm \text{ and } R_2=\infty$

so $\frac{1}{30}=(\mu-1)\left(\frac{1}{10}-\frac{1}{\infty}\right)$

i.e., $3\mu-3=1 \text{ or } \mu=(4/3)$

W.E-76: ***A concave lens of glass, refractive index 1.5, has both surface of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as lens***

Sol: When glass lens is immersed in a medium, its refractive index is ${}^m\mu_g$.

$${}^m\mu_g=\frac{{}^a\mu_g}{{}^a\mu_m}=\frac{1.50}{1.75}=\frac{6}{7} \quad \therefore \text{ By lens makers' formula}$$

$$\frac{1}{f}=\left({}^m\mu_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \text{ or } \frac{1}{f}=\left(\frac{6}{7}-1\right)\left(-\frac{1}{R}-\frac{1}{R}\right)$$

or $\frac{1}{f}=\left(-\frac{1}{7}\right)\left(-\frac{2}{R}\right)$ or $f=\frac{7R}{2}=3.5R$

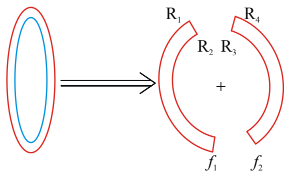
Hence, the given lens in medium behaves like convergent lens of focal length 3.5R

WE-77: ***A hollow equi convex lens of glass will be have like a glass plate***

Sol: Hollow convex lens is as shown in figure

$$\frac{1}{f_1}=(\mu_g-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=0$$

$\left(as\ R_1 = R_2\right)$

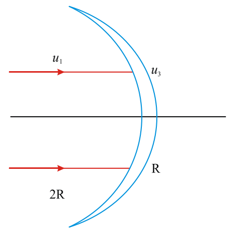


Hollow glass lens

or $f_1 = \infty$, similarly $f_2 = \infty$

Therefore, a hollow equi convex lens of any material will behave like a glass plate.

W.E-78: ***The diagram shows a concavo -convex lens. What is the condition on the refractive indices so that the lens is diverging?***



The refractive index of the lens is μ_2

Sol: $\frac{\mu_3}{v} + \frac{\mu_1}{u} = \frac{\mu_1 - \mu_2}{2R} + \frac{\mu_2 - \mu_3}{R}$ $\therefore \frac{\mu_1 - \mu_2}{2} < \mu_3 - \mu_2$

$\Rightarrow \mu_1 - \mu_2 < 2\mu_3 - 2\mu_2 \Rightarrow \mu_1 + \mu_2 < 2\mu_3$

W.E-79: ***The magnification of an object placed in front of a convex lens of focal length 20cm is +2. To obtain a magnification of -2, the object will have to be moved a distance equal to***

Sol: When magnification is +2 then the image is virtual. Both the image and the object are on the same side of the lens.

$u = -x;\ v = -2x;\ f = +20$

Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have $\frac{1}{-2x} + \frac{1}{x} = \frac{1}{20}$ or

$x = 10cm$.

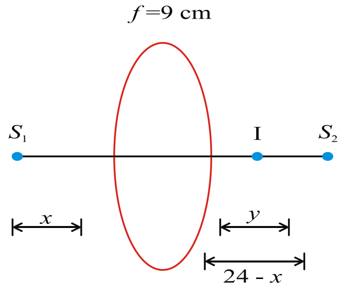
To have a magnification of -2 the image must be real.

$u = -y,\ v = +2y\ and\ f = +20$

$\therefore \frac{1}{2y} + \frac{1}{y} = \frac{1}{20}$ or $y = 30cm$ $\quad \therefore y - x = 20cm$

W.E-80: ***Two point sources S_1 and S_2 are 24cm apart. Where should a convex lens of focal length 9 cm be placed in between them so that the images of both sources are formed at same place?***

Sol: In this case one of the image will be real and other virtual. Let us assume that image of S_1 is real and that of S_2 is virtual.



Applying $\frac{1}{y}+\frac{1}{x}=\frac{1}{9}$ For $S_1:\frac{1}{y}+\frac{1}{x}=\frac{1}{9}\rightarrow(1)$

for $S_2:-\frac{1}{y}+\frac{1}{24-x}=\frac{1}{9}\rightarrow(2)$

Solving eqs. (1) and (2) , we get $x=6cm$

W.E-81: ***An object placed at A $(OA>f)$. Here, f is the focal length of the lens. The image is formed at B. A perpendicular is errected at 0 and C is chosen such that $\angle BCA=90^0$. Let $OA=a, OB=b$ and $OC=c$. Then the value of f is***

Sol: From $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have $\frac{1}{b}+\frac{1}{a}=\frac{1}{f}$

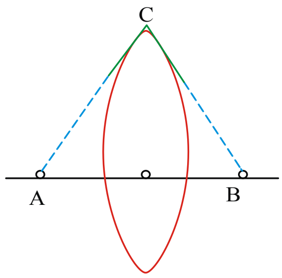
or $f=\frac{ab}{a+b}\rightarrow(1)$

Further $AC^2+BC^2=AB^2$

or $(a^2+c^2)+(b^2+c^2)=(a+b)^2$

or $a^2+b^2+2c^2=a^2+b^2+2ab$

$ab=c^2$

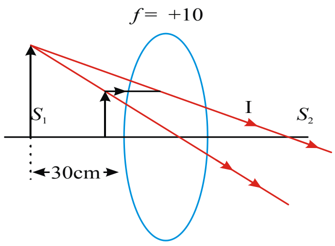


Substituting this in Eq. (1) we get $f=\frac{c^2}{a+b}$

W.E-82: ***Convex lens has a focal length of 10cm.***

a) Where should the object be placed if the image is to be 30cm from the lens on the same side as the object?

b) What will be the magnification?

Sol: 

a) In case of magnifying lens, the lens is convergent and the image is erect, enlarged, virtual , between infinity and object and on the same side of lens as shown in figure. So here $f = 10cm$ and $v = -30cm$ and hence from lens-formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

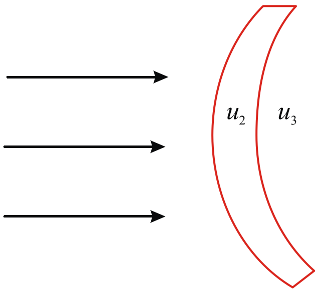
we have $\frac{1}{-30} - \frac{1}{u} = \frac{1}{10}$ $i.e\ u = -7.5cm$

So the object must be placed in front of lens at a distance of 7.5cm (which is <f) from it.

b) $m = \left[\frac{I}{O}\right] = \frac{v}{u} = \frac{-30}{-7.5} = 4$

i.e, image is rect, virtual and four times the size of object.

W.E-83: ***In the figure, light is incident on the thin lens as shown. The radius of curvature for both the surface is R. Determine the focal length of this system.***

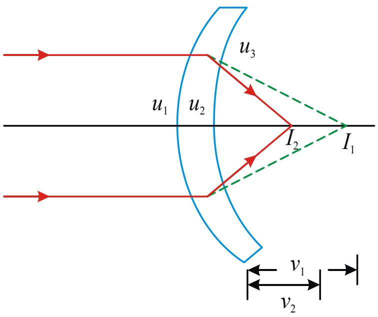


Sol: For refraction at first surface,

$$\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{+R} \rightarrow (i)$$

For refraction at 2nd surface

$$\frac{\mu_3}{v_2} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{+R} \rightarrow (ii)$$



Adding equations (i) and (ii) we get

$$\frac{\mu_3}{v_2} = \frac{\mu_3 - \mu_1}{R} or\ v_2 = \frac{\mu_3 R}{\mu_3 - \mu_1}$$

Therefore, focal length of the given lens system is $\dfrac{\mu_3 R}{\mu_3 - \mu_1}$

W.E-84: ***The linear magnification of an object placed on the principal axis of a convex lens of focal length 30cm is found to be +2. In order to obtain a magnification of -2, by how much distance should the object be moved?***

Sol: In the first case, the magnification is positive which implies that the image is erect, virtual and on the same side of the lens as the object. If a is the object distance then $u = -a$ and $v = -2a$. From the lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \; or \; \frac{1}{-2a} - \frac{1}{-a} = \frac{1}{30} \Rightarrow a = 15cm$$

So the object is at a distance of 15cm from the lens. In the second case, the magnification is negative, the image is real, inverted and on the other side of the lens as the object. If b is the object distance, then $u = -b$ and $v = +2b$. Hence

$$\frac{1}{2b} - \frac{1}{-b} = \frac{1}{30} \Rightarrow b = 45cm$$

Thus the object has to be moved through a distance of $(45-15) = 30cm$ away from the lens.

W.E-85: ***The distance between the object and the real image formed by a convex lens is d. If the linear magnification is m, find the focal length of the lens in terms of d and m.***

Sol: The convex lens formula for a real image is

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \rightarrow (i)$$

Where no sign conventions are to be used. Multiplying we get $\frac{u}{v} + 1 = \frac{u}{f}$ or $\frac{1}{m} = \frac{u}{f} - 1 = \frac{u-f}{f}$

or $m(u-f) = f \; or \; u = \dfrac{(1+m)f}{m} \rightarrow (ii)$

Multiplying (i) by v we get

$$1 + \frac{v}{u} = \frac{v}{f} \; or \; 1+m = \frac{v}{f} or \, v = f(1+m) \rightarrow (iii)$$

Now $u + v = d$. Using (ii) and (iii) we have

$$d = \frac{(1+m)f}{m} + f(1+m)$$

which gives $f = \dfrac{md}{(1+m)^2}$

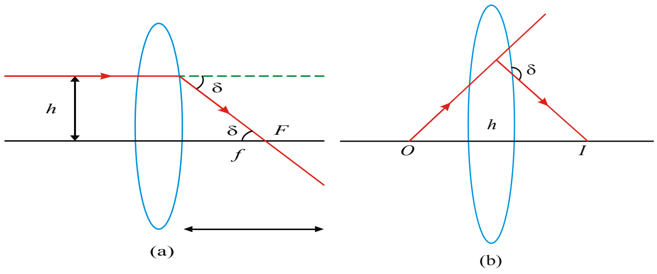
W.E-86: ***A concave lens of focal length f forms an image which is n times the size of the object. What is the distance of the object from the lens in terms of f and n?***

Sol: The concave lens formula is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ where no sign conventions are to be used. Thus

$$\frac{u}{v} - 1 = \frac{u}{f} or \frac{1}{n} - 1 = \frac{u}{f}\left(\because \frac{v}{u} = n\right)$$

or $u = \left(\frac{1-n}{n}\right) f$

* **Power of A Lens :** The power of lens is the measure of its ability to produce convergence or divergence of a parallel beam of light. The power P of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distant from the optical centre.



$\tan\delta = \frac{h}{f}$;   if $h = 1,\ \tan\delta = \frac{1}{f}$

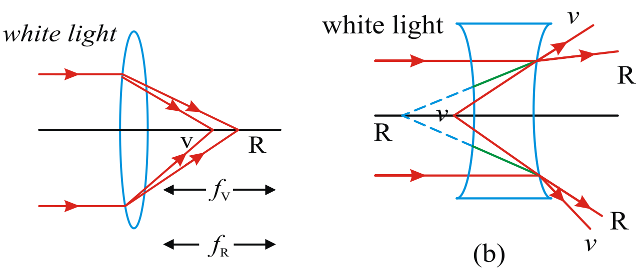
As per definition power (P) = $\tan\delta = \frac{1}{f}$

* If lens is placed in a medium other than air of refractive index μ. Then power $P = \frac{\mu}{f}$

The S.I unit of power is diopter (D) and $1D = 1m^{-1}$

i.e. $P = \frac{1}{f(in\,m)} = \frac{100}{f(in\ cm)} D$

* A convex lens converge the incident rays. Due to this reason, the power of a convex lens is taken as positive. On the other hand, a concave lens diverge the incident rays. Therefore its power is taken as negative.
* **Some important points regarding lens:**
* Every part of a lens forms complete image. If a portion of lens is obstructed, full image will be formed but the intensity will be reduced.
* The focal length of a lens depends on its refractive index i.e $\frac{1}{f} \propto (\mu - 1)$, so the focal length of a given lens is different for different wave lengths and maximum for red and minimum for violet whatever the nature of the lens as shown in figure.

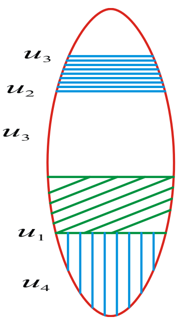


Filling up of a lens:

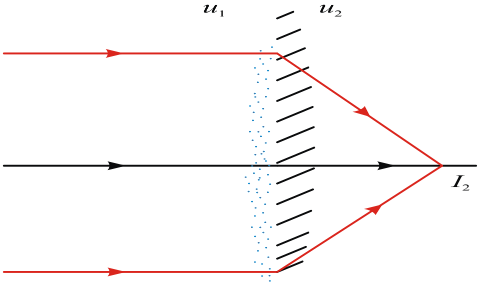
* If a lens made a number of layers of different refractive indices as shown in figure, for a given

wave length of light it will have many focal lengths as $\frac{1}{f} \propto (\mu - 1)$

Hence it will form many images as there are different $\mu's$ .According to given diagram number of images formed by lens is 4.



* If a lens is made of two or more materials and are placed side by side as shown in below, then there will be one focal length and hence one image



* **Lens immersed in a liquid:**

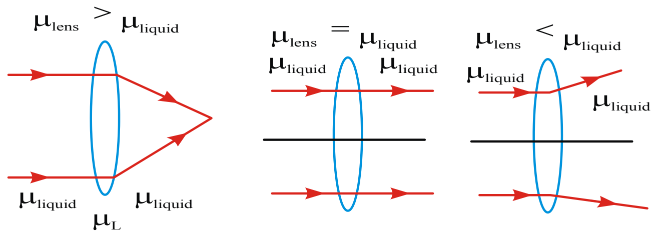
If a lens made of material of refractive index μ_{lens} is immersed in a liquid of refractive index μ_{liquid} ,if f_a is the focal length of a lens placed in air, then

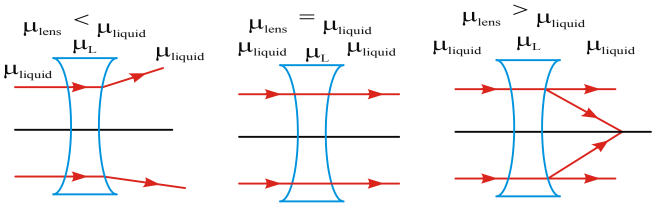
$$\frac{1}{f_a} = (\mu_{lens} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow (1)$$

If f_l is the focal length of lens immersed in a liquid then $\frac{1}{f_l} = \left(\frac{\mu_{lens}}{\mu_{liquid}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \rightarrow (2)$

$$\frac{(1)}{(2)} \Rightarrow \boxed{\frac{f_l}{f_a} = \frac{(\mu_{lens} - 1)}{\left(\frac{\mu_{lens}}{\mu_{liquid}} - 1\right)}}$$

* Depending upon the values of μ_{lens} and μ_{liquid} , we have three cases





(a) (b) (c)

Case(a): If $\mu_{lens} > \mu_{liquid}$, then f_l and f_a are of same sign and $f_l > f_a$ i.e. the nature of lens remains unchanged, but it's focal length increases and hence power of lens decreases.

Case(b): If $\mu_{lens} = \mu_{liquid}$ then $f_l = \infty$,ie. the lens behaves as a plane glass plate and becomes invisible in the medium.

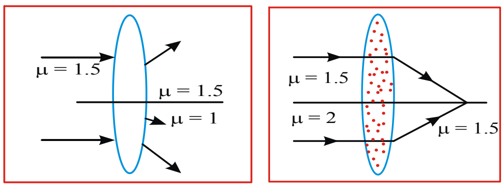
Case(c): If $\mu_{lens} < \mu_{liquid}$, then f_l and f_a have opposite sign and the nature of lens changes ie. a convex lens diverges the light rays and concave lens converge the light rays.

W.E-87: ***A glass convex lens of refractive index (3/2) has a focal length equal to 0.3m. Find the focal length of the lens if it is immersed in water of refractive index (4/3)?***

Sol: As according to lens-makers' formula

$$\frac{f_w}{f_a} = \frac{(\mu_g - 1)}{\left(\frac{\mu_g}{\mu_w} - 1\right)}; \frac{f_w}{0.3} = \frac{\left(\frac{3}{2} - 1\right)}{\left(\frac{3/2}{4/3} - 1\right)} \Rightarrow f_w = 1.2m$$

W.E-88: ***As shown in figure a spherical air lens of radii*** $R_1 = R_2 = 10cm$ ***is cut in a glass*** $(\mu = 1.5)$ ***cylinder. Determine the focal length and nature of air lens. If a liquid of refractive index 2 is filled in the lens, what will happen to its focal length and nature?***

A) 

Sol: According to lens-maker's formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \text{with } \mu = \frac{\mu_L}{\mu_M}$$

Initially, $\mu = \frac{\mu_A}{\mu_G} = \frac{1}{(3/2)} = \frac{2}{3}; R_1 = +10cm$

and $R_2 = -10cm$

So $\frac{1}{f} = \left[\frac{2}{3} - 1\right]\left[\frac{1}{+10} - \frac{1}{-10}\right] = -\frac{2}{30}$ ie $f = -15cm$

ie. the air lens in glass behaves as divergent lens of focal length 15cm.

When the liquid of $\mu = 2$ is filled in the air cavity, $\mu = \frac{\mu_L}{\mu_M} = \frac{2}{1.5} = \frac{4}{3}$

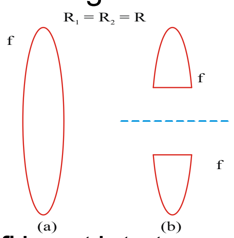
So that now $\frac{1}{f'}=\left[\frac{4}{3}-1\right]\left[\frac{1}{10}-\frac{1}{-10}\right]=\frac{2}{30}$

$f'=15cm$ ie. the liquid lens in glass will behave as a convergent lens of focal length 15cm.

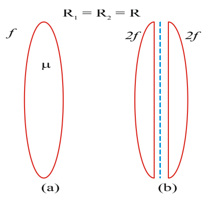
* If the two radii of curvature of a thin lens are not equal, the focal length remains unchanged, where the light is incident on either of two surfaces.

Cutting of a lens:

* If an equi convex lens of focal length 'f' is cut into two equal parts along its principal axis, as shown in figure , as none of μ, R_1 and R_2 will change, the focal length of each part will be equal that of initial lens, but intensity of image formed by each part will reduced to half.



* If an equi convex lens of focal length 'f' is cut into two equal parts transverse to principal axis, as shown in figure, the focal length of each part will become double that of initial value, but intensity of image remains same.



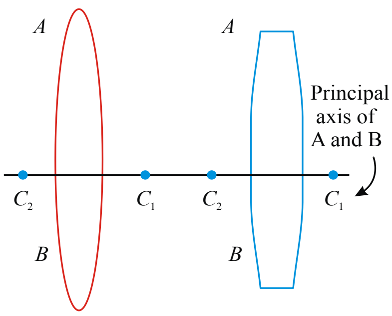
For original lens $\frac{1}{f}=\frac{2(\mu-1)}{R}\rightarrow(1)$

For each part of cut lens

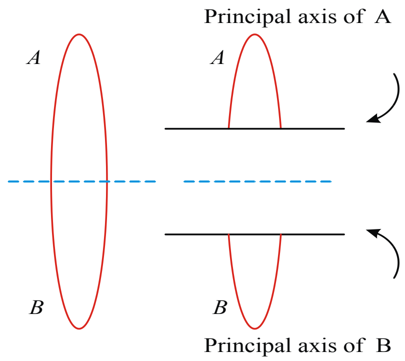
$\frac{1}{f'}=\frac{(\mu-1)}{R}\rightarrow(2)$

From (1) and (2) we get $f'=2f$

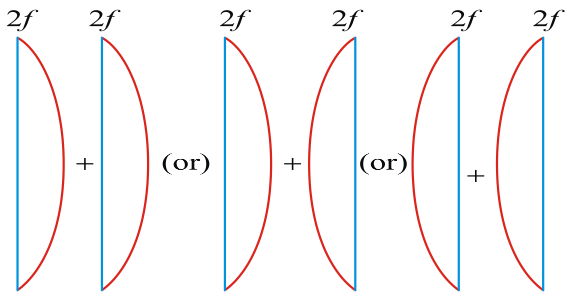
* a) On removing a part of lens with out disturbing remaining part, the principal axis position of the remaining part is same as earlier, but intensity of image is reduced



b) If given lens is cut along the principal axis and the separation between them increase in a direction transverse to principal axis, each part has own principal axis.

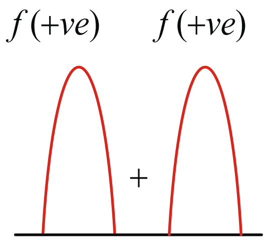


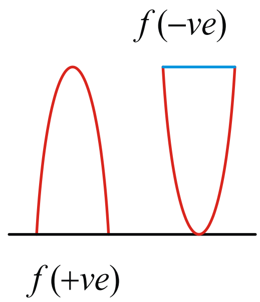
* If the equi convex lens of focal length ‘f’ is divided into two equal parts transverse to the principal axis as shown in figure, the focal length of each part is 2f. If these two parts are put in contact in different combinations as shown in figure



$\frac{1}{F_{net}}=\frac{1}{2f}+\frac{1}{2f}, \frac{1}{F_{net}}=\frac{2}{2f}$ and $F_{net}=f, P_{net}=\frac{1}{f}$

* If an equi convex lens of focal length ‘f’ is divided into two equal parts along its principal axis as shown in figure, the focal length of each part is f. If these two parts are put in contact in different combinations as shown in figure





For the first combined $\frac{1}{f_{net}}=\frac{1}{f}+\frac{1}{f}$

$f_{net}=\frac{f}{2} \quad \therefore P_{net}=\frac{2}{f}$

For the second combination as shown in figure, first part will behave as convergent lens of focal length f while the other divergent of same focal length (being thinner near the axis), so

in this case, $\frac{1}{F_{net}}=\frac{1}{f}-\frac{1}{f'}$; $\quad F_{net}=\infty, P_{net}=0$

* To a lens $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

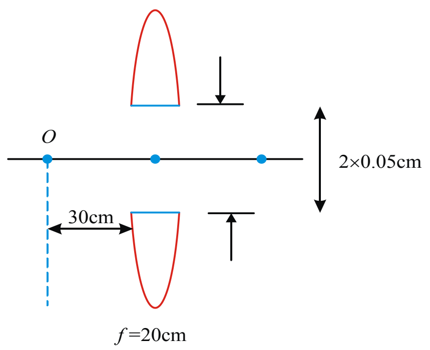
On differentiating above equation, we get

$$-\frac{1}{v^2}.\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0$$

(or) $V_i=\left(\frac{v}{u}\right)^2 V_0$ where V_i =velocity of image with respect to lens, V_0=velocity of object with respect to lens. ; i.e. $V_i=m^2.V_0=\left[\frac{f}{u+f}\right]^2.V_0$

If an object is moved at constant speed towards a convex lens from infinity to focus, the image will move other side of the lens slower in the beginning and faster later on away from the lens. If the object moves from F to optical centre, the image moves with greater speed on the same side of object from infinity to towards lens.

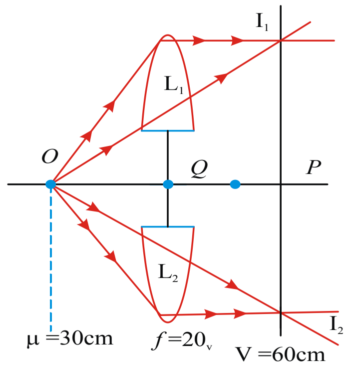
W.E-89: ***A point object O is placed at a distance of 30 cm from a convex lens of focal length 20cm cut into two halves each of which is displaced by 0.05cm as shown in figure. Find the position of the image? If more than one image is formed, find their number and distance between them?***



Sol: Considering each part as separate lens with $u=-30cm$ and $f=20cm$, from lens formula

$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we have $\frac{1}{v}-\frac{1}{-30}=\frac{1}{20}$

ie. $v=60cm$



So each part will form a real image of the point object at 60cm from the lens as shown in figure. As there are two pieces, two images are formed. Now in similar triangles $\left(OI_1I_2 \ and \ OL_1L_2\right)$

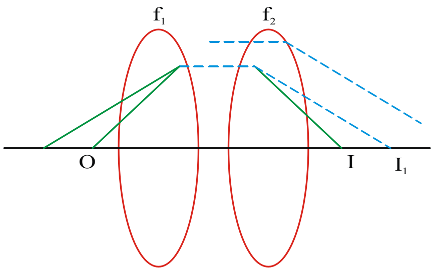
$$\frac{I_1I_2}{L_1L_2}=\frac{OP}{OQ}=\frac{(u+v)}{u}$$

ie $I_1I_2 = \frac{90}{30} \times (2 \times 0.05) = 0.3cm$

So the two images formed are 0.3cm apart.

Ø Combination of Lenses :

Consider two thin lenses kept in contact as shown in figure. Let a point object 'O' is placed on the axis as shown in figure.



First lens of focal length f_1 from the image I_1 of the object 'O' at a distance v_1 from it.

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \to (1)$$

Now the image I_1 will act as an object for second lens and the second lens forms image I at a distance 'v' from it, then

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \to (2)$$

So adding (1) and (2) equations we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} (or) \frac{1}{v} - \frac{1}{u} = \frac{1}{F}$$

so $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

i.e., the combination behaves as a single lens of equivalent focal length "F" given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

This derivation is valid for any number of thin lenses in contact co-axially.

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ... \frac{1}{f_n}$$

In terms of power $P_{net} = P_1 + P_2 + P_3 + ...P_n$

Here focal length values are to be substituted with sign.

Note: If the two thin lens are separated by a distance 'd' , then $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ and

$P_{net} = P_1 + P_2 - dP_1P_2$

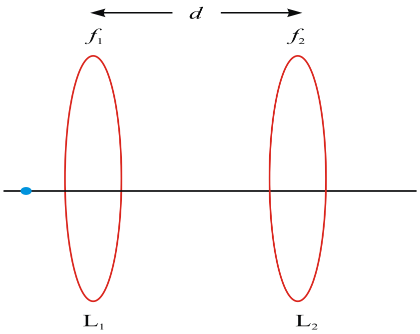
Note: If the medium on either side of the lens is air and the medium between the lens is one having refractive index μ, we can imagine that the rays emerging from the first lens are incident on the second lens as if they have traversed a thickness $\frac{d}{\mu}$ in air.

Hence $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{(d/\mu)}{f_1 f_2}$

$\therefore P=p_1+p_2-\left(\frac{d}{\mu}\right)P_1P_2$

Note: If two thin lenses of equal focal length but of opposite nature are pair in contact, the resultant focal length of the combination will be $\frac{1}{F}=\frac{1}{f}+\left(-\frac{1}{f}\right)=0$ i.e. $F=\infty \; and \; P=0$

Note: If f_1 and f_2 are focal lengths of two lenses $(L_1 \; and \; L_2)$ are separated by a distance 'd' on common principal axis and 'F' is the equivalent focal length of the system.

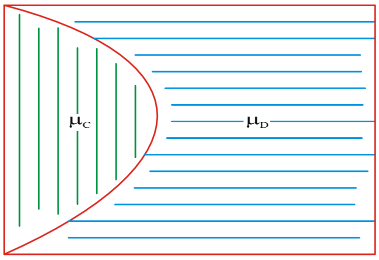


Then

i) The distance of equivalent lens from second lens L_2 is $\frac{Fd}{f_1}$ towards the object if the value is positive and away from the object if the value is negative

ii) The distance of equivalent lens from the first lens L_1 is $\frac{Fd}{f_2}$ away from the object, if the value is positive and towards the object if the value is negative. It is note that F, $f_1 \; and \; f_2$ are to be substituted according to sign convention.

Note: A plane glass plate is constructed by combining a plano-convex lens and a plano-concave lens of different materials as shown in figure. (μ_C is the refractive index of convergence lens, μ_D is the refractive index of divergent lens and R is the radius of curvature of common interface).



by lens maker's formula

$$\frac{1}{f_C}=(\mu_C-1)\left[\frac{1}{\infty}-\frac{1}{-R}\right]=\frac{(\mu_C-1)}{R}\rightarrow(1)$$

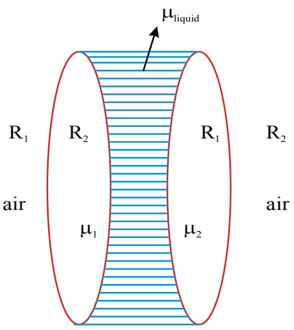
and $\frac{1}{f_D}=(\mu_D-1)\left[\frac{1}{-R}-\frac{1}{\infty}\right]=\frac{-(\mu_D-1)}{R}\rightarrow(2)$

Now as the lenses are in contact,

$$\frac{1}{F}=\frac{1}{f_C}+\frac{1}{f_D}=\frac{(\mu_C-\mu_D)}{R};F=\frac{R}{(\mu_C-\mu_D)}$$

As $\mu_C\neq\mu_D$, the system will act as lens. The system will behave as convergent lens if $\mu_C>\mu_D$ (as its focal length will be positive) and as divergent lens if $\mu_C<\mu_D$ (as F will be negative)

Note: Two convex lenses made of materials of refractive indices $\mu_1\ \&\ \mu_2$, they are placed as shown in figure, the gap between them is filled with a liquid of refractive index μ_{liquid}. This combination is placed in air then



The system is equal to combination of three thin lenses in contact so

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_{liquid}}+\frac{1}{f_2}$$

where $\frac{1}{f_1}=(\mu_1-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

$$\frac{1}{f_{liquid}}=(\mu_{liquid}-1)\left(\frac{1}{R_2}+\frac{1}{R_1'}\right)$$

$$\frac{1}{f_2}=(\mu_2-1)\left[\frac{1}{R_1'}+\frac{1}{R_2'}\right]$$

If the effective focal length F of the combination is +ve then the combination behaves like converging lens, if F is -ve then the combination behaves like diverging lens.

Note: If two convex lenses made of materials of refractive indices $\mu_1\ \&\ \mu_2$ are kept in contact and the whole arrangement is placed in a liquid of refractive index μ_{liquid} then this is equivalent to combination of two lenses kept in contact in a medium.

In this case $\frac{1}{F}=\frac{1}{f_1(m)}+\frac{1}{f_2(m)}$

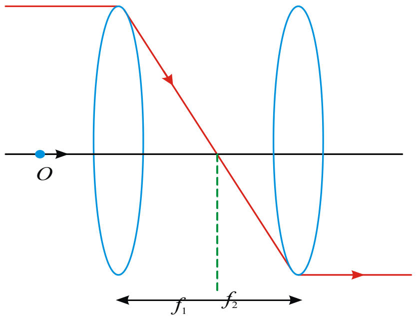
where $\frac{1}{f_1(m)}=\left(\frac{\mu_1}{\mu_{liquid}}-1\right)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$

$$\frac{1}{f_2(m)}=\left(\frac{\mu_1}{\mu_{liquid}}-1\right)\left(\frac{1}{R_1'}+\frac{1}{R_2'}\right)$$

Note: If parallel incident ray on first lens emerges parallel from the second lens, then $f_e=\infty$

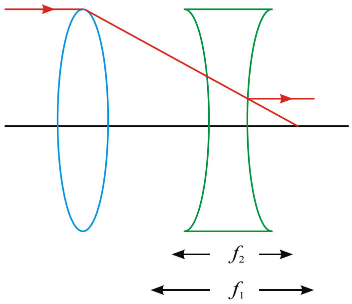
$$\frac{1}{\infty}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1f_2}\Rightarrow d=f_1+f_2$$

(i) If both the lenses are convex, then $d=f_1+f_2$



(ii) If second lens is concave, then

$$d=f_1+(-f_2)=f_1-f_2$$



W.E-90: ***Two thin lenses, when in contact, produce a combination of power +10+diopter. When they are 0.25m part, the power reduces to +6 diopter. The focal length of the lenses are.......m and ....m***

Sol: When the lenses in contact,

$P=P_1+P_2$ or $P_1+P_2=10\rightarrow(1)$

When lenses have d separation,

$P_1+P_2-dP_1P_2=P$ or $P_1+P_2-\frac{P_1P_2}{4}=6$

or $10-\frac{P_1P_2}{4}=6$ or $P_1P_2=16\rightarrow(2)$

From (1) and (2) . we get $P_1=8D,\ P_2=2D$

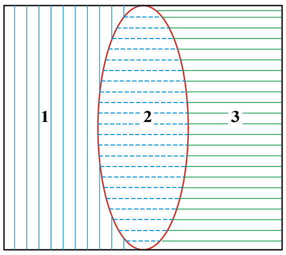
$\therefore\ f_1=\frac{1}{8}=0.125m,\ f_2=\frac{1}{2}=0.5m$

W.E-91: ***Two plano-concave lenses of glass of refractive index 1.5 have radii of curvature of 20 and 30 cm . They are placed in contact with curved surfaces towards each other and the space between them is filled with a liquid of refractive index (4/3). Find the***

focal length of the system.

Sol: As shown in figure the system is equivalent to combination of three thin lenses in contact.

ie. $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\frac{1}{f_3}$



But by lens-maker's formula

$$\frac{1}{f_1}=\left[\frac{3}{2}-1\right]\left[\frac{1}{\infty}-\frac{1}{20}\right]=-\frac{1}{40}$$

$$\frac{1}{f_2}=\left[\frac{4}{3}-1\right]\left[\frac{1}{20}+\frac{1}{30}\right]=\frac{5}{180}$$

$$\frac{1}{f_3}=\left[\frac{3}{2}-1\right]\left[\frac{1}{-30}-\frac{1}{-\infty}\right]=-\frac{1}{60}$$

So $\frac{1}{F}=-\frac{1}{40}+\frac{5}{180}-\frac{1}{60}$

i.e., the system will behave as a divergent lens of focal length 72cm.

W.E-92: ***Two thin symmetrical lenses of different nature and of different material have equal radii of curvature $R=15cm$. The lenses are put close together and immersed in water $\left(\mu_w=\frac{4}{3}\right)$. The focal length of the system in water is 30cm. The difference between refractive indices of the two lenses is***

Sol: Let f_1 and f_2 be the focal lengths in water. Then

$$\frac{1}{f_1}=\left(\frac{\mu_1}{\mu_w}-1\right)\left(\frac{1}{R}+\frac{1}{R}\right)\Rightarrow\frac{1}{f_1}=\left(\frac{\mu_1}{\mu_w}-1\right)\left(\frac{2}{R}\right)\rightarrow(1)$$

$$\frac{1}{f_2}=\left(\frac{\mu_2}{\mu_w}-1\right)\left(-\frac{1}{R}-\frac{1}{R}\right)\ \frac{1}{f_2}=\left(\frac{\mu_2}{\mu_w}-1\right)\left(-\frac{2}{R}\right)\rightarrow(2)$$

Adding Eqs. (1) and (2) we get

$$\frac{1}{f_1}+\frac{1}{f_2}=\frac{2(\mu_1-\mu_2)}{\mu_w R}$$

But the given system is equal to combination of two lens kept in contact in liquid so

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\ \ or\ \ \frac{1}{30}=\frac{2(\mu_1-\mu_2)}{\mu_w R}$$

$\therefore (\mu_1 - \mu_2) = \frac{\mu_w R}{60}$; substituting the values we get $(\mu_1 - \mu_2) = \frac{4 \times 1}{3 \times 60} = \frac{1}{3}$

W.E-93: ***A converging lens of focal length 5.0cm is placed in contact with a diverging lens of focal length 10.0cm. Find the combined focal length of the system.***

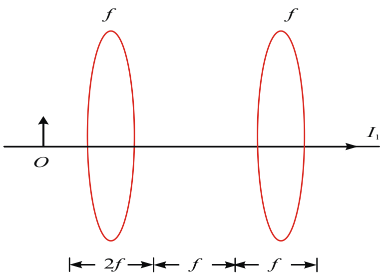
Sol: Here $f_1 = +5.0cm$ and $f_2 = -10.0cm$

Therefore, the combined focal length F is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{5.0} - \frac{1}{10.0} = +\frac{1}{10.0}$

$\therefore\ F = +10.0cm$ ie. the combination behaves as a converging lens of focal length 10.0cm.

W.E-93: ***Two thin converging lenses are placed on a common axis, so that the centre of one of them coincides with the focus of the other. An object is placed at a distance twice the focal length from the left-side lens. Where will its image be? What is the lateral magnification? The focal length of each lens is f.***

Sol:



The image formed by frist lens will be at distance 2f with lateral magnification $m_1 = -1$.For second lens this image will behave as a virtual object. Using the lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ we have, $\frac{1}{v} - \frac{1}{f} = \frac{1}{f}$

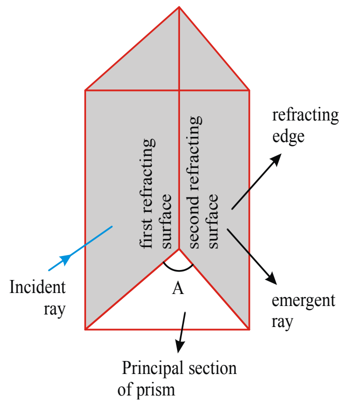
$\therefore v = \frac{f}{2}; m_2 = \frac{v_2}{u_2} = \frac{(f/2)}{f} = \frac{1}{2}$

Therefore, final image (real) is formed at a distance f/2 right side of the second lens wiht total lateral magnification,

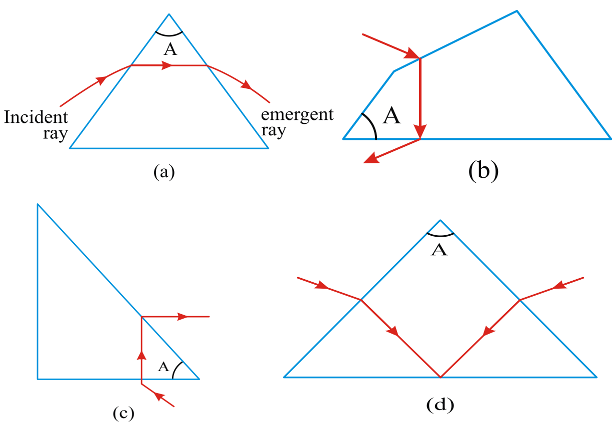
$m = m_1 \times m_2 = (-1) \times \left(\frac{1}{2}\right) = -\frac{1}{2}$

Refraction through Prism

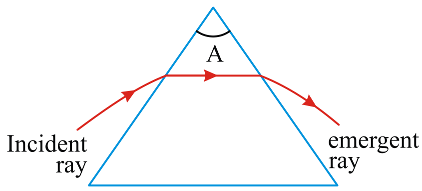
* Prism is a transparent medium bounded by any number of surfaces in such a way that the surface on which light is incident and the surface from which the light emerges are plane and non-parallel as shown in figure.



* The plane surface on which light is incident and emerges are called refracting faces.
* The angle between the faces on which light is incident and from which it emerges is called refracting angle or apex angle or angle of prism (A).
* The two refracting surfaces meet each other in a line called refracting edge.
* A section of the prism by a plane perpendicular to the refracting edge is called principal section



* Angle of deviation (δ) means the angle between emergent and incident rays. While measuring the deviation value in anticlock wise direction is taken as positive and clock wise direction is negative.

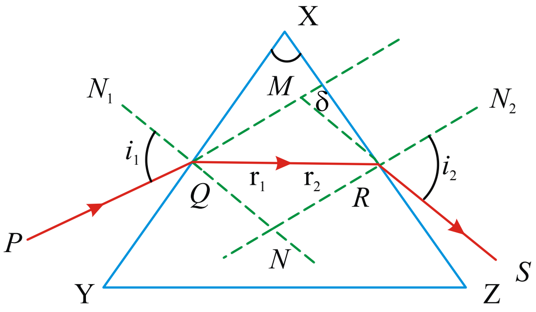


δ = deviation angle

Note: If refractive index of the material of the prism is equal to that of sorroundings, no refraction at its surfaces will takes place and light will pass through it undeviated ie. $[\delta = 0]$.

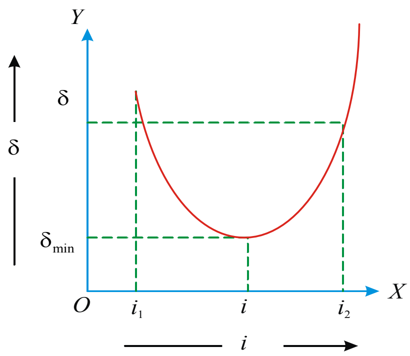
vii) Generally we use equilateral or right angled or Isosceles prism.

Determination of Refrative index of material of the prism for minimum deviation



* **Minimum Deviation**

From the equation $\delta = (i_1 + i_2) - A$, the angle of deviation δ depends upon angle of incidence (i_1). If we determine experimentally, the angle of deviation corresponding to different angles of incidence and then plot a graph by taking angle of incidence (i) on x-axis , angle of deviation (δ) on y-axis, we get the curve as shown in figure.



by snell's law $\mu = \dfrac{\sin i}{\sin r} = \dfrac{\sin i_1}{\sin r_1} = \dfrac{\sin i_2}{\sin r_2}$

$$\boxed{\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}} \quad \boxed{\frac{\mu_p}{\mu_m} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}}$$

Note: Deviaiton produced by small angled prism for small angle, from equation above

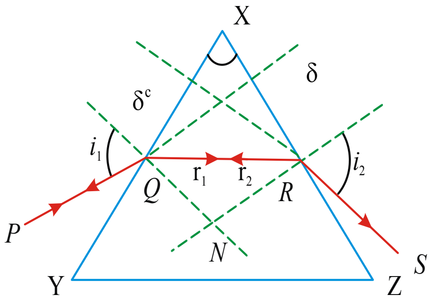
$\mu = \dfrac{i_1}{r_1} = \dfrac{i_2}{r_2}; i_1 = \mu r_1, i_2 = \mu r_2$ But $\delta = (i_1 + i_2) - A$

$\delta = \mu r_1 + \mu r_2 - A; \delta = \mu(r_1 + r_2) - A$ But $r_1 + r_2 = A$

For a prism immersed in a medium of refractive index μ_m

$$\delta = (\mu - 1)A \Rightarrow \delta = \left(\frac{\mu_p}{\mu_m} - 1\right)A$$

Note: There are two values of angle of incidence for same angle of deviation:



When a light ray is incident at an angle i_1 at the surface (xy), it emerges at an angle i_2 from the surface (zx) with a deviation angle δ .As the path of light is reversible, therefore if angle of incidence is i_2, at the face (xy), then the angle of emergence will be i_1, with the same angle of deviation (δ)

Note:

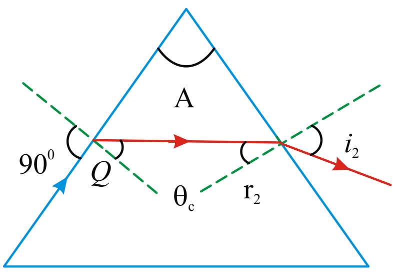
i) For a given material of prism, wave length of light and angle of incidence. When the angle of prism increases angle of deviation also increases as $\delta \propto A$.

ii) With increase in wavelength , deviation decreases ie. deviation for red is least while maximum for violet as $\delta \propto (\mu - 1)$ $\left\{\mu \alpha \frac{1}{\lambda}\right\}$

iii) When a given prism is immersed in liquid, the angle of deviation changes as $\delta \propto (\mu_r - 1)$

* **Maximum deviation:**

Deviation of ray will be maximum when the angle of incidence is maximum i.e $i = 90^0$. Therefore the maximum deviation $\delta_{max} = 90 + i_2 - A$



To find the angle of emergence in this case let us apply Snell's law at second surface.

$$\frac{\mu_a}{\mu} = \frac{\sin r_2}{\sin i_2} = \frac{1}{\mu}$$

As $i_1 = 90^0, r_1 = \theta_C$

Also $r_1 + r_2 = A, \theta_C + r_2 = A$

So, $r_2 = A - \theta_C$

$$\mu \sin(A - \theta_C) = 1 \sin i_2$$

$$i_2 = \sin^{-1}\left[\mu \sin(A - \theta_C)\right]$$

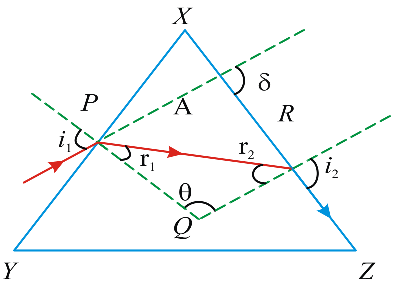
Maximum deviation is

$$\delta_{max} = 90^0 + i_2 - A$$

$\delta_{max} = 90^0 + \sin^{-1}\left[\mu \sin\left(A - \theta_C\right)\right] - A$

Note: Since the law of reversibility always true, then for an angle of incidence $i = \sin^{-1}\left[\mu \sin\left(A - \theta_C\right)\right]$, the ray grazes at the other surface.

* **Condition of grazing emergence:** If a ray can emerge out of a prism, the value of angle of incidence i_1 for which angle of emergence $i_2 = 90^0$ is called condition of grazing emergence. In this situation as the ray emerges out of face XZ, i.e., TIR does not take place at it.



$r_2 < \theta_C \rightarrow (1)$

But as in a prism $r_1 + r_2 = A; r_1 = A - r_2$

So $r_1 = A - (< \theta_C)$ ie. $r_1 > A - \theta_C \rightarrow (2)$

Now from snell's law at face XY, we have $1 \sin i_1 = \mu \sin r_1$

But inview of equation (2)

$\sin r_1 > \sin\left(A - \theta_C\right)$; $\quad \dfrac{\sin i_1}{\mu} > \sin\left(A - \theta_C\right)$

$\sin i_1 > \mu \sin\left(A - \theta_C\right)$

ie. $\sin i_1 > \mu\left[\sin A \cos\theta_C - \cos A \sin\theta_C\right]$

ie $\sin i_1 > \mu\left[(\sin A)\sqrt{\left(1 - \sin^2\theta_C\right)} - \cos A \sin\theta_C\right]$

i.e $\sin i_1 > \left[\sqrt{\left(\mu^2 - 1\right)} \sin A - \cos A\right]$

(as $\sin\theta_C = \left(\dfrac{1}{\mu}\right)$)

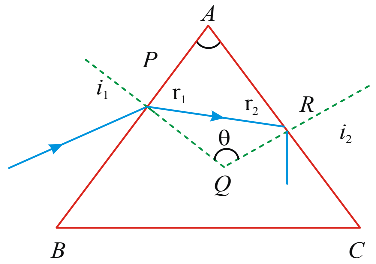
or $i_1 > \sin^{-1}\left[\sqrt{\left(\mu^2 - 1\right)} \sin A - \cos A\right]$

or $\left(i_1\right)_{min} = \sin^{-1}\left[\left(\sqrt{\mu^2 - 1}\right) \sin A - \cos A\right] \rightarrow (3)$

i.e light will emerge out of prism only if angle of incidence is greater than $\left(i_1\right)_{min}$ given by Eq. (3). In this situation deviation will be given by $\delta = \left(i_1 + 90^0 - A\right)$ with i_1 given by Eq.(3)

* **Condition of no emergence:**
The light will not emerge out of a prism for a values of anlge of incidence if at face AB for $i_{1(max)} = 90^0$ at face AC

$r_2 > \theta_C \rightarrow (1)$



Now for Snell's law at face AB, we have $1 \times \sin 90^0 = \mu \sin r_1$

ie. $r_1 = \sin^{-1}\left(\frac{1}{\mu}\right)$; or $r_1 = \theta_C \rightarrow (2)$

From eq. (1) and (2) ; $r_1 + r_2 > 2\theta_C \rightarrow (3)$

However in prism ; $r_1 + r_2 = A \rightarrow (4)$

So from eq. (3) and (4) ; or $A > 2\theta_C \rightarrow (5)$

$\frac{A}{2} > \theta_C \; or \sin\left[\frac{A}{2}\right] > \sin\theta_C \Rightarrow \sin\left(\frac{A}{2}\right) > \frac{1}{\mu}$

i.e. $\mu > \left[\cos ec\left(\frac{A}{2}\right)\right] \rightarrow (6)$

i.e., A ray of light will not emerge out of a prism (what ever be the angle of incidence) if $A > 2\theta_C$, i.e if $\mu > \cos ec\left(\frac{A}{2}\right)$ (or)$\mu = \sqrt{\cot^2(A/2) + 1}$

Note: Limiting Angle : In order to have an emergent ray, the maximum angle of the prism is $2\theta_C$, where θ_C is the critical angle of the prism w.r.t the surrounding medium $2\theta_C$ is called the limiting angle of the prism.

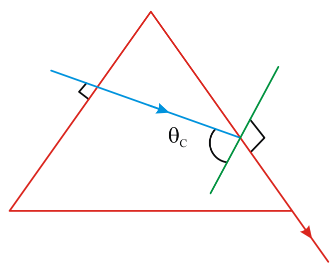
Note: If the angle of incidence at first surface i is such that

a) If $i = \sin^{-1}\left[\mu \sin(A - \theta_C)\right]$, the ray grazes at the other surface.

b) If $i > \sin^{-1}\left[\mu \sin(A - \theta_C)\right]$, then the ray emerges out of a prism from the other surface.

c) If $i < \sin^{-1}\left[\mu \sin(A - \theta_C)\right]$, the ray under go TIR at the other surface.

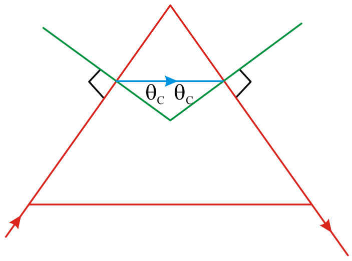
Note: Normal incidence- grazing emergence: If the incident ray falls normally on the prism and grazes from the second surface, then



a) $i_1 = r_1 = 0, i_2 = 90^0 \; and \; r_2 = \theta_C = A$

b) $A = \theta_C = \sin^{-1}\left(\frac{1}{\mu}\right)$ c)Deviation $d = 90 - \theta_C$

Note: Grazing incidence - grazing emergence: If the incident ray falls on the prism with grazing incidence and grazes from the second surface, then



(i) $i_1 = i_2 = 90^0$ (ii) $r_1 = r_2 = \theta_C$

(iii) Angle of prism $A = 2\theta_C$

(iv) Deviation $d = 180 - 2\theta_C = 180 - A$

W.E-94: ***An equilateral glass prism is made of a material of refractive index 1.5. Find its angle of minimum deviation.***

Sol: $A = 60^0, \mu = 1.5, \delta_{\min} = ?$

Substituting $\mu = \dfrac{\sin\left(\dfrac{A+\delta_{\min}}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$1.5 = \dfrac{\sin\left(\dfrac{60^0+\delta_{\min}}{2}\right)}{\left(\sin\dfrac{60^0}{2}\right)}$ $1.5\sin 30^0 = \sin\left(\dfrac{60^0+\delta_{\min}}{2}\right)$

$\sin\left(\dfrac{60^0+\delta_{\min}}{2}\right) = 1.5\times 0.5000 = 0.75$

$\dfrac{60^0+\delta_{\min}}{2} = 48^0 35'$

$60^0 + \delta_{\min} = 97^0 10' \Rightarrow \delta_{\min} = 37^0 10'$

W.E-95: ***A prism of refracting anlge 4^0 is made of a material of refractive index 1.652, Find its angle of minimum deviation.***

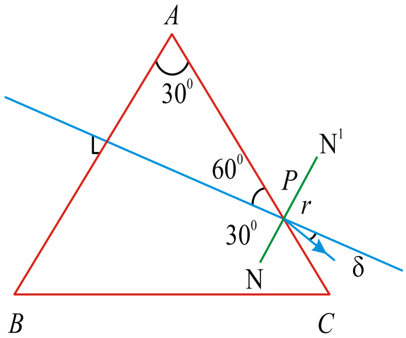
Sol: $A = 40^0,\ \mu = 1.652, \delta = ?$

Substituting in $\delta = A(\mu - 1) = 4^0(1.652 - 1)$

$= 4^0 \times 0.652 = 2.608^0$

W.E-96: ***A ray of light is incident normally on one of the faces of a prism of apex angles 30^0 and refractive index $\sqrt{2}$. The angle of deviation of the ray is......degree***

Sol: Apply Snell's law of refraction at P



$$\frac{\sin 30^0}{\sin r} = \frac{1}{\sqrt{2}}$$

or $\sin r = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}} = \sin 45^0$

or $r = 45^0$

$\therefore \delta = r - 30^0 = 45^0 - 30^0 = 15^0$

$\therefore$ Deviation of ray = 15^0

W.E-97: ***A ray of light is incident normally on one of the refracting surfaces of a prism of refracting angle A,. The emergent ray grazes the other refracting surface. Find the refractive index of the material of prism.***

Sol: For normal incidence on one of the refracting faces of the prism, $i_1 = 0$ *and* $r_1 = 0$. But $r_1 + r_2 = A$ when light undergoes refraction through a prism.

$\therefore 0 + r_2 = A;\ r_2 = A$

When the emergent light grazes the second surface , r_2 becomes the critical angle (C)

i.e. $C = A$ and $\mu = \frac{1}{\sin C} = \frac{1}{\sin A}$

W.E-98: ***A ray of light passing through a prism having $\mu = \sqrt{2}$ suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. What is the angle of prism.***

Sol: As the prism is in 'the position of minimum deviation $\delta_m = (2i - A)$ with $r = A/2$

According to given problem, $i = 2r = A[as\ r = A/2]$

$\delta_m = 2A - A = A$ and hence from

$\mu = \frac{\sin(A+\delta)/2]}{\sin(A/2)}$ i.e $\sqrt{2} = \frac{\sin A}{\sin A/2}(or)$

$\sqrt{2}\sin\frac{A}{2} = 2\sin\frac{A}{2}\cos\frac{A}{2}$ i.e, $\cos\frac{A}{2} = \frac{1}{\sqrt{2}}(or)$

$\frac{A}{2} = \cos^{-1}\left[\frac{1}{\sqrt{2}}\right] = 45^0$ ie. $A = 90^0$

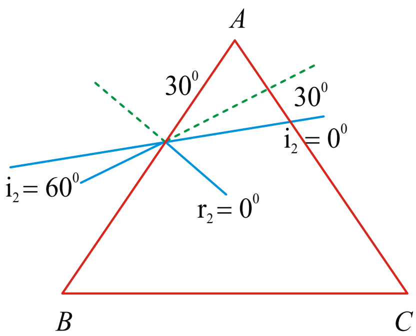
W.E-99: ***A ray of light is incident at an angle of 60^0 on one face of prism of angle 30^0. The***

ray emerging out of the prism makes an angle of 30^0 with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism.

Sol: According to given problem,

$A = 30^0, i = 60^0 \; and \; \delta = 30^0$

and as in prism ,



$\delta = (i_1 + i_2) - A \; ; 30^0 = (60^0 + i_2) - 30^0$; i.e $i_2 = 0^0$

So the emergent ray is perpendicular to the face from which it emerges.

Now as $i_2 = 0, \; r_2 = 0;$ But as $r_1 + r_2 = A, r_1 = A = 30^0$

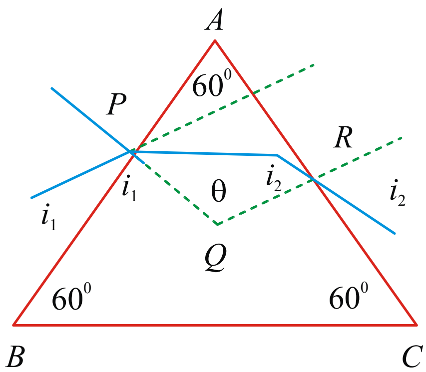
So at first face $1 \sin 60^0 = \mu \sin 30^0$ ie $\mu = \sqrt{3}$

W.E-100: ***A ray of light undergoes deviation of 30^0 when incident on an equilateral prism of refractive index $\sqrt{2}$. What is the angle subtended by the ray inside the prism with the base of the prism?***

Sol: Here $\delta = 30^0$ and $A = 60^0$. So if the prism had been in minimum deviation.

$$\mu = \frac{\sin\left[\left(30^0 + 60^0\right)/2\right]}{\sin\left(60^0/2\right)} = \frac{\sin 45^0}{\sin 30^0} = \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

And as μ of the prism is given to be $\sqrt{2}$



The prism position is in the minimum diviation position implies that

$$r_1 = r_2 = r = \left(\frac{A}{2}\right) = \left(\frac{60^0}{2}\right) = 30^0$$

Therefore, angle subtended by the ray inside the prism with the surface AB, $\left(90^0 - r\right) = \left(90^0 - 30^0\right) = 60^0$ and as base also subtends an angle of 60^0 with the face AB,the ray inside the prism is parallel to the base, ie. the angle subtended by the ray inside the prism with base is zero.

W.E-101: ***A 60^0 prism has a refractive index of 1.5. Calculate (a) the angle of incidence for minimum deviation, (b) the angle of emergence of light at maximum deviation.***

Sol: (a) As the prism is in the position of minimum of deviation, $r=(A/2)=(60^0/2)=30^0$, so that at either face $\sin i=1.5\sin 30^0=0.75$ or $i=\sin^{-1}(0.75)=49^0$

Note: In this situation angle of emergence is equal to angle of incidence = 49^0 and deviation $\delta_m=(2i-A)=(2\times 49-60)=38^0$

b) For maximum deviation, $i_1=90^0$ so that $r_1=\theta_C=\sin^{-1}\left(\frac{2}{3}\right)=42^0$, But as in a prism

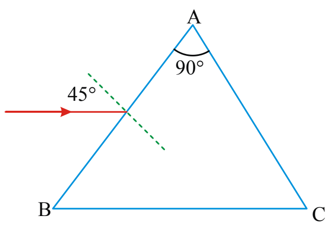
$r_1+r_2=A$ so $r_2=A-r_1=60^0-42^0=18^0$

Now applying Snell's law at the second face,

$\mu\sin r_2=\sin i_2,\ i.e.,\frac{3}{2}\sin 18^0=\sin i_2$

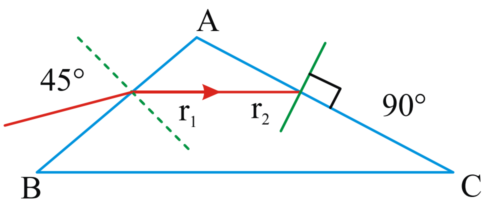
ie. $i_2=\sin^{-1}[1.5\times 0.31]=\sin^{-1}(0.465)\cong 28^0$

W.E-102: ***Monochromatic light falls on a right angled prism at an angle of incidence 45^0. The emergent light is found to slide along the face AC. Find the refractive index of material of prism.***



Sol: Since the emergent light slides along the face AC , angle of emergence is 90^0, as shown . It implies that angle of incidence ray of the ray that falls on face AC is equal to the critical angle $\theta_C \quad \therefore r_2=\theta_C\rightarrow(1)$

From the prism theory, we know



$r_1+r_2=A=90^0 \quad \therefore r_2=90^0-r_1\rightarrow(2)$

From the equations (1) and (2) $90^0-r_1=\theta_C$

$\therefore \sin(90^0-r_1)=\sin\theta_C\,(or)\cos r_1=\sin\theta_C$

But $\sin\theta_C=\frac{1}{\mu}\ \therefore \cos r_1=\frac{1}{\mu}$

Applying Snell's law at the boundary AB, $1\sin 45^0 = \mu \sin r_1 = \mu\sqrt{1-\frac{1}{\mu^2}}$

$\therefore \frac{1}{\sqrt{2}} = \sqrt{\mu^2 - 1}$ or $\mu^2 = 3/2 = 1.5 \Rightarrow \mu = \sqrt{1.5}$

W.E-103: ***The refractive index of a prism is 2. This prism can have what maximum refracting angle?***

Sol: Critical angle

$$\theta_C = \sin^{-1}\left(\frac{1}{\mu}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30^0$$

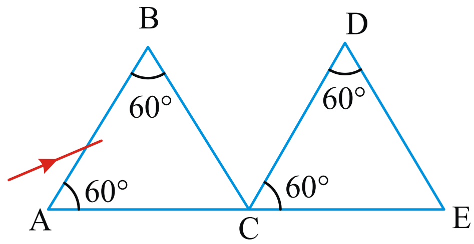
If $A > 2\theta_C$ the ray does not emerge from the prism. So, maximum refracting angle can be 60^0.

W.E-104: ***For an equilateral prism, it is observed that when a ray strikes grazingly at one face it emerges grazingly at the other. Its refractive index will be***

Sol: $i_1 = i_2 = 90^0, r_1 = r_2 = \frac{A}{2} = 30^0$

$$\Rightarrow \mu = \frac{\sin i_1}{\sin r_1} = 2$$

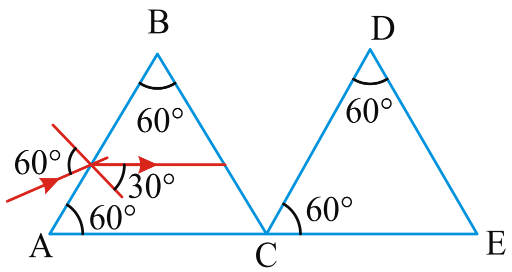
W.E-105: ***Two identical prisms of refractive index $\sqrt{3}$ are kept as shown in figure. A light ray strikes the first prism at face AB. Find,***



i) The angle of incidence, so that the emergent ray from the first prism has minimum deviation

ii) Through what angle of prism DCE should be rotated about C so that the final emergent ray also has minimum deviation.

Sol:



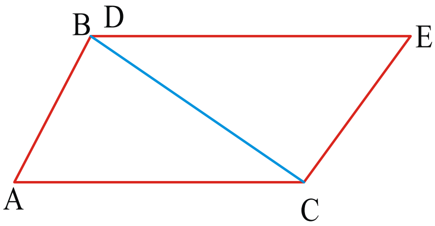
i) At minimum deviation $r_1 = r_2 = 30^0$

For Snell's law, $\mu = \frac{\sin i_1}{\sin r_1}$

or $\sqrt{3} = \frac{\sin i_1}{\sin 30^0}$ or $\sin i_1 = \frac{\sqrt{3}}{2} = \sin 60^0$

$\therefore i_1 = 60^0$

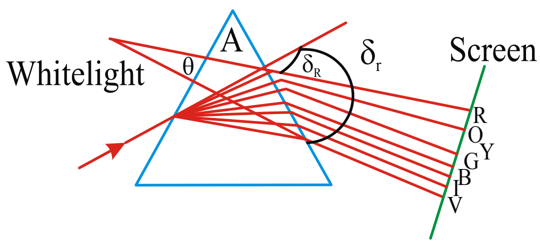
ii) Rotation of DCE about C for final emergent ray to have minimum deviation:



The figure displays the position in which net deviation suffered by the ray of light is minimum. This is achieved when the second prism is rotated anticlockwise by 60^0 about C.

* **Dispersion by A Prism :**

When white light passes through a prism it splits up into different component colours. This phenomenon is called dispersion and arises due to the fact that refractive index of prism is different for different wave lengths. So different wave lengths in passsing through a prism are deviated through different angles and as $\delta \propto (\mu - 1)$, violet is deviated most while red is least deviated giving rise to display of colours known as spectrum. The spectrum consists of visible and invisible regions.



In visibile spectrum the deviation and the refractive index for the yellow ray are taken as the mean values. If the dispersion in a medium takes place in the order given by "VIBGYOR" it is called normal dispersion. If however, the dispersion does not follow the rule "VIBGYOR", it is said to be anamalous dispersion. A medium which brings about dispersion is called dispersive medium. Prism that separated light accordance to wavelength are known as dispersive prisms. Dispersive prism are mainly used in spectrometers to separate closely adjacent spectral lines. Prisms made of glass used in the visible region for dispersion. Dispersion also occurs in U.V and I.R regions, but materials used for the dispersion are different.

* **Angular dispersion**

The difference in the angles of deviations of any pair of colours is called angular dispersion (θ) for those two colours. If the refractive indices of violet, red and yellow are indicated by $\mu_v, \mu_R and \mu_y$. The deviation δ_y corresponding to yellow colour is taken as mean deviation. The deviations $\delta_v, \delta_R and \delta_y$ can be written as

$\delta_v = (\mu_v - 1)A,\ \delta_R = (\mu_R - 1)A$

and $\delta_y = (\mu_y - 1)A$

Angular dispersion for violet and red

$\theta = (\delta_v - \delta_R) = (\mu_v - \mu_R)A$

Thus the angular dispersion depends on the nature of the material of prism and upon the angle of the prism.. In general the angular dispersion means we consider angular dispersion of violet and red i.e the total angle through which the visible spectrum is spread out.

* **Dispersive Power :**

Dispersive power indicates the ability of the material of the prism to disperse the light rays. It is the ratio of angular dispersion of two extreme colours to their mean deviation

$$\omega = \frac{Angular\ dispersion}{Mean\ deviation}$$

$$\omega = \frac{\delta_v - \delta_R}{\left(\frac{\delta_v + \delta_R}{2}\right)}$$

But the mean colour of red and violet colours is yellow colour, so $\frac{\delta_v + \delta_R}{2} = \delta_y$

So, $\omega = \frac{\theta}{\delta_y} = \frac{\delta_v - \delta_R}{\delta_y}$

where δ_y is the deviation for yellow light

$$\omega = \frac{\mu_v - \mu_R}{(\mu_y - 1)} = \frac{d\mu}{(\mu - 1)}$$

It is seen that the dispersive power is independent of the angle of prism and angle of incidence, but depends on material of prism.

The dispersive power more precisely expressed with reference to C, D and F Fraunhoffer's lines in the solar spectrum. The C,D and F lines lies in the red , yellow and blue regions of the spectrum and their wavelengths are $6563\ \overset{0}{A}$, $5893\ \overset{0}{A}$ and $4861\ \overset{0}{A}$ respectively. Then the dispersive power may be expressed as $\omega = \frac{\mu_F - \mu_C}{\mu_D - 1}$

Where $\mu_D = \frac{\mu_F + \mu_C}{2}$

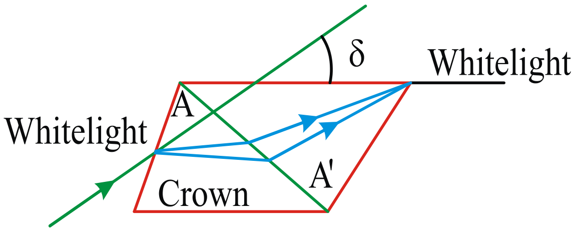
It is noted that a single prism produces both deviation and dispersion simultaneously. However if two prisms (crown and flint) are combined together we can get deviation without dispersion or dispersion without deviation. The dispersive power of flint glass prism is greater than that of crown glass prism for same refracting angle . i.e the angular separation of spectral colours in flint glass is more than crown glass. If two prisms of prism angles A and A' and refractive indices $\mu\ and\ \mu'$ respectively are placed together then the Total deviation

$$\delta = \delta_1 + \delta_2 = (\mu_y - 1)A + (\mu'_y - 1)A'$$

and total dispersion

$$\theta = \theta_1 + \theta_2 = (\mu_V - \mu_R)A + (\mu'_v - \mu'_R)A'$$

Deviation without Dispersion Or achromatic Prism :



An achromatic prism is a combination of two appropriate prisms so constructed that it shows no colours. Flint glasses have higher dispersive power than crown glass. Hence, it is possible to combine two prisms of different materials and specified angles such that ray of white light may pass through the combination without dispersion, though it may suffer deviation. Such a combination is called achromatic combination.

i.e $\delta \neq 0 \ and \ \theta = 0$

$$\therefore (\mu_v - \mu_R) A + (\mu'_v - \mu'_R) A' = 0$$

$$\frac{(\mu_v - \mu_R) A}{(\mu_y - 1)}(\mu_y - 1) + \frac{(\mu'_v - \mu'_R) A}{(\mu'_y - 1)}(\mu'_y - 1) = 0$$

i.e $\omega_C \delta_C + \omega_f \delta_f = 0$

In this case as the deviation produced by flint prism is opposite to crown prism. Therefore the net deviation $\delta = \delta_C - \delta_f$

$$\delta = (\mu_y - 1) A - (\mu'_y - 1) A'$$

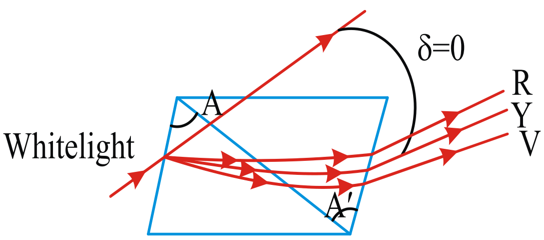
$$\delta = \frac{(\mu_y - 1)}{(\mu_V - \mu_R)}(\mu_V - \mu_R) A$$

$$- \frac{(\mu'_y - 1)}{(\mu'_v - \mu'_R)}(\mu'_v - \mu'_R) A'$$

$$\delta = \frac{\theta_C}{\omega_C} - \frac{\theta_f}{\omega_f}$$

Dispersion without Deviation OR Direct Vision Prism :

If the angles of crown and flint glass prism are so adjusted that the deviation produced for the mean rays by the first prism is equal and opposite to that produced by the second prism, then the final beam will be parallel to the incident beam. Such combination of two prism will produce dispersion of the incident beam without deviation.



i.e $\delta = 0 \ and \ \theta \neq 0$

$$\therefore (\mu_y - 1) A + (\mu'_y - 1) A' = 0$$

$$\frac{(\mu_y - 1)}{(\mu_v - \mu_R)}(\mu_v - \mu_R)A +$$

$$\frac{(\mu'_y - 1)}{(\mu'_v - \mu'_R)}(\mu'_v - \mu'_R)A' = 0$$

ie. $\frac{\theta_C}{\omega_C} + \frac{\theta_f}{\omega_f} = 0$

In this case as the dispersion produced by flint glass prism is opposite to crown glass prism.

Therefore the net angular dispersion $\theta = \theta_C - \theta_f$

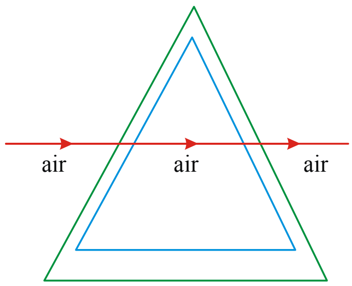
$$\theta = (\mu_V - \mu_R)A - (\mu'_V - \mu'_R)A' (or)$$

$$\theta = \frac{(\mu_V - \mu_R)A}{(\mu_y - 1)}(\mu_y - 1) - \frac{(\mu'_V - \mu'_R)A'}{(\mu'_y - 1)}(\mu'_y - 1)$$

$$\theta = \omega_C \delta_C - \omega_f \delta_f$$

W.E-106: ***A beam of white light passing through a hollow prism gives no spectrum why?***

Sol: ***Light travels from air to air in case of hollow prism. No refraction and no dispersion occur.***



The glass slabs forming the prism are very thin and permit the rays to pass undeviated. Hence a hollow prism gives no spectrum.

W.E-107: ***White light is passed through a prism of angle 5^0. If the refractive indices for red and blue colours are 1.641 and 1.659 respectively. Calculate the angle of dispersion between them.***

Sol: As for small angle of prism $\delta = (\mu - 1)A$

$\delta_b = (1.659 - 1) \times 5^0 = 3.295^0$ and

$\delta_r = (1.641 - 1) \times 5^0 = 3.205^0$ so

$\theta = \delta_b - \delta_r = 3.295^0 - 3.205^0 = 0.090^0$

W.E-108: ***The refractive indices of flints glass prism for C,D and F lines are 1.790, 1.795 and 1.805 respectively. Find the dispersive power of the flint glass prism.***

Sol: $\mu_C = 1.790,\ \mu_v = 1.795\ and\ \mu_F = 1.805$

$$\omega = \frac{\mu_F - \mu_C}{\mu_v - 1} = \frac{1.805 - 1.790}{1.795 - 1} = \frac{0.015}{0.795} = 0.1887$$

W.E-109: ***A thin prism P_1 with angle 4^0 and made from glass of refractive index 1.54 is combined with another prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. What is the angle of the prism P_2?***

Sol: In case of thin prism $\delta = (\mu - 1)A$, when two prisms are combined together.

$$\delta = \delta_1 + \delta_2 = (\mu - 1)A + (\mu' - 1)A'$$

For producing dispersion without deviation

$\delta = 0$, ie. $(\mu' - 1)A' = -(\mu - 1)A$ or

$$A' = -\frac{1.54 - 1}{1.72 - 1} \times 4^0 = -3^0$$

So the angle of the other prism is 3^0 and opposite to the first.

W.E-110: ***A crown glas prism of refracting angle 8^0 is combined with a flint glass prism to obtain deviation without dispersion. If the refractive indices for red and violet rays for crown glass are 1.514 and 1.524 and for the flint glass are 1.645 and 1.665 respectively, find the angle of flint glass prism and net deviation.***

Sol: The condition for deviation without dispersion is $(\mu_v - \mu_R)A = (\mu'_v - \mu'_R)A'$

$$\therefore A' = \frac{(1.524 - 1.514) \times 8^0}{(1.665 - 1.645)} = \frac{0.08^0}{0.02} = 4^0$$

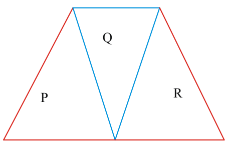
For crown glass $\mu = \frac{1.514 + 1.524}{2} = 1.519$

For flint glass $\mu = \frac{1.645 + 1.665}{2} = 1.655$

$\therefore$ The net deviation $(\delta - \delta') = (\mu - 1)A - (\mu' - 1)A'$

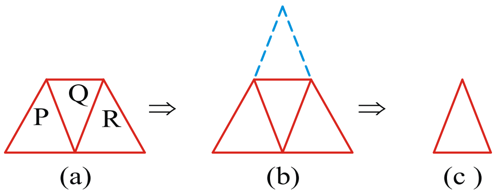
$= 0.159 \times 8^0 - 0.655 \times 4^0 = 1.53^0$

W.E-111: ***A given ray of light suffers minimum deviation in an equilateral prism P. Addtional prism Q and R of identical shape and of the same material as P are now added as shown in figure. The ray will suffer***



1) The greater deviation
2) no deviation
3) same deviation as before
4) total internal reflection

Sol: Figure (a) is part of an equilateral prism of figure
b) as shown in figure which is a magnified image of figure
c) Therefore, the ray will suffer the same deviation in figure(a) and figure (c)



W.E-112: ***Calculate (a) the refracting angle of a flint glass prism which should be combined with a crown glass prism of refracting angle 6^0 so that the combination may not have deviation for D line and (b) the angular seperation between C and F lines,given that the refractive indices of the materials are as follows:***

	C	D	F
Flint	1.790	1.795	1.805
Crown	1.527	1.530	1.535

Sol: Let A_1 and A_2 be the refracting angles of the flint and crown glass prisms respectively. $\mu_1 and \mu_2$ be th refractive indices for the D line of flint and crown glasses respectively. (a) If δ_1 *and* δ_2 be the angles of deviations due to the flint and crown glass prisms respectively, then for no deviation of D line

$$\delta_1 + \delta_2 = 0; A_1\left(\mu_1 - 1\right) + A_2\left(\mu_2 - 1\right) = 0$$

$$\frac{A_1}{A_2} = -\left(\frac{\mu_2 - 1}{\mu_1 - 1}\right)$$

The negative sign indicates that A_1 and A_2 are oppositely directed.

$$\frac{A_1}{6^0} = \left(\frac{1.530 - 1}{1.795 - 1}\right); A_1 = 6^0 \times \frac{0.530}{0.795} = -4^0$$

b) Angular dispersion due to the flint glass prism

$$= A_1\left(\mu_F - \mu_C\right) = -4^0\left(1.805 - 1.790\right) = -0.060$$

Angular dispersion due to the crown glass prism

$$= A_2\left(\mu_F - \mu_C\right) = 6^0\left(1.535 - 1.527\right) = 0.048$$

Net angular dispersion = $0.048 - 0.060 = -0.012$

The negative sign indicated that the resultant dispersion is in the direction of the deviation produced by the flint prism.

* **Optical Instruments :** Optical instruments are used primarily to assist the eye in viewing the object. Optical instruments are classified into three groups, they are

a) visual instruments

Ex: microscope, telescope

b) photographing and projecting instruments

Ex: cameras

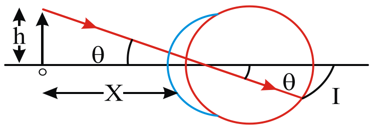
c) analysing and measuring instruments

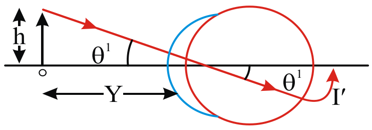
Ex: spectrometer

Optical instruments such as telescope and microscopes have one object lens and one eye lens. The lens towards the object is called objective and the lens towards eye is called eye piece. Single lens forms images with defects (aberrations). If the eye is placed near to the eye lens it will not recieve marginal rays of the eye lens. This reduces the field of view and the intensity is not uniform in the field of view, the central part being brighter than the marginal part.

So in designing telescopes and microscopes for practical purposes, combination of lenses are used for both objective and eye lenses to minimize aberrations. A combination of lenses used as an eye lens is known as eyepiece. In any eyepiece that lens near to the objective is called field lens and the lens near to the eye is called eye lens. The field lens increase the field of view and the eye lens acts as a magnifier. We consider two eyepieces namely, Ramsden's eyepiece and Huygen's eyepiece.

* **The Eye:** The light enters the eye through a curved front surface, called cornea and passes through the pupil which is the central hole in the iris. The size of pupil can change under control muscles. The cornea-lens-fluid system isequivalent to single converging lens.
The light focused by the lens on retina which is a film of nerve fibres. The retina contains rods and cones which sense the light intensity and colour respectively. The retina transmit electrical signals to the brain through optic nerve.
The shape (curvature) and focal length of the eye lens may be adjusted by the ciliary muscles. The image formed by this eye lens is real, inverted and diminished at the retina.
The size of the image on the retina is roughly proportional to the angle subtended by the object on the eye. This angle is known as the visual angle.Therefore it is known as the angular size.



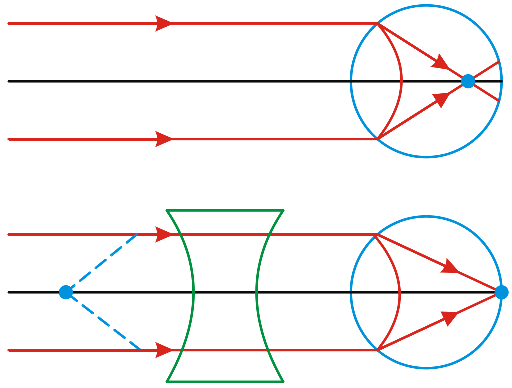


When the object is distant, its visual angle θ and hence image at retina is small and object looks smaller.
When the object is brought near to the eye its visual angle θ and hence size of image will increase and object looks larger as shown in figure (b)
Optical instruments are used to increase this visual angle artificially in order to improve the clarity.
Eg : Microscope, Telescope

When the eye is focussed on a distant object $(\theta \approx 0)$ the ciliary muscles are relaxed so that the focal length of the eye-lens has maximum value which is equal to its distance from the retina.
When the eye is focussed on a closer object (θ increases) the ciliary muscles of the eye are strained and focal length of eye lens decreases. The ciliary muscles adjust the focal length in such a way that the image is again formed on the retina and we see the object clearly. This process of adjusting focal length is called accomodation.
If the object is brought too close to the eye the focal length cannot be adjusted to form the image on the retina.Thus there is a minimum distance for the clear vision of an object.
The nearest point at which an object is seen clearly by the eye is called the near point of the eye and distance of near point from the eye is called the **least distance of distinct vision**, It is equal to 25cm for normal eye and it is denoted by D.
The farthest point from an eye at which an object is distinctly seen is called **far point** and for a normal eye it is theoretically at infinity.
Deffects of Vision: Our eyes are marvellous organs that have the capability to interpret incoming electromagnetic waves as images through a complex process. But over eye may develop some defects due to various reasons. Some common optical defects of the eye are a)myopia b) hypermetropia c) presbyopia

* **Myopia:** The light from a distant object arriving at the eye lens may be converged at a point

infront of the retina. This defect is called **Myopia (or) shortsightedness.** In thi s defect, the far point of the eye is at a distance lesser than infinity, and distant objects are not clearly visible.



This defect is rectified by using spectacles having divergent lens (concave lens) which forms the image of a distant object at the far point of defected eye.

From lens formula

$$\frac{1}{F.P} - \frac{1}{-(distance\ of\ object)} = \frac{1}{f} = P$$

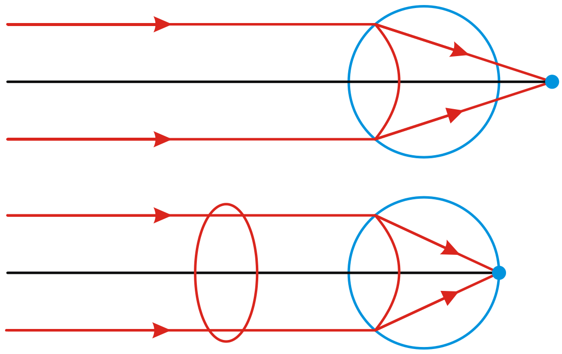
Where F.P= Far point of the defective eye. If the object is at infinity

Power of lens (p)= $\frac{1}{f} = \frac{1}{F.P}$

* **Hypermetropia: (or) Long-sightedness.**

The light from an object at the eye lens may be converged at a point behind the retina. This defect is called

In this type of defect, near point is at a distance greater than 25cm and near objects are not clearly visible.



This defect is rectified by using spectacles having convergent lens(i.e convex lens) which forms the image of near objects at the near point of the defected eye (which is more than 25cm)

$$\frac{1}{-N.P.} - \frac{1}{-(distance\ of\ object)} = \frac{1}{f} = P$$

N.P.= Near Point of defected eye.

If the objective is placed at D=25cm=0.25m

$$P = \frac{1}{f} = \left(\frac{1}{0.25} - \frac{1}{N.P.}\right)$$

* **Presbyopia:** The power of accomodation of eyelens may change due to the decreasing effectiveness of ciliary muscles.So, far point is lesser than infinity and near point is greater than 25cm and both near and far objects are not clearly visible. This defect is called **presbyopia.**This defect is rectified by using bifocal lens.
* **Astigmatism:** This defect arises due to imperfect spherical nature of lens, the focal length of eye lens in two orthogonal directions becomes different, eye cannot see objects in two orthogonal directions clearly simultaneously. This defect is remedied by cylindrical lens in a particular direction.

W.E-113: ***A person cannot see distinctly any object placed beyond 40cm from his eye. Find the power of lens which will enable him to see distant stars clearly is?.***

Sol: The person cannot see objects clearly beyond 0.4m.

so his far point = $0.4m$ distance of object = ∞.

He should use lens which forms image of distant object $(u=\infty)$ at a distance of 40cm infront of it.

$$-\frac{1}{0.40}-\frac{1}{-\infty}=\frac{1}{f}=p\,; \qquad \Rightarrow P=\frac{-10}{4}=-2.5D$$

W.E-114: ***A far sighted person cannot focus distinctly on objects closer than 1m. What is the power of lens that will permit him to read from a distance of 40cm?***

Sol: As near point is 1m and distance of objects is 0.40m both in front of lens.

$$P=\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{-1}-\frac{1}{-0.40}\Rightarrow P=1.5D$$

Simple Microscope :

To view an object with naked eye, the object must be placed between D and infinity. The maximum angle is subtended when it is placed at D.

* **Magnifying power of simple microscope:**

The magnifying power or angular magnification of a simple microscope is defined as the ratio of visual angle with instrument to the maximum visual angle for unaided eye when the object is at least distance of distinct vision.

$$M=\frac{visual\ angle\ with\ instrument}{\max imum\ visual\ angle\ for\ unaided\ eye}$$

$$M=\frac{\theta}{\theta_0}$$

Case(1): When the final image is formed at far point (or) When the fimal image is formed at infinity

In this case $u=f, V=\infty$; So $M_\infty=\frac{D}{f}$

As here u is maximum, magnifying power is minimum and as in this situation parallel beam of light enters the eye, eye is least strained and is said to be normal, relaxed and unstrained.

Case(2): When the final image is formed at near point (or) When the final image is formed at D

$v=-D, u\ \ is-ve$

$$\frac{1}{f}=-\frac{1}{D}-\frac{1}{-u};\ \frac{1}{u}=\frac{1}{f}+\frac{1}{D}$$

$$M_D=D\left(\frac{1}{f}+\frac{1}{D}\right);M_D=\left(1+\frac{D}{f}\right)$$

As the minimum value of v(=D) in this situation u is minimum and magnifying power is maximum and eye is under maximum strain.

Note: If lens is kept at a distance 'a' from the eye then D is replaced by $(D-a)$

$$\boxed{M_D=1+\left(\frac{D-a}{f}\right)};\boxed{M_\infty=\frac{D-a}{f}}$$

* **Some important points regarding microscope:**
* As $M_D=1+\frac{D}{f};M_\infty=\frac{D}{f}$, so $M_D>M_\infty$
* As $M_D=1+\frac{D}{f};M_\infty=\frac{D}{f}$, so smaller the focal length of the lens greater the magnifying power of the simple microscope.
* With increasing wave length of light used, focal length of microscope will increase and hence magnifying power will decrease.
* The maximum possible magnifying power of a simple microscope for a defect-free image is about 4.
* As we use single lens in microscope, the image formed by a single lens possesses several defects like spherical aberration and astigmatism, at larger magnifications the image becomes too defective.
* For higher magnifying power , we cannot use simple microscope, this is because, at larger magnifications the image becomes too defective.So we use compound microscope for higher magnifying power.
* Simple magnifier is an essential part of most of optical instruments such as microscope or telescope in the form of an eye piece.

W.E-115: ***A graph sheet divided into squares each of size $1mm^2$ is kept at a distance of 7cm from a magnifying glass of focal length of 8cm. The graph sheet is viewed through the magnifying lens keeping the eye close to the lens. Find (i) the magnification produced by the lens, (ii) the area of each square in the image formed (iii) the magnifying power of the magnifying lens.Why is the magnification found in (i) different from the magnifying power?***

Sol: i) $u=-7cm;f=+8cm;v=?$

For a lens, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$

$$\frac{1}{+8}=\frac{1}{v}-\frac{1}{-7};\frac{1}{v}=\frac{1}{8}-\frac{1}{7}=\frac{-1}{56};v=-56cm$$

Magnification, $M=\frac{v}{u}=\frac{-56}{-7}=+8$

ii) Each square is of size $1mm^2$ ie. its length and breadth are each to 1mm. The virtual image formed has linear magnification 8. So its length and breadth are each equal to 8mm, The area of the image of each square= $8\times 8mm^2 = 64mm^2$

iii) Magnifying power of the magnifying glass ie. simple microscope.

$$m = 1+\frac{D}{f} = 1+\frac{25}{8} = 4.125\left(\therefore D = 25cm\right)$$

The magnification found in (i) is different from the magnifying power because the image distance in (i) is different from the least distance of distinct vision D.

W.E-116: ***If the focal length of a magnifier is 5cm calculate***

a) the power of the lens

b) the magnifying power of the lens for relaxed and strained eye.

Sol: As power of a lens is reciprocal of focal length in $P = \frac{1}{\left(5\times 10^{-2}m\right)} = \frac{1}{0.05} diopter = 20D$

b) For relaxed eye, MP is minimum and will be

$$MP = \frac{D}{f} = \frac{25}{5} = 5$$

While for strained eye, MP is maximum and will be

$$MP = 1+\frac{D}{f} = 1+5 = 6$$

W.E-117: ***A man with normal near point (25cm) reads a book with small print using a magnifying glass, a thin convex lens of focal length 5cm.***

a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass?

b) What is the maximum and minimum magnifying powe r possible using the above simple microscope?

Sol: a) As for normal eye far and near points are ∞ and 25cm respectively, so for magnifier $v_{max} = \infty$ and $v_{min} = -25cm$.. However , for a lens as $\frac{1}{v}-\frac{1}{u} = \frac{1}{f}$

ie $u = \frac{f}{\left(f/v\right)-1}$ so u will be minimum when

$$v_{min} = -25cm$$

ie. $(u)_{min} = \frac{5}{-\left(5/25\right)-1} = \frac{-25}{6} = -4.17cm$

and u will be maximum when $v_{max} = \infty$

ie $(u)_{max} = \frac{5}{\left(5/\infty\right)-1} = 5cm$

so the closest and farthest distances of the book from the magnifier (or eye) for clear viewing are 4.17cm and 5cm respectively. (b) As in case of simple magnifier MP=(D/u). So MP will be minimum when

$u_{\max} = 5cm$

i.e $(MP)_{\min} = \frac{-25}{-5} = 5\left[\because M = \frac{D}{f}\right]$

and MP will be maximum when $u = \min = (25/6)cm$

$(MP)_{\max} = \frac{-25}{-(25/6)} = 6\left[= 1 + \frac{D}{f}\right]$

Compound Microscope

A simple magnifying lens is not useful where large magnification is required. A highly magnified image must be produced in two stages. A compound microscope is used for that purpose.

* **Magnifying power:**

$$M = \frac{Visual\ angle\ with\ instrument}{Max.visual\ angle\ for\ unaided\ eye} = \frac{\theta}{\theta_0}$$

$$\boxed{M = -\left(\frac{v}{u}\right)\left(\frac{D}{u_e}\right)}$$

Where u is the object distance for the objective lens, v is image distance for the objective lens, u_e is the object distance for the eye piece.

i.e $M = m_0 \times m_e$

The length of the tube $L = v + u_e$

Case(i): If the final image is formed at infinity (far point):

In this case $u_e = f_e$

$\therefore M_\infty = -\frac{v}{u}\left[\frac{D}{f_e}\right]$ with $L_\infty = v + f_e$

A microscope is usually considered to operate in this mode unless stated other wise. In this mode u_e is maximum and hence magnifying power is minimum.

Note: When the object is very close to the principal focus F_0 of the objective, the image due to the objective becomes very close to the eyepiece. Then replace u with f_0 and v_0 with L so the expression for magnifying power. $\boxed{M_\infty \approx -\frac{L}{f_0}\left(\frac{D}{f_e}\right)}$

Case-ii: If the final image is formed at D (Near point):

In this case, for eye piece $V_e = -D, u_e is - ve$

$$-\frac{1}{D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

i.e $\frac{1}{u_e} = \frac{1}{D}\left[1 + \frac{D}{f_e}\right]$; $m = m_o m_e$

$$\therefore M_D = -\frac{v}{u}\left[1 + \frac{D}{f_e}\right] \text{with } L_D = v + \frac{f_e D}{f_e + D}$$

In this situation as u_e is minimum magnifying power is maximum and eye is most strained.

When the object is very close to the principal focus F_0 of the objective,the image due to the objective becomes very close to the eyepiece. Then replace u with f_0 and *v* with L so the expression for magnifying power.

$$\boxed{M_D \approx -\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)}$$

* **Some important points regarding compound microscope:**
* As magnifying power of a compound microscope is negative, the image seen is always truly inverted.
* For a microscope magnifying power is minimum when final image is at ∞ a nd maximum when final image is at least distance of distinct vision D, $i.e$ and $M_{\max} = -\frac{v}{u}\left(1 + \frac{D}{f_e}\right)$
* For a given microscope magnifying power for normal setting remain practically unchanged if field and eye lens are interchanged as $M = \frac{LD}{f_0 f_e}$
* In an actual compound microscope each of the objective and eye piece consists of a combination of several lenses instead of a single lens to eliminate the aberrations and to increase the field of view.
* In low power microscopes, the magnifying power is about 20 to 40, while in high power microscopes, the magnifying power is about 500 to 2000.

W.E-118: ***A microscope consists of two convex lenses of focal lenghts 2cm and 5cm placed 20cm apart. Where must the object be placed so that the final virtual image is at a distance of 25cm from the eye?***

Sol: For the eyepiece, focal length $f = f_e = +5cm$; $v = v_e = -25cm$, $u = u_e = ?$ substituting in

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} ; \frac{1}{5} = \frac{1}{-25} - \frac{1}{u_e}$$

$$\frac{1}{u_e} = -\frac{1}{25} - \frac{1}{5} = \frac{-6}{25}$$

$$u_e = -\frac{25}{6}\text{ cm}$$

object for the eyepiece is to be at a distance of $\frac{25}{6}$ cm to its left.

But $v_0 + u_e = 20cm$ where $u_e = \frac{25}{6}$ cm

$$v_0 = 20 - u_e = 20 - \frac{25}{6} = \frac{95}{6}\text{ cm}$$

For the objective, $v = v_0 = +\frac{95}{6}$ cm

$$f = f_0 = +2cm;\ u = ?$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u};\ \frac{1}{2} = \frac{6}{95} - \frac{1}{u}$$

$$\frac{1}{u} = \frac{6}{95} - \frac{1}{2} = -\frac{83}{190};\ u = -\frac{190}{83} = -2.29cm$$

The object is to be placed at a distance of 2.29cm to the left side of the objective.

W.E-119: ***Find the magnifying power of a compound microscope whose objective has a focal power of 100Dand eye piece has a focal power of 16D when the object is placed at a distance of 1.1cm from the objective. Assume that the final image is formed at the least distance of distinct vision (25cm)***

Sol: The magnifying power of a compound microscope when the final image forms at the least distance of distinct vision,

$$m = \frac{v_0}{u}\left(1 + \frac{D}{f_e}\right)$$

To find v_0; power of the objective $p_0 = 100D$. Focal length of the objective,

$$f = f_0 = \frac{1}{p_0} = \frac{1}{100}m = \frac{100}{100}cm = 1cm$$

$$u = u_0 = -1.1cm;\ v = v_0 = ?$$

For a lens, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{1} = \frac{1}{v_0} - \frac{1}{-1.1};\ \frac{1}{v_0} = \frac{1}{1} - \frac{1}{1.1} = \frac{0.1}{1.1}$$

$$v_0 = 11cm$$

Power of the eyepiece, $p_e = 16D$;focal length of the eye piece.

$$f_e = \frac{1}{p_e} = \frac{1}{16}m = \frac{100}{16}cm = 6.25cm$$

Least distance of distinct vision, $D = 25cm$

$$\therefore m=\frac{11}{-1.1}\left(1+\frac{25}{6.25}\right)=-10\times 5=-50$$

W.E-120: ***In a compound microscope, the object is 1cm from the objective lens. The lenses are 30 cm apart and the intermediate image is 5cm from the eye piece. What magnification is produced?***

Sol: As the lenses are 30cm apart and intermediate image is formed 5cm in front of eye lens,

$u_e=5cm$ and $v=L-u_e=30-5=25cm$

Now as in case of compound microscope,

$$M=m_0\times m_e=-\frac{v}{u}\times\left[\frac{D}{u_e}\right]$$

here $u=1cm$ and $D=25cm$

So $M=-\frac{25}{1}\times\left[\frac{25}{5}\right]=-125$

Negative sign implies that final image is inverted.

W.E-121: ***A compound microscope has a magnifying power 30. The focal length of its eyepiece is 5cm. Assuming the final image to be at the least distance of distinct vision (25cm), calculatethe magnification produced by objective.***

Sol: In case of compound microscope,

$M=m_0\times m_e\rightarrow(1)$

And in case of final image at least distance of distinct vision,

$$m_e=\left[1+\frac{D}{f_e}\right]\rightarrow(2)$$

so, from eqs. (1) and (2), $M=m_0\left[1+\frac{D}{f_e}\right]$

Here $M=-30;\ D=25cm$ and $f_e=5cm$

So, $-30=m\left[1+\frac{25}{5}\right]$ ie $m_0=\frac{-30}{6}=-5$

Negative sign implies that image formed by objective is inverted.

W.E-122: ***A compound microscope is used to enlarge an object kept at a distance 0.03m from its objective which consists of several convex lenses in contact and has focal length 0.02m. If a lens of focal length 0.1m is removed from the objective, find out the distance by which the eyepiece of the microscope must be moved to refocus the image?***

Sol: If initially the objective forms the image at distance v_1. $\frac{1}{v_1}-\frac{1}{-3}=\frac{1}{2}$ ie $v_1=6cm$

Now as in case of lenses in contact

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}+\text{..........}\ \text{or}\ \frac{1}{F}=\frac{1}{f_1}+\frac{1}{F'}$$

with $\frac{1}{F'}=\frac{1}{f_2}+\frac{1}{f_3}+\text{........}$

So if one of the lenses is removed, the focal length of the remaining lens system

$\frac{1}{F'}=\frac{1}{F}-\frac{1}{f'}=\frac{1}{2}-\frac{1}{10}$ ie $F'=25cm$

This lens will form the image of same object at a distance v_2 such that

$\frac{1}{v_2}-\frac{1}{-3}=\frac{1}{2.5}$ ie v_2=15cm

So to refocus the image, eyepiece must be moved by the same distance through which the image formed by the objective has shifted ie. 15-6=9cm away from the objective.

W.E-123: ***The focal lengths of the objective and the eyepiece of a compound microscope are 2.0cm and 3.0cm respectively. The distance between the objective and the eye-piece is 15.0cm. The final image formed by the eyepiece is at infinity. Find the distance of object and image produced by the objective, from the objective lens.***

Sol: As final image is at infinity, the distance of intermediate image from eye lens u_e will be given by

$\frac{1}{-\infty}-\frac{1}{u_e}=\frac{1}{f_e}$ ie $u_e=-f_e=-3cm$

and as the distance between the lenses is 15.0cm , the distance of intermediate image (formed by objective) from the objective will be

$v=L-u_e=L-f_e=15-3=12cm$

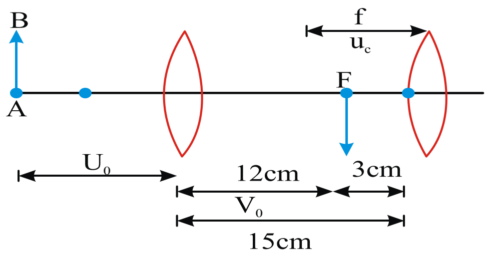
and if u is the distance of object from objective,

$\frac{1}{12}-\frac{1}{u}=\frac{1}{2}$ ie $u=-24cm$

So object is at a distance of 2.4cm in front of objective.

W.E-124: ***The focal lengths of the objective and the eye piece of a compound microscope are 2.0cm and 3.0cm respectively. The distance between the objective and the eye-piece is 15.0cm. The final image formed by the eyepiece is at infinity. The two lenses are thin. Find the distance, incm, of the object and the image produced by the objective, measured form the objective lens, are respectively.***

Sol: The eyepiece forms the final image at infinity. Its object should therefore lie at its focus. 'F' denotes focus of eyepiece . 'I' denotes image formed by the objective lens which serves as object for eyepiece. It should be at 3cm from eyepiece.



$\therefore v_0$ for objective lens = 15-3-12cm(1)

$\therefore \frac{1}{v_0}-\frac{1}{u_0}=\frac{1}{f_0}$ or

$\frac{1}{12}-\frac{1}{u_0}=\frac{1}{2}\Rightarrow\frac{1}{u_0}=\frac{1}{12}-\frac{1}{2}=\frac{-5}{12}$ or

$u_0=-2.4cm$

From objective lens $u_0=2.4cm$ (to left)

$v_0=12cm$ (to right)

Telescopes : A microscope is used to view the objects placed close to it. To look at distant objects such as star, a planet or a cliff etc, we use another optical instrument called telescope, which increases the visual angle of distant object.

The telescope that uses a lens as an objective is called refracting telescope. However, may telescopes use a curved mirror as an objective such telescopes are known as reflecting telescopes. There are three types of refracting telescopes in use.

i) Astronomical telescope
ii) Terrestrial telescope
iii) Galilean telescope

* **Astronomical Telescope :**

Magnifying power (M):

Magnifying power of a telescope is given by

$$M = \frac{Visual\ angle\ with\ instrument}{Visual\ angle\ for\ unaided\ eye} = \frac{\theta}{\theta_0}$$

From the above figure, $\theta_0 = \frac{h}{f_0}$

and $\theta = \frac{h}{-u_e}; M = \frac{\theta}{\theta_0} = \frac{-\left(\frac{h}{u_e}\right)}{\left(\frac{h}{f_0}\right)} = -\frac{f_0}{u_e}$

The length of the tube $L = f_0 + u_e$

Case-i If the final image is at infinity (far point): In this case, for eyepiece $v_e = -\infty$, $u_e = -ve$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e}$$

Hence $u_e = f_e$

Hence $M_\infty = -\frac{f_0}{f_e}$ and $L_\infty = f_0 + f_e$

Usually a telescope is operated in this mode unless stated other wise. In this mode u_e is maximum, hence magnifying power is minimum, while length of tube is maximum.
This case is also called normal adjustment because in this case eye is least strained are relaxed.

Case-ii:If the final image is at D (Near point): In this situation for eyepiece $v_e = -D$

$\frac{1}{-D} - \frac{1}{u_e} = \frac{1}{f_e}$ ie $\frac{1}{u_e} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$

$$M_D = \frac{-f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

In this case length of the tube $L_D = f_0 + \frac{f_e D}{f_e + D}$

In this situation u_e is minimum , hence magnifying power is maximum while the length of the tube is minimum and eye is most strained.

* **Some important points regarding astronomical telescope:**

* In case of telescope if object and final image are at infinity and total light entering the telescope leaves it, parallel to its axis.

$$\therefore magnifying\ power = \frac{f_0}{f_e} = \frac{A_0}{A_e}$$

where A_0 and A_e are the apertures of objectives and eyepiece.

* As magnifying power is negative, the image seen in astronomical telescope is truly inverted i.e left is turned right with upside down simultaneously. However as most of the astronomical objects are symmetrical this inversion does not effect the observations.
* For given telescope, magnifying power is minimum when final image is at infinity (Far point) and maximum when it is at least distance of distinct vision (Near point) ie.

$M_{min} = -\left(\frac{f_0}{f_e}\right)$ and $M_{max} = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$

* In case of a telescope when the final image is at ∞, now if field and eye lenses are interchanged magnifying power will change from $\left(\frac{f_0}{f_e}\right)$ to $\left(\frac{f_e}{f_0}\right)$ ie it will change from m to $\left(\frac{1}{m}\right)$ ie will become $\left(\frac{1}{m^2}\right)$ times of its initial value.

* * As magnifying power for normal setting as $\left(\frac{f_0}{f_e}\right)$ to have large magnifying power f_0 must be as large as practically possible and f_e is small. This is why in a telescope, objective is of large focal length while eyepiece of smaller focal length.

* Larger aperture of objective helps in improving the brightness of image by gathering more light from the distant object. However it increase aberrations particularly spherical.
* If a fly is sitting on the objective of a telescope and we take a photograph of distant astronomical object through it, the fly will not be seen but the intensity of the image will be slightly reduced as the fly will act as obstruction to light and will reduce the aperture of the objective.
* A telescope produces angular magnification whereas a microscope produces linear magnification. The image due to a telescope appears to be near to the eye increasing the visual angle.
* **Terrestrial Telescope:** The magnifying power and length of telescope for relaxed eye will be

$$M_\infty = \frac{-f_0}{f_e}(-1) = \frac{f_0}{f_e}, L_\infty = f_0 + f_e + 4f$$

* The magnifying power and the length of telescope for image at D will be

$$M_D = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right),\ L_D = f_0 + 4f + \frac{Df_e}{D+f_e}$$

W.E-125: ***An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and eye piece is 36cm and the final image is formed at infinity. Determine the focal length of objective and eye piece.***

Sol: For final image at infinity,

$M_\infty = \frac{f_0}{f_e}$ and $L_\infty = f_0 + f_e$ $\quad \therefore 5 = \frac{f_0}{f_e} \to (1)$

and $36 = f_0 + f_e \to (ii)$

Solving these two equations , we have

$f_0 = 30cm \, and \;\; f_e = 6cm$

W.E-126: ***A telescope has an objective of focal length 50cm and an eyepiece of focal length 5cm. The least distance of distinct vision is 25cm. The telescope is focused for distinct vision on a scale 2m away from the objective. Calculate***

a) magnification produced

b) separation between objective and eye piece,

Sol: Given $f_0 = 50cm$ and $f_e = 5cm$

For objective

$$\frac{1}{v_0} - \frac{1}{-200} = \frac{1}{50} \therefore v_0 = \frac{200}{3}cm$$

$$m_0 = \frac{v_0}{u_0} = \frac{(200/3)}{-200} = -\frac{1}{3}$$

For eyepiece:

$$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$$

$\therefore u_e = -\frac{25}{6}cm$ **and** $m_e = \frac{v_e}{u_e} = \frac{-25}{-(25/6)} = 6$

a)Magnification, $m = m_0 \times m_e = -2$

b) Seperation between objective and eyepiece.

$$L = v_0 + |u_e| = \frac{200}{3} + \frac{25}{6} = \frac{425}{6} - 70.83cm$$

W.E-127: ***A telescope objective of focal length 1m forms a real image of the moon 0.92cm in diameter. Calculate the diameter of the moon taking its mean distance from the earth to be $38 \times 10^4 km$. If the telescope uses an eyepiece of 5cm focal length, what would be the distance between the two lenses for (i) the final image to be formed at infinity (ii) the final image(virtual) at 25 cm form eye.***

Sol: $f_0 = 1m$

object distance from the objective

=distance of the moon from the earth

$= 3.8 \times 10^5 km = 3.8 \times 10^8 m$

image distance from the objective

= focal length of the objective =1m

image size = image diameter= $0.92 \times 10^{-2} m$

object size = object diameter

ie diameter or moon=?

We know that $\frac{Object \;\; diameter}{\operatorname{Im} age \, diameter} = \frac{Object \, dis \tan ce}{\operatorname{Im} age \, dis \tan ce}$

$$\frac{Diameter\ of\ moon}{Image\ diameter} = \frac{3.8\times10^8}{1}$$

$\therefore$ Diameter of moon= $3.8\times10^8 \times$ Image diameter

$= 3.8\times10^8 \times 0.92\times10^{-2} m = 3.946\times10^6 m = 3496km$

i) For normal adjustment, the distance between the two lenses $f_0 + f_e = 100+5 = 105cm$

ii) For the final image at 25cm form the eye, the distance between the two lenses

$$= f_0 + \left(\frac{Df_e}{D+f_e}\right) = 100 + \left(\frac{25\times5}{25+5}\right) = 104.2cm$$

W.E-128: ***In an astronomical telescope, the focal lengths of the objective and the eye piece are 100cm and 5cm respectively. If the telescope is focussed on a scale 2m from the objective, the final image is formed at 25cm from the eye. Calculate (i) the magnification and (ii) the distance between the objective and the eyepiece***

Sol: $f_0 = 100cm ; f_e = 5cm$

To find the image distance due to objective

$u_0 = -2m = -200cm; v_0 = ?$

For a lens $\frac{1}{f} = \frac{1}{u} - \frac{1}{v}; \frac{1}{+100} = \frac{1}{v_0} - \frac{1}{-200}$

$\frac{1}{v_0} = \frac{1}{100} - \frac{1}{200} = \frac{1}{200}; v_0 = 200cm$

Magnifying of objective, $m_0 = \frac{v_0}{v} = \frac{200}{-200} = -1$

To find the object distance for the eyepiece

$v_e = -25cm , u_e = ?$

For a lens $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}; \frac{1}{+5} = \frac{1}{-25} - \frac{1}{u_e}$

$\frac{1}{u_e} = -\frac{1}{25} - \frac{1}{5} = \frac{-6}{25}$ $\qquad u_e = -\frac{25}{6}cm$

Magnification of the eyepiece, $m_e = \frac{v_e}{u_e} = \frac{-25\times6}{-25} = 6$

i) Magnification of the eyepiece, $m_o \times m_e = -1\times6 = -6$

ii) Distance between the objective and the eyepiece $= v_0 + |u_e| = 200 + \frac{25}{6} = 204.2cm$

W.E-129: ***A tower 100m tall at a distance of 3km is seen through a telescope having objective of focal length 140cm and eyepiece of focal length 5cm. What is the size of final image if it is at 25cm from the eye?***

Sol: For objective lens

$\frac{1}{v} - \frac{1}{3\times10^5} = \frac{1}{140}$ ie $v = 140cm = f_0$

so $m_0 = \frac{v}{u} = \frac{140}{-3\times10^5} = -\frac{14}{3}\times10^{-4}$ and as final image is at least distance of distinct vision, so for eye lens, we have

$\frac{1}{-25} - \frac{1}{u_e} = \frac{1}{5}$ ie $u_e = \frac{-25}{6}$

so $m_e = \frac{v_e}{u_e} = \frac{-25}{\left(-\frac{25}{6}\right)} = 6$

and hence, $m = m_0 \times m_e = -\frac{14}{3}\times10^{-4}\times6$

But as $m = \left(\frac{I}{O}\right)$

$I = m\times O = -28\times10^{-4}\left(100\times10^2\right) = -28cm$

Negative sign implies that image is inverted.

W.E-130: ***The diameter of the moon is 3.5×10^3 km and its distance from the earth 3.8×10^5 km. It is seen through a telescope having focal lengths of objective and eye-piece as 4m and 10cm respectively. Calculate (a) magnifying power of telescope(b) angular size of image of moon***

Sol: For normal adjustment

a) $|M| = \frac{f_0}{f_e} = \frac{4\times100}{10} = 40$

b) $L = f_0 + f_e = 400 + 10 = 410cm = 4.10m$

c) As the angle subtended by moon on the objective of telescope $\theta_0 = \frac{3.5\times10^3}{3.8\times10^5} = \frac{3.5}{3.8}\times10^{-2}$ rad

and as $|M| = \left|\frac{\theta}{\theta_0}\right|$, the angular size of final image

$|\theta| = |M|\times\theta_0 = 40\times\frac{3.5}{3.8}\times10^{-2} = 0.3684$ rad

i.e $|\theta| = 0.368\times\frac{180^0}{\pi} = 21^0$

W.E-131: ***An astronomical telescope consisting of an objective of focal lenght 60cm and eyepiece of focal length 3cm is focused on the moon so that the final image is formed at least distance of distinct vision ie 25cm from the eye piece. Assuming the angular diameter of moon is $(1/2)^0$ at the objective, calculate (a) angular size and (b) linear size of image seen through the telescope.***

Sol: As final image is at least distance of distinct vision,

$|M| = \frac{f_0}{f_e}\left[1+\frac{f_e}{D}\right] = \frac{60}{3}\left[1+\frac{3}{25}\right] = 22.4$

Now as by definition $M = \left(\frac{\theta}{\theta_0}\right)$, so the angular size of image

$$\theta = M \times \theta_0 = 22.4 \times \left[\frac{1}{2}\right]^0 = 11.2^0$$

$$= \frac{\pi}{180} \times 11.2 = 0.2 rad$$

And if I is the size of final image which is at least distance of distinct vision $\theta = \left(\frac{I}{25}\right)$

$i.e\ I = 25 \times \theta = 25 \times 0.2 = 5 cm$

PREVIOUS MAINS QUESTIONS

1. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of $9 cms^{-1}$, the speed (in cms^{-1}) with which image moves at that instant is [NA Sep. 03, 2020 (I)]

SOLUTION: (1) Distance of object, $u = -30$ cm

Distance of image, $v = 10$ cm

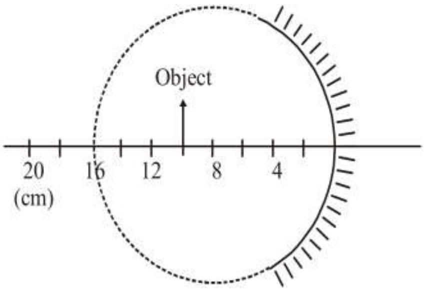
Magnification, $m = \frac{-v}{u} = \frac{(-10)}{-30} = \frac{1}{3}$

Speed of image $= m^2 \times$ speed of object $= \frac{1}{9} \times 9 = 1$ cm s^{-1}

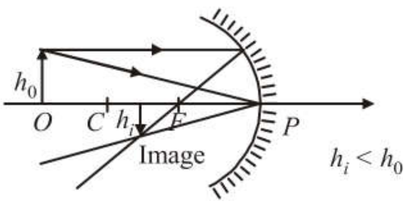
2.A spherical mirror is obtained as shown in the figure from a hollow glass sphere. If an object is positioned in front of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale) [Sep. 02, 2020 (I)]

(a) Inverted, real and magnified (b) Erect, virtual and magnified

(c) Erect, virtual and unmagnified (d) Inverted, real and unmagnified



SOLUTION: (d) Object is placed beyond radius of curvature (*R*) of concave mirror hence image formed is real, inverted and diminished or unmagnified.

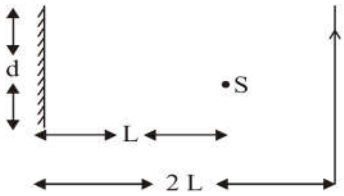


3. A concave mirror for face viewing has focal length of 0.4m.The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is: [9 April 2019 I]

(a) 0.24m (b) 1.60m (c)0.32m (d) 0.16m

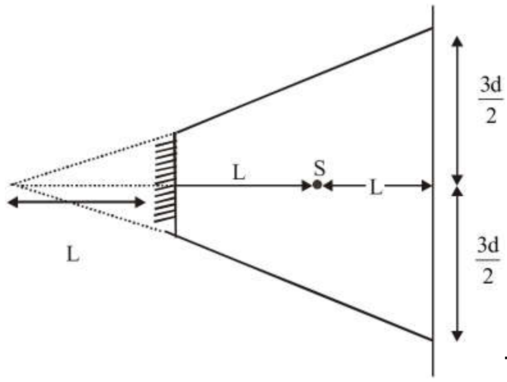
SOLUTION: (c) $+5 = -\frac{v}{u} \Rightarrow v = -5u$ Using $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{-5u} + \frac{1}{u} = \frac{1}{0.4} \Rightarrow u = 0.32m$

4. A point source of light, S is placed at a distance L in front of the center of plane mirror of width d which is hangingvertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below. The distance over which the man can see the image of the light source in the mirror is: [12 Jan. 2019 I]



(a) d (b) $2d$ (c) $3d$ (d) $\frac{d}{2}$

SOLUTION: (c)

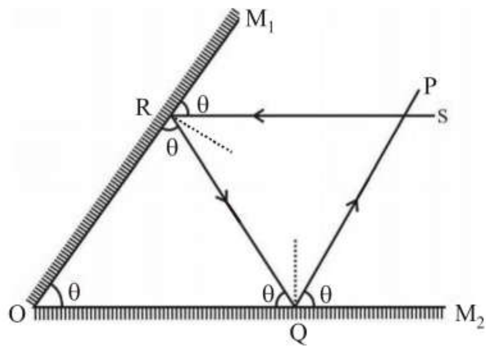


Total distance $=\frac{3d}{2}+\frac{3d}{2}=3d$

5. Two plane mirrors are inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror (M_2) is finallyreflected fiiom the second mirror (M_2) parallel to the first mirror (M1). The angle between the two mirrors will be: [9 Jan. 2019 II]

(a) $45°$ (b) $60°$ (c) $75°$ (d) $90°$

SOLUTION: b)



Let angle between the two mirrors be θ. Ray PQ || mirror M_1 and Rs || mirror M_2

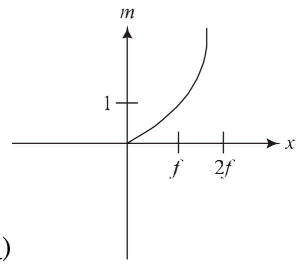
$$M_1Rs=\angle ORQ=\angle M_1OM_2=\theta$$

Similarly, $\angle M_2QP=\angle OQR=\angle M_2OM_1=\theta$

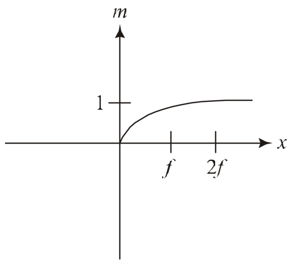
In ΔORQ, $3\theta=180° \Rightarrow \theta=\frac{180°}{3}=60°$

6. An object is gradually moving away from the focal point of a concave mirror along the axis of the mirror. The graphical representation of the magnitude of linear magnification (m) versus distance of the object from the mirror (x) is correctly given by (Graphs are drawn schematically and are not to scale) [8 Jan. 2020 II]

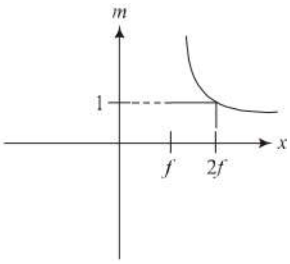
a)



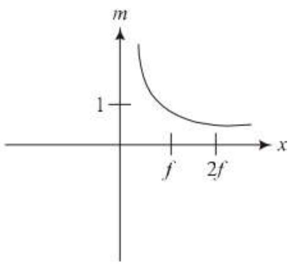
(b)



(c)



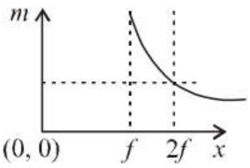
(d)



SOLUTION: (c) Using mirror formula, magnification is given by $m = \frac{f}{u-f} = \frac{-1}{1-\frac{u}{f}}$

At focus magnification is ∞ And at $u = 2f$, magnification is 1.

Hence graph (d) correctly depicts 'm' versus distance of object 'x' graph.



7. A particle is oscillating on the X-axis with an amplitude2 cm about the pointt $x_0 = 10$ cm with a frequency ω. Aconcavemirror offocal length 5 cm is placed at the origin (see figure) Identify the correct statements: [Online Apri115, 2018]

(A) The image executes periodic motion

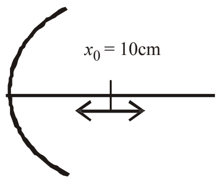
(B) The image executes non-periodic motion

(C) The turning points of the image are asymmetric w.r.to the image ofthe point at $x = 10$ cm

(D) The distance between the turning points of the oscillations of the image is $\frac{100}{21}$

(a) (B), (D) (b) (B), (C)

(c) (A), (C), (D) (d) (A), (D)



SOLUTION: (c) When object is at 8 cm Image $V_1 = \frac{f \times u}{u-f} = \frac{5\times 8}{8-5} = -\frac{40}{3}$ cm

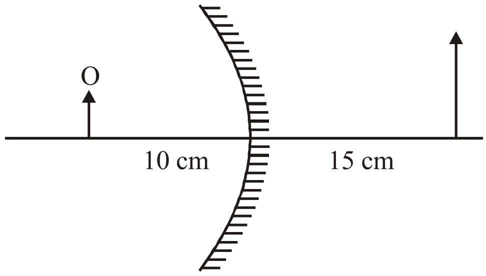
When object is at 12 cm Image $V_2 = \frac{f \times u}{u-f} = \frac{5\times12}{12-5} = -\frac{60}{7}$ cm

Separation $= |V_1 - V_2| = \frac{40}{3} - \frac{60}{7} = \frac{100}{21}$ cm. So A, C and D are correct statements.

8. You are asked to design a shaving mirror assuming that a person keeps it 10 cm from his face and views the magnified image of the face at the closest comfortable distance of 25cm. The radius of curvature of the mirror would then be: [Online Apri110, 2015]

(a) 60 cm (b) -24 cm (c) -60 cm (d) 24 cm

SOLUTION: 8. (c) Convex mirror is used as a shaving mirror.



From question: $v = 15$ cm, $u = -10$ cm

Radius of curvature, $R = 2f = ?$ Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{15} + \frac{1}{(-10)} = \frac{1}{f} \Rightarrow f = -30$ cm Therefore radius of curvature, $R = 2f = -60$ cm

9. A car is fitted with a convex side-view mirror of focal length20 cm. A second car 2.8m behind the first car is overtaking the first car at a relative speed of15 m/s. The speed of the image of the second car as seen in the mirror of the first one is: [2011]

(a) $\frac{1}{15}$ m/s (b) 10 m/s (c) 15 m/s (d) $\frac{1}{10}$ m/s

SOLUTION: (a) From mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating the above equation, we get $\frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$ Also, $\frac{v}{u} = \frac{f}{u-f}$

$$\Rightarrow \frac{dv}{dt} = -\left(\frac{f}{u-f}\right)^2 \frac{du}{dt}$$

$$\Rightarrow \frac{dv}{dt} = \left(\frac{0.2}{2.8-0.2}\right)^2 \times 15 = \frac{1}{15}\text{m/s}$$

10. To get three images of a single object, one should have two plane mirrors at an angle of [2003]

(a) 60^o (b) 90^o (c) 120^o (d) 30^o

SOLUTION: 10. (b) The number of images formed is given by $n = \frac{360}{\theta} - 1$

$$\Rightarrow \frac{360}{\theta} - 1 = 3 \qquad \Rightarrow \theta = \frac{360°}{4} = 90°$$

11. If two plane mirrors are kept at $60°$ to each other, then the number of images formed by them is [2002]

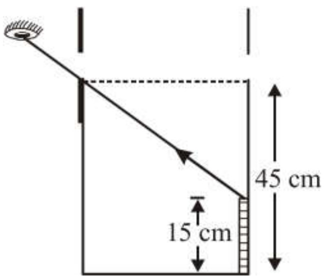
(a) 5 (b) 6 (c) 7 (d) 8

SOLUTION: (a) When two plane mirrors are inclined at each other at an angle θ then the number of the images (n) of a point object kept between the plane mirrors is $n = \frac{360°}{\theta} - 1$, (if $\frac{360°}{\theta}$ is even integer)

Number of images formed $= \frac{360°}{60^o} - 1 = 5$

REFRACTION AT PLANE SURFACE

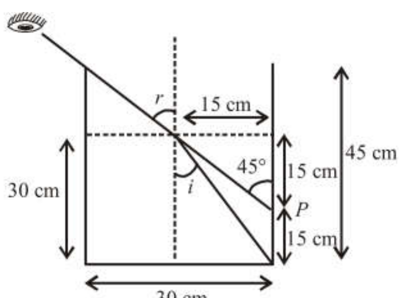
12. An observer can see through a small hole on the side of ajar (radius 15 cm) at a point at height of15 cm from thebottom (see figure). The hole is at a height of45 cm. When the jar is filled with a liquid up to a height of30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N/100, where N is an integer, the value of$N$ is [Sep. 03, 2020 (I)]



……15 cm...

SOLUTION: (158) From figure, $\sin i = \frac{15}{\sqrt{15^2+30^2}}$ and $\sin r = \sin 45°$

From Snell's law, $\mu \times \sin i = 1 \times \sin r \quad \Rightarrow \mu \times \frac{15}{\sqrt{15^2+30^2}} = 1 \times \sin 45° = \frac{1}{\sqrt{2}}$



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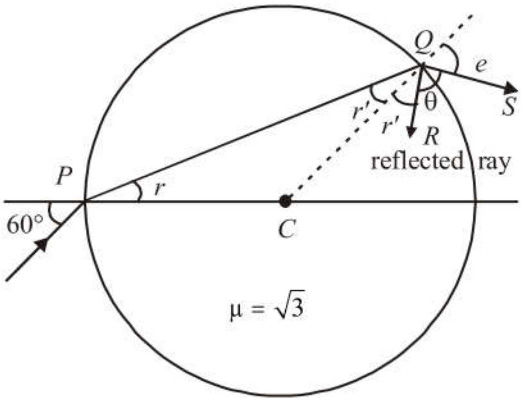
$\mu == 158 \times 10^{-2} = \frac{N}{100}$Hence, value of $N = 158$.

13. A light ray enters a solid glass sphere of refractive index$\mu = \sqrt{3}$ at an angle of incidence $60°$.

The ray is both reflected and refracted at the farther surface of the sphere. The angle (in degrees) between the reflected and refracted rays at this surface is [Sep. 02, 2020 (II)]

SOLUTION: (90.00) In the figure, QR is the reflected ray and QS is refracted ray. CQ is normal.



Apply Snell's law at P $1 \sin 60° = \sqrt{3} \sin r \quad \Rightarrow \sin r = \frac{1}{2} \Rightarrow r = 30°$

From geometry, $CP = CQ \quad r' = 30°$

Again, apply smell's law at Q, $\sqrt{3} \sin r' = 1 \sin e \quad \Rightarrow \frac{\sqrt{3}}{2} = \sin e \Rightarrow e = 60°$

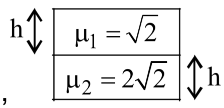
From geometry $\quad r' + \theta + e = 180°$ (As angles lies on a straight line)

$$\Rightarrow 30° + \theta + 60° = 180° \Rightarrow \theta = 90°.$$

14. A vessel of depth $2h$ is halffilled with a liquid of refractive index $2\sqrt{2}$ and the upper half with another liquid refractive index $\sqrt{2}$. The liquids are immiscible. The apparent depth of the inner surface of the bottom of vessel will be: [9 Jan. 2020 I]

(a) $\frac{h}{\sqrt{2}}$ (b) $\frac{h}{2(\sqrt{2}+1)}$ (c) $\frac{h}{3\sqrt{2}}$ (d) $\frac{3}{4} h\sqrt{2}$

SOLUTION: (d) Apparent depth,

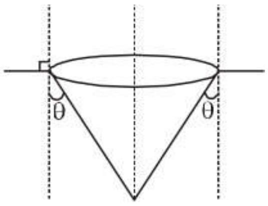


$$D = \frac{t_1}{\mu_1} + \frac{t_2}{\mu_2} = \frac{h}{\sqrt{2}} + \frac{h}{2\sqrt{2}} = \frac{3h}{2\sqrt{2}} = \frac{3h\sqrt{2}}{4}$$

15. There is a small source of light at some depth below the surface of water (refractive index $= \frac{4}{3}$) in a tank of large cross-sectional surface area. Neglecting any reflection from the bottom and absorption by water, percentage of light that emerges out of surface is nearly [Use the fact that surface area of spherical cap of height h and radius ofcurvature r is $2\pi rh$] [9 Jan. 2020 II]

(a) 21% (b) 34% (c) 17% (d) 50%

SOLUTION: (c) Given, Refractive index, $\mu = \frac{4}{3}$ $\quad \frac{4}{3} \sin \theta = 1 \sin 90°$



$$\Rightarrow \sin\theta = \frac{3}{4} \Rightarrow \cos\theta = \frac{\sqrt{7}}{4}$$

Solid angle, $\Omega = 2\pi(1 - \cos\theta) = 2\pi\left(1 - \sqrt{7}/4\right)$

Fraction of energy transmitted$= \frac{2\pi(1-\cos\theta)}{4\pi} = \frac{1-\sqrt{7}/4}{2} = 0.17$

Percentage of light emerges out of surface $= 0.17 \times 100 = 17\%$

16. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be: [8 Jan. 2020 I]

(a) $15°$ (b) $30°$ (c) $45°$ (d) $60°$

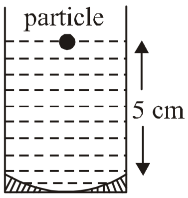
SOLUTION: (b) Here, fiiom question, relative permittivity $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = 3 \Rightarrow \varepsilon = 3\varepsilon_0$

Relative permeability $\mu_r = \frac{\mu}{\mu_0} = \frac{4}{3} \Rightarrow \mu = \frac{4}{3}\mu_0 \;\; \mu\varepsilon = 4\mu_0\varepsilon_0$

$$\sqrt{\frac{\mu_0\varepsilon_0}{\mu\varepsilon}} = \frac{v}{c} = \frac{1}{2}\left(\because c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\right) \quad n = \sqrt{\mu_r\varepsilon_r} = \sqrt{\frac{4}{3} \times 3} = 2$$

And $n = \frac{1}{\sin\theta_c}$ $\Rightarrow \sin\theta_c = \frac{1}{n} = \frac{1}{2}$ Critical angle, $\theta_c = 30°$

17. A concave mirror has radius of curvature of40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure). If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is ate distance d from the surface of water. The value of d isclose to: [12 Apr. 2019 I] (Refractive index of water $= 1.33$)



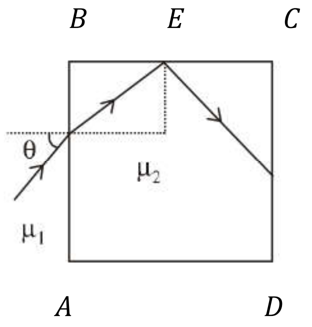
(a) 6.7 cm (b) 13.4 cm (c) 8.8 cm (d) 11.7 cm

SOLUTION:. (c) If v is the distance of image formed by mirror, then $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{v} + \frac{1}{-5} = \frac{1}{-20}$

$\therefore v = \frac{20}{3}$ cm Distance of this image from water surface$= \frac{20}{3} + 5 = \frac{35}{3}$ cm

Using, $\frac{RD}{AD} = \mu \;\; \therefore AD = d = \frac{RD}{\mu} = \frac{(35/3)}{1.33} = 8.8$ cm

18. A transparent cube offside d, made of a material of refractive index μ_2, is immersed in a liquid of refiactive index $\mu_1 (\mu_1 < \mu_2)$. Aray is incident on the face AB at an angle θ(shown in the figure). Total internal reflection takes place at point E on the face BC.



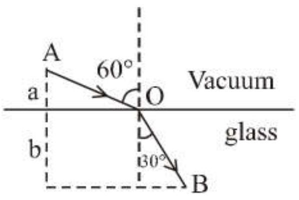
Then θ must satisfy: [12 Apr. 2019 II]

(a) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (c) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$ (d) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

SOLUTION:. (c) Using, $\sin\theta_{\max} = \mu_1\sqrt{\mu_2^2-\mu_1^2} = \sqrt{\frac{\mu_2^2}{\mu_1^2}-1}$

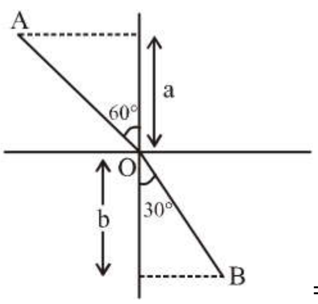
or $\theta_{\max} = \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right)$ For $T_1R,\ \theta < \sin^{-1}\left(\sqrt{\frac{\mu_2^2}{\mu_1^2}-1}\right)$

19. A ray of light AO in vacuum is incident on a glass slab at angle 60^o and refiacted at angle 30^o along OB as shown inthe figure. The optical path length of light ray from A to Bis: [10 Apr. 2019 I]



(a) $\frac{2\sqrt{3}}{a}+2b$ (b) $2a+\frac{2b}{3}$ (c) $2a+\frac{2b}{\sqrt{3}}$ (d) $2a+2b$

SOLUTION:. (d) From the given figure As $\sin 60^o = \mu\sin 30^o$

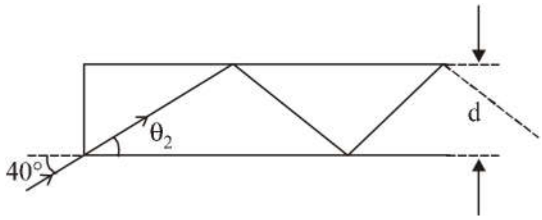


$$\Rightarrow \mu = \frac{\sin 60^o}{\sin 30^o} = \sqrt{3} \quad \frac{a}{AO} = \cos 60^o \Rightarrow AO = 2a$$

$$\frac{b}{BO} = \cos 30^o \Rightarrow BO = \frac{2b}{\sqrt{3}}$$

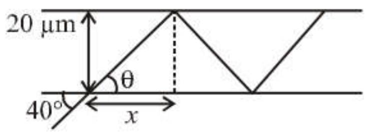
Optical path length $= AO + \mu BO \quad = 2a + (\sqrt{3})\frac{2b}{\sqrt{3}} = 2a + 2b$

20. In figure, the optical fiber is $l=2\mathrm{m}$ long and has a diameter of $d=20\mu\mathrm{m}$. Ifa ray oflight is incident on one end of the fiber at angle $\theta_1=40^\circ$, the number ofreflections it makesbefore emerging from the other end is close to:(refiactive index offiber is 1.31 and $\sin 40^\circ=0.64$) [8 April 2019 I]



(a) 55000 (b) 66000 (c) 45000 (d) 57000

SOLUTION: (d) Using Snell's law of refraction, $1\times\sin 40^\circ=1.31\sin\theta$



$\Rightarrow \sin\theta=\frac{0.64}{1.31}=0.49\approx 0.5 \Rightarrow \theta=30^\circ$ $x=20\mu\mathrm{m}\times\cot\theta$

Number of reflections $=\frac{2}{20\times10^{-6}\times\cot\theta}=\frac{2\times10^{6}}{20\times\sqrt{3}}=57735\approx 57000$

21. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 $\mathrm{V/m}$, then the amplitude of the electric field for the wave propagating in the glass medium will be: [12 Jan. 2019 I]

(a) $30\mathrm{V/m}$ (b) 10 $\mathrm{V/m}$(c) $24\mathrm{V/m}$ (d) 6 $\mathrm{V/m}$

SOLUTION: (c) As 4% of light gets reflected, so only (100-4=96%) of light comes after refraction so,

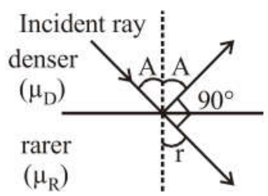
$$\mathrm{P_{refracted}}=\frac{96}{100}\mathrm{P_I} \Rightarrow \mathrm{K_2A_t^2}=\frac{96}{100}\mathrm{K_1A_i^2} \Rightarrow \mathrm{r_2A_t^2}=\frac{96}{100}\mathrm{r_1A_i^2}$$

$\Rightarrow \mathrm{A_t^2}=\frac{96}{100}\times\frac{1}{}\times\frac{3}{2}$ (30)2 $\therefore \mathrm{A_t}\sqrt{\frac{64}{100}\times(30)^2}=24$

22. Let the refractive index of a denser medium with respect toa rarer medium be $\mathrm{n_{12}}$ and its critical angle be θ_C. At an angle of incidence, A when light is travelling from denser medium to rarer medium, a part of the light is reflected and the rest is refracted and the angle between reflected and refracted rays is 90°. Angle A is given by: [Online April 8, 2017]

(a) $\frac{1}{\cos^{-1}(\sin\theta_C)}$ (b) $\frac{1}{\tan^{-1}(\sin\theta_C)}$ (c) $\cos^{-1}(\sin\theta_C)$ (d) $\tan^{-1}(\sin\theta_C)$

SOLUTION: (d)



:

From Snell's law, $\frac{\mu_R}{\mu_D} = \frac{\sin i}{\sin r}$ (i)

$\angle i = A$ and $\angle r = (90° - A)$ We also know that, $\sin\theta_C = \underline{\mu_R}$

From $eq^n(i)$, $\sin\theta_C = \frac{\sin A}{\sin(90°-A)}$ $\sin\theta_C = \frac{\sin A}{\cos A}$ $\sin\theta_C = \tan A$ or $A = \tan^{-1}(\sin\theta_C)$

23. A diver looking up through the water sees the outside world contained in a circular horizon. The refractive index of water is $\frac{4}{3}$, and the diver's eyes are 15 cm below the surface of water. Then the radius of the circle is: [Online April 9, 2014]

(a) $15 \times 3 \times \sqrt{5}$ cm (b) $15 \times 3\sqrt{7}$ cm (c) $\frac{15\times\sqrt{7}}{3}$ cm (d) $\frac{15\times 3}{\sqrt{7}}$ cm

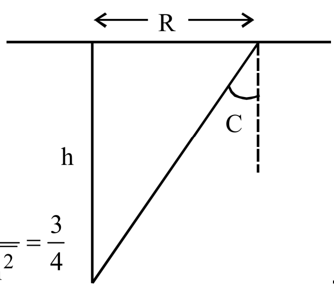
SOLUTION: **(d)** Given, $\mu = \frac{4}{3}$

$h = 15$ cm

$R = ?$

$\frac{\sin 90°}{\sin C} = \mu$

$\Rightarrow \sin C = \frac{1}{\mu} = \frac{R}{\sqrt{R^2 + h^2}} = \frac{3}{4}$

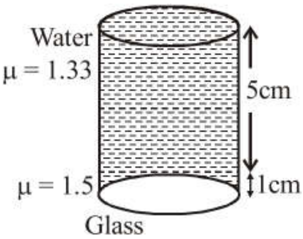


$\Rightarrow 16R^2 = 9R^2 + 9h^2$ O or, $7R^2 = 9h^2$ or, $R = \frac{3}{\sqrt{7}}h = \frac{3}{\sqrt{7}} \times 15\text{cm}$

24. A printed page is pressed by a glass of water. The refractive index of the glass and water is 1.5 and 1.33, respectively. If the thickness of the bottom of glass is 1 cm and depth of water is 5 cm, how much the page will appear to be shifted if viewed *from* the top? [Online April 25, 2013]

(a) 1.033 cm (b) 3.58lcm (c) 1.3533 cm (d) 1.90 cm

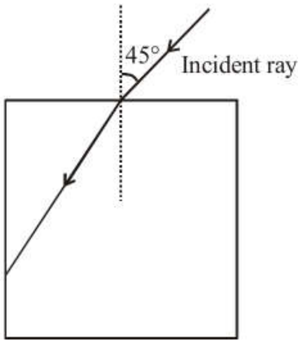
SOLUTION: (c) Real depth $= 5\text{cm} + \text{lcm} = 6$ cm



Apparent depth $= \frac{d_1}{\mu_1} + \frac{d_2}{\mu_2} = \frac{5}{1.33} + \frac{1}{1.5} = 3.8 + 0.7 = 4.5$ cm

Shift $= 6$ cm-4.5cm $\cong 1.5$ cm

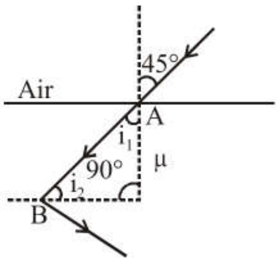
25. A light ray falls on a square glass slab as shown in the diagram. The index refraction of the glass, if total internal reflection is to occur at the vertical face, is equal to: [Online April 23, 2013]



(a) $\frac{(\sqrt{2}+1)}{2}$ (b) $\sqrt{\frac{5}{2}}$ (c) $\frac{3}{2}$ (d) $\sqrt{\frac{3}{2}}$

SOLUTION: (d) At point A by Snell's law $\mu = \frac{\sin 45^\circ}{\sin r} \Rightarrow \sin r = \frac{1}{\mu\sqrt{2}}$ ... (i)

At point B, for total internal reflection, $\sin i_1 = \frac{1}{\mu}$



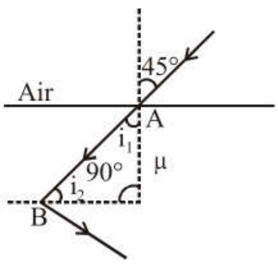
From figure, $i_1 = 90^\circ - r \ \ (\sin 90^\circ - r) = \frac{1}{\mu}$

$\Rightarrow \ \cos r = \frac{1}{\mu}$ ... ... (ii)

Now $\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{1}{2\mu^2}} \ = \sqrt{\frac{2\mu^2 - 1}{2\mu^2}}$ ... (iii)

From equations(ii) and(iii) $\frac{1}{\mu} = \sqrt{\frac{2\mu^2-1}{2\mu^2}}$

Squaring both sides and then solving, we get $\mu = \sqrt{\frac{3}{2}}$



26. Light is incident from a medium into air at two possible angles of incidence (A) 20° and (B) 40°. In the medium light travels 3.0 cm in 0.2 ns. The ray will: [Online April 9, 2013]

(a) suffer total internal reflection in both cases (A) and(B)

(b) suffer total internal reflection in case (B) only

(c) have partial reflection and partial transmission in case(B)

(d) have 100% transmission in case (A)

SOLUTION: (b) Velocity of light in medium$V_{med} = \frac{3cm}{0.2ns} = \frac{3\times10^{-2}m}{0.2\times10^{-9}s} = 1.5$ ims

Refractive index of the medium $\mu = \frac{V_{air}}{V_{med}} = \frac{3\times10^8}{1.5} = \frac{2m}{s}$As $\mu = \frac{1}{\sin C}$

$$\sin C = \frac{1}{\mu} = \frac{1}{2} = 30°$$

Condition of TIR is angle of incidence i must be greater than critical angle. Hence ray will suffer TIR in case of(B) $(i = 40° > 30°)$ only.

27. Let the x-z plane be the boundary between two transparent media. Medium 1 in $z \geq 0$ has a refiactive index of $\sqrt{2}$ and medium 2 with $z < 0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{\imath} + s\sqrt{3}\hat{\jmath} - 10\hat{k}$is incident on the plane of separation. The angle of refraction in medium 2 is: [2011]

(a) $45°$ (b) $60°$ (c) $75°$ (d) $30°$

SOLUTION: (a) As refractive index for $z > 0$ and $z \leq 0$ is different XY plane should be the boundary between two media. Angle of incidence is given by $\cos(\pi - i) = \frac{(6\sqrt{3}\hat{\imath}+8\sqrt{3}\hat{\jmath}-10\hat{k}).\hat{k}}{?n}$

$$-\cos i = -\frac{1}{2}$$

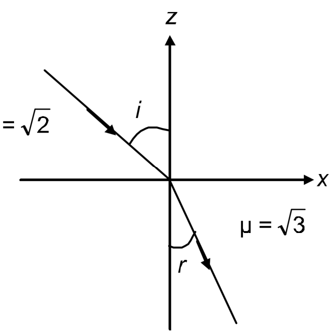
$$\Rightarrow \angle i = 60°$$

From Snell's law,

$$\frac{\sin i}{\sin r} = \frac{u_2}{u_1}$$

$$\Rightarrow \frac{\sin i}{\sin r} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \sqrt{2}\sin i = \sqrt{3}\sin r$$



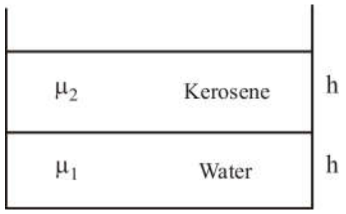
$$\Rightarrow \sqrt{2}\sin 60° = \sqrt{3} \quad \Rightarrow \sqrt{2}\times\frac{\sqrt{3}}{2} = \sqrt{3}\sin r \qquad \Rightarrow \angle r = 45°$$

28. A beaker contains water up to a height h_1 and kerosene of height h_2 above water so that the total height of(water +kerosene) is $(h_1 + h_2)$. Refractive index of water is$\mu_1$and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is [2011 RS]

(a) $\left(1+\frac{1}{\mu_1}\right)h_1 - \left(1+\frac{1}{\mu_2}\right)h_2$ (b) $\left(1-\frac{1}{\mu_1}\right)h_1 + \left(1-\frac{1}{\mu_2}\right)h_2$

(C) $\left(1+\frac{1}{\mu_1}\right)h_2 - \left(1+\frac{1}{\mu_2}\right)h_1$ (d) $\left(1-\frac{1}{\mu_1}\right)h_2 + \left(1-\frac{1}{\mu_2}\right)h_1$

SOLUTION: (b)

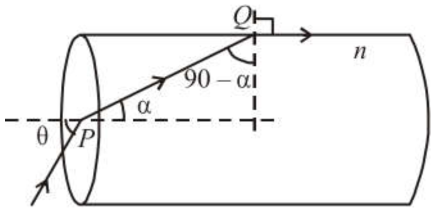


Apparent shift of the bottom due to water, $\Delta h_1 = h_1\left[1-\frac{1}{\mu_1}\right]$

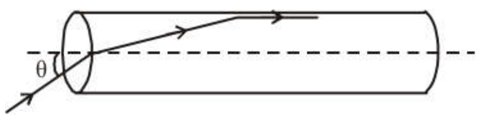
Apparent shift of the bottom due to kerosene, $\Delta h_2 = h_2\left[1-\frac{1}{\mu_2}\right]$

Thus, total apparent shift: $= \Delta h_1 + \Delta h_2$

$$= h_1\left[1-\frac{1}{\mu_1}\right] + h_2\left[1-\frac{1}{\mu_2}\right]$$



29. A transparent solid cylindrical rod has refractive index of $\frac{2}{\sqrt{3}}$ It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.



The incident angle θ for which the light ray grazes along the wall of the rod is: [2009]

(a) $\sin^{-1}(\sqrt{3}/2)$ (b) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$ (c) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (d) $\sin^{-1}(1/2)$

SOLUTION: (c)Applying Snell's law for medium inside the cylinder and air at Q we get

$$n = \frac{\sin 90^\circ}{\sin(90^o - \alpha)} = \frac{1}{\cos\alpha} \therefore \cos\alpha = \frac{1}{n}$$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{1}{n^2}} = \frac{\sqrt{n^2-1}}{n} \ldots \text{(i)}$$

Applying Snell's Law for air and medium inside the cylinder rat P we get $n = \frac{\sin\theta}{\sin\alpha}$

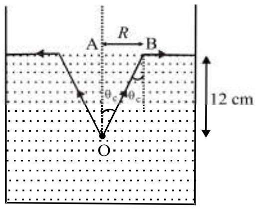
$\Rightarrow \sin\theta = n \times \sin\alpha = \sqrt{n^2-1}$; [from (i)] $\sin\theta = \sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 - 1} = \sqrt{\frac{4}{3}-1} = \frac{1}{\sqrt{3}}$

or $\theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

30. A fish looking up through the water sees the outside world contained in a circular horizon. If the refractive index of water is $\frac{4}{3}$ and the fish is 12 cm below the surface, the radius of this circle in cm is [2005]

(a) $\frac{36}{\sqrt{7}}$ (b) $36\sqrt{7}$ (c) $4\sqrt{5}$ (d) $36\sqrt{5}$

SOLUTION: (a) From the figure it is clear that $\tan\theta_c = \frac{AB}{OA}$ $\Rightarrow R = OA\tan\theta_c$



$\Rightarrow R = \frac{OA\sin\theta_c}{\cos\theta_c}$ $\Rightarrow R = \frac{OA\sin\theta_c}{\sqrt{1-\sin^2\theta_c}}$

$$\Rightarrow \tan\theta_c = \frac{R}{12} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} \therefore \sin\theta_c = \frac{1}{\mu} = \frac{3}{4}$$

$$\Rightarrow \tan\theta_c = \frac{3}{\sqrt{16-9}} = \frac{3}{\sqrt{7}} = \frac{R}{12} \Rightarrow R = \frac{36}{\sqrt{7}} \text{ cm}$$

31. Consider telecommunication through optical fibers. Which of the following statements is not true? [2003]

(a) Optical fibers can be of graded refractive index

(b) Optical fibers are subject to electromagnetic interference $\mathrm{fi_{iO}m}$ outside

(c) Optical fibers have extremely low transmission loss

(d) Optical fibers may have homogeneous core with a suitable cladding.

SOLUTION: (b) Optical fibers form a dielectric wave guide and are free from electromagnetic interference or radio frequency interference. There is extremely low transmission loss in optical fiber.

32. Which of the following is used in optical fibers? [2002]

(a) total internal reflection (b) scattering (c) diffraction (d) refraction.

SOLUTION: (a) In an optical fiber, light is sent through the fiber without any loss by the phenomenon of total internal reflection. Total internal reflection of light waves confines the light rays inside the optical fiber.

REFRACTION AT CURVED SURFACES

33. A point like object is placed at a distance of 1 m in front of a convex lens of focal length $0.5\mathrm{m}$. Alane mirror is placed at a distance of 2 m behind the lens. The position and nature of the final

image formed by the system is: [Sep. 06, 2020 (D)]

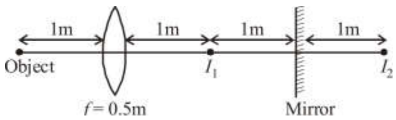
(a) $2.6 \mathrm{m} \mathrm{from}$ the mirror, real

(b) 1 m from the mirror, virtual

(c) 1 m from the mirror, real

(d) $2.6 \mathrm{m} from$ the mirror, virtual

SOLUTION: (d) Focal length of the convex lens, $f=0.5 \mathrm{m}$ Object is at $2 f$ so, image $\left(I_1\right)$ will also be at $2 f$ Image $\mathrm{of} I_1$ i. e., I_2 will be 1 m behind mirror. Now I_2 will be object for lens.



$$u=(-1)+(-1)+(-1)=-3 \mathrm{m}$$

Using lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{+0.5}+\frac{1}{-3}$ or $v=\frac{3}{5}=0.6 \mathrm{m}$

Hence, distance of image $\mathrm{fi:om}$ mirror $=2+0.6=2.6 \mathrm{m}$ and real.

34. A double convex lens has power P and same radii of curvature R ofboth the surfaces. The radius of curvature of surface of a planoconvex lens made of the same material with power $1.5 P$ is: [Sep. 06, 2020 (II)]

(a) $2 R$ (b) $\frac{R}{2}$ (c) $\frac{3 R}{2}$ (d) $\frac{R}{3}$

SOLUTION: (d) Given, using lens maker's formula Here, $R_1=R_2=R$ (For double convex lens)

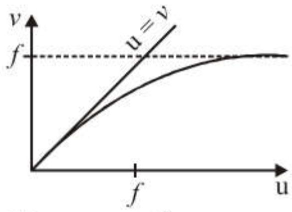
$\frac{1}{f}=(\mu-1)\left(\frac{1}{R}-\frac{1}{-R}\right) \Rightarrow P=\frac{1}{f}=(\mu-1) \frac{2}{R}$ ....(i)

For Plano convex lens,$R_1=R^{\uparrow}$, $R_2=\infty \therefore$ Using lens maker's formula again, we have

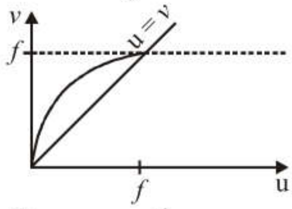
$1.5 P=(\mu-1)\left(\frac{1}{R} 1-\frac{1}{\infty}\right)$.........(ii) $\Rightarrow \frac{3}{2} P=\frac{\mu-1}{R}$, From(i) and(ii), $\frac{3}{2}=\frac{R^{\prime}}{2 R} \Rightarrow R^{\prime}=\frac{R}{3}$

35. For a concave lens of focal length f, the relation betweenobject and image distances u and v, respectively, from its pole can best be represented by ($u=v$is the reference line):[Sep. 05, 2020 (I)]

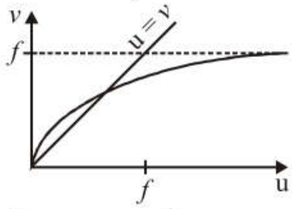
(a)



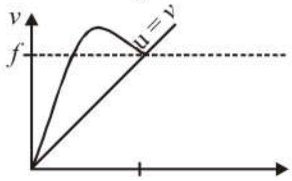
(b)



(c)



(d)



SOLUTION: (d) From lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow v=\frac{uf}{u+f}$

Case-I: If v $=u \Rightarrow f+u=f \Rightarrow u=0$

Case-II: If $u=\infty$ then $v=f$.Hence, correct u versus v graph, that satisfies this condition is (a).

36. The distance between an object and a screen is 100 cm. A lens can produce real image ofthe object on the screen fortwo different positions between the screen and the object. The distance between these two positions is 40 cm. If the power of the lens is close to $\left(\frac{N}{100}\right) D$ where N is an integer, the value of N is [Sep. 04, 2020 (I]

SOLUTION: (476.19)Given, Distance between an object and screen, $D=100$ cm

Distance between the two position of lens, $d=40$ cm

Focal length of lens,$f=\frac{D^2-d^2}{4D}=\frac{100^2-40^2}{4(100)}=\frac{(100+40)(100-40)}{4(100)}=21$ cm

Power, $P=\frac{1}{f}=\frac{100}{21}=\frac{N}{100}$ $\quad N=476.19.$

37. A point object in air is in front of the curved surface of p*lanoconvex* lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is [8 Jan. 2020 I]

SOLUTION: (60)

Given : $\mu=1.5; R_{\text{curved}}=30$ cm Using, Lens-maker formula $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

For plano-convex lens $R_1 \rightarrow \infty$ then $R_2=-R$ $f=\frac{R}{\mu-1}=\frac{30}{1.5-1}=60$ cm

38. A thin lens made of glass (refractive index $= 1.5$) offocal length $f = 16$ cm is immersed in a liquid ofrefiiactive index 1.42. Ifits focal length in liquid isf_l, then the ratio f_l/f isclosest to the integer: [7 Jan. 2020 II]

(a) 1 (b) 9 (c) 5 (d) 17

SOLUTION: (b) Using lens maker's formula $\frac{1}{f} = \left(\frac{\mu_g}{\mu_a} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

Here, μ_g and μ_a are the refractive index of glass and air respectively

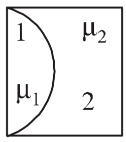
$\Rightarrow \frac{1}{f} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.......... (i)

When immersed in liquid $\frac{1}{f_l} = \left(\frac{\mu_g}{\mu_l} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ [Here, $\mu_l =$ refiactive index ofliquid]

$\Rightarrow \frac{1}{f_l} = \left(\frac{1.\cdot 5}{142} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ....(ii)

Dividing (i) by(ii) $\Rightarrow \frac{f_l}{f} = \frac{(1.5-1)1.42}{0.08} = \frac{1..42}{016} = \frac{142}{16} \approx 9$

39. One Plano-convex and one Planoconcave lens of same radius of curvature'R' but of different materials are joined side by side as shown in the figure. If the refractive index of the material ofl is μ_1 and that of 2 is μ_2, then the focal length of the combination is: [10 Apr. 2019 I]



(a) $\frac{R}{\mu_1 - \mu_2}$ (b) $\frac{2R}{\mu_1 - \mu_2}$ (c) $\frac{2R}{2(\mu_1 - \mu_2)}$ (d) $\frac{R}{2 - (\mu_1 - \mu_2)}$

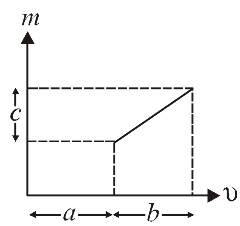
SOLUTION: (a) Focal length of Plano-convex lens- $\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right) = \frac{\mu_1 - 1}{R} \Rightarrow f_1 = \frac{R}{(\mu_1 - 1)}$

Focal length of Plano-concave lens- $\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{\mu_2 - 1}{-R} \Rightarrow f_2 = \frac{-R}{(\mu_2 - 1)}$

For the combination of two lens- $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R} = \frac{\mu_1 - \mu_2}{R}$

$$\Rightarrow f_{eq} = \frac{R}{\mu_1 - \mu_2}$$

40. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focallength of the lens used? [10 Apr. 2019 II]



(a) $\frac{b^2}{ac}$ (b) $\frac{b^2 c}{a}$ (c) $\frac{a}{c}$ (d) $\frac{b}{c}$

SOLUTION: (d) From the equation of line $m = k_1 v + k_2$ $(\because y = mx + c)$

$$\Rightarrow \frac{v}{u} = k_1 v + k_2 \left(\because m = \frac{v}{u}\right)$$

$\Rightarrow \frac{1}{u} = k_1 + \frac{k_2}{v}$ (Dividing both sides by v) $\Rightarrow \frac{k_2}{v} - \frac{1}{u} - k_1$

Comparing with lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, we get $k_1 = \frac{1}{-f}$ and $\mathrm{k}_2 = 1$

$$f = \frac{1}{\text{slopeof}\, m - v\text{graph}} = -\frac{b}{c}$$

41. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and $x_2 (x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is: [9 Apr. 2019 II]

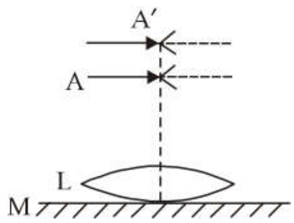
(a) 2: 1 (b) 3: 1 (c) 5: 3 (d) 4: 3

SOLUTION: (b) Using, $M = \frac{v}{u}$ or $-2 = \frac{v_1}{x_1} \Rightarrow v_1 = -2x_1$

We have $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ or $\frac{1}{-2x_1} - \frac{1}{x_1} = \frac{1}{20}$ $\therefore x_1 = 30$ cm And $\frac{1}{2x_2} - \frac{1}{x_2} = \frac{1}{20}$

or $x_2 = -10$ cm So, $\frac{x_1}{x_2} = \frac{30}{10} = 3$

42. A thin convex lens L (refractive index $= 1.5$) is placed on a plane mirror M. When a pin is placed at A, such that OA $= 18$ cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_i is put between the lens and the mirror, the pin has to be moved to A', suchthat OA' $= 27$ cm, to get its inverted real image at A' itself. The value of μ_i will be:



[9 Apr. 2019 II] (a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\sqrt{3}$ (d) $\sqrt{2}$

SOLUTION: (a) $\frac{1}{f_1} = \frac{2}{f_p}$ Here $2f_1 = 18$ cm or $f_1 = 9$ cm So, $\frac{1}{9} = \frac{2}{f_p}$ or $f_l = 18$ cm

Using, $\frac{1}{f_p} = (\mu - 1)\left(\frac{2}{R}\right)$ or $\frac{1}{18} = (1.5 - 1)\left(\frac{2}{R}\right)$ $R = 18$ cm

when liquid is put between, then $\frac{1}{f_2} = \frac{2}{f_p} + \frac{2}{f}$ or $\frac{1}{(27/2)} = \frac{2}{18} + \frac{2}{f}$ or $f = -54$ cm

Now $-\frac{1}{54} = (\mu_1 - 1) \times \frac{1}{R} = (\mu_1 - 1) \times \left(\frac{1}{-18}\right) \therefore \mu_1 = \frac{1}{3} + 1 = \frac{4}{3}$

43. An upright object is placed at a distance of40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of60cm on the other side of the lens. The position and size of the final image will be: [8 April 2019 I]

(a) 20 cm from the convergent mirror, same size as the object

(b) 40 cm from the convergent mirror, same size as the object

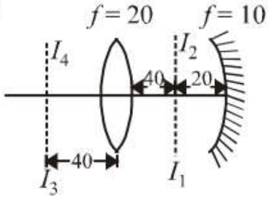
(c) 40 cm from the convergent lens, twice the size of the object

(d) 20 cm from the convergent mirror, twice the size of the object

SOLUTION: **(Bouns)** $v_1 = \frac{40\times 20}{(40-20)} = 40\,cm$

$u_2 = 60 - 40 = 20$ cm

$\therefore v_2 = \frac{20\times 10}{(20-10)} = 20\,cm$

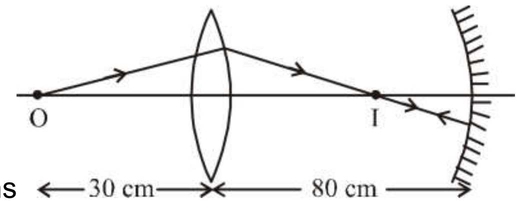


44. A convex lens (of focal length 20 cm) Anda concave mirror, having their principal axes along the same lines, are kept80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be: [8 Apr. 2019 II]

(a) 30 cm (b) 25 cm (c) 10 cm (d) 20 cm

Image traces back to object itself as image formed by lens is a center of curvature of mirror.

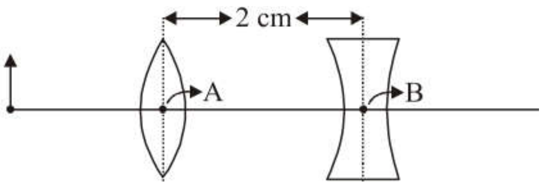
SOLUTION: (c) For lens



$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \text{or } \frac{1}{v}-\frac{1}{-30}=\frac{1}{20} \quad \therefore v=+60 \text{ cm}$$

According to the condition, image formed by lens should be the center of curvature of the mirror, and so $2f' = 20$ or $f' = 10$ cm

45. What is the position and nature of image formed by lens combination shown in figure? (r_1, f_2 are focal lengths)S[12 Jan. 2019 I]



$\leftrightarrow^{20cm}$: $f_1 = +5$ cm $f_2 = -5$ cm

(a) 70 cm from point B at left; virtual

(b) 40 cm from point B at right; real

(c) $\frac{20}{3}$ cm from point B at right, real

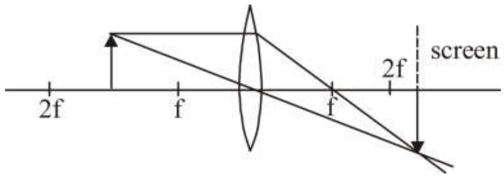
(d) 70 cm from point B at right; real

SOLUTION: (d) By lens's formula, $\frac{1}{V}-\frac{1}{u}=\frac{1}{f}$ For first lens, $[u_1=-20]$ $\therefore \frac{1}{V_1}-\frac{1}{-20}=\frac{1}{5} \Rightarrow V_I=\frac{20}{3}$

Image formed by first lens will behave as an object for second lens so, $u_2=\frac{20}{3}-2=\frac{14}{3}$

$\frac{1}{V_2}-\frac{1}{\frac{14}{3}}=\frac{1}{-5}\Rightarrow V_2=70\ \text{cm}$

46. Formation of real image using a biconvex lens is shown below: [12 Jan. 2019 II]



If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen?

Image disappears (b) Magnified image(c) Erect real image (d) No change

SOLUTION: (a) According to lens maker's formula, $\frac{1}{f}=(\mu_{re1}-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ Focal length of lens will change due to change in refractive index μ_{re1}. So, image will be formed at new position. Hence image disappears

47. A Plano-convex lens (focal lengthf_2, refractive index μ_2,radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be: [12 Jan. 2019 II]

(a) f_1-f_2 (b) $\frac{R}{\mu_2-\mu_1}$ (c) $\frac{2f_1f_2}{f_1+f_2}$ (d) f_1+f_2

SOLUTION: (b) $\frac{1}{f_2}=(\mu_2-1)\left(\frac{+1}{R}\right)$ $\frac{1}{f_1}=(\mu_1-1)\frac{(-1)}{R}$

Now when combined the focal length is given by $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

$$=(\mu_1-1)\frac{(-1)}{R}+(\mu_2-1)\frac{+1}{R}=\frac{1}{R}[\mu_2-I-\mu_1+l]=\frac{\mu_2-\mu_1}{R}$$

$$\Rightarrow f=\frac{R}{\mu_2-\mu_1}$$

48. An object is at a distance of 20 m from a convex lens of focal length 0.3m. The lens forms an image ofthe object. Ifthe object moves away $\text{fi}_{\text{iO}}\text{m}$ the lens at a speed of 5m/s,the speed and direction ofthe image will be: [11 Jan. 2019 I]

(a) $2.26\times10^{-3}\text{m/s}$ away iiiom the lens

(b) $0.92\times10^{-3}\text{m/s}$ away $\text{fi}_{\text{i}}\text{om}$ the lens

(c) $3.22\times10^{-3}\text{m/s}$ towards the lens

(d) $1.16\times10^{-3}\text{m/s}$ towards the lens

SOLUTION: (d) By lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\therefore\frac{1}{v}-\frac{1}{(-20)}=\frac{10}{3}$

$$\frac{1}{v}=\frac{10}{3}-\frac{1}{20}=\frac{197}{60};v=\frac{60}{197}$$

Magnification of lens (m) is given by $m=\left(\frac{v}{u}\right)=\frac{\left(\frac{60}{197}\right)}{20}$

velocity of image wrto. to lens is given by $v_{I/L}=m^2v_{O/L}$
direction of velocity of image is same as that of object $v_{O/L}=5m/s$

$$v_{I/L}=\left(\frac{60\times 1}{197\times 20}\right)^2(5)=1.16\times 10^{-3}m/s \text{ towards the lens}$$

49. A Plano convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another Plano concave lens of refractive index μ_2 and focal length f_2 If the radius of curvature of their spherical faces is R eachand $f_1=2f_2$, then μ_1 and μ_2 are related as: [10 Jan. 2019 I]
(a) $\mu_1+\mu_2=3$ (b) $2\mu_1-\mu_2=1$ (c) $3\mu_2-2\mu_1=1$ (d) $2\mu_2-\mu_1=1$

SOLUTION: (b) From lens maker's formula, $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(\mu_1-1)\left(\frac{1}{\infty}-\frac{1}{-R}\right)=\frac{1}{2f_2}$

Similarly, for Plano-concave lens $\frac{1}{f_2}=(\mu_2-1)\left(\frac{1}{-R}-\frac{1}{\infty}\right)$

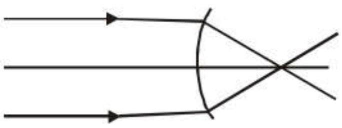
Dividing $\frac{1}{f_1}$ by $\frac{1}{f_2}$ we get, $\frac{(\mu_1-1)}{R}=\frac{(\mu_2-1)}{2R}$ or, $2\mu_1-\mu_2=1$

50. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus.

[10 Jan. 2019 II]

(a) 1 cm (b) 2 cm (c) 4.0 cm (d) 3.1 cm

SOLUTION: (d) using, $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$ $R=7.8mm$



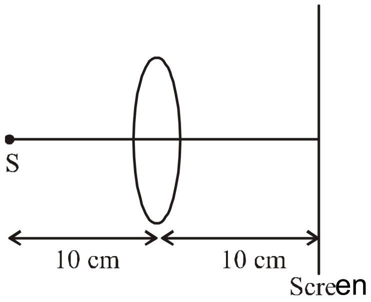
$$\mu_1=1\mu_2=1.34\Rightarrow\frac{1.34}{V}-\frac{1}{\infty}=\frac{1.34-1}{7.8}[\because u=\infty]\therefore\ V=30.7mm=3.07cm=3.1\text{ cm}$$

51. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm fi¡om the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance d. Then dis: [9 Jan. 2019 I]
(a) 1.1 cm away from the lens
(b) 0

(c) 0.55 cm towards the lens

(d) 0.55 cm away from the lens

SOLUTION: (d)



Using lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{10}-\frac{1}{-10}=\frac{1}{f} \Rightarrow f=5\mathrm{cm}$

Shift due to slab, $=t\left(1-\frac{1}{\mu}\right)$ in the direction of incident ray or, $d=1.5\left(\frac{1-2}{3}\right)=0.5$

Now, $u=-9.5$ Again using lens formulas $\frac{1}{v}-\frac{1}{-9.5}=\frac{1}{5} \Rightarrow \frac{1}{v}=\frac{1}{5}-\frac{2}{19}=\frac{9}{95}$ or, $v=\frac{95}{9}=10.55\mathrm{cm}$

Thus, screen is shifted by a distance $d=10.55-10=0.55$ cm away from the lens.

52. A planoconvex lens becomes an optical system of 28cm focal length when its plane surface is silvered and illuminated from left to right as shown in Fig-A. If the same lens is instead silvered on the curved surface and illuminated from other side as in Fig. B, it acts like an optical system of focal length 10 cm. The refractive index of the material of lens if: [Online Apri1 15, 2018]

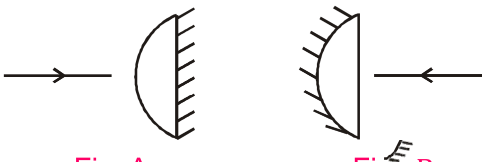
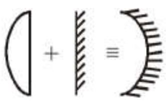


Fig. A Fig. B

(a) 1.50 (b) 1.55 (c) 1.75 (d) 1.51

SOLUTION: (b) Case-1



$\frac{1}{f_1}=\left(\frac{\mu-1}{R}\right)$ f $=-28$

$$P=2P_1+P_2 \Rightarrow \frac{1}{28}=2\left(\frac{\mu-1}{R}\right) \text{(Power, } P=\frac{1}{f} \text{ \& } f_{\text{planemirror}}=\infty)$$

Case-2 $\frac{1}{f_1}=\left(\frac{\mu-1}{R}\right)$ $f_2=-\frac{R}{2}$ $f=-10$ cm $P=2P_1+P_2 \Rightarrow \frac{1}{10}=2\left(\frac{\mu-1}{2}\right)+\frac{2}{R}$

or, $\frac{1}{10}=\frac{1}{28}+\frac{2}{R} \Rightarrow \frac{2}{R}=\frac{1}{10}-\frac{2}{28}=\frac{18}{280}$ or, $R=\frac{280}{9}$cm or, $\frac{1}{28}=2\left(\frac{\mu-1}{280}\right)9 \Rightarrow \mu-1=\frac{5}{9}$

$$\mu=1+\frac{5}{9}=\frac{14}{9}=1.55$$

53. A convergent doublet of separated lenses, corrected for spherical aberration, has resultant focal

length of $10\mathrm{cm}$.The separation between the two lenses is $2\mathrm{cm}$. The focal lengths of the component lenses [Online Apri115, 2018]

(a) $18\mathrm{cm}, 20\mathrm{cm}$ (b) $10\mathrm{cm},\ 12\mathrm{cm}$(c) $12\mathrm{cm},\ 14\mathrm{cm}$ (d) $16\mathrm{cm},\ 18\mathrm{cm}$

SOLUTION: (a) For minimum spherical aberration separation,$d=f_1-f_2=2$ cm

Resultant focal length $=\Gamma=10$ cm Using $\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}-\frac{d}{f_1 f_2}$ and solving,

we get $\mathrm{f}_1=16cm$, $\mathrm{f}_2=18$cm and 20 cm respectively.

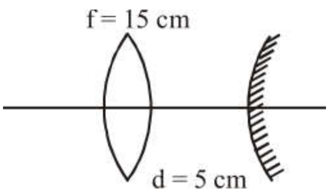
54. In an experiment a convex lens of focal length 15 cm is placed coaxially on an optical bench in front of a convex mirror at a distance of 5 cm from it. It is found that an object and its image coincide, if the object is placed at a distance of 20 cm $\mathrm{fi_iom}$ the lens. The focal length of the convex mirror is: [Online April 9, 2017]

(a) 27.5 cm (b) 20.0 cm (c) 25.0 cm (d) 30.5 cm

SOLUTION: (a) Given, focal length of lens $(\mathrm{f})=15$ cm

object is placed at a distance $(\mathrm{u})=-20$ cm

By lens formula,$\frac{1}{\mathrm{f}}=\frac{1}{\mathrm{v}}-\frac{1}{\mathrm{u}}$

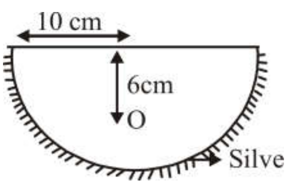


$$\frac{1}{\mathrm{v}}=\frac{1}{\mathrm{f}}+\frac{1}{\mathrm{u}}=\frac{1}{15}-\frac{1}{20}=\frac{4-3}{60}$$

$\mathrm{v}=60$ cm

The image I gets formed at 60 cm to the right of the lens and it will be inverted. The rays $\mathrm{fi_iom}$ the image (I) formed further falls on theconvex mirror forms another image. This image should form in such a way that it coincides with object at the same point due to reflection takes place by convex mirror. Distance between lens and mirror will be $\mathrm{d}=$ image distance $(\mathrm{v})-$ radius of curvature of convex mirror.$5=60-2\mathrm{f} \therefore 2\mathrm{f}=60-5 \therefore \mathrm{f}=\frac{55}{2}=27.5$ cm (convex mirror)

55. A hemispherical glass body of radius 10 cm and refractive index 1.5 is silvered on its curved surface. A small air bubble is 6 cm below the flat surface inside it along the axis. The position of the image of the air bubble made by the mirror is seen: [Online Apri110, 2016]



red

(a) 14 cm below flat surface (b) 20 cm below flat surface

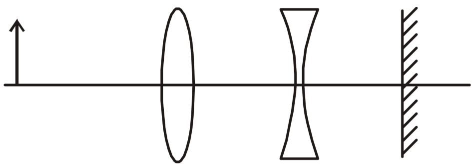
(c) 16 cm below flat surface (d) 30 cm below flat surface

SOLUTION: (b) Given, radius of hemispherical glass $R = 10$ cm Focal length $f = \frac{10}{2} = -5$ cm

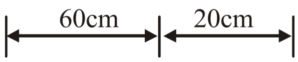
$u = (10 - 6) = -4$cm.By using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-4} = \frac{1}{-5} \Rightarrow v = 20$ cm.

Apparent height, $h_a = h_r \frac{\mu_1}{\mu_2} = 30 \times \frac{1}{1.5} = 20$ cm below flat surface.

56. A convex lens, of focal length 30 cm, a concave lens of focal length 120 cm, and a plane mirror are arranged as shown. For an object kept at a distance of60 cm from the convex lens, the final image, formed by the combination, is a real image, at a distance of: [Online April 9, 2016]



$|Focal\ length|$ $= 30$ cm $|Focal\ length|$ $= 120$ cm



I………...70cm………I

(a) 60 cm from the convex lens

(b) 60 cm from the concave lens

(c) 70 cm from the convex lens

(d) 70 cm from the concave lens

SOLUTION: (a) Len's formula is given by $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

For convex lens,$\frac{1}{30} = \frac{1}{v} + \frac{1}{60} \Rightarrow \frac{1}{60} = \frac{1}{v}$

Similarly, for concave lens$\frac{1}{-120} = \frac{1}{v} - \frac{1}{40} \Rightarrow \frac{1}{v} = \frac{1}{60}$

Virtual object 10 cm behind plane mirror. Hence real image 10 cm Infront of mirror or, 60 cm from convex lens.

57. To find the focal length of a convex mirror, a student records the following data : [Online April 9, 2016]

Object Pin	Convex Lens	Convex Mirror	Image Pin
22.2cm	32.2cm	45.8cm	71.2cm

The focal length of the convex lens is f_1 and that ofmirror is f_2. Then taking index correction to be negligibly small, f_1 and f_2 are close to:

(a) $f_1 = 7.8cm \ \ f_2 = 12.7cm$

(b) $f_1 = 12.7 \text{ cm } \ f_2 = 7.8 \text{ cm}$

(c) $f_1 = 15.6 \text{ cm } \ f_2 = 25.4 \text{ cm}$

(d) $f_1 = 7.8cm \ \ f_2 = 25.4cm$

SOLUTION: (a) Taking $f_2 = 12.07$Using Mirror's formula $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

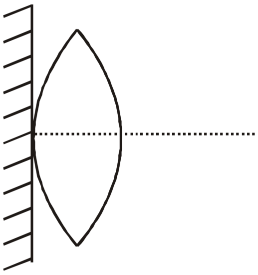
$$\Rightarrow \frac{1}{12.7} = \frac{1}{25.4} + \frac{1}{u} \Rightarrow \frac{1}{12.7} - \frac{1}{25.4} = \frac{1}{u}$$

$$u = 25.4 = v'$$

Now using Len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f_1} = \frac{1}{25.4+13.6} + \frac{1}{10} = \frac{1}{39} + \frac{1}{10} \Rightarrow f_1 = \frac{390}{49} = 7.96$

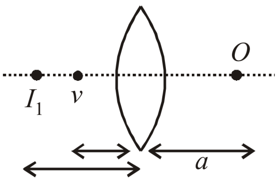
The closest answers is (a) as option (c) and (d) are not possible.

58. A thin convex lens of focal length 'f' is put on a plane mirror as shown in the figure. When an object is kept at a distance 'a' from the lens - mirror combination, its image is formed at a distance $\frac{a}{3}$ in front of the combination. The value of a' is: [Online April 11, 2015]



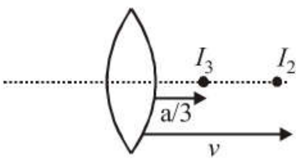
3f (b) $\frac{3}{2}f$ (c) f (d) $2f$

SOLUTION: (d) When object is kept at a distance '*a*' from thin convex lens



By lens formula: $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ $\frac{1}{V} - \frac{1}{(-a)} = \frac{1}{f}$ or, $\frac{1}{v} = \frac{1}{f} - \frac{1}{a}$

(i)Mirror forms image at equal distance from mirror Now, again from lens formula



$$\frac{3}{a} - \frac{1}{V} = \frac{1}{f}$$

$\frac{3}{a} - \frac{1}{f} + \frac{1}{a} = \frac{1}{f}$ [From eqn. (i)] Hence, $a = 2f$

59. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length f. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$, it has the focal lengths f_1 and f_2 respectively. The correct relation between the focal lengths is: [2014]

(a) $f_1 = f_2 < f$

(b) $f_1 > f$ and f_2 becomes negative

(c) $f_2 > f$ and f_1 becomes negative

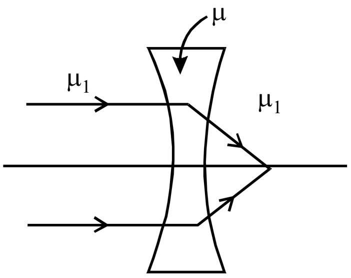
(d) f_1 and f_2 both become negative

SOLUTION: (b) By Lens maker's formula for convex lens

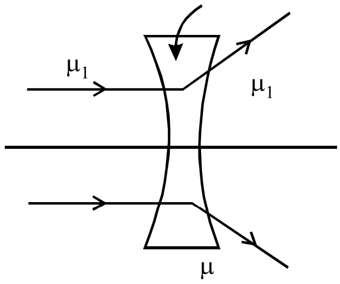
$$\frac{1}{f} = \left(\frac{\mu}{\mu_L} - 1\right)\left(\frac{2}{R}\right)$$

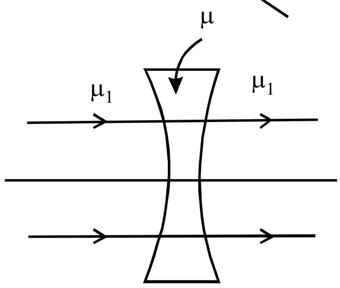
for, $\mu_{L_1} = \frac{4}{3}$, $f_1 = 4R$ for $\mu_{L_2} = \frac{5}{3}$, $f_2 = -5R$ $\Rightarrow f_2 = (-)$ ve

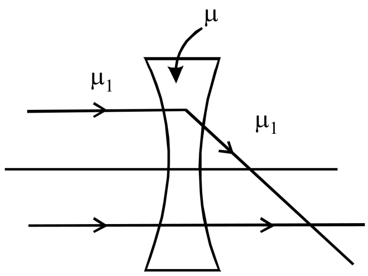
60. The refractive index of the material of a concave lens is μ.It is immersed in a medium refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is:[Online Apri112, 2014]



(a)

(b)


(c)


(d)


SOLUTION: (a) If a lens of refractive index μ is immersed in a medium of refractive index μ_1,

then its focal length in medium is given by $\frac{1}{\mathrm{f_m}} = (\mu_l - 1)\left(\frac{1}{\mathrm{R_1}} - \frac{1}{\mathrm{R_2}}\right)$

If $\mathrm{f_a}$ is the focal length oflens in air, then $\frac{1}{\mathrm{f_a}} = (\mu_l - 1)\left(\frac{1}{\mathrm{R_1}} - \frac{1}{\mathrm{R_2}}\right)$ $\Rightarrow \frac{\mathrm{f_m}}{\mathrm{f_a}} = \frac{(\mu_l - 1)}{(\mu_l - 1)}$

If $\mu_1 > \mu$, then $\mathrm{f_m}$ and $\mathrm{f_a}$ have opposite signs and the nature of lens changes i.e. a convex lens diverges the light rays and concave lens converges the light rays. Thus, given option (a) is correct.

61. An object is located in a fixed position in front of a screen. Sharp image is obtained on the screen for two positions of a thin lens separated by 10 cm. The size of the images in two situations are in the ratio 3: 3. What is the distance between the screen and the object? [Online April 11, 2014]

(a) 124.5cm (b) 144.5cm (c) 65.0 cm (d) 99.0 cm

SOLUTION: (d) Given: Separation of lens for two of its position, $d = 10$ cm

Ratio of size of the images in two positions $\frac{I_1}{I_2} = \frac{3}{2}$

Distance of object from the screen, $D = ?$

Applying formula, $\frac{I_1}{I_2} = \frac{(D+d)^2}{(D-d)^2} \Rightarrow \frac{3}{2} = \frac{(D+10)^2}{(D-10)^2} \Rightarrow \frac{3}{2} = \frac{D^2+100+20D}{D^2+100-20D}$

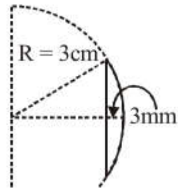
$\Rightarrow 3D^2 + 300 - 60D = 2D^2 + 200 + 40D \Rightarrow D^2 - 100D + 100 = 0$ On solving, we get $D = 99$ cm

Hence the distance between the screen and the object is 99 cm.

62. Diameter of a plano-convex lens is 6 cm and thickness at the center is 3 mm. If speed of light in material of lens is $2 \times 10^8 \mathrm{m/s}$, the focal length of the lens is [2013]

(a) 15 cm (b) 20 cm (c) 30 cm (d) 10 cm

SOLUTION: (c) $n = \frac{Ve1ocityof1ightinvacuum}{Ve1ocityof1ightinmedium} \therefore n = \frac{3}{2}$



$3^2 + (R - 3mm)^2 = R^2 \quad \Rightarrow 3^2 + R^2 - 2R(3mm) + (3mm)^2 = R^2 \Rightarrow R \approx 15$ cm

$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right) \Rightarrow f = 30$ cm

63. The image of an illuminated square is obtained on a screen with the help of a converging lens. The distance of the square from the lens is 40 cm. The area of the image is 9 times that of the square. The focal length of the lens is: [Online April 22, 2013]

(a) 36 cm (b) 27 cm (c) 60 cm (d) 30 cm

SOLUTION: (d) If side of object square $= \ell$

and side of image square $= \ell'$

From question, $\frac{p\prime 2}{p} = 9$

or $\frac{\ell'}{\ell} = 3$ i. e., magnification $m = 3$ V $u = -40$ cm V $= 3 \times 40 = 120$ cm

f =?

From formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \therefore \frac{1}{120} - \frac{1}{-40} = \frac{1}{f}$ or, $\frac{1}{f} = \frac{1}{120} + \frac{1}{40} = \frac{1+3}{120}$ $f = 30$ cm

64. An object at $2.4m$ in fiiont ofa lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus of film? [2012]

(a) $7.2m$ (b) $2.4m$ (c) $3.2m$ (d) $5.6m$

SOLUTION: (d) The focal length of the lens $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} + \frac{1}{240} = \frac{20+1}{240} = \frac{21}{240} \quad \therefore f = \frac{240}{21}$ cm

When glass plate is interposed between lens and film, so shift produced will be

Shift $= t\left(1 - \frac{1}{\mu}\right) = 1\left(1 - \frac{1}{3/2}\right) = 1 \times \frac{1}{3}$

Now image should be form at$v' = 12 - \frac{1}{3} = \frac{35}{3}$ cm

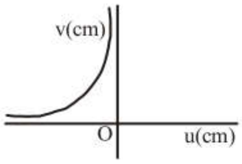
Now the object distance u. Using lens formula again $\frac{1}{f} = \frac{1}{v'} - \frac{1}{u} \Rightarrow \frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left[\frac{3}{7} - \frac{21}{48}\right]$

$$\Rightarrow \frac{1}{u} = \frac{1}{5}\left[\frac{48 - 49}{7 \times 16}\right] \Rightarrow u = -7 \times 16 \times 5 = -560cm = -5.6m$$

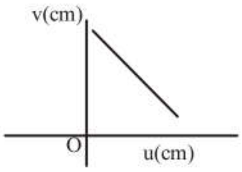
65. When monochromatic red light is used instead of blue light in a convex lens, its focal length will [2011 RS]

(a) increase

(b) decrease

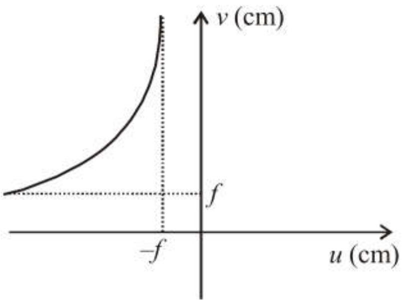


(d)



SOLUTION: (c) From the lens formula $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}$ This graph suggests that when

$u=-f,\ v=+\infty$ When u is at- $\propto,\ v=f$

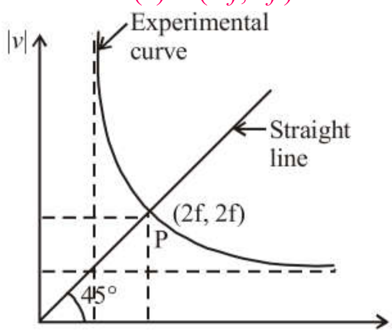


When the object is moved further away from the lens, v decreases but remains positive.

66. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance *u* and the image distance *v*, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the *x*- axis meets the experimental curve at *P*. The coordinates of *P* will be **[2009]**

(a) $(f/2, f/2)$ (b) (f, f) (c) $(4f, 4f)$ (d) $(2f, 2f)$

SOLUTION: **(d)**



$|u|$

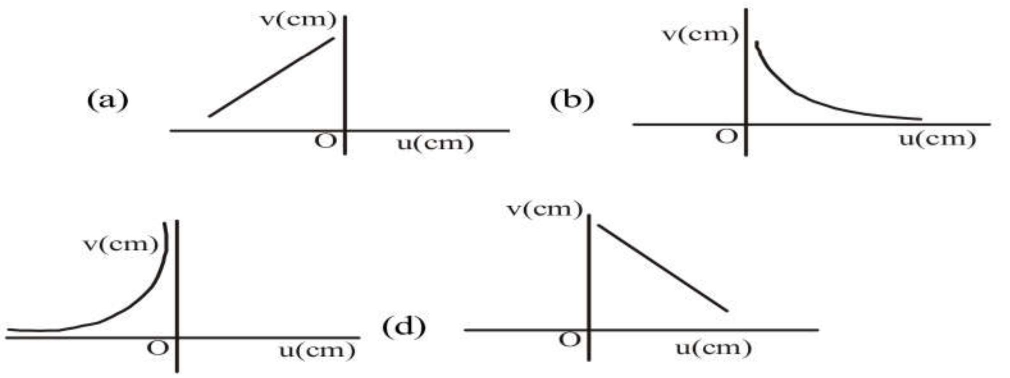
For the graph to intersect $y=x$ line. The value of $|v|\text{and}|u|$ must be equal.

From lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

When $u = -2f, v = 2f$ Also $v = \frac{f}{1+\frac{f}{u}}$

As $|u|$ increases, v decreases for $|u| > f$ The graph between $|v|$ and $|u|$ is shown in the figure. A straight line passing through the origin and making an angle of 45^0 with the x-axis meets the experimental curve at $P(2f, 2f)$.

67. A student measures the focal length of a convex lens by putting an object pin at a distance 'u' from the lens and measuring the distance 'v' of the image pin. The graph between 'u' and 'v' plotted by the student should look like **[2008]**



(a)

SOLUTION: (a) From the Cauchy Formula, $\mu = A + \frac{B}{\lambda^2} + \frac{C}{\lambda^1} \therefore \mu \propto \frac{1}{\lambda}$

As, $\lambda_{\text{blue}} < \lambda_{red} \lambda_{\text{blue}} > \mu_{red}$ From lens maker's formula and $\frac{1}{f} \alpha (\mu - 1) \Rightarrow \frac{1}{f_B} > \frac{1}{f_R} \Rightarrow$

$f_R > f_B$.

68. Two lenses of power $-15D$ and $+5D$ are in contact witheach other. The focal length of the combination is [2007]

(a) $+10$ cm (b) -20 cm (c) -10 cm (d) $+20$ cm

SOLUTION: (c) When two thin lenses are in contact coaxially, power of combination is given by

$$P = P_1 + P_2 = (-15 + 5)D = -10D.$$

Also, $P = \frac{1}{f} \Rightarrow f = \frac{1}{P} = \frac{1}{-10}$ meter $\alpha f = -\left(\frac{1}{10} \times 100\right)$ cm $= -10$ cm.

69. A thin glass (refractive index 1.5) lens has optical power of $-5D$ in air. Its optical power in a liquid medium with refractive index 1.6 will be [2005]

(a) $-1D$ (b) $1D$ (c) $-25D$ (d) 25 D

SOLUTION: (b) According to lens maker's formula in air $\frac{1}{f_a} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{f_a} = \left(\frac{1.5}{1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ .... (i)}$$

Using lens maker's formula in liquid medium, $\frac{1}{f_m} = \left(\frac{\mu_g}{\mu_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \frac{1}{f_m} = \left(\frac{1.5}{1.6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad .... \text{(ii)}$$

Dividing (i) by(ii), $\frac{f_m}{f_a} = \left(\frac{1.\cdot 5 - 1}{\frac{15}{1.6} - 1}\right) = -8 \therefore P_a = -5 = \frac{1}{f_a} \Rightarrow f_a = -\frac{1}{5}$

$$\Rightarrow f_m = -8 \times f_a = -8 \times -\frac{1}{5} = \frac{8}{5} \therefore P_m = \frac{\mu}{f_m} = \frac{1.6}{8} \times 5 = 1\text{D}$$

70. A Plano convex lens of refractive index 1.5 and radius of curvature 30 cm. Is silvered at the curved surface. Now this lens has been used to form the image of an object. At what distance from this lens an object be placed in order to have a real image of size of the object [2004]

(a) 60 cm (b) 30 cm (c) 20 cm (d) 80 cm

SOLUTION: (c) Here, $\mathrm{R}_1 = \infty, \mathrm{R}_2 = 30$ cm Using lens maker's formula Here $\frac{1}{f_l} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$= (1.5 - 1)\left[\frac{1}{\infty} - \frac{1}{-30}\right] = \frac{1}{60}$$

The focal length (F) of the final mirror is $\frac{1}{F} = \frac{2}{\mathrm{f}_l} + \frac{1}{\mathrm{f_m}} = 2 \times \frac{1}{60} + \frac{1}{30/2} = \frac{1}{10}$

$F = 10$ cm Real image will be equal to the size of the object if the object distance $u = 2F = 20$ cm

PRISM AND DISPERSION OF LIGHT

71. The surface of a metal is illuminated alternately with photons of energies $E_1 = 4\text{eV}$ and $E_2 = 2.5\text{eV}$ respectively. The ratio of maximum speeds of the photoelectrons emitted in the two cases is 2. The work function of the metal in $(\text{eV})$is [Sep. 05, 2020 $(\text{ID}]$

SOLUTION: 2 From the Einstein's photoelectric equation

Energy of photon= Kinetic energy ofphotoelectrons + Work function

⇒ Kinetic energy = Energy of Photon- Work Function. Let φ_0 be the work function ofmetal and v_1 and v_2 be the velocity of photoelectrons. Using Einstein's photoelectric equation, we have

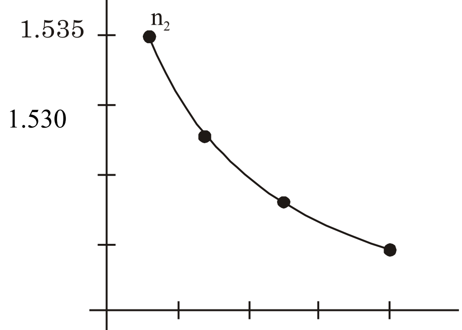
$\frac{1}{2}mv_1^2 = 4 - \varphi_0$ (i)

$\frac{1}{2}mv_2^2 = 2.5 - \varphi_0$ (ii) $\quad \Rightarrow \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{4-\varphi_0}{2.5-\varphi_0} \Rightarrow (2)^2 = \frac{4-\varphi_0}{2.5-\varphi_0}$

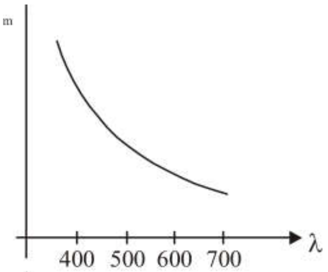
$$\Rightarrow 10 - 4\varphi_0 = 4 - \varphi_0 \therefore \varphi_0 = 2eV$$

72.The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if $\mathrm{D_m}$ is the angle of minimum deviation? **[11 Jan. 2019, I]**

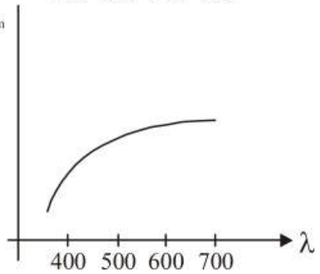
1.530



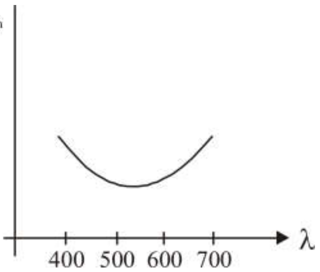
a)



b)



c)



SOLUTION: (a) When angle of prism is small, then angle of deviation is given by $D_m = (\mu - 1)A$

So, if wavelength of incident light is increased, μ decreasesand hence D_m decreases

73. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation If the refractive index of the material of the prism is $\sqrt{3}$,then the angle of incidence is

: [11 Jan. 2019 II]

(a) $90°$ (b) $30°$ (c) $60°$ (d) $45°$

SOLUTION: (c) For minimum deviation:$r = r == 30^o \underline{A}$

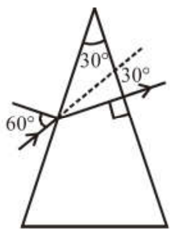
by Snell's law $\mu_1 \sin i = \mu_2 \sin r \therefore 1 \times \sin i = \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2} \Rightarrow i = 60$

74. A ray of light is incident at an angle of $60°$ on one face of a prism of angle $30°$. The emergent ray oflight makes anangle of $30°$ with incident ray. The angle made by the1.52 emergent ray with second face of prism will be: [Online Apri116, 2018]

(a) $30°$ (b) $90°$ (c) $0°$ (d) $45°$

SOLUTION:

(c) Angle of prism, $A = 30°, i = 60°$,
angle of deviation, $\delta = 30°$
Using formula, $\delta = i + e - A$
$\Rightarrow e = \delta + A - i$
$= 30° + 30° - 60° = 0°$



75. In an experiment for determination of refractive index of glass of a prism by i-δ , plot it was found that a ray incident at angle $35°$, suffers a deviation of$40°$ and that it emerges at angle $79°$. In

that case which of the following is closest to the maximum possible value of the refractive index?D [2016]

(a) 1.7 (b) 1.8 (c) 1.5 (d) 1.6

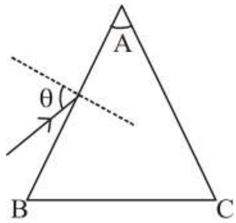
SOLUTION: (c) We know that $i + e - A = 6 \therefore \quad 35^\circ + 79^\circ - A = 40^\circ \quad A = 74^\circ$

But $\mu = \frac{\sin\left(\frac{A+D_m}{2}\right)}{\frac{\sin A}{2}} = \frac{\sin\left(\frac{74+D_m}{2}\right)}{\sin\frac{74}{2}} = \frac{5}{3}\sin\left(37^\circ + \frac{D_m}{2}\right)$

μ_{max} can be $\frac{5}{3}$. That is μ_{max} is less than $\frac{5}{3} = 1.67$

But 6_m will be less than 40° so $\mu < \frac{5}{3}\sin 57^\circ < \frac{5}{3}\sin 60^\circ \Rightarrow \mu = 1.5$

76. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ,a ray, incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided: [2015]



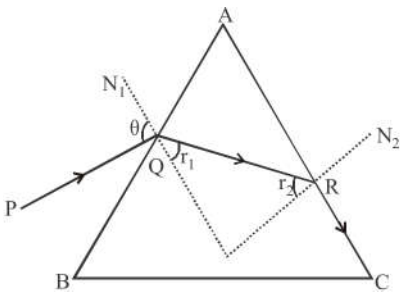
(a) $\theta > \cos^{-1}[\mu \sin(A + \sin^{-1}\left(\frac{1}{\mu}\right)]$

(b) $\theta < \cos^{-1}[\mu \sin(A + \sin^{-1}\left(\frac{1}{\mu}\right)]$

(c) $\theta > \sin^{-1}[\mu \sin(A - \sin^{-1}\left(\frac{1}{\mu}\right)]$

(d) $\theta < \sin^{-1}[\mu \sin(A - \sin^{-1}\left(\frac{1}{\mu}\right)]$

SOLUTION: (c) When $r_2 = C$, $\angle N_2RC = 90^\circ$ Where $C =$ critical angle As $\sin C = \frac{1}{\mu} = \sin r_2$



Applying Snell’s law at R’ $\mu \sin r_2 = 1 \sin 90^\circ$ (i)

Applying Snell’s law at Q’ $1 \times \sin\theta = \mu \sin r_1$ (ii)

But $r_1 = A - r_2$ So, $\sin\theta = \mu \sin$ (A - r2)

$\sin\theta = \mu \sin$ A $\cos r_2 -$ cos A (iii) [using (i)]

From(1) $\cos r_2 = \sqrt{1 - \sin^2 r_2} = \sqrt{1 - \frac{1}{\mu^2}}$ (iv)

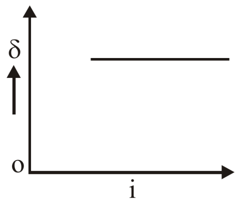
By eq. (iii) and (iv) $\sin\theta = \mu \sin A\sqrt{1-\frac{1}{\mu^2}} - \cos A$

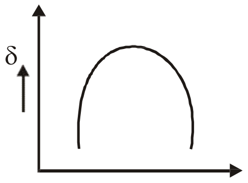
on further solving we can show for ray not to transmitted through face AC

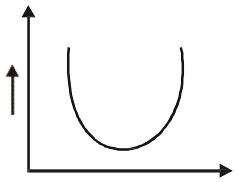
$$\theta = \sin -1[\mu \sin (A - \sin^{-1}\left(\frac{1}{\mu}\right)]$$

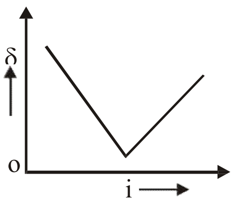
So, for transmission through face AC $\theta > \sin -1[\mu \sin (A - \sin^{-1}\left(\frac{1}{\mu}\right)]$

77. The graph between angle of deviation (6) and angle of incidence(i) for a triangular prism is represented by [2013]

a) 

b) 

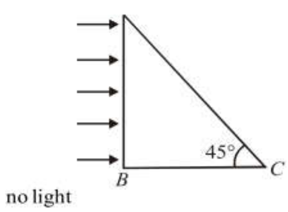
c) 

d) 

SOLUTION: (c) For the prism as the angle of incidence (i) increases, the angle of deviation (6) first decreases goes to minimum value and then increases.

78. A beam of light consisting of red, green and blue colors is incident on a right-angled prism on face AB. The refractive indices of the material for the above red, green and blue colors are 1.39, 1.44 and 1.47 respectively. Aperson looking on surface AC of the prism will see

Online May 26, 2012]

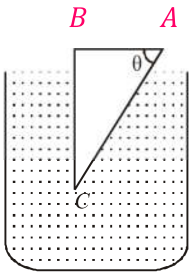


(a)no light (b) green and blue colors (c) red and green colors (d) red color only

SOLUTION: (d) For light to come out through face AC′, total internal reflection must not take place.

i.e., $\theta < c \Rightarrow \sin\theta < \sin c \Rightarrow \sin\theta < \frac{1}{\mu}$ or $\mu < \frac{1}{\sin\theta} \Rightarrow \mu < \frac{1}{\sin 45°} \Rightarrow \mu < \sqrt{2} \Rightarrow \mu < 1.414$

79. A glass prism of refractive index 1.5 is immersed in water (refractive index $\frac{4}{3}$) as shown in figure. A light beam incident normally on the face AB is totally reflected to reach the face BC, if [Online May 19, 2012]



(a) $\sin\theta > \frac{5}{9}$ (b) $\sin\theta > \frac{2}{3}$ (c) $\sin\theta > \frac{8}{9}$ (d) $\sin\theta > \frac{1}{3}$

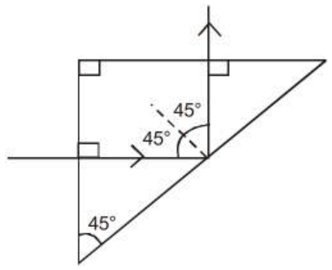
SOLUTION: (c) For total internal reflection on face AC $\theta >$ critical angle (C)

and $\sin\theta \geq \sin C$ $\sin\theta \geq \frac{1}{{}_w\mu_g}$ $\sin\theta \geq \frac{\mu_w}{\mu_g} \Rightarrow \sin\theta \geq \frac{3}{2}\frac{4}{3}$ $\therefore \sin\theta \geq \frac{8}{9}$

80. Which of the following processes play a part in the formation of a rainbow? [Online May 7, 2012]
(i) Refraction (ii) Total internal reflection(iii) Dispersion (iv) Interference
(a) (i), (ii) and(iii) (b) (i) and(ii) (c) (i), (ii) and(iv) (d) (iii) and(iv)
SOLUTION: (a) Rainbow is formed due to the dispersion of light suffering refraction and total internal reflection (TIR) in the droplets present in the atmosphere.

81. The refractive index of a glass is 1.520 for red light and1.525 for blue light. Let D_1 and D_2 be angles of minimum deviation for red and blue light respectively in a prism of this glass. Then, [2006]
(a) $D_1 < D_2$ (b) $D_1 = D_2$
(c) D_1 can be less than or greater than D_2 depending upon the angle of prism(d) $D_1 > D_2$
SOLUTION: (a) When angle of prism is small, Angle of deviation, $D = (\mu - 1)A$
Since $\lambda_b < \lambda_r \Rightarrow \mu_r < \mu_b \Rightarrow D_1 < D_2$

82. Alight ray is incident perpendicularly to one face of a $90°$ prism and is totally internally reflected at the glass-air interface. If the angle of reflection is $45°$, we conclude that the refractive index n [2004]



(a) $n > \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$ (c) $n < \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

SOLUTION: (b) For total internal reflection Incident angle $(i) >$ critical angle

$\sin i > \sin i_c \quad \Rightarrow \sin 45° > \sin i_c \Rightarrow \sin i_c = \frac{1}{n} \qquad \therefore \sin 45° > \frac{1}{n} \quad \Rightarrow \frac{1}{\sqrt{2}} > \frac{1}{n} \Rightarrow n > \sqrt{2}$

OPTICAL INSTRUMENTS

83. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10cm.The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is. [Sep. 05, 2020 (I)]

SOLUTION: (50) Given: Length of compound microscope, $L = 10$ cm Focal length of objective $f_0 = 1$ cm and of eye - piece, $f_e = 5$ cm $\quad u_0 = f_e = 5$ cm

Final image formed at infinity(∞), $v_e = \infty \qquad v_0 = 10 - 5 = 5$

Using lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \therefore \frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{5} - \frac{1}{u_0} = \frac{1}{1} \Rightarrow u_0 = -\frac{5}{4}$ cm

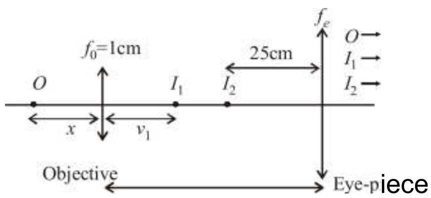
or, $\frac{5}{4} = \frac{n}{40} \qquad \therefore n = \frac{200}{4} = 50$ cm.

84. In a compound microscope, the magnified virtual image is formed at a distance of25 cm from the eye-piece. The focal length of its objective lens is 1 cm. If the magnification is100 and the tube length of the microscope is 20 cm, then the focal length of the eye-piece lens (in cm) is

[Sep. 04,2020(I)]

SOLUTION: (4.48)According to question, final image i. e., $v_2 = 25$ cm,

$f_0 = 1$ cm, magnification, $m = m_1 m_2 = 100$ object image formed by 1st lens image formed by 2nd lens



Using lens formula, For first lens or objective $= \frac{1}{v_1} - \frac{1}{-x} = \frac{1}{1} \Rightarrow v_1 = \frac{x}{x-1}$

Also magnification $|m_1| = \left|\frac{v_1}{u_1}\right| = \frac{1}{x-1}$

For 2nd lens or eye-piece, this is acting as object $u_2 = -(20 - v_1) = -\left(20 - \frac{x}{x-1}\right)$ and $v_2 = -25$ cm

Angular magnification $|m_A| = |\frac{D}{u_2}| = \frac{25}{|u_2|}$

Total magnification $m = m_1 m_A = 100$ $\left(\frac{1}{x-1}\right)\left(\frac{25}{20-\frac{x}{x-1}}\right) = 100 \Rightarrow \frac{25}{20(x-1)-x} = 100 \Rightarrow 1 = 80(x-1) - 4x$

$$\Rightarrow 76x = 81 \Rightarrow x = \frac{81}{76} \Rightarrow u_2 = -\left(20 - \frac{\frac{81}{76}}{\frac{81}{76} - 1}\right) = \frac{-19}{5}$$

Again, using lens formula for eye-piece $\frac{1}{-25} - \frac{1}{-\frac{19}{5}} = \frac{1}{fe} \Rightarrow f_e = \frac{25\times19}{106} \approx 4.48$ cm

85. The magnifying power of a telescope with tube length 60cm is 5. What is the focal length of its eye piece? [8 Jan. 2020 I]

(a) 20 cm (b) 40 cm (c) 30 cm (d) 10 cm

SOLUTION: (d) For telescope Tube length $(\mathrm{L}) = f_o + f_e = 60$ and

magnification $(m) = \frac{f_0}{fe} = 5 \Rightarrow f_0 = 5f_e$

$f_o = 50$ cm and $f_e = 10$ cm Hence focal length of eye-piece, $f_e = 10$ cm

86. If we need a magnification of 375 from a compoundmicroscope of tube length 150 mm and an objective of focal length 5 mm, the focal length of the eye-piece, should be close to: [7 Jan. 2020 I]

(a) 22mm (b) 12mm (c) 2mm (d) 33mm

SOLUTION: (a) According question, $M = 375$ $L = 150\text{mm}, f_0 = 5$ mm and $f_e = ?$

Using, magnification, $M = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right) \Rightarrow 375 = \frac{150}{5}\left(1 + \frac{250}{f_e}\right)$ $(\because D = 25\text{cm} = 250$ mm)

$$\Rightarrow 12.5 = 1 + \frac{250}{fe} \Rightarrow f_e = \frac{25.0}{115} = 21.7 \approx 22 \ mm$$

87. An observer looks at a distant tree of height 10 m with a telescope of magnyfing power of 20. To the observer the tree appears: [2016]

(a) 20 times taller (b) 20 times nearer (c) 10times taller (d) 10 times nearer

SOLUTION: (b) A telescope magnifies by making the object appearing closer.

88. To determine refractive index of glass slab using a travelling microscope, minimum number of readings required are: [Online April 10, 2016]

(a) Two (b) Four (c) Three (d) Five

SOLUTION: (c) Reading one ⇒without slab

Reading two ⇒with slab

Reading three ⇒with saw dust

Minimum three readings are required to determine refractive index of glass slab using a travelling microscope.

89. A telescope has an objective lens of focal length 150cm and an eyepiece of focal length 5 cm. If a 50 m tall tower a ta distance of l km is observed through this telescope in normal setting, the angle formed by the image of the tower is θ, then θ is close to: [Online Apri110, 2015]

(a) 30° (b) 15° (c) 60° (d) 1°

SOLUTION: (c) Magnifying power of telescope, $MP=\frac{\beta(\text{ang1esubtendedbyimageateyepiece})}{\alpha(\text{ang1esubtendedbyobjectonobjective})}$

Also, $MP=\frac{f_0}{fe}=\frac{150}{5}=30$

$\alpha=\frac{50}{1000}=\frac{1}{20}$ rad

$\beta=\theta=\mathrm{MP}\times\alpha=30\times\frac{1}{20}=\frac{3}{2}=1.5$ rad

or, $\beta=1.5\times\frac{180^{\circ}}{\pi}=84^{\circ}$

90. In a compound microscope, the focal length of objective lens is 1.2 cm and focal length ofeye piece is 3.0 cm. When object is kept at 1.25 cm in front of objective, final image is formed at infinity. Magnifying power of the compound microscope should be: [Online April 11, 2014]

(a) 200 (b) 100 (c) 400 (d) 150

SOLUTION: . (a) Given: $f_0=1.2$ cm; $f_e=3.0$ cm $u_0=1.25$ cm; $M_{\infty}=?$

From $\frac{1}{f_0}=\frac{1}{v_0}-\frac{1}{u_0}\Rightarrow\frac{1}{1.2}=\frac{1}{v_0}-\frac{1}{(-1.25)}\Rightarrow\frac{1}{v_0}=\frac{1}{1.2}-\frac{1}{1.25}\Rightarrow v_0=30$ cm

Magnification at infinity,$M_{\infty}=-\frac{v_0}{u_0}\times\frac{D}{fe}=\frac{30}{1.25}\times\frac{25}{3}$

($D=25$ cm least distance ofdistinct vision) $=200$Hence the magnifying power of the compound microscope is 200

91. The focal lengths of objective lens and eye lens of a Galilean telescope are respectively 30 cm and 3.0 cm. telescope produces virtual, erect image of an object situated far away fiiom it at least distance of distinct vision from the eye lens. In this condition, the magniђing power of the Galilean telescope should be: [Online April 9, 2014]

(a) $+11.2$ (b) -11.2 (c) -8.8 (d) $+8.8$

SOLUTION: . (d) Given, Focal length of objective, $\mathrm{f}_0=30$ cm

focal length of eye lens, $\mathrm{f}_{\mathrm{e}}=3.0$ cm Magniђing power, $\mathrm{M}=?$

Magnifying power of the Galilean telescope,$\mathrm{M}_{\mathrm{D}}=\frac{\mathrm{f}_0}{\mathrm{f}_{\mathrm{e}}}\left(1-\frac{\mathrm{f}_{\mathrm{e}}}{\mathrm{D}}\right)=\frac{30}{3}\left(1-\frac{3}{25}\right)=10\times\frac{22}{25}=8.8$ cm

[$D = 25$ cm]

92. This question has Statement-I and Statement-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.
Statement 1: Very large size telescopes are reflecting telescopes instead of refracting telescopes.
Statement 2: It is easier to provide mechanical support to large size mirrors than large size lenses.
[Online April 23, 2013]

(a) Statement-I is true and Statement-2 is false.
(b) Statement-I is false and Statement-2 is true.
(c) Statement-I and statement-2 are true and Statement-2 is correct explanation for statement-I.
(d) Statements-I and statement-2 are true and Statement-2 is not the correct explanation for statement-I.

SOLUTION: (c) One side of mirror is opaque and another side is reflecting this is not in case of lens hence, it is easier to provide mechanical support to large size mirrors than large size lenses. Reflecting telescopes are based on the same principle except that the formation of images takes place by reflection instead of refraction.

93. The focal length of the objective and the eyepiece of a telescope are 50 cm and 5 cm respectively. If the telescope is focused for distinct vision on a scale distant 2 m fromits objective, then its magnifying power will be: [Online April 22, 2013]

(a) -4 (b) -8 (c) $+8$ (d) -2

SOLUTION: (d) Given: $f_0 = 50$ cm, $f_e = 5\text{cm}$ $d = 25$ cm, $u_0 = -200$ cm

Magnification $M = ?$ As $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{v_0} = \frac{1}{f_0} + \frac{1}{u_0} = \frac{1}{50} - \frac{1}{200} = \frac{4-1}{200} = \frac{3}{200}$

or $v_0 = \frac{200}{3}\text{cm}$ Now $v_e = d = -25$ cm

From, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e} \Rightarrow -\frac{1}{u_e} = \frac{1}{f_e} - \frac{1}{v_e}$

$= \frac{1}{5} + \frac{1}{25} = \frac{6}{25}$ or, $u_e = \frac{-25}{6}$ cm

Magnification $M = M_0 \times M_e = \frac{v_0}{u_0} \times \frac{v_e}{u_e} = \frac{-200/3}{200} \times \frac{-25}{-25/6} = -\frac{1}{3} \times 6 = -2$

94. A telescope of aperture $3 \times 10^{-2}\text{m}$ diameter is focused on a window at 80 m distance fitted with a wire mesh of spacing $2 \times 10^{-3}\text{m}$. Given: $\lambda = 5.5 \times 10^{-7}\text{m}$, which of the following is true for observing the mesh through the telescope? [Online May 26, 2012]
(a) Yes, it is possible with the same aperture size.
(b) Possible also with an aperture half the present diameter.
(c) No, it is not possible.

(d) Given data is not sufficient.

SOLUTION: (a) Given: $d = 3 \times 10^{-2}\text{m}$ $\lambda = 5.5 \times 10^{-7}\text{m}$

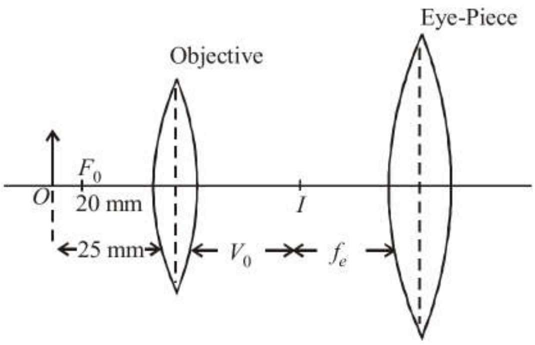
Limit of resolution, $\Delta\theta = \frac{1.22\lambda}{d} = \frac{1.22 \times 5.5 \times 10^{-7}}{3 \times 10^{-2}} = 2.23 \times 10^{-5}$ rad.

At a distance of 80 m , the telescope is able to resolve between two points which are separated by $2.23 \times 10^{-5} \times 80\text{m} = 1.78 \times 10^{-3}\text{m}$

95. We wish to make a microscope with the help of two positive lenses both with a focal length of 20 mm each and the object is positioned 25 mm $from$ the objective lens. How far apart the lenses should be so that the final image is formed at infinity? [Online May 12, 2012]

(a) 20mm (b) $1\alpha)\text{mm}$ (c) 120mm (d) 80mm

SOLUTION: (c)



To obtain final image at infinity, object which is the image formed by objective should be at focal distance of eye-piece. By lens formula (for objective) $\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0}$ or, $\frac{1}{v_0} - \frac{1}{-25} = \frac{1}{20}$

$\Rightarrow \frac{1}{v_0} = \frac{1}{20} - \frac{1}{25} = \frac{5-4}{100} = \frac{1}{100}$ mm $\Rightarrow v_0 = 100\text{mm}$

Therefore, the distance between the lenses= $v_0 + f_e = 100\text{mm} + 20\text{mm} = 120$ mm

96. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment distances are measured by [2008]

(a) a Vernier scale provided on the microscope

(b) a standard laboratory scales

(c) a meter scale provided on the microscope

(d) a screw gauge provided on the microscope

SOLUTION: (a) To find the refractive index of glass using a travelling microscope, a Vernier scale is provided on the microscope

97. The image formed by an objective of a compound microscope is [2003]

(a) virtual and diminished

(b) real and diminished

(c) real and enlarged

(d) virtual and enlarged

SOLUTION: (c) A real, inverted and enlarged image of the object is formed by the objective lens of a compound microscope.

98. An astronomical telescope has a large aperture to [2002]

(a) reduce spherical aberration

(b) have high resolution

(c) increase span of observation

(d) have low dispersion

SOLUTION: (b) The resolving power of a telescope is $R.P=\frac{D}{1.22\lambda}$

where $D=$ diameter of the objective lens and $\lambda=$ wavelength of light. Clearly, $R.P\propto\frac{D}{\lambda}$

Resolving power of telescope resolution will be high if its objective is of large aperture.

wave optics (physical optics)

In geometrical optics, light is represented as a ray which travels in a straight line in a homogeneous medium. The phenomenon like Interference and Dffraction cannot be explained on the basis of particle nature of light. These phenomenon can only be explained on the basis of wave nature of light. This part of optics is called physical optics or wave optics. The wave theory of light was presented by Christian Huygen. It should be pointed out that Huygen did not know whether the light waves were longitudinal or transverse and also how they propagate through vaccum. It was then explained by Maxwell by introducing electromagnetic wave theory in nineteenth century.

Geometrical optics

1. In geometrical optics, light is assumed to be travelling in a straight line. This property is known as rectilinear propagation.
2. By using rectilinear propagation of light, laws of reflection, refraction, total internal reflection etc. are explained geometrically.

Physical optics or wave optics:

1. In physical optics, light is considered as a wave
2. Huygen's wave principle and principle of superposition are used to explain interference and diffraction
3. Electromagnetic wave nature of light is used to explain the concept of polarisation.

Condition for applicability of geometrical optics and wave optics: When the size of the object incteracting with light, is much larger than the wavelength of light, we can apply geometrical optics.

When the wavelength of light is comparable to or less than the size of the object interacting with light, we can apply wave optics.

If 'b' is the size of the object interacting with light, 'l' is the distance between the object and the screen and 'λ' is the wavelengh of light then,

i) The condition for applicability of geometrical optics is $\frac{b^2}{l\lambda} >> 1$

ii) The condition for applicability of wave optics is

$$\frac{b^2}{l\lambda} \approx 1 \; or \; \frac{b^2}{l\lambda} << 1$$

Note: The object interacting with light may be a mirror, a lens, a prism, an aperture (pin hole), a slit and a straight edge.

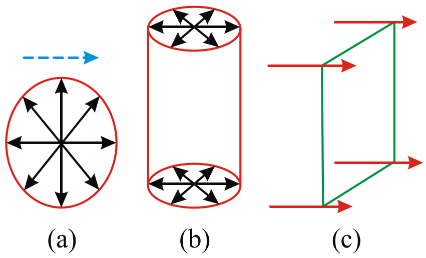
WAVE FRONT

According to wave theory of light, a source of light sends out disturbance in all directions. In a homogeneous medium, the disturbance reaches to all those particles of the medium in phase, which are located at the same distance from the source of light and hence at any instant, all such particles must be vibrating in phase with each other.

The locus of all the particles of the medium, which at any instant are vibrating in the same phase, is called the wavefront.

Depending upon the shape of the source of light, wavefront can be of the following types

1. **Spherical wavefront:** A spherical wavefront is produced by a point source of light. It is because, the locus of all such points, which are equidistant from the point source, is a sphere



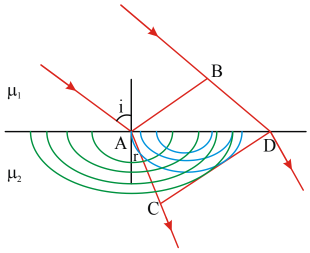
2. **Cylindrical wavefront:** When the source of light is linear in shape (such as a slit), a cylinderical wavefront is produced. It is because, all the points, which are equidistant from the linear source, lie on the surface of a cylinder (b).

3. **Plane wavefront:** A small part of a spherical or a cylindrical wavefront originating from a distant source will appear plane and hence it is called a plane wavefront (c).

Huygen's Principle: Every point on the wave front becomes a source of secondary disturbance and generates wavelets which spread out in the medium with the same velocity as that of light in the forward direction only.

→ The envelope of these secondary waves at any instant of time gives the position of the new wave front at that instant.

→ The wave front in medium is always perpendicular to the direction of wave propagation.



AB is width of incident beam
CD is width of refracted beam

$$\frac{\text{width of incident beam}}{\text{width of refracted beam}} = \frac{\cos i}{\cos r}$$

The Doppler Effect:

i) When any source emitting light (like sun, moon, star, atom etc) is approaching or receding from the observer then the frequency or wavelength of light appears to be changing to the observer. This apparent change in frequency or waveelength of light is called Doppler effect in light.

Blue Shift: When the distance between the source and observer is decreasing (i.e. the source is approaching the observer) then frequency of light appears to be increasing or wavelength appears to be decreasing i.e. the spectral line in electromangetic spectrum gets displaced towards blue end, hence it is known as blue shift.

Red Shift: When the distance between the source and observer is increasing (i.e. the source is receding from the observer) then frequency of light appears to be decreasing or wavelength appears to be increasing i.e. the spectral line in electromangetic spectrum gets displaced towards red end, hence it is known as red shift.

Doppler Shift, $\frac{\Delta v}{v} = \frac{V}{C}$ (where V is the speed of source and C is the speed of light)

W. E-1 ***What speed should a galaxy move with respect to us so that the sodium line at 589.0 nm is observed at 589.6 nm?***

Sol. $\frac{\Delta\lambda}{\lambda} = \frac{V}{C}$;

$$V = +c\left(\frac{0.6}{589.0}\right) = 3\times10^{8}\left(\frac{0.6}{589.0}\right) = +3.06\times10^{5}\,ms^{-1}$$

Therefore. the galaxy is moving away from us with speed 306 km/s.

Principle of superposition of waves:

If two or more waves meet at a place simultaneously in the same medium, the particles of the medium undergo displacements due to all the waves simultaneously. The resultant wave is due to the resultant displacement of the particles.

Principle of superposition of waves states that when two or more waves are simultaneously impressed on the particles of the medium, the resultant displacement of any particle is equal to the sum of displacements of all the waves. (or)

"When two or more waves overlap, the resultant displacement at any point and at any instant is the vector sum of the instantaneous displacements that would be produced at the point by individual waves, if each wave were present alone".

If $y_1, y_2, ...y_n$ denote the displacements of 'n' waves meeting at a point, then the resultant displacement is given by $y = y_1 + y_2 + ... + y_n$.

a) Superposition of coherent waves: Consider two waves travelling in space with an angular frequency ω. Let the two waves arrive at some point simultaneously. Let y_1 and y_2 represent the displacements of two waves at this point.

$\therefore\ y_1 = A_1\sin(\omega t + \phi_1)\ \&\ y_2 = A_2\sin(\omega t + \phi_2)$

Then according to the principle of superposition the resultant displacement at the point is given by,

$y = y_1 + y_2$ or $y = A_1\sin(\omega t + \phi_1) + A_2\sin(\omega t + \phi_2)$

$$= A_1(\sin\omega t\cos\phi_1 + \cos\omega t\sin\phi_1)$$
$$+A_2(\sin\omega t\cos\phi_2 + \cos\omega t\sin\phi_2)$$
$$= A\cos\phi.\sin\omega t + A\sin\phi.\cos\omega t$$
$$= A\sin(\omega t + \phi)$$

where $A\cos\phi = A_1\cos\phi_1 + A_2\cos\phi_2$.......(1)

and $Aisn\phi = A_1\sin\phi_1 + A_2\sin\phi_2$........(2)

Here A and ϕ are respectively the amplitude and initial phase of the resultant displacement

Squaring and adding equations (1) & (2), we get

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2)}$$
$$= \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}.....(3)$$

Where $\phi = \phi_1 - \phi_2$, phase difference between the two waves.

Dividing equation (2) by equation (1), we get

$$\tan\phi = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}........(4)$$

Since the intensity of a wave is proportional to square of the amplitude, the resultant intensity I of the wave from equation (3) may be written as

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$ ....(5)

where I_1 and I_2 be the intensities of the two waves.

It can be seen that the amplitude (intensity) of the resultant displacement varies with phase difference of the constituent displacements.

Case I : When $\phi = \phi_1 - \phi_2 = 0, 2\pi, 4\pi ...... 2n\pi$

where $n = 0, 1, 2, ...$

$\Rightarrow \cos\phi = 1$

$\therefore A = A_1 + A_2$ from (3)

and $\sqrt{I} = \sqrt{I_1} + \sqrt{I_2}$ from (5)

Hence the resultant amplitude is the sum of the two individual amplitudes. This condition refers to the constructive interference.

Case II: When $\phi = \phi_1 - \phi_2 = \pi, 3\pi, 5\pi ..... (2n-1)\pi$ where $n = 1, 2, 3, ...$; $\Rightarrow \cos\phi = -1$

$\therefore A = |A_1 - A_2|$ and $\sqrt{I} = |\sqrt{I_1} - \sqrt{I_2}|$

Hence the resultant amplitude is the difference of the individual amplitudes and is referred to as destructive interference.

b) Superspposition of incoherent waves:

Incoherent waves are the waves which do not maintain a constant phase diference. The phase of the waves fluctuates irregularly with time and independently of each other. In case of light waves the phase fluctuates randomly at a rate of about 10^8 per second. Light detectors such as human eye, photographic film etc, cannot respond to such rapid changes. The detected intensity is always the average intensity, averaged over a time interval which is very much larger than the time of fluctuations. Thus

$I_{av} = I_1 + I_2 + 2\sqrt{I_1 I_2} < \cos\phi >$. The average value of the $\cos\phi$ over a large time interval will be zero and hence $I_{av} = I_1 + I_2$

This implies that the superposition of incoherent waves gives uniform illumination at every point and is simply equal to the sum of the intensities of the component waves.

Interference:

→ The variation in intensity occurs due to the redistribution of the total energy of the interfering waves is called interference.

→ Interference of light is a wave phenomenon.

→ The source of light emitting wave of same frequency and travelling with either same phase or constant phase difference are called Coherent Sources.

Ex: Two virtual sources derived from a single source can be used as Coherent Sources.

→ The source producing the light wave travelling with rapid and random phase changes are called Incoherent Sources.

Ex: 1. Light emitted by two candles.

2. Light emitted by two lamps.

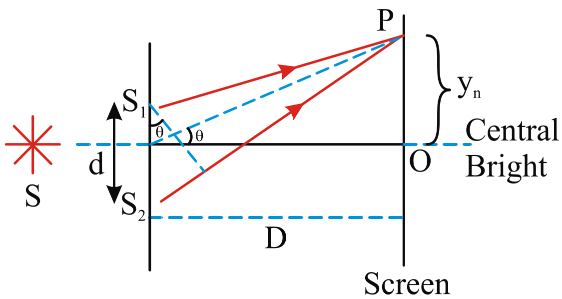
Conditions for Steady Interference

→ The two sources must be coherent.
→ Two sources must be narrow.
→ Two sources must be close together.

NOTE: The two sources must be mono chromatic, otherwise the fringes of different colours overlap and hence interference cannot be observed.

Young's Double Slit Experiment

→ Young with his experiment measured the most important characteristic of the light wave i.e wavelength (λ)
→ Young's experiment conclusively established the wave nature of light.



s_1s_2=d

→ When source illuminates the two slits, the pattern observed on the screen consists of large number of equally spaced bright and dark bands called "interference fringes"

a) Bright fringes :

Bright fringes occur whenever the waves from S_1 and S_2 interfere constructively. i.e. on reaching 'P', the waves with crest (or trough) superimpose at the same time and they are said to be in phase.

The condition for finding a bright fringe at 'P' is that $S_2P - S_1P = n\lambda$,

Where $n = 0, \pm 1, \pm 2, \pm 3,$ .... and n is called the order of bright fringe. Hence for n^{th} order bright fringe, the path difference is

$$d \sin\theta = n\lambda$$

$$\Rightarrow d\left(\frac{y_n}{D}\right) = n\lambda$$

$$\therefore \; y_n = \frac{n\lambda D}{d}$$

Where y_n is the position of n^{th} maximum from O.

The bright fringe corresponding to n = 0, is called the zero - order fringe or central maximum. It means it is the fringe with zero path difference between two waves on reaching the point P. The bright fringe corresponding to n = 1 is called first order bright fringe i.e., if the path difference between the two waves on reaching 'P' is λ. Similarly second order bright fringe $n = 2$ is located where the path difference is 2λ and so on.

From $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

For maximum intensity $\cos\frac{\phi}{2} = 1$

$i.e. \frac{\phi}{2} = 0, \pm, \pi, \pm 2\pi$ ....

(or) Phase difference between the waves $\phi = \pm 2\ \pi n$ with n = 0, 1, 2, 3 ....

The corresponding path difference, $\Delta x = n\lambda$

Hence $I_{\max} = 4I_0$.

b) Dark fringes :

Dark fringes occur whenever the waves from S_1 and S_2 interfere destructively. i.e., on reaching 'P' one wave with its crest and another wave with its trough superimpose. Then the phase difference between the waves is π and the waves are said to be in opposite phase.

Destructive interference occurs at P, if S_1P and S_2P differ by a odd integral multiple of $\frac{\lambda}{2}$.

Thus the condition for finding dark fringe at P is that $S_2P - S_1P = (2n-1)\frac{\lambda}{2}$.

Where $n = \pm 1, \pm 2, \pm 3,$..........., and n is called order of dark fringe. Hence for n^{th} order dark fringe, the path difference, $d \sin\theta = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow d\left(\frac{y_n}{D}\right) = (2n-1)\frac{\lambda}{2} \therefore y_n = \left(\frac{2n-1}{2}\right)\frac{\lambda D}{d}$$

Where y_n is the position of n^{th} minima from O.

The first dark fringe occurs when

$S_2P - S_1P = \frac{\lambda}{2}$.This is called first order dark (n = 1) fringe and similarly for $S_2P - S_1P = \frac{3\lambda}{2}$ second order dark fringe (n = 2) occurs and so on.

From $I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$

For minimum intensity $\cos\frac{\phi}{2} = 0$

i.e., $\frac{\phi}{2} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}$......

(or) $\phi = \pm\pi, \pm 3, \pm 5\pi$......

(or) $\phi = \pm(2n-1)\pi$ with n = 1, 2, 3 ......

The corresponding path difference, $\Delta x = (2n-1)\frac{\lambda}{2}$

Hence $I_{\min} = O$

c) Fringe width (β):

The distance between two adjacent bright (or dark) fringes is called the fringe width. It is denoted by β.

The n^{th} order bright fringe occurs from the central maximum at $y_n = \frac{n\lambda D}{d}$

The $(n+1)^{th}$ order bright fringe occurs from the central maximum at $y_{n+1} = \frac{(n+1)\lambda D}{d}$

$\therefore$ The fringe separation, β is given by

$$\beta = y_{n+1} - y_n = \frac{\lambda D}{d}$$

In a similar way, the same result will be obtained for the dark fringes also.

$\therefore$ Fringe width, $\beta = \frac{\lambda D}{d}$

Thus fringe width is same every where on the screen and the width of bright fringe is equal to the width of dark fringe.

$$\therefore \; \beta_{bright} = \beta_{dark} = \beta = \frac{\lambda D}{d}$$

d) The locus of the point P lying in the xy-plane such that $S_2P - S_1P = (\Delta x)$ (path difference) is a constant, is a hyperbola. If the distance D is very large compared to the fringe width, the fringes will be very nearly straight lines.

Note:

Constructive Interference

i) a) If the phase difference is $\phi = (2n)\pi$ (even multiples of π). Where n = 0, 1,2, 3,......

i.e. when $\phi = 0, 2\pi, 4\pi........2n\pi$

b) If the path difference $x = 2n\left(\frac{\lambda}{2}\right)$ (even multiples of half wavelength).

i.e when $x = 0, \lambda, 2\lambda........n\lambda$

The amplitude and intensity are maximum.

$$\boxed{A_{\max} = (A_1 + A_2)}$$

$$\boxed{I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = (A_1 + A_2)^2}$$

Note: If $A_1 = A_2 = a$ then $\boxed{A_{\max} = 2a}$

If $I_1 = I_2 = I_0$ then $\boxed{I_{\max} = 4I_0}$

Destructive Interference

ii) a) If the phase difference $\phi = (2n-1)\pi$ (odd multiples of π) where n = 1, 2, 3.....

i.e. when $\phi = \pi, 3\pi, 5\pi.......(2n-1)\pi$

b) If the path difference $x = (2n-1)\lambda/2$ (odd multiples of $\lambda/2$)

i.e. when $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}........\frac{(2n-1)\lambda}{2}$

The amplitude and Intensity are minimum.

$$\boxed{A_{\min} = (A_1 - A_2)}$$

$$\boxed{I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = (A_1 - A_2)^2}$$

Note: If $A_1 = A_2 = a$ then $\boxed{A_{\min} = 0}$

If $I_1 = I_2 = I_0$ then $\boxed{I_{\min} = 0}$

$$\boxed{\frac{I_{\max}}{I_{\min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}}$$

iii) phase difference = $\frac{2\pi}{\lambda}$ (path difference).

$$\boxed{\phi = \frac{2\pi}{\lambda} x}$$

iv) Since $\beta \alpha \lambda, \beta_{\mathrm{Re}d} > \beta_{voilet}, as\, \lambda_{red} > \lambda_{voilet}$

v) In YDSE, if blue light is used instead of red light then β decreases $(\therefore \lambda_B < \lambda_R)$

vi) If YDSE is conducted in vaccum instead of air, then β increases $(\therefore \lambda_{vaccum} > \lambda_{air})$

vii) In certain field of view on the screen, if n_1 fringes are formed when light of wavelength λ_1 is used and n_2 fringes are formed when light of wavelength λ_2 is used, then

$y = \frac{n\lambda D}{d}$ = constant $\Rightarrow n\lambda$ = constant

$\therefore\ n_1\lambda_2 = n_2\lambda_2$ (or) $n_1\beta_1 = n_2\beta_2$

viii) The distance of n^{th} bright fringe from central maximum is $(y_n)_{bri} = \frac{n\lambda D}{d} = n\beta$

The distance of m^{th} dark fringe from central maximum is

$$(y_m)_{dark} = \frac{(2m-1)}{2}\frac{\lambda D}{d} = \frac{(2m-1)}{2}\beta$$

$\therefore$ The distance between n^{th} bright and m^{th} dark fringes is

$$(y_n)_{bri} - (y_m)_{dark} = n\beta - \frac{(2m-1)}{2}\beta$$

ix) When white light is used in YDSE the inteference patterns due to different component colours of white light overlap (incoherently). The central bright fringes for different colours are at the same position. Therefore, the central fringe is white. For a point P for whih $S_2P - S_1P = \frac{\lambda_b}{2}$ where $\lambda_b (\approx 4000 A^0)$ represents the wavelength for the blue colour, the blue component will be absent and the fringe will appear red in colour.

Slightly farther away where $S_2Q - S_1Q = \frac{\lambda_r}{2}$ where $\lambda_r (\approx 8000 A^0)$ is the wavelength for the red colour, the fringe will be predominantly blue.

Thus, the fringe closest on either side of the central white fringe is red and fathest will appear blue. After a few fringes, no clear fringe pattern is seen.

x) To know maximum number of possible maxima on the screen

If $d\sin\theta = n\lambda\,(or)\sin\theta = \frac{n\lambda}{d}$

As $\sin\theta \le 1, \frac{n\lambda}{d} \le 1 \qquad \therefore\ n \le \frac{d}{\lambda}$

Therefore the maximum number of complete maxima on the screen will be $2(n)+1$

Ex: If $d = 3\lambda\ then\ \sin\theta = \frac{n\lambda}{3\lambda} = \frac{n}{3}$ As $\sin\theta \le 1$,

n can take values $\quad -3,-2,-1,0,1,2,3$

$\therefore$ Maximum number of maxima is 7.

xi) Fringe visibility (or) band visibility (V) :

It is the measure of contrast between the bright and dark fringes

Fringe visibility, $V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

where $I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$\therefore\ V = \frac{4\sqrt{I_1 I_2}}{2(I_1 + I_2)} = \frac{2\sqrt{I_1 I_2}}{(I_1 + I_2)}$

V has no unit and no dimensional formula.

Generally, 0 < V < 1.

Fringe visibility is maximum, if $I_{min} = 0$, then

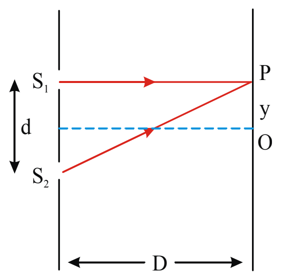
V = 1

For poor visibility , $I_{max} = I_{min}$, then V = 0

i.e., if V = 1, then the fringes are very clear and contrast is maximum and if V = 0, then there will be no fringes and there will be uniform illumination i.e., the contrast is poor.

xii) When one slit is fully open and another one is partially open then the contrast between the fringes decreases. i.e., if the slit widths are unequal, the minima will not be completely dark.

xiii) Missing wavelengths in front of one slit in YDSE:



Suppose P is a point of observation in front of slit S_1 as shown in figure. Path difference between the two waves from S_1 and S_2 is

$\Delta x = S_2P - S_1P = \sqrt{D^2 + d^2} - D$

$$= D\left(1+\frac{d^2}{D^2}\right)^{1/2} - D = D\left(1+\frac{d^2}{2D^2}\right) - D = \frac{d^2}{2D}$$

$$\therefore \Delta x = \frac{d^2}{2D} ....(1)$$

But for missing wavelengths, intensity will be zero. i.e., the corresponding path difference,

$$\Delta x = (2n-1)\frac{\lambda}{2} \qquad ...... (2)$$

From equations (1) and (2)

$$\frac{d^2}{2D} = (2n-1)\frac{\lambda}{2}$$

$\therefore$ Missing wavelength, $\lambda = \dfrac{d^2}{(2n-1)D}$

By putting $n = 1, 2, 3, ...,$ the wavelengths at P are

$$\lambda = \frac{d^2}{D}, \frac{d^2}{3D}, \frac{d^2}{5D}, ...$$

In the above case, if **bright fringes** are to be formed exactly opposite to S_1 then

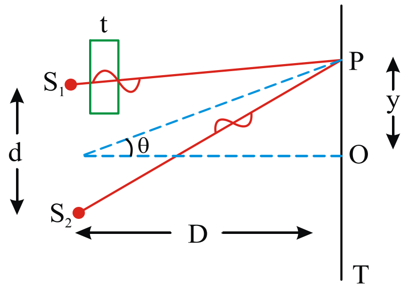
$$\frac{d^2}{2D} = n\lambda \Rightarrow \lambda = \frac{d^2}{2Dn}$$

By putting n = 1, 2, 3 ,,, the possible wavelengths at P are

$$\lambda = \frac{d^2}{2D}, \frac{d^2}{4D}, \frac{d^2}{6D}, ...$$

xiv) Lateral displacement of fringes:

To determine the thickness of a given thin sheet of transparent material such as glass or mica, that transparent sheet is introduced in the path of one of the two interfering beams. The fringe pattern gets displaced towards the beam in whose path the sheet is introduced. This shift is known as lateral displacement or lateral shift.



The optical path from S_1 to $P = (S_1P - t) + \mu t$. The optical path from S_2 to $P = S_2P$.

To get central zero fringe at P, $\Delta_{s_1p} = \Delta_{s_2p}$

$$\Rightarrow S_1P - t + \mu t = S_2P$$

$$\therefore \; S_2P - S_1P = (\mu - 1)t$$

Since $\mu > 1$, this implies $S_2P > S_1P$ hence the fringe pattern must shift towards the beam from S_1.

But $S_2P - S_1P = d\sin\theta = d\frac{y}{D}$, where 'y' is the lateral shift.

$\therefore\ (\mu-1)t = d\frac{y}{D}$

$\therefore$ Lateral shift $(y) = \frac{D}{d}(\mu-1)t = \frac{\beta}{\lambda}(\mu-1)t$

(or) Thickness of sheet

$$t = \frac{yd}{(\mu-1)D} = \frac{y\lambda}{(\mu-1)\beta}$$

From the above it is clear that

a) For a given colour, shift is independent of order of the fringe i.e. shift in zero order maximum = shift in 9th minima (or) shift in 6th maxima = shift in 2nd minima. Since the refractive index depends on wavelength hence lateral shift is different for different colours.

b) The number of fringes shifted = $\frac{lateral\ shift}{fringe\ width}$

$\therefore n = \frac{y}{\beta} = \frac{(\mu-1)t}{\lambda}$ (or) $n\lambda = (\mu-1)t$

Therefore, number of fringes shifted is more for shorter wavelength.

c) If a transparent sheet of thickness 't' and its relative refractive index μ_r (w.r.t. surroundings) be introduced in one of the beams of interference, then

1) the lateral shift $y = \frac{(\mu_r-1)tD}{d}$

2) the number of fringes shifted $n = \frac{(\mu_r-1)t}{\lambda}$

d) Due to the presence of transparent sheet, the phase difference between the interfering waves at a given point is given by $= \frac{2\pi}{\lambda}(\mu-1)t$.

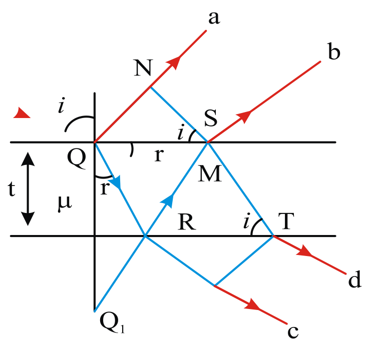
e) If YDSE is performed with two different colours of light of wavelengths $\lambda_1\ \&\ \lambda_2$ but by placing the same transparent sheet in the path of one of the interfering waves then $n_1\lambda_1 = n_2\lambda_2$. where n_1 and n_2 are the number of fringes shifted with wavelengths $\lambda_1\ \&\ \lambda_2$.

vi) When two different transperent sheets of thickness t_1, t_2 and refractive index μ_1, μ_2 are placed in the paths of two interfering waves in YDSE, if the central bright fringe position is not shifted, then $(\mu_1-1)t_1 = (\mu_2-1)t_2$.

Important Concepts :

→ **Formation of colours in thin films** :

a) Interference due to reflected light



Reflected system :

→ Path difference between the rays Qa and QRSb. (PD) = QRS in medium - QN in air

$\therefore P.D = 2\mu t\cos r$ This is the path lag

due to reflection on film additional path lag of $\lambda/2$ exists. (stoke's theorem)

Total path difference $= 2\mu t\cos r + \dfrac{\lambda}{2}$

Condition for maximum

→ $2\mu t\cos r + \dfrac{\lambda}{2} = n\lambda$

OR $\boxed{2\mu t\cos r = (2n-1)\dfrac{\lambda}{2}}$ For all values of n is equal to 1, 2, 3............... n.

→ **Condition for Minimum**

$2\mu t\cos r + \dfrac{\lambda}{2} = (2n-1)\dfrac{\lambda}{2}$

$\boxed{2\mu t\cos r = n\lambda}$ for values of n = 0,1,2,3... n = 0 gives the central minima.

For normal incident $i = o = r$

$2\mu t = n\lambda$ for dark ; $2\mu t = (2n-1)\dfrac{\lambda}{2}$ for bright.

Transmitted system

→ Interference of two rays Rc and Td. By symmetry it can be concluded that the path difference between the rays in $2\mu t\cos r$.

But there would not be any extra phase lag because either of the two rays suffers reflection at denser surface.

→ **Condition for maxima :** $\boxed{2\mu t\cos r = n\lambda}$

→ **Condition for minimum :**

$\boxed{2\mu t\cos r = (2n-1)\dfrac{\lambda}{2}}$

If YDSE is conducted with white light,

→ Central fringe is always achromatic (white)

→ When path difference is small, then some coloured fringes are obtained on two sides of the central fringe. The outer edge of the fringe is violet and inner edge is red.

→ The fringe width is different for different colours

→ The number of fringes obtained is less than that with monochromatic light source.

WE-2: ***Light waves from two coherent sources having intensity ratio 81 : 1 produce interference. Then, the ratio of maxima and minima in the interference pattern will be***

Sol. Given, $\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = \frac{81}{1}$

$\therefore \frac{A_1}{A_2} = \frac{9}{1}$ **or** $A_1 = 9A_2$ .... (1) $\quad \therefore \frac{I_{\max}}{I_{\min}} = \frac{(A_1 + A_2)^2}{(A_1 - A_2)^2}$

From Eq. (i), we get

$$\frac{I_{\max}}{I_{\min}} = \frac{(9A_2 + A_2)^2}{(9A_2 - A_2)^2} = \frac{(10)^2}{(8)^2} = \frac{25}{16}$$

WE-3: ***Two slits are made one millimetre apart and the screen is placed one metre away. When blue-green light of wavelength 500 nm is used, the fringe separation is***

Sol. Fringe separation, $\beta = \frac{D\lambda}{d}$

Given, $D = 1m, \lambda = 500\,nm = 5 \times 10^{-7}\,m$ **and**

$d = 1mm = 1 \times 10^{-3}\,m$

$\therefore$ Fringe separation, $\beta = \frac{1 \times 5 \times 10^{-7}}{1 \times 10^{-3}}\,m$

$$= 5 \times 10^{-4}\,m = 0.5\,mm$$

WE-4: ***In YDSE, the two slits are separated by 0.1 mm and they are 0.5 m from the screen. The wavelengh of light used is 5000 Å . What is the distance between 7th maxima and 11th minima on the screen?***

Sol. Here, d = 0.1 mm = 10^{-4} m,

$D = 0.5\,m, \lambda = 5000\,\text{Å} = 5.0 \times 10^{-7}\,m$

$\therefore \quad \Delta x = (X_{11})_{dark} - (X_7)_{bright} = \frac{(2 \times 11 - 1)\lambda D}{2d} - \frac{7\lambda D}{d}$

$\Delta x = \frac{7\lambda D}{2d} = \frac{7 \times 5 \times 10^{-7}}{2 \times 10^{-4}}$

$= 8.75 \times 10^{-3}\,m$

$= 8.75\,mm$

WE-5: ***In Young's double slit experiment interference fringes 1^0 apart are produced on the screen, the slit separation is*** $(\lambda = 589\ nm)$

Sol. The fringe width, $\beta = \dfrac{\lambda D}{d}$

The angualr separation of the fringes is given by

$$\theta = \frac{\beta}{D} = \frac{\lambda}{d}$$

Given, $\theta = 1^0 = \dfrac{\pi}{180} rad$

$\lambda = 589\ nm$

$$\therefore\ d = \frac{\lambda}{\theta} = \frac{589 \times 180 \times 10^{-9}}{\pi}$$

$= 0.0337\ mm$

WE-6: ***In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of n for which nth bright band due to red light coincides with (n + 1)th bright band due to blue light, is***

Sol. $\dfrac{n_R \lambda_R D}{d} = \dfrac{n_B \lambda_B D}{d}$ **or** $\dfrac{n_R}{n_B} = \dfrac{\lambda_B}{\lambda_R} = \dfrac{5200}{7800} = \dfrac{2}{3}$

Therefore 2^{nd} of red coincides with 3^{rd} of blue.

WE-7: ***Young's double slit experiment is made in a liquid. The 10th bright fringe in liquid lies where 6th dark fringe lies in vacuum. The refractive index of the liquid is approximately***

Sol. Fringe width $\beta = \frac{\lambda D}{d}$. When the apparatus is immersed in a liquid, λ and hence β is reduced μ (refractive index) times.

$10\beta' = (5.5)\beta$ or $10\lambda'\left(\dfrac{D}{d}\right) = (5.5)\dfrac{\lambda D}{d}$

or $\dfrac{\lambda}{\lambda'} = \dfrac{10}{5.5} = \mu$ or $\mu = 1.8$

WE-8: ***In Young's double slit experiment, how many maximas can be obtained on a screen (including the central maximum) on both sides of the central fringe if*** $\lambda = 2000$ ***Å and*** $d = 7000$ ***Å?***

Sol. For maximum intensity on the screen $d\sin\theta = n\lambda$ or $\sin\theta = \dfrac{n\lambda}{d}$; $= \dfrac{(n)(2000)}{(7000)} = \dfrac{n}{3.5}$

maximum value of $\sin\theta = 1$

$\therefore n = -3, -2, -1, 0, 1, 2, 3$; $\therefore 7$ maximas.

WE-9: ***In a double slit experiment the angular width of a fringe is found to be*** 0.2^0 ***on a screen placed l m away. The wavelength of light used in 600 nm. What will be the angular width of the fringe if the entire experimental apparatus is immersed in water? Take refractive index of water to be 4/3.***

Sol. Angular fringe separation,

$\theta = \frac{\lambda}{d} ord = \frac{\lambda}{\theta}$; In water, $d = \frac{\lambda'}{\theta'}$

$\therefore \frac{\lambda}{\theta} = \frac{\lambda'}{\theta'} or \frac{\theta'}{\theta} = \frac{\lambda'}{\lambda} = \frac{1}{\mu} = \frac{3}{4}$

or $\theta' = \frac{3}{4}\theta = \frac{3}{4} \times 0.2^0 = 0.15^0$

WE-10: ***In a Young's experiment, one of the slits is covered with a transparent sheet of thickness $3.6 \times 10^{-3} cm$ due to which position of central fringe shifts to a position orginally occupied by 30th fringe.If $\lambda = 6000$ Å, then find the refractive index of the sheet.***

Sol. The position of 30th bright fringe,

$y_{30} = \frac{30\lambda D}{d}$ Now position shift of central fringe is

$y_0 = \frac{30\lambda D}{d}$; But we know, $y_0 = \frac{D}{d}(\mu - 1)t$

$\frac{30\lambda D}{d} = \frac{D}{d}(\mu - 1)t$

$\Rightarrow \quad (\mu - 1) = \frac{30\lambda}{t} = \frac{30 \times (6000 \times 10^{-10})}{(3.6 \times 10^{-5})} = 0.5$

$\therefore \quad \mu = 1.5$

WE-11: ***The maximum intensity in the case of n identical incoherent waves each of intensity $2\frac{W}{m^2}$ is $32\frac{W}{m^2}$ the value of n is***

Sol. I = n I_0, 32 = n 2, n = 16

WE-12: ***Compare the intensities of two points located at respective distance $\frac{\beta}{4}$ and $\frac{\beta}{3}$ from the central maxima in a interference of YDSE (β is the fringe width)***

Sol. $\Delta\theta = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\left(\frac{d}{D}\frac{\beta}{4}\right) = \frac{2\pi}{\lambda}\left(\frac{d}{D}\frac{\lambda D}{4d}\right)$

$\therefore \quad \Delta\theta = \frac{2\pi}{4} = \frac{\pi}{2} \Rightarrow I = 4I_0 \cos^2\left(\frac{\pi}{4}\right)$

Similarly $\Delta\theta = \frac{2\pi}{3} \Rightarrow I = 4I_0 \cos^2\left(\frac{2\pi}{2 \times 3}\right) = I_0$

$\therefore$ required ratio = $\boxed{2:1}$

WE-13: ***In Young's double slit experiment intensity at a point is (1/4) of the maximum intensity. Angular position of this points is***

Sol: $I = I_{max} \cos^2\left(\frac{\phi}{2}\right); \therefore \frac{I_{max}}{4} = I_{max} \cos^2\left(\frac{\phi}{2}\right)$

$\cos\frac{\phi}{2} = \frac{1}{2} or \frac{\phi}{2} = \frac{\pi}{3}$

$\therefore \phi = \frac{2\pi}{3} = \left(\frac{2\pi}{\lambda}\right).\Delta x$ where $\Delta x = d\sin\theta$

$\frac{\lambda}{3} = d\sin\theta, \sin\theta = \frac{\lambda}{3d}, \theta = \sin^{-1}\left(\frac{\lambda}{3d}\right)$

WE-14: ***In Young's double slit experiment the y co-ordinates of central maxima and 10th maxima are 2 cm and 5 cm respectively. When the YDSE apparatus is immersed in a liquid of refractive index 1.5 the corresponding y co-ordinates will be***

Sol. Fringe width $\beta \propto \lambda$. Therefore, λ and hence β will decrease 1.5 times when immersed in liquid. The distance between central maxima and 10th maxima is 3 cm in vacuum. When immersed in liquid it will reduce to 2 cm. Position of central maxima will not change while 10th maxima will be obtained at y = 4 cm.

WE-15: ***In YDSE, bi-cromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1m. The minimum distance between two successive regions of complete darkness is:***

Sol. Let nth minima of 400 nm coincides with mth minima of 560nm, then

$(2n-1)\left(\frac{400}{2}\right) = (2m-1)\left(\frac{560}{2}\right)$ or

$\frac{2n-1}{2m-1} = \frac{7}{5} = \frac{14}{10} = ...$

i.e., 4th minima of 400 nm coincides with 3rd minima of 560 nm. Location of this minima is,

$Y_1 = \frac{(2\times 4-1)(1000)(400\times 10^{-9})}{2\times 0.1} = 14\,mm$

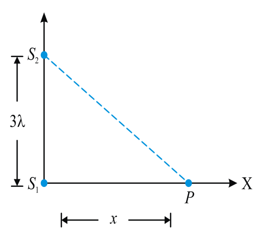
Next 11th minima of 400 nm will coincide with 8th minima of 560 nm.
Location of this minima is,

$Y_2 = \frac{(2\times 11-1)(1000)(400\times 10^{-9})}{2\times 0.1} = 42\,mm$

$\therefore$ Required distance $Y_2 - Y_1 = 28\ mm$.

WE-16: ***An interference is observed due to two coherent sources S_1 placed at origin and S_2 placed at $(0, 3\lambda, 0)$. Here λ is the wavelength of the sources. A detector D is moved along the positive x-axis. Find x-coordinates on the x-axis (excluding x = 0 and $x = \infty$) where maximum intensity is observed.***

Sol: At x = 0, path difference is 3λ. Hence, third order maxima will be obtained. At $x = \infty$, path difference is zero. Hence, zero order maxima is obtained. In between first and second order maxima will be obtained.



First order maxima:

$S_2P - S_1P = \lambda\ (or)\ \sqrt{x^2 + 9\lambda^2} - x = \lambda$

or $\sqrt{x^2+9\lambda^2}=x+\lambda$ Squaring both sides, we get $x^2+9\lambda^2=x^2+\lambda^2+2x\lambda$. Solving this, we get $x=4\lambda$. Second order maxima:

$S_2P-S_1P=2\lambda$; (or) $\sqrt{x^2+9\lambda^2}-x=2\lambda$ (or)

$\sqrt{x^2+9\lambda^2}=(x+2\lambda)$ Squaring both sides, we get

$x^2+9\lambda^2=x^2+4\lambda^2+4x\lambda$

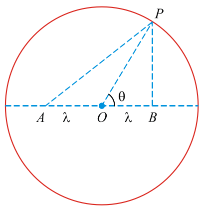
Solving, we get $x=\frac{5}{4}\lambda=1.25\lambda$

Hence, the desired x coordinates are,

$x=1.25\lambda$ and $x=4\lambda$.

WE-17: ***Two coherent light sources A and B with separation 2λ are placed on the x-axis symmetrically about the origin. They emit light of wavelength λ. Obtain the positions of maxima on a circle of large radius, lying in the x-y plane and with centre at the origin.***

Sol:



For P to have maximum intensity, $d\cos\theta=n\lambda$

$2\lambda\cos\theta=n\lambda$ $\cos\theta=\frac{n}{2}$ where n is integer

For $n=0,\ \theta=90^0,270^0$

$n=\pm1,\theta=60^0,120^0,\ 240^0,300^0$

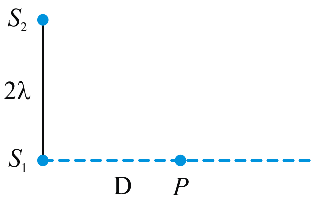
$n=\pm2,\ \theta=0^0,180^0$

So, positions of maxima are at

$\theta=0^0,60^0,90^0,120^0,180^0,240^0,270^0$ and 300^0 ; i.e., 8 positions will be obtained.

Short cut : In $d=n\lambda$ then number of maximum on the circle is 4n.Note: For minima; $\Delta x=(2n-1)\frac{\lambda}{2}$

WE-18: ***Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between the sources is 2λ. Consider a line passing through S_2 and perpendicular to the line S_1S_2. Find the position of farthest and nearest minima***



Sol: $\Delta x_{\min}=(2n-1)\frac{\lambda}{2}$ The farthest minima has path difference $\lambda/2$ while nearest minima has path difference $(3/2)\lambda$. For the nearest minima.

$S_1P - S_2P = \frac{3}{2}\lambda$; [as maximum path difference is 2λ]

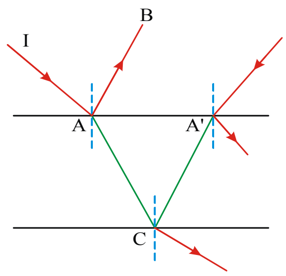
$\Rightarrow \sqrt{(2\lambda)^2 + D^2} - D = \frac{3}{2}\lambda \Rightarrow (2\lambda)^2 + D^2 = \left(\frac{3}{2}\lambda + D\right)^2 \quad \Rightarrow 4\lambda^2 + D^2 = \frac{9}{4}\lambda^2 + D^2 \times 2 \times \frac{3}{2}\lambda \times D$

$\Rightarrow 3D = 4\lambda - \frac{9\lambda}{4} = \frac{7\lambda}{4} \Rightarrow D = \frac{7}{12}\lambda$

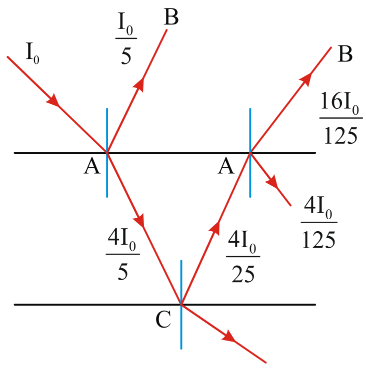
For the farthest minima, $S_1P - S_2P = \frac{\lambda}{2}$

$\Rightarrow \sqrt{4\lambda^2 + D^2} - D = \frac{\lambda}{2} \quad \Rightarrow 4\lambda^2 + D^2 = \frac{\lambda^2}{4} + D^2 + D\lambda \Rightarrow D = 4\lambda - \lambda/4 = \frac{15\lambda}{4}$

WE 19: ***A ray of light of intensity I is incident on a parallel glass slab at a point A as shown. It undergoes partial reflection and refraction. At each reflection 20% of incident energy is reflected. The rays AB and A' B' undergo interference. The ratio I_{max}/I_{min} is***



Sol: According to the question, Intensity of ray AB, $I_1 = \frac{I_0}{5}$ and Intensity of ray $A'B'$,

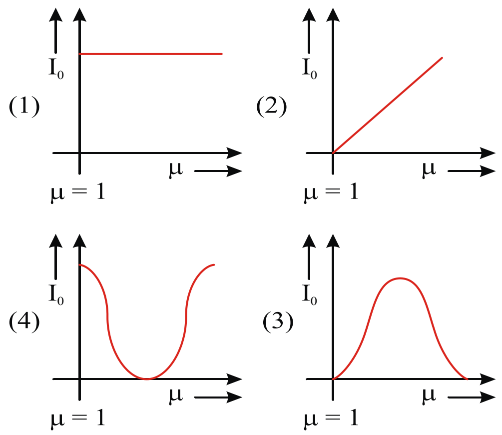


$I_2 = \frac{16I_0}{125}, I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 = \frac{81}{125}I_0,$

$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 = \frac{I_0}{125}, \frac{I_{max}}{I_{min}} = 81$.

WE 20: ***In a YDSE experiment if a slab whose refractive index can be varied is placed in front of one of the slits, then the variation of resultant intensity at mid-point of screen with 'μ' will be best represented by*** $(\mu \geq 1)$.

[Assume slits of equal width and there is no absorption by slab]



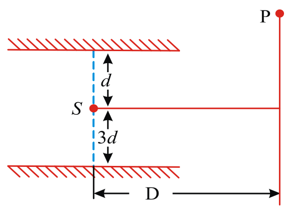
Sol. $\Delta x = (\mu - 1)t$;   For $\mu = 1, \Delta x = 0$

$\therefore$ I = maximum = I_0 ; As μ increases path difference Δx also increases.; For $\Delta x = 0$ to $\frac{\lambda}{2}$, intensity will decrease from I_0 to zero.

Then for $\Delta x = \frac{\lambda}{2} \, to \, \lambda,$ intensity will increase from zero to I_0.

Hence option 3 is correct

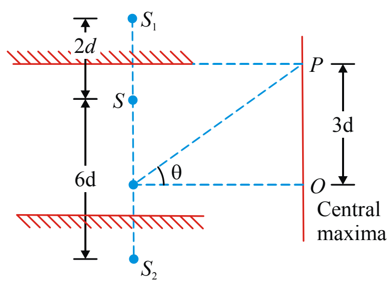
WE 21: ***Consider the optical system shown in fig. The point source of light S is having wavelength equal to λ. The light is reaching screen only after reflection. For point P to be 2nd maxima, the value of λ would be*** $(D >> d \ \& \ d >> \lambda)$



1) $\frac{12d^2}{D}$ 2) $\frac{6d^2}{D}$ 3) $\frac{3d^2}{D}$ 4) $\frac{24d^2}{D}$

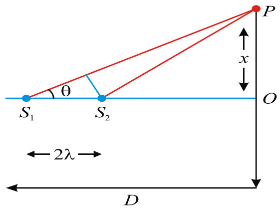
Sol: a. At P, $\Delta x = \frac{(8d) \times 3d}{D}$;

For 2nd maxima, $\Delta x = 2\lambda$; $\Rightarrow \frac{24d^2}{D} = 2\lambda$



$\Rightarrow \lambda = \frac{12d^2}{D}$

WE 22: ***Two coherent point sources S_1 and S_2 vibrating in phase emit light of wavelength λ. The separation between them is 2λ as shown in figure. The first bright fringe is formed at 'P' due to interference on a screen placed at distance 'D' from $S_1 (D >> \lambda)$, then OP is***



1) $\sqrt{3}$ D 2) 1.5 D 4) $\sqrt{2}$ D 4) 2 D

Sol: $\Delta x = d\cos\theta = \lambda\,;\cos\theta = \dfrac{\lambda}{d} = \dfrac{\lambda}{2\lambda} = \dfrac{1}{2}$

$\theta = 60^0 \tan 60 = \dfrac{x}{D}\ x = \sqrt{3}D$

Diffraction

→ The bending of light around edges of an obstacle on the enchroachment of light within geometrical shadow is known as *"diffraction of light"*

→ Diffraction is a characteristic wave property.

→ Diffraction is an effect exhibited by all electro-magnetic waves, water waves and sound waves

→ Diffraction takes place with very small moving particles such as atoms, neutrons and electrons which show wavelike properties.

→ When light passes through a narrow aperture some light is found to be enchroached into shadow regions.

→ When slit width is larger, the enchroachment of light is small and negligible.

→ When slit width is comparable to wavelength of light the enchroachment of light is more

→ If the size of obstacle or aperture is comparable with the wavelength of light, light deviates from rectilinear propagation near edges of obstacle or aperture and enchroaches into geometrical shadow.

→ Diffraction phenomenon is classified into two types, a) Fresnel diffraction b) Fraunhoffer diffraction

Fresnel Diffraction

→ The source or screen or both are at finite distances from diffracting device (obstacle or aperture)

→ In Fresnel diffraction, the effect at any point on the screen is due to exposed wave front which may be spherical or cylindrical in shape.

→ Fresnel diffraction does not require any lens to modify the beam.

→ Fresnel diffraction can be explained in terms of "half period zones or strips"

Fraunhoffer Diffraction:

The source and the screen are at infinite distance from diffracting device (aperture or obstacle).

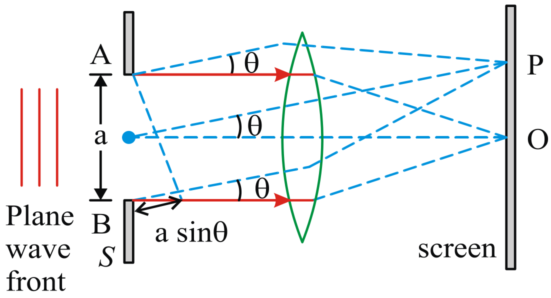
→ In Fraunhofer diffraction the wave front meeting the obstacle is plane wave front.

→ Fraunhofer diffraction requires lenses to modify the beam.

Diffraction Due to Single Slit

→Diffraction is supposed to be due to interference of secondary wavelets from the exposed portion of wavefront from the slit.

Whereas in interference, all bright fringes have same intensity. In diffraction, bright bands are of decreasing intensity.



i) Condition for minimum intensity is

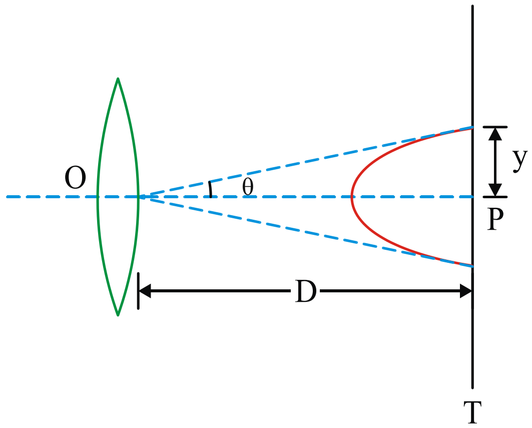
$$\boxed{a\sin\theta = n\lambda} \quad (n = 1, 2, 3, ...)$$

Where 'a' is the width of the slit, θ is the angle of diffraction

ii) Condition for maximum intensity

$$\boxed{a\sin\theta = (2n+1)\frac{\lambda}{2}} \quad (n = 1, 2, 3, ...)$$

The intensity decreases as we go to successive maxima away from the centre, on either side. The width of central maxima is twice as that of secondary maxima.



For first minia $a\sin\theta = \lambda$

$$a\frac{y}{D} = \lambda \; (\because \; \sin\theta \approx \tan\theta) \; \therefore \; y = \frac{\lambda D}{a}$$

Width of central maxima $w = 2y = \dfrac{2\lambda D}{a}$

Note: If lens is placed close to the slit, then D = f. Hence 'f' be the focal length of lens, then width of the central maximum $w = \dfrac{2f\lambda}{a}$.

Note: If this experiment is performed in liquid other than air, width of diffraction maxima will decrease and becomes $\frac{1}{\mu}$ times. With while light, the central maximum is white and the rest of the diffraction bands are coloured.

→ **Interference and diffraction bands**

If N interference bands are contained by the width of the central bright.

width $= N\beta = N\left(\frac{D\lambda}{d}\right); \therefore \frac{2D\lambda}{a} = \frac{ND\lambda}{d}$

therefore width of the slit $a = \frac{2d}{N}$

WE-23: ***A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit.***

Sol: $\theta = \frac{y}{D}, \theta = \frac{2.5\times10^{-3}}{1} radian$

Now, $a\sin\theta = n\lambda$

Since θ is very small, therefore $\sin\theta = \theta$.

or $a = \frac{n\lambda}{\theta} = \frac{1\times500\times10^{-9}}{2.5\times10^{-3}} m$

$= 2\times10^{-4} m = 0.2\, mm$

WE-24: ***A screen is placed 50 cm from a single slit, which is illuminated with 6000 Å light, If distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?***

Sol: In case of diffraction at single slit, the position of minima is given by $a\sin\theta = n\lambda$.

Where d is the aperture size and for small θ:

$\sin\theta = \theta = (y/D)$

$\therefore\ a\left(\frac{y}{D}\right) = n\lambda,\ i.e., y = \frac{D}{a}(n\lambda)$

So that, $y_3 - y_1 = \frac{D}{a}(3\lambda - \lambda) = \frac{D}{a}(2\lambda)$ and hence, $a = \frac{0.50\times(2\times6\times10^{-7})}{3\times10^{-3}} = 2\times10^{-4} m$

$= 0.2\, mm$

WE-25: ***In a single slit diffraction experiment first minimum for $\lambda_1 = 660\, nm$ coincides with first maxima for wavelength λ_2. Calculate λ_2.***

Sol: Position of minima in diffraction pattern is given by; $a\sin\theta = n\lambda$

For first minima of λ_1, we have

$a\sin\theta_1 = (1)\lambda_1$ or $\sin\theta_1 = \frac{\lambda_1}{a}$ .....(i)

The first maxima approximately lies between first and second minima. For wavelength λ_2 its position will be

$a\sin\theta_2 = \frac{3}{2}\lambda_2 \therefore \quad \sin\theta_2 = \frac{3\lambda_2}{2a}$ ...... (ii)

The two will coincide if,

$\theta_1 = \theta_2$ or $\sin\theta_1 = \sin\theta_2$

$\therefore \quad \frac{\lambda_1}{a} = \frac{3\lambda_2}{2a}$ or

$\lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3} \times 660\,nm = 440\,nm$

WE-26: ***Two slits are made one millimeter apart and the screen is placed one meter away. What should the width of each slit be to obtain 10 maxima of the double slit pattern within the central maximum of the single slit pattern.***

Sol: We have $a\theta = \lambda$ (or) $\theta = \frac{\lambda}{a}$

(a = width of each slit)

$10\frac{\lambda}{d} = 2\frac{\lambda}{a}$

$\therefore \quad a = \frac{d}{5} = \frac{1}{5} = 0.2\,mm$

The Validity of Ray Optics:

The distance of the screen from the slit, so that spreading of light due to diffraction from the centre of screen is just equal to size of the slit, is called Fresnel distance. It is denoted by Z_F. The diffraction pattern of a slit consists of secondary maximum and minima on the two sides of the central maximum. Therefore, one can say that on diffraction from a slit, light spreads on the screen in the form of central maximum. The angular position of first secondary minimum is called half angular width of the central maximum and it is given by

$\theta = \frac{\lambda}{a}$ (provided θ is small)

If the screen is placed at a distance D from the slit, then the linear spread of the central maximum is given by

$y = D\theta = \frac{D\lambda}{a}$

It is, in fact, the distance of first secondary minimum from the centre of the screen. It follows that as the screen is moved away (D is increased), the linear size of the central maximum i.e., spread distance, when $D = Z_F$,

y = a (size of the slit)

Setting this condition in the above equation, we have

$a = \frac{Z_F\lambda}{a}$ or $\boxed{Z_F = \frac{a^2}{\lambda}}$

It follows that if screen is placed at a distance beyond Z_F, the spreading of light due to diffraction will be quite large as compared to the size of the slit. The above equation shows that the ray -optics is valid in the limit of wavelength tending to zero.

WE-27: ***For what distance is ray optics a good approximation when the aperture is 3 mm wide and the wavelength is 500 nm?***

Sol: For distance $Z \le Z_F$,

ray optics is the good appropriate

Fresnel distance $Z_F = \frac{a^2}{\lambda} = \frac{(3\times10^{-3})^2}{5\times10^{-7}} = 18\ m$

Limit of resolution:

→ The smallest linear or angular separation between two point objects at which they can be just separately seen or resolved by an optical instrument is called the limit of resolution of the instrument.

Resolving Power:

→ The resolving power of an optical instrument is reciprocal of the smallest linear or angular separation between two point objects, whose images can be just resolved by the instrument.

Resolving power = $\frac{1}{Limit\ of\ resolution}$

The resolving power of an optical instument is inversely propotional to the wavelength of light used.

Diffraction as a limit on resolving power:

→ All optical instuments like lens, telescope, microscope, etc, act as apertures. Light on passing through them undergoes diffraction. This puts the limit on their resolving power.

Rayleigh's criterion for resolution:

→ The images of two point objects are resolved when the central maximum of the diffraction pattern of one falls over the first minimum of the diffraction pattern of the other.

Resolving Power of a Microscope:

→ T he resolving power of a microscope is defined as the reciprocal of the smallest distance d between two point objects at which they can be just resolved when seen in the microscope.

→ Resolving power of microscope $= \frac{1}{d} = \frac{2\mu\sin\theta}{1.22\lambda}$

Clearly, the resolving power of a microscope depends on:

i) the wave length (λ) of the light used

ii) Half the angle (θ) of the cone of light from each point object.

iii) the refractive index (μ) of the medium between the object and the objective of the microscope

Resolving Power of a Telescope:

→ The resolving power of a telescope is defined as the reciprocal of the smallest angular separation '$d\theta$' between two distant objects whose images can be just resolved by it.

→ Resolving power of telescope $\frac{1}{d\theta} = \frac{D}{1.22\lambda}$

Clearly, the resolving power of telescope de pends on : (i) the diameter (D) of the telescope objective (ii) The wavelength (λ) of the light used.

WE- 28: ***Assume that light of wavelength 6000 Å is coming from a star. What is the limit of resolution of a telescope whose objective has a diameter of 100 inch?***

Sol: A 100 inch telescope implies that

a = 100 inch = 254 cm. Thus if,

$\lambda \approx 6000$ Å $= 6\times10^{-5}\,cm$ then,

$$\Delta\theta = \frac{1.22\lambda}{a} \approx 2.9\times10^{-7} \text{ radians}$$

POLARIZATION

→ The properties of light, like interference and diffraction demonstrate the wave nature of light.

Ø Both longitudinal and transverse waves can exhibit interference and diffraction effects.

Ø The properties like polarization can be exhibited only by transverse waves.

Ø The peculiar feature of polarized light is that human eye cannot distinguish between polarised and unpolarised light.

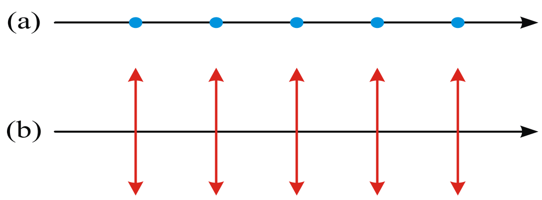
Ø As light is an electromagnetic wave, among its electric and magnetic vectors only electric vector is mainly responsible for optical effects.

Ø The electric vector of wave can be identified as a "light vector"

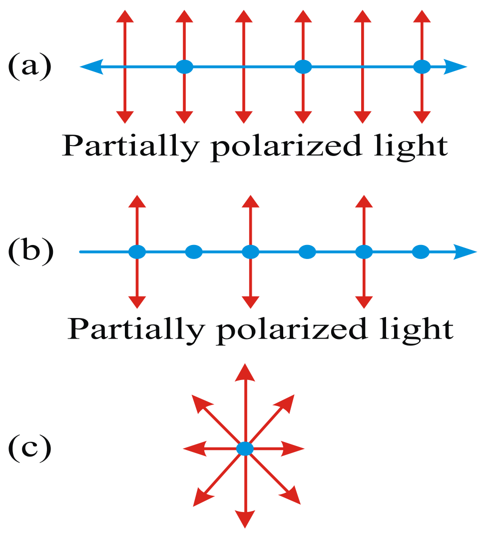
Ø Ordinary light is unpolarised light in which electric vector is oriented randomly in all directions perpendicular to the direction of propagation of light.

Ø The phenomena of confining the vibrations of electric vector to a particular direction perpendcular to the direction of propagation of light is called "Polarization". Such polarised light is called linearly polarised or plane polarised light.

Ø The plane in which vibrations are present is called "plane of polarization."



Polarized light



Partially polarized light

Partially polarized light

Partially polarized light

→ Plane polarised light can be produced by different methods like

i. Reflection ii. Refraction

iii. Double refraction iv. Polaroids.

Polarization by Reflection

→ The ordinary light beam is incident on transparent surface like glass or water. Both reflected and refracted beams get partially polarised.

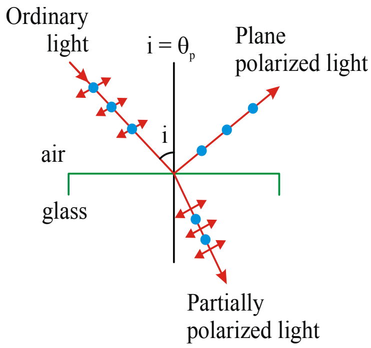
→ The degree of polarization changes with angle of incidence.

→ At a particular angle of incidence called "polarising angle" the reflected beam gets completely plane polarised. The reflected beam has vibrations of electric vector perpendicular to the plane of paper.

→ The polarising angle depends on the nature of reflecting surface.

Brewster's Law: When angle of incidence is equal to "polarising angle" the reflected and refracted rays will be perpendicular to each other.

→ Brewster's law states that " The refractive index of a medium is equal to the tangent of polarising angle θ_p ".



→ The refractive index of the medium changes with wavelength of incident light and so polarising angle will be different for different wavelengths.

→ The complete polarization is possible when incident light is monochromatic.

$$\mu = \frac{\sin\theta_p}{\sin r} = \frac{\sin\theta_p}{\sin(90^0 - \theta_p)} = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p$$

→ From **Brewster's law,** μ **=** $\tan\theta_p$.

→ If i= θ_p , the reflected light is completely polarised and the refracted light is partially polarised.

→ If i< θ_p or i> θ_p , both reflected and refracted rays get partially polarised.

→ For glass θ_p = $\tan^{-1}(1.5) \approx 57^0$

For water θ_p = $\tan^{-1}(1.33) \approx 53^0$

WE-29: ***When light of a certain wavelength is incident on a plane surface of a material at a glancing angle 30⁰, the reflected light is found to be completely plane polarized determine.***

a) refractive index of given material and

b) angle of refraction.

Sol: a) Angle of incident light with the surface is 30^0. The angle of incidence = $90^0 - 30^0 = 60^0$. Since reflected light is completely polarized, therefore incidence takes place at polarizing angle of incidence θ_p.

$\therefore \theta_p = 60^0$

Using Brewster's law

$\mu = \tan\theta_p = \tan 60^0 \quad \mu = \sqrt{3}$

b) From Snell's law

$$\mu = \frac{\sin i}{\sin r} \quad \therefore \sqrt{3} = \frac{\sin 60^0}{\sin r}$$

or $\sin r = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = \frac{1}{2}$, $r = 30^0$.

Polarisation by Refraction

→ The unpolarised light when incident on a glass plate at an angle of incidence equal to the polarising angle, the reflected light is completely plane polarised, but the refracted light is partially polarised.

→ The refracted light gets completely plane polarised if incident light is allowed to pass through number of thin glass plates arranged parallel to each other. Such an arrangement of glass plates is called "pile of plates".

Polarisation by Double Refraction (Additional)

→ Bartholinus discovered that when light is incident on a calcite crystal two refracted rays are produced. It is called "double refraction" or "birefringence"

→ An ink dot made on the paper when viewed through calcite crystal two images are seen due to double refraction. On rotating the crystal one image remains stationary and the other image rotates around the stationary image.

→ The rotating image revolves round the stationary image in circular path.

→ The stationary image is formed due to ordinary ray and revolving image is formed by extraordinary ray.

→ A plane which contains the optic axis and is perpendicular to the two opposite faces is called the principal section of crystal.

→ The ordinary ray emerging from the calcite crystal obey the laws of refraction and vibrations are perpendicular to the principal section of the crystal.

→ The extra ordinary ray does not obey the laws of refraction and the vibrations are in the plane of principal section of crystal.

→ Both ordinary and extraordinary rays are plane polarised.

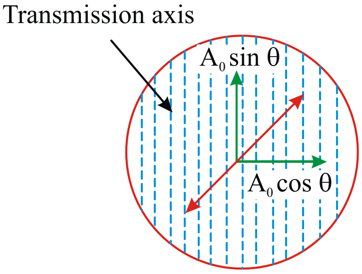
Polaroid : Polaroid is an optical device used to produce plane polarised light making use of the phenomenon of "selective absorption".

→ More recent type of polaroids are H-polaroids.

→ H-polaroids are prepared by stretching a film of polyvinyl alcohol three to eight times to original length.

Effect of polarizer on natural light:

If one of waves of an unpolarized light of intensity I_0 is incident on a polaroid and its vibration amplitude A_0 makes an angle θ with the transmission axis, then the component of vibration parallel to transmission axis will be $A_0 \cos\theta$ while perpendicular to it $A_0 \sin\theta$. Now as polaroid will pass only those vibrations which are parallel to its transmission axis, the intensity I of emergent light wave will be



$I = KA_0^2 \cos^2\theta$ (or)

$I = I_0 \cos^2\theta [as\ I_0 = KA_0^2]$ In unpolarized light, all values of θ starting from 0 to 2π are equally probable, therefore

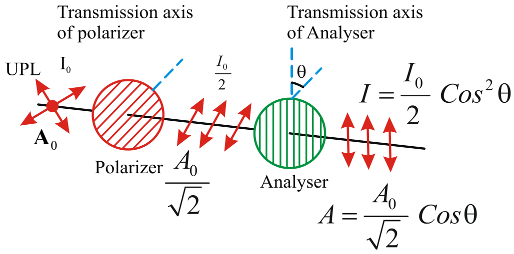
$$I = I_0 < \cos^2\theta > \Rightarrow I = \frac{I_0}{2\pi}\int_0^{2\pi}\cos^2\theta d\theta = \frac{I_0}{2} \qquad \therefore\ I = \frac{I_0}{2}$$

Thus, if unpolarized light of intensity I_0 is incident on a polarizer, the intensity of light transmitted through the polarizer is $\frac{I_0}{2}$. The amplitude of polarized light is $\frac{A_0}{\sqrt{2}}$.

Effect of Analyser on plane polarized light:

When unpolarized light is incident on a polarizer, the transmitted light is linearly polarized. If this light further passes through analyser, the intensity varies with the angle between the transmission axes of polarizer and analyser.

Malus states that "the intensity of the polarized light transmitted through the analyser is proportional to cosine square of the angle between the plane of transmission of analyser and the plane of transmission of polarizer." This is known as **Malus law**.



Therefore the intensity of polarized light after passing through analyser is $\boxed{I = \frac{I_0}{2}\cos^2\phi}$

Where I_0 is the intensity of unpolarized light. The amplitude of polarized light after passing through analyser is $A = \frac{A_0}{\sqrt{2}}\cos\theta$.

Case (i) : If $\theta = 0^0$ axes are parallel then $I = \frac{I_0}{2}$

Case (ii): If $\theta = 90^0$ axes are perpendicular, then $I = 0$.

Case (iii): If $\theta = 180^0$ axes are parallel then $I = \frac{I_0}{2}$

Case (iv): If $\theta = 270^0$ axes are perpendicular then I = 0 Thus for linearly polarized light we obtain two positions of maximum intensity and two positions of minimum (zero) intensity, when we rotate the axis of analyser w.r.t to polarizer by an angle 2π. In the above cases if the polariser is rotated with respect to analiser then there is no change in the outcoming intensity.

Note: In case of three polarizers P_1, P_2 and P_3: If θ_1 is the angle between transmission axes of P_1 and P_2, θ_2 is the angle between transmission axes of P_2 and P_3. Then the intensity of emerging light from P_3 is

$$I = \frac{I_0}{2}\cos^2\theta_1\cos^2\theta_2.$$

WE-30: ***Unplarized light falls on two polarizing sheets placed one on top of the other. What must be the angle between the characteristic directions of the sheets if the intensity of the tramsitted light is one third of intensity of the incident beam?***

Sol: Intensity of the light transmitted through the first polarizer $I_1 = I_0/2$, where I_0 is the intensity of the incident unpolarized light.

Intensity of the light transmitted through the second polarizer is $I_2 = I_1 \cos^2\theta$ where θ is the angle between the characteristic directions of the polarizer sheets.

But $I_2 = I_0/3$ (given)

$$\therefore\ I_2 = I_1\cos^2\theta = \frac{I_0}{2}\cos^2\theta = \frac{I_0}{3}$$

$$\therefore \cos^2\theta = 2/3 \quad \Rightarrow \quad \theta = \cos^{-1}\sqrt{\frac{2}{3}}$$

WE-31: ***Unpolarized light of intensity 32 Wm^{-2} passes through three polarizers such that the transmission axis of the last polarizer is crossed with the first. If the intensity of the emerging light is 3 Wm^{-2}, what is the angle between the transmission axes of the first two polarizers? At what angle will the transmitted intensity be maximum?***

Sol: If θ is the angle between the transmission axes of first polaroid P_1 and second P_2 while ϕ between the transmission axes of second polaroid P_2 and third P_3, then according to given problem.

$\theta + \phi = 90^0$ or $\phi = (90^0 - \theta)$.....(1)

Now if I_0 is the intensity of unpolarized light incident on polaroid P_1, the intensity of light transmitted through it,

$$I_1 = \frac{1}{2}I_0 = \frac{1}{2}(32) = 16\frac{W}{m^2}......(2)$$

Now as angle between transmission axes of polaroids P_1 and P_2 is θ, in a accordance with Malus law, intensity of light transmitted through P_2 will be

$I_2 = I_1\cos^2\theta = 16\cos^2\theta$ ....(3)

And as angle between transmission axes of P_2 and P_3 is ϕ, light transmitted through P_3 will be

$I_3 = I_2\cos^2\phi = 16\cos^2\theta\cos^2\phi$.........(4)

According to given problem, $I_3 = 3W/m^2$

So, $4(\sin 2\theta)^2 = 3$ *i.e.*, $\sin 2\theta = (\sqrt{3}/2)$ or

$2\theta = 60^0$, i.e., $\theta = 30^0$.

WE-32: ***Discuss the intensity of transmitted light when a polaroid sheet is rotated between two crossed polaroids?***

Sol: Let I_0 be the intensity of polarised light after passing through the first polariser P_1. Then the intensity of light after passing through second polariser P_2 will be

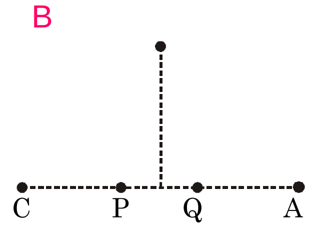
$I = I_0\cos^2\theta$, where θ is the angle between pass axes of P_1 and P_2. Since P_1 and P_2 are crossed the angle between the pass axes of P_2 and P_3 will be $(\pi/2 - \theta)$. Hence the intensity of light emerging from P_3 will be $I = I_0\cos^2\theta\cos^2\left(\frac{\pi}{2} - \theta\right)$

$= I_0\cos^2\theta\sin^2\theta = (I_0/4)\sin^2 2\theta$

Therefore, the transmitted intensity will be maximum when $\theta = \pi/4$.

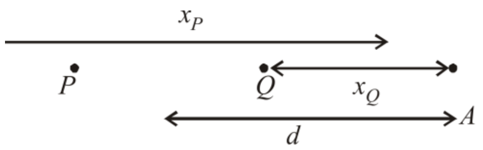
PREVIOUS MAINS QUESTIONS

1. In the figure below, P and Q are two equally intense coherent sources emitting radiation of wavelength 20 m.The separation between P and Q is 5 m and the phase ofPis ahead ofthat ofQ by 90^0. A, B and C are three distinct points of observation, each equidistant from the midpoint of PQ. The intensities of radiation at A, B, C will be in theratio: [Sep. 06, 2020 (D)]



(a) 0: 1 : 4 (b) 2: 1 : 0
(c) 0: 1 : 2 (d) 4: 1 : 0

SOLUTION. (b) For (A)



$$x_P - x_Q = (d + 2.5) - (d - 2.5) = 5\text{m}$$

Phase difference $\Delta\varphi$ due to path difference= $\frac{2\pi}{\lambda}(\Delta x) = \frac{2\pi}{20}(5) = \frac{\pi}{2}$.

At A, Q is ahead of P by path, as wave emitted by Q reaches before wave emitted by P.

Total phase difference at $A \frac{\pi}{2} - \frac{\pi}{2} = 0$

(due to P being ahead of Q by $90°$) $I_A = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos \Delta(|)$

$$= I + I + 2\sqrt{I}\sqrt{I}\cos(0) = 4I$$

For C,Path difference, $x_Q - x_P = 5\text{m}$

Phase difference $\Delta\varphi$ due to path difference= $\frac{2\pi}{\lambda}(\Delta\kappa) = \frac{2\pi}{20}(5) = \frac{\pi}{2}$

Total phase difference at $C = \frac{\pi}{2} + \frac{\pi}{2} = \pi$ $\quad I_{\text{net}} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos(\Delta(|))$

$$= I + I + 2\sqrt{I}\sqrt{I}\cos(\pi) = 0$$

For B,Path difference, $x_P - x_Q = 0$

Phase difference, $\Delta\varphi = \frac{\pi}{2}$

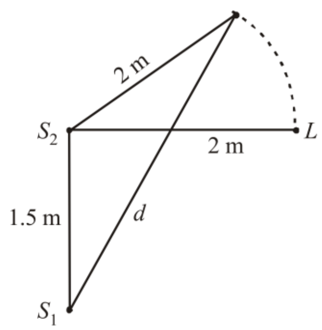
(due to P being ahead of Q by $90°$) $\quad I_B = I + I + 2\sqrt{I}\sqrt{I}\cos\frac{\pi}{2} = 2I$

Therefore intensities of radiation at A, B and C will be in the ratio

$$I_A : I_B : I_C = 4I : 2I : 0 = 2 : 1 : 0.$$

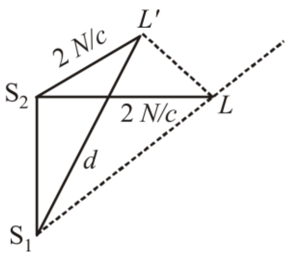
2. Two coherent sources of sound, S_1 and S_2, produce soundwaves of the same wavelengths, $\lambda = 1\text{m}$, in phase. S_1 and $S_2$are placed 1.5m apart (see fig.). A listener, located at L,directly in front ofS_2 finds that the intensity is at a minimum when he is 2 m away from S_2. The listener moves away from S_1, keeping his distance from S_2 fixed. The adjacent maximum of intensity is observed when the listener is at a distance d from S_1. Then, d is: [Sep. 05, 2020 (II)]

(a) 12m (b) 5m (c) 2m (d) 3m



SOLUTION. (d) Initially, $S_2L = 2\text{m}$ $S_1L = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2} = 2.5\text{m}$

Path difference, $\Delta x = S_1L - S_2L = 0.5\text{m} = \frac{\lambda}{2}$



When the listener move from L, first maxima will appear if path difference is integral multiple of wavelength. For example$\Delta x = n\lambda = 1\lambda$ $(n = 1$ for first maxima$)\Delta x = \lambda = S_1L' - S_2L$

$$\Rightarrow 1 = d - 2 \Rightarrow d = 3\text{m}$$

3. Two light waves having the same wavelength $\lambda \text{invacuum}$are in phase initially. Then the first wave travels a path L_1through a medium rarefactive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is : [Sep. 03, 2020 (II)]

(a) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$ (b) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$ (c) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$ (d) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$

SOLUTION. (c) The distance traversed by light in a medium refractive index m in time t is given by$d = vt$ (i)where v is velocity oflight in the medium. The distance traversed by light in a vacuum in this time,$\Delta = ct = c \times \frac{d}{v}$ [from equation (i)]= $d\frac{c}{v} = \mu d$ (ii) $\left(\mu = \frac{c}{v}\right)$

This distance is the equivalent distance in vacuum and is called optical path.
Optical path for first ray which travels a path L_1 through a medium of refractive index $n_1 = n_1L_1$
Optical path for second ray which travels a path L_2 through a medium refractive index $n_2 = n_2L_2$

Path difference $= n_1L_1 - n_2L_2$ Now, phase difference= $\frac{2\pi}{\lambda} \times$ path difference $= \frac{2\pi}{\lambda} \times (n_1L_1 - n_2L_2)$

4. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of

maximum and minimum intensities fringes will be: [8 April 2019 I]

(a) 2 (b) 18 (c) 4 (d) 9

SOLUTION: As we know, $\frac{A_1}{A_2}=\frac{3}{1}$ $\frac{I_{max}}{I_{m\ln}}=\left(\frac{A_1+A_2}{A_1-A_2}\right)^2=\left(\frac{4}{2}\right)^2=\frac{4}{1}=4$

5. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: [9 Jan. 2019 I]

(a) 16: 9 (b) 25: 9 (c) 4: 1 (d) 5: 3

SOLUTION: (b) As we know, $\frac{I_{max}}{I_{m\ln}}=\left(\frac{A_1+A_2}{A_1-A_2}\right)^2$ and $\sqrt{\frac{I_1}{I_2}}=\frac{A_1}{A_2}$

$$\frac{I_{max}}{I_{mn}}=16\Rightarrow\frac{A_{max}}{A_{min}}=4\Rightarrow\frac{A_1+A_2}{A_1-A_2}=\frac{4}{1}$$

$$.\frac{A_1}{A_2}=\frac{5}{3}\Rightarrow\frac{I_1}{I_2}=\left(\frac{5}{3}\right)^2=\frac{25}{9}$$

6. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the lightkam: [2015]

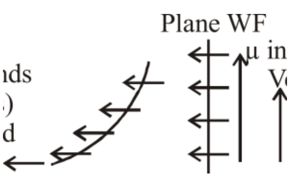
(a) bends downwards (b) bends upwards

(c) becomes narrower (d) goes horizontally without any deflection

SOLUTION: (b) (Light ber $\vee\simeq 1$ decreasescreases

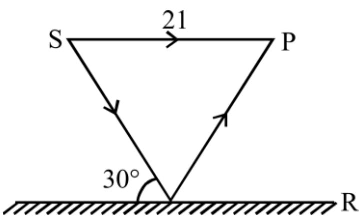
upwardsRefracte

WΓ



7. Interference pattern is observed at P 'due to superimposition of two rays coming out fi$_{i}$om a source S'as shown in the figure. The value of' 1' for which maxima is obtained at P' is:

(R is perfect reflecting surface) [Online Apri112, 2014]



(a) $1=\frac{2n\lambda}{\sqrt{3}-1}$ (b) $1=\frac{(2n-1)\lambda}{2(\sqrt{3}-1)}$ (c) $1=\frac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$ (d) $1=\frac{(2n-1)\lambda}{\sqrt{3}-1}$

SOLUTION: (c)

8. Two monochromatic light beams of intensity 16 and 9unitsare interfering. The ratio of intensities

of bright and dark parts of the resultant pattern is: [Online April 11, 2014]

(a) $\frac{16}{9}$ (b) $\frac{4}{3}$ (c) $\frac{7}{1}$ (d) $\frac{49}{1}$

SOLUTION: (d) Intensity $\propto (amplitude)^2$ $\frac{I_1}{I_2} = \frac{16}{9} = \frac{a_1^2}{a_2^2} \Rightarrow a_1 = 4; a_2 = 3$

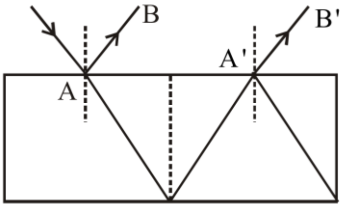
Therefore the ratio of intensities of bright and dark parts $\frac{I_{B1:1ght}}{I_{Dark}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2} = \frac{(4+3)^2}{(4-3)^2} = \frac{49}{1}$

9. n identical waves each of intensity I_0 interfere with eachother. The ratio ofmaximum intensities ifthe interferenceis (i) coherent and(ii) incoherent is:[Online April 23, 2013]

(a) n^2 (b) $\frac{1}{n}$ (c) $\frac{1}{n^2}$ (d) n

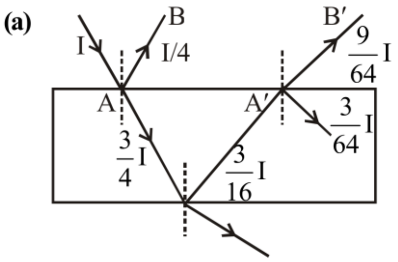
SOLUTION: (d) $\frac{(\text{Max imumintensity})\text{coherentinterference}}{(\text{Max imumintensity})\text{incoherentinterference}} = \frac{n^2 I_0}{n I_0} = n$

10. A ray oflight of intensity I is incident on a parallel glass slab at point A as shown in diagram. It undergoes partial reflection and refraction. At each reflection, 25% ofincidentenergy is reflected. The rays AB and $A'B^{\uparrow}$ undergointerference. The ratio ofI_{max} and I_{min} is: [Online April 9, 2013]



(a) 49:1 (b) 7:1 (c) 4:1 (d) 8:1

SOLUTION:



From figure $I_1 = \frac{I}{4}$ and $I_2 = \frac{9I}{64} \Rightarrow \frac{I_2}{I_1} = \frac{9}{16}$

By using $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_2}{I_1}}+1}{\sqrt{\frac{I_2}{I_1}}-1}\right) = \left(\frac{\sqrt{\frac{9}{16}}+1}{\sqrt{\frac{9}{16}}-1}\right) = \frac{49}{1}$

11. Two coherent plane light waves of equal amplitude makes a small angle $\alpha(<< 1)$ with each other. They fall almostnormally on a screen. Ifλ is the wavelength oflight waves, the fiinge width Δx ofinterference patterns of the two sets of waves on the screen is [Online May19, 2012]

(a) $\frac{2\lambda}{\alpha}$ (b) $\frac{\lambda}{\alpha}$ (c) $\frac{\lambda}{(2\alpha)}$ (d) $\frac{\lambda}{\sqrt{\alpha}}$

SOLUTION: (c) $\Delta x = \frac{\lambda}{(2\alpha)}$

12. A thin air film is formed by putting the convex surface of plane-convex lens over a plane glass

plate. With monochromatic light, this film gives an interference pattetndue to light reflected from the top (convex) surface andthe bottom (glass plate) surface ofthe film.

Statement-l : When light reflects fiom the air-glass plateinterface, the reflected wave suffers a phase change of π.

Statement-2: The centre ofthe interference pattern isdark. [2011]

(a) Statement-l is true, Statement-2 is true, Statement -2 is the correct explanation of Statement-l.

(b) Statement-l is true, Statement-2 is true, Statement -2 is not the correct explanation ofStatement -1.

(c) Statement-l is false, Statement-2 is true.

(d) Statement-l is true, Statement-2 is false.

SOLUTION: b) A phase change ofπ rad appears when the rayreflectsat the glass-air interface. As a result, there will be a destructive interference at the center. So, the center of the interference pattern is dark

Directions: Questions number 13-15 are based on the followingparagraph.

An initially parallel cylindrical beam travels in a medium ofrefractive index $\mu(1)=\mu_0+\mu_2 I$, where μ_0 and μ_2 are positiveconstants and I is the intensity ofthe light beam. The intensityofthe beam is decreasing with increasing radius

13. As the beam enters the medium , it will [2010]

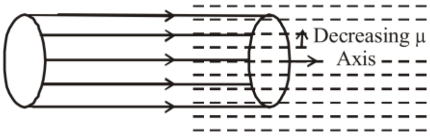
(a) diverge

(b) converge

(c) diverge near the axis and converge near the periphery

(d) travel as a cylindrical beam

SOLUTION: b) When light beam is moving and as it enters the medium,the refractive index will decrease fiom the axis towards the periphery of the beam. Therefore, the beam will converge less distance as one move fiom the axis to the periphery and hence the beam will converge.



14. The initial shape ofthe wavefiont ofthe beam is [2010]

(a) convex

(b) concave

(c) convex near the axis and concave near the periphery

(d) planar

SOLUTION: (d) Initially the parallel beam is cylindrical. Therefore, the waterfront will be planar.

15. The speed oflight in the medium is [2010]

(a) minimum on the axis ofthe beam

(b) the same everywhere in the beam

(c) directly proportional to the intensity I

(d) maximum on the axis ofthe beam

SOLUTION: . (a) The speed of light (v) in a medium of refriactive index (μ) is given by$\mu=\frac{c}{v}$, where c is the speed oflight in vacuum

$$v=\frac{c}{\mu}=\frac{c}{\mu_0+\mu_2(I)}$$

As I is decreasing with increasing radius, it is maximum onthe axis ofthe beam. Therefore, v is minimum on the axis ofthe beam.

16. To demonstrate the phenomenon of interference, werequire two sources which emit radiation [2003]

(a) of nearly the same frequency

(b) of the same frequency

(c) of different wavelengths

(d) of the same frequency and having a definite phase relationship

SOLUTION: (d) To demonstrate the phenomenon of interference we require two sources of light which emit radiation of same frequency and having a definite phase relationship (a phase relationship that does not change with time)

17. A young's double-slit experiment is performed using monochromatic light of wavelength λ. The inntensity oflight at a point on the screen, where the path difference isλ, is K units. The intensity oflight at a point where thepath difference is $\frac{\lambda}{6}$ is given by $\frac{\mathrm{nK}}{12}$, where n is an integer. The value of n is . [Sep. 06, 2020 (II)]

SOLUTION: (9)In young's double slit experiment, intensity at a point isgiven by$I=I_0\cos^2\frac{\varphi}{2}$ (i)where, $\varphi=$ phase difference, Using phase difference, $\varphi=\frac{2\pi}{\lambda}\times$ path differenceFor path difference λ,

phase difference $\varphi_1=2\pi$

For path difference, $\frac{\lambda}{6}$, phase difference $\varphi_2=\frac{\pi}{3}$Using equation (i),

$$\frac{I_1}{I_2}=\frac{\cos^2(\varphi_1/2)}{\cos^2(\varphi_2/2)}\Rightarrow\frac{K}{I_2}=\frac{3}{4}=\frac{4}{3}\Rightarrow I_2=\frac{3K}{4}=\frac{9K}{12}\qquad n=9.$$

18. In a Young's double slit experiment, light of500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree)of the fringes formed on the distance screen is close to:[Sep. 03, 2020 (I)]

(a) $0.17°$ (b) $0.57°$ (c) $1.7°$ (d) $0.07°$

SOLUTION: b) Given : Wavelength oflight, $\lambda=500$ nmDistance between the slits, $d=0.05$ mm

Angular width of the fringe formed,$\theta=\frac{\lambda}{d}=\frac{500\times10^{-9}}{0.05\times10^{-3}}=0.01$ rad $=0.57°$.

19. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source $(\lambda = 632.8 \text{ nm})$. The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at a distance of 1.27 mm fiom the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to:

[Sep. 02, 2020 (I)]

(a) $1.27\mu n$ (b) 2.87nm(c) 2nm (d) $2.05\mu n$

SOLUTION: (a) Path difference, $\Delta P = d\sin\theta = d\theta$

$d =$ distance between slits $= 1$ mm $= 10^{-3}$ mm

$D =$ distance between the slits and screen $= 100$ cm $= 1\text{m}$

$y =$ distance between central bright fringe and observed fringe $= 1.27\text{mm}$

$$\Delta P = \frac{dy}{D} = \frac{10^{-3} \times 1.270\text{mm}}{1\text{m}} = 1.27\mu\text{m}$$

20. In a Young's double slit experiment, 16 fringes are observed in a certain segment of the screen when light ofwavelength700 nm is used. If the wavelength of light is changed to400 nm, the number of fringes observed in the same segment of the screen would be: [Sep. 02, 2020 (II)]

(a) 24 (b) 30 (c) 18 (d) 28

SOLUTION: (d) Let n_1 fiinges are visible with light of wavelength λ_1

and n_2 with light ofwavelength λ_2. Then$\beta = \frac{n_1 D\lambda_1}{d} = \frac{n_2 D\lambda_2}{d}$ $\left(\because \beta = \frac{n\lambda D}{d}\right)$

$$\Rightarrow \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \qquad \Rightarrow n_2 = \frac{700}{400} \times 16 = 28$$

21. In a Young's double slit experiment 15 fringes are observed on a small portion of the screen when light ofwavelength500 nm is used. Ten fringes are observed on the same section of the screen when another light source of wavelength λ is used. Then the value of λ is (in nm)

[9 Jan 2020 II]

SOLUTION: (750) Fringe width, $\beta = \frac{\lambda D}{d}$ where, λ =wavelength, D =distance of screen from slits,

$d =$ distance between slits $\qquad 15 \times \frac{\lambda_1 D}{d} = 10 \times \frac{\lambda_2 D}{d} \qquad \Rightarrow 15\lambda_1 = 10\lambda_2$

$$\Rightarrow \lambda_2 = 1.5\lambda_1 = 1.5 \times 500\text{nm} \Rightarrow \lambda_2 = 750\text{nm}$$

22.In a double-slit experiment, at a certain point on the screen the path difference between the two interfering waves is$\frac{1}{8}$ th of a wavelength. The ratio of the intensity of light at that point to that at the center of a bright fringe is:[8 Jan 2020 II]

(a) 0.853 (b) 0.672 (c) 0.568 (d) 0.760

SOLUTION: (a) Given, Path difference, $\Delta x = \frac{\lambda}{8}$

Phase differences, $\Delta\varphi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \frac{\lambda}{8} = \frac{\pi}{4} \qquad I = I_0\cos^2\left(\frac{\Delta\varphi}{2}\right)$

$$\Rightarrow \frac{I}{I_0} = \cos^2\left(\frac{\frac{\pi}{4}}{2}\right) = \cos^2\left(\frac{\pi}{8}\right) \qquad \Rightarrow \frac{I}{I_0} = 0.853$$

23. In a Young's double slit experiment, the separation between the slits is 0.15 mm. In the experiment, a source of light of wavelength 589 nm is used and the interference pattern is observed on a screen kept 1.5 m away. The separation between the successive bright fringes on the screen is: [7 Jan 2020 II]

(a) 6.9mm (b) 3.9mm (c) 5.9mm (d) 4.9mm

SOLUTION: (c) Given, distance between screen and slits, $D = 1.5\text{m}$

Separation between slits, $\mathrm{d} = 0.15$ mm

Wavelength of source of light, $\lambda = 589$ nm

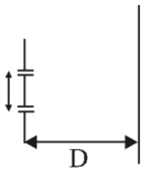
Fringe-width $\beta = \frac{D}{d}\lambda = \frac{1.5}{0.15\times10^{-3}} \times 589 \times 10^{-9} m = 589 \times 10^{2}\text{mm} = 5.89\text{mm} \approx 5.9\text{mm}$

24. In a double slit experiment, when a thin film of thickness $t$having refiactive index μ is introduced in front ofone of the slits, the maximum at the center of the fringe pattern shifts by one fiiinge width. The value of t is ($\lambda \approx$ is the wavelength of the light used): [12 April 2000 I]

(a) $\frac{2\lambda}{(\mu-1)}$ (b) $\frac{\lambda}{2(\mu-1)}$(c) $\frac{\lambda}{(\mu-1)}$ (d) $\frac{\lambda}{(2\mu-1)}$

SOLUTION: (c) Given, $\Delta = \beta$ or $\frac{D(\mu-1)t}{d} = \frac{D\lambda}{d} t = \frac{\lambda}{(\mu-1)}$

25. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index 1/4 is put in front ofone ofthe slits, the central maximum gets shifted by a distance equal to n fringe widths. Ifthe wavelength oflight used is λ, t will be: [9 April 2019 I]



(a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{nD\lambda}{a(\mu-1)}$(c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{2D\lambda}{a(\mu-1)}$

SOLUTION: . (Bonus) Shift $= \mathrm{n}\beta$ (given)

$D\frac{(\mu-1)t}{a} = \frac{n\lambda D}{a}$[$\because$ Shift $= \frac{D(\mu-1)\mathrm{t}}{a}$] or $t = \frac{n\lambda}{(\mu-1)}$

26. In a Young's double slit experiment, the path difference, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$ th of wavelength. The ratio of the intensity at this point to that at

the center of a bright fringe is close to:[11 Jan 2019 I]

(a) 0.74 (b) 0.85 (c) 0.94 (d) 0.80

SOLUTION: b) Given, path difference, $\Delta x=\frac{\lambda}{8}$

Phase difference $(\Delta\varphi)$ is given by$\Delta\varphi=\frac{2\pi}{\lambda}(\Delta x)$ $\Delta\varphi=\frac{(2\pi)}{\lambda}\frac{\lambda}{8}=\frac{\pi}{4}$For two sources in different phases

$$I=I_0\cos^2\left(\frac{\pi}{8}\right) hence \frac{I}{I_0}=\cos^2\left(\frac{\pi}{8}\right)$$

$$=\frac{1+\cos\frac{\pi}{4}}{2}=\frac{1+\frac{1}{\sqrt{2}}}{2}=0.85$$

27. In a Young's double slit experiment with slit separation0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ radianby using light of wavelength λ_1 When the light of wavelength λ_2 is used a bright fiiinge is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740 nm), their values are:[10 Jan. 2019 I]

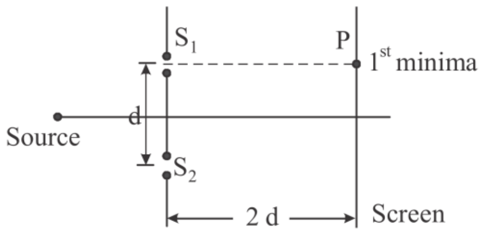
(a) 625 nm, 500 nm (b) 380 nm, 525 nm (c) 380 nm, 500 nm (d) 400 nm, 500 nm

SOLUTION: (a) Path difference $=d\sin\theta\approx d\theta=0.1\times\frac{1}{40}mm=2500nm$

For bright fringe, path difference must be integral multiple$of\lambda. 2500=n\lambda_1=m\lambda_2$

$\lambda_1=625$ (for $n=4$), $\lambda_2=500$ (for $m=5$)

28. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (SI)? [10 Jan 2019 II]



(a) $\frac{\lambda}{2(\sqrt{5}-2)}$ (b) $\frac{\lambda}{(\sqrt{5}-2)}$ (c) $\frac{\lambda}{2(5-\sqrt{2})}$ (d) $\frac{\lambda}{(5-\sqrt{2})}$

SOLUTION: (a) Here, $x_1=2d$ and $x_2=\sqrt{5}d$For, first minima, $\Delta x=\frac{\lambda}{2}$

$$\Delta x=x_2-x_1=\sqrt{5}d-2d=\frac{\lambda}{2} \quad \Rightarrow d=\frac{\lambda}{2(\sqrt{5}-2)}$$

29. In a Young's double slit experiment, the slits are placed0.320 mm apart. Light of wavelength $\lambda=500$ nm isincident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ\leq\theta\leq30^\circ$is [9 Jan 2019 II]

(a) 640 (b) 320 (c) 321 (d) 641

SOLUTION: (d) For 'n' number maxima'sd $\sin\theta=n\lambda$ $\quad 0.32\times10^{-3}\sin30^\circ=n\times500\times10^{-9}$

$$n = \frac{0.32 \times 10^{-3}}{500 \times 10^{-9}} \times \frac{1}{2} = 320$$

Hence total no. of maxima's observed in angular range $-30^{\circ} \leq \theta \leq 30^{\circ}$

$$= 320 + 1 + 320 = 641$$

30. In a Young↑s double slit experiment, slits are separated by0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance fiᵢom the common central maximum to thepoint where the bright fringes due to both the wavelengths coincide is : [2017]

SOLUTION: . (d) For common maxima, $n_1\lambda_1 = n_2\lambda_2$ $\Rightarrow \frac{n_1}{n_2} = \frac{\lambda_2}{\lambda_1} = \frac{520\times10^{-9}}{650\times10^{-9}} = \frac{4}{5}$

For λ_1 $y = \frac{n_1\lambda_1 D}{d}$, $\lambda_1 = 650$ nm

$y = \frac{4\times650\times10^{-9}\times1.5}{0.5\times10^{-3}}$ or, $y = 7.8$mm

31. In a Young' s double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that D >> d >> λ. If the fringe width is β, the distance from point ofmaximum intensityto thepoint where intensity falls to half of maximum intensity on either side is: [Online April 11, 2015]

(a) $\frac{\beta}{6}$ (b) $\frac{\beta}{3}$ (c) $\frac{\beta}{4}$ (d) $\frac{\beta}{2}$

SOLUTION: (c) $2I_0 = 4I_0\cos^2\left(\frac{\Delta\varphi}{2}\right)$ here, $\Delta\varphi = \frac{\pi}{2}$But, $\Delta\varphi = \frac{2\pi}{\lambda}\Delta$x so, $\Delta x = \frac{\lambda}{4}$

$\frac{dy}{D} = \frac{\lambda}{4}$ (i)$\frac{\lambda D}{d} = \beta$ (ii)

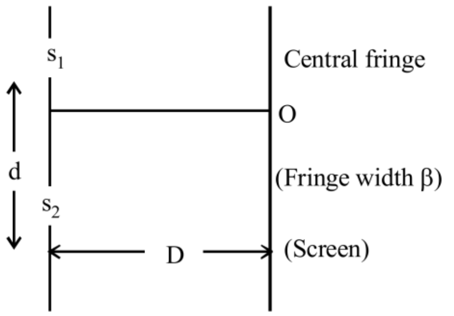
Multiplying equation (i) and (ii) we get,$y = \frac{\beta}{4}$

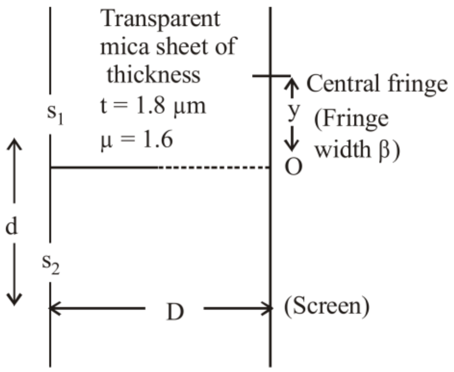
32. In a Young'sdouble slit experiment, the distance between the two identical slits is 6.1 times larger than the slit width .Then the number of intensity maxima observed within the central maximum of the single slit diffraction pattern is:[Online Apri119, 2014]

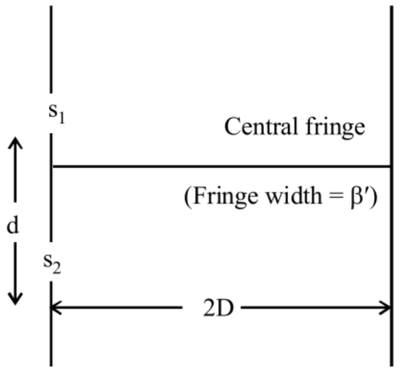
(a) 3 (b) 6 (c) 12 (d) 24

SOLUTION: (c)

33. Using monochromatic light of wavelength λ, anexperimentalist sets up the Young' s double slit experimentin three ways as shown.If she observes that $y = \beta'$, the wavelength of light usedis: [Online April 9, 2014]







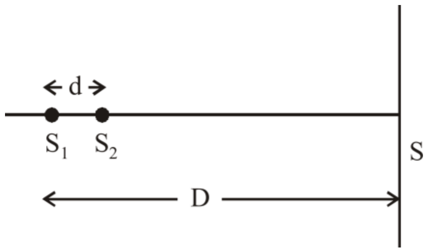
(a) 520nm (b) 540nm (c) 560nm (d) 580nm

SOLUTION: b) Given $t = 1.8 \times 10^{-6}$m $\mu = 1.6$

n = 2 (from figure) Applying formula $(\mu - 1)t = n\lambda$

$$(1.6 - 1) \times 1.8 \times 10^{-6} = 2\lambda \text{or, } \lambda = \frac{1.8 \times 10^{-6} \times 0.6}{2} = 540 \text{ nm}$$

34. Two coherent point sources S_1 and S_2 are separated by asmall distance d' as shown. The fringes obtained on thescreen will be [2013]



creen

(a) points (b) straight lines(c) semi-circles (d) concentric circles

SOLUTION: (d) It will be concentric circles.

35. The source that illuminates the double-slit in 'double-slit interference experiment' emits two distinct monochromatic waves of wavelength 500 nm and 600 nm, each ofthemproducing its own pattern on the screen. At the central point of the pattern when path difference is zero, maximaofboth the patterns coincide and the resulting interference pattern is most distinct at the region of zero path difference. But as one moves out of this central region, the two fringe systems are gradually out of step such that maximum due to on wavelength coincides with the minimum due to the other and the combined fringe system becomes completely indistinct. This may happen when path difference in nm is: [Online April 25, 2013]

(a) 2000 (b) 3000(c) 1000 (d) 1500

SOLUTION: (d)

36. A thin glass plate of thickness is $\frac{2500}{3}\lambda(\lambda$ is wavelengthof light used) and refiactive index $\mu = 1.5$ is inserted between one of the slits and the screen in Young's double slit experiment. At a point on the screen equidistant from the slits, the ratio of the intensities before and after the introduction of the glass plate is:[Online April 25, 2013]

(a) 2: 1 (b) 1: 4 (c) 4: 1 (d) 4: 3

SOLUTION: (c)

37. This question has Statement - 1 and Statement - 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement - 1: In Young' s double slit experiment, the number of fringes observed in the field ofview is small with longer wavelength of light and is large with shorter wavelength of light.

Statement - 2: In the double slit experiment the fringe width depends directly on the wavelength of light.[Online April 22, 2013]

(a) Statement - 1 istrue, Statement - 2 is trueandthe Statement - 2 is correct explanation ofthe Statement - 1.

(b) Statement - 1 is false and the Statement - 2 is true.

(c) Statement - 1 is true Statement - 2 is true and theStatement - 2 is not correct explanation of theStatement - 1.

(d) Statement - 1 is true andthe Statement - 2 is false.

SOLUTION: (c) Fringe width $B = \frac{D}{d}\lambda$ And number of fringes observed in the field of view is obtained by $\frac{d}{\lambda}$

38. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that other slit. IfI_m be the maximum intensity, the resultant intensity I when they interfere at phase difference φ is given by: [2012]

(a) $\frac{I_m}{9}(4 + 5\cos\varphi)$ (b) $\frac{I_m}{3}\left(1 + 2\cos^2\frac{\varphi}{2}\right)$

(c) $\frac{I_m}{5}\left(1 + 4\cos^2\frac{\varphi}{2}\right)$ (d) $\frac{I_m}{9}\left(1 + 8\cos^2\frac{\varphi}{2}\right)$

SOLUTION: (d) Let a, be the amplitude oflight from first slit and a_2 be the amplitude of light from second slit.

$a_1 = a$, Then $a_2 = 2a$

Intensity $I \propto$ (amplitude) $I_1 = a_{1^2} = a^2$ and $I_2 = a_{2^2} = 4a^2 = 4I$

$$I_r = a_{1^2} + a_{2^2} + 2a_1 a_2 \cos\varphi = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi = I_1 + 4I_1 + 2\sqrt{4I_1^2}\cos\varphi$$

$$\Rightarrow I_r = 5I_1 + 4I_1\cos\varphi \quad ...(1)$$

Now, $I_{\max} = (a_1 + a_2)^2 = (a + 2a)^2 = 9a^2$ then $I_{\max} = 9I_1 \Rightarrow I_1 = \frac{I_{\max}}{9}$

Substituting in equation (1)$I_\gamma = \frac{5I_{\max}}{9} + \frac{4I_{\max}}{9}\cos\varphi$

$$I_r = \frac{I_{\max}}{9}[5 + 4\cos\varphi]$$

$$I_r = \frac{I_{\max}}{9}\left[5 + 8\cos^2\frac{\varphi}{2} - 4\right]$$

$$I_r = \frac{I_{\max}}{9}\left[1 + 8\cos^2\frac{\varphi}{2}\right]$$

39. In Young's double slit interference experiment, the slit widths are in the ratio 1 : 25. Then the ratio of intensity at the maxima and minima in the interference pattern is[Online May 26, 2012]

(a) 3:2 (b) 1:25 (c) 9:4 (d) 1:5

SOLUTION: $\frac{I_{\max}}{I_{m\ln}} = \left(\frac{w_1+w_2}{w_1-w_2}\right)^2$ $here \frac{w_1}{w_2} = \frac{1}{25}$ on solving

$$\frac{I_{\max}}{I_{\min}} == \frac{9}{4} = 9:4$$

40. The maximum number ofpossible interference maxima forslit separation equal to 1.8λ, where λ is the wavelength oflight used, in a Young's double slit experiment is[Online May 12, 2012]

(a) zero (b) 3 (c) infinite (d) 5

SOLUTION: b) As $\sin\theta = \frac{n\lambda}{d}$ and $\sin\theta$ cannot be $\}\rhd 1$

$1 = \frac{n\lambda}{1.8\lambda}$ or $= 1.8$ Hence maximum number of possible interference maxima's,0, ± 1 i.e. 3

41. In a Young's double slit experiment with light of wavelengthλ, fiiinge pattern on the screen has fiiinge width β. When two thin transparent glass (refractive index μ) plates of thickness t_1 and $t_2 (t_1 > t_2)$ are placed in the path of the two beams respectively, the fringe pattern will shift by a distance [Online May 7, 2012]

(a) $\frac{\beta(\mu-1)}{\lambda}\left(\frac{t_1}{t_2}\right)$ (b) $\frac{\mu\beta t_1}{\lambda t_2}$(c) $\frac{\beta(\mu-1)}{\lambda}(t_1 - t_2)$ (d) $(\mu - 1)\frac{\lambda}{\beta}(t_1 + t_2)$

SOLUTION: (c) Shift $= \frac{\beta(\mu-1)}{\lambda}t_1 - \frac{\beta(\mu-1)}{\lambda}t_2 = \frac{\beta(\mu-1)}{\lambda}(t_1 - t_2)$

42. At two points P and Q on screen in Young's double slit experiment, waves fi¡om slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$, respectively. The ratio of intensities at P and Q will be:

[2011 RS]

(a) 2: 1 (b) $\sqrt{2}$: 1 (c) 4: 1 (d) 3: 2

SOLUTION: (a) Path difference at $P \Delta x_1 = 0$

Phase difference at P will be $\Delta\varphi_1 = \frac{2\pi}{\lambda}\Delta x_1 = \frac{2\pi}{\lambda} \times 0 = 0°$

Resultant Intensity at P $\quad I_1 = I_0 + I_0 + 2I_0 \cos 0° = 4I_0$

Path difference at $Q \Delta x_2 = \frac{\lambda}{4}$

Phase difference at Q $\quad \Delta\varphi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \left(\frac{\pi}{2}\right)$

Resultant intensity at $Q. I_2 = I_0 + I_0 + 2I_0 \cos\frac{\pi}{2} = 2I_0$

Thus, $\frac{I_1}{I_2} = \frac{4I_0}{2I_0} = \frac{2}{1}$

43. In a Young's double slit experiment, the two slits act as coherent sources of wave of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherents ources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case is I_2, then the ratio $\frac{I_1}{I_2}$ is[2011 RS]

(a) 2 (b) 1 (c) 0.5 (d) 4

SOLUTION: (a) For coherent sources, intensity at mid point $I_1 \propto (a+a)^2 \Rightarrow I_1 \propto (2a)^2$

For incoherent sources, intensity of mid-point is $I_2 \propto 2a^2$ $\quad \frac{I_1}{I_2} = \frac{2}{1}$

44. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincides. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength ofthe unknown lights: [2009]

(a) 885.0nm (b) 442.5nm(c) 776.8nm (d) 393.4nm

SOLUTION: b) Let λ be the wavelength ofunknown light. Third bright fringe of known light coincides with the 4th bright fringe of the unknown light. $\frac{3\lambda_1 D}{d} = \frac{4\lambda D}{d}$

$$\frac{3(590)D}{d} = \frac{4\lambda D}{d} \quad \Rightarrow \lambda = \frac{3}{4} \times 590 = 442.5\text{mn}$$

45. In a Young's double slit experiment the intensity at a point where the path difference is $\frac{\lambda}{6}$

(λ being the wavelength of light used) is *I*. If I_0 denotes the maximum intensity, $\frac{I}{I_0}$ is equal to

[2007]

(a) $\frac{3}{4}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{2}$

SOLUTION: The intensity of light at any point of the screen where the phase difference due to light

coming from the two slits is φ isgiven by $\mathrm{I} = \mathrm{I_0} \cos^2\left(\frac{\varphi}{2}\right)$

Where I_0 is the maximum intensity . This formula is applicable when $I_1 = I_2$.

Phase difference $\varphi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$ $\qquad$ $\frac{I}{I_0} = \cos^2\frac{\pi}{6} = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$

46. AYoungs double slit experiment uses a monochromaticsource. The shape ofthe interference fringes formed on ascreen is [2005]

(a) circle (b) hyperbola(c) parabola (d) straight line

SOLUTION: (d) The light passing through the slits interfere and produce dark and bright band one screen. The shape of interference fringes formed on a screen in case of a monochromatic source is a straight line.

47. The maximum number ofpossible interference maxima forslit-separation equal to twice the wavelength in Young'sdouble-slit experiment is [2004]

(a) three (b) five (c) infinite (d) zero

SOLUTION: b) For constructive interference path difference (As $\sin\theta \leq 1$) $\qquad d\sin\theta = n\lambda$

Given $d = 2\lambda$ $\qquad$ $2\lambda\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{n}{2}$

$n = 0, 1, -1, 2, -2$ hence five maxima are possible.

48. A beam of plane polarized light of large cross-sectional area and uniform intensity of $3.3\mathrm{Wm^{-2}}$ falls normally on a polarizer (cross sectional area $3 \times 10^{-4}\mathrm{m^2}$) which rotates about its axis with an angular speed of $31.4\ \mathrm{rad/s}$. The energy of light passing through the polarizer per revolution, is close to: [Sep. 04, 2020 (I)]

(a) $1.0 \times 10^{-5}\mathrm{J}$ $\qquad$ (b) $1.0 \times 10^{-4}\mathrm{J}$ $\qquad$ (c) $1.5 \times 10^{-4}\mathrm{J}$ $\qquad$ (d) $5.0 \times 10^{-4}\mathrm{J}$

SOLUTION: (d) Given:Intensity, $I_0 = 3.3\mathrm{Wm^{-2}}$

Area, $A = 3 \times 10^{-4}\mathrm{m^2}$

Angular speed, $(j)^{=31.4\mathrm{rad/s}}$

Average energy $= I_0 A\langle\cos^2\theta\rangle$

$\langle\cos^2\theta\rangle = \frac{1}{2}$ per revolution

Average energy $= \frac{(3.3)(3\times10^{-4})}{2} = 5 \times 10^{-4}\mathrm{J}$

49. Orange light of wavelength $6000 \times 10^{-10}\mathrm{m}$ illuminates a single slit of width $0.6 \times 10^{-4}\mathrm{m}$. The maximum possible number of diffraction minima produced on both sides of the central maximum is [NA Sep. 04, 2020(II)]

SOLUTION: (198)

For obtaining secondary minima at a point path difference

should be integral multiple of wavelength $d \sin\theta = n\lambda$ $\sin\theta = \frac{n\lambda}{d}$

For n to be maximum $\sin\theta = 1$ $n = \frac{d}{\lambda} = \frac{6\times10^{-5}}{6\times10^{-7}} = 100$

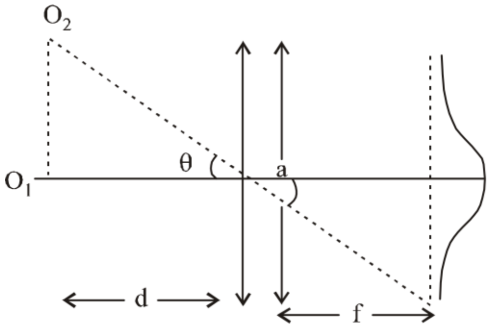
Total number of minima on one side $= 99$

Total number of minima $= 198.$

50. The aperture diameter of telescope is $5\mathrm{m}$. The separation between the moon and the earth is 4×10^{5} km. With light of wavelength of5500 A, the minimum separation between objects on the surface of moon, so that they are just resolved, is close to: [9 Jan. 2020 I]

(a) 60 m (b) 20 m (c) 200 m (d) 600 m

SOLUTION: (a)



Smallest angular separation between two distant objects

here moon and earth, $\theta = 1.22\frac{\lambda}{a}$

a $=$ aperture diameter oftelescope

Distance $O_1O_2 = (\theta)d$

Minimum separation between objects on the surface of moon, $= \left(1.22\frac{\lambda}{a}\right)d$

$$= \frac{(1.22)(5500\times10^{-10})\times4\times10^{5}\times10^{3}}{5} = 5368\times10^{2}\mathrm{m} = 53.68\mathrm{m} \approx 60\mathrm{m}$$

51. A polarizer - analyzer set is adjusted such that the intensity of light coming out of the analyzer is just 10% of the original intensity. Assuming that the polarizer -analyzer set does not absorb any light, the angle by which the analyzer need to be rotated further to reduce the output intensity to be zero, is: [7 Jan. 2020 I]

(a) 71.6° (b) 18.4° (c) 90° (d) 45°

SOLUTION:

SOLUTION:

SOLUTION:

SOLUTION:
SOLUTION:
SOLUTION:
SOLUTION:b) According to question, the intensity of light coming out of the analyzer is just 10%

ofthe original intensity (I_0)Using, $I = I_0 \cos^2\theta$ $\Rightarrow \frac{I_0}{10} = I_0 \cos^2 0 \Rightarrow \frac{1}{10} = \cos^2\theta$

$$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}} = 0.316 \Rightarrow \theta \approx 71.6^\circ$$

Therefore, the angle by which the analyzer need to be rotated further to reduced the output intensity to be zero $\varphi = 90^\circ - \theta = 90^\circ - 71.6^\circ = 18.4^\circ$

52. The value of numerical aperture of the objective lens of a microscope is 1.25. Iflight ofwavelength 5000 A is used, the minimum separation between two points, to be seen as distinct, will be: [12 Apri12019 I]

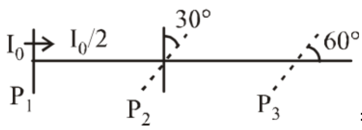
(a) $0.24\mu n$ (b) $0.38\mu m$ (c)$0.12\mu m$ (d) $0.48\mu m$

SOLUTION: (a) $x = \frac{1.22\lambda}{2\mu \sin\theta} = \frac{1.22\times 5000}{2\times 1.25} = 0.24\mu m$

53. A system of three polarizers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis ofP_2 is inclined at 60^o to the pass axis ofP_3. When a beam of unpolarized light of intensity I_o is incident on P_1,the intensity oflight transmitted by the three polarizers is I. The ratio (I_o/I) equals (nearly): [12 April 2019 II]

(a) 5.33 (b) 16.(X) (c) 10.67 (d) 1.80

SOLUTION: (c) $I = \left(\frac{I_0}{2}\right)\cos^2 30^\circ \cos^2 60^\circ$



$= \frac{I_0}{2} \times \frac{3}{4} \times \frac{1}{4}$ $\frac{I_0}{I} = \frac{32}{3} = 10.67$

54. Diameter of the objective lens ofa telescope is 250 cm. For light of wavelength 600 nm. Coming fi$_{\text{i}}$om a distant object, the limit of resolution of the telescope is close to:[9 April 2019 II]

(a) 1.5×10^{-7} rad(b) 2.0×10^{-7} rad (c) 3.0×10^{-7} rad (d) 4.5×10^{-7} rad

SOLUTION: (c) $\theta = \frac{1.22\lambda}{d} = \frac{1.22\times 600\times 10^{-9}}{250\times 10^{-2}} = 3.0 \times 10^{7}$ rad

55. Calculate the limit of resolution of a telescope objective having a diameter of200 cm, if it has to detect light of wavelength 500 nm coming from a star. [8 April 2019 II]

(a) 305×10^9 radian (b) 610×10^9 radian(c) 152.5×10^9 radian (d) 457.5×10^9 radian

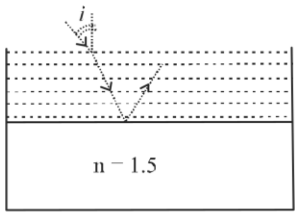
SOLUTION: (a) $\theta = \frac{1.22\lambda}{d} = \frac{1.22\times 500\times 10^{-9}}{2} = 305 \times 10^9$ rad.

56. In a double-slit experiment, green light (5303A) falls on a double slit having a separation of $19.44\mu m$ and a width of4.05 μm. The number ofbright fringes between the first and the second diffraction minima is: [11 Jan 2019 II]

(a) 10 (b) 05 (c) 04 (d) (P

SOLUTION: b)

57. Consider a tank made of glass (refractive index 1.5) with athick bottom. It is filled with a liquid rarefactive index μ.A student finds that, irrespective of what the incident angle i (see figure) is for a beam oflight entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value ofμ is: [9 Jan. 2019 I]



(a) $\sqrt{\frac{5}{3}}$ (b) $\frac{3}{\sqrt{5}}$ (c) $\frac{5}{\sqrt{3}}$ (d) $\frac{4}{3}$

SOLUTION: b) According to Brewster's law, refractive index of material (μ) is equal to tangent ofpolarising angle $\tan i_b = \mu = \underline{1.5}\mu$ $\frac{1}{\mu} < \frac{1.5}{\sqrt{\mu^2+(15)^2}} (\because \sin i_c < \sin i_b)$

$$\sin i_b = \frac{1.5}{\sqrt{\mu^2 + (15)^2}}$$

or, $\sqrt{\mu^2 + (15)^2} < 1.5 \times \mu$

$\Rightarrow \mu^2 + (1.5)^2 < (\mu \times 1.5)^2$

$\Rightarrow \mu < \frac{3}{\sqrt{5}}$ i.e. minimum value of μ should be $\frac{3}{\sqrt{5}}$

58. The angular width of the central maximum in a single slit diffraction pattern is $60°$. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is
slit separation distance?(i.e. distance between the centers of each slit.) [2018]

(a) 25 μn (b) 50μm (c) 75 μn (d) 1(X) μn

SOLUTION: (a) Angular width of central maxima $= \frac{2\lambda}{d}$ or, $\lambda = \frac{d}{2}$; Fringe width, $\beta = \frac{\lambda \times D}{d'}$

$$10^{-2} = \frac{d}{2} \times \frac{50 \times 10^{-2}}{d'} = \frac{10^{-6} \times 50 \times 10^{-2}}{2 \times d'}$$

Therefore, slit separation distance, d' $= 25\mu m$

59. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{I}{8}$. The angle between polarizer A and C is: [2018]

(a) $0°$ (b) $30°$ (c) $45°$ (d) $60°$

SOLUTION: $\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ or, $\theta = 45°$

60. Light of wavelength 550 nm falls normally on a slit of width22.0×10^{-5}cm. The angular position of the second minima from the central maximum will be (in radians) [Online Apri115, 2018]

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{12}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

SOLUTION: (a) If angular position of2nd maxima from central maxima

is θ then $\sin\theta = \frac{(2n-1)\lambda}{2a} = \frac{3\lambda}{20} = \frac{3\times550\times10^{-9}}{2\times22\times10^{-7}}$ $\theta = \frac{\pi}{8}$ rad

61. Unpolarized light of intensity I is incident on a system of two polarizers, A followed by B. The intensity of emergent light is $I/2$. Ifa third polarizer C is placed between A and B,the intensity ofemergent light is reduced to $I/3$. The angle between the polarizers A and C is θ. Then [Online Apri116, 2018]

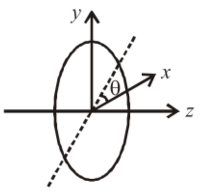
(a) $\cos 0 = \left(\frac{2}{3}\right)^{1/4}$ (b) $\cos 0 = \left(\frac{1}{3}\right)^{1/4}$ (c) $\cos 0 = \left(\frac{1}{3}\right)^{1/2}$ (d) $\cos 0 = \left(\frac{2}{3}\right)^{1/2}$

SOLUTION: (a) Polarizer A and B have same alignment of transmission axis .Lets assume

polarizer c is introduced at θ angle $\frac{1}{2}\cos^2\theta \times \cos^2\theta = \frac{1}{3}$ or, $\cos^4\theta = \frac{2}{3} \Rightarrow \cos\theta = \left(\frac{2}{3}\right)^{1/4}$

62. A plane polarized light is incident on a polarizer with its pass axis making angle θ with x-axis, as shown in the figure. At four different values of$\theta, \theta = 8°, 38°, 188°$ and $218°$, the observed intensities are same. What is the angle between the direction of polarization and x-axis?

[Online Apri115, 2018]



(a) $203°$ (b) $45°$ (c) $98°$ (d) $128°$

SOLUTION: (a)

63. An observer is moving with half the speed of light towards stationary microwave source emitting waves at fiiequency10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $= 3 \times 10^8 ms^{-1}$) [2017]

(a) 17.3 GE (b) 15.3 GHz(c) 10.1 GHz (d) 12.1 GHz

SOLUTION: (a) Use relativistic doppler's effect as velocity of

observer is not small as compared to light $f = f_0\sqrt{\frac{c+v}{c+v}}$; $V =$ relative speed ofapproach

$$f = 10\sqrt{\frac{c+\frac{c}{2}}{c-\frac{c}{2}}} = 10\sqrt{3} = 17.3 \text{ GHz}$$

$f_0 = 10$ GHz

64. A single slit of width 0.1 mm is illuminated by a parallel beam of light of wavelength 6000 A and diffraction bands are observed on a screen $0.5\mathrm{m}$ from the slit. The distance of the third dark band $\mathrm{fi_{i}om}$ the central bright band is: [Online April 9, 2017]

(a) 3mm (b) $9\mathrm{mm}$ (c) 4.5mm (d) 1.5mm

SOLUTION: b) $\mathrm{a}=0.1\mathrm{mm}=10^{\triangleleft}$ cm,$\lambda=6000\times 10^{10}\mathrm{cm}=6\times 10$ 7cm, $\mathrm{D}=0.5\mathrm{m}$

for 3^{rd} dark band, a $\sin\theta=3\lambda$ or $\sin\theta=\frac{3\lambda}{\mathrm{a}}=\frac{\mathrm{x}}{\mathrm{D}}$

The distance of the third dark band $from$ the central bright band

$\mathrm{x}=\frac{3\lambda\mathrm{D}}{\mathrm{a}}=\frac{3\times 6\times 10^{-7}\times 0.5}{10^{\triangleleft}}=9$ mm

65. A single slit of width b is illuminated by a coherent monochromatic light of wavelength λ. If the second and fourth minima in the diffraction pattern at a distance 1 $\mathrm{mfi_{i}om}$ the slit are at 3 cm and 6 cm respectively $from$ thecentral maximum, what is the width of the central maximum?

(i.e. distance between first minimum on either side of the central maximum) [Online April 8, 2017]

(a) 1.5 cm (b) $3.0\mathrm{cm}$ (c) 4.5 cm (d) $6.0\mathrm{cm}$

SOLUTION: . b) For secondary minima, $\mathrm{b}\sin\theta=\mathrm{n}\lambda\Rightarrow\sin\theta=\frac{\mathrm{n}\lambda}{\mathrm{b}}$

Distance of nth secondary minima $\mathrm{x}=\mathrm{D}\sin\theta$ or $\sin\theta_{1}=\frac{\mathrm{x}_{1}}{\mathrm{D}}$ $\sin\theta_{1}=\frac{2\lambda}{\mathrm{b}}$

$$\mathrm{n}=4 \qquad \sin\theta_{2}=\frac{4\lambda}{\mathrm{b}}=\frac{\mathrm{x}_{2}}{\mathrm{D}}$$

$$\mathrm{x}_{2}-\mathrm{x}_{1}=\frac{4\lambda}{\mathrm{b}}-\frac{2\lambda}{\mathrm{b}}=\frac{2\lambda}{\mathrm{b}}$$

$3=\frac{2\lambda}{\mathrm{b}}\Rightarrow\mathrm{b}=\frac{2\lambda}{3}$ ..... (i) Width of central maxima $=\frac{2\lambda}{\mathrm{b}}=\frac{2\lambda}{\frac{2\lambda}{3}}=3$ cm... $from$ eq. (i)

66. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated bya parallel beam of light of wavelength λ the spread ofthespot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size(say $\mathrm{b}_{\min}$) when : [2016]

(a) $\mathrm{a}=\sqrt{\lambda\mathrm{L}}\mathrm{and}\mathrm{b}_{\min}=\sqrt{4\lambda\mathrm{L}}$

(b) $\mathrm{a}=\frac{\lambda^{2}}{\mathrm{L}}\mathrm{and}\mathrm{b}_{\min}=\sqrt{4\lambda\mathrm{L}}$

(c) $\mathrm{a}=\frac{\lambda^{2}}{\mathrm{L}}\mathrm{and}\mathrm{b}_{\mathrm{m\,ln}}=\left(\frac{2\lambda^{2}}{\mathrm{L}}\right)$

(d) $\mathrm{a}=\sqrt{\lambda 1}\mathrm{and}\mathrm{b}_{\min}=\left(\frac{2\lambda^{2}}{\mathrm{L}}\right)$

SOLUTION: (a) Given geometrical spread $=a$ Diffraction spread $=\frac{\lambda}{a}\times L=\frac{\lambda L}{a}$

The sum $b=a+\frac{\lambda L}{a}$ For b to be minimum $\frac{db}{da}=0$ $\frac{d}{da}\left(a+\frac{\lambda L}{a}\right)=0$

$a=\sqrt{\lambda L}$ $\quad b \min =\sqrt{\lambda L}+\sqrt{\lambda L}=2 \sqrt{\lambda L}=\sqrt{4 \lambda L}$

67. Two stars are 10 light years away fi: om the earth. They are seen through a telescope of objective diameter 30 cm. The wavelength of light is 600 nm. To see the stars just resolved by the telescope, the minimum distance between them should be (1 light year $=9.46 \times 10^{15} \mathrm{m}$) of the order of:[Online Apri110, 2016]

(a) 10^{8} km (b) 10^{10} km (c) 10^{11} km (d) 10^{6} km

SOLUTION: (a) We know that $\Delta\theta=\frac{0.61 \lambda}{4}=\frac{l}{\mathrm{R}}$

The minimum distance between them $l=\frac{\mathrm{R}}{9} 0.61 \times \lambda=\frac{9.46 \times 10^{15} \times 10 \times 0.61 \times 600 \times 10^{-9}}{0.3}=1.15 \times 10^{11} \mathrm{m}$ $\Rightarrow 1.115 \times 10^{8}$ km.

68. In Young's double slit experiment, the distance between slits and the screen is $1.0 \mathrm{m}$ and monochromatic light of600 nm is being used. A person standing near the slits is looking at the fringe pattern. When the separation between the slits is varied, the interference pattern disappears for a particular distance d_{0} between the slits. If the angular resolution of the eye is $\frac{1^{\circ}}{60}$, the value of d_{0} is close to:[Online April 9, 2016]

(a) I mm (b) $3 \mathrm{mm}$ (c) $2 \mathrm{mm}$ (d) $4 \mathrm{mm}$

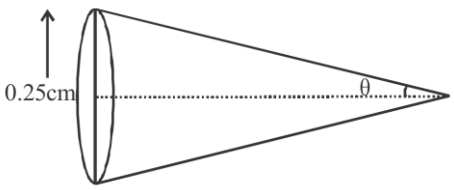
SOLUTION: (c) Given $\mathrm{D}=1.0 \mathrm{m}$, wavelength ofmonochromatic light $\lambda=600 \mathrm{nm}.$

$$\mathrm{d}: \mathrm{D} \theta=1 \times \frac{\pi}{180} \times \frac{1}{60}$$

$$\mathrm{d}_{0}=2 \times 10^{-3}=2 \mathrm{mm}$$

69. Assuming human pupil to have a radius of0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is : [2015]

(a) $100 \mu \mathrm{m}$ (b) $300 \mu \mathrm{m}$(c)$1 \mu n$ (d) $30 \mu \mathrm{m}$



SOLUTION: $\sin \theta=\frac{0.25}{25}=\frac{1}{100}$

Resolving power $=\frac{1.22 \lambda}{2 \mu \sin \theta}=30 \mu \mathrm{m}.$

70. Unpolarized light of intensity I_{0} is incident on surface of a block of glass at Brewster's angle. In that case, which one of the following statements is true?[Online April 11, 2015]

(a) reflected light is completely polarized with intensity less than $\frac{\mathrm{I}_{0}}{2}$

(b) transmitted light is completely polarized with intensity less than $\frac{I_0}{2}$

(c) transmitted light is partially polarized with intensity $\frac{I_0}{2}$

(d) reflected light is partially polarized with intensity $\frac{I_0}{2}$

SOLUTION: (a) When unpolarized light is incident at Brewster's angle then reflected light is completely polarized and the intensity of the reflected light is less than half of the incident light.

71. Two beams, A and B, of plane polarized light with mutually perpendicular planes of polarization are seen through a polaroid. From the position when the beam A has maximum intensity (and beam B has zero intensity), a rotation ofpolaroid through $30°$ makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then $\frac{I_A}{I_B}$ equals: [2014]

(a) 3 (b) $\frac{3}{2}$ (c) 1 (d) $\frac{1}{3}$

SOLUTION: (d) According to malus law, intensity of emerging beam is given by, $I = I_0 \cos^2\theta$

Now, $I_A = I_{A^1} COS^2 30^0$ AND $I_B = I_{B^1} COS^2 60^0$ As $I_{A'} = I_{B'}$

$\Rightarrow I_A \times \frac{3}{4} = I_B \times \frac{1}{4}; \frac{I_A}{I_B} = \frac{1}{3}$

72. The diameter of the objective lens of microscope makes an angle β at the focus ofthe microscope. Further, the medium between the object and the lens is an oil of refractive index n. Then the resolving power of the microscope [Online Apri119, 2014]

(a) increases with decreasing value o fn

(b) increases with decreasing value of β

(c) increases with increasing value ofn $\sin 2\beta$

(d) increases with increasing value of $\frac{1}{n \sin 2\beta}$

SOLUTION: (c) Resolving power of microscope, R.P. $= \frac{2n \sin\theta}{\lambda}$

$\lambda =$ Wavelength of light used to illuminate the object

$n =$ Refiactive index of the medium between object and objective

$\theta =$ Angle

73. A ray of light is incident fi_iom a denser to a rarer medium. The critical angle for total internal reflection is θ_{iC} and Brewster's angle of incidence is θ_{iB}, such that $\sin\theta_{iC}/\sin\theta_{iB} = \eta = 1.28$. The relative refractive index of the two media is: [Online April 19, 2014]

(a) 0.2 (b) 0.4 (c) 0.8 (d) 0.9

SOLUTION: (c) Here, $\sin\theta_{ic}/\sin\theta_{iB} = 1.28$

As we know, $\mu=\frac{\sin\theta_{iB}}{\sin\left(\frac{\pi}{2}-\theta_{iB}\right)}$ where, θ_{iB} is Brewster's angle of incidence,

And, $\mu=\frac{1}{\sin\theta_{ic}}$ On solving we get, relative refractive index of the two media.

74. In an experiment of single slit diffraction pattern, first minimum for red light coincides with first maximum of someother wavelength. If wavelength of red light is 6600 A,then wavelength of first maximum will be: [Online Apri112, 2014]

(a) 3300A (b) 4400Â (c) 5500A (d) 6600A

SOLUTION: (b) In a single slit experiment, For diffraction maxima, a $\sin\theta=(2n+1)\frac{\lambda}{2}$

and for diffraction minima, a $\sin\theta=n\lambda$ According to question,

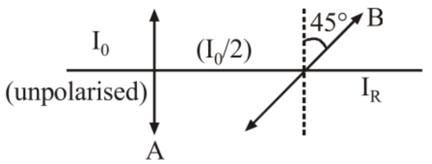
$$(2\times1+1)\frac{\lambda}{2}=1\times6600$$

$$(\lambda_R=6600A) \qquad \lambda=\frac{6600\times2}{3}=4400A$$

75. Abeam of unpolarized light of intensity I_0 is passed through a polaroid and then through another polaroid Bwhich is oriented so that its principal plane makes an angleof $45°$ relative to that ofA. The intensity of the emergent light is [2013]

(a) I_0 (b) $I_0/2$ (c) $I_0/4$ (d) $I_0/8$

SOLUTION: (c) Relation between intensities



$$I_r=\left(\frac{I_0}{2}\right)\cos^2(45°)=\frac{I_0}{2}\times\frac{1}{2}=\frac{I_0}{4}$$

76. This question has Statement-I and Statements-2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement-I : Out of radio waves and microwaves, the radio waves undergo more diffraction.

Statement-2 : Radio waves have greater frequency compared to microwaves.

[Online April 25, 2013]

(a) Statement-I is true, Statement-2 is true and Statement-2 is the correct explanation

(b) of Statement-I

(b) Statement-I is false , Statement-2 is true.

(c) Statement- 1 is true, Statement-2 is false.

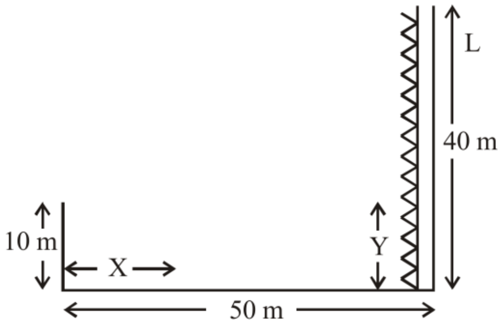
(d) Statement-I is true, Statement-2 is true but Statement-2 is not the correct explanation of Statement-I

SOLUTION: (c) Wavelength of radio waves is greater than microwaves hence frequency of radio waves is less thanmicrowaves. The degree of diffraction is greater whose wavelength is greater.

77. A person lives in ahigh-rise building on the bank ofariver50 m wide. Across the river is a well-lit

tower ofheight 40 m.When the person, who is at a height of10 m, looks through a polarizer at an appropriate angle at light of the tower reflecting from the river surface, he notes that intensity of light coming $\mathrm{fi_{i}om}$ distance X from his building is the least and this corresponds to the light coming from light bulbs at height $\mathrm{Y}^{\uparrow}$ on the tower. The values of X and Y are respectively close to

(refractive index of water $=\frac{4}{3}$)[Online April 9, 2013]



(a) $25\mathrm{m}, 10\mathrm{m}$ (b) $13\mathrm{m}, 27\mathrm{m}$ (c) $22\mathrm{m}, 13\mathrm{m}$ (d) $17\mathrm{m}, 20\mathrm{m}$

SOLUTION: b)

78. The first diffraction minimum due to the single slit diffraction is seen at $\theta=30^{\circ}$ for a light of wavelength 5000A falling perpendicularly on the slit. The width of the slit is [Online May 12, 2012]

(a) 2.5×10^{-5} cm (b) 1.25×10^{-5} cm(c) 10×10^{-5} cm (d) 5×10^{-5} cm

SOLUTION: (c) For first minimum,$d\sin\theta=\lambda\Rightarrow d=\frac{\lambda}{\sin\theta}=\frac{5000\times 10^{-8}\mathrm{cm}}{\sin 30^{\circ}}=\frac{5000\times 10^{-8}\mathrm{cm}}{/^{1}2}=10\times 10^{-5}$ cm

79. Two polaroid's have their polarizing directions parallel so that the intensity of a transmitted light is maximum. The angle through which either polaroid must be turned if the intensity is to drop by one-half is [Online May 7, 2012]

(a) 135° (b) $Reject)^{\circ}$ (c) 120° (d) 180°

SOLUTION: (a) For $I=\frac{I_{0}}{2}$ and , $I=I_{0}\cos^{2}\theta$ $\quad hence\ \theta=45^{\circ}$

Therefore the angle through which either polaroid's turned is $135^{\circ}(=180^{\circ}-45^{\circ})$

80. Statement-I: On viewing the clear blue portion of the sky through a Calcite Crystal the intensity of transmitted light varies as the crystal is rotated.

Statement-2: The light coming $\mathrm{fi_{i}om}$ the sky is polarized due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light. [2011 RS]

(a) Statement-I is true, statement-2 is false.

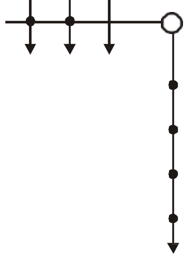
(b) Statement- 1 is true, statement-2 is true, statement-2 is the correct explanation of statement-I

(c) Statement- 1 is true, statement-2 is true, statement-2 is

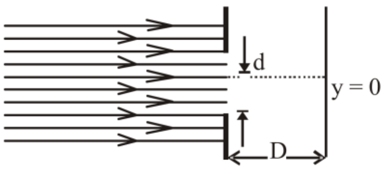
not the correct explanation of statement-IStatement-I is false, statement-2 is true.

SOLUTION: b) When viewed through a polaroid which is rotatedthen the light $\mathrm{fi_{i}om}$ a clear blue portion ofthe sky shows arise and fall of intensity The light coming from the sky is polarized due to scattering of sunlight by particles in the atmosphere

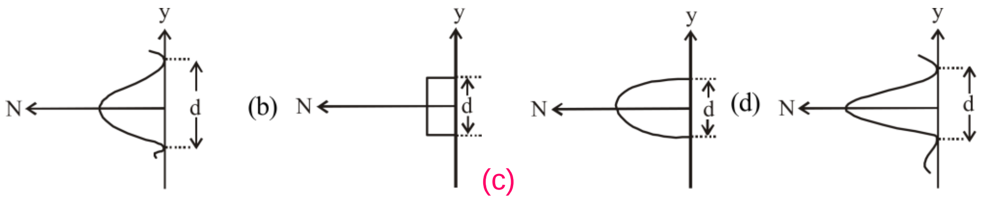
cattered light



81. In an experiment, electrons are made to pass through a narrow slit of width ‘*d*’ comparable to their de Broglie wavelength. They are detected on a screen at a distance ‘*D*’ from the slit (see figure). Which of the following graphs can be expected to represent the number of electrons ‘*N*’ detected as afunction of the detector position $y'(y=0$ corresponds to the middle of the slit) [2008]



(a)



SOLUTION: (d) The electron beam will be diffracted and the maxima is obtained at $y=0$.
Also, the diffraction pattern, should be wider than the slit width.

82. If I_0 is the intensityofthe principal maximum in the single slit diffraction pattern, then what will be its intensity when the slit width is doubled? [2005]

(a) 4 I_0 (b) 2 I_0 (c) $\frac{I_0}{2}$ (d) I_0

SOLUTION: (a) $\mathrm{I}=\mathrm{I}_0\left(\frac{\sin\varphi}{\varphi}\right)^2$ and $\varphi=\frac{\pi}{\lambda}(\mathrm{b}\sin\theta)$When the slit width is doubled, the amplitude of the wave at the center of the screen is doubled, so the intensity at the center is increased by a factor 4.

83. When an unpolarized light of intensity I_0 is incident on a polarizing sheet, the intensity of the light which does not get transmitted is [2005]

(a) $\frac{1}{4}I_0$ (b) $\frac{1}{2}I_0$ (c) I_0 (d) zero

SOLUTION: b) From the law ofMalus, $I=I_0\cos^2\theta$When an unpolarised light is converted into planepolarized light by passing through polaroid, its intensity become half.

$$\Rightarrow D\le\frac{yd}{(1.22)\lambda}=\frac{10^{-3}\times3\times10^{-3}}{(1.22)\times5\times10^{-7}}=\frac{30}{6.1}\approx5\mathrm{m}\qquad D_{\max}=5\mathrm{m}$$

84. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye?[Take wavelength oflight $=500$ nm] [2005]

(a) lm (b) $5\mathrm{m}$ (c) $3\mathrm{m}$ (d) $6\mathrm{m}$

SOLUTION:

$$\frac{y}{D} \geq \frac{1.22\lambda}{d} \quad \therefore \text{Dmax} = 5\text{m}$$

85. The angle of incidence at which reflected light is totally polarized for reflection from air to glass (refractive index is n), is [2004]z

(a) $\tan^{-1}(1/n)$ (b) $\sin^{-1}(1/n)$ (c) $\sin^{-1}(n)$ (d) $\tan^{-1}(n)$

SOLUTION:

(d) From the Brewster's law, angle of incidence for total polarization is given by $\tan\theta = n$ $\Rightarrow \theta = \tan^{-1}n$ Where n is the refractive index of the glass.

86. Wavelength of light used in an optical instrument are$\lambda_1 = 4000\mathrm{A}$ and $\lambda_2 = 5000\mathrm{A}$, then ratio of their respective resolving powers (corresponding to λ_1 andλ_2) is [2002]

(a) 16: 25 (b) 9: 1 (c) 4: 5 (d) 5: 4

SOLUTION: (d) The resolving power of an optical instrument is inversely proportional to the wavelength of light used. $\frac{(R.P)_1}{(R.P)_2} = \frac{\lambda_2}{\lambda_1} = \frac{5}{4}$

ELECTRIC CHARGES AND FIELDS

Charge and its properties

- Study of characteristics of electric charges at rest is known as electrostatics.
- Electric charge is the property associated with a body or a particle due to which it is able to produce as well as experience the electric and magnetic effects.
- Charge is a fundamental property of matter and never found free.
- The excess or deficiency of electrons in a body gives the concept of charge.
- There are two types of charges namely positive and negative charges.
- The deficiency of electrons in a body is known as positively charged body.
- The excess of electrons in a body is known as negatively charged body.
- If a body gets positive charge, its mass slightly decreases.
- If a body is given negative charge, its mass slightly increases.
- Charge is relativistically invariant, i.e. it does not change with motion of charged particle and no change in it is possible, whatsover may be the circumstances. i.e. $q_{static} = q_{dynamic}$
- Charge is a scalar. S.I. unit of charge is coulomb(C).

  One electrostatic unit of charge

  (esu) $= \frac{1}{3 \times 10^9}$ coulomb.

  One electromagnetic unit of charge (emu) = 10coulomb
- Charge is a derived physical quantity with dimensions [AT].

Quantization of Charge : The electric charge is discrete. It has been verified by Millikan's oil drop experiment.

- Charge is quantised. The charge on any body is an integral multiple of the minimum charge or electron charge, i.e if q is the charge then $q = \pm ne$ where n is an integer, and e is the charge of electron = $1.6 \times 10^{-19} C$.
- The minimum charge possible is $1.6 \times 10^{-19} C$.
- If a body possesses n_1 protons and n_2 electrons, then net charge on it will be $(n_1 - n_2)e$,

  i.e. $n_1(e) + n_{2(-e)} = (n_1 - n_2)e$

Law of conservation of charge

- The total net charge of an isolated physical system always remains constant,

  i.e. $q = q_+ + q_- =$ constant.
- In every chemical or nuclear reaction, the total charge before and after the reaction remains constant.
- This law is applicable to all types of processes like nuclear, atomic, molecular and the like.
- Charge is conserved. It can neither be created nor destroyed. It can only be transferred from one object to the other.
- Like charges repel each other and unlike charges attract each other.
- Charge always resides on the outer surface of a charged body. It accumulates more at sharp points.
- The total charge on a body is algebric sum of the charges located at different points on the body.

Electrification:

A body can be charged by friction, conduction and induction.

By Friction:

When two bodies are rubbed together, equal and opposite charges are produced on both the bodies.

By Conduction:

An uncharged body acquiring charge when kept in contact with a charged body is called conduction.

Conduction preceeds repulsion.

By Induction:

If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel like charge present in the neutral body. Opposite charge is induced at the near end and like charge at the farther end. Inducing body neither gains nor loses charge. Induction always preceeds attraction.

- Repulsion is the sure test of electrification.
- Induced charge $q^1 = -q\left[1-\frac{1}{K}\right]$ where K is Dielectric constant

Coulomb's Law: '

The force of attraction or repusion between two stationary electric charges is directly proportional to the product of magnitude of the two charges and is inversely proportional to the square of the distance between them and this force acts along the line joining those two charges'

$$F = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2}$$

$\in_0$ - permittivity of free space or vacuum or air.

$\in_r$ - Relative permittivity or dielectric constant of the medium in which the charges are situated.

$$\in_0 = 8.857\times10^{-12}\frac{C^2}{Nm^2} \text{ or } \frac{farad}{metre}, \quad \text{and} \quad \frac{1}{4\pi \in_0} = 9\times10^9 \, Nm^2/C^2$$

Permitivity of Medium:

Permitivity is the measure of degree of the medium which resist the flow of charges

In SI. for medium other than free space, the constant

$K_0^1 = \frac{1}{4\pi \in}$ so that we can write the equation for the force between the charges as

$$F = \frac{1}{4\pi \in} \frac{q_1 q_2}{r^2}$$

$$\therefore \frac{F_0}{F} = \frac{\in}{\in_0} = \in_r$$

$\in_r$ is known as the relative permitivity of the medium.

It is a constant for a given medium and it charges separated by a medium decreases compared with the force between the same charges in free space separated by the same distance.

Relative permitivity $\in_r$ is also known as dielectric constant K of the medium or specific inductive capacity.

Relative permitivity of a medium is defined as the ratio of permitivity of the medium to permitivity of free space (or) air

(or)

Relative permitivity of a medium is defined as the electrostatic force (F_0) between two charges in air to the force (F) between the same two charges kept in the medium at same distance.

Dielectric constant (or) Relative permitivity

$$K = \frac{Pemitivity\ of\ the\ medium}{Permitivity\ of\ free\ space}$$

It has no units and no dimesions

Hence, the mathematical form of inverse square law is given as

$$F = \frac{1}{4\pi \in} \frac{q_1 q_2}{r^2} = \frac{1}{K} \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$$

For force or vacuum or air K=1 and for a good conductor like metals, $K = \infty$

Conclusion :

1) The introduction of a glass slab between two charges will decrease the magnitude of rorce between them.

2) The introduction of a metallic slab between two charges will decrase the magnitude of force to zero.

Note:1 When the some charges are separated by the some distance in two different media,

$$F_1 = \frac{1}{K_1} \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2} \text{ --------(1)}$$

and $F_2 = \frac{1}{K_2} \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}$ ------(2)

from (1) and (2) $\Rightarrow F_1 K_1 = F_2 K_2$

Note:2 When the same charges are separated by different distance in the same medium

Fd^2 = constant (or) $F_1 d_1^2 = F_2 d_2^2$

Note : 3 If different charges are at the same separation in a given medium $\frac{F^1}{F} = \frac{q_1^1 q_2^1}{q_1 q_2}$

Note : 4 If the force between two charges in two different media is the same for different separations.

$$F = \frac{1}{K} \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2} = \text{constant}$$

Kr^2=constant or $K_1 r_1^2 = K_2 r_2^2$

If the force between two charges separated by a distance $'r_0'$ in vacuum or air is same as the force between the same charges separated by a distance 'r' in a medium.

$$Kr^2 = r_0^2 \Rightarrow r = \frac{r_0}{\sqrt{K}}$$

Here K is dielectric constant of the medium.

The effective distance 'r' in medium for a distance r_0 in vaccum = $\frac{r_0}{\sqrt{K}}$.

Similarly, the effective distance in vaccum for a dielectric slab of thickness 'x' and dielectric constant K is

$$x_{eff} = f\sqrt{K}$$

Coulomb's Law in Vector Form

$$\overrightarrow{F_{12}} = \frac{1}{4\pi \in_0} \frac{q_1 q_2}{r_2^2} \hat{r}_{12} \text{ and } \overrightarrow{F_{21}} = -\overrightarrow{F_{12}}$$

Here F_{12} is force exerted by q_1 on q_2 and F_{21} is force exerted by q_2 on q_1

↳ Suppose the position vector of two charges q_1 and q_2 are $\vec{r_1}$ and $\vec{r_2}$, then electric force on charge q_1 due to q_2 is,

$$\vec{F_1} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left|\vec{r_1} - \vec{r_2}\right|^3}\left(\vec{r_1} - \vec{r_2}\right)$$

Similarly, electric force on q_2 due to charge q_1 is $\vec{F_2} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{\left|\vec{r_2} - \vec{r_1}\right|^3}\left(\vec{r_2} - \vec{r_1}\right)$

Here q_1 and q_2 are to be substitued with sign.

$\vec{r_1} = x_1 i + y_1 j + z_1 k$ and $\vec{r_2} = x_2 i + y_2 j + z_2 k$ where (x_1, y_1, z_1) and (x_2, y_2, z_2) are the co-ordinates of charges q_1 and q_2.

Limitations of Colulomb's Law

- Coulomb's law holds for stationary charges only which are point sized.

  This law is valid for all types of charge distributions.

  This law is valid at distances greater than $10^{-15} m$.

  This law obeys Newton's third law.

  This law represents central forces.

  This law is analogous to Newton law of gravitation in mechanics.
- The electric force is an action reaction pair, i.e the two charges exert equal and opposite forces on each other.
- The electric force is conservative in nature.
- Coulomb force is central.
- Coulomb force is much stronger than gravitational force. $\left(10^{36} F_g = F_E\right)$

Forces between multiple charges :

Force on a charged particle due to a number of point charges is the resultant of forces due to individual point charges $\vec{F} = \vec{F_1} + \vec{F}_2 + \vec{F}_3 + ...$

Test charge:

That small positive charge, which does not influence the other charges and by the help of which we determine the effect of other charges, is defined as test charge.

Linear charge density (λ) is defined as the charge per unit length.

$$\lambda = \frac{dq}{dl}$$

where dq is the charge on an infinitesimal length dl.

Units of λ are Coulomb / meter (C/m)

Examples:-Charged straight wire, circular charged ring

Surface charge density (σ) is defined as the charge per unit area.

$$\sigma = \frac{dq}{ds}$$

where dq is the charge on an infinitesimal surface area ds. Units of σ are $coulomb / meter^2$ (C / m^2).

Examples:-Plane sheet of charge, conducting sphere.

Volume charge density (ρ) is defined as charge per unit volume.

$$\rho = \frac{dq}{dv}$$

where dq is the charge on an infinitesimal volume element dv. Units of (ρ) are $coulomb / meter^3$ (C/ m^3)

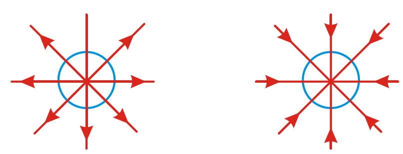
Examples:- Charge on a dielectric sphere etc.,

- Charge given to a conductor always resides on its outer surface.
- If surface is uniform then the charge distributes uniformly on the surface.
- In conductors having nonspherical surfaces, the surface charge density (σ) will be larger when the radius of curvature is small
- The working of lightening conductor is based on leakage of charge through sharp point due to high surface charge density.

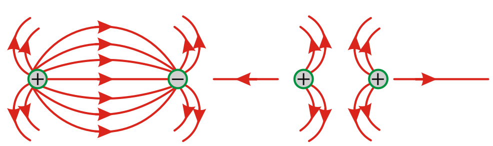
Lines of Force:

Line of force is an imaginary path along which a unit +ve test charge would tend to move in an electric field.

- Lines of force start from +ve charge and end at –ve charge.
- Lines of force in the case of iSol...ated +ve charge are radially outwards and in the case of iSol...ated –ve charge are radially inwards.
- The tangent at any point to the curve gives the direction of electric field at that point.
- Lines of force do not intersect.
- Lines of force tend to contract longitudinally and expand laterally.



(A) Radially outward (B) Radially inward



(C) (D)

Electric Field:

The space around electric charge upto which its influence is felt is known as electric field.

Electric field is a conservative field.

Intensity of Electric Field:

The intensity of electric field or electric field strength E at a point in space is defined as the force experienced by unit positive test charge placed at that point".

The intensity of electric field is also ofted called as electric field strength.

Consider an electric field in a given region. Bring a charge q_0 to a given point in that field without disturbing any other charge that has produced the field.

Let $\vec{F}$ be the electric force experienced by q_0 and it is found to be proportional to q_0

$\vec{F}$ $\vec{F} \propto q_0 \Rightarrow \vec{F} = \vec{E}q_0$. Here $\vec{E}$ is proportionality constant called electric field strength

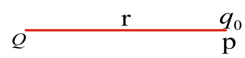
$$\vec{E} = \frac{\vec{F}}{q_0}$$

Electric field strength is a vector quantity. Its direction is the direction along which a free positive charge experiences the force in the electric field.

The S.I unit of elctric field strength is newton per coulomb (NC^{-1}). It can also be expressed in volt per metre (Vm^{-1}).

field internsity due to an iSol..ated point charge :

Consider a point charge 'Q' placed at point A as shown. Let us find the electric field $\vec{E}$ at a point P at a distance 'r' from charge Q. Imagine a positive test charge q_0 P. The charge Q produces a field $\vec{E}$ at P.



The force applied by Q and q_0 is given by

$$F = \frac{1}{4\pi \in_0} \frac{Qq_0}{r^2}$$. This acts along Ap.

According to definition

$$\vec{E} = \frac{F}{q_0} \Rightarrow \vec{E} = \frac{1}{4\pi \in_0} \frac{Q}{r^2} \hat{r}$$

If 'q' is positive, E is along $\overline{AP}$ and if 'q' is negative E will be along $\overline{PA}$.

If the charge 'q' is in a medium of p is in medium of permititivity ε, and dielectric constnat K, $\left(K = \frac{\varepsilon}{\varepsilon_0}\right)$ the intensity of electic field in a medium (E_{med}) is given by

$$E_{med} = \frac{1}{4\pi\varepsilon} \frac{Q}{r^2} \qquad \therefore E_{mec} = \frac{E_{free\,space}}{K}$$

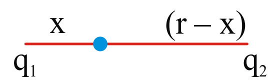
NULL POINT OR NEUTRAL POINT

In the case of a system of charges if the net electric field is zero at a point, it is knwon as null point.

Application :

Two point (like) charge q_1 and q_2 are separated by a distance 'r' and fixed, We can locate the point on the line joining those charges where resultant or net field is zero.

Case 1: If the charges are like, the neutral point will be between the charges.



Let P be the null point where $\vec{E}_{net} = 0$

$\Rightarrow \vec{E}_1 + \vec{E}_2 = 0$ (due to those charges)

or $\vec{E}_1 = -\vec{E}_2$ and $E_1 = E_2$

$$\Rightarrow \frac{1}{4\pi \in_0} \frac{q_1}{x^2} = \frac{1}{4\pi \in_0} \frac{q_2}{(r-x)^2}$$

or $\frac{q_1}{x^2} = \frac{q_2}{(r-x)^2}$

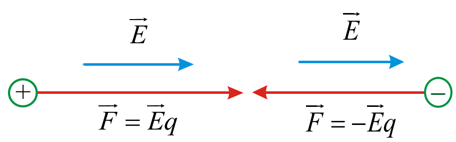
on Sol...ving we get $x = \frac{r}{\sqrt{\frac{q_2}{q_1}}+1}$

Case 2 : If the charges are unlike, the neutral point will be outside the charge on the lime joining them.

In this case $\frac{q_1}{x^2} = \frac{q_2}{(r+x)^2}$

On Sol...ving we get $x = \frac{r}{\sqrt{\frac{q_2}{q_1}}-1}$

- If instead of a single charge, field is produced by no.of charges, by the principle of super position resultant electric field intensity $\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + ...$
- If q_0 is positive charge then the force acting on it is in the direction of the field.
- If q_0 is negative then the direction of this force is opposite of the field direction.



Motion of a charged particle in a uniform electric field :

a) A charged body of mass 'm' and charge 'q' is initially at rest in a uniform electric field of intensity E. The force acting on it, $F = Eq$.

↳ Here the direction of F is in the direction of field if 'q' is $+ve$ and opposite to the field if 'q' is $-ve$.

↳ The body travels in a straight line path with uniform acceleration, $a = \frac{F}{m} = \frac{Eq}{m}$, initial velocity, $u = 0$.

At an instant of time t.

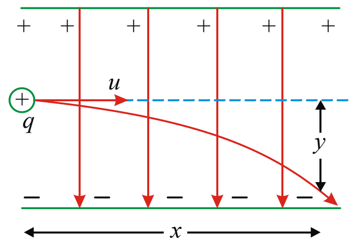
Its final velocity, $v = u + at = \left(\frac{Eq}{m}\right)t$

Displacement $s = ut + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{Eq}{m}\right)t^2$

Momentum, $P = mv = (Eq)t$

Kinetic energy, $$K.E = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{E^2q^2}{m}\right)t^2$$

- When a charged particle enters perpendicularly into a uniform electric field of intensity E with a velocity 'v' then it describes parabolic path as shown in figure.



- Along the horizontal direction, there is no acceleration and hence $x = ut$.
  Along the vertical direction, acceleration

  $a = \frac{F}{m} = \frac{Eq}{m}$ (here gravitational force is not considered)

  Hence vertical displacement, $y = \frac{1}{2}\left(\frac{Eq}{m}\right)t^2$

  $$y = \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{x}{u}\right)^2 = \left(\frac{qE}{2mu^2}\right)x^2$$

- At any instant of time t, horizontal component of velocity, $v_x = u$
- vertical componet of velocity

  $$v_y = at = \left(\frac{Eq}{m}\right)t$$

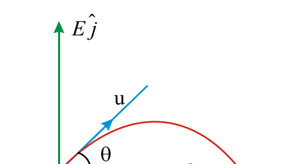
  $$\therefore v = |\bar{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + \frac{E^2q^2t^2}{m^2}}$$

- Two charges +Q each are separated by a distance 'd'. The intensity of electric field at the mid point of the line joining the charges is zero.

Oblique projection of charged particle in an uniform elctric field (Neglecting gravitational force) :

Consider a uniform electric field E in space along Y-axis. A negative charged particle of mass 'm' and charge 'q' be projected in the XY plane from a point 'O' with a velocity u making an angle θ with the X-axis. (Neglecting gracitational force).

Initial velocity of the particle is



$$\vec{u} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$$

Force acting on the particle is

$\vec{F} = q\vec{E}$ (along-ve Y axis)

$$\vec{a} = -\frac{qE}{m}\hat{j}$$

Velocity of the particle after time 't' is

$$\vec{v} = \vec{u} + \vec{a}t;\ \vec{v} = u\cos\theta\hat{i} + (u\sin\theta - at)\hat{j}$$

If the point of projection is taken as origin, its position vector after time 't' is

$\vec{r} = x\hat{i} + y\hat{j}$ where x=(ucos θ) t

$$y = (u\sin\theta)t - \frac{1}{2}at^2$$

If the charged particle is projected along the x-axis, then $\theta = 0^0$

$$\Rightarrow \vec{v} = u\hat{i} - \frac{Eq}{m}t\hat{j}$$

Here $x = ut$ and $y = \frac{1}{2}\frac{Eq}{m}t^2$

Direction of motion of particle after time 't' makes an angle α with x-axis, where $\tan\alpha = \frac{Eqt}{mu}$

- A charged particle of charge $\pm Q$ is projected with an initial velocity u in a vertically upward electric field making an angle θ to the horizontal. Then
  If gravitational force is considered

  Net force $m\vec{g} \mp \vec{F} = mg \mp Eq$

  Net acceleration $= g \mp \frac{Eq}{m}$

  The negative sign is used when electric field is in upward direction where as positive sign is used when electric field is in downward direction for positively charged projected particle.

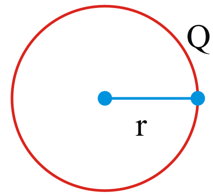
  a. Time of flight $= \dfrac{2u\sin\theta}{g \mp \frac{EQ}{m}}$

  b. Maximum height $= \dfrac{u^2\sin^2\theta}{2\left(g \mp \frac{EQ}{m}\right)}$

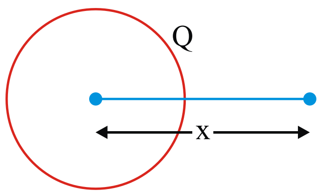
  c. Range $= \dfrac{u^2\sin 2\theta}{g \mp \frac{EQ}{m}}$

- Intensity of electric field inside a charged hollow conducting sphere is zero.
- A hollow sphere of radius r is given a charge Q.
  Intensity of electric field at any point inside it is zero.

Intensity of electric field on the surface of the sphere is $\frac{1}{4\pi \in_0} \frac{Q}{r^2}$

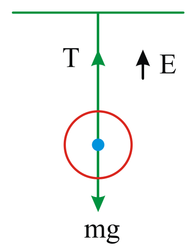


Intensity of electric field at any point outside the sphere is (at a distance 'x' from the centre) $\frac{1}{4\pi \in_0} \frac{Q}{x^2}$

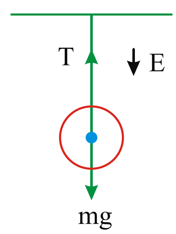


Time period of oscillation of a charged body

↳ The bob of a simple pendulum is given a +ve charge and it is made to oscillate in a vertically upward electric field, then the time period of oscillation is $2\pi\sqrt{\dfrac{l}{g-\dfrac{EQ}{m}}}$



↳ In the above case, if the bob is given a -ve charge then the time period is given by $2\pi\sqrt{\dfrac{l}{g+\dfrac{EQ}{m}}}$



↳ A sphere is given a charge of 'Q' and is suspended in a horizontal electric field. The angle made by the string with the vertical is, $\theta = \tan^{-1}\left(\frac{EQ}{mg}\right)$

↳ The tension in the string is $\sqrt{(EQ)^2 + (mg)^2}$

Hence effective acceleration

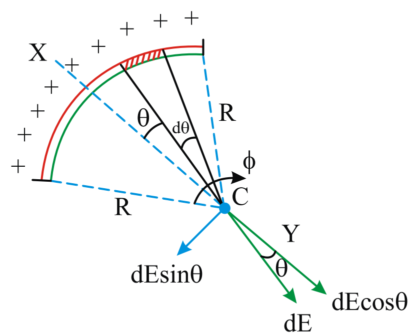
$$g_{eff} = \frac{F}{m} = \sqrt{g^2 + \left(\frac{Eq}{m}\right)^2}$$

$\therefore$ Time period of oscillation is given by

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{1}{\sqrt{g^2 + \left(\frac{Eq}{m}\right)^2}}}$$

Electric field strength due to a charged circular arc at its centre

Consider a circular arc of radius R which subtends an angle ϕ at its centre. Let us calculate the electric field strength at C.



Consider a polar segment on arc of angular width $d\theta$ at an angle θ from the angular bisector XY as shown. The length of elemental segment is Rdθ. The charge on this element dq is

$$dq = \frac{Q}{\phi} d\theta$$

Due to this dq, electric field at centre of arc C is given as

$$dE = \frac{dq}{4\pi\varepsilon_0 R^2}$$

The electric field component dE to this segment dE sin θ which is perpendicular to the angle bisector gets cancelled out on integration.

The net electric field at centre will be along angle bisector which can be calculated by integrating dEcos θ within limits from $-\phi/2$ to $\phi/2$

Hence net elctric field strength at centre C is $E_c = \int dE \cos\theta$

$$= \int_{-\phi/2}^{\phi/2} \frac{Q}{4\pi\varepsilon_0 \phi R^2} \cos\theta d\theta = \frac{Q}{4\pi\varepsilon_0 R^2 \phi} \int_{-\phi/2}^{\phi/2} \cos\theta d\theta$$

$$= \frac{Q}{4\pi\varepsilon_0 R^2 \phi} [\sin\theta]_{-\phi/2}^{\phi/2}$$

$$\frac{Q}{4\pi\varepsilon_0 R^2 \phi} [\sin\phi/2 + \sin\phi/2]$$

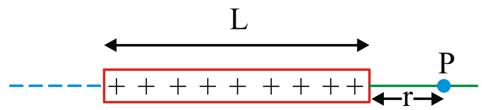
$$E_c = \frac{2Q\sin(\phi/2)}{4\pi\varepsilon_0 R^2 \phi}$$

for a semi circular ring $\phi = \pi$. So at centre

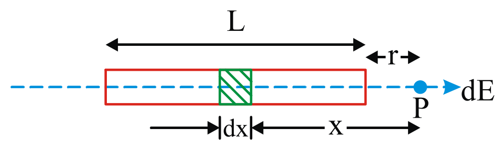
$$E_c = \frac{2Q\sin(\phi/2)}{4\pi\varepsilon_0 R^2 \phi} = \frac{2Q\sin(\pi/2)}{4\pi\varepsilon_0 R^2 \pi} = \frac{2Q}{4\pi^2 \varepsilon_0 R^2}$$

Electric field strength due to a uniformly charged rod

At an axial point :



Consider a rod of length L, uniformly charged with a charge Q. To calculate the electric field strength at a pont P situated at a distance 'r' from one end of the rod, consider an element of length dx on the rod as shown in the figure.



Charge on the elemental length dx is $dq = \frac{Q}{L}dx$

$$dE = \frac{dq}{4\pi\varepsilon_0 x^2} = \frac{Qdx}{4\pi\varepsilon_0 Lx^2}$$

The net electric field at point P can be given by integrating this expression over the length of the rod.

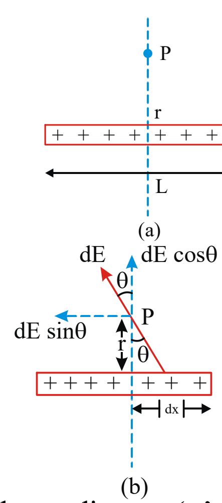
$$E_p = \int dE = \int_r^{r+L} \frac{Q}{Lx^2 4\pi\varepsilon_0} dx = \frac{Q}{4\pi\varepsilon_0 L}\int_r^{r+L} \frac{1}{x^2} dx$$

$$E_p = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{-1}{x}\right]_r^{r+L}$$

$$E_p = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{1}{r} - \frac{1}{r+L}\right] = \frac{Q}{4\pi \in_0 r(r+L)}$$

At an equatorial point :

To find the electric field due to a rod at a point P situated at a distance 'r' from its centre on its equatorial line



Consider an element of length dx at a distance 'x' from centre of rod as in figure (b). Charge on the element is $dq = \frac{Q}{L}dx$.

The strength of electric field at P due to this point charge dq is dE.

$$\Rightarrow dE = \frac{dq}{4\pi\varepsilon_0 \left(r^2 + x^2\right)}$$

The component dEsin θ will get cancelled and net electric field at point P will be due to integration of $dE\cos\theta$ only.

Net electric field strength at point P can be given as

$$E_p = \int dE\cos\theta = \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{Qdx}{L\left(r^2 + x^2\right)} \times \frac{r}{\sqrt{r^2 + x^2}} \times \frac{1}{4\pi\varepsilon_0}$$

$$E_p = \frac{Qr}{4\pi\varepsilon_0 L} \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{\left(r^2 + x^2\right)^{3/2}}$$

From the diagram $\tan\theta = \frac{x}{r}$

$x = r\tan\theta$; On differentiation; $dx = r\sec^2\theta d\theta$

$$E_p = \frac{Qr}{4\pi\varepsilon_0 L}\int \frac{r\sec^2\theta d\theta}{r^3\sec^3\theta} ; = \frac{Q}{4\pi\varepsilon_0 Lr}\int \frac{r\sec^2 d\theta}{r^3\sec^3\theta}$$

$$= \frac{Q}{4\pi\varepsilon_0 Lr}\int \cos\theta d\theta = \frac{Q}{4\pi\varepsilon_0 Lr}\left[\sin\theta\right]$$

Substituting $\theta = \tan^{-1}\frac{x}{r} = \sin^{-1}\frac{x}{\sqrt{x^2 + r^2}}$

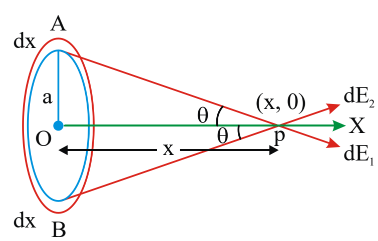
$$E_P = \frac{Q}{4\pi\varepsilon_0 L}\left[\frac{x}{\sqrt{x^2 + r^2}}\right]_{-\frac{L}{2}}^{\frac{L}{2}} ; \quad = \frac{Q}{4\pi\varepsilon_0 r}\left(\frac{1}{\sqrt{\frac{L^2}{4} + r^2}}\right)$$

$$E_p = \frac{Q}{4\pi \in_0 r}\left\{\frac{2}{\sqrt{L^2 + 4r^2}}\right\}$$

Electric field due to a uniformly chaged ring :

The intensity of electric field at a distance 'x' meters from the centre along the axis:

Consider a circular ring of radius 'a' having a charge 'q' uniformly distributed over it as shown in figure. Let 'O' be the cetnre of the ring .



Consider an element dx of the ring at point A. The charge on this element is given by

$dq = dx \times$ charge density $dq = dx \frac{q}{2\pi a} = \frac{qdx}{2\pi a}$

a) The intensity of electric field dE_1 at point P due to the element dx at A is given by

$$dE_1 = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}$$

The direction of dE_1 is as shown in figure. The component of intensity along x-axis will be

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta = dE_1\cos\theta$$

The component of intensity along y-axis will be

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\sin\theta = dE_1\sin\theta$$

Similarly if we consider an element dx of the ring opposite to A which lies at B, the component of intensity perpendicular to the axis will be equal and opposite perpendicular to the axis will be equal and opposite to the component of intensity perpendicular to the axis due to element at A. Hence they cancel each other. Due to symmetry of ring the component of intensity due to all elements of the ring perpendicular to the axis will cancel.

So the resultant intensity is only along the axis of the ring. The resultant intensity is given by

$$E = \int \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2}\cos\theta$$

$$E = \frac{1}{4\pi\varepsilon_0}\int \frac{qdx}{2\pi a r^2} \times \frac{x}{r} \text{ (where } \cos\theta = x/r)$$

$$E = \frac{1}{4\pi\varepsilon_0} \times \frac{qx}{(2\pi a)} \times \frac{1}{(a^2+x^2)^{\frac{3}{2}}}\int dx$$

$$\left[\therefore r^3 = (a^2+x^2)^{3/2}\right] \quad E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{2\pi a}\frac{1}{(a^2+x^2)^{3/2}} \times 2\pi a$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(a^2+x^2)^{3/2}}$$

At its centre x = 0

$\therefore$ Electric field at centre is zero.

By symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the element exactly opposite to it. As in the figure the electric field at centre due to segment A is cancelled by that due to segment B. Thus net electric field strength at the centre of a uniformly charged ring is $E_{centre} = 0$.

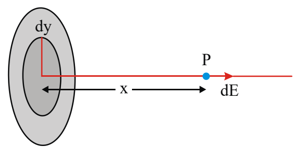
Electric field strength due to a uniformly surface charged disc

Consider a disc of radius R, charged on its surface with a charge density σ.

Let us find electric field strength due to this disc at a distance 'x' from the centre of disc on its axis at point P as shown in figure.

Consider an elemental ring of radius 'y' and width dy in the disc as shown in figure. The charge on this elemental ring dq can be given as $dq = \sigma 2\pi y dy$

{Area of elemental ring ds= $dy = 2\pi y dy$ }



Electirc field strength due to a ring of radius Y, charge Q at a distance x from its centre on its axis can be given as

$$E = \frac{Qx}{4\pi\varepsilon_0 \left(x^2 + y^2\right)^{3/2}}$$

Due to the lemental ring electric field strength dE at point P can be given as

$$dE = \frac{xdq}{4\pi\varepsilon_0 \left(x^2 + y^2\right)^{3/2}} = \frac{\sigma 2 y \pi dyx}{4\pi\varepsilon_0 \left(x^2 + y^2\right)^{3/2}}$$

Net electric field at point P due to whole disc is given by integrating above expression within the limits from 0 to R

$$E = \int dE = \int_0^R \frac{\sigma 2\pi xydy}{4\pi\varepsilon_0 \left(x^2 + y^2\right)^{3/2}}$$

$$= \frac{\sigma\pi x}{4\pi\varepsilon_0} \int_0^R \frac{2ydy}{\left(x^2 + y^2\right)^{3/2}} = \frac{2\sigma\pi x}{4\pi\varepsilon_0} \left[\frac{-1}{\sqrt{x^2 + y^2}}\right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

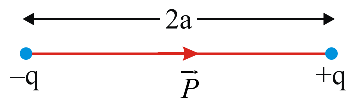
Electric field strength due to a uniformly charged disc at a distance x from its surface is given as

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

If we put x = 0 we get $E = \dfrac{\sigma}{2\varepsilon_0}$

Electric dipole:

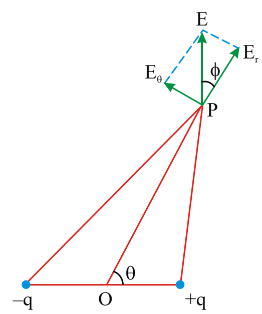
A system of two equal and opposite point charges fixed at a small distance constitutes an electric dipole. Electric dipole is analogous to bar magnet or magnetic dipole in magnetism. Every dipole has a characteristic property called dipole moment, which is similar to magnetic moment of a bar magnet. If 2a is the distance between the charges +q and –q, then electric dipole moment is p = q.2a.



Dipole moment is a vector quantity and its direction is from negative charge to positive charge as shown.

Electric field at any point due to a dipole :

We know that the electric field is the -ve gradiant of potential. In polar form if V is the potential at (r,θ) the electric field will have two components radial and transverse components which are represented by E_r & E_θ respectively.



Then $E_r = -\left(\frac{\partial V}{\partial r}\right) = -\frac{p\cos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)$

$$E_r = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3} \quad \left[\begin{array}{l} E_r = -\frac{\partial V}{\partial r} \\ E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) \end{array}\right]$$

The tranverse component of electric field

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} \quad = -\frac{1}{r}\left(-\frac{p\sin\theta}{4\pi\varepsilon_0 r^2}\right)$$

$$E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

$$E = \sqrt{E_\theta^2 + E_R^2}$$

$$E = \sqrt{\frac{p^2\sin^2\theta}{\left(4\pi\varepsilon_0 r^3\right)^2} + \frac{4p^2\cos^2\theta}{\left(4\pi\varepsilon_0 r^3\right)^2}}$$

$$E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$\Rightarrow E = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{\left[1+3\cos^2\theta\right]}$$

Field at a point on the axial line : ($\theta = 0^0$)

$$E_{axial} = \frac{2p}{4\pi\varepsilon_0 r^3}$$

Field at a point on the equitorial line ($\theta = 90^0$)

$$E_{equitorial} = \frac{p}{4\pi\varepsilon_0 r^3}$$

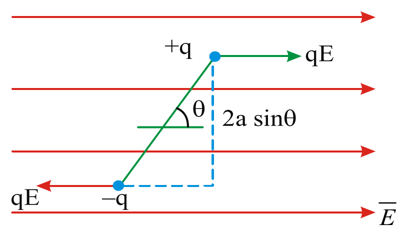
The direction of E at any point is given by

$$\tan\phi = \frac{E_\theta}{E_r} = \frac{\dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}}{\dfrac{2p\cos\theta}{4\pi\varepsilon_0 r^3}} \Rightarrow \tan\phi = \frac{1}{2}\tan\theta$$

$$\phi = \tan^{-1}\left[1/2\tan\theta\right]$$

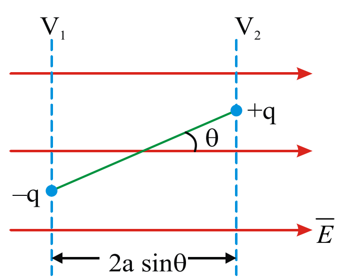
Note : Electric dipole placed in an uniform electric field experiences torque is given by

$\tau = pE\sin\theta$ in vector form $\overline{\tau} = \overline{p}\times\overline{E}$



The torque on the dipole tends to align the dipole along the direction of electric field.
The net force experienced by it is zero.

Note : The potential energy of dipole in an electric field is



$U = -pE\cos\theta$.

In vector form $U = -\vec{p}.\vec{E}$

if $\theta = 0^o$; $\tau = 0$ and $U = -pE$

if $\theta = 90^o$; $\tau = pE$ and $U = 0$

if $\theta = 180^o$; $\tau = 0$ and $U = pE$

So, if $\vec{p}$ is parallel to $\vec{E}$ then, potential energy is minimum and torque on the dipole is zero, and the dipole will in stable equilibrium.

If $\vec{p}$ is anti parallel to $\vec{E}$ then, potential energy is maximum and again torque is zero, but it is in unstable equilibrium

Note : Work done in rotating a dipole in electric field from an initial angle θ_1 with field to final angle θ_2 with field is $W = pE(\cos\theta_1 - \cos\theta_2)$

Note : Force on dipole in non-uniform electric field:

The force on the dipole due to electric field is given by $F = -\Delta U$ (Force = negative potential energy gradient). If the electric field is along $\vec{r}$, we can write

$$\vec{F} = -\frac{d}{dr}(\vec{p}.\vec{E})$$

If $\vec{p}$ and $\vec{E}$ are along the same direction we can write $\vec{F} = \frac{-d}{dr}(pE\cos\theta)$ or $F = -p\left(\frac{dE}{dr}\right)$.

Oscillatory Motion of Dipole in an Electric Field

When dipole is displaced from its position of equilibrium. The dipole will then experience a torque given by

$\tau = -pE\sin\theta$

For small value of θ, $\tau = -pE\theta$ ----------(1)

Where negative sign shows that torque is acting against increasing value of θ

Also, $\tau = I\alpha$,

Where, I = moment of inertia and

α = angular acceleration.

$$= \frac{d^2\theta}{dt^2\left(\omega = \frac{d\theta}{dt}\right)} \quad \tau = I\frac{d^2\theta}{dt^2} \text{ ---------(ii)}$$

Hence, from eqs (i) and (ii), we have

$$I\frac{d^2\theta}{dt^2} = -pE\theta \text{ or } \frac{d^2\theta}{dt^2} = \frac{-pE}{I}\theta \text{ ----(iii); } \frac{d^2\theta}{dt^2} \propto -\theta$$

This equation represents simple harmonic motion (SHM). when dipole is displaced from its mean position by small angle, then it will have SHM.

Eq (iii) can be written as $\frac{d^2\theta}{dt^2} + \frac{pE}{I}\theta = 0$

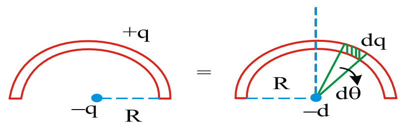
On comparing above equation with standard equation of SHM.

$\frac{d^2\theta}{dt^2} + \omega^2 y = 0$, we have ; $\omega^2 = \frac{pE}{I} \Rightarrow \omega = \sqrt{\frac{pE}{I}}$

$T = 2\pi\sqrt{\frac{I}{pE}}$, where T is the time period of oscillations.

Distributed dipole:

Consider a half ring with a charge +q uniformly distributed and another equal negative charge –q placed at its centre. Here –q is point charge while +q is distributed on the ring. Such a system is called distributed dipole.



The net dipole moment is $p_{net} = \frac{2qR}{\pi}$

If $\theta = \phi$ $\quad p_{net} = 2\int_0^{\phi/2} dp\cos\theta \; ; \; = \frac{2qR}{\pi}\sin\phi/2$

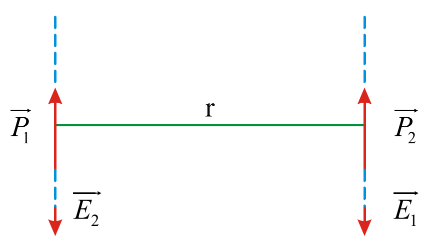
If the arrangement is a complete circle,

$\frac{\phi}{2} = \pi \;\Rightarrow p_{net} = 0$.

Force between two short dipoles

Consider two short dipoles seperated by a distance r. There are two possibilities.

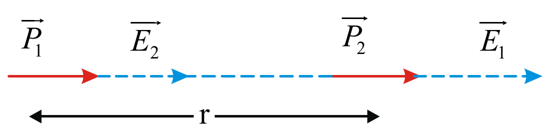
a) If the dipoles are parallel to each other.



$$F = \frac{1}{4\pi\epsilon_0}\frac{3p_1p_2}{r^4}$$

As the force is positive, it is repulsive. Similarly if $\overrightarrow{p_1} \parallel -\overrightarrow{p_2}$ the force is attractive.

b) If the dipoles are on the same axis

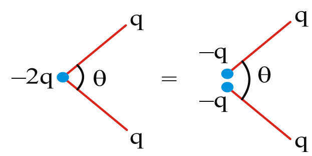


$$F = -\frac{1}{4\pi\epsilon_0}\frac{6p_1p_2}{r^4}$$

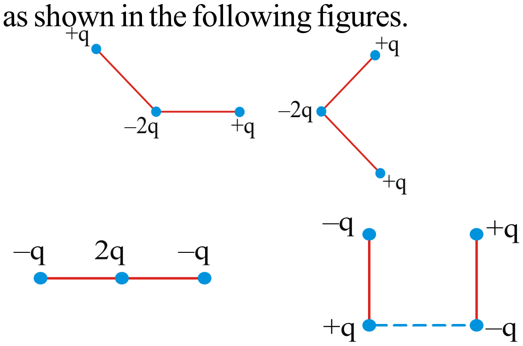
As the force is negative, it is attractive.

Quadrapole:

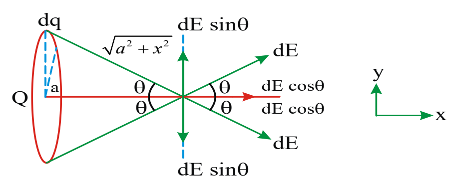
We have discussed about elecric dipole with two equal and unlike point charges separated by a small distance. But in some cases the two charges are not concentrated at its ends. (Like in water molecule) consider a situation as shown in the figure. Here three charges –2q, q and q are arranged as shown. It can be visualised as the combination of two dipoles each of dipole moment p = qd at an angle θ between them. The arrangment of two electric dipoles are called quadrapole. As dipole moment is a vector the resultant dipole moment of the system is $p^{|} = 2p\cos\theta/2$.



Few other quadrapoles are also as shown in the following figures.



Electric filed at the axis of a circular uniformly charged ring



Intensity of electric field at a point P that lies on the axis of the ring at a distance x from its centre is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{\left(x^2+R^2\right)^{3/2}}$$

where $\left\{\cos\theta = \frac{x}{\sqrt{a^2+x^2}}\right\}$

Where R is the radius of the ring. From the above expression E = 0 at the centre of the ring.

E will be maximum when $\frac{dE}{dx} = 0$.

Differentating E w.r.t x and putting it equal to zero we get $x = \frac{R}{\sqrt{2}}$ and $E_{\max} = \frac{2}{3\sqrt{3}}\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\right)$

Electric field due to a Charged Spherical Conductor (Spherical Shell)

'q' amount of charge be uniformly distributed over a spherical shell of radius 'R'

$\sigma =$ Surface charge density, $\sigma = \frac{q}{4\pi R^2}$

When point 'P' lies outside the shell :

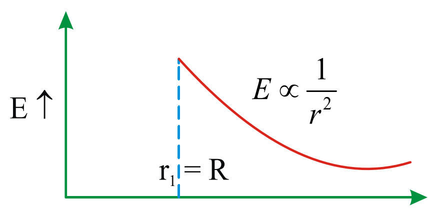
$$E = \frac{1}{4\pi \epsilon_0} \times \frac{q}{r^2}$$

↳ This is the same expression as obtained for electric field at a point due to a point charge. Hence a charged spherical shell behave as a point charge concentrated at the centre of it.

$$E = \frac{1}{4\pi \epsilon_0} \frac{\sigma.4\pi R^2}{r^2} \quad \because \sigma = \frac{q}{4\pi r^2} \; ; \; E = \frac{\sigma.R^2}{\epsilon_0 \, r^2}$$

When point 'P' lies on the shell : $E = \frac{\sigma}{\epsilon_0}$

When Point 'P' lies inside the shell: E = 0



→ Distance from the centre

Note : The field inside the cavity is always zero this is known as **elctro static shielding**

Electric filed due to a Uniformly charged non – conducting sphere

Electric field intensity due to a uniformly charged non-conducting sphere of charge Q, of radius R at a distance r from the centre of the sphere

q is the amount of charge be uniformly distributed over a Sol...id sphere of radius R.

ρ = Volume charge density $\rho = \frac{q}{\frac{4}{3}\pi R^3}$

When point 'P' lies inside sphere :

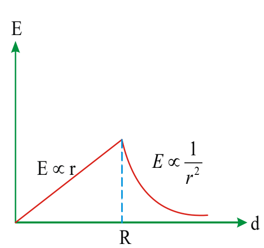
$$E = \frac{1}{4\pi\varepsilon_0} \frac{Qr}{R^3} \quad \textbf{for} \quad r < R \quad E = \frac{\rho.r}{3 \epsilon_0}$$

When point 'P' lies on the sphere:

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \; ; \qquad E = \frac{\rho.\, R}{3 \epsilon_0}$$

When point 'P' lies outside the sphere:

$$E = \frac{1}{4\pi \epsilon_0} \frac{q}{r^2} \; ; \; E = \frac{\rho.\, R^3}{3 \epsilon_0 \, r^2}$$



Electric Field due to a charged Disc:

Electric field due to a uniformly charged disc with surface charge density σ of radius at a distance x from the centre of the disc is

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

If Q is the total charge on the disc, then

$$E = \frac{2Q}{4\pi\varepsilon_0 R^2}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

Electric Potential:

Work done to bring a unit positive charge from infinite distance to a point in the electric field is called electric potential at that point .

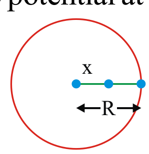
it is given by $V = \frac{W}{q}$

- It represents the electrical condition or state of the body and it is similar to temperature.
- +vely charged body is considered to be at higher potential and -vely charged body is considered to be at lower potential.
- Electric potential at a point is a relative value but not an absol.ute value.
- Potential at a point due to a point charge $= \frac{1}{4\pi \in_0} \frac{Q}{r}$
- Potential due to a group of charges is the algebraic sum of their individual potentials.

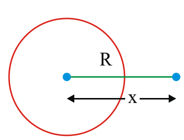
  i.e. $V = V_1 + V_2 + V_3 + \ldots\ldots$
- Two charges +Q and -Q are separated by a distance d, the potential on the perpendicular bisector of the line joining the charges is zero.
- When a charged particle is accelerated from rest through a p.d. ' V ' , work done,

  $$W = Vq = \frac{1}{2}mv^2 \ (or)\ v = \sqrt{\frac{2Vq}{m}}$$
- The work done in moving a charge of q coulomb between two points separated by p.d. $V_2 - V_1$ is $q(V_2 - V_1)$.
- The work done in moving a charge from one point to another point on an equipotential surface is zero.
- A hollow sphere of radius R is given a charge Q the potential at a distance x from the centre is

  $$\frac{1}{4\pi \in_0}.\frac{Q}{R}\ (x \le R)$$



- The potential at a distance when x>R is $\frac{1}{4\pi \in_0}.\frac{Q}{x}$.



- A sphere is charged to a potential. The potential at any point inside the sphere is same as that of the surface.
- Inside a hollow conducting spherical shell, E=0, $V \neq 0$.

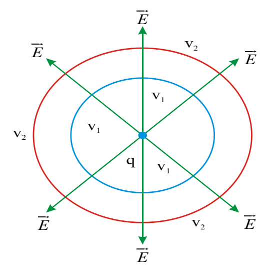
- Relation among E, V and d in a uniform electric field is $E = \frac{V}{d}$ (or) $E = -\frac{dV}{dx}$
- Electric field is always in the direction of decreasing potential .

  The component of electric field in any direction is equal to the negative of potential gradient in that direction.

  $$\vec{E} = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- An equipotential surface has a constant value of potential at all points on the surface .

  For single charge q



- Electric field at every point is normal to the equipotential surface passing through that point
- No work is required to move a test charge on unequipotential surface.

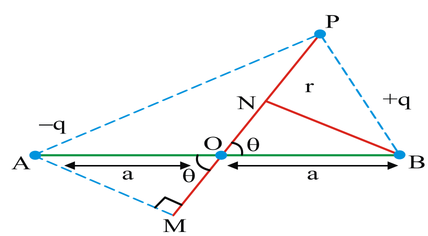
Zero Potential Point

Two unlike charges Q_1 and $-Q_2$ are seperated by a distance 'd'. The net potential is zero at two points on the line joining them, one (x) in between them and the other (y) outside them

$$\frac{Q_1}{x} = \frac{Q_2}{d-x} \text{ and } \frac{Q_1}{y} = \frac{Q_2}{d+y}$$

Potential due to a dipole:

An electric dipole consists of two equal and opposite charges seperated by a very small distance. If 'q' is the charge and 2a the length of the dipole then electric dipole moment will be given by p = (2a)q.



Let AB be a dipole whose centre is at 'O' and 'P' be the point where the potential due to dipole is to be determined. Let r, θ be the position co-ordinates of 'P' w.r.t the dipole as shown in figure. Let BN & AM be the perpendiculars drawn on to OP and the line produced along PO. From geometry $ON = a\cos\theta = OM$.
Hence the distance ,BP from +q charge is $r - a\cos\theta$
[because PB = PN as AB is very small in comparsion with r].
For similar reason

$AP = r + a\cos\theta\,[\because AP = PM]$.

Hence potential at P due to charge +q situated at B is $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r - a\cos\theta)}$.

Similarly potential at P due to charge -q at A is

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{-q}{(r + a\cos\theta)}.$$

Hence the total potential at P is

$V = V_1 + V_2$

$$V = \frac{q}{4\pi\varepsilon_0(r - a\cos\theta)} - \frac{q}{4\pi\varepsilon_0(r + a\cos\theta)}$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r - a\cos\theta} - \frac{1}{r + a\cos\theta}\right]$$

$$V = \frac{q(2a\cos\theta)}{4\pi\varepsilon_0(r^2 - a^2\cos^2\theta)}$$

But r >> a $\therefore r^2 - a^2\cos^2\theta \approx r^2$ $\qquad \therefore V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$.

Hence potential varies inversely as the square of the distance from the dipole.

Special Casses

On the axial line : For a point on the axial line $\theta = 0^0$ $\therefore V_{axial} = p/4\pi\varepsilon_0 r^2$ volts for a dipole.

Point on the equitorial line : For a point on the equitorial line $\theta = 90^0$. $\therefore V_{equitorial} = 0$ Volts.

Equitorial line is a line where the potential is zero at any point.

Equipotential surfaces

: Equipotential surface in an electric field is a surface on which the potential is same at every point. In other words, the locus of all points which have the same electric potential is called equipotential surface. An equipotential surface may be the surface of a material body or a surface drawn in an electric field. The importantpropertiesof equipotential surfaces are as given below.

a) As the potential difference between any two points on the equipotential surface is zero, no work is done in taking a charge from one point to another.

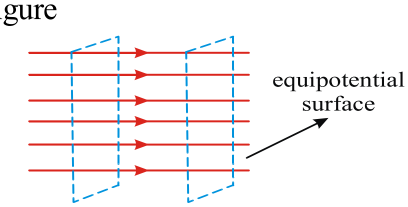
b) The electric field is always perpendicular to an equipotential surface. In other words electric field or lines of force are perpendicular to the equipotential surface.

c) No two equipotential surfaces intersect. If they intersect like that, at the point of intersection field will have two different directions or at the same point there will be two different potentials which is impossible.

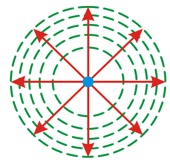
d) The spacing between equipotential surfaces enables to identify regions of strong and weak fields $E = -\frac{dV}{dr}$.

So $E \propto \frac{1}{dr}$ (if dV is constant).

e) At any point on the equipotential surface component of electric field parallel to the surface is zero.
In uniform field , the lines of force are straight and parallel and equipotential surfaces are planes perpendicular to the lines of force as shown in figure



The equipotential surfaces are a family of concentric spheres for a uniformly charged sphere or for a point charge as shown in figure



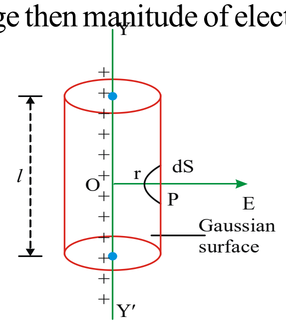
Equipotential surfaces in electrostatics are similar to wave fronts in optics. The wave fronts in optics are the locus of all points which are in the same phase. Light rays are normal to the wave fronts. On the other hand the equipotential surfaces are perpendicular to the lines of force.

Note :

1) In case of non-uniform electric field, the field lines are not straight, and in that case equipotential surfaces are curved but still perpendicular to the field.
2) Electric potential and potential energy are always defined relative to a reference. In general we take zero reference at infinity. The potential at a point P in an electric field is V if potential at infinity is taken as zero. If potential at infinity is V_0, the potential at P is $(V-V_0)$.
3) The potential difference is a property of two points and not of the charge q_0 being moved.

Electric potential potential due to a linear charge distribution

Consider a thin infinitely long line charge having a uniform linear charge density λ placed along YY^1. Let P is a point at distance 'r' from the line charge then manitude of electri field at point P is given by $E = \frac{\lambda}{2\pi \in_0 r}$



We know that $V(r) = -\int \overline{E}.d\overline{r}$

Here $E = \frac{\lambda}{2\pi \in_0 r}$ and $\overline{E}.d\overline{r} = Edr$

So $V(r) = -\int Edr = -\int \frac{\lambda}{2\pi \in_0 r} dr$

$$\therefore V(r) = \left(\frac{-\lambda}{2\pi \in_0} \log_e r \right) + C$$

Where C is constant of integration and V(r) gives electric potential at a distance 'r' from the linear charge distribution

Electric potential due to infinite plane sheet of charge (Non conducting)

Consider an infinite thin plane sheet of positivive charge having a uniform surface charge density σ on both sides of the sheet. by symmetry , it follws that the electric filed is perpendicular to the plane sheet of charge and directed in out ward direction.

The electric field intensity is $E = \dfrac{\sigma}{2\varepsilon_0}$

Electrostatic potential due to an infinite plane sheet of charge at a perpendicular distance r from the sheet given by $V(r) = -\int \overline{E}.d\overline{r} = -\int Edr$

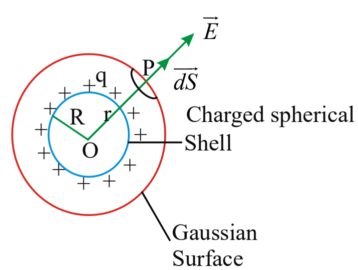
$$V(r) = -\int \frac{\sigma}{2 \in_0} dr = \left(\frac{-\sigma}{2 \in_0} r \right) + C$$

where C is constant ofintegration similarly the electric pontential due to an infinite plane conducting plate at a perpendicular distance r from the plate is given by $V(r) = -\int \overline{E}.d\overline{r} = -\int Edr$

$$V(r) = -\int \frac{\sigma}{\in_0} dr = \left(\frac{-\sigma}{\in_0} r \right) + c$$

where C is constant of intergration

Electric potential due to a chared sperical shell (or conducting sphere):



Consider a thin spherical shell of radius R and having charge+q on the spherical shell.

Case (i): When point P lies outside the spherical shell. The electric field at the point is

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \text{ (for r > R)}$$

The potential $V(r) = -\int \overline{E}.d\overline{r} = -\int Edr$

$$= -\int \frac{1}{4\pi \in_0} \frac{q}{r^2} dr = \frac{1}{4\pi \in_0} \frac{q}{r} + C$$

Where C is constant of integration

If $r \to \infty$, $V(\infty) \to 0$ and $C = 0$

$$V(r) = \frac{1}{4\pi\varepsilon_0} \frac{q}{r} (r > R)$$

Case (ii) : When point P lies on the surface of spherical shell then r = R

electrostatice potential at P on the surface is

$$V = \frac{1}{4\pi \in_0} \frac{q}{R}$$

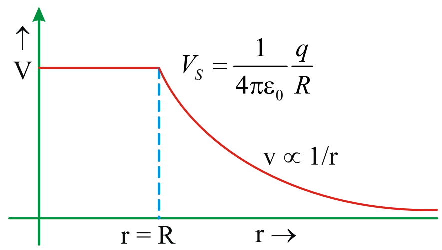
Case (iii) : For points inside the charged spherical shell (r < R), the electric field E = 0

So we can write $-\frac{dV}{dr} = 0$

$\Rightarrow V$ is constant and is equal to that on the surface

So, $V = \frac{1}{4\pi \in_0}\frac{q}{R}$ for $r \le R$

The varitaion of V with distance 'r' from centre is as shown in the graph.



Electric potential due to a uniformly charged Non-conducting solid sphere :

Consider a charged sphere of radius R with total charge q uniformly distributed on it.

Case (i) : For points Outside the sphere (r > R)

The electric field at any point is

$$E = \frac{1}{4\pi\varepsilon_0}, \frac{q}{r^2} \qquad \text{(for r > R)}$$

The potential at any point outside the shell is

$$V(r) = -\int \bar{E}.d\bar{r} = -\int Edr$$

$$= -\int \frac{1}{4\pi \in_0}\frac{q}{r^2}dr = \frac{1}{4\pi \in_0}\frac{q}{r} + C$$

Where C is constant of integration

If $r \to \infty, V(\infty) \to 0$ and C=0

$$V(r) = \frac{1}{4\pi \in_0}\frac{q}{r} \qquad \text{(r > R)}$$

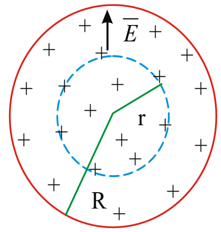
Case (ii) : When point P lies on the surface of spherical shell then r = R

The electrostatic potential at P on the surface is

$$V = \frac{1}{4\pi \in_0}\frac{q}{R}$$

Case (iii) : FOr points inside the sphere (r < R)

The electric field is $E = \frac{1}{4\pi \in_0}\frac{qr}{R^3}$



$$dV = \overline{E}.d\overline{r} = -Edr$$

$$\int_{v_s}^{v} dV = -\int_{R}^{r} E\,dr = -\int_{R}^{r} \frac{1}{4\pi \in_0} \frac{qr}{R^3} dr$$

$$V - V_s = -\frac{1}{4\pi \in_0} \frac{q}{R^3} \left(\frac{r^2}{2}\right)_R^r$$

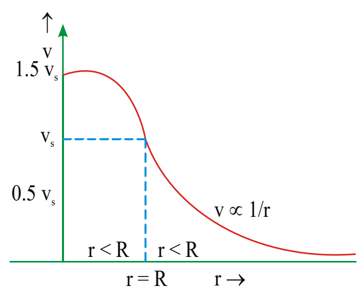
$$V - \frac{1}{4\pi \in_0} \frac{q}{R} = -\frac{1}{4\pi \in_0} \frac{q}{R^3} \left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$\Rightarrow V = \frac{1}{4\pi \in_0} \frac{q}{R} \times \left[\frac{3}{2} - \frac{r^2}{2R^2}\right]$$

At the centre r = 0 then

Potential at centre $V_C = \frac{1}{4\pi \in_0} \frac{3q}{2R} = \frac{3}{2} \frac{1}{4\pi \in_0} \frac{q}{R}$

The variation of V with distance 'r' from centre is as shown in the graph.



Potential of a charged ring:

A charge q is distributed over the circumference of ring (either uniformly or non-uniformly) , then

electric potential at the centre of the ring is $V = \frac{1}{4\pi\varepsilon_o} . \frac{q}{R}$.

At distance 'r' from the centre of ring on its axis would be $V = \frac{1}{4\pi\varepsilon_o} . \frac{q}{\sqrt{R^2 + r^2}}$

Electric potential of a uniformly charged disc

Consider a uniformly charged circular disc having surface charge density σ .

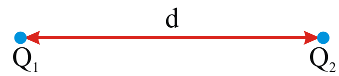
↳ Potential a at point on its axial line at distance x from the centre is $V = \frac{\sigma}{2\varepsilon_o}\left[\sqrt{R^2 + x^2} - x\right]$

- At the centre of disc $x = 0$ $\quad V = \dfrac{\sigma R}{2\varepsilon_o}$
- For $x >> R$, $V = \dfrac{q}{4\pi\varepsilon_o x}$
- Potential on the edge of the disc is $V = \dfrac{\sigma R}{\pi\varepsilon_o}$

Potential Energy of System of Charges

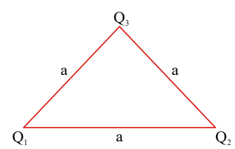
- Two charges Q_1 and Q_2 are separated by a distance 'd'. The P.E. of the system of charges is $U = \dfrac{1}{4\pi\epsilon_0}.\dfrac{Q_1Q_2}{d}$

  from U=W=Vq



- Three charges Q_1, Q_2, Q_3 are placed at the three vertices of an equilateral triangle of side 'a'. The P.E. of the system of charges is

$$U = \frac{1}{4\pi\epsilon_0}\left[\frac{Q_1Q_2}{a} + \frac{Q_2Q_3}{a} + \frac{Q_3Q_1}{a}\right] \text{ or } U = \frac{1}{4\pi\epsilon_0}\frac{\sum Q_1Q_2}{a}$$



- A charged particle of charge Q_2 is held at rest at a distance 'd' from a stationary charge Q_1. When the charge is released, the K.E. of the charge Q_2 at infinity is $\dfrac{1}{4\pi\epsilon_0}.\dfrac{Q_1Q_2}{d}$.
- If two like charges are brought closer, P.E of the system increases.
- If two unlike chargtes are brought closer, P.E of the system decreses.

  For an attractive system U is always NEGATIVE.

  For a repulsive system U is always POSITIVE.

  For a stable system U is MINIMUM.

  i.e. $F = -\dfrac{dU}{dx} = 0$ (for stable system)

POTENTIAL ENERGY OF A SYSTEM OF TWO CHARGES IN AN EXTERNAL FIELD: Consider two charges q_1 and q_2 located at two points A and B having position vectgors r_1 and r_2 respectively. Let V_1 ang V_2 be the potentials due to external sources at the two points respectively.

The work done in bringing the charge q_1 from infinity to the point A is $W_1 = q_1V_1$

In bringing charge q_2, the work to be done not only against the external field but also against the filed due to q_1.

The work done in bringing the charge q_2 from infinity to the point B is $W_2 = q_2V_2$.

The workdone on q_2 against the field due to q_1 is $W_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$ where r_{12} is the distance between q_1 and q_2.

The total work done in bringing the charge q_2 against the two fields from infinity to the point B is

$$W_2 = q_2 V_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

The total work done in assembling the configuration or the potential energy of the system is

$$W = q_1 V_1 + q_2 V_2 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

PROBLEMS

1. Can two similarly charged bodies attract each other?

SOLUTION :

Yes, when the charge on one body (q_1) is much greater than that on the other (q_2) and they are close enough to each other so that force of attraction between q_1 and induced charge on the other exceeds force of repulsion between q_1 and q_2

2. Two point sized identical spheres carrying charges q_1 and q_2 on them are seperated by a certain distance. The mutual force between them is F. These two are brought in contact and kept at the same separation. Now, the force between them is F^1. Then $\frac{F^1}{F} = \frac{(q_1+q_2)^2}{4q_1q_2}$.

SOLUTION :

When charges seperated by certain distance the force is given by

$$\text{Then } F = \frac{1}{4\pi \in_0}\frac{q_1q_2}{r^2} \text{------------(1)}$$

When charges brought in contanct and kept at the same distance the force is given by

$$F^1 = \frac{1}{4\pi \in_0}\frac{(q_1+q_2)^2}{4r^2} \text{----------(2)}$$

$$\text{from (1) and (2) ; } \therefore \frac{F^1}{F} = \frac{(q_1+q_2)^2}{4q_1q_2}$$

3. The force of attraction between two charges separated by certain distance in air is F_1. If the space between the charges is completely filled with dielectric of constant 4 the force becomes F_2. If half of the distance between the charges is filled with same dielectric the force between the charges is F_3. Then $F_1 : F_2 : F_3$ is

1) 16 : 9 : 4 2) 9 : 36 : 16 3) 4 : 1 : 2 4) 36 : 9 : 16

SOLUTION :

. $$F = \frac{1}{4\pi \in_0 \in_r}\frac{q_1q_2}{d^2}$$

4. A ball of mass m = 0.5 kg is suspended by a thread and a charge q = 0.1 μC is supplied. When a ball with diameter 5cm and a like charge of same magnitude is brought close to the first ball, but below it, the tension decreases to 1/3 of its initial value. The distance between centres of the balls is

1) $0.12\times10^{-2}m$ 2) $0.51\times10^{-4}m$

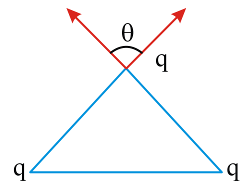
3) $0.2\times10^{-5}m$ 4) $0.52\times10^{-2}m$

SOLUTION :

$$T + \frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r^2} = mg$$

5. Consider three charges q_1,q_2 and q_3 each equal to q at the vertices of an equilateral triangle of side 'l' what is the force on any charge due to remaining charges.

Sol.. The forces acting on the charge 'q' are



$\bar{F}_1 = \frac{1}{4\pi \in_0} \frac{q^2}{l^2}$

$\bar{F}_2 = \frac{1}{4\pi \in_0} \frac{q^2}{l^2}$

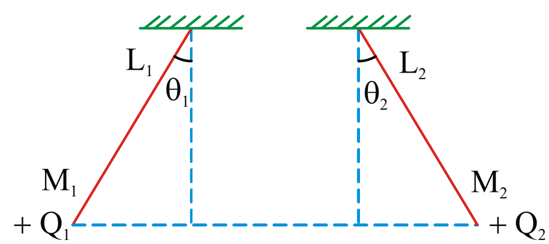
clearly $\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = |F|$

The resultant force is

$F^1 = \sqrt{F^2 + F^2 + 2FF\cos 60^0}$

$= \sqrt{3}F = \sqrt{3}\frac{1}{4\pi \in_0} \frac{q^2}{l^2}$

6. Two small spheres of masses, M_1 and M_2 are suspended by weightless insulating threads of lengths L_1 and L_2. the sphere carry charges Q_1 and Q_2 respectively. The spheres are suspended such that they are in level with another and the threads are inclined to the vertical at angles of θ_1 and θ_2 as shown below, which one of the following conditions is essential, if $\theta_1 = \theta_2$.



1) $M_1 \neq M_2$ but $Q_1 = Q_2$ 2) $M_1 = M_2$

3) $Q_1 = Q_2$ 4) $L_1 = L_2$

SOLUTION :

There are three forces acting on each sphere are

(i) tension (ii) weight(w) (iii) electrostatic force of repulsion for sphere

In equailibirum, from figure

$\tan\theta_1 = F_1 / M_1 g$

From sphere 2, in equilibirum from figure

$\tan\theta_2 = F_2 / M_2 g$

for $F_1 = F_2 C$

or $\theta_1 = \theta_2$ only for $\frac{F_1}{M_1 g} = \frac{F_2}{M_2 g}$

But, $F_1 = F_2$ and then $M_1 = M_2$

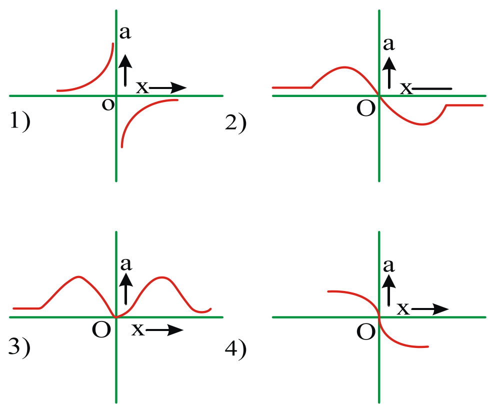
7. Five point charges each +q, are placed on five vertices of a regular hexagon of side L, The magnitude of the force on a point charge of value – q placed at the centre of the hexagon (in newton) is

1)Zero 2) $\dfrac{\sqrt{3}q^2}{4\pi\in_0 L^2}$ 3) $\dfrac{q^2}{4\pi\in_0 L^2}$ 4) $\dfrac{q^2}{4\sqrt{3}\,\pi\in_0 L^2}$

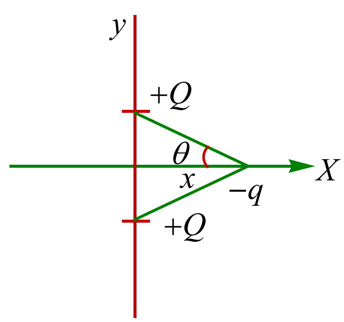
SOLUTION :

$$F=\frac{1}{4\pi\in_0}.\frac{q_1q_2}{d^2}$$

8. Two identical positive charges are fixed on the y-axis, at equal distance from the origin O, A partical with a negative charge starts on the negative x-axis at a large distance from O, moves along the x-axis passed through O and moves far away from O. Its acceleration a is taken as positive along its direction of motion. The particle's acceleration a is plotted against its x-co-ordinate. Which of the following best represents the plot?



SOLUTION :



At a distance x from 'O'. If particle exists, it's acceleration is

$$a=\frac{F}{m}=\frac{1}{4\pi\in_0}\frac{Q_q}{\left(a^2+x^2\right)}\frac{x}{3/2}$$

a is always directed along +ve x-axis.

9 A particle of mass 'm' carrying a charge $-q_1$ is moving around a fixed charge $+q_2$ along a circular path of radius 'r' find time period of revolution of charge q_1

SOLUTION :

Electrostatic force on -q_1 to +q_2 will provide the necessary centripetal force

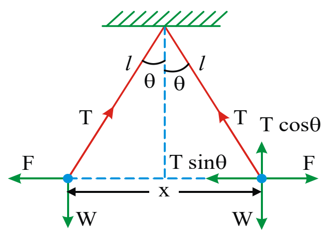
Hence $\frac{Kq_1q_2}{r^2} = \frac{mv^2}{r}$; $v = \sqrt{\frac{Kq_1q_2}{mr}}$

$$T = \frac{2\pi r}{v} = \sqrt{\frac{16\pi^3 \in_0 mr^3}{q_1q_2}}$$

10. Two identical small charged spheres each having a mass 'm' hang in equilibrium as shown in fig. The length of each string is 'l' and the angle made by any string with vertical is θ. Find the magnitude of the charge on each sphere.

SOLUTION :

The forces acting on the sphere are tension in the string T, force of gravity 'mg' and repulsice force F_e.



$T\cos\theta = mg$ ----(1)

$T\sin\theta = F_e = \frac{Kq^2}{r^2}$ ---(2)

From (1) and (2)

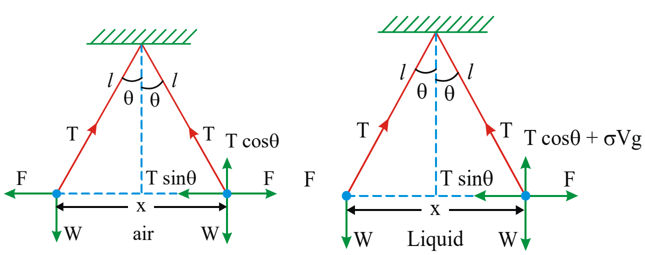
$F_e = mg\tan\theta$; $\frac{Kq^2}{r^2} = mg\tan\theta$

from fig $r = 2l\sin\theta$; $\frac{1}{4\pi \in_0}\frac{q^2}{(2l\sin\theta)^2} = mg\tan\theta$

$q = \sqrt{16\pi \in_0 l^2 mg\tan\theta\sin^2\theta}$

11. Two identical balls each having density ρ are suspended from a common point by two insulating strings of equal length Both the balls have equal mass and charge. In equilibrium, each string makes an angle θ with the vertical. Now both the ball are immersed in a liquid. As a result, the angle θ does not change. The density of the liquid is σ. Find the dielectric constant of the liquid.

SOLUTION :



Let v is the volume of each ball, then mass of each ball is $m = \rho v$; When balls are in air

$T\cos\theta = mg$; $T\sin\theta = F$

$F = mg\tan\theta = \rho vg\tan\theta$ ---------(1)

When balls are suspended in liquid. The coulumbic force is reduced to $F^1 = \frac{F}{K}$ and apparent weight = weight - upthrust ; $W^1 = \rho vg - \sigma vg$

According to the problem, angle θ is uncharged-Therefore

$F^1 = W^1\tan\theta = (\rho vg - \sigma vg)\tan\theta$ ------(2)

From (1) and (2) ; $\frac{F}{F^1} = K = \frac{\rho}{\rho - \sigma}$

12 Two small objects X and Y are permanently separated by a distance 1 cm. Object X has a charge of + 1.0 μC and object Y has a charge of - 1.0 μC. A certain number of electrons are removed from X and put onto Y to make the electrostatic force between the two objects an attractive force whose magnitude is 360 N. Number of electrons removed is

1) 8.4×10^{13} 2) 6.25×10^{12} 3) 4.2×10^{11} 4) 3.5×10^{10}

SOLUTION :

$q_1 = 1.0\mu c$

$q_2 = -1.0\mu c$

$q_1^1 = (+1.0\mu - ne)$

$F = \frac{1}{4\pi\epsilon_0}\cdot\frac{q_1^1 + q_2^1}{r^2} = 360N$

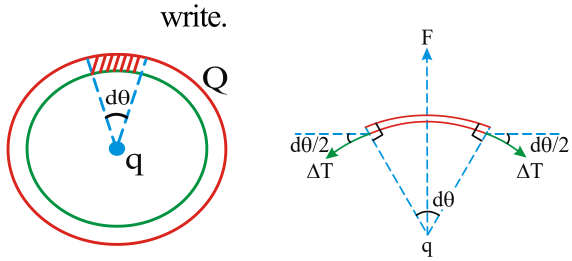
$e = 1.6\times10^{-19}c$

After calculation $n = 6.25\times10^{12}$

13. A ring of radius R is with a uniformly distributed charge Q on it. A charge q is now placed at the centre of the ring. Find the increment in tension in the ring

SOLUTION :

. Consider an element of the ring. Its enlarged view is as shown. For equilibrium of this segment, we can write.



$$F = 2\Delta T\sin\left(\frac{d\theta}{2}\right)$$

Here F is the repulsive force between q and elemental charge dQ

$$\left[\because dQ = \frac{Q}{2\pi R}(Rd\theta)\right]$$

The electric outward force on element is $F = \frac{1}{4\pi \in_0} \frac{qdQ}{R^2}$

From the above three equations, we can write

$$\frac{1}{4\pi \in_0} \frac{q}{R^2} \frac{QRd\theta}{2\pi R} = 2\Delta T\left(\frac{d\theta}{2}\right)$$

$$(\because \sin\alpha = \alpha \;\; for \;\; small \; angle)$$

14. A electric field of $1.5 \times 10^4 NC^{-1}$ exists between two parallel plates of length 2 cm. An electron enters the region between the plates at right angles to the field with a kinetic energy of E_k=2000eV The deflection that the electron experiences at the deflecting plates is

1) 0.34 mm 2) 0.57 mm 3) 7.5 mm 4) 0.75 mm

SOLUTION :

$$y = \frac{eE\alpha^2}{4k}(K = K.E)$$

15. Two equal negative charges –q each are fixed at points (0, –a) and (0,a) on y-axis. A positive charge Q is released from rest at the point (2a, 0) on the x-axis. The charge Q will

1) execute simple harmonic motion about the origin

2) move to the origin and remain at rest

3) move to infinity

4) execute oscillatory but not simple harmonic motion

SOLUTION :

$$a = \frac{F}{m} = \frac{1}{4\pi \in_0} \frac{Qq_x}{(a^2 + x^2)^{3/2}}$$

a not directly proportional to (-x)

$\therefore$ it executes oscillatory but not SHM.

16. **If the electric field between the plates of a cathode ray oscilloscope be $1.2 \times 10^4 N/C$, the deflection that an electron will experience if it enters at right angles to the field with kinetic energy 2000 eV is (The deflection assembly is 1.5cm long.)**

1) 0.34 cm 2) 3.4 cm

3) 0.034 mm 4) 0.34 mm

SOLUTION :

Deflection $y = \frac{eEx^2}{4(K)}$ where K is kinetic energy.

17. A thin fixed ring of radius 'a' has a positive charge 'q' uniformly distributed over it. A particle of mass 'm' having a negative charge 'Q' is placed on the axis at a distance of $x(x << a)$ from the centre of the ring. Show that the motion of the negatively charged particle is approximately simple harmonic. Calculate the time period of oscillation.

SOLUTION :

The force on the point charge Q due to the element dq of the ring is

$$dF = \frac{1}{4\pi \in_0} \frac{dqQ}{r^2} \text{ along AB}$$

For every element of the ring, there is symmetrically situated diametrically opposite element, the components of forces along the axis will add up while those perpendicular to it will cancel each other. Hence, net force on the charge $-Q$ is -ve sign shows that this force will be towards the centre of ring.

$$F = \int dF \cos\theta = \cos\theta \int dF$$

$$= \frac{x}{r}\int \frac{1}{4\pi \epsilon_0}\left[-\frac{Qdq}{r^2}\right]$$

so,

$$F = -\frac{1}{4\pi \epsilon_0 \frac{Qx}{r^3}}\int dq = -\frac{1}{4\pi \epsilon_0}\frac{Qqx}{\left(a^2+x^2\right)^{\frac{3}{2}}} \text{ ----(1)}$$

(as $r = \left(a^2+x^2\right)^{\frac{1}{2}}$ and $r = \left(a^2+x^2\right)^{\frac{1}{2}}$)

As the restoring force is not linear, the motion will be oscillatory. However, if $x << a$, then

$$F = -\frac{1}{4\pi \epsilon_0}\frac{Qq}{a^3}x = -kx$$

with $k = \frac{Qq}{4\pi \epsilon_0 a^3}$

i.e., the restoring force will become linear and so the motion is simple harmonic with time period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\pi \epsilon_0 ma^3}{qQ}}$$

18. A sphere carrying charge 0.01 C is kept at rest without falling down, touching a wall by applying an electric field 100 N/C.If the coeffcient of friction between the sphere and the wall is 0.2 , the weight of the sphere is

1) 4N 2) 2 N 3) 20 N 4) 0.2 N

SOLUTION :

$$mg = \mu qE$$

19. A bob of a simple pendulum of mass 40gm with a positive charge 4×10^{-6}**C is oscillating with a time period T_1 .An electric field of intensity 3.6×10^4N/C is applied vertically upwards.Now the time period is T_2 the value of $\frac{T_2}{T_1}$ is ($g = 10m/s^2$)**

1)0.16 2) 0.64 3)1.25 4)0.8

SOLUTION :

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$$g_{eff} = g - \frac{qE}{m}$$

20. In a liquid medium of dielectric constant K and of specific gravity 2, two identically charged spheres are suspended from a fixed point by threads of equal lengths. The angle between them is 90º. In another medium of unknown dielectric constant K[1], and specific gravity 4, the angle between them becomes 120º. If density of material of spheres is 8 gm/cc then K[1] is :

1) $\frac{K}{2}$ 2) $\frac{\sqrt{3}}{K}$ 3) $\frac{\sqrt{3}}{2}K$ 4) $\frac{K}{\sqrt{3}}$

SOLUTION :

$$F = mgTan\theta$$

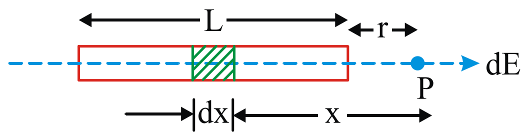
$$F^1 = mg\left(1-\frac{\rho_1}{\rho_s}\right)\tan\theta^1$$

$$F = \frac{1}{4\pi \in_0}.\frac{q_1 q_2}{r^2}$$

$$F^1 = \frac{1}{4\pi \in_0}.\frac{q_1 q_2}{\left(kr^1\right)^2}$$

21. A point charge q is situated at a distance 'r' from one end of a thin conducting rod of length L having a charge Q (uniformly distributed a long its length). find the magnitude of electric force between the two.

SOLUTION :



Consider a small element of the rod of length dx, at a distance 'x' from the point charge q. Trating the element as a point charge, the force between 'q' and charge element will be $dF = \frac{1}{4\pi \in_0}\frac{qdQ}{x^2}$; But,

$$dQ = \frac{Q}{L}dx$$

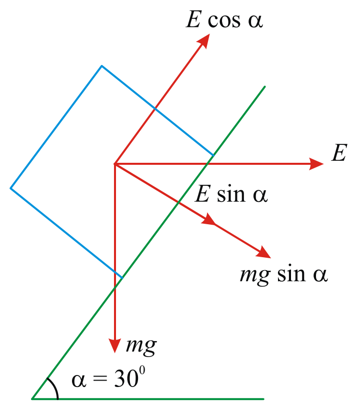
So, $dF = \frac{1}{4\pi \in_0}\frac{qQdx}{Lx^2}$

$$F = \int dF = \frac{1}{4\pi \in_0}\frac{qQ}{L}\int_r^{r+L}\frac{dx}{x^2}$$

$$= \frac{1}{4\pi \in_0}\frac{qQ}{L}\left[-\frac{1}{x}\right]_r^{r+L} = \frac{1}{4\pi \in_0}\left[\frac{1}{r}-\frac{1}{r+L}\right]$$

$$F = \frac{1}{4\pi \in_0}\frac{qQ}{r(r+L)}$$

22. A particle of mass 1kg and carrying positive charge 0.01 C is sliding down an inclined plane of angle 30^0 with the horizontal. An electric field E is applied to stop the particle. If the coefficient of friction between the particle and the surface of the plane is $\frac{1}{2\sqrt{3}}$, E must be



1) 1260 V/m 2) 245 V/m 3) $140\sqrt{3}$ V/m 4) $\frac{490}{\sqrt{3}}$ V/m

SOLUTION :

$$mg\left(\sin\alpha-\mu\cos\alpha\right)-\mu qE\sin\alpha=qE\cos\alpha$$

23. A particle of mass m and charge q is placed at rest in a uniform electric field E and then released. The kinetic energy attained by the particle after moving a distance y is

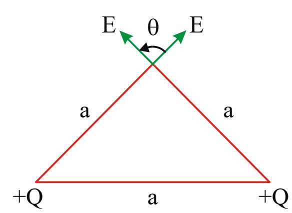
1) qEy^2 2) qE^2y 3) qEy 4) q^2Ey

SOLUTION :

K.E =FS; K.E = qEy

24. Two charges +Q each are placed at the two vertices of an equilateral triangle of side a. The intensity of electric field at the third vertex is

SOLUTION :



$$E^1=\sqrt{E^2+E^2+2EEC\text{os}\,\theta}$$

$$=\sqrt{2E^2+2E^2Cos\theta}$$

$$=\sqrt{2E^2\left(1+\cos\theta\right)}$$

$$=2ECos\frac{\theta}{2};\quad E=\sqrt{3}\frac{1}{4\pi\in_0}\frac{Q}{a^2}$$

25. A particle of charge $-q$ and mass m moves in a circular orbit of radius r about a fixed charge $+Q$. The relation between the radius of the orbit r and the time period T is

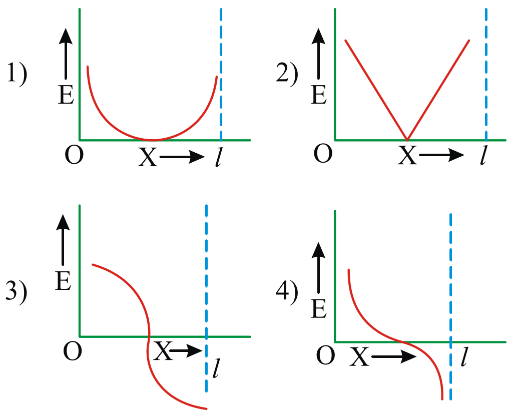
1) $r=\frac{Qq}{16\pi^2\in_0 m}T^3$ 2) $r^3=\frac{Qq}{16\pi^3\in_0 m}T^2$

3) $r^2 = \frac{Qq}{16\pi^3 \epsilon_0 m} T^3$ 4) $r^2 = \frac{Qq}{16\pi \epsilon_0 m} T^3$

SOLUTION :

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2} = mr\omega^2 \; ; \; \omega = \frac{2\pi}{T}$$

26. Two identical point charges are placed at a separation of l. P is a point on the line joining the charges, at a distance x from any one charge. The field at P is E. E is plotted against x for values of x from close to zero to slightly less than l. Which of the following best represents the resulting curve?



SOLUTION :

$$E = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{x^2} - \frac{1}{(l-x)^2}\right]$$

at E = 0

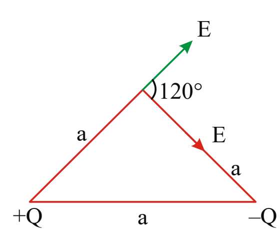
$x > \frac{l}{2}$ E-towards left

$x < \frac{l}{2}$ E-towards right

27. Two charges +Q, -Q are placed at the two vertices of an equilateral triangle of side 'a', then the intensity of electric field at the third vertex is

SOLUTION :

$$E^1 = 2E\cos\frac{\theta}{2} = E \;(\theta = 120^0)$$

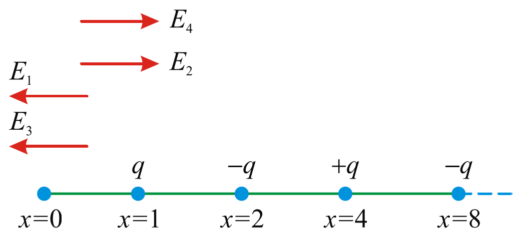


$$E^1 = \frac{1}{4\pi \epsilon_0}\frac{Q}{a^2}.$$

28. An infinite number of charges each 'q' are placed in the x-axis at distances of 1,2,4,8...meter from the origin. If the charges are alternately positive and negative find the intensity of electric field at origin.

SOLUTION :

The electric field intensities due to positive charges and due to -ve charges the field intensity is towards the charges



The resultant intensity at the origin

$$E = E_1 + E_3 - E_4 -----$$

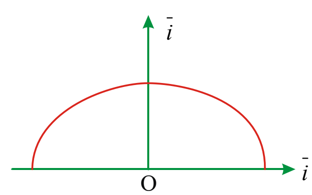
$$E = \frac{Q}{4\pi \in_0}\left(1 - \frac{1}{2^2} + \frac{1}{4^2} - \frac{1}{8^2} + ...\right)$$

Since the expression in the bracket is in GP with a common ratio = $= \frac{-1}{2^2} = \frac{-1}{4}$

$$E = \frac{Q}{4\pi \in_0}\frac{1}{\left[1 - \left(\frac{-1}{4}\right)\right]} = \frac{Q}{4\pi \in_0}\frac{4}{5}$$

$$E = \frac{4}{5}\frac{Q}{4\pi \in_0} \qquad E = \frac{Q}{5\pi \in_0}$$

29. A thin semicircular ring of radius 'r' has a positive charge distributed uniformly over it. The net field E at the centre 'O' is (AIEEE 2010)



1) $\frac{q}{2\pi^2 \in_0 r^2}\bar{j}$ 2) $\frac{q}{4\pi^2 \in_0 r^2}\bar{j}$

3) $-\frac{q}{4\pi^2 \in_0 r^2}\bar{j}$ 4) $-\frac{q}{2\pi^2 \in_0 r^2}\bar{j}$

SOLUTION :

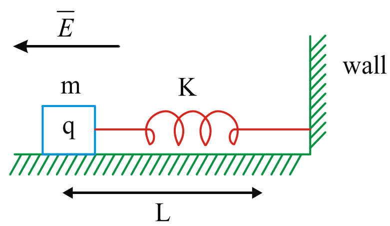
$$E = \frac{1}{4\pi \in_0}\frac{q\sin\pi/2}{r^2\,\pi/2};\; E = \frac{q\sin\pi/2}{2\pi^2 \in_0 r^2}\left(-\bar{j}\right)$$

30 A point mass 'm' and charge 'q' is connected with a spring of negligible mass with natural length L., Initially spring is in natural length. Now a horizontal uniform electric field E is switched on as shown. Find

a) The maximum separation between the mass and the wall

b) Find the separation of the point mass and wall at the equilibrium position of mass

c) Find the energy stored in the spring at the equilibrium position of the point mass.



SOLUTION :

At maximum separation, velocity of point mass is zero. From work energy theorem,

$W_{spring} + W_{field} = 0$

$qEx_0 - \frac{1}{2}kx_0^2 = 0$ (x_0 is maximum elongation)

$\Rightarrow x_0 = \frac{2qE}{K}$; $\therefore$ separation $= L + \frac{2qE}{k}$

b) At equilibrium position. Eq $Eq = kx \Rightarrow x = \frac{qE}{k}$

$\Rightarrow$ separation $= L + \frac{qE}{k}$

c) $U = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{qE}{k}\right)^2 = \frac{q^2E^2}{2k}$

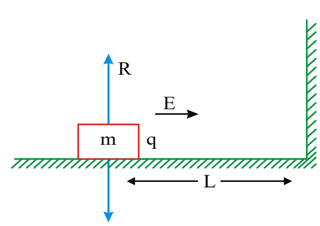
31. A block having mass 'm' ad carge 'q' is resting on a frctionless plane at distance L from the wall as shown inf fig. Discuss the motion of the block when a uniform electric field E is applied horizontally towards the wall assuming that collision of the block with the wall is perfectly elastic.

SOLUTION :

The situation is shown in fig. Electric forece $\vec{F} = q\vec{E}$ will accelerate the block towards the wall producing an acceleration

$a = \frac{F}{m} = \frac{qE}{m}$ $L = \frac{1}{2}at^2$

i.e., $t = \sqrt{\frac{2L}{a}} = \sqrt{\frac{2mL}{qE}}$

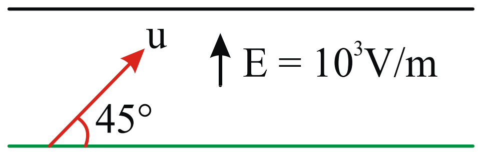


As collision with the wall is perfectly elastic, the block will rebound with same speed and as now is motion is oppisite to the acceleration,, it will come to rest after travelling same distance L in same time t. After stopping it will beagain accelerated towards the wall and so the block will execute oscillatory motion with 'spain' L and time period

$T = 2t = \sqrt{\dfrac{2mL}{qE}}$

However, as the restoring force F(=qE) when the block is moving away from the wall is constance and not proportional to displacement x, the motion is not simple harmonic.

32. A particle having charge that on an electron and mass 1.6 x 10^{-30} kg is projected with an initial speed 'u' to the horizontal from the lower plate of a parallel plate capacitor as shown. The plates are sufficiently long and have separation 2cm. Then the maximum value of velocity of particle not to hit the upper plate. (E=10^3 V/m upwards).

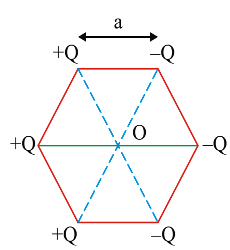


1) 2 x 10^6 m/s   2) 4 x 10^6 m/s
3) 6 x 10^6 m/s   4) 3x 10^6 m/s

SOLUTION :

Maximum height $= \dfrac{u^2 \sin^2\theta}{2\left(g \mp \dfrac{EQ}{m}\right)}$

33. Six charges are placed at the vertices of a regular hexagon as shown in thg figure. The electric field on the line passing through point O and perpendicular to the plane of the figure at a distance of $x(>>a)$ from O is



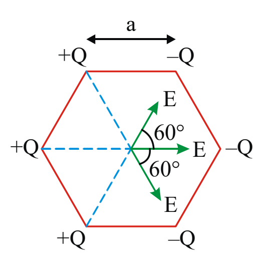
SOLUTION : This is basically a problem of finding the electric field due to three dipoles. The dipole moment of each dipole is $P = Q(2a)$

Electric field due to each dipole will be $E = \dfrac{KP}{x^3}$

The direction of electric field due to each dipole is as shown below:

$E_{net} = E + 2E\cos 60^0 = 2E$

$= 2\left(\dfrac{1}{4\pi\varepsilon_0}\right)\left(\dfrac{2Qa}{x^3}\right) = \dfrac{Qa}{\pi\varepsilon_0 x^3}$



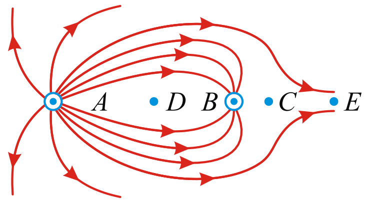
34. The field lines for two point charges are shown in fig.

i. Is the field uniform?

ii. Datermine the ratio q_A / q_B.

iii. What are teh sing of q_A and q_B ?

iv. If q_A and q_B are separated by a distance $10\left(\sqrt{2}-1\right)$ cm, find the position of neutral point.



SOLUTION :

i. No

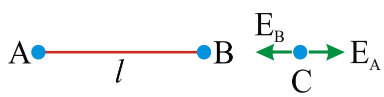
ii. Number of lines coming from or coming to a charge is proportional to magnitude of charge, so $\frac{q_A}{q_B}=\frac{12}{6}=2$

iii. q_A is positive and q_B is negative

iv. C is the other neutral point.

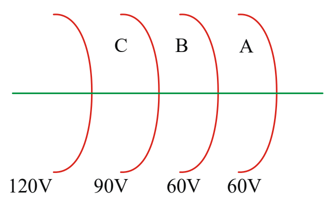
v. For neutral point $E_A = E_B$

$$\frac{1}{4\pi\varepsilon_0}\frac{q_A}{(1+x)^2}=\frac{1}{4\pi\varepsilon_0}\frac{q_B}{x^2}$$



$$\left(\frac{l+x}{x}\right)^2=\frac{q_A}{q_B}=2 \Rightarrow x=10 \text{ cm}$$

35. Four equipotential curves in an electric field are shown in the figure. A,B,C are three points in the field.If electric intensity at A,B,C are E_A, E_B, E_C then



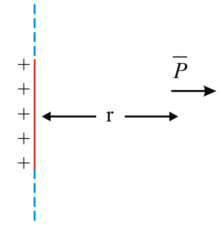
1) $E_A = E_B = E_C$ 2) $E_A > E_B > E_C$

3) $E_A < E_B < E_C$ 4) $E_A > E_B < E_C$

SOLUTION :

$$E=-\frac{dV}{dx}$$

36. An electric dipole of dipole moment p is kept at a distance r from an infinite long charged wire of linear charge density λ as shown. Find the force acting on the dipole ?



SOLUTION : Field intensity at a distance r from the line of charge is $E = \dfrac{\lambda}{2\pi \in_0 r}$

The force on the dipole is $F = -p\dfrac{dE}{dr}$

$$= -p\left[\frac{-\lambda}{2\pi \in_0 r^2}\right] = \frac{p\lambda}{2\pi \in_0 r^2}$$

Here the net force on dipole due to the wire will be attractive.

37. Electric field on the axis of a small electric dipole at a distance r is $\vec{E}_1$ and $\vec{E}_2$ at a distance of 2r on a line of perpendicular bisector. Then

1) $\overrightarrow{E_2} = -\overrightarrow{E_1}/8$ 2) $\overrightarrow{E_2} = -\overrightarrow{E_1}/16$

3) $\overrightarrow{E_2} = -\overrightarrow{E_1}/4$ 4) $\overrightarrow{E_2} = \overrightarrow{E_1}/8$

SOLUTION :

$$E_{axis} = \frac{2kp}{r^3} \text{ and } E_{bicector} = \frac{kp}{2r^3}$$

38. A thin copper ring of radius 'a' is charged with q units of electricity. An electron is placed at the centre of the copper ring. If the electron is displaced a little, it will have frequency.

1) $\dfrac{1}{2\pi}\sqrt{\dfrac{eq}{4\pi \in_0 ma^3}}$ 2) $\dfrac{1}{2\pi}\sqrt{\dfrac{q}{4\pi \in_0 ema^3}}$

3) $\sqrt{\dfrac{eq}{4\pi \in_0 ma}}$ 4) $\sqrt{\dfrac{q}{4\pi \in_0 ema^3}}$

SOLUTION :

$$E = \frac{1}{4\pi \in_0}\frac{qx}{\left(a^2 + x^2\right)^{3/2}} = \frac{qx}{4\pi \in_0 a^3}$$

$$\therefore m\frac{d^2x}{dt^2} = -\frac{1}{4\pi \in_0}\frac{qex}{a^3}$$

So motion is S.H.M.

$$\omega^2 = \frac{1}{4\pi \in_0}\frac{qe}{ma^3}$$

39: A charge Q is distributed over two concentric hollow spheres of radii 'r' and R (> r) such that the surface densities are equal. Find the potential at the common centre.

SOLUTION : If q_1 and q_2 are the charges on spheres of radii 'r' and R respectively, then in accordance with conservation of charge

$q_1 + q_2 = Q$ ------(1)

And according to given problem $\sigma_1 = \sigma_2$,

i.e., $\frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$ or $\frac{q_1}{q_2} = \frac{r^2}{R^2}$ ------(2)

So from Eqs (1) and (2)

$q_1 = \frac{Qr^2}{(r^2 + R^2)}$ and $q_2 = \frac{QR^2}{(r^2 + R^2)}$ -----(3)

Now as potential inside a conducting sphere is equal to that at its surface, so potential at the common centre,

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$$

Substituting the value of q_1 and q_2 from Eq.(3)

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{(R^2 + r^2)} + \frac{Qr}{(R^2 + r^2)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q(R + r)}{(R^2 + r^2)}$$

40. A thin fixed ring of radius 1 metre has a positive charge $1\times10^{-5}C$ uniformly distributed over it. A particle of mass 0.9gm and having a negative charge of $1\times10^{-6}C$ is placed on the axis at a distance of 1 cm from the centre of the ring. Assuming that the oscillations have small amplitude, the time period of oscillations is

1) 0.23s 2) 0.39s 3) 0.49 s 4) 0.63s

SOLUTION :

$$F = \frac{Qq}{4\pi\epsilon_0}\frac{x}{R^3} = -kx \text{ and } T = 2\pi\sqrt{\frac{m}{k}}$$

41. If electric potential V at any point (x, y, z) all in metres in space is given by V = 4x² volt. Calculate the electric field at the point (1m, 0m, 2m).

SOLUTION : As electric field E is related to potential V through the relation

$$E = -\frac{dV}{dr}$$

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}(4x^2) = -8x$$

$$E_y = -\frac{dV}{dy} = -\frac{d}{dy}(4x^2) = 0$$

And, $E_z = -\frac{dV}{dz} = -\frac{d}{dz}(4x^2) = 0$

So, $\vec{E} = \hat{i}\ E_x + \hat{j}\ E_y + \hat{k}\ E_z = -8x\hat{i}$

i.e., it has magnitude 8 V/m and is directed along negative x-axis.

42. A conducting spherical bubble of radius r and thickness t (t >> r) is charged to a potential V. Now it collapses to form a spherical droplet. Find the potential of the droplet.

SOLUTION :

Here charge and mass are conserved. If R is the radius of the resulting drop formed and ρ is density of soap

..ution, $\frac{4}{3}\pi R^3 \rho = 4\pi r^2 t\rho \Rightarrow R = (3r^2 t)^{1/3}$

Now potential of the bubble is $V = \frac{1}{4\pi \in_0} \frac{q}{r}$

or $q = 4\pi \in_0 rV$

Now potential of resulting drop is

$$V^| = \frac{1}{4\pi \in_0} \frac{q}{R} \quad = \left(\frac{r}{3t}\right)^{1/3} V.$$

43. A spherical charged conductor has surface charge density σ. The intensity of electric field and potential on its surface are E and V. Now radius of sphere is halved keeping the charge density as constant. The new electric field on the surface and potential at the centre of the sphere are

1) 4E, V 2) E, V/2 3) E, V 4) 2E, 4V

SOLUTION :

$$E = \frac{\sigma}{\varepsilon_0} \text{ and } V = \frac{\sigma R}{\varepsilon_0}$$

44. Two thin rings each having a radius R are placed at distance d apart with their axes coinciding.The charges on the two rings are +q, -q. The potential difference between the rings

1) $\frac{Q.R}{4\pi\varepsilon_0 . d^2}$ 2) $\frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right)$ 3) $\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right)$ 4) 0

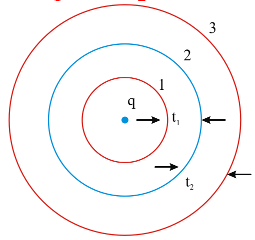
SOLUTION :

$$V_1 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right)$$

$$V_2 = \frac{-Q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right)$$

$$\Delta V = V_1 - V_2$$

45. Figure shows three spherical and equipotential surfaces 1,2 and 3 round a point charge q. The potential difference $V_1 - V_2 = V_2 - V_3$. If t_1 and t_2 be the distance between them. Then



1) $t_1 = t_2$ 2) $t_1 > t_2$ 3) $t_1 < t_2$ 4) $t_1 \leq t_2$

SOLUTION :

$$V_1 - V_2 = kq\left(\frac{1}{r_1} - \frac{1}{r_2}\right);\ r_2 - r_1 = \frac{(V_1 - V_2)r_1 r_2}{kq}$$

but $(r_2 - r_1) = t$

$\therefore t \alpha r_1 r_2$

if P.D is constant then $(r_2 - r_1) = t$

46. A half ring of radius ' r ' has a linear charge density λ. The potential at the centre of the half ring is

1) $\frac{\lambda}{4\varepsilon_0}$ 2) $\frac{\lambda}{4\pi^2\varepsilon_0 r}$ 3) $\frac{\lambda}{4\pi\varepsilon_0 r}$ 4) $\frac{\lambda}{4\pi\varepsilon_0 r^2}$

SOLUTION :

potential due to small element ' p ' at the centre

$$v = \int dv = \int k\frac{\lambda}{r}dl = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\int dl$$

$$dv = K.\lambda\frac{dl}{r};\quad = \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{r}\pi r \Rightarrow \frac{\lambda}{4\varepsilon_0}$$

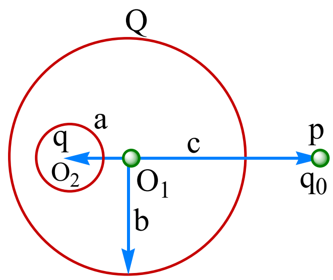
47. Two metal sphres A and B have their capacities in th ratio 3:4. They are put in contact with each other and an amount of charge $7\times10^{-6}C$ is given to the combination. Next, the two spheres are separated and kept wide the apart so that one has no electrical infuence on the other. The potential due to the smaller sphere at a distance of 50m from it is

1) 540V 2) 270V 3) 1180V 4) zero

SOLUTION :

$$q_1 = \left(\frac{r_1}{r_1 + r_2}\right)q;\ V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

48. A conducting sphere fo radius b has a spherical cavity with its centre displaced by "a" from centre fo sphere. A point charge q is placed at the centre fo cavity, 'Q' charge is given to conducting sphere and charge q_0 is placed at a distance c from centre (O_1) of sphere such that O_1, O_2 and P are collinear then which of the following is in correct



1) charge distribution on inner surface of cavity is uniform

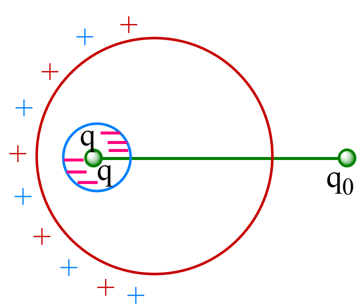
2) potential of conductor is $\left(\dfrac{q_0}{4\pi \in_0 c}+\dfrac{Q+q}{4\pi \in_0 b}\right)$

3) charge distribution of outer surface of conducting sphere is non uniform

4) Intensity of electric field at O_1 is $\dfrac{1}{4\pi \in_0}\dfrac{q_0}{c^2}$

SOLUTION :

-1 is uniform an linear surface (Q+q)



And non-uniform on outer surface

$\rightarrow$ Vcenter = V conductor =

$$\frac{K(Q+q)}{b}+\frac{K(q_0)}{c}$$

49. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting spherical shell. The potential difference between the surface of solid sphere and the shell is V. The shell is now given a charge –3Q. The new potential difference between the same surfaces will be

1) –2V 2) 4V 3) V 4) 2V

SOLUTION :

Pd between the two spheres is independent of charge on outer shell.

50. A particle of mass 1Kg and carrying 0.01C is at rest on an inclined plane of angle 30^0 with horizontal when an electric field of $\frac{490}{\sqrt{3}} NC^{-1}$ applied parllel to horizontal .The coefficient of friction is

1) 0.5 2) $\frac{1}{\sqrt{3}}$ 3) $\frac{\sqrt{3}}{2}$ 4) $\frac{\sqrt{3}}{7}$

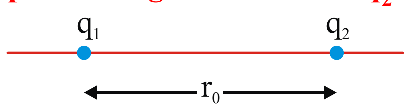
SOLUTION :

$N = mg \sin\theta + qE \sin\theta$

$mg \sin\theta = \mu N + qE\cos\theta$

51: Charge q_1 is fixed and another point charge q_2 is placed at a distance r_0 from q_1 on a frictionless horizontal surface. Find the velocity of q_2 as a function of seperation r between them (treat the changes as point charges and mass of q_2 is m)

SOLUTION : .



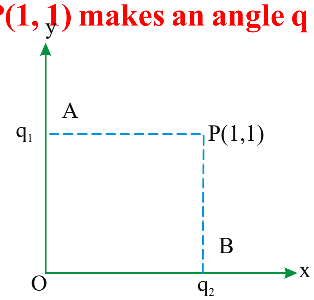
According law of concervation of energy

$U_1 + K_1 = U_2 + K_2$

$\frac{1}{4\pi \in_0}\frac{q_1 q_2}{r_0} + 0 = \frac{1}{4\pi \in_0}\frac{q_1 q_2}{r} + \frac{1}{2}mv^2$

$\frac{1}{2}mv^2 = \frac{q_1 q_2}{4\pi \in_0}\left[\frac{1}{r_0} - \frac{1}{r}\right]$; $v = \sqrt{\frac{q_1 q_2}{2\pi \in_0 m}\left[\frac{1}{r_0} - \frac{1}{r}\right]}$

52. Two points charges q_1 and q_2 (=q_1/2) are placed at points A(0, 1) and B (1, 0) as shown in the figure. The electric field vector at point P(1, 1) makes an angle q with the x–axis, then the angle q is



1) $\tan^{-1}\left(\frac{1}{2}\right)$ 2) $\tan^{-1}\left(\frac{1}{4}\right)$ 3) $\tan^{-1}(1)$ 4) $\tan^{-1}(0)$

SOLUTION :

$$\theta = \tan^{-1}\left[\frac{E_2}{E_1}\right]$$

$$E_1 = \frac{1}{4\pi \in_0}.\frac{q_1}{1^2}$$

$$E_2 = \frac{1}{4\pi \in_0}.\frac{q_2^2}{1^2}$$

$$q_2 = \frac{q_1}{2}$$

53: A proton moves with a speed of 7.45 x 10^5 m/s directly towards a free proton originally at rest. Find the distance of closest approach for the two protons.

Given $(1/4\pi\varepsilon_0) = 9\times 10^9\, m/F;\ m_P = 1.67\times 10^{-27}\, kg$ **and e = 1.6 x 10^{-19} coulomb.**

SOLUTION :

As here the particle at rest is free to move, when one particle approaches the other, due to electrostatic repulsion other will also start moving and so the velocity of first particle will decrease while of other will increase and at closest approach both will move with same velocity. So if v is the common velocity of each particle at closest approach, by 'conservation of momentum'.

$mu = m\upsilon + m\upsilon$ i.e., $\upsilon = \frac{1}{2}u$

And by 'conservation of energy'

$$\frac{1}{2}mu^2 = \frac{1}{2}m\upsilon^2 + \frac{1}{2}m\upsilon^2 + \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

So, $r = \frac{4e^2}{4\pi\varepsilon_0 mu^2}$ $\left[\text{as } \upsilon = \frac{u}{2}\right]$

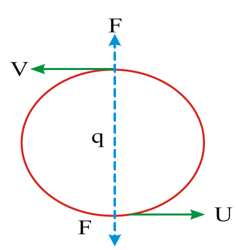
And hence substituting the given data,

$$r = 9\times 10^9 \times \frac{4\times(1.6\times 10^{-19})^2}{1.67\times 10^{-27}\times(7.45\times 10^5)^2} = 10^{-12}\,m$$

54: A small ball of mass 2 x 10^{-3} kg having a charge of $1\mu C$ is suspended by a string of length 0.8m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be impacted to the lower ball so that it can make complete revolution :

SOLUTION :

To complete the circle at top most point $T_2 = 0$



$$Mg - \frac{q^2}{4\pi\in_o \ell^2} = \frac{MV^2}{\ell}$$

$$\Rightarrow V^2 - g\ell = \frac{-q^2}{4\pi\in_o M\ell} \quad \textbf{...(1)}$$

from law of conservation of energy

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mg\,2\ell \;....\;(2)$$

from (1) and (2);

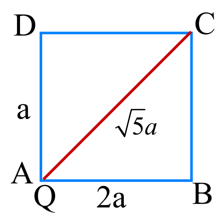
$$u = \sqrt{4g\ell - \frac{q^2}{4\pi\in_o m\ell}} = 5\cdot 86\,m/s$$

55. **The longer side of a rectangle is twice the length of its shorter side. A charge q is kept at one vertEX. The maximum electric potential due to that charge at any other vertex is V, then the minimum electric potential at any other vertex will be**

1) 2V 2) $\sqrt{3}\ V$ 3) $V/\sqrt{5}$ 4) $\sqrt{5}\ V$

SOLUTION :

long and short diagonal lengths are $\sqrt{p^2+q^2\pm 2pq\cos\theta}$



$$V_D=\frac{1}{4\pi\in_0}\frac{Q}{a}$$

$$V_c=\frac{1}{4\pi\in_0}\frac{Q}{\sqrt{5}a}=\frac{V_0}{\sqrt{5}}$$

56. **A small electric dipole is placed at origin with its dipole moment directed along positive x -axis. The direction of electric field at point $\left(2,2\sqrt{2},0\right)$ is**

1) along z - axis 2) along y - axis
3) along negative y-axis 4) along negative z-axis

SOLUTION :

$\theta=\tan^{-1}\left(\sqrt{2}\right)$

$$\tan\phi=\frac{E_0}{E_r}=\frac{1}{2}\tan\theta$$

$$\theta+\phi=\frac{\pi}{2}\ \Rightarrow\tan\theta=\sqrt{2}$$

57. **Two electric dipoles each of dipolemoment $p=6.2\times10^{-30}C-m$ are placed with their axis along the same line and their centres at a distanced=$10^{-8}cm$. The force of attraction between dipoles is**

1) $2.1\times10^{-16}N$ 2) $2.1\times10^{-12}N$
3) $2.1\times10^{-10}N$ 4) $2.1\times10^{-8}N$

SOLUTION :

$$F=\frac{1}{4\pi\in_0}\frac{6P_1P_2}{d^4}$$

58. **Two charges $+3.2\times10^{-19}C$ and $-3.2\times10^{-19}C$ placed $2.4\,A^0$ apart form an electric dipole. It is placed in a uniform electric field of intensity $4\times10^5\,V/m$ the work done to rotate the electric dipole from the equilibrium position by 180^0 is**

1) $3\times10^{-23}J$ 2) $6\times10^{-23}J$

3) $12\times10^{-23}J$ 4) Zero

SOLUTION :

$P = 2ql \qquad l = 2.4A^0$

$w = PE(\cos\theta_1 - \cos\theta_2)$

$\theta_1 = 90^0 \qquad \theta_2 = 180^0$

59. Two opposite and equal charges 4 x 10^{-8} **coulomb when placed 2 x 10^{-2} cm away, from a dipole. If this dipole is placed in an external electric field 2 x 10^{-2} newton/coulomb, the value of maximum torque and the work done in rotating it through 180^0 will be**

1) $32x10^{-4}\,Nm$ and $32x10^{-4}\,J$
2) $64x10^{-4}\,Nm$ and $64x10^{-4}\,J$
3) $64x10^{-4}\,Nm$ and $32x10^{-4}\,J$
4) $32x10^{-4}\,J$ and $64x10^{-4}\,Nm$

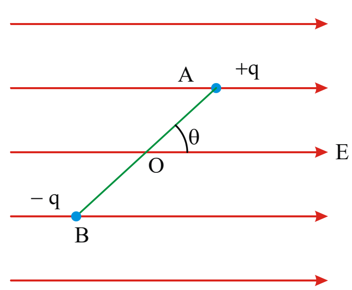
SOLUTION :

$\tau = pESin\theta$

$\tau_{\max} = pE$

$q = pE(\cos\theta_1 - \cos\theta_2)$

60. A point particle of mass M is attached to one end of a massless rigid non-conducting rod of length L. Another point particle of the same mass is attached to the other end of the rod. The two particle carry charges + q and – q respectively. This arrangement is held in a region of a uniform electric field E such that the rod makes a small angle θ (say of about 5^0) with the field direction (see figure). The expression for the minimum time needed for the rod to become parallel to the field after it is set free.



1) $t = \frac{\pi}{2}\sqrt{\frac{mL}{2qE}}$
2) $t = \frac{\pi}{2}\sqrt{\frac{mL}{qE}}$
3) $t = \frac{\pi}{2}\sqrt{\frac{2mL}{qE}}$
4) $t = \frac{\pi}{2}\sqrt{\frac{3mL}{2qE}}$

SOLUTION :

$t = \frac{T}{4}$

$T = 2\pi\sqrt{\frac{T}{pE}}, I = \frac{ml^2}{2}$

$p = qL$

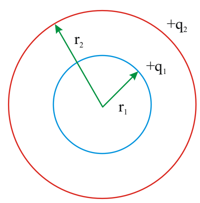
61: If an electron enters into a space between the plates of a parallel plate capacitor at an an angle α with the plates and leaves at an angle β to the plates, find the ratio of its kinetic energy while entering the capacitor to that while leaving.

SOLUTION : . Let u be the velocity of electron while entering the field and v be the velocity when it leaves the plates. Component of velocity parallel to the plates will remain unchanged.

Hence $u\cos\alpha = u\cos\beta \quad \therefore \frac{u}{v} = \frac{\cos\beta}{\cos\alpha}$

$$\therefore \frac{\left(\frac{1}{2}mu^2\right)}{\left(\frac{1}{2}mv^2\right)} = \left(\frac{u}{v}\right)^2 = \left(\frac{\cos\beta}{\cos\alpha}\right)^2$$

62: Figure shows two concentric conductiong shells of radii r_1 and r_2 carrying uniformly distributed characges q_1 and q_2. respectively. Find out an expression for the potential of each shell.



SOLUTION : The potential of each sphere consists of two points:

One due to its own charge, and

Second due to the charge on the other sphere.

Using the principle of superposition, we have

$V_1 = V_{r_1,surface} + V_{r_2,inside}$ and

$V_2 = V_{r_1,outside} + V_{r_2,surface}$

Hence, $V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

and $V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2}$

63. Here is a special parallelogram with adjacent side lengths 2a and a and the one of the possible angles between them as 60°. Two charges are to be kept across a diagonal only. The ratio of the minimum potential energy of the system to the maximum potential energy is

1) $\sqrt{3} : \sqrt{7}$ 2) 3 : 7 3) 1 : 2 4) 1 : 4

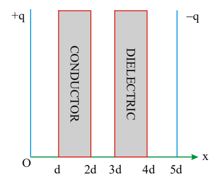
SOLUTION :

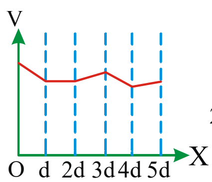
$$r = \sqrt{(2a)^2 + a^2 \pm 2(2a)a.\cos 60}$$

$$U_{\max/\min} = \frac{1}{4\pi \in_0}.\frac{Q_1Q_2}{r}$$

$$U_1 : U_2 = \sqrt{3} : \sqrt{7}$$

64. The distance between plates of a parallel plate capacitor is 5d. The positively charged plate is at x=0 and negativily charged plates is at x=5d. Two slabs one of conducotor and the other of a dielectric of same thickness d are inserted between the plates as shown in figre. Potential (V) versus distance x graph will be



1)


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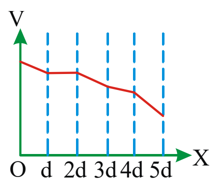

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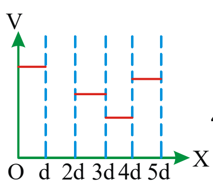

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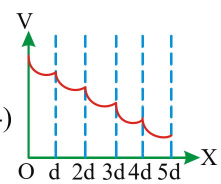


SOLUTION :

. E inside the conductor is zero.

V = constant between d and 2d

E inside dielectric is non zero

$dv \neq 0 \qquad dv = -\left(\overline{E}.d\vec{r}\right)$

65. Two concentric spherical conducting shells of radii R and 2R carry charges Q and 2Q respectively. Change in electric potential on the outer shell when both are connected by a conducting wire is :

$\left(k = \frac{1}{4\pi\varepsilon_0}\right)$

1) zero 2) $\frac{3kQ}{2R}$ 3) $\frac{kQ}{R}$ 4) $\frac{2kQ}{R}$

SOLUTION :

On outer shell

$$V_1 = K\left(\frac{Q}{2R}, \frac{2Q}{2R}\right) = K\left(\frac{3Q}{2R}\right)$$

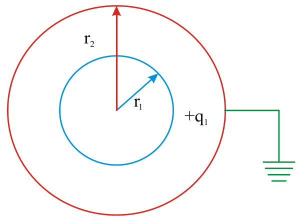
$$V_2 = K\left(\frac{3Q}{2R}\right)$$

$$V_1 - V_2 = 0$$

66: In the previous example, if the charge $q_1 = +q_0$ and the outer shell is earthed, then

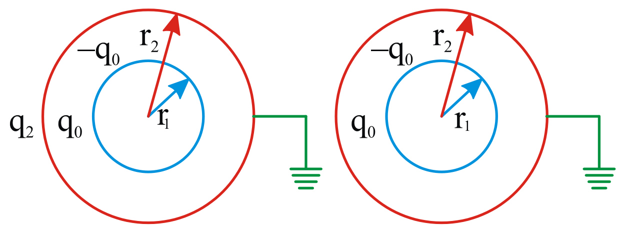
a) determine the charge on the outer shell, and

b) find the potential of the inner shell.



SOLUTION : a) We know that charge on facing surfaces is equal and opposite. So, if charge on inner sphere is q_0, then charge on inner surface of shell should be $-q_0$. Now, let charge on outer surface of shell be q_2.

As the shell is earthed. So its potential should be zero. So,



$$V_{shell} = \frac{kq_0}{r_2} + \frac{k(-q_0)}{r_2} + \frac{kq_2}{r_2} = 0 \Rightarrow q_2 = 0$$

Hence, charge on outer surface of shell is zero. Final charges appearing are shown in fig

b) Potential of inner sphere:

$$V_1 = \frac{kq_0}{r_1} + \frac{k(-q_0)}{r_2} = \frac{q_0}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

67. An electron travelling from infinity with velocity 'v' into an electric field due to two stationary electrons separated by a distance of 2m. If it comes to rest when it reaches the mid point of the line joining the stationary electrons.The initial velocity ' v ' of the electron is

1) 16m/s 2) 32m/s 3) $16\sqrt{2}m/s$ 4) $32\sqrt{2}m/s$

SOLUTION :

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}, \text{ r} = 1\text{m}$$

68. Work performed when a point charge 2×10^{-8} C is transformed from infinity to a point at a distance of 1cm from the surface of the ball with a radius of 1cm and a surface charge density $\sigma = 10^{-8}C/cm^2$

1) $1.1\times10^{-4}J$ 2) $11\times10^{-4}J$ 3) $0.11\times10^{-4}J$ 4) $113\times10^{-4}J$

SOLUTION :

Potential at a distance 2cm from its centre

$$= \frac{Q}{4\pi\epsilon_0 r} = \frac{4\pi r^2\sigma}{4\pi\epsilon_0 r'} \Rightarrow \frac{r^2\sigma}{\epsilon_0 r'} = \frac{\sigma}{2\epsilon_0}\times\frac{1}{100}$$

since r=1 cm and r' =2 cm

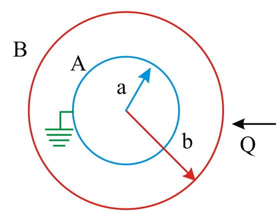
PD b/w the two points is equal to $\frac{\sigma}{200 \in_0}$

work done =VQ= $\frac{\sigma}{200 \in_0} X 2\times10^{-8} = 11\times10^{-4} J$

69: Consider two concentric spherical metal shells of radii 'a' and b > a. The outer shell has charge Q, but the inner shell has no charge, Now, the inner shell is grounded. This means that the inner shell will come at zero potential and that electric field lines leave the outer shell and end on the inner shell.

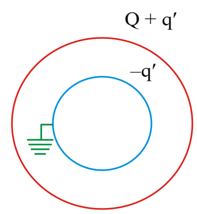
a) Find the charge on the inner shell.

b) Find the potential on outer sphere.



SOLUTION : a) When an object is connected to earth (grounded), its potential is reduced to zero.

Let q' be the charge on A after it is earthed as shown in fig



The charge q' on A induces $-q'$ on inner surface of B and $+q'$ on outer surface of B. In equilibrium, the charge distribution is as shown in fig

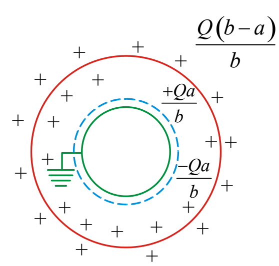
Potential of inner sphere = potential due to charge on A + potential due to charge on B = 0

$$V_A = \frac{q'}{4\pi\varepsilon_0 a} - \frac{q'}{4\pi\varepsilon_0 b} + \frac{Q+q'}{4\pi\varepsilon_0 b} = 0$$

or $q' = -Q\left(\frac{a}{b}\right)$

This implies that a charge $+Q(a/b)$ has been transferred to the earth leaving negative charge on A.

Final charge distribution will be as shown in fig..



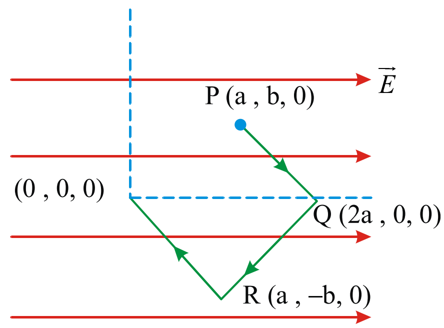
As b>a, so charge on the outer surface of outer shell will be $\frac{Q(b-a)}{b} > 0$.

b) Potential of outer surface V_B = potential due to charge on A + potential due to charge on B.

$$V_B = V_{a,out} + V_{b,both\ surface} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\left(-Q\frac{a}{b}\right)}{b} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{b} = \frac{Q(b-a)}{4\pi\varepsilon_0 b^2}$$

70



A point charge q moves from point P to point S along the path PQRS in a unifrom electric field $\vec{E}$ pointing parallel to the positive direction of the x-axis. The coordinates of the points P, Q, R and S are (a,b,0), (2a,0,0), (a, –b, 0) and (0,0,0) respectively. The work done by the field in the above process is given by the expression

1) qaE 2) –qaE

3) $q(\sqrt{a^2+b^2})E$ 4) $3qE\sqrt{a^2+b^2}$

SOLUTION :

$$W = \overline{F}.d\overline{r}, d\overline{r} = \left(a\hat{i} + b\hat{j}\right)$$

$$= q\overline{E}.d\vec{r} \qquad \vec{E} = E\hat{i}$$

$$W = -qaE$$

71. The potential at a point x (measured in μ m) due to some charges situated on the x-axis is given by

$V(x) = \frac{20}{x^2-4}$ volt. The electric field E at x = 4 μ m is given by

1) $\frac{5}{3}\frac{V}{\mu m}$ and in the positive x - direction

2) $\frac{10}{9}\frac{V}{\mu m}$ and in the negative x - direction

3) $\frac{10}{9}\frac{V}{\mu m}$ and in the positive x-direction

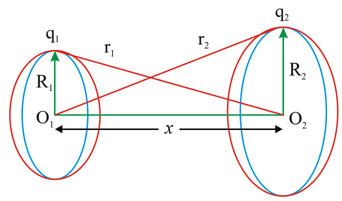
4) $\frac{5}{3}\frac{V}{\mu m}$ and in the negative x-direction

SOLUTION :

$$V(x)=\frac{20}{x^2-4}$$

$$E=-\frac{dv}{dx} \text{ at } x=4\mu m$$

72. Two circular loops of radii 0.05 and 0.09m, respectively, are put such that their axes coincide and their centres are 0.12 m apart. Charge of 10^{-6} coulomb is spread uniformly on each loop. Find the potential difference between the centres of loops.



SOLUTION : The potential at the centre of a ring will be due to charge on both the rings and as every element of a ring is at a constant distance from the centre, so

$$V_1=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{R_1}+\frac{q_2}{\sqrt{R_2^2+x^2}}\right]$$

$$=9\times10^9\left[\frac{10^{-4}}{5}+\frac{10^{-4}}{\sqrt{9^2+12^2}}\right]$$

$$=9\times10^5\left[\frac{1}{5}+\frac{1}{15}\right]=2.40\times10^5V$$

similarly, $$V_2=\frac{1}{4\pi\varepsilon_0}\left[\frac{q_2}{R_2}+\frac{q_1}{\sqrt{R_1^2+x^2}}\right]$$

or $$V_2=9\times10^3\left[\frac{1}{9}+\frac{1}{13}\right]=\frac{198}{117}\times10^5$$

$$V_2=1.69\times10^5V$$

So, $$V_1-V_2=(2.40-1.69)\times10^5=71kV$$

73. Two spherical conductors A and B of radii 1 mm and 2mm are seperated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in the equilibrium condition the ratio of electric fields at surfaces of A and B is

1) 4: 1 2) 1 : 2 3) 2 : 1 4) 1 : 4

SOLUTION :

$$V=K.\frac{Q}{R};\ \frac{V}{2}=K.\frac{Q}{2R} \quad =K.\frac{Q}{R}$$

$$\frac{1}{2}\left(K.\frac{Q}{R}\right)=K.\frac{Q}{d};\ d=2R$$

When the two conducting spheres are connected by a conducting wire, charge will flow from one sphere (having higher potential) to other (having lower potential) till both acquire the same potential.

There fore, $E=\frac{V}{r}\Rightarrow \frac{E_1}{E_2}=\frac{r_2}{r_1}=\frac{2}{1}=2:1$

74. **A charge +q is fixed at each of the points $x=x_0, x=3x_0, x=5x_0.....\infty$ on the x - axis and a charge-q is fixed at each of the points $x=2x_0, x=4x_0, x=6x_0.......\infty$. Here x_0 is a positive constant. Take the electric potential at a point due to a charge Q at a distance r from it to be $\frac{Q}{4\pi\in_0 r}$. Then the potential at the origin due to the above system of charges is**

1) 0 2) $\frac{q}{8\pi\in_0 x_0 \log_e(2)}$ 3) ∞ 4) $\frac{q\log_e(2)}{4\pi\in_0 x_0}$

SOLUTION :

$$V=\frac{q}{4\pi\in_0}\left[\frac{1}{x_0}-\frac{1}{2x_0}+\frac{1}{3x_0}-\frac{1}{4x_0}+---\right]$$

$$=\frac{q}{4\pi\in_0 x_0}\left[1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+---\right]$$

$$=\frac{q}{4\pi\in_0 x_0}\log(2)$$

75. **A sphere of radius R carries a charge density = kr (where k is a constant). The electrostatic energy of the within the sphere is**

1) $\frac{\pi k^2}{8\pi\varepsilon_0}\frac{R^7}{7}$ 2) $\frac{\pi k^2}{4\pi\varepsilon_0}\frac{R^7}{7}$

3) $\frac{\pi k^2}{8\pi\varepsilon_0}\frac{R^5}{5}$ 4) $\frac{\pi k^2}{4\pi\varepsilon_0}\frac{R^5}{5}$

SOLUTION :

$$q=\int_0^r kr4\pi r^2 dr = k\pi r^4$$

$$E=\frac{1}{4\pi\in_0}\frac{q}{r^2}$$

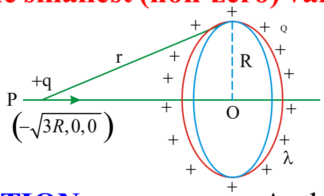
Energy within the sphere

$$\int\left(\frac{1}{2}\varepsilon_0 E^2\right)dV$$

$$\int_0^R \frac{1}{2}\varepsilon_0\left(\frac{kr^2}{4\varepsilon_0}\right)^2 4\pi r^2 dr$$

$$U_{total}=U_{within}+U_{outside}$$

76. A circular ring of radius R with uniform positive charge density λ per unit length is located in the y - z plane with its centre at the origin O. A particle of mass 'm' and positive charge 'q' is projected from the point $p\left[-\sqrt{3}R,0,0\right]$ on the negative x-axis directly towards O, with initial speed v. Find the smallest (non-zero) value of the speed such that the particle does not return to P?



SOLUTION : . As the electric field at the centre of a ring is zero, the particle will not come back due to repulsion if it crosses the centre fig.

$$\frac{1}{2}mv^2+\frac{1}{4\pi\varepsilon_0}\frac{qQ}{r}>\frac{1}{4\pi\varepsilon_0}\frac{qQ}{R}$$

But here, $Q=2\pi R\lambda$ and $r=\sqrt{\left(\sqrt{3}R\right)^2+R^2}=2R$

So, $\frac{1}{2}mv^2>\frac{1}{4\pi\varepsilon_0}\frac{2\pi R\lambda q}{R}\left[1-\frac{1}{2}\right]$ or $v>\sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}$

So, $V_{\min}=\sqrt{\left(\frac{\lambda q}{2\varepsilon_0 m}\right)}$

77. The electric potential in a region of space is given by $V=3x^2y$ volt; where x and y are in meters. Find the electric field strength along the three principle directions and the magnitude of the absolute strength

1) $3x^2\sqrt{4y^2-x^2}\,V/m$

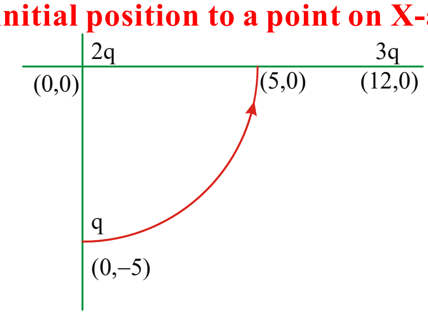
2) $3x^2\sqrt{4y^2+x^2}\,V/m$

3) $\sqrt{4y^2-x^2}\,V/m$

4) $\sqrt{4y^2+x^2}\,V/m$

SOLUTION :

78. 2q and 3q are two charges separated by a distance 12 cm on x-axis. A third charge q is placed at 5 cm on y-axis as shown in figure. Find the change in potential energy of the system if 3q is moved from initial position to a point on X-axis in circular path



1) $\frac{q^2}{4\pi\varepsilon_o}$ 2) $\frac{6q^2}{4\pi\varepsilon_o(91)}$ 3) $\frac{18q^2}{4\pi\varepsilon_o(91)}$ 4) $\frac{3q^2}{4\pi\varepsilon_o}$

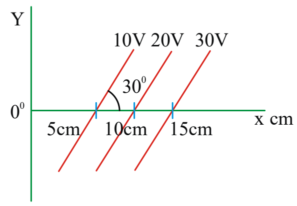
SOLUTION :

$$U_{ini} = \frac{1}{4\pi\epsilon_0}\left[\frac{2q^2}{5} + \frac{3q^2}{13} + \frac{6q^2}{12}\right]$$

$$U_{final} = \frac{1}{4\pi\epsilon_0}\left[\frac{2q^2}{5} + \frac{3q^2}{7} + \frac{6q^2}{12}\right]$$

$$\left|U_f - U_i\right| = \frac{3q^2}{4\pi\epsilon_0}\left[\frac{1}{13} - \frac{1}{7}\right] \quad \Delta w = \left(U_f - U_i\right)\frac{18q^2}{4\pi\epsilon_0(91)}$$

79. Some equipotential surfaces are shown in figure. The electric field strength is



1) 100 V/m along x-axis 2) 100 V/m along y-axis

3) 400 V/m at an angle 120^0 with x-axis 4) $\frac{400}{\sqrt{3}}$ V/m an angle 120^0 with x-axis

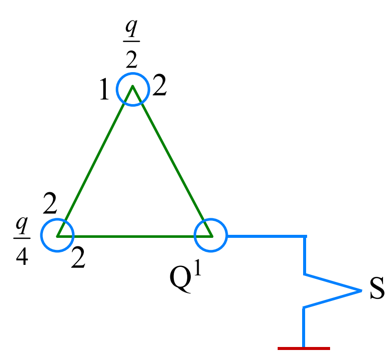
SOLUTION :

$\Delta V = -\int \vec{E}.\vec{dr}$ and find will be in such a sirection that V decreasing and perpendicular to equipotential surface.

80. There are three uncharged identical metallic spheres 1,2 and 3 each of radius r and are placed at the vertices of an equilateral triangle of side d. A charged metallic sphere having charge q of same radius r is touched to sphere 1, after some time it is taken to the location of sphere 2 and is touched to it, then it is taken far away from spheres 1,2 and 3. After that the sphere 3 is grounded, the charge on sphere 3 is (neglect electrostatic induction by assuming d>> 2r)

1) Zero 2) $\frac{-3qr}{4d}$ 3) $\frac{-qr}{2d}$ 4) $\frac{-qr}{4d}$

SOLUTION :



$$V_3 = \frac{1}{4\pi \in_0}\left(\frac{q}{2d} + \frac{q}{4d} + \frac{Q^1}{r}\right) = 0$$

$$Q^1 = -\frac{3qr}{4d}$$

81. The electric field potential in space has the form V(x, y, z) = -2xy+3yz^{-1}. The electric field intensity $\vec{E}$ magnitude at the point (-1, 1, 2) is

1) $2\sqrt{86}$ units 2) $2\sqrt{163}$ units

3) $\sqrt{163}$ units 4) $\sqrt{86}$ units

SOLUTION :

Let the required electric field $\vec{E}$ he written as $\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

so that E_x, E_y and E_z are the components of field strength along X, Y and Z axes respectively.

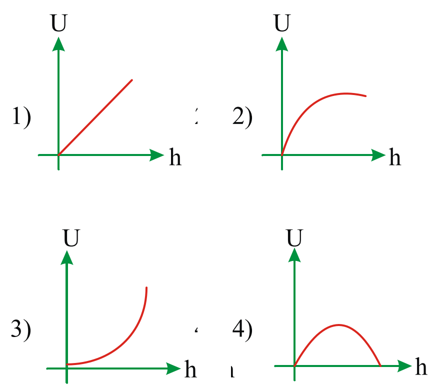
$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(-2x + 3yz^2\right) = 2y \qquad E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial t}\left(-2xy + 3yz^2\right) = 2x - 3z^2$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left(-2xy + 3yz^2\right) = -6yz \quad \vec{E} = (qy)\hat{i} + \left(2x - 3z^2\right)\hat{j} + (-6yz)\hat{k}\,\Sigma$$

At the point (-1, 1, 2) $\vec{E} = 2\hat{i} - 14\hat{j} - 12\hat{k}$

The magnitude of $\vec{E}$ is $E = \sqrt{(2)^2 + (-14)^2 + (-12)^2} = 2\sqrt{86}$ unit

82. A particle of mass m and charge q is projected vertically upwards. A uniform electric field $\vec{E}$ is acted vertically downwards. The most appropiate graph between potential energy U (gravitational plus electrostatic) and height h(<< radius of earth) is : (assume U to be zero on surface of earth)



SOLUTION :

$$g_{eff} = g + \left(\frac{qE}{m}\right)$$

$$\Delta w = U_f = m\left(g + \frac{qE}{m}\right)h$$

$$U \propto h$$

83. The electric potential at a point (x, 0, 0) is given by $V = \left[\frac{1000}{x} + \frac{1500}{x^2} + \frac{500}{x^3}\right]$ then the electric field at x = 1 m is (in volt/m)

1) $-5500\hat{i}$ 2) $5500\hat{i}$ 3) $\sqrt{5500}\hat{i}$ 4) zero

SOLUTION :

$$V = \left[\frac{1000}{x} + \frac{1500}{x^2} + \frac{500}{x^3}\right] \quad E_x = \frac{dv}{dx}$$

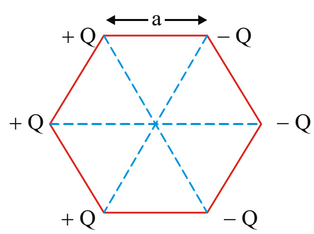
$$E_x = \frac{1000}{x^2} + \frac{2(1500)}{x^3} + \frac{3(500)}{x^4}$$

x=1

E_x = 1000+3000+1500

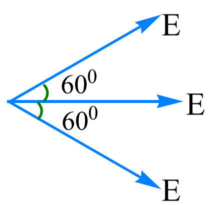
$$E_x = 5500\hat{i}$$

84. Six charges are placed at the vertices of a regular hexagon as shown in the figure. The electric field on the line passing through point O and perpendicular to the plane of the figure at a distance of x (>> a) from O is



1) $\frac{Qa}{\pi\varepsilon_0 x^3}$ 2) $\frac{2Qa}{\pi\varepsilon_0 x^3}$ 3) $\frac{\sqrt{3}Qa}{\pi\varepsilon_0 x^3}$ 4) zero

SOLUTION :



$$E_{equitorial} = \frac{1}{4\pi \epsilon_0}\frac{P}{x^3}$$

$$P = 2Qa$$

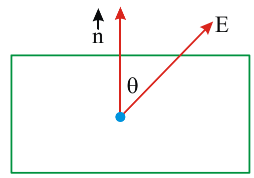
$$E_R = 2E_{eq} = \frac{1}{4\pi \epsilon_0}.\frac{2P}{x^3} = \frac{Qa}{\pi \epsilon_0 x^3}$$

GAUSS LAW

Electric flux:

It is the measure of total number of electric lines of force crossing normally the given area.

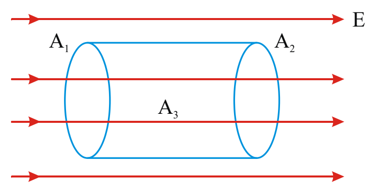
↳ The total flux passes through the given surface is given by $\phi = \vec{E}.\vec{A}$



$\therefore \phi = EA\cos\theta$

where θ is the angle made by the normal with the electric field.

For a closed body outward flux is taken to be positive while inward flux is taken to be negative.



a) Flux through A_1 : negative

b) Flux through A_2 : positive

c) Flux through A_3 : 0

Gauss Law

i) According to this law, the total flux linked with a closed surface called Gaussian surface is $(1/\varepsilon_0)$ times the net charge enclosed by the closed surface.

ii) Alternatively, Gauss law can be stated as the surface integral of electric field $\vec{E}$ over a closed surface is equal to $1/\epsilon_0$ times the charge (q) enclosed by that closed surface.

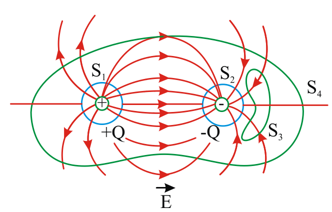
i.e., $\phi = \oint \vec{E}.\overline{dA} = \frac{q}{\epsilon_0}$

q is the total charge enclosed by the Gaussian surface.

iii) Coulomb's law can be derived from Gauss law.

iv) The electric field $\vec{E}$ is the resultant field due to all charges, both those inside and those outside the Gaussian surface.

v) The electric field due to a charge outside the Gaussian surface contributes zero net flux through the surface, Because as many lines due to that charge enter the surface as leave it.



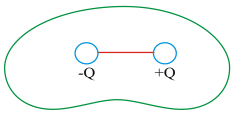
a) Flux from surface $S_1 = +\frac{Q}{\varepsilon_0}$

b) Flux from surface $S_2 = \frac{-Q}{\varepsilon_0}$

c) Flux from S_3 = flux from surface $S_4 = 0$

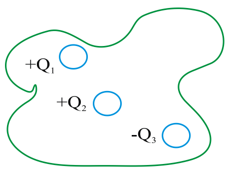
Applications of Gauss Law :

i) If a dipole is enclosed by a surface



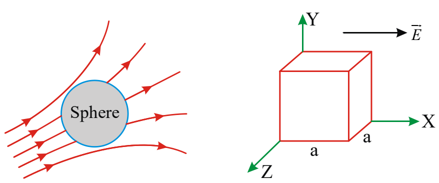
$\because Q_{enc} = 0 \quad \Rightarrow \phi = 0$

ii) The net charge Q_{enc} is the algebraic sum of the enclosed positive and negative charges. If Q_{enc} is positive then the net flux is outwards. If Q_{enc} is negative then the net flux is inwards.



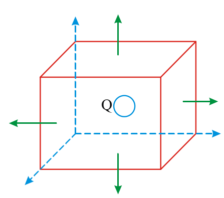
$$\phi = \frac{1}{\varepsilon_0}(Q_1 + Q_2 - Q_3)$$

iii) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non - uniform) total flux linked with it will be zero



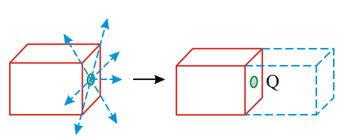
(A) $\phi_T = 0$ (B) $\phi_{in} = \phi_{out} = Ea^2 \Rightarrow \phi_T = 0$

iv) If charge is kept at the centre of cube



$$\phi_{total} = \frac{1}{\varepsilon_0}(Q) \; ; \; \phi_{face} = \frac{1}{6\varepsilon_0}(Q)$$

v) If charge is kept at the centre of a face, first we should enclose the charge by assuming a Gaussian surface (an identical imaginary cube)

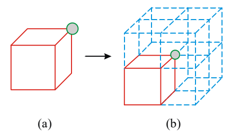


(a) (b)

Total flux emerges from the system (Two cubes) $\phi_{total} = \frac{Q}{\varepsilon_0}$

Flux from given cube (i.e., from left side 5 faces only) $\phi_{cube} = \frac{Q}{2\varepsilon_0}$

vi) If a charge is kept at the corner of a cube



For enclosing the charge seven more cubes are required so total flux from the 8 cube system is $\phi_T = \frac{Q}{\varepsilon_0}$.

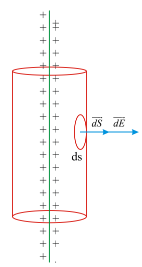
Flux from given cube $\phi_{cube} = \frac{Q}{8\varepsilon_0}$.

Flux from one face opposite to the charge, of the given cube

$\phi_{face} = \frac{Q/8\varepsilon_0}{3} = \frac{Q}{24\varepsilon_0}$ (Because only three faces are seen).

Electric field at a point due to a line of charge: A thin straight wire over which 'q' amount of charge be uniformly distributed. λ be the linear charge density i.e, charge present per unit length of the wire.

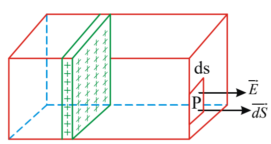
$$E = \frac{q}{2\pi \in_0 rl} \quad E = \frac{\lambda}{2\pi \in_0 r}$$



↳ This implies electric field at a point due to a line charge is inversely proportional to the distance of the point from the line charge.

Electric field intensity at a point due to a thin infinite charged sheet [Non conducting plate]

'q' amount of charge be uniformly distributed over the sheet. Charge present per unit surface area of the sheet be σ. i.e surface charge density σ

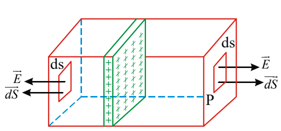


$$E = \frac{q}{2A \in_0}; \; E = \frac{\sigma}{2 \in_0} \; where \; \sigma = \frac{q}{A}$$

↳ E is independent of the distance of the point from the charged sheet.

Electric field intensity at a point due to a thick infinite charged sheet [Conducting plate] :

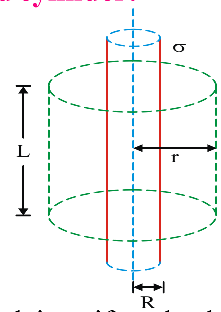
'q' amount of charge be uniformly distributed over the sheet. Charge present per unit surface area of the sheet be σ.



$$E = \frac{q}{A \in_0} = \frac{\sigma}{\in_0}$$

Electric field at a point due to a thick charged sheet is twice that produced by the thin charged sheet of same charge density.

Electric field due to long uniformly charged cylinder:



Consider a long cylinder of radius R which is uniformly charged on its surface with charge density σ. We know that at the interior points of a metal body electric field strength is zero. Let us find the electric field at a point and at a distance r from the axis of the cylinder. Consider a cylindrical Gaussian surface of radius r and length L as shown in the figure.

From Gauss's law, we can write

$$\oint \bar{E}.d\bar{s} = \frac{1}{\epsilon_0}(q_{en})$$

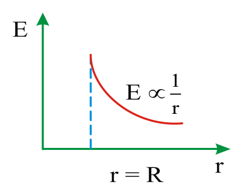
Here $q_{enclosed} = \sigma 2\pi RL$

Here electric flux through the circular faces is zero. So, from Gauss law

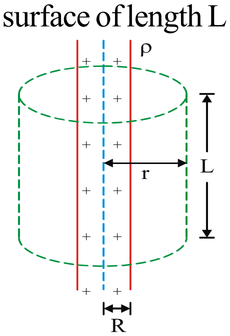
$$\oint \bar{E}.d\bar{s} = \frac{\sigma 2\pi RL}{\epsilon_0} \text{ or } E2\pi rL = \frac{\sigma 2\pi RL}{\epsilon_0}$$

$$\Rightarrow E = \frac{\sigma R}{\epsilon_0 r}$$

The variation of E with distance r from the axis is as shown in the graph.



Electric field due to uniformly charged non-conducting cylinder: Consider a long cylinder of radius R charged with volume charge density ρ uniformly. Let us find electric field at a distance r from the axis of the cylinder. Consider a cylindrical Gaussian surface of length L and radius r as shown,



$$\oint \bar{E}.d\bar{s} = \frac{q_{encl}}{\epsilon_0}; \text{ where } q_{encl} = \rho\pi R^2 L$$

Here electric flux through the circular faces is zero.

Case (i): If $r > R$, then from Gauss's law

$$\oint \bar{E}.d\bar{s} = \frac{\rho\pi R^2 L}{\epsilon_0} \Rightarrow E2\pi rL = \frac{\rho\pi R^2 L}{\epsilon_0}$$

$$\text{or } E = \frac{\rho R^2}{2\epsilon_0 r} \Rightarrow E_{out} \propto \frac{1}{r}$$

Case (ii): If $r = R$, then $E = \dfrac{\rho R}{2\epsilon_0}$

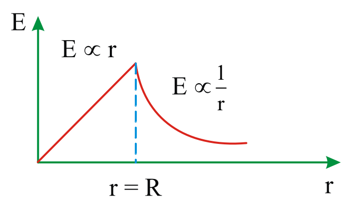
Case (iii): If $r < R$, $q_{encl} = \rho\pi r^2 L$

from Gauss law $\oint \bar{E}.d\bar{s} = \dfrac{q_{encl}}{\epsilon_0}$

$$E2\pi rL = \frac{\rho\pi r^2 L}{\epsilon_0} \text{ (or) } E = \frac{\rho r}{2\epsilon_0} \Rightarrow E_{in} \propto r$$

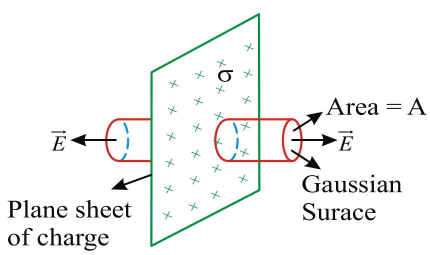
In vector form $\bar{E} = \dfrac{\rho\vec{r}}{2\epsilon_0}$

The variation of E with distance r from the axis is as shown in the graph.



Electric Intensity and electric potential due to infinite plane sheet of charge (nonconducting):

If E is the magnitude of electric field at point P, then electric flux crossing through the gaussian surface is given by



ϕ = E x area of the end face (circular caps) of the cylinder

or ϕ = E x 2A ....(i)

According to Gauss's theorem, we have

$$\phi = \frac{q}{\varepsilon_0}$$

Here, the charge enclosed by the gaussian surface, $q = \sigma A$

$$\therefore \phi = \frac{\sigma A}{\epsilon_0} \qquad ....(ii)$$

From equations (i) and (ii), we have

$$E \times 2A = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad E = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

Where $\hat{n}$ is unit vector normal to the plane and away from it.

Thus, we find that the magnitude of the electric field at a point due to an infinite plane sheet of charge is independent of its distance from the sheet of charge.

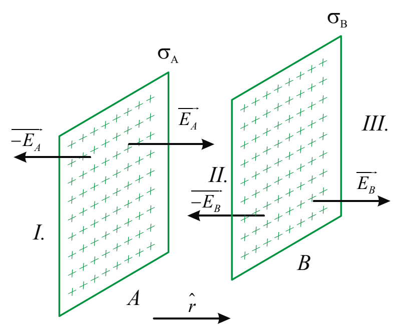
Electric intensity due to two thin parallel charged sheets:

Two charged sheets A and B having uniform charge densities σ_A and σ_B respectively.

In region I :

$$E = \frac{1}{2\epsilon_0}(\sigma_A + \sigma_B)$$

In region II:

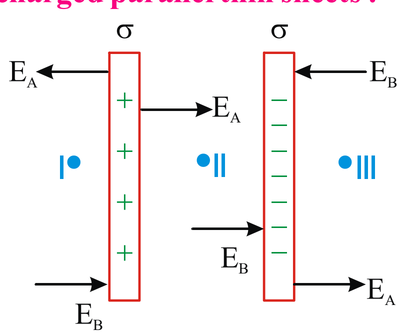


$$E_{II} = \frac{1}{2\epsilon_0}(\sigma_A - \sigma_B)$$

In region III :

$$E_{III} = \frac{1}{2\epsilon_0}(\sigma_A + \sigma_B)$$

Electric field due to two oppositely charged parallel thin sheets :



$$E_I = -\frac{1}{2\epsilon_0}\left[\sigma + (-\sigma)\right] = 0$$

$$E_{II} = \frac{1}{2\epsilon_0}[\sigma - (-\sigma)] = \frac{\sigma}{\epsilon_0}$$

$$E_{III} = \frac{1}{2 \in_0}(\sigma - \sigma) = 0$$

Electric field due to a charged Spherical shell

'q' amount of charge be uniformly distributed over a spherical shell of radius 'R'

$\sigma =$ Surface charge density, $\sigma = \frac{q}{4\pi R^2}$

When point 'P' lies outside the shell (r>R):

$$E = \frac{1}{4\pi \in_0} \times \frac{q}{r^2}$$

↳ This is the same expression as obtained for electric field at a point due to a point charge. Hence a charged spherical shell behaves as a point charge concentrated at the centre of it.

$$E = \frac{1}{4\pi \in_0} \frac{\sigma . 4\pi R^2}{r^2} \left(\because \sigma = \frac{q}{4\pi R^2} \right)$$

$$E = \frac{\sigma . R^2}{\in_0 r^2}$$

When point 'P' lies on the shell (r =R) :

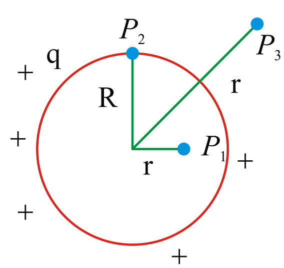
$$E = \frac{\sigma}{\in_0}$$

When Point 'P' lies inside the shell (r <R):

E = 0

↳ The electric intensity at any point due to a charged conducting solid sphere is same as that of a charged conducting sperical shell.

Electric Potential (V) due to a Uniformly Charged sphe-rical c onducting shell (Hollow sphere)

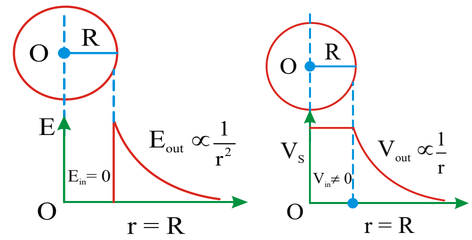


↳ When point (P_3) lies outside the sphere $(r > R)$, the electric potential, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

↳ When point (P_2) lies on the surface $(r = R)$, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

↳ When point (P_1) lies inside the surface $(r < R)$, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

↳ Note: The electric potential at any point inside the sphere is same and is equal to that on the surface.

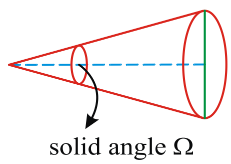


(A) (B)

Note: The electric potential at any point due to a charged conducting sphere is same as that of a charged conducting spherical shell

Solid angle

Solid angle is the three dimensional angle subtended by the lateral surface of a cone at its vertex



Let us calculate the solid angle subtended by a surface X at a point O. Join all the points of the periphery of the surface X to the point O by straight lines as shown. It gives a cone with vertex at O.

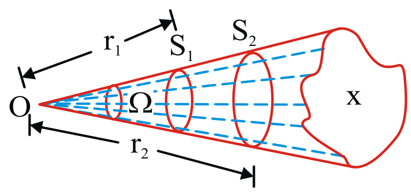


Fig. (b)

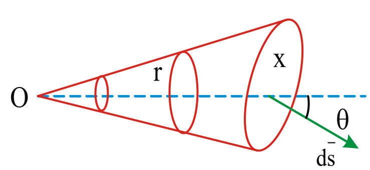
By taking centre at O, we draw several spherical sections on this cone of different radii as shown. Let the area of spherical section which is of radius r_1 be s_1 and the area of section of radius r_2 be s_2. **The ratios of area of any surface intersected by cone to the square of radius of that sphere is a constant** and it gives actually the solid angle Ω From the figure, solid angle subtended by surface X at the point O is given by

$$\Omega = \frac{s_1}{r_1^2} = \frac{s_2}{r_2^2}.$$

Note: SI unit of solid angle is steradian and it is a dimensionless quantity.

one steradian is the solid angle subtended at the centre of the sphere by the surface of the sphere having area equal to square of the radius of the sphere.

The surface subtending solid angle need not be normal to the axis of the cone. For example consider a surface X of area $d\bar{s}$ as shown. The axis of cone formed by the surface at O is not normal to the surface. In this cone solid angle Ω subtended at point O can be given as $\Omega = \frac{ds\cos\theta}{r^2}$



Here θ is the angle between $d\bar{s}$ and axis of the cone.

Relation between semi- vertex angle of a cone and solid angle substended

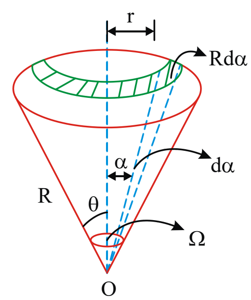
Consider a spherical surface of radius R. Let X be a surface on that sphere which substends a semi vertex angle θ (in radian) at the centre of the sphere. Now consider an elemental strip of this section of radius $r = R \sin\alpha$ and angular width $d\alpha$ as shown. Then surface area of this strip is given by $ds = (2\pi R \sin\alpha) R d\alpha$.

The total area of spherical section can be obtained by integrating this elemental area from 0 to θ.

Total area of spherical section is

$$S = \int ds = \int_0^\theta 2\pi R^2 \sin\alpha \; d\alpha$$

$$= 2\pi R^2 (-\cos\alpha)_0^\theta \qquad = 2\pi R^2 (1 - \cos\theta)$$



If Ω is solid angle subtended by this section at the centre O, then its area is given by $S = \Omega R^2$ (as discussed earlier) So, we can write $\Omega R^2 = 2\pi R^2 (1 - \cos\theta)$ and $\Omega = 2\pi(1 - \cos\theta)$

Note : The solid angle substended by a hemispherical surface at its centre is given by

$$\Omega = 2\pi\left(1 - \cos 90^0\right) = 2\pi \text{ steradians}$$

If $\theta = 180^0$ in the previous case, we get the solid angle substended by a closed surface

$$\Omega = 2\pi\left(1 - \cos 180^0\right) = 4\pi \text{ steradians}$$

The total solid angle substanded by a closed surface is always 4π steradians, irrespective of the size and shape of the closed surface.

Cavity in the conductor

We have discussed that there will be no electric field inside a charged conductor and all the charge resides on its outer surface only. Suppose that charged conductor has a cavity or cavities and there are no charges within the cavity or cavities, even then charge resides on the outer surface of the conductor. There will be no charge on the walls of the cavity or cavities. This can be verfied very easily using Gauss's law by enclosing the cavity with a Gaussian surface.

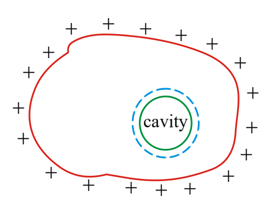


Fig.

$\oint \bar{E}.d\bar{s} = 0$ For the dotted surface.

$\Rightarrow q = 0$ inside cavity.

Consider a conductor with spherical cavity inside it. There is no charge on the conductor. Now a point charge +q is kept at the centre of the cavity. Due to this charge, a charge -q is induced on the inner surface of cavity . The total flux originated by +q will terminate on the cavity walls and no field lines enter into the conductor body

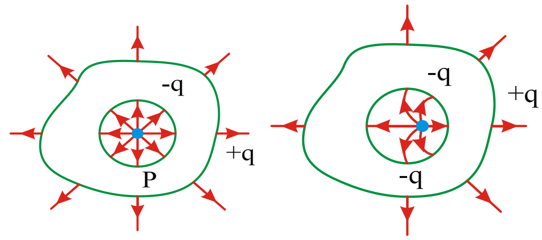


Fig. (a) Fig. (b)

We can consider a Gaussian surface around the cavity and prove that induced charge on the cavity walls is -q. The reason is electric field $\left(\overline{E}\right)$ is zero inside the material of the conductor. The Total enclosed charge within the Gaussian surface is zero. Here the conductor is initially uncharged. From conservation of charge, we can say that on the outer surface of the conductor a charge +q will be induced. At any point inside the material of conductor, say at P, the electric field produced by +q in the cavity is cancelled by the field produced by charges induced on the walls of cavity and on the outer surface of the conductor. If the point charge is not at the centre of the spherical cavity, even then induced charges on the cavity walls and on the outer surface of the conductor are -q and +q only.

But the distribution of induced charges will change in such a way that at any point P in the material of the conductor resultant electric field is zero.

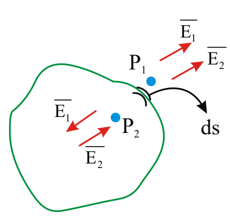
Suppose the conductor has charge q_0 on it initially. This charge resides on the outer surface of the conductor. If point charge q is kept inside the cavity, induced charges on the walls of cavity and on the outer surface of the conductor are the same as before. i.e., – q and + q. But the total charge on the outer surface of the conductor is $(q_0 + q)$ now.

If the charge inside the cavity is displaced, the induced charge distribution on inner surface of the body changes such that at any point inside the material of the conductor resultant field is zero. In this case the charge distribution on outer surface of the conductor does not change and only the charge distribution on the cavity walls will change.

Now the charge inside the cavity is fixed. If another charge is brought towards the conductor from outside., it will not affect the charge distribution inside the cavity and only the distribution of charge on the outer surface will be affected.

Mechanical force on the charged conductor

We know that like charges repel each other. So, when a conductor is charged, the charge on any point of the conductor is repelled by the charge on its remaining part. It means surface of a charged conductor experiences mechanical force.



Consider a charged conductor as shown. Let ds be the surface area of a small element on the conductor. The electric field at point P_1 near the conductor surface can be considered as the superposition of fields $\overline{E}_1$ and $\overline{E}_2$. Here $\overline{E}_1$ is the field produced by that elemental surface and $\overline{E}_2$ is the field due to the remaing surface of the conductor.

$$\overline{E} = \overline{E}_1 + \overline{E}_2$$

But we know that $E = \frac{\sigma}{\epsilon_0}$ at P_1 which is just outside the conductor and is zero at P_2 which is just inside the conductor

So at P_1 we have $E_1 + E_2 = \frac{\sigma}{\epsilon_0}$

and at P_2 we have $E_1 - E_2 = 0$

$$\Rightarrow E_1 = E_2 = \frac{\sigma}{2\epsilon_0}$$

Now the force experienced by small surface ds due to the charge on the rest of the surface is

$$F = (dq)E_2 = (\sigma ds)E_2 = \frac{\sigma^2 ds}{2\epsilon_0}$$

and $\frac{\text{Force}}{\text{Area}} = \frac{F}{ds} = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2$

Electric pressure on a charged surface

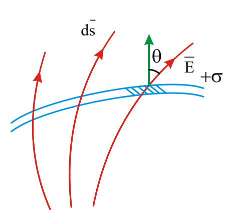
From the above derivation we observed that a small surface of a charged conductor will experience a force by the remaining surface. The force per unit area of the surface is $\frac{1}{2}\epsilon_0 E^2$ or $\frac{\sigma^2}{2\epsilon_0}$

This is known as electric pressure on the charged metal surface.

$$\Rightarrow P_e = \frac{1}{2}\epsilon_0 E^2$$

Suppose a charged body is in an external electric field. Let us find out the electric pressure on the surface of that charged body.

Consider a surface uniformly charged with charge density σ. On that surface 'ds' is the surface area of a small element. The charge on that element is $dq = \sigma ds$



The given surface is in an external electric field represented by the field lines as shown.

Let E be the intensity of electric field on the elemental surface. Here angle between $\overline{E}$ and $d\overline{s}$ is θ. In this case $\overline{E}$ has two comperments.

Component parallel to the surface is

$E_{11} = E\ \sin\theta$

and component normal to the surface is

$E_{\perp} = E\ \cos\theta$

Here force due to E_{11} on the surface is tangential which tries to stretch the surface. Where as the force due to $E_{\perp}$ applies outward pressure on the surface. Now outward force on the elemental surface is

$dF = (dq)E_{\perp} = \sigma\, ds\, E_{\perp}$

So, the outwards electric pressure on the surface is

$$P_e = \frac{dF}{ds} = \sigma E_{\perp} \Rightarrow P_e = \sigma E \cos\theta$$

PROBLEMS

1: A particle that carries a charge '–q' is placed at rest in uniform electric field 10 N/C. It experiences a force and moves. In a certain time 't', it is observed to acquire a velocity $10\bar{i} - 10\bar{j}$ m/s. The given electric field intersects a surface of area $1m^2$ in the x–z plane. Find the Electric flux through the surface.

SOLUTION :

Force on charge $\bar{F} = q\bar{E}$

$\therefore$ particle moves opposite to $\bar{E}$ with $\bar{V}$

unit vector in the direction of $\bar{V}$ is $\frac{\bar{i}}{\sqrt{2}} - \frac{\bar{j}}{\sqrt{2}}$

unit vector in the direction of $\bar{E}$ is $\frac{\bar{i}}{\sqrt{2}} - \frac{\bar{j}}{\sqrt{2}}$

$\bar{E} = 10\left[\frac{-i}{\sqrt{2}} + \frac{j}{\sqrt{2}}\right]$ ie. $\bar{A} = 1 \times \bar{j}$

Electric flux $\phi = \bar{E} \cdot \bar{A} = 5\sqrt{2}\ \ Nm^2/C$

2. A solid metallic sphere has a charge +3Q. Concentric with this sphere is a conducting spherical shell having charge +Q. The radius of the sphere is a and that of the spherical shell is b, (b>a). What is the electric field at a distance R (a<R<b) from the centre.

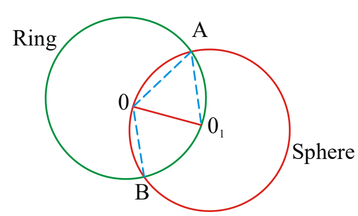
1) $\frac{Q}{2\pi\varepsilon_0 R}$ 2) $\frac{3Q}{2\pi\varepsilon_0 R}$ 3) $\frac{3Q}{4\pi\varepsilon_0 R^2}$ 4) $\frac{4Q}{2\pi\varepsilon_0 R^2}$

SOLUTION :

$$\int E.d\bar{l} = \frac{Q_{encl}}{\epsilon_0}$$

$$E = \frac{3Q}{4\pi \epsilon_0 R^2}$$

3. A charge Q is distributed uniformly on a ring of radius r. A sphere of equal radius r is constructed with its centre at the periphery of the ring as shown in figure. Find the flux of the electric field through the surface of the sphere.



1) $\frac{Q}{3\varepsilon_0}$ 2) $\frac{q}{\varepsilon_0}$ 3) $\frac{q}{2\varepsilon_0}$ 4) zero

SOLUTION :

From the geometry of the figure. $OA = OO_1$ and $O_1A = O_1O$. Thus, OAO_1 is equilateral triangle. Hence $\angle AOO_1 = 60^0$ or $\angle AOB = 120^0$.

The are AO_1B of the ring subtends an angle 120^0 at the centre O. Thus, third of the ring is inside the sphere.

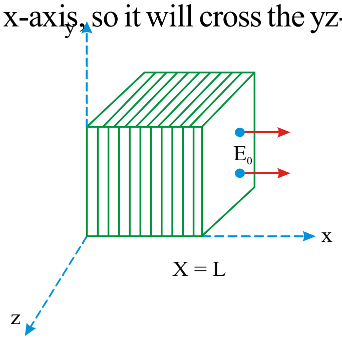
The charge enclosed by the sphere $= \frac{Q}{3}$. From Gauss's law, the flux of the electric field through the surface of the sphere is $\frac{Q}{3\varepsilon_0}$

4: The electric field in a region is given by $\vec{E} = E_0 \frac{x}{L}\hat{i}$. Find the charge contained inside a cubical volume bounded by the surface x = 0, x = L, y =0 , y = L, z = 0 and z = L.

SOLUTION :

At x = 0, E = 0 and at x = l, $\vec{E} = E_0\hat{i}$

The direction of the field is along the x-axis, so it will cross the yz-face of the cube. The flux of this field



$\phi = \phi_{\text{left face}} + \phi_{\text{right face}}$; $\quad = 0 + E_0L^2 = E_0L^2$

By Gauss's law, $\phi = \frac{q}{\epsilon_0}$ $\therefore q = \epsilon_0 \phi = \epsilon_0 E_0L^2$

5. A charge 'q' is distributed over two concentric hollow conducting spheres of radii a and b (b > a) such that their surface charge densites are equal. The potential at their common centre is

1) Zero

2) $\frac{q}{4\pi\epsilon_0}\frac{(a+b)}{(a^2+b^2)^2}$

3) $\frac{q}{4\pi\epsilon_0}\left[\frac{1}{a}+\frac{1}{b}\right]$

4) $\frac{q}{4\pi\epsilon_0}\left[\frac{a+b}{(a^2+b^2)}\right]$

SOLUTION :

$\sigma = \frac{q_1}{4\pi a^2} = \frac{q_2}{4\pi b^2}$; $\frac{q_1}{q_2} = \frac{a^2}{b^2}$, $q_1 + q_2 = q$

$q_1 + q_1\left[\frac{b^2}{a^2}\right] = q$

$q_1\left[1+\frac{b^2}{a^2}\right] = q$; $q_1\left[\frac{a^2+b^2}{a^2}\right] = q$

$q_1 = \frac{qa^2}{a^2+b^2}$; $q_2 = \frac{qb^2}{a^2+b^2}$

potential at commoncentre

$V = \frac{q\,a}{4\pi\epsilon_0\left(a^2+b^2\right)} \times \frac{q\,b}{4\pi\epsilon_0\left(a^2+b^2\right)}$

$$V=\frac{q}{4\pi\in_0}\left[\frac{a}{a^2+b^2}+\frac{b}{a^2+b^2}\right]=\frac{q}{4\pi\in_0}\left[\frac{a+b}{a^2+b^2}\right]$$

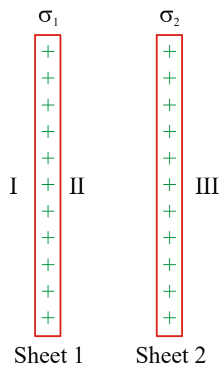
6. Two parallel plane sheets 1 and 2 carry uniform charge densities σ_1 and σ_2 as shown in fig. electric field in the region marked II is $(\sigma_1>\sigma_2)$

1) $-\frac{(\sigma_1+\sigma_2)}{2\varepsilon_0}$

2) $\frac{-(\sigma_1\sigma_2)}{2\varepsilon_0}$

3) $\frac{(\sigma_1+\sigma_2)}{2\varepsilon_0}$

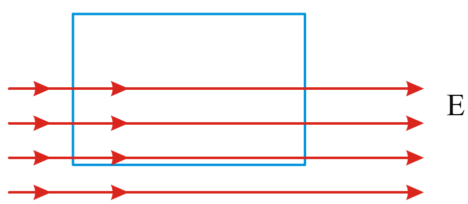
4) $\frac{(\sigma_1-\sigma_2)}{2\varepsilon_0}$



SOLUTION :

$$E_{net}=E_1-E_2=\frac{\sigma_1}{2\in_0}-\frac{\sigma_2}{2\in_0};\ E_{net}=\frac{\sigma_1-\sigma_2}{2\in_0}$$

7. A square surface of side lm in the plane of the paper. A uniform electric field E (V/m) also in the plane of the paper, is limited only to the lower half of the square surface, the electric flus (in SI units) associated with the surface is.



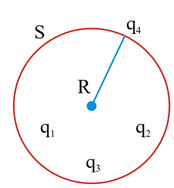
SOLUTION :

A, electric flux, $\phi_E=\int E.ds$

$=\int E\,ds\cos\theta=\int E\,ds\cos 90^0=0$

Thus, the lines are parallel to the surface.

8. q_1,q_2,q_3 and q_4 are point charges located at points as shown in the figure as S is a spherical Gaussian surface of radius R. Which of the following is true according to the Gauss's law



1) $\oint(\vec{E}_1+\vec{E}_2+\vec{E}_3).d\vec{A}=\frac{q_1+q_2+q_3}{2\varepsilon_0}$

2) $\oint(\vec{E}_1+\vec{E}_2+\vec{E}_3+\vec{E}_4).d\vec{A}=\frac{(q_1+q_2+q_3)}{\varepsilon_0}$

3) $\oint\left(\vec{E}_1+\vec{E}_2+\vec{E}_3+\vec{E}_4\right).d\vec{A}=\dfrac{\left(q_1+q_2+q_3+q_4\right)}{\varepsilon_0}$

4) None of the above

SOLUTION :

$$\oint\left(\overline{E}_1+\overline{E}_2+\overline{E}_3+\overline{E}_4\right).\overline{ds}=\frac{q}{\in_0}=\frac{q_1+q_2+q_3}{3}$$

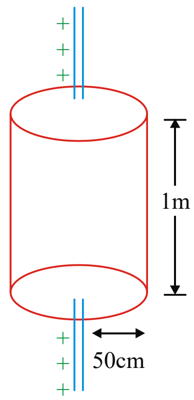
9. Electric charge is uniformly distributed along a long straight wire of radius 1 mm. The charge per cm length of the wires is Q coulomb. Another cylindrical surface of radius 50 cm and length 1 m symmetrically encloses the wire as shown in the figure. The total electric flux passing through the cylindrical surface is

1) $\dfrac{Q}{\varepsilon_0}$

2) $\dfrac{100Q}{\varepsilon_0}$

3) $\dfrac{10Q}{(\pi\varepsilon_0)}$

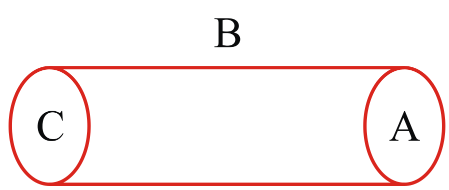
4) $\dfrac{100Q}{(\pi\varepsilon_0)}$



SOLUTION :

The total flux passing through cylindrical surface

10: A hollow cylinder has a chartge 'q' coulomb within it. If ϕ is the electric flux in unit of V-m, associated with the curved surface B, the electirc flux linked with the plane surface A in unit of V-m, will be

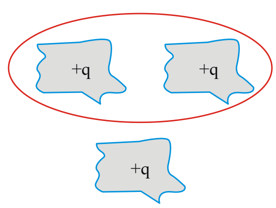


SOLUTION :

We have, $\phi_{total}=\phi_A+\phi_B+\phi_c=\dfrac{q}{\varepsilon_0}$

$2\phi^{'}+\phi=\dfrac{q}{\varepsilon_0}$ $\quad\Rightarrow \phi^{'}=\dfrac{1}{2}\left(\dfrac{q}{\varepsilon_0-\phi}\right)$

11. Shown below is a distribution of charges. The flux of electric field due to these charges through the surface S is

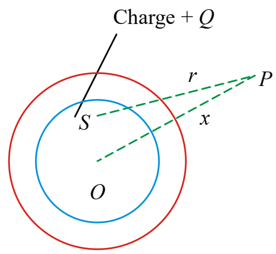


1) $3q/\varepsilon_0$ 2) $2q/\varepsilon_0$ 3) q/ε_0 4) Zero

SOLUTION :

The $\oint \cdot ds = \frac{q+q}{\in_1} = \frac{2q}{\in_0} = \frac{Q/cm}{\in_0} = \frac{100Q}{\in_0}$

12. The adjacent diagram shows a charge +Q held on an insulating suppot S and enclosed by a hollow spherical conductor, O represents the centre of the spherical conductor and P is a point such that OP=x and SP=r. The electric field at point, P will be



SOLUTION :

. According to Gauss's' theorem,

$$\oint E.ds = \frac{Q_{in}}{\varepsilon_0}; \Rightarrow E.4\pi x^2 = \frac{Q}{\varepsilon_0} \text{ or } E = \frac{Q}{4\pi\varepsilon_0 x^2}$$

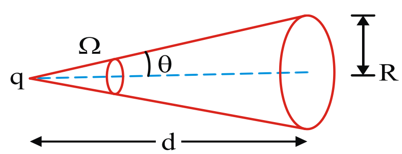
13. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where, 'r' is the distance from the centre, a and b are constants. Then the charge density the ball is

SOLUTION :

Here, $\phi = ar^2 + b$; As $\phi = ar^2 + b$ $\therefore \oint E.ds = \frac{q}{\varepsilon_0}$; $-2ar.4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow q = -8\varepsilon_0 a\pi r^3$

$$\rho = \frac{q}{\frac{4}{3}\pi r^3}; \Rightarrow \rho = -6a\varepsilon_0$$

14. A point charge q is placed at a distance d from the centre of a circular disc of radius R. Find electric flux through the disc due to that charge



SOLUTION :

Sol : We know that total flux originated from a point charge q in all directions is $\frac{q}{\in_0}$. This flux is originated in a solid angle 4π. In the given case solid angle subtended by the cone subtended by the disc at the point charge is $\Omega = 2\pi(1-\cos\theta)$

So, the flux of q which is passing through the surface of the disc is

$$\phi = \frac{q}{\in_0}\frac{\Omega}{4\pi} = \frac{q}{2\in_0}(1-\cos\theta)$$

From the figure, $\cos\theta = \dfrac{d}{\sqrt{d^2 + R^2}}$ so

$$\phi = \frac{q}{2\in_0}\left\{1 - \frac{d}{\sqrt{d^2+R^2}}\right\}$$

15. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance R/2 from the centre of the shell is

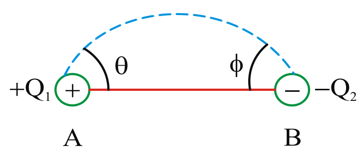
1) $\dfrac{2Q}{4\pi\varepsilon_0 R}$ 2) $\dfrac{2Q}{4\pi\varepsilon_0 R} - \dfrac{2q}{4\pi\varepsilon_0 R}$

3) $\dfrac{2Q}{4\pi\varepsilon_0 R} + \dfrac{q}{4\pi\varepsilon_0 R}$ 4) $\dfrac{(q+Q)}{4\pi\varepsilon_0}\dfrac{2}{R}$

SOLUTION :

$$V = \frac{1}{4\pi\in_0}\left[\frac{2Q}{R} + \frac{q}{R}\right]$$

16. Two point charges $+Q_1$ and $-Q_2$ are placed at A and B respectively. A line of force emanates from Q_1 at an angle θ with the line joining A and B. At what angle will it terminate at B?



SOLUTION :

We know that number of lines of force emerge is proportional to magnitude of the charge. The field lines emanating from Q_1, spread out equally in all directions. The number of field lines or flux through cone of half angle θ is $\frac{Q_1}{4\pi}2\pi(1-\cos\theta)$. Similarly the number of lines of force terminating on $-Q_2$ at an angle ϕ is $\frac{Q_2}{4\pi}2\pi(1-\cos\phi)$. The total lines of force emanating from Q_1 is equal to the total lines of force terminating on Q_2

$$\Rightarrow \frac{Q_1}{4\pi}2\pi(1-\cos\theta) = \frac{Q_2}{4\pi}2\pi(1-\cos\phi)$$

or $\dfrac{Q_1}{2}(1-\cos\theta) = \dfrac{Q_2}{2}(1-\cos\phi)$; $Q_1\sin^2\theta/2 = Q_2\sin^2\phi/2$

$$\sin\phi/2 = \sqrt{\frac{Q_1}{Q_2}}\sin\theta/2 \quad \Rightarrow \phi = 2\sin^{-1}\left\{\sqrt{\frac{Q_1}{Q_2}}\sin\theta/2\right\}$$

17. Two concentric sphere of radii a_1 and a_2 carry charges q_1 and q_2 respectively. If the surface charge density (σ) is same for both spheres, the electric potential at the common centre will be

1) $\dfrac{\sigma}{\in_0}\dfrac{a_1}{a_2}$ 2) $\dfrac{\sigma}{\in_0}\dfrac{a_2}{a_1}$ 3) $\dfrac{\sigma}{\in_0}(a_1 - a_2)$ 4) $\dfrac{\sigma}{\in_0}(a_1 + a_2)$

SOLUTION :

$$V_1 = \frac{\sigma a_1}{\in_0}, V_2 = \frac{\sigma a_2}{\in_0}$$

$$v = \frac{\sigma a_1}{\in_0}(a_1 + a_2)$$

18. A long string with a charge of λ per unit length passes through an imaginary cube of edge a. The maximum flux of the electric field through the cube will be

1) $\lambda a / \in_0$ 2) $\sqrt{2}\lambda a / \in_0$ 3) $6\lambda a^2 / \in_0$ 4) $\sqrt{3}\lambda a / \in_0$

SOLUTION :

Max. Flux exists when max length of charged wire is enclosed in cube.

$\therefore$ Max. length of wire inside cube = $\sqrt{3}l$

$$\therefore \phi = \left(\frac{\sqrt{3}l.\lambda}{\in_0}\right)$$

19. A rod with linear charge density λ is bent in the shape of circular ring. The electric potential at the centre of the circular ring is

1) $\frac{\lambda}{4\varepsilon_0}$ 2) $\frac{\lambda}{2\varepsilon_0}$ 3) $\frac{\lambda}{\varepsilon_0}$ 4) $\frac{2\lambda}{\varepsilon_0}$

SOLUTION :

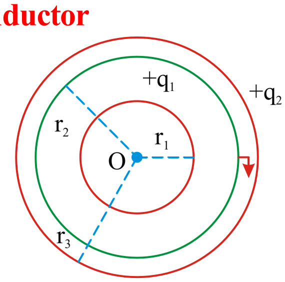
$$V = \frac{1}{4\pi \in_0}.\frac{Q}{r} \Rightarrow \lambda = \frac{Q}{2\pi r}$$

$$V = \frac{2\pi r\lambda}{4\pi \in_0 r} = \frac{1}{2\in_0}$$

20. Assume three concentric conducting spheres where charge q_1 *and* q_2 have been placed on inner and outer sphere where as middle sphere has been earthed. Find the charge on the outer surface of middle spherical conductor

1) $-\frac{r_2}{r_3}q_2$ 2) $-q_1$

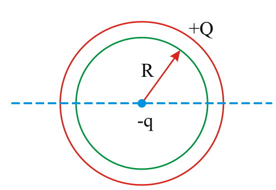
3) $-q_2$ 4) $\frac{r_2}{r_3}q_1$



SOLUTION :

$$\frac{q_1}{r_2} + \frac{q - q_1}{r_2} + \frac{q_2}{r_3} = 0 \quad \therefore \quad q = \frac{-r_2}{r_3}q_2$$

21. A thin spherical shell radius of r has a charge Q uniformly distributed on it. At the centre of the shell, a negative point charge -q is placed. If the shell is cut into two identical hemispheres, still equilibrium is maintained. Then find the relation between Q and q?



SOLUTION :

Sol : Here the outward electric pressure at every point on the shell due to its own charge is

$$P_1 = \frac{\sigma^2}{2\in_0} = \frac{1}{2\in_0}\left(\frac{Q}{4\pi r^2}\right)^2 ; \; P_1 = \frac{Q^2}{32\pi^2 \in_0 r^4}$$

Due to -q, the electric field on the surface of the shell is

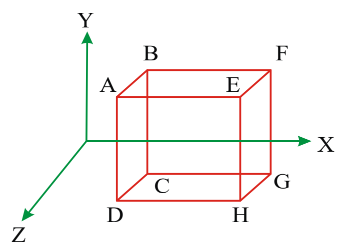
$$E = \frac{1}{4\pi\in_0}\frac{q}{r^2}$$

This electric field pulls every point of the shell in inward direction. The inward pressure on the surface of the shell due to the negative charge is $P_2 = \sigma E \; ; = \left(\frac{Q}{4\pi r^2}\right)\left(\frac{1}{4\pi\in_0}\frac{q}{r^2}\right) = \frac{Qq}{16\pi^2\in_0 r^4}$

For equlibrium of the hemispherical shells $P_2 \geq P_1$ or $\frac{Qq}{16\pi^2\in_0 r^4} \geq \frac{Q^2}{32\pi^2\in_0 r^4}$; $\quad q \geq \frac{Q}{2}$

COMPREHENSION

The electric field in a region is given by $\overline{E} = (\alpha x)\bar{i}$. Here is α is a constant of proper dimensions.



22. Find the total flux passing through a cube bounded by surfaces $x = l, x = 2l, y = 0, y = l, z = 0, z = l.$

1) αl^3 2) $2\alpha l^3$ 3) $3\alpha l^3$ 4) $4\alpha l^3$

SOLUTION :

$$\phi = \phi_1 + \phi_2 = \phi_{ABCD} + \phi_{EFGH}$$

$$\phi = + \int_{ABCD} E.d\bar{s} + \int_{EFGH} E.d\bar{s} E = \alpha x\hat{i}$$

$$\phi = -\alpha l l^2 + 2\alpha l.l^2$$

$$\phi = \alpha l^3$$

23. The charge contained inside the above cube is

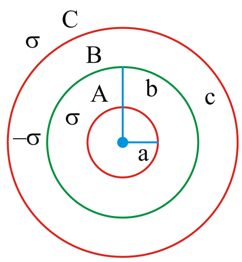
1) $2\alpha \in_0 l^3$ 2) $\alpha\in_0 l^3$ 3) $4\alpha\in_0 l^3$ 4) $3\alpha\in_0 l^3$

SOLUTION :

. $\phi = \frac{Q_{encl}}{\in_0}$

$$\phi_{end} = \alpha l^3 \in_0$$

24. Three concentric metallic spheres A, B and C have radii a,b and c (a < b < c) and surface charge densities on them are $\sigma, -\sigma$ and σ respectively. The values of V_A and V_B will be



1) $\frac{\sigma}{\epsilon_0}(a-b+c), \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b}-b+c\right)$ 2) $(a-b+c), \frac{a^2}{c}$

3) $\frac{\epsilon_0}{\sigma}(a-b+c), \frac{\epsilon_0}{\sigma}\left(\frac{a^2}{b}-b+c\right)$ 4) $\frac{\sigma}{\epsilon_0}\left(\frac{a^2}{c}-\frac{b^2}{c}+c\right), \frac{\sigma}{\epsilon_0}(a-b+c)$

SOLUTION :

$$V_A = \frac{1}{4\pi\epsilon_0}\left(\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right)$$

$$\Rightarrow V_A = \frac{\sigma}{\epsilon_0}(a-b+c) \text{ and}$$

$$\Rightarrow V_B = \frac{1}{4\pi\epsilon_0}\left(\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right)$$

$$\Rightarrow V_B = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b}-b+c\right)$$

25. If r and T are radius and surface tension of a spherical soap bubble respectively then find the charge needed to double the radius of bubble

SOLUTION :

For smaller bubble

$P_1 = \left(P_0 + \frac{4T}{r}\right)$ and $V_1 = \frac{4}{3}\pi r^3$ For larger bubble

$P_2 = P_0 + \frac{4T}{R} - \frac{\sigma^2}{2\epsilon_0}$ and $V_2 = \frac{4}{3}\pi R^3$ where $\sigma = \frac{q}{4\pi R^2}$

for air in the bubble, $P_1V_1 = P_2V_2$

$$\left(P_0 + \frac{4T}{r}\right)r^3 = \left[\left(P_0 + \frac{4T}{R}\right) - \frac{q^2}{16\pi^2 R^4 \times 2\epsilon_0}\right]R^3$$

$$P_0\left[R^3 - r^3\right] + 4T\left[R^2 - r^2\right] - \frac{q^2}{32\pi^2\epsilon_0 R} = 0$$

But R = 2r

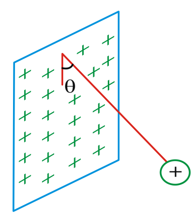
$$P_0\left[7r^3\right] + 4T\left[3r^2\right] - \frac{q^2}{32\pi^2\epsilon_0(2r)} = 0$$

$$\frac{q^2}{64\pi^2 \in_0 r} = 7P_0r^3 + 12Tr^2$$

$$q^2 = 64\pi^2 \in_0 r^3\left[7P_0r + 12T\right]$$

$$q = 8\pi r\left[\in_0 r\left(7P_0r + 12T\right)\right]^{1/2}$$

26. A charged ball hangs from silk thread which makes an angle 'θ' with large charged conducting sheet 'P' as shown. The surface charge density (σ) of the sheet is proportional to



1) $\cos\theta$ 2) $\cot\theta$ 3) $\sin\theta$ 4) $\tan\theta$

SOLUTION :

$$E = \frac{\sigma}{\in_0}$$

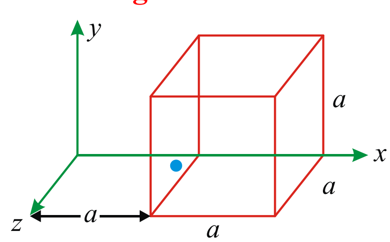
$$F = mg\tan\theta$$

$$q \times \frac{\sigma}{\in_0} = ma\tan\theta$$

$$\sigma = \left(\frac{mg \in_0}{q}\right)\tan\theta$$

$$\sigma \;\alpha\; \tan\theta$$

27. The electric field components in the figure are $E_x = \alpha x^{1/2}, E_y = 0, E_z = 0$ where α = 800 N / m². If a = 0.1 m is the side of cube then the charge within the cube is



1) $9.27\times10^{-12}C$ 2) $6\times10^{-12}C$

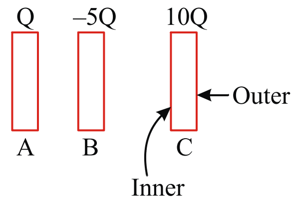
3) $2.5\times10^{-12}C$ 4) Zero

SOLUTION :

Magnitude of E at the left face $E_L = \alpha a^{1/2}$ at right face $E_R = \alpha\left(2a\right)^{1/2}$

$\phi = \left(E_R - E_L\right)a^2$ and $q = \phi \in_0$

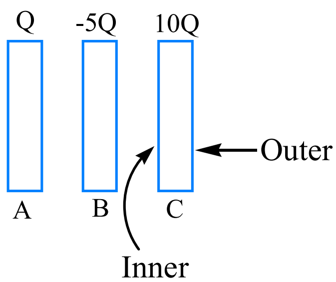
28. Three very large plates are given charges as shown in the figure. If the cross-sectional area of each plate is the same, the final charge distribution on plate C is



1) +5Q on the inner surface , +5Q on the outer surface
2) +6Q on the inner surface, +4Q on the outer surface
3) +7Q on the inner surface, +3Q on the outer surface
4) +8Q on the inner surface, +2Q on the outer surface

SOLUTION :

Inside the conductor field E=0.



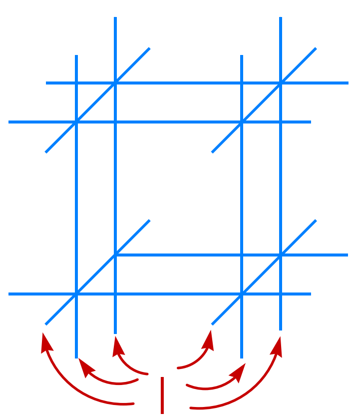
Field due to induced charges will be reduced to $\vec{E}_{inside} = \frac{Q}{\epsilon_0 A} + \frac{-5Q}{\epsilon_0 A} + \frac{q}{\epsilon_0 A} - \left(\frac{10-q}{\epsilon_0 A}\right) = 0$

$Q = -5Q + q - 10 + q = 0$

q = 7Q inside surface

10 - q = 3Q outside surface

29. Twelve infinite ling wire of uniform linear charge density (1) are passing along the twelve edges of a cube. Find electric flux through any face of cube.



1) $\left(\frac{\lambda\ell}{2\varepsilon_0}\right)$ 2) $\left(\frac{\lambda\ell}{\varepsilon_0}\right)$ 3) $\left(\frac{\lambda\ell}{3\varepsilon_0}\right)$ 4) $\left(\frac{3\lambda\ell}{\varepsilon_0}\right)$

SOLUTION :

ϕ_{cube} = Flux due to single wire from whole cube

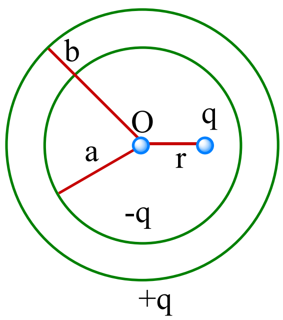
$\phi_{cube} = \dfrac{\lambda \ell}{8\varepsilon_0}$ similarly four wires out of twelve will have same contribution and eight will have zero

$\phi_{face_1} = \dfrac{\lambda \ell}{8\varepsilon_0} \times 4 = \dfrac{\lambda \ell}{2\varepsilon_0}$

30. A point charge q is a distance r from the centre O of an uncharged spherical conducting layer, whose inner and outer radii equal to a and b respectively. The potential at the point

O if $r < a$ is $\dfrac{q}{4\pi \in_0}$ times

1) $\left(\frac{1}{r}-\frac{1}{a}+\frac{1}{b}\right)$ 2) $\left(\frac{1}{a}-\frac{1}{r}+\frac{1}{b}\right)$ 3) $\left(\frac{1}{b}-\frac{1}{c}-\frac{1}{r}\right)$ 4) $\left(\frac{1}{a}-\frac{1}{b}-\frac{1}{r}\right)$



if $e < q$

$$= V = \frac{q}{4\pi \in_0}\left[\frac{1}{r}-\frac{1}{a}+\frac{1}{b}\right]$$

31. One-fourth of a sphere of radius R is removed as shown in fig. An electric field E exists parallel to x-y plane. Find the flux through the remaining curved part.

1) $\pi R^2 E$ 2) $\sqrt{2}\pi R^2 E$ 3) $\pi R^2 E/\sqrt{2}$ 4) $2\pi R^2 E$

SOLUTION :

$\phi = \vec{E}.\left(\overrightarrow{A_1} + \overrightarrow{A_2}\right)$

CONCEPTUAL BITS

1. Dimensions of ε_0 are

   1) $\left[M^{-1}L^{-3}T^{4}A^{2}\right]$ 2) $\left[M^{0}L^{-3}T^{3}A^{3}\right]$ 3) $\left[M^{-1}L^{-3}T^{3}A\right]$ 4) $\left[M^{-1}L^{-3}TA^{2}\right]$

KEY:1

2. A soap bubble is given a negative charge, then its radius.
   1) Decreases
   2) Increases
   3) Remanins unchanged
   4) Nothing can be predicted as information is insufficient

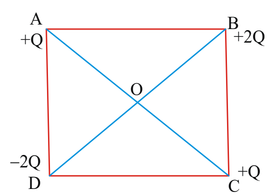
KEY:2

3. Two charges are placed at a distance apart. If a glass slab is placed between them, force between them will
   1) be zero 2) increase
   3) decrease 4) remains the same

KEY:3

4. The p.d $\left(V_B - V_c\right)$ between two points from C to B
   1) does not depend on the path
   2) depends on the path
   3) depends on test charge
   4) independent of electric field

5. Four charges are arranged at the corners of a square ABCD as shown in the figure. The force on the positive charge kept at the centre 'O' is



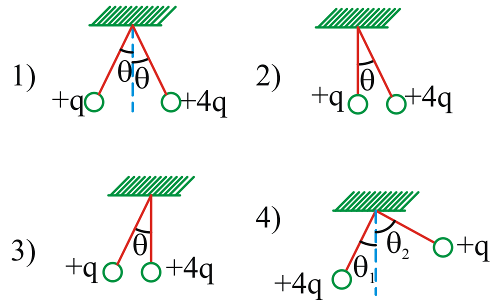
   1) zero
   2) along the diagonal AC
   3) along the diagonal BD
   4) perpendicular to side AB

KEY:3

6. Two identical +ve charges are at the ends of a straight line AB. Another identical +ve charge is placed at 'C' such that AB=BC. A, B and C being on the same line. Now the force on 'A'
   1) increases 2) decreases
   3) remains same 4) we cannot say

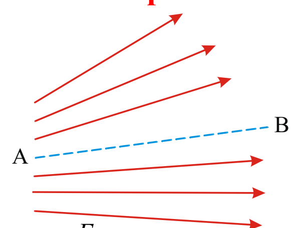
KEY:1

7. Two metal spheres of same mass are suspended from a common point by a light insulating string. The length of each string is same. The spheres are given electric charges +q on one end and +4q on the other. Which of the following diagram best shows the resulting positions of spheres?



KEY:1

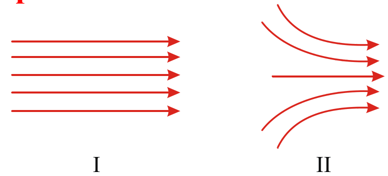
8. Figure shows the electric lines of force emerging from a charged body. If the electric field at 'A' and 'B' are E_A and E_B respectively and if the displacement between 'A' and 'B' is 'r' then



1) $E_A > E_B$ 2) $E_A < E_B$ 3) $E_A = \frac{E_B}{r}$ 4) $E_A = \frac{E_B}{r^2}$

KEY:1

9. Drawings I and II show two samples of electric field lines



1) The electric fields in both I and II are produced. by negative charge located somewhere on the left and positive charges located somewhere on the right
2) In both I and II the electric field is the same every where
3) In both cases the field becomes stronger on moving from left to right
4) The electric field in I is the same everywhere, but in II the electric field becomes stronger on moving from left to right

KEY:4

10. An electron and proton are placed in an electric field. The forces acting on them are F_1 and F_2 and their accelerations are a_1 and a_2 respectively then

1) $\overline{F}_1 = \overline{F}_2$ 2) $\overline{F}_1 + \overline{F}_2 = 0$
3) $|\overline{a}_1| = |\overline{a}_2|$ 4) $|\overline{a}_1| \geq |\overline{a}_2|$

KEY:2

11. The bob of a pendulum is positively charged. Another identical charge is placed at the point of suspension of the pendulum. The time period of pendulum
1) increases 2) decreases
3) becomes zero 4) remains same.

KEY:4

12. Intensity of electric field inside a uniformly charged hollow sphere is
1) zero 2) non zero constant
3) change with r

4) inversely proportional to r

KEY:1

13. A positive charge q_0 placed at a point P near a charged body experiences a force of repulsion ofmagnitude F, the electric field E of the charged body at P is

1) $\frac{F}{q_0}$ 2) $< \frac{F}{q_0}$ 3) $> \frac{F}{q_0}$ 4) F

KEY:2

14. A cube of side b has charge q at each of its vertices. The electric field at the centre of the cube will be (KARNATAKA CET 2000)

1) zero 2) $\frac{32q}{b^2}$ 3) $\frac{q}{2b^2}$ 4) $\frac{q}{b^2}$

KEY:1

15. Two copper spheres of the same radii, one hollow and the other solid, are charged to the same potential, then

1) hollow sphere holds more charge
2) Sol...id sphere holds more charge
3) both hold equal charge
4) we can't say

KEY:3

16. A charged bead is capable of sliding freely through a string held vertically in tension. An electric field is applied parallel to the string so that the bead stays at rest of the middle of the string. If the electric field is switched off momentarily and switched on

1) the bead moves downwards and stops as soon as the field is switched on
2) the bead moved downwards when the field is switched off and moves upwards when the field is switched on
3) the bead moves downwards with constant acceleration till it reaches the bottom of the string
4) the bead moves downwards with constant velocity till it reaches the bottom of the string

KEY:4

17. $E = -\frac{dV}{dr}$, here negative sign signified that

1) E is opposite to V 2) E is negative
3) E increases when V decreases
4) E is directed in the direction of decreasing V

KEY:4

18. A charged particle is free to move in an electric field

1) It will always move perpendicular to the line of force
2) It will always move along the line of force in the direction of the field.
3) It will always move along the line of force opposite to the direction of the field.
4) It will always move along the line of force in the direction of the field or opposite to the direction of the field depending on the nature of the charge

KEY:4

19. Two parallel plates carry opposite charges such that the electric field in the space between them is in upward direction. An electron is shot in the space and parallel to the plates. Its deflection from the original direction will be

1) Upwards 2) Downwards
3) Circular 4) does not deflect

KEY:2

20. Potential at the point of a pointed conductor is
1) maximum 2) minimum
3) zero 4) same as at any other point
KEY:4
21. Two point charges $-q$ and $+2q$ are placed at a certain distance apart. Where should a third point charge be placed so that it is in equilibrium?
1) on the line joining the two charges on the right of $+2q$
2) on the line joining the two charges on the left of $-q$
3) between $-q$ and $+2q$
4) at any point on the right bisector of the line joining $-q$ and $+2q$.
KEY:2

22. When a positively charged conductor is placed near an earth connected conductor, its potential
1) always increases 2) always decreases
3) may increase or decrease 4) remains the same
KEY:2
23. An electron moves with a velocity $\vec{v}$ in an electric field $\vec{E}$. If the angle between $\vec{v}$ and $\vec{E}$ is neither 0 nor π, then path followed by the electron is
1) straight line 2) circle
3) ellipse 4) parabola
KEY:4
24. If a unit charge is taken from one point to another over an equipotential surface, then
1) work is done on the charge
2) work is done by the charge
3) work on the charge is constant
4) no work is done
KEY:4
25. Electric potential at some point in space is zero. Then at that point
1) electric intensity is necessarily zero
2) electric intensity is necessarily non zero.
3) electric intensity may or may not be zero
4) electric intensity is necessarily infinite.
KEY:3
26. When an electron approaches a proton, their electro static potential energy
1) decreases 2) increases
3) remains unchanged 4) all the above
KEY:1
27. Two charges +q and –q are kept apart. Then at any point on the right bisector of line joining the two charges.
1) the electric field strength is zero
2) the electric potential is zero
3) both electric potential and electric field strength are zero
4) both electric potential and electric field strength are non - zero
KEY:2
28. When 'n' small drops are made to combine to form a big drop, then the big drop's
1) Potential increases to $n^{1/3}$ times original potential and the charge density decreases to $n^{1/3}$ times original charge

2) Potential increases to $n^{2/3}$ times original potential and charge density increases to $n^{1/3}$ times original charge density
3) Potential and charge density decrease to $n^{1/3}$ times original values
4) Potential and charge density increases to 'n' times original values

KEY:2

29. A hollow metal sphere of radius 5cm is charged such that the potential on its surface is 10V. The potential at the centre of the sphere is
1) 0 V 2) 10 V
3) same as at point 5cm away from the surface
4) same as at point 25cm from the surface

KEY:2

30. Which of the following pair is related as in work and force
1) electric potential and electric intensity
2) momentum and force
3) impulse and force
4) resistance and voltage

KEY:1

31. The equipotential surfaces corresponding to single positve charge are concentric spherical shells with the charge at its origin. The spacing between the surfaces for the same change in potential
1) is uniform throughout the field
2) is getting closer as $r \to \infty$
3) is getting closer as $r \to 0$
4) can be varied as one wishes to

KEY:3

32. Four identical charges each of charge q are placed at the corners of a square. Then at the centre of the square the resultant electric intensity E and the net electric potential V are
1) $E \neq 0, V = 0$ 2) $E = 0, V = 0$
3) $E = 0, V \neq 0$ 4) $E \neq 0, V \neq 0$

KEY:3

33. Equipotential surfaces are shown in figure a and b. The field in

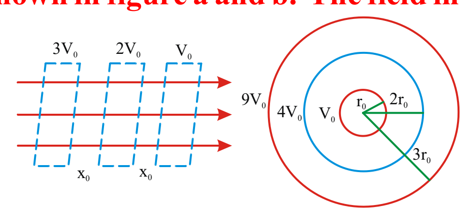


FIGURE (A) FIGURE (B)

1) a is uniform only 2) b is uniform only
3) a and b is uniform 4)both are nonuniform

KEY:1

34.. Due to the motion of a charge, its magnitude
1) changes
2) does not changes
3) increases (or) decreases depends on its speed
4) can not be predicted

KEY:2

35. The coulomb electrostatic force is defined for
1) two spherical charges at rest
2) two spherical charges in motion

3) two point charges in motion
4) two point charges at rest

KEY:4

36. A ring with a uniform charge Q and radius R, is placed in the yz plane with its centre at the origin

a) The field at the origin is zero

b) The potential at the origin is $k\frac{Q}{R}$

c) The filed at the point (x, 0, 0) is $k\frac{Q}{x^2}$ d) The field at the point (x, 0, 0) is $k\frac{Q}{R^2+x^2}$

Choose the correct answer

1) a and b are true   2) c is true
3) a,b,c are true   4) a,b,c,d are true

KEY:1

37. Match List-I with List-II

List-I   List-II

a) proton and electron   e) gains same velocity in an elctric field for same time
b) proton and positron   f) gains same KE in an electric field for same time.
c) Deutron and α - particle   g) experience same force in electric field
d) electron and positron   h) gains same KE when accelerated by same potential difference.

1) $a-h, b-g, c-e, d-f$
2) $a-h, b-g, c-f, d-e$
3) $a-g, b-h, c-e, d-f$
4) $a-e, b-f, c-g, d-h$

KEY:1

38. Match List-I with List-II

List-I   List-II

a) Two like charges are brought nearer   e) the force between them decreases.
b) Two unlike charge of some brought nearer   f) potential energy of the system increases
c) When a third charge of same nature is placed equidistance from two like charges   g) mutual forces are not affected
d) When a dielectric medium is introduced between two charges   h) potential energy of the system decreases

1) $a-h, b-f, c-g, d-e$
2) $a-f, b-h, c-g, d-e$
3) $a-h, b-f, c-e, d-g$
4) $a-g, b-e, c-f, d-h$

KEY:2

39. Match the following :

a) Electric field outside a conducting charged sphere — e) Constant

b) Electric potential out side the conducting charged sphere — f) directly propor national to distance from centre

c) Electric field inside a non-conducting charged sphere — g) inversely propor tional to the distance

d) Electric potential in side a charged conducting sphere — h) inversely proportional to the square of the distance

1) $a-h, b-g, c-e, d-f$

2) $a-e, b-f, c-h, d-g$

3) $a-h, b-g, c-f, d-e$

4) $a-g, b-h, c-f, d-e$

KEY:3

40. An electron and proton are sent into an electric field. The ratio of force experienced by them is

1) 1 : 1 2) 1 : 1840

3) 1840 : 1 4) 1 : 9.11

KEY:1

41. The angle between electric dipolemoment p and the electric field E when the dipole is in stable equilibrium

1) 0 2) $\pi/4$ 3) $\pi/2$ 4) π

KEY:1

42. 'Debye' is the unit of

1) electric flux 2) electric dipolemoment

3) electric potential 4) electric field intensity

KEY:1

43. An equipotential line and a line of force are

1)perpendicular to each other

2)parallel to each other

3) in any direction

4) at an angle of 45^0

KEY:1

44. The electric field at a point at a distance r from an electric dipole is proportional to

1) $\frac{1}{r}$ 2) $\frac{1}{r^2}$ 3) $\frac{1}{r^3}$ 4) r^2

KEY:3

45. An electric dipole placed with its axis in the direction of a uniform electric field experiences

1) a force but not torque

2) a torque but no force

3) a force as well as a torque

4) neither a force nor a torque

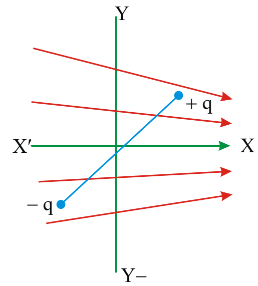
KEY:4

46. An electron enters an electric field with its velocity in the direction of the electric lines of force.

Then
1) the path of the electron will be a circle
2) the path of the electron will be a parabola
3) the velocity of the electron will decrease
4) the velocity of the electron will increase

KEY:3

47. An electric dipole is placed in a non uniform electric field increasing along the +ve direction of X - axis. In which direction does the dipole



1) move along + ve direction of X - axis, rotate clockwise
2) move along - ve direction of X - axis, rotate clockwise
3) move along + ve direction of X - axis, rotate anti clockwise
4) move along - ve direction of X - axis, rotate anti clockwise

KEY:1

48. An electric dipole placed in a nonuniform electric field experiences
1) a force but no torque
2) a torque but no force
3) a force as well as a torque
4) neither a force nor a torque

KEY:3

49. If E_a be the electric field intensity due to a short dipole at a point on the axis and E_r be that on the perpendicular bisector at the same distance from the dipole, then
1) $E_a = E_r$ 2) $E_a = 2E_r$
3) $E_r = 2E_a$ 4) $E_a = \sqrt{2E_r}$

KEY:2

50. A negatively charged particle is situated on a straight line joining two other stationary particles each having charge +q. The direction of motion of the negatively charged particle will depend on
1) the magnitude of charge
2) the position at which it is situated
3) both magnitude of charge and its position
4) the magnitude of +q

KEY:2

51. The electric potential due to an extremely short dipole at a distance r from it is proportional to
1) $\frac{1}{r}$ 2) $\frac{1}{r^2}$ 3) $\frac{1}{r^3}$ 4) $\frac{1}{r^4}$

KEY:2

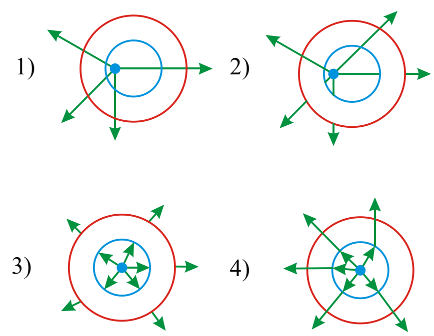
52. The angle between the electric dipole moment and the electric field strength due to it, on the equatorial line is
1) 0^0 2) 90^0 3) 180^0 4) 60^0

KEY:3

53. The acceleration of a charged particle in a uniform electric field is
1) proportional to its charge only
2) inversely proportional to its mass only
3) proportional to its specific charge
4) inversely proportional to specific charge

KEY:3

54. A metallic shell has a point charge q kept inside its cavity. Which one of the following diagrams correctly represents the electric lines of forces ?



ASSERTION & REASON

In each of the following questions, a statement of Assertion (A) is given followed by a corresponding statement of Reason (R) just below it. Mark the correct answer.
1) Both 'A' and 'R' are true and 'R' is the correct explanation of 'A'
2) Both 'A' and 'R' are true and 'R' is not the correct explanation of 'A'
3) 'A' is true and 'R' is false
4) 'A' is false and 'R' is true

KEY:3

55. Assertion(A) : Force between two point charges at rest is not changed by the presence of third point charge between them.
Reason(R): Force depends on the magnitude of the first two charges and seperation between them

KEY:1

56. Assertion (A): Electric potential at any point on the equatorial line of an electric dipole is zero
Reason (R): Electric potential is scalar

KEY:1

57. Assertion (A) : Electrons always move from a region of lower potential to a region of highe potential
Reason (R) : Electrons carry a negative charge

KEY: 1

58. Assertion(A): A metallic shield in form of a hollow shell may be built to block an electric field.
Reason (R): In a hollow spherical shield, the electric field inside it is zero at every point.

KEY: 1

59. Assertion (A): For practical purpose, the earth is used as a reference for zero potential in electrical circuits.
Reason (R): The electrical potential of a sphere of radius R with charge Q uniformly distributed on the surface is given by $\frac{Q}{4\pi\varepsilon_0 R}$

KEY:2

60. Assertion(A): Coulomb force between charges is central force
Reason (R): Coulomb force depends on medium between charges

KEY:2

61. Assertion(A): Electric and gravitational fields are acting along same line. When proton and α -

particle are projected up veritically along that line, the time of flights is less for proton.
Reason (R): In the given electric field acceleration of a charged particle is directly proportional to specific charge

KEY:1

62. Assertion(A): When a proton with certain energy moves from low potential to high potential then its KE decreases.
Reason (R): The direction of electric field is opposite to the potential gradient and work done against it is negative.

KEY:2

63. Assertion(A): In bringing an electron towards a proton electrostatic potential energy of the system increases.
Reason (R): Potential due to proton is positive

KEY: 4

64. Assertion(A): The surface of a conductor is an equipotential surface
Reason (R): Conductor allows the flow of charge

KEY:2

65. Assertion (A) : A charge ' q_1 ' exerts some force on a second charge ' q_2 ' . If a third charge ' q_3 ' is brought near , the force exerted by q_1 on q_2 does not change
Reason (R): The elecrtostatic force between two charges is independent of presence of third charge

KEY:2

66. Assertion (A) : A point charge 'q' is rotated along a circle around another point charge Q. The work done by electric field on the rotating charge in half revolution is zero.
Reason (R) : No work is done to move a charge on an equipotential line or surface.

KEY: 1

67. Assertion: (A): Work done by electric force is path independent.
Reason: (R): Electric force is conservative

KEY:1

68. Assertion (A): In bringing an electron towards a proton electrostatic potential energy of the system increases.
Reason (R): Potential due to proton is positive.

KEY:4

69. Assertion(A): Two particles of same charge projected with different velocity normal to electric field experience same force
Reason (R): A charged particle experiences force, independent of velocity in electric field

KEY:1

70. Assertion(A): The coulomb force is the dominating force in the universe
Reason (R): The coulomb force is stronger than the gravitational force.

KEY:4

71. Assertion(A): A circle is drawn with a point positive charge $(+q)$ at its centre. The work done in taking a unit positive charge once around it is zero
Reason (R): Displacement of unit positive charge is zero

KEY:2

72. Assertion(A): Electric potential at any point on the equatorial line of electric dipole is zero.
Reason (R): Electric potential is scalar

KEY: 2

73. Assertion(A): The potential at any point due to a group of $'N'$ point charges is simply arrived at by the principle of superposition

Reason (R): The potential energy of a system of two charges is a scalar quantity

KEY:2

74. Assertion (A): The electrostatic potential energy is independent of the manner in which the cofiguration is achieved

Reason (R): Electrostatic field is conservative field

KEY:1

STATEMENT QUESTIONS

75. Statement-1:- For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement-2:- The net work done by a conservative force on an ojecte moving along a closed loop is zero

1) Statement-1 is true, statement-2 is true, Statement-2 is the correct explanation of statement-1.

2) Statement-1 is true, statement-2 is true, Statement-2 is not the correct explanation of statement-1.

3) Statement-1 is false, Statement-2 is true.

4) Statement-1 is true, Statement-2 is false

KEY:1

76. Out of the following statements

A. Three charge system can not have zero mutual potential energy

B. The mutual potential energy of a system of charges is only due to positive charges

1) A is wrong and B is correct

2) A is correct and B is wrong

3) Both A and B are correct

4) Both A and B are wrong

KEY:4

77. Statement A: Electrical potential may exist at a point where the electrical field is zero

Statement B : Electrical Field may exist at a point where the electrical potential is zero.

Statement C : The electric potential inside a charge conducting sphere is constant.

1) A, B are true 2) B,C are true

3) A,C are true 4) A,B,C are true

KEY:4

78. Statement A: If an electron travels along the direction of electric field it gets accelerated

Statement B: If a proton travels along the direction of electric field it gets retarded

1) Both A & B are true 2) A is true, B is false

3) A is false, B is true 4) Both A & B are false

KEY:4

79. Choose the wrong statement

1) Work done in moving a charge on equipotential surface is zero.

2) Electric lines of force are always normal to an equipotential surface

3) When two like charges are brought nearer, then electrostatic potential energy of the system gets decreased.

4) Electric lines of force diverge from positive charge and converge towards negative charge.

KEY:3

80. A : Charge cannot exist without mass but mass can exist without charge.

B : Charge is invariant but mass is variant with velocity

C : Charge is conserved but mass alone may not be conserved.

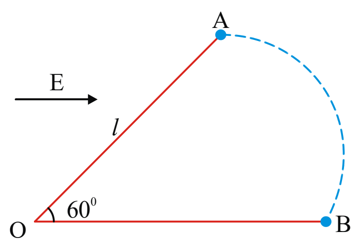
1) A, B, C are true 2) A, B, C are not true
3) A, B are only true 4) A, B are false, C is true

KEY:1

81. A dielectric slab of thickness d is inserted in a parallel plate capacitor whose negative plate is at x = 3d. The slab is equidistant from the plates. The capacitor is given some charge. As 'x' goes from 0 to 3d
1) the magnitude of the electric field remains the same
2) the direction of the electric field remains the same
3) the electric potential increases continuously
4) the electric potential dicreases at first, then increases and again dicreases

KEY:2

82. A particle of mass m and charge q is fastened to one end of a string fixed at point O. The whole system lies on a frictionless horizontal plane. Initially, the mass is at rest at A. A uniform electric field in the direction shown is then switched on. Then



1) the speed of the particle when it reaches B is $\sqrt{\frac{2qE\ell}{m}}$

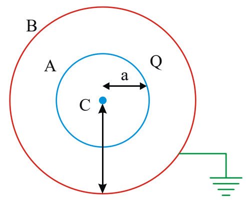
2) the speed of the particle when it reaches B is $\sqrt{\frac{qE\ell}{m}}$

3) the tension in the string when particles reaches at B is $\frac{Eq}{2}$.

4) the tension in the string when the particle reaches at B is qE.

KEY:2

83. A conducting sphere A of radius a, with charge Q, is placed concentrically inside a conducting shell B of radius b. B is earthed. C is the common centre of the A and B



p) The field at a distance r from C, where $a \leq r \leq b$, is $k\frac{Q}{r^2}$

q) The potential at a distance r from C, where $a \leq r \leq b$, is $k\frac{Q}{r}$

r) The potential difference between A and B is $kQ\left(\frac{1}{a}-\frac{1}{b}\right)$

s) The potential at a distance r from C, where $a \le r \le b$, is $kQ\left(\frac{1}{r}-\frac{1}{b}\right)$

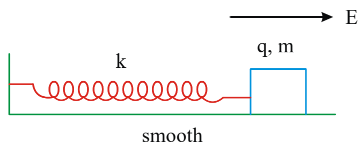
Choose the correct answer

1) p and r are true 2) q is true

3) p,r,s are true 4) p,q,r,s are true

KEY:3

84. A block of mass m is attached to a spring of force constant k. Charge on the block is q. A horizontal electric field E is acting in the direction as shown. Block is released with the spring in unstretched position



a) block will execute SHM

b) Time period of oscillation is $2\pi\sqrt{\frac{m}{k}}$

c) amplitude of oscillation is $\frac{qE}{k}$

d) Block will oscillate but not simple harmonically

Choose the correct answer

1) a and b are true 2) d is true

3) a,b,c are true 4) a,b,c,d are true

KEY:3

85. The Electric field is given by $\vec{E}=\frac{\vec{F}}{q_0}$, here the test charge '$q_0$' should be

a) Infinitesimally small and positive

b) Infinitesimally small and negative

1) only a 2) only 'b'

3) a (or) b 4) neither 'a' or 'b'

KEY:4

86. A charge is moved against repulsion. Then there is

A) decreasing its kinetic energy

B) increasing its potential energy

C) increasing both the energies

D) decreasing both the energies.

1) A, B, C, D are true 2) A, B, C are true

3) A, B are true 4) A only true

KEY:3

87. Which of the following statements are correct?

a) The electrostatic force does not depend on medium in which the charges are placed

b) The electrostatic force between two charges does not exist in vacuum

c) The gravitational force between masses can be usually neglected in comparision with electrostatic force

d) Any excess charge given to a conductor, not always resides on the outer surfaceof the conductor.
1) both a & c 2) only 'c' 3) both c & d 4) all

KEY:2

88. The property of the electric line of force
a) The tangent to the line of force at any point is parallel to the directio of 'E' at the point
b) No two lines of force intersect each other
1) both a & b 2) only a 3) only b 4) a or b

KEY:1

89. Which of the following statements are correct.
a) Electric lines of force are just imaginary lines
b) Electric lines of force will be parallel to the surface of conductor
c) If the lines of force are crowded, them field is strong
d) Electric lines of force are closed loops
1) both a & c 2) both b & d
3) only a 4) all

KEY:1

90. Statement(A): Negative charges always move from a higher potential to lower potential point
Statement (B): Electric potential is vector.
1) A is true but B is false 2) B is true but A is false
3) Both A and B false 4) Both A and R are true

KEY:3

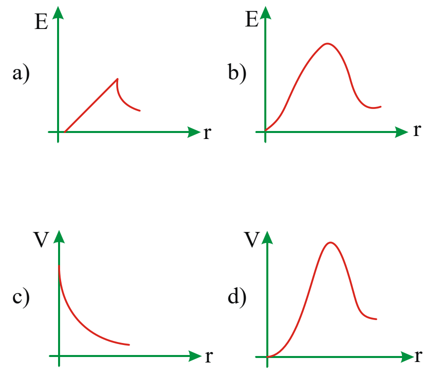
91. Statement (A): A solid conducting sphere holds more charge than a hollow conducting sphere of same radius
Statement (B) : Two spheres A and B are connected by a conducting wire. No charge will flow from A to B, when their radii are R and 2R and charges on them are 2q and q respectively
1) A is true, B is false 2) A is false B is true
3) Both A and B are true 4) Both A and B are false

KEY:4

92. A positively charged thin metal ring of radius R is fixed in the xy plane, with its centre at the origin O. A negatively charged particle P is released from rest at the point (0, 0, z_0), where z_0>0. Then the motion of P is
a) Periodic, for all value of z_0 satisfying $0 < z_0 < \infty$
b) Simple harmonic, for all values of z_0 satisfying $0 < z_0 \leq R$
c) Approximately simple harmonic, provided z_0<<R
d) Such that P crosses O and continues to move along the negative z-axis towards $z = -\infty$
Choose the correct answer
1) a and b are true 2) c is true
3) a,c,d are true 4) a,b,c,d are true

KEY:1

93. A circular ring carries a uniformly distributed positive charge. The electric field (E) and potential (V) varies with distance (r) from the centre of the ring along its axis as



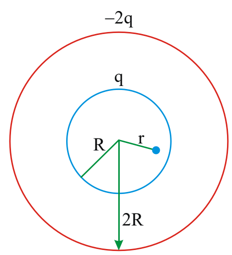
Choose the correct answer

1) b and c are true 2) a is true

3) a,b,c are true 4) a,b,c,d are true

KEY:1

94. Two concentric shells of radii R and 2R have given charges q and – 2q as shown in figure. In a region r < R



a) E = 0 b) $E \neq 0$ c) V = 0 d) $V \neq 0$

Choose the correct answer

1) a and c are true 2) c is true

3) a,d,c are true 4) a,b,c,d are true

KEY:1

95. Two identical metallic spheres A and B of exactly equal masses are given equal positive and negative charges respectively. Then 1) mass of A > Mass of B

2) mass of A < Mass of B

3) mass of A = Mass of B

4) mass of A $\geq$ Mass of B

KEY:2

96. An electron of mass M_e, initially at rest , moves through a certain distance in a uniform electric field in time t_1, proton of mass M_p also initially at rest, takes time t_2 to move through an equal distance in this uniform electric field. Neglecting the effect of gravity the ratio t_2/t_1 is nearly equal to

1) 1 2) $\sqrt{M_p / M_e}$ 3) $\sqrt{M_e / M_p}$ 4) 1836

KEY:2

97. Match the following

List-I	List-II
a) Fluid flow	d) Temperature difference
b) Heat flow	e) Pressure difference
c) Charge flow	f)Potential difference

1) $a-e, b-d, c-f$ 2) $a-d, b-e, c-f$

3) $a-f, b-e, c-d$ 4) $a-e, b-f, c-d$

KEY:1

98. An electric dipole when placed in a uniform electric field will have minimum potential energy, if the angle between dipole moment and electric field is

1) zero 2) $\pi/2$ 3) π 4) $3\pi/2$

KEY:1

99. Match List-I with List-II

List-I	List-II
a) Electric potential inside a charged conducting sphere	e) inversly proportional to square of the distance (r^2)
b) Electric potential outside the conducting charged sphere	f) directly proportional to distance (r) from the centre
c) Electric field inside the non conducting charged sphere	g) constant
d) Electric field outside a conducting charged sphere	h) inversly proportional to distance (r)

1) $a-f, b-e, c-g, d-h$

2) $a-e, b-f, c-h, d-g$

3) $a-h, b-g, c-e, d-f$

4) $a-g, b-h, c-f, d-e$

KEY:1

100. Electric potential at the centre of a charged hollow spherical conductor is

1) zero

2) twice as that on the surface

3) half of that on the surface

4) same as that on the surface

KEY:4

Electric charge and coulombs Law:

1. Three charges $+Q$, q, $+Q$ are placed respectively, atdistance, $d/2$ and d from the origin, on the x - axis. Ifthe net force experienced by $+Q$, placed at $x=0$, is zero, then value ofq is: [9 Jan. 2019 I]

(a) $-Q/4$ (b) $+Q/2$ (c) $+Q/4$ (d) $-Q/2$

Solution : **(a)**

$$\text{Force due to charge } +Q,\ F_a = \frac{KQQ}{d^2}$$

$$\text{Force due to charge } q, F_b = \frac{KQq}{\left(\frac{d}{2}\right)^2}$$

$$\text{For equilibrium}, \vec{F}_a + \vec{F}_b = 0$$

$$\Rightarrow \frac{kQQ}{d^2} + \frac{kQq}{(d/2)^2} = 0\ q = -\frac{Q}{4}$$

2. Charge is distributed within a sphere ofradius R with a volume charge density $p(r) = \frac{A}{r^2} e^{-2}r/_a$ where A and aare constants. If Q is the total charge of this charge distribution, the radius R is: [9 Jan. 2019, II]

(a) a $\log\left(1 - \frac{Q}{2\pi aA}\right)$ (b) $\frac{a}{2}\log\left[\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

(c) a $\log\left[\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (d) $\frac{a}{2}\log\left(1 - \frac{Q}{2\pi aA}\right)$

Solution : **(b)**

$$Q = \int\ p\,dv = \int_0^R \frac{A}{r^2} e^{2r/a}(4\pi r^2 dr)$$

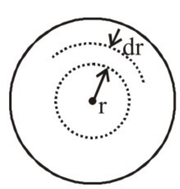
$$= 4\pi A \int_0^R e^{2r/a}\,dr = 4\pi A\left(\frac{e^{2r/a}}{-}\frac{2}{a}\right)_0^R$$

t'c$_1^\tau$b$_k^1$( )1$_1$ιti( )11\

$$=4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a}-1\right)$$

$$Q = 2\pi aA(1 - e^{-2R/a})$$

$$R = \frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$$



3. Two identical conducting spheres A and B, carry equal charge. They are separated bya distance much larger than their diameter, and the force between them is F. A third identical conducting sphere, C, is uncharged. Sphere C is first touched to A, then to B, and then removed. As a result, the force between A and B would be equal to

[Online Apri116, 2018]

(a) $\frac{3F}{4}$ (b) $\frac{F}{2}$ (c) F (d) $\frac{3F}{8}$

Solution : (d)

Spheres A and B carry equal charge say' q'

Force between them, $F = \frac{kqq}{r^2}$

When A and C are touched, charge on both $q_A = q_C = \frac{q}{2}$

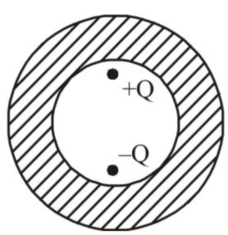
Then when B and C are touched, charge on B $q_B = \frac{\frac{q}{2}+q}{2} = \frac{3q}{4}$

Now, the force between charge q_A and q_B

$$F^{\dagger} = \frac{kq_Aq_B}{r^2} = \frac{k \times \frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}\frac{kq^2}{r^2} = \frac{3}{8}F$$

4. Shown in the figure are two point charges $+Q$ and $-Q$ inside the cavity ofa spherical shell. The charges are kept near the surface of the cavity on opposite sides of the centre ofthe shell. If o_1 is the surface charge on the inner surface and Q_1 net charge on it and 0_2 the surface charge on the outer surface and Q_2 net charge on it then:

[Online Apri110, 2015]



(a) $0_1 \neq 0, Q_1 = 0$ (b) $0_1 \neq 0, Q_1 = 0$ (b) $o_2 = 0, Q_2 = 0$ $0_2 \neq 0, Q_2 = 0$

(c) $0_1 = 0, Q_1 = 0$ (d) $0_1 \neq 0, Q_1 \neq 0$ $(d) o_2 = 0, Q_2 = 0$ $0_2 \neq 0, Q_2 \neq 0$

Solution : (c)

Inside the cavity net charge is zero.

$Q_1 = 0$ and $0_1 = 0$

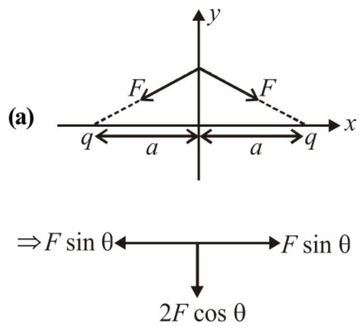
There is no effect of point charges $+Q, -Q$ and inducedcharge on inner surface on the outer surface.

$$Q_2 = 0 \text{ and } 0_2 = 0$$

5. Two charges, each equal to q, are kept at $x = -a$ and $x = a$on the x - axis. A particle ofmass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement $(y << a)$ along the y - axis, the net force acting on the particle is proportional to [2013]

(a) y (b) $-y$ (c) $\underline{1}$ (d) $-\underline{1}$

Solution :



$$\Rightarrow F_{\text{net}} = 2F\cos\theta$$

$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2 + a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2 + a^2}}$$

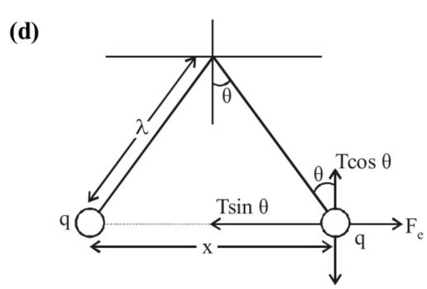
$$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \; (\because y << a)$$

$$\Rightarrow \frac{kq^2y}{a^3} \text{ So, } F \propto y$$

6. Two balls of same mass and cayrrying equal chyarge are hung from a fixed support of length l. At electrostatic equilibrium, assuming that angles made by each thread is small, the separation, x between the balls is proportional to : [Online April 9, 2013]

(a) l (b) l^2 (c) $l^{2/3}$ (d) $l^{1/3}$

Solution :



mg

In equilibrium, $F_e = T \sin\theta$ $mg = T\cos 0$

$\tan\theta = \frac{F_e}{mg} = \frac{q^2}{4\pi\in 0x^2 \times mg}$ also $\tan\theta \approx \sin = \frac{x/2}{l}$

Hence, $\frac{x}{2l} = \frac{q^2}{4\pi\in 0x^2\times mg} \Rightarrow x^3 = \frac{2q^2\ell}{4\pi\in 0mg}$

$$x = \left(\frac{q^2 l}{2\pi \in 0mg}\right)^{1/3}$$

Therefore $x \propto P^{1/3}$

7. Two identical charged spheres suspended from acommon point by two massless strings of length l are initially a distance $d(d << l)$ apart because oftheir mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result charges approach each otherwith a velocity v. Then as a function ofdistance x between them, [2011]

(a) $v \propto x^{-1}$ (b) $v \propto x^1/_2$ (c) $v \propto x$ (d) $v \propto x^{-/2}1$

Solution : (d)

From figure $T\cos\theta = mg$ (i)

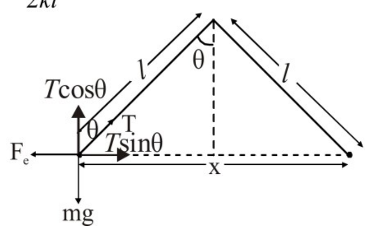
$T\sin\theta = F_e$ (ii)

Dividing equation (ii) by(i), we get$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{F_e}{mg} \Rightarrow F_e = mg\tan\theta$

$\Rightarrow \frac{kq^2}{x^2} = mg\tan\theta \Rightarrow q^2 = \frac{x^2 mg\tan\theta}{k}$

Since 6 is small $\tan\theta \approx \sin\theta = \frac{x}{2l}$

$\therefore q^2 = \frac{x^3 mg}{2kl} \Rightarrow q^2 \propto x^{3/2}$



$$\Rightarrow \frac{dq}{dt}\alpha\frac{3}{2}\sqrt{x}\frac{dx}{dt} = \frac{3}{2}\sqrt{x}V$$

Since $\frac{dq}{dt}$ =const.

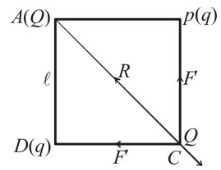
$$\Rightarrow v \propto x^{-1/2} \; [q^2 \propto x^3]$$

8. A charge Q is placed at each ofthe opposite corners ofa square. A charge q is placed at each of the other two corners. Ifthe net electrical force on Q is zero, then Qlq equals: [2009]

(a) -1 (b) 1 (c) $-\frac{1}{\sqrt{2}}$ (d) $-2\sqrt{2}$

Solution : (d)

Let F be the force between Q and Q. The forcebetween q and Q should be attractive for net force on Q to be zero. Let F' be the force between Q and q. The resultantof F' and F is R. For equilibrium



F

Net force on Q at C is zero.

$$\vec{R} + \vec{F} = 0 \;\Rightarrow\; \sqrt{2}F' = -F$$

$$\Rightarrow \sqrt{2} \times k\frac{Qq}{p^2} = -k\frac{Q^2}{(\sqrt{2}p)^2}$$

$$\Rightarrow \frac{Q}{} = -2\sqrt{2}$$

9. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio

$\frac{\text{e1ectronicchargeonthemoon}}{\text{e1ectronicchargeontheearth}}$ to be [2007]

(a) $g_M l g_E$ (b) 1 (c) 0 (d) $g_E l g_M$

Solution : (b)

It is obvious that by charge conservaiton law,electronic charge does not depend on acceleration due to gravity as it is a universal constant.So,

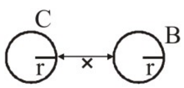
electronic charge on earth= electronic charge on moon

Required ratio $= 1$.

10. Two spherical conductors B and C having equal radii and carrying equal charges on them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that B but uncharged is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is [2004]

(a) $Fl8$ (b) 3 $Fl4$ (c) $Fl4$ (d) 3 $Fl8$

Solution :

10. (d) 

Initial force, $F = K\frac{Q_B Q_C}{x^2}$

x is distance between the spheres. When third spherical conductor comes in contact with B charge on B is halved $i.e.$, $\frac{Q}{2}$ and charge on third sphere becomes $\frac{Q}{2}$. Now it is touched to C, charge then equally distributes themselves to make potential same, hence charge on C becomes

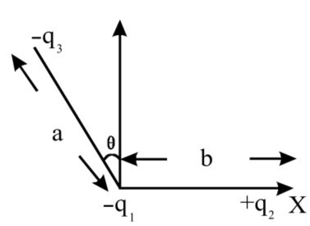
$$\left(Q+\frac{Q}{2}\right)\frac{1}{2}=\frac{3Q}{4}.$$

$$F_{new}=k\frac{Q_{\acute{C}}Q_B'}{x^2}=k\frac{\left(\frac{3Q}{4}\right)\left(\frac{Q}{2}\right)}{x^2}=k\frac{3}{8}\frac{Q^2}{x^2}$$

or $F_{new}=\frac{3}{8}F$

11. Three charges $-q_1$, $+q_2$ and $-q_3$ are place as shown in the figure. The x - component of the force on $-q_1$ is proportional to [2003]

Y



(a) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2}+\frac{q_3}{a^2}\cos 0$ (d) $\frac{q_2}{b^2}-\frac{q_3}{a^2}\sin\theta$

Solution : **(b)**

Force applied by charge q_2 on q_1

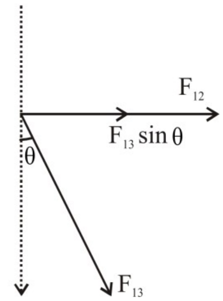
$$F_{12} = k\frac{q_1 q_2}{b^2}$$

Force applied by charge q_3 on q_1

$$F_{13} = k\frac{q_1 q_3}{a^2}$$

The X-component of net force (F_x) on q_1 is $F_{12} + F_{13} \sin\theta$

$$\therefore\ F_x = k\frac{q_1 q_2}{b^2} + k\frac{q_1 q_2}{a^2}\sin\theta$$



$\mathbf{F_{13} \cos\theta}$

$$F_x \propto \frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$$

12. **If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium then the value of q is [2002]**

(a) $Ql2$ **(b) $-Ql2$** **(c) $Ql4$** **(d) – $Ql4$**

Solution :**(d)**

At equilibrium net force is zero,

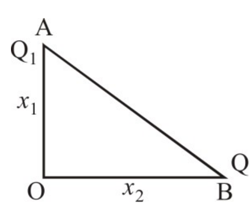
$$k\frac{Q \times Q}{(2x)^2} + k\frac{Qq}{x^2} = 0$$

$\overline{\mathbf{QqQ}}$

$$\Rightarrow q = -\frac{Q}{4}$$

Electric field and field lines :

13. **Charges Q_1 and Q_2 are at points A and B of a right angle triangle OAB (see figure). The resultant electric field at point O is perpendicular to the hypotenuse, then $Q_1 l Q_2$ is proportional to: [Sep. 06, 2020 (D]]**

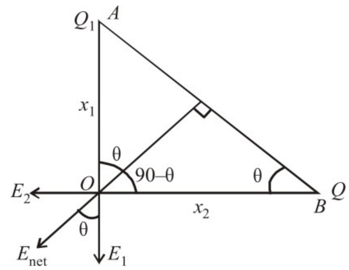


(a) $\frac{x_1 3}{3}$ (b) $\frac{x_2}{x_1}$ (c) $\frac{x_1}{x_2}$ (d) $\frac{x_2 2}{2} x_2 x_1$

Solution : (c)

Electric field due charge $Q_2, E_2 = \frac{kQ_2}{x_2^2}$

Electric field due charge $Q_1, E_1 = \frac{kQ_1}{x_1^2}$



From figure, $\tan\theta = \frac{E_2}{E_1} = \frac{x_1}{x_2} \Rightarrow \frac{kQ_2}{x_2^2 \times \frac{kQ_1}{x_1^2}} = \frac{x_1}{x_2}$

$\Rightarrow \frac{Q_2 x_1^2}{Q_1 x_2^2} = \frac{x_1}{x_2} \Rightarrow \frac{Q_2}{Q_1} = \frac{x_2}{x_1}$ or, $\frac{Q_1}{Q_2} = \frac{x_1}{x_2}$

14. Consider the force F on a charge 'q' due to a uniformly charged spherical shell ofradius R carrying charge Q dis - tributed uniformly over it. Which one ofthe following state - ments is true for F, if q' is placed at distance r from the centre ofthe shell? [Sep. 06, 2020 (II)]

(a) $\mathrm{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r \langle R$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2} \rangle \mathrm{F} > 0$ for $r < R$

(c) $\mathrm{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for $r > R$ (d) $\mathrm{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{R^2}$ for all r

Solution : (c)

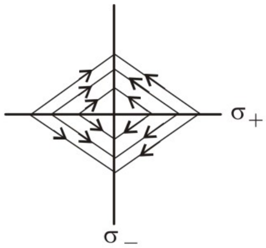
For spherical shell $E = \frac{1Q}{4\pi\varepsilon_0 r^2}$ (if $r \geq R$) $= 0$ (ifr $< R$)

Force on charge in electried field, $F = qEF = 0$ (For $r < R$)

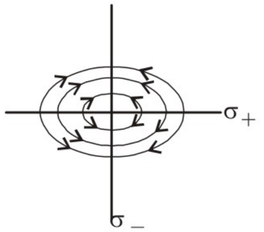
$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$ (For $r > R$)

15. **Two charged thin infinite plane sheets ofuniform surfacecharge density 0_+ and 0_-, where $|0_+| > |0_-|$, intersect at right angle. Which ofthe following best represents theelectric field 1ines foro_- this system? [Sep. 04, 2020 (I)]**

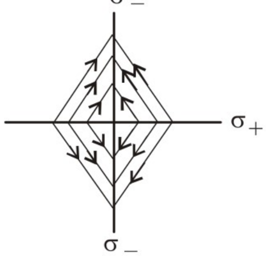
(a)



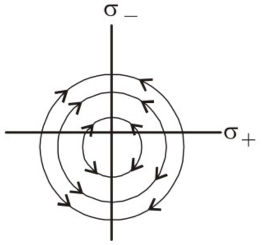
(b)



(c)



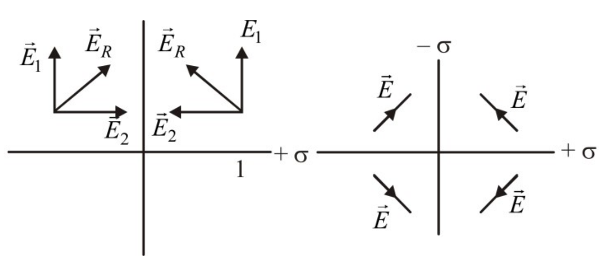
(d)

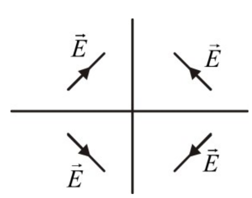


Solution : **(c)**

The electric field produced due to uniformly chargedinfinite plane is uniform. So option (b) and(d) are wrong.

And $+$ve charge density o_+ is bigger in magnitude so itsfield along ydirection will be bigger than field of - ve charge density o_-in$X-0$ direction. Hence option (c) is correct.





16. Aparticle ofcharge q and mass m is subjected to an electricfield $E = E_0(1 - ax^2)$ in the x - direction, where a and E_0 are constants. Initially the particle was at rest at $x = 0$. Other than the initial position the kinetic energy ofthe particle becomes zero when the distance of the particle from the origin is: [Sep. 04, 2020 (II)]

(a) a (b) $\sqrt{\frac{2}{a}}$ (c) $\sqrt{\frac{3}{a}}$ (d) $\sqrt{\frac{1}{a}}$

Solution : **(c)**

Given,Electric field, $E = E_0(1 - x^2)$

Force, $F = qE = qE_0(1 - x^2)$

Also, $F = ma = mv\frac{dv}{dx}\left(\because a = v\frac{dv}{dx}\right)$

$$mv\frac{dv}{dx} = qE_0(1 - x^2)$$

$$\Rightarrow vdv = \frac{qE_0(1 - x^2)dx}{m}$$

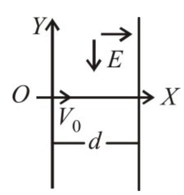
Integrating both sides we get,

$$\Rightarrow \int_0^v v\,dv = \int_0^x \frac{qE_0(1 - x^2)dx}{m}$$

$$\Rightarrow \frac{v^2}{2} = \frac{qE_0}{m}\left(x - \frac{9x^3}{3}\right) = 0$$

$$\Rightarrow x = \sqrt{\frac{3}{a}}$$

17. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto $x = d$. Equation of path of electron in the region $x > d$ is: [Sep. 02, 2020 (I)]



(a) $y = \frac{qEd}{mV_0^2}(x - d)$ (b) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$

(c) $y = \frac{qEd}{mV_0^2}x$ (d) $y = \frac{qEd^2}{mV_0^2}x$

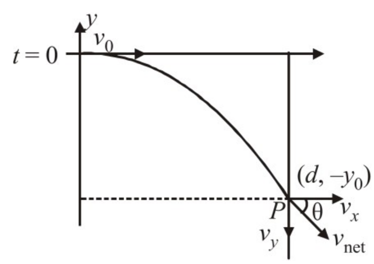
Solution : **(b)**

$$F_x = 0, a_x = 0, (v)_x = \text{constant}$$

Time taken to reach at 'P' $= \underline{d} = t_0$ (let) ...(i)

$$v_0$$

(Along $-y$), $y_0 = 0 + \frac{1}{2} \cdot \frac{qE}{m} \cdot t_0^2$ (ii)



$$\tan\theta = \frac{v_y}{v_x} = \frac{qEt_0}{m \cdot v_0}, \left(t = \frac{d}{v_0}\right)$$

$$\tan\theta = \frac{qEd}{m \cdot v_0^2}, \text{Slope} = \frac{-qEd}{mv_0^2}$$

No electric field $\Rightarrow F_{net} = 0, \vec{v} = \text{const.}$

$$y = mx + c, \begin{Bmatrix} qEd \\ m = \overline{2} \\ mv_0 \\ (d - y_0) \end{Bmatrix}$$

$$-y_0 = \frac{-qEd}{mv_0^2}, d + c \Rightarrow c = -y_0 + \frac{qEd^2}{mv_0^2}$$

$$y = \frac{-qEd}{mv_0^2} x - y_0 + \frac{qEd^2}{mv_0^2}$$

$$y_0 = \frac{1}{2} \cdot \frac{qE}{m} (\quad)(\quad)^2 = \frac{1}{2} \frac{qEd^2}{mv_0^2}$$

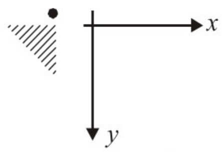
$$y = \frac{-qEdx}{mv_0^2} - \frac{1}{2} \frac{qEd^2}{mv_0^2} + \frac{qEd^2}{mv_0^2}$$

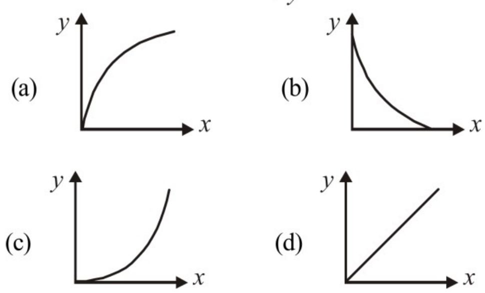
$$y = \frac{-qEd}{mv_0^2} + \frac{1}{2} \frac{qEd^2}{mv_0^2} \Rightarrow y = \frac{\text{qEd}}{\text{mv}_0^2}\left(\frac{d}{2} - x\right)$$

18. **A small point mass carrying some positive charge on it, is released from the edge ofa table. There is a uniform electric field in this region in the horizontal direction. Which of the following options then correctly describe the trajectory of the mass? (Curves are drawn schematically and are not to scale).**

[Sep. 02, 2020 (ID]

$\rightarrow \boldsymbol{E}$

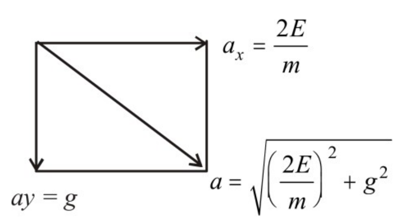




Solution : **(d)**

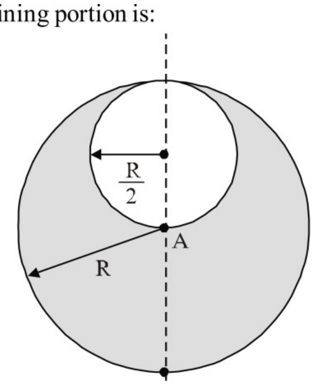
Net force acting on the particle,$\vec{F} = qE\hat{i}+mg\hat{i}$

Net acceleration ofparticle is constant, initial velocity is zero therefore path is straight line.



19. **Consider a sphere ofradius R which carries a uniform charge density p. Ifa sphere ofradius $\frac{R}{2}$ is carved out of $|\overline{E}_A|$ it, as shown, the ratio $|\overline{E}_B|$ of magnitude of electric field $\overline{E}_A$ and $\overline{E}_B$, respectively, at points A and B due to**

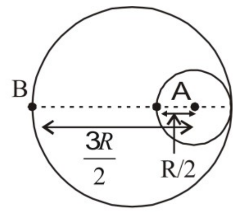
the remaining portion is: **[9 Jan. 2020, I]**



(a) $\frac{21}{34}$ (b) $\frac{18}{34}$ (c) $\frac{17}{54}$ (d) $\frac{18}{54}$

Solution

Electric field at A $\left(R^{\dagger} = \frac{R}{2}\right)$



$$E_A.ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \vec{E}_A = \frac{\rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0 \cdot 4\pi\left(\frac{R}{2}\right)^2}$$

$$\Rightarrow \vec{E}_A = \frac{o(R/2)}{3\varepsilon_0} = \left(\frac{oR}{6\varepsilon_0}\right)$$

Electric fields at 'B' $\vec{E}_B = \frac{k\times \text{p}\times\frac{4}{3}\pi R^3}{R^2} - \frac{k\times\text{p}\times\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$

$$\Rightarrow \vec{E}_B = \frac{oR}{3\varepsilon_0} - \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(0)}{\left(\frac{3R}{2}\right)^2}\frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$\Rightarrow \vec{E}_B = \frac{oR}{3\varepsilon_0} - \frac{oR}{54\varepsilon_0}$$

$$\Rightarrow E_B = \frac{17}{54}\left(\frac{oR}{\varepsilon_0}\right)$$

$$\left|\frac{E_A}{E_B}\right| = \frac{1\times 54}{6\times 17} = \left(\frac{9}{17}\right) = \frac{9}{17}\times\frac{2}{2} = \frac{18}{34}$$

20. An electricdipole ofmoment $\overline{p} = (\hat{i} - 3\hat{j} + 2\hat{k}) \times 10^{-29}$ C.m is at the origin $(0, 0, 0)$. The electric field due to this dipole at $\vec{r} = +\hat{\imath}+3j+5k$ (note that $r.p = 0$) is parallel to: [9 Jan. 2020, I]

(a) $(+\hat{\imath} - 3j - 2k)$ (b) $(-\hat{\imath}+3j - 2k)$

(c) $(+\hat{\imath}+3\hat{j} - 2k)$ (d) $(-\hat{\imath} - 3j+2k)$

Solution : (c)

Since $\vec{r}\cdot\vec{p} = 0$

$\vec{E}$ must be antiparallel to $\vec{p}$ $\hat{E}$ is parallelto $(\hat{\imath}+3j - 2\hat{k})$

25. A particle ofmass m and charge q has an initial velocity $\vec{v} = v_0 \oint$. Ifan electric field $E1 = E_0^1 i$ and magnetic field $B1 = B_0\hat{\imath}$ act on the particle, its speed will double after a time: ``` [7 Jan 2020, II]

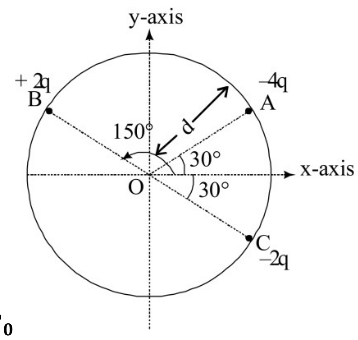
(a) $\frac{2mv_0}{qE_0}$ (b) $\frac{3mv_0}{qE_0}$ (c) $\frac{\sqrt{3}mv_0}{qE_0}$ (d) $\frac{\sqrt{2}mv_0}{qE_0}$

Solution :**(c)**

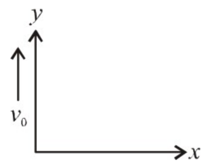
In the x direction $F_x = qE$

$$\Rightarrow ma_x = qE \Rightarrow a_x = \underline{E_0 q}$$

For speed to be double, $\rightarrow E_0$



$\rightarrow B_0$



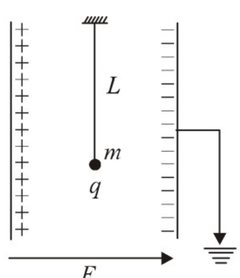
$$v_0^2 + v_x^2 = (2v_0)^2$$

$$\Rightarrow v_{x=\sqrt{3}}v_0 = a_x t$$

$$\Rightarrow \sqrt{3}v_0 = 0 + \frac{qE_0 t}{m} \Rightarrow t = \frac{\sqrt{3}v_0 m}{E_0 q}$$

26. A simple pendulum of length L is placed between the plates ofa parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period ofthe pendulum is given by:

[10 April 2019, II]



(a) $2\pi\sqrt{\frac{L}{\left(g+\frac{qE}{m}\right)}}$ (b) $2\pi\sqrt{\frac{L}{\sqrt{g^2-\frac{q^2E^2}{m^2}}}}$ (c) $2\pi\sqrt{\frac{L}{\left(g-\frac{qE}{m}\right)}}$ (d) $2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$

Solution : **(d)**

Time period ofthe pendulum (T) is given by $T=2\pi\sqrt{\frac{L}{g_{\text{eff}}}}$

$$g_{\text{eff}}=\frac{\sqrt{(mg)^2+(qE)^2}}{m}$$

$$\Rightarrow g_{\text{eff}}=\sqrt{g^2+\left(\frac{gE}{m}\right)^2}\Rightarrow T=2\pi\sqrt{\frac{L}{\sqrt{g^2+\left(\frac{qE}{m}\right)^2}}}$$

27. Four point charges $-q,+q,+q$ and $-q$ are placed ony - axis at $y=-2d, y=-d, y=+d$ and $y=+2d$ respectively. The magnitude ofthe electric field E at a point on the x - axis at $x=$ D, with D $>> d$, will behave as:

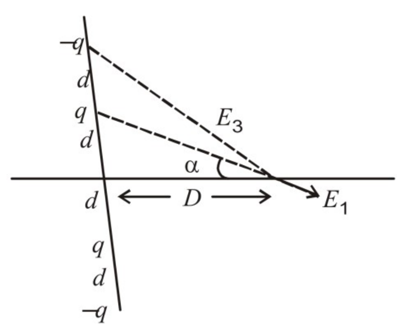
[9 April 2019, II]

(a) $E\propto\frac{1}{D^3}$ (b) $E\propto\frac{1}{D}$ (c) $E\propto\frac{1}{D^4}$ (d) $E\propto\frac{1}{D^2}$

Solution : **(d)**

$$\rightarrow E=(E_1+E_2)+\rightarrow\rightarrow(E_3+E_4)\rightarrow\rightarrow$$

$$\text{or } E=2E\cos\alpha-2E\cos\beta$$



$$=\frac{2kq}{(D^2+d^2)}\times\frac{D}{\sqrt{D^2+d^2}}-\frac{2kq}{(D^2+(2d)^2}\times\frac{D}{\sqrt{D^2+(2d)^2}}$$

$$=\frac{2kqD}{(D^2+d^2)^{3/2}}-\frac{2kqD}{[D^2+(2d)^2]^{3/2}}$$

For d $<< D$

$$E\propto\frac{D}{D^3}\propto\frac{1}{D^2}$$

28. The bob of a simple pendulum has mass 2 g and a charge of $5.01/\ _{4}$C. It is at rest in a uniform horizontal electric field ofintensity2000 $\mathrm{V/m}$. At equilibrium, the angle that the pendulum makes with the vertical is: [8 April 2019 I]

(take $g=10\mathrm{m/s^{2}}$)

(a) $\tan^{1}(2.0)$ (b)$\tan^{1}(0.2)$ (c) $\tan^{1}(5.0)$ (d)$\tan^{1}(0.5)$

Solution : (d)

At equilibrium resultant force on bob must be zero, so

$T\cos\theta=mg$ (i)

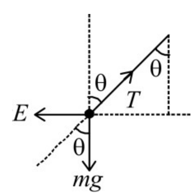
$T\sin\theta=qE$

Solving (i) and (ii) we get (ii) $Y.\wedge_{1}$,

$\tan\theta=\underline{qE}$

mg

$\tan\theta=\frac{5\times10^{-6}\times2000}{2\times10^{-3}\times10}=\frac{1}{2}q$ - - - $\cdot$ - - - - -> X



[Here, $q=5\times10^{-6}$C,

$\mathrm{E}=2000\mathrm{v/m},\mathrm{m}=2\times10^{-3}\mathrm{kg}]$

$\Rightarrow\tan^{-1}\left(\frac{1}{2}\right)$

29. For auniformly charged ring ofradius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value ofh is: [9 Jan. 2019 I]

(a) $\frac{\mathrm{R}}{\sqrt{5}}$ (b) $\frac{\mathrm{R}}{\sqrt{2}}$ (c) R (d) $\mathrm{R}\sqrt{2}$

Solution :

Electric field on the axis of a ring of radius R at a distance h from the centre,$\mathrm{E}=\frac{\mathrm{kQh}}{(\mathrm{h}^{2}+\mathrm{R}^{2})^{3/2}}$

Condition: for maximum electric field $\frac{\mathrm{dE}}{\mathrm{dh}}=0$

$$\Rightarrow \frac{d}{dh}\left[\frac{kQh}{(R^2+h^2)^{3/2}}\right] = 0$$

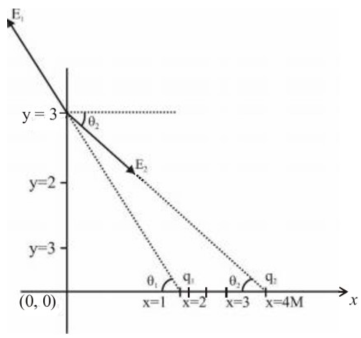
By using the concept ofmaxima and minima we get,h $= \frac{R}{\sqrt{2}}$

30. Two point charges $q_1(\sqrt{10}\mu C)$ and $q_2(-25\mu C)$ are placed on the x - axis at $x = 1m$ and $x = 4m$ respectively. The electric field (in V/m) at a point $y = 3m$ on y - axis is, [take $\frac{1}{4\pi\epsilon 0} = 9\times 10^9 Nm^2C^{-2}$] [9 Jan 2019, II]

(a) $(63\hat{i} - 27\hat{j})\times 10^2$ (b) $(-63\hat{i} + 27\hat{j})\times 10^2$

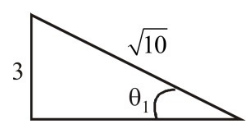
(c) $(81\hat{i} - 81\hat{j})\times 10^2$ (d) $(-81\hat{i} + 81\hat{j})\times 10^2$

Solution : (a)



Let $\vec{E}_1$ and $\vec{E}_2$ are the vaues ofelectric field due to charge, q_1 and q_2 respectively magnitude of $E_1 = \frac{1q_1}{4\pi\epsilon 0 r_1^2}$

$$= \frac{1\sqrt{10}\times 10^{-6}}{4\pi\in 0(1^2+3^2)}$$



$$= (9\times 10^{9)}\times\sqrt{10}\times 10^{-7}$$

1

$$= 9\sqrt{10}\times 10^2$$

$$\vec{E}_1 = 9\sqrt{10}\times 10^2[\cos\theta_1(-i\rightarrow) + \sin\theta_1\vec{j}] \Rightarrow E_1 = 9\times\sqrt{10}\times 10^2\left[\frac{1}{\sqrt{10}}(-\hat{\imath}) + \frac{3}{\sqrt{10}}\hat{j}\right] \Rightarrow E_1 = 9\times 10^2[-\hat{\imath}+3j]$$

$= [-9\hat{\imath} + 27\hat{\jmath}]10^2$ Similarly, $E_2 = \frac{1q_2}{4\pi\epsilon 0 r^2}$

$$E_2 = \frac{9\times 10^9\times(25)\times 10^{-6}}{(4^2+3^2)} E_2 = 9\times 10^3 V/m$$

$$\vec{E}_2 = 9 \times 10^3(\cos\theta_2\hat{i} - \sin\theta_2\hat{j}) \ \tan\theta_2 = \frac{3}{4}$$

$$\vec{E}_2 = 9 \times 10^3\left(\frac{4}{5}\hat{\imath} - \frac{3}{5}j\right) = (72\hat{\imath} - 54j) \times 102$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = (63\hat{\imath} - 27j) \times 10^2 Vlm$$

31. A body ofmass M and charge q is connected to a spring ofspring constant k. It is oscillating along x - direction about its equilibrium position, taken to be at $x = 0$, with an amplitude A. An electric field E is applied along the x - direction. Which ofthe following statements is correct? [Online Apri115, 2018]

(a) The total energy ofthe system is $\frac{1}{2}mc0^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$

(b) The new equilibrium position is at a distance: $\frac{2qE}{k}$from $x = 0$

(c) The new equilibrium position is at a distance: $\frac{qE}{2k}$ ffom$x = 0$

(d) The total energy ofthe system is $\frac{1}{2}mo)^{2}A^2 - \frac{1}{2}\frac{q^2E^2}{k}$

Solution : (a)

Equilibrium position will shift to point where resultant force $= 0$

$$kx_{eq} = qE \Rightarrow x_{eq} = \frac{qE}{k}$$

$$\text{Total energy} = \frac{1}{2}mc0^2A^2 + \frac{1}{2}kx_{eq}^2$$

$$\text{Total energy} = \frac{1}{2}mo)^2A^2 + \frac{1}{2}\frac{q^2E^2}{k}$$

32. A solid ball ofradius R has a charge density p given by $p = p_0\left(1 - \frac{r}{R}\right)$ for $0 \leq r \leq R$. The electric field outside the ball is: [Online Apri115, 2018]

(a) $\frac{p_0R^3}{\varepsilon_0r^2}$ (b) $\frac{4p_0R^3}{3\varepsilon_0r^2}$ (c) $\frac{3p_0R^3}{4\varepsilon_0r^2}$ (d) $\frac{p_0R^3}{12\varepsilon_0r^2}$

Solution : (d)

$$\text{Charge density, } p = p_0\left(1 - \frac{r}{R}\right)$$

$$dq = pdv$$

$$q_{in} = \int dq = pdv$$

$$= p_0\left(1-\frac{r}{R}\right)4\pi r^2 dr\ (dv = 4\pi r^2 dr)$$

$$= 4\pi p_0 \int_0^R \left(1-\frac{r}{R}\right) r^2 dr$$

$$= 4\pi p_0 \int_0^R r^2\, dr - \frac{r^2}{R} dr$$

$$= 4\pi p_0 \left[\left[\frac{r^3}{3}\right]_0^R - \left[\frac{r^4}{4R}\right]_0^R\right] = 4\pi p_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right]$$

$$= 4\pi p_0 \left[\frac{R^3}{3} - \frac{R^3}{4}\right] = 4\pi p_0 \left[\frac{R^3}{12}\right]$$

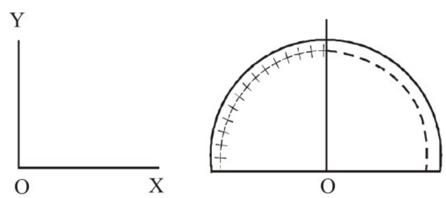
$$q = \frac{\pi p_0 R^3}{3}$$

$$E.4\pi r^2 = (\quad)(\quad)$$

Electric field outside the ball, $E = \frac{p_0 R^3}{2}$

$$12 \in 0r$$

34. A wire oflength L (= 20 cm), is bent into a semicircular arc. If the two equal halves of the arc were each to be uniformly charged with charges $\pm Q$, $[|Q| = 10^3 \varepsilon_0$ Coulomb where ε_0 is the permittivity (in SI units) of free space] the net electric field at the centre O of the semicircular arc would be: [Online April 11, 2015]



(a) $(50 \times 10^3 \mathrm{N/C})\hat{j}$ **(b)** $(50 \times 10^3 \mathrm{N/C})\hat{i}$ **(c)** $(25 \times 10^3 \mathrm{N/C})\hat{j}$ **(d)** $(25 \times 10^3 \mathrm{N/C})\hat{i}$

Solution : **(d)**

Given: Length of wire $L = 20$ cm

charge $Q = 10^3 \varepsilon_0$

We know, electric field at the centre of the semicircular arc

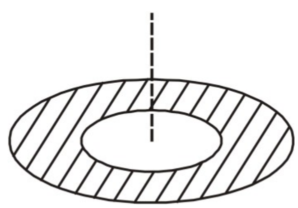
$$E = \underline{2K\lambda}$$

$$r$$

or, $E = \frac{2K\left(\frac{2Q}{\pi r}\right)}{r}\left[As\lambda = \frac{2Q}{\pi r}\right]$

$$= \frac{4KQ}{\pi r^2} = \frac{4KQ\pi^2}{\pi L^2} = \frac{4\pi KQ}{L^2} = 25 \times 10^3 Nl\, C\hat{\imath}$$

35. A thin disc ofradius $b = 2$a has a concentric hole ofradius 'a' in it (see figure). It carries uniform surface charge 0' on it. Ifthe electric field on its axis at height $h'(h << a)$ from its centre is given as Ch' then value of C' is: [Online Apri110, 2015]



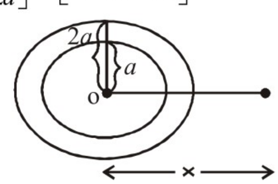
(a) $\frac{o}{4a\in 0}$ (b) $\frac{o}{8a\in 0}$ (c) $\frac{o}{a\in 0}$ (d) $\frac{o}{2a\in 0}$

Solution : (a)

Electric field due to complete disc $(R = 2a)$ at a distance x and on its axis

$$E_1 = \frac{o}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{R^2 + x^2}}\right] E_1 = \frac{o}{2\varepsilon_0}\left[1 - \frac{h}{\sqrt{4a^2 + h^2}}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{h}{2a}\right] \quad \left[\begin{array}{l}\text{here } x = h \\ \text{and, } R = 2a\end{array}\right]$$



Similarly, electric field due to disc $(R = a) E_2 = \frac{o}{2\varepsilon_0}\left(1 - \frac{h}{a}\right)$

Electric field due to given disc$E = E_1 - E_2$

$$\frac{o}{2\varepsilon_0}\left[1 - \frac{h}{2a}\right] - \frac{o}{2\varepsilon_0}\left[1 - \frac{h}{a}\right] = \frac{oh}{4\varepsilon_0 a}$$

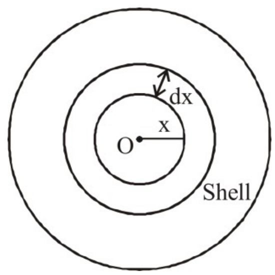
Hence, $c = \frac{o}{4a\varepsilon_0}$

36. Aspherically symmetric charge distribution is characterized by a charge density having the following variations: $p(r) = p_0\left(1-\frac{r}{R}\right)$ for $r < R$ $p(r) = 0$ for $r \geq R$ Where r is the distance from the centre of the charge distribution p_o is a constant. The electric field at an internalpoint $(r < R)$ is: [Online Apri112, 2014]

(a) $\frac{p_0}{4\varepsilon_o}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$ (b) $\frac{p_0}{\varepsilon_o}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$ (c) $\frac{p_0}{3\varepsilon_o}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$ (d) $\frac{p_0}{12\varepsilon_o}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$

Solution : (b)

Let us consider a spherical shell of radius x and thickness dx.



Charge on this shell$dq = p.4\pi x^2 dx = p_0\left(1-\frac{x}{R}\right).4\pi x^2 dx$

Total charge in the spherical region from centre to $r(r < R)$ is

$$q = \int dq = 4\pi p_0 \int_0^r \left(1-\frac{x}{R}\right) x^2 dx$$

$= 4\pi p_0\left[\frac{x^3}{3}-\frac{x^4}{4R}\right]_0^r = 4\pi p_0\left[\frac{r^3}{3}-\frac{r^4}{4R}\right] = 4\pi p_0 r^3\left[\frac{1}{3}-\frac{r}{4R}\right]$ Electric field at r, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$

$$= \frac{1}{4\pi\varepsilon_0}.\frac{4\pi p_0 r^3}{r^2}\left[\frac{1}{3}-\frac{r}{4R}\right] = \frac{p_0}{\varepsilon_0}\left[\frac{r}{3}-\frac{r^2}{4R}\right]$$

37. The magnitude of the average electric field normally present in the atmosphere just above the surface of the Earth is about 150 N/C, directed inward towards the center ofthe Earth. This gives the total net surface charge carried by the Earth to be: [Given $\varepsilon_o = 8.85 \times 10^{-12} C^2/N - m^2, R_E = 6.37 \times 10^6 m$] [Online April 9, 2014]

(a) $+670kC$ (b) $-670kC$ (c) $-680kC$ (d) $+680kC$

Solution : (c)

Given,Electric field $E = 150N/C$

Total surface charge carried by earth $q = ?$

or, $q^{=} \in EA0$

$= \in E\pi r^2 0.$

$$= 8.85 \times 10^{-12} \times 150 \times (6.37 \times 10^{6})^{2}. = 680 \text{ Kc}$$

As electric field directed inward hence

$$q = -680 \text{ Kc}$$

38. The surface charge densityofa thin charged disc ofradius R is 0. The value ofthe electric field at the centre of the disc is $\frac{o}{2\epsilon 0}$. With respect to the field at the centre, the electric field along the axis at a distance R from the centre ofthe disc : [Online April 25, 2013]

(a) reduces by 70.7% (b) reduces by 29.3%

(c) reduces by 9.7% (d) reduces by 14.6%

Solution : (a)

Electric field intensity at the centre ofthe disc. $E = \frac{o}{2\epsilon 0}$ (given)

Electric field along the axis at any distance x from thecentre ofthe disc

$$E^{\dagger} = \frac{o}{2 \in 0}\left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

From question, $x = R$(radius ofdisc)$E^{\dagger} = \frac{o}{2\epsilon 0}\left(1 - \frac{R}{\sqrt{R^2+R^2}}\right)$

$$= \frac{o}{2 \in 0}\left(\frac{\sqrt{2}R - R}{\sqrt{2}R}\right) = \frac{4}{14}E$$

% reduction in the value of electric field$= \frac{\left(E-\frac{4}{14}E\right)\times 100}{E} = \frac{1000}{14}\% = 70.7\%$

39. Aliquid drop having6excess e1ectrons is kept stationary under a uniform electric field of 25.5kVm^{-1}. The density of liquid is 1.26×10^{3} kg m^{-3}. The radius ofthe drop is (neglect buoyancy). [Online April 23, 2013]

(a) $4.3 \times 10^{-7}\text{m}$ (b) $7.8 \times 10^{-7}\text{m}$ (c) $0.078 \times 10^{-7}\text{m}$ (d) $3.4 \times 10^{-7}\text{m}$

Solution :

$$F = qE = mg(q = 6e = 6 \times 1.6 \times 10^{-19})$$

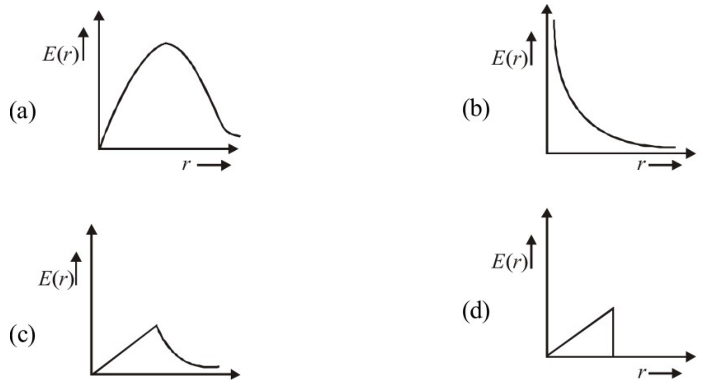
Density $(d) = \frac{\text{mass}}{\text{volume}} = \frac{m}{\frac{4}{3}\pi r^3}$

or $r^3 = \frac{m}{\frac{4}{3}\pi d}$

Putting the value ofd and $m\left(=\left(\frac{qE}{g}\right)\right)$ and solving we get r

$$= 7.8 \times 10^{-7}\mathrm{m}$$

40. **In a uniformly charged sphere oftotal charge Q and radius R, the electric field E is plotted as function of distance from the centre, The graph which would correspond to the above will be: [2012]**



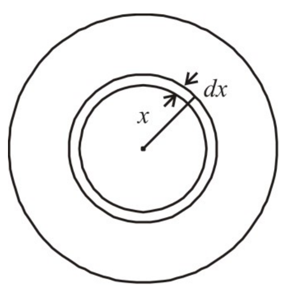
Solution : **(a)**

Let us consider a spherical shell of radius x andthickness dx.

Charge on this shell $dq = \mathrm{p}.4\tau.\mathrm{r}^2 dx^{=\mathrm{p}_0} \left(\frac{5}{4} - \frac{x}{R}\right).4\pi\kappa^2 dx$

Total charge in the spherical region from centre to $r(r < R)$ is

$$q = \int \ d\,q = 4\pi\mathrm{p}_0 \int_0^r \left(\frac{5}{4} - \frac{x}{R}\right) x^2 dx$$



$$= 4\pi\mathrm{p}_0 \left[\frac{5}{4}.\frac{r^3}{3} - \frac{1}{R}.\frac{r^4}{4}\right] = \pi\mathrm{p}_0 r^3 \left(\frac{5}{3} - \frac{r}{R}\right)$$

Electric field at $r, E = \frac{1}{4\pi\in 0}.\frac{q}{r^2}$

$$= \frac{1}{4\pi \in 0}.\frac{\pi\mathrm{p}_0 r^3}{r^2}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{\mathrm{p}_0 r}{4 \in 0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

41. Three positive charges of equal value q are placed atvertices of an equilateral triangle. The resulting lines of force should be sketched as in [Online May26, 2012]

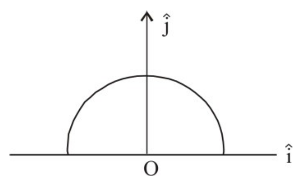
(a) 

(b) 

(c) 

(d) 

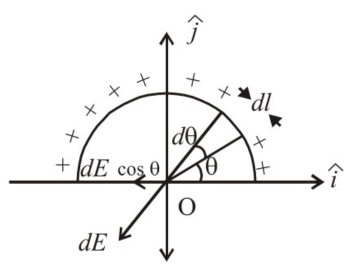
42. A thin semi - circular ring ofradius r has a positive charge q distributed uniformly over it. The net field $\bar{E}$ at the centre O is [2010]



(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (b) $-\frac{q}{4\pi^2\varepsilon_\cap r^2}j$ (c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Solution : **(c)**

Letus consider a differential element dl subtending at angle dQ at the centre Q as shown in the figure. Linear charge density $\lambda = \frac{q}{Qr}$



$dE \sin\theta$

Charge on the element, $dq = \left(\frac{q}{\pi r}\right) dl = \frac{q}{\pi r}(rd\theta)\ (dl = rd\theta) = \left(\frac{q}{\pi}\right) d\theta$

Electric field at the center O due to dq is$dE = \frac{1}{4\pi\in 0}\cdot\frac{dq}{r^2} = \frac{1}{4\pi\in 0}\cdot\frac{q}{\pi r^2}d0$

Resolving dE into two rectangular component, we fmd the component $dE\cos\theta$ will be counter balanced by another element on left portion. Hence resultant field at O is the resultant ofthe component $dE\sin\theta$ only.

$$E = \int dE\sin\theta = \int_0^\pi \frac{q}{4\pi^2 r^2 \in 0}\sin 0 d0 = \frac{q}{4\pi^2 r^2 \in 0}[-\cos\theta]_0^\pi$$

$$= \frac{q}{4\pi^2 r^2 \in 0}(+1+1) = \frac{q}{2\pi^2 r^2 \in 0}$$

The direction of E is towards negative y - axis. $\vec{E} = -\frac{q}{2\pi^2 r^2 \in 0}\hat{j}$

43. Let there be a spherically symmetric charge distribution with charge densityvarying as $\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $= R$, and $p(r) = 0$ for $> R$, where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by [2010]

(a) $\frac{p_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (b) $\frac{4\pi p_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (c) $\frac{\rho_0 \gamma}{4\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Solution : (a)

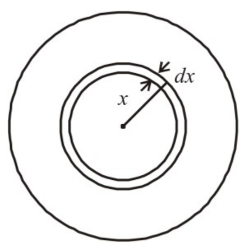
Let us consider a spherical shell of radius x and thickness dx.

Due to shpherically symmetric charge distribution, the chrge on the spherical surface ofradius x is

$$dq = dV p \cdot 4\pi x^2 dx = p_0\left(\frac{5}{4} - \frac{x}{R}\right) \cdot 4\pi x^2 dx$$

Total charge in the spherical region from centre to $r(r < R)$ is

$$q = \int dq = 4\pi p_0 \int_0^r \left(\frac{5}{4} - \frac{x}{R}\right) x^2 dx$$



$$= 4\pi p_0\left[\frac{5}{4}\cdot\frac{r^3}{3} - \frac{1}{R}\cdot\frac{r^4}{4}\right] = \pi p_0 r^3\left(\frac{5}{3} - \frac{r}{R}\right)$$

Electric field intensity at a point on this spherical surface

$$E = \frac{1}{4\pi \in 0}\cdot\frac{q}{r^2}$$

$$= \frac{1}{4\pi \in 0}\cdot\frac{\pi p_0 r^3}{r^2}\left(\frac{5}{3} - \frac{r}{R}\right) = \frac{p_0 r}{4 \in 0}\left(\frac{5}{3} - \frac{r}{R}\right)$$

44. This question contains Statement - I and Statement - 2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement - I : For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent ofthe path connecting point P to point Q.

Statement - 2 : The net work done by a conservative force on an object moving along a closed loop is zero. [2009]

(a) Statement - 1 is true, Statement - 2 is true; Statement - 2 is the correct explanation ofStatement - I.

(b) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not the correct explanation ofStatement - I.

(c) Statement - 1 is false, Statement - 2 is true.

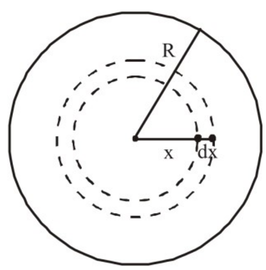
(d) Statement - 1 is true, Statement - 2 is false.

Solution : (a)

45. Let $\rho(r) = \frac{Q}{\pi R^4} r$ be the charge density distribution for a solid sphere ofradius R and total charge Q. For a point 'P' inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is : [2009]

(a) $\frac{Q}{4\pi\in r}$ (b) $\frac{Qr_1^2}{4\pi\in 0R^4}$ (c) $\frac{Qr_1^2}{4}$ (d) 0

Solution : (b)



Let us consider a spherical shell ofthickness dx and radius x. The area ofthis spherical shell $= 4\pi x^2$.

The volume ofthis spherical shell $= 4\pi x^2 dx$. The charge enclosed within shell

$$dq = \left[\frac{Q.x}{\pi R^4}\right][4\pi x^2 dr] = \frac{4Q}{R^4} x^3 dx$$

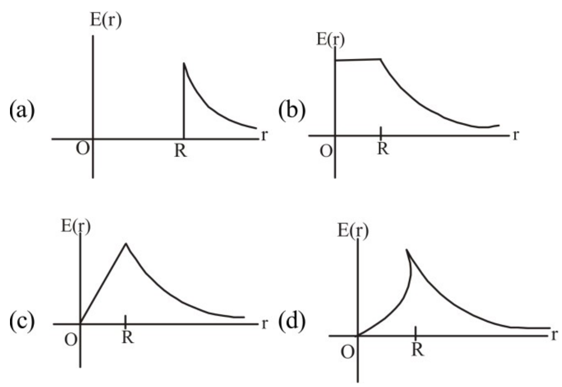
The charge enclosed in a sphere of radius r_1 can be calculated by

$$Q = \int dq = \frac{4Q}{R^4}\int_0^{r_1} x^3\, dx = \frac{4Q}{R^4}\left[\frac{x^4}{4}\right]_0^{r_1} = \frac{Q}{R^4} r_1^4$$

The electric field at point P inside the sphere at a distance r_1 from the centre ofthe sphere is $E = \frac{1Q}{4\pi E_{r_1^2}}$

$$\Rightarrow E = \frac{1}{4\pi \epsilon_0} \frac{\left[\frac{Q}{R^4} r_1^4\right]}{r_1^2} = \frac{1}{4\pi \in 0} \frac{Q}{R^4} r_1^2$$

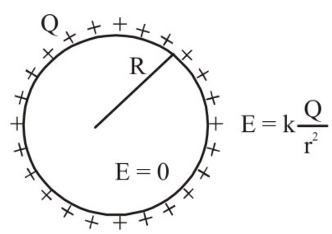
46. A thin spherical shell of radus R has charge Q spreaduniformly over its surface. Which ofthe following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \leq r < \infty$, where r is the distance fi: omthe centre ofthe shell? [2008]



Solution : **(a)**

The electric field inside a thin spherical shell ofradius

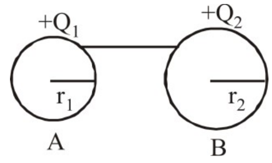
R has charge Q spread uniformly over its surface is zero.



47. Two spherical conductors A and B ofradii 1 mm and2 mmare separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the magnitude ofthe electric fields at the surfaces ofspheres A and B is [2006]

(a) 4: 1 (b) 1: 2 (c) 2: 1 (d) 1: 4

Solution : **(c)**



When the two conducting spheres are connected by a conducting wire, charge will flow from one to other till both acquire same potential.

After connection, $V_1 = V_2$

$$\Rightarrow k\frac{Q_1}{r_1} = k\frac{Q_2}{r_2} \Rightarrow \frac{Q_1}{r_1} = \frac{Q_2}{r_2}$$

The ratio of electric fields $\frac{E_1}{E_2} = \frac{k\frac{Q_1}{r_1^2}}{k\frac{Q_2}{r_2^2}} \Rightarrow \frac{E_1}{E_2} = \frac{Q_1}{r_1^2} \times \frac{r_2^2}{Q_2}$

$$\Rightarrow \frac{E_1}{E_2} = \frac{r_1 \times r_2^2}{r_1^2 \times r_2} \Rightarrow \frac{E_1}{E_2} = \frac{r_2}{r_1} = \frac{2}{1}$$

Outside the shell the electric field is $E = k\frac{Q}{r^2}$. These characteristics are represented by graph (a).

48. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$ respectively. The location ofa point on the x axis at which the net electric field due to these two point charges is zero is [2005]

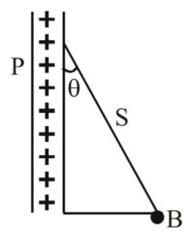
(a) $\frac{L}{4}$ (b) $2L$ (c) $4\ L$ (d) $8\ L$

Solution : At $P\frac{-K2q}{(x-L)^2} + \frac{K8q}{x^2} = 0$

$$\Rightarrow \frac{1}{(x-L)^2} = \frac{4}{x^2}$$

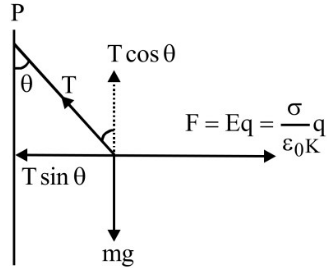
$$\Rightarrow x = 2x - 2L \text{ or } x = 2L$$

49. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density 0 of the sheet is proportional to [2005]



(a) $\cot\theta$ (b) $\cos\theta$ (c) $\tan\theta$ (d) $\sin\theta$

Solution : (c)



$$T\sin\theta = qE \text{ (i)}$$

$$T\cos\theta = mg \text{ (ii)}$$

Dividing (i) by(ii), $\tan 6 = \frac{qE}{mg} = \frac{q}{mg}\left(\frac{o}{\varepsilon_0 K}\right)\frac{oq}{\varepsilon_0 K.mg}$

$$o \propto \tan\theta$$

50. Four charges equal to $-Q$ are placed at the four corners ofa square and a charge q is at its centre. Ifthe system is in equilibrium the value ofq is [2004]

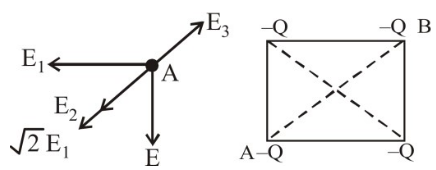
(a) $-\frac{Q}{2}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$ (c) $-\frac{Q}{4}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

Solution :

For the system to be equilibrium, net field at A should be zero

$$\sqrt{2}E_1 + E_2 = E_3$$

$$\frac{kQ\times\sqrt{2}}{a^2} + \frac{kQ}{(\sqrt{2}a)^2} = \frac{kq}{\left(\frac{a}{\sqrt{2}}\right)^2}$$



$$\Rightarrow \frac{Q\sqrt{2}}{1} + \frac{Q}{2} = 2q \Rightarrow q = \frac{Q}{4}(2\sqrt{2}+1)$$

51. A charged oil drop is suspended in auniform field of3×10^4 v/m so that it neither falls nor rises. The charge on the drop will be (Take the mass ofthe charge $= 9.9\times10^{-15}$ kg and $g = 10\text{m/s}^2$) [2004]

(a) 1.6×10^{-18}C (b) 3.2×10^{-18}C (c) 3.3×10^{-18}C (d) 4.8×10^{-18}C

Solution : (c)

Given, Electric field, $E = 3\times10^4$

Mass ofthe drop, $m = 9.9\times10^{-15}$ kg

At equilibnum, coulomb force on drop balances weight ofdrop $qE = mg$

$$\Rightarrow q = \frac{mg}{E} \Rightarrow q = \frac{9.9\times10^{-15}\times10}{3\times10^4} = 3.3\times10^{-18}\text{C}$$

Electric dipole , electric flux and Gauss Law :

52. **Two identical electric point dipoles have dipole moments $\overrightarrow{P_1} = P\hat{\imath}$ and $\overrightarrow{P_2} = -P\hat{\imath}$ and are held on the x axis at distance a' from each other. When released, they move along x - axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm' , their speed when they are infinitely far apart is:**

[Sep. 06, 2020 (ID]

(a) $\frac{P}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$ (b) $\frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$ (c) $\frac{P}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$ (d) $\frac{P}{a}\sqrt{\frac{2}{2\pi\varepsilon_0 ma}}$

Solution :

Let v be the speed ofdipole.

Using energy conservation$K_i + U_i = K_f + U_f$

$$\Rightarrow 0 - \frac{2k \cdot p_1}{r^3} p_2 \cos(180^\circ) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 0$$

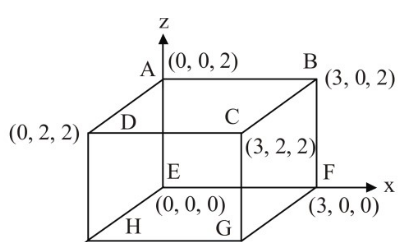
$(\because$ **Potential energy ofinteraction between dipole**$= \frac{-2p_1 p_2 \cos\theta}{4\pi\in r^3,0})$

$$\Rightarrow mv^2 = \frac{2\Phi_1 p_2}{r^3} \Rightarrow v = \sqrt{\frac{2\Phi_1 p_2}{mr^3}}$$

When $p_1 = p_2 = p$ and $r = a$

$$v = \frac{p}{a}\sqrt{\frac{1}{2\pi \in ma0}}$$

53. **An electric field $\overline{E} = 4x\hat{\imath} - (y^2 + 1)\hat{\jmath}N/C$ passes through the box shown in figure. The flux of the electric field through surfaces ABCD and BCGF are marked as φ_1 and φ_{11} respectively. The difference between $(\varphi_1 - \varphi_{11})$ is (inNm^2/C) .** **[9 Jan 2020, II]**



$\swarrow^{(0,2,0)}$ $(3, 2, 0)$

Solution : (-48)

Flux ofelectric field $\vec{E}$ through any area $\vec{A}$ is defined as$\varphi = \int E \,.\, A\cos\theta$

Here, θ = angle between electric field and area vector ofa surface

For surface *ABCD* Angle, $\theta = 90^{\circ}$

$$\varphi_1 = \int E.A\cos 90^{\circ} = 0$$

For surface *BCGF* $\varphi_n = \int \vec{E}.\overline{dA}$

$$\varphi_{11} = [4\times\hat{i} - (y^2+1)j].4\hat{i} = 16x$$

$$(|)11 = 48\frac{Nm^2}{C}$$

$$\varphi_1 - \varphi_{11} = -48$$

54. In finding the electric field using Gauss law the formula $|\vec{E}| = \frac{q_{enc}}{\epsilon 0|A|}$ is applicable. In the formula ϵ_0 is permittivity offree space, A is the area ofGaussian surface and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which ofthe following situation? [8 Jan 2020, I]

(a) Only when the Gaussian surface is an equipotential surface. Only when the Gaussian surface is an

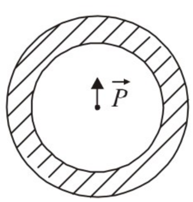
(b) equipotential surface and $|\vec{E}|$ is constant on the surface.

(c) Only when $|\vec{E}|$ = constant on the surface.

(d) For any choice ofGaussian surface.

Solution : (a)

55. Shown in the figure is a shell made ofa conductor. It hasinner radius a and outer radius b, and carries charge Q. At its centre is a dipole p as shown. In this case:



[12 April 2019, I]

(a) surface change density on the inner surface is uniformand equal to $\frac{Q/2}{4\pi a^2}$

b) electric field outside the shell is the same as that ofa point charge at the centre ofthe shell.

(c) surface charge density on the outer surface depends on $|\vec{p}|$

(d) surface charge density on the inner surface of the shell is zero everywhere. -

Solution :

Surface charge density depends only due to Q. Also

$$\oint \to E.d \to A = \underline{q_{1_0}\lambda\mathcal{E}}$$

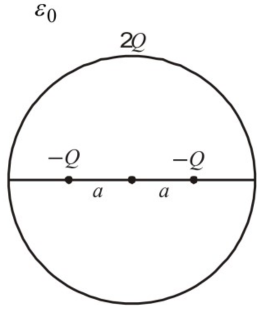
or $E \times 4\pi r^2 = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{1Q}{4\pi\varepsilon_{0r^2}}, r \geq R$

56. Let a total charge 2 Q be distributed in a sphere ofradiusR, with the charge density given by $\mathrm{p}(r) = kr$, where r is the distance from the centre. Two charges A and B, of - Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. IfA and B do not experienceany force, then. [12 April 2019, II]

(a) $a = 8^{1/4}\mathrm{R}$ (b) $a = \frac{3R}{2^{1/4}}$ (c) $a = 2^{1/4}\mathrm{R}$ (d) $a = R/\sqrt{3}$

Solution :

56. (a) $\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$



or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int \; S(4\pi r^2)dr$

or $\mathrm{E} \times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r (kr)\,(4\pi r^2)dr$

or $E \times 4\pi r^2 = \frac{4\pi k}{\varepsilon_0}\left(\frac{r^4}{4}\right)$

$k\,2$

$E = \overline{4\varepsilon_{0}}_{r}$ (i)

Also $2Q = \int_0^R (kr)\,(4\pi r^2)dr = 4\pi k|\frac{r^4}{4}|_0^R$

$Q = \frac{\pi k R^4}{2}$ (ii)

From above equations,$E = \frac{Qr^2}{2\pi\varepsilon_0 R^4}\,(Reject)$

According to given condition$= EQ \frac{Q^4}{4\pi\varepsilon_0(20)^2}$ (iv)

From equations (iii) and (iv), we have

$a = 8^{-1/4}$ R.

57. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. Ifit is slightly rotated from its equilibrium orientation, then its angular frequency o) is : [8 April 2019, II]

(a) $\sqrt{\frac{qE}{md}}$ (b) $\sqrt{\frac{2qE}{md}}$ (c) $2\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

Solution :

$$\tau = -PE \sin\theta \text{ or } I\alpha = -PE(\theta)$$

$$\alpha = \frac{PE}{I}(-\theta)$$

On comparing with $\alpha = -0)^2\theta$

$$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$$

58. An electric field of1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole? [11 Jan 2019, II]

(a) -20×10^{-18}J (b) -7×10^{-27}J (c) -10×10^{-29}J (d) -9×10^{-20}J

Solution : Potential energy ofa dipole is given by$U = -\vec{P}.\vec{E}$

$$= -\text{PE} \cos\theta$$

[WhereO = angle between dipole and perpendicular to thefield]

$$= -(10^{-29})(10^3)\cos 45^\circ$$

$$= -0.707\times10^{-26}\text{J} = -7\times10^{-27}\text{J}$$

59. Charges–q and + q located at A and B, respectively,constitute an electric dipole. Distance AB = 2a, O is the mid point of the dipole and OP is perpendicular to AB.A charge Q is placed at P where OP $= y$ and $y >> 2a$. The charge Q experiences an electrostatic force F. IfQ is now moved along the equatorial line to P$'$ such that OP' $= \left(\frac{y}{3}\right)$, the force on Q will be close to: $(\frac{y}{3} >> 2a) \uparrow$ P [10 Jan 2019, II]

(a) 3 F (b) $\frac{F}{3}$ (c) 9 F (d) 27 F

Solution : (d)

Electric field ofequitorial plane ofdipole$= -\overline{K\vec{P}/r^3}$

At point P, $= +\frac{KP}{y^3}Q$

At Point P^1 , $F^1 = +\frac{KPQ}{(y/3)^3} = 27$ F.

61. An electric dipole has a fixed dipole moment , which makes angle 6 with respect to x - axis. When subjected to an electric field $\overline{E_1} = E\hat{\imath}$, it experiences a torque $\overline{T_1} = \tau\hat{i}$. When subjected to another electric field $\overline{E_2} = \sqrt{3E_1}\hat{j}$ it experiences torque $\overrightarrow{T_2} = -\overline{T_1}$. The angle θ is: [2017]

(a) 60° (b) 90° (c) 30° (d) 45°

Solution : (a)

$T = PE\sin\theta$ Torque experienced by the dipole in an electric field, $\vec{T} = \vec{P} \times \vec{E}$

$$\vec{p} = p\cos\theta\,\hat{\imath} + p\sin\theta j$$

$$\vec{E}_1 = E\vec{i}$$

$$\overrightarrow{T_1} = \vec{p} \times \vec{E}_1 = (p\cos\theta\,\hat{\imath} + p\sin\theta\,j) \times E(\hat{\imath})$$

$$\tau\hat{k} = pE\sin\theta(-\hat{k}) \text{ (i)}$$

$$\vec{E}_2 = \sqrt{3}E_1\hat{j}$$

$$\vec{T}_2 = p\cos O\hat{\imath} + p\sin\theta\hat{j}) \times \sqrt{3}E_1\hat{j}$$

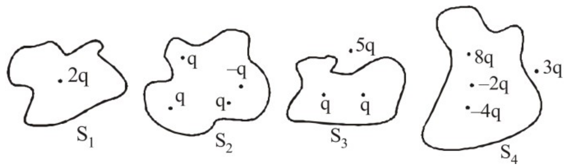
$$\tau\hat{k} = \sqrt{3}pE_1\cos\theta\hat{k} \text{ (ii)}$$

From eqns. (*i*) and (*ii*)

$$pE\sin\theta = \sqrt{3}pE\cos\theta$$

$$\tan\theta = \sqrt{3}\ \theta = 60^\circ$$

62. **Four closed surfaces and corresponding charge distributions are shown below. [Online Apri19, 2017]**



Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3, and Φ_4. Then:

(a) $\Phi_1 \langle \Phi_2 = \Phi_3 \rangle \Phi_4$ **(b)** $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$

(c) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ **(d)** $\Phi_1 > \Phi_3; \Phi_2 < \Phi_4$

Solution : **(c)**

The net flux linked with closed surfaces S_1, S_2, S_3 & S_4 are

For surface S_1 , $\varphi_1 = (2q)/\varepsilon_0$

For surface S_2, $\varphi_2 = \frac{1}{\varepsilon_0}(q + q + q - q) = \frac{1}{\varepsilon_0}2q$

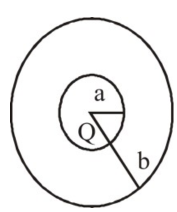
For surface S_3, $\varphi_3 = \frac{1}{\varepsilon_0}(q + q) = \frac{1}{\varepsilon_0}(2q)$

For surface S_4, $\varphi_4 = (8q - 2q - 4q)/\varepsilon_0 = (2q)/\varepsilon_0$

Hence, $\varphi_1 = \varphi_2 = \varphi_3 = \varphi_4$ i.e. net electric flux is same for allsurfaces.

Keep in mind, the electric field due to a charge outside $(S_3 \text{and } S_4)$, the Gaussian surface contributes zero net flux through the surface, because as many lines due to thatcharge enter the surface as leave it.

63. **The region between two concentric spheres ofradii ↑ a↑ and b1, respectively (see figure), have volume charge density $p = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre ofthe spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: [2016]**



(a) $\frac{2Q}{\pi(a^2-b^2)}$ **(b)** $\frac{2Q}{\pi a^2}$ **(c)** $\frac{Q}{2\pi a^2}$ **(d)** $\frac{Q}{2\pi(b^2-a^2)}$

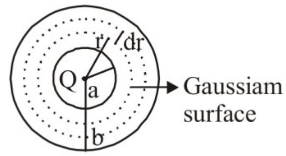
Solution : **(c)**

Applying Gauss’ s law

$$\oint_S \vec{E}\cdot\overline{ds} = \in \underline{Q0}$$

$$E \times 4\pi r^2 = \underline{Q + 2\pi A r_0^2 - 2\pi A a^2} \in$$

$$\rho = \frac{dr}{dV}$$



$$Q = \rho 4\pi r^2$$

$$Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A[r^2 - a^2]$$

$$E = \frac{1}{4\pi \in 0}\left[\frac{Q - 2\pi A a^2}{r^2} + 2\pi A\right]$$

For E to be independent of r' $Q - 2\pi A a^2 = 0$

$$A = \frac{Q}{2\pi a^2}$$

64. The electric field in a region of space is given by, $\vec{E} = E_o\hat{i} + 2E_o\hat{j}$ where $E_o = 100\mathrm{N/C}$. The flux ofthe field through a circular surface ofradius $0.02\mathrm{m}$ parallel to the Y - Z plane is nearly: [Online Apri119, 2014]

(a) $0.125\mathrm{Nm}^2/\mathrm{C}$ (b) $0.02\mathrm{Nm}^2/\mathrm{C}$ (c) $0.005\mathrm{Nm}^2/\mathrm{C}$ (d) $3.14\mathrm{Nm}^2/\mathrm{C}$

Solution : (a)

$$\rightarrow \mathrm{E} = \mathrm{E}_0\hat{\mathrm{i}} + 2\mathrm{E}_0\hat{\mathrm{j}}$$

Given, $\mathrm{E}_0 = 100\mathrm{N/c}$

So, $\rightarrow \mathrm{E} = 100\hat{\mathrm{i}} + 200\mathrm{j}$

Radius ofcircular surface $= 0.02\mathrm{m}$

Area $= \pi \mathrm{r}^2 = \frac{22}{7} \times 0.02 \times 0.02$

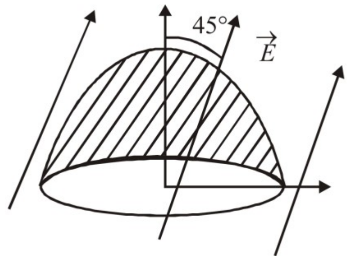
$= 1.25 \times 10^{-3}\,\hat{\mathrm{i}}\,\mathrm{m}^2$ [Loop is parallel to Y - Z plane] Now, flux $(\varphi) = \mathrm{EA}\cos\theta$

=(100î+200j). $1.25 \times 10^{-3}\hat{\mathrm{i}}\cos\theta^\circ[\theta = 0^\circ]$

$$= 125 \times 10^{-3}\mathrm{Nm}^2/\mathrm{c}$$

$$= 0.125\mathrm{Nm}^2/\mathrm{c}$$

66. The flat base ofa hemisphere ofradius a with no charge inside it lies in a horizontal plane. A uniform electric field $\vec{E}$ is applied at an angle $\frac{\pi}{4}$ with the vertical direction. The electric flux through the curved surface ofthe hemisphere is [Online May 19, 2012]



(a) $\pi a^2 E$ (b) $\frac{\pi a^2 E}{\sqrt{2}}$ (c) $\frac{\pi a^2 E}{2\sqrt{2}}$ (d) $\frac{(\pi+2)\pi a^2 E}{(2\sqrt{2})^2}$

Solution : (b)

We know that, $\varphi = \oint E . dS = E \oint dS \cos 45^\circ$

In case of hemisphere $\varphi_{\text{curved}} = \varphi_{\text{circular}}$

Therefore, $\varphi_{\text{curved}} = E\pi a^2 . \frac{1}{\sqrt{2}} = \frac{E\pi a^2}{\sqrt{2}}$

67. An electric dipole is placed at an angle of 30° to a non - uniform electric field. The dipole will experience [2006]

(a) a translational force only in the direction ofthe field

(b) a translational force only in a direction normal to the direction of the field

(c) a torque as well as a translational force

(d) a torque only

Solution : (c)

As the dipole is placed in non - uniform field, so the force acting on the dipole will not cancel each other. This will result in a force as well as torque.

68. Ifthe electric flux entering and leaving an enclosed surface respectively is φ_1 and φ_2, the electric charge inside the surface will be [2003]

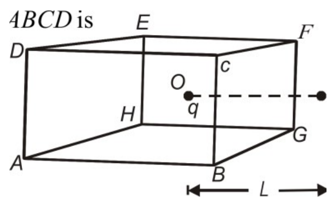
(a) $(\varphi_2 - \varphi_1)\varepsilon_o$ (b) $(\varphi_1 - \varphi_2)/\varepsilon_o$ (c) $(\varphi_2 - \varphi_1)/\varepsilon_o$ (d) $(\varphi_1 - \varphi_2)\varepsilon_o$

Solution : (a)

The electric flux φ_1 entering an enclosed surface istaken as negative and the electric flux φ_2 leaving the surface is taken as positive, by convention. Therefore the net flux leaving the enclosed surface, $\varphi = \varphi_2 - \varphi_1$

According to Gauss theorem $\varphi = \in \underline{q}0 \Rightarrow q^{=} \in 0\varphi = \in 0(\varphi_2 - \varphi_1)$

69. A charged particle q is placed at the centre O of cube of length L ($A\ BCDEFGH$) . Another same charge q is placed at a distance L from O. Then the electric flux through [2002]



q

(a) $ql4\pi \in_0 L$ (b) zero (c) $ql2\pi \in_0 L$ (d) $ql3\pi \in_0 L$

Solution : (None)

Electric flux due to charge placed outside is zero.

But for the charge inside the cube, flux due to each face is

$\frac{1}{6}\left[\underline{q_0}\right]$ which is not in option.

ELECTRIC POTENTIAL & CAPACITANCE

ELECTRIC CAPACITY

The ratio of charge to potential of a conductor is called its capacity. $C = \frac{Q}{V}$

Unit : farad (F)

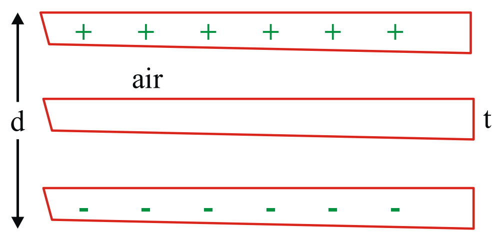
Parallel Plate Capacitor:

If two plates each of area A are seperated by a distance 'd' then its capacity

$$C=\frac{\varepsilon_0 A}{d} \text{ (air as medium)},$$

$$C=\frac{k\varepsilon_0 A}{d} \text{ (dielectric medium)}$$

- When a dielectric medium is introduced between the plates of a parallel plate capacitor, its capacity increases to 'k' times the original capacity.
- When a dielectric slab of thickness 't' is introduced between the plates of a parallel plate capacitor,

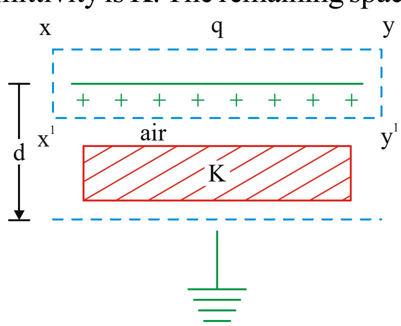


$$new\ \ capacity = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{k}\right)} = \frac{\epsilon_0\ A}{(d-t) + \frac{t}{k}}$$

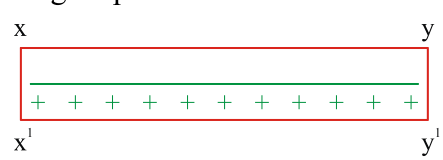
GAUSS METHOD

Let us consider a case of parallel plate capacitor in which a medium of dielectric constant K is partially filled as shown in figure.

Then the field is uniform in air as well as in medium but they will have different values. let 't' be the thickness of the medium whose relative permittivity is K. The remaining space of (d - t) thickness be occupied by air.



Imagine a Gaussian surface enclosing the plate as shown.



If E_0 is the field in air, then from Gauss law

$$\int E_0 ds = \frac{q}{\varepsilon_0} \Rightarrow E_0 A = \frac{q}{\varepsilon_0}$$

$$E_0 = \frac{q}{A\varepsilon_0} \quad .... \text{(a)}$$

Similarly by considering a Gaussian surface through the medium, then by Gauss law,

$$\int E.ds = \frac{q}{\varepsilon_0 K} \Rightarrow EA = \frac{q}{\varepsilon_0 K}$$

where E is a field in the medium

$$\therefore E = \frac{q}{A\varepsilon_0 K} \quad .... \text{(b)}$$

The P.D. between the two plates of the capacitor.

$$V = E_0(d-t) + E.t$$

$$V = \frac{q}{A\varepsilon_0}(d-t) + \frac{q}{A\varepsilon_0 K} t$$

$$= \frac{q}{A\varepsilon_0}\left[(d-t) + \frac{t}{K}\right]$$

$$\text{or } C = \frac{q}{V} = \frac{q}{\frac{q}{A\varepsilon_0}\left[d-t+t/K\right]}$$

- When a metal slab of thickness 't' is introduced between the plates of a parallel plate capacitor,

  new capacity $= \dfrac{\varepsilon_o A}{d-t}$.

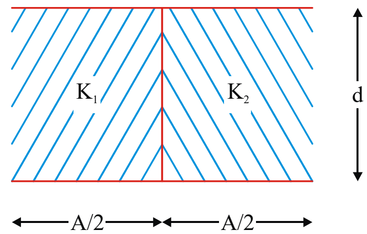
  (for metal $k = \infty$)
- The method for the calculation of capacitance requires integration of the electric field between two conductors or the plates which are separated with a potential difference V_{ab}

  $$\text{i.e. } V_{ab} = -\int_b^a \bar{E}.d\bar{r}$$

  $$\text{or } V_+ - V_- = -\int \bar{E}.d\bar{r} \quad \text{from this } C = \frac{q}{V_{ab}}$$

- When a thin metal sheet $(t \approx 0)$ is introduced between the plates of a parallel plate capacitor, then capacity remains unchanged.
- A dielectric slab of thickness 't' is introduced between the plates, to restore the original capacity, if the distance between the plates is increased by x, then $x = t\left(1 - \frac{1}{k}\right)$.
- Two dielectric slabs of equal thickness are introduced between the plates of a capacitor as shown in figure, then new capacity

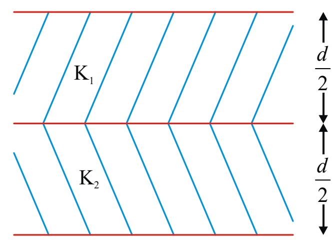
$$=\frac{C}{2}(K_1+K_2).$$



If the two dielectrics are of different face areas A_1 and A_2 but of same thickness, then capacity,

$$C=\frac{\in_0}{d}(K_1A_1+K_2A_2)$$

- If two dielectric slabs of constants k_1 and k_2 are introduced as shown in figure, new capacity $=\frac{2k_1k_2}{(k_1+k_2)}.C$



- If number of dielectric slabs of same cross sectional area 'A' and of thicknesses $t_1, t_2, t_3, ...t_n$ and constants $k_1, k_2.......k_n$ are introduced between the plates, effective capacity

$$C=\frac{\varepsilon_0 A}{d-(t_1+t_2+.....t_n)+\left(\frac{t_1}{k_1}+......+\frac{t_n}{k_n}\right)}$$

- In the above case if the dielectric media are completely filled between the plates, effective capacity

$$C=\frac{\varepsilon_0 A}{\left(\frac{t_1}{k_1}+.....+\frac{t_n}{k_n}\right)}$$

- The capacity of a parallel plate capacitor is independent of the charge on it, potential difference between the plates and the nature of plate material.
- In a capacitor, the energy is stored in the electric field between the two plates.
- Capacity of a spherical conductor = $4\pi\in_0 r$, where r is the radius of the sphere.
- If we imagine earth to be a uniform solid sphere then capacity of earth is $4\pi\in_0 R$
- Where R = Radius of the earth = $6400\times10^3 m$

Note : For the earth, $R=6.4\times10^6$ m

The capacity of earth is

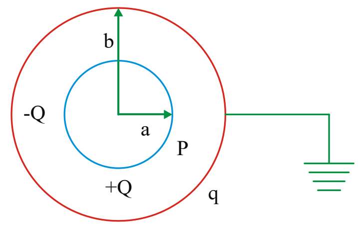
$$C=4\pi\in_0 R=\frac{1}{9\times10^9}\times6.4\times10^6=711\mu F$$

Spherical condenser

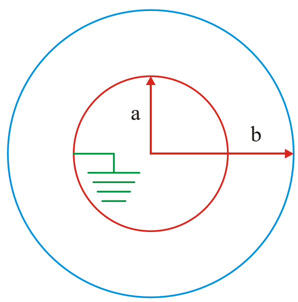
$$V = V = V_p - V_q = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{a} + \frac{-q}{b}\right) - 0$$

$$= \frac{1}{4\pi\varepsilon_0} q\left(\frac{b-a}{ab}\right)$$

$$C = \frac{q}{v}$$



a) $C = 4\pi \in_0 \dfrac{ab}{b-a}$, if inner sphere is charged and outer sphere is earthed.

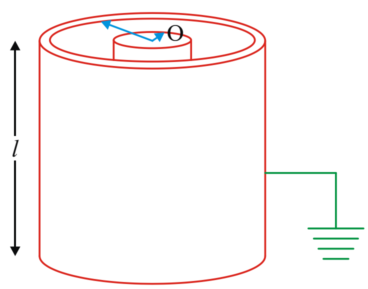


b) $C = 4\pi \in_0 \dfrac{b^2}{b-a}$,

If inner sphere is earthed and outer sphere is charged.

Cylindrical Capacitor:

A cylinjdrical capacitor consists of two coaxial cylinders and its capacitance is given by

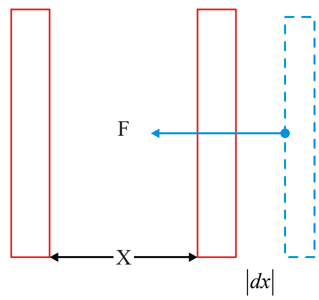


$$c = \frac{2\pi\varepsilon_0 l}{\log_e\left(\dfrac{b}{a}\right)}$$

Where l is the length of each of cylinder and a and b are the radii of the inner and outer cylinders.

Force between the plates of a capacitor

Consider a parallel plate capacitor with plate area A. Let Q and –Q be the charges on the plates of capacitor. Let F be the force of attraction between the plates. Let E be the field between the capacitor plates.The expression for the force can be derived by energy method. Let the distance between the plates be x.
So electric field energy between the plates is



$$U = \frac{1}{2} \epsilon_0 E^2 (Ax)$$

$$\frac{dU}{dx} = \frac{1}{2} \epsilon_0 E^2 A$$

By definition

$$F = -\frac{dU}{dx} = \frac{-1}{2} \epsilon_0 E^2 A$$

(Conservative force)

So the force of attraction between the plates is $F = \frac{1}{2} \epsilon_0 E^2 A$

Note : For an isolated charged capacitor $F = \frac{Q^2}{2 \epsilon_0 A}$. This force does not depend on the separation between the plates, and so the constant amount of force is needed to change the separation.

Note : For a capacitor having constant potential difference across the plates the force

$$F = \frac{C^2V^2}{2 \epsilon_0 A} = \frac{\epsilon_0^2 A^2}{d^2} \frac{V^2}{2 \epsilon_0 A};$$

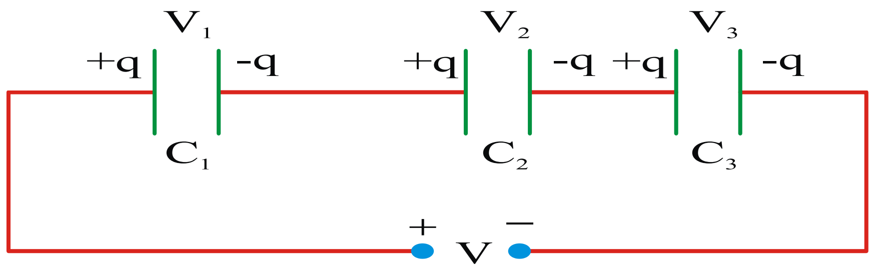
$$F = \frac{1}{2} \epsilon_0 \frac{V^2}{d^2} A$$

In this case force depends on the separation between the plates. Thus to change the separation variable force is needed.

CAPACITORS IN SERIES

In series combination, the capacitors are first arranged in a series order such that the second plate of first capacitor is connected to the first plate of second capacitor, the second plate of second capacitor is connected to first plate of third capacitor and so on. And finally the first plate of first capacitor and second plate of last capacitor are connected to opposite terminals of battery.

Let us consider three capacitors of capacities C_1, C_2 and C_3 connected in series across a source of potential difference 'V' as shown in figure.



At the moment, the system is connected to the source, left plate of first condenser acquires positive charge due to conduction. This inturn will produce negative charge of equal magnitude, on the left face of second plate of first condenser due to induction. The process continues for the remaining two condensers. Hence the charge acquired by all the three capacitors will be same.

As the capacitors are different, the potentials developed across them will be different.

$$q = C_1V_1 = C_2V_2 = C_3V_3$$

$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, V_3 = \frac{q}{C_3}$$

$$\text{But } V = V_1 + V_2 + V_3$$

$$V = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \text{.... (1)}$$

If a single capacitor when connected across the same source draws the same charge, that capacitance is said to be the equivalent capacitance of the three capacitors. If C_S is the equivalent capacitance.

$$C_S = \frac{q}{V}$$

$$V = \frac{q}{C_S} \text{-----(2)}$$

Substituting (2) in (1)

$$\frac{q}{C_S} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3};$$

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

In general $$\frac{1}{C_S} = \sum \frac{1}{C_n}$$

- The resultant capacity of series combination is smaller than the least capacity of the capacitors of the combination.
- In series, ratio of charges on three capacitors is 1 : 1 :1.
- The ratio of potential differences across three capacitors is

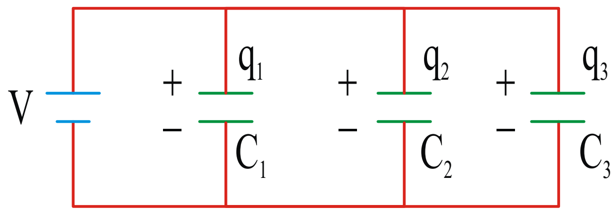
$$V_1 : V_2 : V_3 = \frac{Q}{C_1} : \frac{Q}{C_2} : \frac{Q}{C_3} = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

- P.D across first capacitor is $V_1 = \dfrac{\dfrac{1}{C_1}}{\left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}\right)} V$

similary we can find V_2 and V_3.

Capacitors in parallel

Capacitors are said to be connected in parallel if the two plates of any capacitor are connected one to positive terminal and the other to negative terminal of the source, then the connection is said to be parallel connection.



Let us consider three capacitors of capacities C_1, C_2 and C_3 connected in parallel across a source 'V' as shown

The moment capacitors are connected, charge is drawn from the voltage source and this charge is drawn along three branches and thus gets shared. As all capacitors are connected in parallel, the potential across any of the capacitors is same . Here charge gets shared depending upon their capacitances for maintaining same potential.

$$V = \frac{q_1}{C_1} = \frac{q_2}{C_2} = \frac{q_3}{C_3}$$

$$\therefore q_1 + q_2 + q_3 = C_1V + C_2V + C_3V$$

$$q = V(C_1 + C_2 + C_3)$$

$$\frac{q}{V} = C_1 + C_2 + C_3 \qquad \dots (1)$$

If a single capacitor when connected to the same source draws a charge q then that capacitor is said to be the effective or equivalent capacitor for the three parallel capacitors.

If the effective capactiance is C_p,

$$C_P = \frac{q}{V} \dots (2)$$

from (1) and (2)

$$C_P = C_1 + C_2 + C_3; \text{ In general } C_P = \sum C_n$$

- The resultant capacity of parallel combination is greater than the largest capacity of the capacitors of the combination.
- In parallel, ratio of P.D. on three capacitors is 1 : 1 :1.
- The ratio of charges on three capacitors is $Q_1 : Q_2 : Q_3 = C_1V : C_2V : C_3V = C_1 : C_2 : C_3$
- The charge on first capacitor is

  $Q_1 = \frac{C_1}{C_1 + C_2 + C_3} Q$ similarly we can find Q_2 and Q_3.
- When n identical capacitors each of capacity **C** are first connected in series and next connected in parallel then the ratio of their effective capacities

$$C_s = \frac{C}{n}; \; C_p = nC$$

$$\frac{C_s}{C_p} = n^2 : 1$$

Types of Dielectrics :

- A dielectric is an insulating material in which electrons are tightly bound to the nuclei of the atoms.
  Ex: glass, mica, paper etc.
- There are two types of dielectrics
  1) Non-polar dielectrics
  2) Polar dielectrics
- In non polar dielectrics the centre of positive charge and centre of negative charge of each molecule coincide
- Under ordinary conditions Non-polar molecule will have zero dipole moment.
- When a Non-polar dielectric is subjected to electric field, the positive charge of each molecule is shifted in the direction of electric filed and negative charge in the opposite direction.
  Ex: oxygen, nitrogen
- In polar dielectrics the centre of positive charges and centre of negative charges of each molecule do not coincide.
- Each molecule has a permanent dipole moment.
- When polar dielectric is subjected to external electric field, the electric field exerts torque on the dipoles, tending to align them in the direction of the field.
  Ex: CO_2, NH_3,HCl, etc.
- If a dielectric is charged by induction then induced charge q^1 is less than inducing charge q. Induced charge, $q^1 = -q\left[1 - \frac{1}{K}\right]$
  where K is dielectric constant.
- Electric field due to induced charges on the dielectric is $E_{ind}\ or\ E_p = E_0 - \frac{E_0}{K} = E_0\left(1 - \frac{1}{k}\right)$.

Dielectric Strength of Air : A conducting sphere cannot hold very large quantity of charge. It can hold a maximum charge Q such that the electric intensity on the surface is equal to dielectric strength of air $\left(3\times10^6 Vm^{-1}\right)$

i.e. $\frac{1}{4\pi\in_0}\frac{Q}{R^2} = 3\times10^6 Vm^{-1}$

Energy stored in a condenser : Energy stored in a charged condenser $U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{q^2}{2C}$

- If a condenser is connected across a battery and U is the energy stored in the condenser then the work done by the battery in charging the condenser is $2U$ (W = qV = 2U)
  For a parallel plate capacitor
  $$U = \frac{1}{2}(Ad)\frac{\sigma^2}{\in_0}\left(as\ E = \frac{\sigma}{\in_0}\right)$$
  Energy density
  $$\frac{U}{V} = \frac{\sigma^2}{\in_0} = \frac{1}{2}\in_0 E^2$$ (here V is volume i.e. Ad)

a) When three capacitors are in **series**, the ratio of energies is

$$U_1 : U_2 : U_3 = \frac{Q^2}{2C_1} : \frac{Q^2}{2C_2} : \frac{Q^2}{2C_3} = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$$

b) When three capacitors are in **parallel**, the ratio of energies is

$$U_1 : U_2 : U_3 = \frac{1}{2}C_1V^2 : \frac{1}{2}C_2V^2 : \frac{1}{2}C_3V^2 = C_1 : C_2 : C_3$$

c) Energy density (μ) = energy/ volume

$$\mu = \frac{1}{2}\varepsilon E^2 = \frac{1}{2}k\varepsilon_0 E^2$$

(Where K in the dielectric constant of medium between the plates)

Effect of Dielectric:

- A parallel plate capacitor is fully charged to a potential V. **Without disconnecting the battery** if the gap between the plates is completely filled by a dielectric medium,capacity increases to k times the original capacity.
- P.D. between the plates remains same.
- Charge on the plates increases to k times the original charge.
- Energy stored in the capacitor increases to k times the original energy.
- **After disconnecting the battery** if the gap between the plates of the capacitor is filled by a dielectric medium,capacity increases to k times the original capacity.
- P.D. between the plates decreases to $\frac{1}{k}$ times the original potential.
- Charge on the plates remains same.
- Energy stored in the capacitor decreases to $\frac{1}{k}$ times the original energy.
- A capacitor is fully charged to a potential 'v'. After disconnecting the battery, the distance between the plates of capacitors is increased by means of insulating handles. Potential difference between the plates increases. ($V = \frac{Q}{C}$, Q remains same, and C decreases)
- A capacitor with a dielectric is fully charged. Without disconnecting the battery if the dielectric slab is removed, then some charge flows back to the battery.

Mixed Grouping of Capacitors:

- Number of capacitors in a row $n = \frac{desired\ \ potential}{given\ \ potential}$
- Number of such rows $m = \frac{desired\ capacity}{original\ capacity} \times n$
- Total number of capacitors = $m \times n$.

Coalesence of a Charged Oil Drops:

There are ‘n’ charged drops of radius ‘r’ and charge ‘q’. The drops are merge to form a bigger drop. If capicity of small drop is ‘C’ then

1) capacity of bigger drop is $C^1 = n^{\frac{1}{3}} \times C$
2) Potential of bigger drop is

$$V^1 = \frac{Q}{C^1} = \frac{nq}{n^{1/3}.C} = \frac{n^{2/3}q}{C} = n^{2/3}V.$$

3) Energy of bigger drop is

$$U = \frac{Q^2}{2C^1} = \frac{n^2 q^2}{2n^{1/3}.C} = \frac{n^{5/3} q^2}{2C} = n^{5/3}U.$$

4) Surface charge density of bigger drop is

$$\sigma^1 = \frac{Q}{4\pi R^2} = \frac{nq}{4\pi n^{2/3}.r^2} = \frac{n^{1/3} q}{4\pi r^2} = n^{1/3}.\sigma$$

S.No	**Quantity**	**For each charged small drop**	**For the big drop**
1.	Radius	r	$R=n^{1/3}r$
2.	Charge	Q	Q=nxq
3.	Capacity	C	$C^1=n^{1/3}xC$
4.	Potential	V	$V^1=n^{2/3}xV$
5.	Energy	U	$U^1=n^{5/3}U$
6.	Surface charge	σ	$\sigma=n^{1/3}.\sigma$

Introduction of dielectric in a charged capacitor

A dielectric slab (K) is introduced between the plates of the capacitor

S.No	**Physical quantity**	**With battery permanently connected**	**With battery disconnected**
1.	Capacity	K times increases	K times increases
2.	Charge	K times increases	Remains constant
3.	P.D.	Remains Constant	K times increases
4.	Electric Intensity	Remains Constant	K times increases
5.	Energy stored in condenser	K times increases	K times increases

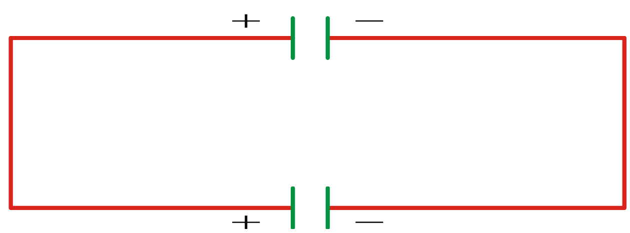
The distance between the plates of condenser is increased by n times.

S.No	Physical quantity	With battery permanently connected	With battery disconnected
1.	Capacity	n times decreases	n times decreases
2.	Charge	n times decreases	Remains constant
3.	P.D.	Remains Constant	n times increases
4.	Electric Intensity	n times decrease	Remains constant
5.	Energy stored in condenser	n times decreases	n times increases

redistribution of charge, Common potential and Loss of energy

Two capacitors of capacities C_1 and C_2 are charged to potentials V_1 and V_2 separately and they are connect so that charge flows. Here charge flows from higher potential to lower potential till both capacitors get the same potential

a) Two capacitors are connected in parallel such that positive plate of one capacitor is connected to positive plate of other capacitor



Let V be the common potential

Then $Q = Q_1 + Q_2$ (charge conservation)

$$(C_1 + C_2)\, V = C_1V_1 + C_2\, V_2 \;;$$

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

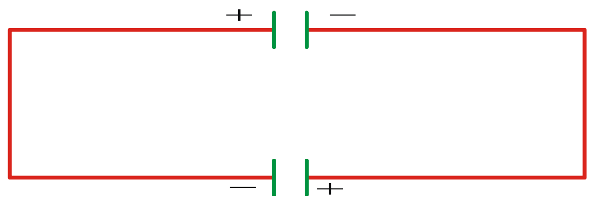
In this case there will be loss in energy of the system $\Delta U = U_f - U_i$;

$$\text{where } U_f = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2$$

$$U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 \;;$$

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

b) If positive plate of one capacitor is connected to negative plate of other capacitor, common potential is given by



$$V = \frac{C_1V_1 \sim C_2V_2}{C_1 + C_2}$$

Here charge flow takes place if $V_1 \neq V_2$

In this case, the loss of energy

$$\Delta U = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$$

Charge transfered is $= q_1 - q_1^1 \ (or)\left(q_2 - q_2^1\right)$

$= C_1V_1 - C_1V \ (or) \ C_2V_2 - C_2V \ = C_1(V_1 - V)\ (or)\ C_2(V_2 - V)$

Application :

a) Redistribution of charges when two conductors are connected by conduting wire

In charging a conductor, work is required to be done. This work done is stored up as the potential energy of the conductor.

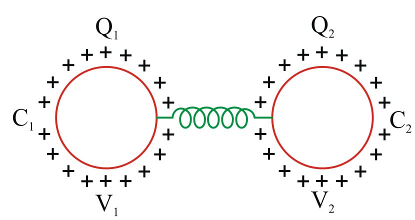
Energy of a charged conductor,

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

When two charged bodies are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential until their potentials are equal.

Let the amounts of charge on two conductors A and B are Q_1 and Q_2 their capacities are C_1 and C_2 and their potentials are V_1 and V_2 respectively, then $Q_1 = C_1V_1$ and $Q_2 = C_2V_2$

Let the amount of charge after the conductors are connected, are $Q_1^{|}$ and $Q_2^{|}$ respectively, then $Q_1^1 = C_1V; Q_2^1 = C_2V$



Charges are redistributed in the ratio of their capacities.

$\therefore \ Q^I_1 : Q^I_2 = C_1 : C_2$ (since V is same)

In case of spherical conductors, $C = 4\pi\varepsilon_0 r$

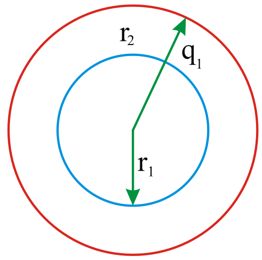
so, $Q^I_1 : Q^I_2 = r_1 : r_2$

Van De Graff Generator

Van De Graaff generator is used to develop very high voltages and resulting large electric fields and used to accelerate charged particles to high energies

Principle : Whenever a charge is given to a metal body it will spread on the outer surface of it. if we put a charged metal body inside the hollow metal body and the two are connected by a wire, whole of the charge of innner body will flow to the outer surface of the hallow body. No matter how large the charge is on the inner body.

Consider a sperical conductor 1 of radius r_1 holding charge q_1 uniformly distributed on it. it is kept inside a hollow conductor 2 of radius r_2 which is uncharged.



Electric potential of inner sphere is

$$V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_1}$$

Electric potential of outer sphere is

$$V_2 = \frac{1}{4\pi\varepsilon_o}\frac{q_1}{r_2}$$

potential difference between the two conductors

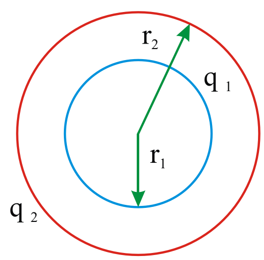
$$V_1 - V_2 = \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

If 'q_2' charge is on the outer shell

$$V_1 = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_2 = \frac{1}{4\pi\varepsilon_o}\left(\frac{q_1 + q_2}{r_2}\right)$$

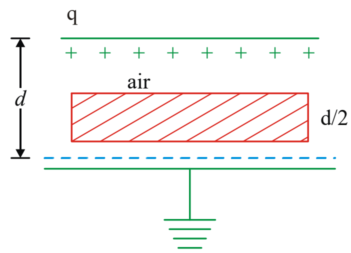
$$V_1 - V_2 = \frac{q_1}{4\pi\varepsilon_o}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$



potential difference $(V_1 - V_2)$ will remain the same for any value of q_2

PROBLEMS

1: A metal slab of thickness, equal to half the distance between the plates is introduced between the plates of a parallel plate capacitor as shown. Find its capacity.



SOLUTION :

SOLUTION : Sol. When capacitor is partially filled with dielectric capacity $C = \frac{\varepsilon_0 A}{\left[d - t\left(1 - \frac{1}{k}\right)\right]}$

For metal slab of thickness t = d/2,

$$C = \frac{\epsilon_0 A}{d - t} \; (K = \infty \text{ for metal slab}) = \frac{\epsilon_0 A}{d - \frac{d}{2}} = 2\frac{\epsilon_0 A}{d}$$

2. A parallel plate capacitor of capacity $5\mu F$ and plate separation 6cm is connected to a 1V battery and is charged. A dielectric of dielectric constant 4 and thickness 4 cm is introduced into the capacitor. The additional charge that flows into the capacitor from the battery is

1) $2\mu C$ 2) $3\mu C$ 3) $5\mu C$ 4) $10\mu C$

SOLUTION :

$$q_1 = cv;$$

$$q_2 = C^{/}V, \frac{c}{c^{/}} = \frac{d - t\frac{t}{K}}{d};$$

$$\Delta q = q_2 - q_1$$

3. Two conductors carrying equal and opposite charges produce a non uniform electric field along X - axis given by $E = \frac{Q}{\epsilon_0 A}\left(1 + Bx^2\right)$ where A and B are constants. Separation between the conductors along X-axis is X. Find the capacitance of the capacitor formed.

SOLUTION : Potential difference between the conductors is given by $V = V_+ - V_- = \int_0^X E dx$

$$\Rightarrow V = \int_0^X \frac{Q}{\epsilon_0 A}\left(1 + Bx^2\right) dx$$

or $V = \frac{Q}{\epsilon_0 A}\left(x + \frac{Bx^3}{3}\right)_O^X = \frac{Q}{\epsilon_0 A}\left(X + \frac{BX^3}{3}\right)$

Capacity $C = \frac{Q}{V} = \frac{\in_0 A}{X\left(1 + \frac{BX^2}{3}\right)}$

4. A capacitor is filled with an insulator and a certain potential difference is applied to its plates. The energy stored in the capacitor is U. Now the capacitor is disconnected from the source and the insulator is pulled out of the capacitor. The work performed against the forces of electric field in pulling out the insulator is 4U. Then dielectric constant of the insulator is

1) 4 2) 8 3) 5 4) 3

SOLUTION :

$$U = \frac{1}{2}\frac{q^2}{C} \; ; \; U + 4U = \frac{1}{2}\frac{q^2}{C_0}$$

$$5U = \frac{1}{2}\frac{q^2}{C_0} \; ; \; k = \frac{C}{C_0} = 5$$

5. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge +Q is now given to its positive plate. The potential difference across the capacitor is now

1) V 2) $V + \frac{Q}{C}$ 3) $V + \frac{Q}{2C}$ 4) $V - \frac{Q}{C}$, $if V < CV$

SOLUTION :

$$E = \frac{\sigma}{\in_0} = \frac{\frac{Q}{2} + CV}{A \in_0} ; \; V' = Ed = \frac{\frac{Q}{2} + CV}{\frac{A \in_0}{d}}$$

$$= \frac{\frac{Q}{2} + CV}{C} = V + \frac{Q}{2C}$$

6. A parallel plate capacitor of capacitance C is connected to a battery and is charged to a potential difference V. Another capacitor of capacitance 2C is similarly charged to a potential difference 2V. The charging battery is now disconnected and the capacitors are connected in parallel to each other in such a way that the positive terminal of one is connected to the negative terminal of the other. The final energy of the configuration is

1) zero 2) $\frac{3}{2}CV^2$ 3) $\frac{35}{6}CV^2$ 4) $\frac{9}{2}CV^2$

SOLUTION :

Net charge $Q = Q_2 - Q_1$ potential is V_l

$$\therefore V_1 = \left(\frac{C_0}{C + C_0}\right)V_0$$

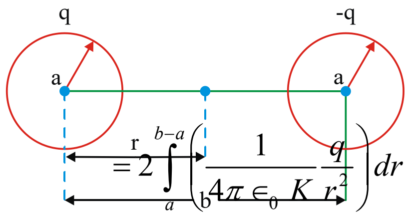
Similarly after nth operation ; $E = 1/2C^1V^2$

7. Find the capacitance of a system of two identical metal balls of radius a if the distance between their centres is equal to b, with b>>a. The system is located in a uniform dielectric with permittivity K.

SOLUTION : Let q and -q be the charges on two balls. Then

$V_1 = V_{ball} - V_\infty = V \qquad V_2 = V_{ball} - V_\infty = -V$

The potential difference between the balls



$$V_1 - V_2 = 2V$$

$$= 2\int_a^{b-a} E\,dx = 2\int_a^{b-a}\left(\frac{1}{4\pi \in_0 K}\frac{q}{r^2}\right)dr = \frac{2q}{4\pi \in_0 K}\left[\frac{1}{a} - \frac{1}{b-a}\right]$$

$$C = \frac{q}{V_1 - V_2} = \frac{q}{\left[\frac{2q}{4\pi \in_0 K}\frac{(b-2a)}{a(b-a)}\right]} = \frac{2\pi \in_0 K\, a(b-a)}{(b-2a)}$$

For b >> a, we can write $C = 2\pi \in_0 K$ a .

8. A capacitor of capacitance 10 μ F is charged to a potential 50 V with a battery. The battery is now disconnected and an additional charge 200 μ C is given to the positive plate of the capacitor. The potential difference across the capacitor will be

1) 50 V 2) 80 V 3) 100V 4) 60 V

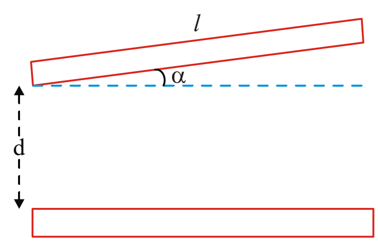
SOLUTION :

$$q_0 = CV = 500\mu C$$

$$\frac{700-q}{2A\in_0} + \frac{q}{2A\in_0} + \frac{500-q}{2A\in_0} = \frac{q}{2A\in_0}$$

$$q = 600\mu C\ ;\ \Delta V = \frac{q}{C} = 60\text{V}$$

9. Capacitor has square plates each of side 'l' making an angle 'α' with each other as shown. Then for small value of α, the capacitance 'C' is given by



SOLUTION : . At one side, distance between plates d,

At another side,

distance $d + l\sin\alpha \simeq d + l\alpha$

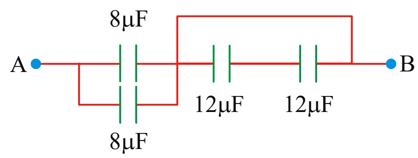
Mean distance between the plates

$$= \frac{d(d + l\alpha)}{2} = d + \frac{l\alpha}{2}$$

Capacity $C = \frac{\in_0 A}{d} = \frac{\in_0 l^2}{d + \frac{l\alpha}{2}}$

$$= \frac{\in_0 l^2}{d}\left[1 + \frac{l\alpha}{2d}\right]^{-1} = \frac{\in_0 l^2}{d}\left[1 - \frac{l\alpha}{2d}\right]$$

10. The equivalent capacity between A and B in the given circuit is



SOLUTION : Here $12\,\mu F$ and $12\,\mu F$ are short circuited. Hence they are not charged.

$\therefore$ Take only $8\,\mu F$ and $8\,\mu F$ parallel combination.

$C = 8 + 8 = 16\,\mu F$

11. When the space between the plates of a parallel plate condenser is completely filled with two slabs of dielectric constants K_1 and K_2 and each slab having area A and thickness equal to d/2 as shown in the figure

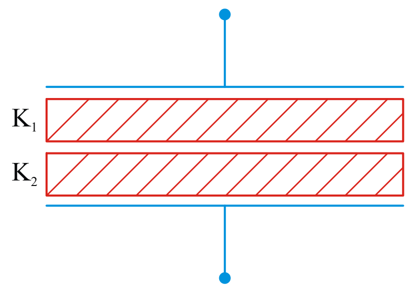
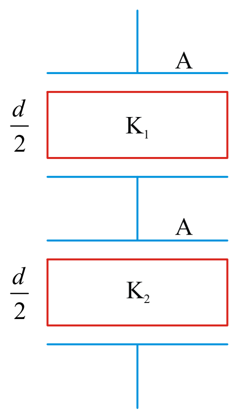


Fig. The equivalent circuit is as shown



a) Capacity of the upper half $C_1 = \dfrac{2K_1 \in_o A}{d}$

b) Capacity of the lower half $C_2 = \dfrac{2K_2 \in_o A}{d}$

c) C_1 and C_2 may be supposed to be conencted in series.

d) Effective capacity

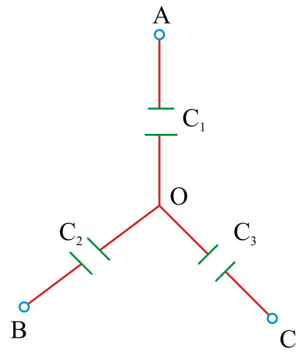
$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{\varepsilon_0 A}{d}\left(\frac{2K_1K_2}{K_1 + K_2}\right) = C_0\left(\frac{2K_1K_2}{K_1 + K_2}\right)$$

Here C_0 is the capacity of the condenser with air medium.

e) Effective dielectric constant $K = \left(\dfrac{2K_1K_2}{K_1 + K_2}\right)$

12. Three uncharged capacitors of capacities C_1, C_2 and C_3 are connected as shown in the figure to one another and the point. A, B and C are at potentials V_1, V_2 and V_3 respectively. Then the potential

at O will be



1) $\frac{V_1C_1+V_2C_2+V_3C_3}{C_1+C_2+C_3}$ 2) $\frac{V_1+V_2+V_3}{C_1+C_2+C_3}$ 3) $\frac{V_1(V_2+V_3)}{C_1(C_2+C_3)}$ 4) $\frac{V_1V_2V_3}{C_1C_2C_3}$

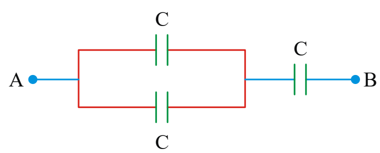
SOLUTION :

$$q_1 = q_2 + q_3$$

$$\Rightarrow (V_1 - V_0)C_1 = (V_0 - V_2)C_2 + (V_0 - V_3)C_3$$

$$\therefore V_0 = \frac{C_1V_1 + C_2V_2 + C_3V_3}{C_1 + C_2 + C_3}$$

13: In the net work three identical capacitors are connected as shown. Each of them can withstand to a maximum 100 V potential difference. What is the maximum voltage that can be applied across A and B so that no capacitor gets spoiled.



SOLUTION :

Sol. Let q_{max} be the max-charge supplied by the battery between A and B so that no capacitor gets spoiled.

For each capacitor

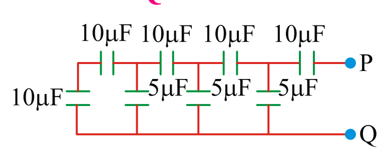
$q_{max} = CV_0 = C(100) = 100\,C$

For the combination $q_{max} = C_{equivalent}(V_{max})$

$100\,C = \frac{2}{3}C(V_{max}) \Rightarrow V_{max} = 150\,V$

Among 150V, potential difference across parallel combination is 50V and the potential difference across the other capacitor is 100V.

14. The equivalent capacitance between P and Q is



1) 10 μF 2) 20 μF 3) 5 μF 4) 15 μF

SOLUTION :

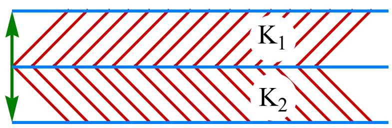
From Left $C = \frac{10 \times 10}{10+10} = 5\mu F$

$C^1 = 5+5 = 10\mu F$

$C^{11} = \frac{10 \times 10}{10+10} = 5\mu F$ and so on

finally $C_{eff} = \frac{10 \times 10}{10+10} = 5\mu F$

15. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be throught as a dielectric slab of thickness d(d_1+d_2) and effective dielectric constant K. The K is



1) $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$ 2) $\frac{K_1 d_1 + K_2 d_2}{K_1 + K_2}$

3) $\frac{K_1 K_2 (d_1 + d_2)}{K_2 d_1 + K_1 d_2}$ 4) $\frac{2K_1 K_2}{K_1 + K_2}$

SOLUTION :

The given capacitor is equivalent to two capacitors joined in series, where $C_1 = \frac{K_1 \varepsilon_1 \text{A}}{d_1}$, $C_2 = \frac{K_2 \varepsilon_0 \text{A}}{d_2}$

$$\frac{1}{C_{eq}} = \frac{1}{\text{C}_1} + \frac{1}{\text{C}_2} = \frac{d_1}{K_1 \varepsilon_0 A} + \frac{d_2}{K_2 \varepsilon_0 A} = \frac{d_1 K_2 + K_1 d_2}{\varepsilon_0 A K_1 K_2}$$

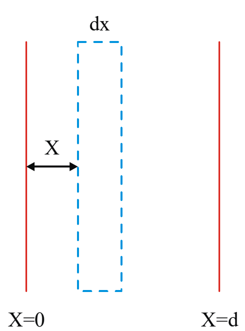
or $C_{eq} = \frac{\varepsilon_0 A K_1 K_2}{d_1 K_2 + K_1 d_2}$

but $C_{eq} = \frac{\varepsilon_0 AK}{d_1 + d_2}$; $K = \frac{K_1 K_2 (d_1 + d_2)}{d_1 K_2 + K_1 d_2}$

16. Calculate the capacitance of a parallel plate capacitor, with plate area A and distance between the plates d, when filled with a dielectic whose permittivity varies as

$$\in(x) = \in_0 + kx\left(0 < x < \frac{d}{2}\right) \quad ; \quad \in(x) = \in_0 + k(d-x)\left(\frac{d}{2} < x \le d\right)$$

SOLUTION :



The given capacitor is equivalent to two capacitors in series. Let C_1 and C_2 be their capacities. Then

$$\frac{1}{\text{C}} = \int \left[\frac{1}{\text{dC}_1} + \frac{1}{\text{dC}_2} \right]$$

Consider an element of width dx at a distance x from the left plate. Then

$$dC_1 = \frac{(\varepsilon_0 + kx)A}{dx} \text{ for } 0 < x < \frac{d}{2}$$

and $dC_2 = \frac{\{\epsilon_0 + k(d-x)\}A}{dx}$ for $\frac{d}{2} < x \le d$

on substituting these two values we get

$$\frac{1}{C} = \int \frac{1}{dC} = \frac{2}{KA}\ell n\left(\frac{2\epsilon_0 + Kd}{2\epsilon_0}\right) \Rightarrow C = \frac{KA}{2}\ell n\left(\frac{2\epsilon_0 + Kd}{2\epsilon_0}\right)$$

17. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is

1) $\frac{1}{2}(K-1)CV^2$ 2) $CV^2(K-1)/K$

3) $(K-1)CV^2$ 4) zero

SOLUTION :

On introduction and removal and again on introduction, the capacity and potential remain same. So, net work done by the system in this process.

$$W = U_f - U_i \; ; \; \frac{1}{2}CV^2 - \frac{1}{2}CV^2 = 0$$

18. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitor is

1) $\sqrt{\frac{2mC\Delta T}{s}}$ 2) $\frac{mC\Delta T}{s}$ 3) $\frac{ms\Delta T}{C}$ 4) $\sqrt{\frac{2ms\Delta T}{C}}$

SOLUTION :

$E = \left(\frac{1}{2}\right)CV^2$ The energy stored in capacitor is lost in form of heat energy. $H = ms\Delta T$ $\therefore ms\Delta T = \left(\frac{1}{2}\right)CV^2$;

$$C = \frac{q_1 + q_2}{V_A - V_B} = \frac{q_1 + q_2}{\frac{q_2}{C_2} + \frac{q_1}{C_1}}$$

$$\therefore C = \frac{2C_1C_2 + C_3(C_1 + C_2)}{C_1 + C_2 + 2C_3} \; ; \quad \therefore \quad V = \sqrt{\frac{2ms\Delta T}{C}}$$

19: When the space between the plates of a parallel plate condenser is completely filled with two slabs of dielectric constants K_1 and K_2 and each slab having area $\frac{A}{2}$ and thickness equal to distance of seperation d as shown in the figure.

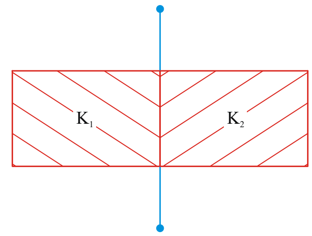
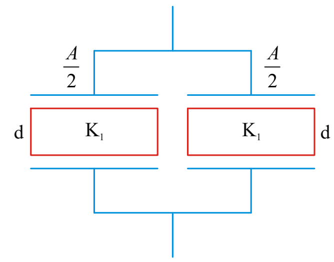


Fig. The equivalent circuit is as shown



a) Capacity of the left half $C_1 = K_1 \dfrac{\varepsilon_0 A}{2d}$

b) Capacity of the right half $C_2 = K_2 \dfrac{\varepsilon_0 A}{2d}$

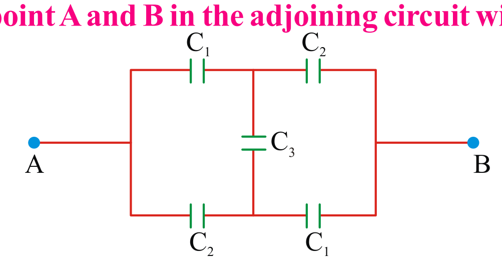
c) C_1 and C_2 may be supposed to be connected in parallel then effective capacity

$$C = C_1 + C_2 = \frac{\varepsilon_0 A}{d}\left(\frac{K_1 + K_2}{2}\right)$$

$C = C_0\left(\dfrac{K_1 + K_2}{2}\right)$ where C_0 is capacity of capacitor without dielectric.

d) Effective dielectric constant $K = \dfrac{K_1 + K_2}{2}$

20. The capacity between the point A and B in the adjoining circuit wil be

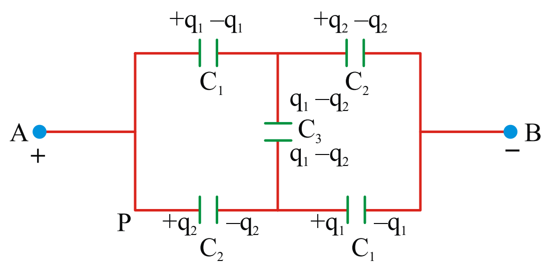


1) $\dfrac{2C_1C_2 + C_3(C_1 + C_2)}{C_1 + C_2 + 2C_3}$ 2) $\dfrac{C_1C_2 + C_2C_3 + C_3C_1}{C_1 + C_2 + C_3}$

3) $\dfrac{C_1(C_2 + C_3) + C_2(C_1 + C_3)}{C_1 + C_2 + 3C_3}$ 4) $\dfrac{C_1C_2C_3}{C_1C_2 + C_2C_3 + C_3C_1}$

SOLUTION :

According to the symmetry of the circuit charges on two condensers fo capacity C_1 will be same and charges on condensers of capacity C_2 will be same.



$$\frac{q_2}{C_2}+\frac{q_2-q_1}{C_3}-\frac{q_1}{C_1}=0\ ;\ \therefore \frac{q_2}{q_1}=\frac{C_1(C_2+C_3)}{C_2(C_1+C_3)}$$

Capacity of whole circuit

21. A parallel plate capacitor has area of each plate A, the separation between the plates is d . It is charged to a potential V and then disconnected from the battery. The amount of work done in the filling the capacitor Completely with a dielectric constant k is

1) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left[1-\frac{1}{k^2}\right]$ 2) $\frac{1}{2}\frac{V^2\varepsilon_0 A}{kd}$ 3) $\frac{1}{2}\frac{V^2\varepsilon_0 A}{k^2 d}$ 4) $\frac{1}{2}\frac{\varepsilon_0 AV^2}{d}\left[1-\frac{1}{K}\right]$

SOLUTION :

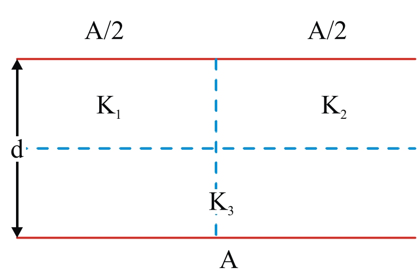
Work done $=$ decrease in energy

$$\text{ie } w = E_1-E_2=\frac{1}{2}\frac{\varepsilon_0 A}{d}v^2-\frac{\varepsilon_0 Av^2}{2d}\left[1-\frac{1}{k}\right]$$

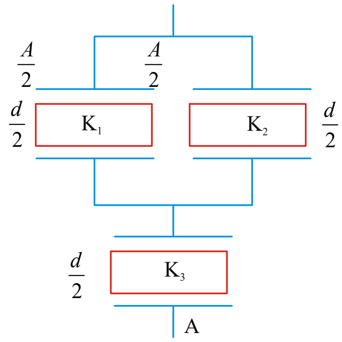
22: A parallel plate capacitor of area A, plate separation d and capacitance C is filled with three different dielectric materials having dielectric constants K_1, K_2 and K_3 as shown in fig. If a single dielectric material is to be used to have the same effective capacitance as the above combination then its dielectric constant K is given by :

SOLUTION : Let $C=\frac{\epsilon_0 A}{d}$; $C_1=K_1\frac{\epsilon_0\frac{A}{2}}{\frac{d}{2}}=K_1C$

$$C_2=K_2\frac{\epsilon_0\frac{A}{2}}{\frac{d}{2}}=K_2C\,;\,C_3=\frac{K_3\ \epsilon_0 A}{\frac{d}{2}},=2K_3C$$



The equivalent circuit as shown

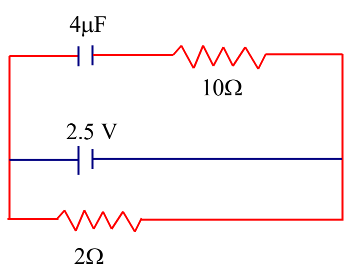


$$\frac{1}{C}=\frac{1}{C_3}+\frac{1}{C_1+C_2}, \quad \frac{1}{KC}=\frac{1}{2K_3C}+\frac{1}{(K_1+K_2)C}$$

$$\frac{1}{K}=\frac{1}{K_1+K_2}+\frac{1}{2K_3}$$

Note : dielectric 1 and 2 are not in parallel

23. A capacitor of $4\,\mu$F is connected as shown in the circuit. The internal resistance of the battery is $0.5\,\Omega$. The amount of charge on the capacitor plates will be



1) 0 2) $4\mu C$ 3) $16\mu C$ 4) $8\mu C$

SOLUTION :

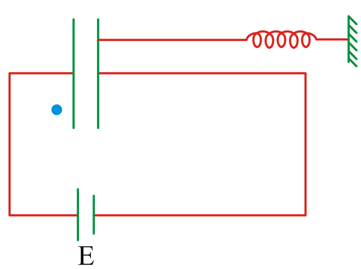
No current flows in upper arm of the circuit. Current in lower arm of the circuit.

$$I=\frac{E}{R+r}=\frac{2.5}{2+0.5}=1\,\text{A}$$

Termial potential difference of battery, $V=E-I=2.5-1\times0.5=2\,V$

So, charge on the capacitor plates, $Q=CV=4\,\mu\,\text{F}\times 2\,V=8\,\mu\,\text{C}$

24.. One plate of a capacitor is connected to a spring as shown in figure. Area of both the plates is A. In steady state; separation between the plates is 0.8d (spring was unstretched and the distance between the plates was d, when the capacitor was uncharged). The force constant of the spring is approximately



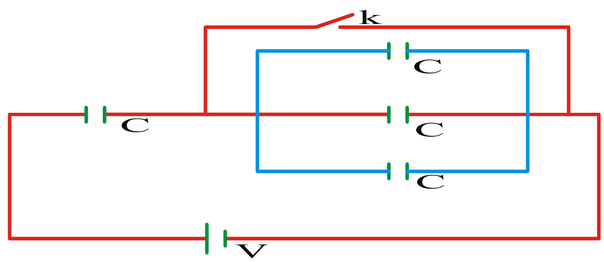
1) $\frac{4\in_0 AE^2}{d^3}$ 2) $\frac{2\in_0 AE}{d^2}$

3) $\frac{6\in_0 E^2}{Ad^3}$ 4) $\frac{\in_0 AE^3}{2d^3}$

SOLUTION :

$\frac{q^2}{2A\in_0} = kx$; $\frac{(CE)^2}{2A\in_0} = k(d-0.8d)$, $C = \frac{\in_0 A}{0.8d}$ $K = \frac{4\in_0 AE^2}{d^3}$

25. The charge flowing through the cell on closing the key k is equal to



1) $\frac{CV}{4}$ 2) 4 CV 3) $\frac{4}{3}CV$ 4) $\frac{3}{4}CV$

SOLUTION :

$C_{net} = \frac{3}{4}C$; when key was open $q = \frac{3}{4}CV$

when key was closed 3C becomes short circuited. Net charge on C is now $q' = CV$

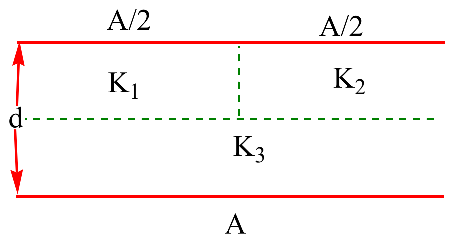
$\Delta q = q' - q = \frac{CV}{4}$

26: Solve the above problem when a thin metal sheet is inserted, separating dielectric 1 and 2 from 3.

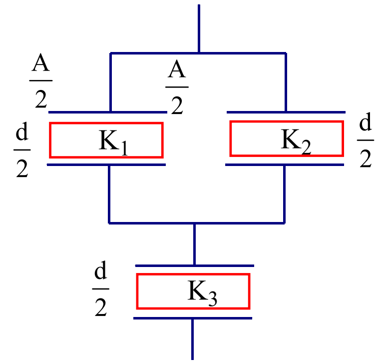
SOLUTION : . Let $C = \frac{\in_0 A}{d}$; $C_1 = K_1 \frac{\in_0 \frac{A}{2}}{\frac{d}{2}} = K_1 C$

$C_2 = K_2 \frac{\in_0 \frac{A}{2}}{\frac{d}{2}} = K_2 C$

$C_3 = \frac{K_3 \in_0 A}{\frac{d}{2}} = 2K_3 C$



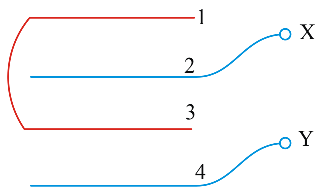
The equivalent circuit as shown



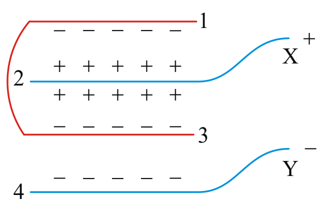
$$\frac{1}{C}=\frac{1}{C_3}+\frac{1}{C_1+C_2};\ \frac{1}{KC}=\frac{1}{2K_3C}+\frac{1}{(K_1+K_2)C}$$

$$\frac{1}{K}=\frac{1}{K_1+K_2}+\frac{1}{2K_3}$$

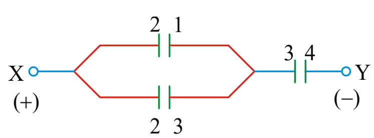
27: Four identical metal plates are located in air at equal distance d from one another. The area of each plate is A. Find the equivalent capacitance of the system between X and Y.



SOLUTION : Let us give numbers to the four plates. Here X and Y are connected to the positive and negative terminals of the battery (say), then the charge distribution will be as shown

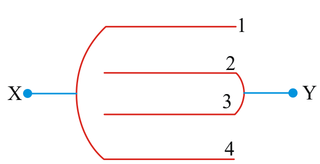


Here the arrangement can be represented as the grouping of three identical capacitors each of capacity $\frac{\epsilon_0 A}{d}$. The arrangement will be as shown

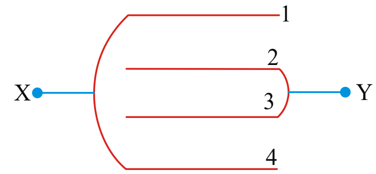


Now the equivalent capacitance between X and Y is $C_{XY}=\frac{(C+C)C}{C+C+C}=\frac{2C}{3}=\frac{2\epsilon_0 A}{3d}$

28: Find equivalent capacity between X and Y

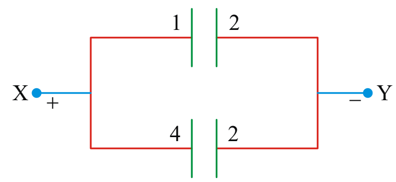


SOLUTION : Let us give numbers to the four plates. Here X and Y are connected to the positive and negative terminals of the battery (say) ,



Here the arrangement can be represented as the grouping of two identical capacitors each of capacity $\frac{\epsilon_0 A}{d}$.

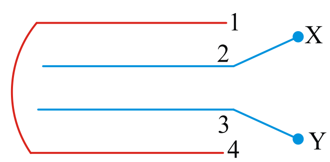
The arrangement will be as shown



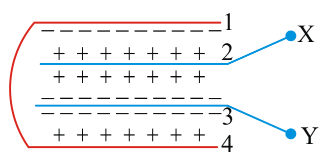
Now the equivalent capacitance between X and Y is

$$C_{XY} = (C + C) = 2C = 2\frac{\varepsilon_0 A}{d}$$

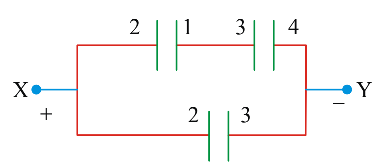
29. Find equivalent capacity X and Y



SOLUTION : . Let us give numbers to the four plates. Here X and Y are connected to the positive and negative terminals of the battery (say).



Here the arrangement can be represented as the grouping of three identical capacitors each of capacity $\frac{\epsilon_0 A}{d}$. The arrangement will be as shown



Now the equivalent capacitance between X and Y is $C_{XY} = \left(\frac{(C)(C)}{C+C}\right) + C = \frac{C}{2} + C = \frac{3}{2}C$

$$= C_{XY} = \frac{3}{2}C = \frac{3}{2}\frac{\varepsilon_0 A}{d}$$

30: A capacitor of capacitance C_o is charged to a potential V_o and then isolated. A small capacitor C is then charged from C_o, dischar-ged and charged again, the process being repeated n times. Due to this, potential of the large capacitor is decreased to V. Find the capacitance of the small capacitor:

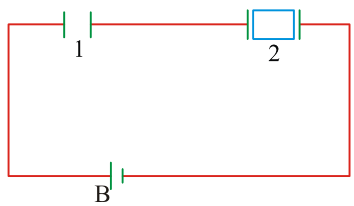
SOLUTION : When key is closed, common potential $V_1 = \frac{C_o V_o}{C_o + C}$ charge left on large capacitor after first sharing of charges $Q_o^1 = C_o V_1$

common potential after second sharing of charges in $V_2 = \frac{C_0}{C_0 + C} V_1$; $V_2 = \frac{C_o^2 V_0}{(C_o + C)^2}$

after n^{th} sharing charges $V_n = \left(\frac{C_o}{C_o + C}\right)^n V_0$

But $V_n = V$; $V = \left(\frac{C_0}{C_0 + C}\right)^n V_0$; $\therefore C = C_o\left[\left(\frac{V_o}{V}\right)^{1/n} - 1\right]$

31. Two identical capacitors 1 and 2 are connected in series to a battery as shown in figure. Capacitor 2 contains a dielectric slab of dielectric constant K as shown. Q_1 and Q_2 are the charges stored in the capacitors. Now the dielectric slab is removed and the corresponding charges are Q'_1 and Q'_2. Then



1) $\frac{Q'_1}{Q_1} = \frac{K+1}{K}$

2) $\frac{Q'_2}{Q_2} = \frac{K+1}{2}$

3) $\frac{Q'_2}{Q_2} = \frac{K+1}{2K}$

4) $\frac{Q'_2}{Q_2} = \frac{K}{2}$

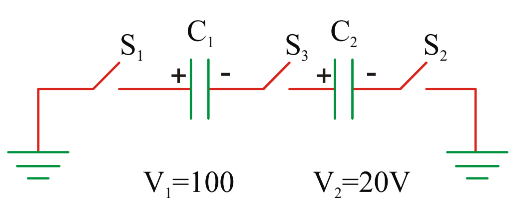
SOLUTION :

$Q_1' = Q_2' = \frac{CE}{2}$; Before the slab is removed

$C_1 = C \text{ and } C_2 = kC$; $C_{net} = \left(\frac{k}{k+1}\right)C$

$\frac{Q_2'}{Q_2} = \frac{k+1}{2k}$

32: In the circuit shown in figure $C_1 = 1\mu F$ and $C_2 = 2\mu F$. The capacitor C_1 is charged to 100V and the capacitor C_2 is charged to 20V. After charging then are connected as shown. When the switches S_1, S_2 *and* S_3 are closed, the charge flowing through S_1 is



SOLUTION : When S_1, S_2 and S_3 are closed, both the capacitors are in parallel with unlike charged plates together. So, they attain a common potential.

Before closing the switches,

Charge on C_1 is $q_1 = 100 \times 1 = 100\,\mu C$

Charge on C_2 is $q_2 = 20 \times 2 = 40\,\mu C$

After closing the switches

Common potential $V = \frac{q_1 - q_2}{C_1 + C_2} = \frac{100 - 40}{3} = 20\,V$

Now final charges $q_1^1 = C_1 V = 1 \times 20 = 20\,\mu C$ $\qquad$ $q_2^1 = C_2 V = 2 \times 20 = 40\,\mu C$

The charge that flows through S_1 is

$\Delta q = 100 - 20 = 80\,\mu C$

33. Two capacitors of capacites 1 μF and C μF are connected in series and the combination is charged to a potential difference of 120 V. If the charge on the combition is $80\mu C$, the energy stored in the capacitor C in microjoules is :

1) 1800 2) 1600 3) 14400 4) 7200

SOLUTION :

Since $1\mu F$ and C are connected in series $V_1 = CV_2 = \frac{C(120)}{1+C} = 80$

on solving $C = 2\mu F$ $\therefore 2V_2 = 80$

$V_2 = 40V$; $\therefore U = \frac{1}{2}CV^2 = 1600\mu J$

34. A positively charged particle is released from rest in a uniform electric field. The electric potential energy of the charge.

1) remains a constant because the electric field is uniform.

2) increases because the charge moves along the electric field.

3) decreases because the charge moves along the electric field.

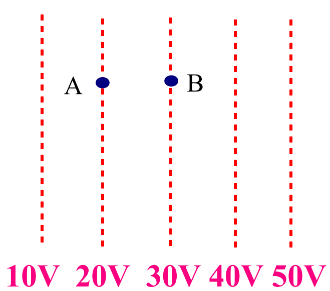
4) decreases because the charge moves opposite tot he electric field.

SOLUTION :

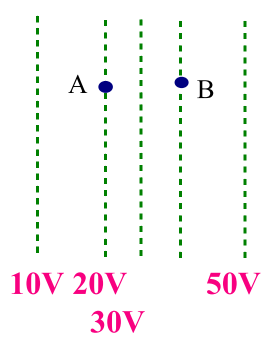
When a positively charged particle is released from rest in a uniform electric field, it moves along the electric field, i.e.,from higher potential to lower potential.That is electric potential energy of the charge decreases.

35. Figure shows some equipotential lines distrubuted in space. A charged object is moved from point A to point B.

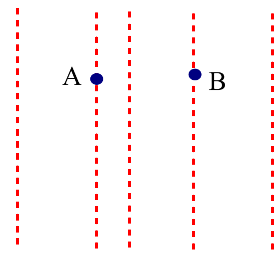
i)



ii)



iii)



10V 20V 40V 50V

1) The work done in figure (i) is the greatest.
2) The work done in figure (ii) is the least.
3) The work done is the same in figure (i) , (ii) and (iii)
4) the work done in figure (iii) is greater than figure (ii) but equal to that in figure (i).

SOLUTION :

Work done in carrying a charge q from point A to point B

$W = q\,(V_B - V_A)$.

In all figures, $V_A = 20$ V and $V_B = 40$ V

$W = q\,(40 - 20) = 20\,q$.

36. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard :

S_1 : At any point inside the sphere, electric intensity is zero.
S_2 : At any point inside the sphere, the electrostatic potential is 100 V.
Which of the following is a correct statement ?
1) S_1 is true but S_2 is false.
2) Both S_1 and S_2 are false.
3) S_1 is true , S_2 is also true and S_1 is the cause of S_2 .
4) S_1 is true , S_2 is also true but the statements are independent.

SOLUTION :

S_1 it true, S_2 is also true as potential inside the charged conducting sphere is equal to the potential on its surface.

Now, As $E = -\frac{dV}{dr} = \frac{d}{dr}(100\text{ V}) = 0$

So, S_1 is the cause of S_2.

37. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

1) spheres
2) planes
3) paraboloids
4) ellipsoids

SOLUTION :

Collection of charges whose total sum is not zero can be considered as a point charge from a great distance.So, equipotentials should be spheres.

More than one option correct

38. Consider a uniform electric field in the $\hat{z}$ direction. The potential is a constant

1) in all space
2) for any x for a given z.
3) for any y for a given z.
4) on the x-y plane for a given z.

SOLUTION :

Electric field is along the z-axis, sp

$E_X = E_Y = 0$. As $E = \frac{dV}{dr}$, e.e., V is constant

when $E = 0$. So, equipotential surfaces are in x - y Plane, i.e., for a given z, potential is constant for aby x , and y and on the x - y plane.

39. Equipotential surfaces

1) are closer in regions of large electric fields compared to regions of lower electric fields.

2) will be more crowded near sharp edges of a conductor.

3) will be more crowded near regions of large charge densities.

4) will always be equally spaced.

SOLUTION :

$E = -\frac{dV}{dr}$ or $dr = -\frac{dV}{E}$

For a fixed value of dV, $dr \propto -\frac{1}{E}$; which implies that spacing between two equipotential surfaces decreases as E increases. So,. equipotential surfaces are closer in regions of large electric fields compared to regions of lower electric fields. At sharp edges a=of conductor, charge density is more. So - electric field is large and hence the equipotential surfaces will be more crowded.

40. The work done to move a charge along an equipotential from A to B

1) cannot be defined as $-\int_A^B \overline{E}.\overrightarrow{dl}$

2) must be defined as $-\int_A^B \overline{E}.\overrightarrow{dl}$

SOLUTION :

Work done to move a charge q from A to B along an equipotential surface,

$$W = q(V_B - V_a) q \int_A^B dV = -q \int_A^B \overline{E}.\overrightarrow{dl}$$

Also, for an equipotential surface, $V_A = V_B =$ Constant

$\therefore W = 0$

3) is zero 4) can have a non - zero value.

41. In a region of constant potential

1) the electric field is uniform

2) the electric field is zero

3) there can be no charge inside the region

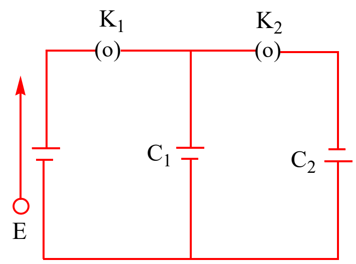
4) the electric field shall necessarily change if a charge is placed outside the region.

SOLUTION :

In a region of constant potential , the electric field is zero as $E = -\frac{dV}{dr} = -\frac{d}{dr}$ (Constant t) =0

Further , according to Gauss's law, charge inside the region should be zero if $E = 0$.

42. In the circuit shown in figure initialy key K_1 **is closed and key K_2 is open. Then is K opened and K_2 is closed (order is important). [Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively.]**



Then

1) charge on C_1 gets redistributed such that $V_1=V_2$

2) charge on C_1 gets redistributed such that $Q_1'=Q_2'$

3) charge on C_1 gets redistributed such that $C_1V=C_2V_2=C_1E$

4) charge on C_1 gets redistributed such that $Q_1'+Q_2'=Q$

SOLUTION :

Initially, when K_1 is closed and K_2 is opened, C_1 gets charged to potential E possessing a charge $Q=C_1E$. When K_1 is opened and K_2 is closed, battery gets disconnect from the circuit and C_1 and C_2 gets connected in parallel. Charge on C_1 gets redistributed such that $V_1=V_2$. Also, as there in no loss of charge, $Q_1'+Q_2'=Q$.

43 If a conductor has a potential $v \neq 0$ and there are no charges any where else outside, then

1) there must be charges on the surface or inside itself.

2) there cannot be any charge in the body of the conductor.

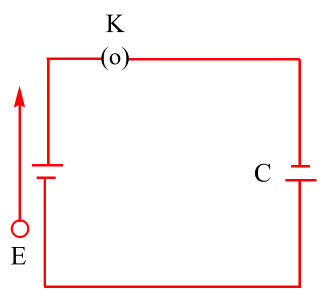
3) there must be charges only on the surface.

4) there must be charges inside the surface.

SOLUTION :

For a conductor having potential, $V \neq 0$, there mustbe charges on the surface of conductor if there are no charges outside it. The interior of a charged conductor does not have any net charge.

44. A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations :



A : Key K is kept closed and plates of capacitors are moved a part using insulting handle

B : Key K is opened and plates of capacitors are moved apart using insulting handle.

Choose the correct option (s).

1) In A : Q remains same but C changes

2) In B : V remains same but C changes.

3) In A : V remains same and hence Q changes

4) In B : Q remains same and hence V changes

SOLUTION :

Situation A : Since Key K is kept closed, so V remains same. As the plates of capacitors are moved apart, C changes and hence Q changes.

Situation B : Since key K is opened, the charge on the plates Q remains same. But C changes as the plates are moved apart and hence V changes.

THEORY BITS

1. A metal plate of thickness half the separation between the capacitor plates of capacitance C is inserted. The new capacitance is

   1) C 2) C/2 3) zero 4) 2C

KEY;4

2. If an earthed plate is brought near positively charged plate, the potential and capacity of charged plate

   1) increases, decreases 2) decreases, increases
   3) decreases, decreases 4) increases, increases

KEY;2

3. A parallel plate capacitor is first charged and then isolated, and a dielectric slab is introduced between the plates. The quantity that remains unchanged is

   1) Charge Q 2) Potential V
   3) Capacity C 4) Energy U

KEY;1

4. The plates of charged condenser are connected by a conducting wire. The quantity of heat produced in the wire is

   1) Inversely proportional to the capacity of the condenser.
   2) Inversely proportional to the square of the potential of the condenser.
   3) proportional to the length of wire
   4) independent of the resistance of the wire

KEY;4

5. One plate of parallel plate capacitor is smaller than the other, the charge on the smaller plate will be

   1) less than other 2) more than other
   3) equal to other
   4) will depend upon the medium between them

KEY;3

6. In a parallel plate capacitor, the capacitance 1) increases with increase in the distance between the plates

   2) decreases if a dielectric material is put between the plates
   3) increases with decrease in the distance between the plates
   4) increases with decrease in the area of the plates

KEY;1

7. If an uncharged capacitor is charged by connecting it to a battery, then the amount of energy lost as heat is

   1) 1/2QV 2) QV 3) $1/2QV^2$ 4) QV^2

KEY;1

8. When a dielectric material is introduced between the plates of a charged condenser, after disconnecting the battery the electric field between the plates

   1) decreases 2) increases
   3) does not change 4) may increase or decrease

KEY;1

9. A parallel plate capacitor is charged and the charging battery is then disconnected. If the plates of the capacitor are moved further apart by means of insulating handles
   1) the charge in the capacitor becomes zero
   2) the capacitance becomes infinite
   3) the charge in the capacitor increases
   4) the voltage across the plates increases

KEY;4

10. The ratio of charge to potential of a body is known as
   1) conductance 2) capacitance
   3) inductance 4) reactance

KEY;2

11. A parallel plate capacitor filled with a materail of dielectric constant K is charged to a certain voltage and is isolated. The dielectric material is removed. Then
   a)The capacitance decreases by a factor K
   b) The electric field reduces by a factor K
   c) The voltage across the capacitor increases by a factor K
   d) The charge strored in the capacitor increases by a factor K
   1) a and b are true 2) a and c are true
   3) b and c are true 4) b and d are true

KEY;2

12. When air is replaced by a dielectric medium of constant K, the capacity of the condenser 1) increases K times 2) increases K^2 times
   3) remains unchanged 4) decreases K times

KEY;1

13. For metals the value of dielectric constant (K) is
   1) One 2) Infinity 3) Zero 4) Two

KEY;2

14. If we increase the distance between two plates of the capacitor, the capacitance will
   1) decrease 2) remain same
   3) increase 4) first decrease then increase

KEY;1

15. The magnitude of electric field E in the annular region of a charged cylindrical capacitor
   1) is same throughout
   2) is higher near the outer cylinder than near the inner cylinder
   3) varies as 1/r where r is the distance from the axis
   4) varies as r where r is the distance from the axis

KEY;3

16. In a charged capacitor the energy is stored in (r) is less than at B
   1) both in positive and negative charges
   2) positive charges
   3) the edges of the capacitor plates
   4) the electric field between the plates

KEY;4

17. In order to increase the capacity of a parallel plate condenser one should introduce between the plates a sheet of (assume that the space is completely filled)

1) Mica 2) Tin

3) Copper 4) Stainless steel

KEY;1

18. Two condensers of unequal capacities are connected in series across a constant voltage d.c. source. The ratio of the potential differences across the condensers will be

1) direct proportion to their capacities

2) inverse proportion to their capacities

3) direct proportion to the square of their capacities

4) inverse proportion to the square root of their capacities

KEY;2

19. The condenser used in the tuning circuit of radio receiver is

1) paper condenser 2) electrolytic condenser 3) leyden jar 4) gang condenser

KEY;4

20. Force acting upon a charged particle kept between the plates of a charged condenser is F. If one of the plates of the condenser is removed, force acting on the same particle will become

1) zero 2) F/2 3) F 4) 2F

KEY;2

21. Space between the plates of a parallel plate capacitor is filled with a dielectric slab. The capacitor is charged and then the supply is disconnected to it. If the slab is now taken out then

1) work is not done to take out the slab

2) energy stored in the capacitor reduces

3) potential difference across the capacitor is decreased

4) potential difference across the capacitor is increased

KEY;4

22. Read the following statements

a) Non polar molecules have uniform charge distribution

b) Polar molecules have non - uniform charge distribution

c) Polar molecules are already polarized

d) Molecules are not already polarized without electric field in Non - polar molecules

1) only a & b are correct 2) only c & d are correct

3) only c is wrong 4) all are correct

KEY;4

23. A parallel plate capacitor of capacity C_o is charged to a potential V_o.

A) The energy stored in the capacitor when the battery is disconnected and the plate separation is doubled is E_1

B) The energy stored in the capacitor when the charging battery is kept connected and the separation between the capacitor plates is doubled is E_2. Then $\frac{E_1}{E_2}$ value is

1) 4 2) 3/2 3) 2 4) 1/2

KEY;1

24. Select correct Statements

a) Charge cannot be isolated

b) Repulsion is the sure test to know the presence of charge

c) Waxed paper is dielectric in paper capacitor

d) Variable capacitor is used in tuning circuits in radio

1) a, b only 2) a, c only

3) a, b, c only 4) b,c,d only

KEY;4

25. A variable parallel plate capacitor and an electroscope are connected in parallel to a battery. The reading of the electroscope would be decreased by

1) increasing the area of overlap of the plates

2) placing a block of paraffin wax between the plates

3) decreasing the distance between the plates

4) decreasing the battery potential

KEY;4

26. Two identical capacitors are joined in parallel, charged to a potential V, separated and then connected in series i.e., the positive plate of one is connected to the negative plate of other.

1) the charges on the free plates are enhanced

2) the charges on the free plates are decreased

3) the energy stored in the system increases

4) the potential difference between the free plates is 2V

KEY;4

27. A parallel plate condenser is charged by connecting it to a battery. The battery is disconnected and a glas slab is introduced between the plates. Then

1) potential increases

2) electric intensity increases

3) energy decreases 4) capacity decreases

KEY;3

28. Select correct statement for a capacitor having capacitance C, is connected to a source of constant emf E

1) Almost whole of the energy supplied by the battery will be stored in the capacity, if resistance of connecting wire is negligibly small

2) Energy received by the capacitor will be half of energy supplied by the battery only when the capacitor was initially uncharged

3) Strain energy in the capacitor must increases even if the capacitor had an initial charge

4) Energy stored depends on type of the source of emf

KEY;3

29. Van de Graff genetor is used to

1) supply electricity for industrial use

2) produce intense magnetic fields

3) generate high voltage

4) obtain highly penetrating X-rays

KEY;3

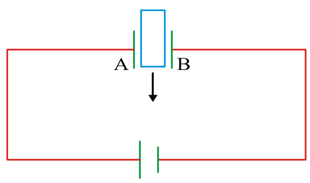
30. A number of spherical conductors of different radii have same potential. Then the surface charge density on them
1) is proportional to their radii
2) is inversely proportional to their radii
3) are equal
4) is proportional to square of their radii

KEY;2

31. Three charged particles are initially in position 1. They are free to move and they come in position 2 after some time. Let U_1 and U_2 be the electrostatic potential energies in position 1 and 2. Then
1) $U_1 > U_2$ 2) $U_2 > U_1$ 3) $U_1 = U_2$ 4) $U_2 \geq U_1$

KEY;1

32. An insulator plate is passed between the plates of a capacitor. Then current



1) always flows from A to B

2) always flows from B to A
3) firs t flows from A to B and then from B to A
4) first flows from B to A and then from A to B

KEY;4

33. A condenser is charged and then battery is removed. A dielectric plate is put between the plates of condenser, then correct statement is
1) Q constant V and U decrease
2) Q constant V increases U decreases
3) Q increases V decreases U increases
4) Q, V and U increase

KEY;1

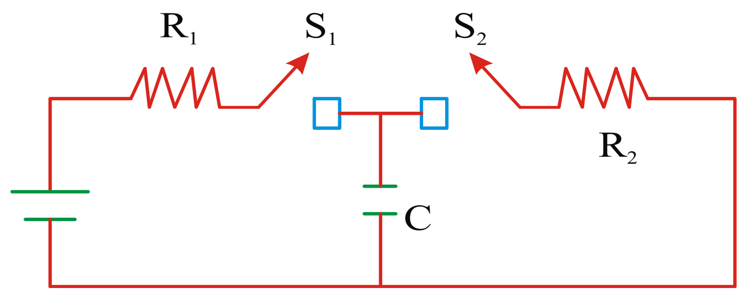
34. The capacitance of a capacitor depends on
1) the geometry of the plates
2) separation between plates
3) the dielectric between the plates
4) all the above

KEY;4

35. The electric field $\left(\vec{E}\right)$ between two parallel plates of a capacitor will be uniform if
1) the plate separation (d) is equal to area of the plate (A)
2) the plate separation (d) greater when compared to area of the plate (A)
3) the plate separation (d) is less when compared to area of the plate (A)
4) 2 (or) 3

KEY;3

36. A capacitor C is connnected to a battery circuit having two switches S_1 and S_2 and resistors R_1 and R_2. The capacitor will be fully charged when



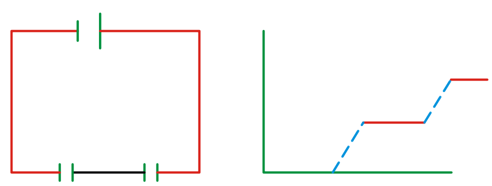
1) both S_1 and S_2 are closed
2) S_1 is closed and S_2 is open
3) S_1 is open and S_2 is closed
4) any one of the above

KEY;2

37. A condenser stores
1) potential 2) charge 3) current 4) energy in magnetic field

KEY;2

38. Figure shows two capacitors connected in series and joined to a cell. The graph shows the variation in potential as one moves from left to right on the branch containing capacitors.



1) $C_1 > C_2$
2) $C_1 = C_2$
3) $C_1 < C_2$
4) data insufficient to conclude the answer

KEY;3

39. Two condensers of unequal capacities are connected in parallel across a constant voltage d.c. source. The ratio of the charges stored in the condensers will be
1) direct proportion to their capacities
2) inverse proportion to their capacities
3) direct proportion to the square root of their capacities
4) inverse proportion to the square of their capacities

KEY;1

40. Two parallel plate air capacitors are constructed, one by a pair of iron plates and the second by a pair of copper plates of same area and same spacings. Then
1) the copper plate capacitor has a greater capacitance than the iron one
2) both capacitors will have equal non zero capacitances, in the uncharged state
3) both capacitors will have equal capacitances only if they are charged equally
4) the capacitances of the two capacitors are unequal even they are unequally charged

KEY;2

41. A parallel plate capacitor is charged and then isolated. Regarding the effect of increasing the plate separation, select the appropriate alternative.

1) decreases constant decreases
2) increases increases increases
3) constant decreases decreases
4) constant increases increases

KEY;4

42. A parallel plate capacitor is charged by connecting its plates to the terminals of a battery. The battery remains connected to the condenser plates and a glass plate is interposed between the plates of the capacitor, then
1) the charge increases while the potential difference remains constant
2) the charge decreases while the potential difference remains constant
3) the charge decreases while the potential difference increases
4) the charge increases while the potential difference decreases

KEY;1

43. A parallel plate capacitor is charged to a fixed potential and the charging battery is then disconnected. If now, the plates of the capacitor are moved further apart, then
1) the charge on the capacitor increases
2) the voltage across the capacitor increases
3) the energy stored in the capacitor decreases
4) the capacitance increases

KEY;2

44. In a parallel-plate capacitor, the region between the plates is filled by a dielectric slab. The capacitor is connected to a cell and the slab is taken out. Then
1) some charge is drawn from the cell
2) some charge is returned to the cell
3) the potential difference across the capacitor is reduced
4) no work is done by an external agent in taking the slab out

KEY;2

45. A parallel plate air condenser is charged and then disconnected from the charging battery. Now the space between the plates is filled with a dielectric then, the electric field strength between the plates
1) increases while its capacity increases
2) increases while its capacity decreases
3) decreases while its capacity increases
4) decreases while its capacity decreases

KEY;3

46. A capacitor works in
1) A.C. circuits only 2) D.C. circuits only 3) both A,C & D.C
4) neither A.C. nor in D.C. circuit.

KEY;3

47. When two identical condensers are connected in series choose the correct statement regarding the working voltage (the maximum p.d. that can be applied to a condenser) and the capacity
1) working voltage increases, capacity increases
2) working voltage increases, capacity decreases
3) working voltage decreases, capacity increases
4) working voltage decreases, capacity decreases

KEY;2

48. Two unequal capacitors, initially uncharged, are connected in series across a battery. Which of the following is true

1) The potential across each is the same 2) The charge on each is the same

3) The energy stored in each is the same

4) The equivalent capacitance is the sum of the two capacitances

KEY;2

49. Which of the following will not increase the capacitance of an air capacitor?

1) adding a dielectric in the space between the plates

2) increasing the area of the plates

3) moving the plates closer together 4) increasing the voltage

KEY;4

50. Three identical condensers are connected together in four different ways. First all of them are connected in series and the equivalent capacity is C_1. Next all of them are connected in parallel and the equivalent capacity is C_2. Next two of them are connected in series and the third one connected in parallel to the combination and the equivalent capacity is C_3. Next two of them are connected in parallel and the third one connected in series with the combination and the equivalent capacity is C_4. Which of the following is correct ascending order of the equivalent capacities?

1) $C_1 < C_3 < C_4 < C_2$ 2) $C_1 < C_4 < C_3 < C_2$

3) $C_2 < C_3 < C_4 < C_1$ 4) $C_2 < C_4 < C_3 < C_1$

KEY;2

51. On a capacitor of capacitance C_0 following steps are performed in the order as given in column I.

A) Capacitor is charged by connecting it across a battery of emf E_0

B) Dielectric of dielectric constant K and thickness d is inserted

C) Capacitor is disconnected from battery

D) Separation between plates is doubled

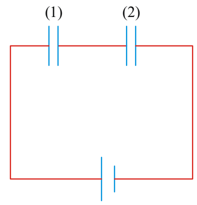
Column-I (Steps performed)	Column-II (Final value of Quantity (Symbols have usual meaning)
a) (A)(D)(C)(B)	p) $Q = \frac{C_0 E_0}{2}$
b) (D)(A)(C)(B)	q) $Q = \frac{KC_0 E_0}{K+1}$
c) (B)(A)(C)(D)	r) $C = \frac{KC_0}{K+1}$
d) (A)(B)(D)(C)	s) $V = \frac{E_0(K+1)}{2K}$

1) a-p,r,s, b-p,r,s, c-r, d-q,r 2) a-p, b-p,r c-r, d-q,

3) a-p,s, b-r,s, c-r, d-q, 4) a-r,s, b-s, c-r, d-q,r

KEY;1

52. In the circuit, both capacitors are indentical. Column I indicates action done on capacitors 1 and Column II indicates effect on capacitor 2



Column-I	Column-II
a) Plates are moved further apart	p)Amount of charge on left plate increases
b) Area increased	q) Potential difference increases
c) Left plate is earthed	r) Amount of charge on right plate decreases
d) It's plates are short circuited	s) None of the above effects

1.a-r, b-p,q, c-s, d-p,q 2.a-r, b-p, c-s, d-q
3.a-r, b-p, c-r, d-q 4.a-s, b-,q, c-s, d-q

KEY;1

53. Three identical capacitors are connected together differently. For the same voltage to every combination, the one that stores maximum energy is
1) the three in series 2) the three in parallel
3) two in series and the third in parallel with it
4) two in parallel and the third in series with it

KEY;2

54. The potential across a 3 μF capacitor is 12 V when it is not connected to anything. It is then connected in parallel with an uncharged 6 μF capacitor. At equilibrium, the charge and potential difference across the capacitor 3 μF and 6 μF are listed in column I. Match it with column III.

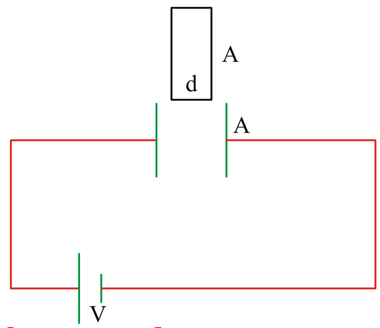
Column-I	Column-II
a) charge on 3 μF capacitor	p) 12 μC
b) charge on 6 μF capacitor	q)24 μF
c) potential difference across 3 μF capacitor	r) 8 V
d) potential difference across 6 μF capacitor	s) 4 V

1) a-r, b-p, c-s, d-q 2) a-p, b-q, c-s, d-s
3) a-r, b-p, c-q, d-q 4) a-r, b-q, c-s, d-q

KEY;2

55. Some events related to a capacitor are listed in column-I. Match these with their effects) in column - II

Column-I	Column-II
a) Insertion of dielectric while battery remain attached	p) Eelctric field between plates changes
b) Removal of dielectric while battery is not present	q) Charge present on plates changes
c) Slow decrease in separation between plates while battery is attached	r) Energy stored in capacitor increases
d) Slow increase of separation between plates while battery positive is not present	s) Work done by capacitor agent is



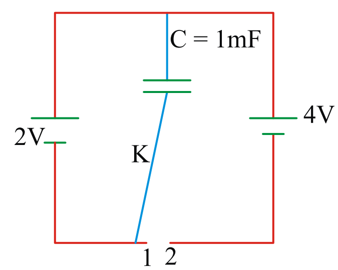
1) a-r, b-p,, c-p,q,s, d-q   2) a-p, b-p,, c-r,s, d-s

3) a-q,r, b-p,r,s c-p,q,r, d-r,s

4) a-r, p,b-q,, c-s, d-q

KEY;3

56. The circuit involves two ideal cells connected to a 1 μ F capacitor via key K. Initially the key K is in position 1 and the capacitor is charged fully by 2V cell. The key is pushed to position 2. Column I gives physical quantities involving the circuit after the key is pushed from position 1. Column.II gives corresponding results. Match the column-I with Column-II



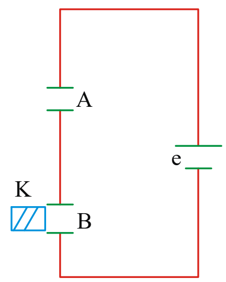
Column-I	Column-II
a) The net charge crossing the 4 volt cell in μC is	p) 2
b) The magnitude of work done by 4 volt cell in μJ is	q) 6
c) The gain in potential energy of capacitor in μJ is	r) 8
d) The net heat produced in circuit in μJ is	s) 16

1) a-r, b-p, c-s, d-q   2) a-p, b-r, c-q, d-p

3) a-r, b-p, c-q, d-q   4) a-r, b-q, c-s, d-q

KEY;2

57. Two identical capacitors A and B are connected to a battery of emf E as shown in figure. Now a dielectric slab is inserted between the plates of capacitor B while battery remains connected. Due to this inserting some physical quantities may change which are mentioned in Column-I and the effect is mentioned in Column-II.Match the Column I with Column-II



Column-I	Column-II
a) Charge on A	p) Increases
b) Charge on B	q) Decreases
c) Potential difference across A	r) Remains constant
d) Potential difference across B	s) Will change

1) a-r, b-p, c-s, d-q 2) a-p,s b-q,s, c-q,s d-q,s
3) a-r, b-p, c-q, d-q 4) a-r, b-q, c-s, d-q

KEY;2

58.

Column - I	Column - II
A) electrical potential	p) vector
B) energy stored in a condenser	q) $\frac{1}{2}CV^2$
C) force between two capacitor plates	r) scalar
D) electric capacity	s) $\frac{1}{2}\epsilon_0 E^2 A$

	A	B	C	D
1)	r	q,r	p,s	r
2)	r	q,r	p,q	s
3)	q,r	p,q	r,s	s
4)	p,q	r	q,r	s

KEY;1

ASSERTION & REASONING

1) Both A and R false
2) Both A and R true and R is not correct reason for A
3) A is true and R is false
4) Both A and R are true and R is correct reason of A.

59. Assertion (A) : The strength of electric filed in the charged and isolated capacitor is decreased when the dielectric slab is inserted.
Reason(R): When the dielctric slab is inserted between the plates of a charged capacitor, electricfield produced due to induced charge, opposite to the external field.

KEY;4

60. Assertion: If temperature is increased, the dielectric constant of a polar dielectric decreases whereas that of a non-polar dielectric does not change significantly
Reason: The magnitude of dipole moment of individual polar molecule decreases significantly with increase in temperature.

KEY;3

61. Assertion: The heat produced by a resistor in any time t during the charging of a capacitor in a series circuit is half the energy stored in the capacitor by that time.
Reason: Current in the circuit is equal to the rate of increase in charge on the capacitor.

KEY;4

62. Assertion: A dielectric is inserted between the plates of an isolated fully-charged capacitor. The dielectric completely fills the space between the plates. The magnitude of electrostatic force on either metal plate decreases, as it was before the insertion of dielectric medium.
Reason: Due to insertion of dielectric slab in an isolated parallel plate capacitor (the dielectric completely fills the space between the plates), the electrostatic potential energy of the capacitor decreases.

KEY;4

63. Assertion: If the potential difference across a plane parallel plate capacitor is doubled then the potential energy of the capacitor is doubled then the potential energy of the capacitor becomes four times under all conditions
Reason: The potential energy U stored in the capacitor is $U = \frac{1}{2}CV^2$, where C and V have usual meaning.

KEY;4

64. Assertion: A parallel plate capacitor is charged to a potential difference of 100V, and disconnected from the voltage source. A slab of dielectric is then slowly inserted between the plates. Compared to the energy before the slab was inserted, the energy stored in the capacitor with the dielectric is decreased.
Reason: When we insert a dielectric between the plates of a capacitor, the induced charges tend to draw in the dielectric into the field (just as neutral objects are attracted by charged objects due to induction). We resist this force while slowly inserting the dielectric, and thus do negative work on the system, removing electrostatic energy from the system.

KEY;1

65. Statement $'A'$: The energy stored gets reduced by a factor $'K'$ when the battery is disconnected after charging the capacitor and then the dielectric is introduced
Statement $'B'$: The energy stored in the capacitor increases by a factor $'k'$ when a dielectric is introduced between the plates with the battery present in the circuit

KEY;4

66. Assertion (A): A metallic sheild in form of a hollow shell may be built to block an electric field.
Reason (R): In a hollow spherical sheild, the electric field inside it is zero at every point.

KEY;1

67. Assertion (A): When two spheres carrying same charge but a different radii are connected by a conducting wire, the charge flows from smaller sphere to large sphere.
Reason (R): Smaller sphere is at high potential when equal charges are imparted to both the spheres

KEY;1

68. Assertion (A): Two capacitors are connected in par allel to a battery. If a dielectric medium is inserted between the plates of one of the capacitors then the energy stored in the system will increase
Reason (R): On inserting dielectric medium between the plates of a capacitors, its capacity increases

KEY;1

69. Assertion (A): When a charged capacitor is discharged through a resistor, heat is produced in the resistor
Reason (R): In charging a capacitor, energy is stored in the capacitor.

KEY;2

70. Assertion (A): A capacitor of capacitance C is connected across a battery of potential difference V. The energy stored in the capacitor is $\frac{1}{2}CV^2$

Reason (R): The energy supplied by the battery is $\frac{1}{2}CV^2$

KEY;3

71. Assertion (A): Two metal plates each of area A form a parallel plate capacitor. Now one plate is displaced up, then the capacitance of capacitor decreases.

Reason (R): Due to displacing one plate, the overlapping area decreases, capacitance $C = \frac{\varepsilon_0 A}{d}$ decreases.

KEY;1

72. Assertion (A): Two plates of a parallel plate capacitor are drawn apart, keeping them connected to a battery. Next the same plates are drawn apart from the same initial condition, keeping the battery disconnected, then the work done in both cases are same.
Reason (R): Capacitor plates have same charge in both cases and displacements of plates in both cases are also same.

KEY;4

73. Assertion (A) : Two metallic plates placed side by side form three capacitors.
Reason (R) : The infinity and first face of first plate is one capacitor, the second face of first plate and first face of second plate forms second capacitor and the second face of second plate and infinity forms the third capacitor, but the capacitance of first and third capacitance are extremely small

KEY;1

74. Statement (A): The energy stored gets reduced by a factor 'K' when the battery is disconnected after charging the capacitor and then the dielectric is introduced
Statement (R): The energy stored in the capacitor increases by a factor 'k' when a dielectric is introduced between the plates with the battery present in the circuit

KEY;4

75. Out of the following statements
A) The capacity of a conductor is affected due to the presence of an uncharged isolated conductor
B) A conductor can hold more charge at the same potential if it is surrounded by dielectric medium.
1) Both A and B are correct
2) Both A and B are wrong
3) A is correct and B is wrong
4) A is wrong and B is correct

KEY;1

76. A parallel plate condenser is charged by connecting it to a battery. Without disconnecting the battery, the space between the plates is completely filled with a medium of dielectric constant k. Then
1) potential becomes 1/k times
2) charge becomes k times
3) energy becomes 1/k times
4) electric intensity becomes k times.

KEY;2

77. Which of the following statements are correct?

a) When capacitors are connected in parallel the effective capacitance is less than the individual capacitances

b) The capacitances of a parallel plate capacitor can be increased by decreasing the separation of plates

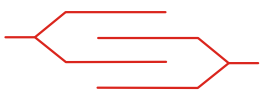
c) When capacitors are connected in series the effective capacitance is less than the least of the individual capacities

d) In a parallel plate capacitor the electrostatic energy is stored on the plates

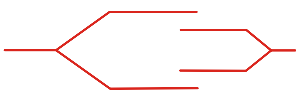
1) (a) and (b) 2) (a) and (c) 3) (c) and (d) 4) (b) and (c)

KEY;4

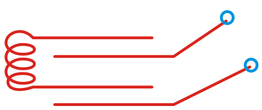
78. The effective capacity of the following capacitors is ________

a) 

e) $\frac{2c}{3}$

b) 

f) 2C

c) 

g) 3C

d) 

h) $\frac{5C}{2}$

i) $\frac{3C}{2}$

1) $a-g, b-f, c-e, d-i$ 2) $a-g, b-h, c-e, d-i$

3) $a-i, b-h, c-e, d-g$ 4) $a-g, b-e, c-h, d-i$

PRACTICE BITS

1. **The capacity of a parallel plate condenser consisting of two plates each 10 cm square and are seperated by a distance of 2 mm is (Take air as the medium between the plates)**

   1) $8.85\times10^{-13}F$ 2) $4.42\times10^{-12}F$

   3) $44.25\times10^{-12}F$ 4) $88.5\times10^{-13}F$

KEY : 2

SOLUTION :

$$C=\frac{\varepsilon_0 A}{d}$$

2. **Sixty four spherical drops each of radius 2 cm and carrying 5C charge combine to form a bigger drop. Its capacity is**

   1) $\frac{8}{9}\times10^{-11}F$ 2) $90\times10^{-11}F$ 3) $1.1\times10^{-11}F$ 4) $9\times10^{11}F$

KEY : 1

SOLUTION :

$$C^1=n^{\frac{1}{3}}C$$

3. **A parallel plate condenser has initially air medium between the plates. If a slab of dieletric constant 5 having thickness half the distance of seperation between the plates is introduced, the percentage increase in its capacity is**

   1) 33.3% 2) 66.7% 3) 50% 4) 75%

KEY : 2

SOLUTION :

$$C_0=\frac{\varepsilon_0 A}{d};\qquad C=\frac{\in_0 A}{d-t\left(1-1/k\right)}\qquad \Delta C\%=\frac{C-C_0}{C_0}\times100\%$$

4. **When a dielectric slab of thickness 4 cm is introduced between the plates of parallel plate condenser, it is found the distance between the plates has to be increased by 3cm to restore to capacity to original value. The dielectric constant of the slab is**

   1) 1/4 2) 4 3) 3 4) 1

KEY : 2

SOLUTION :

$$C=\frac{\in_0 A}{d-t\left(1-1/k\right)}=\frac{\in_0 A}{d-d'}$$

5. **The area of the positive plate is A_1 and the area of the negative plate is $A_2\,(A_2<A_1)$. They are parallel to each other and are separated by a distance d . The capacity of a condenser with air as dielectric is**

   1) $\frac{\varepsilon_0 A_1}{d}$ 2) $\frac{\varepsilon_0 A_2}{d}$ 3) $\frac{\varepsilon_0 A_1 A_2}{d}$ 4) $\frac{\varepsilon_0 A_1}{A_2 d}$

KEY : 2

SOLUTION :

Effective area only $\therefore A_2$

6. The cross section of a cable is shown in fig. The inner conductor has a radius of 10 mm and the dielectric has a thickness of 5 mm. The cable is 8 km long. Then the capacitance of the cable is

$[\log_e 1.5 = 0.4]$

1) 3.8 μF 2) 1.1 μF 3) $4.8\times10^{-10}\mu F$ 4) 3.3 μF

KEY : 1

SOLUTION :

$$C = \frac{K\varepsilon_o 2\pi l}{\ln(b/a)}$$

7. A highly conducting sheet of aluminium foil of negligible thickness is placed between the plates of a parallel plate capacitor. The foil is parallel to the plates. If the capacitance before the insertion of foil was 10 μ F, its value after the insertion of foil will be

1) 20 μ F 2) 10 μ F 3) 5 μ F 4) Zero

KEY : 2

SOLUTION :

$V = Ed$ without foil

$V' = \frac{Ed}{2} + \frac{Ed}{2} = Ed$ with foil

Hence $C = \frac{Q}{V} = C' = 10\mu F$

8. Two metal plates are separated by a distance d in a parallel plate condenser. A metal plate of thickness t and of the same area is inserted between the condenser plates. The value of capacitance increases by ...... times

1) $\frac{d-t}{d}$ 2) $\left(1-\frac{t}{d}\right)$ 3) $\left(t-\frac{t}{d}\right)$ 4) $\frac{1}{\left(1-\frac{t}{d}\right)}$

KEY : 4

SOLUTION :

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}}\ ;k = \infty$$

9. A radio capacitor of variable capacitance is made of n parallel plates each of area A and separated from each other by a distance d. The alternate plates are connected together. The capacitance of the combination is

1) $\frac{nA\epsilon_o}{d}$ 2) $\frac{(n-1)A\epsilon_o}{d}$ 3) $\frac{(2n-1)A\epsilon_o}{d}$ 4) $\frac{(n-2)A\epsilon_o}{d}$

KEY : 2

SOLUTION :

Due to n plates n-1 capacitors are formed

10. The radius of the circular plates of a parallel plate condenser is 'r'. Air is there as the dielectric. The distance between the plates if its capacitance is equal to that of an isolated sphere of radius r' is

1) $\frac{r^2}{4r'}$ 2) $\frac{r^2}{r'}$ 3) $\frac{r}{r'}$ 4) $\frac{r^2}{4}$

KEY : 1

SOLUTION :

$$\frac{\epsilon_o\left(\pi r^2\right)}{d} = 4\pi \epsilon_o r^1 \quad \therefore d = \frac{r^2}{4r^1}$$

11. Two condensers of capacity C and 2C are connected in parallel and these are charged upto V volt. If the battery is removed and dielectric medium of constant K is put between the plates of first condenser, then the potential at each condenser is

1) $\frac{V}{k+2}$ 2) $2+\frac{k}{3V}$ 3) $\frac{2V}{k+2}$ 4) $\frac{3V}{k+2}$

KEY : 4

SOLUTION :

Q = constant , CV + 2CV = $KCV^{l} + 2CV^{l}$

12. Given a number of capacitors labelled as C,V. Find the minimum number of capacitors needed to get an arrangement equivalent to C_{net}, V_{net} 1) $n = \frac{C_{net}}{C} \times \frac{V_{net}{}^2}{V^2}$ 2) $n = \frac{C}{C_{net}} \times \frac{V^2}{V_{net}{}^2}$

3) $n = \frac{C}{C_{net}} \times \frac{V}{V_{net}}$ 4) $n = \frac{C_{net}}{C} \times \frac{V_{net}}{V}$

KEY : 4

SOLUTION :

$$F = \frac{\sigma Q}{2\varepsilon_0} = \frac{Q^2}{2\varepsilon_0 A}$$

13. Two capacitors of capacities $3\mu F$ and $6\mu F$ are connected in series and connected to 120V. The potential difference across $3\mu F$ is V_0 and the charge here is q_0. We have

A) $q_0 = 40\mu C$ **B)** $V_0 = 60V$

C) $V_0 = 80V$ **D)** $q_0 = 240\mu C$

1) A, C are correct 2) A, B are correct 3) B, D are correct 4) C, D are correct

KEY : 4

SOLUTION :

$$Q = \left(\frac{C_1C_2}{C_1+C_2}\right)V$$

14. n Capacitors of 2 μ F each are connected in parallel and a p.d of 200v is applied to the combination. The total charge on them was 1c then n is equal to

1) 3333 2) 3000 3) 2500 4) 25

KEY : 3

SOLUTION :

$$Q = nCV$$

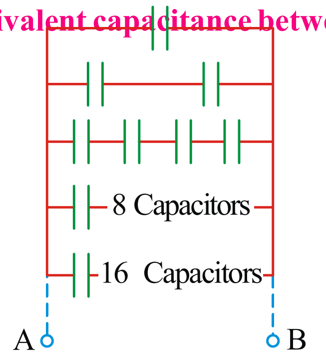
15. An infinite number of identical capacitors each of capacitance 1 mF are connected as shown in the figure. Then the equivalent capacitance between A and B is

1) 1 mF

2) 2 mF

3) ½ mF

4) ∞



KEY : 2

SOLUTION :

$$C_R = C + \frac{C}{2} + \frac{C}{4} + ...$$

16. When two capacitors are joined in series the resultance capacity is $2.4\mu F$ and when the same two are joined in parallel the resultant capacity is $10\mu F$. Their individual capacities are

1) $7\mu F, 3\mu F$ 2) $1\mu F, 9\mu F$

3) $6\mu F, 4\mu F$ 4 $8\mu F, 2\mu F$

KEY : 3

SOLUTION :

$$C_S = \frac{C_1C_2}{C_1 + C_2};\ C_1 + C_2 = C_P$$

17. Three condensers $1\mu F$, $2\mu F$ and $3\mu F$ are connected in series to a p.d. of 330 volt. The p.d across the plates of $3\mu F$ is

1) 180 V 2) 300 V 3) 60 V 4) 270 V

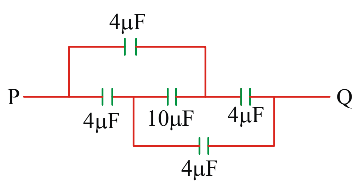
KEY : 3

SOLUTION :

$$Q = C_{eff}V\ ;\ Q = C_1V$$

18. The effective capacitance between the point P and Q in the given figure is

1) 4 μF 2) 16 μF



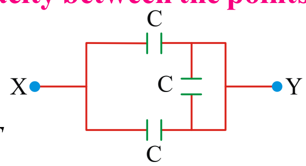
3) 26 μF 4) 10 μF

KEY : 1

SOLUTION :

$$C^1 = \frac{C_1C_2}{C_1 + C_2};\ C^{11} = \frac{C_3C_4}{C_3 + C_4};\ C_{eff} = C^1 + C^{11}$$

19. The equivalent capacity between the points X and Y in the circuit with $C = 1\mu F$ (2007M)



1) $2\mu F$ 2) $3\mu F$

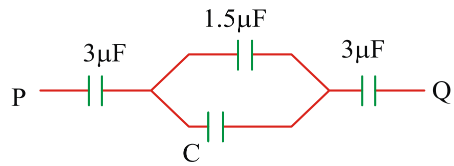
3) $1\mu F$ 4) $0.5\mu F$

KEY : 1

SOLUTION :

$C_{eff} = C_1 + C_2$

20. The equivalent capacitance of the network given below is 1 μ F. The value of 'C' is



1) 3 μ F 2) 1.5 μ F 3) 2.5 μ F 4) 1 μ F

KEY : 2

SOLUTION :

$1.5\mu c, c$ are in parallel ;

its effective capacitance 1.5 + c

1.5+c, $3\mu F, 3\mu F$ are in series

21. Three capacitors of $3\mu F, 2\mu F$ and $6\mu F$ are connected in series. When a battery of 10V is connected to this combination then charge on $3\mu F$ capacitor will be

1) $5\mu C$ 2) $10\mu C$ 3) $15\mu C$ 4) $20\mu C$

KEY : 2

SOLUTION :

In series charge constant $Q = C_{eff}V$

22. Two spheres of radii 12 cm and 16 cm have equal charge. The ratio of their energies is

1) 3 : 4 2) 4 : 3 3) 1 : 2 4) 2 : 1

KEY : 1

SOLUTION :

$U = \frac{q^2}{2C}, U \alpha \frac{1}{r}$

23. A parallel capacitor of capacitance C is charged and disconnected from the battery. The energy stored in it is E. If a dielectric slab of dielectric constant 6 is inserted between the plates of the capacitor then energy and capacitance will become

1) 6E, 6C 2) E, C 3) E/6, 6C 4) E, 6C

KEY : 3

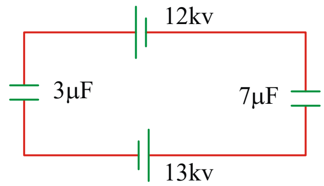
SOLUTION :

$C_k = KC \Rightarrow C_K = 6C$

at battery is disconnected 'Q' does not change

$\therefore U_k = \frac{Q^2}{2KC} = \frac{U}{K} = \frac{E}{6}$

24. In the circuit diagram given below, the value of the potential difference across the plates of the capacitors are



1) 17.5 KV, 7.5 KV 2) 10 KV, 15 KV
3) 5 KV, 20 KV 4) 16.5 KV, 8.5KV

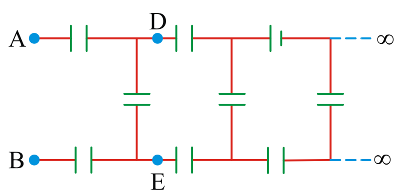
KEY : 1

SOLUTION :

By Kirchoff loop theorem

$$12-\frac{2}{3}+13-\frac{q}{7}=0 \quad ; \quad V_3=\frac{q}{3}, V_4=\frac{q}{7}$$

25. The equivalent capacity of the infinite net work shown in the figure (across AB) is (Capacity of each capacitor is 1 μ F)



1) ∞ 2) $1\mu F$ 3) $\left(\frac{\sqrt{3}-1}{2}\right)\mu F$ 4) $\left(\frac{\sqrt{3}+1}{2}\right)\mu F$

KEY : 3

SOLUTION :

Between D & F effective capacitance is x

$$\frac{1}{x+1}+1+1=x$$

26. A condenser of capacity 10 μ F is charged to a potential of 500 V. Its terminals are then connected to those of an uncharged condenser of capacity 40 μ F. The loss of energy in connecting them together is

1) 1J 2) 2.5J 3) 10J 4) 12 J

KEY : 1

SOLUTION :

$$\Delta E=\frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1-V_2)^2$$

27. A $2\mu F$ condenser is charged to 500V and then the plates are joined through a resistance. The heat produced in the resistance in joule is

1) 50 x 10^{-2} Joule 2) 25 x 10^{-2} Joule
3) 0.25 x 10^{-2} Joule 4) 0.5 x 10^{-2} Joule

KEY : 2

SOLUTION :

$$\text{Energy Stored}=\frac{1}{2}cv^2$$

28. **The time in seconds required to produce a P.D at 20V across a capacitor at $1000\ \mu F$ when it is charged at the steady rate of $200\ \mu C/\sec$ is**

1) 50 2) 100 3) 150 4) 200

KEY :2

SOLUTION :

$$\frac{dq}{dt} = \frac{c.dv}{dt}$$

29. **The force between the plates of a parallel plate capacitor of capacitance C and distance of separation of the plates d with a potential difference V between the plates, is**

1) $\frac{CV^2}{2d}$ 2) $\frac{C^2V^2}{2d^2}$ 3) $\frac{C^2V^2}{d^2}$ 4) $\frac{V^2d}{C}$

KEY :1

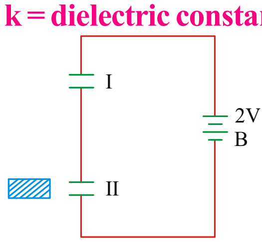
SOLUTION :

$$F = \frac{QE}{2} = \frac{CV}{2}\left[\frac{V}{d}\right]$$

30. **Two identical capacitors are connected as show in the figure. A dielectric slab is introduced between the plates of one of the capacitors so as to fill the gap, the battery remaining connected. The charge on each capacitor will be (charge on each condenser is q_0 ; k = dielectric constant)**

1) $\frac{2q_0}{1+1/k}$ 2) $\frac{q_0}{1+1/k}$

3) $\frac{2q_0}{1+k}$ 4) $\frac{q_0}{1+k}$



KEY :1

SOLUTION :

$$C_{eff} = \frac{C_0 k}{1+k}\ ;\ q = C_{eff}V$$

31. **A capacitor of capacitance 1 μF withstands a maximum voltage of 6 kV, while another capacitor of capacitance 2 μF withstands a maximum voltage of 4 kV. If they are connected in series, the combination can withstand a maximum voltage of**

1) 3 kV 2) 6 kV 3) 10 kV 4) 9 kV

KEY :4

SOLUTION :

$V_1 \le V_{max1}, V_2 \le V_{max2}$

32. **Energy 'E' is stored in a parallel plate capacitor 'C_1'. An identical uncharged capacitor 'C_2' is connected to it, kept in contact with it for a while and then disconnected, the energy stored in C_2 is**

1) E/2 2) E/3 3) E/4 4) Zero

KEY : 3

SOLUTION :

$$U_2^{/} = \frac{1}{2}C_2V^2{}_{\text{common}}$$

33. A parallel plate capacitor with plates separated by air acquires 1 μ C of charge when connected to a battery of 500V. The plates still connected to the battery are then immersed in benzene [k=2.28]. Then a charge that flows from the battery is

1) 1.28 μ C 2) 2.28 μ C 3) 1 / 4 μ C 4) 4.56 μ C

KEY : 1

SOLUTION :

$$Q_0 = C_0V_0 \; ; \; Q = CV_0 = KC_0V_0$$

$$\Delta Q = Q - Q_0 = (K-1)C_0V_0$$

34. An air capacitor with plates of area 1 m^2 and 0.01 metre apart is charged with 10^{-6} C of electricity. When the capacitor is submerged in oil of relative permittivity 2, then the energy decreases by

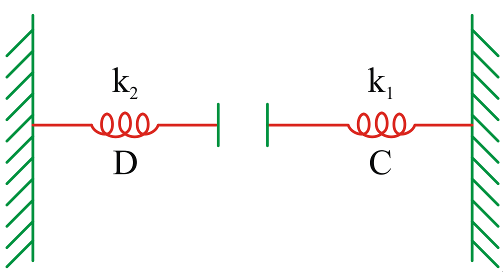
1) 20 % 2) 50 % 3) 60 % 4) 75 %

KEY : 2

SOLUTION :

$$U^{/} = \frac{U}{K}$$

35. In the given figure the capacitor of plate area A is charged upto charge q. The ratio of elongations (neglect force of gravity) in springs C and D at equilibrium position is



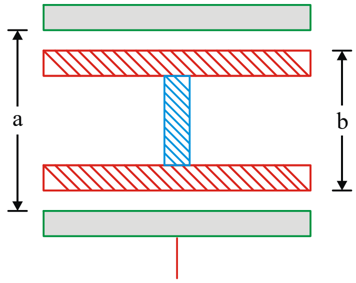
1) $\frac{k_1}{k_2}$ 2) $\frac{k_2}{k_1}$ 3) k_1k_2 4) $\sqrt{\frac{k_1}{k_2}}$

KEY : 2

SOLUTION :

$$F_e = k_1x_1 = k_2x_2 \quad \therefore \frac{x_1}{x_2} = \frac{k_2}{k_1}$$

36. If metal section of shape H is inserted in between two parallel plates as shown in figure and A is the area of each plate then the equivalent capacitance is



1) $\frac{A\in_0}{a}-\frac{A\in_0}{b}$ 2) $\frac{A\in_0}{a+b}$

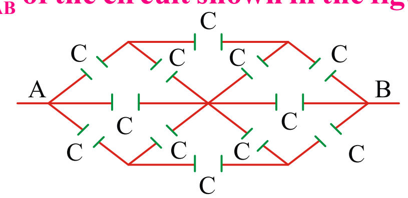
3) $\frac{A\in_0}{a}+\frac{A\in_0}{b}$ 4) $\frac{A\in_0}{a-b}$

KEY : 4

SOLUTION :

Net space between metal plates is a-b

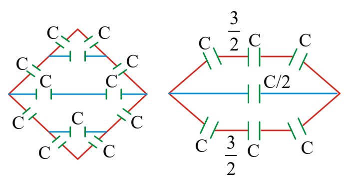
37. The equivalent capacitance C_{AB} of the circuit shown in the figure is



1) $\frac{5}{4}C$ 2) $\frac{4}{5}C$ 3) 2 C 4) C

KEY : 1

SOLUTION :



38. A solid conducting sphere of radius 10cm is enclosed by a thin metallic shell of radius 20cm. A charge q=20 μC is given to the inner sphere. The heat generated in the process is

1) 12 J 2) 9 J 3) 24 J 4) zero

KEY :2

SOLUTION :

$$H=U_i-U_f=\frac{q^2}{2C_1}-\frac{q^2}{2C_2}$$

$$C_1=4\pi\in_0 R_1, C_2=4\pi\in_0 R_2$$

39. A condenser of capacity $500\mu F$ is charged at the rate of $400\mu C$ per second. The time required to raise its potential by $40V$ is

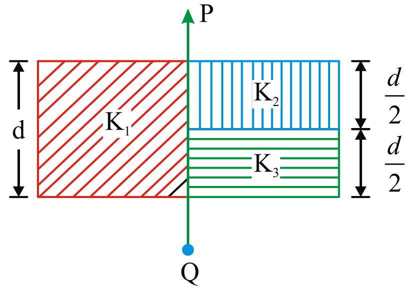
1) 50 s 2) 100 s 3) 20 s 4) 10 s

KEY :1

SOLUTION :

$$\frac{dq}{dt}=\frac{c.dv}{dt}$$

20. In the figure shown the effective capacity across P and Q is (the area of each plate is 'a')



1) $\frac{a\in_0}{d}\left[\frac{K_1}{2}+\frac{K_2K_3}{K_2+K_3}\right]$ 2) $\frac{a\in_0}{2d}\left[\frac{K_2}{2}+\frac{K_1K_3}{K_1+K_3}\right]$

3) $\frac{a\in_0}{3d}\left[\frac{K_3}{2}+\frac{K_1K_2}{K_1+K_2}\right]$ 4) $\frac{a\in_0}{d}\left[\frac{K_1}{2}+\frac{K_1+K_2}{K_2K_3}\right]$

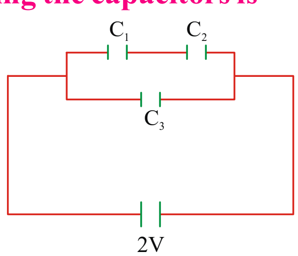
KEY : 1

SOLUTION :

$$C_1=\frac{K_1\in_o\left(\frac{A}{2}\right)}{d};\ C_2=\frac{K_2\in_o A}{d};\ C_3=\frac{K_3\in_o A}{d}$$

$$\therefore C=\frac{C_2C_3}{C_2+C_3}+C_1$$

40. Two capacitors $C_1=2\mu F$ and $C_2=6\mu F$ in series, are connected in parallel to a third capacitor $C_3=4\mu F$. This arrangement is then connected to a battery of e.m.f.=2 V, as shown in figure. The energy lost by the battery in charging the capacitors is



1) $22\times10^{-6}J$ 2) $11\times10^{-6}J$

3) $\left(\frac{32}{3}\right)\times10^{-6}J$ 4) $\left(\frac{16}{3}\right)\times10^{-6}J$

KEY : 2

SOLUTION :

$$E_{lost}=\frac{1}{2}C_{eff}V^2$$

41. A capacitor is connected with a battery and stores energy U. After removing the battery, it is connected with another similar capacitor in parallel. The new stored energy in each capacitor will be

1) $\frac{U}{2}$ 2) U 3) $\frac{U}{4}$ 4) $\frac{3U}{2}$

KEY : 3

SOLUTION :

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2};\ U = \frac{1}{2}(C_1 + C_2)V^2$$

42. A parallel plate capacitor of capacity $100\mu F$ is charged by a battery at 50 volts. The battery remains connected and if the plates of the capacitor are separated so that the distance between them is halved the original distance, the additional energy gives by the battery to the capacitor in Joules is ......

1) 125×10^{-3} 2) 12.5×10^{-3}

3) 1.25×10^{-3} 4) 0.125×10^{-3}

KEY :1

SOLUTION :

$$U_i = \frac{1}{2}CV^2 \qquad U_f = \frac{1}{4}CV^2$$

additional energy supplied by the battery

$$E = 2\left(U_i - U_f\right) = 2\left[\frac{1}{4}CV^2\right] = \frac{1}{2}CV^2$$

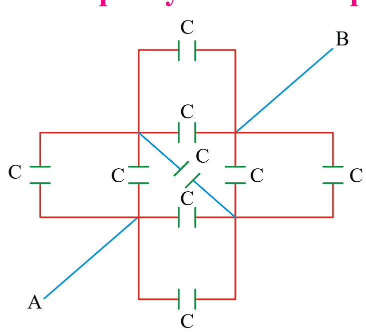
43. The equivalent capacity between the points A and B in the adjoining circuit will be

1) C

2) 2C

3) 3C

4) 4



KEY : 2

SOLUTION :

Use Wheat stone's Bridge principle

44. A parallel plate capacitor with air as medium between the plates has a capacitance of 10 μ F. The area of the capacitor is divided into two equal halves and filled with two media having dielectric constant K_1=2 and K_2=4. The capacitance will now be

1) 10 μ F 2) 20 μ F 3) 30 μ F 4) 40 μ F

KEY :3

SOLUTION :

$$c = \frac{c_0}{2}(k_1 + k_2)$$

45. The capacity of a parallel plate condenser with air medium is $60\mu F$ having distance of seperation *d*. If the space between the plates is filled with two slabs each of thinckness $d/2$ and dielectric constants 4 and 8,

the effective capacity becomes

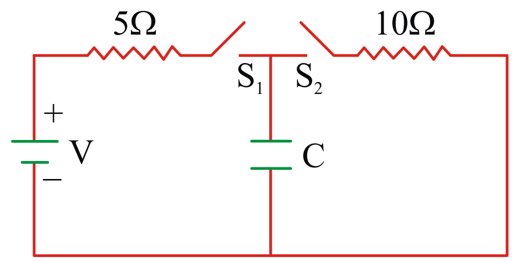
1) $160\mu F$ 2) $320\mu F$ 3) $640\mu F$ 4) $360\mu F$

KEY :2

SOLUTION :

$$C = \frac{2K_1K_2}{K_1+K_2}C_0$$

46. In the adjoining diagram, the condenser C will be fully charged to potential V if



1) S_1 and S_2 both are open
2) S_1 and S_2 both are closed
3) S_1 is closed and S_2 is open
4) S_1 is open and S_2 is closed.

KEY :3

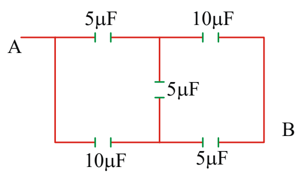
SOLUTION :

If S_1 and S_2 both are closed then charge and discharge processes with simoultaneously take place. Hence to charge the condenser fully the key S_1 must be closed and S_2 must remain open

47. The capacitance C_{AB} in the given network

1) $7\ \mu F$ 2) $\frac{50}{7}\mu F$

3) $7.5\mu F$ 4) $\frac{7}{50}\mu F$

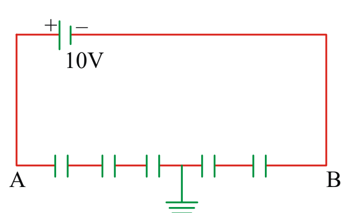


KEY : 1

SOLUTION :

$$\therefore C = \frac{2C_1C_2 + C_3\left(C_1+C_2\right)}{C_1+C_2+2C_3}$$

48. In the following circuit; find the potentials at points A and B is



1) 10V, 0V
2) 6 V, -4V
3) 4V, -6V
4) 5V, -5V

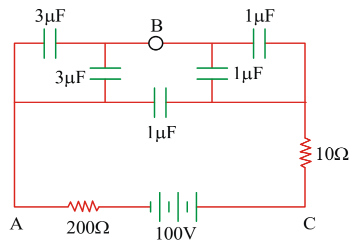
KEY : 2

SOLUTION :

P.D across each condenser = 2V

Potential at earth = 0V ; $\therefore V_A = +6V \quad V_B = -4V$

49. The potential difference between the points A and B in the following circuit in steady state will be



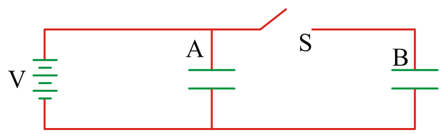
1) $V_{AB} = 100$ volt 2) $V_{AB} = 75$ volt 3) $V_{AB} = 25$ volt 4) $V_{AB} = 50$ volt

KEY :3

SOLUTION :

$$V_{AB} = \frac{VC_2}{C_1 + C_2} ; \therefore V_{AB} = \frac{1}{4} \times 100 = 25V$$

50. In the following circuit two identical capacitors, a battery and a switch(s) are connected as shown. the switch(s) is opened and dielectric of constant $(K = 3)$ are inserted in the condensers. The ratio of electrostatic energies of the system before and after filling the dielectric will be



1) 3: 1 2) 5 : 1 3) 3:5 4) 5 : 3

KEY :3

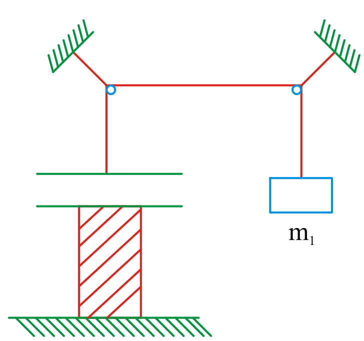
SOLUTION :

$$\therefore U_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

$$\therefore U_2 = \frac{1}{2} \times 3 \times CV^2 + \frac{1}{2} \times 3C\frac{V^2}{9} = \frac{5}{3}CV^2$$

$$\therefore \frac{U_1}{U_2} = \frac{3}{5}$$

51. In the given figure a capacitor of plate area A is charged upto charge q. The mass of each plate is m_2. The lower plate is rigidly fixed. The value of m_1 if the system remains in equilibrium is



1) $m_2 + \frac{q^2}{\in_0 Ag}$ 2) m_2 3) $\frac{q^2}{2\in_0 Ag} + m_2$ 4) $2m_2$

KEY :3

SOLUTION :

$T = m_1 g$ and $T = m_2 g + F_e$; $\therefore m_1 g = m_2 g + F_e$

Here, $F_e = \frac{q^2}{2\in_0 A}$

52. A capacitor of capacitance C is fully charged by a 200V supply. It is then discharged through a small coil through a small coil of resistance wire embedded in a thermally insulated block of specific heat 250 $Jkg^{-1}K^{-1}$ and of mass 100 gram. If the temperature of the block rises by 0.4^0C, the capacitacne C is ____ μF

1) 500 2) 1000 3) 1200 4) 1225

KEY :1

SOLUTION :

As the energy stored in the capacitor is dissipated as heat energy of the resistance wire, Use the relation

$$\frac{1}{2}CV^2 = ms\Delta t$$

53. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2.The negative ends are also connected, the decrease in energy of the combined system is

1) $1/4C\left(V_1^2 - V_2^2\right)$ 2) $1/4C\left(V_1^2 + V_2^2\right)$ 3) $1/4C\left(V_1 - V_2\right)^2$ 4) $1/4C\left(V_1 + V_2\right)^2$

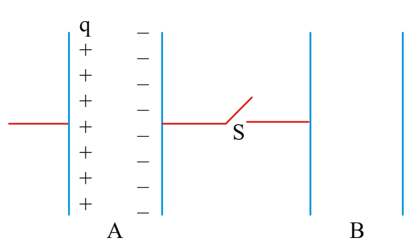
KEY :3

SOLUTION :

$Q = CV$; $U_i = 1/2CV^2$

$U_f = 1/2CV^2$ work done $= U_i - U_f$

54. Consider the situation shown in the figure. The capacitor A has a charge q on it whereas B is uncharged. The charge appearing on the capacitor B a long 7 time after the switch is closed is:



1) Zero 2) q /2 3) q 4) 2q

KEY :1

SOLUTION :

Due to attraction with positive charge, the negative charge on capacitor A will not flow thrugh the switch S.

55. Find the capacitance of a system of two identical metal balls of radius a if the distance between their centres is equal to b, with b>>a. The system is located in a uniform dielectric with permittivity K.

1) $\pi \in_0 Ka$ 2) $4\pi \in_0 Ka$

3) $2\pi \in_0 Ka$ 4) $2/3\pi \in_0 Ka$

KEY :3

SOLUTION :

$$V_1 - V_2 = 2V = 2\int_a^{b-a} E\ dr\ ;\ C = \frac{q}{V_1 - V_2}$$

Previous JEE MAINS Questions

ELECTRIC POTENTIAL & CAPACITANCE

1.Ten charges are placed on the circumference ofa circl ofradius R with constant angular separationbetween successive charges. Alternate charges 1, 3, 5, 7, 9 have charge $(+q)$ each, while 2, 4, 6, 8, 10 have charge $(-q)$ each. The potential V and the electric field E at the centre of the circle are respectively: (Take $V=0$ at infinity)

[Sep. 05, 2020 (II)]

(a) $V=\frac{10q}{4\pi\varepsilon_0 R}; E=0$ (b) $V=0; E=\frac{10q}{4\pi\varepsilon_0 R^2}$

(c) $V=0; E=0$ (d) $V=\frac{10q}{4\pi\varepsilon_0 R}; E=\frac{10q}{4\pi\varepsilon_0 R^2}$

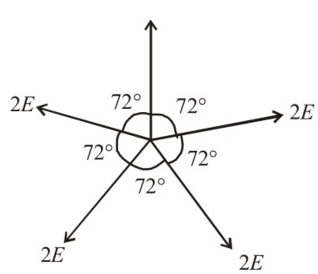
SOLUTION : (c)

Potential at the centre, $V_c=\frac{KQ_{\text{net}}}{\text{R}}$

$$Q_{\text{net}}=0$$

$$V_c=0$$

Let E be electric field produced by each charge at the centre, then resultant electric field will be $E_C=0$, since equal electricfield vectors are acting at equal angle so their resultant isequal to zero.



2. Two isolated conducting spheres S_1 and S_2 ofradius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and -3μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively:

[Sep. 03, 2020 (I)]

(a) 4.5 μC on both (b) $+4.5\mu$C and -4.5μC

(c) 3 μC and6 μC (d) 6 μC and3 μC

SOLUTION : (d)

Total charge $Q_1+Q_2=Q_1^1+Q_2^1$

$$=12\mu C-3\mu C=9\mu C$$

Two isolated conducting sphres S_1 and S_2 are now connected by a conducting wire.

$$V_1=V_2=\frac{KQ_1^1}{\frac{2}{3}R}=\frac{KQ_2^1}{\frac{R}{3}}=12-3=9\mu\text{C}$$

$$Q_1^1=2Q_2^{'}\Rightarrow 2Q_2^1+Q_2^1=9\mu C$$

$$Q_1^1=6\mu C \text{ and } Q_2^1=3\mu C$$

3. Concentric metallic hollow spheres ofradiiR and $4R$ hold charges Q_1 and Q_2 respectively. Given that surface charge densities ofthe concentric spheres are equal, the potential difference $V(R)-V(4R)$is: [Sep. 03, 2020 (II)]

(a) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (b) $\frac{3Q_2}{4\pi\varepsilon_0 R}$

(c) $\frac{Q_2}{4\pi\varepsilon_0 R}$ (d) $\frac{3Q_1}{4\pi\varepsilon_0 R}$

SOLUTION : (a)

We have given two metallic hollow spheres ofradii R

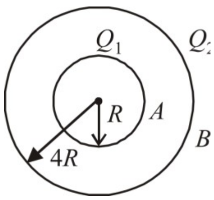
and $4R$ having charges Q_1 and Q_2 respectively.

Potential on the surface ofinner sphere (at A)

$$V_A = \frac{kQ_1}{R} + \frac{kQ_2}{4R}$$

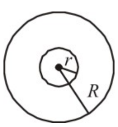
Potential on the surface of outer sphere (at B)

$$V_B = \frac{kQ_1}{4R} + \frac{kQ_2}{4R} \text{ (Here, k} = \frac{1}{4\pi\varepsilon_0})$$



Potential difference, $\Delta V = V_A - V_B = \frac{3}{4} \cdot \frac{kQ_1}{R} = \frac{3}{16\pi\epsilon 0} \cdot \frac{Q_1}{R}$

4. A charge Q is distributed over two concentric conducting thin spherical shells radii r and $R(R > r)$. Ifthe surface charge densities on the two shells are equal, the electric potential at the common centre is: [Sep. 02, 2020 (II)]

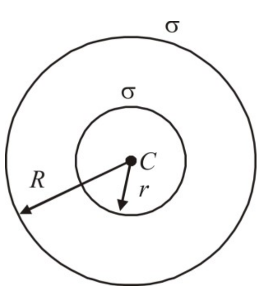


(a) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{2(R^2+r^2)}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(2R+r)}{(R^2+r^2)}Q$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{(R+2r)Q}{2(R^2+r^2)}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)}{(R^2+r^2)}Q$

SOLUTION : (d)

Let 0 be the surface charge density ofthe shells.



Charge on the inner shell, $Q_1 = 04\pi r^2$

Charge on the outer shell, $Q_2 = 04\pi R^2$

Total charge, $Q = 04\pi(r^2 + R^2)$

$$\Rightarrow 0 = \frac{Q}{4\pi(r^2 + R^2)}$$

Potential at the common centre,

$$V_c = \frac{KQ_1}{r} + \frac{KQ_2}{R} \text{ (where } K = \frac{1}{4\pi\varepsilon_0})$$

$$= \frac{Ko4\pi r^2}{r} + \frac{Ko4\pi R^2}{R}$$

$$= Ko4\pi(r + R)$$

$$= \frac{KQ4\pi(r + R)}{4\pi(r^2 + R^2)}$$

$$= \frac{1(r + R)Q}{4\pi\varepsilon_0(r^2 + R^2)}$$

5. A point dipole $= p - p_o\hat{x}$ kept at the origin. The potential and electric field due to this dipole on the y - axis at a distance d are, respectively: (Take $\text{V} = 0$ at infinity) [12 April 2019 I]

(a) $\frac{|\overline{p}|}{4\pi\varepsilon_0 d^2}, \frac{\overline{p}}{4\pi\varepsilon_0 d^3}$ (b) $0, \frac{-\overline{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \frac{|\overline{p}|}{4\pi\varepsilon_0 d^3}$ (d) $\frac{|\overline{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\overline{p}}{4\pi\varepsilon_0 d^3}$

SOLUTION : (b)

The electric potential at the bisector is zero and electric field is antiparallel to the dipole moment.

$$V = 0 \text{ and } \vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{-\vec{P}}{d^3}\right)$$

6. A uniformly charged ring ofradius 3a and total charge q is placed in xy - plane centred at origin. Apoint charge q is moving towards the ring along the z - axis and has speed v at z = 4a. The minimum value ofv such that it crosses the origin is: [10 April 2019 I]

(a) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

SOLUTION : (c)

Potential at any point ofthe charged ring

$$V_p = \frac{Kq}{\sqrt{R^2 + Z^2}}$$

$$R = 3a \ , Z = 4a$$

$$l = \sqrt{R^2 + Z^2} = 5a$$

The minimum velocity (v_0) shouldjust sufficient to reach the point charge at the center, therefore

$$\frac{1}{2}mv_0^2 = q[V_C - V_P]$$

$$= q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$v_0^2 = \frac{4Kq^2}{15ma} = \frac{4}{15}\frac{1}{4\pi\varepsilon_0}\frac{q^2}{ma}$$

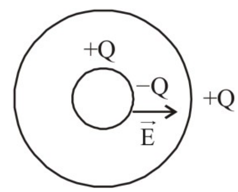
$$\Rightarrow v_0 = \sqrt{\frac{2}{m}}^{(\ )^{\frac{1}{2}}} (\ )$$

7. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that ofthe outer surface ofthe hollow shell be V. Ifthe shell is now given a charge of - 4 Q, the new potential difference between the same two surfaces is: [8 April 2019 I]

(a) $-2V$ (b) $2V$ (c) $4V$ (d) V

SOLUTION : . (d)

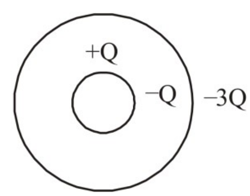
When charge Q is on inner solid conducting sphere



Electric field between spherical surface

$$\vec{E} = \frac{KQ}{r^2} So \int \ \vec{E}.d\vec{r} = V \ given$$

Now when a charge $-4Q$ is given to hollow shell



Electric field between surface remain unchanged. $\vec{E} = \frac{KQ}{r^2}$

as, field inside the hollow spherical shell $= 0$ Potential difference between them remain unchanged

i.e. $\int \ \vec{E}.d\vec{r} = V$

8. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{\imath}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is: [8 Jan. 2019 II]

(a) 320V (b) -48V (c) 180V (d) -520V

SOLUTION : . (c)

Given, $\overline{E} = (Ax + B)i^{\wedge}$

or $E = 20x + 10$

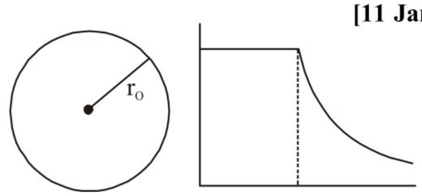
Using $= \int E\,dx$, we have

$$V_2 - V_1 = \int_{-5}^{1} (20x + 10)\,dx = -180\text{V}$$

or $V_1 - V_2 = 180\text{V}$

9. The given graph shows vanation(with distance rfrom centre)

of: **[11 Jan. 2019 I]**



r_o

r

(a) Electric field ofa uniformly charged sphere

(b) Potential ofa uniformly charged spherical shell

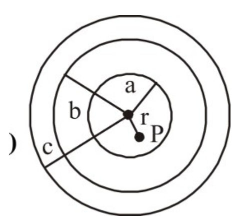
(c) Potential ofa uniformly charged sphere

(d) Electric field ofa uniformlycharged spherical shell

SOLUTION : (b)

Electric potential is constant inside a charged spherical

shell.



10. A charge Q is distributed over three concentric spherical

shells of radii a, b, c($a < b < c$) such that their surface

charge densities are equal to one another.The total potential at a point at distance r from theircommon centre, where $r < a$, would be: [10 Jan. 2019 I]

(a) $\frac{Q}{12\pi\epsilon 0}\frac{ab+bc+ca}{abc}$ (b)

(C) $\frac{Q}{4\pi\epsilon 0(a+b+c)}$ (d) $\frac{Q(a+b+c)}{4\pi\epsilon 0(a^2+b^2+c^2)}$

SOLUTION : (d

Potential at point P, $V = \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$

Since surface charge densities are equal to one another

i.e., $0_a = 0_b = 0_c$

$$Q_a : Q_b : Q_c :: a^2 : b^2 : c^2$$

$$Q_a = \left\{\frac{a^2}{a^2 + b^2 + c^2}\right\} Q$$

$$Q_b = \left\{\frac{b^2}{a^2 + b^2 + c^2}\right\} Q$$

$$Q_c = \left\{\frac{c^2}{a^2 + b^2 + c^2}\right\} Q$$

$$V = \frac{Q}{4\pi \in 0}\left[\frac{(a+b+c)}{a^2 + b^2 + c^2}\right]$$

11. Two electric dipoles, A, B with respective dipole

moments $\vec{d}_A = -4qa\,\hat{\imath}$ and $\vec{d}_B = -2qa\,\hat{\imath}$ are placed on

the x - axis with a separation R, as shown in the figure

$\rightarrow$ R $-$

AB x

The distance from A at which both of them produce the

same potential is: [10 Jan. 2019 I]

(a) $\frac{R}{\sqrt{2}+1}$ (b) $\frac{\sqrt{2}R}{\sqrt{2}+1}$ (c) $\frac{R}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}R}{\sqrt{2}-1}$

SOLUTION : (d)

Let at a distance ‘x’ from point B, both the dipoles produce same potential

$\rightarrow \mathrm{R} \leftarrow$

4qa 2qa

$$\frac{4qa}{(R+x)} = \frac{2qa}{(x^2)}$$

$$\Rightarrow \sqrt{2x} = R + x \Rightarrow x = \frac{R}{\sqrt{2}-1}$$

Therefore distance from A at which both ofthem produce

the same potential

$$= \frac{R}{\sqrt{2}-1} + R = \frac{\sqrt{2}R}{\sqrt{2}-1}$$

12. Consider two charged metallic spheres S_1 and S_2 ofradii R_1 and R_2, respectively. The electric fields E_1 (on S_1) and E_2 (on S_2) on their surfaces are such that $E_1 l E_2 = R_1 l R_2$. Then the ratio $V_1(\text{on } S_1)\, l V_2(\text{on} S_2)$ of the electrostatic potentials on each sphere is: [8 Jan. 2019 II]

(a) $R_1 l R_2$ (b) $(R_1 l R_2)^2$(c) $(R_2 l R_1)$ (d) $\left(\frac{R_1}{R_2}\right)^3$

SOLUTION : . (b)

Electric field at a point outside the sphere is given by

$$E = \frac{1Q}{4\pi\in 0 r^2} \text{ But } p = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$E = \frac{pR^3}{3 \in 0 r^2}$$

At surface $r = R$ $E = \frac{pR^3}{3\in 0}$

Let p_1 and p_2 are the charge densities oftwo sphere.

$$E_1 = \frac{pR_1}{3\varepsilon_0} \text{ and } E_2 = \frac{p_2 R_2}{3\varepsilon_0}$$

$$\frac{E_1}{E_2} = \frac{p_1 R_1}{p_2 R_2} = \frac{R_1}{R_2}$$

This gives $p_1 = p_2 = p$

Potential at a point outside the sphere

$$V = \frac{1Q}{4\pi\varepsilon_0 r} = \frac{pR^3}{3\varepsilon_0 r}\left(\because p = \frac{Q}{\frac{4}{3}\pi R^3}\right)$$

At surface, $r = R$

$$V = \frac{pR^2}{3\varepsilon_0} \text{ so, } V_1 = \frac{pR_1^2}{3\varepsilon_0} \text{ and } V_2 = \frac{pR_2^2}{3\varepsilon_0}$$

$$\frac{V_1}{V_2} = (\quad)(\quad)^2$$

13. Three concentric metal shells A, B and C of respective radii a, b and $c (a < b < c)$ have surface charge densities $+0, -0$ and $+0$ respectively. The potential of shell B is:

[2018]

(a) $\in \underline{o}0 \left[\frac{a^2-b^2}{a} + c\right]$ (b) $\in \underline{o}0 \left[\frac{a^2-b^2}{b} + c\right]$

(c) $\in \underline{o}0 \left[\frac{b^2-c^2}{b} + a\right]$ (d) $\in \underline{o}0 \left[\frac{b^2-c^2}{c} + a\right]$

SOLUTION : (b)

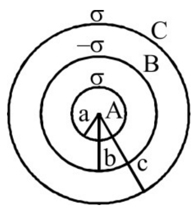
Potential outside the shell, $V_{\text{outside}} = \underline{KQ}$

r

where r is distance ofpoint from the centre of shell

Potential inside the shell, $V_{\text{inside}} = \frac{KQ}{R}$

where R" is radius ofthe shell



$$V_B = \underline{Kq_A} + \frac{Kq_B}{} + \frac{Kq_C}{}$$

$r_b\ r_b\ r_c$

$$v_B = \frac{1}{4\pi \in 0}\left[\frac{04\pi a^2}{b} - \frac{04\pi b^2}{b} + \frac{04\pi c^2}{c}\right]$$

$$V_B = \in \underline{\sigma}0\left[\frac{a^2 - b^2}{b} + c\right]$$

14. There is a uniform electrostatic field in a region. The potential at various points on a small sphere centred at P, in the region, is found to vary between in the limits 589.0 V to 589.8 V. What is the potential at a point on the sphere whose radius vector makes an angle of60° with the direction ofthe field? [Online April 8, 2017]

(a) 589.5 V (b) 589.2V (c) 589.4V (d) 589.6V

SOLUTION : (c)

Potential gradient is given by,

$$\Delta V = E.d$$

$$0.8 = Ed(\max)$$

$$\Delta V = Ed\cos\theta = 0.8\times\cos 60 = 0.4$$

Hence, maximum potential at a point on the sphere

$$= 589.4V$$

15. Within a spherical charge distribution of charge density $p(r)$, N equipotential surfaces ofpotential VO, $V_0 + \Delta V, V_0 + 2\Delta V, \ldots\ldots\ldots V_0 + N\Delta V (\Delta V > 0)$, are drawn and haveincreasing radii $r_0, r_1, r_2\ r_N$, respectively. If thedifference in the radii ofthe surfaces is constant for allvalues ofV_0 and ΔV then: [Online Apri110, 2016]

(a) $p(r) = \text{constant}$ (b) $p(r) \propto \frac{1}{r^2}$

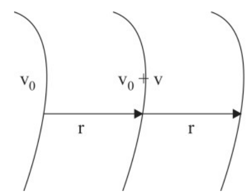
(c) $p(r) \propto \frac{1}{r}$ (d) $p(r) \propto r$

SOLUTION : . (c)

As we know electric field, $E = \frac{-dv}{dr} E = \text{constant}$ dv and dr same

$$E = \frac{K\varphi}{r^2} = c$$

$$\Rightarrow (|) \propto r^2 + 2\Delta v$$



$$\varphi = \int_0^r p\,4\pi r^2 dr \Rightarrow p \propto \frac{1}{r}$$

16. The potential (in volts) ofa charge distribution is given by $V(z) = 30 - 5z^2 \text{for} |z| \leq lm\ \ V(z) = 35 - 10|z| \text{for} |z| \geq lm.$ V(z) does not depend on x and y. If this potential isgenerated by a constant charge per unit volume p_0 (inunits of ε_0) which is spread over a certain region, thenchoose the correct statement. [Online Apri19, 2016]

(a) $p_0 = 20\varepsilon_0$ in the entire region

(b) $p_0 = 10\varepsilon_0 \text{for} |z| \leq 1m$ and $p_0 = 0$ elsewhere

(c) $p_0 = 20\varepsilon_0 \text{for} |z| \leq 1m$ and $p_0 = 0$ elsewhere

SOLUTION : . (b)

$$\Sigma_1 = \frac{-dv}{dr} = 10|z|$$

$$\Sigma_2 = \frac{-dv}{dr} = 10 \text{ (constant: E)}$$

The source is an infinity large non conducting thick plate ofthickness 2 m.

$$10Z \cdot 10A = \frac{p \cdot A \propto Z}{\varepsilon_0}$$

$r_0 = 10e_0$ for $|z| \leq 1m.$

17. Auni(d)p40εintheentire region Rhaspotential $V_{(h}$(measuredwithrespectto∞)onitssurface.Forthis speretheequipotentia1 surf aces withpotentials $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then [2015]

(a) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(b) $2R = R_4$

(c) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(d) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

SOLUTION : . (a)

We know, $V_0 = \frac{Kq}{R} = V$ surface

Now, $V_i = \frac{Kq}{2R^3}(3R^2 - r^2)$ [For $r < R$] At the centre ofsphare $r = 0$. Here

$$V = \frac{3}{2}V_0$$

Now, $\frac{5}{4}\frac{Kq}{R} = \frac{Kq}{2R^3}(3R^2 - r^2)$

$$\Rightarrow R_2 = \frac{R}{\sqrt{2}}$$

$$\frac{3}{4}\frac{Kq}{R} = \frac{Kq}{R^3}$$

$$\frac{1}{4}\frac{Kq}{R} = \frac{Kq}{R_4}$$

$$R_4 = 4R$$

Also, $R_1 = 0$ and $R_2 < (R_4 - R_3)$

18. An electric field $\vec{E} = (25\hat{i} + 30\hat{j})NC^{-1}$ exists in a region of space. If the potential at the origin is taken to be zero then the potential at $x = 2m, y = 2m$ is : [Online April 11, 2015]

(a) $-110J$ (b) $-140J$ (c) $-120J$ (d) $-130J$

SOLUTION : . (a)

As we know, $E = -$

dx

Potential at the point $x = 2m, y = 2m$ is given by:

$$\int_0^V dV = -\int_0^{2,2}(25dx + 30dy)$$

on solving we get, $V = -110$ volt.

19. Assume that an electric field $\vec{E} = 30x^2i$ exists in space. Then the potential difference $V_A - V_O$, where V_o is the potential at the origin and V_A the potential at $x = 2m$ is:

(a) $120J/C$ (b) $-120J/C$ [2014](c) $-80J/C$ (d) 80 J/C

SOLUTION : . (c)

Potential difference between any two points in an electric field is given by,

$$dV = -\vec{E} \cdot \overline{dx}$$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -[10x^3]_0^2 = -80J/C$$

20. Consider a fmite insulated, uncharged conductor placed near a finite positively charged conductor. The uncharged body must have a potential: [Online April 23, 2013]

(a) less than the charged conductor and more than at infinity.

(b) more than the charged conductor and less than at infinity.

(c) more than the charged conductor and more than at infinity.

(d) less than the charged conductor and less than at infinity.

SOLUTION : (a)

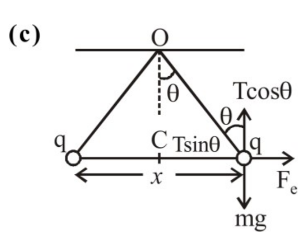
The potential ofuncharged body is less than that of the charged conductor and more than at infinity.

21. Two small equal point charges of magnitude q are suspended from a common point on the ceiling by insulating mass less strings ofequal lengths. They come to equilibrium with each string making angle θfi: om the vertical. If the mass of each charge is m, then the electrostatic potential at the centre oflinejoining them will be $\left(\frac{1}{4\pi\in 0}=k\right)$. [Online April 22, 2013]

(a) $2\sqrt{kmg\tan\theta}$ (b) $\sqrt{kmg\tan\theta}$

(c) $4\sqrt{kmgltan\theta}$ (d) $\sqrt{kmgl\tan\theta}$

SOLUTION :



In equilibrium, $F_e=T\sin\theta$

mg=$T\cos\theta$

$$\tan\theta=\frac{F_e}{mg}=\frac{q^2}{4\pi\in 0x^2\times mg}$$

$$x=\sqrt{\frac{q^2}{4\pi\in 0\tan\theta mg}}$$

Electric potential at the centre ofthe line

$$V=\frac{kq}{x/2}+\frac{kq}{x/2}=4\sqrt{kmg/\tan\theta}$$

22. Apoint charge ofmagnitude $+1\mu C$ is fixed at $(0,0,0)$. An isolated uncharged spherical conductor, is fixed with its center at $(4,0,0)$. The potential and the induced electric field at the centre ofthe sphere is: [Online April 22, 2013]

(a) $1.8\times 10^5 Vand -5.625\times 10^6 V/m$

(b) 0 Vand $0V/m$

(c) $2.25\times 10^5 Vand -5.625\times 10^6 V/m$

(d) $2.25\times 10^5 V$ and $0V/m$

SOLUTION : (c)

$$q=1\mu C=1\times 10^{-6}C$$

$$r=4cm=4\times 10^{-2}m$$

Potential $V=\frac{kq}{r}=\frac{9\times 10^9\times 10^{-6}}{4\times 10^{-2}}=2.25\times 10^5$ V.

Induced electric field $E=-\frac{kq}{r^2}$

$$=\frac{9\times 10^9\times 1\times 10^{-6}}{16\times 10^{-4}}=-5.625\times 10^6 V/m$$

23. A charge of total amount Q is distributed over two concentric hollow spheres ofradii r and $R(R>r)$ such that the surface charge densities on the two spheres are equal. The electric potential at the common centre is

[Online May 19, 2012]

(a) $\frac{1}{4\pi\varepsilon_0}\frac{(R-r)Q}{(R^2+r^2)}$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{(R+r)Q}{2(R^2+r^2)}$

(c) $\frac{1}{4\pi\varepsilon 0}\frac{(R+r)Q}{(R^2+r^2)}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{(R-r)Q}{2(R^2+r^2)}$

SOLUTION : . (c)

Let q_1 and q_2 be charge on two spheres of radius

$||r$ and $1R'$ respectively

As, $q_1 + q_2 = Q$

and $0_1 = 0_2$ [Surface charge density are equal]

$$\frac{q_1}{r\pi r^2} = \frac{q_2}{4\pi R^2}$$

So, $q_1 = \frac{Qr^2}{R^2+r^2}$ and $q_2 = \frac{QR^2}{R^2+r^2}$

Now, potential, $V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r} + \frac{q_2}{R}\right]$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Qr}{R^2+r^2} + \frac{QR}{R^2+r^2}\right]$$

$$= \frac{Q(R+r)}{R^2+r^2}\frac{1}{4\pi\varepsilon_0}$$

24. The electric potential $V(x)$ in a region around the origin is given by $V(x) = 4x^2$ volts. The electric charge enclosed in a cube ofl m side with its centre at the origin is (in coulomb) [Online May 7, 2012]

(a) $8\varepsilon_0$ (b) $-4\varepsilon_0$ (c) 0 (d) $-8\varepsilon_0$

SOLUTION : . (c)

Charges reside only on the outer surface of a conductor with cavity.

25. The electrostatic potential inside acharged spherical ball is given by $\varphi = ar^2 + b$ where r is the distance fi$_{\text{IO}}$m the centre and a, b are constants. Then the charge density inside the ballis: [2011]

(a) $-6a\varepsilon_0 r$ (b) $-24\pi a\varepsilon_0$

(c) $-6a\varepsilon_0$ (d) $-24\pi a\varepsilon_{0r}$

SOLUTION : (c)

Electric field$E = -\frac{d\varphi}{dr} = -2ar$ (i)

By Gauss[1]s theorem $E = \frac{1q}{4\pi 8_0 r^2}$ (ii)

From(i) and(ii),

$$Q = -8\pi\varepsilon_0 ar^3$$

$$\Rightarrow dq = -24\pi\varepsilon_0 ar^2 dr$$

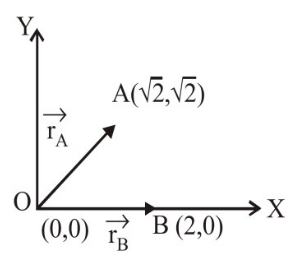
Charge density, p $= \frac{dq}{4\pi r^2 dr} = -6\varepsilon_0 a$

26. An electric charge $10^{-3}\mu$C is placed at the origin $(0,0)$ of $X - Y$ co - ordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and $(2, 0)$ respectively. The potential difference between the points A and B will be [2007]

(a) 4.5 volts (b) 9 volts (c) Zero (d) 2 volt

SOLUTION :

(c)



The distance ofpoint $A(\sqrt{2}, \sqrt{2})$ from the origin,

$$r_A = \sqrt{(\sqrt{2})^2 + (\sqrt{2})^2} = \sqrt{4} = 2\text{units.}$$

The distance ofpoint $B(2,0)$ from the origin,

$$r_B = \sqrt{(2)^2 + (0)^2} = 2 \text{ units.}$$

Now, potential at , due to charge $\theta = 10^{-3}\mu C$

$$V_A = \frac{1}{4\pi \in 0} \cdot \frac{Q}{(r_A)}$$

Potential at B, due to charge $Q = 10^{-3} QCV_B = \frac{1}{4\pi\in 0} \cdot \frac{Q}{(r_B)}$

Potential difference between the points A and B is given by

$$V_A - V_B = \frac{1}{4\pi \in 0} \cdot \frac{10^{-3}}{r_A} - \frac{1}{4\pi \in 0} \cdot \frac{10^{-3}}{r_B}$$

$$= \frac{10^{-3}}{4\pi \in 0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right) = \frac{10^{-3}}{4\pi \in 0}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$= \frac{Q}{4\pi \in 0} \times 0 = 0.$$

27. Charges are placed on the vertices ofa square as shown.

Let $\vec{E}$ be the electric field and V the potential at the

centre. Ifthe charges on A and B are interchanged with

those on D and C respectively, then [2007]

(a) $\frac{\overline{E}}{E}$remains unchanged,

(b)Vchangeschanges,Vremains unchanged

(c) both $\overline{E}$ and V change

(d) $\overline{E}$ and Vremain unchanged

SOLUTION : (a)

As shown in the figure, the resultant electric fields

before and after interchanging the charges will have the

same magnitude, but opposite directions.

As potential is a scalar quantity, So the potential will be

same in both cases.

28. The potential at a point x (measured in μm) due to some

charges situated on the x - axis is given by $V(x) = 20/(x^2 - 4)$

volt. The electric field E at $x = 4\mu m$ is given by [2007]

(a) (10/9) volt/μm and in the +ve$x$ direction

(b) (5/3) volt/μm and in the - ve x direction

(c) (5/3) volt/μm and in the +ve$x$ direction

(d) (10/9) volt/μm and in the - ve x direction

SOLUTION : (a)

Given, potential $V(x) = \frac{20}{x^2-4}$ volt

Electric field $E = -\frac{dV}{dx} = -\frac{d}{dx}\left(\frac{20}{x^2-4}\right)$

$$\Rightarrow E = +\frac{40x}{(x^2-4)^2}$$

At $x = 4\mu m$,

$$E = +\frac{40\times 4}{(4^2-4)^2} = +\frac{160}{144} = +\frac{10}{9}\text{volt}/\mu m.$$

Positive sign indicates that $\vec{E}$ is in +ve$x$ - direction.

29. Two thin wire rings each havinga radiusR are placed at a

distance d apart with their axes coinciding. The charges

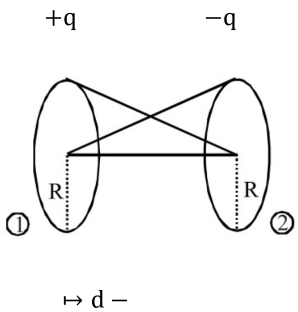
on the two rings are $+q$ and - q. The potential difference

between the centres ofthe two rings is [2005]

(a) $\frac{q}{2\pi\in 0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ (b) $\frac{qR}{4\pi\in\dagger]d^2}$

(c) $\frac{q}{4\pi\in 0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$ (d) zero

SOLUTION : (a)



Potential at the center ofring ofcharge $+q$ = potential due to iteself+ potential due to other ring of charge $-q$.

$$\Rightarrow V_1=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

Potential at the centre ofring ofcharge $-q$ = potential due to itself + potential due to other ring ofcharge $+q$.

$$\Rightarrow V_2=\frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{R}+\frac{q}{\sqrt{R^2+d^2}}\right]$$

$$\Delta V=V_1-V_2$$

$$=\frac{1}{4\pi\varepsilon_0}\left[\frac{q}{R}+\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

$$=\frac{1}{2\pi\varepsilon_0}\left[\frac{q}{R}-\frac{q}{\sqrt{R^2+d^2}}\right]$$

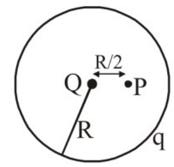
30. A thin spherical conducting shell ofradius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P, a distance $\frac{R}{2}$ from the centre of the shell is [2003]

(a) $\frac{2Q}{4\pi\varepsilon_o R}$ (b) $\frac{2Q}{4\pi\varepsilon_o R}-\frac{2q}{4\pi\varepsilon_o R}$

(c) $\frac{2Q}{4\pi\varepsilon_o R}+\frac{q}{4\pi\varepsilon_o R}$ (d) $\frac{(q+Q)2}{4\pi\varepsilon_o R}$

SOLUTION : . (c)

Electric potential due to charge Q at point P is



$$V_1=\frac{1}{4\pi\varepsilon_o}\frac{Q}{R/2}=\frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}$$

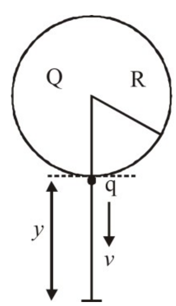
Electric potential due to charge q inside the shell is

$$V_2=\frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

The net electric potential at point P is

$$V=V_1+V_2=\frac{1}{4\pi\varepsilon_o}\frac{2Q}{R}+\frac{1}{4\pi\varepsilon_o}\frac{q}{R}$$

31. A solid sphere ofradiusR carries a charge $Q+q$ distributeduniformaly over its volume. Avery small point like piece ofit ofmass m gets detached from the bottom ofthe sphereand falls down verticallyunder gravity. This piece carriescharge q. Ifit acquires a speed v when it has fallen througha vertical height y(see figure), then: (assume the remainingportion to be spherical). [Sep. 05, 2020 (I)]



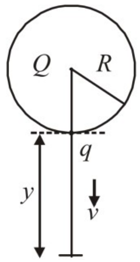
(a) $v^2=y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym}+g\right]$ (b) $v^2=y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m}+g\right]$

(c) $v^2=2y\left[\frac{QqR}{4\pi\varepsilon_0 (R+y)^3 m}+g\right]$(d) $v^2=2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m}+g\right]$

SOLUTION : . (d)

By using energy conservation,

$$\Delta KE+(\Delta PE)_{\text{Electro}}+(\Delta PE)_{\text{gravitationa1}}=0$$



$$\frac{1}{2}mV^2+\left(k\frac{Qq}{R+y}-k\frac{Qq}{R}\right)+(-mgy)=0$$

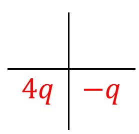
$$\Rightarrow\frac{1}{2}mV^2=mgy+kQq(\quad)(\quad)$$

$$\Rightarrow V^2=2gy+\frac{2kQq}{m}\frac{y}{R(R+y)}$$

or, $V^2=2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m}+g\right]$

32. A two point charges $4q$ and $-q$ are fixed on the x - axis at $x=-\frac{d}{2}$ and $=\frac{d}{2}$, respectively. Ifa third point charge ‘ q’ is taken ffom the origin to $x=d$ along the semicircle asshown in the figure, the energy ofthe charge will:

[Sep. 04, 2020 (I)]



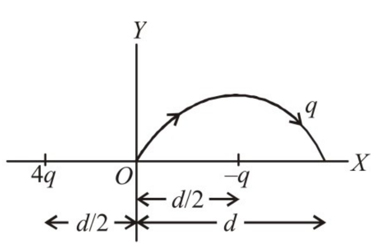
(a) increase by $\frac{3q^2}{4\pi\varepsilon_0 d}$ (b) increase by $\frac{2q^2}{3\pi\varepsilon_0 d}$

(c) decrease by $\frac{q^2}{4\pi\varepsilon_0 d}$ (d) decrease by $\frac{4q^2}{3\pi\varepsilon_0 d}$

SOLUTION : . (d)

Change in potential energy, $\Delta u=q(V_f-V_i)$

Potential of $-q$ is same as initial and final point ofthe path.



$$\Delta u=q\left(\frac{k4q}{3d/2}-\frac{k4q}{d/2}\right)=-\frac{4q^2}{3\pi\varepsilon_0 d}$$

- ve sign shows the energy ofthe charge is decreasing.

33. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to: [Sep. 03, 2020 (II)]

(a) 1: 2 (b) 10: 7 (c) 2: 1 (d) 5: 7

SOLUTION : (c)

According to work energy theorem, gain in kinetic energy is equal to work done in displacement ofcharge.

$$\frac{1}{2}mv^2=q\Delta V$$

Here, $\Delta V=$ potential difference between two positions of charge q.

For same q and $\Delta V v\propto\frac{1}{\sqrt{m}}$

Mass ofhydrogen ion $m_H=1$ Mass ofhelium ion $m_{He}=4$

$$\frac{v_H}{v_{He}}=\sqrt{\frac{4}{1}}=2:1.$$

34. In free space, a particle A ofcharge 1 μC is held fixed at a point P. Another particle B ofthe same charge and mass 4 μg is kept at a distance ofl mm from P. IfB is released, then its velocity at a distance of9 mm from P is:

[Take $\frac{1}{4\pi\varepsilon_0}=9\times10^9 Nm^2C^{-2}$] [10 April 2019 II]

(a) 1.0ms (b) 3.0×10^4m/s

(c) 2.0×10^3m/s (d) 1.5×10^2m/s

SOLUTION : (c)

Using conservation of energy

$$U_i = U_F + \frac{1}{2}mv^2$$

$$\frac{kq_1q_2}{r_1} = \frac{kq_1q_2}{r_2} + \frac{1}{2}mv^2$$

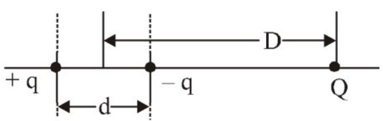
$$\Rightarrow \frac{1}{2}mv^2 = kq_1q_2\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$v^2 = \frac{2kq_1q_2}{m}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$= \frac{2\times 9\times 10^9\times 10^{-1}}{4\times 10^{-6}\times 10^{-3}}\left\{1 - \frac{1}{9}\right\} = 4\times 10^{+6}$$

$$v = 2\times 10^3 \mathrm{m/s}$$

35. A system of three charges are placed as shown in the figure:



If $D >> d$, the potential energy ofthe system is best given by [9 April 2019 I]

(a) $\frac{1}{4\pi\epsilon_0}\left[\frac{-q^2}{d} \frac{-qQd}{2D^2}\right]$ (b) $\frac{1}{4\pi\in 0}\left[\frac{-q^2}{d} + \frac{2qQd}{D^2}\right]$

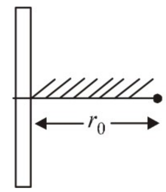
(c) $\frac{1}{4\pi\in 0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$ (d) $\frac{1}{4\pi\in 0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

SOLUTION : . (d)

$$U = \frac{1}{4\pi\in 0}\left[\frac{q(-q)}{d} + \frac{qQ}{\left(D+\frac{d}{2}\right)} + \frac{(-q)Q}{\left(D-\frac{d}{2}\right)}\right]$$

$$= \frac{1}{4\pi\in 0}\left\{\frac{q^2qQd}{dD^2}\right\}$$, Ignoring $\frac{d^2}{4}$

36. Apositive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function ofinstantaneous distance r from line charge, is proportionalto: [8 April 2019 II]



(a) $v \propto e^{+r/r_0}$ (b) $v \propto \sqrt{ln(\ \ )(\ \ )}$

$(\ \ )(\ \ )$

(c) $v \propto ln\left(\frac{}{r_0}\right)$ (d) $v \propto \left(\frac{}{r_0}\right)$

SOLUTION : (b)

Using, $[K+U]_i = [K+U]_f$

or $0 + Vq = mv^2 + v'q$

or $mv^2 = (V - V')q$

$$= -q\int_{r_0}^{r} E\,dr = q\int_{r_0}^{r}\frac{\lambda}{2\pi\in 0r}dr = \frac{\lambda q}{2\pi\in 0}(\ \ )(\ \ )$$

$$\Rightarrow v \propto \sqrt{\ln\frac{r}{r_0}}$$

39. Four equal point charges Q each are placed in the xy plane at $(0,2)$, $(4,2)$, $(4,-2)$ and $(0,-2)$. The work required to put a fifth charge Q at the origin of the coordinate system will be: [10 Jan. 2019 II]

(a) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}$

SOLUTION : (b)

Potential at origin

$$v=\frac{KQ}{2}+\frac{KQ}{2}+\frac{KQ}{\sqrt{20}}+\frac{KQ}{\sqrt{20}}$$

and potential at $\infty=0=KQ\left(1+\frac{1}{\sqrt{5}}\right)$

Work required to put a fifth charge Q at origin W =

$$VQ=\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$$

40. Statement 1 : No work is required to be done to move a test charge between any two points on an equipotential surface.

Statement 2 : Electric lines of force at the equipotential surfaces are mutually perpendicular to each other.

[Online April 25, 2013]

(a) Statement 1 is true, Statement2 is true, Statement2 is the correct explanation ofStatement 1.

(b) Statement 1 is true, Statement2 is true, Statement2 is not the correct explanation of Statement 1.

(c) Statement 1 is true, Statement2 is false.

(d) Statement 1 is false, Statement2 is true.

SOLUTION : . (c)

The work done in moving a charge along an equipotential surface is always zero.

The direction of electric field is perpendicular to the equipotential surface or lines.

41. An insulating solid sphere of radius R has a uniformly positive charge density p. As a result ofthis uniform charge distribution there is a fmite value of electric potential at the centre ofthe sphere, at the surface ofthe sphere and also at a point outside the sphere. The electric potential at infinite is zero. [2012]

Statement - 1 When a charge q is taken from the centre to the surface of the sphere its potential energy changes by $\frac{q\text{p}}{3\varepsilon_0}$

Statement −2 The electric field at a distance $r(r<R)$ from the centre ofthe sphere is $\frac{\text{p}r}{3\varepsilon_0}$

(a) Statement 1 is true, Statement2 is true; Statement2 is not the correct explanation of statement 1.

(b) Statement 1 is true Statement2 is false.

(c) Statement 1 is false Statement 2 is true.

(d) Statement 1 is true, Statement2 is true, Statement2 is the correct explanation ofStatement 1

SOLUTION : (c)

The potential energy at the centre ofthe sphere $U_c=\frac{3}{2}\frac{KQq}{R}$

The potential energy at the surface ofthe sphere $U_s=\frac{KqQ}{R}$

Now change in the energy $\Delta U=U_c-U_s$

$=\frac{KQq}{R}\left[\frac{3}{2}-1\right]=\frac{KQq}{2R}$ Where $Q=\text{p}.V=\text{p}.\frac{4}{3}\pi R^3$

$$\Delta U=\frac{2K\pi R^3\text{p}q}{3R}$$

$$\Delta U=\frac{2}{3}\times\frac{1}{4\pi\in 0}\frac{\pi R^3\text{p}q}{R}$$

$$\Delta U=\frac{R^2 pq}{6\in 0}$$

Using Gauss' s law

$$\int \vec{E}\cdot\overline{dA}=\frac{q_{en}}{E_0}=\frac{\beta\times\frac{4}{3}\pi R^3}{E_0}$$

$$\Rightarrow\int E\,dA(\cos\theta)=\frac{\beta\times 4\pi R^3}{3E_0}$$

$$\Rightarrow E(4\pi R^2)=\beta\times\frac{4}{3}\pi R^3\times\frac{1}{E_0}$$

$$\Rightarrow E=\frac{\beta r}{3E_0}(r<R)$$

42. Two positive charges of magnitude 'q' are placed, at theends ofa side (side 1) ofa square ofside '2a'. Two negativecharges ofthe same magnitude are kept at the other corners.Starting from rest, ifa charge Q moves from the middle ofside 1 to the centre of square, its kinetic energy at thecentre of square is [2011 RS]

(a) zero

(b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

(d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

SOLUTION : (d)

Initial potential ofthe charge, $V_A=\frac{2kq}{a}-\frac{2kq}{a\sqrt{5}}$

$$\Rightarrow V_A=\frac{1}{4\pi E}\frac{2q}{a}\left(1-\frac{1}{\sqrt{5}}\right)$$

(Here potential due to each $q=\frac{kq}{a}$ and potential due to each $-q=\frac{-kq}{a\sqrt{5}}$)

Final potential ofthe charge

$$V_B=0$$

(Point B is equidistant from all the four charges) Using work energy theorem,

$$(W_{AB})_{\text{electric}}=Q(V_A-V_B)$$

$$=\frac{2qQ}{4\pi E_0 a}\left[1-\frac{1}{\sqrt{5}}\right]$$

$$=\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{2Qq}{a}\left[1-\frac{1}{\sqrt{5}}\right]$$

43. Two points P and Q are maintained at the potentials of10 V and - 4 V, respectively. The work done in moving 100 electrons from P to Q is: [2009]

(a) 9.60×10^{-17}J

(b) -2.24×10^{-16}J

(c) 2.24×10^{-16}J

(d) -9.60×10^{-17}J

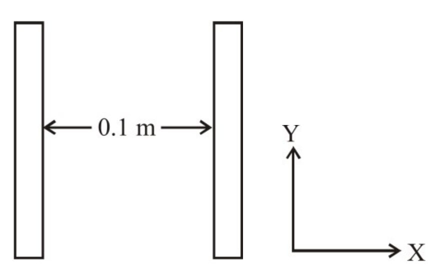
SOLUTION : . (c)

Work done, $W_{PQ}=q(V_Q-V_P)$

$$=(-100\times 1.6\times 10^{-19})(-4-10)$$

$$=+2.24\times 10^{-16}\text{J}$$

44. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2-V_1=20$ V. (i. e., plate 2 is at a higher potential). The plates are separated by $d=0.1$m and can be treated as infinitely large. An electron is released from rest on the inner surface ofplate 1. What is its speed when it hits plate 2? ($e=1.6\times 10^{-19}$C, $m_e=9.11\times 10^{-31}$kg) [2006]



(a) 2.65×10^{6}m/s

(b) 7.02×10^{12}m/s

(c) 1.87×10^{6}m/s

(d) 32×10^{-19}m/s

SOLUTION : (a)

Gain in kinetic energy = work done by potential

Difference $eV=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{\frac{2eV}{m}}$

$$=\sqrt{\frac{2\times 16\times 10^{-19}\times 20}{91\times 10^{-31}}}=2.65\times 10^6 \mathrm{m/s}$$

45. A charged particle 'q' is shot towards another charged particle 'Q' which is fixed, with a speed 'v'. It approaches 'Q' upto a closest distance r and then returns. If q weregiven a speed of $2v$' the closest distances of approachwould be [2004]

(a) $rl2$ (b) $2r$ (c) r (d) $rl4$

SOLUTION : . (d)

$$\frac{1}{2}mv^2=\frac{kQq}{r}$$

$$\Rightarrow \frac{1}{2}m(2v)^2=\frac{kqQ}{r'}\Rightarrow r'=\frac{r}{4}$$

46. On moving a charge of20 coulomb by2 cm, 2 J ofwork is done, then the potential difference between the points is [2002]

(a) 0.1V (b) 8V (c) 2V (d) 0.5V

SOLUTION : (a)

By using $W=q(V_B-V_A)$

$$V_B-V_A=\frac{2\mathrm{J}}{20\mathrm{C}}=0.1\mathrm{J/C}=0.1\mathrm{V}$$

47. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal ofone is connected to the negative terminal ofthe other. The final energy ofthis configuration is: [Sep. 05, 2020 (I)]

(a) $\frac{25}{6}CV^2$ (b) $\frac{3}{2}CV^2$

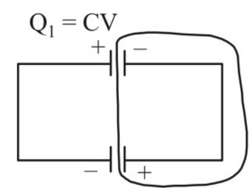
(c) zero (d) $\frac{9}{2}CV^2$

SOLUTION : . (b)

When capacitors C and $2C$ capacitance are charged to V and 2V respectively.

$$Q_1=CV\ Q_2=2C\times 2V=4CV$$

When connected in parallel



$$Q_2=4CV$$

By conservation of charge

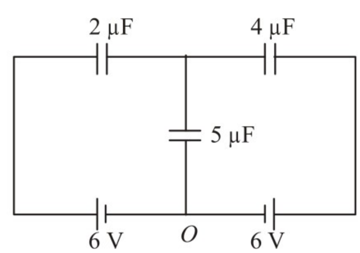
$$4CV-CV=(C+2C)V_{\text{common}}$$

$$V_{\text{common}}=\frac{3CV}{3C}=V$$

Therefore final energy ofthis configuration,

$$U_f=\left(\frac{1}{2}CV^2+\frac{1}{2}\times 2CV^2\right)=\frac{3}{2}CV^2$$

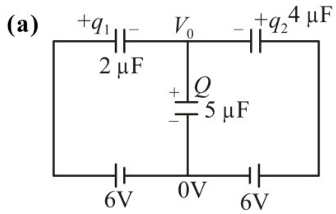
48. In the circuit shown, charge on the 5 μF capacitor is: [Sep. 05, 2020 (II)]



(a) 18.00μC (b) 10.90μC

(c) 16.36μC (d) 5.45μC

SOLUTION :



Let q_1 and q_2 be the charge on the capacitors of2μF and4μF.

Then charge on capacitor of5μF $\quad Q = q_1 + q_2$

$$\Rightarrow 5V_0 = 2(6 - V_0) + 4(6 - V_0)$$

$$\Rightarrow 5V_0 = 12 - 2V_0 + 24 - 4V_0$$

$$\Rightarrow 11V_0 = 36 \Rightarrow V_0 = \frac{36}{11}V$$

$$\Rightarrow Q = 5V_0 = \frac{180}{11}\mu C$$

49. A capacitor C is fully charged with voltage V_0. After disconnecting the voltage source, it is connected in parallel with another uncharged capacitor of capacitance $\frac{C}{2}$. The energy loss in the process after the charge is distributed between the two capacitors is: [Sep. 04, 2020 (II)]

(a) $\frac{1}{2}CV_0^2$ (b) $\frac{1}{3}CV_0^2$

(c) $\frac{1}{4}CV_0^2$ (d) $\frac{1}{6}CV_0^2$

SOLUTION : . (d)

When two capacitors with capacitance C_1 and C_2 at potential V_1 and V_2 connected to each other by wire, charge begins to flow from higher to lower potential till they acquire common potential. Here, some loss ofenergytakes place which is given

Heat loss, $H = \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$

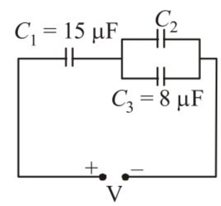
In the equation, put $V_2 = 0, V_1 = V_0$

$$C_1 = C, C_2 = \frac{C}{2}$$

Loss of heat $= -2\left(\frac{C+^{C}}{2}\right)C \times \frac{C}{2} - (V_0 - 0)^2 = \frac{C}{6}V_0^2$

$$H = \frac{1}{6}CV_0^2$$

50. In the circuit shown in the figure, the total charge is 750 μCand the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is: [Sep. 03, 2020 (I)]

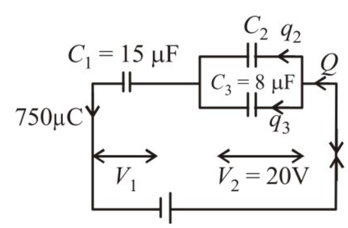


(a) 450 μC (b) 590 μC

(c) 160 μC (d) 650 μC

SOLUTION : . (b)

According to question, $Q = 750\mu C = q_2 + q_3$



Capacitors C_2 and C_3 are in parallel hence, Voltage across $C_2 =$ voltage across $C_3 = 20$V Change on capacitor C_3,

$$q_3 = C_3 \times V_3 = 8 \times 20 = 160\mu C$$

$$q_2 = 750\mu C - 160\mu C = 590\mu C$$

51. A5 μF capacitor is charged fullyby a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5μF capacitor. Ifthe energy change during the charge redistribution is $\frac{X}{100}$J then value of$X$ to the nearest integer is . [NA Sep. 02, 2020 (I)]

SOLUTION : (4)

Given, $C_1 = 5\mu\text{F}$ and $V_1 = 220$ Volt

When capacitor C_1 fully charged it is disconnected from the supply and connected to uncharged capacitor C_2.

$$C_2 = 2.5\mu\text{F}, V_2 = 0$$

Energy change during the charge redistribution,

$$\Delta U = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$= \frac{1}{2} \times \frac{5 \times 2..5}{(5 + 25)}(220 - 0)^2 \mu\text{J}$$

$$= \frac{5}{2 \times 3} \times 22 \times 22 \times 100 \times 10^{-6}\text{J}$$

$$= \frac{5 \times 11 \times 22}{3} \times 10^{-4}\text{J} = \frac{55 \times 22}{3} \times 10^{◁}\text{J}$$

$$= \frac{1210}{3} \times 10^{-4}\text{J} = \frac{1210}{3} \times 10^{-3}\text{J} = 4 \times 10^{-2}\text{J}$$

According to questions, $\frac{x}{100} = 4 \times 10^{-2}$

$$x = 4$$

52. A $10\mu\text{F}$ capacitor is fully charged to a potential difference of50 V. After removing the source voltage it is connected to an uncharged capacitor in parallel. Now the potential difference across them becomes 20 V. The capacitance of the second capacitor is: [Sep. 02, 2020 (II)]

(a) 15 [ffi (b) 30[ffi

(c) 20 [ffi (d) 10[ffi

SOLUTION : . (a)

Given,Capacitance of capacitor, $C_1 = 10\mu\text{F}$

Potential difference before removing the source voltage,

$$V_1 = 50\text{V}$$

If C_2 be the capacitance of uncharged capacitor, then

common potential is $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$

$$\Rightarrow 20 = \frac{10 \times 50 + 0}{20 + C} \Rightarrow C = 15\mu F$$

53. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 μF. When these capacitors are individually connected to a voltage source of 1 V, the energy stored in the capacitor C_2 is 4 times that of C_1. If these capacitors are connected in series, their effective capacitance will be: [8 Jan. 2020 I]

(a) 4.2 ↓Ψ (b) 3.2 ↓Ψ

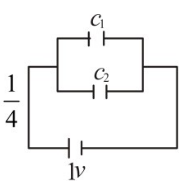
(c) 1.6μF (d) 8.4μF

SOLUTION : . (c)

In parallel combination, $C_{eq} = C_1 + C_2 = 10\mu F$

When connected across 1 V battery, then

$$\frac{U_1}{U_2} = \frac{\left(\frac{1}{2}C_1V^2\right)}{\left(\frac{1}{2}C_2V^2\right)} = \frac{1}{4} \Rightarrow \frac{C_1}{C_2} = \frac{1}{4}$$

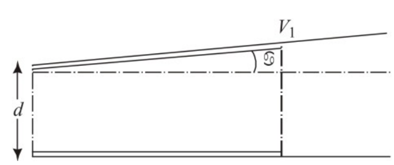


$C_2 = 8\mu$Fand $C = 2\mu F$

Now C_1 and C_2 are connected in series combination,

Cequivalent $= \frac{C_1C_2}{C_1 + C_2} = \frac{2 \times 8}{2 + 8} = \frac{16}{10} = 16\mu F$

54. A capacitor is made oftwo square plates each of side a' making a very small angle a between them, as shown in figure. The capacitance will be close to: [8 Jan. 2020 II]



$a V_2$

(a) $\frac{\epsilon a^2{}_0}{d}\left(1-\frac{\alpha a}{2d}\right)$

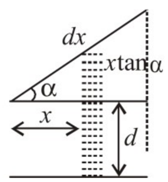
(b) $\frac{\epsilon a^2{}_0}{d}\left(1-\frac{\alpha a}{4d}\right)$

(c) $\frac{\epsilon a^2{}_0}{d}\left(1+\frac{\alpha a}{d}\right)$

(d) $\frac{\epsilon a^2{}_0}{d}\left(1-\frac{3\alpha a}{2d}\right)$

SOLUTION :

(a)



$x = 0 \ \to a$

Consider an infinitesimal strip ofcapacitor ofthickness dx at a distance x as shown.

Capacitance ofparallel plate capacitor ofarea A is given by $C = \frac{\varepsilon_0 A}{t}$

[Here t = seperation between plates]

So, capacitance ofthickness dx will be $dC = \frac{\varepsilon_0 a dx}{d + x\tan\alpha}$

Total capacitance of system can be obtained by integrating with limits x = 0 to x = a

$$C_{eq} = \int \ d\,C = a\varepsilon_0 \int_{x=0}^{x=a} \frac{dx}{x\tan\alpha + d}$$

[By Binomial expansion]

$$\Rightarrow C_{eq} = \frac{a\varepsilon_0}{d}\int_0^a \left(1 - \frac{x\tan\alpha}{d}\right)dx = \frac{a\varepsilon_0}{d}(\quad)_0(\quad)^a$$

$$\Rightarrow C_{eq} = \frac{a^2\varepsilon_0}{d} = \left(1 - \frac{a\tan\alpha}{2d}\right) = \frac{\varepsilon_0 a^2}{d}\left(1 - \frac{\alpha a}{2d}\right)$$

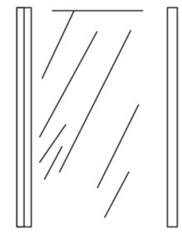
55. Aparallel plate capacitor has plates of area A separated by distance 'd' between them. It is filled with a dielectric which has a dielectric constant that varies as $k(x) = K(1 + \alpha x)$ where 'x' is the distance measured from one of the plates. If $(\alpha d) << 1$, the total capacitance ofthe system is best given by the expression: [7 Jan. 2020 I]

(a) $\frac{AK\in_0}{d}\left(1+\frac{\alpha d}{2}\right)$

(b) $\frac{A\in_0 K}{d}\left(1+\left(\frac{\alpha d}{2}\right)^2\right)$

(c) $\frac{A\in_0 K}{d}\left(1+\frac{\alpha^2 d^2}{2}\right)$

(d) $\frac{AK\in_0}{d}(1+\alpha d)$



SOLUTION : (a)

Given, $K(x) = K(1 + \alpha x)$

Capacitance of element, $C_{el} = \frac{K\varepsilon_0 A}{dx}$

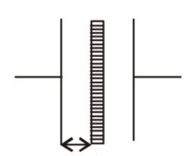
$$\Rightarrow C_{el} = \frac{\varepsilon_0 K(1 + \text{o}x)A}{dx}$$

$$\int \ d\left(\frac{1}{C}\right) = \frac{1}{C_{el}} = \int_0^d \left(\frac{dx}{\varepsilon_0 KA(1+\alpha x)}\right)$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha}[ln(1 + \text{Gi}\mathcal{K})]_0^d$$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 KA\alpha} ln(1 + \alpha d)[\alpha d << 1]$$

$$= \frac{1}{\varepsilon_0 K4\alpha}\left[\alpha d - \frac{\alpha^2 d^2}{2}\right]$$



$$= \frac{1}{\varepsilon_0 K4}\left[1 - \frac{\alpha d}{2}\right]$$

$$C = \frac{\varepsilon_0 K4}{d\left(1 - \frac{\alpha d}{2}\right)} \Rightarrow C = \frac{\varepsilon_0 KA}{d}\left(1 + \frac{\alpha d}{2}\right)$$

56. A $60 pF$ capacitor is fully charged by a 20 V supply. It is then disconnected from the supply and is connected to another uncharged 60 pF capacitor in parallel. The electrostatic energy that is lost in this process by the time the charge is redistributed between them is (in nJ)

[NA 7 Jan. 2020 II]

SOLUTION : . (6)

In the first condition, electrostatic energy is

$$U_j = \frac{1}{2}CV_0^2 = \frac{1}{2} \times 60 \times 10^{-12} \times 400 = 12 \times 10^{-9} J$$

In the second condition $U_F = \frac{1}{2}C'V'^2$

$$U_f = \frac{1}{2}2C.\left(\frac{V_0}{2}\right)^2 \left(\because C' = 2C, V' = \frac{V_0}{2}\right)$$

$$= \frac{1}{4} \times 60 \times 10^{-12} \times (20)^2 = 6 \times 10^9 J$$

Energy lost $= U_i - U_f = 12 \times 10^9 J - 6 \times 10^9 J = 6nJ$

57. The parallel combination oftwo air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference ofthe combined system is: [9 April 2020 II]

(a) $\frac{n\mathrm{V}}{\mathrm{K}+n}$ (b) V

(c) $\frac{\mathrm{V}}{\mathrm{K}+n}$ (d) $\frac{(n+1)\mathrm{V}}{(\mathrm{K}+n)}$

SOLUTION : . (d)

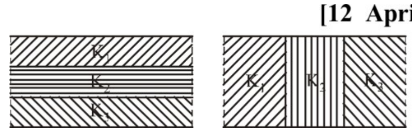
$$V \not\subset = \frac{C\overline{V} + (nC)V}{kC + nC}$$

$$\frac{(n+1)V}{k+n}$$

58. Two identical parallel plate capacitors, ofcapacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, ofequal thickness and dielectric constants K_1 , K_2 and K_3. The first capacitors is filled as shown in Fig. I, and the second one is filled as shown in Fig. II.Ifthese two modified capacitors are charged by the samepotential V, the ratio ofthe energy stored in the two, wouldbe (E_1 refers to capacitors (I) and E_2 to capacitors (II):

2019 I]

[12 April



(1) (11)

(a) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2}$

(b) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2}{K_1K_2K_3}$

(c) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(d) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2}{9K_1K_2K_3}$

SOLUTION : . (c)

$$\frac{1}{C_1} = \frac{d/3}{k_1\varepsilon_0 A} + \frac{d/3}{k_2\varepsilon_0 A} + \frac{d/3}{k_3\varepsilon_0 A}$$

$$\text{or } C_1 = \frac{3k_1k_2k_3\varepsilon_0 A}{d(k_1k_2+k_2k_3+k_3k_1)}$$

$$C_2 = \frac{k_1\varepsilon_0(A/3)}{d} + \frac{k_2\varepsilon_0(A/3)}{d} + \frac{k_3\varepsilon_0(A/3)}{d}$$

$$= \frac{(k_1+k_2+k_3)\varepsilon_0 A}{3d}$$

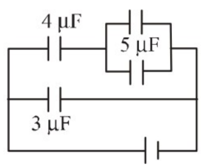
$$\frac{U_1}{U_2} = \frac{\frac{1}{2}C_1V^2}{\frac{1}{2}C_2V^2}$$

$$= \frac{E_1}{E_2} = \frac{9k_1k_2k_3}{(k_1+k_2+k_3)(k_1k_2+k_2k_3+k_3k_1)}$$

59. In the given circuit, the charge on4 μF capacitor will be:

[12 April 2019 II]

1}^{1}F



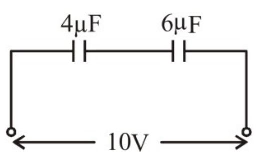
10V

(a) 5.4μC (b) 9.6μC

(c) 13.4μC (d) 24 μC

SOLUTION : (d)

$$V_1 + V_2 = 10$$



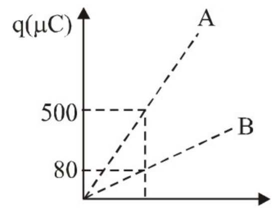
and4 $V_1 = 6V_2$

On solving above equations, we get

$$V_1 = 6\text{V}$$

Charge on 4 μf, $q = CV_1 = 4 \times 6 = 24\mu\text{C}.$

60. Figure shows charge (q) versus voltage (V) graph for series and parallel combination oftwo given capacitors. The capacitances are: [10 April 2019 I]



10V V (Volt)

(a) 40 μF and 10 μF (b) 60 μF and40 μF

(c) 50 μF and 30 μF (d) 20 μF and30 μF

SOLUTION : . (a)

Equivalent capacitance in series combination (C') is

given by $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow C' = \frac{C_1C_2}{C_1+C_2}$

For parallel combination equivalent capacitance

$$C'' = C_1 + C_2$$

For parallel combination $q = 10(C_1 + C_2)$

$$q_1 = 500\mu C$$

$$500 = 10(C_1 + C_2)$$

$C_1 + C_2 = 50\mu F$ (i)

For Series Combination - $q_2 = 10\frac{C_1C_2}{(C_1+C_2)}$

$80 = 10\frac{C_1C_2}{50}$ From equation

$$C_1C_2 = 400$$

From equation (i) and (ii)

$$C_1 = 10\mu F \; C_2 = 40\mu F$$

61. A capacitor with capacitance $5\mu F$ is charged to 5 μC. If the plates are pulled apart to reduce the capacitance to 2 $1/_{4}F$, how much work is done? [9 April 2019 I]

(a) $6.25 \times 10^{-6} J$ (b) $3.75 \times 10^{-6} J$

(c) $2.16 \times 10^{-6} J$ (d) $2.55 \times 10^{-6} J$

SOLUTION : (b)

$$(0 = (0_f - v_j = \frac{q}{2}\left(\frac{1}{c_f} - \frac{1}{C_i}\right)$$

....(ii)

....(iii)

$$= \frac{(5 \times 10)^2}{2}\left(\frac{1}{2} - \frac{1}{5}\right) \times 10^6$$

$$= 3.75 \times 10^{\lhd} J$$

62. (b) Capacitance ofa capacitor with a dielectric ofdielectric constant k is given by$C = \frac{k\epsilon A0}{d}$

$$E = \frac{V}{d}\ C = \frac{k \in AE0}{V}$$

$$15 \times 10^{-12} = \frac{k \times 8.86 \times 10^{-12} \times 10^{-4} \times 10^6}{500}$$

$$k = 8.5$$

62. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of106 V/ m. The plate area is $10^{-4} m^2$. What is the dielectric constant ifthe capacitance is 15 pF ? [8 April 2019 I]

(given $'0 = 8.86 \times 10^{-1}\ C^2/Nm^2$)

(a) 3.8 (b) 8.5

(c) 4.5 (d) 6.2

SOLUTION :

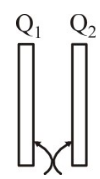
63. A parallel plate capacitor has $1\mu F$ capacitance. One ofits two plates is given $+2\mu C$ charge and the other plate, $+4\mu C$ charge. The potential difference developed across the capacitor is: [8 April 2019 II]

(a) 3V (b) 1V

(c) 5V (d) 2V

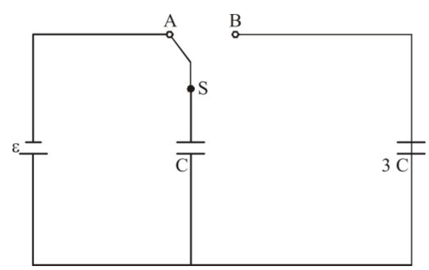
SOLUTION : (b)

$$V = \frac{Q}{C} = \left(\frac{Q_1 - Q_2}{2C}\right)$$



$$= \left(\frac{4-2}{2 \times 1}\right) = 1V\ \frac{Q_1 - Q_2}{2} - \frac{(Q_1 - Q_2)}{2}$$

64. In the figure shown, after the switch 'S' is turned from position 'A' to position 'B', the energy dissipated in thecircuit in terms ofcapacitance 'C' and total charge Q' is: [12 Jan. 2019 I]



(a) $\frac{1}{8}\frac{Q^2}{C}$ (b) $\frac{3}{8}\frac{Q^2}{C}$

(c) $\frac{5}{8}\frac{Q^2}{C}$ (d) $\frac{3}{4}\frac{Q^2}{C}$

SOLUTION : (b)

Energy stored in the system initially $U_i = \frac{1}{2} CE^2$

$$U_f = \frac{1}{2}\frac{Q^2}{C_{eq}} = \frac{(CE)^2}{2 \times 4C} = \frac{1}{2}\frac{CE^2}{4}$$

$$[\text{As } Q = CE, \text{ and } C_{eq} = 4C]$$

$$\Delta U = \frac{1}{2}CE^2 \times \frac{3}{4} = \frac{3}{8}CE^2 = \frac{3}{8}\frac{Q^2}{C}$$

65. Aparallel plate capacitor with plates ofarea 1 m^2 each, are at a separation of 0.1 m. Ifthe electric field between the plates is 100 N/C, the magnitude ofcharge on each plate is:

(Take $\in 0 = 8.85 \times 10^{-12} \frac{C^2}{N-M^2}$) [12 Jan. 2019 II]

(a) 7.85×10^{-1} C
(b) 6.85×10^{-1} C
(c) 8.85×10^{-10}C
(d) 9.85×10^{-1} C

SOLUTION : . (c)

$$E = \frac{o}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

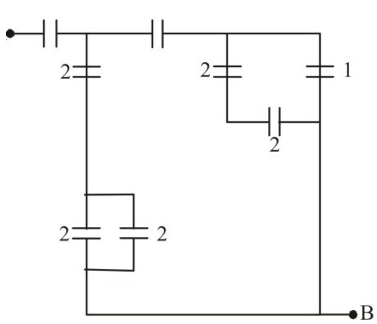
$$Q = \varepsilon_0 . E. A = 8.85 \times 10^{-12} \times 100 \times 1$$

$$= 8.85 \times 10^{-10}C$$

66. In the circuit shown, find C ifthe effective capacitance of the whole circuit is to be 0.5μF. All values in the circuit are inμF. [12 Jan. 2019 II]

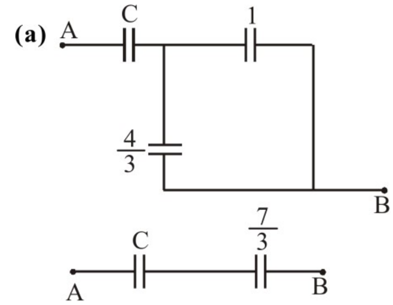
C 2

A



(a) $\frac{7}{11}\mu F$
(b) $\frac{6}{5}\mu F$
(c) 4 μF
(d) $\frac{7}{10}\mu F$

SOLUTION :



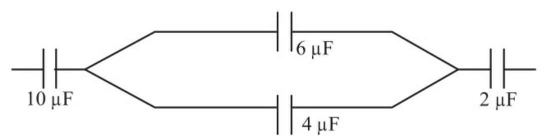
For series combination $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\Rightarrow \frac{}{\frac{7}{3} + c} = \frac{1}{2}\frac{7C}{3}$$

$$\Rightarrow 14C = 7 + 3C$$

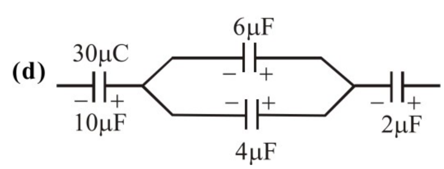
$$\Rightarrow C = \frac{7}{11}\mu F$$

67. In the figure shown below, the charge on the leit plate of the 10 μF capacitor is -30μC. The charge on the right plate ofthe 6μF capacitor is: [11 Jan. 2019 I]



(a) $-12\mu C$
(b) $+12\mu C$
(c) $-18\mu C$
(d) $+18\mu C$

SOLUTION :

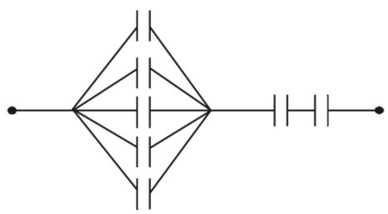


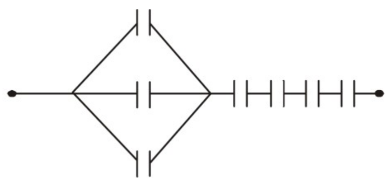
As given in the figure, 6μF and 4μF are in parallel. Now using charge conservation

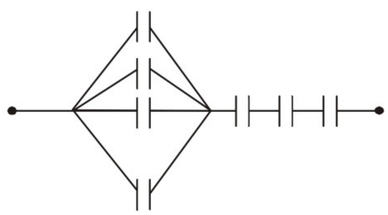
Charge on 6μF capacitor $= \frac{6}{6+4} \times 30 = 18\mu C$

Since charge is asked on right plate therefore is $+ 18\mu C$

68. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu$F. Which of the combinations, shown in figures below, will achieve the desired value? [11 Jan. 2019 II]

(a)


(b)


(c)


(d)


SOLUTION : . (b)

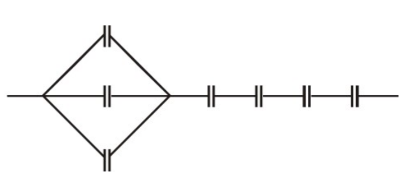
As required equivalent capacitance should be $C_{eq} = \frac{6}{13}\mu F$

Therefore three capacitors must be in parallel and 4 must be in series with it.

$$\frac{1}{C_{eq}} = \left[\frac{1}{3C}\right] + \left\{\frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \frac{1}{C}\right\}$$

$$C_{eq} = \frac{3C}{13} = \frac{6}{13}\mu F \text{ [as } C = 2\mu F]$$

So, desired combination will be as below:



69. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is: [10 Jan. 2019 II]

(a) 692 pJ (b) 508 pJ

(c) 560 pJ (d) 600 pJ

SOLUTION : . (b)

$$W = -\Delta u = (-1)\left|\frac{(c\varepsilon)^2}{2kc} - \frac{(c\varepsilon)^2}{2c}\right|$$

$$= \frac{\varepsilon^2 c}{2}\frac{k-1}{k} = 508J$$

70. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10, K_2 = 12$ and $K_3 = 1(4)$ The dielectric constant of a material which when fully inserted in above capacitor, gives same capacitance would be: [10 Jan. 2019 I]

(a) 4 (b) 14 (c) 12 (d) 36

SOLUTION : (c) Let dielectric constant ofmaterial used be K.

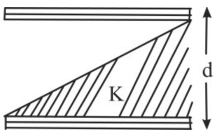
$$\frac{k_1 \in_0 A_1}{d} + \frac{k_2 \in_0 A_2}{d} + \frac{k_3 \in_0 A_3}{d} = \frac{k \in_0 A}{d}$$

$$\text{or } \frac{10\in 0A/3}{d} + \frac{12\in 0A/3}{d} + \frac{14\in 0A/3}{d} = \frac{K\in 0A}{d}$$

$$\frac{\in 0^A}{d}\left(\frac{10}{3} + \frac{12}{3} + \frac{14}{3}\right) = \frac{K \in 0A}{d}$$

$$K = 12$$

71. A parallel plate capacitor is made oftwo square plates of side 'a' , separated by a distance $d(d << a)$. The lower triangular portion is filled with a dielectric ofdielectric constant K, as shown in the figure. Capacitance of this capacitor is: [9 Jan. 2019 I]



$-a \rightarrow$

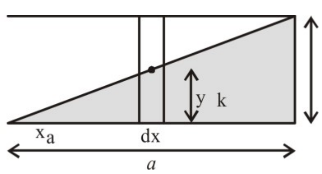
(a) $\frac{K\epsilon a^2 0}{2d(K+1)}$ (b) $\frac{K\epsilon a^2 0}{d(K-1)}$ In K

(c) $\frac{K\epsilon a^2 0}{d}$ In K (d) $\frac{1}{2}\frac{K\epsilon a^2 0}{d}$

SOLUTION : (b

d

)



From figure, $\frac{y}{x} = \frac{d}{a} \Rightarrow y = \frac{d}{a}x$

$dy = \frac{d}{a}(dx) \Rightarrow \frac{1}{dc} = \frac{y}{K\varepsilon_0 adx} + \frac{(d-y)}{\varepsilon_0 adx}$

$$\frac{1}{dc} = \frac{y}{\varepsilon_0 abx}\left(\frac{y}{k} + d - y\right)$$

$$\int dc = \int \frac{\varepsilon_0 adx}{\frac{y}{k} + d - y}$$

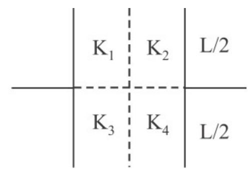
or, $c = \varepsilon_0 a.\frac{a}{d}\int_{0 d+y}^{d} \frac{d}{}\left(\frac{1}{k} \quad -1\right) y -$

$$= -\left(\frac{1}{k} \quad -1\right) - \varepsilon_0 a^2 d\left[ln\left(d + y\left(\frac{1}{k}-1\right)\right)\right]_0^d$$

$$= \frac{k \in 0a^2}{(1-k)d} ln\left(\frac{d + d\left(\frac{1}{k} - 1\right)}{d}\right)$$

$$= \frac{k \in 0a^2}{(1-k)d} ln\left(\frac{1}{k}\right) = \frac{k \in 0a^2 lnk}{(k-1)d}$$

72. Aparallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1 , K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be: [9 Jan. 2019 II]



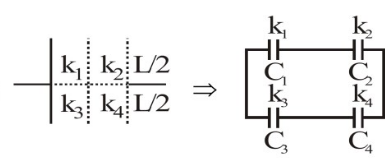
$+d/2 \cdot\rightarrow +d/2 \rightarrow$

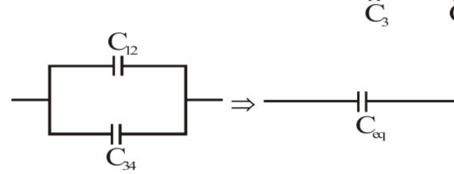
(a) $K = \frac{(K_1+K_3)(K_2+K_4)}{K_1+K_2+K_3+K_4}$ (b) $K = \frac{(K_1+K_2)(K_3+K_4)}{2(K_1+K_2+K_3+K_4)}$

(c) $K = \frac{(K_1+K_2)(K_3+K_4)}{K_1+K_2+K_3+K_4}$ (d) $K = \frac{(K_1+K_4)(K_2+K_3)}{2(K_1+K_2+K_3+K_4)}$

SOLUTION :

(Bonus)





$$\underline{k_1\epsilon_0\frac{L}{2}\times L}.\underline{k_2\left[\in 0\frac{L}{2}\times L\right]}$$

$$C_{12} = \frac{C_1C_2}{C_1 + C_2} = \frac{d/2d/2}{(k_1 + k_2)[\quad]}$$

$$C_{12} = \frac{k_1k_2 \in {}_0L^2}{k_1 + k_2 d}$$

in the same way we get, $C_{34} = \frac{k_3k_4\in {}_0L^2}{k_3+k_4 d}$

$$C_{eq} = C_{12} + C_{34} = \left[\frac{k_1k_2}{k_1+k_2} + \frac{k_3k_4}{k_3+k_4}\right]\frac{\in_0 L^2}{d}$$ .. (i) Now if $k_{eq} = K$,

$$C_{eq} = \frac{k\in_0 L^2}{d}$$ ...(ii)

on comparing equation (i) to equation (ii), we get

$$k_{eq} = \frac{k_1k_2(k_3+k_4) + k_3k_4(k_1+k_2)}{(k_1+k_2)(k_3+k_4)}$$

This does not match with any ofthe options so this must be a bonus.

73. Aparallel plate capacitor ofcapacitance 90 pF is connected to a battery ofemf20V. Ifa dielectric material ofdielectric constant $k = \frac{5}{3}$ is inserted between the plates, the magnitude ofthe induced charge will be: [2018]

(a) 1.2nC (b) 0.3nC

(c) 2.4nC (d) 0.9nC

SOLUTION : (a)

Charge on Capacitor, $Q_i = CV$

After inserting dielectric ofdielectric constant

$$= KQ_f = (kC)V$$

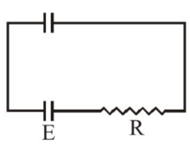
Induced charges on dielectric

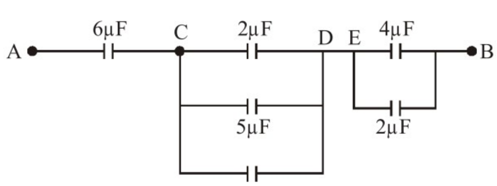
$$Q_{ind} = Q_f - Q_j = KCV - CV$$

$$(K-1)CV = \left(\frac{5}{3} - 1\right) \times 90\text{pF} \times 2\text{V} = 1.2\text{nc}$$

$$Q = C_{eq}E\left[1 - e^{-t/RC_{eq}}\right]$$

$(\because\ Q_0 = C_{eq}E)$





5μF

The equivalent capacitance between C &D capacitors of 2μF, 5μF and 5 μF are in parallel.

$C_{CD} = 2 + 5 + 5 = 12\mu F$ (In parallel grouping

$$C_{eq} = C_1 + C_2 + \cdots + C_n)$$

Similarly equivalent capacitance between E & BC_{EB}

$$= 4 + 2 = 6\mu F$$

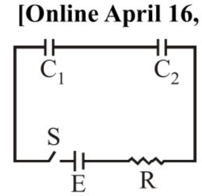
Now equivalent capacitance between A& B

$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{12} + \frac{1}{6} = \frac{5}{12}$$

$\Rightarrow C_{eq} = \frac{12}{5} = 2.4\mu F$ (In series grouping,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots \ldots + \frac{1}{C_n})$$

74. In the following circuit, the switch S is closed at t = 0. The charge on the capacitor C_1 as a function of time will be given by $\left(C_{eq} = \frac{C_1C_2}{C_1+C_2}\right)$. **[Online April 16, 2018]**



(a) $C_{eq}E[1 - \exp(-t/RC_{eq})]$
(b) $C_1E[1 - \exp(-tR/C_1)]$
(c) $C_2E[1 - \exp(-t/RC_2)]$
(d) $C_{eq}E\exp(-t/RC_{eq})$

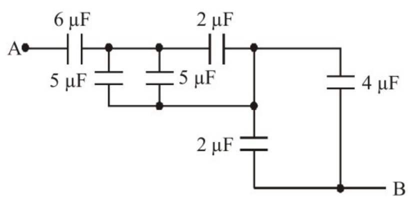
SOLUTION : . (a)

During charging charge on the capacitor increases with time. Charge on the capacitor C_1 as a function oftime,

$$Q = Q_0(1 - e^{-t/RC})\ c_{eq}$$

Both capacitor will have charge as they are connected in series

75. The equivalent capacitance between A and B in the circuit given below is:



[Online Apri115, 2018]

(a) $4.9\mu F$ (b) $3.6\mu F$

(c) $5.4\mu F$ (d) $2.4\mu F$

SOLUTION : (d)

The simplified circuit ofthe circuit given in question

as follows:

76. Aparallel plate capacitor with area $2\ 00cm^2$ and separationbetween the plates $1.5cm$, is connected across a battery of emfV. Ifthe force ofattraction between the plates is 25×10^-6N, the value ofV is approximately.

[Online April 15, 2018]

$$\left(\left(= 8.85 \times 10^{-12} \frac{C^2}{N.m^2}\right)\right)$$

(a) $150V$ (b) $1\alpha)V$

(c) $250V$ (d) $3\alpha)V$

SOLUTION : (c)

Given area ofParallel plate capacitor, $A = 200cm^2$

Separation between the plates, $d = 1.5$ cm

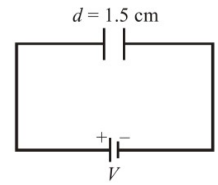
Force of attraction between the plates, $F = 25 \times 10^{-6}N$

$$F = QE$$

$F = \frac{Q^2}{2A\in 0}$(E due to parallel plate $= \frac{o}{2\in 0} = \frac{Q}{A2\in 0}$)

But $Q = CV = \frac{\in 0A(V)}{d}$

$$F = \frac{(\in 0AV)^2}{d^2 \times 2A \in 0}$$

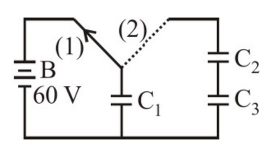


$$= \frac{(\in 0A)^2 \times V^2}{d^2 \times 2 \times (A \in 0)} = \frac{(\in 0A) \times V^2}{d^2 \times 2}$$

or, $25 \times 10^{-6} = \frac{(8.85\times10^{-12})\times(200\times10^{-4})\times V^2}{2.25\times10^{-4}\times2}$

$$\Rightarrow V = \sqrt{\frac{25\times10^{-6}\times225\times10^{-4}\times2}{885\times10^{-12}\times200\times10^{-4}}} \approx 250V$$

77. A capacitor C_1 is charged up to a voltage $V = 60V$ by connecting it to battery B through switch (1), Now C_1 is disconnected from battery and connected to a circuit consisting oftwo uncharged capacitors $C_2 = 3.0\mu F$ and $C_3 = 6.0\mu F$ through a switch (2) as shown in the figure. The sum offinal charges on C_2 and C_3 is: [Online Apri115, 2018]



(a) $36\mu C$ (b) $20\mu C$ (c) $54\mu C$ (d) $40\mu C$

SOLUTION : (a)

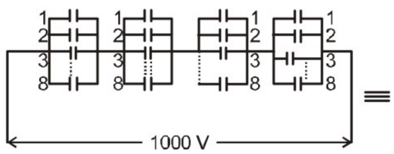
The sum offinal charges on C_2 and C_3 is 36 μC.

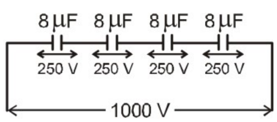
78. A capacitance of $2\mu F$ is required in an electrical circuit across a potential difference of $1.0kV$. A large number of $1\mu F$capacitors are available which can withstand a potential difference ofnot more than 300 V. The minimum number of capacitors required to achieve this is [2017]

(a) 24 (b) 32 (c) 2 (d) 16

SOLUTION : . (b)

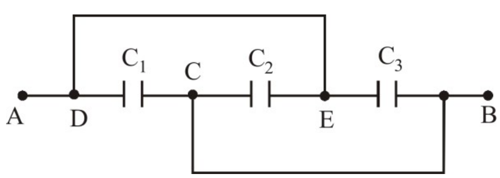
To get a capacitance of 2μF arrangement of capacitors of capacitance 1μF as shown in figure 8 capacitors of 1μF in parallel with four such branches in series i. e., 32 such capacitors are required.





$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} C_{eq} = 2\mu F$$

79. A combination ofparallel plate capacitors is maintained at a certain potential difference.



When a 3 mm thick slab is introduced between all the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant ofthe slab. [Online April 9, 2017]

(a) 3 (b) 4 (c) 5 (d) 6

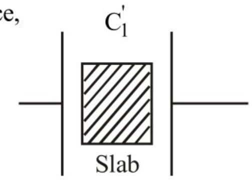
SOLUTION : (c)

Before introducing a slab capacitance ofplates $C_1 = \frac{\varepsilon_0 A}{3}$

If a slab of dielectric constant K is introduced between plates then

$C = \frac{K\varepsilon_0 A}{d}$ then $C_1' = \frac{\varepsilon_0 A}{2.4}$

C_1 and C_1' are in series hence,



$$\frac{\varepsilon_0 A}{3} = \frac{k\frac{\varepsilon_0 A}{3} \cdot \frac{\varepsilon_0 A}{2.4}}{k\frac{\varepsilon_0 A}{3} + \frac{\varepsilon_0 A}{2.4}}$$

$3k = 2.4k + 3$

$$0.6k = 3$$

Hence, the dielectric constant ofslap is given by,

$$k = \frac{30}{6} = 5$$

80. The energy stored in the electric field produced by ametal sphere is 4.5 J. If the sphere contains 4 μC charge, its radius will be: [Take : $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 N - m^2/C^2$]

[Online April 8, 2017]

(a) 20mm (b) 32mm (c) 28mm (d) 16mm

SOLUTION : (d)

Energy of sphere $= \frac{Q^2}{2C}$

$$4.5 = \frac{16 \times 10^{-12}}{2C}$$

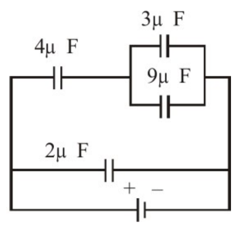
$$C = \frac{16 \times 10^{-1}}{9} = 4\pi\varepsilon_0 R$$

(capacity of spherical conductor)

$$R = \frac{16\times10^{-1}}{9} \times \frac{1}{4\pi\varepsilon_0} \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

$$= 9 \times 10^9 \times \frac{16}{9} \times 10^{-12} = 16 \text{ mm}$$

81. A combination of capacitors is set up as shown in the figure. The magnitude ofthe electric field, due to a point charge Q (having a charge equal to the sum ofthe charges on the 4 μF and 9 μF capacitors), at a point distance 30 m from it, would equal: [2016]

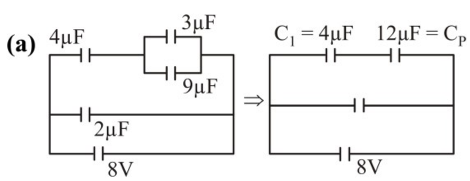


8V

(a) 420N/C (b) 480N/C

(c) 240N/C (d) 360N/C

SOLUTION :



Charge on C_1 is $q_1 = \left[\left(\frac{12}{4+12}\right) \times 8\right] \times 4 = 24\mu C$

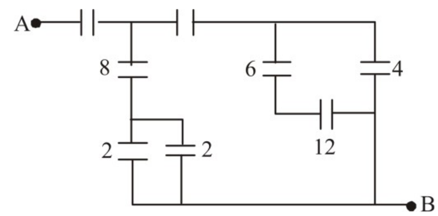
The voltage across C_P is $V_P = \frac{4}{4+12} \times 8 = 2V$ Voltage across $9\mu F$ is also $2V$

Charge on $9\mu F$ capacitor $= 9 \times 2 = 18\mu C$ Total charge on 4 μF and $9\mu F = 42\mu C$

$$E = \frac{KQ}{r^2} = 9 \times 10^9 \times \frac{42 \times 10^{-6}}{30 \times 30} = 420NC^{-1}$$

82. Figure shows a network ofcapacitors where the numbers indicates capacitances in micro Farad. The value of capacitance C ifthe equivalent capacitance between point A and B is to be 1 μF is: [Online Apri110, 2016]

c 1



(a) $\frac{32}{23}\mu F$ (b) $\frac{31}{23}\mu F$ (c) $\frac{33}{23}\mu F$ (d) $\frac{34}{23}\mu F$

SOLUTION : . (a)

Capacitors $2\mu F$ and $2\mu F$ are parallel, their equivalent $= 4\mu F$

$6\mu F$ and 12 μF are in series, their equivalent $= 4\mu F$

Now $4\mu F$ (2 and 2 μF) and $8\mu F$ in series $= \frac{3}{8}\mu F$

And $4\mu F(12\ \&6\mu F)$ and $4\mu F$ in parallel $= 4 + 4 = 8\mu F$

$8\mu F$ in series with $1\mu F = \frac{1}{8} + 1 \Rightarrow \frac{8}{9}\mu F$

Now $C_{eq} = \frac{8}{9} + \frac{8}{3} = \frac{32}{9}$

C_{eq} ofcircuit $= \frac{32}{9}$

With $C - \frac{1}{C_{eq}} = \frac{1}{C} + \frac{9}{32} = 1 \Rightarrow C = \frac{32}{23}$

83. Three capacitors each of4 μF are to be connected in such a way that the effective capacitance is 6μF. This can be done by connecting them : [Online April 9, 2016]

(a) all in series

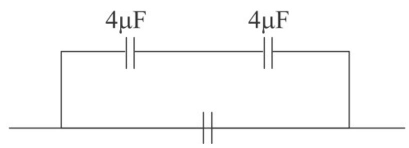
(b) all in parallel

(c) two in parallel and one in series

(d) two in series and one in parallel

SOLUTION : . (d)

To get effective capacitance of 6 μF two capacitors of 4 μF each connected in sereies and one of4 μF capacitor in parallel with them.



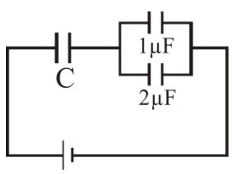
$4\mu F$

Two capacitances in series $\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$

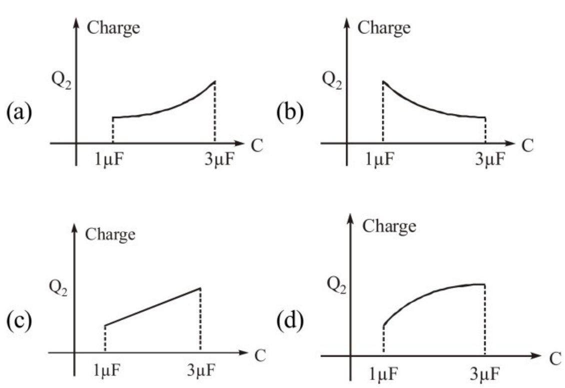
1 capacitor in parallel

$$C_{eq}=C_3+C=4+2=6\mu F$$

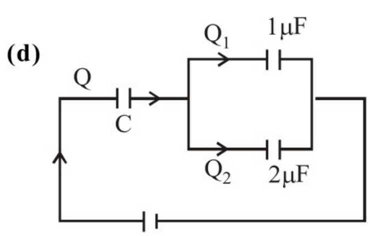
84. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of $1C^1$ is given properly by: (*figures are drawn schematically and are not to scale*) [2015]



E



SOLUTION :

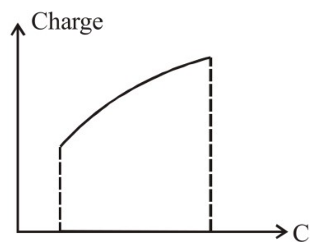


From figure, $Q_2=\frac{2}{2+1}Q=\frac{2}{3}Q$

$$Q=E\left(\frac{C\times 3}{C+3}\right)$$

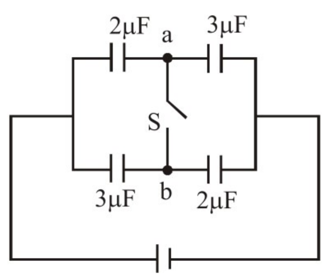
$$Q_2=\frac{2}{3}\left(\frac{3CE}{C+3}\right)=\frac{2CE}{C+3}$$

Therefore graph d correctly dipicts.



$1\mu F\ 3\mu F$

85. In figure a system of four capacitors connected across a 10V battery is shown. Charge that will flow from switch S when it is closed is: [Online April 11, 2015]



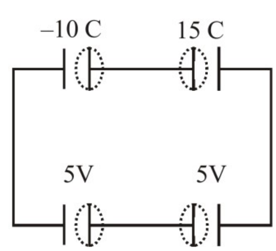
10 V

(a) 5 μC from b to a (b) 20 μC from a to b

(c) zero (d) 5 μC from a to b

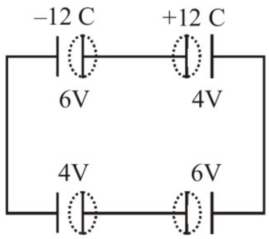
SOLUTION : . (a)

when switch is closed



$-15C\ +10C$

When switch is open



$-12C \ +12C$

Charge of $5\mu c$ flows from b to a

$$= \frac{o}{\lambda} \ln\left(1+\frac{\lambda d}{K_0}\right)$$

Now it is given that capacitance ofvacuum $= C_0$. Thus, $C = \frac{Q}{V}$

$= \frac{o.s}{v}$ (Let surface area of plates $= s$) $= \frac{o.s}{\frac{o}{\lambda}\ln\left(1+\frac{\lambda d}{K_0}\right)}$

$= s\lambda.\frac{d}{d}\frac{1}{\ln\left(1+\frac{\lambda d}{K_0}\right)}$ (in vacuum $\varepsilon_0 = 1$) $c = \frac{\lambda d}{\ln\left(1+\frac{\lambda d}{K_0}\right)} \cdot C_0$ (here,

$C_0 = \frac{s}{d}$)

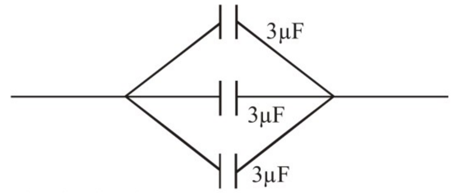
86. A parallel plate capacitor is made oftwo circular plates separated by a distance 5 mm and with a dielectric of dialectric constant 2.2 between them. When the electric field in the dielectric is $3 \times 10^4 V/m$ the charge density of the positive plate will be close to: [2014]

(a) $6 \times 10^{-7} C/m^2$ (b) $3 \times 10^{-7} C/m^2$

(c) $3 \times 10^4 C/m^2$ (d) $6 \times 10^4 C/m^2$

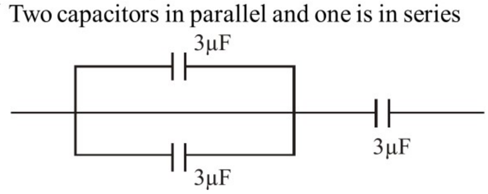
SOLUTION : . (a) Electric field in presence ofdielectric between the two

plates ofa parallel plate capaciator is given by,



$C_{eq} = 3 + 3 + 3 = 9\ \mu F$

(iii) Two capacitors in parallel and one is in series



$$E = \frac{o}{K\varepsilon_0}$$

Then, charge density

$$o = K\varepsilon_C E$$

$$= 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$$

$$\approx 6 \times 10^{-7} C/m^2$$

87. The gap between the plates ofa parallel plate capacitor of area A and distance between plates d, is filled with a dielectric whose permittivityvaries linearly from ∈ 1 at one plate to ∈ 2 at the other. The capacitance of capacitor is: [Online Apri119, 2014]

(a) $\in 0(\in +\in)A/d$

(b) $\in 0(\in +\in)A/2d$

(c) $\in 0A/[d\ln(\in/\in)]$

(d) $\in 0(\in -\in)A/[d\ln(\in/\in)]$

SOLUTION : . (d)

88. The space between the plates ofa parallel plate capacitor

is filled with a dielectric' whose 'dielectric constant'

varies with distance as per the relation:

$K(x)=K_{o}+\lambda x(\lambda=$ a constant) The capacitance C, ofthe capacitor, would be related to its vacuum capacitance C_{o} for the relation :

[Online Apri112, 2014]

(a) $c=\frac{\lambda d}{ln(1+K_{o}\lambda d)}c_{o}$ (b) $C=\frac{\lambda}{d.ln(1+K_{o}\lambda d)}C_{o}$

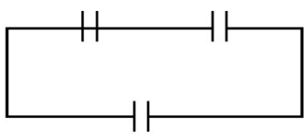
(c) $c=\frac{\lambda d}{ln(1+\lambda d/K_{o})}c_{o}$ (d) $C=\frac{\lambda}{dJn(1+K_{o}/\lambda d)}C_{o}$

SOLUTION : (c)

The value of dielectric constant is given as, $C_{eq}=2\mu F$

(iv) Two capacitors in series and one is in parallel

$$K=K_{0}+\lambda x$$



And, $V=\int_{0}^{d}E\,dr\Rightarrow V=\int_{0}^{d}\frac{o}{K}dx$

$$=0\int_{0}^{d}\frac{1}{(K_{0}+\lambda x)}dx=\frac{o}{\lambda}[\ln(K_{0}+\lambda d)-\ln K_{0}]$$

$$C_{eq}=4.5\mu F$$

89. Aparallel plate capacitor is made oftwo plates oflength 1,

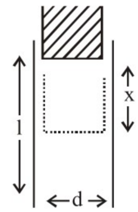
width w and separated by distance d. A dielectric slab

(dielectric constant K) that fits exactly between the plates

is held near the edge of the plates. It is pulled into the

capacitor by a force $F=-\frac{\partial U}{\partial x}$ where U is the energy of the capacitor when dielectric is inside the capacitor up to distance x (See figure). Ifthe charge on the capacitor is Q then the force on the dielectric when it is near the edge is:

[Online April 11, 2014]



(a) $\frac{Q^{2}d}{2col^{2}\varepsilon_{0}}K$ (b) $\frac{Q^{2}to}{2dl^{2}\varepsilon_{0}}(K-1)$

(c) $\frac{Q^{2}d}{2wl^{2}\varepsilon_{0}}(K-1)$ (d) $\frac{Q^{2}W}{2dl^{2}\varepsilon_{0}}K$

SOLUTION : (c)

90. Three capacitors, each of3 μF, are provided. These cannot

be combined to provide the resultant capacitance of:

[Online April 9, 2014]

(a) 1 μF (b) 2 μF (c) 4.5μF (d) 6 μF

SOLUTION : (d)

Possible combination of capacitors

(i) Three capacitors in series combination

$3\mu F$ $3\mu F$ $3\mu F$

$$\frac{1}{C_{eq}}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$$

$$\frac{1}{C_{eq}}=1\mu F$$

(ii) Three capacitors in parallel combination

91. Aparallel plate capacitor havinga separation between the plates d, plate area A and material with dielectric constant K has capacitance C_0. Now one - third of the material is replaced by another material with dielectric constant 2K, so that effectively there are two capacitors one with area $\frac{1}{3}$A, dielectric constant 2K and another with area $\frac{2}{3}$A and dielectric constant K. If the capacitance of this new capacitor is C then $\frac{C}{C_0}$ is [Online April 25, 2013]

(a) 1 (b) $\frac{4}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

SOLUTION : . (b)

$$C_0 = \frac{k \in 0A}{d}$$

$$C = \frac{k \in 02}{3d} + \frac{2k \in 0A}{3d} = \frac{4}{3}\frac{k \in 0A}{d}$$

$$\frac{C}{C_0} = \frac{\frac{4}{3}\frac{k \in 0A}{d}}{\frac{k \in 0A}{d}} = \frac{4}{3}$$

92. To establish an instantaneous current of 2 A through a 1 μF capacitor; the potential difference across the capacitor plates should be changed at the rate of:

[Online April 22, 2013]

(a) 2×10^4V/s (b) 4×10^6V/s

(c) 2×10^6V/s (d) 4×10^4V/s

SOLUTION : (c)

$$\text{As, } C = \frac{Q}{V} = \frac{It}{V}$$

$$\Rightarrow \frac{V}{t} = \frac{I}{C} = \frac{2}{1 \times 10^{-6}} = 2 \times 10^6 \text{V/s}$$

93. A uniform electric field $\overline{E}$ exists between the plates of a charged condenser. A charged particle enters the space between the plates and perpendicular to $\overline{E}$. The path of the particle between the plates is a: [Online April 9, 2013]

(a) straight line (b) hyperbola

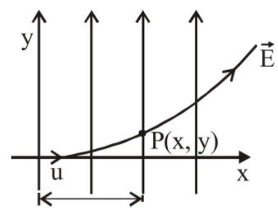
(c) parabola (d) circle

SOLUTION : . (c)

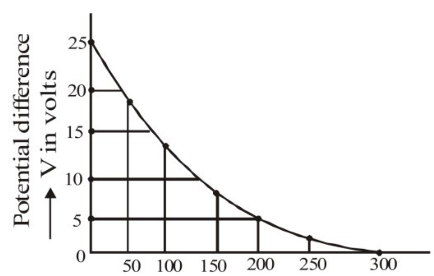
When charged particle enters perpendicularly in an electric field, it describes a parabolic path

$$y = \frac{1}{2}\left(\frac{QE}{m}\right)\left(\frac{x}{4}\right)^2$$

This is the equation ofparabola.



94. The figure shows an experimental plot discharging of a capacitor in an RC circuit. The time constant τ ofthis circuit lies between : [2012]



Time in seconds →

(a) 150 sec and200 sec (b) 0 sec and 50 sec

(c) 50 sec and 100 sec (d) 100 sec and 150 sec

SOLUTION : (d)

The discharging ofa capacitor is given as$q = q_0 exp[-t/RC]$

$$RC = \text{time constant} = \tau$$

If e is the capacitance ofthe capacitor

$q=CV$ and $q=CV_0$

Thus, $CV=CV_0e^{t/\tau}$

$V=V_0e^{-t/\tau}$ (i)

From the graph (given in the problem

when $t=0.5, V=25$ i.e.,

$V_0=25$ volt.

and when $t=200, V=5$ volt

Thus equation (i) becomes

$5=25e^{-200/\tau}$

$\Rightarrow 1/5=e^{-200/\tau}$

Taking $\log_e$ on both sides

$$\log_e\frac{1}{5}=-200/\tau\Rightarrow-\frac{200}{\tau}=\log e^5$$

$$\tau=\frac{200}{\log_e 5}$$

or $\tau=\frac{200}{\log_e\left(\frac{10}{2}\right)}=\frac{200}{\log_e 10-\log_e 2}$

$$\tau=\frac{200}{2.302-0.693}=\frac{200}{1.609}=124.300$$

Which lies between 100 s and 150 s

95. The capacitor of an oscillatory circuit is enclosed in a container. When the container is evacuated, the resonance frequency ofthe circuit is 10 kHz. When the container is filled with a gas, the resonance frequency changes by50 Hz. The dielectric constant ofthe gas is

[Online May 26, 2012]

(a) 1.001 (b) 2.001 (c) 1.01 (d) 3.01

SOLUTION : . (c)

The dielectric constant ofthe gas is 1.01

96. Statement 1: It is not possible to make a sphere of capacity 1 farad using a conducting material.

Statement 2: It is possible for earth as its radius is $6.4\times10^6\mathrm{m}$. [Online May 26, 2012]

(a) Statement 1 is true, Statement2 is true, Statement2 is the correct explanation ofStatement 1.

(b) Statement 1 is false, Statement2 is true.

(c) Statement 1 is true, Statement2 is true, Statement2 is not the correct explanation of Statement 1.

(d) Statement 1 is true, Statement 2 is false.

SOLUTION : (d)

Capacitance of sphere is given by : $C=4\pi\in 0r$

If, $C=1\mathrm{F}$ then radius of sphere needed:

$$r=\frac{C}{4\pi\in 0}=\frac{1}{4\pi\times8.85\times10^{-12}}$$

or, $r=\frac{10^{12}}{4\pi\times8.85}=9\times10^9\mathrm{m}$

$9\times10^9\mathrm{m}$ is very large, it is not possible to obtain such a large sphere. Infact earth has radius $6.4\times10^6\mathrm{m}$ only and capacitance of earth is $711\mu\mathrm{F}$.

97. A series combination of n_1 capacitors, each of capacity C_1 is charged by source ofpotential difference 4 V. When another parallel combination of n_2 capacitors each of capacity C_2 is charged by a source ofpotential difference V, it has the same total energy stored in it as the first combination has. The value of C_2 in terms of C_1 is then

[Online May 12, 2012]

(a) $16\frac{n_2}{n_1}C_1$ (b) $\frac{2C_1}{n_1 n_2}$

(c) $2\frac{n_2}{n_1}C_1$ (d) $\frac{16C_1}{n_1 n_2}$

SOLUTION : (d)

Equivalent capacitance of n_2 number of capacitors

each ofcapacitance C_2 in parallel $= n_2C_2$

Equivalent capacitance of n_1 number of capacitors each

ofcapacitances C_1 in series.

Capacitance ofeach is $C_1 = \frac{C_1}{n_1}$

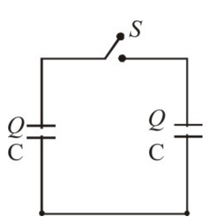
According to question, total energy stored in both the

combinations are same

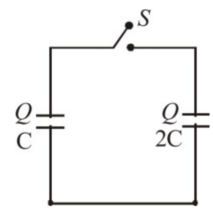
i.e., $\frac{1}{2}(\quad)(\quad)(4V)^2 = \frac{1}{2}(n_2C_2)V^2$

$$C_2 = \frac{16C_1}{n_1 n_2}$$

98. Two circuits (a) and (b) have charged capacitors of capacitance C, 2Cand3C with open switches. Charges on each ofthe capacitor are as shown in the figures. On closing the switches [Online May 7, 2012]



2 2 3 2



L R L R

Circuit (a) Circuit (b)

(a) No charge flows in(a) but charge flows from R to L in(b)

(b) Charges flow from L to R in both (a) and (b)

(c) Charges flow from R to L in (a) and from L to R in (b)

(d) No charge flows in(a) but charge flows from L to R in(b)

SOLUTION : . (c)

Charge (or current) always flows from higher potential

to lower potential.

Potential $= \frac{\text{Charge}}{\text{Capacitance}}$

99. Let C be the capacitance ofa capacitor discharging througha resistor R. Suppose t_1 is the time taken for the energystored in the capacitor to reduce to halfits initial value andt_2 is the time taken for the charge to reduce to one - fourth its initial value. Then the ratio t_1/t_2 will be [2010]

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2

SOLUTION : (c)

Initial energyofcapacitor, $E_1 = \frac{q_1^2}{2C}$

Final energy ofcapacitor,2 $(q_1)^2$

$$E_2 = \frac{1}{2}E_1 = \frac{q_1}{4C} = \left(\frac{\sqrt{2}}{2C}\right)$$

t_1 = time for the charge to reduce to $\frac{1}{\sqrt{2}}$ of its initial value

and t_2 = time for the charge to reduce to $\frac{1}{4}$ of its initial value

We have, $q_2 = q_1 e^{-t/CR} \Rightarrow \ln\left(\frac{q_2}{q_1}\right) = -\frac{t}{CR}$

$\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{-t_1}{CR}$ (1)

and $\ln\left(\frac{1}{4}\right) = \frac{-t_2}{CR}$ (2)

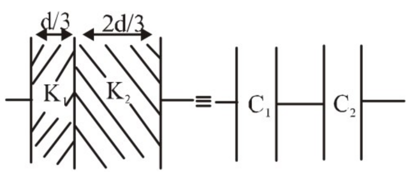
By(1) and (2) , $\frac{t_1}{t_2} = \frac{\ln\left(\frac{1}{\sqrt{2}}\right)}{\ln\left(\frac{1}{4}\right)} = \frac{1}{2}\frac{\ln\left(\frac{1}{2}\right)}{2\ln\left(\frac{1}{2}\right)} = \frac{1}{4}$

100. A parallel plate capacitor with air between the plates hascapacitance of 9 pF. The separation between its plates is 'd'. The space between the plates is now filled with two dielectrics. One ofthe dielectrics has dielectric constant $k_1 = 3$ and thickness $\frac{d}{3}$ while the other one has dielectric constant $k_2 = 6$ and thickness $\frac{2d}{3}$. Capacitance of the capacitor is now [2008]

(a) $1.8pF$ (b) 45 pF (c) $40.5pF$ (d) $20.25pF$

SOLUTION :

100. (c)



The capacitance with air between the plates

$$C = \frac{\varepsilon_0 A}{d} = 9\text{pF}$$

On introducing two dielectric between the plates, the given capacitance is equal to two capacitances connected in series where

$$C_1 = \frac{k_1 \epsilon_0 A}{d/3} = \frac{3 \in 0A}{d/3}$$

$$= \frac{3 \times 3 \in 0A}{d} = \frac{9 \in 0A}{d}$$

and

$$C_2 = \frac{k_2 \epsilon_0 A}{2d/3} = \frac{3k_2 \epsilon_0 A}{2d}$$

$$= \frac{3 \times 6 \in 0A}{2d} = \frac{9 \in 0A}{d}$$

The equivalent capacitance C_{eq} is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$= \frac{d}{9 \in 0A} + \frac{d}{9 \in 0A} = \frac{2d}{9 \in 0A}$$

$$C_{eq} = \frac{9 \in 0A}{2\ \ d} = \frac{9}{2} \times 9pF = 40.5pF$$

101. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential Vvolt. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is [2007]

(a) zero (b) $\frac{1}{2}(K-1)CV^2$

(c) $CV^2(K-1)$ (d) $(K-1)CV^2$

SOLUTION : (a)

The potential energy of a charged capacitor is given by

$U = \frac{Q^2}{2C}$.

When a dielectric slab is introduced between the plates

the energy is given by $\frac{Q^2}{2KC}$,

where K is the dielectric constant.

Again, when the dielectric slab is removed slowly its

energy increases to initial potential energy. Thus, work

done is zero.

102. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. Ifthe capacitance between any two adjacent plates is 'C' then the resultant capacitance is [2005]

(a) $(n+1)C$ (b) $(n-1)C$

(c) nC (d) C

SOLUTION : . (b)

As n plates arejoined alternatelypositive plate ofall

$(n-1)$ capacitor are connected to one point and negative

plate ofall $(n-1)$ capacitors are connected to other point.

It means $(n-1)$ capacitorsjoined in parallel.

Resultant capacitance $=(n-1)C$

103. A fully charged capacitor has a capacitance 'C'. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity 's' and mass m'. Ifthe temperature ofthe block is raised by $\Delta\Gamma$, the potential difference 'V across the capacitance is [2005]

(a) $\frac{mC\Delta T}{s}$ (b) $\sqrt{\frac{2mC\Delta T}{s}}$ (c) $\sqrt{\frac{2ms\Delta T}{C}}$ (d) $\frac{ms\Delta T}{C}$

SOLUTION : . (c)

Applying conservation of energy,

Electric potential energy ofcapacitor = heat absorbed

$$\frac{1}{2}CV^2=m.s\Delta t; \mathrm{V}=\sqrt{\frac{2ms\Delta t}{C}}$$

104. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor [2003]

(a) decreases (b) remains unchanged

(c) becomes infinite (d) increases

SOLUTION : . (b)

The capacitance without aluminium foil is $C=\frac{\varepsilon_0 A}{d}$

Here, d is distance between the plates of a capacitor

$A=$ Area ofplates of capacitor

When an aluminium foil of thickness t is introduced between the plates.

Capacitance, $C^{'}=\frac{\varepsilon_0 A}{d-t}$

Ifthickness offoil is negligible 50 $d-t\sim d$. Hence, $C=C'$.

105. The work done in placing a charge of 8×10^{-18} coulomb on a condenser ofcapacity 100 micro - farad is [2003]

(a) 16×10^{-32} joule (b) 3.1×10^{-2} joule

(c) 4×10^{-1} joule (d) 32×10^{-32} joule

SOLUTION : (d)

The work done is stored in the form ofpotential energy

which is given by $U=\frac{1}{2}\frac{Q^2}{C}$

$$U=\frac{1}{2}\times\frac{(8\times 10^{-18})^2}{100\times 10^{-6}}=32\times 10^{-3}\ \mathrm{J}$$

106. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to [2002]

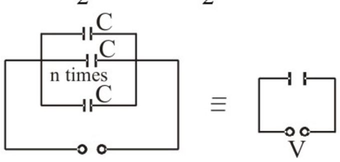
(a) CV (b) $\frac{1}{2}nCV^2$ (c) CV^2 (d) $\frac{1}{2n}CV^2$

SOLUTION : (b)

In parallel, equivalent capacitance of n capacitor of capacitance C $C' = nC$

Energy stored in this capacitor $E = \frac{1}{2}C^1V^2$

$$\Rightarrow E = \frac{1}{2}(nC)V^2 = \frac{1}{2}nCV^2$$



V

Alternatively

Each capacitor has a potential difference of V between the plates.

So, energy stored in each capacitor$= \frac{1}{2}CV^2$

Energy stored in n capacitor$= \left[\frac{1}{2}CV^2\right] \times n$

107. Capacitance (in F) ofa spherical conductor with radius 1 m is [2002]

(a) 1.1×10^{-10} (b) 10^{-6}

(c) 9×10^{-9} (d) 10^{-3}

SOLUTION : . (a)

Capacitance ofspherical conductor $= 4\pi E_0 R$

Here, R is radius of conductor

$$C = 4\pi \in 0R = \frac{1}{9 \times 10^9} \times 1 = 1.1 \times 10^{-10}F$$

CURRENT ELECTRICITY

Strength of Electric Current

The strength of electric current is defined as rate of flow of charge through any cross section of a conductor.

. The instantaneous current is defined by the equation,

$$I = \underset{\Delta t \to 0}{\text{Lt}} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

$$\text{Average current } i = \frac{q}{t}$$

Ampere :

If one coulomb of charge passes through a cross-section of the conductor per second then the current is one ampere.

$$1 \text{ ampere} = \frac{1\,\text{coulomb}}{1\,\text{second}}$$

current is a scalar quantity.

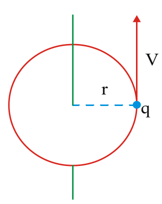
Applications on electric current

1. If the current is varying with time t, then the charge flowing in a time interval from t_1 to t_2 is $q = \int_{t_1}^{t_2} I dt$

2. If n particles, each having a charge q, pass through a given cross sectional area in time t, then average current is $i = \frac{nq}{t}$

3. If a point charge q is revolving in a circle of radius r with speed v then its time period is

$T = (2\pi r / v)$



4. The average current associated with this revolving charge is

$$I = \frac{q}{T} = fq = \frac{\omega}{2\pi} q = \frac{vq}{2\pi r}$$

Where f is the frequency of revolution in Hz.
ω is the angular frequency in rad/sec
v is linear velocity of the charge q
r is radius of the circular path

5. If in a discharge tube n_1 protons are moving from left to right in t seconds and n_2 electrons are moving simultaneously from right to left in t seconds, then the net current in any crossection of the discharge tube is

$I = \frac{(n_1 + n_2)e}{t}$ (from left to right)

here e is the magnitude of charge of electron (or) proton.

Drift Velocity:

Drift velocity is the average velocity acquired by free electrons inside a metal by the application of an electric field which results in current.

Drift velocity $v_d = \frac{J}{ne} = \frac{I}{Ane}$

where, J = I/A is current density
n is number of free electrons per unit volume
e is charge of electron

The drift velocity is related to relaxation time is $v_d = \frac{eE}{m}\tau$

Note :

1.The drift velocity of electrons is of the order of $10^{-4}\ ms^{-1}$.

2. Greater the electric field, greater will be the drift velocity $v_d \propto E$

3. The direction of drift velocity for electrons in a metal is opposite to that of electric field applied $\vec{E}$

Current Density $(\vec{J})$:

Current density at a point is defined as a vector having magnitude equal to current per unit area.

$$\vec{J} = \underset{\Delta s \to 0}{Lt} \frac{\Delta I}{\Delta s} = \frac{dI}{ds}\hat{n}$$

If the normal to the area makes an angle θ with the direction of the current, then the current density is

$$J = \frac{\Delta I}{\Delta s \cos\theta},$$

$$dI = Jds\cos\theta$$

$$dI = \vec{J}.\vec{ds}$$

i.e.,

$$I = \int \vec{J}.\vec{ds}$$

SI unit of $\vec{J}$ is Am^{-2}

Dimensional formula of J is $[AL^{-2}]$

Current is the flux of current density.

Relaxation time (τ) :

1. It is the time interval between two successive collisions of electrons with +ve ions in the metallic lattice.

The resistance of a conductor is given by $R = \frac{2ml}{ne^2\tau A}$

where n = number density of electrons
e = electron charge
m = mass of electron
τ = relaxation time.

Mobility (μ) :

Mobility (μ) of a charge carrier (like electron) is defined as the average drift velocity resulting from the application of unit electric field strength.

$$\mu = \frac{drift\ velocity}{electric\ field}$$

$$\therefore \quad \mu = \frac{|v_d|}{E}$$

Mobility depends on pressure and temperature.

OHM's LAW :

For a given conductor, at a given temperature the strength of electric current through it is directly proportional to the potential difference applied across at its ends".

i.e.

$$I \alpha V \Rightarrow I = \frac{V}{R}$$

$$V = IR$$

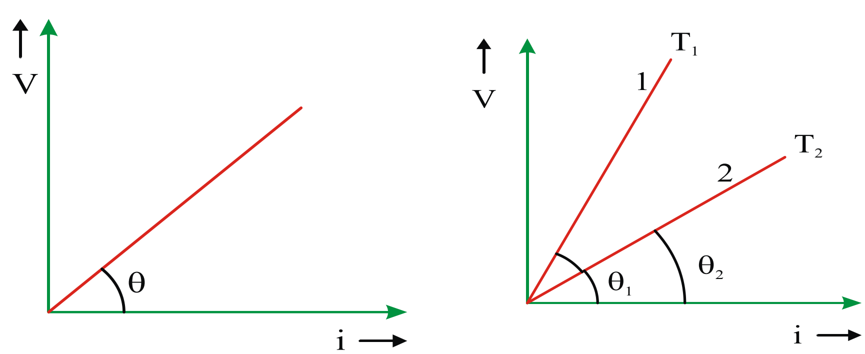
Where R is electrical resistance of the conductor

Note :

- ohm's law is neither a basic law nor a deriavable one
- ohm's law is just an empherical relation.
- Microscopically Ohm's law is expressed as

  $J = nev_d \Rightarrow J = \sigma E$ where σ is the electrical conductivity of the material.
- The conductors which obey Ohm's law are called Ohmic conductors.

  Ex : all metals
- For Ohmic conductors V – i graph is a straight line passing through origin (metals).



(A) Slope of the line

(B) Here $\tan\theta_1 > \tan\theta_2$

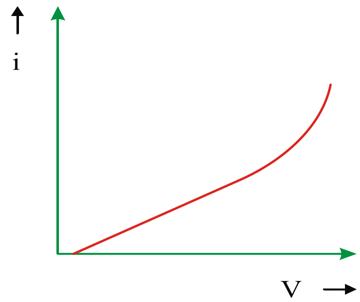
$$\tan\theta = v / i = R$$

$$T_1 > T_2$$

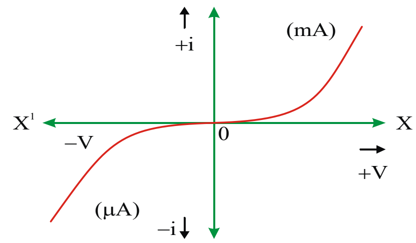
$$\text{So } R_1 > R_2 \text{ i.e}$$

- The substances which do not obey Ohm's law are called non-Ohmic conductors.

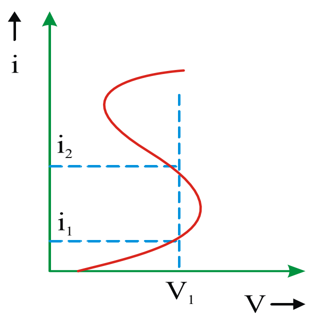
  Ex: Thermistor, Electronic Valve, Semi-conductor devices, gases, crystal rectifier etc.,
- The V – i graph for a non – Ohmic conductor is non-linear.



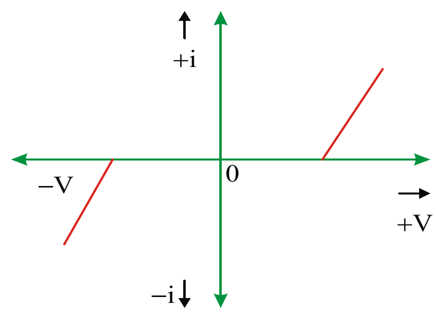
vaccum tube



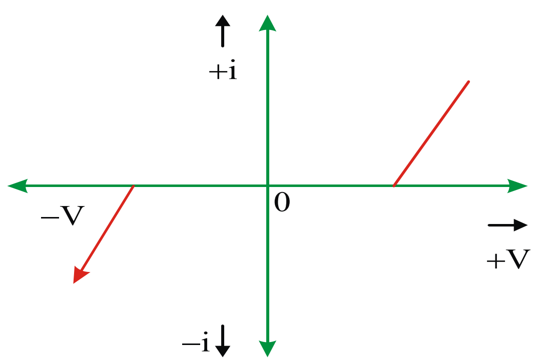
semi conductors



Thermistor



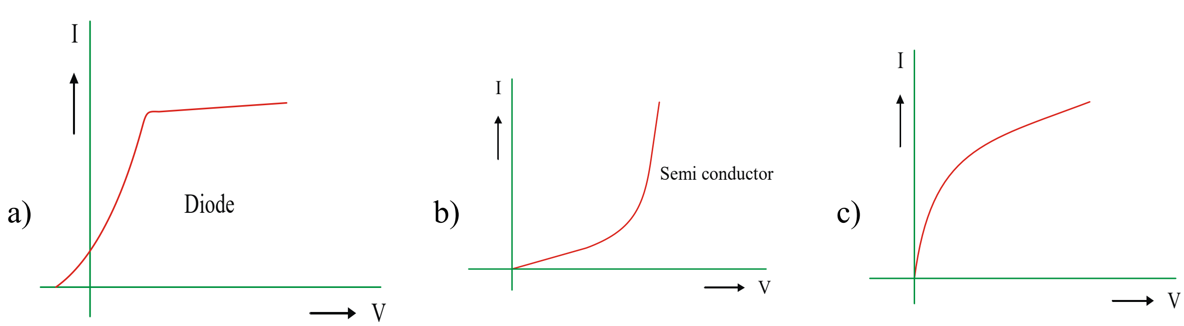
dilH_2SO_4(Platinum Electrode)



Neon Gas(With tungsten Electrode)

Non - Ohmic Circuits :

The circuits in which Ohm's law is not obeyed are called non-ohmic circuits. The V-I graph is a curve, e.g. torch bulb, electrolyte, semiconductors, thermonic valves etc. as shown by curves (a), (b), (c).



Resistance-Definiton :

The resistance of a conductor is defined as the ratio of the potential difference 'V' across the condutor to the current 'i' flowing through the conductor.

Resistance $R = \frac{V}{i}$

- The resistance of a conductor depends upon
  1) shape (dimensions) 2) nature of material 3) impurities 4) Temperature
- The resistance of a conductor increases with impurities.
- The resistance of a semi conductor decreases with impurities.

Factors Effecting the Resistance of A Conductor

1. The resistance of the conductor is directly proportional to the length (l) of the conductor i.e.

$$R \alpha l \text{ (or) } \frac{R_1}{R_2} = \frac{l_1}{l_2}$$

For small changes in the length, $\frac{\Delta R}{R} = \frac{\Delta l}{l}$

2. The resistance of a conductor is inversely proportional to the area of cross-section (A)

i.e., $R \alpha \frac{1}{A}$ (or) $R \propto \frac{1}{r^2}$; $\frac{R_1}{R_2} = \left(\frac{A_2}{A_1}\right) = \left(\frac{r_2^2}{r_1^2}\right)$

For small changes in area (or) radius we have $\frac{\Delta R}{R} = \frac{\Delta A}{A} = -\frac{2\Delta r}{r}$

3. As the temperature increases resistance of metallic conductors increases and that of semiconductors decreases.

Conductance:

The reciprocal of resistance (R) is called conductance.

$$\text{conductance, } G = \frac{1}{R}.$$

The S.I unit of conductance is mho or siemen or ohm^{-1}.

Resistivity:

As we know, that the resistance of the conductor is directly proportional to its length and inversely proportional to its area of cross section, we can write

$$R \propto \frac{l}{A} \Rightarrow R = \frac{\rho l}{A}$$

where ρ is specific resistance or resisitivity of the material of the conductor.

Note:

1. Resistitivity is the specific property of a material but Resistance is the bulk property of a conductor.
2. Resistivity is independent of dimensions of the conductor such as length, area of the cross section.
3. Resistivity depends on the nature of the material of the conductor, temperature and impurities.
4. Resistivity of any alloy is more than resistivity of its constituent elements.

i) $R_{alloys} > R_{conductors}$ ii) $\alpha_{metals} > \alpha_{alloys}$

Special Cases :

1. The alternate forms of resistance is

$$R = \rho\frac{l^2}{V} = \rho\frac{l^2 d}{m} = \frac{\rho V}{A^2} = \frac{\rho m}{d\,A^2}$$

Where d is density of material of conductor
V is volume of the conductor
m is mass of the conductor

2. If a conductor is streched or elongated or drawn or twisted, then the volume of the conductor is constant. Hence

a. $R = \frac{\rho l^2}{V} \Rightarrow R\alpha l^2$

b. $R = \frac{\rho V}{A^2} \Rightarrow R\alpha \frac{1}{A^2}\alpha\frac{1}{r^4}$

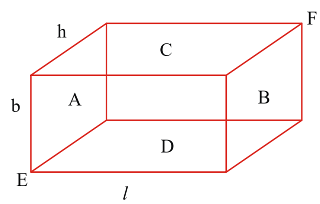
c. Interms of mass of the wire $R\alpha\frac{l^2}{m}$

and $R\alpha\frac{m}{A^2}\alpha\frac{m}{r^4}$

3. For small changes in the length or radius during the stretching

$$\frac{\Delta R}{R} = 2\frac{\Delta l}{l} \quad ; \quad \frac{\Delta R}{R} = -2\frac{\Delta A}{A} = -4\frac{\Delta r}{r}$$

4. In case of a cuboid of dimensions $l \times b \times h$ is



Resistance across AB, $R_{AB} = \frac{\rho l}{b \times h}$

Resistance across CD, $R_{CD} = \frac{\rho b}{l \times h}$

Resistance across EF, $R_{EF} = \frac{\rho h}{l \times b}$

If $l > b > h$, then

$$R_{\max} = \frac{\rho l}{b \times h} \qquad R_{\min} = \frac{\rho h}{l \times b}$$

5. If a wire of resistance R is stretched to 'n' times its original length, its resistance becomes n^2 R.

6. If a wire of resistance R is stretched until its radius becomes $\frac{1}{n}$ th of its original radius then its resistance

becomes $\mathbf{n^4R}$.

7. When a wire is stretched to increase its length by x% (where x is very small) its resistance increases by 2x %.
8. When a wire is stretched to increase its length by x% (where x is large) its resistance increases by $\left(2x + \frac{x^2}{100}\right)$.
9. When a wire is stretched to reduce its radius by x% (where x is very small), its resistance increases by 4x%.

Conductivity:

Conductivity is the measure of the ability of a material to conduct electric current through it. It is reciprocal of resistivity.

$$\sigma = \frac{1}{\rho} = \frac{l}{RA}$$

S.I unit : sieman / m : (Sm^{-1})

For perfect insulators $\sigma = 0$

For perfect conductors, σ is infinity.

Temperature dependence of resistance:

For conductors i.e metals resistance increases with rise in temperature

$R_t = R_o(1 + \alpha t + \beta t^2)$ for t > 300°C

$R_t = R_o(1 + \alpha t)$ for t < 300°C or $\qquad \alpha = \frac{R_t - R_o}{R_o t} \, /^0 C$

If R_0 = resistance of conductor at 0°C

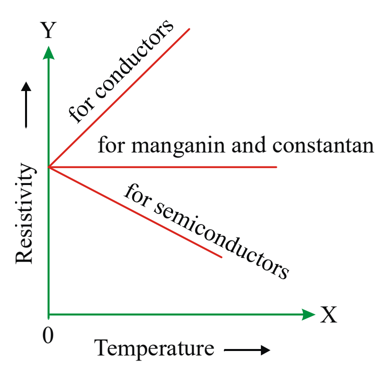
If R_t = resistance of conductor at t°C And α, β = temperature co-efficients of resistance

If R_1 and R_2 are the resistances at t_1°C to t_2°C respectively then $\frac{R_1}{R_2} = \frac{1 + \alpha t_1}{1 + \alpha t_2}$

$$\therefore \alpha = \frac{R_2 - R_1}{R_1 t_2 - R_2 t_1}$$

The value of α is different at different temperatures.

At a given temperature $\alpha = \frac{1}{R_t}\left(\frac{dR}{dt}\right)$ at t^0C



Graph shows the variation of resistivity with temperature for conductors, semiconductors and for alloys like manganin and constantan.
Since the resistivity of manganin and
constantan remains constant with respect to change in temperature, these materials are used for the bridge wires and resistance coils.

- The resistivity of manganin and constantan is almost independent of temperature.
- **Two resistors having resistances R_1 and R_2 at $0^o C$ are connected in series. The condition for the effective resistance in series in same at all temperatures**

$$R_1 + R_2 = R_1^{'} + R_2^{'}$$

$$R_1 + R_2 = R_1(1+\alpha_1 t) + R_2(1+\alpha_2 t)$$

$$R_1\alpha_1 = -R_2\alpha_2$$

Variation of resistance of some materials

Material	Temp. coefficient of resistance (α)	Variation of resistance with temperature rise
Metals	Positive	Increases
Solid non-metal	Zero	independents
Semi-conductor	Negative	Decreases
Electrolyte	Negative	Decreases
Ionized gases	Negative	Decreases
Alloys	Small positive value	Almost constant

Variation of Resistivity with Temperature:

If ρ_1 is the resistivity of a material at temperature t_1 and ρ_2 is the resistivity of the same material at temperature t_2 ,then

$$\rho_2 = \rho_1\left[1+\alpha\left(t_2 - t_1\right)\right]$$

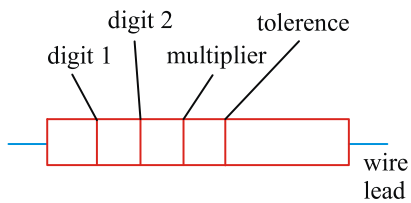
Thermistor:

A thermistor is a heat sensitive and non-ohmic device.

- This is made of semiconductor compounds as oxides of Ni, Fe, Co etc.
- This will have high +ve (or) -ve temperature coefficient of resistance.
- Thermistor with -ve 'α' are used as resistance
  themometers which can measure low temperature of order of 10K and small changes of in the order of 10^{-3}K.
- Having -ve α, these are widely used in measuring the rate of energy flow in micro wave beam.
- Thermistor can also be used to serve as thermostat.

Resistor Colour codes

Colour	Number	Multiplier	Tolerance(%)
Black	0	$\times 10^0$	
Brown	1	$\times 10^1$	
Red	2	$\times 10^2$	
Orange	3	$\times 10^3$	
Yellow	4	$\times 10^4$	
Green	5	$\times 10^5$	
Blue	6	$\times 10^6$	
Violet	7	$\times 10^7$	
Gray	8	$\times 10^8$	
White	9	$\times 10^9$	–
Gold	–	$\times 10^{-1}$	$\pm 5\%$
Silver	–	$\times 10^{-2}$	$\pm 10\%$
No clour	–		$\pm 20\%$



Colour bands on a resistor:

B.B.ROY of **G**reat **B**ritain having **V**ery **G**ood **W**ife with **G**old and **S**ilver

Resistors in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits.

Super Conductor :

There are certain metals for which the resistance suddenly falls to zero below certain temp. Called critical temperature.

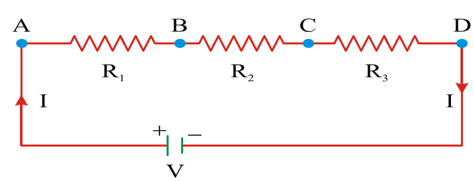
↳ Critical temperature depends on the nature of material. The materials in this state are called super conductors.

↳ Without any applied emf steady current can be maintained in super conductors.

Ex:

Hg below 4.2 K or Pb below 8.2K

Resistances In Series:

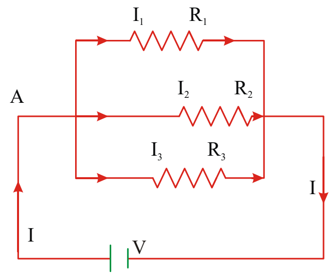


1. If resistors of resistances R_1, R_2, R_3, ... are connected in series, the resultant resistance $R = R_1 + R_2 + R_3 + \ldots\ldots$
2. When resistances are connected in series, same current passes through each resistor. But the potential differences are in the ratio $V_1 : V_2 : V_3 \ldots\ldots = R_1 : R_2 : R_3 \ldots\ldots$
3. When resistors are joined in series, the effective resistance is greater than the greatest resistance in the circuit.

4. When two resistances are connected in series then

$$V_1 = \frac{VR_1}{R_1 + R_2} \text{ and } V_2 = \frac{VR_2}{R_1 + R_2}$$

Resistances in Parallel



1. If resistors of resistance R_1, R_2, R_3 ....are connected in parallel, the resultant resistance R is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\dots\dots\dots$$

2. If resistances R_1 and R_2 are connected in parallel, the resultant resistance. $R = \frac{R_1 R_2}{R_1 + R_2}$

3. When resistors are joined in parallel the potential difference across each resistor is same. But the currents are in the ratio $i_1 : i_2 : i_3 :$............

$$= \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \dots\dots\dots\dots$$

4. When two resistances are parallel then

$$I_1 = \frac{IR_2}{R_1 + R_2} \text{ and } I_2 = \frac{IR_1}{R_1 + R_2}$$

Note:

1. When resistors are joined in parallel, the effective resistance is less than the least resistance in the circuit.
2. A wire of resistance 'R' is cut into 'n' equal parts and all of them are connected in parallel, equivalent resistance becomes $\frac{R}{n^2}$.
3. In 'n' wires of equal resistances are given, the number of combinations that can be made to give different resistances is 2^{n-1}.
4. If 'n' wire of unequal resistances are given, the number of combinations that can be made to give different resistances is 2^n (If n >2).
5. If R_s and R_p be the resultant resistances of R_1 and R_2 when connected in series and parallel then

$$R_1 = \frac{1}{2}\left(R_s + \sqrt{R_s^2 - 4R_sR_p}\right)$$

$$R_2 = \frac{1}{2}\left(R_s - \sqrt{R_s^2 - 4R_sR_p}\right)$$

6. If a uniform wire of resistance R is, stretched to 'm' times its initial length and bent into a regular polygon of 'n' sides

a) Resistance of the wire after stretching is

$R_1 = m^2 R (R' \alpha l^2)$

b) Resistance of each side $R_2 = \frac{m^2 R}{n}$

c) Resistance across diagonally opposite points $R_0 = \left(\frac{\frac{n}{2} R_2}{2} \right) \Rightarrow R_0 = \frac{m^2 R}{4}$

d) Resistance across one side

$R_3 = \frac{(n-1)}{n} R_2 = \frac{(n-1) m^2 R}{n^2}$

7. 12 wires each of resistance 'r' are connected to form a cube. Effective resistance across

a) Diagonally opposite corners $= \frac{5r}{6}$.

b) face diagonal $= \frac{3r}{4}$.

c) two adjacent corners $= \frac{7r}{12}$.

8. If two wires of resistivities ρ_1 and ρ_2, lengths l_1 and l_2 are connected in series, the equivalent resistivity

$\rho = \frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$.

If $l_1 = l_2$ then $\rho = \frac{\rho_1 + \rho_2}{2}$.

If $l_1 = l_2$ then conductivity $\sigma = \frac{2\sigma_1 \sigma_2}{\sigma_1 + \sigma_2}$.

9. If two wires of resistivities ρ_1 and ρ_2, Areas of cross section A_1 and A_2 are connected in parallel, the equivalent resistivity

$\rho = \frac{\rho_1 \rho_2 (A_1 + A_2)}{\rho_1 A_2 + \rho_2 A_1}$.

If $A_1 = A_2$ then $\rho = \frac{2\rho_1 \rho_2}{\rho_1 + \rho_2}$.

and conductivity $\sigma = \frac{\sigma_1 + \sigma_2}{2}$.

10. If 'n' wires each of resistance 'R' are connected to form a closed polygon, equivalent resistance across two adjacent corners is $R_{eff} = \left(\frac{n-1}{n} \right) R$

JOULE's LAW:

According to **Joule's law**, the current passing through a conductor produces heat.

$W = vit$

Now, work done, $W = (iR)\, i\, t$

$W = i^2 R t = \frac{v^2}{R} t = v\, i\, t$

This work is converted into energy in the conductor.

$\therefore$ Thermal energy produced, $Q = i^2 Rt$ in Joules

Or $Q = \frac{i^2 Rt}{4.2}$ in cal.

As $H \propto i^2$, heating effect of current is common to both A.C and D.C.

Joule's effect is irreversible.

Electrical Energy:

↳ The electric energy consumed in a circuit is defined as the total workdone in maintaing the current in an electric circuit for a given time.

Electrical Energy $= Vit = Pt = i^2 Rt = \frac{V^2 t}{R}$

S.I. unit of electric energy is **joule**

1 K.W.H. = 36×10^5 J

Electrical Power:

↳ The rate at which work is done in maintaining the current in electric circuit. Electrical power

$P = \frac{W}{t} = Vi = i^2 R = \frac{V^2}{R}$ watt (or) joule / sec

↳ Heat energy produced due to the electric current $H = \frac{W}{J} = \frac{Pt}{J} = \frac{E\,it}{J} = \frac{i^2 Rt}{J} = \frac{E^2 t}{RJ}$

$H = ms\Delta t$

Where $s = 4200 J/Kg^0C$

where J is mechanical equivalent of heat.

Fuse wire: A fuse wire generally prepared from tin - lead alloy (63% tin + 37% lead). **It should have high resistivity, low melting point.**

Let R be the resistance of fuse wire.

We know that $R = \frac{\rho L}{\pi r^2}$

(L and r denote length and radius)

The heat produced in the fuse wire is $H = i^2 R = \frac{i^2 \rho L}{\pi r^2}$

If H_0 is heat loss per unit surface area of the fuse wire, then heat radiated per second is $= H_0 2\pi r L$ At thermal equilibrium,

$$\frac{i^2 \rho^2 L}{\pi r^2} = H_0 2\pi r L$$

$$\text{(or) } H_0 = \frac{i^2\rho}{2\pi^2 r^3}$$

According to Newton's law of cooling.

$$H_0 = K\theta$$

Where θ is the increase in temperature of fuse wire and K is a constant.

$$\theta = \frac{i^2\rho}{2\pi^2 r^3 K}$$

Here θ is independent of length L of the fuse wire provided i remains constant.

For a given material of fuse wire $i^2 \alpha\, r^3$.

↳ **If radiation losses are neglected, due to heating effect of current the temperature of fuse wire will increase continuously, and it melt in time 't' such that**

$$H = ms\,\Delta\theta; \frac{I^2 Rt}{J} = ms(\theta_{mp} - \theta_r)$$

$$t = \frac{\pi^2 r^4 s\left(\theta_{mp} - \theta_r\right)J}{I^2\rho}; t\,\alpha\,r^4$$

i.e., in absence of radiation lossess, the time in which fuse will melt is also independent on length and varies with radius as r^4.

Note :

a) If resistances are connected in series , i.e.., I is same

$P\alpha R$ with $V\alpha R$ [$as\ V = IR$]

i.e.., in series potential difference and power consumed will be more in larger resistance.

However, if resistances are connected in parallel, i.e.., V is same

$P\alpha\frac{1}{R}$ with $I\alpha\frac{1}{R}$ [$as\ V = IR$]

i.e.., in parallel current and power consumed will be more in smaller resistance. This in turn implies that more power is consumed in larger resistance if reistances are in series and in smaller reistance if reistances are in parallel.

b) A reistance R under a potential difference V dissipates power.

$$P = \left(V^2/R\right)$$

So If the resistance is changed from R to (R/n) keeping V same, the power consumed will be

$$P^1 = \frac{V^2}{(R/n)} = n\frac{V^2}{R} = nP$$

i.e.., if for a given voltage, resistance is changed from R to (R/n) , power consumed changes from P to nP.

c) If n equal resistances are connected in series with a voltage source, the power dissipated will be

$$P_s = \frac{V^2}{nR} \quad [\ as\ R_s = nR\ ]$$

And if the same resistances are connected in parallel with the same voltage source

$$P_p = \frac{V^2}{(R/n)} = \frac{nV^2}{R} \quad [as\, R_p = (R/n)]$$

So, $\frac{P_p}{P_s} = n^2$ i.e.., $P_P = n^2 P_S$.

i.e.., power consumed by n equal resistors in parallel is n^2 times that of power consumed in series if V remains same.

d) As resistance of a given electric appliance (e.g.., bulb , heater, geyser or press) is constant and is given by,

$$R = \frac{V_S}{I} = \frac{V_S}{(W/V_S)} = \frac{V_s^2}{W} \quad [as\ I = \frac{W}{V}]$$

Where V_s and W are the voltage and wattage specified on the appliance. So if the applied voltage is different from specified, the ' actual power consumption' will be

$$P = \frac{V_A^2}{R} = \left(\frac{V_A}{V_S}\right)^2 \times W \quad [\text{ as } R = \frac{V_S^2}{W}].$$

Bulbs connected in Series:

↳ If Bulbs (or electrical appliances) are connected in series, the current through each resistance is same. Then power of the electrical appliance

$P \propto R$ & $V \propto R\left[\therefore P = i^2Rt\right]$

i.e. In series combination; the potential difference and power consumed will be more in larger resistance.

↳ When the appliances of power P_1, P_2, P_3.... are in series, the effective power consumed (P) is

$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + \ldots\ldots\ldots$ i.e. effective power is less than the power of individual appliance.

↳ If 'n' appliances, each of equal resistance 'R' are connected in series with a voltage source 'V', the power dissipated ' P_s ' will be $P_s = \frac{V^2}{nR}$.

Bulbs connected in parallel:

↳ If Bulbs (or electrical appliances) are connected in parallel, the potential difference across each resistance is same. Then $P \propto \frac{1}{R}$ and $I \propto \frac{1}{R}$.

i.e. The current and power consumed will be more in smaller resistance.

↳ When the appliances of power P_1, P_2, P_3.... are in parallel, the effective power consumed(P) is

$P = P_1 + P_2 + P_3 + \ldots\ldots\ldots$

i.e. the effective power of various electrical appliance is more than the power of individual appliance.

↳ If 'n' appliances, each of resistance 'R' are connected in parallel with a voltage source 'V', the power dissipated 'Pp' will be

$$P_P = \frac{V^2}{(R/n)} = \frac{nV^2}{R}$$

$$\frac{P_P}{P_S} = n^2 \left(or\right) P_P = n^2 P_S$$

This shows that power consumed by 'n' equal resistances in parallel is n^2 times that of power consumed in series if voltage remains same.

- In parallel grouping of bulbs across a given source of voltage, the bulb of greater wattage will give more brightness and will allow more current through it, but will have lesser resistance and same potential difference across it.
- For a given voltage V, if resistance is changed from 'R' to $\left(\frac{R}{n}\right)$, power consumed changes from 'P' to 'nP'

$P' = \frac{V^2}{R'}$ where $R' = \frac{R}{n}$, then

$$P' = \frac{V^2}{(R/n)} = \frac{nV^2}{R} = nP.$$

- If t_1, t_2 are the time taken by two different coils for producing same heat with same supply, then

  If they are connected in series to produce same heat, time taken $t = t_1 + t_2$

  If they are connected in parallel to produce same heat, time taken is $t = \frac{t_1 t_2}{t_1 + t_2}$.

Consumption of Electrical Energy:

- Units of electrical energy consumed by an electrical appliance =

$$\frac{\text{Number of watts} \times \text{Number of hours}}{1000}$$

It is in KWH.

CELLS

Primary Cells:

Voltaic, Leclanche, Daniel and Dry cells are primary cells. They convert chemical energy into electrical energy. They can't be recharged. They supply small currents.

Secondary Cells (or) Storage Cells:

Electrical energy is first converted into chemical energy and then the stored chemical energy is converted into electrical energy due to these cells.

- These cells can be recharged.
- The internal resistance of a secondary cell is low where as the internal resistance of a primary cell is large.

EMF of a Cell:

The energy supplied by the battery to drive unit charge around the circuit is defined as electro motive force of the cell.

- EMF is also defined as the absolute potential difference between the terminals of a source when no energy is drawn from it. i.e., in the open circuit of the cell. It depends on the nature of electrolyte used in the cell. Unit :J/C (or) Volt

emf of a cell depends on

a) metal of electrodes

b) nature of electrolyte

c) temperature

emf of the cell is independent of

a) area of plate
b) quantitiy of electrolyte
c) distance between plate
d) size of the cell

Internal Resistance of a Cell

↳ It is the resistance offered by the electrolyte of the cell.
It depends on

↳ area of the electrodes used ($r \alpha \frac{1}{A}$)

↳ nature of electrolyte , concentration($r \alpha C$)

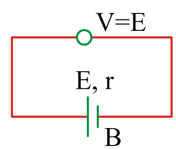
↳ area of cross section of the electrolyte through which the current flows and

↳ age of the cell.

↳ Internal resistance of an ideal cell is zero.

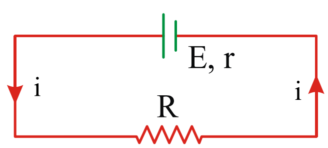
Terminal Voltage:

When no current flows through the cell, the circuit is said to be an open circuit. This is shown in figure.



In such a case, the potential difference (p.d) across the terminals of the cell, called the terminal voltage (V) will be equal to the emf (E) of the cell.

If an external resistance R is connected across the two terminals of the cell, as in figure then current flows in the closed circuit.,



$$i = \frac{V}{R} \qquad \text{........... (1)}$$

and also $$i = \frac{E}{(R+r)} \qquad \text{........... (2)}$$

iR + ir = E, V +ir = E, V = E – ir

Lost volts:

It is the difference between emf and P.D. of a cell It is used in driving the current between terminals of the cell.

Lost volts **E - V =** i **r**

Note: Formulae related with cells

$$i = \frac{E-V}{r} \qquad \text{..................(A)}$$

$$r = \frac{E-V}{i} \qquad \text{....................(B)}$$

$$r=\left(\frac{E-V}{V/R}\right)=\left(\frac{E-V}{V}\right)R=\left(\frac{E}{V}-1\right)R \quad .... \text{(C)}$$

↳ $V = iR = \dfrac{ER}{(R+r)}$

↳ Fractional energy useful $=\dfrac{V}{E}=\dfrac{R}{R+r}$

↳ % of fractional useful energy

$$=\left(\frac{V}{E}\right)100=\left(\frac{R}{R+r}\right)100$$

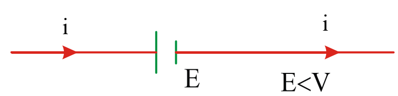
↳ Fractional energy lost, $\dfrac{V'}{E}=\dfrac{r}{R+r}$

↳ % of lost energy, $\left(\dfrac{V'}{E}\right)100=\left(\dfrac{r}{R+r}\right)100$

↳ internal resistance, $r=\left[\dfrac{E-V}{V}\right]R$

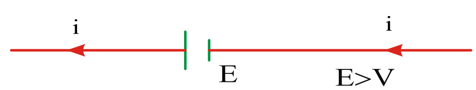
Different Concepts with cell

↳ When the cell is charging, the EMF is less than the terminal voltage ($E < V$) and the direction of current inside the cell is from +ve terminal to the –ve terminal.



$V = E + i\,r$

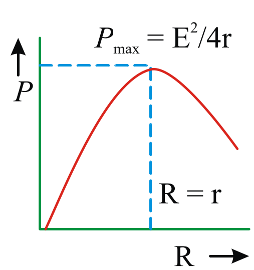
↳ When the cell is discharging, the EMF is greater than the terminal voltage ($E > V$) and the direction of current inside the cell is from –ve terminal to the +ve terminal.



$V = E - i\,r$; Hence $E > V$

↳ Power delivered will be maximum when $R = r$. So $P_{max} = \dfrac{E^2}{4r}$

↳ This statement in generalized form is called 'maximum power transfer theorem'



Here the % of energy lost and energy useful are each equal to 50%

Back EMF:

When current flows through the electrolyte solution, electrolysis takes place with a layer of hydrogen and this hinders the flow of current. In the neighbourhood of both electrodes, the concentrations of ions get altered. This opposing EMF is called back EMF and the phenomenon is called Electrolytic polarisation.

To reduce back emf manganese dioxide (or) potassium dichromate is added to electrolyte of cell.

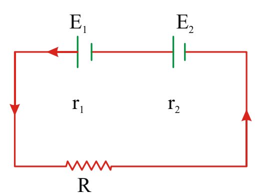
Grouping of Cells

Electric Cells in Series:

When 'n' identical cells each of EMF 'E' and internal resistance 'r' are connected in series to an external resistance 'R', then

- total emf of the combination $= nE$
- effective internal resistance $= nr$
- Current through external resistance $i = \dfrac{nE}{R + nr}$
- If $R << nr$ then $i = \dfrac{E}{r}$ = current from one cell
- If $R >> nr$ then $i = \dfrac{nE}{R}$
- If two cells of different emf's are in series

$$E_{eq} = E_1 + E_2 \;;\; r_{eq} = r_1 + r_2 \;;\; i = \frac{E_1 + E_2}{r_1 + r_2 + R}$$

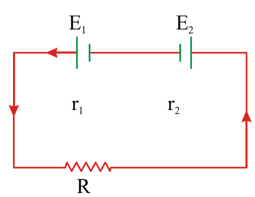


T.P.D across the first cell $V_1 = E_1 - ir_1$

T.P.D across the second cell $V_2 = E_2 - ir_2$

- If one of the cell is in reverse connection $(E_1 > E_2)$ then $E_{eq} = E_1 - E_2$

$$r_{eq} = r_1 + r_2 \;;\; i = \frac{E_1 - E_2}{r_1 + r_2 + R}$$



First cell is discharging then $V_1 = E_1 - ir_1$

Second cell is charging then $V_2 = E_2 + ir_2$

cell having less emf in charging state.

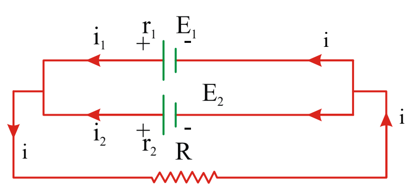
Wrongly Connected Cells

- By mistake if 'm' cells out of 'n' cells are wrongly connected to the external resistance 'R'
  a) total emf of the combination = (n – 2m) E
  b) total internal resistance = n r
  c) total resistance = R + n r
  d) current through the circuit (i) = $\frac{(n-2m)E}{R+nr}$

Electric Cells in Parallel

When 'n' identical cells each of EMF 'E' and internal resistance 'r' are connected in parallel to an external resistance 'R', then

- total emf of the combination = E
- effective internal resistance = $\frac{r}{n}$
- total resistance in the circuit = R + $\frac{r}{n}$
- **current through the external resistance** $i = \frac{E}{R+\frac{r}{n}} = \frac{nE}{nR+r}$
- If R >> $\frac{r}{n}$, then i = $\frac{E}{R}$ = current from one cell.
- If R << $\frac{r}{n}$, then i = $\frac{nE}{r}$
- If two cells of emf E_1 and E_2 having internal resistances r_1 and r_2 are connected in parallel to an external resistance 'R', then



the effective emf, $E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$

the effective internal resistance, $r_{eff} = \frac{r_1 r_2}{r_1 + r_2}$

Current through the circuit, $i = \frac{E}{r_{eff} + R}$

$i = i_1 + i_2$

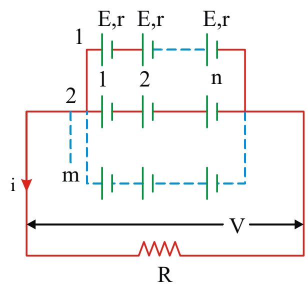
$i_1 = \frac{E_1 - iR}{r_1}$ and $i_2 = \frac{E_2 - iR}{r_2}$

Potential difference across R, i.e terminal potential of the cells is $V = iR = \frac{ER}{R + r_{eff}}$

↳ When the cell E_2 is reversed in polarity then we should use - E_2 in all the above equations.

Mixed Grouping:

If n identical cell's are connected in a row and such m rows are connected in parallel as then



↳ Equivalent emf of the combination $\mathbf{E_{eq} = nE}$

↳ Equivalent internal resistance of the combination $\mathbf{r_{eq}} = \frac{nr}{m}$

↳ Main current flowing through the load $i = \frac{nE}{R + \frac{nr}{m}} = \frac{nmE}{mR + nr}$

↳ Condition for maximum power $R = \frac{nr}{m}$ and $P_{max} = (mn)\frac{E^2}{4r}$

↳ **Condition for maximum current**

$$\frac{R}{n} + \frac{r}{m} = \text{minimum}$$

$$\frac{d}{dm}\left[\frac{mR}{N} + \frac{r}{m}\right] = 0 \; ; \left[n = \frac{N}{m}\right]$$

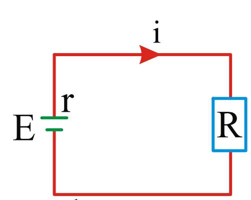
$$\frac{R}{N} - \frac{r}{m^2} = 0; i.e., \frac{R}{n} = \frac{r}{m} \qquad \mathbf{(N = n \times m)}$$

So in case of mixed grouping of cells, current in the circuit will be maximum when $\left(\frac{R}{n} = \frac{r}{m}\right)$

$$I_{max} = \frac{nE}{2R} = \frac{mE}{2r}$$

↳ Total number of cells = $m \times n$

Maximum power transfer theorem



Consider a device of resistance R connected to a source of e.m.f E and internal resistance r as shown.

Current in the circuit is $i = \left(\dfrac{E}{R+r}\right)$.

Power dissipated in the device is $P = i^2R$

$$\Rightarrow P = \frac{E^2R}{(R+r)^2}$$

For maximum power dissipated in the device

$$\frac{dP}{dR} = 0 \Rightarrow \frac{d}{dR}\left[\frac{E^2R}{(R+r)^2}\right] = 0$$

On simplification, we can get $R = r$

So, the power dissipated in an external resistance is maximum if that resistance is equal to internal resistance of the source supplying the current to that device.

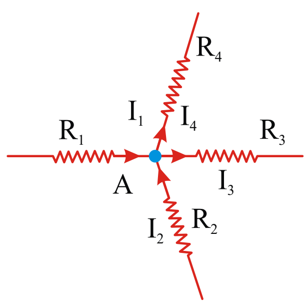
KIRCHHOFF'S LAWS

When the circuit is complecated to find current kirchhoff's laws are formulated.

i)Kirchhoff s First Law (Junction Law or Current law) : It states that the sum of the currents flowing into a junction is equal to the sum of the currents flowing out of the junciton.

Or

"The algebraic sum of currents at a junction is zero".



Distribution of current at a junction in the circuit

$I_1 + I_2 = I_3 + I_4$ or $I_1 + I_2 - I_3 - I_4 = 0$

If we take currents approaching point A in Fig as positive and that leaving the point as negative, then the above relation may be written as

$$I_1 + I_2 + (-I_3) + (-I_4) = 0$$

$$\Sigma I = 0$$

Note:

Thus, Kirchhoff's first law is accordance with **law of conservation of charge**, since no charge can accumulate at a junction.

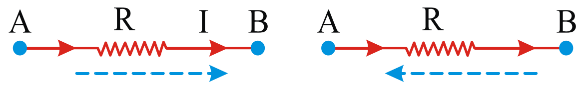
(ii) Kirchhoff s Second Law (Loop Law or Potential law) : Kirchhoff's second law states that the algebraic sum of changes in potential around any closed loop is zero.

(Kirchhoff's second law) can be expressed as $\Sigma V = 0$.

In terms of potential drops and emfs, the law is expressed as $\Sigma(iR) + \Sigma E = 0$

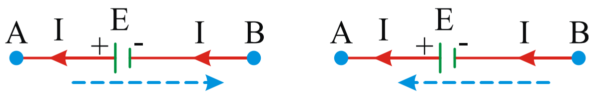
Sign conventions:

(a) The change in potential in traversing a resistance in the direction of current is –IR while in the opposite direction +IR as shown in the figure.



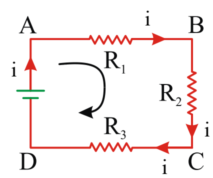
$V_A - IR = V_B$	$V_B + IR = V_A$
$V_A - V_B = IR$	$V_A - V_B = IR$

(b) The change in potential in traversing an emf source from negative to positive is +E while in the opposite direction –E irrespective of the direction of current in the circuit as shown in the figure.



$V_A - E = V_B$	$V_B + E = V_A$
$V_A - V_B = E$	$V_A - V_B = E$

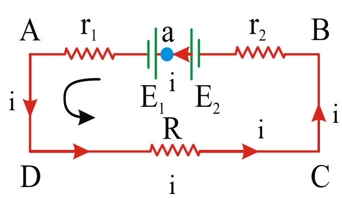
Example 1:



Apply the kirchhoff's second law to the loop ABCDA, then

$$-iR_1 - iR_2 - iR_3 + E = 0\ ;\ \therefore i = \frac{E}{(R_1 + R_2 + R_3)}$$

Example 2:



Apply the kirchhoff's second law to the loop ADCBA, then

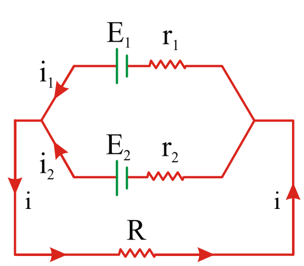
$$-iR - ir_2 + E_2 - E_1 - ir_1 = 0$$

$$i(r_1 + R + r_2) = E_2 - E_1 \Rightarrow i = \frac{E_2 - E_1}{r_1 + r_2 + R}$$

Note:

1) This law represents "**conservation of energy**"
2) If there are n meshes in a circuit, the number of independent equations in accordance with loop rule will be (n – 1).

Application : This is the most general case of parallel grouping in which E and r of different cells are different and the positive terminals cells are connected as shown



Kirchhoff's second law in diferent loops gives the following equations,

$E_1 - iR - i_1 r_1 = 0$

or $i_1 = \frac{E_1}{r_1} - \frac{iR}{r_1}$ **................ (1)**

$E_2 - iR - i_2 r_2 = 0$

$i_2 = \frac{E_2}{r_2} - \frac{iR}{r_2}$ **............... (2)**

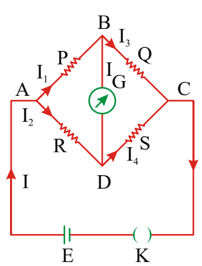
Adding Eqs. (1), (2) we get

$i_1 + i_2 = (E_1 / r_1) + (E_2 / r_2) - iR(1/r_1 + 1/r_2)$

or $i[1 + R(1/r_1 + 1/r_2)] = (E_1 / r_1) + (E_2 / r_2)$

$\therefore i = \frac{(E_1 / r_1) + (E_2 / r_2)}{1 + R(1/r_1 + 1/r_2)}$

WHEATSTONE BRIDGE



Condition for balancing of bridge :

Applying Kirchhoff's first law at junction B and D we get $I_1 - I_3 - I_G = 0$; and $I_2 + I_G - I_4 = 0$

Applying Kirchhoff's second law for closed loop ABDA, $-I_1 P - I_G G + I_2 R = 0$

Applying Kirchhoff's second law for closed loop BCDB , $-I_3 Q + I_4 S + I_G G = 0$

The values of P, Q, R, S are adjusted such that I_G becomes zero. At this stage the bridge is set to be in balance condition.

i.e., In balanced condition of bridge $I_G = 0$

$\Rightarrow$ In balanced condition the above equations respectively become

$I_1 = I_3$ ....(1)

and $I_2 = I_4$ ....(2)

$I_1 P = I_2 R$ ....(3)

$I_3 Q = I_4 S$ ....(4)

Dividing equation (3) by equation (4)

$$\frac{I_1 P}{I_3 Q} = \frac{I_2 R}{I_4 S}$$

Using eqns (1) and (2) we get $\frac{P}{Q} = \frac{R}{S}$ ....(5)

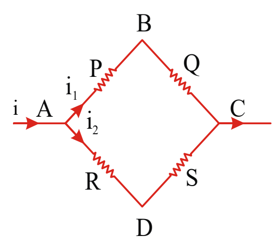
This is the balancing condition for Wheat stone bridge.

Applications of Wheats one Bridge

1. We can compare two unknown resistances R and S from $\frac{P}{Q} = \frac{R}{S}$
2. In place of resistances we can use capacitors to form a D.C. Wheatstone bridge with four capacitors of capacitances C_1, C_2, C_3 and C_4. The balancing condition will be $\frac{C_1}{C_2} = \frac{C_3}{C_4}$
3. It has been found that the bridge has the greatest sensitivity when the resistances are as nearly equal as possible.
   The bridge is most sensitive if P=Q=R=S.
4. Equivalent resistance of balanced bridge across the ends of battery when the bridge is balanced is given by
   $$\frac{(P+Q)(R+S)}{P+Q+R+S}$$
5. There are seven closed meshes in wheatstone's bridge

Application :

Direction of current in an unbalanced wheatstone's bridge :



$$V_{AB} = V_A - V_B = i_1 P = i\frac{(R+S)P}{P+Q+R+S}$$

$$V_{AD} = V_A - V_D = i_2 R = i\frac{(P+Q)R}{P+Q+R+S}$$

$$(V_B - V_D) = \frac{i}{P+Q+R+S}\left[(P+Q)R - (R+S)P\right]$$

$$= \frac{i}{P+Q+R+S}[QR - PS]$$

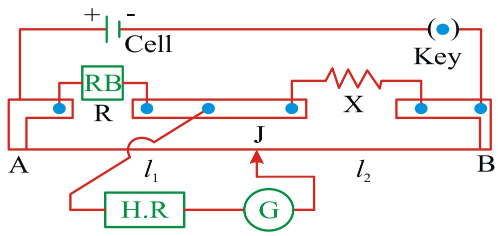
$if\ QR > PS,\ V_B > V_D \Rightarrow$ current flows from B to D

$QR < PS, V_B < V_D \Rightarrow$ current flows from D to B

$QR = PS,\ V_B = V_D \Rightarrow$ Balanced bridge

Metre bridge:

It works on the principle of Wheatstone Bridge $\left(\frac{P}{Q}=\frac{R}{S}\right)$



↳ When the Meter bridge is balanced then

$$\frac{R}{X}=\frac{l_1}{l_2}=\frac{l_1}{100-l_1}$$

Where l_1 *is* the balancing length from the left end.

Note:

1. If resistance in the left gap increases or resistance in the right gap decreases, balancing point shifts towards right side.
2. If resistance in the left gap decreases or resistance in the right gap increases, balancing point shifts towards left.
3. If a cm, b cm are the end corrections at A and B, then $\frac{R}{X}=\frac{l_1+a}{l_2+b}$
4. Meter bridge is more sensitive if $l_1 = 50$ cm
5. The resistance of copper strip is called end resistance

Potentiometer:

Potentiometer is an instrument which can measure accurately the emf of a source or the potential difference across any part of an electric circuit without drawing any current.

a) Principle :

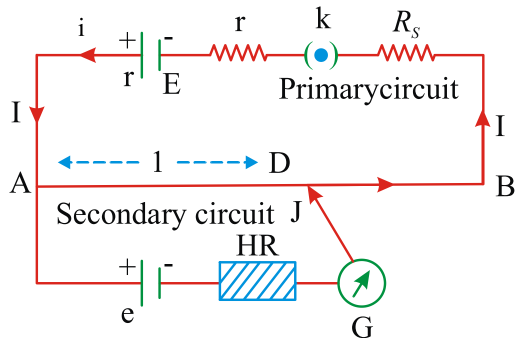
The principle of potentiometer states that when a constant current is passed through a wire of uniform area of cross-section, the potential drop across any portion of the wire is directly proportional to the length of that portion.

The principle of potentiometer require that

i) potentiometer wire should be of uniform area of cross-section and

ii) current through the wire should remain constant.

b) Theory of potentiometer : The end of the potentiometer wire AB are connected to a standard cell of emf E or a source of emf E that supplies constant current. The current through the potentiometer wire can be varied by means of a series resistance R_s which is adjustable.



Let r be the internal resistance of the cell of emf E connected across the potentiometer wire of length L and resistance R. The current through the potentiometer wire is

$$I = \frac{E}{r + R + R_s}$$

The potential of the wire decreases from the end A to the end B. The potential fall or potential drop across a length l of the potentiometer wire is

V = Current x Resistance of length l of the potentiometer wire $= I \times \left(\frac{R}{L}\right) l$

If the resistance per unit length of the wire , $\frac{R}{L}$ is denoted by ρ , the potential drop across the wire is

$$V = I \times \rho \times \ell$$

$\frac{V}{l}$ is called potential drop per unit length of the potentiometer wire or potential gradient of the wire. It is given by

$$\frac{V}{l} = I\rho \quad = \left(\frac{E}{r + R + R_S}\right)\frac{R}{L}$$

Thus, the unknown voltage V is measured when no current is drawn from it.

1) When specific resistance (S) of potentiometer wire is given then potential gradiant

$$X = \frac{IS}{A} = \frac{IS}{\pi r^2}$$

where A = area of cross - section of potentiometer wire r = Radius of potentiometer wire.

2) When two wires of length L_1 *and* L_2 and

resistance R_1 *and* R_2 are joined together to form the potentiometer wire, then $\frac{x_1}{x_2} = \frac{R_1 L_2}{L_1 R_2}$

Potential gradient depends on

a) Resistance per unit length of the potentiometer wire (ρ =R/L)

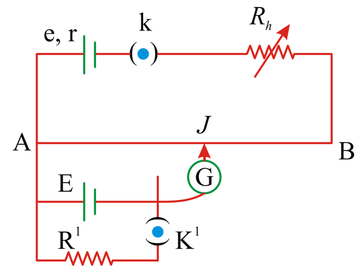
b) Radius of crosssection of the potentiometer wire, when the series resistance is included in the circuit and cell in the primary circuit is not ideal.

c) Current flowing through potentiometer wire.

d) emf of the cell in primary circuit

e) Series resistance in the primary circuit
f) Total length (L) and resistance (R) of the potentiometer wire.
g) If cell in primary circuit is ideal and in the absence of series resistance potential gradient **only depends on emf of cell** in primary circuit and length of potentiometer wire

To determine the internal resistance of a primary cell:



↪ Initially in secondary circuit key K' remains open and balancing length (l_1) is obtained. Since cell E is in open circuit so it's emf balances on length l_1

i.e $E = xl_1$ .....(i)

↪ Now key K ' is closed so cell E comes in closed circuit. If the process of balancing is repeated again keeping constant then potential difference V balances on length l_2

i.e $V = xl_2$ .....(ii)

↪ **By using formula internal resistance** $r = \left(\frac{E}{V} - 1\right).R'$

Where E = emf of cell in secondary circuit
V = Terminal voltage

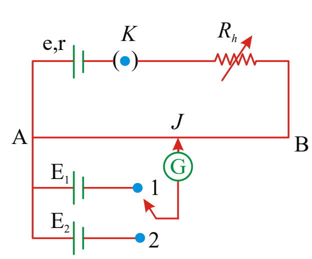
i.e p.d on R, $r = \left(\frac{l_1 - l_2}{l_2}\right)R'$

$\because \frac{E}{V} = \frac{l_1}{l_2}$, $\frac{E}{V} - 1 = \frac{l_1}{l_2} - 1$

Comparison of emf's of two cells

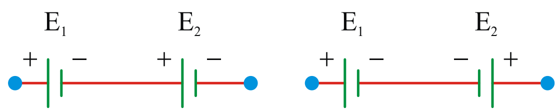
↪ Let l_1 and l_2 be the balancing length with the cell E_1 and E_2 respectively, then $E_1 = xl_1$ and

$E_2 = xl_2 \Rightarrow \frac{E_1}{E_2} = \frac{l_1}{l_2}$



↪ Let $E_1 > E_2$ and both are connected in series. If balancing length is l_1 when cells assist each other and it is

l_2 when they oppose each other as shown then:

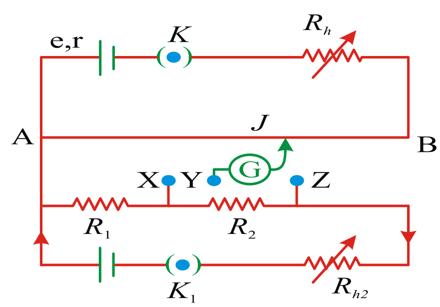


$(E_1 + E_2) = xl_1 \qquad (E_1 - E_2) = xl_2$

$$\Rightarrow \frac{E_1 + E_2}{E_1 - E_2} = \frac{l_1}{l_2} \quad \text{(or)} \quad \frac{E_1}{E_2} = \frac{l_1 + l_2}{l_1 - l_2}$$

Comparison of resistances:

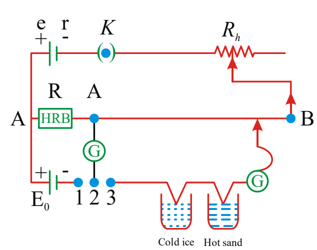
Let the balancing length for resistance R_1 (when XY is connected) be l_1 and let balancing length for resistance $R_1 + R_2$ (when YZ is connected) be l_2. keeping X constant



Then $iR_1 = xl_1$ and

$$i(R_1 + R_2) = xl_2 \Rightarrow \frac{R_2}{R_1} = \frac{l_2 - l_1}{l_1}$$

To determine thermo emf:



- The value of thermo-emf in a thermocouple for ordinary temperature difference is very low $(10^{-6}\, volt)$. For this the potential gradient x must be also very low $(10^{-4} V/m)$. Hence a high resistance (R) is connected in series with the potentiometer wire in order to reduce current in the primary circuit
- The potential difference across R must be equal to the emf of standard cell

  i.e $iR = E_0 \qquad \therefore i = \frac{E_0}{R}$
- **The small thermo emf produced in the** thermocouple $e = xl$
- $x = i\rho = \frac{iR^{|}}{L} \qquad \therefore \; e = \frac{iR^{|} l}{L}$

where L = Length of potentiometer wire,

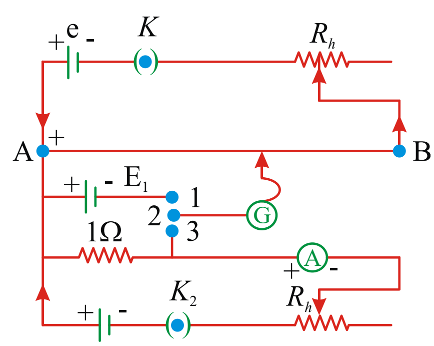
ρ =resistance per unit length, l = balancing length of e and

R^{l} = Resistance of potentiometer wire

SENSITIVITY OF POTENTIO METER

1. Sensitivity of potentio meter is estimated by its potential gradient.
2. Sensitivity is inversly proportional to potential gradient so lower the potential gradient higher will be the sensitivity.
3. The best instrument for accurate measurement of e.m.f. of a cell is potentiometer, because it does not draw any current from the cell.

Calibration of ammeter: Checking the correctness of ammeter readings with the help of potentiometer is called calibration of ammeter.



↳ For the calibration of an ammeter, 1Ω resistance coil is specifically used in the secondary circuit of the potentiometer, because the potential difference across 1Ω is equal to the current following through it i.e $V = i$

↳ If the balancing length for the emf

E_0 is l_0 then $E_0 = xl_0 \Rightarrow x = \frac{E_0}{l_0}$ (Process of standardisation)

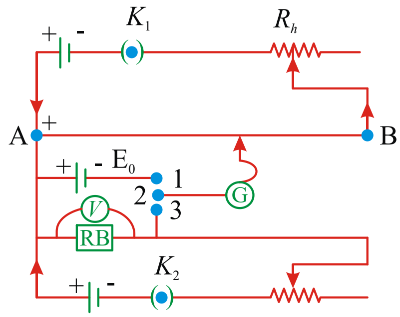
↳ Let i' current flows through 1Ω resistance giving potential difference as $V' = i'(1) = xl_1$ where l_1 is the balancing length. so error can be found as $\Delta i = i - i' = i - xl_1 = i - \frac{E_0}{l_0} \times l_1$

Here i is ammeter reading

Calibration of voltmeter:

↳ Checking the correctness of voltmeter readings with the help of potentiometer is called calibration of voltmeter.

↳ If l_0 is balancing length for E_0 the emf of standard cell by connecting 1 and 2 of bi-directional key, then $x = E_0 / l_0$



↳ The balancing length l_1 for unknown potential difference V' is given by (closing 2 and 3) $V' = xl_1 = \left(\frac{E_0}{l_0}\right)l_1$

If the voltmeter reading is V then the error will be $\left(V - V'\right)$ which may be +ve, -ve or zero

PROBLEMS

1. In a hydrogen atom, electron moves in an orbit of radius 5 × 10^{-11} m with a speed of 2.2 × 10^6 m/s. Calculate the equivalent current.

SOLUTION :

$$\text{Current } i = f.e = \frac{v}{2\pi r}.e$$

$$= \frac{2.2\times10^{6}}{2\pi\times5\times10^{-11}} \times 1.6 \times 10^{-19}$$

$$= 1.12 \times 10^{-3} \text{ amp} = 1.12 \text{ mA.}$$

2. The current through a wire depends on time as $i = i_0 + \alpha t$, where $i_0 = 10A$ and $\alpha = 4A/s$. Find the charge that crossed through a section of the wire in 10 seconds.

SOLUTION :

$$i = i_0 + \alpha t \text{ ; but } i = \frac{dq}{dt}$$

$$\Rightarrow dq = (i_0 + \alpha t)dt$$

$$q = \int_{t=0}^{t=10} dq \Rightarrow q = \left[i_0 t + \frac{\alpha t^2}{2} \right]_0^{10}$$

$$= (10 i_0 + 50\alpha) = 300 \text{ coloumb}$$

3. Consider a wire of length 4m and cross-sectional area I mm^2 carrying a current of 2A. If each cubic metre of the material contains 10^{29} free electrons, find the average time taken by an elctron to cross the length of the wire.

SOLUTION :

$$v_d = \frac{i}{nAe} = \frac{2}{10^{29}\times10^{-6}\times1.6\times10^{-19}} ms^{-1}$$

$$= 12.5 \times 10^{-4} \text{ ms}^{-1}$$

Average time taken by an electron to cross the length of wire

$$t = \frac{l}{v_d} = \frac{4}{1.25\times10^{-4}} \text{ s} = 3.2 \times 10^4 \text{ s}$$

4. The electron of hydrogen atom is considered to be revolving around the proton in circular orbit of radius $\frac{\hbar^2}{me^2}$ with velocity $\frac{e^2}{\hbar}$, where $\hbar = \frac{h}{2\pi}$. The current I is

1) $\frac{4\pi^2 me^2}{h^2}$ 2) $\frac{4\pi^2 me^2}{h^3}$ 3) $\frac{4\pi^2 m^2 e^2}{h^3}$ 4) $\frac{4\pi^2 me^5}{h^3}$

KEY : 4

SOLUTION :

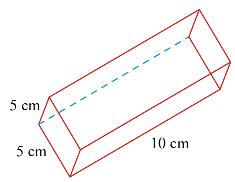
$$I=\frac{e}{t}=\frac{e}{2\pi r/v}=\frac{ev}{2\pi r}$$

5. A rectangular block has dimensions 5 cm × 5 cm × 10 cm. Calculate the resistance measured between (a) two square ends and (b) the opposite rectangular ends. Specific resistance of the material is $3.5\times10^{-5}\,\Omega m$.

SOLUTION :

a) Resistance between two square ends $R_1=\frac{\rho\ell}{A}$

$$R_1=\frac{3.5\times10^{-5}\times10\times10^{-2}}{5\times5\times10^{-4}}=1.4\times10^{-3}\,\Omega$$



b) Resistance between the opposite rectangular ends $R_2=\frac{\rho\ell}{A}$

$$R_2=\frac{3.5\times10^{-5}\times5\times10^{-2}}{5\times10\times10^{-4}}=1.4\times10^{-4}\,\Omega$$

6. In a straight conductor of uniform cross-section charge q is flowing for time t. Let s be the specific charge of an electron. The momentum of all the free electrons per unit length of the conductor, due to their drift velocity only is

1) $\frac{q}{ts}$ **2)** $\left(\frac{q}{ts}\right)^2$ **3)** $\sqrt{\frac{q}{ts}}$ **4) qts**

KEY : 1

SOLUTION :

$$I=nAev_d \text{ or } v_d=\frac{I}{nAe}=\frac{q/t}{nAe}$$

No. of free electrons per unit length of conductor

$$N=nA\times1$$

$\therefore$ Momentum of all the free electrons is

$$p=Nmv_d$$

7. The temperature coefficient of resistance of platinum is $\alpha=3.92\times10^{-3}K^{-1}$ at 0° C. Find the temperature at which the increase in the resistance of platinum wire is 10% of its value at 0^0C.

SOLUTION :

$$R_2=\frac{110R_1}{100}=1.1R_1;\ \alpha=3.92\times10^{-3}K^{-1}$$

$$\Delta t=\frac{R_2-R_1}{R_1\alpha}\Rightarrow=\frac{1.1R_1-R_1}{R_1\alpha}$$

$$=\frac{R_1(1.1-1)}{R_1\alpha}=\frac{0.1R_1}{R_1\alpha}=\frac{0.1}{3.92\times10^{-3}}$$

$$\Delta t=25.51^0C;\ t_2=25.51+20=45.51^0C$$

8. Potential difference of 100 V is applied to the ends of a copper wire one metre long. Find the ratio of average drift velocity and thermal velocity of electrons at 27^oC. (Consider there is one conduction electron per atom. The density of copper is 9.0×10^3; Atomic mass of copper is 63.5 g. $N_A = 6.0\times10^{23}$ per gram-mole, conductivity of copper is $5.81\times10^7\ \Omega^{-1}$. $K=1.38\times10^{-23}\ JK^{-1}$)

1) 3.67×10^{-6} **2)** 4.3×10^{-6} **3)** 6×10^{-5} **4)** 5.6×10^{-6}

KEY : 1

SOLUTION :

$$\therefore v_d=\frac{\sigma E}{ne};\ v_{rms}=\sqrt{\frac{3K_BT}{m_e}}$$

9. The resistance of iron wire is 10Ω and $\alpha=5\times10^{-3}/^\circ C$. If a current of 30A is flowing in it at $20^\circ C$, keeping the potential difference across its length constant, if the temperature is increased to $120^\circ C$, what is the current flowing through that wire ?

SOLUTION :

$$\alpha=\frac{R_{120}-R_{20}}{R_{20}(120-20)};\quad 5\times10^{-3}=\frac{R_{120}-10}{10\times100}$$

$$\therefore\ R_{120}=15\Omega;\ \text{But } V=IR$$

Here V is constant. Hence,

$$\frac{I_{120}}{I_{20}}=\frac{R_{20}}{R_{120}};\ \frac{I_{120}}{30}=\frac{10}{15};\ \therefore I_{120}=20A$$

10. Resistance of a wire at temperature t^0C is $R=R_0(1+at+bt^2)$

Here, R_0 is the temperature at 0^0C. Find the temperature coefficient of resistance at temperature t is

SOLUTION :

$$\alpha=\frac{1}{R}.\frac{dR}{dt}=\frac{1}{R_0(1+at+bt^2)}[R_0(a+2bt)]$$

$$\therefore\alpha=\left(\frac{a+2bt}{1+at+bt^2}\right)$$

11. A silver wire has a resistance of 2.1 Ω at $27.5^\circ C$ & 2.7Ω at 100^0C. Determine the temperature coefficient of resistivity of silver.

SOLUTION :

$$R_t=R_0(1+\alpha\theta)$$

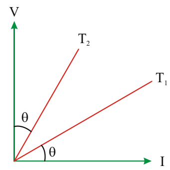
$$2.1=R_0\ (1+\alpha\times27.5)......(1)$$

$$2.7 = R_0 (1+\alpha \times 100)......(2)$$

Solve equation (1) and (2) $\alpha = 0.0039^0 C^{-1}$

12. V-I graph of a conductor at temperature T_1 **and T_2 are shown in the figure $(T_2 - T_1)$ is proportional to**

SOLUTION :



SOLUTION :

Slope of line gives resistance

$$\text{So, } R_1 = \tan\theta = R_0 (1+\alpha T_1)$$

$$R_2 = \tan(90-\theta) = \cot\theta = R_0 (1+\alpha T_2)$$

$$\cot\theta - \tan\theta = R_0\alpha (T_2 - T_1)$$

$$\text{or} \quad \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = R_0\alpha (T_2 - T_1)$$

$$R_0\alpha (T_2 - T_1) = \frac{\cos 2\theta}{\frac{(\sin 2\theta)}{2}} \text{; or } T_2 - T_1 \ \alpha \ \cot 2\theta$$

13. A heating element using nichrome conne-cted to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0°C? Temperature coefficient of resi-stance of nichrome averaged over the temperature range involved is 1.70×10^{-4} C^{-1}?

1) $680^0 C$ **2)** $867^0 C$ **3)** $920^0 C$ **4)** $750^0 C$

KEY : 2

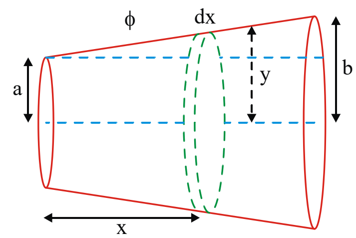
SOLUTION :

$$R_{27} = \frac{V}{I} = \frac{230}{3.2}, \ R_\theta = \frac{230}{2.8}$$

$$\frac{R_{27}}{R_\theta} = \frac{R_0 (1+\alpha \times 27)}{R_0 (1+\alpha \times \theta)}$$

14. Figure shows a conductor of length l having a circular cross -section. The radius of cross-section varies linearly from a to b. The resistivity of the material is ρ. Assuming that b - a <<l, find the resistance of the conductor.

SOLUTION :



$$\tan\phi = \frac{b-a}{l} = \frac{y-a}{x}$$

$$\text{yl - al} = \text{bx - ax}$$

$$l\left(\frac{dy}{dx}\right) = (b-a) \Rightarrow dx = \left(\frac{l}{b-a}\right)dy \rightarrow (1)$$

Resistance across the elemental disc under consideration $dR = \rho\frac{dx}{A} \rightarrow (2)$

from (1) and (2) $dR = \rho\left(\frac{l}{b-a}\right)\frac{dy}{\pi y^2}$

$\Rightarrow$ Resistance across the given conductor,

$$R = \int_{y=a}^{b} dR$$

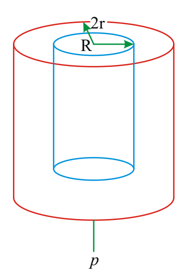
$$\Rightarrow R = \rho\frac{l}{\pi(b-a)} \cdot \int_{y=a}^{y=b}\frac{dy}{y^2}$$

$$\therefore R = \rho\frac{l}{\pi ab}$$

15. A hollow cylinder of specific resistance ρ, inner radius R, outer radius 2R and length l is as shown in figure. What is the net resistance between the inner and outer surfaces ?

SOLUTION :

Consider a ring of width 'dr' and radius 'r'.



Resistance accross the ring is

$$dR = \frac{\rho\, dr}{dA} = \frac{\rho\, dr}{2\pi rl}$$

Net resistance $= \int_R^{2R}\frac{\rho(dr)}{(2\pi rl)} = \left(\frac{\rho}{2\pi l}\right)ln(2)$

16. **There are two concentric spheres of radius a and b respectively. If the space between them is filled with medium of resistivity ρ, then the resistance of the intergap between the two spheres will be (Assume b > a)**

SOLUTION :

Consider a concentric spherical shell of radius x and thickness dx, its resistance is

$$dR,\ dR = \frac{\rho dx}{4\pi x^2}$$

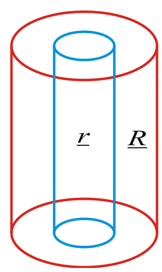
Total resistance

$$\mathbf{R} = \int_a^b dR = R = \left(\frac{\rho}{4\pi}\right)\int_a^b \frac{dx}{x^2} = \frac{\rho}{4\pi}\left[\frac{1}{a}-\frac{1}{b}\right]$$

17. **A hollow copper cylinder is of inner radius 4cm and outer radius 5cm. Now hollow portion is completely filled with suitable copper wires. Find percentage change in its electric resistance.**

SOLUTION :

A hollow cylidner of inner radius 'r' and outer radius 'R' has specific resistance 'ρ'. If its length is 'l' then its resistance



$$= \frac{\rho l}{\pi\left(R^2 - r^2\right)}$$

$$R_1 = \frac{\rho l}{\pi\left(5^2 - 4^2\right)} = \frac{\rho l}{9\pi} = \frac{k}{9}$$

Final Resistance

$$R_2 = \frac{\rho l}{\pi\left(5^2\right)} = \frac{\rho l}{25\pi} = \frac{k}{25}$$

$$\text{Percentage of change} = \frac{R_2 - R_1}{R_1} \times 100$$

$$= \frac{\frac{k}{25} - \frac{k}{9}}{\frac{k}{9}} \times 100 = -64\%$$

18. **The sides of rectangular block are 2cm, 3cm and 4cm. The ratio of the maximum to mini m u m resistance between its parallel faces is**

1) 3 2) 4 3) 2 4) 1

KEY : 2

SOLUTION :

$$R_{max} = \frac{\rho a}{bc},\ R_{min} = \frac{\rho c}{ab}$$

$$a > b > c = 4 > 3 > 2$$

$$\frac{R_{max}}{R_{main}} = \frac{Pa}{bc} \times \frac{ab}{Pc} = \frac{a^2}{c^2} = \frac{4^2}{2^2} = 4.$$

19. If resistivity of the material of a conductor of uniform area of cross-section varies along its length as $\rho = \rho_0 [1 + \alpha x]$. Find then the resistance of the conductor if its lengths is 'L' and area of cross-section is 'A'

$$\frac{\rho_0}{A}\left[L + \frac{1}{2}\alpha L^2\right]$$

SOLUTION :

$$dR = \rho \frac{dx}{A} = \rho_0 (1 + \alpha x)\frac{dx}{A}; \quad \therefore R = \int_0^L dR\ ]$$

20. How many number of turns of nichrome wire of specific resistance $10^{-6}\Omega m$ and diameter 2mm that should be wound on a cylinder of diameter 5cm to obtain a resistance of 40Ω?

SOLUTION :

.

If R is the radius of the cylinder

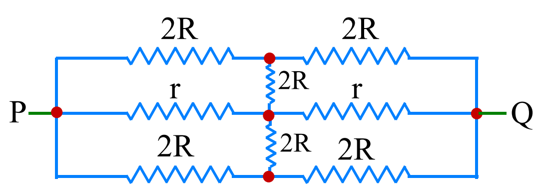
r is the radius of the wire

N is the number of turns

then $R' = \frac{\rho \ell}{A} \quad \therefore \quad R' = \frac{\rho(2\pi R)N}{\pi r^2}$

$$40 = \frac{10^{-6}\left(2 \times 2.5 \times 10^{-2} \times N\right)}{1 \times 10^{-6}} = \therefore\ N = 800$$

21. The effective resistance between poins P and Q of the electrical circuit shown in the figure is [2002, 2M]

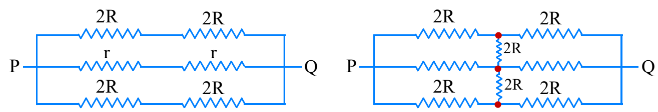


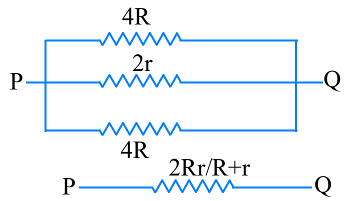
1) $\frac{2Rr}{R+r}$ 2) $\frac{8R(R+r)}{3R+r}$ 3) $2r + 4R$ 4) $\frac{5R}{2} + 2r$

KEY : 1

SOLUTION :

The circuit can be redrawn as follows





22. Suppose the colours on the resistor as shown in Figure are brown, yellow, green and gold as read from left to right. Using the table, find the resistance of the resistor

SOLUTION :

Brown Yellow Green Gold

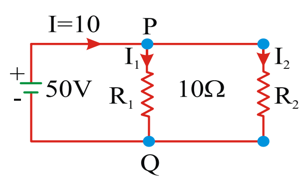
1 $4 \longrightarrow \quad \times 10^5 \longrightarrow \quad +5\% \longrightarrow$

$$= 14 \times 10^5 \left(1 \pm \frac{5}{100}\right)\Omega$$

$$= (1.4 \pm 0.07)10^6\,\Omega = (1.4 \pm 0.07)\text{M}\Omega$$

Some times tolerance is missing from the code and there are only three bands. Then the tolerance is 20%.

23. For a circuit shown in Fig find the value of resistance R_2 and current I_2 flowing through R_2



SOLUTION :

If equivalent resistance of parallel combination of R_1 and R_2 is R, then

$$R = \frac{R_1 R_2}{R_1 + R_2} = \frac{10R_2}{10 + R_2}$$

According to Ohm's law, $R = \frac{V}{I}$

$$R = \frac{50}{10} = 5\Omega \Rightarrow \frac{10R_2}{10 + R_2} = 5 \Rightarrow R_2 = 10\Omega\,.$$

The current is equally divided into R_1 and R_2.

Hence $I_2 = 5A$.

24. Two wires of equal diameters of resisti-vities ρ_1 and ρ_2 and length x_1 and x_2 respecti-vely are joined in series. Find the equivalent resistivity of the combination.

SOLUTION :

Resistance, $R_1 = \frac{\rho_1 \ell_1}{A_1}; R_2 = \frac{\rho_2 \ell_2}{A_2}$

$\ell_1 = x_1, \ell_2 = x_2$

As the wires are of equal diameters $A_1 = A_2 = A$.

$$R_1 = \frac{\rho x_1}{A}, R_2 = \frac{\rho x_2}{A_2}; \ R = \frac{\rho x}{A}$$

where $x = x_1 + x_{2;}$ $R = R_1 + R_2$

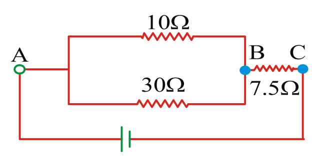
$$\frac{\rho x}{A} = \frac{\rho_1 x_1}{A} + \frac{\rho_2 x_2}{A}; \quad \rho x = \rho_1 x_1 + \rho_2 x_2$$

$$\rho(x_1 + x_2) = \rho_1 x_1 + \rho_2 x_2 \quad [\because x = x_1 + x_2]$$

$$\therefore \rho = \frac{\rho_1 x_1 + \rho_2 x_2}{x_1 + x_2} \text{ also } \frac{1}{\sigma} = \frac{\frac{x_1}{\sigma_1} + \frac{x_2}{\sigma_2}}{x_1 + x_2}$$

25. Find equivalent resistance of the network in Fig. between points (i) A and B and (ii) A and C.

SOLUTION :

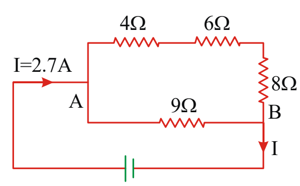


(i)The 10Ω and 30Ω resistors are connected in parallel between points A and B. The equivalent resistance between A and B is

$$R_1 = \frac{10 \times 30}{10 + 30} \text{ohm} = 7.5\Omega$$

(ii) The resistance R_1 is connected in series with resistor of 7.5Ω, hence the equivalent resistance between points A and C is, R_2=(R_1+7.5) ohm = (7.5+7.5) ohm = 15 Ω.

26. Find potential difference between points A and B of the network shown in Fig. and distribution of given main current through different resistors.



SOLUTION :

. Between points A and B resistors of 4Ω, 6Ω and 8Ω resistances are in series and these are in parallel to 9Ω resistor.

Equivalent resistance of series combinaiton is

$R_1 = (4 + 6 + 8)$ ohm = 18

If equivalent resistance between A and B is

$R = 9 \times 18 / (9 + 18)$ ohm = 6 Ω

Potential difference between A and B is

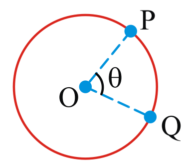
$$V = IR = 2.7 \times 6V = 16.2V$$

Current through $9\,\Omega$ resistor = 16.2/9=1.8A

Current through $4\,\Omega$, $6\,\Omega$ and $8\,\Omega$ resistors =

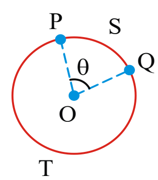
$$2.7 - 1.8 = 0.9A.$$

27. P and Q are two points on a uniform ring of resistance R. The equivalent resistance between P and Q is



SOLUTION :

Resistance of section PSQ



$$R_1 = \frac{R}{2\pi r}.r\theta = \frac{R\theta}{2\pi}$$; Resistance of section PTQ

$$R_2 = \frac{Rr(2\pi - \theta)}{2\pi r};$$

$$R_2 = \frac{R(2\pi - \theta)}{2\pi}$$

As R_1 and R_2 are in parallel

So, $$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R\theta}{4\pi^2}(2\pi - \theta)$$

28. Two wires of the same material have length 6cm and 10cm and radii 0.5 mm and 1.5 mm respectively. They are connected in series across a battery of 16V. The p.d. across the shorter wire is

1) 5V 2) 13.5 V 3) 27 V 4) 10 V

KEY : 2

SOLUTION :

$$l_1 = 6cm \;\; , l_2 = 10cm,$$

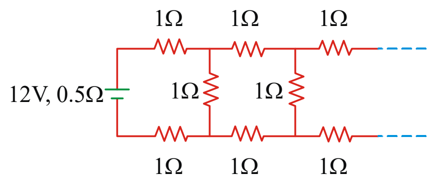
$$r_1 = 0.5 \times 10^{-3}, r_2 = 1.5 \times 10^{-3}$$

In series combination i = constant

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} = \frac{\frac{\rho l_1}{A_1}}{\frac{\rho l_2}{A_2}} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} \quad V_1 + V_2 = 16V$$

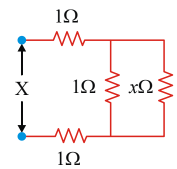
Solving for $V_1 = 13.5V$

29. Determine the current drawn from a 12V supply with internal resistance 0.5 Ω. by the infinite network shown in Fig. Each resistor has 1 Ω. resistance.



SOLUTION :

First calculate net resistance of ∞ network



$$x=2+\frac{x}{x+1};\ x^2-2x-2=0;$$

on solving, $x = 1+\sqrt{3} = 2.73\Omega$

Total resistance $= 2.73 + 0.5 = 3.23\ \Omega$

$$I=\frac{12}{3.23}=3.73A$$

30. Three ammeters P,Q and R with internal resistances r, 1.5r ,3r respectively . Q and R parallel and this combination is in series with P , The whole combination concted between X and Y . When the battery connected between X and Y , the ratio of the readings of P,Q and R is

1) 2:1:1 2) 3:2:1 3) 3:1:2 4) 1:1:1

KEY : 2

SOLUTION :

$$i=\frac{V}{R}$$

31. A fuse wire with radius of 0.2mm blows off with a current of 5 Amp. The fuse wire of same material, but of radius 0.3mm will blow off with a current of

1) $5\times\frac{3}{2}$ Amp 2) $\frac{5\sqrt{3}}{2}$ Amp 3) $5\sqrt{\frac{27}{8}}$ Amp 4) 5 Amp

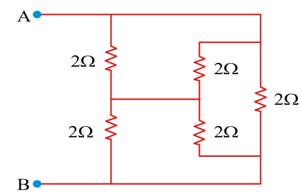
SOLUTION :

$$\because i^2 \propto r^3$$

$$\frac{i_1}{i_2}=\left(\frac{r_1}{r_2}\right)^{3/2}=\left(\frac{0.2}{0.3}\right)^{3/2}$$

$$i_2 = 5\sqrt{\frac{27}{8}}\ \text{Amp}$$

32. Find the equivalent resistance across AB:



1) 1Ω 2) 2Ω 3) 3Ω 4) 4Ω

KEY : 1

SOLUTION :

Apply series and parallel combinations

33. A 1 kW heater is meant to operate at 200 V. (a) What is its resistance ? (b) How much power will it consume if the line voltage drops to 100 V ? (c) How many units of electrical energy will it consume in a month (of 30 days) if it operates 10 hr daily at the specified voltage ?

SOLUTION :

.a) The resistance of an electric appliance is given by , $R = \frac{V_S^{\,2}}{W}$ so, $R = \frac{(200)^2}{1000} = 40\Omega$

b) The ' actual power ' consumed by an electric appliance is given by ,

$$P = \left(\frac{V_A}{V_S}\right)^2 \times W$$

$$\text{so, } P = \left(\frac{100}{200}\right)^2 \times 1000 = 250W$$

c) The total electrical energy consumed by an electric appliance in a specified time is given by,

$$E = \frac{\Sigma W_1 h_1}{1000} kWh$$

$$\text{so, } E = \frac{1000 \times (10 \times 30)}{1000} = 300kWh$$

34. A cell of emf 12 V and internal resistance 6 Ω is connected in parallel with another cell of emf 6 V and internal resistance 3 Ω, such that the positive of the first cell joins the positive of the second cell and similarly the negative of first cell joins the negative of the second cell. A bulb of filament resistance 14 Ω is connected across the combination. The power delivered to be bulb is

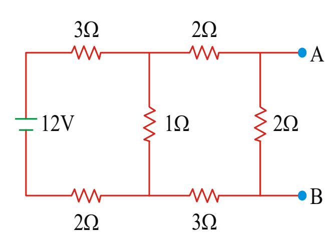
1) 4.0 W 2) 3.5 W 3) 8.5 W 4) 2.5 W

KEY : 2

SOLUTION :

$$i = \frac{E_{eff}}{R_{eff}}, p = i^2 R$$

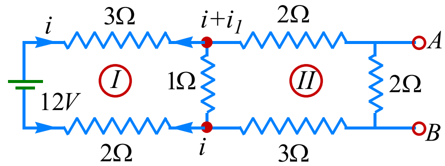
35. The potential difference between the points A and B is



1) 1.50 V 2) 2.50 V 3) 1.00 V 4)0.50 V

KEY : 4

SOLUTION :



For the first loop $12 = 5i + i_1$

For the second loop $0 = 7(i - i_1) - i_1$

or $8i_1 = 7i$ $or,$ $i_1 = (7/8)i$

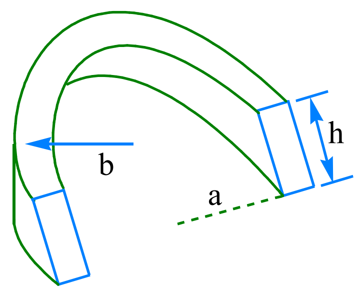
Therefore, we obtain $12 = 5i + \frac{7}{8}i = \frac{47i}{8}$

or, $i = \frac{12 \times 8}{47} A = 2.04 A$

$i_1 = \frac{7}{8} \times 2.04 A = 1.79 A$

Thus, the p.d across A and B is $V_A - V_B = (i - i_1) \times 2 = 0.25 \times 2 = 0.50V$

36. A conductor having resistivity ρ is bent in the shape of a half cylinder as shown in the figure. The inner and outer radii of the cylinder are a and b respectively adn the height of the cylinder is h. A potential differenced is applied across the two rectangular faces of the conductor. Calculate the resistance offered by the conductor.

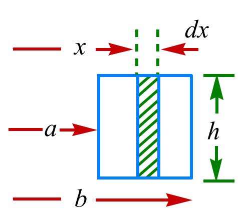


1) $\frac{\rho h}{\pi b a}$ **2)** $\frac{\rho a}{\pi h^2}$ **3)** $\frac{\rho \pi}{h \ln(b/a)}$ **4)** $\frac{\rho \log b/a}{\pi h}$

KEY : 3

SOLUTION :

Consider a strip of with dx shown on the rectanguar face in the figure



Think of a half cylindrical conductor of radius x and inifinitesimally sma thickness dx.

Length of this conductor $= \pi x$

Cross sectional area of this conductor = $h\ dx$

$\therefore$ Resistance of this thin cylindrical conductor $dR = \dfrac{\rho\pi x}{h\ dx}$

The given conductor is made of countless number of such thin conductors, all connected in parallel.

$$\therefore \frac{1}{R} = \int \frac{1}{dR} = \frac{h}{\rho\pi}\int_a^b \frac{dx}{x} = \frac{h}{\rho\pi}\ln\left(\frac{b}{a}\right)$$

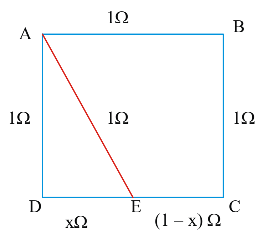
$$\therefore R = \frac{\rho\pi}{h\ln(b/a)}$$

37. ABCD is a square where each side is a uniform wire of resistance 1Ω. A point E lies on CD such that if a uniform wire of resistance 1Ω is connected across AE and constant potential difference is applied across A and C, then B and E are equi-potential.

1) $\dfrac{CE}{ED} = 1$ **2)** $\dfrac{CE}{ED} = \dfrac{1}{\sqrt{2}}$ **3)** $\dfrac{CE}{ED} = \dfrac{1}{2}$ **4)** $\dfrac{CE}{ED} = \sqrt{2}$

KEY : 4

SOLUTION :



Equivalent resistance between A and E is

$$y = \frac{(x+1)}{x+2}$$

For B and E to be at equal potential, we get

$$\frac{R_{AE}}{R_{AB}} = \frac{R_{EC}}{R_{BC}} \Rightarrow \frac{x+1}{(x+2)1} = \frac{1-x}{1}$$

Solving $x = \sqrt{2} - 1$

Now $\dfrac{CE}{ED} = \dfrac{1-x}{x} = \sqrt{2}$

38. A lamp of 100W works at 220 volts. What is its resistance and current capacity ?

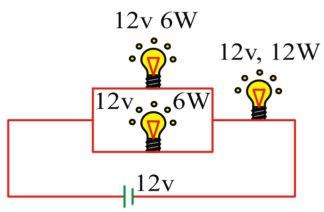
SOLUTION :

Power of the lamp, P = 100W
Operating voltage, V = 220V
Current capacity of the lamp,

$$i = \frac{P}{V} = \frac{100}{220} = 0.455A$$

Resistance of the lamp, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\Omega$

39. Three bulbs with their power and working voltage are connected as shown in the circuit diagram to a 12 V battery. The total power consumed by the bulbs is (ignore the internal resistance of the battery shown)



1) 24 W   2) 12 W   3) 6 W   4) 15 W

KEY : 3

SOLUTION :

$$P = \frac{V^2}{R}$$

40. A 100W – 220V bulb is connected to 110V source. Calculate the power consumed by the bulb.

SOLUTION :

Power of the bulb, P = 100W

Operating voltage, V = 200V

Resistance of the bulb, $R = \frac{V^2}{P} = \frac{(220)^2}{100} = 484\Omega$

Actual operating voltage, $V^1 = 110$ V

Therefore, power consumed by the bulb,

$$P^1 = \frac{(V^1)^2}{R} = \frac{(110)^2}{484} = 25W.$$

41. A 100W and a 500W bulbs are joined in series and connected to the mains. Which bulb will glow brighter ?

SOLUTION :

Let R_1 and R_2 be the resistances of the two bulbs. If each bulb is connected separately to the mains of voltage V,

$$\text{then } P_1 = \frac{V^2}{R_1} \text{ and } P_2 = \frac{V^2}{R_2}$$

$$\therefore \frac{P_1}{P_2} = \frac{R_2}{R_1} \text{ (or) } \frac{R_1}{R_2} = \frac{P_2}{P_1} = \frac{500}{100} = 5$$

If the two bulbs are in series with the mains, the same current 'i' flows through each of them.

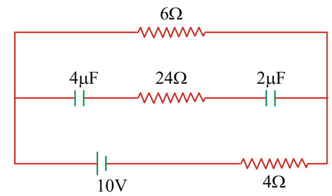
Let P_1 and P_2 be the powers dissipated by two bulbs, turn

$$P_1^1 = i^2R_1 \text{ and } P_2^1 = i^2R_2$$

$$\therefore \frac{P_1^1}{P_2^1} = \frac{R_1}{R_2} = 5 \text{ or } P_1^1 = 5P_2^1$$

Since 100 watt bulb dissipates more power, it

42. The charge developed on 4 μ F condenser is



1) 18 μ C2) 4 μ C

3) 8 μ C 4) Zero

KEY : 3

SOLUTION :

current i = 1 amp

P.D across $6\,\Omega = 6\,Volt$

P.D across $4\,\Omega = 2\,Volt$

$\therefore$ Charges on $4\mu F = 8\,\mu C$

43. A cell develops the same power across two resistances R_1 and R_2 separately. The internal resistance of the cell is

SOLUTION :

. Let r be the internal resistance of the cell and E its EMF. When connected across the resistance R_1 in the circuit, current passing through the resistance is

$$i = \frac{E}{R_1 + r}; \quad \therefore \quad P_1 = i^2 R_1 = \left(\frac{E}{R_1 + r}\right)^2 R_1$$

Similarly $P_2 = \left(\frac{E}{R_2 + r}\right)^2 R_2$; Given that $P_1 = P_2$

Substituting the values, we get ; $r = \sqrt{R_1 R_2}$

44. Same mass of copper is drawn into 2 wires of 1mm thick and 3mm thick. Two wires are connected in series and current is passed. Heat produced in the wires is the ratio of

1) 3 : 1 2) 9 : 1 3) 81 : 1 4) 1 : 81

KEY : 3

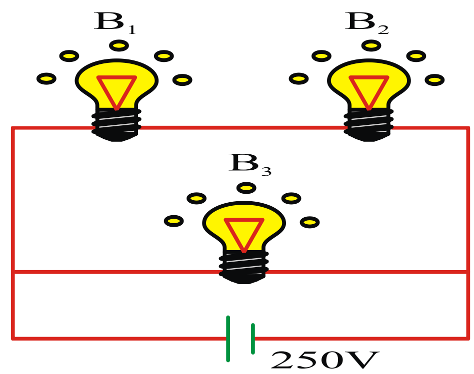
SOLUTION :

$JQ = i^2 Rt$

$Q \alpha R \alpha \frac{1}{A^2}$ when wire is stretched

$$\frac{Q_1}{Q_2} = \frac{r_2^4}{r_1^4} = \frac{3^4}{1^4} = 81.1$$

45. A 100 W bulb B_1 **and two 60 W bulbs B_2 and B_3, are connected to a 250V source, as shown in the figure. Now W_1, W_2 and W_3 are the output powers of the bulbs B_1, B_2 and B_3 respectively. Then**



SOLUTION :

A bulb is essentially a resistance $R = \frac{V^2}{P}$ where P denotes the power of the bulb.

$\therefore$ Resistance of $B_1(R_1) = V^2/100$

Resistance of $B_2(R_2) = V^2/60$

Resistance of $B_3(R_3) = V^2/60$

$$\therefore I_1 = \text{Current in } B_1 = \frac{250}{(R_1 + R_2)} = \frac{250 \times 300}{8V^2}$$

$$I_2 = \text{Current in } B_2 = \frac{250}{(R_1 + R_2)} = \frac{250 \times 300}{8V^2}$$

I_3 = Current in $B_3 = I_1$ as B_1, B_2 are in series

$\therefore$ W_1 output power of $B_1 = I_1^2 R_1$

$$\therefore W_1 = \left(\frac{250 \times 300}{8V^2}\right)^2 \times \frac{V^2}{100}$$

$$W_2 = I_2^2 R_2 \text{ or } W_2 = \left(\frac{250 \times 300}{8V^2}\right)^2 \times \frac{V^2}{60}$$

$$W_3 = I_2^2 R_3 \text{ or } W_3 = \left(\frac{250 \times 300}{8V^2}\right)^2 \times \frac{V^2}{60}$$

$\therefore W_1 : W_2 : W_3 = 15 : 25 : 64$ or $W_1 < W_2 < W_3$

46. Masses of three are in the ratio 1:3:5. Their lengths are in the ratio 5:3:1. When they are connected in series to an external source, the amounts of heats produced in them are in the ratio

1) 125 : 15 : 1 **2) 1 : 15 : 125**

3) 5 : 3 : 1 **4) 1 : 3 : 5**

KEY : 1

SOLUTION :

$m_1 : m_2 : m_3 = 1:3:5$

$l_1 : l_2 : l_3 = 5:3:1$

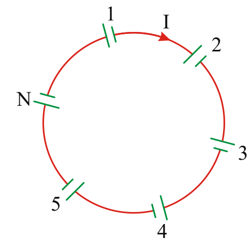
$Q = i^2 Rt \quad R = \frac{\rho d l^2}{m}$

$Q \,\alpha\, R = \frac{\rho d l^2}{m}$

$\frac{Q_1}{Q_2} = \frac{l_1^2}{l_2^2} \times \frac{m_2}{m_1} \Rightarrow$

$Q_1 : Q_2 : Q_3 = 125:15:1$

47. A group of N cells where e.m.f. varies directly with the internal resistance as per the equation E_N = 1.5 r_N are connected as shown in the figure. The current I in the circuit is:



1) 0.51 A 2) 5.1 A 3) 0.15 A 4) 1.5 A

KEY : 4

SOLUTION :

$i = \frac{E_N}{r_N} = 1.5$

48. A heater coil rated at 1000W is connected to a 110V mains. How much time will take to melt 625 gm of ice at 0^0C. (for ice L = 80 cal/gm)

1) 100s 2) 150s 3) 200s 4) 210s

KEY : 4

SOLUTION :

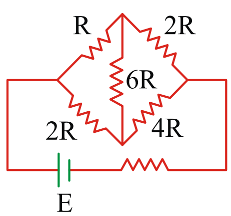
$JQ = P \times t$

$J \times m L = P \times t$

$1 \times 625 \times 10^{-3} \times \frac{80 \times 4.2}{10^{-3}} = 1000 \times t$

$t = 210s$

49. A battery if internal resistance 4Ω is connected to the network of ressitances as shown. What must be the value of R so that maximum power is delivered to the network ? Find the maximum power ?



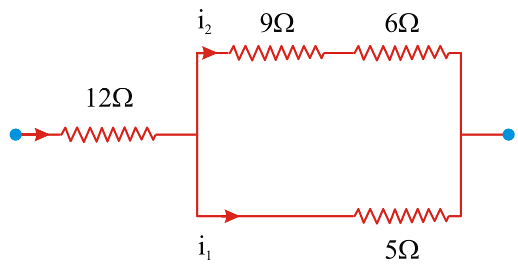
SOLUTION :

i) According to maximum power transfer theorem

$$R_{ext} = R_{int} \quad \frac{3R \times 6R}{9R} = 4 \Rightarrow R = \frac{4}{2} = 2\Omega$$

$$\text{ii)} \quad P_{max} = i^2 R_{ext} = \left(\frac{E}{4+4}\right)^2 \times 4 = \frac{E^2}{16}$$

50. In the following circuit, 5Ω resistor develops 45 J/s due to current flowing through it. The power developed across 12Ω resistor is



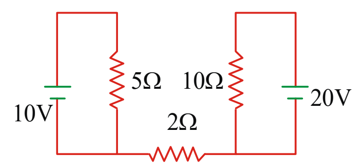
1) 16 W   2) 192 W   3) 36 W   4) 64 W

KEY : 2

SOLUTION :

$P = i^2 R = 192W$

51. Find out the value of current through 2Ω resistance for the given circuit.



1) 0   2) 1.6 A

3) 2.4 A   4) 3A

KEY : 1

SOLUTION :

2Ω resistor is in open circut so current is 0

52. When a current drawn from a battery is 0.5A, its terminal potential difference is 20V. And when current drawn from it is 2.0A, the terminal voltage reduces to 16 V. Find out. e.m.f and internal resistance of the battery.

SOLUTION :

We know

$V = E - Ir$; $I = 0.5$ A, $V = 20$ Volt, we have

$20 = E - 0.5r$....... (i)

$I = 2$ A, $V = 16$ Volt, we have

$16 = E\text{-}0.2r$....... (ii)

From eqs (i) and (ii)

$2E - r = 40$ and $E - 2r = 16$

Solving we get $E = 21.3V$, $r = 2.675\,\Omega$.

53. **Cell A has emf 2E and internal resistance 4r. Cell B has emf E and internal resistance r. The negative of A is connected to the positive of B and a load resistance of R is connected across the battery formed. If the terminal potential difference across A is zero, then R is equal to**

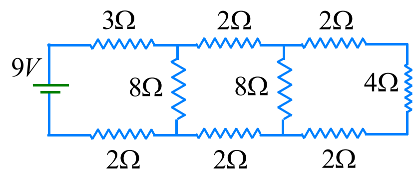
1) 3r 2) 2r 3) r 4) 5r

KEY : 3

SOLUTION :

$$i = \frac{e_1 + e_2}{R + r_1 + r_2}, V_1 = E_1 - ir_1 = 0$$

54. **In the circuit shown in the figure, the current through [1998, 2M]**



1) the 3Ω resistor is 0.50 A
2) the 3Ω resistor is 0.25 A
3) the 4Ω resistor is 0.50 A
4) the 4Ω resistor is 0.25 A

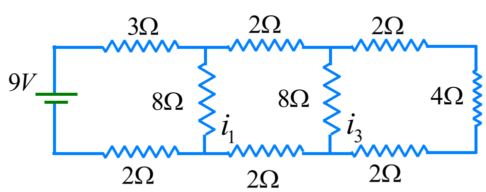
KEY : 4

SOLUTION :

Net resistance of the circuit is Ω.

$\therefore$ Current drawn from the battery

$i = \frac{9}{9} = 1A$ = current through 3Ω resistor



Potential difference between A and B is

$V_A - V_B = 9 - 1(3+2) = 4V = 8i_1$

$i_1 = 0.5$ A

$\therefore$ $i_2 = 1 - i_1 = 0.5$ A

Similarity potential difference between C and D

$V_C - V_D = (V_A - V_B) - i_2(2+2)$

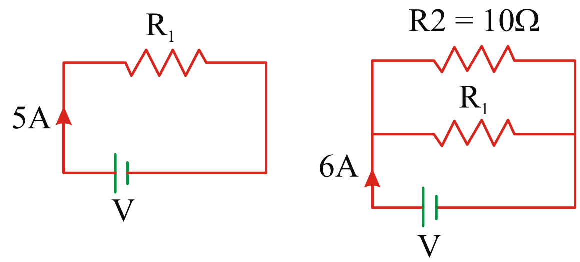
$= 4 - 4i_2 = 4 - 4(0.5) = 2V = 8i_3$

$i_3 = 0.25$ A

Therefore $i_4 - i_2 - i_3 = 0.5 - 0.25 \quad i_2 = 0.25A$

55. An ideal battery passes a current of 5A through a resistor. When it is connected to another resistance of 10Ω in parallel, the current is 6A. Find the resistance of the first resistor.

SOLUTION :



Current through R_1 in the first case $i_1 = 5A$

Current in the second case $i_2 = 6A$

Effective resistance in the second case $R = \frac{R_1R_2}{R_1+R_2}$; $V = I_1R_1$ and $V = I_2\frac{R_1R_2}{R_1+R_2}$

$$I_1R_1 = I_2\frac{R_1R_2}{R_1+R_2} \Rightarrow I_1 = I_2\frac{R_2}{R_1+R_2}$$

$$5 = 6\times\frac{10}{R_1+10} \Rightarrow 5(R_1+10) = 60$$

$$5R_1 + 50 = 60,\ 5R_1 = 10$$

$$R_1 = \frac{10}{5} = 2\Omega \Rightarrow R_1 = 2\Omega \cdot$$

56. A cell develops the same power across two resistances $R_1 \& R_2$ separately. The internal resistance of the cell is

1) $\sqrt{R_1R_2}$ 2) $\sqrt{2R_1R_2}$ 3) R_1+R_2 4) R_1-R_2

KEY : 1

SOLUTION :

$$P_1 = i^2 R_1 = \left(\frac{E}{R_1+r}\right)^2 R_1$$

$$P_2 = i^2 R_2 = \left(\frac{E}{R_2+r}\right)^2 R_2$$

57. When a battery is connected to the resistance of 10Ω the current in the circuit is 0.12A. The same battery gives 0.07A current with 20Ω. Calculate e.m.f. and internal resistance of the battery.

SOLUTION :

We know that $E = Ir + IR$

$$I_1r + I_1R_1 = I_2r + I_2R_2\ ;\ r = \frac{I_2R_2 - I_1R_1}{I_1 - I_2}$$

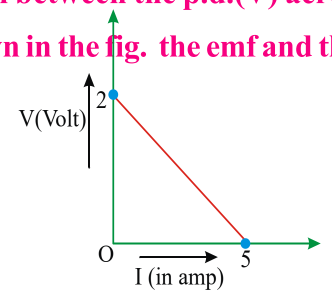
$$r = \frac{0.07\times20 - 0.12\times10}{0.12-0.07} = \frac{1.4-1.2}{0.05} = \frac{0.2}{0.05} = 4\Omega$$

Internal resistance $r = 4\Omega$

e. m. f $E = Ir + IR$

$0.12\times 4 + 0.12\times 10 = 0.48 + 1.2$; $E = 1.68$ volt.

58. For a cell, the graph between the p.d.(V) across the terminals of the cell and the current I drawn from the cell is shown in the fig. the emf and the internal resistance of the cell is E and r repectively.



1) $E = 2V,\ r = 0.5\Omega$ 2) $E = 2V,\ r = 0.4\Omega$

3) $E > 2V,\ r = 0.5\Omega$ 4) $E > 2V,\ r = 0.4\Omega$

KEY : 2

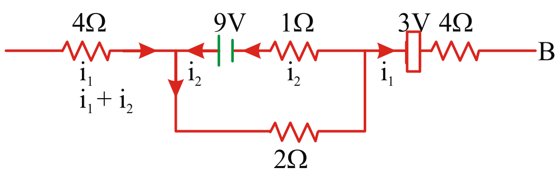
SOLUTION :

$V = E - ir$

$i = 0,\ V = E = 2V$

$V = 0,\ r = \frac{E}{i} = 0.4\Omega$

59. For the circuit shown in the figure, potential difference between points A and B is 16V. Find the current passing through 2Ω



1) 3.5A 2) 3A 3) 4.5A 4) 5.5A

KEY : 1

SOLUTION :

$V_A - V_B = 16$

$4i_1 + 2(i_1 + i_2) - 3 + 4i_1 = 16 .... (1)$

$9 - i_2 - 2(i_1 + i_2) = 0$

Solving eqs (1) and (2) $i_1 = 1.5\,A\ and\ i_2 = 2A$

60. Two wires 'A' and 'B' of the same material have their lengths in the ratio 1 : 2 and radii in the ratio 2 : 1. The two wires are connected in parallel across a battery. The ratio of the heat produced in 'A' to the heat produced in 'B' for the same time is

1) 1 : 2 2) 2 : 1 3) 1 : 8 4) 8 : 1

KEY : 4

SOLUTION :

$Q = \frac{V^2}{R};\ \frac{Q_1}{Q_2} = \frac{R_2}{R_1} = \frac{l_2}{l_1}\times\frac{r_1^2}{r_2^2}.$

$\frac{Q_1}{Q_2} = \frac{2}{1}\times\frac{(2)^2}{1^2} = \frac{8}{1}$

61. Two cells A and B with same e.m.f of 2 V each and with internal resistances $r_A = 3.5\Omega$ and $r_B = 0.5\Omega$ are connected in series with an external resistance $R = 3\Omega$. Find the terminal voltages across the two cells.

SOLUTION :

Current through the circuit $i = \frac{\varepsilon}{(R+r)} = \frac{2+2}{(3+3.5+0.5)} = \frac{4}{7}$

i) $R = 3\Omega, r_A = 3.5\Omega, E = 2V$

Terminal voltages A, $V_A = E - ir$

$= 2 - \frac{4}{7} \times 3.5 = 0$ volt

ii) $r_B = 0.5\,\Omega, R = 3\,\Omega, E = 2V$

Terminal voltage at B, $V_B = E - ir$

$= 2 - \frac{4}{7} \times 0.5 = 1.714$ *volts*.

62. The minimum number of cells in mixed grouping required to produce a maximum current of 1.5 A through an external resistance of $30\,\Omega$, given the emf of each cell is 1.5 V and internal resistance is 1Ω is

1) 30 2) 120 3) 40 4) 60

KEY : 2

SOLUTION :

$i_{max} = \frac{mE}{2r} = \frac{nE}{2R}$

n = number of cells in each row.

m = number of rows.

$1.5 = \frac{1.5 \times n}{2 \times 30}$

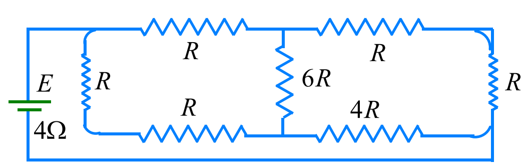
$n = 60$

$1.5 = \frac{1.5 \times m}{2 \times 1}$

$m = 2$

$\therefore$ total no of cells; $= n \times m = 2 \times 60 = 120$

63. A battery of internal resistance 4Ω is connected to the network of resistances as shown in figure. In order that the maximum power can be delivered to the network, the value of R in Ω should be

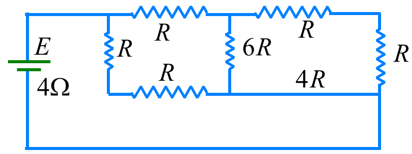


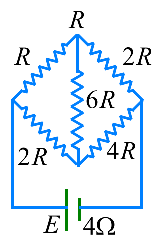
1) $\frac{4}{9}$ 2) 2 3) $\frac{8}{3}$ 4) 18

KEY : 2

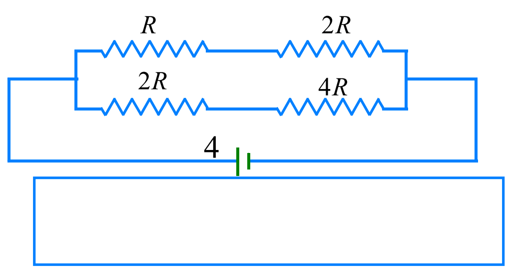
SOLUTION :

The given circuit is a balanced wheatstone's bridge





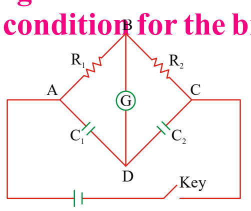
Thus, no current will flow across 6R of the side CD. The given circuit will now be equivalent to



For maximum power, net external resistance = Total internal resistance

$R = 2\Omega$

64. In Wheat stone's bridge shown in the adjoining figure galvanometer gives no deflection on pressing the key, the balance condition for the bridge is :



1) $\dfrac{R_1}{R_2} = \dfrac{C_1}{C_2}$

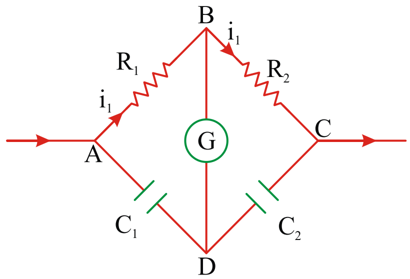
2) $\dfrac{R_1}{R_2} = \dfrac{C_2}{C_1}$

3) $\dfrac{R_1}{R_1 + R_2} = \dfrac{C_1}{C_1 - C_2}$ 4) $\dfrac{R_1}{R_1 - R_2} = \dfrac{C_1}{C_1 + C_2}$

KEY : 3

SOLUTION :

At balance, the potentials of point B and D are same and there will be no current in the arm BD. Thus,



$$i_1R_1 = \frac{q}{C_1} \quad .... (i) \quad V_A - V_B = V_A - V_D$$

where q is the charge on both the capacitor plates connected in series.

Quite similarly $V_B - V_C = V_D - V_C$

or $$i_1R_2 = \frac{q}{C_2} \quad .... (ii)$$

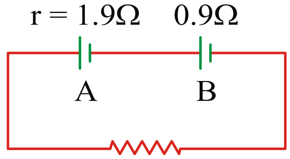
Dividing eqs. (i) and (ii), we get

$$\frac{R_1}{R_2} = \frac{C_2}{C_1}$$

65. Two cells A and B each of 2 V are connected in series to an external resistance R=1 ohm . The internal resistance of A is r_A =1.9 ohm and B is r_B = 0.9 ohm. Find the potential difference between the terminals of A.

SOLUTION :

Total current through the circuit $i = \frac{voltage}{Total\ resistance}$

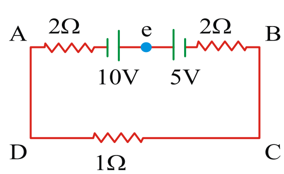


$$= \frac{4}{(1+1.9+0.9)} = \frac{4}{3.8} A.$$

potential difference at $A, V_A = \varepsilon - ir$,

$$= 2 - \frac{4}{3.8} \times 1.9 = 2 - 2 = 0.$$

66. he given circuit as shown below, calculate the magnitude and direction of the current



SOLUTION :

Effective resistance of the circuit is

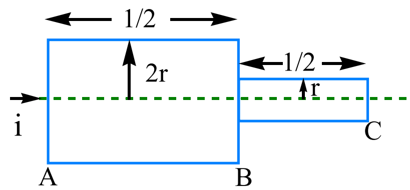
$$R_{eff} = 2 + 2 + 1 = 5\Omega$$

$\therefore$ total current in the circuit is $i = \dfrac{V_1 - V_2}{R_{eff}}$

$$\mathbf{i} = \frac{10-5}{5} = 1A$$

Since the cell of larger emf decides the direction of flow of current, the direction of current in the circuit is **from A to B through e**

67. Two bars of radius r and 2r are kept in contact as shown. An electric current I is passed through the bars. Which one of the following is correct? [2006, 3M]



1) Heat produced in bar BC is 4 times the heat produced in bar AB
2) Electric field in both halves is equal
3) Current density across AB is double that of across BC
4) Potential difference across AB is 4 times that of across BC

KEY : 1

SOLUTION :

Current flowing through both the bars is equal. Now the heat produced is given by

$H = I^2Rt$ or $H \alpha R$ $H_{BC} = 4H_{AB}$

68. A voltmeter resistance 500Ω is used to measure the emf of a cell of internal resistance 4Ω. The percentage error in the reading of the voltmeter will be

SOLUTION :

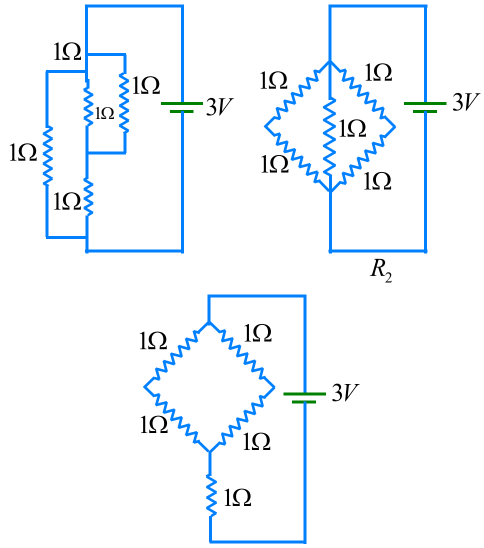
$$V = E - ir$$

$$\therefore \text{Percentage error} = \frac{\Delta E}{E} \times 100 = \frac{ir}{E} \times 100$$

$$= \frac{\left(\frac{E}{R+r}\right) r}{E} \times 100 = \left(\frac{r}{R+r}\right) \times 100$$

$$= \left(\frac{4}{500+4}\right) \times 100 = 0.8\%$$

69. Figure shows three resistor configurations R_1, R_2 and R_3 connected to 3V battery. If the power dissipated by teh configuration R_1, R_2 and R_3 is P_1, P_2 and P_3, respectively, then [2008, 3M]



1) $P_1>P_2>P_3$ 2) $P_1>P_3>P_2$
3) $P_2>P_1>P_3$ 4) $P_3>P_2>P_1$

KEY : 3

SOLUTION :

Applying

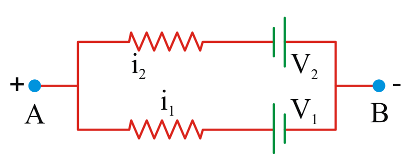
$$P=\frac{V^2}{R}, R_1=1\Omega, R_2=0.5\Omega$$

$$\text{and } R_1=\frac{(3)^2}{1}=9\Omega \ \therefore P_1=\frac{(3)^2}{2}=18W$$

$$P_2=\frac{(3)^2}{2}=4.5W \qquad \therefore P_2>P_1>P_3$$

$\therefore$ Correct option is (c)

70. Find the emf (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of emfs V_1 and V_2 and internal resistances r_1 and r_2 respectively, with polarities as shown in figure

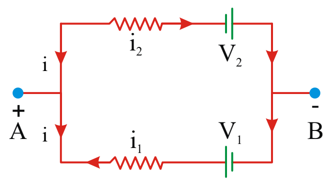


SOLUTION :

EMF of battery is equal to potential difference across the terminals, when no current is drawn from battery (for external circuit) [Here, all the elements in the circuit are in series]

Current in internal circuit = i

$$\therefore i=\frac{\text{Net emf}}{\text{Total resistance}} \text{ or } i=\frac{V_1+V_2}{r_1+r_2}$$



$$\therefore V_A - V_B = V_1 - ir_1 \; [\because V_1 \text{ cell is discharging}]$$

$$\text{or} \quad V_A - V_B = V_1 - \left(\frac{V_1 + V_2}{r_1 + r_2}\right) r_1$$

$$\text{or} \quad V_A - V_B = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

$$\therefore \text{Equivalent emf of the battery} = V$$

$$\therefore V = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

(ii) Internal resistance of equivalent battery. r_1 and r_2 are in parallel.

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} \quad \text{or} \quad r = \frac{r_1 r_2}{r_1 + r_2}$$

71. An electric motor operating on 50 volt D.C. supply draws a current of 10 amp. If the efficiency of motor is 40%, then the resistance of the winding of the motor is

1) 1.5Ω 2) 3Ω 3) 4.5Ω 4) 6Ω

KEY : 2

SOLUTION :

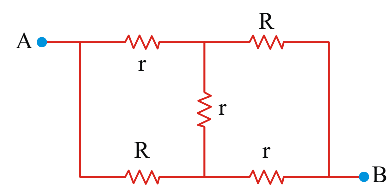
Input power $P = VI = 50 \times 10$

Power dissipated as heat $= I^2 R = 100 R$

$$\text{Efficiency} = \frac{\textit{Out put power}}{\textit{Input power}}$$

$$\frac{40}{100} = \frac{500 - 100R}{500}$$

72. Equivalent resistance across A and B in the given circuit if r = 10 Ω, R = 20 Ω is



1) 7Ω 2) 14Ω 3) 35Ω 4) 20/3Ω

KEY : 2

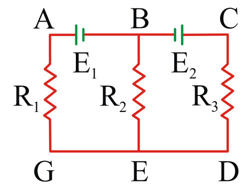
SOLUTION :

$$R^1 = r\left[\frac{3R + r}{3r + R}\right]$$

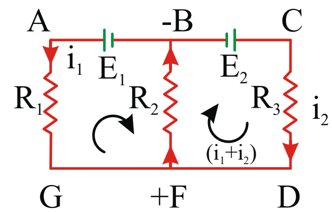
73. In the given circuit values are as follows

$\varepsilon_1 = 2V, \varepsilon_2 = 4V, R_1 = 1\Omega$ **and** $R_2 = R_3 = 1\Omega$.

Calculate the Currents through R_1, R_2 and R_3.



SOLUTION :



Let i_1, i_2 are currents across R_1 and R_3.

$(i_1 + i_2)$ is current across R_2.

Their direction are taken as shown

From Kirchhoff's second law for AGFBA loop

$$-i_1R_1 - (i_1+i_2)R_2 + E_1 = 0; \; i_1 + i_1 + i_2 = 2$$

$$2i_1 + i_2 = 2 \rightarrow (1)$$

From Kirchhoff's second law for BCDEB loop

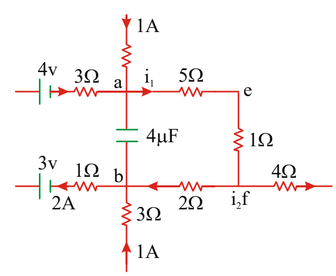
$$-i_2R_3 - (i_1+i_2)R_2 + E_2 = 0; \; i_2 + i_1 + i_2 = 4$$

$$i_2 + 2i_2 = 4 \rightarrow (2)$$

Solving equation (1) and (2) we get $i_1 = 0A$, $i_2 = 2A$

Thus currents across R_1 is 0, while across R_3 and R_2 are 2A each.

74. A part of circuit in steady state along with the currents flowing in the branches, the value of resistances is shown in figure. Calculate the energy stored in the capacitor.



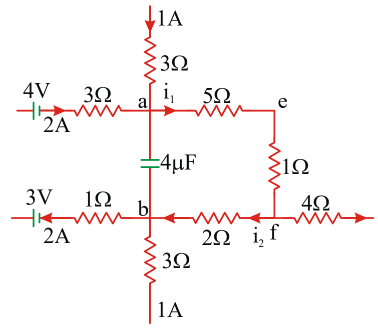
1) $8\times10^{-1}J$ **2)** $8\times10^{-2}J$ **3)** $8\times10^{-3}J$ **4)** $8\times10^{-4}J$

KEY : 4

SOLUTION :

When the capacitor plates get fully charged, there will be no current in branch ab, Remember capacitance acts as the open circuit since capacitance offers infinite resistance to d.c. The capacitance simply collects the charge. Applying Kirchooff's first law to the junctions a and b, we find $i_1 = 3A \; and \; i_2 = 1A$. Now applying Kirchhoff's second law to the closed mash aefba, we get $3\times5 + 3\times1 + 1\times2 = V_a - V_b$

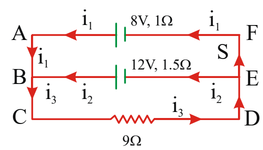
$$V_a - V_b = 20V$$



Energy stored in the capacitor $U = \frac{1}{2}C(V_a - V_b)^2 = \frac{1}{2} \times 4 \times 10^{-6} \times (20)^2$

$= 8 \times 10^{-4} J$

75. Solve for current values in figure.



SOLUTION :

Applying Kirchhoff's first law at the junction B we have $i_1 + i_2 = i_3$ .... (1)

Applying Kirchhoff's second law to loop ABEFA

$-12 + i_2 \times 1.5 - i_1 \times 1 + 8 = 0$

$i_1 - 1.5\, i_2 = -4$........ (2)

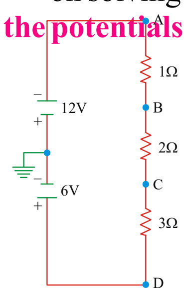
From loop BCDEB

$-(i_2 \times 1.5) - (i_3 \times 9) + 12 = 0$

$1.5\, i_2 + 9i_3 = 12$....... (3)

on solving $i_1 = -1$A and $i_3 = 1$ A

76. In the circuit shown in figure, the potentials of B,C and D are :



1) $V_B = 6V; V_C = 9V; V_D = 11V$

2) $V_B = 11V; V_C = 9V; V_D = 6V$

3) $V_B = 9V; V_C = 11V; V_D = 6V$

4) $V_B = 9V; V_C = 6V; V_D = 11V$

KEY : 2

SOLUTION :

Potential at O is zero being earthed.

Applying Kirchhoff's second law

$i(1+2+3) = 12 - 6 \text{ or } i = 1A$

$V_A - V_D = (1+2+3) \times 1 = 6V$

$V_A - V_B = 1 \times 1 = 1V$

$V_A - V_C = (1+2) \times 1 = 3V$

Also, $V_A - V_O = 12V$ or $V_A = 12V$

Thus, $V_D = 12 - 6 = 6V$,

$V_B = 12 - 1 = 11V$, $V_C = 12 - 3 = 9V$

77. 'n' identical resistors are taken. 'n/2' resistors are connected in series and the remaining are connected in parallel. The series connected group is kept in the left gap of a meter bridge and the parallel connected group in the right gap. The distance of the balance point from the left end of the wire is

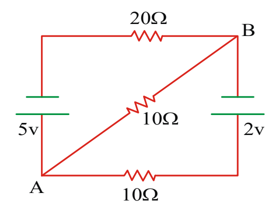
1) $\frac{100n^2}{n^2+4}$ **2)** $\frac{100n^2}{n^2+1}$ **3)** $\frac{400}{n^2+4}$ **4)** $\frac{400}{n^2+1}$

KEY : 1

SOLUTION :

$$\frac{X}{100-X} = \frac{nr/2}{2r/n}$$

78. The p.d between the terminals A & B is



1) 2V 2) 3V 3) 3.6 V 4) 1.8 V

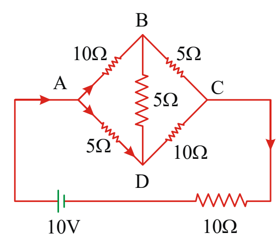
KEY : 4

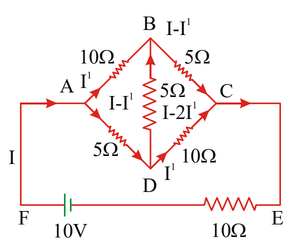
SOLUTION :

$$i = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{1 + R\left(\frac{1}{r_1} + \frac{1}{r_2}\right)} = \frac{\frac{5}{20} + \frac{2}{10}}{1 + 10\left(\frac{1}{20} + \frac{1}{10}\right)}$$

$$V = iR = i \times 10 = 1.8v$$

79. Determine the current in each branch of the network shown in fig.





SOLUTION :

Apply KVL in loop ABDA

$$-10I^1 + 5\left(I - 2I^1\right) + 5\left(I - I^1\right) = 0$$

$$2I = 5I' \text{.......(1)}$$

Apply KVL in ADCEFA loop

$$-5\left(I-I^1\right)-10I^1+10-10I=0$$

$$5I^1 + 15I = 10.......(2)$$

From equation (1) and (2)

$$I=\frac{10}{17}$$

$$I^1=\frac{21}{5}=\frac{4}{17}A$$

$$\text{Current in AB branch } =\frac{4}{17}$$

$$=I-I^1=\frac{10}{17}-\frac{4}{17}=\frac{6}{17}A$$

Current in DB branch

$$I-2I'=\frac{10}{17}-\frac{8}{17}=\frac{2}{17}A$$

80. In a metere bridge, the balance length from left end (standard resistance of 1Ω is in the right gap) is found to be 20 cm, the length of resistance wire in left gap is 1/2 m and radius is 2mm its specific resistance is

1) $\pi\times10^{-6}ohm-m$ **2)** $2\pi\times10^{-6}ohm-m$

3) $\frac{\pi}{2}\times10^{-6}ohm-m$ **4)** $3\pi\times10^{-6}ohm-m$

KEY : 2

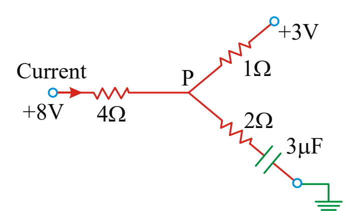
SOLUTION :

$X=\frac{sl}{A}$ $\frac{X}{R}=\frac{20}{80}$

$\frac{1}{4}=\frac{S\times\frac{1}{2}}{\pi\times\left(2\times10^{-3}\right)^2}$; $X=\frac{R\times2}{8}$

$X=\frac{1}{4}$; $S=2\pi\times10^{-6}\Omega-m$

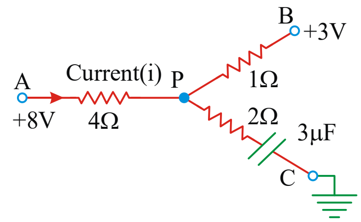
81. The energy stored in the capcitor is



1) $12\mu J$ **2)** $24\mu J$ **3)** $36\mu J$ **4)** $48\mu J$

KEY : 2

SOLUTION :



We have

$$V_A - V_B = i \times (4+1)$$

$$8-3 = i \times 5$$

$$5 = i \times 5$$

$$i = 1A$$

$$V_A - V_p = 4 \times 1$$

$$8 - V_P = 4;\ V_P = 4\,volt$$

Now $V_C = 0$. So, the energy stored in the capacitor is $\xi = \frac{1}{2} \times 3 \times 16 = 24\mu J$

82. When a conducting wire is connected in the right gap and known resistance in the left gap, the balancing length is 60cm. The balancing length becoms 42.4 cm when the wire is stretched so that its length increases by

1) 10% 2) 20% 3) 25% 4) 42.7%

KEY : 4

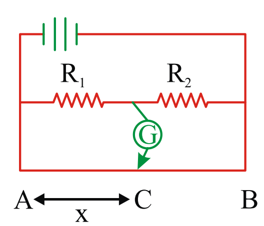
SOLUTION :

$$\frac{X}{R} = \frac{60}{40};\qquad \frac{X}{R'} = \frac{42.4}{57.6};\ \frac{R'}{R} = \frac{60}{40} \times \frac{57.6}{42.4}$$

$$R \alpha l^2$$

$$\frac{l^1 - l}{l} \times 100 = \left(\frac{\sqrt{R^1} - \sqrt{R}}{\sqrt{R}}\right) \times 100$$

83. In the shown arrangement of the experiment of the meter bridge if AC corresponding to null deflection of galvanometer is x, what would be its value if the radius of the wire AB is doubled?



SOLUTION :

For null deflection of galvanometer in a metrebridge experiment,

$$\frac{R_1}{R_2} = \frac{R_{AC}}{R_{CB}} \text{ or } \frac{R_1}{R_2} = \frac{x}{(100-x)}$$

Since R_1 / R_2 remains constant, x / (100-x) also remains constant. The value of x remains as such.

$\therefore$ Length of AC = x

84. A metallic conductor at $10^{\circ}C$ connected in the left gap of meter bridge gives balancing length 40 cm. When the conductor is at $60^{\circ}C$, the balancing point shifts by---cm, (tempera-ture coefficeint of resistance of the material of the wire is (1/220)/°C)

1) 4.8 2) 10 3) 15 4) 7

KEY :1

SOLUTION :

$$\frac{X}{R}=\frac{40}{60}=\frac{2}{3};\ \frac{X_0(1+\alpha t_1)}{R}=\frac{2}{3}$$

$$\frac{X_0(1+\alpha t_2)}{R}=\frac{l}{100-l}$$

$$\frac{1+\alpha t_1}{1+\alpha t_2}=\frac{2}{3}\times\left[\frac{100-l}{l}\right]$$

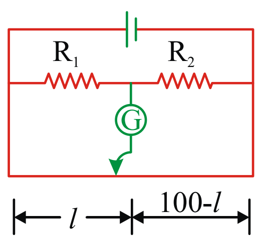
$$l=44.8$$

Balancing point shifts by =44.8-40=4.8.

85. A resistance of 2Ω is connected across one gap of a metre-bridge (the length of the wire is 100 cm) and an unknown resistance, greater than 2Ω, is connected across the other gap. When these resistances are interchanged, the balance point shifts by 20 cm. Neglecting any corrections, the unknown resistance is

SOLUTION :

Refer to the diagram Apply the conditions of the balanced Wheatstone's bridge for the two cases.

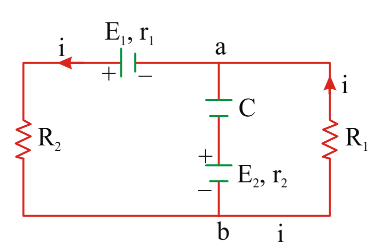


$$\frac{2}{x}=\frac{\ell}{100-\ell}\ ....(i)$$

$$\frac{x}{2}=\frac{\ell+20}{80-\ell}\ ....(ii)$$

Equations (i) and (ii) give $x=3\Omega$

86. In the steady state, the energy stored in the capacitor is :



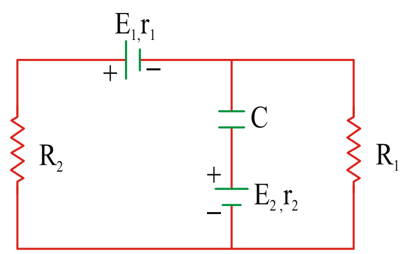
1) $\frac{1}{2}C(E_1+E_2)^2$ 2) $\frac{1}{2}C(E_1-E_2)^2$

3) $\frac{1}{2}C\left(\frac{E_1R_1+E_1R_2}{r_1+r_2+R_1+R_2}\right)^2$ 4) $\frac{1}{2}C\left(E_2+\frac{E_1R_1}{r_1+R_1+R_2}\right)^2$

KEY : 2

SOLUTION :

When the capacitor plate acquire full cahrge q_0, there will be no current in the capacitor arm. Applying Kirchhoff's second law to the current carrying circuit



$$i(R_1 + R_2) = E_1 - ir_1 \text{ or } i = \frac{E_1}{r_1 + R_1 + R_2}$$

Now $V_a - V_b = -iR_1 = -\frac{E_1 R_1}{r_1 + R_1 + R_2}$

and $V_C = \frac{q_0}{C} = E_2 + iR_1 = E_2 + \frac{E_1 R_1}{r_1 + R_1 + R_2}$

Now energy stored in the capacitor

$$U = \frac{1}{2} C V_c^2 \ ; \ = \frac{1}{2} C \left[E_2 + \frac{E_1 R_1}{r_1 + R_1 + R_2} \right]^2$$

87. The length of a potentiometer wire is 1m and its resistance is 4Ω. A current of 5 mA is flowing in it. An unknown source of e.m.f is balanced on 40 cm length of this wire, then find the e.m.f of the source.

SOLUTION :

$$x = I\rho = I\frac{R}{L} = \frac{5 \times 4}{1} = 20\, mV$$

E = 1 x = 0. 40 x 20 = 8 mV

88. 1 Ω resistance is in series with an Ammeter which is balanced by 75 cm of potentiometer wire. A standard cell of 1.02V is balanced by 50 cm. The Ammeter shows a reading of 1.5A. The error in the Ammeter reading is

1) 0.002A 2) 0.03A 3) 1.01A 4) no error

KEY : 2

SOLUTION :

$1.02\text{V} \rightarrow 50\text{cm}$

? $\rightarrow 75\text{cm}$

$$\text{V} = \frac{75 \times 1.02}{50} = 1.53 \ ; \ \text{error} = 1.53 - 1.5 = 0.03$$

89. A cell of e.m.f 2 volt and internal resistance 1.5Ω is connected to the ends of 1m long wire. The resistance of wire is $0.5\Omega/m$. Find the value of potential gradient on the wire.

SOLUTION :

$$X=\frac{IR}{L}=\left(\frac{E}{R+r}\right)\frac{R}{L}=\frac{2\times 0.5}{0.5+1.5}=\mathbf{0.5\ V/m}$$

90. An ideal battery of emf 2V and a series resistance R are connected in the primary circuit of a potentio meter of length 1m and resistance 5Ω. The value of R to give a potential difference of 5mV across the 10cm of potentiometer wire is

1) 180 Ω 2) 190 Ω 3) 195 Ω 4) 200 Ω

KEY : 3

SOLUTION :

$$5\times 10^{-3}=\frac{V}{L}l=\frac{iR}{L}l=\left(\frac{2}{R+5}\right)\frac{5}{1}\times 10\times 10^{-2}$$

91. In a potentiometer experiment the balancing length with a cell is 560 cm. When an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. Find the internal resistance of the cell.

SOLUTION :

Balancing length $\ell_1 = 560$ cm

Change in balancing length $(\ell_1-\ell_2)=60$ cm

$560-\ell_2=60$

$\therefore \ell_2 = 500$ cm

$$r=R\left(\frac{\ell_1-\ell_2}{\ell_2}\right)\Rightarrow r=10\times\frac{60}{500}=\frac{6}{5}=1.2\Omega .$$

92. A wire of length L and 3 identical cells of negligible internal resistances are connected in series. Due to the current, the temperature of the wire is raisied by ΔT in a time t. A number N of similar cells is now connected in series with a wire of the same material and cross-section but the length 2L The temperature of the wire is raised by the same amount ΔT in the same time t. The value of N is :

1) 3 2) 2 3) 6 4) 4

KEY :3

SOLUTION :

In the first case, three identical cells are connected in seires with a wire of length L. Let the terminal potential difference of each cell is V and resistance of the wire is R. Then heat developed in the wire in time t is

$$H=\frac{(3V)^2}{R}t=ms\Delta T$$

where m is the mass of the wire, s-the specific heat of its material and ΔT is the rise in its temperature. When N such indentical cells are connected in series, the effective terminal otential is NV volt and if the length of the wire is doubled, its resistance and mass also doubled. Then heat develope in the wire is

$$H^{`}=\frac{(NV)^2}{2R}t=(2m)s.\Delta T$$

Dividing both the equations, we get

$$\frac{N^2}{2\times 9}=2\Rightarrow N=6$$

93. In an experiment for calibration of voltmeter, a standard cell of emf 1.5V is balanced at 300cm length of potentiometer wire. The P.D.across a resistance in the circuit is balanced at 1.25m. If a voltmeter is connected across the same resistance, it reads 0.65V. The error in the volt meter is

1) 0.05V 2) 0.025V 3) 0.5V 4) 0.25V

KEY :2

SOLUTION :

$$300\times 10^{-3}\text{m}\Rightarrow 1.5\text{V}\ ;\ 1.25\text{m}\rightarrow ?$$

$$\text{V}=0.625\text{V}\ ;$$

Error in ammeter reading

$$=0.625-0.65=0.025\text{v}\cdot$$

94. In a potentiometer experiment when a battery of e.m.f. 2V is included in the secondary circuit, the balance point is 500cm. Find the balancing length of the same end when a cadimum cell of e.m.f. 1.018V is connected to the secondary circuit.

SOLUTION :

$$\text{E}\propto \ell$$

$$\frac{\text{E}_1}{\text{E}_2}=\frac{\ell_1}{\ell_2}$$

$$\ell_2=\frac{\text{E}_2}{\text{E}_1}\times \ell_1=\frac{1.018}{2}\times 500=254.5\,\text{cm}$$

95. A potentiometer wire of length 100cm has a resistance 5Ω. It is connected in series with a resistance and a cell of emf 2v and of negligible internal resistance. A source of emf 5mv balanced by 10 cm length of potentiometer wire. The value of external reistance is _____

1) 540Ω **2)** 195Ω **3)** 190Ω **4)** 990Ω

KEY : 2

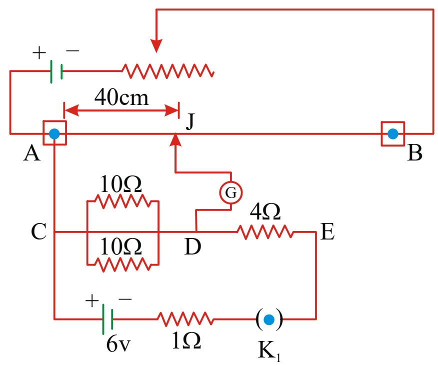
SOLUTION :

$$\text{E}'=\text{i}\rho\, l=\left[\frac{\text{E}}{\text{R}+\text{R}_3}\right]\frac{\text{R}}{\text{L}}\cdot l$$

$$5\times 10^{-3}=\left[\frac{2}{5+\text{R}_3}\right]\frac{5}{100}\times 10$$

$$\text{R}_s=195\Omega$$

96. In the circuit shown in fig., the potential difference between the points C and D is balanced against 40 cm length of poten-tiometer wire of total length 100 cm. In order to balance the potential difference between the points D and E. The jockey to be pressed on potentiometer wire at a distance of



1) 16 cm 2) 32 cm 3) 56 cm 4) 80 cm

KEY : 2

SOLUTION :

$$\frac{V_1}{V_2}=\frac{iR_1}{iR_2}=\frac{l_1}{l_2}$$
$$\therefore \frac{5}{4}=\frac{40}{l_2}$$

97. The resistance of a 240 V – 200 W electric bulb when hot is 10 times the resistance when cold. The resistance at room temperature and the temperature coefficient of the filament are (given working temperature of the filament is $2000°C$)

1) $28.8\Omega, 4.5\times10^{-3}/°C$ 2) $14.4\Omega, 4.5\times10^{-3}/°C$

3) $28.8\Omega, 3.5\times10^{-3}/°C$ 4) $14.4\Omega, 3.5\times10^{-3}/°C$

KEY : 1

SOLUTION :

Resistance of the hot bulb $R_2=\frac{V^2}{P}=\frac{240\times240}{200}$

Resistance of the bulb at room temperature

$$R_1=\frac{R_2}{10}=\frac{288}{10};\ \alpha=\frac{R_2-R_1}{R_1 t}.$$

98. In an experiment with potentiometer to measure the internal resistance of a cell, when the cell is shunted by 5Ω, the null point is obtained at 2m. When cell is shunted by 20Ω the null point is obtained at 3m. The internal resistance of cell is

1) 2Ω 2) 4Ω 3) 6Ω 4) 8Ω

KEY : 2

SOLUTION :

$$\frac{V_1}{V_2}=\frac{l_1}{l_2}$$

$$V_1=\left[\frac{E}{R_1+r}\right]R_1, V_2=\left[\frac{E}{R_2+r}\right]R_2$$

$$\frac{R_1(R_2+r)}{R_2(R_1+r)}=\frac{l_1}{l_2}\Rightarrow r=4\Omega$$

THEORY BITS

1. **Material used for heating coils is**
   1) Nichrome 2) Copper3) Silver 4) Manganin

KEY:1

2. **Among the following dependences of drift velocity v_d on electric field E, Ohm's Law obeyed is**
   1) $v_d \alpha E$ 2) $v_d \alpha E^2$ 3) $v_d \alpha \sqrt{E}$ 4) v_d = constant

KEY:1

3. **A heater coil is cut into two equal parts and only one part is used in the heater. Then the heat generated becomes**
   1) become one fourth 2) halved 3) doubled 4) become four times

KEY:3

4. **A steady current is passing through a linear conductor of nonuniform cross-section. The net quantity of charge crossing any cross section per second is**
   1) independent of area of cross-section
   2) directly proportional to the length of the conductor
   3) directly proportional to the area of cross section.
   4) inversely proportional to the area of the conductor

KEY:1

5. **Fuse wire is a wire of**
   1) low melting point and low value of α
   2) high melting pointand high value of α
   3) high melting point and low value of α
   4) low melting point and high value of α

KEY:4

6. **The drift speed of an electron in a metal is of the order of**
   1) 10^{-13} m/s 2) 10^{-3} mm/s 3) 10^{-4} m/s 4) 10^{-30} m/s

KEY:3

7. **Two electric bulbs rated P_1 watt and V volt, are connected in series, across V-volt supply. The total power consumed is**
   1) $\frac{P_1 + P_2}{2}$ 2) $\sqrt{P_1 \cdot P_2}$ 3) $\frac{P_1 \cdot P_2}{P_1 + P_2}$ 4) $(P_1 + P_2)$

KEY:3

8. **In metals and vacuum tubes charge carriers are**
   1) electrons 2) protons 3) both 4) positrons

KEY:1

9. **At absolute zero silver wire behaves as**
   1) Super conductor 2) Semi conductor 3) Perfect insulator 4) Semi insulator

KEY:2

10. **The electric intensity E, current density j and conductivity σ are related as :**
    1) $j = \sigma E$ 2) $j = E/\sigma$ 3) $jE = \sigma$ 4) $j = \sigma^2 E$

KEY:1

11. **Electric field (E) and current density (J) have relation**
    1) $E \propto J^{-1}$ 2) $E \propto J$ 3) $E \propto \frac{1}{J^2}$ 4) $E^2 \propto \frac{1}{J}$

KEY:2

12. A steady current flows in a metallic conductor of non-uniform cross-section. The quantity/quantities constant along the length of the conductor is/are
1) current, electric field and drift speed
2) drift speed only
3) current and drift speed 4) current only

KEY:4

13. In an electric circuit containg a battery, the charge (assumed positive) inside the battery
1) always goes form the positive terminal to the negative terminal
2) may move from the positive terminal to the negative terminal
3) always goes from the negative terminal to the positive terminal
4) does not move .

KEY:2

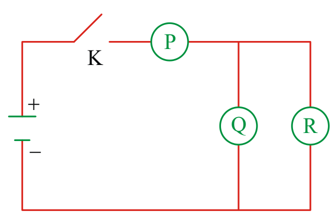
14. The resistance of an open circuit is
1) Infinity 2) Zero
3) Negative 4) cann't be predicted

KEY:1

15. From the following the quantity which is analogous to temperature in electricity is
1) potential 2) resistance
3) current 4) charge

KEY:1

16. Three identical bulbs P, Q and R are connected to a battery as shown in the figure. When the circuit is closed



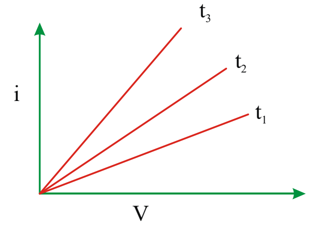
1) Q and R will be brighter than P
2) Q and R will be dimmer than P
3) All the bulbs will be equally bright
4) Q and R will not shine at all

KEY:2

17. For making standard resistance, wire of following material is used
1) Nichrome 2) Copper
3) Silver 4) manganin

KEY:4

18. i-v graph for a metal at temperatures t_1, t_2, t_3 are as shown. The highest temperature is



1) t_1 2) t_2 3) t_3 4) $t_1 = t_2 = t_3$

KEY:1

19. With the increase of temperature, the ratio of conductivity to resistivity of a metal conductor
    1) Decreases 2) Remains same
    3) Increases 4) May increase or decrease

KEY:1

20. Temperature coefficient of resistance 'α' and resistivity 'ρ' of a potentiometer wire must be
    1) high and low 2) low and high
    3) low and low 4) high and high

KEY:2

21. Metals have
    1) Zero resistivity 2) High resistivity
    3) Low resistivity 4) Infinite resistivity

KEY:1

22. Kirchoff's law of meshes is in accordance with law of conservation of
    1) charge 2) current
    3) energy 4) angular momentum

KEY:3

23. Consider a rectangular slab of length L, and area of cross-section A. A current I is passed through it, if the length is doubled the potential drop across the end faces
    1) Becomes half of the initial value 2) Becomes one-forth of the initial value
    3) Becomes double the initial value
    4) Remains Same

KEY:3

24. A metallic block has no potential difference applied across it, then the mean velocity of free electrons is (T = absolute temperature of the block)
    1) Proportional to T 2) Proportional to $\sqrt{T}$
    3) Zero
    4) Finite but independent of temperature.

KEY:3

25. The resistance of a metal increases with increasing temperature because
    1) The collisions of the conducting electrons with the electrons increases.
    2) The collisions of the conducting electrons with the lattice consisting of the ions of the metal increases
    3) The number of the conduction electrons decreases.
    4) The number of conduction electrons increase.

KEY:2

26. In the absence of applied potential, the electric current flowing through a metallic wire is zero because
    1) The average velocity of electron is zero
    2) The electrons are drifted in random direction with a speed of the order of 10^{-2}cm/s.
    3) The electrons move in random direction with a speed of the order close to that of velocity of light.
    4) Electrons and ions move in opposite direction.

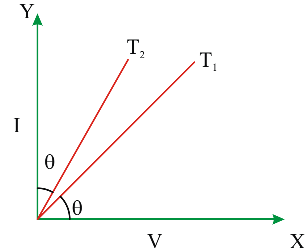
KEY:1

27. Ohm's law is not applicable for
    1) insulators 2) semi conductors
    3) vaccum tube 4) all the above

KEY:4

28. V - I graphs for two materials is shown in the figure. The graphs are drawn at two different

temperatures.



1) $T_1 - T_2 \propto \cot 2\theta$ 2) $T_1 - T_2 \propto \sin 2\theta$

3) $T_1 - T_2 \propto \tan 2\theta$ 4) $T_1 - T_2 \propto \cos 2\theta$

KEY:1

29. Wires of Nichrome and Copper of equal dimensions are connected in series in electrical electrical circuit. Then.
 1) More current will flow in copper wire
 2) More current will flow in Nichrome wire
 3) Copper wire will get heated more
 4) Nichrome wire will get heated more

KEY:4

30. The balancing lengths of potentiometer wire are l_1 and l_2 when two cells of emf E_1 and E_2 are connected in the secondary circuit in series first to help each other and next to oppose each other $\frac{E_1}{E_2}$ is equal to ($E_1 > E_2$).

 1) $\frac{l_1}{l_2}$ 2) $\frac{l_1 - l_2}{l_1 + l_2}$ 3) $\frac{l_1 + l_2}{l_1 - l_2}$ 4) $\frac{l_2}{l_1}$

KEY:1

31. Assertion : Material used in the construction of a standard resistance is constantan or manganin.
 Reason : Temperature coefficient of constantan is very small.
 1) Both (A) and (R) are true and (R) is the correct explanation of A.
 2) Both (A) and (R) are true but (R) is not the correct explanation of A.
 3) (A) is true but (R) is false
 4) (A) is false but (R) is true

KEY:1

32. The thermistors are usually made of
 1) metals with low temperature coefficient of resistivity
 2) metals with high temperature coefficient of resistivity.
 3) metal oxides with high temperature coefficient of resistivity
 4) semiconducting materials having

KEY:3

33. A piece of copper and another of germanium are cooled from room temperature to 80K. The resistance of
 1) each of them increases
 2) each of them decreases
 3) copper increases and germanium decreases
 4) copper decreases and germanium increases

KEY:4

34. Read the following statements carefully
 Y: The resistivity of semiconductor decreases with increase of temperature
 Z: In a conducting solid, the rate of collisions between free electrons and ions increases with increases

of temperature.
Select the correct statement(s) from the following
1) Y is true but Z is false
2) Y is false but Z is true
3) Both Y and Z are true
4) Y is true and Z is the correct reason for Y

KEY:3

35. Assertion : Potentiometer is much better than a voltmeter for measuring emf of cell
Reason : A potentiometer draws no current while measuring emf of a cell
1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:1

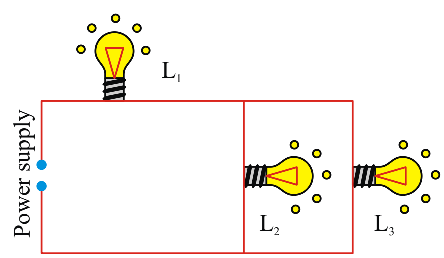
36. Two lamps have resistance r and R, R being greater than r. If they are connected in parallel in an electric circuit, then
1) the lamp with resistance R will shine more brightly
2) the lamp with resistance r will shine more brightly
3) the two lamps will shine equal brightly
4) the lamp with resistance R will not shine at all

KEY:2

37. Two bulbs are fitted in a room in the domestic electric installation. If one of them glows brighter than the other, then
1) the brighter bulb has smaller resistance
2) the brighter bulb has larger resistance
3) both the bulbs have the same resistance
4) nothing can be said about the resistance unless other factors are known

KEY:1

38. Figure shows three similar lamps L_1, L_2, L_3 connected across a power supply. If the lamp L_3 fuses. The light emitted by L_1 and L_2 will change as



1) no change
2) brilliance of L_1 decreases and that of L_2 increases
3) brilliance of both L_1 and L_2 increases
4) brilliance of both L_1 and L_2 decreases

KEY:2

39. The potential difference across a conductor is doubled, the rate of generation of heat will
1) become one fourth 2) be halved
3) be doubledtimes 4) become four times

KEY:4

40. The flow of the electric current through a metallic conductor is
1) only due to electrons
2) only due to +ve charges

3) due to both nuclei and electrons.

4) can not be predicted.

KEY:1

41. Two metallic wires of same material and same length have different diameters. When the wires are connected in parallel across an ideal battery the rate of heat produced in thinner wire is Q_1 and that in thicker wire is Q_2. The correct statement is

1) $Q_1 = Q_2$ 2) $Q_1 < Q_2$ 3) $Q_1 > Q_2$

4) It will depend on the emf of the battery

KEY:2

42. There are two metalic wires of same material, same length but of different radii. When these are connected to an ideal battery in series, heat produced is H_1 but when connected in parallel, heat produced is H_2 for the same time. Then the correct statement is

1) $H_1 = H_2$ 2) $H_1 < H_2$

3) $H_1 > H_2$ 4) No relation

KEY:2

43. When light falls on semiconductors, their resistance

1) decreases 2) increases

3) does not change 4) can't be predicted

KEY:1

44. In above question, if the bulbs are connected in parallel, total power consumed is

1) $\frac{P_1 + P_2}{2}$ 2) $\sqrt{P_1 \cdot P_2}$ 3) $\frac{P_1 \cdot P_2}{P_1 + P_2}$ 4) $(P_1 + P_2)$

KEY:4

45. If n, e, τ, m, are representing electron density, charge, relaxation time and mass of an electron respectively then the resistance of wire of length l and cross sectional area A is given by

1) $\frac{ml}{ne^2\tau A}$ 2) $\frac{2mA}{ne^2\tau}$ 3) $ne^2\tau A$ 4) $\frac{ne^2\tau A}{2m}$

KEY:1

46. Which of the following causes production of heat, when current is set up in a wire

1) Fall of electron from higher orbits to lower orbits

2)Inter atomic collisions

3)Inter electron collisions

4)Collisions of conduction electrons with atoms

KEY:4

47. A constant voltage is applied between the two ends of a metallic wire. If both the length and the radius of the wire are doubled, the rate of heat developed in the wire

1) will be doubled 2) will be halved

3) will remain the same 4) will be quadrupled

KEY:1

48. Back emf of a cell is due to

1) Electrolytic polarization

2) Peltier effect

3) Magnetic effect of current

4) Internal resistance

KEY:1

49. The direction of current in a cell is
1) (–) ve pole to (+) ve pole during discharging
2) (+) ve pole to (–) ve pole during discharging
3) Always (–) ve pole to (+) ve pole
4) always flows from (+) ve ploe to (–) ve pole

KEY:1

50. When an electric cell drives current through load resistance, its Back emf,
1) Supports the original emf
2) Opposes the original emf
3) Supports if internal resistance is low
4) Opposes if load resistance is large

KEY:1

51. The terminal voltage of a cell is greater than its emf. when it is
1) being charged 2) an open circuit
3) being discharged 4) it never happens

KEY:1

52. What is constant in a battery (also called a source of emf) ?
1) current supplied by it
2) terminal potential difference
3) internal resistance 4) emf

KEY:4

53. A cell is to convert
1) chemical energy into electrical energy
2) electrical energy into chemical energy
3) heat energy into potential energy
4) potential energy into heat energy

KEY:1

54. 'n' identical cells, each of internal resistance (r) are first connected in parallel and then connected in series across a resistance (R). If the current through R is the same in both cases, then
1) R = r/2 2) r = R/2 3) R = r 4) r = 0

KEY:3

55. The value of internal resistance of ideal cell is
1) Zero 2) infinite 3) 1 Ω 4) 2 Ω

KEY:1

56. When electric field ($\vec{E}$) is applied on the ends of a conductor, the free electrons starts moving in direction
1) similar to $\vec{E}$ 2) Opposite to $\vec{E}$
3) Perpendicular to $\vec{E}$ 4) Cannot be predicted

KEY:2

57. In a circuit two or more cells of the same emf are connected in parallel in order
1) Increases the pd across a resistance in the circuit
2) Decreases pd across a resistance in the circuit
3) Facilitate drawing more current from the battery system
4) Change the emf across the system of batteries

KEY:3

58. The sensitivity of potentiometer wire can be increased by
1) decreasing the length of potentiometer wire
2) increasing potential gradient on its wire
3) increasing emf of battery in the primary circuit

4) decreasing the potential gradient on its wire

KEY:4

59. According to joule's law if potential difference across a conductor having a material of specific resistance ρ, remains constant, then heat pro-duced in the conductor is directly proportional to

1) ρ 2) ρ^2 3) $\frac{1}{\sqrt{\rho}}$ 4) $\frac{1}{\rho}$

KEY:4

60. Internal resistance of a cell depends on
1) concentration of electrolyte
2) distance between the electrodes
3) area of electrode
4) all the above

KEY:4

61. When cells are arranged in series
1) the current capacity decreases
2) The current capacity increases
3) the emf increases 4) the emf decreases

KEY:3

62. On increasing the resistance of the primary circuit of potentiometer, its potential gradient will
1) become more 2) become less
3) not change 4) become infinite

KEY:2

63. To supply maximum current, cells should be arrange in
1) series 2) parallel 3) Mixed grouping
4) depends on the internal and external resistance

KEY:4

low temperature coefficient of resistivity

64. For a chosen non-zero value of voltage, there can be more than one value of current in
1) copper wire 2) thermistor
3) zener diode 4) manganin wire

KEY:2

65. The terminal Pd of a cell is equal to its emf if
1) external resistance is infinity
2) internal resistance is zero
3) both 1 and 2
4) internal resistance is 5Ω

KEY:3

66. The electric power transfered by a cell to an external resistance is maximum when the external resistance is equal to ...(r internal resistance)

1) r/2 2) $2r$ 3) r 4) r^2

KEY:3

67. Which depolarizers are used to neutralizes hydrogen layer in cells
1) Potassium dichromite 2) Manganese dioxide
3) 1 or 2 4) hydrogen peroxide

KEY:3

68. Assertion : A current flows in a conductor only when there is an electric field within the conductor.
Reason : The drift velocity of electron in presence of electric field decreases.
1) Both (A) and (R) are true and (R) is the correct explanation of A.

2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:3

69. Assertion : Series combination of cells is used when their internal resitance is much smaller than the external resistance.

Reason : $I = \frac{n\varepsilon}{R+nr}$ where the symbols have their standard meaning,in series connection

1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:1

70. Assertion (A) : To draw more current at low P.d; parallel connection of cells is preferred.

Reason (R) : In parallel connection, current $i = \frac{nE}{r}$, if r >> R.

1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:1

71. When a piece of aluminium wire of finite length is drawn through a series of dies to reduce its diameter to half its original value, its resistance will become

1) Two times 2) Four times
3) Eight times 4) Sixteen times

KEY:4

72. Kirchoff's law of junctions is also called the law of conservation of

1) energy 2) charge 3) momentum
4) angular momentum

KEY:2

73. Wheatstones's bridge cannot be used for measurement of very ——— resistances.

1) high 2) low 3) low(or) high 4) zero

KEY:2

74. In a balanced Wheatstone's network, the resi-stances in the arms Q and S are interchan-ged. As a result of this :

1)galvanometer and the cell must be interchanged to balance
2) galvanometer shows zero deflection
3) network is not balanced
4) network is still balanced

KEY:3

75. If galvanometer and battery are interchanged in balanced wheatstone bridge, then

1) the battery discharges
2) the bridge still balances
3) the balance point is changed
4) the galvanometer is damaged due to flow of high current

KEY:2

76. The conductivity of a super conductor, in the super conducting state is

1) Zero 2) Infinity
3) Depends on temp
4) Depends on free election

KEY:2

77. Wheatstone bridge can be used
1) To compare two unknown resistances.
2) to measure small strains produced in hardmetals
3) as the working principle of meterbridge
4) All the above

KEY:4

78. In a wheatstone's bridge three resistances P,Q,R connected in three arms and the fourth arm is formed by two resitances S_1,S_2 connected in parallel.The condition for bridge to be balanced will be
1) $\frac{P}{Q}=\frac{R}{S_1+S_2}$ 2) $\frac{P}{Q}=\frac{2R}{S_1+S_2}$
3) $\frac{P}{Q}=\frac{R(S_1+S_2)}{S_1S_2}$ 4) $\frac{P}{Q}=\frac{R(S_1+S_2)}{2S_1S_2}$

KEY:3

79. A piece of silver and another of silicon are heated from room temperature. The resistance of
1) each of them increases
2) each of them decreases
3) Silver increases and Silicon decreases
4) Silver decreases and Silicon increases

KEY:3

80. Assertion : At any junction of a network, algebraic sum of various currents is zero
Reason : At steady state there is
no accumulation of charge at the junction.
1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:1

81. A metre bridge is balanced with known resistance in the right gap and a metal wire in the left gap. If the metal wire is heated the balance point.
1) shifts towards left
2) shifts towards right
3) does not change
4) may shift towards left or right depending on the nature of the metal.

KEY:2

82. In metre bridge experiment of resistances, the known and unknown resistances are inter-changed . The error so removed is
1) end correction
2) index error
3) due to temperature effect
4) random error

KEY:1

83. In a metre-bridge experiment, when the resistances in the gaps are interchanged, the balance-point did not shift at all. The ratio of resistances must be
1) Very large 2) Very small
3) Equal to unity 4) zero

KEY:3

84. A certain piece of copper is to be shaped into a conductor of minimum resistance. Its length and cross sectional area should be

1) L and A 2) 2L and A/2
3) L/2 and 2A 4) 3L and A/3

KEY:3

85. Assertion (A) : Meterbridge wire is made up of manganin
Reason (R) : The temperature coeffiecient of resistance is very small for manganin
1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false 4) (A) is false but (R) is true

KEY:1

86. Assertion (A) : When the radius of a copper wire is doubled, its specific resistance gets increased.
Reason (R):Specific resistance is independent of cross-section of material used
1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:4

87. In comparing emf's of 2 cells with the help of potentiometer, at the balance point, the current flowing through the wire is taken from
1) Any one of these cells. 2) both of these cells
3) Battery in the primary circuit
4) From an unknown source

KEY:3

88. A potentiometer wire is connected across the ideal battery now, the radius of potentiometer wire is doubled without changing its length. The value of potential gradient
1) increases 4 times 2) increases two times
3) Does not change 4) becomes half

KEY:3

89. A resistor R_1 dissipates the power P when connected to a certain generator. If the resistor R_2 is put in series with R_1, the power dissipated by R_1
1) Decreases 2) Increases
3) Remains the same
4) Any of the above depending upon the relative values of R_1 and R_2

KEY:1

90. In a potentiometer of ten wires, the balance point is obtained on the sixth wire. To shift the balance point to eighth wire, we should
1) increase resistance in the primary circuit.
2) decrease resistance in the primary circuit.
3) decrease resistance in series with the cell whose emf. has to be measured.
4) increase resistance in series with the cell whose emf. has to be measured.

KEY:1

91. If the emf of the cell in the primary circuit is doubled, with out changing the cell in the secondary circuit, the balancing length is
1) Doubled 2) Halved
3) Uncharged 4) Zero

KEY:2

92. The potential gradients on the potentiometer wire are V_1 and V_2 with an ideal cell and a real cell of same emf in the primary circuit then
1) $V_1 = V_2$ 2) $V_1 > V_2$ 3) $V_1 < V_2$ 4) $V_1 \leq V_2$

KEY:2

93. A potentiometer is superior to voltmeter for measuring a potential because

1) voltmeter has high resistance
2) resistance of potentiometer wire is quite low
3) potentiometer does not draw any current from the unknown source of emf. to be measured.
4) sensitivity of potentiometer is higher than that of a voltmeter.

KEY:3

94. A series high resistance is preferable than shunt resistance in the galvanometer circuit of potentiometer. Because
1) shunt resistances are costly
2) shunt resistance damages the galvanometer
3) series resistance reduces the current through galvanometer in an unbalanced circuit
4) high resistances are easily available

KEY:3

95. A cell of emf 'E' and internal resistance 'r' connected in the secondary gets balanced against length 'ℓ' of potentiometer wire. If a resistance 'R' is connected in parallel with the cell, then the new balancing length for the cell will be

1) $\left(\frac{R}{R-r}\right)l$ 2) $\left(\frac{R-r}{R}\right)l$ 3) $\left(\frac{R}{r}\right)$ 4) $\left(\frac{R}{R+r}\right)l$

KEY:4

96. Given a current carrying wire of non-uniform cross section. Which of the following quantity or quantities are constant throughout the length of the wire?
1) current, electric field and drift speed
2) drift speed only
3) current and drift speed
4) current only

KEY:4

97. Potentiometer is an ideal instrument, because
1) no current is drawn from the source of unknown emf
2) current is drawn from the source of unknown emf
3) it gives deflection even at null point
4) it has variable potential gradient

KEY:1

98. If the value of potential gradient on potentiometer wire is decreased, then the new null point will be obtained at
1) lower length 2) higher length
3) same length 4) nothing can be said

KEY:2

99. A cell of negligible internal resistance is connected to a potentiometer wire and potential gradient is found. Keeping the length as constant, if the radius of potentiometer wire is increased four times, the potential gradient will become (no series resistance in primary)
1) 4 times 2) 2 times 3) half 4) constant

KEY:4

100. For the working of potentiometer, the emf of cell in the primary circuit (E) compared to the emf of the cell in the secondary circuit (E^1) is
1) $E > E^1$ 2) $E < E^1$
3) Both the above 4) $E = E^1$

KEY:1

101. A long constan wire is connected across the terminals of an ideal battery. if the wire is cut in to two equal pieces and one of them is now connected to the same battery, what will be the mobility of free electrons now in the wire compared to that in the first case?
1) same as that of previous value

2) double that of previous value
3) half that of previous value
4) four times that of previous value

KEY:1

102. At the moment when the potentiometer is balanced,
1) Current flows only in the primary circuit
2) Current flows only in the secondary circuit
3) Current flows both in primary and secondary circuits
4) current does not flow in any circuit

KEY:1

103. The quantity that cannot be measured by a potentiometer is ....
1) Resistance 2) emf
3) current in the wire 4) Inductance

KEY:4

104. A : The emf of the cell in secondary circuit must be less than emf of cell in primary circuit in potentiometer.
R : Balancing length cannot be more than length of potentiometer wire.
1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:1

105. From the following the standard cell is
1) Daniel cell 2) Cadmium cell
3) Leclanche cell 4) Lead accumulator

KEY:2

106. Metal wire is connected in the left gap, semi conductor is connected in the right gap of meter bridge and balancing point is found. Both are heated so that change of resistances in them are same. Then the balancing point
1) will not shift
2) shifts towards left
3) shifts towards right
4) depends on rise of temperature

KEY:3

107. Assertion (A) : Bending of a conducting wire effects electrical resistance.
Reason (R) : Resistance of a wire depends on resistivity of that material.
1) Both (A) and (R) are true and (R) is the correct explanation of A.
2) Both (A) and (R) are true but (R) is not the correct explanation of A.
3) (A) is true but (R) is false
4) (A) is false but (R) is true

KEY:2

108. If the current in the primary circuit is decreased, then balancing length is obtained at
1) Lower length 2) Higher length
3) Same length 4) 1/3rd length

KEY:2

PREVIOUS JEE MAINS QUESTIONS

CURRENT ELECTRICITY

1. A circuit to veri5r Ohm' s law uses ammeter and voltmeterin series or parallel connected correctly to the resistor.In the circuit: [Sep. 06, 2020 (II)]

(a) ammeter is always used in parallel and voltmeter is series

(b) Both ammeter and voltmeter must be connected in

parallel

(c) ammeter is always connected in series and voltmeter in parallel

(d) Both, ammeter and voltmeter must be connected in series

SOLUTION : (c)

Ammeter: In series connection, the same currentflows through all the components. It aims at measuring the current flowing through the circuit and hence, it isconnected in series.Voltmeter: A voltmeter measures voltage change betweentwo points in a circuit. So we have to place the voltmeter inparallel with the cicuit component.

2. Consider four conducting materials copper, tungsten, mercury and aluminium with resistivity p_C, p_T, p_M and $p_A$respectively. Then : [Sep. 02, 2020 (I)]

(a) $p_C > p_A > p_T$ (b) $p_M > p_A > Pc$

(c) $p_A > p_T > Pc$ (d) $p_A > p_M > Pc$

SOLUTION : . (b)

$$p_M = 98 \times 10^{-8}$$

$$p_A = 2.65 \times 10^{-8}$$

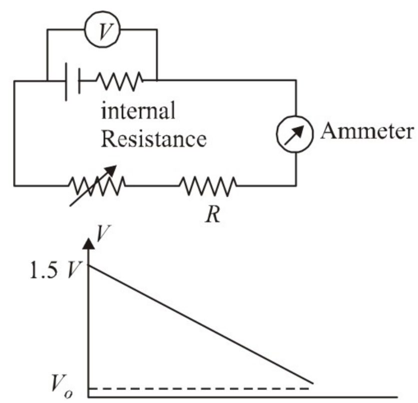
$$p_C = 1.724 \times 10^{-8}$$

$$p_T = 5.65 \times 10^{-8}$$

$$p_M > p_T > p_A > p_C$$

3. To verify Ohm' s law, a student connects the voltmeteracross the battery as, shown in the figure. The measuredvoltage is plotted as a function of the current, and thefollowing graph is obtained:

[12 Apr. 2019 I]



$I \rightarrow$ 1000 mA

IfV_0 is almost zero, identifythe correct statement:

(a) The emf of the battery is 1. 5 V and its internal

resistance is 1.5Ω

(b) The value ofthe resistance R is 1.5Ω

(c) The potential difference across the batteryis 1. 5 V whenit sends a current of 1000 mA

(d) The emfofthe batteryis 1. 5 V and the value of R is 1.5Ω

SOLUTION : (a)

When $i = 0, V = \varepsilon = 1.5$ volt

4. A current of 5 A passes through a copper conductor

(resistivity) $= 1.7 \times 10^{-8}\Omega\text{m})$ of radius of cross - section5 mm. Find the mobility of the charges if their driftvelocity is $1.1 \times 10^{-3}\text{m/s}$. [10 Apr. 2019 I]

(a) $1.8\text{m}^2/\text{Vs}$ (b) $1.5\text{m}^2/\text{Vs}$

(c) $1.3\text{m}^2/\text{Vs}$ (d) 1. 0n ≠/∨ s

SOLUTION : (d)

Charge mobility

$(\mu) = \frac{V_d}{E}$ [Where V_d = driit velocity]

and resistivity $(p) = \frac{E}{j} = \frac{EA}{I} \Rightarrow E = \frac{I(p)}{A}$

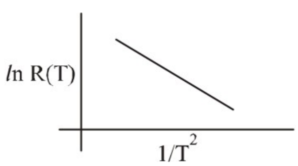
$$\Rightarrow \mu = \frac{V_d}{E} = \frac{V_d A}{I_p}$$

$$= \frac{1.1 \times 10^{-3} \times . \pi \times (5 \times 10^{-3})^2}{5 \times 17 \times 10^{-8}}$$

$$\mu = 1.0 \frac{m^2}{V_s}$$

5. In an experiment, the resistance ofa material is plotted asa function of temperature (in some range). As shown inthe figure, it is a straight line.

[10 Apr. 2019 I]



One may canclude that:

(a) $R(T) = \frac{R_0}{T^2}$ (b) $R(T) = R_0 e^{-T_0^2/T^2}$

(c) $R(T) = R_0 e^{-T^2/T_0^2}$ (d) $R(T) = R_0 e^{T^2/T_0^2}$

SOLUTION : (b)

Equation ofstraight line from graph

$$y = -mx + c$$

$$\Rightarrow lnR = -m\left(\frac{1}{T^2}\right) + c$$

here, m & c are constants

$$R = e\left[-m\left(\frac{1}{T^2}\right) + c\right] = e^{-m\left(\frac{1}{T^2}\right)} \times e^c$$

$$\frac{-T_0^2}{2}$$

$$R(T) = R_0 e^T$$

6. Space between two concentric conducting spheres ofradii a and $b(b > a)$ is filled with a medium ofresistivity ρ. Theresistance between the two spheres will be: [10Apr. 2019 II]

(a) $\frac{p}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$ (b) $\frac{p}{2\pi}\left(\frac{1}{a} - \frac{1}{b}\right)$

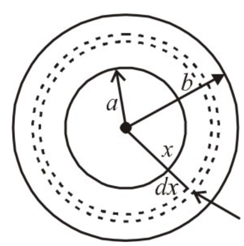
(c) $\frac{p}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$ (d) $\frac{p}{4\pi}\left(\frac{1}{a} + \frac{1}{b}\right)$

SOLUTION : (a)

$$dR = \frac{(p)(dx)}{4\pi\kappa^2}$$

$$R = \int dR$$

$$\int dR = p\int_a^b \frac{dx}{4\pi x^2}$$



$$\Rightarrow R = \frac{p}{4\pi}\left[\frac{-1}{x}\right]_a^b$$

$$R = \left(\frac{p}{4\pi}\right).\left(\frac{1}{a} - \frac{1}{b}\right)$$

7. In a conductor, ifthe number of conduction electronsper unit volume is $8.5 \times 10^{28} m^{-3}$ and mean free time is25 fs (femto second), it' s approximate resistivity is:$\left(m_e = 9.1 \times 10^{-31} kg\right)$ [9Apr. 2019II]

(a) $10^{-6} \Omega m$ (b) $10^{-7} \Omega m$

(c) $10^{-8} \Omega m$ (d) $10^{-5} \Omega m$

SOLUTION : . (c)

$$p = \frac{m}{ne^{2}\tau}$$

$$= \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-1}}$$

$$= 10^{8} \Omega - \mathrm{m}$$

8. A 200Ω resistor has a certain color code. Ifone replacesthe red color by green in the code, the new resistance will be: [8 April 2019 I]

(a) 100 Ω (b) 400 Ω (c) 300 Ω (d) 500 Ω

SOLUTION : . (d)

Number 2 is associated with the red colour. This

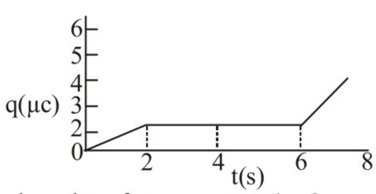
colour is replaced by green.

Colour code figure for green is 5

New resistance $= 500\Omega$

9. The charge on acapacitor plate in acircuit, as afunction oftime, is shown in the figure:

[12 Jan. 2019 II]



What is the value of current at t = 4 s ?

(a) Zero (b) 3 μA (c) 2 μA (d) 1.5μA

SOLUTION : . (a)

Clearly, from graph

Current, $I = \frac{dq}{dt} = 0$ at $t = 4$s [Since q is constant]

10. A resistance is shown in the figure. Its value and tolerance are given respectively by:

[9 Jan. 2019 I]

RED ↘ ↙ORANGE

VIOLET ↑ SILVER ↖

(a) 270 Ω, 10% (b) 27 kΩ, 10%

(c) 27 kΩ,20% (d) 270 Ω, 5%

SOLUTION : (b)

Color code: Bl, Br, R, O, Y, G, B, V, Gr, W

0,1, 2, 3, 4, 5, 6, 7, 8 9

R=AB×C±D% where D = tolerance

$D_{gold} = \pm 5\%, D_{silver} = \pm 10\%; D_{nocolour} = \pm 20\%$

Red voilet orange silver

$R = 27 \times 10^{3}\ \Omega \pm 10\% = 27\ \mathrm{k}\Omega \pm 10\%$

11. Drift speed ofelectrons, when 1.5 A ofcurrent flows in acopper wire of cross section 5 mm^2, is v. If the electrondensity in copper is $9 \times 10^{28}/\mathrm{m}^3$ the value of v in mm/sclose to (Take charge of electron to be $= 1.6 \times 10^{-19}$C) [9 Jan. 2019 I]

(a) 0.02 (b) 3 (c) 2 (d) 0.2

SOLUTION : (a)

Using, $I = neAv_d$

Drift speed $v_d = \frac{1}{neA}$

$$\frac{1.5}{9\times10^{28}\times1.6\times10^{-19}\times5\times10^{-6}} = 0.02 \text{ mms } 1$$

12. A copper wire is stretched to make it 0.5% longer. Thepercentage change in its electrical resistance ifits volumeremains unchanged is: [9 Jan. 2019 I]

(a) 2.0% (b) 2.5% (c) 1.0% (d) 0.5%

SOLUTION : (c)

Resistance, $R = \frac{pl}{A}$

$R = p\frac{l}{A} \times \frac{l}{l} = \frac{pl^2}{V}$ [≫Volume $(V) = A$ ≫.]

Since resistivity and volume remains constant therefore % change in resistance

$$\frac{\Delta R}{R} = \frac{2\Delta l}{l} = 2 \times (0.5) = 1\%$$

13. A carbon resistance has following colour code. What isthe value of the resistance? [9 Jan. 2019 II]

GOY Golden

(a) 530 kΩ $\pm$5% (b) 5.3 kΩ $\pm$5%

(c) 6.4 MΩ $\pm$5%(d) 64 MΩ $\pm 10\%$

SOLUTION : (a)

Colour code for carbon resistor

Bl, Br, R, O, Y, QBlue, V, Gr, W

0 1 2345 6 789

Resistance, $R = AB \times C \pm D$

Bands A and B are the first two significant figures of resistance

B and C indicates the decimal multiplier or the number of zeros that follow A and B

B and D is tolerance: Gold= $\pm$5%,

Silver = ± 10 % No colour= $\pm$20%

$R = 53 \times 10^4 \pm 5\% = 530\text{k}\Omega \pm 5\%$

14. A heating element has a resistance of 100Ω at roomtemperature. When it is connected to a supply of 220 V,a steady current of 2 A passes in it and temperature is$500^{\circ}C$ more than room temperature. What is thetemperature coefficient of resistance of the heatingelement? [Online April 16, 2018]

(a) $1 \times 10^{-4^{o}}C^{-1}$ (b) $5 \times 10^{-4^{o}}C^{-1}$

(c) $2 \times 10^{-4^{o}}C^{-1}$ (d) $0.5 \times 10^{-4^{o}}C^{-1}$

SOLUTION : (c)

Resistance after temperature increases by $500^{\circ}C$ i.e.,

$$R_t = \frac{V}{I} = \frac{220}{2} = 110\Omega$$

$R_0 = 100$ (given) temperature coefficient ofresistance, $\alpha = ?$

using $R_1 = R_0(1 + \alpha t)$

$$110 = 100(1 + \alpha 500)$$

$$\alpha = \frac{10}{100 \times 500}$$

or, $\alpha = 2 \times 10^{-4\circ}C^{-1}$

15. A copper rod of cross - sectional area A carries a uniformcurrent I through it. At temperature T, ifthe volume chargedensity of the rod is p, how long will the charges take totravel a distance d ?

[Online April 15, 2018]

(a) $\frac{2pdA}{IT}$ (b) $\frac{2pdA}{I}$ (c) $\frac{pdA}{I}$ (d) $\frac{pdA}{IT}$

SOLUTION : (c)

Charge density $p = \frac{\text{charge}}{\text{vo1u}} = \frac{q}{Ad} \Rightarrow q = pAd$

Also, $q = IT \Rightarrow T = \frac{q}{I} = \frac{pAd}{I}$

16. A uniform wire oflength l and radius r has a resistance of100 Ω. It is recast into a wire ofradius $\frac{r}{2}$. The resistanceofnew wire will be:

[Online April 9, 2017]

(a) 1600 Ω (b) 400 Ω (c) 200 Ω (d) 100 Ω

SOLUTION : . (a)

Given, $R_1 = 100\Omega, r^1 = r/2, R_2 = ?$

Resistivity ofwire, $R = \frac{pl}{A} .\cdot$ Area $\times$ length $=$ volume

Hence, $R = \frac{pV}{A^2}$

Since, p $\rightarrow$constant, V $\rightarrow$constant

$$R \propto \frac{1}{A^2}$$

or $R \propto \frac{1}{r^4} A = \pi r^2$

$\frac{R_2}{R_1} = 16 \Rightarrow R_2 = 16 \times 100 = 1600\Omega$, Resistance ofnewwire.

17. When 5V potential difference is applied across a wire oflength 0.1m, the drift speed ofelectrons is $2.5 \times 10^{-4}\text{ms}^{-1}$.If the electron density in the wire is $8 \times 10^{28}\text{m}^{-3}$, theresistivity ofthe material is close to:

[2015]

(a) $1.6 \times 10^{-6}\Omega\text{m}$ (b) $1.6 \times 10^{-5}\Omega\text{m}$

(c) $1.6 \times 10^{-8}\Omega\text{m}$(d) $1.6 \times 10^{-7}\Omega\text{m}$

SOLUTION : .(b)

$$V = IR = (neAv_d)p\frac{l}{A}$$

$$p = \frac{V}{V_d 1ne}$$

Here V = potential difference

1 = length ofwire

n = no. of electrons per unit volume of conductor.

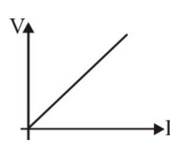
e = no. ofelectrons

Placing the value of above parameters we get resistivity

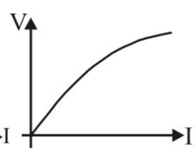
$$p = \frac{5}{8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4} \times 0.1}$$

$$= 1.6 \times 10^{-5}\Omega\text{m}$$

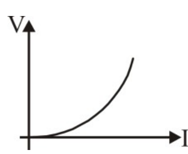
18. Suppose the drift velocity v_d in a material varied with theapplied electric field E as $v_d \propto \sqrt{E}$. Then V - I graph fora wire made of such a material is best given by: [Online April 10, 2015]



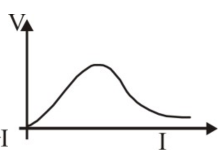
(a)



(b)



(c)



(d)

SOLUTION : . (c)

$i = neAV_d$ and $V_d \propto \sqrt{E}$ (Given)

or, $i \propto \sqrt{E}$

$$i^2 \propto E$$

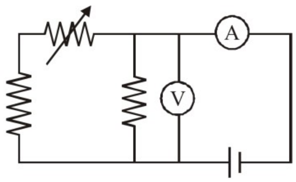
$$i^2 \propto V$$

Hence graph (c) correctly dipicts the V - I graph for a wire made of such type of material.

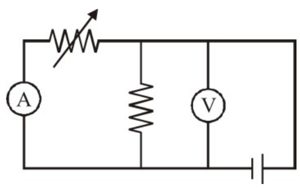
19. Correct set up toveriђy Ohm’ slaw is:

[Online April 23, 2013]

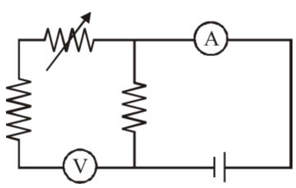
(a)



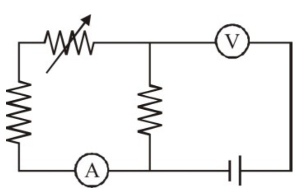
(b)



(c)



(d)



SOLUTION : . (a)

In ohm’ s law, we check V = IR where I is the corrent

flowing through a resistor and V is the potential difference across that resistor. Only option (a) fits the above criteria.Remember that ammeter is connected in series withresistance and voltmeter parallel with the resistance.

20. The resistance of a wire is R. It is bent at the middle by 180° and tx)th the ends are twisted together to make a shorter wire.The resistance ofthe new wire is [Online May 26, 2012]

(a) 2 R (b) $Rl2$ (c) $Rl4$ (d) $Rl8$

SOLUTION : . (c)

Resistance of wire $(R)=\mathrm{p}\frac{l}{A}$

If wire is bent in the middle then

$$l'=\frac{l}{2}, A'=2A$$

New resistance, $\mathrm{R}'=\mathrm{p}\frac{l'}{\mathrm{A}'}=\frac{\mathrm{p}\frac{l}{2}}{2A}=\frac{\mathrm{p}l}{4A}=\frac{R}{4}.$

21. Ifa wire is stretched to make it 0.1% longer, its resistancewill: [2011]

(a) increase by 0.2% (b) decrease by 0.2%

(c) decrease by 0.05% (d) increase by 0.05%

SOLUTION : . (a)

Resistance ofwire$R=\frac{\mathrm{p}l}{A}=\frac{\mathrm{p}l^{2}}{V}(V=AP)$

Hence, $R=\mathrm{p}\frac{l^{2}}{V}$ =constant $\times P^{2}$

Fractional change in resistance

$$\frac{\Delta R}{R}=2\frac{\Delta l}{l}$$

$$100\times\frac{\Delta R}{R}=200\times\left(\frac{d\ell}{\ell}\right)$$

$dPIP=0.1\%$

% change in $R=[200\times(\frac{0.1}{100}\}]=0.2\%$

Resistance will increase by 0.2%.

DIRECTIONS: Question No. 22 and 23 are based on the following paragraph.

Consider a block ofconducting material ofresistivity ’ p’ shown in the figure. Current ‘I’ enters at ‘A’ and leaves from ‘D’ . We apply superposition

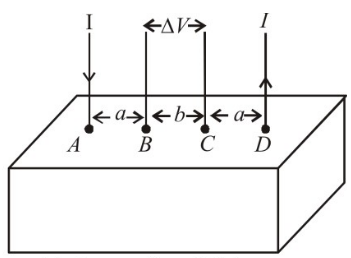
principle to find voltage ΔV' developed between 'B' and C'. The calculation is done in the following steps: (i) Take current 'I' entering from 'A' and assume it to spread

over a hemispherical surface in the block.

(ii) Calculate field $\mathrm{E(r)}$ at distance r' from Abyusing Ohm's

law $\mathrm{E = pj}$, wherej is the current per unit area at r'.

$(Reject)$ From the 'r' dependence of$\mathrm{E(r)}$, obtain the potential V(r) at r. (iv) Repeat (i), (ii) and (iii) for current 'I' leaving 'D' andsuperpose results for 'A' and 'D'.



sured between B and C is

22. ΔV mea [2008]

(a) $\frac{pI}{\pi a}-\frac{pI}{\pi(a+b)}$ (b) $\frac{pI}{a}-\frac{pI}{(a+b)}$

(c) $\frac{pI}{2\pi a}-\frac{pI}{2\pi(a+b)}$ (d) $\frac{pI}{2\pi(a-b)}$

SOLUTION : . (a)

Letj be the current density.

Then $j \times 2\pi r^2 = I \Rightarrow j = \frac{I}{2\pi r^2}$

$$E = pj = \frac{pI}{2\pi r^2}$$

Now, $V_B - V_C$

$$= -\int_{a+b}^{a} \vec{E} \cdot \overline{dr} = -\int_{a+b}^{a} \frac{pI}{2\pi r^2} dr$$

$$= -\frac{pI}{2\pi}\left[-\frac{1}{r}\right]_{a+b}^{a} = \frac{pI}{2\pi a} - \frac{pI}{2\pi(a+b)}$$

On applying superposition as mentioned we get

$$\Delta V_{BC} = 2 \times \Delta V_{\acute{B}C} = \frac{pI}{\pi a} - \frac{pI}{\pi(a+b)}$$

23. For current entering at A, the electric field at a distance r' from A is [2008]

(a) $\frac{pI}{8\pi r^2}$ (b) $\frac{pI}{r^2}$ (c) $\frac{pI}{2\pi r^2}$ (d) $\frac{pI}{4\pi r^2}$

SOLUTION : . (c)

As shown in Answer (a) $E = \frac{pI}{2\pi r^2}$

24. The resistance of a wire is 5 ohm at $50°C$ and 6 ohm at$100°C$. The resistance ofthe wire at $0°C$ will be [2007]

(a) 3 ohm (b) 2 ohm (c) 1 ohm (d) 4 ohm

SOLUTION : (d)

Resistance ofa metal conductor at temperature $t°C$

is given by

$R_t = R_0(1 + \alpha t)$,

R_0 is the resistance ofthe wire at 0^oC and α is the temperature coefficient ofresistance.

Resistance at $50°C$, $R_{50} = R_0(1 + 50\alpha)$ .. (i)

Resistance at $100°C$, $R_{1\alpha)} = R_0(1 + 100\alpha)$ (ii)

From(i), $R_{50} - R_0 = 50\alpha R_0$ $(Reject)$

From(ii), $R_{1\alpha)} - R_0 = 100\alpha R_0$ (iv)

Dividing (iii) by (iv), we get

$$\frac{R_{50} - R_0}{R_{100} - R_0} = \frac{1}{2}$$

Here, $R_{50} = 5\Omega$ and $R_{100} = 6\Omega$

$$\frac{5-R_0}{6-R_0}=\frac{1}{2}$$

or, $6-R_0=10-2R_0$ or, $R_0=4\Omega$.

25. A material B^1 has twice the specific resistance of A^1. Acircular wire made of† B^1 has twice the diameter of a wiremade of A^1. then for the two wires to have the sameresistance, the ratio $l_B l l_A$ oftheir respective lengths mustbe [2006]

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{4}$ (d) 2

SOLUTION : . (d)

Let d_A and d_B are the diameter ofwireA and B respectively.

Let p_B and $\mathrm{p}_A k$ the resistivityofwire A and B. We have given

$$\mathrm{p}_B=2\mathrm{p}_A$$

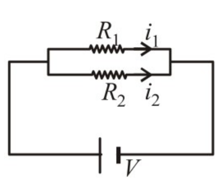
$$d_B=2d_A$$

Ifboth resistances are equal

$$R_B=R_A$$

$$\Rightarrow \frac{\mathrm{p}_B l_B}{A_B}=\frac{\mathrm{p}_A l_A}{A_A}$$

$$\frac{p_B}{p_A}=\frac{\mathrm{p}_A}{\mathrm{p}_B}\times\frac{d_B^2}{d_A^2}=\frac{\mathrm{p}_A}{2\mathrm{p}_A}\times\frac{4d_A^2}{d_A^2}=2$$



26. An electric current is passed through a circuit containingtwo wires ofthe same material, connected in parallel. Ifthelengths and radii are in the ratio of $\frac{4}{3}$ and $\frac{2}{3}$, then the ratioofthe current passing through the wires will be [2004]

(a) 8/9 (b) 1/3 (c) 3 (d) 2

SOLUTION : . (b)

Given,

$$\frac{l_1}{l_2}=\frac{4}{3} \text{ and } \frac{r_1}{r_2}=\frac{2}{3}$$

$$R_1=\frac{\mathrm{p}l_1}{\pi r_1^2}; R_2=\frac{\mathrm{p}l_2}{\pi r_2^2}$$

When wires are in parallel to the circuit potential difference across each wire is same

$$i_1R_1=i_2R_2$$

$$\frac{i_1}{i_2}=\frac{R_2}{R_1}=\frac{\mathrm{p}l_2}{\pi r_2^2}\times\frac{\pi r_1^2}{\mathrm{p}r_1}=\frac{l_2}{l_1}\times\frac{r_1^2}{r_{2^2}}$$

$$=\frac{3}{4}\times\frac{4}{9}=\frac{1}{3}$$

27. The length ofa given cylindrical wire is increased by 100%.Due to the consequent decrease in diameter the change inthe resistance ofthe wire will be [2003]

(a) 2α)% (b) 1α)% (c) 50% (d) 3α)%

SOLUTION : . (d)

Since volume of wire remains unchanged on increasing length, hence

$$A\times P==A'\times P'$$

$$\Rightarrow p'=2l$$

$$A'=\frac{A\times l}{l'}=\frac{A\times l}{2l}=\frac{A}{2}$$

Percentage change in resistance

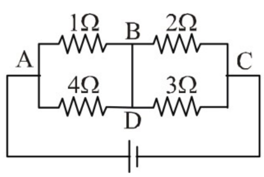
$$= \frac{R_f - R_i}{R_i} \times 100 = \frac{\text{p}\frac{l'}{A'} - \beta\frac{l}{A}}{\text{p}\frac{l}{A}} \times 100$$

$$= \left[\left(\frac{p'}{A'} \times \frac{A}{l}\right) - 1\right] \times 100$$

$$= \left[\left(\frac{2l}{A/_2} \times \frac{A}{\ell}\right) - 1\right] \times 100 = (4-1) \times 100$$

$$= 300\%$$

28. In the given circuit diagram, awire isjoining points BandD. The current in this wire is: [9 Jan. 2020 I]



20V

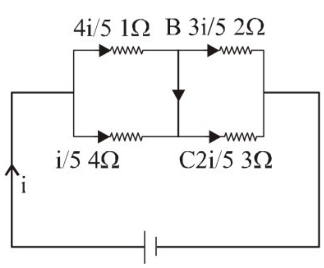
(a) 0.4A (b) 2A (c) 4A (d) zero

SOLUTION : . (b)

From circuit diagram,

$$\frac{1}{R_1} = \frac{1}{1} + \frac{1}{4} \Rightarrow R_1 = \frac{4}{5}$$

$$\frac{1}{R_2} = \frac{1}{2} + \frac{1}{3} \Rightarrow R_2 = \frac{6}{5}$$



20

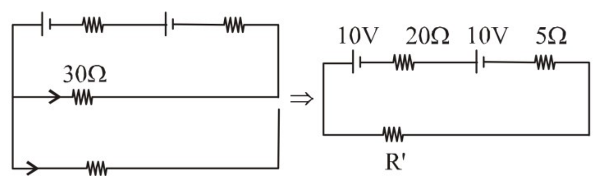
$$R_{\text{eff}} = R_1 + R_2 = \frac{4}{5} + \frac{6}{5} = 2\Omega$$

$$i = \frac{v}{R_{\text{eff}}} = \frac{20}{2} = 10A$$

$$I_{BC} = \frac{4i}{5} - \frac{3i}{5} = \frac{i}{5} = 2A$$

29. The series combination oftwo batteries, both ofthe sameemf 10 V, but different internal resistance of20 Ω and 5Ω, is connected to the parallel combination of tworesistors 30 Ω and RΩ. The voltage difference acrossthe battery ofinternal resistance 20 Ω is zero, the valueof R (in Ω) is [NA. 8 Jan. 2020 II]

SOLUTION : (30.00)



R

The resistance of30Ω is in parallel with R. Their effective resistance

$$\frac{1}{R'} = \frac{1}{30} + \frac{1}{R}$$

$$R' = \frac{30}{30+R} \quad \text{(i)}$$

$$\text{Also, V} = \text{IR} \Rightarrow 10 = \frac{20\times20}{R+25}1$$

$$\Rightarrow R' + 25 = 40 \Rightarrow R' = 15$$

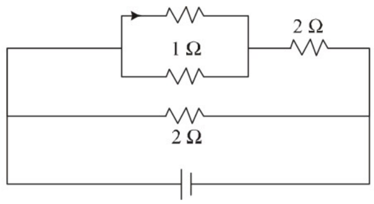
$$R^{Reject} = 15 = \frac{30}{30+R} \text{ Using (i)}$$

$$\Rightarrow 30 + R = 2R$$

$$\Rightarrow R = 30\Omega$$

30. The current I_1 (in A) flowing through 1 Ω resistor in thefollowing circuit is: [7 Jan. 2020 I]

I_1 1Ω



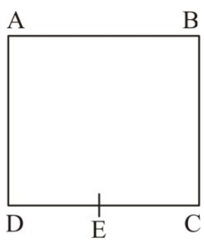
1V

(a) 0.4 (b) 0.5 (c) 0.2 (d) 0.25

SOLUTION : . (c)

31. Awire ofresistance R is bent to form a square ABCD asshown in the figure. The effective resistance between Eand C is: (E is mid - point ofarm CD

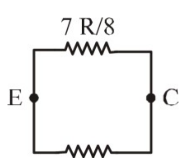
) [9 April 2019 I]



(a) R (b) $\frac{7}{64}$R (c) $\frac{3}{4}$R (d) $\frac{1}{16}$R

SOLUTION : . (b)

$$\mathrm{R}_{\mathrm{eq}}=\frac{\left(\frac{7R}{8}\right)\left(\frac{R}{8}\right)}{R}=\frac{7R}{64}$$



R/8

32. Ametal wire ofresistance 3 Ω is elongated to make auniformwire of double its previous length. This new wire is nowbent and the endsjoined to make a circle. Iftwo points onthe circle make an angle 60° at the centre, the equivalentresistance between these two points will be: [9 Apr. 2019 II]

(a) $\frac{12}{5}\Omega$ (b) $\frac{5}{2}\Omega$ (c) $\frac{5}{3}\Omega$ (d) $\frac{7}{2}\Omega$

SOLUTION : (c)

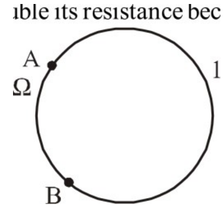
When length becomes do‥ omes

$(R \propto l^2)$

0Ω

$R=4\times 3=12\Omega$ 2

ıble ıts resıstance bec

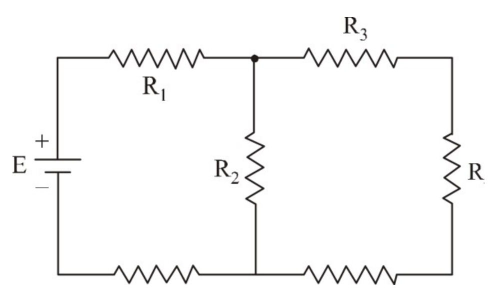


$$R_{\mathrm{eq}}=\frac{2\times 10}{12}=\frac{5}{3}\Omega$$

33. In the figure shown, what is the current (in Ampere) drawnfrom the battery? You are given :

[8 Apr. 2019 II]

$\mathrm{R}_1=15\Omega$, $\mathrm{R}_2=10\Omega$, $\mathrm{R}_3=20\Omega$, $\mathrm{R}_4=5\Omega$, $\mathrm{R}_5=25\Omega$, $\mathrm{R}_6=30\Omega$, $\mathrm{E}=15\mathrm{V}$



4

R_6 R_5

(a) 13/24 (b) 7/18 (c) 9/32 (d) 20/3

SOLUTION : . (c)

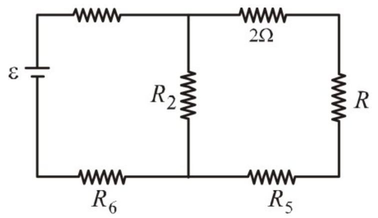
R_3, R_4 and R_5 are in series so their equivalent

$$R=20+5+25=50\Omega$$

This is parallel with R_2, and so net resistance ofthe circuit

R_1 R_3

4



$$R_{eq} = \left(\frac{10 \times 50}{10 + 50}\right) + 15 + 30 = \frac{160}{3}\Omega$$

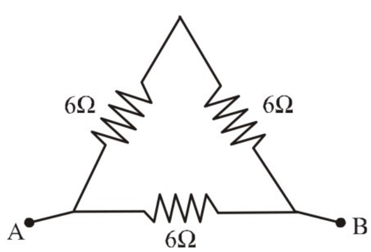
So, $j = \frac{\varepsilon}{R_{eq}} = \frac{15}{(100/3)} = \frac{9}{32}A$

34. A uniform metallic wire has a resistance of 18 Ω and isbent into an equilateral triangle. Then, the resistancebetween any two vertices of the triangle is:

[10 Jan. 2019 I]

(a) 4 Ω (b) 8 Ω (c) 12 W (d) 2 W

SOLUTION : (a)



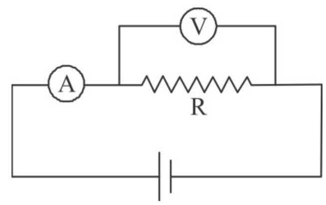
Resistance, $R \propto l$ so resistance of each side of the equilateral triangle $= 6\Omega$

Resistance R_{eq} between any two vertices

$$\frac{1}{R_{eq}} = \frac{1}{12} + \frac{1}{6} \Rightarrow R_{eq} = 4\Omega$$

35. The actual value ofresistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using thestandard formula $= \frac{V}{I}$, where V andI are the reading ofthe voltmeter and ammeter, respectively. Ifthe measuredvalue of R is 5% less, then the internal resistance of thevoltmeter is:

[10 Jan. 2019 II]



(a) 600 Ω (b) 570 Ω (c) 35 W (d) 350 W

SOLUTION : . (b)

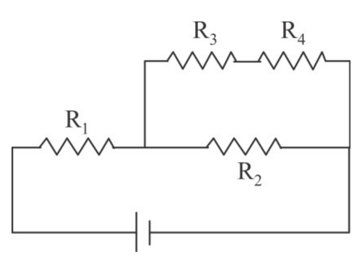
Using, R_{eq} R_1 R_2

$0.95R = \frac{RRo}{R+Ro}$ (measured value 5% less then internal resistance ofvoltmeter) or, $0.95 \times 30 = 0.05Ro$

$Ro^{=19\times30=57}$

36. In the given circuit the internal resistance ofthe 18 V cellis negligible. If$R_1 = 400\Omega$, $R_3 = 100\Omega$ and $R_4 = 500\Omega$and the reading of an ideal voltmeter across R_4 is 5 V,then the value of R_2 will be:

[9 Jan. 2019 II]

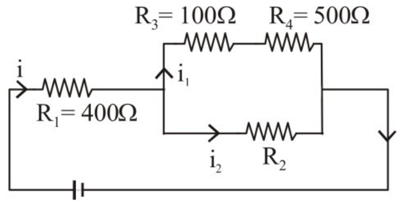


18 V

(a) 300 W (b) 450 W

(c) 550 W (d) 230 W

SOLUTION : (a)



18V

Across R_4 reading ofvoltmeter, $V_4 = 5V$

Current, $i_4 = \frac{V_4}{R_4} = 0.01A$

$$V_3 = i_1 R_3 = 1V$$

$$V_3 + V_4 = 6V = V_2$$

$$V_1 + V_3 + V_4 = 18V$$

$$\Rightarrow V_1 = 12V$$

$$i = \frac{V_1}{R_1} = 0.03A$$

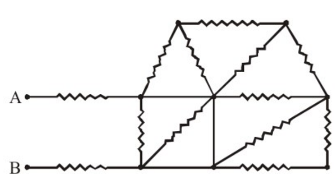
$$i = i_1 + i_2 \Rightarrow i_2 = i - i, = 0.03 - 0.01A = 0.02A$$

$$R_2 === 300\Omega \underline{V_2 6}$$

$$i_2 \ 0.02$$

37. In the given circuit all resistances are ofvalueR ohm each.The equivalent resistance between A and B is:

[Online Apri115, 2018]



(a) $2R$ (b) $\frac{5R}{2}$ (c) $\frac{5R}{3}$ (d) 3 R

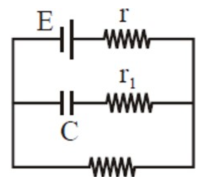
SOLUTION : (a)

$$R_{series} = R_1 + R_2 + \cdots .. + R_n$$

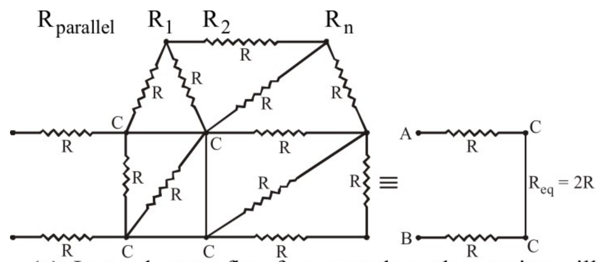
38. In the given circuit diagram when the current reaches steadystate in the circuit, the charge on the capacitor ofcapacitance C will be: [2017]

(a) $CE\frac{r_2}{(r+r_2)}$ (b) $CE\frac{r_1}{(r_1+r)}$

(c) CE (d) $CE\frac{r_1}{(r_2+r)}$



SOLUTION :

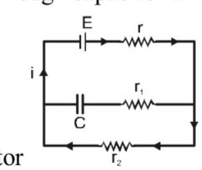


(a) In steady state, flow fo current through capacitor will be zero.

Current through the circuit,

$$i = \frac{E}{r + r_2}$$

Potential difference through capacitor

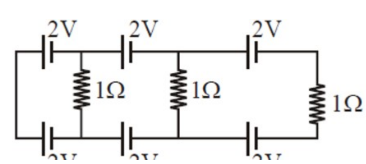


1

$$V_c = \frac{Q}{C} = E - ir = E - (\ )^r(\ )$$

$$Q = CE\frac{r_2}{r + r_2}$$

39.



In the above circuit the current in each resistance is

[2017]

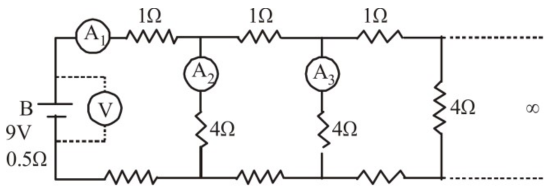
(a) 0.5A (b) 0A (c) 1 A (d) 0.25A

SOLUTION : . (b)

The potential difference in each loop is zero.

No current will flow or current in each resistance is Zero.

40.



1Ω 1Ω 1Ω

A 9V batterywith internal resistance of0.5Ω is connected across an infinite network as shown in the figure. All ammeters A_1 , A_2, A_3 and voltmeter V are ideal.Choose correct statement. [Online April 8, 2017]

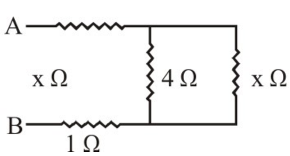
(a) Reading of A_1 is 2 A (b) Reading of A_1 is 18 A

(c) Reading of V is 9 V (d) Reading of V is 7 V

SOLUTION : (a)

The given circuit can be redrawn as,

1Ω



as 4 Ω and xΩ are parallel $x^{\uparrow} = \frac{1}{4} + \frac{1}{x} = \frac{(4+x)}{4x}$

$$x' = \frac{4x}{4+x}$$

& 1Ω and 1 Ω are also parallel $x'' = 2\Omega$ Now equivalent resistance ofcircuit

$$x = \frac{4x}{4+x} + 2 = \frac{8+6x}{4+x}$$

$$4x + x^2 = 8 + 6x$$

$$x^2 - 2x - 8 = 0$$

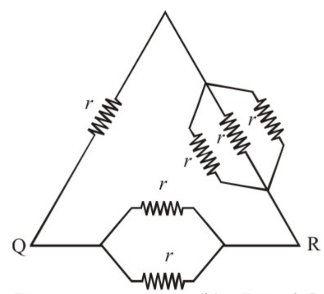
$$x = \frac{2 \pm \sqrt{4 - 4(1)(-8)}}{2} = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2} = 4\Omega$$

Reading ofAmmeter $A_1 = \frac{V}{(R+r)}$

$A_1 = \frac{9}{4+0.5} = 2$ Ampere

41. Six equal resistances are connected between points P, Q and R as shown in figure. Then net resistance will bemaximumbetween : P

[Online April 25, 2013]



R (b) P and Q

(a) P and (c) Q and R (d) Any two points

SOLUTION : .

Resistance between P and Q

$$r_{PQ} = r \| \left(\frac{r}{3} + \frac{r}{2}\right) = \frac{r \times \frac{5}{6}r}{r + \frac{5}{6}r} = \frac{5}{11}r$$

Resistance between Q and R

$$r_{QR} = \frac{r}{2} \| \left(r + \frac{r}{3}\right) = \frac{\frac{r}{2} \times \frac{4}{3}r}{\frac{r}{2} + \frac{4}{3}r} = \frac{4}{11}r$$

Resistance between P and R

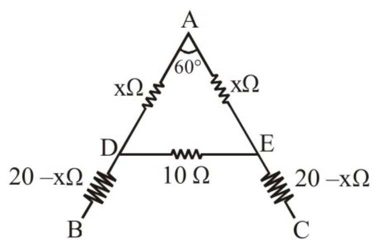
$$r_{PR} = \frac{r}{3} \| \left(\frac{r}{2} + r\right) = \frac{\frac{r}{3} \times \frac{3}{2}r}{\frac{r}{3} + \frac{3}{2}r} = \frac{3}{11}r$$

Hence, it is clear that r_{PQ} is maximum

42. A letter 1A† is constructed ofa uniform wire with resistance1.0Ω per cm, The sides ofthe letter are 20 cm and the crosspiece in the middle is 10 cm long. The apex angle is 60. Theresistance between the ends ofthe legs is close to: [Online April 9, 2013]

(a) 50.0Ω (b) 10 Ω (c) 36.7Ω (d) 26.7Ω

SOLUTION : (d)



For ADE $\frac{1}{\mathrm{R}'}=\frac{1}{2\mathrm{x}}+\frac{1}{10}$

or $\mathrm{R}^{Reject}=\frac{20\mathrm{x}}{10+2\mathrm{x}}$

$$\mathrm{R}_{\mathrm{BC}}=\frac{20\mathrm{x}}{10+2}+20-\mathrm{x}+20-\mathrm{x}\ldots \text{(i)}$$

or $\frac{20\mathrm{x}}{10+2\mathrm{x}}+40=2\mathrm{x}$

Solving we get$\mathrm{x}=10\Omega$

Putting the value of$\mathrm{x}=10\Omega$ in equation (i)

We get $\mathrm{R}_{\mathrm{BC}}=\frac{20\times10}{10+2\times10}+20-10+20-10$

$$=\frac{80}{3}=26.7\Omega$$

43. Two conductors have the same resistance at $0^{\mathrm{o}}\mathrm{C}$ but theirtemperature coefficients ofresistance are α_1 and α_2. Therespective temperature coefficients of their series andparallel combinations are nearly [2010]

(a) $\frac{\alpha_1+\alpha_2}{2},\alpha_1+\alpha_2$ (b) $\alpha_1+\alpha_2,\frac{\alpha_1+\alpha_2}{2}$ (c) $\alpha_1+\alpha_2,\frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (d) $\frac{\alpha_1+\alpha_2}{2},\frac{\alpha_1+\alpha_2}{2}$

SOLUTION : (d)

Let R_1 and R_2 be the resistances oftwo conductors, then

$$R_1=R_0[1+\alpha_1\Delta t]$$

$$R_2=R_0[1+\alpha_2\Delta t]$$

Here, R_0 is the resistance ofconductor at 0°C

In Series, $R=R_1+R_2=R_0[2+(\alpha_1+\alpha_2)\Delta t]$

$$=2R_0\left[1+\left(\frac{\alpha_1+\alpha_2}{2}\right)\Delta t\right]$$

$$\alpha_{eq}=\frac{\alpha_1+\alpha_2}{2}$$

In Parallel, $\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{R_0[1+\alpha_1\Delta t]}+\frac{1}{R_0[1+\alpha_2\Delta t]}$

$$\Rightarrow\frac{1}{\frac{R_0}{2}(1+\alpha_{eq}\Delta t)}$$

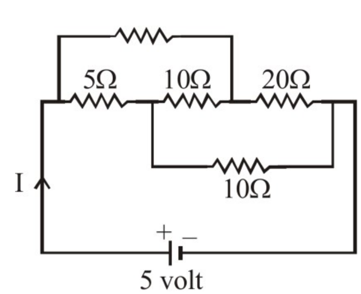
$$=\frac{1}{R_0(1+\alpha_1\Delta t)}+\frac{1}{R_0(1+\alpha_2\triangle t)}$$

$$2(1-\alpha_{eq}\Delta t)=(1-\alpha_1\Delta t)(1-\alpha_2\Delta t)$$

$$\alpha_{eq}=\frac{\alpha_1+\alpha_2}{2}$$

44. The current Idrawn from the5volt source willbe [2006]

10Ω



(a) $0.33\mathrm{A}$ (b) $0.5\mathrm{A}$ (c) $0.67\mathrm{A}$ (d) $0.17\mathrm{A}$

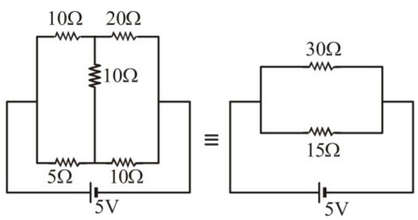
SOLUTION :

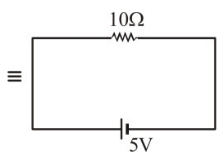
The network of resistors is a balanced wheatstone

bridge. Hence, no current will flow through centre

resistor.

The equivalent circuit is

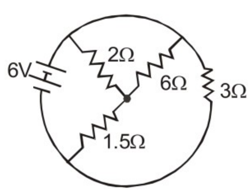




$$R_{eq} = \frac{15 \times 30}{15 + 30} = 10\Omega \Rightarrow I = \frac{V}{R} = \frac{5}{10} = 0.5\text{A}$$

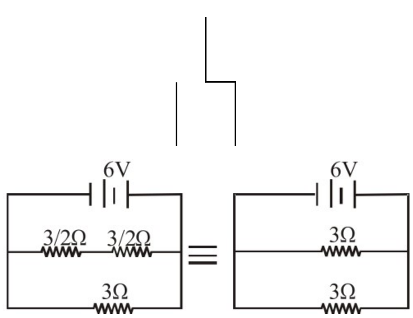
45. The total current supplied to the circuit by the battery is

[2004]



(a) 4A (b) 2 A (c) 1A (d) 6A

SOLUTION : (a)



hence $R_{eq} = 3/2; I = \frac{6}{3/2} = 4\text{A}$

46. The resistance ofthe series combination oftwo resistancesis S. when they arejoined in parallel the total resistance isP. If $S = nP$ then the minimum possible value ofn is [2004]

(a) 2 (b) 3 (c) 4 (d) 1

SOLUTION : . (c)

Let R_1 and R_2 be the two given resistances

Resistance ofthe series combination,

$$S = R_1 + R_2$$

Resistance ofthe parallel combination,

$$P = \frac{R_1 R_2}{R_1 + R_2}$$

As per question $S = nP$

$$\Rightarrow R_1 + R_2 = \frac{n(R_1 R_2)}{(R_1 + R_2)}$$

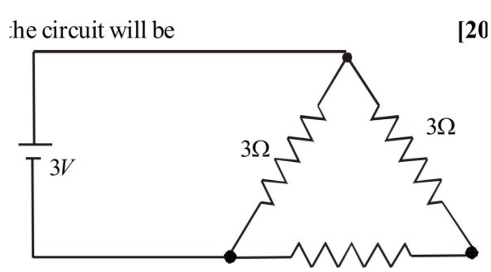
$$\Rightarrow (R_1 + R_2)^2 = nR_1 R_2$$

Minimum value ofn is 4 for that

$$(R_1 + R_2)^2 = 4R_1 R_2$$

$$\Rightarrow (R_1 - R_2)^2 = 0$$

47. A 3 volt battery with negligible internal resistance isconnected in a circuit as shown in the figure. The currentI, in 1 103]



3Ω

(a) 1A (b) 1.5A (c) 2 A (d) 1/3 A

SOLUTION : (b)

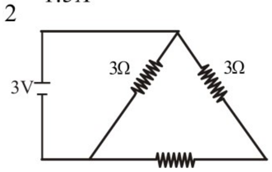
In the given circuit, resistance of 3Ω is in parallel

with series combination oftwo 3Ω resistance.

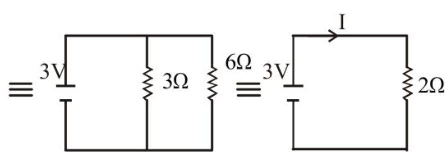
$$R_p = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$$

Using ohm' s law $V = IR$

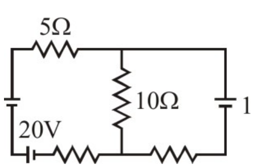
$$\Rightarrow I = \frac{V}{R} = \frac{3}{2} = 1.5A$$



3Ω



48.



2Ω 4Ω

In the figure shown, the current in the 10 V battery is close to : [Sep. 06, 2020 (II)]

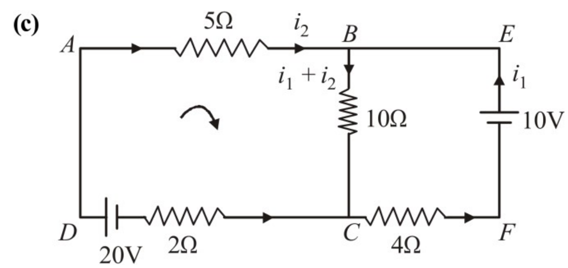
(a) 0.71 A from positive to negative terminal

(b) 0.42 A from positive to negative terminal

(c) 0.21 A from positive to negative terminal

(d) 0.36A ffom negative to positive terminal

SOLUTION :



Using Kirchoffs loop law in loop *ABCD*

$$-5i_2 - 10(i_1 + i_2) - 2i_2 + 20 = 0$$

$\Rightarrow -10i_1 - 17i_2 + 20 = 0$ (i) Using Kirchoffs loop law in loop *BEFC*

$$\Rightarrow -10 + 4i_1 + 10(i_1 + i_2) = 0$$

$\Rightarrow 14i_1 + 10i_2 + 10 = 0$ (ii) Multiplying equation (i) by 10, we have

$$(10i_1 + 17i_2 = 20) \times 10$$

$\Rightarrow 100i_1 - 170i_2 = 200$ (iii) Multiplying equation (ii) by 17, we have

$$(14i_1 + 10i_2 = 10) \times 17$$

$\Rightarrow 238i_1 - 170i_2 = 170$ (iv)

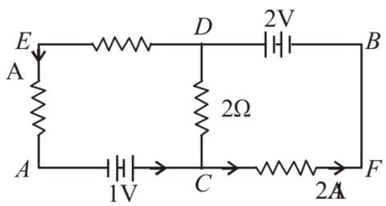
On solving equations (iii) and (iv), we get

$$-138i_1 = 30 \Rightarrow i_1 = -\frac{30}{138} = -0.217$$

i_1 is negative it means current flows from positive to negative terminal.

49. In the circuit, given in the figure currents in different branchesand value ofone resistor are shown. Then potential at point B with respect to the point A is: [Sep. 05, 2020 (II)]

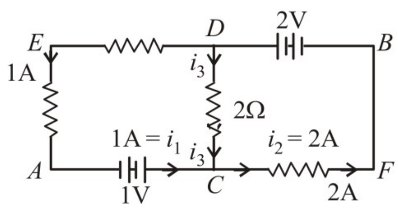
1



D

a) $+2\mathrm{V}$ (b) $-2\mathrm{V}$ (c) $-1\mathrm{V}$ (d) $+1\mathrm{V}$

SOLUTION : . (d)



Let us assume the potential at $A = V_A = 0$ Using Kirchoffs junction rule at C, we get

$$i_1 + i_3 = i_2$$

$$1\mathrm{A} + i_3 = 2\mathrm{A} \Rightarrow i_3 = 2A$$

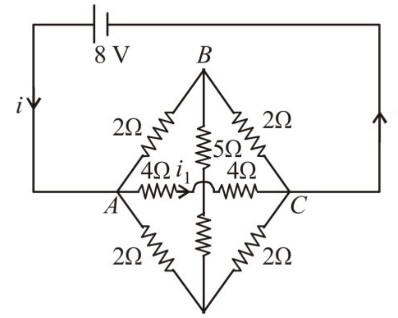
Now using Kirchoffs loop law along *ACDB*

$$V_A + 1 + i_3(2) - 2 = V_B$$

$$\Rightarrow V_A + 1 + i_3(1) - 2 = V_B$$

$$\Rightarrow V_B - V_A = 3 - 2 = 1 \text{ volt}$$

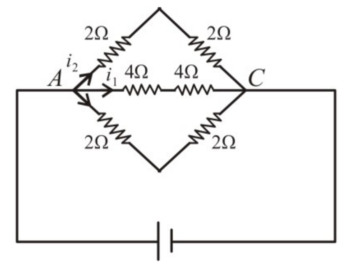
50.The value of current i_1 flowing from A to C in the circuitdiagram is: [Sep. 04, 2020 (ID]



(a) 2A (b) 4A (c) 1A (d) 5 A

SOLUTION : . (c)

The equivalent circuit can be drawn as

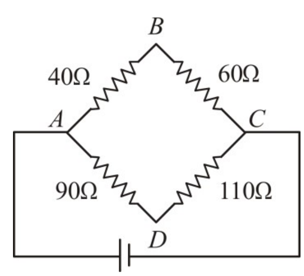


8 V

Voltage across $AC = 8\mathrm{V}$ Resistance $R_{AC} = 4 + 4 =$

$$8\Omega i_1 = \frac{V}{R_{AC}} = \frac{8}{4+4} = 1 \text{ Amp}$$

51.



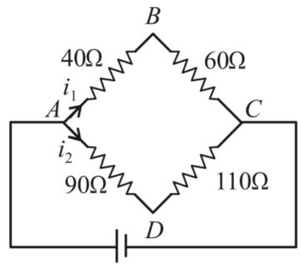
40 V

Four resistances 40 Ω, 60Ω, 90Ω and 110 Ω make the arms of a quadrilateral $ABCD$. Across AC is a battery of emf40 V and internal resistance negligible. The potential difference across BD in V is

[NA. Sep. 04, 2020 (II)]

SOLUTION :

(2)



40 V

Current through AB, $i_1 = \frac{40}{40+60} = 0.4$ Current through

AD, $i_2 = \frac{40}{90+110} = \frac{1}{5}$ Using KVL in BAD loop

We have the current distribution as shown in the figure.

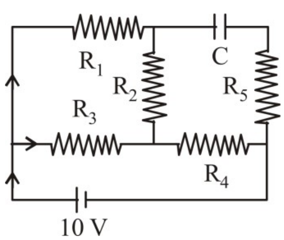
Equivalent resistance, $R_{eq} = \left(\frac{4\times2}{4+2}\right) + 2$

$$V_B + i_1(40) - i_2(90) = V_D$$

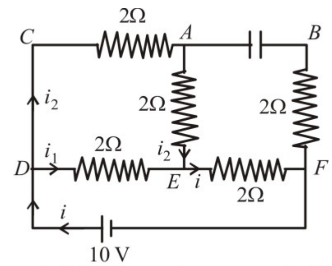
$$\Rightarrow V_B - V_D = \frac{1}{5}(90) - \frac{4}{10}(40)$$

$$\Rightarrow V_B - V_D = 18 - 16 = 2\text{V}$$

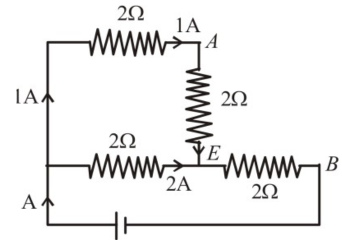
52. An ideal cell ofemf10 V is connected in circuit shown infigure. Each resistance is 2 *Ω*. The potential difference (inV) across the capacitor when it is fully charged is [Sep. 02, 2020 (II)]



SOLUTION : (08.00)

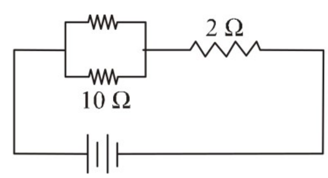


or is fully charged no current will



53. In the given circuit, an ideal voltmeter connected acrossthe 10 *Ω* resistance reads 2V. The internal resistance r, ofeach cell is: [10 Apr. 2019 I]

15 *Ω*



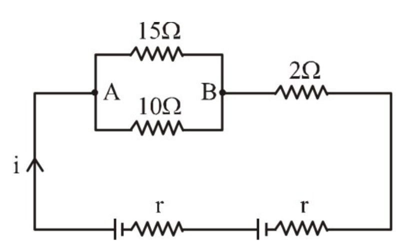
1.5 V, 1.5 V

rΩ rΩ

(a) 1 *Ω* (b) 0.5*Ω* (c) 1.5*Ω* (d) 0*Ω*

SOLUTION : (b)

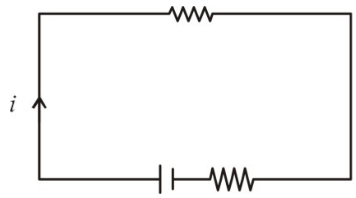
For the given circuit



1.5V 1.5V

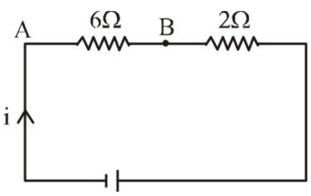
$$i = \frac{3}{8+2r}$$

Now voltage across AB



$$i \times 6 = \frac{3}{8+2r} \times 6 = 2$$

$$\Rightarrow 9 = 8 + 2r$$



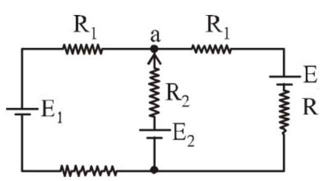
$$\Rightarrow r = \frac{1}{2}\Omega$$

P to be maximum, $\frac{dP}{dR} = 0$ or $\frac{d}{dR}\left[\left(\frac{\varepsilon}{R+r}\right)^2 R\right] = 0$ or

$$R = r$$

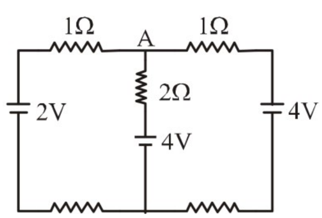
54. For the circuit shown, with $R_1 = 1.0\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2V$and $E_2 = E_3 = 4V$, the potential difference between thepoints a' and b ' is approximately(in V): [8 April 2019 I]

3



(a) 2.7 (b) 2.3 (c) 3.7 (d) 3.3

SOLUTION : (d)



Applying parallel combination ofbatteries

$$\frac{\frac{E_1}{1+1} + \frac{E_2}{2} + \frac{E_3}{1+1}}{\frac{1}{1+1} + \frac{1}{2} + \frac{1}{1+1}}$$

$$\frac{\frac{2}{2} + \frac{4}{2} + \frac{4}{2}}{\frac{1}{2} + \frac{1}{2} + \frac{1}{2}} = \frac{5 \times 2}{3}$$

$$= \frac{10}{3} = 3.3 \text{ Volt}$$

55. A cell of internal resistance r drives current through anexternal resistance R. The power delivered by the cell tothe external resistance will be maximum when : [8Apr. 2019 II]

(a) $R = 0.001r$ **(b) $R = 1000r$**

(c) $R = 2r$ **(d) $R = r$**

SOLUTION : (d)

$$j = \left(\frac{\varepsilon}{R+r}\right)$$

Power delivered to R.

$$P = i^2 R = \left(\frac{\varepsilon}{R+r}\right)^2 R$$

$$R$$

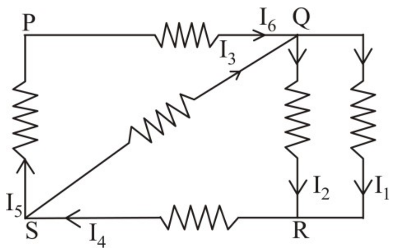
Net current, $i = \frac{10}{\frac{4}{3}+2} = \frac{10 \times 3}{10} = 3$ Amp $i_1 = 2$ A and

$$i_2 = 1\text{A}$$

$$V_{AEB} = 1 \times 2 + 3 \times 2 = 8\text{V}$$

56. In the given circuit diagram, the currents, $I_1 = -0.3A$, $I_4 = 0.8A$ and $I_5 = 0.4A$, are flowing as shown. The currents I_2, I_3 and I_6, respectively, are:

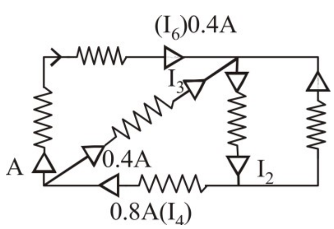
[12 Jan. 2019 II]



(a) $1.1A, -0.4A, 0.4A$ (b) $1.1A, 0.4A, 0.4A$

(c) $0.4A, 1.1A, 0.4A$ (d) $-0.4A, 0.4A, 1.1A$

SOLUTION : (b)



From KCL, $I_3 = 0.8 - 0.4 = 0.4A$

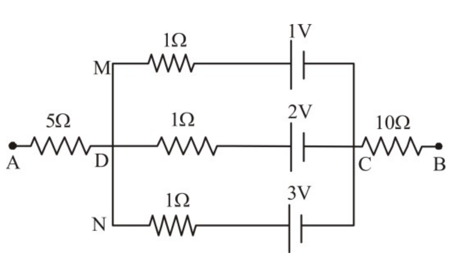
$$I_2 = 0.4 + 0.4 + 0.3$$

$$= 1.1A$$

and $I_6 = 0.4A$

57. In the circuit shown, the potential difference between A and B is: [11 Jan. 2019 II]

(a) 1V (b) 2V (c) 3V (d) 6V



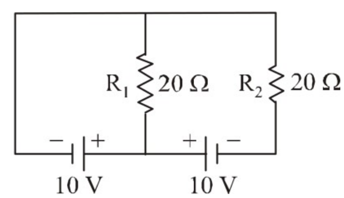
SOLUTION : (b)

Given, $E_1 = 1V, E_2 = 2V, E_3 = 3V, r_1 = 1\Omega,$

$r_2 = 1\Omega$ and $r_3 = 1\Omega$

$$V_{AB} = V_{CD} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3}}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}} = \frac{\frac{1}{1} + \frac{2}{1} + \frac{3}{1}}{\frac{1}{1} + \frac{1}{1} + \frac{1}{1}} = \frac{6}{3} = 2V$$

58. In the given circuit the cells have zero internal resistance. The currents (in Amperes) passing through resistance R_1 and R_2 respectively, are: [10 Jan. 2019 I]



(a) 1, 2 (b) 2, 2 (c) $0.5, 0$ (d) $0, 1$

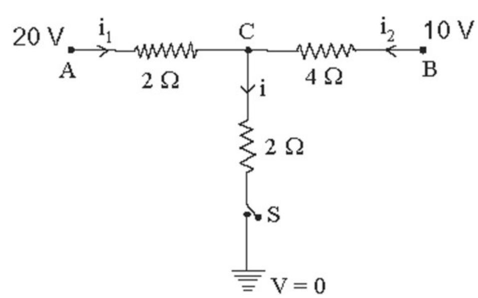
SOLUTION : (c)

Current passing through resistance R_1,

$$i_1 = \frac{v}{R_1} = \frac{10}{20} = 0.5A$$

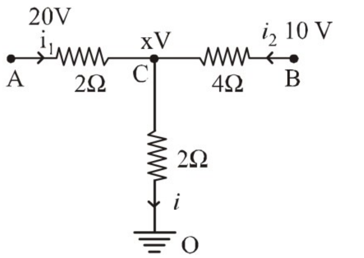
and, $i_2 = 0$

59. When the switch S, in the circuit shown, is closed thenthe valued ofcurrent i will be: [9 Jan. 2019 I]



(a) 3A (b) 5A (c) 4A (d) 2A

SOLUTION : .(b)



Let voltage at C = xV From kirchhoffs current law,

$$\text{KCL: } i_1 + i_2 = i$$

$$\frac{20-x}{2}+\frac{10-x}{4}=\frac{x-0}{2} \Rightarrow x = 10$$

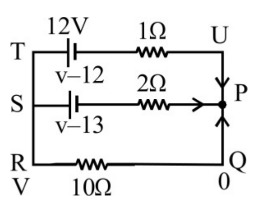
$$i=\frac{V}{R}=\frac{X}{R}=\frac{10}{2}=5A$$

60. Two batteries with e.m.f. 12V and 13 V are connected inparallel across a load resistor of 10Ω. The internalresistances ofthe two batteries are 1Ω and 2Ω respectively.The voltage across the load lies between: [2018]

(a) 11.6Vandll.7V (b) 11.5Vandll.6V

(c) 11.4Vandll.5V (d) 11.7Vandll.8V

SOLUTION : . t!))



Using Kirchhoff’ s law at P we get

$$\frac{V-12}{1}+\frac{V-13}{2}+\frac{V-0}{10}=0$$

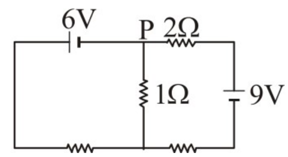
[Let potential at P, Q, U = 0 and at R = V

$$\Rightarrow \frac{V}{1}+\frac{V}{2}+\frac{V}{10}=\frac{12}{1}+\frac{13}{2}+\frac{0}{10}$$

$$\Rightarrow \frac{10+5+1}{10}V=\frac{24+13}{2} \Rightarrow v\left(\frac{16}{10}\right)=\frac{37}{2}$$

$$\Rightarrow V=\frac{37\times10}{16\times2}=\frac{370}{32}=11.56 \text{ volt}$$

61. In the circuit shown, the current in the 1Ω resistor is: [2015]



3Ω Ω 3Ω

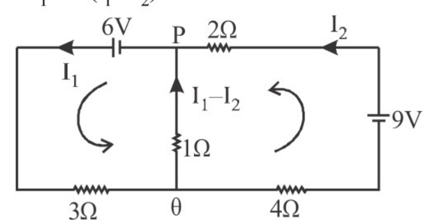
(a) 0.13A, ffom Qto P **(b) 0.13A, from Pto Q**

(c) 1.3A from P to Q **(d) OA**

SOLUTION : (a)

FromKVL

$$-6+3I_1+1\,(I_i-I_2)=0$$



$$6=3I_1+I_1-I_2; 4I_1-I_2=6\ (1)$$

$$-9+2I_2-(I_1-I_2)+3I_2=0$$

$$-I_1+6I_2=9\ (2) \text{ On solving (1) and (2)}$$

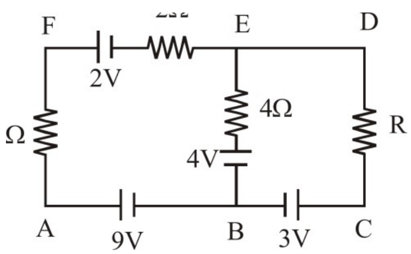
$$I_1=0.13A$$

Direction Q to P, since $I_1 > I_2$.

62. In the electric network shown, when no current flowsthrough the 4Ω resistor in the arm EB, the potentialdifference between the points A and D will be : [Online April 11, 2015]

9 ∩

2



(a) 6 V (b) 3 V (c) 5 V (d) 4 V

SOLUTION : (c)

As no current flows through arm EB then

$$V_D = 0V$$

$$V_E = 0V$$

$$V_B = -4V$$

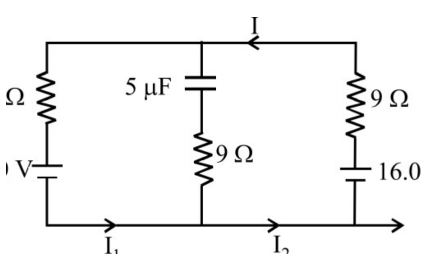
$$V_A = 5V$$

So, potential difference between the points A and D

$$V_A - V_D = 5V$$

63. The circuit shown here has two batteries of 8.0 V and 16.0V and three resistors $3\ \Omega$, 9Ω and $9\ \Omega$ and a capacitor of$5.0\mu F$. [Online Apri112, 2014]

3



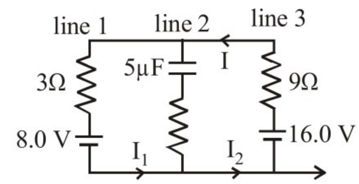
How much is the current I in the circuit in steady state?

(a) $1.6A$ (b) $0.67A$

(c) $2.5A$ (d) $0.25A$

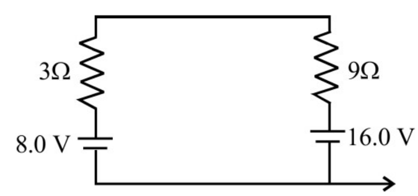
SOLUTION :

(b)



In steady state capacitor is fully charged hence no current will flow through line 2.

By simplifing the circuit



Hence resultant potential difference across resistances will be 8.0 V.

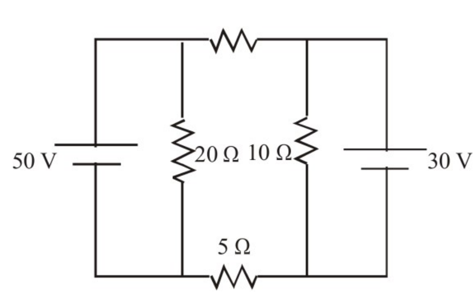
Thus current $I = \frac{V}{R}$

$$= \frac{8.0}{3+9} = \frac{8}{12} \text{ or, } I = \frac{2}{3} = 0.67A$$

64. In the circuit shown, current (in A) through 50 V and 30 Vbatteries are, respectively.

[Online April 11, 2014]

5Ω



(a) 2. 5 and3 (b) 3. 5 and2

(c) 4. 5 and 1 (d) 3 and 2. 5

SOLUTION : (a)

Current through 50 V and 30 V batteries are

respectively 2.5 A and 3 A.

65. A d. c. main supply of e.m.f. 220V is connected across astorage battery of e.m.f. 200V through a resistance of 1Ω.The batteryterminals are connected to an extemal resistance 'R' . The minimum value of 'R' , so that a current passesthrough the battery to charge it is: [Online April 9, 2014]

(a) 7 Ω (b) 9 Ω (c) 11 Ω (d) Zero

SOLUTION : . (c)

Given, emfofcell $E = 200V$

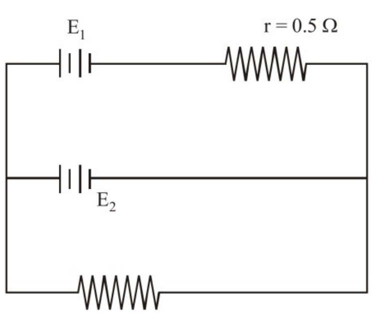
Internal resistance ofcells $= 1\Omega$ D. C. main supplyvoltage $V = 220V$ External resistance $R = ?$

$$r = \left(\frac{E - V}{V}\right) R$$

$$1 = \left(\frac{20}{220}\right) \times R \; R = 11\Omega.$$

66. Adc source ofemf$E_1 = 100V$ and internal resistancer $= 0.5\Omega$, a storage battery of emf $E_2 = 90V$ and an externalresistance R are connected as shown in figure. For whatvalue of R no current will pass through the battery?

[Online April 22, 2013]



R

(a) 5. 5Ω (b) 3. 5Ω(c) 4. 5Ω (d) 2. 5Ω

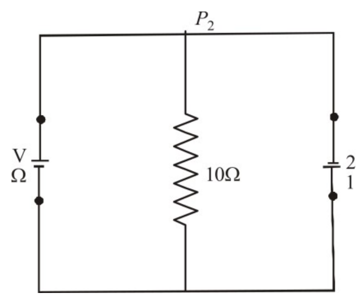
SOLUTION : (c)

$$\frac{100}{R+r} = \frac{90}{R} \Rightarrow \frac{R+r}{R} = \frac{10}{9} \Rightarrow 1 + \frac{0.5}{R} = \frac{10}{9}$$

$$\Rightarrow \frac{0.5}{R} = \frac{1}{9} R = 4.5\Omega$$

67. A5Vbatterywith internal resistance 2 Ωanda2Vb atterywith internal resistance 1Ω are connected to a 10Ω resistoras shown in the figure. [2008]

5 2



ΩV

The current in the 10Ω resistor is

(a) $0.27AP_2$ to P_1 (b) $0.03AP_1$ to P_2

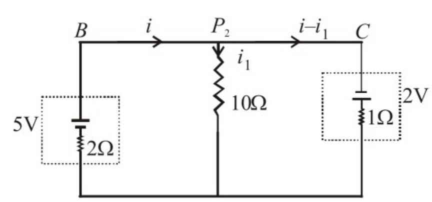
(c) $0.03AP_2$ to P_1 (d) $0.27AP_1$ to P_2

SOLUTION : (c)

Applying Kirchoff' s second law in $AB\ P_2P_1A$, we get

$$-2i + 5 - 10i_1 = 0$$

$$2i + 10i_1 = 5 \text{ (i)}$$



Again app^{AP}1ying Kirchoffs second 1aw^{D}in$P_2CDP_1P_2$

we get,

$$10\, i_1+2-i+i_1=0$$

$$2_i-22i_1=4 \text{ (ii) From(i) and(ii)}$$

$$32i_1=1$$

$$\Rightarrow i_1=\frac{1}{32}\text{A from } P_2 \text{ to } P_1$$

68. Abattery is used to chargea parallel plate capacitor till thepotential difference between the plates becomes equal tothe electromotive force of the battery. The ratio of theenergy stored in the capacitor and the work done by thebatterywill be [2007]

(a) 1/2 (b) 1 (c) 2 (d) ¼

SOLUTION : . (a)

$$\text{Energy in capacitor}=\frac{1}{2}CV^2$$

$$\text{Work done by battery}=QV=CV^2$$

where $C=$ Capacitance ofcapacitor

$V=$ Potential difference,

$e=$ emf ofbattery

$$\text{Required ratio}=\frac{\frac{1}{2}CV^2}{CV^2}=\frac{1}{2}\,(V=e)$$

69. TheK rchhoffs ffist l aw$(\Sigma i=0)$ andsecondlaw $(\Sigma iR=\Sigma E)$,where the symbols have their usual meanings, arerespectively based on [2006]

(a) conservation of charge, conservation ofmomentum

(b) conservation of energy, conservation of charge

(c) conservation ofmomentum, conservation of charge

(d) conservation of charge, conservatrion of energy

SOLUTION : . (d)

Note: Kirchhoffs first law is based on conservation of charge and Kirchhoffs second law is based on conservation of energy.

70. A thermocouple is made fi: om two metals, Antimony andBismuth. Ifone junction ofthe couple is kept hot and theother is kept cold, then, an electric current will [2006]

(a) flow fi: om Antimony to Bismuth at the hotjunction

(b) flow fi: om Bismuth to Antimony at the coldjunction

(c) now flow through the thermocouple

(d) flow fi: om Antimony to Bismuth at the coldjunction

SOLUTION : . (d)

At cold junction, current flows from Antimony to Bismuth because current flows from metal occurring later in the series to metal occurring earlier in the thermoelectricseries. In thermoelectric series, Bismuth comes earlier thanAntimony so at coldjunction, current. Flow fi: om Antimonyto Bismuth.

71. Two sources of equal emf are connected to an externalresistance R. The internal resistance ofthe two sources are$R_1$and$R_2(R_1>R_1)$. Ifthe potential difference across thesource having internal resistance R_2 is zero, then [2005]

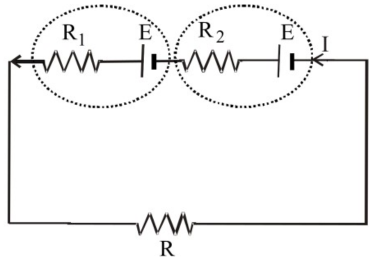
(a) $R=R_2-R_1$

(b) $R=R_2\times(R_1+R_2)/(R_2-R_1)$

(c) $R=R_1R_2/(R_2-R_1)$

(d) $R=R_1R_2/(R_1-R_2)$

SOLUTION : (a)



Let E be the emfof each source of current

Current in the circuit $I = \frac{2E}{R+R_1+R_2}$

Potential difference across cell having internal resistance R_2

$$V = E - iR_2 = 0$$

$$E - \frac{2E}{R+R_1+R_2} \cdot R_2 = 0$$

$$\Rightarrow \mathrm{R} + \mathrm{R_1} + \mathrm{R_2} - 2\mathrm{R_2} = 0$$

$$\Rightarrow \mathrm{R} + \mathrm{R_1} - \mathrm{R_2} = 0$$

$$\Rightarrow \mathrm{R} = \mathrm{R_2} - \mathrm{R_1}$$

72. Two voltameters, one ofcopper and another of silver, arejoined in parallel. When a total charge q flows through thevoltameters, equal amount ofmetals are deposited. Iftheelectrochemical equivalents of copper and silver are $Z_1$and Z_2 respectively the charge which flows through thesilver voltameter is [2005]

(a) $\frac{q}{1+\frac{Z_2}{Z_1}}$ \(b) $\frac{q}{1+\frac{Z_1}{Z_2}}$ (c) $q\frac{Z_2}{Z_1}$ (d) $q\frac{Z_1}{Z_2}$

SOLUTION : (a)

From Faraday's first law of electrolysis, mass deposited

$$m = Zq$$

$$\Rightarrow Z \propto \frac{1}{q} \Rightarrow \frac{Z_1}{Z_2} = \frac{q_2}{q_1}$$

Also $q = q_1 + q_2$ (ii)

$\Rightarrow \frac{q}{q_2} = \frac{q_1}{q_2} + 1$ (Dividing (ii) by q_2)

$$\Rightarrow q_2 = \frac{q}{1+\frac{q_1}{q_2}} (Reject)$$

From equation (i) and(iii),

$$q_2 = \frac{q}{1+\frac{Z_2}{Z_1}}$$

73. An energy source will supply a constant current into theload ifits internal resistance is [2005]

(a) very large as compared to the load resistance

(b) equal to the resistance ofthe load

(c) non - zero but less than the resistance ofthe load

(d) zero

SOLUTION : (d)

Current is given by

$$I = \frac{E}{R+r},$$

Ifinternal resistance (r) is zero,

$I = \frac{E}{R}$ =constant.

Thus, energy source will supply a constant current if its internal resistance is zero.

74. The thermo emf of a thermocouple varies with the temperature θ ofthe hot junction as $E = a\theta + b\theta^2$ in voltswhere the ratio a/b is $700^\circ\mathrm{C}$. Ifthe coldjunction is kept at$0^\circ\mathrm{C}$, then the neutral temperature is [2004]

(a) $14\alpha)^\circ\mathrm{C}$ (b) $350^\circ\mathrm{C}$ (c) $7\alpha)^\circ\mathrm{C}$

(d) No neutral temperature is possible for this termocouple.

SOLUTION : (d)

Given $E = a\theta + b\theta^2 \Rightarrow \frac{dE}{d\theta} = a + 2b\theta$

At neutral temperature $\theta = \theta_n : \frac{dE}{d\theta} = 0$

$$\Rightarrow \theta_n = \frac{-a}{2b} = -350 \Rightarrow \frac{d^2E}{d\theta^2} = 2b$$

hence no 6 is possible for E to be maximum no neutral temperature is possible.

75. The electrochemical equivalent ofa metal is 3.35×10^{-7} kgper Coulomb. The mass ofthe metal liberated at the cathodewhen a 3A current is passed for 2 seconds will be[2004]

(a) 6.6×10^{57}kg (b) 9.9×10^{-7} kg

(c) 19.8×10^{-7} kg (d) 1.1×10^{-7} kg

SOLUTION : . (c)

From the Faraday's first law ofelectrolysis,

$$m = Zit$$

$$\Rightarrow m = 3.3 \times 10^{-7} \times 3 \times 2$$

$$= 19.8 \times 10^{-7}\text{kg}$$

76. The thermo e.m.f. ofa thermo - couple is 25 μV/°C at roomtemperature. A galvanometer of 40 ohm resistance, capableof detecting current as low as 10^{-5}A, is connected withthe thermo couple. The smallest temperature difference thatcan be detected by this system is [2003]

(a) 16°C (b) 12°C (c) 8°C (d) 20°C

SOLUTION : (a)

Let the smallest temperature difference be θ°C that can be detected by the thermocouple, then Thermo

$$\text{emf} = (25 \times 10^{-6})\theta$$

Let I is the smallest current which can be detected by the galvanometer ofresistance R.

Potential difference across galvanometer

$$IR = 10^{-5} \times 40$$

$$10^{-5} \times 40 = 25 \times 10^{-6} \times \theta$$

$$\Rightarrow \theta = 16°\text{C}.$$

77. The negative Zn pole ofa Daniell cell, sending a constantcurrent through a circuit, decreases in mass by O. 13g in 30minutes. Ifthe electeochemical equivalent of Zn and Cuare 32.5 and 31.5 respectively, the increase in the mass ofthe positive Cu pole in this time is [2003]

(a) 0.180g (b) 0.14lg (c) 0.126g (d) 0.242g

SOLUTION : . (c)

According to Faraday's first law of electrolysis

$$m = Z \times I \times t$$

When I and t is same, $m \propto Z$

$$\frac{m_{\text{Cu}}}{m_{\text{Zn}}} = \frac{Z_{\text{Cu}}}{Z_{\text{Zn}}} \Rightarrow m_{\text{Cu}} = \frac{Z_{\text{Cu}}}{Z_{\text{Zn}}} \times m_{\text{Zn}}$$

$$\Rightarrow m_{Cu} = \frac{31.5}{32.5} \times 0.13 = 0.126\text{g}$$

78. The mass of product liberated on anode in an electrochemical cell depends on [2002]

(a) $(It)^{1/2}$ (b) It (c) Ilt (d) I^2t

(where t is the time period for which the current is passed).

SOLUTION : (b)

From the Faraday's first law ofelectrolysis

$$m = ZIt \Rightarrow m \propto It$$

79. An electrical power line, having a total resistance of2 Ω,delivers 1 kW at 220 V. The efficiency ofthe transmissionline is approximately: [Sep. 05, 2020 (I)]

(a) 72% (b) 91% (c) 85% (d) 96%

SOLUTION : . (b)

Given: Power, $P = 1\text{kW} = 1000\text{W}$

$R = 2\Omega, V = 220\text{V}$

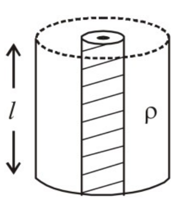
Current, $I = \frac{P}{V} = \frac{1000}{220}$

$$P_{1\text{oss}} = I^2 R = \left(\frac{1000}{220}\right)^2 \times 2$$

Efficiency $= \frac{1000}{1000+P_{1\text{oss}}} \times 100 = 96\%$

80. Model a torch battery of length l to be made up of a thincylindrical bar ofradius 'a' and a concentric thin cylindricalshell ofradius 'b' filled in between with an electrolyte ofresistivityp (see figure). If the battery is connected to a resistance ofvalue R, the maximum Joule heating in R will take place for:

[Sep. 03, 2020 (I)]



$\vec{a}$

$\rightarrow b$

(a) $R = \frac{\text{p}}{2\pi l}\left(\frac{b}{a}\right)$ (b) $R = \frac{\text{p}}{2\pi l} ln \left(\frac{b}{a}\right)$

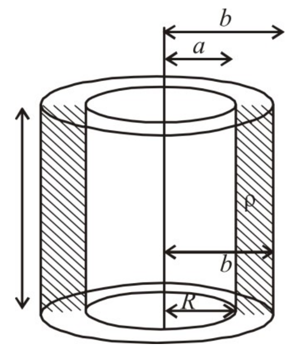
(c) $R = \frac{\text{p}}{\pi l} ln \left(\frac{b}{a}\right)$ (d) $R = \frac{2\text{p}}{\pi l} ln \left(\frac{b}{a}\right)$

SOLUTION : (b)

Maximum power in external resistance is generated when it is equal to internal resistance ofbattery i.e., P_R maximum when $r = R$

The maximum Joule heating in R will take place for, the resistance ofsmall element

$$\Delta R = \frac{\text{p}dr}{2\pi r l} \Rightarrow R = \frac{\text{p}}{2\pi l}\int_a^b \frac{dr}{r}$$



l

or, $R = \frac{\text{p}}{2\pi} \ln \frac{b}{a}$

81. In a building there are 15 bulbs of45 W, 15 bulbs of100 W,15 small fans of10 W and2 heaters ofl kW. The voltageofelectric main is 220 V. The minimum ffise capacity(ratedvalue) ofthe building will be: [7 Jan. 2020 II]

(a) 10A (b) 25A (c) 15A (d) 20A

SOLUTION : (d)

Net Power, P

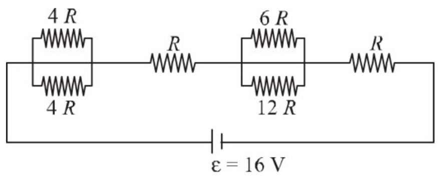
$= 15 \times 45 + 15 \times 100 + 15 \times 10 + 2 \times 1000$

$= 15 \times 155 + 2000\text{W}$

Power, $P = VI \Rightarrow I = \frac{P}{V}$

$$I_{\text{main}} = \frac{15 \times 155 + 2000}{220} = 19.66A \approx 20A$$

82. The resistive network shown below is connected toaD.C.source of 16 V. The power consumed by the network is 4Watt. The value of$R$ is: [12 Apr. 2019 I]



(a) 6Ω (b) 8Ω (c) 1Ω (d) 16Ω

SOLUTION : (b)

Equivalent resistance,

$$R_{\text{eq}} = \frac{4R \times 4R}{4R + 4R} + R + \frac{6R \times 12R}{6R + 12R} + R$$

$$= 2R + R + 4R + R = 8R.$$

Using, $P = \frac{V^2}{R_{\text{eq}}} \Rightarrow 4 = \frac{16^2}{8R}$

$$R = \frac{16^2}{4 \times 8} = 8\Omega$$

83. One kg ofwater, at 20°C, is heated in an electric kettlewhose heating element has a mean (temperature averaged)resistance of20 Ω. The rms voltage in the mains is 200 V.Ignoring heat loss from the kettle, time taken for water toevaporate fully, is close to:[Specific heat ofwater $= 4200$ J/(kgoC), Latent heat ofwater= 2260kJ/kg] [12 Apr. 2019 II]

(a) 16 minutes (b) 22 minutes

(c) 3 minutes (d) 3 minutes

SOLUTION : (b)

98. Three resistors of 4 Ω, 6Ω and 12 Ω are connected inparallel and the combination is connected in series with a 1.5 V battery ofl Ω internal resistance. The rate ofJouleheating in the 4 Ω resistor is [Online May 12, 2012]

(a) 0.55W (b) 0.33W (c) 0.25W (d) 0.86W

SOLUTION : . (c)

Resistors 4 Ω, 6Ω and 12 Ω are connected in parallel,

its equivalent resistance (R) is given by

$$\frac{1}{R} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} \Rightarrow R = \frac{12}{6} = 2\Omega$$

Again R is connected to 1.5 V battery whose internal

resistance $r = 1\Omega$.

Equivalent resistance now,$R' = 2\Omega + 1\Omega = 3\Omega$

Current, $I_{\text{total}} = \frac{V}{R'} = \frac{1.5}{3} = \frac{1}{2}\text{A}$

$$I_{\text{total}} = \frac{1}{2} = 3x + 2x + x = 6x$$

$$\Rightarrow x = \frac{1}{12}$$

Current through 4Ω resistor $= 3x$

$$= 3 \times \frac{1}{12} = \frac{1}{4}A$$

Therefore, rate ofJoule heating in the 4Ω resistor

$$= I^2R = \left(\frac{1}{4}\right)^2 \times 4 = \frac{1}{4} = 0.25\text{W}$$

99. This question has Statement 1 and Statement 2. Of thefour choices given after the Statements, choose the onethat best describes the two Statements.

Statement 1: The possibility ofan electric bulb fusing ishigher at the time of switching ON.

Statement 2: Resistance ofan electric bulb when it is not litup is much smaller than when it is lit up.

[Online May 7, 2012]

(a) Statement 1 is true, Statement 2 is false

(b) Statement 1 is false, Statement 2 is true, Statement

2 is not a correct explanation of Statement 1.

(c) Statement 1 is true, Statement 2 is true, Statement 2is a correct explanation of Statement 1.

(d) Statement 1 is false, Statement2 is true.

SOLUTION : . (c)

100. The resistance ofa bulb filmanet is 100Ω at a temperatureof $100^{\circ}\mathrm{C}$. If its temperature coefficient of resistance be0.005 per ∘ c, its resistance will become 200 Ω at atemperature of [2006]

(a) $3\alpha)^{\circ}\mathrm{C}$ (b) $4(\mathrm{D}^{0}\mathrm{C}$ (c) $5\alpha)^{\circ}\mathrm{C}$ (d) $2\alpha)^{\circ}\mathrm{C}$

SOLUTION : . (b)

Let resistance ofbulb filament be R_0 at $0^{\circ}\mathrm{C}$ using $R =$

$R_0(1+\alpha\Delta t)$ we have

$$R_1 = R_0[1+\alpha\times 100] = 100\ (1)$$

$$R_2 = R_0[1+\alpha\times \mathrm{T}] = 200\ (2)$$

On dividing we get

$$\frac{200}{100} = \frac{1+\alpha T}{1+100\alpha} \Rightarrow 2 = \frac{1+0.005.T}{1+100\times 0005}$$

$$\Rightarrow T = 400^{\circ}\mathrm{C}$$

Note: We may use this expression as an approximation because the difference in the answers is appreciable.

For accurate results one should use $R = R_0e^{\alpha\Delta T}$

101. An electric bulb is rated 220 volt – 100 watt. The powerconsumed by it when operated on 110 volt will be [2006]

(a) 75 watt (b) 40 watt (c) 25 watt (d) 50 watt

SOLUTION : . (c)

The resistance ofthe electric bulb is

$$R = \frac{V^2}{P} = \frac{(220)^2}{100}$$

The power consumed when operated at 110 V is

$$P' = \frac{\nabla^2}{R}$$

$$\Rightarrow P = \frac{(110)^2}{(220)^2/100} = \frac{100}{4} = 25\mathrm{W}$$

102. A heater coil is cut into two equal parts and only one partis now used in the heater. The heat generated will nowbe [2005]

(a) four times (b) doubled

(c) halved (d) one fourth

SOLUTION : (b)

Heat generated,$H = \frac{V^2t}{R}$

After cutting equal length ofheater coil will become half. As $R \propto P$

Resistance ofhalfthe coil $= \frac{R}{2}$

$$H = \frac{V^2t}{R} = 2H$$

As R reduces to half, 'H' will be doubled.

103. The resistance ofhot tungsten filament is about 10 timesthe cold resistance. What will be the resistance of100 Wand 200 V lamp when not in use? [2005]

(a) 20Ω (b) 40Ω (c) 200Ω (d) 400Ω

SOLUTION : . (b)

Power, $P = Vi = \frac{V^2}{R}$

Resistance oftungsten filament when in use

$$R_{\mathrm{hot}} = \frac{V^2}{P} = \frac{200\times 200}{100} = 400\Omega$$

Resistance when not in use *i.e.*, cold resistance

$$R_{\text{cold}} = \frac{400}{10} = 40\Omega$$

104. The thermistors are usuallymade of [2004]

(a) metal oxides with high temperature coefficient of resistivity

(b) metals with high temperature coefficient of resistivity

(c) metals with low temperature coefficient ofresistivity

(d) semiconducting materials having low temperature coefficient of resistivity

SOLUTION : . (a)

Thermistors are usually made of metaloxides with

high temperature coefficient ofresistivity

105. Time taken by a 836 W heater to heat one litre ofwaterfrom 10°C to 40°C is [2004]

(a) 150 s (b) 100 s (c) 50 s (d) 200 s

SOLUTION : . (a)

Heat supplied in time t for heating 1L water from

$$10^{\circ}\text{C to } 40^{\circ}\text{C}$$

$$\Delta Q = mC_p \times \Delta T$$

$$= 1 \times 4180 \times (40 - 10) = 4180 \times 30$$

But $\Delta Q = P \times t = 836 \times t$

$$\Rightarrow t = \frac{4180 \times 30}{836} = 150\text{s}$$

106. A 220 volt, 1000 watt bulb is connected across a 110 voltmains supply. The power consumed will be [2003]

(a) 750 watt **(b) 500 watt**

(c) 250 watt **(d) l(Knwatt**

SOLUTION : . (c)

We know that resistance, $R = \frac{V_{\text{rated}}^2}{P_{\text{rated}}} = \frac{(220)^2}{1000} = 48.4\Omega$

When this bulb is connected to 110 volt mains

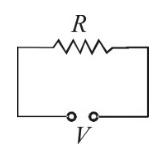
supplywe Get $P = \frac{V^2}{R} = \frac{(110)^2}{48.4} = 250\text{W}$

107. A wire when connected to 220 V mains supply has powerdissipation P_1. Now the wire is cut into two equal pieceswhich are connected in parallel to the same supply. Powerdissipation in this case is P_2. Then $P_2 : P_1$ is [2002]

(a) 1 (b) 4 (c) 2 (d) 3

SOLUTION : (b

) Case 1 Initialpower dissipation,



$$P_1 = \frac{V^2}{R}$$

Case 2 Balancinglength ffomP $= 100 - 49$

When wire is cut into two equal pieces, the resistance

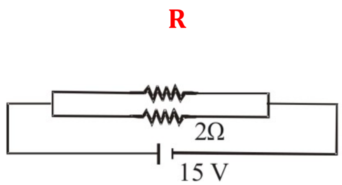
of $x = \underline{102} = 0.02\text{volt/cm}$

each piece is $\underline{R}$. When they are connected in parallel

100–49

2

108. Ifin the circuit, power dissipation is 150 W, then R is[2002]



(a) 2Ω (b) 6Ω (c) 5Ω (d) 4Ω

SOLUTION : . (b)

The equivalent resistance ofparallel combination of

$$\frac{R'}{S}=\frac{l_2}{100-l_2}\Rightarrow\frac{2R}{S}=\frac{l_2}{100-l_2}$$

$$R_{eq}=\frac{2\times R}{2+R}\Rightarrow 2\times\frac{1}{3}=\frac{l_2}{100-l_2}\text{ Using (i)}$$

Power dissipation $P=\frac{V^2}{\mathrm{R}_{eq}}150=\frac{(15)^2}{\mathrm{R}_{eq}}\Rightarrow p_2=40$ cm

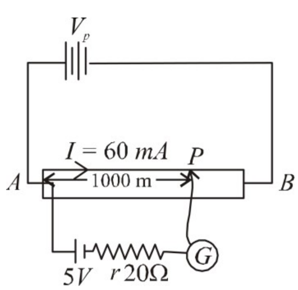
109. Two resistors 400Ω and 800Ω are connected in series acrossa 6V battery. The potential difference measured by avoltmeter of10 $\mathrm{k}\Omega$ across 400Ω resistor is close to:[Sep. 03, 2020 (II)]

(a) $2\mathrm{V}$ (b) $1.8\mathrm{V}$ (c) $2.05\mathrm{V}$ (d) $1.95\mathrm{V}$

SOLUTION : . (d)

The voltmeter of resistance $10\mathrm{k}\Omega$ is parallel to the

resistance of400Ω. So, their equivalent resistance is



$$\Rightarrow V=\frac{150}{77}=1.95\text{ volt}$$

110. Which of the following will NOT be observed when amultimeter (operating in resistance measuring mode)probes connected across a component, are just reversed? [Sep. 03, 2020 (II)]

(a) Multimeter shows an equal deflection in both cases i. e. before and after reversing the probes ifthe chosen component is resistor.

(b) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

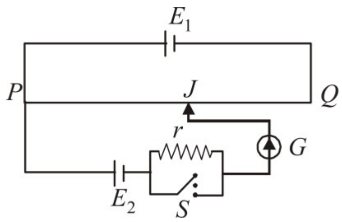
(c) Multimeter shows a deflection, accompanied by a splash of light out of connected and NO deflection on reversing the probes if the chosen component is LED.

(d) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.

SOLUTION : (b)

Multimeter shows deflection in both cases i.e. before and after reversing the probes ifthe chosen component is capacitor.

III. A potentiometer wire PQ of1 m length is connected to astandard cell E_1. Another cell E_2 of emf 1.02 V is connected with a resistanc e'r^1 and switch S (as shown infigure). With switch S open, the null position is obtainedat a distance of49 cm ffom Q. The potential gradient inthe potentiometer wire is : [Sep. 02, 2020 (II)]



(a) $0.02\mathrm{V/cm}$ (b) $0.0\mathrm{lV/cm}$

(c) $0.03\mathrm{V/cm}$ (d) $0.04\mathrm{V/cm}$

SOLUTION : . (a)

Potential gradient, $x = \frac{\text{Potentia1dro}}{\text{1eng}}$

Here, Potential drop $= 1.02$

$$\frac{1}{R'} = \frac{1}{10k\Omega} + \frac{1}{400\Omega} = \frac{1}{10000} + \frac{1}{400}$$

$$\Rightarrow \frac{1}{R}1 = \frac{1+25}{10000} = \frac{26}{10000}$$

$$\Rightarrow R^{Reject} = \frac{10000}{26}\Omega$$

Using Ohm[1]s law, current in the circuit

$$I = \frac{\text{Voltage}}{\text{Net Res istance}} = \frac{6}{\frac{10000}{26} + 800}$$

Potential difference measured by voltmeter

$$V = IR^{\dagger} = \frac{6}{\frac{10000}{26} + 800} \times \frac{10000}{26}$$

Let R be the resistance ofthe whole wire Potential

gradient for the potentiometer wire $A\,B' = -\frac{dV}{dl} =$

$$\frac{I\times R}{l} = \left[\frac{60\times R}{l_{AB}}\right] mvlm$$

$$(dV)$$

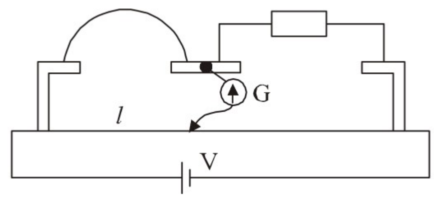
$$V_{AP} = (\quad)^{l_{AP}} = \frac{60\times R}{1200} \times 1000mV$$

$$\Rightarrow V_{AP} = 50\mathrm{R}mV$$

Also, $V_{AP} = 5V$(for balance point at P)

$$R = \frac{V_{AP}}{50\times 10^{-3}} = \frac{5}{50\times 10^{-3}} = 100\Omega$$

112. In a meter bridge experiment S is a standard resistance. Ris a resistance wire. It is found that balancing length is$l = 25$ cm. IfR is replaced by a wire ofhalflength and halfdiameter that of R of same material, then the balancingdistance l' (in cm)R will nowbe $\overline{\mathrm{S}}$. [NA. 9 Jan. 2020 II]

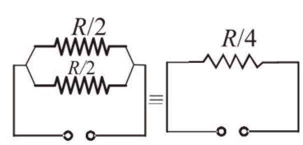


SOLUTION : . (40)

For the given meter bridge

Equivalent resistance, $R_{eq} = \frac{\mathrm{R}/2}{2} = \frac{\mathrm{R}}{4}\frac{R}{S} = \frac{l_1}{100-l_1}$ Where,

P_1 = balancing length



$$\Rightarrow \frac{R}{S} = \frac{25}{75} = \frac{1}{3} \quad \text{(i)}$$

New resistance,

$$P_2 = \frac{V^2}{R/4} = 4\left(\frac{V^2}{R}\right) = 4P_1$$

Power dissiVpated, $V \Rightarrow R' = \frac{\mathrm{p}_{\frac{l}{2}}}{R'--} = \mathrm{p}\frac{l\times 2}{A}\frac{A}{4}2R$

$$\left(\because \mathrm{R} = \mathrm{p}\frac{\ell}{\mathrm{A}}\right)$$

2Ω and R is

113. The length ofa potentiometer wire is 1200 cm and it carriesa current of 60 mA. For a cell of emf 5 V and internalresistance of 20 Ω, the null point on it is found to be at1000 cm. The resistance ofwhole wire is:

[8 Jan. 2020 I]

(a) 80 Ω (b) 120 Ω (c) 60 Ω (d) 100 Ω

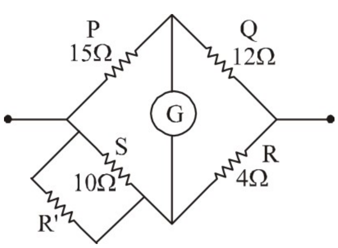
SOLUTION : . (d)

$$\Rightarrow 150=\frac{225\times(R+2)}{2R}\Rightarrow\frac{2R}{2+R}=\frac{3}{2}$$

$$\Rightarrow 4R=6+3R\Rightarrow R=6\Omega$$

114. Four resistances of15 Ω, 12Ω, 4Ω and 10 Ω respectivelyin cyclic order to form Wheatstone’s network. Theresistance that is to be connected in parallel with theresistance of10 Ω to balance the network is Ω.[NA. 8 Jan. 2020 I]

SOLUTION : . (10)



As per Wheatstone bridge balance condition $\frac{P}{Q}=\frac{S}{R}$

Let resistance R’ is connected in parallel with

resistance S

of 10Ω

$$\frac{15}{12}=\frac{10R^1}{}\frac{10+R^1}{4}\Rightarrow 5=\frac{10R'}{10+R'}$$

$$\Rightarrow 50+5R'=10R'$$

$$R'=\frac{50}{5}=10\Omega$$

115. The balancing length for a cell is 560 cm in a potentiometerexperiment. When an external resistance of 10 Ω isconnected in parallel to the cell, the balancing lengthchanges by60 cm. Ifthe internal resistance ofthe cell is $\frac{N}{10}\Omega$, where N is an integer then value ofN is[NA. 7 Jan. 2020 II]

SOLUTION : . (12)

We know that

$E\propto l$ where l is the balancing length

$\mathrm{E}=k(560)$ (i)

When the balancing length changes by 60 cm

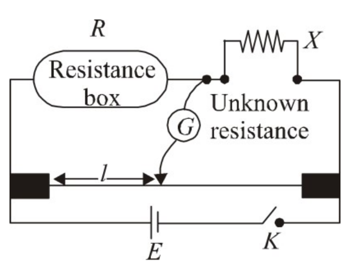
$\frac{E}{r+10}10=k(500)$ (ii)

Dividing (i) by (ii) we get

$$\Rightarrow\frac{r+10}{10}=\frac{56}{50}\Rightarrow 50r+500=560$$

$$\Rightarrow r=\frac{6}{5}\Omega=\frac{N}{10}\Omega\Rightarrow N=12$$

116. In a meter bridge experiment, the circuit diagram and thecorresponding observation table are shown in figure.[10 Apr. 2019 I]



Sl.No.	RΩ	1 (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the reading is consistent?

(a) 3 (b) 2 (c) 4 (d) 1

SOLUTION : . (c)

For a balanced bridge

$$\frac{R_1}{R_2}=\frac{l_2}{l_1}$$

So $\frac{R}{X}=\frac{l}{100-}$

Using the above expression

$$X=\frac{R(100-l)}{l}$$

for observation (1) $X=\frac{100\times 40}{60}=\frac{2000}{3}\Omega$

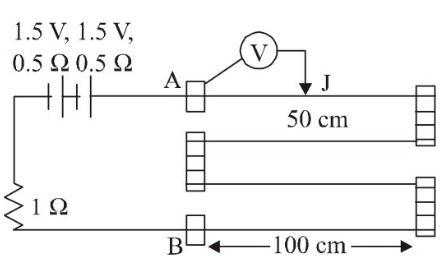
for observation (2) $X=\frac{100\times 87}{13}=\frac{8700}{13}\Omega$

for observation (3) $X=\frac{10\times 98.5}{15}=\frac{1970}{3}\Omega$

for observation (4) $X=\frac{1\times 99}{1}=99\Omega$

Clearly we can see that the value of x calculated in observation (4) is inconsistent than other.

117. In the circuit shown, a four - wire potentiometer is made ofa400 cm long wire, which extends between Aand B. Theresistance per unit length of the potentiometer wire is $r=0.01\Omega/cm$. Ifan ideal voltmeter is connected as shownwithjockey J at 50 cm from end A, the expected reading ofthe voltmeter will be: [8 Apr. 2019 II]



(a) $0.50V$ (b) $0.75V$ (c) $0.25V$ (d) $0.20V$

SOLUTION : (c)

The resistance ofpotentiometer wire

$$R=0.01\times 400=4\Omega$$

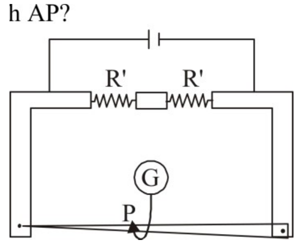
Current in the wire

$$i=\frac{V}{R_T}=\frac{3}{4+0.5+0.57+1}=\frac{1}{2}A$$

Now $V=iR_{AJ}=\frac{1}{2}\times(0.01\times 50)=0.25$ V.

118. In a meter bridge, the wire oflength 1 m has a non - uniformcross - section such that, the variation $\frac{dR}{d1}$ ofits resistanceR with length l is $\frac{dR}{d1}\propto\frac{1}{\sqrt{l}}$. Two equal resistances areconnected as shown in the figure. The galvanometer haszero deflection when thejockey is at point P. What is thelengt [12 Jan. 2019 I]

h AP?



$\leftarrow l\!-\!\!\rightarrow\leftarrow 1-l\rightarrow$

(a) $0.2m$ (b) $0.3m$ (c) $0.25m$ (d) $0.35m$

SOLUTION : . (c)

We have given

$\frac{dR}{d\ell}\propto\frac{1}{\sqrt{\ell}}\Rightarrow\frac{dR}{d\ell}=k\times\frac{1}{\sqrt{\ell}}$ (where k is constant)

$$dR=k\frac{d\ell}{\sqrt{\ell}}$$

Let R_1 and R_2 be the resistance ofAP and PB respectively. Using wheatstone bridge principle

$\frac{R'}{R'} = \frac{R_1}{R_2}$ or $R_1 = R_2$

Now, $\int dR = k\int \frac{d\ell}{\sqrt{\ell}}$

$$R_1 = k\int_0^{\ell} l^{-1/2}\, dl = k.2.\sqrt{l}$$

$$R_2 = k\int_p^{1} l^{-1/2}\, dl = k.(2 - 2\sqrt{l})$$

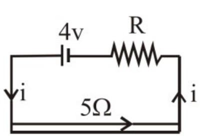
Putting $R_1 = R_2$

$$k2\sqrt{\ell} = k(2 - 2\sqrt{\ell})$$

$$2\sqrt{\ell} = 1$$

$$\sqrt{l} = \frac{1}{2}$$

i.e., $l = \frac{1}{4}\text{m} \Rightarrow 0.25\text{m}$



119. An ideal battery of4 V and resistance R are connected inseries in the primary circuit ofa potentiometer oflength1 m and resistance 5Ω. The value ofR, to give a potentialdifference of5 mV across 10 cm ofpotentiometer wire is:

[12 Jan. 2019 I]

(a) 490Ω (b) 480Ω (c) 395Ω (d) 495Ω

SOLUTION : . (c)

Current flowing through the circuit (I) is given by

$$I = \left(\frac{4}{R+5}\right)A$$

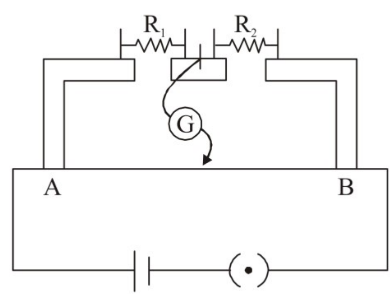
Resistance oflength 10 cm ofwire= $5 \times \frac{10}{100} = 0.5\Omega$

According to question, $5 \times 10^{-3} = \left(\frac{4}{R+5}\right).(0.5)$

$\frac{4}{R+5} = 10^{-2}$ or $R + 5 = 400\Omega$

$$R = 395\Omega$$

122. In the experimental set up of metre bridge shown in thefigure, the null point is obtaine data distance of40 cm fromA. Ifa 10 Ω resistor is connected in series with R_1, the nullpoint shifts by 10 cm. The resistance that should beconnected in parallel with $(R_1 + 10)\Omega$ such that the nullpoint shifts back to its initial position is:[11 Jan. 2019 II]



(a) 20 Ω (b) 40 Ω (c) 60 Ω (d) 30 Ω

SOLUTION : (c)

Initially at null deflection $\frac{R_1}{R_2} = \frac{2}{3}$ (i)

Finally at null deflection, when null point is shifted

$\frac{R_1+10}{R_2} = 1 \Rightarrow R_1 + 10 = R_2$ (ii)

Solving equations (i) and (ii) we get

$$\frac{2R_2}{3} + 10 = R_2$$

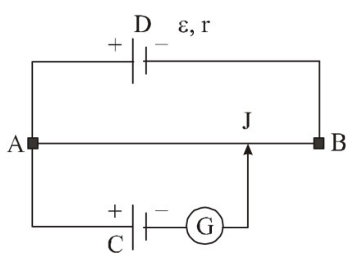
$$10 = \frac{R_2}{3} \Rightarrow R_2 = 30\Omega$$

$$\& \; R_1 = 20\Omega$$

Now ifrequired resistance is R then $\frac{\frac{30\times R}{30+R}}{30} = \frac{2}{3}$

$$R = 60\Omega$$

123. A potentiometer wire AB having length L and resistance12 r isjoined to a cellD ofemfe and internal resistance r. A cell C having emf $\varepsilon/2$ and internal resistance 3r isconnected. The length AJ at which the galvanometer asshown in fig. shows no deflection is: [10 Jan. 2019 I]



$\frac{\varepsilon}{2}, 3r$

(a) $\frac{11}{12}L$ (b) $\frac{11}{24}L$ (c) $\frac{13}{24}L$ (d) $\frac{5}{12}L$

SOLUTION : . (c)

Let x be the length AJ at which galvanometer shows

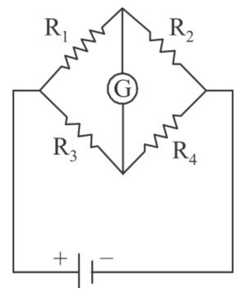
null deflection current,

$$i=\frac{\varepsilon}{12r+r}=\frac{3}{13} \text{ or, } i\left(\frac{x}{L}12r\right)=\frac{\varepsilon}{2}$$

$$\Rightarrow \frac{\varepsilon}{13r}\left[\frac{x}{L}\cdot 12r\right]=\frac{\varepsilon}{2} \Rightarrow \frac{\varepsilon}{13r}\left[\frac{x}{L}\cdot 12r\right]=\frac{\varepsilon}{2}$$

$$\text{or, } x=\frac{13L}{24}$$

124. The Wheatstone bridge shown in Fig. here, gets balancedwhen the carbon resistor used as R_1 has the colour code(Orange, Red, Brown). The resistors R_2 and R_4 are 80 $\Omega$and 40 Ω, respectively.Assuming that the colour code for the carbon resistorsgives their accurate values, the colour code for the carbon resistor, used as R_3, would be: [10 Jan. 2019 II]



(a) Brown, Blue, Brown (b) Brown, Blue, Black

(c) Red, Green, Brown (d) Grey, Black, Brown

SOLUTION : (a)

Given, colour code ofresistance,

R_1 =Orange, Red and Brown

$$R_1=32\times 10=320$$

using balanced wheatstone bridge principle,

$R_3=160$ i.e. colour code for R_3 Brown, Blue and Brown

25. In a potentiometer experiment, it is found that no currentpasses through the galvanometer when the terminals ofthe cell are connected across 52 cm ofthe potentiometerwire. Ifthe cell is shunted by a resistance of5 Ω, a balanceis found when the cell is connected across 40 cm of thewire. Find the internal resistance ofthe cell. [2018]

(a) 1 Ω (b) 1.5Ω (c) 2 Ω (d) 2.5Ω

SOLUTION :

Using formula, internal resistance,

$$r=\left(\frac{l_1-l_2}{l_2}\right)s=\left(\frac{52-40}{40}\right)\times 5=1.5\Omega$$

126. On interchanging the resistances, the balance point of ameter bridge shifts to the left by 10 cm. The resistance oftheir series combination is 1kΩ. How

much was the resistance on the left slot before interchanging the resistances? [2018]

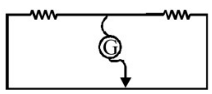
(a) 990 Ω (b) 505 Ω (c) 550 Ω (d) 910 Ω

SOLUTION : . (c)

$$R_1 + R_2 = 1000$$

$$\Rightarrow R_2 = 1000 - R_1$$

$$R_1\ R_2 = 1000 - R_1$$

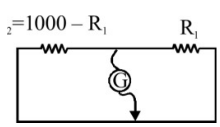


(l) $100 - l$

On balancing condition

$R_1(100 - l) = (1000 - R_1)l$ (i)

On Interchangin g resistance balance point shifts left by 10 cm R



$$(l - 10)\ (100 - l + 10) = (110 - t)$$

On balancing condition

$$(1000 - R_1)(110 - l) = R_1(l - 10)$$

or, $R_1(l - 10) = (1000 - R_1)(110 - f)$ (ii) Dividing eqn (i) by(ii)

$$\frac{100 - l}{l - 10} = \frac{l}{110 - l}$$

$$\Rightarrow (100 - l)(110 - l) = l(l - 10)$$

$$\Rightarrow 11000 - 100l - 110l + l^2 = l^2 - 10l$$

$$\Rightarrow 11000 = 200l or, l = 55$$

Putting the value of l' in eqn (i)

$$R_1(100 - 55) = (1000 - R_1)55$$

$$\Rightarrow R_1(45) = (1000 - R_1)55$$

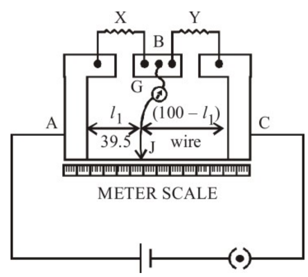
$$\Rightarrow R_1(9) = (1000 - R_1)11$$

$$\Rightarrow 20R_1 = 11000$$

$$R_1 = 550K\Omega$$

127. In a meter bridge, as shown in the figure, it is given thatresistance $Y = 12.5\Omega$ and that the balance is obtained at adistance 39.5 cm from end A (by jockey J). Afterinterchanging the resistances X and Y, a new balance pointis found at a distance l_2 from end A. What are the values of$X$ and l_2 ?

[Online Apri115, 2018]



(a) 19.15Ω and 39.5 cm (b) 8.16Ω and 60.5 cm

(c) 19.15Ω and 60.5 cm (d) 8.16Ω and 39.5 cm

SOLUTION : (b)

For a balanced meter bridge,

$$\frac{x}{39.5} = \frac{Y}{(100 - 39.5)} \Rightarrow Y = 39.5 = X \times (100 - 39.5)$$

$$\text{or, } X = \frac{12.5 \times 39.5}{60.5} = 8.16\Omega$$

When X and Y are interchanged l_1 and $(100 - l_1)$ will also interchange so, $l_2 = 60.5$ cm

128. Which ofthe following statements is false? [2017]

(a) A rheostat can be used as a potential divider

(b) Kirchhoffs second law represents energy conservation

(c) Wheatstone bridge is the most sensitive when all thefour resistances are ofthe same order ofmagnitude

(d) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

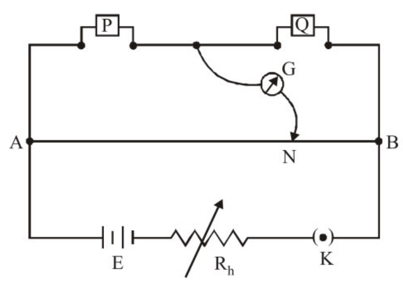
SOLUTION : . (d)

There is no change in null point, ifthe cell and the galvanometer are exchanged in a balanced wheatstone bridge.

On balancing condition $\frac{R_1}{R_3}=\frac{R_2}{R_4}$ After exchange

On balancing condition $\frac{R_1}{R_2}=\frac{R_3}{R}$

129. In a meter bridge experiment resistances are connected asshown in the figure. Initially resistance $\mathrm{P}=4\Omega$ and theneutral point N is at 60 cm fromA. Now an unknown resis - tance R is connected in series to P and the new position ofthe neutral point is at 80 cm from A. The value ofunknownresistance R is : [Online April 9, 2017]



(a) $\frac{33}{5}\Omega$ (b) 6 Ω (c) 7 Ω (d) $\frac{20}{3}\Omega$

SOLUTION : (d)

In balance position ofbridge, $\frac{\mathrm{P}}{\mathrm{Q}}=\frac{l}{(100-l)}$

Initiallyneutral position is 60 cm. ffom A, so

$$\frac{4}{60}=\frac{\mathrm{Q}}{40}\Rightarrow \mathrm{Q}=\frac{16}{6}=\frac{8}{3}\Omega$$

Now, when unknown resistance R is connected in series to P, neutral point is 80 cm from Athen,

$$\frac{4+\mathrm{R}}{80}=\frac{\mathrm{Q}}{20}$$

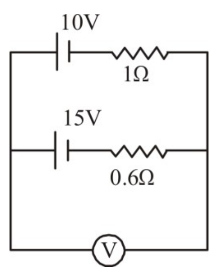
$$\frac{4+\mathrm{R}}{80}=\frac{8}{60}$$

$$\mathrm{R}=\frac{64}{6}-4=\frac{64-24}{6}=\frac{40}{6}\Omega$$

Hence, the value ofunknown resistance R is $=\frac{20}{3}\Omega$

130. A potentiometer PQ is set up to compare two resistances asshown in the figure. The ammeter A in the circuit reads 1.0Awhen two way key K_3 is open. The balance point is at a lengthl_1 cm from P when two waykey K_3 is plugged in between 2and 1, while the balance point is at a lengh l_2 cm from P whenkey K_3 is plugged in between 3 and 1. The ratio of two resistances $\frac{\mathrm{R}_1}{\mathrm{R}_2}$, is found to be: [Online April 8, 2017]

(a) $\frac{l_1}{l_1+l_2}$ (b) $\frac{l_2}{l_2-l_1}$ (c) $\frac{l_1}{l_1-l_2}$ (d) $\frac{l_1}{l_2-l_1}$



(a) 12.5V (b) 24.5V (c) 13.1V (d) 11.9V

SOLUTION : . (d)

When key is at point (1)

$$\mathrm{V}_1=\mathrm{iR}_1=\mathrm{x}l_1$$

When key is at (3)

$$V_2 = i(R_1 + R_2) = xl_2$$

$$\frac{R_1}{R_1 + R_2} = \frac{l_1}{l_2} \Rightarrow \frac{R_1}{R_2} = \frac{l_1}{l_2 - l_1}$$

132. In an experiment ofpotentiometer for measuring the internalresistance ofprimary cell a balancing length P is obtained onthe potentiometer wire when the cell is open circuit. Now thecell is short circuited by a resistance R. If R is to be equal tothe internal resistance ofthe cell the balancing length on thepotentiometer wire will be [Online May 26, 2012]

(a) ℓ (b) 2ℓ (c) $\ell/2$ (d) $\ell/4$

SOLUTION : (c)

As the two cells oppose each other hence, the

effective emfin closed circuit is $15 - 10 = 5\text{V}$ and net

resistance is $1 + 0.6 = 1.6\Omega$ (because in the closed

circuit the internal resistance of two cells are in series.

Current in the circuit,

$$I = \frac{\text{effectiveemf}}{\text{tota1resistance}} = \frac{5}{1.6}A$$

The potential difference across voltmeter will be same asthe terminal voltage of either cell.

Since the current is drawn from the cell of 15 V

$$V_1 = E_1 - Ir_1$$

$$= 15 - \frac{5}{1.6} \times 0.6 = 13.1\text{V}$$

133. It is preferable to measure the e.m.f. of a cell by potentiometer than by a voltmeter because ofthe followingpossible reasons. [Online May 12, 2012]

(i) In case of potentiometer, no current flows through the cell.

(ii) The length ofthe potentiometer allows greater precision.

(iii) Measurement by the potentiometer is quicker.

(iv) The sensitivity of the galvanometer, when using a potentiometer is not relevant.

Which of these reasons are correct?

(a) (i), (iii), (iv) (b) (i), (iii), (iv)

(c) (i),(ii) (d) (i), (ii), (iii),(iv)

SOLUTION : . (c)

Balancing length l will give emf of cell $E = Kl$

Here K is potential gradient.

If the cell is short circuited by resistance $\uparrow R^1$

Let balancing length obtained be l' then $V = kl'$

$$r = \left(\frac{E - V}{V}\right)R$$

$$\Rightarrow V = E - V \; [r = R \text{ given}]$$

$$\Rightarrow 2V = E$$

$$\text{or, } 2Kl' = Kl \quad l' = \frac{l}{2}$$

134. In a sensitive meter bridge apparatus the bridge wire shouldpossesses [Online May 12, 2012]

(a) high resistivity and low temperature coefficient.

(b) low resistivity and high temperature coefficient.

(c) low resistivity and low temperature coefficient.

(d) high resistivity and high temperature coefficient.

SOLUTION : (c)

To measure the emfof a cell we prefer potentiometer

rather than voltmeter because

(i) the length of potentiometer which allows greater

precision.

(ii) in case ofpotentiometer, no current flows through

the oeL ofhigh sensitivity.

Bridge wire in a sensitive meter bridge wire should be

ofhigh resistivity and low temperature coefficient

135. In a metre bridge experiment null point is obtained at40 cmfrom one end of the wire when resistance X is balancedagainst another resistance Y. IfX $< Y$, then the new positionof the null point fi$_{\mathrm{i}}$om the same end, if one decides tobalance a resistance of 3X against Y, will be close to:[Online April 9, 2013]

(a) 80 cm (b) 75 cm (c) 67 cm (d) 50 cm

SOLUTION : . (c)

From question, $\frac{x}{y}=\frac{40}{100-40}=\frac{2}{3}$

$$\Rightarrow x=\frac{2}{3}y$$

Again, $\frac{3x}{y}=\frac{Z}{100-Z}$

or $\frac{3\times\frac{2y}{3}}{y}=\frac{Z}{100-Z}$

Solving we get $Z=67$ cm

Therefore new position ofnull point $\cong 67$ cm

. (a)

.

136. The current in the primary circuit ofa potentiometer is 0.2A. The specific resistance and cross - section of thepotentiometer wire are 4×10^{-7} ohm metre and $8\times 10^{-7}\mathrm{m}^{2}$, respectively. The potential gradient will be equal to [2011 RS]

(a) 1 V/m (b) 0.5V/m (c) 0.1V/m (d) 02 V/m

SOLUTION : . (c)

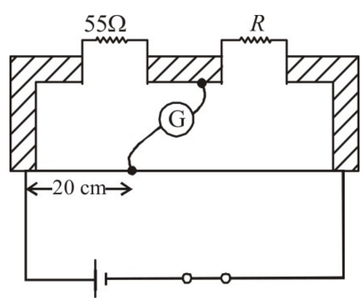
Potential gradient

$$\Rightarrow k=\frac{V}{l}=\frac{IR}{l}=\frac{I}{l}\left(\frac{pl}{A}\right)=\frac{Ip}{A}$$

$$k=\frac{0.2\times 4\times 10^{-7}}{8\times 10^{-7}}=\frac{0.8}{8}=0.1\mathrm{V/m}$$

137. Shown in the figure belowis a meter - bridge set up with nulldeflection in the galvanometer.131. A 10V batterywith internal resistance 1Ω anda 15Vbattery

with internal resistance 0.6Ω are connected in parallel toa voltmeter (see figure). The reading in the voltmeter willbe close to: [OnlineApril I0, 2015]



The value ofthe unknown resistor R is [2008]

(a) 13.75Ω (b) 220 Ω (c) 110 Ω (d) 55 Ω

SOLUTION : (b)

Given,

Balance point from one end, $P_1=20$ cm

From the condition for balance ofmetre bridge, we have

$$\frac{55}{R}=\frac{l_1}{100-l_1}$$

$$\frac{55}{R}=\frac{20}{80}$$

$$\Rightarrow R=220\Omega$$

138. In a Wheatstone[1]s bridge, three resistances P, Q and Rconnected in the three arms and the fourth arm is formedby two resistances S_1 and S_2 connected in parallel. Thecondition for the bridge to be balanced will be [2006]

(a) $\frac{P}{Q}=\frac{2R}{S_1+S_2}$ (b) $\frac{P}{Q}=\frac{R(S_1+S_2)}{S_1S_2}$

(c) $\frac{P}{Q}=\frac{R(S_1+S_2)}{2S_1S_2}$ (d) $\frac{P}{Q}=\frac{R}{S_1+S_2}$

SOLUTION : . (b)

From balanced wheat stone bridge $\frac{P}{Q}=\frac{R}{S}$ where

$$S=\frac{S_1S_2}{S_1+S_2}$$

139. In a potentiometer experiment the balancing with a cell is atlength 240 cm. On shunting the cell with a resistance of2Ω,the balancing length becomes 120 cm. The internalresistance of the cell is [2005]

(a) 0.5Ω (b) 1Ω (c) 2Ω (d) 4Ω

SOLUTION : (c)

Initial balancing length, $P_1=240$ cm Newbalancing

length, $l_2=120$ cm.

The internal resistance ofthe cell,

$$r=\left(\frac{l_1-l_2}{l_2}\right)\times R=\frac{240-120}{120}\times 2=2\Omega$$

140. In a meter bridge experiment null point is obtained at 20 cm.from one end of the wire when resistance X is balancedagainst another resistance Y. If$X<Y$, then where will be thenew position of the null point from the same end, if onedecides to balance a resistance of 4 X against Y [2004]

(a) 40 cm (b) 80 cm (c) 50 cm (d) 70 cm

SOLUTION : . (c)

From the balanced wheat stone bridge $\frac{R_1}{R_2}=\frac{l_1}{l_2}$

where $P_2=100-P_1$ In the first case $\frac{X}{y}=\frac{20}{80}$

$$y=4X$$

In the second case$\frac{4X}{Y}=\frac{l}{100-l}$

$$\Rightarrow\frac{4X}{4X}=\frac{l}{100-l}$$

$$\Rightarrow p=50$$

141. The length of a wire of a potentiometer is 100 cm, and the e. m.f. ofits standard cell is E volt. It is employed to measurethe e.m.f. ofabatterywhose internal resistance is 0.5Ω. Ifthe balance point is obtained at $l=30$ cm from the positive

end, the e.m.f. ofthe battery is [2003]

(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{(100-0.5)}$

(c) $\frac{30(E-0.5i)}{100}$ (d) $\frac{30E}{100}$

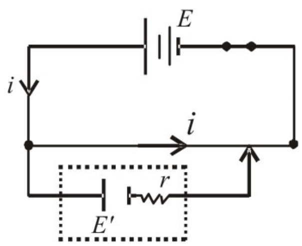
SOLUTION : . (d)

From the principle ofpotentiometer, $V\propto l$

If a cell of emf$E$ is employed in the circuit between the ends ofpotentiometer wire oflength L, then

$$\frac{V}{E}=\frac{l}{L};$$

$$\Rightarrow \mathrm{V}=\frac{\mathrm{E}l}{\mathrm{L}}=\frac{30\mathrm{E}}{100}$$



Note: In this arrangement, the internal resistance ofthe battery E does not play any role as current is not passing through the battery.

142. An ammeter reads upto 1 ampere. Its internal resistance is0.81ohm. To increase the range to 10 A the value of therequired shunt is [2003]

(a) 0.03Ω (b) 0.3Ω (c) 0.9Ω (d) 0.09Ω

SOLUTION : . (d)

$$i_g \times G = (i - i_g)S$$

$$S = \frac{i_g \times G}{i - i_g} = \frac{1 \times 0.81}{10 - 1} = 0.09\Omega$$

143. Ifan ammeter is to be used in place ofa voltmeter, then wemust connect with the ammeter a [2002]

(a) low resistance in parallel

(b) high resistance in parallel

(c) high resistance in series

(d) low resistance in series.

SOLUTION : (c)

To use an ammeter in place of voltmeter, we must

connect a high resistance in series with the ammeter.

Connecting high resistance in series makes its resistance much higher.

MOVING CHARGES AND MAGNETISM

i) A current carrying wire produces a magnetic field of its own. This was first observed by Oersted.

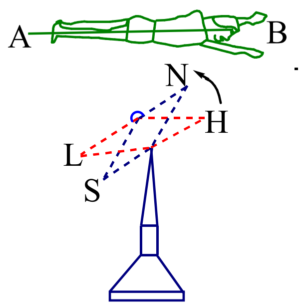
ii) When current is flowing through a conductor, only magnetic field is produced around it, which is non conservative.

iii) The direction of magnetic lines of force due to straight current carrying conductor will be concentric circles around the conductor in a plane which is always perpendicular to the length of the conductor.

The direction of magnetic field can be found by using

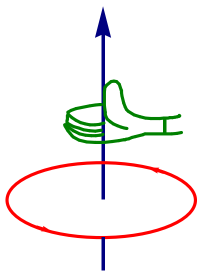
i) Ampere's Swimming Rule:

Imagine a person swimming along a current carrying wire in the direction of the current facing a magnetic needle below the wire, then the magnetic north pole of the needle deflects towards his left hand.

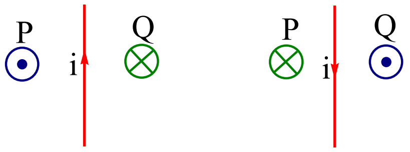


ii) Ampere's Right Hand Thumb Rule:

When a straight conductor carrying current is held in the right hand such that the thumb is pointing along the direction of current, then the direction in which fingers curl round it gives the direction of magnetic lines of force.



The direction of magnetic field for current carrying conductor is as given below.

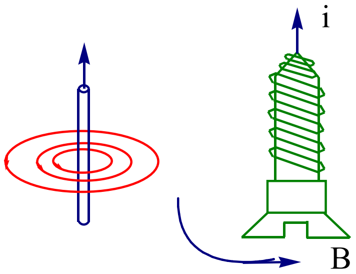


⊗ indicates $\overline{B}$ into the plane of paper

⊙ indicates $\overline{B}$ out of the plane of paper

iii) Maxwell's cork screw rule:

Imagine a right handed cork screw advancing in the direction of current, then the direction of rotation of the screw head gives the direction of magnetic lines of force,

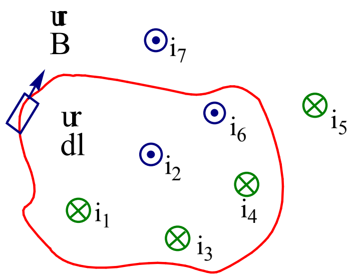


Ampere's Circuital Law:

STATEMENT :

The line integral of the magnetic induction field (B) along any closed path in air (or) vacuum is equal to μ_0 times the net current across the area bounded by this path.

$$\oint \vec{B}.\vec{dl} = \mu_0 i_{net}$$



Consider a closed plane curve as shown in figure. $\vec{dl}$ is a small length element on the curve. Let $\vec{B}$ be the resultant magnetic field at the position of $\vec{dl}$. If the scalar product $\vec{B}.\vec{dl}$ is integrated by varying $\vec{dl}$ on the closed curve it is called line integral of $\vec{B}$ along the curve and it is represented by $\oint \vec{B}.\vec{dl}$

The rule for deciding whether an enclosed current is positive or negative : The fingers of the right hand are to be taken in the direction of integration around the path. If a current pierces the membrane stretched across the area in the direction of the thumb, then it is positive current. If the current pierces the membrane in the opposite direction, then it is negative.

For the above closed path $\oint \vec{B}.\vec{dl} = \mu_0(i_1 - i_2 + i_3 + i_4 - i_6)$

Points to remember regarding Ampere's Law

a) The line integral does not depend on the shape of the closed path or on the position of the current carrying wire in the loop.

b) If a conductor carrying current is outside the closed path, the line integral of B due to that conductor is zero i.e., we need not consider the currents that do not pierce the area of the closed path.

c) Ampere's circuital law is always true no matter how distorted the path or how complicated may be the magnetic field. In most cases even though Ampere's circuital law is true it is inconvenient because it is impossible to perform the path integral. However in few special symmetric cases it is easy to perform path integral using ampere's law.

d) Ampere's circuital law is applicable for conductors carrying steady current.

e) Ampere's circuital law is analogous to Gauss law.

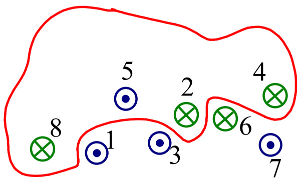
f) Ampere's circuital law is not independent of Biot-Savart's law. It can be derived from Biot-Savart's law. Its relation with Biot-Savart's Law is similar to the relation between Gauss Law and Coulomb's Law in electrostatics.

Ex:1: Eight wires cut the page perpendicular to the points shown. Each wire carries current i_0. Odd currents are out of the page and even current into the page. Find the line integral $\oint \vec{B}.\vec{dl}$ along the loop.

Sol. According to Ampere's, circuital law

$$\oint \vec{B}.\vec{dl} = \mu_0 i_{enclosed}$$

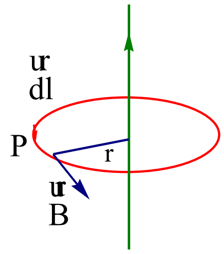
$$\oint \vec{B}.\vec{dl} = \mu_0 \left[i_8 - i_5 + i_2 + i_4 \right]$$



since, all the wires carry same current of i_0, we have $\oint \vec{B}.\vec{dl} = 2\mu_0 i_0$.

Intensity Of Magnetic Induction (B) Near A Long Straight Conductor :

Consider an infinitely long wire carrying current i as shown in figure. P is a point at a perpendicular distance r from the conductor. The magnetic induction field produced by the conductor is radially symmetric i.e., magnetic lines of force are concentric circles centred at the conductor. The tangent drawn to the line of force at any point gives the direction of magnetic induction field $\vec{B}$ at that point. $\vec{dl}$ is a small element on the circle of radius r and angle between $\vec{B}$ and $\vec{dl}$ is 0^0 every where on this path. From Ampere's circuital law



$\oint \vec{B}.\vec{dl} = \mu_0 i \qquad \oint B dl \cos 0^0 = \mu_0 i \qquad B\oint dl = \mu_0 i \qquad B(2\pi r) = \mu_0 i \Rightarrow B = \frac{\mu_0 i}{2\pi r}$

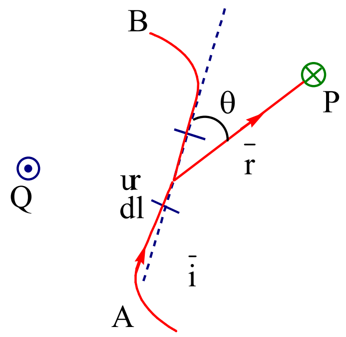
Here r must be much less than the length of conductor.

Magnetic induction at any point along the axis of ocnductor is zero.

Magnetic Field Due To a Current Element - Biot-Savart Law :

All magnetic fields are due to currents (or moving charges) and due to intrinsic magnetic moments of particles. Here, the relation between current and the magnetic field produced by the current is given by the Biot-Savart's law.

Biot and Savart conducted several experiments and established the relation between magnetic induction $\left(\vec{B}\right)$ and current(i).



The above figure shows a finite conductor AB carrying current 'i'. Consider an infinitesimal element $\vec{dl}$ of the conductor. The magnetic field $\vec{dB}$ due to this element is to be determined at point 'P' which is at a distance 'r' from it. Let θ be the angle between $i\vec{dl}$ and the radius vector $\vec{r}$.

According to Biot-Savart's law, the magnitude of magnetic induction dB.

a) is directly proportional to the current(i) flowing through the element i.e., $dB \alpha i \rightarrow (i)$

b) is directly proportional to the length (dl) of the element i.e., $dB \alpha dl \rightarrow (ii)$

c) is directly proportional to the sine of the angle (θ) between length of the element and the line joining the element to the point P.

$dB \alpha \sin\theta \rightarrow (iii)$

d) is inversely proportional to the square of the distance (r) of the point from the element.

$$dB\alpha\frac{1}{r^2}\rightarrow(iv)$$

↳ If the conductor is in vacuum (or) air then

$$dB=\frac{\mu_0}{4\pi}\frac{idl\sin\theta}{r^2}$$

↳ Here $\frac{\mu_0}{4\pi}$ is the proportionality constant and μ_0 is called as permeability of free space or air.

The value of μ_0 is $4\pi\times10^{-7}\,tesla-m/A$

↳ The above equation gives the magnitude of the magnetic field produced due to small current element at a distance 'r' from it.

↳ If current flows in the direction as shown in the figure, the direction of dB at P is directed perpendicular to the plane of the paper in the inward direction.

↳ In vector form the above equation can be written

$$\vec{dB}=\frac{\mu_0}{4\pi}\frac{i\vec{dl}\times\bar{r}}{r^3}$$

↳ The resultant field at P due to the entire conductor can be obtained by integrating the above equation.

$$B=\int_A^B\frac{\mu_0 i}{4\pi}\frac{\vec{dl}\times\vec{r}}{r^3}$$

Magnetic Field Due To A Straight Current Carrying Wire :

Consider a straight conductor carrying current 'i'. Let 'P' be a point at a perpendicular distance 'd' from the conductor.

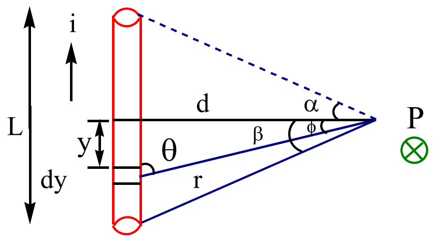
Let 'dy' be a small current element at a distance 'r' from 'P'.

↳ According to Biot-Savart's law, the magnetic induction at P due to the small element is

$$dB=\frac{\mu_0}{4\pi}\frac{idy\sin\theta}{r^2}$$

↳ As every element of the wire contributes to $\vec{B}$ in the same direction, the magnetic induction due to the entire conductor is

$$B=\int dB=\frac{\mu_0 i}{4\pi}\int\frac{dy.\sin\theta}{r^2}$$



$$\tan\varnothing=y/d$$

$$y = d\tan\phi \Rightarrow dy = d(\sec^2\phi)d\phi$$

$$\frac{r}{d} = \sec\phi \qquad\qquad r = d\sec\phi$$

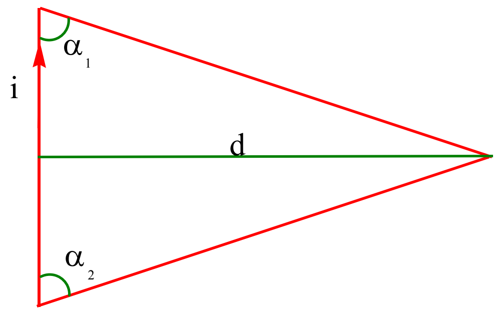
$$B = \frac{\mu_0 i}{4\pi}\int \frac{d(\sec^2\phi).d\phi\sin(90^0-\phi)}{d^2\sec^2\phi} \qquad\qquad [\because \theta = (90-\phi)]$$

$$B = \frac{\mu_0 i}{4\pi}\int_{-\beta}^{\alpha} \frac{d(\sec^2\phi)d\phi\cos\phi}{d^2\sec^2\phi} \qquad\qquad B = \frac{\mu_0 i}{4\pi d}\int_{-\beta}^{\alpha}\cos\phi\, d\phi$$

($-\beta$ is taken because the angle is measured anti clockwise)

$$B = \frac{\mu_0 i}{4\pi d}(\sin\alpha + \sin\beta)$$

Similary B is given as



$$B = \frac{\mu_0 i}{4\pi d}[\cos\alpha_1 + \cos\alpha_2]$$

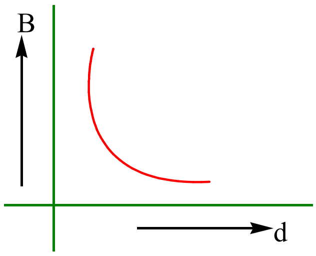
Special Cases :

i) If the point is along the length of the wire (but not on it then as $\vec{dl}$ and $\vec{r}$ will be either parallel (or) antiparallel i.e. $\theta = 0\ (or)\ \pi$

so $\vec{dl}\times\vec{r} = 0$ and hence $\vec{B} = \int_A^B \vec{dB} = \vec{0}$

ii) If a point is at a perpendicular distance d from the wire then the magnetic field B varies inversely with distance d.

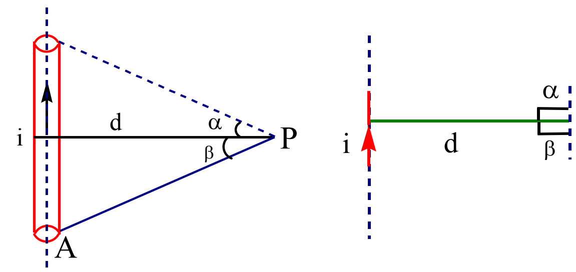
$B \propto \frac{1}{d}$



iii) If the wire is of finite length 'L' and the point is on its perpendicular bisector, at a distance 'd' from the wire, i.e $\alpha = \beta$

$$B = \frac{\mu_0}{4\pi}\frac{2i}{d}\sin\alpha \quad \text{with} \quad \sin\alpha = \frac{L}{\sqrt{L^2+4d^2}}$$

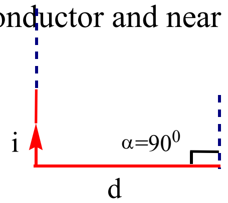
iv) If wire is of infinite length and the point P lies at a distance 'd' from the wire which is at a large distance from its ends as shown in figure, $\alpha = \beta = \pi / 2$



$$B = \frac{\mu_0}{4\pi}\frac{i}{d}(2) = \frac{\mu_0}{4\pi}\frac{2i}{d} = \frac{\mu_0 i}{2\pi d}$$

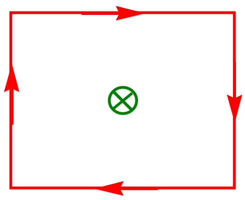
v) At a point away from the conductor and near the edge of conductor

$\alpha = 90^0,\ \beta = 0^0\ B = \frac{\mu_0}{4\pi}\frac{i}{d}$



vi)a)Magnetic induction at the centre of current carrying wire bent in the form of square of side 'a' is

$$B_{net} = 4B_{side}$$

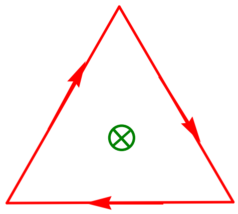


$$B_{net} = 4\frac{\mu_0}{4\pi} \times \frac{i}{a/2}(\sin 45^0 + \sin 45^0)$$

$$B = 8\sqrt{2}\left(\frac{\mu_0 i}{4\pi a}\right) \otimes$$

b) Magnetic induction at the centroid of current carrying wire bent in the form of equilateral triangle of side 'a' is

$$B_{net} = 3B_{\ eachside}$$



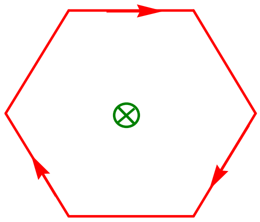
$$B_{net} = 3\frac{\mu_0}{4\pi} \times \frac{i}{r}(\sin 60^0 + \sin 60^0)$$

$$\left(where\ \ r = \frac{a}{2\sqrt{3}}\right)$$

$$B = 18\frac{\mu_0 i}{4\pi a}$$

c) Magnetic induction at the centre of current carrying wire bent in the form of hexagon of side 'a' is given by

$B_{net} = 6B_{eachside}$



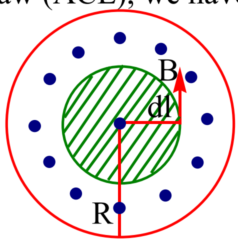
Here $\alpha = \beta = 30^0$

$B = 4\sqrt{3}\,\dfrac{\mu_0 i}{4\pi a}$

The Magnetic Field due to a long straight Current Carrying Conductor.

a) Taking a circular ampere loop centered to the wire of radius r<R. To find B inside the conductor using ampere's circuital law (ACL), we have

$\oint \vec{B}.\vec{dl} = \mu_0 i'$,



Where $J = \dfrac{i}{\pi R^2}$

Here $i' = J.\pi r^2$

$\Rightarrow \oint B dl \cos 0 = \mu_0 J \pi r^2$

or $B \oint dl = \mu_0 J \pi r^2$

or $B 2\pi r = \mu_0 J \pi r^2$

(or) $B = \dfrac{\mu_0 J}{2} r; \qquad r \le R$

$B \alpha r$

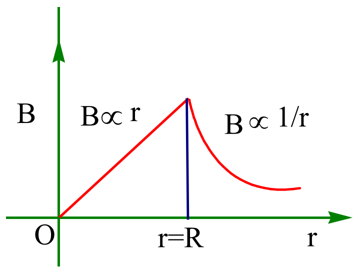
b) At a point outside the wire (r>R) $\oint B.dl \cos 0^0 = \mu_0 i'$,

Where $i' = i$ because the amperian encloses total current or

$B \oint dl = \mu_0 i$ (or) $B 2\pi r = \mu_0 i$

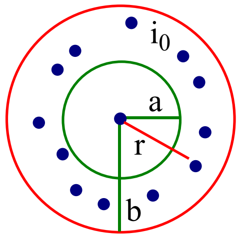
$\Rightarrow B = \dfrac{\mu_0 i}{2\pi r}; \qquad r \ge R \qquad B \alpha \dfrac{1}{r}$

c) B varies linearly inside the conductor and hyperbolically outside the conductor.



Magnetic induction is maximum at the periphery of the wire

d) The variation of $\vec{B}$ as the function of radial distance r due to a hollow cylinder carrying a current i_0.



Taking a circular amperian loop of radius r(>a) and applying ACL,

$\oint \vec{B}.\vec{dl} = \mu_0 i; \qquad B2\pi r = \mu_0 i,$

Where $i = \frac{i_0}{\pi(b^2 - a^2)}.\pi(r^2 - a^2)$

$= \frac{i_0(r^2 - a^2)}{b^2 - a^2}$

then $B = \frac{\mu_0 i_0 (r^2 - a^2)}{2\pi(b^2 - a^2)r} \qquad a \le r \le b$

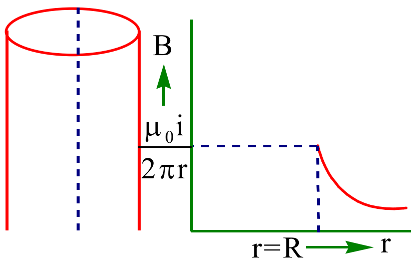
B=0 for $r \le a$ (as because i=0)

for $r > b \qquad B = \frac{\mu_0 i_0}{2\pi r}$

e) **For thin hollow cylinder**

i) $B_{inside} = 0$ ii) $B_{surface} = \frac{\mu_0 i}{2\pi R}\ (r = R)$

iii) $B_{outside} = \frac{\mu_0 i}{2\pi r}\ (r > R)$



WORKDONE :

f) **Work done to move a unit north pole through a small distance dl' along the tangent at a distance 'r' away from current carrying conductor**

$\Rightarrow dw = \vec{F}.\vec{dl}$

$\vec{F} = m\vec{B} = \vec{B} (\because m = 1)$

but $dw = \vec{F}.\vec{dl} \Rightarrow dw = \vec{B}.\vec{dl}$

Total work done in moving it once around the conductor. $W = \oint dw$

$W = \oint \vec{B}.\vec{dl}$

But from Ampere's circuital law

$\oint \vec{B}.\vec{dl} = \mu_0 i \qquad \Rightarrow W = \mu_0 i$

If a pole of strength 'm' is rotated for 'n' times around the current carrying conductor, then the work done is

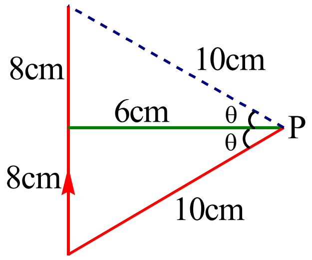
$W = \mu_0 i \times nm$

Here $W \neq 0$, the magnetic field produced by crrent carrying conductor is a non-conservative field.

Ex:2 **Find the magnetic induction due to a straight conductor of length 16cm carrying current of 5A at a distance of 6cm from the midpoint of conductor.**

Sol.

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}(\sin\theta + \sin\theta)$$



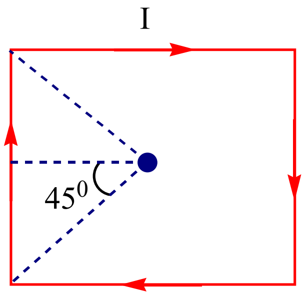
but $\sin\theta = \frac{8}{10} = \frac{4}{5}$

$$B = 10^{-7} \times \frac{5}{6\times 10^{-2}} \times 2 \times \frac{4}{5}$$

Ex:3 **If a straight conductor of length 40cm bent in the form of a square and the current 2A is allowed to pass through square, then find the magnetic induction at the centre of the square loop**

Sol.

$B_{net} = 4B_{side}$



$$B_{net} = 4\frac{\mu_0}{4\pi} \times \frac{I}{L/2}(\sin 45^0 + \sin 45^0)$$

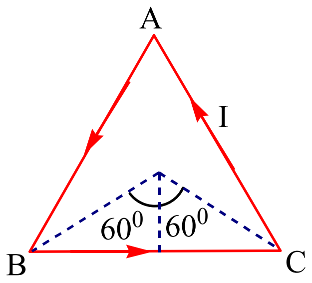
$$= 4X\frac{\mu_0}{4\pi} \times \frac{I}{L/2}(\sqrt{2}) = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}I}{L}$$

$$= 10^{-7} \times 8\sqrt{2} \times 2 \times 10 = 16\sqrt{2}\mu T$$

Ex:4 **If a thin uniform wire of length 1m is bent into an equilateral traiangle and crries a current of $\sqrt{3}A$ in anitclockwise direction, find the net magnetic induction at the centroid**

Sol. $B_{net} = 3B_{eachside}$ $\qquad B_{net} = 3\dfrac{\mu_0}{4\pi} \times \dfrac{I}{r}(\sin 60^0 + \sin 60^0)$

$\left(\because r = \dfrac{a}{2\sqrt{3}}\right)$



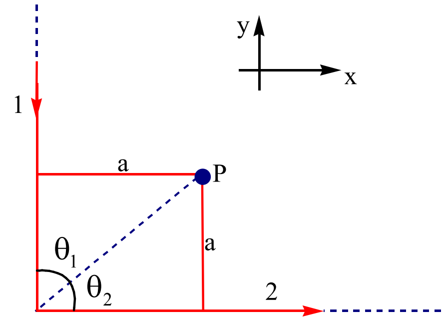
$= 3 \times \dfrac{\mu_0}{4\pi}\dfrac{I}{r}(2\sin 60^0)$

$= 3 \times \dfrac{\mu_0}{4\pi}\dfrac{I(2\sqrt{3})}{a}2 \times \dfrac{\sqrt{3}}{2} = 18\dfrac{\mu_0}{4\pi}\dfrac{I}{a}$

$B = 18 \times 10^{-7} \times \dfrac{\sqrt{3}}{1/3} = 54\sqrt{3} \times 10^{-7}T$

Ex:5 **A large straight current carrying conductor is bent in the form of L shape. Find $\vec{B}$ at P.**

Sol. Let us divide the conductor into two semi infinite segments 1 and 2. Then, induction at P is



$\vec{B} = \vec{B}_1 + \vec{B}_2$ ..i

$\vec{B}_1 = \dfrac{\mu_0 i}{4\pi a}(\sin(90^0 - \theta_1) + \sin 90^0)\hat{k}$ ..ii

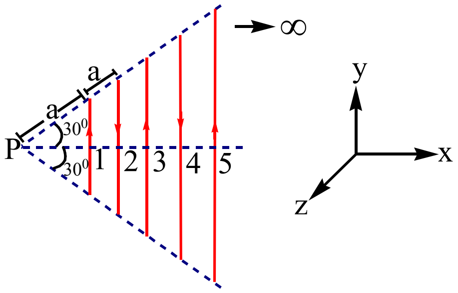
$\vec{B}_2 = \dfrac{\mu_0 i}{4\pi a}(\sin(90^0 - \theta_2) + \sin 90^0)\hat{k}$ ..iii

then $\vec{B} = \dfrac{\mu_0 i}{4\pi a}(\cos\theta_1 + \cos\theta_2 + 2)\hat{k}$,

where $\cos\theta_1 = \cos\theta_2 = \dfrac{1}{\sqrt{2}}$

Hence, $\vec{B} = (2 + \sqrt{2})\dfrac{\mu_0 i\hat{k}}{4\pi a}$

Ex:6 **Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Find net magnetic field at point P ?**



Sol.

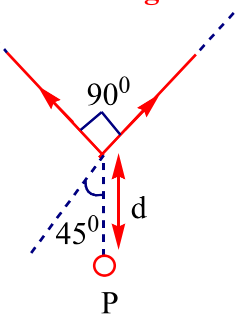
$$B_{net} = \frac{\mu_0 I}{4\pi}(\sin 30^0 + \sin 30^0)\hat{k}\left[\frac{1}{d} - \frac{1}{2d} + \frac{1}{3d} - \frac{1}{4d} ..... + \infty\right]$$

$$\left(Where\ d = a\cos 30 = \frac{\sqrt{3}a}{2}\right)$$

$$\therefore B_{net} = \frac{\mu_0 I}{2\sqrt{3}\pi a}\hat{k}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\infty\right]$$

$$= \frac{\mu_0 I}{2\sqrt{3}\pi a}\ln 2\,\hat{k} = \frac{\mu_0 I}{4\pi}\frac{\ln 4}{\sqrt{3}a}\hat{k}$$

Ex:7 **Find the magnetic field at P due to the arrangement shown**

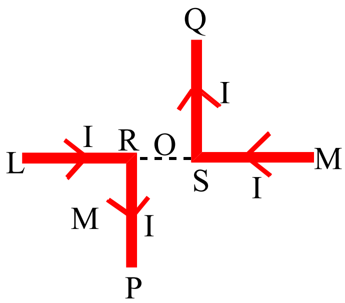


Sol. $$B_{net} = 2 \times \frac{\mu_0 I}{4\pi r}\left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right)$$

here $r = \frac{d}{\sqrt{2}}$

$$B_{net} = \frac{\mu_0 I}{\sqrt{2}\pi d}\left(1 - \frac{1}{\sqrt{2}}\right)$$

Ex:8 **A pair of stationary and infinitely long bent wires are placed in the x-y plane as shown in figure. The wires carry current of 10 ampere each as shown. The segment L and M are along the x-axis. The segment P and Q are parallel to the Y-axis such that OS = OR = 0.02m. Find the magnitude and direction of the magnetic induction at the origion O.**



Sol. Since point O is along the length of segment L and M the field at O due to these two segments will be zero

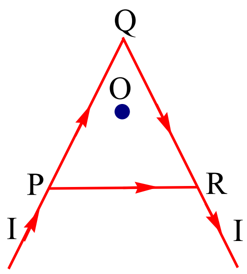
$\therefore$ Magnetic field at O is due to QS and RP.

$$\therefore B_{SQ} = \frac{\mu_0}{4\pi} \times \frac{I}{OS} = 10^{-7} \times \frac{10}{0.02} \text{ e}$$

$$B_{RP} = \frac{\mu_0}{4\pi} \times \frac{I}{OR} = 10^{-7} \times \frac{10}{0.02} \text{ e}$$

$$\therefore B_0 = B_{SQ} + B_{RP} = 10^{-7} \times \frac{10}{0.02} \times 2 = 10^{-4} T \text{ e}$$

Ex:8 **An equilateral triangle of side length *l* is formed from a piece of wire of uniform resistance. The current I is as shown in figure. Find the magnitude of the magnetic field at its centre O.**



Sol. The magnetic field induction at O due to current through PR is

$$B_1 = \frac{\mu_0}{4\pi} \frac{2l/3}{r} \left[\sin 30^0 + \sin 30^0\right]$$

$$= \frac{\mu_0}{4\pi} \frac{2l}{3r} \text{ e} \qquad \text{(directed outside)}$$

The magnetic field induction at O due to current through PQR is

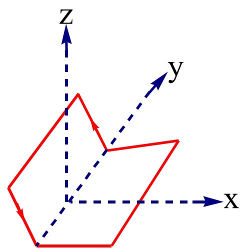
$$B_2 = 2 \times \frac{\mu_0 (l/3)}{4\pi r}\left[\sin 30^0 + \sin 30^0\right]$$

$$= \frac{\mu_0}{4\pi} \frac{2l}{3r} \otimes \qquad \text{(directed inside)}$$

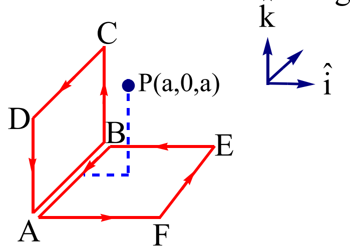
$\therefore$ Resultant magnetic induction at O

$\Rightarrow B_1 - B_2 = 0$

Ex:10 A non planar loop of conducting wire carrying a current I is placed as shown in the figure each of the straight sections of the loop is of length 2a. Find the direction of magnetic field due to this loop at the point P (a,0,a)



Sol. The magnetic field at P (a,0,a) due to the loop is equal to the vector sum of the magnetic fields produced by loops ABCDA and AFEBA as shown in the figure



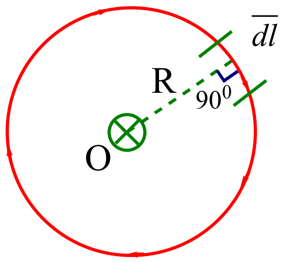
Magnetic field due to ABCDA will be along $\hat{i}$ and due to loop AFEBA, along $\hat{k}$. Magnitude of magnetic field due to both the loops will be equal.

There fore, direction of resultant magnetic field at P will be $\frac{1}{\sqrt{2}}(\hat{i}+\hat{k})$.

Magnetic Field At The Centre Of A Circular Coil Carrying Current

Consider a circular coil of radius R carrying a current i in clockwise direction. Consider any small element dl of the wire. The magnetic field at the centre O due to the current element $i\vec{dl}$ is

$$dB = \frac{\mu_0}{4\pi}\frac{i\vec{dl}\times\vec{R}}{R^3}$$



Where $\vec{R}$ is the vector joining the element to the centre O. The direction of this field is perpendicular to the plane of the diagram and is going into it.

The magnitude of the magnetic field is $dB = \frac{\mu_0}{4\pi}\frac{idl}{R^2}$

As the fields due to all such elements have the same direction, the net field is also in this direction. It can, therefore, be obtained by integrating equation

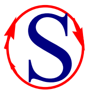
i) under proper limits. Thus,

$$B = \int dB = \int \frac{\mu_0 i}{4\pi R^2} dl$$

If the coil has N turns $\int dl = 2\pi RN$

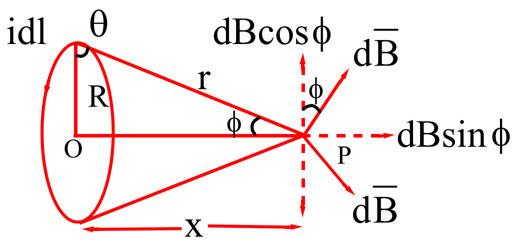
$$= B = \frac{\mu_0 i}{4\pi R^2}\int dl = \frac{\mu_0 i}{4\pi R^2} \times 2\pi RN = \frac{\mu_0 iN}{2R} \otimes$$

If the current is in clock wise direction, then the magnetic field produced is normally inwards and the face of the coil behaves as south pole.



If the current is in anti clock wise direction, then the magnetic field produced is normally outwards and the face of the coil behaves as north pole.

Field At An Axial Point Of A Circular Loop :



Consider a circular poop of radius R, carrying current in in yz plane with centre at origin O. Let P be a point on the axis of the loop at a distance 'x' from the centre 'O' of the loop.
Consider a conducting element dl of loop. According to Biot-Savart's law, the magnitude of magnetic field due to the current element is

$$\left|\vec{dB}\right| = \frac{\mu_0}{4\pi}\frac{\left|i\vec{dl} \times \vec{r}\right|}{r^3} \text{ where } r = \sqrt{x^2 + R^2}$$

Here the element dl is in yz plane where as the displacement vector $\bar{r}$ from $\overline{dl}$ to the point p is in xy plane. So $\left|i\vec{dl} \times \vec{r}\right| = idl \times r$

$$\left|\vec{dB}\right| = \frac{\mu_0}{4\pi}\frac{idl \times r}{r^3} = \frac{\mu_0}{4\pi}\frac{idl}{r^2}$$

The direction of $\vec{dB}$ is perpendicular to the plane formed by $\bar{r}$ and $\overline{dl}$.

In case of a point P on the axis of circular coil, for every current element 'idl' there is a symmetrically situated opposite element. The componenet of the field **dB** perpendicular to the axis cancel each other while component of the field **dB** along the axis add up and contributes to the net magnetic field.

i.e., $B = \int dB \sin\phi = \frac{\mu_0}{4\pi}\int \frac{idl \sin\theta}{r^2}\sin\phi$

Here angle θ between the element $\vec{dl}$ and $\vec{r}$ is $\pi/2$ every where and r is same for all elements and also $\sin\phi = (R/r)$ so,

$$B = \frac{\mu_0}{4\pi}\int \frac{idl \sin\theta}{r^2}\sin\phi = \frac{\mu_0}{4\pi}\int \frac{idl \sin 90^0}{r^2}\frac{R}{r}$$

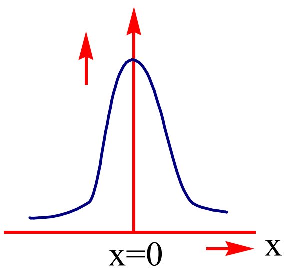
$$= \frac{\mu_0}{4\pi}\frac{iR}{r^3}\int dl$$

for a loop $\int dl = 2\pi R$ and as $r^3 = (x^2 + R^2)^{3/2}$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi iR^2}{(x^2 + R^2)^{3/2}} = \frac{\mu_0 iR^2}{2(x^2 + R^2)^{3/2}}$$

The direction of magnetic field B is along the axis of the loop.

i) The magnetic field B varies non linearly with distance **x** from centre as shown in figure.



For a coil having N turns, $\int dl = 2\pi RN$

so, $B = \frac{\mu_0 NiR^2}{2(x^2 + R^2)^{3/2}}$.

It is maximum when $x^2 = 0$, i.e., at the centre of the coil whose value is given by

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NI}{R} = \frac{\mu_0 Ni}{2R}$$

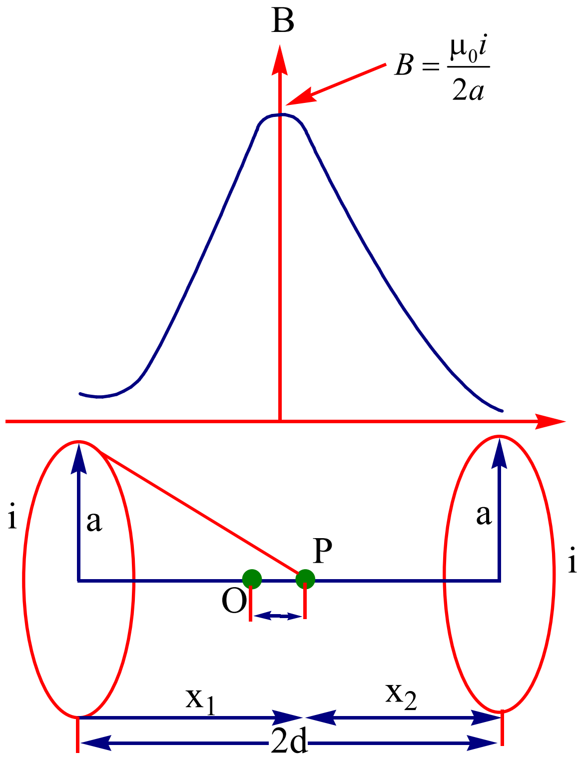
ii) if $x >> R$

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NIR^2}{x^3} = \frac{\mu_0}{4\pi}\frac{2NIA}{x^3}$$

Where $A = \pi R^2$, area of the coil.

Field between two similar coaxial circular loops

Let us consider two loops, each having N turns and carrying current i are place at a distance 2d apart.



Assuming the current is flowing in the same direction in each coil, the magnetic field at a short distance x from midway point O.

$$B = \frac{\mu_0 Nia^2}{2}\left[\frac{1}{\left(a^2 + x_1^2\right)^{3/2}} + \frac{1}{\left(a^2 + x_2^2\right)^{3/2}}\right]$$

$$= \frac{\mu_0 Nia^2}{2}\left[\frac{1}{\left[a^2 + (d+x)^2\right]^{3/2}} + \frac{1}{\left[a^2 + (d-x)^2\right]^{3/2}}\right]$$

The field will be uniform between the loops, if $\frac{dB}{dx}$=0 i.e,

$$\frac{\mu_0 Nia^2}{2}\left[(-3)\frac{(d+x)}{\left[a^2 + (d+x)^2\right]^{5/2}} + 3\frac{(d-x)}{\left[a^2 + (d-x)^2\right]^{5/2}}\right] = 0$$

$$\Rightarrow (d+x)\left[a^2+(d-x)^2\right]^{5/2}$$

$$=(d-x)\left[a^2+(d+x)^2\right]^{5/2} \quad ......(i)$$

Now

$$\left[a^2+(d+x)^2\right]^{5/2}=\left[a^2+d^2+x^2+2xd\right]^{5/2}$$

$=\left[a^2+d^2+x^2+2xd\right]^{5/2}$ [since x is small, so neglecting x^2]

$$=\left(a^2+d^2\right)^{5/2}\left[1+\frac{2xd}{\left(a^2+d^2\right)}\right]^{5/2}$$

$$=\left(a^2+d^2\right)^{5/2}\left[1+\frac{5xd}{\left(a^2+d^2\right)}\right]$$

$$\text{Q } \frac{2xd}{\left(a^2+d^2\right)} << 1$$

Similarly

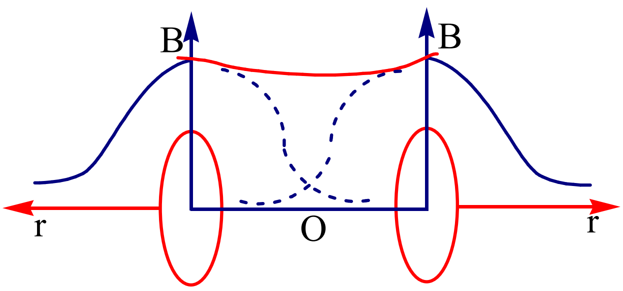
$$\left[a^2+(d-x)^2\right]^{5/2}=\left(a^2+d^2\right)^{5/2}\left[1+\frac{5xd}{\left(a^2+d^2\right)}\right]$$

Substituting these value in equation (i), we got

$$(d+x)\left[1-\frac{5xd}{\left(a^2+d^2\right)}\right]=(d-x)\left[1+\frac{5xd}{\left(a^2+d^2\right)}\right]$$

$$\frac{5d^2}{\left(a^2+d^2\right)}=1, \text{Q } d=a/2, \text{or } a=2d$$

and $$B=2\left[\frac{\mu_0}{2}\frac{Nia^2}{\left[a^2+\left(\frac{a}{2}\right)^2\right]^{3/2}}\right] \qquad B=\frac{8\mu_0 Ni}{5\sqrt{5}a}$$



Circular Current Loop As Magnetic Dipole :

From the above expression $B = \frac{\mu_0}{4\pi}\frac{2NIA}{x^3}$

Comparing with $B = \frac{\mu_o}{4\pi}\frac{2M}{x^3}$

a) Magnetic moment of the circular current carrying coil is M = NiA;

b) M is independent of shape of the coil

$\therefore$ Current loop behaves like a magnetic dipole with poles on either side of its face and it is known as "magnetic shell".

c) SI unit of magnetic moment (M) is $A-m^2$ and dimensional formula is IL^2.

d) Magnetic moment of a curent loop is a vector perpendicular to the plane of the loop and the direction is given by right hand thumb rule.

Magnetic Dipole Moment of a Revolving Electron:

Consider an electron revolving in a circular path of radius r around a nucleus with uniform speed v.

The current in the orbit is

$$i = \frac{e}{T} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

Magnetic dipole moment of a revolving electron is $\mu = iA = \frac{ev}{2\pi r}\times \pi r^2 = \frac{evr}{2}$

Magnetic dipole moment of a revolving electron in the first orbit of hydrogen atom is called Bohr magneton (μ).

From Bohr second postulates, for an electron revolving in first orbit of hydrogen atom.

$$m_e vr = \frac{h}{2\pi}(n=1)$$

Where h = Planck's constant, m_e = mass of electron

$$\mu = \frac{evr}{2} = \frac{e}{2}\frac{h}{2\pi m_e} = \frac{eh}{4\pi m_e}$$

$$(\mu)_{\min} = \frac{e}{4\pi m_e}h$$

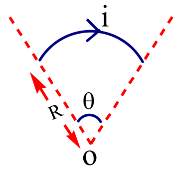
$$= \frac{1.60\times10^{-19}\times6.63\times10^{-34}}{4\times3.14\times9.11\times10^{-31}} = 9.27\times10^{-24}\,Am^2$$

This value is called the . **Bohr magneton**.

Special cases :

i) For an arc shaped conductor carrying current subtending an angle θ at the centre.

$$B = \frac{\mu_0 i}{2R}\frac{\theta}{2\pi}\otimes$$



$\therefore$ Magnetic induction at the centre $B = \frac{\mu_0 i\theta}{4\pi R}\otimes$

ii) For a quadrant circular wire carrying current.

$\theta = 90^0$

Magnetic induction at the centre $B = \frac{\mu_0 i}{8R}\otimes$

iii) If B_0 is magnetic induction at the centre of a circular current carrying coil of radius R having N turns and B_A is magnetic induction at a point on the axis of it at a distance x from centre then

$$B_A = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

Proof : $B_0 = \frac{\mu_0 Ni}{2R}$ and $B_A = \frac{\mu_0 NiR^2}{2(R^2+x^2)^{3/2}}$

$$\Rightarrow B_A = \frac{\mu_0 Ni}{2R\left(1+\frac{x^2}{R^2}\right)^{3/2}} \Rightarrow B_A = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

$$B_A = B_0\left[1-\frac{3x^2}{2R^2}\right]$$

iv) If a particle of charge q moves in a circular path of radius r with a velocity v, then the magnetic induction at the centre of circular loop

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0}{2r}\times\frac{q\text{v}}{2\pi r} = \frac{\mu_0}{4\pi}\frac{q\text{v}}{r^2}$$

If f is the frequency of rotation

$$B = \frac{\mu_0}{2r}\times qf$$

If ω is the angular velocity, then

$$B = \frac{\mu_0}{2r}\times\frac{q\omega}{2\pi} = \frac{\mu_0}{4\pi}\frac{q\omega}{r}$$

v) A charge ‘q’ is moving with a velocity of ‘v’. Then the expression of magnetic induction due to this charge at a position vector $\bar{r}$ from the charge is

Biot - Savart Law for a current element is

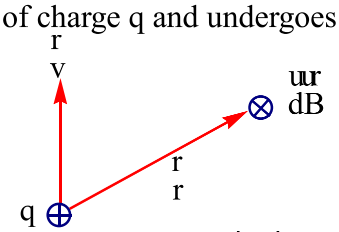
$$d\vec{B} = \frac{\mu_0 i\, \vec{dl} \times \vec{r}}{4\pi r^3}$$

If a charged particle of charge q and undergoes a displacement $\vec{dl}$ during a time dt put $i = \frac{q}{dt}$.

or $i\vec{dl} \times \vec{r} = \frac{q\vec{dl}}{dt} \times \vec{r}$

Putting $\frac{dl}{dt} = v$

$i\vec{dl} \times \vec{r} = q(\vec{v} \times \vec{r})$



Using the above equations, $d\vec{B} = \frac{\mu_0 q(\vec{v} \times \vec{r})}{4\pi r^3}$.

vi) a)When a wire of length *'l'* carrying current 'i' is bent in a circular loop of 'n' turns then the magnetic induction at the centre of the loop is $B = \frac{\mu_0 n i}{2r} = \frac{\mu_0 \pi n^2 i}{l}$ $\quad (\because n \times 2\pi r = l)$

b) The same wire of length *'l'* carrying current 'i' is first bent into a circular coil with n_1 turns and then into another circular coil with n_2 turns. If B_1, B_2 are magnetic inductions at their centres in the two cases, then

c) $\frac{B_1}{B_2} = \left(\frac{n_1}{n_2}\right)^2$

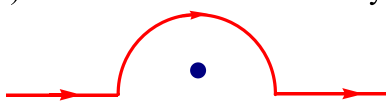
d) If r_1 *and* r_2 are radii of turns of the coil in the above case, then ratio of magnetic induction is

$\frac{B_1}{B_2} = \left(\frac{r_2}{r_1}\right)^2$

e) If two circular coils are connected in series, then the ratio of magnetic induction at their centres is $\frac{B_1}{B_2} = \left(\frac{n_1}{n_2}\right)\left(\frac{r_2}{r_1}\right)$

f) If the two coils are made up of same wire and connected in parallel, then the ratio of the magnetic induction at their centres is $\frac{B_1}{B_2} = \left(\frac{r_2}{r_1}\right)^2$.

vii) a) For semi circular wire carrying current.

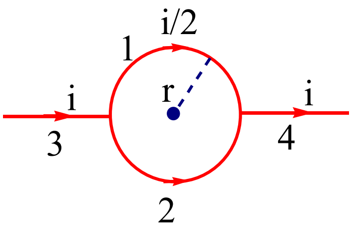


$\theta = 180^0$

Magnetic induction at the centre $B = \frac{\mu_0 i}{4R} \otimes$

b) To a circular wire, two straight wires are attached as shown. When current is passed through it the magnetic field at the centre is zero.

$$B_1 = \frac{\mu_0\left(\frac{i}{2}\right)}{4R} \otimes$$

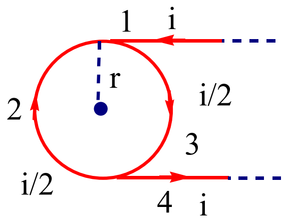


$$B_3 = B_4 = 0$$

$$B_2 = \frac{\mu_0\left(\frac{i}{2}\right)}{4R} e \qquad \therefore B_{net}\, at\, O = Zero$$

c) To a circular wire, two straight wires are attached as shown. When current is passed through it the magnetic field at the centre.

$$B_1 = \frac{\mu_0 i}{4\pi r} e$$



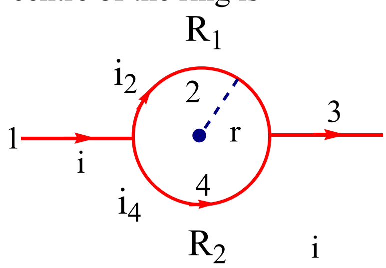
$$B_2 = \frac{\mu_0\left(\frac{i}{2}\right)}{4R} e$$

$$B_3 = \frac{\mu_0\left(\frac{i}{2}\right)}{4R} \otimes$$

$$B_4 = \frac{\mu_0 i}{4\pi r} e \qquad \vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4$$

$$B_{net} = \frac{\mu_0 i}{2\pi r} e$$

d) The upper and lower halves of the ring have resistances R_1 and R_2. Two straight wires are connected to it as shown. The magnetic induction at the centre of the ring is



$$B_1 = B_3 = 0 \qquad B_2 = \frac{\mu_0 i_2}{4r} \otimes$$

$$B_4 = \frac{\mu_0 i_4}{4r} e$$

Since R_1 and R_2 are parallel to each other

$i_2 R_1 = i_4 R_2$; $\quad i_2 = \dfrac{i}{R_1 + R_2} \times R_2$

$i_4 = \dfrac{i}{R_1 + R_2} \times R_1$

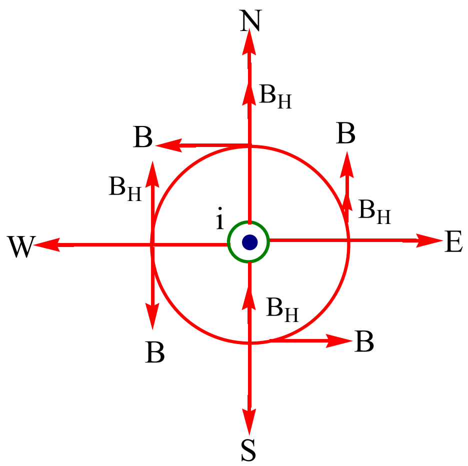
$\vec{B}_{net} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 \quad \vec{B}_{net} = \dfrac{\mu_0}{4r}(i_2 : i_4)$

e) A straight current carrying conductor is held vertically in earth's magnetic field. It carries current in the upward direction, then the direction of magnetic filed (B) due to it

a) due north of the conductor is towards west $B_{net} = \sqrt{B^2 + B_H^2}$.

b) due west of the conductor is towards south $B_{net} = B - B_H$

c) due south of the conductor is towards east $B_{net} = \sqrt{B^2 + B_H^2}$.



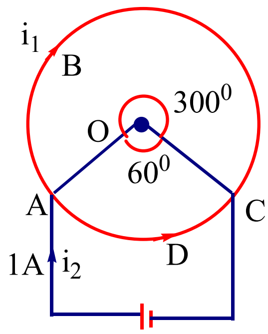
Ex:11 A 2A current is flowing through a circular coil of radius 10cm containing 100 turns. Find the magnetic flux density at the centre of the coil.

Sol. $B = N\dfrac{\mu_0 i}{2r}$

$= 100 \times \dfrac{2\pi \times 10^{-7} \times 2}{10 \times 10^{-2}}$

$= 1.26 \times 10^{-3} Wb/m^2$

Ex:12 **A cell is connected between the points A and C of a circular conductor ABCD of centre O with angle AOC=60°, If B_1 and B_2 are the magnitudes of the magnetic fields at O due to the currents in ABC and ADC respectively, the ratio B_1/B_2 is**

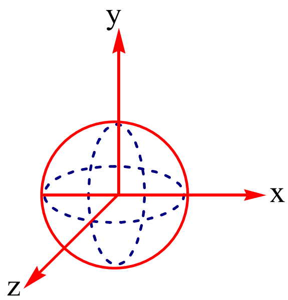


Sol: $B = \frac{\mu_0}{4\pi}\frac{\theta i}{r} \Rightarrow B \propto \theta i \left(but\, \frac{i_1}{i_2} = \frac{l_2}{l_1} = \frac{\theta_2}{\theta_1} \right)$

$$\Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2}.\frac{i_1}{i_2} \quad \Rightarrow \frac{B_1}{B_2} = \frac{\theta_1}{\theta_2} \times \frac{\theta_2}{\theta_1} = 1$$

Ex:13 **Three rings, each having equal radius R, are placed mutuallly perpendicular to each other and each having its centre at the origin of coordinate system. If current I is flowing through each ring then find the magnitude of the magnetic filed at the common centre.**

Sol.



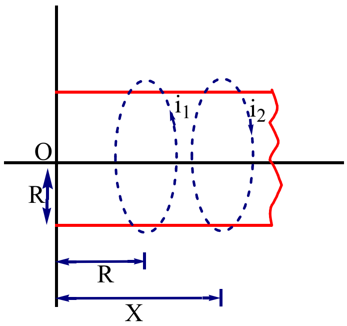
B due to the ring lying in XY-plane is $B_{xy} = \frac{\mu_0 I}{2R}$ along Z-axis.

B due to the ring lying in YZ-plane is $B_{yz} = \frac{\mu_0 I}{2R}$ along X-axis and

B due to the ring lying in XZ-plane is $B_{xz} = \frac{\mu_0 I}{2R}$ along Y-axis.

$$\therefore \vec{B}_{net} = \frac{\mu_0 I}{2R}\left(\hat{i} + \hat{j} + \hat{k}\right) \quad \Rightarrow B_{net} = \sqrt{3}\,\frac{\mu_0 I}{2R}$$

Ex:14 Two wires are wrapped over a wooden cylinder to form two co-oxial loops carrying currents i_1 and i_2. If $i_2 = 8i_1$ then find the value of x for B=0 at the origin O.



Sol. Magnetic induction at 'O' due to 1st loop

$$\vec{B}_1 = \frac{\mu_0 i_1 R^2}{2(R^2+R^2)^{3/2}} \text{ to left}$$

Magnetic induction at 'O' due to 2nd loop.

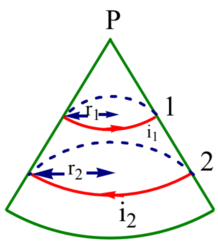
$$\vec{B}_2 = \frac{\mu_0 i_2 R^2}{2(R^2+R^2)^{3/2}} \text{ to right}$$

$$\vec{B}_1 + \vec{B}_2 = 0$$

$$\Rightarrow \frac{i_1}{(2R^2)^{3/2}} - \frac{i_1}{(R^2+x^2)^{3/2}} \text{ and } i_2 = 8i_1$$

$$\Rightarrow x = \sqrt{7}R$$

Ex:15 Two wires wrapped over a conical frame form the loops I and 2. If they produce no net magnetic filed at the apex P, Find the value of i_1/i_2.



Sol. Magnetic induction due to a loop at apex,

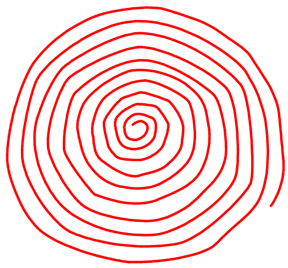
$$B = \frac{\mu_0 i r^2}{2(r^2+x^2)^{3/2}}$$

But $r^2 + x^2 = i^2 \Rightarrow (r^2+x^2)^{3/2} = l^3$ where 'l' is slant length $B = \frac{\mu_0 i}{2r}\left(\frac{r}{l}\right)^3$

But $\frac{r}{l} = \sin\phi$ where ϕ is apex angle, same for both the loops

$\vec{B}_1 + \vec{B}_2 = 0$ (given) $\quad \Rightarrow B_1 = B_2 \Rightarrow \frac{i_1}{i_2} = \frac{r_1}{r_2}$

Ex:16 **A thin insulated wire form a spiral of N=100 turns carrying a current of i=8mA. The inner and outer radii are equal to a=5cm and b=10cm. Find the magnetic field at the centre of the coil.**



Sol. Let n = no. of turns per unit length along the radial of spiral. Consider a ring of radii x and x + dx.

No.of turns in the ring = ndx.

$$n = \frac{N}{(b-a)}$$

Magnetic field at the centre due to the ring is

$$dB = \frac{\mu_0 (ndx) i}{2x}$$

So net field

$$B = \int dB = \int_a^b \frac{\mu_0 nidx}{2x} = \frac{\mu_0 ni}{2}\int_a^b \frac{dx}{x}$$

or $B = \frac{\mu_0 ni}{2} \ln \frac{b}{a}$ or $B = \frac{\mu_0 Ni}{2(b-a)} \ln \frac{b}{a}$

$$= \frac{4\pi \times 10^{-7} \times 100 \times 8 \times 10^{-3}}{2(10-5)\times 10^{-2}} \ln \frac{10}{5}.$$

$$B = 6.9 \times 10^{-6} T$$

Ex:17 **A plastic disc of radius 'R' has a chanrge 'q' uniformly distributed over its surface. If the disc is rotated with a frequency 'f' about its axis, then the magnetic induction at the centre of the disc is given by**

Sol. $dB = \frac{\mu_0 di}{2x}, dq = \frac{q}{\pi R^2}(2\pi x)dx$

$$di = (dq)f = \frac{2qxdx}{R^2} f$$

$$dB = \frac{\mu_0 2qxdx}{2xR^2} \Rightarrow B = \int_0^R \frac{\mu_0 2q.dx}{2R^2}(f)$$

$$B = \frac{\mu_0 qf}{R^2}(R) \Rightarrow B = \frac{\mu_0 qf}{R}$$

Ex:18 **A charge of 1C is placed at one end of a non conducting rod of length 0.6m. The rod is rotated in a vertical plane about a horizontal axis passing through the other end of the rod with angular frequency$10^4\pi$ rad/s. Find the magnetic field at a point on the axis of rotation at a distance of 0.8m from the centre of the path.**

Sol. $B = \dfrac{\mu_0 i r^2}{2(r^2+x^2)^{3/2}}$, $i = \dfrac{q\omega}{2\pi}$

$$B = \frac{\mu_0}{4\pi}\frac{q\omega r^2}{(r^2+x^2)^{3/2}}$$

Ex:19 **Two circular coils made of same material having radii 20 cm & 30 cm have turns 100 & 50 respectively. If they are connected**
a) in series
b) in parallel
c) separately across a source of emf find the ratio of magnetic inductions at the centre of circles in each case

Sol. a) $B = \dfrac{\mu_0 n i}{2r}$

coils are in series $\Rightarrow$ i is same in both

$$B \alpha \frac{n}{r}$$

$$\frac{B_1}{B_2} = \frac{100}{50}\times\frac{30}{20} = 3:1$$

b) coils are parallel $\Rightarrow$potential difference is same $i\alpha\dfrac{1}{R}$

Where $R = \dfrac{\rho(n\pi r)}{A}$

Where A is area of cross section of wire which is same for both

$$\Rightarrow R\alpha nr;\ i\alpha\frac{1}{nr}$$

but $B_0 = \dfrac{\mu_0 n i}{2r} \Rightarrow B\alpha\dfrac{n}{r}\times\dfrac{1}{nr} \Rightarrow B\alpha\dfrac{1}{r^2}$

$$\therefore \frac{B_1}{B_2} = \left(\frac{30}{20}\right)^2 = \frac{9}{4}$$

c) For the coils, potential difference is same

$i\alpha\dfrac{1}{R}$ where $R = \dfrac{\rho(n\pi r)}{A}$; $R\alpha nr$

$I\alpha\dfrac{1}{nr} \Rightarrow B_0\alpha\dfrac{1}{r^2}$ $\qquad \therefore \dfrac{B_1}{B_2} = \dfrac{9}{4}$

Ex:20 **Two circular coils are made from a uniform wire the ratio of radii of circular coils are 2:3 & no.of turns is 3:4. If they are connected in parallel across a battery.**

A : Find ratio of magnetic inductions at their centres

B : Find the ratio magnetic moments of 2 coils.

Sol. When connected in parallel

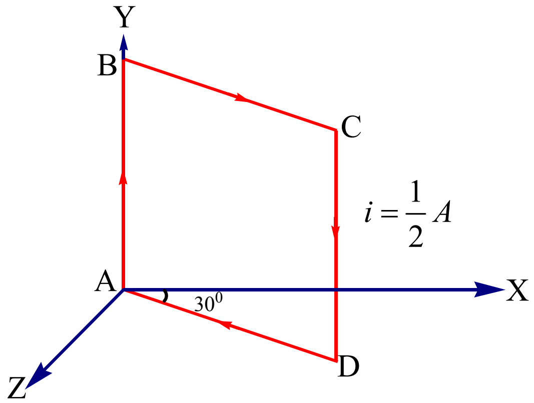
$$a) B \alpha \frac{1}{r^2}; \frac{B_1}{B_2} = \left(\frac{r_2}{r_1}\right)^2 \quad = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$b) M = ni\, A_{coil} \quad \text{but } i = \frac{V}{R} = \frac{V}{\rho l} A_{wire}$$

$$\Rightarrow M = \frac{Va_{wire}}{\rho(2\pi r_{coil})}(\pi r^2{}_{coil});\ M = \frac{VA_{wire}}{\rho X 2} r_{coil}$$

$$\frac{M_1}{M_2} = \frac{r_1}{r_2} = \frac{2}{3}.$$

Ex:21 **Figure shows a square current carrying loop ABCD of side 2m and current $i = \frac{1}{2}A$. The magnetic moment $\vec{M}$ of the loop is**



Sol. $DA = 2\cos 30^0 \hat{i} - 2\sin 30^0 \hat{k} = (-\sqrt{3}\hat{i} - \hat{k})$

$\vec{AB} = 2\hat{j} \therefore \vec{M} = i(\vec{DA} \times \vec{AB})$

$$= \frac{1}{2}\left[(-\sqrt{3}\hat{i} - \hat{k}) \times (2\hat{j})\right]$$

$$= -\sqrt{3}k + \hat{i} = (i - \sqrt{3}\hat{k})A - m^2.$$

Ex:22 If two charged particles each of charge q mass m are connected to the ends of a rigid massless rod and is rotated about an axis passing through the centre and $\perp$ to length. Then find the ratio of magnetic moment to angular momentum.

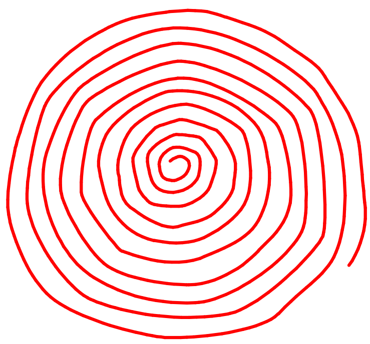
Sol. $M = niA = 2 \times \frac{q}{t} \pi \left(\frac{l}{2}\right)^2$

$= 2 \times \frac{q\omega}{2\pi} \frac{\pi l^2}{4} = \frac{q\omega l^2}{4}$

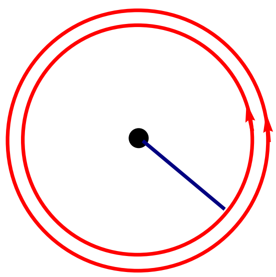
$L = 2(mr^2\omega) = 2\left(m\frac{l^2}{4}\omega\right) = \frac{ml^2\omega}{2};$

$\frac{M}{L} = \frac{q}{2m}.$

Ex:23 Find the magnetic dipole moment of the spiral of total number of turns N, carrying current i having inner and outer radii a and b respectively.



Sol. Let us take a thin coil of thickness dr. Then the number of turns of the coil is



$dN = \frac{N}{b-a}.dr$

the dipole moment of the coil is

$M = (dN)(i)(A) = \left(\frac{Ndr}{b-a}\right)(i)(\pi r^2) = \frac{\pi Ni}{b-a}\int_a^b r^2 dr$

$M = \frac{\pi iN}{3}(a^2 + ab + b^2).$

Ex:24 Consider a non conducting plate of radius a and mass m which has a charge q distributed uniformly over it, The plate is rotated about its own axis with an angular speed ω. Show that the magnetic moment M and the angular momentum L of the plate are related as $\frac{M}{L}=\frac{q}{2m}$.

Sol. If σ is the surface charge density, then $q=\sigma\pi a^2$

Current $i=\sigma\omega r\ \ dr$

The magnetic moment of the element ring

$dM=(idA)=\sigma\omega\, dr(\pi r^2)=\pi\sigma\omega r^3 dr$

and $M=\pi\sigma\omega\int_0^a r^2\, dr=\frac{\pi\sigma\omega^4}{4}$

$=M(\pi a^2\sigma)\frac{\omega a^2}{4}=\frac{q\omega a^2}{4}$

The angular momentum of the disc about its axis

$L=\frac{ma^2}{2}\omega$

The ratio $\frac{M}{L}=\frac{4}{\frac{\omega m a^2}{2}}=\frac{q}{2m}$

Tangent Galvanometer

i) Tangent galvanometer works on the principle of Tangent law i.e., $B=B_H Tan\theta$

Here B = Magnetic induction at the centre of the current carrying coil $=\frac{\mu_0 ni}{2r}$

ii) It is a moving magnet type galvanometer

iii) During experiement, plane of the coil should be along the magnetic meridian [to fulfill the requirement of tangent law]

iii) current measured by Tangent galvanometer is $i=\left(\frac{2rB_H}{\mu_0 n}\right)Tan\theta=KTan\theta$

r = Radius of coil, K = reduction factor

n = number of turns of coil

iv) SI unit of reduction factor is ampere

v) Reading is more accurate when $\theta=45^0$ since relative error $\frac{di}{i}\alpha\frac{1}{\sin 2\theta}$ and it is minimum for 45^0

vi) Sensitivity is maximum when $\theta=0^0$ since $\frac{d\theta,}{di}\alpha\cos 2\theta,$ which is maximum for $\theta=0^0$

vii) Reduction factor K depends on horizontal component of earth's magnetic field.

viii) T.G gives different readings at different places for same current.

ix) T.G cannot be used at magnetic poles, since $B_H=0$ at magnetic poles.

x) T.G is used to measure the current of the order of $10^{-6} A$.

Ex:25 A magnetic needle is arranged at the centre of a current carrying coil having 50 turns with radius of coil 20cm arranged along magnetic meridian. When a current of 0.5mA is allowed to pass through the coil the deflection is observed to be 30^0. Find the horizontal component of earth's magnetic field

Sol. $B = B_H \tan\theta \qquad \frac{\mu_0 ni}{2r\tan\theta} = B_H$

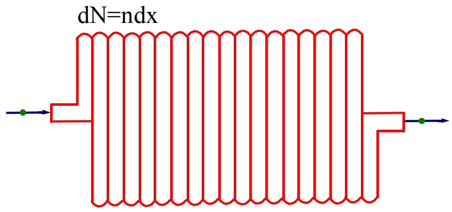
$$B_H = \frac{4\pi\times10^{-7}\times50\times5\times10^{-4}\times\sqrt{3}}{2\left(10^{-1}\right)(1)}$$

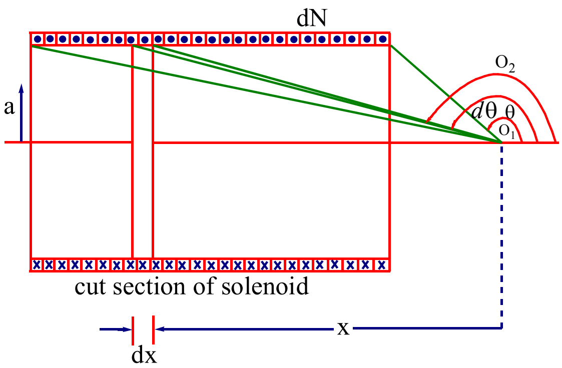
$$= 5\sqrt{3}\pi\times10^{-8}T = 26.35\times10^{-8}T = 2.635\times10^{-7}T$$

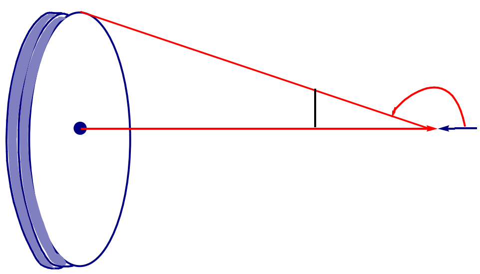
Solenoid And Toroid :

Solenoid

A solenoid is a wire wound in a closely spaced spiral over a hollow cylindrical non-conducting core. The wire is coated with an insulating material so that the adjacent turns physically touch each other, but they are electrically insulated







If n is tha number of turns per unit length, each carrying a current i, uniformly wound round a cylinder of radius a , then the number of turns in length dx is ndx. Thus the magnetic field at the axial point P due to the element

$$dB = \frac{\mu_0 (ndx) i}{2(a^2 + x^2)^{3/2}}$$

The direction of magnetic field is along the axis of the solenoid and the sense of advance of a right handed screw. From geometry, we have

$$x = a\cot(180^0 - \theta) = -a\cot\theta$$

and $dx = a\ \text{cosec}^2\theta d\theta$

$$dB = \frac{\mu_0 ni \sin\theta d\theta}{2}$$

$$B = \frac{\mu_0 ni}{2}\int_{\theta_1}^{\theta_2} \sin\theta d\theta$$

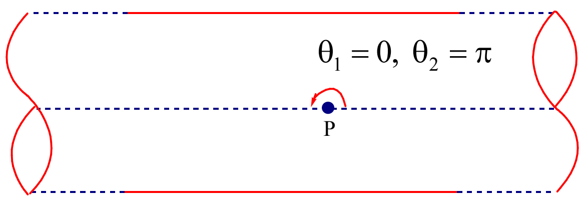
$$B = \frac{\mu_0 ni}{2}[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$B = \frac{\mu_0 ni}{2}[\cos\theta_1 - \cos\theta_2]$$

Special cases:

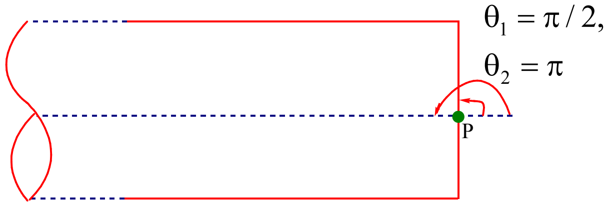
Case 1 : Solenoid is of infinite length and the point chosen is at the middle $\theta_1 = 0, \theta_2 = \pi$

$$\therefore B = \mu_0 ni$$



Case 2 : Solenoid is of inifinite length and the point is at the end fo the solenoid $\theta_1 = \pi/2, \theta_2 = \pi$

$$\therefore B = \frac{\mu_0 ni}{2}$$



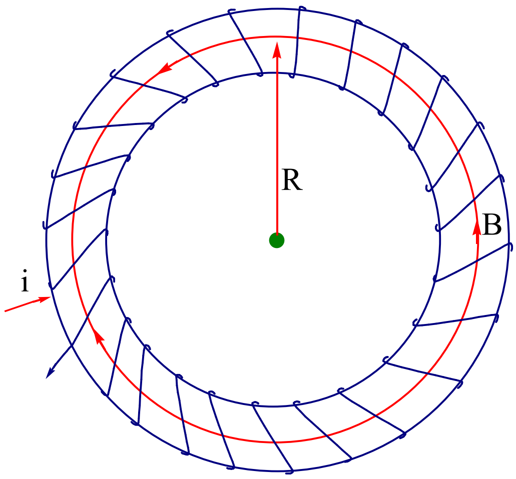
Toroid or Anchor Ring

It is a solenoid of small radius bent round to form a toroid. In an ideal toroid, the field is confined entirely within the core and is uniform. The value of magnetic field at any point on the mean circumferential line is given by

$$B = \mu_0 ni$$

If N is the total turns in the toroid, then

$$n = \frac{N}{2\pi r}$$



$$\therefore B = \mu_0\left(\frac{N}{2\pi R}\right)i \text{ or } B = \frac{\mu_0 Ni}{2\pi R}$$

Ex:26 A solenoid of length 8cm has 100 turns in it. If radius of coil is 3cm and if it is carrying a current of 2A, find the magnetic induction at a point 4cm from the end on the axis of the solenoid.

Sol. $B = \frac{\mu_0 ni}{2}(\sin\alpha + \sin\beta)$

$$= \frac{4\pi \times 10^{-7} \times 100 \times 2}{2} \times 2 \times \frac{4}{5} = 64\pi\mu T$$

Ex:27 A solenoid 60cm long and of radius 4.0cm has 3 layers of windings of 300 turns each. A 2.0cm long wire of mass 2.5g lies inside the solenoid (near its centre) normal to its axis, both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire ? g=9.8 ms^{-2}.

Sol. $mg = Bi_{wire}l$ but $B = \mu_0 ni_{solenoid}$

$$\Rightarrow mg = \mu_0 ni_{solenoid} \times i_{wire} Xl$$

$$i_{solenoid} = \frac{mg}{\mu_0 ni_{wire}l} = 108A.$$

Ex:28 A toroid of non ferromagnetic has core of inner radius 25cm and outer radius 26cm. It has 3500 turns & carries a current of 11A, then find the magnetic field at a point

i) In the internal cavity of toroid

ii) At the midpoint of the windings

iii) At a point which is at a distance of 30cm from the centre of toroid

Sol. i) B = 0.

ii) $B = \frac{\mu_0}{2\pi}\frac{ni}{r} = 2\times10^{-7}\times\frac{3500\times11}{51\times10^{-2}}\times2$

$= \frac{88}{3}\times10^{-3} = 29.3\times10^{-3}T$

iii) B = 0

Based on magnetism for solenoid and toroid.

Ex:29 A solenoid of 2m long & 3cm diameter has 5 layers of winding of 500 turns per metre length in each layer & carries a current of 5A. Find intensity of magnetic field at the centre of the solenoid.

Sol. For long solenoid at the centre

$B = \mu_0 ni$

$H = \frac{B}{\mu_0} = ni = (500\times2)5\times5 = 2.5\times10^4\frac{A}{m}$.

Force Acting On A Charged Particle Moving In A Uniform Magnetic Field:

i) If charge +q is moving with velocity $\vec{v}$, making an angle θ with the direction of field. force acting on the charge is, $\vec{F} = q(\vec{v}\times\vec{B})$

Magnitude of force is F=Bqv $Sin\theta$, direction of $\vec{F}$ is perpendicular to plane containing both $\vec{v}$ and $\vec{B}$.

ii) If $\theta = 0^0$ or 180^0, then the force acting on the particle is zero. And the particle keeps moving in the same path. i.e, **undeviated**.

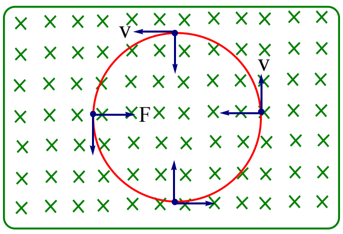
iii) If the charged particle enters normal to the magnetic field, the force acting on it is maximum. ie $F_{max} = Bqv$

iv) This force acts right angles to $\vec{B}$ and $\vec{v}$. It acts as centripetal force and the path of particle will be **circular**.

Then the radius of the circular path is given by

$r = \frac{mv}{Bq} \Rightarrow r = \frac{P}{Bq}$ (from $Bqv = \frac{mv^2}{r}$)

Where p = momentum.



v) $r = \frac{\sqrt{2mK}}{qB}$ where K is kinetic energy of the particle.

vi) If charged particle is accelerated through a potential difference of V volts before it enters into the magnetic field normally then $r = \frac{\sqrt{2mqV}}{qB}$.

vii) Speed, kinetic energy remains constant, but velocity, acceleration, momentum and force are variable since their directions are continuously changing.

viii) The time period of rotation is

$$T = \frac{2\pi r}{v} \qquad \therefore T = \frac{2\pi m}{qB}$$

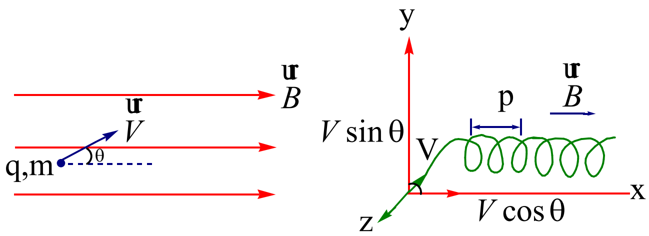
Angular frequency of rotation is $\omega = \frac{Bq}{m}$

$\therefore$ T and ω are independent of v and r of charged particle.

ix) When the particle enters the magnetic field at angle θ with $\vec{B}$,(such that $\theta \neq 0^0, \theta \neq 90^0, \theta \neq 180^0$), then the path followed by the particle will be **helical.**

x) Radius of circular path of the helix is given by

$$r = \frac{mv\sin\theta}{qB}.$$



xi) Time period of rotation is $T = \frac{2\pi m}{qB}$

xii) Distance travelled by the particle along magnetic field in one complete rotation or pitch of helix is given by $P = (v\cos\theta)T$

$$P = \frac{2\pi mv\cos\theta}{qB}$$

xiii) Work done by the magnetic field on the charged particle is zero.

Ex:30 **A magnetic field of $\left(4.0\times10^{-3}\hat{k}\right)T$ exerts a force $\left(4.0\hat{i}+3.0\hat{j}\right)\times10^{-10}N$ on a particle having a charge $10^{-9}C$ and moving in the x-y plane. Find the velocity of the particle.**

Sol. Magnetic force $\vec{F}_m=\left(4.0\hat{i}+3.0\hat{j}\right)\times10^{-10}N$

Let velocity of the particle in x-y plane be.

$\vec{v}=v_x\hat{i}+v_y\hat{j}$ Then from the relation $\vec{F}_m=q\left(\vec{v}\times\vec{B}\right)$

We have $\left(4.0\hat{i}+3.0\hat{j}\right)\times10^{-10}=$

$10^{-9}\left[\left(v_x\hat{i}+v_y\hat{j}\right)\times\left(4\times10^{-3}\hat{k}\right)\right]$

$=\left(4v_y+10^{-12}\hat{i}-4v_x\times10^{-12}\hat{j}\right)$

comparing the coefficient of $\hat{i}$ and $\hat{j}$ we have,

$4\times10^{-10}=4v_y\times10^{-12}$

$\therefore v_y=10^2m/s=100m/s$ and

$3.0\times10^{-10}=-4v_x\times10^{-12}$

$\therefore v_x=-75m/s \quad \therefore \vec{v}=-75\hat{i}+100\hat{j}$

Ex:31 **If a particle of charge $1\mu C$ is projected into a magnetic field $\vec{B}=\left(2\hat{i}+y\hat{j}-z\hat{k}\right)$T with a velocity $\vec{V}=\left(4\hat{i}+2\hat{j}-6\hat{k}\right)ms^{-1}$, then it passes undeviated. If it is now projected with a velocity $\vec{V}=\hat{i}+\hat{j}$, then find the force experienced by it**

Sol. Charged particle moves in a magnetic field undeviated when $\vec{V}$ is parallel or anti parallel to $\vec{B}$

$$\frac{V_x}{B_x}=\frac{V_y}{B_y}=\frac{V_z}{B_z}=k;\frac{4}{2}=\frac{2}{y}=\frac{-6}{-z}$$

$y=1\ z=3$

$\therefore \vec{B}=\left(2\hat{i}+\hat{j}-3\hat{k}\right)\quad \vec{F}=q\left(\vec{V}\times\vec{B}\right)$

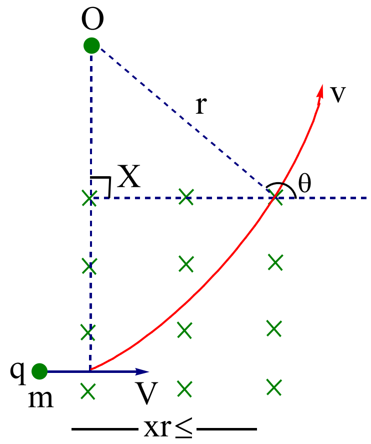
$$\vec{F}=10^{-6}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&0\\2&1&-3\end{vmatrix}$$

$$\vec{F}=10^{-6}\left[i(-3)-\hat{j}(-3)+\hat{k}(-1)\right]$$

$$\left|\vec{F}\right|=10^{-6}\left|\left(-3\hat{i}+3\hat{j}-\hat{k}\right)\right|N=\sqrt{19}\mu N$$

Deviation Of Charged Particle In Uniform Magnetic Field:

Case 1: Suppose a charged particle enters perpendicular to the uniform magnetic field if the magnetic field extends to a distance 'x' which is less than or equal to radius of the path.



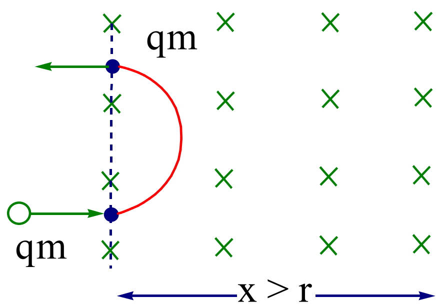
In this case, $r = \frac{mv}{Bq}$

Angle of deviation 'θ' can be determined by using the formula $\boxed{\sin\theta = \frac{x}{r}} = \frac{xqB}{mv}$

$\therefore \theta = \sin^{-1}\left(\frac{xqB}{mv}\right)$

The above relation can be used only when $x \leq r$.

Case 2: For x > r,



In this case, $r = \frac{mv}{Bq}$,

In this case, deviation $\theta = 180^0$.

Note: If particle moves for a time 't' in the field, then in such a case,

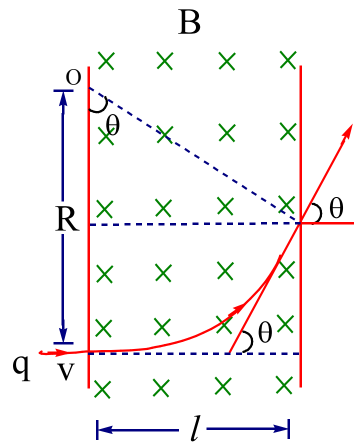
$\theta = \omega t$. $\boxed{\theta = \frac{Bq}{m}t}$

Ex:32 An α - particle is accelerated by a potential difference of 10^4 V. Find the change in its direction of motion, if it enters normally in a region of thickness 0.1 m having transverse magnetic induction of 0.1 tesla. (Given : mass of α -particle 6.4×10^{-27} kg).

Sol. The situation is shown in Fig.

When a charged particle with charge q is accelerated through a potential difference V volt, then

$\frac{1}{2}mv^2 = qV$ .....(i) or $v = \sqrt{\left(\frac{2qV}{m}\right)}$ ....(i)



α - particle in magnetic field moves in a circle of radius R which is given by

$R = \frac{mv}{qB}$ or $R = \frac{1}{B}\sqrt{\left(\frac{2mV}{q}\right)}$ ....(ii)

The change in direction of α -particle (θ) from figure is given by

$$Sin\theta = \frac{l}{R} = lB\sqrt{\left(\frac{q}{2mV}\right)}$$

Here $l = 0.1m, B = 0.1$ tesla, $V = 10^4$ volt

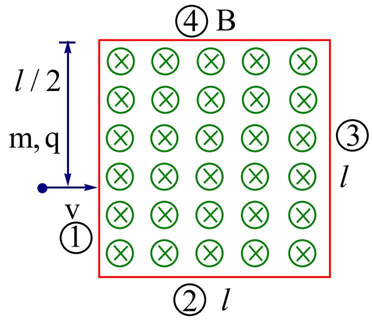
$q = 2e = 2\times1.6X10^{-19} = 3.2\times10^{-19}C$

and $m = 6.4\times10^{-27}kg$

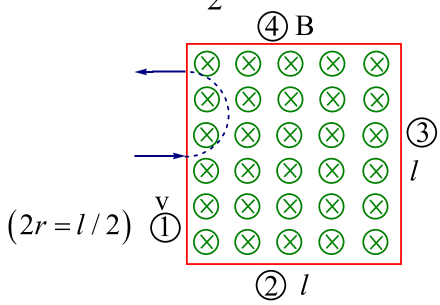
$\therefore \sin\theta = 0.1\times0.1\times\sqrt{\left(\frac{3.2\times10^{-19}}{2\times6.4\times10^{-27}\times10^4}\right)} = \frac{1}{2}$

or $\theta = 30^0$.

Ex:33 The magnetic field (B) is confined in a square region. A positive charged particle of charge q and mass m is projected as shown in fig. Find the limiting velocities of the particle so that it may come out of face (1),(2),(3) and (4).



Sol. For the positive charge coming out from face (1), the radius of the path in magnetic field should be less than or equal to $l/4$. For limiting case $(2r = \frac{l}{2})$.



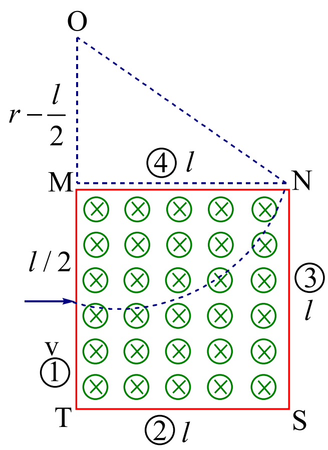
$$r_{max} = \frac{l}{4} = \frac{mv}{qB} \Rightarrow v_{max} = \frac{qBl}{4m}$$

Hence, if the velocity is $< \frac{qBl}{4m}$, the charge particle comes out of face (1).

We can observe from right palm rule that the particle cannot come out from face (1)

For a positive charge coming out of face (4) let particle come out at point N from ΔOMN

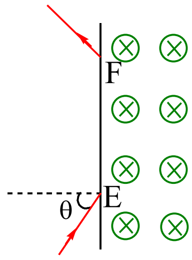
$$(ON)^2 = (OM)^2 + (MN)^2$$



$$r^2 = \left(r - \frac{1}{2}\right)^2 + t^2 \Rightarrow r = \frac{5}{4}l$$

If the particle comes out from face (4), $r < \frac{5}{4}l \Rightarrow \frac{mv}{qB} < \frac{5}{4}l$ (or) $v < \frac{5}{4}\frac{qBl}{m}$. If velocity $v > \frac{5}{4}\frac{qBl}{m}$, the particle will come out from face (3).

Ex:34 A particle of mass m and change +q enters a region of magnetic filed with a velocity v, as shown in fig.



a) Find the angle subtended by the circular arc described by it in the magnetic field.

b) How long does the particle stay inside the magnetic filed ?

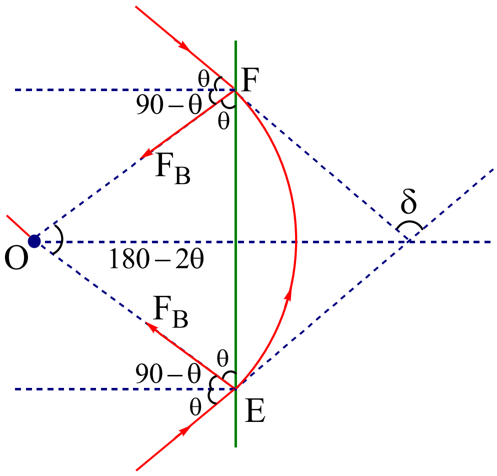
c) If the particle enters at E, what is the intercept EF ?

Sol. a) The particle circulates under the influence of magnetic field. As the magnetic field is uniform, the charge comes out symmetrically. The angle subtended at the centre is $(180-2\theta)$

b) The length of the arc traced by the particle, $l = R(\pi - 2\theta)$

Time spent in the field, $t = \frac{l}{v} = \frac{R(\pi-2\theta)}{v}$ and $R = \frac{mv}{Bq}$

which gives $t = \frac{m}{Bq}(\pi - 2\theta)$



As time period: $T = \frac{2\pi n}{Bq}$, hence

$t = \frac{T}{2\pi}(\pi - 2\theta)$

We can generalize this reslut. If $\varnothing$ is the angle subtended by the arc traced by the charged particle in the magnetic field, the time spent is $t = T\left(\frac{\phi}{2\pi}\right)$

c. Intercept EF = $2R\cos\theta$.

Fleming's Left Hand Rule :

Stretch the fore finger, central finger and thumb of left hand in mutually perpendicular directions, such that if fore finger indicates direction of magnetic field, Central finger indicates direction of current, then thumb indicates direction of force on conductor.

Force On A Current Carrying Conductor Kept In Uniform Magnetic Field.

i) A conductor carrying current i is placed in a uniform magnetic field of induction B at an angle θ with the field direction. The force acting on it is given by

$\overline{F} = i(\overline{l} \times \overline{B})$. $\quad |\overline{F}| = BilSin\theta$

ii) If B and l are parallel or anti-parallel F = 0

iii) If B and l are perpendicular, then $F_{Max} = Bil$.

iv) Direction of force can be found using Fleming's left hand rule.

Lorentz Force :

i) When a charge enters a region where both electric and magnetic fields exists simulataneously, force acting on it is called Lorentz force and is given by $\overline{F} = \overline{F}_e + \overline{F}_m = q\left[\overline{E} + (\overline{V} \times \overline{B})\right]$.

ii) Cyclotron:

a) The cyclotron is a machine to accelerate charged particles or ions to high energies using both electric and magnetic fields in combination.

b) Cyclotron uses the fact that the frequency of revolution of the charged particle in a magnetic field is independent of its energy.

c) Centripetal force is provided by the magnetic force $\frac{mv^2}{r} = Bqv$

d) Radius of circular path is $r = \frac{mv}{Bq}$

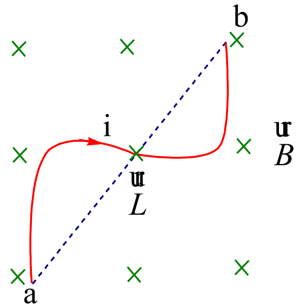
e) Time period of charged particle is $T = \frac{2\pi r}{v}$

$T = \frac{2\pi m}{Bq}$ $f = \frac{1}{T} = \frac{Bq}{2\pi m}$ = cyclotron frequency.

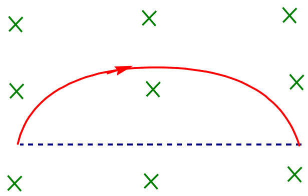
f) K.E of charged particles is $\quad$ K.E=1/2 $mv^2 = \frac{1}{2}m\left(\frac{Bqr}{m}\right)^2 = \frac{B^2q^2r^2}{2m}$

iii) Special Cases :

a). The force acting on a curved wire joining points a and b as shown in the figure is the same as that on a straight wire joining these points. It is given by $\vec{F} = i\vec{L} \times \vec{B}$ $\quad$ where $\overline{L} = ab$

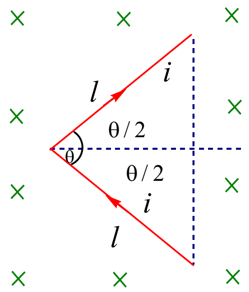


b) The force experienced by a semi circular wire of radius ‘r’ when it is carrying a current ‘i’ and is placed in a uniform external magnetic field of induction B as shown in the figure is given by F=BI(2r).



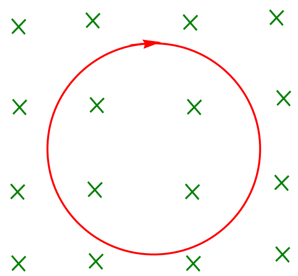
Force directed vertically upwards

c) The force on the wire shown $F = Bil\sin\frac{\theta}{2}$ towards left



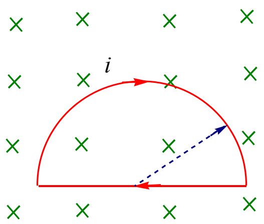
$$= l_{eff} = 2l\sin\frac{\theta}{2}$$

d) The force on a closed loop of any shape carrying current in a uniform magnetic field is always zero.

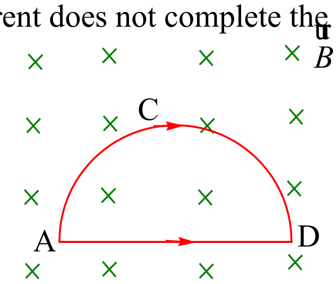


since $l_{eff} = 0$

e) The net force experienced by a closed current loop and current completes the loop in a uniform field is zero.



f) In case of a closed loop but current does not complete the loop the net force is not zero.

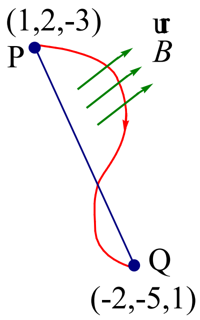


$\vec{F}_{ACD} = \vec{F}_{AD} \qquad \therefore \vec{F}_{loop} = \vec{F}_{ACD} + \vec{F}_{AD} = 2\vec{F}_{AD}$

$\therefore \left|\vec{F}_{loop}\right| = 2\left|\vec{F}_{AD}\right|$

Ex:35 **Find the force experienced by the wire carrying a current 2A if the ends P and Q of the wire have coordinates (1, 2, -3) m and (-2, -5, 1) m respectively when it is placed in a magnetic field $\vec{B} = \left(\hat{i} + \hat{j} + \hat{k}\right)T$**

Sol. The force acting on the wire is

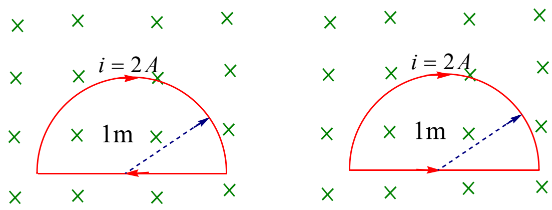


$\vec{F} = i\vec{r}_{21} \times \vec{B} \qquad = i\left(\vec{r}_2 - \vec{r}_1\right) \times \vec{B}$

$= 2\left(-3\hat{i} - 7\hat{j} + 4\hat{k}\right) \times \left(\hat{i} + \hat{j} + \hat{k}\right)$

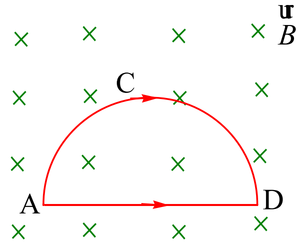
$= 2\left(-11\hat{i} + 7\hat{j} + 4\hat{k}\right) N$

Ex:36 **In Fig. a semicircular wire loop is placed in uniform magnetic field B=1.0 T. The plane of the loop is perpendicular to the magnetic field. Current i=2A flows in the loop in the direction shown. Find the magnitude of the magnetic force in both the cases (a) and (b). The radius of the loop is 1m.**



(a) (b)

Sol. It forms a closed loop and the current completes the loop. Therefore, net force on the loop in uniform field should be zero. From the figure, net force on the loop in uniform field shoud be zero. In case (b) although it forms a closed loop, but current does not complete the loop. Hence, net force is not zero.



$$\vec{F}_{ACD} = \vec{F}_{AD}$$

$$\therefore \vec{F}_{loop} = \vec{F}_{ACD} + \vec{F}_{AD} = 2\vec{F}_{AD}$$

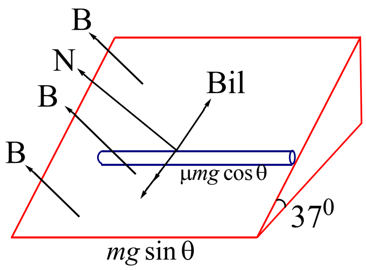
$$\therefore \left|\vec{F}_{loop}\right| = 2\left|\vec{F}_{AD}\right|$$

$$= 2ilB\sin\theta (l = 2r = 2.0m)$$

$$= (2)(2)(2)(1)\sin 90^0 = 8N$$

Ex:37 A rough inclined plane inclined at angle of 37^0 with horizontal has a metallic wire of length 20cm with its length $\perp^r$ to length of inclined plane (μ = 0.1) When a current of its passignthrough the wire and a magnetic field is applied normal to the plane upwards, the wire starts moving up with uniform velocity for B = 0.5T. Then find the mag-nitude of current i, (mass of the wire = 50g)

Sol.



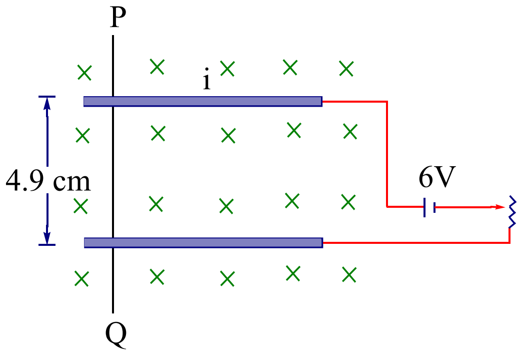
When the wire is in equilibrium

$$Bil = mg\sin\theta + f$$

$$Bil = mg\left(\sin\theta + \mu\cos\theta\right)$$

$$Bil = 5\times10^{-2}\times10\left(\frac{3}{5}+0.1\times\frac{4}{5}\right)i \quad = \frac{10^{-1}\times3.4}{10^{-1}} = 3.4A$$

Ex:38 **A wire PQ of mass 10g at rest on two parallel metal rails. The separation between the rails is 4.9cm. A magnetic field of 0.80 tesla is applied perpendicular to the plane of the rails, directed in wards. The resistance of the circuit is slowly decreased. When the resistance decreases to below 20 ohm, the wire PQ begins to slide on the rails. Calculate the coefficient of friction between the wire and the rails.**



Sol. Wire PQ begins to slide when magnetic force is just equal to the force of friction, i.e.

$$\mu mg = il\,B\sin\theta \quad (\theta = 90^0)$$

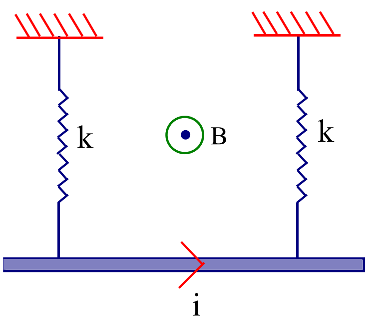
Here, $i = \frac{E}{R} = \frac{6}{20} = 0.3A \qquad \mu = \frac{ilB}{mg}$

$$= \frac{(0.3)(4.9\times10^{-2})(0.8)}{(10\times10^{-3})(9.8)} = 0.12$$

Ex:39 **A current carrying conductor of mass m, length l carrying a current i hangs by two identical springs each of stiffness k. For an outward magnetic field B find the deformation of the springs. Put** $m = 50gm.$

$g = 10m/s^2, l = \frac{1}{2}m, i = 1A$ **and** $B = 1T$ **and**

$k = 50N/m$

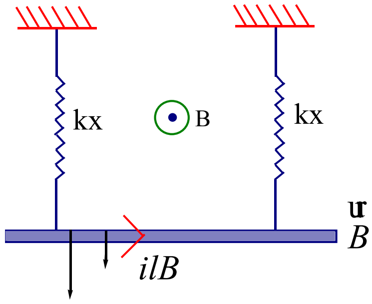


Sol. The forces acting on the rod are 'mg' downwards, $F_{mag} = ilB$ downwards

and $F_{spring} = 2kx$ upwards

Under the action of these forces the rod is in equilibrium. Then, $F_{net} = 0$

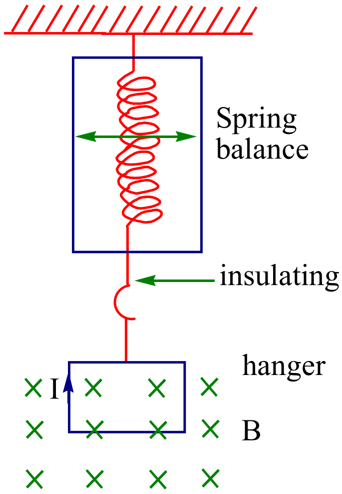
or $mg + ilB = 2kx$



or $x = \frac{mg + ilB}{2k} = \frac{\left(\frac{1}{20}\right)(10) + (1)\left(\frac{1}{2}\right)(1)}{2 \times 50}$

$= \frac{1}{200} m = 0.5 cm$

Ex:40 **A square loop of side a hangs from an insulating hanger of spring balance. The magnetic field of strength B occurs only at the lower edge. It carries a current I. Find the change in the reading of the spring balance if the direction of current is reversed**



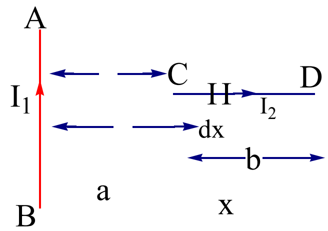
Sol : Initially F_1=mag + IaB (down wards)

when the direction of current is reversed

F_2 = mg - IaB (down wards) $\Rightarrow \Delta F = 2IaB$

Ex:41 **A rod CD of length b carrying a current I_2 is placed in a magnetic field due to a thin long wire AB carrying current I_1 as shown in fig. Then find the net force experienced by the wire**

Sol.



Magnetic induction due to a straight wire at a position of small element dx at a distance x from the conductor AB is $\frac{\mu_0}{2\pi}\frac{I_1}{x}$

Force on the current element is $\vec{dF} = dBI_2 dx \sin 90^0$

$$dF = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{x}dx$$

Net force on conductor is $F = \int_a^{a+b} \frac{\mu_0}{2\pi}\frac{I_1 I_2}{x}dx$

$$= \frac{\mu_0}{2\pi} I_1 I_2 [\log x]_a^{a+b} = \frac{\mu_0}{2\pi} I_1 I_2 \log\left(1+\frac{b}{a}\right)$$

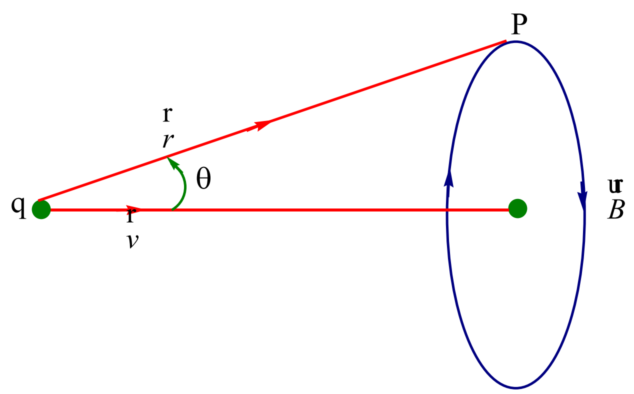
Magnetic field of moving charge

1. We know that a point charge q, at rest in the observer's inertial frame, produces an electric field along the radius vectro and is given by

$$\vec{E} = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{q}{r^3}\vec{r}$$

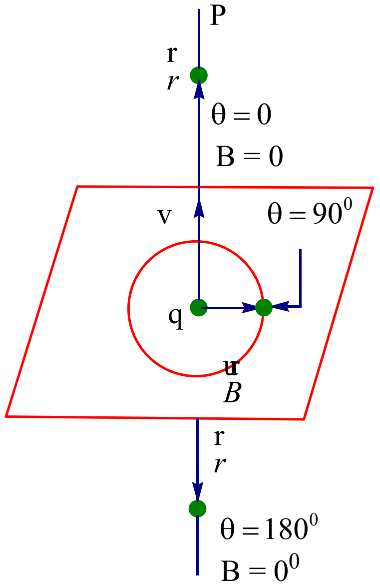
If the charge is moving relative to the observer's inertial frame, it produces a magnetic field in addition to electric field. The magnitude of which is proportional to the speed of the charge relative to the observer provided (v<c). The magnetic field vector $\vec{B}$ at the point P, a distant $\vec{r}$ from the charge q moving with velocity $\vec{v}$ is found to be

$$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q}{r^3}\left(\vec{v}\times\vec{r}\right)....(1)$$



The direction of $\vec{B}$ is thus perpendicular to the plane of $\vec{v}$ and $\vec{r}$. It is in the direction of advance of a right handed screw rotated from $\vec{v}$ to $\vec{r}$. It s magnitude is given by

$$B=\left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}....(2)$$



The following points should be remembered regarding with magnetic field

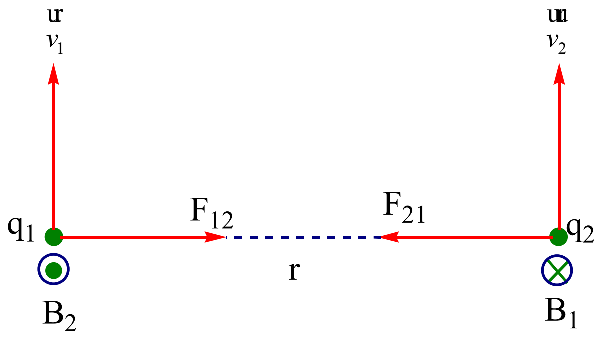
1. The magnetic field $\vec{B}$ is zero at all points on a line on which charge moves. That is when S θ=0 or θ= 180^0, $\vec{B}$=0
2. It is maximum in the plane perpendicular to and through the charge, as sin θ= 1, at all points in this plane.
3. $\vec{B}$ remains unaltered in magnitude at all points on the circumference of circle passing through P and lying in a plane perpendicular $\vec{v}$ with its centre on the velocity direction.

Force between moving charges :

The force acting on a charge q_2, moving with velocity v_2 in a magnetic field produced by charge q_1 moving with a velocity v_1 is

$$\vec{F}_{21}=q_2\left(\vec{v}_2\times\vec{B}_1\right)$$

$$=q_2\left(\vec{v}_2\times\frac{\mu_0 q_1}{4\pi r^3}\left(\vec{v}_1\times\vec{r}\right)\right) \qquad =\frac{\mu_0 q_1 q_2}{4\pi r^3}\left[\vec{v}_2\left(\vec{v}_1\times\vec{r}\right)\right]$$



The magnitude of force which they exert on each other

$$F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2 v_1 v_2}{r^2} \qquad \text{for } v_1 = v_2 = v;\ F_m = \frac{\mu_0}{4\pi}\frac{q_1 q_2}{r^2}v^2 .....(i)$$

In addition to the magnitude force, there is an electric force between them, whose magnitude is given by

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} .....(ii)$$

This force is of repulsive nature, On dividing equation (i) by (ii), we have $\frac{F_m}{F_e} = v^2\mu_0\varepsilon_0$

As $c = \sqrt{\frac{1}{\mu_0\varepsilon_0}}$ $\quad \therefore \frac{F_m}{F_e} = \frac{v^2}{c^2} ......(iii)$

Since $v < c_1$, and so $F_m < F_e$. As $F_m < F_e$, so the net force between the charges is of repulsive nature.

Force Between Two Parallel Current Carrying Long Straight Conductors

i) Force per unit length on each wire is given by $\frac{F}{l} = \frac{\mu_0}{2\pi}\frac{i_1 i_2}{r}$

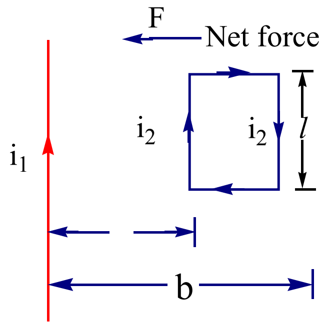
If $i_1 = i_2 = 1 amp, r = 1m,$ then force per unit length of the conducter is $2\times10^{-7} N/m$

ii) If currents in the two wires are in **same direction,** then the force of **attraction** takes place between them.

iii) If currents in the two wires are in **opposite direction,** then the force of **repulsion** takes place between them

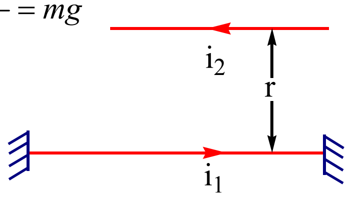
iv) A stright and very long wire carries current i_1 and rectangular loop of wire carrying current i_2 is placed nearby it. The force on the loop is

$$F = \frac{\mu_0 i_1 i_2 l}{2\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$



v) A very long horizontal wire carries a current i_1 is rigidly fixed. Another wire is placed directly above and parallel to it carries a current i_2. r is the perpendicular distance of seperation between the wires and currents are in opposite directions for the second wire remains stationary, the condition is

$$F = mg \Rightarrow \frac{\mu_0 i_1 i_2 l}{2\pi r} = mg$$

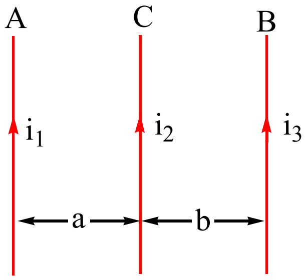


$$\Rightarrow \boxed{\frac{m}{l} = \frac{\mu_0 i_1 i_2}{2\pi r g}}$$

vi) Three long parallel conductors carry currents as shown

a) Resultant force per unit length on the wire **‘C’** is

$$F = \frac{\mu_0}{2\pi}\left[\frac{i_1 i_2}{a} : \frac{i_2 i_3}{b}\right]$$



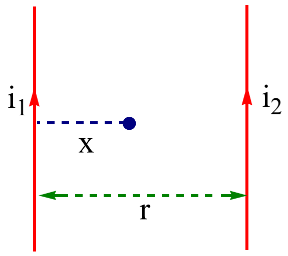
b) If the resultant force on the wire ‘C’ is zero, the condition is $\frac{i_1 i_2}{a} = \frac{i_2 i_3}{b} \Rightarrow \frac{i_1}{a} = \frac{i_3}{b}$

Note: Here the resultant force per unit length on the A and B wire can be also determined in the similar way. The currents can be along different directions.

Null Points Due To Two Current Carrying Parallel Wires.

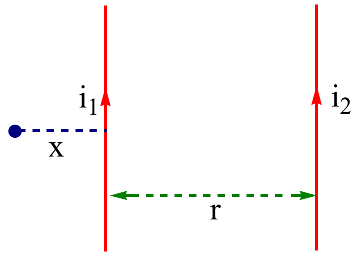
i) Two straight parallel conductors are carrying currents $i_1, i_2 \left(i_1 < i_2\right)$ in the **same direction,** and are seperated by a distance r, the null point is formed in between them. The distance of the null point from the conductor carrying smaller current is

$$x = \frac{r}{\frac{i_2}{i_1} + 1}$$



ii) Two straight parallel conductors are carrying currents $i_1, i_2 (i_1 < i_2)$ in **opposite directions,** and are seperated by a distance r, then the null point is formed **out side** the conductors, the distance of the null point from the conductor carrying smaller current is given by

$$x = \frac{r}{\frac{i_2}{i_1} - 1}$$



Ex:42 **A long straight conductor carrying a current of 2A is in parallel to another conductor of length 5cm. and carrying a current 3A. They are separated by a distance of 10cm. Calculate (a) B due to first conductor at second conductor (b) the force on the short conductor.**

Sol. Given $i_1 = 2A; i_2 = 3A$

$r = 10cm = 10 \times 10^{-2} m; l_2 = 5cm$

a) $B = \frac{\mu_0 i_1}{2\pi r} = 2 \times 10^{-7} \times \frac{2}{10 \times 10^{2}} = 4 \times 10^{-6}$ Tesla

b) $F = \frac{\mu_0 i_1 i_2}{2\pi r} \times 1_2$

$= 2 \times 10^{-7} \times \frac{2 \times 3}{10 \times 10^{-2}} \times 5 \times 10^{-2} \quad = 6 \times 10^{-7} N$

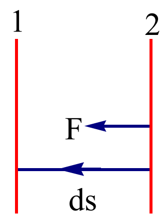
Ex:43 **Two long stright parallel current carrying conductors each of length *l* and current *i* are placed at a distance r_0. Show that the total work done by an external agent in slowly reducing their distance of seperation to $\frac{r_0}{2}$ is $\frac{\mu_0}{2\pi} i^2 l\,n(2)$**

Sol. The force acting on the conductor 2 is F = ilB

$= il \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i^2 l}{2\pi r}$

This force does a work dW in displacing the conductor 2 by a distance dr

$dW = \vec{F}.\vec{dr}$



$= \frac{\mu_0 i^2 l}{2\pi r}(-dr) \quad (\because \theta = 180^0)$

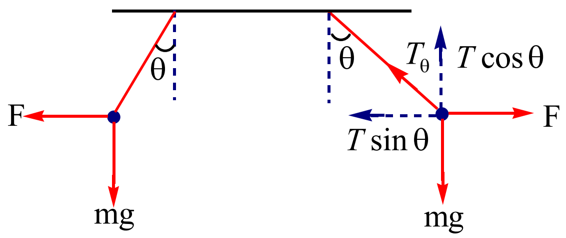
Then, the total work done is

$W = \int dW$

$= -\frac{\mu_0 i^2 l}{2\pi} \int_{r_0}^{\frac{r_0}{2}} \frac{dr}{r} = \frac{\mu_0 i^2 l}{2\pi} \ln 2$

Ex:44 **Two parallel horizontal conductors are suspended by light vertical threads 75.0 cm long. Each conductor has a mass of 40.0gm per metre, and when there is no current they are 0.5 cm apart. Equal magnitude current in the two wires result in a separation of 1.5cm. Find the values and directions of currents**

Sol.



The situation is shown in figure

Here, we have $T\cos\theta = mg$

$T\sin\theta = F = \frac{\mu_0}{4\pi}.1.\frac{2i_1i_2}{d}$ or $T\sin\theta = \frac{\mu_0}{4\pi}.1.\frac{2i^2}{d}$

from the above equations $\tan\theta = \frac{\mu_0}{4\pi}.l.\frac{2i^2}{d}.\frac{1}{mg}$

where θ is small, $\tan\theta \approx \sin\theta$

From figure $\sin\theta = \frac{0.5\times10^{-2}}{75\times10^{-2}}$

$m = 40.0\times10^{-3}1kg$ Where $l =$ length of conductor in meter

Substituting We get $\frac{0.5\times10^{-2}}{75\times10^{-2}} =$

$10^{-7}.I.\frac{2i^2}{\left(1.5\times10^{-2}\right)}\times\frac{1}{\left(40\times10^{-3}\right)1\times9.8}$ Solving, we get $i = 14amp.$

As conductgors are repelled, the currents in them are in opposite directions.

Ex:45 A conductor AB of length 10cm at a distance of 10cm from an infinitely long parallel conductor carrying a current 10A. What work must be done to move AB to a distance of 20cm if it carries 5A?

Sol. Force on a conductor at a distance X is $F = \frac{\mu_0 i_1 i_2 l}{2\pi x}$

Wone doen to displace it through a small distance

$dx = dW = \vec{F}.\vec{dx}$ $dW = \frac{\mu_0 i_1 i_2 l}{2\pi x}dx$

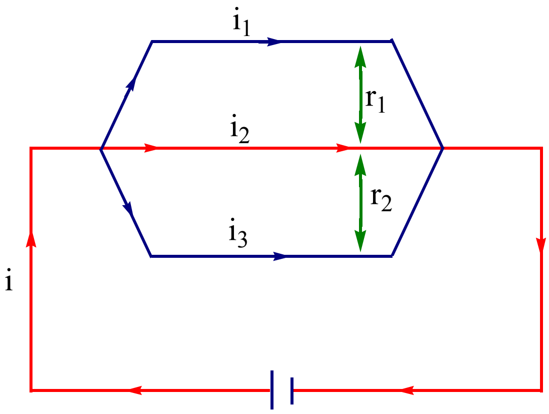
$W = \int_{0.1}^{0.2}\frac{\mu_0 i_1 i_2 l}{2\pi x}dx$ $W = \frac{\mu_0 i_1 i_2 l}{2\pi}[\log_e x]_{0.1}^{0.2}$

$$W = \frac{4\pi\times10^{-7}\times10\times5\times10\times10^{-2}}{2\pi}\log e^2$$

$W = 0.693\times10^{-6}J$

Ex:46 **Three long straight wires are connected parallel to each other across a battery of negligible internal resistance. The ratio of their resistances are 3:4:5. What is the ratio of distances of middle wire from the others if the net force experienced by it is zero**

Sol: The wires are in parallel and ratio of their resistances are 3:4:5, Hence currents in wires are in the ratio $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$



$$i_1 = \frac{k}{3},\ i_2 = \frac{k}{4},\ i_3 = \frac{k}{5}$$

Force between top and middle wire is

$$F_1 = \frac{\mu_0}{4\pi}.\frac{2i_1i_2}{r_1} = \frac{\mu_0}{4\pi} \times \frac{2\left(\frac{1}{3}\right)\left(\frac{1}{4}\right)k^2}{r_1}$$

Force between bottom and middle wire

$$F_2 = \frac{\mu_0}{4\pi} \times \frac{\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)k^2}{r_2}$$

As the forces are equal and opposite so $F_1 = F_2 \Rightarrow \frac{r_1}{r_2} = \frac{5}{3}$

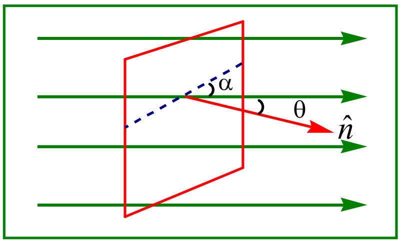
Force Between Two Streams Of Electric Charges:

i) If two streams of electrons or protons are moving with velocity ‘v’ in parallel and same directions, there will be both electric repulsive force and magnetic attractive force. Since electric force predominates the magnetic force, there will be repulsion.

ii) If they move parallel and opposite directions, there will be electric repulsive force and magnetic repulsive force and hence there will be repulsion again.

Torque Acting On A Current Loop Kept In Uniform Magnetic Fied :

i) When a coil carrying current is placed in unifrom magnetic field, the net force on it is zero but it experiences a torque or couple.



ii) Torque acting on a current carrying coil placed in uniform magnetic field is $\vec{\tau} = \vec{M} \times \vec{B}$

iii) Torque acting on the coil is $\tau = BiNA\sin\theta$

$= BiNA\cos\alpha$ Here A = are of coil carrying current i

N = number of turns of the coil

B = Magnetic induction of the field

$\alpha =$ Angle made by the plane of the coil with $\vec{B}$

$\theta =$ Angle made by the normal to the plane of the coil with $\vec{B}$

iv) If the plane of coil is parallel to the direction of magnetic field $\tau = \tau_{max} = BiNA$

v) If the plane of coil is perpendicular to the direction of magnetic field, $\tau = 0$

vi) If current carrying coil is placed in a non-uniform magnetic field it experiences both force and torque.

vii) For a given area, torque is independent of shape of the coil

viii) Torque is directly proportional to area of the coil.

Special Cases

i) When a current carrying coil is placed in uniform magnetic field, net force on it F = 0. But net torque may acts.

ii) When a current carrying coil is placed in non-uniform magnetic field, net force, net torque both acts.

$\tau_{net} \neq 0 \quad F_{net} \neq 0$.

iii) If the angle made by $\vec{M}$ of the coil with $\vec{B}$ in uniform magnetic field is 'θ', then its potential energy

$P.E = -\vec{M}.\vec{B}$

$\boxed{P.E = -MB\cos\theta}$

iv) If a current carrying coil is rotated in a uniform field such that the angle made by $\vec{M}$ with $\vec{B}$ is changes from θ_1 to θ_2.

$W = MB(\cos\theta_1 - \cos\theta_2)$

v) If ext field is along, the direction of $\vec{M}$, then $\theta = 0°$.

$\underline{\tau = 0}$ $\underline{P.E = -MB}$ (min)

This position corresponds to stable equilibrium.

vi) If external magnetic field is opposite to $\vec{M}$ then, $\theta = 180°$

$\underline{\tau = 0}$. $\underline{\text{P.E.} = +\text{MB}}$ (max)

vii) This corresponds to unstable equilibrium.

Ex:47 A circular loop of area $1cm^2$ carrying a current of 10A is placed in a magnetic field of $2T\hat{i}$. The loop is in xy plane with current in clock wise direction. Find the torque on the loop.

Sol. $\vec{\tau} = \vec{M} \times \vec{B} \qquad = \left(ni\vec{A} \times \vec{B}\right)$

$= \left[10 \times 10^{-4}\left(-\hat{k}\right)\right] \times \left(2\hat{j}\right) = 2 \times 10^{-3}\, Nm\left(\hat{i}\right)$

Ex:48 A metallic wire is folded to form a square loop a side 'a'. It carries a current 'i' and is kept perpendicular to a uniform magnetic field. If the shape of the loop is changed from square to a circle without changing the length of the wire and current, the amount of work done in doing so is

Sol. $W = Find\ \ P.E - \text{initial P.E}$

$W = -M_f B - \left(M_i B\right)$

$W = iB\left(A_i - A_f\right)$

$W = iBa^2\left[1 - \dfrac{4}{\pi}\right]$

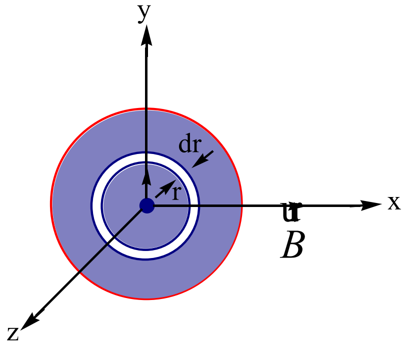
Ex:49 A flat insulating disc of radius 'a' carries an excess charge on its surface is of surface charge density $\sigma C/m^2$. Consider disc to rotate around the axis passing through its centre and perpendicualr to its plane with angular speed ω rad/s. If magnetic field $\vec{B}$ is directed perpendicualr to the rotation axis, then find the torque acting on the disc.

Sol. Suppose the disc is placed in xy-plane and is rotated about the z-axis. Consider an annular ring of radius r and of thickness dr, the charge on this ring.

$dq = \sigma\left(2\pi r dr\right)$

As the ring rotates with angular velocity ω, so the current

$$i = \frac{dq}{dt} = \frac{\sigma\left(2\pi r dr\right)}{\dfrac{2\pi}{\omega}} = \sigma\omega r dr$$



The torque on the current loop $\vec{\tau} = i\vec{A} \times \vec{B}$

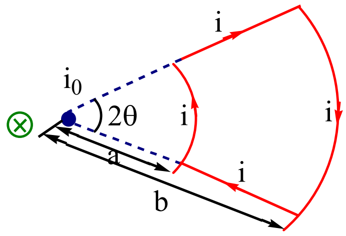
Hence the torque on this annular ring

$d\vec{\tau} = i\left(d\vec{A} \times \vec{B}\right) \qquad = \sigma\omega r dr\left(\pi r^2 B \sin 90^0\right)$

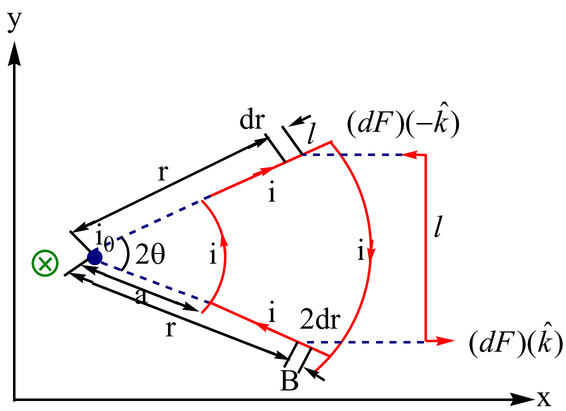
$= \pi\sigma\omega r^3 B dr$

and $\tau = \pi\sigma\omega B\int_0^a r^3 dr = \dfrac{\pi\sigma\omega B a^4}{4}$

Ex:50 **A loop carrying current 'i' is lying inthe plane of the paper. It is the field of a long straight wire with constant current i_0 (inward) as shown in fig. Find the torque acting on the loop.**



Sol. The field due to current carrying wire is tangential to every point on the circular portion of the loop and hence the forces acting on these segments are zero.



Now consider two small elements of length dr at a distance r from the axis symmetrically as shown in fig.

The magnitude of the force experienced by each element is $dF = B\ i\ dr = \left(\dfrac{\mu_0}{2\pi}\dfrac{i_0}{r}\right) i dr$

On element 1 it is into the page and on 2 it out of the page, $d\tau = dF \times 2r\sin\theta$

$= \left(\dfrac{\mu_0 i_0 i}{2\pi r} dr\right) \times 2r\sin\theta$

Now total torque

$\tau = \dfrac{\mu_0 i_0 i \sin\theta}{\pi}\int_a^b dr = \dfrac{\mu_0 i_0 i}{\pi}\sin\theta\left(b-a\right)$

Moving Coil Galvanometer

i) **Principle of moving coil galvanometer:** When a current carrying coil suspended in a unifrom magnetic field, it experiences a torque and hence it rotates.

ii) Poles of magnet are concave is shape, to make the magnetic field radial so that at all orientations the plane of the coil is parallel to the field, and hence torque acting on it is maximum.
This makes the relation between current and deflection linear.

iii) Soft iron cylider is kept at the center of magnetic field to increase the flux.

iv) Phosphor Bronze has
a) high Young's modulus so that the wire will not be stretched easily.
b) low rigidity modulus so that the wire can be twisted easily.
c) small elastic after effect so that it comes back quickly to orginal position after withdrawing current.

v) Small mirror is attached on the phosphor Bronze wire, to measure the deflection using lamp and scale arrangement.

vi) If 'θ' is the deflection for passage of current 'i', then $C\theta = BiAN \Rightarrow i = \left(\frac{C\theta}{BAN}\right)$

where $k = \left(\frac{C}{BNA}\right)$ = Galvanometer constant or figure of merit. It is independent of B_H . Where 'C' is couple per unit twist.

vii) a) **Current sensitivity** of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_I = \frac{d\theta}{di} = \frac{BAN}{C}$$

b) **Voltage sensitivity** of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

$$S_V = \frac{\theta}{V} = \frac{\theta}{iG} \Rightarrow \frac{\theta}{V} = \frac{BAN}{CG}$$

Where G is resistance of galvanometer

i) Increasing B ii) Increasing A
iii) Increasing N iv) Decreasing C

viii) It is used to measure current upto a minimum of 10^{-9} Amp.
a) Plane of coil need not be along the magnetic meridian
b) Galvanometer constant is independent of B_H. So it can be used to measure currents even at poles.
c) External magnetic fields have no effect on deflection. So, it can be used to measure current even in the environment of stray magnetic fields.

The area of the coil in a moving coil galvanometer is 16 cm^2 and has 20 turns. The magnetic induction is 0.2T and the couple per unit twist of the suspended wire is 10^{-6} Nm per degree. If the deflection is 45^0 calculate the current passign through it

Sol. Given, $A = 16cm^2 = 16\times10^{-4}m^2$

$B = 0.2T; N = 20, C = 10^{-6}\,Nm/$ degree; $\theta = 45^0$

From, $C\theta = BiAN$ $\quad i = \frac{C\theta}{BAN} = \frac{10^{-6} \times 45}{0.2 \times 16 \times 10^{-4} \times 20} = 9.94 \times 10^{-1} A.$

Ex:51 A coil area $100 cm^2$ having 500 turns carries a current of 1mA. It is suspended in a uniform magnetic field of induction 10^{-3} Wb/m². Its plane makes an angle of 60^0 with the lines of induction. Find the torque acting on the coil.

Sol. Given $\quad i = 1mA = 10^3 = 10^{-3} A; N = 500; B = 10^{-3} Wb/m^2$

$\theta = 60^0, \tau = ?\ A = 100cm^2 = 100 \times 10^{-4} m^2$

Couple acting on the coil is given by

$\tau = BiAN \sin\phi$

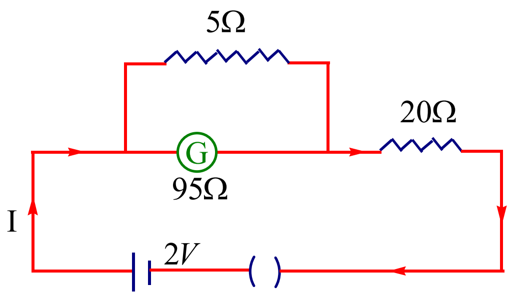
Where ϕ is angle made by normal to the plane of coil with B.

$\phi = 90 - 60 = 30^0$

$\therefore C = 10^{-3} \times 10^{-3} \times 100 \times 10^{-4} \times 500 \times \sin 30$

$= 250 \times 10^{-8} Nm$

Ex:52 A galvanometer of resistance 95Ω, shunted by a resistance of 5 ohm gives a deflection of 50 divisions when joined in series with a resistance of $20k\Omega$ and a 2 volt accumulator. What is the current sensitivity of the galvanometer (in div/ μA)



Sol. In accordance with given problem, the situation is depicted by the circuit diagram in fig. As here $20k\Omega$ is much greater than the resistance of shunted galvanometer $(< 5\Omega)$, the current in the circuit will be

$$I = \frac{2}{20 \times 10^3} = 10^{-4} A = 100\mu A$$

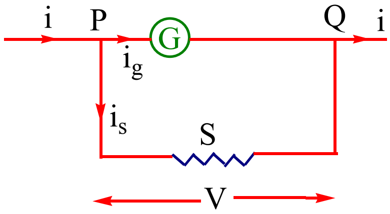
and as this current produces deflection of 50 divisions in the galvanometer

$$CS = \frac{\theta}{I} = \frac{50div}{100\mu A} = \frac{1div}{2\mu A}$$

Shunt

i) A low resistance connected in parallel to galvanometer to protect it from large current is known as shunt.

ii) When shunt is connected range increase but sensitivity decreases.

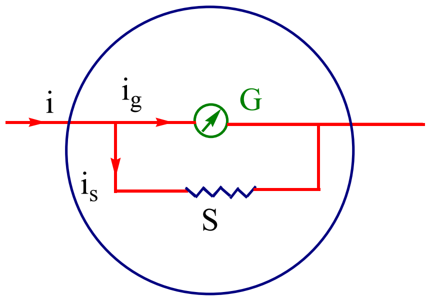


iii) $R_{equivalent} = \frac{GS}{G+S}$

iv) $V = iR_{eq} = i\frac{GS}{G+S}$ v) $V_{PQ} = i_g G = i_s S$

Ammeter

i) Galvanometer can be converted in to Ammeter by connecting low resistance parallel to it.



ii) To increase the range by ‘n’ times or to decrease the sensitivity by ‘n’ times, shunt to be connected across Galvanometer is

$$S = \frac{G}{\left(\frac{i}{i_g} - 1\right)} \Rightarrow S = \frac{G}{n-1}$$

Here $n = \frac{i}{i_g} = \frac{new\ range}{old\ range} = \frac{old\ \ division/amp}{new\ \ division/amp}$

iii) Equivalent resistance of ammeter = $\frac{GS}{G+S}$

iv) The relation between currents is

a) $i = i_q + i_s$

b) $i_g = \frac{iS}{G+S}$ c) $i_s = \frac{iG}{G+S}$

d) $\frac{i_g}{i_s} = \frac{S}{G}$; $\frac{i_g}{i} = \frac{S}{G+S}$; $\frac{I_s}{I} = \frac{G}{G+S}$

v) It is a device used to measure current in electrical circuits.

vi) Resistance of an ammeter is very small and it is zero for an ideal ammeter. Potential drop across ideal ammeter is zero.

vii) Ammeter must always be connected in series to the circuit

viii) Among low range and high range ammeters, low range ammeter has more resistance.

Ex:53: A galvanometer of resitance 20Ω is shunted by a 2Ω resistor. What part of the main current flows through the galvano-meter?

Sol. $\frac{i_g}{i}=\frac{G}{G+S}$. Given $G=20\Omega; S=2\Omega$

$\therefore \frac{i_g}{i}=\frac{2}{22}=\frac{1}{11}; \frac{1}{11}th$ part of current is passing through galvanometer.

Ex:54: A galvanometer has resistance 500 ohm. It is shunted so that its sensitivity decreases by 100 times. Find the shunt resistance.

Sol. Sensitivity $\propto \frac{1}{range}$ $\quad\therefore n=100$

$$S=\frac{G}{(n-1)}=\frac{500}{(100-1)}=\frac{500}{99}\Omega \Rightarrow S=5.05\Omega$$

Ex:55: The resistance of galvanometer is 999Ω. A shunt of 1Ω is connected to it. If the main current is $10^{-2}A$, what is the current flowing through the galvanometer.

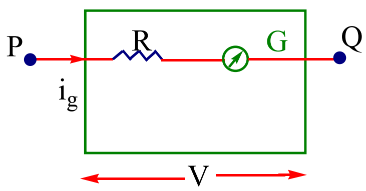
Sol. $G=999\Omega, S=1\Omega\ i=10^{-2}A; i_g=?$

$$i_g=i\left(\frac{S}{G+S}\right)=10^{-2}\times\left(\frac{1}{999+1}\right)=10^{-5}A$$

Ex:56: A galvanometer has a resistance of 98Ω. If 2% of the main current is to be passed through the meter, what should be the value of the shunt?

Sol. $G=98\Omega; \frac{i_g}{i}\times 100=2\%$ $\quad s=\frac{G}{\left(\frac{i}{i_g}-1\right)}; \therefore \frac{i}{i_g}=\frac{100}{2}=50 \quad \therefore S=\frac{98}{(50-1)}=2\Omega$

Voltmeter



i) Galvanometer is converted into voltmeter by connecting high resistance in series to it.
ii) Voltmeter is alwasy connected in parallel to the conductor [P.D. across which is to be measured) in the circuit.
iii) P.D. across the ends of voltmeter is, $V=i_g(G+R)$
iv) Voltmeter is used to measure P.D. across the conductor in electric circuits.
v) Resistance of a voltmeter is very high and that of an ideal volmeter is infinity. Current drawn by an ideal voltmeter is zero.
vi) Among low range and high range voltmeters, high range voltmeter has more resistance.
vii) Equivalent resistance of voltmeter = G+R

viii) Resistance to be connected in series to galvanometer to convert into voltmeter of range $0-V$ volt is $R=\frac{V}{i_g}-G$

ix) To increase the range by n times,

$$n=\frac{new\ range\ V_2}{old\ range\ V_1}=\frac{i_g(G+R)}{i_g(G)}=1+\frac{R}{G}$$

Hence resistance to be conncted in series to galvanometer is $R=G(n-1)$

Ex:57: A maximum current of 0.5mA can be passed through a galvanometer of resistance 20Ω, Calculate the resistance to be connected in series to convert it into a voltmeter of range $(0-5)V$.

Sol. $R=G(n-1), where\ n=\frac{V}{V_g}$

$V=5V; V_g=i_gG=0.5\times10^{-3}\times20=10^{-2}V$

$\therefore n=500$ and $R=20(500-1)=9980\Omega$

Ex:58: A galvanometer has a resistance of 100Ω. A current of 10^{-3} A pass through the galvanometer How can it be converted into (A) ammeter of range 10A and (b) voltmeter of range 10v

Sol. $G=100\Omega; i_g=10^{-3}A$

a) $i=10A;\ n=\frac{i}{i_g}=10^4$

$$S=\frac{G}{(n-1)}=\frac{100}{(10^4-1)}=\frac{100}{999}\Omega$$

b) $V_g=i_gG=10^{-3}\times100=10^{-1}V$

$$V=10V\Rightarrow n=\frac{V}{V_g}=\frac{10}{10^{-1}}=100$$

$\therefore R=G(n-1)=100(100-1)=9900\Omega$

Ex:59: A galvanometer having 30 divisions has current sensitivity of 20 $\mu A/division$. It has a resistance of 25 ohm. How will you convert it into an ammeter measuring voltmeter reading upto 1V?

Sol. The full scale deflection current

$i_g=30\times(20\times10^{-6})=6\times10^{-4}A.$

If S is the required value of the shunt connected in parallel with galvanometer, then

$$i_g=\frac{S}{S+G}i\Rightarrow 6\times10^{-4}=\frac{S}{S+25}\times1$$

After solving, we get $S = \frac{150}{9994}\Omega = 0.0150\Omega$

The resistance of the ammeter

$$R_A = \frac{SG}{S+G} = \frac{0.0150 \times 25}{0.0150 + 25} = 0.0150\Omega$$

To convert this ammeter into the voltmeter, we can use

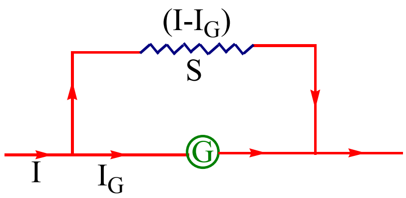
$V = i_g\left(R_A + R_0\right)$ Here $V = 1V, i_g = 1A$

$\therefore 1 = 1\left(0.0150 + R_0\right)$ or $R_0 = 0.985\Omega$

Ex:60: What is the value of shunt which passes 10% of the main current through a galvanometer of 99 ohm?

Sol. As shunt is a small resistance S in parallel with a galvanometer (of resistance G) as shown in fig.

$$\left(I - I_G\right)S = I_G G$$



i.e., $S = \frac{I_G G}{\left(I - I_G\right)}$

And as here, $G = 99\Omega$ and

$$I_G = \left(\frac{10}{100}\right)I = 0.1I$$

$$S = \frac{0.1I \times 99}{\left(I - 0.1I\right)} = \frac{0.1}{0.9} \times 99 = 11\Omega$$

Ex:61 A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.1 T normal to the plane of the coil .If the current in the coil is 5.0 A what is the average force on each electron in the coil due ot the magnetic field (The coil is made of copper wire of cross-sectional area $10^{-5}m^2$ and the free electron density in copper is given to be about $10^{29}m^{-3}$.)

1) $2.5 \times 10^{-25} N$ 2) $7.5 \times 10^{-25} N$

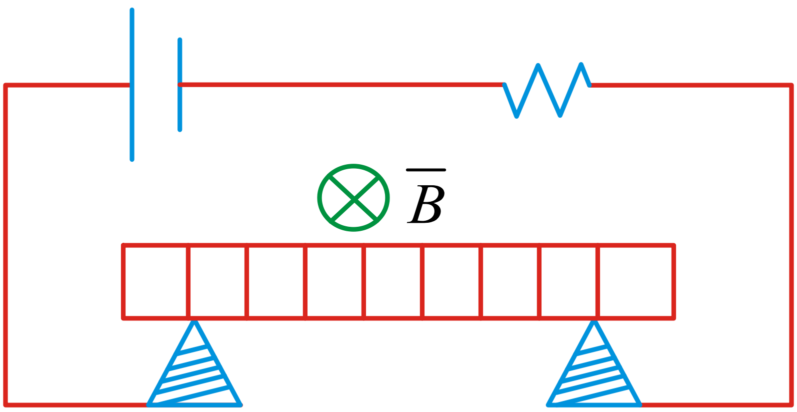
3) $5 \times 10^{-25} N$ 4) $10^{-25} N$

Sol. Key(3) Force acting on each electron, i.e.., Lorentz force, $F_m = ev_d B$

or $F_m = e\left(\frac{1}{Ane}\right)B = \frac{IB}{An}$ $(as I = neAv_d)$ or $F_m = \frac{5 \times 0.10}{10^{-5} \times 10^{29}} N = 5 \times 10^{-25} N$

(as A = cross-sectional area of the wire $= 10^{-5}m^2, n =$ free electron density $= 10^{-29}m^{-3}$)

Ex:62 **A thin 50 cm long metal bar with mass 750 g rests on,but is not attached to , two metallic supports in a uniform 0.45T magnetic field as shown in Fig .A battery and a 25Ω resistor in series are connceted to the supports. The largest voltage the battery can have without breaking the circuit at the supports (units are in"V") is**



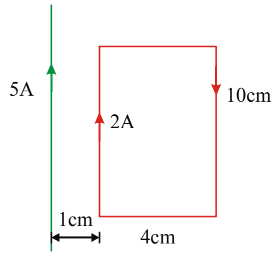
1) 817 2) 718 3) 827 4) 837

Sol Key(1) As F- mg when the bar is just ready to levitate,

$$IIB = mg \text{ or } I = \frac{mg}{lB} = \frac{0.750\times 9.8}{0.5\times 0.45}A = 32.67A$$

$$\varepsilon = IR = (32.67)(25)V = 817V$$

Ex:63 **A rectangular loop of wire of size $4cm\times 10cm$ carries a steady current of 2A. A straight long wire carrying 5A current is kept near the loop (as shown in fig).If the loop and the wire are coplanar, find the net force on the loop**



1) $3.2\times 10^{-5}N$ 2) $1.6\times 10^{-5}N$

3) $0.4\times 10^{-5}N$ 4) $4\times 10^{-5}N$

Sol Key(2). As $\vec{F}_{AB} = -\vec{F}_{DC}, \vec{F}_{AB} + \vec{F}_{DC} = \vec{0}$

$$F_{AD} = k_m\left(\frac{2I_1I_2}{a}\right)(AB)$$

$$= \frac{(10^{-7}N/A)(2\times 5A\times 2A)(10cm)}{(1cm)} = 2\times 10^{-5}N$$

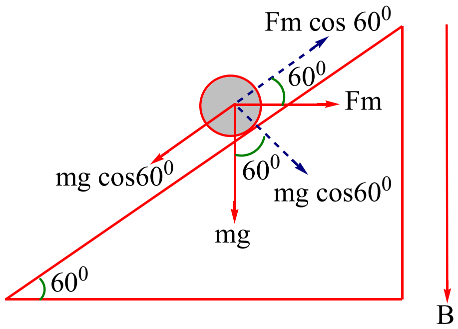
Similarly, $F_{BC} = 0.4\times 10^{-5}N$

Thus . $F_{net} = F_{AD} - F_{BC} = 1.6\times 10^{-5}N$
(towards right)

Ex: 64 **A horizontal rod of mass 10 gm and length 10 cm is placed on a smooth plane inclined at an angle of 60^0 with the horizontal, with the length of the rod parallel to the edge of the inclined plane. A uniform magnetic field of induction B is applied vertically downwards. If the current through the rod is 1.73 ampere, then the value of B for which the rod remains stationary on the inclined plane is**

1) 1.73 Tesla 2) 1/1.73 Tesla
3) 1 Tesla 4) None of the above

Sol Key(3). The given situation can be drawn as follows



$$F = ilB \Rightarrow mg\sin 60^0 = ilB\cos 60^0$$

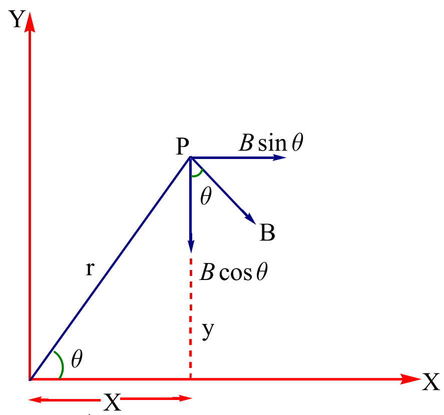
$$\Rightarrow B = \frac{0.01\times 10\times \sqrt{3}}{0.1\times 1.73} = 1T$$

Ex:65. **A long straight wire along the z-axis carries a current I in the negative z direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x,y) in the z=0 plane is**

1) $\frac{\mu_0 I\left(y\hat{i}+x\hat{j}\right)}{2\pi\left(x^2+y^2\right)}$ 2) $\frac{\mu_0 I\left(x\hat{i}+y\hat{j}\right)}{2\pi\left(x^2+y^2\right)}$ 3) $\frac{\mu_0 I\left(x\hat{j}-y\hat{i}\right)}{2\pi\left(x^2+y^2\right)}$ 4) $\frac{\mu_0 I\left(x\hat{i}-y\hat{j}\right)}{2\pi\left(x^2+y^2\right)}$

Sol Key(1) Magnetic field at P is $\vec{B}$, perpendicular to OP in the direction shown in figure

So $\vec{B} = B\sin\theta\hat{i} - B\cos\theta\hat{j}$ Here $B = \frac{\mu_0}{2\pi}\frac{I}{r}$ $\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$



$$\vec{B} = \frac{\mu_0 I}{2\pi}\frac{1}{r^2}\left(y\hat{i} - x\hat{j}\right) = \frac{\mu_0 I\left(y\hat{i} - x\hat{j}\right)}{2\pi\left(x^2+y^2\right)}$$

Moving Charges and Magnetism

(Jee main previous year questions)

Topic 1: Motion of charged particle in Magnetic Field

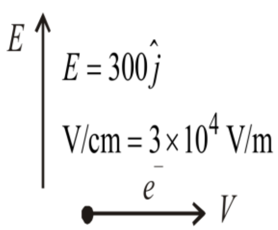
1. **An electron is moving along $+x$ direction with a velocity of $6 \times 10^6 \text{ms}^{-1}$. It enters a region of uniform electric field of 300 V/cm pointing along $+y$ direction. The magnetic and direction of the magnetic field set up in this region such that the electron keeps moving along the x direction will be:**
[Sep. 06, 2020 (I)]

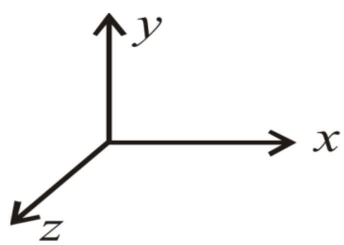
(a) $3 \times 10^{-4}\text{T}$, along $+z$ direction **(b) $5 \times 10^{-3}\text{T}$, along $-z$ direction**

(c) $5 \times 10^{-3}\text{T}$, along $+z$ direction **(d) $3 \times 10^{-4}\text{T}$, along $-z$ direction**

SOL (c) $\vec{E} = 300j \text{ V/cm} = 3 \times 10^4 \text{V/m}$

$\vec{V} = 6 \times 10^6 \hat{\imath}$





$\vec{B}$ must be in $+z$ axis.

$$q\vec{E} + q\vec{V} \times \vec{B} = 0$$

$$E = VB$$

$$B = \frac{E}{V} = \frac{3 \times 10^4}{6 \times 10^6} = 5 \times 10^{-3} T$$

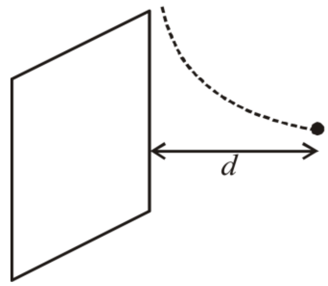
Hence, magnetic field $B = 5 \times 10^{-3}\text{T}$ along $+z$ direction.

2. **A particle of charge q and mass m is moving with a velocity $-v\hat{\imath}$ $(v \neq 0)$ towards a large screen placed in the Y-Z plane at a distance d. If there is a magnetic field $\vec{B} = B_0\hat{k}$, the minimum value of v for which the particle will not hit the screen is:**

[Sep. 06, 2020 (I)]

(a) $\frac{qdB_0}{3m}$ **(b)** $\frac{2qdB_0}{m}$ **(c)** $\frac{qdB_0}{m}$ **(d)** $\frac{qdB_0}{2m}$

SOL. (c) In uniform magnetic field particle moves in a circular path, if the radius of the circular path is 'r', particle will not hit the screen.



$$r = \frac{mv}{qB_0} \qquad \left[\because \frac{mv^2}{r} = qvB_0\right]$$

Hence, minimum value of v for which the particle will not hit the screen.

$$v = \frac{qB_0 d}{m}$$

3. A charged particle carrying charge 1 μC is moving with velocity $(2\hat{i}+3\hat{j}+4\hat{k})$ ms^{-1}. If an external magnetic field of $(5\hat{i}+3\hat{j}-6\hat{k})$ $X10^{-3}$T exists in the region where the particle is moving then the force on the particle is $\overline{F} \times 10^{-9}$ N. The vector $\vec{F}$ is:

[Sep. 03, 2020 (I)]

(a) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$ **(b)** $-30\hat{i}+32\hat{j}-9\hat{k}$

(c) $-300\hat{i}+320\hat{j}-90\hat{k}$ **(d)** $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

SOL. (a) [Given: $q = 1\mu C = 1 \times 10^{-6} C$;

$\vec{V} = (2\hat{\imath} + 3\hat{j} + 4\hat{k})$m/s and $\vec{B} = (5\hat{\imath} + 3\hat{j} - 6\hat{k}) \times 10^{-3}$T]

$$\vec{F} = q(\vec{V} \times \vec{B}) = 10^{-6} \times 10^{-3} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 3 & -6 \end{vmatrix}$$

$$=(-30\hat{i}+32\hat{j}-9\hat{k}) \times 10^{-9}\text{N}$$

$$\vec{F} = (-30\hat{\imath} + 32\hat{j} - 9\hat{k})$$

4. A beam of protons with speed $4\times10^5 ms^{-1}$ enters a uniform magnetic field of $0.3T$ at an angle of 60° to the magnetic field. The pitch of the resulting helical path of protons is close to: (Mass of the proton= 1.67×10^{-27}kg, charge of the proton = 1.69×10^{-19}C)

[Sep. 02, 2020 (I)]

(a) 2 cm **(b) 5 cm** **(c) 12 cm** **(d) 4 cm**

SOL. (d) Pitch $=(v\cos\theta)T$ and $T=\dfrac{2\pi m}{qB}$

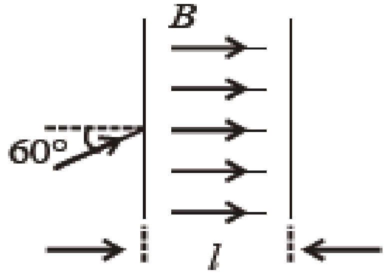
Pitch $=(V\cos\theta)\dfrac{2\pi m}{qB}$

$$=\left(4\times10^5\cos60^\circ\right)\frac{2\pi}{0.3}\left(\frac{1.67\times10^{-27}}{1.69\times10^{-19}}\right)=4\text{ cm}$$

5. The figure shows a region of length 'l' with a uniform magnetic field of $0.3T$ in it and a proton entering the region with velocity $4\times10^5 ms^{-1}$ making an angle 60° with the field. If the proton completes 10 revolutions by the time it cross the region shown, 'l' is close to

(mass of proton = 1.67×10^{-27}kg, charge ofthe proton = 1.6×10^{-19}C)

[Sep. 02, 2020 (II)]



(a) 0.11m **(b) 0.88m** **(c) 0.44m** **(d) 0.22m**

SOL. (c) Time period of one revolution of proton, $T=\frac{2\pi m}{qB}$

Here, m = mass ofproton

q = charge of proton

B = magnetic field.

Linear distance travelled in one revolution,

$p = T(v\cos\theta)$ (Here, v = velocity of proton)

Length of region, $l = 10 \times (v\cos\theta)T$

$$\Rightarrow l = 10 \times v\cos 60^\circ \times \frac{2\pi m}{qB}$$

$$\Rightarrow l = \frac{20\pi m v}{qB} = \frac{20 \times 3.14 \times 1.67 \times 10^{-27} . \times 4 \times 10^5}{1.6 \times 10^{-19} \times 03}$$

$$\Rightarrow l = 0.44\text{m}$$

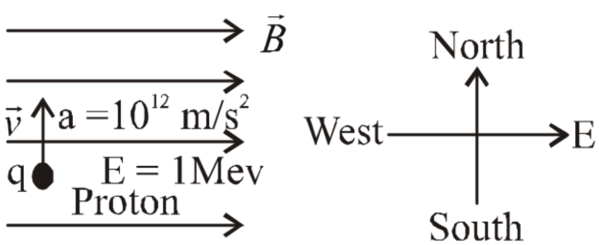
6. Proton with kinetic energy of l MeV moves from south to north. It gets an acceleration of 10^{12}m/s^2 by an applied magnetic field (west to east). The value ofmagnetic field:

(Rest mass of proton is 1.6×10^{-27}kg)

[8 Jan 2020, I]

(a) 0.71mT (b) 7.1mT (c) 0.071mT (d) 71mT

SOL. (a)



As we know, magnetic force $F = qvB = ma, \vec{a} = \left(\frac{qvB}{m}\right)$ perpendicular to velocity.

Also $v = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \times e \times 10^6}{m}}$

$$a = \frac{qvB}{m} = \frac{eB}{m}\sqrt{\frac{2 \times e \times 10^6}{m}}$$

$$10^{12} = \left(\frac{1.\cdot 6 \times 10^{-19}}{167 \times 10^{-27}}\right)^{\frac{3}{2}} . \sqrt{2} \times 10^3 B$$

$B = \frac{1}{\sqrt{2}} \times 10^{-3} T = 0.71\text{mT}$ (approx)

7. **A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of $0.5T$. If an electric field of $100V/m$ makes it to move in a straight path then the mass of the particle is (Given charge of electron $= 1.6 \times 10^{-19}C$)**

[12 April 2019, I]

(a) 9.1×10^{-31} kg **(b) 1.6×10^{-27} kg**

(c) 1.6×10^{-19} kg **(d) 2.0×10^{-24} kg**

SOL. (d) As particle is moving along a circular path $R = \frac{mv}{qB}$ ---(i)

Path is straight line, then

$$qE = qvB$$

$E = vB \Rightarrow v = \frac{E}{B}$ ----(ii)

From equation (i) and(ii)

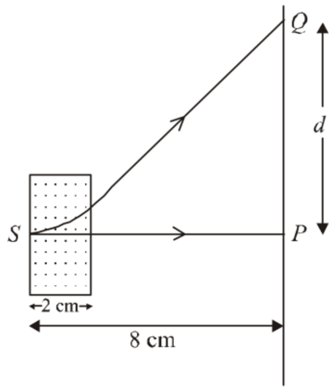
$$m = \frac{qB^2R}{E} = \frac{1.6 \times 10^{-19} \times (0.5)^2 \times 0.5 \times 10^{-2}}{100}$$

$m = 2.0 \times 10^{-24}$ kg

8. **An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}T)_{\hat{k}}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is:**

(Electron's charge $= 1.6 \times 10^{-19}C$, mass of electron $= 9.1 \times 10^{-31}kg$)

[12 April 2019, II]



(a) 11.65 cm **(b) 12.87 cm** **(c) 1.22 cm** **(d) 2.25 cm**

SOL. (b)

9. A proton, an electron, and a Helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane. Let r_p, r_e and r_{He} be their respective radii, then,

[10 April 2019, I]

(a) $r_e > r_p = r_{He}$ **(b) $r_e < r_p = r_{He}$** **(c) $r_e < r_p < r_{He}$** **(d) $r_e > r_p > r_{He}$**

SOL. (b) As $mvr = qvB \Rightarrow r = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$

[As: $\frac{1}{2}mv^2 = K.E.$

$$\Rightarrow m^2v^2 = 2mK.E.$$

$\Rightarrow mv = \sqrt{2mKE}$.]

For proton, electron and α-particle,

$m_{He} = 4m_p$ and $m_p >> m_e$

Also $a_{He} = 2q_p$ and $q_p = q_e$

As KE of all the particles is same then,

$$r\alpha\frac{\sqrt{m}}{q}$$

$$r_{He} = r_p > r_e$$

10. **A proton and an α −particle (with their masses in the ratio of l: 4 and charges in the ratio 1: 2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them**

Will be:

[12 Jan 2019, I]

(a) $1 : \sqrt{2}$ **(b) 1: 2** **(c) 1: 3** **(d) $1 : \sqrt{3}$**

SOL. (a) Radius of the circular path will be $r = \frac{mv}{qB}$

$$\Rightarrow r = \frac{\sqrt{2mKE}}{qB} \qquad \left(p = mv = \sqrt{2mKE}\right)$$

$$KE = q\Delta V$$

$$r = \frac{\sqrt{2mq\Delta V}}{qB} \Rightarrow r \propto \sqrt{\frac{m}{q}}$$

$$\frac{r_p}{r_\alpha} = \frac{1}{\sqrt{2}}$$

11. **In an experiment, electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied.**

[Charge of the electron = 1.6×10^{-19}C, Mass of the electron = 9.1×10^{-31}kg]

[11 Jan 2019, I]

(a) 7.5×10^{-3}m **(b) 7.5×10^{-2}m** **(c) 7.5m** **(d) 7.5×10^{-4}m**

SOL. (d) Radius of the path (r) is given by $r = \frac{mv}{qB}$

$$r = \frac{\sqrt{2mk}}{eB} \qquad \left(p = mv = \sqrt{2mk}\right)$$

$$= \frac{\sqrt{2meV}}{eB} \ (k = eV)$$

$$r = \frac{\sqrt{\frac{2m}{e}V}}{B} = \frac{\sqrt{\frac{2 \times 91 \times 10^{-31}}{16 \times 10^{-19}}(500)}}{100 \times 10^{-3}}$$

$$r = \frac{\sqrt{\frac{91}{016} \times 10^{-10}}}{10^{-1}} = \frac{3}{4} \times 10^{-4}$$

$$= 7.5 \times 10^{-4}$$

12. **The region between $y = 0$ and $y = d$ contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. if $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is:**

[11 Jan 2019, II]

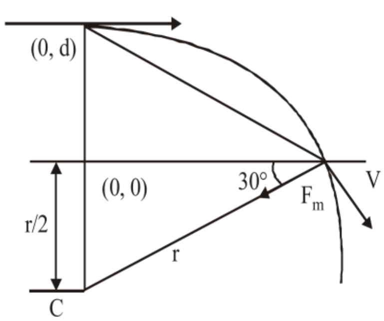
(a) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right)$ **(b)** $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right)$

(c) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$ **(d)** $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$

SOL. (BONUS)

Assuming particle enters from $(0, d)$

$$r = \frac{mv}{qB}, d = \frac{r}{2}$$



$$a = \frac{qVB}{m}\left[\frac{-\sqrt{3}i - j}{2}\right]$$

this option is not given in the all above four choices.

13. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:

[2018]

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$ (c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

SOL. (b) As we know, radius of circular path in magnetic field

$$r = \frac{\sqrt{2Km}}{qB}$$

For electron, $r_e = \frac{\sqrt{2Km_e}}{eB}$ (i)

For proton, $r_p = \frac{\sqrt{2Km_p}}{eB}$ (ii)

For α particle, $r_\alpha = \frac{\sqrt{2Km_a}}{q_\alpha B} = \frac{\sqrt{2K4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB} \ldots(iii)$

$$r_e < r_p = r_\alpha \ (m_e < m_p)$$

14. A negative test charge is moving near a long straight wire carrying a current. The force acting on the test charge is parallel to the direction of the current. The motion of the charge is :

[Online April 9, 2017]

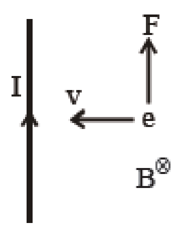
(a) away from the wire (b) towards the wire

(c) parallel to the wire along the current (d) parallel to the wire opposite to the current

SOL. (b) The force is parallel to the direction Of current in magnetic field,

hence $F = q(v \times B)$

According to Fleming's left hand rule,



we have, the direction of motion of charge is towards the wire.

15. In a certain region static electric and magnetic fields exist. The magnetic field is given by $\vec{B} = B_0(i + 2\hat{j} - 4\hat{k})$. If a test charge moving with a velocity $\vec{v} = v_0(3\hat{i} - \hat{j} + 2\hat{k})$ experiences no force in that region, then the electric field in the region, in SI units, is:

[Online April 8, 2017]

(a) $\vec{E} = -v_0B_0(3\hat{i} - 2\hat{j} - 4\hat{k})$ **(b) $\vec{E} = -v_0B_0(\hat{i} + \hat{j} + 7\hat{k})$**

(c) $\vec{E} = v_0B_0(14\hat{j} + 7\hat{k})$ **(d) $\vec{E} = -v_0B_0(14\hat{j} + 7\hat{k})$**

SOL. (d) According to question, as the test charge experiences no net force in that region i.e., sum of electric force $(F_e = q\vec{E})$ and magnetic forces $[F_m = q(\vec{v} \times \vec{B})]$ will be zero. Hence, $F_e + F_m = 0$

$$F_e = -q(\vec{v} \times \vec{B})$$

$$= -B_0v_0\ [(3\hat{i}\text{-}j\text{+}2k)\times(i\text{+}2j\text{-}4k)]$$

$$= -B_0v_0(14\hat{j} + 7\hat{k})$$

16. Consider a thin metallic sheet perpendicular to the plane of the paper moving with speed v′ in a uniform magnetic field B going into the plane ofthe paper (See figure). If charge densities 0_1 and 0_2 are induced on the left and right surfaces, respectively, of the sheet then (ignore fringe effects):

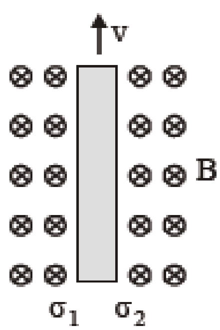
[Online April 10, 2016]

(a) $\sigma_1 = \frac{-\epsilon_0\ vB}{2}, \sigma_2 = \frac{\epsilon_0\ vB}{2}$

(b) $\sigma_1 = \epsilon_0\ vB,\ \sigma_2 = -\epsilon_0\ vB$

(c) $\sigma_1 = \frac{\epsilon_0\ vB}{2}, \sigma_2 = \frac{-\epsilon_0\ vB}{2}$

(d) $\sigma_1 = \sigma_2 = \epsilon_0\ vB$



SOL. (b) $F = qE$ and $F = qvB$

$$E = vB$$

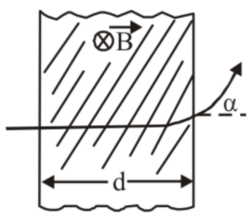
And Gauss's law in Electrostatics $E = \frac{\sigma}{\varepsilon_0}$

$$E = \frac{\sigma}{\varepsilon_0} = vB \Rightarrow \sigma = \varepsilon_0 vB$$

$$\sigma_1 = -\sigma_2$$

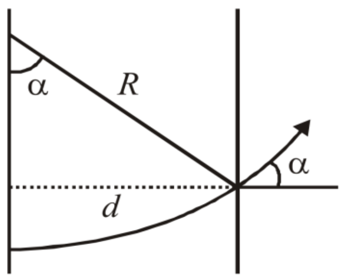
17. A proton (mass m) accelerated by a potential difference V flies through a uniform transverse magnetic field B. The field occupies a region of space by width '*d*'. If α be the angle of deviation of proton from initial direction ofmotion (see figure), the value of *sin* α will be:

[Online Apri110, 2015]



(a) $qV\sqrt{\frac{Bd}{2m}}$ **(b)** $\frac{B}{2}\sqrt{\frac{qd}{mV}}$ **(c)** $\frac{B}{d}\sqrt{\frac{q}{2mV}}$ **(d)** $Bd\sqrt{\frac{q}{2mV}}$

SOL. (d) From figure, $\sin\alpha = d/R$



And we know, $\frac{mv^2}{R} = qvB$

$$\Rightarrow R = \frac{mv}{qB}$$

$$\sin\alpha = \frac{dqB}{mv}$$

$$\sin\alpha = Bd\sqrt{\frac{q}{2mV}}\left[\because qV = \frac{1}{2}mv^2\right]$$

18. A positive charge q′ of mass m' is moving along the + x axis. We wish to apply a uniform magnetic field B for time Δt so that the charge reverses its direction crossing the y axis at a distance d. Then:

[Online Apri112, 2014]

(a) $\mathbf{B=\frac{mv}{qd}}$ **and** $\mathbf{\Delta t=\frac{\pi d}{v}}$ **(b)** $\mathbf{B=\frac{mv}{2qd}}$ **and** $\mathbf{\Delta t=\frac{\pi d}{2v}}$

(c) $\mathbf{B=\frac{2mv}{qd}}$ **and** $\mathbf{\Delta t=\frac{\pi d}{2v}}$ **(*d*)** $\mathbf{B=\frac{2mv}{qd}}$ **and** $\mathbf{\Delta t=\frac{\pi d}{v}}$

SOL. (c) The applied magnetic field provides the required centripetal force to the charge particle, so it can move in circular path of radius $\frac{d}{2}$

$$Bqv=\frac{mv^2}{d/2}$$

$$\text{or, } B=\frac{2mv}{qd}$$

Time interval for which a uniform magnetic field is applied $\Delta t=\frac{\pi\frac{d}{2}}{v}$

(particle reverses its direction after time Δt by covering semi circle).

$$\Delta t=\frac{\pi d}{2v}$$

19. A particle of charge 16×10^{-16}C moving with velocity 10 ms^{-1} along x-axis enters a region where magnetic field of induction $\vec{B}$ is along the y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along x-axis, the

Magnitude of $\vec{B}$ is:

[Online April 23, 2013]

(a) 16×10^3 Wb m^{-2} **(b) 2×10^3 Wb m^{-2}**

(c) 1×10^3 Wb m^{-2} **(d) 4×10^3 Wb m^{-2}**

SOL. (c) Since particle is moving undeflected So, $q_E=qvB$

$$\Rightarrow B=\frac{E}{V}=\frac{10^4}{10}=10^3 wb/m^2$$

20. Proton, deuteron and alpha particle of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relation is correct?

[2012]

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p < r_d$ (c) $r_\alpha > r_d > r_p$ (d) $r_\alpha = r_d > r_p$

SOL. (b) The centripetal force is provided by the magnetic force

$$\frac{mv^2}{R} = qvB \Rightarrow r = \frac{mv}{Bq}\ r \propto \frac{\sqrt{m}}{q}$$

$$r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{m_d}}{q_d} : \frac{\sqrt{m_\alpha}}{q_\alpha}$$

$$= 1 : \sqrt{2} : 1$$

Thus we have, $r_\alpha = r < r$

21. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: A charged particle is moving at right angle to a static magnetic field. During the motion the kinetic energy of the charge remains unchanged.

Statement 2: Static magnetic field exert force on a moving charge in the direction perpendicular to the magnetic field.

[Online May 26, 2012]

(a) Statement 1 is false, Statement2 is true.

(b) Statement 1 is true, Statement2 is true, Statement 2 is not the correct explanation of Statement 1.

(c) Statement 1 is true, Statement 2 is false.

(d) Statement 1 is true, Statement2 is true, Statement 2 is the correct explanation of Statement 1.

SOL. (d) When a charged particle enters the magnetic field in perpendicular direction then it experience a force in perpendicular direction.

i.e. $F = Bqv\ \sin\theta$

Due to which it moves in a circular path.

22. **A proton and a deuteron are both accelerated through the same potential difference and enter in a magnetic field perpendicular to the direction of the field. If the deuteron follows a path of radius *R*, assuming the neutron and proton masses are nearly equal, the radius of the proton's path will be [Online May 19, 2012]**

(a) $\sqrt{2}R$ **(b)** $\frac{R}{\sqrt{2}}$ **(c)** $\frac{R}{2}$ **(d)** R

SOL. (b) As charge on both proton and deuteron is same i.e. 'e'

Energy acquired by both, $E = eV$

For Deuteron.

Kinetic energy, $\frac{1}{2}mV^2 = eV$ [V is the potential difference]

$$v = \sqrt{\frac{2eV}{m_d}}$$

But $m_d = 2m$

Therefore, $v = \sqrt{\frac{2eV}{2m}} = \sqrt{\frac{eV}{m}}$

Radius of path, $R = \frac{mv}{eB}$

Substituting value of 'v' we get

$$R = \frac{2m\sqrt{\frac{ev}{m}}}{eB}$$

$$\frac{R}{2} = \frac{m\sqrt{\frac{ev}{m}}}{eB} \quad \text{(i)}$$

For proton :

$$\frac{1}{2}mV^2 = eV$$

$$V = \sqrt{\frac{2eV}{m}}$$

Radius of path, $R' = \frac{mV}{eB} = \frac{m\sqrt{\frac{2eV}{m}}}{eB}$

$R' = \sqrt{2} \times \frac{R}{2}$ [From eq. (i)]

$$R' = \frac{R}{\sqrt{2}}$$

23. The magnetic force acting on charged particle of charge 2 μC in magnetic field of 2 T acting in y-direction, when the particle velocity is $(2\hat{\imath} + 3\hat{j}) \times 10^6 ms^{-1}$ is

[Online May 12, 2012]

(a) 8 N in z-direction **(b) 8 N in y-direction**

(c) 4 N in y-direction **(d) 4 N in z-direction**

SOL. (a) $\vec{F} = q(\vec{v} \times \vec{B})$

$$= 2 \times 10^{-6} [(2\hat{\imath} + 3j) \times 10^6 \times 2j]$$

$= 2 \times 4\hat{k} = 8N$ in Z-direction.

24. The velocity of certain ions that pass undeflected through crossed electric field $E = 7.7$ kV/m and magnetic field $B = 0.14$T is

[Online May 7, 2012]

(a) 18 km/s **(b) 77 km/s** **(c) 55 km/s** **(d) 1078 km/s**

SOL. (c) As velocity $v = \frac{E}{B} = \frac{7.7 \times 10^3}{0.14} = 55 \text{km/s}$

25. An electric charge $+q$ moves with velocity $\vec{v} = 3\hat{\imath} + 4\hat{j} + \hat{k}$ in an electromagnetic field given by −E=3î+j+2k and $\vec{B} = \hat{\imath}+\hat{j}-3\hat{k}$. The y- component of the force experienced by $+q$ is:

[2011 RS]

(a) $11q$ **(b) $5q$** **(c) $3q$** **(d) $2q$**

SOL. (a) The charge experiences both electric and magnetic force.

Electric force, $F_e = qE$

Magnetic force, $F_m = q(_{\vec{v}\times\vec{B}})$

Net force, $\overline{F} = q[\overline{E} + \vec{v} \times \overline{B}]$

$$= q\left[3\hat{\imath} + \hat{\jmath} + 2\hat{k} + \begin{vmatrix} \hat{\imath}\ \hat{\jmath} & \hat{k} \\ 3 & 41 \\ 1 & 1-3 \end{vmatrix}\right]$$

$$= q[3\hat{\imath} + \hat{\jmath} + 2\hat{k} + \hat{\imath}(-12-1) - \hat{\jmath}(-9-1) + k(3-4)]$$

$$= q[3\hat{\imath}+j+2k\text{-}l3i+l0j\text{-}k]$$

$$= q[-10\hat{\imath} + 11\hat{\jmath} + \hat{k}]$$

$$F_y = 11\text{q j}$$

Thus, the y component of the force.

26. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\vec{E}$ and $\vec{B}$ with a velocity $\vec{v}$ perpendicular to both $\vec{E}$ and $\vec{B}$, and comes out without any change in magnitude or direction of $\vec{v}$. Then

[2007]

(a) $\vec{v} = \overline{B} \times \overline{E}/E^2$ **(b) $\vec{v} = \overline{E} \times \overline{B}/B^2$**

(c) $\vec{v} = \overline{B} \times \overline{E}/ B^2$ **(d) $\vec{v} = \overline{E} \times \overline{B}/E^2$**

SOL. (b) As velocity is not changing, charge particle must go undeflected, then

$$qE = qvB$$

$$\Rightarrow v = \frac{E}{B}$$

Also,

$$\left|\frac{\vec{E} \times \vec{B}}{B^2}\right| = \frac{EB \sin\theta}{B^2}$$

$$= \frac{EB \sin 90^\circ}{B^2} = \frac{E}{B} = |\vec{v}| = v$$

27. A charged particle moves through a magnetic field perpendicular to its direction. Then

[2007]

(a) kinetic energy changes but the momentum is constant

(b) the momentum changes but the kinetic energy is constant

(c) both momentum and kinetic energy of the particle are not constant

(d) both momentum and kinetic energy of the particle are constant

SOL. (b) When a charged particle enters a magnetic field at a direction perpendicular to the direction of motion, the path of the motion is circular. In circular motion the direction of velocity changes at every point (the magnitude remains constant).

Therefore, the tangential momentum will change at every point. But kinetic energy will remain constant as it is given by $\frac{1}{2}mv^2$ and v^2 is the square of the magnitude of velocity which does not change.

28. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a

[2006]

(a) helix (b) straight line (c) ellipse (d) circle

SOL. (b) The charged particle will move along the lines of electric field (and magnetic field). Magnetic field will exert no force. The force by electric field will be along the lines of uniform electric field. Hence the particle will move in a straight line.

29. A charged particle of mass m and charge q travels on a circular path ofradius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is [2005]

(a) $\frac{2\pi q^2 B}{m}$ **(b)** $\frac{2\pi mq}{B}$ **(c)** $\frac{2\pi m}{qB}$ **(d)** $\frac{2\pi qB}{m}$

SOL. (c) Equating magnetic force to centripetal force, $\frac{mv^2}{r} = qvB \sin 90^\circ$

$$\Rightarrow \frac{mv}{r} = Bq \Rightarrow v = \frac{qBr}{m}$$

Time to complete one revolution,

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

30. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity then

[2005]

(a) its velocity will increase

(b) Its velocity will decrease

(c) it will turn towards left of direction of motion

(d) it will turn towards right of direction of motion

SOL. (b) Due to electric field, it experiences force and accelerates i.e. its velocity decreases.

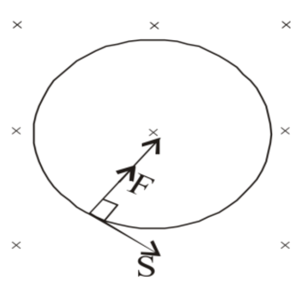
31. A particle of mass M and charge Q moving with velocity $\vec{v}$ describe a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is

[2003]

(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ **(b) zero** **(c)** $BQ2\pi R$ **(d)** $BQv2\pi R$

SOL. (b) The work done, $dW = Fds\cos\theta$

The angle between force and displacement is $90°$. Therefore work done is zero.



32. If an electron and a proton having same momenta enter perpendicular to a magnetic field, then [2002]

(a) curved path of electron and proton will be same (ignoring the sense of revolution)

(b) they will move undeflected

(c) curved path of electron is more curved than that of the proton

(d) path of proton is more curved.

SOL. (a) When a moving charged particle is subjected to a perpendicular magnetic field, then it describes a circular path of radius.

$$r=\frac{p}{qB}$$

where $q=$ Charge of the particle

$p=$ Momentum ofthe particle

$B=$ Magnetic field

Here p, q and B are constant for electron and proton, therefore the radius will be same.

33. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its

[2002]

(a) speed **(b) mass** **(c) charge** **(d) magnetic induction**

SOL. (a) The time period of a charged particle of charge q and mass m moving in a magnetic field(B) is

$$T=\frac{2\pi m}{qB}$$

Clearly time period is independent of speed of the particle.

TOPIC-2 Magnetic Field Lines, Biot-Savart's law and Ampere′s Circuital law

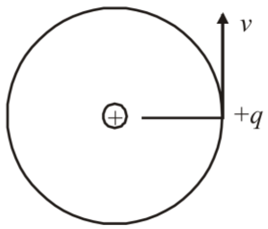
34. A charged particle going around in a circle can be considered to be a current loop. A particle of mass m carrying charge q is moving in a plane with speed v under the influence of magnetic field B. The magnetic moment of this moving particle:

[Sep. 06, 2020 (II)]

(a) $\frac{mv^2\vec{B}}{2B^2}$ **(b)** $-\frac{mv^2\vec{B}}{2\pi B^2}$ **(c)** $-\frac{mv^2\vec{B}}{B^2}$ **(d)** $-\frac{mv^2\vec{B}}{2B^2}$

SOL.

(d)



Length of the circular path, $l = 2\pi r$

Current, $i = \frac{q}{T} = \frac{qv}{2\pi r}$

Magnetic moment M =Current × Area

$$= i \times \pi r^2 = \frac{qv}{2\pi r} \times \pi r^2$$

$$M = \frac{1}{2} q \cdot v \cdot r$$

Radius of circular path in magnetic field, $r = \frac{mv}{qB}$

$$M = \frac{1}{2} qv \times \frac{mv}{qB} \Rightarrow M = \frac{mv^2}{2B}$$

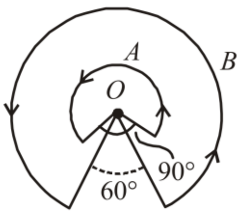
Direction of $\vec{M}$ is opposite of $\vec{B}$ therefore

$$\vec{M} = \frac{-mv^2\vec{B}}{2B^2}$$

(By multiplying both numerator and denominator by B).

35. A wire *A*, bent in the shape ofan arc ofa circle, carrying a current of 2 A and having radius 2 cm and another wire *B*, also bent in the shape of arc of a circle, carrying a current of 3 A and having radius of 4 cm, are placed as shown in the figure. The ratio of the magnetic fields due to the wires *A* and *B* at the common centre *O* is:

[Sep. 04, 2020 (I)]



(a) 4: 6 **(b) 6: 4** **(c) 2: 5** **(d) 6: 5**

SOL. (d) Given: $I_A = 2\text{A}, R_A = 2$ cm, $\theta_A = 2\pi - \frac{\pi}{2} = \frac{3\pi}{2}$

$I_B = 3\text{A}, R_B = 4\text{cm}$, $\theta_B = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

Using, magnetic field, $B = \frac{\mu_0 I\theta}{4\pi R}$

$$\frac{B_A}{B_B} = \frac{I_A}{I_B} \times \frac{\theta_A R_B}{\theta_B R_A} = \frac{2 \times \frac{3\pi}{2} \times 4}{3 \times \frac{5\pi}{3} \times 2} = \frac{6}{5}$$

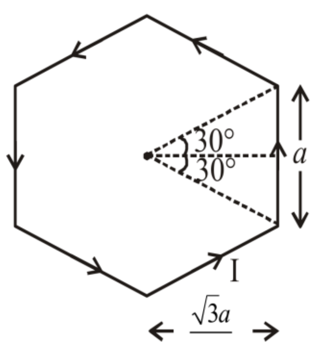
36. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current *I* (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is:

[Sep. 03, 2020 (I)]

(a) $250\sqrt{3}$ **(b) $50\sqrt{3}$** **(c) $500\sqrt{3}$** **(d) $5\sqrt{3}$**

SOL.

(c)



Magnetic field due to one side of hexagon

$$B = \frac{\mu_0 I}{4\pi \frac{\sqrt{3}a}{2}}(\sin 30^\circ + \sin 30^\circ)$$

$$\Rightarrow B = \frac{\mu_0 I}{2\sqrt{3}a}\left(\frac{1}{2}+\frac{1}{2}\right) = \frac{\mu_0 I}{2\sqrt{3}a\pi}$$

Now, magnetic field due to one hexagon coil

$$B = 6 \times \frac{\mu_0 I}{2\sqrt{3}a\pi}$$

Again magnetic field at the centre of hexagonal shape coil of 50 turns,

$$B = 50 \times 6 \times \frac{\mu_0 I}{2\sqrt{3}a\pi}\left[\because a = \frac{10}{100} = 0.1\text{m}\right]$$

or, $B = \dfrac{150\mu_0 I}{\sqrt{3}\times 0.1\times\pi} = 500\sqrt{3}\dfrac{\mu_0 I}{\pi}$

37. A long, straight wire of radius a carries a current distributed uniformly over its cross-section. The ratio of the magnetic fields due to the wire at distance a/3 and $2a$, respectively from the axis of the wire is:

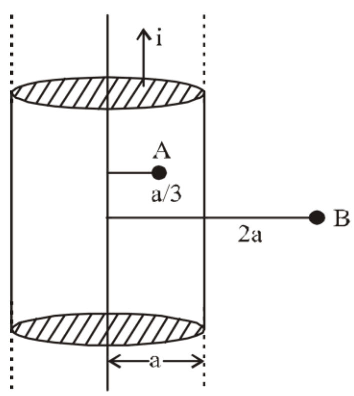
[9 Jan 2020, I]

(a) 2/3 (b) 2 (c) 1/2 (d) 3/2

SOL. (a) Let a be the radius of the wire

Magnetic field at point A (inside)

$$B_A = \frac{\mu_0 i r}{2\pi a^2} = \frac{\mu_0 i \frac{a}{3}}{2\pi a^2} = \frac{\mu_0 i a}{\pi a^2 6} = \frac{\mu_0 i}{6\pi a}$$



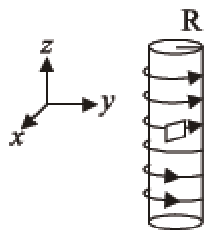
Magnetic field at point B (outside)

$$B_B = \frac{\mu_0 i}{2\pi(2a)}$$

$$\frac{B_A}{B_B}=\frac{\frac{\mu_0 i}{6\pi a}}{\frac{\mu_0 i}{2\pi(2a)}}=\frac{4}{6}=\frac{2}{3}$$

38. An electron gun is placed inside a long solenoid of radius R on its axis. The solenoid has n turns/length and carries a current I. The electron gun shoots an electron along the radius of the solenoid with speed v. If the electron does not hit the surface of the solenoid, maximum possible value of v is (all symbols have their standard meaning):

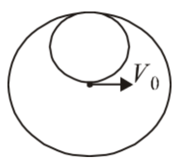
[9 Jan 2020, II]



(a) $\frac{e\mu_0 nIR}{m}$ **(b)** $\frac{e\mu_0 nIR}{2m}$ **(c)** $\frac{e\mu_0 nIR}{4m}$ **(d)** $\frac{2e\mu_0 nIR}{m}$

SOL. (b) Magnetic field inside the solenoid is given by $B=\mu_0 nI$ -----(i)

Here, $n=$ number ofturns per unit length



The path of charge particle is circular. The maximum possible radius of electron $=\frac{R}{2}$

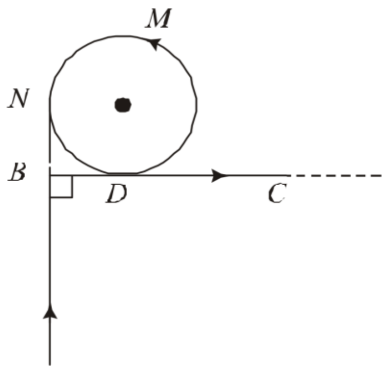
$$\frac{mV_{\max}}{qB}=\frac{R}{2}$$

$\Rightarrow V_{\max}=\frac{qBR}{2m}=\frac{eR\mu_0 nI}{2m}$ (using (i))

39. A very long wire *ABDMNDC* is shown in figure carrying current *I*. *AB* and *BC* parts are straight, long and at right angle. At *D* wire forms a circular turn *DMND* of radius *R*.

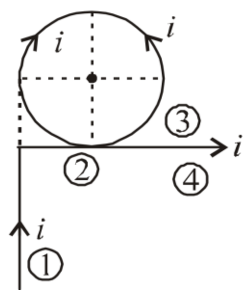
***AB*, *BC* parts are tangential to circular turn at *N* and *D*. Magnetic field at the centre of circle is:**

[8 Jan 2020, II]



(a) $\frac{\mu_0 I}{2\pi R}\left(\pi+\frac{1}{\sqrt{2}}\right)$ **(b)** $\frac{\mu_0 I}{2\pi R}\left(\pi-\frac{1}{\sqrt{2}}\right)$ **(c)** $\frac{\mu_0 I}{2\pi R}(\pi+1)$ **(d)** $\frac{\mu_0 I}{2R}$

SOL. (a)

(a) 

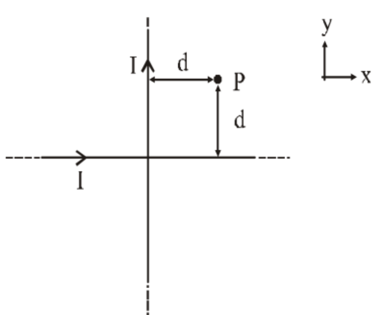
$$B_0 = B_1 + B_2 + B_3 + B_4$$

$$= \frac{\mu_0 l}{4\pi R}[\sin 90^\circ - \sin 45^\circ] + \frac{\mu_0 I}{2R} + \frac{\mu_0 I}{4\pi R}[\sin 45^\circ + \sin 90^\circ]$$

$$= -\frac{\mu_0 \mathrm{I}}{4\pi \mathrm{R}}\left(1-\frac{1}{\sqrt{2}}\right) + \frac{\mu_0 \mathrm{I}}{2\mathrm{R}} + \frac{\mu_0 \mathrm{I}}{4\pi \mathrm{R}}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$\overline{\mathrm{B}_0^{\mathrm{o}}} = \frac{\mu_0 \mathrm{I}}{2\pi \mathrm{R}}\left(\pi + \frac{1}{\sqrt{2}}\right)^{\mathrm{o}}$$

40. Two very long, straight, and insulated wires are kept at 90° angle from each other in *xy*-plane as shown in the figure.



These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be:

[12 April 2019, I]

(a) Zero **(b)** $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$ **(c)** $\frac{+\mu_0 I}{\pi d}(\hat{z})$ **(d)** $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

SOL. (a) $B = B_1 + B_2$

$$= \frac{\mu_0}{2\pi}.\left(\frac{i^{\circ}}{d}.\hat{k} + \frac{i^{\circ}}{d}(-\hat{k})\right) = 0$$

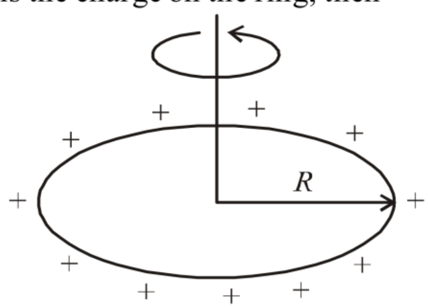
41. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9}T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$N/A^2) .

[12 April 2019, I]

(a) $2\times10^{-6}C$ **(b)** $3\times10^{-5}C$ **(c)** $4\times10^{-5}C$ **(d)** $7\times10^{-6}C$

SOL.

(b) If q is the charge on the ring, then



$$i = \frac{q}{T} = \frac{qw}{2\pi}$$

Magnetic field,

$$B = \frac{\mu_0 i}{2R}$$

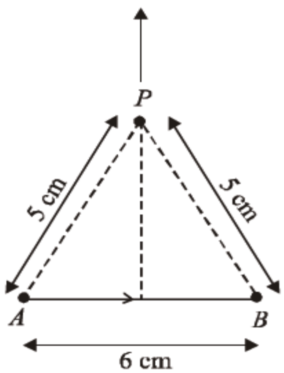
$$= \frac{\mu_0\left(\frac{qw}{2\pi}\right)}{2R}$$

or $3.8\times10^{-9}=\left(\frac{\mu_0}{4\pi}\right)\frac{q\text{w}}{R}=\left(10^{-7}\right)\frac{q\times40\pi}{0.10}$

$$q = 3\times10^{-5}\ \text{C}.$$

42. **Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure) ($\mu_o = 4\pi\times10^{-7}\text{N}-\text{A}^{-2}$)**

[12 April 2019, II]



(a) 2.0×10^{-5}T **(b) 1.5×10^{-5}T** **(c) 3.0×10^{-5}T** **(d) 2.5×10^{-5}T**

SOL. (b) $B=\frac{\mu_0}{4\pi}\frac{i}{r}(\sin\alpha+\sin\beta)$

Here $r=\sqrt{5^2-3^2}=4$ cm

$\alpha=\beta=37°$

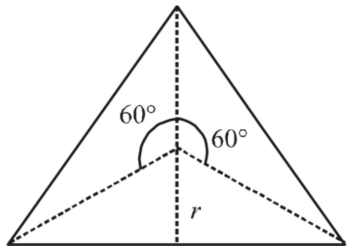
$B=10^{-7}\times\frac{5}{4}2\sin37°=1.5\times10^{-5}\text{T}$

43. **The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is: [Take $\mu_o=4\pi\times10^{-7}\text{NA}^{-2}$]**

[10 April 2019, II]

(a) 18 μT **(b) 9 μT** **(c) 3 μT** **(d) 1 μT**

SOL.

(a) 

$$r = \left(\frac{1}{3}\right)(a\sin 60)$$

$$r = \frac{a}{3} \times \frac{\sqrt{3}}{2} = \left(\frac{a}{2\sqrt{3}}\right)$$

$$B_0 = 3\left[\frac{\mu_0 l}{4\pi r}(\sin 60^\circ + \sin 60^\circ)\right]$$

$$= \frac{3\mu_0 l}{4\pi\left(\frac{a}{2\sqrt{3}}\right)} \times (2)\left(\frac{\sqrt{3}}{2}\right) = \frac{9}{2}\left(\frac{\mu_0 l}{\pi a}\right)$$

$$= \frac{9 \times 2 \times 10^{-7} \times 10}{1} = 18\mu\text{T}$$

44. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be:

[10 April 2019, II]

(a) $\frac{m}{\pi}$ **(b)** $\frac{3m}{\pi}$ **(c)** $\frac{2m}{\pi}$ **(d)** $\frac{4m}{\pi}$

SOL. (d) Let a be the area of the square and r be the radius of circular loop.

$$2\pi r = 4a \Rightarrow r = \left(\frac{2a}{\pi}\right)$$

For square

$$\text{M} = (\text{I})\text{a}^2$$

For circular loop

$$M_1 = I\pi r^2$$

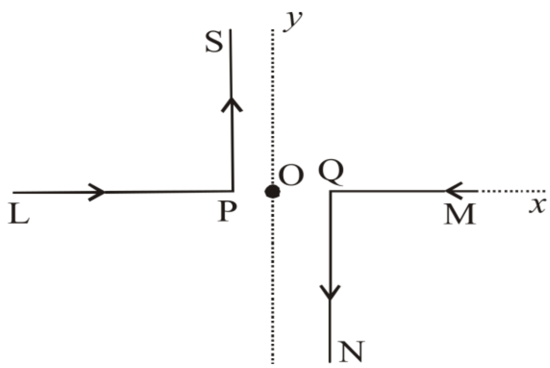
$$M_1 = (I)(\pi)\left(\frac{4a^2}{\pi^2}\right)$$

$$M_1 = \left(\frac{4Ia^2}{\pi}\right)$$

$$M_1 = \frac{4M}{\pi} \quad (\because M = Ia^2)$$

45. As shown in the figure, two infinitely long, identical wires are bent by 90^0 and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4 cm, and the magnitude of the magnetic field at O is 10^{-4}T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7}$NA^{-2}):

[12 Jan 2019, I]



(a) 20 A, perpendicular out of the page **(b) 40 A, perpendicular out of the page**

(c) 20 A, perpendicular into the page **(d) 40 A, perpendicular into the page**

SOL. (c) Let I be the current in each wire. (directed inwards)

Magnetic field at O' due to LP and QM will be zero.

i. e., $B_0 = B_{PS} + B_{QN}$

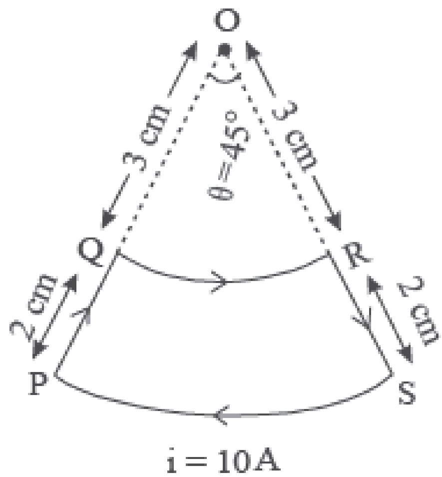
Net magnetic field $B_0 = \frac{\mu_0 i}{4\pi d} + \frac{\mu_0 i}{4\pi d}$

or $10^{-4} = \frac{\mu_0 i}{2\pi d} + \frac{2\times10^{-7}\times i}{4\times10^{-2}}$

i = 20 A and the direction of magnetic field is perpendicular into the plane

46. **A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to:**

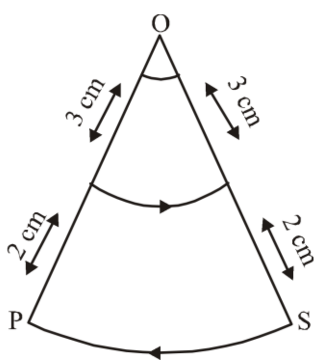
[9 Jan. 2019 I]



(a) 1.0×10^{-7}T **(b) 1.5×10^{-7}T** **(c) 1.5×10^{-5}T** **(d) 1.0×10^{-5}T**

SOL

(d)



There will be no magnetic field at O due to wire PQ and RS

Magnetic field at O' due to arc QR

$$= \frac{\mu)\left(\frac{\pi}{4}\right).I}{4\pi r_1}$$

Magnetic field at O' due to are PS

$$= \frac{\mu)\left(\frac{\pi}{4}\right).I}{4\pi r_2}$$

Net magnetic field at O'

$$B = \frac{\mu_0}{4\pi}(\pi/4) \times 10\left[\frac{1}{(3 \times 10^{-2})} - \frac{1}{(5 \times 10^{-2})}\right]$$

$$\Rightarrow |\vec{B}| = \frac{\pi}{3} \times 10^{-5}T \approx 1 \times 10^{-5}T$$

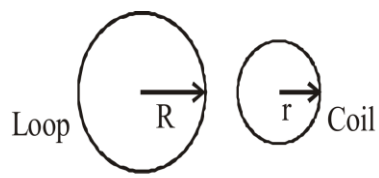
47. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_1) to that at the centre of the coil (Bc), *i.e.*, $\frac{B_L}{B_C}$ will be:

[9 Jan 2019, II]

(a) N **(b) $\frac{1}{N}$** **(c) N^2** **(d) $\frac{1}{N^2}$**

SOL.

(d)



$$L = 2\pi R \qquad L = N \times 2\pi r$$

$$R = Nr \Rightarrow r = \frac{R}{N}$$

$$B_{Loop} = \frac{\mu_0 i}{2R} B_{coi1} = \frac{\mu_0 Ni}{2r} = \frac{\mu_0 Ni}{2\left(\frac{R}{N}\right)} = \frac{\mu_0 N^2 i}{2R}$$

$$\frac{B_L}{B_c} = \frac{1}{N^2}$$

48. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre ofthe loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is:

[2018]

(a) 2 **(b) $\sqrt{3}$** **(c) $\sqrt{2}$** **(d) $\frac{1}{\sqrt{2}}$**

48. (c) Magnetic field at the centre of loop, $B_1 = \frac{\mu_0 I}{2R}$

Dipole moment of circular loop is $m = IA$

$m_1 = I.A = I.\pi R^2 \{R = \text{Radius of the loop}\}$

If moment is doubled (keeping current constant) R becomes $\sqrt{2}R$

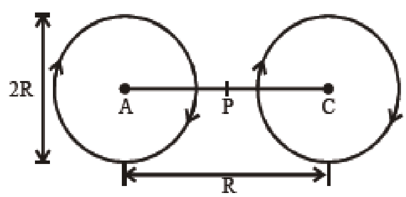
$$m_2 = I.\pi\left(\sqrt{2}R\right)^2 = 2.I\pi R^2 = 2m_1$$

$$B_2 = \frac{\mu_0 I}{2\left(\sqrt{2}R\right)}$$

$$\frac{B_1}{B_2} = \frac{\frac{\mu_0 I}{2R}}{\frac{\mu_0 I}{2\left(\sqrt{2}R\right)}} = \sqrt{2}$$

49. A Helmholtz coil has pair of loops, each with *N* turns and radius *R*. They are placed coaxially at distance *R* and the same current *I* flows through the loops in the same direction. The magnitude of magnetic field at *P*, midway between the centres *A* and *C*, is given by (Refer to figure):

[Online Apri115, 2018]



(a) $\frac{4N\mu_0 I}{5^{3/2}R}$ **(b)** $\frac{8N\mu_0 I}{5^{3/2}R}$ **(c)** $\frac{4N\mu_0 I}{5^{1/2}R}$ **(d)** $\frac{8N\mu_0 I}{5^{1/2}R}$

SOL. (b) Point *P* is situated at the mid-point of the line joining the centres of the circular wires which have same radii (R). The magnetic fields $(\vec{B})$ at P due to the currents in the wires are in same direction.

Magnitude of magnetic field at point, *P*

$$B = 2\left[\frac{\mu_0 NIR^2}{2\left(R^2 + \frac{R^2}{4}\right)^{3/2}}\right] = \frac{\mu_0 NIR^2}{\frac{5^{3/2}}{8}} = \frac{8\mu_0 NI}{5^{3/2}R}$$

50. A current of lA is flowing on the sides of an equilateral triangle of side 4.5×10^{-2}m. The magnetic field at the centre of the triangle will be:

[Online Apri115, 2018]

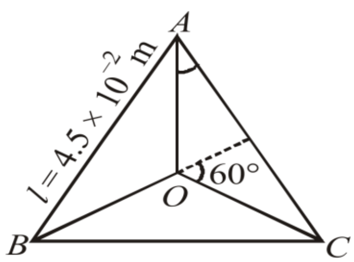
(a) $4 \times 10^{-5} \mathrm{Wb/m^2}$ (b) Zero (c) $2 \times 10^{-5} \mathrm{Wb/m^2}$ (d) $8 \times 10^{-5} \mathrm{Wb/m^2}$

SOL. (a) Here, side of the triangle, $l = 4.5 \times 10^{-2}$m, current, $\mathrm{I} = 1\mathrm{A}$

magnetic field at the centre of the triangle ' O' $B = ?$

From figure, $\tan 60° = \sqrt{3} = \frac{1}{2d}$

$$\Rightarrow d = \frac{l}{2\sqrt{3}} = \left(\frac{4.5 \times 10^{-2}}{2\sqrt{3}}\right) m$$



Magnetic field, $B = \frac{\mu_0 i}{4\pi d}(\cos\theta_1 + \cos\theta_2)$

Putting value of $\mu = 4\pi \times 10^{-7}$ and θ_1 and θ_2

we will get $B = 4 \times 10^{-5} \mathrm{Wb/m^2}$

51. Two identical wires A and B, each of length l', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a$'$. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is:

[2016]

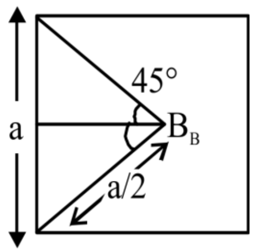
(a) $\frac{\pi^2}{16}$ **(b)** $\frac{\pi^2}{8\sqrt{2}}$ **(c)** $\frac{\pi^2}{8}$ **(d)** $\frac{\pi^2}{16\sqrt{2}}$

SOL. (b) Case (a) :

$$B_A = \frac{\mu_0}{4\pi}\frac{I}{R} \times 2\pi = \frac{\mu_0}{4\pi}\frac{I}{\ell/2\pi} \times 2\pi (\because 2\pi R = \ell)$$

$$= \frac{\mu_0}{4\pi}\frac{I}{l} \times (2\pi)^2$$

Case (b) :



$$B_B = 4 \times \frac{\mu_0 I}{4\pi a/2}[\sin 45° + \sin 45°]$$

$$= 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\ell/8} \times \frac{2}{\sqrt{2}} = \frac{\mu_0}{4\pi}\frac{I}{\ell} \times \frac{64}{\sqrt{2}} = \frac{\mu_0 I}{4\pi\ell} 32\sqrt{2}[4a = 1]$$

$$\Rightarrow \frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

52. **Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle $\theta^{\uparrow}$ with the vertical. If wires have mass λ per unit**

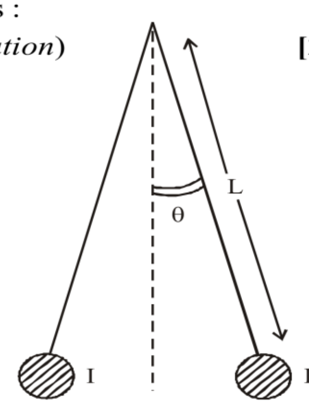
length then the value of I is :
($g = gravitational\ acceleration$) **[2015]**

(a) $2\sqrt{\frac{\pi g L}{\mu_0}\tan\theta}$

(b) $\sqrt{\frac{\pi \lambda g L}{\mu_0}\tan\theta}$

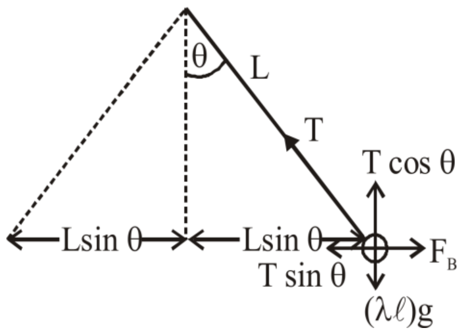
(c) $\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$

(d) $2\sin\theta\sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$



SOL. (d) Let us consider 'ℓ'' length of current carrying wire.

At equilibrium $T\cos\theta = \lambda g \ell$



and $T \sin\theta = \frac{\mu_0}{2\pi}\frac{I'Il}{2L\sin\theta}$ $\left[\because \frac{F_B}{\ell} = \frac{\mu_0 2I \times I}{4\pi 2\ell \sin\theta}\right]$

Therefore, $I = 2\sin\theta\sqrt{\frac{\pi 1gL}{\mu_0 \cos\theta}}$

53. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1, at its centre when a current I passes through it. The ratio $B_1 : B_2$ is:

[Online Apri112, 2014]

(a) 1 : 1 **(b) 1 : 3** **(c) 1 : 9** **(d) 9 : 1**

SOL. (b) For loop $B = \frac{\mu_0 nI}{2a}$

where, a is the radius of loop.

Then, $B_1 = \frac{\mu_0 I}{2a}$

Now, for coil $B = \frac{\mu_0 I}{4\pi}.\frac{2nA}{x^3}$

at the centre x = radius ofloop

$$B_2 = \frac{\mu_0}{4\pi}.\frac{2 \times 3 \times (I/3) \times \pi(a/3)^2}{(a/3)^3} = \frac{\mu_0.3I}{2a}$$

$$\frac{B_1}{B_2} = \frac{\mu_0 I/2a}{\mu_0.3I/2a}$$

$$B_1 : B_2 = 1 : 3$$

54. **A parallel plate capacitor of area 60 cm^2 and separation 3 mm is charged initially to 90 μC. If the medium between the plate gets slightly conducting and the plate loses the charge initially at the rate of 2.5×10^{-8}C/s, then what is the magnetic field between the plates?**

[Online April 23, 2013]

(a) 2.5×10^{-8}T (b) 2.0×10^{-7}T (c) 1.63×10^{-11}T (d) Zero

SOL. (d) Magnetic field between the plates in this case is zero.

55. **A current i is flowing in a straight conductor of length L. The magnetic induction at a point on its axis at a distance $\frac{L}{4}$ from its centre will be:**

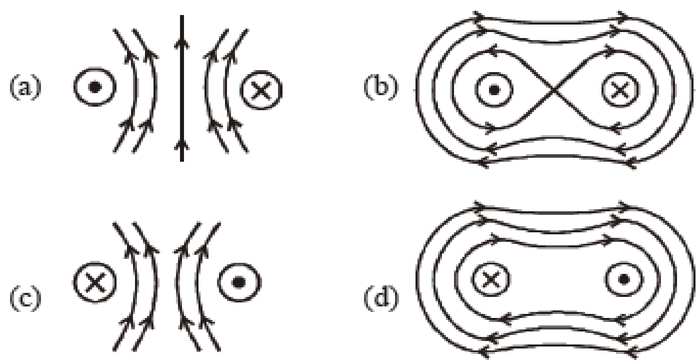
[Online April 22, 2013]

(a) Zero (b) $\frac{\mu_0 i}{2\pi L}$ (c) $\frac{\mu_0 i}{\sqrt{2}L}$ (d) $\frac{4\mu_0 i}{\sqrt{5}\pi L}$

SOL. (a) Magnetic field at any point lies on axial position of current carrying conductor $B = 0$

56. **Choose the correct sketch of the magnetic field lines of a circular current loop shown by the dot and the cross $\otimes$.**

[Online April 22, 2013]



SOL. (a) If magnetic field is perpendicular and into the plane of the paper, it is represented by cross $\otimes$ and if the direction of the magnetic field is perpendicular out of the plane of the paper it is represented by dot .

57. **An electric current is flowing through a circular coil of radius R. The ratio of the magnetic field at the centre of the coil and that at a distance $2\sqrt{2}R$ from the centre of the coil and on its axis is: [Online April 9, 2013]**

(a) $2\sqrt{2}$ **(b) 27** **(c) 36** **(d) 8**

SOL. (b) Given: Radius = R

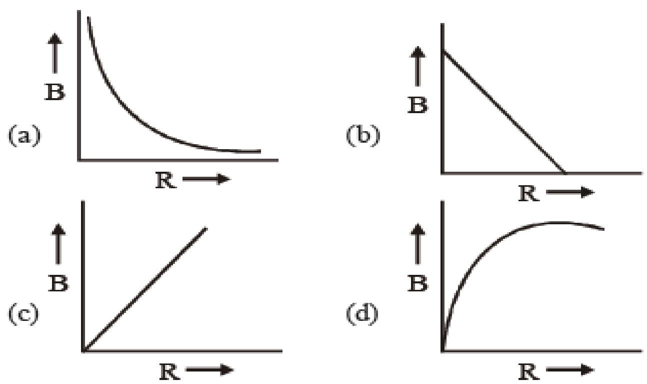
Distance $x = 2\sqrt{2}R$

$$\frac{B_{centre}}{B_{axis}} = \left(1 + \frac{x^2}{R^2}\right)^{3/2} = \left(1 + \frac{\left(2\sqrt{2}R\right)^2}{R^2}\right)^{3/2}$$

$$= (9)^{3/2} = 27$$

58. A charge *Q* is uniformly distributed over the surface of non-conducting disc of radius *R*. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity w. As a result of this rotation a magnetic field of induction B is obtained at the centre ofthe disc. Ifwe keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure:

[2012]



SOL. (a) The magnetic field due to a disc is given as

$$B = \frac{\mu_0 wQ}{2\pi R} \quad \text{i.e., } B \propto \frac{1}{R}$$

59. A current I flows in an infinitely long wire with cross section in the form of a semi-circular ring of radius *R*. The magnitude of the magnetic induction along its axis is:

[2011]

(a) $\frac{\mu_0 I}{2\pi^2 R}$ **(b)** $\frac{\mu_0 I}{2\pi R}$ **(c)** $\frac{\mu_0 I}{4\pi R}$ **(d)** $\frac{\mu_0 I}{\pi^2 R}$

SOL. (d) Let R be the radius of semicircular ring.

Let an elementary length dl is cut for finding magnetic field.

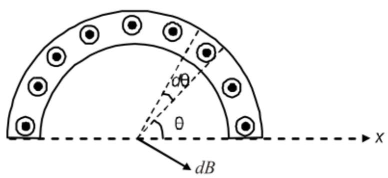
So, $dl = Rd\theta$. Current in a small element, $dI = \frac{d\theta}{\pi} I$

Magnetic field due to the element

$$dB = \frac{\mu_0}{4\pi}\frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R}$$

The component $dB \cos\theta$, ofthe field is cancelled by another opposite component.

Therefore,

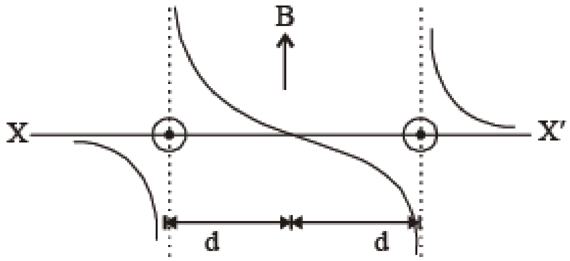


$$B_{net} = \int dB \sin\theta = \frac{\mu_0 I}{2\pi^2 R}\int_0^{\pi} \sin\,\theta d\theta = \frac{\mu_0 I}{\pi^2 R}$$

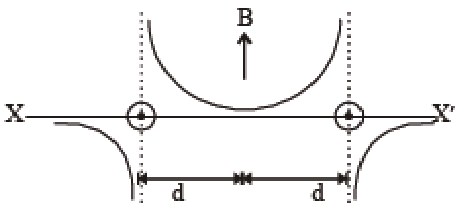
60. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX↑ is given by

[2010]

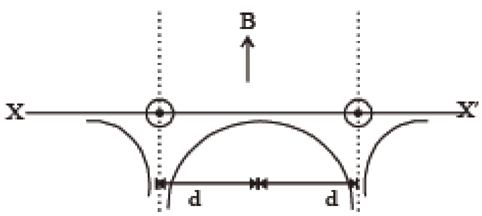
(a)



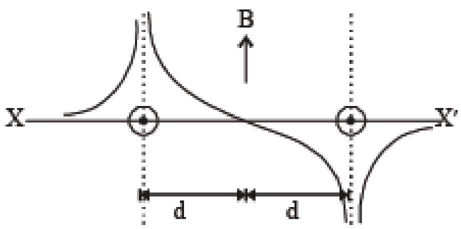
(b)



(c)



(d)



SOL. (a) The magnetic field varies inversely with the distance for a long conductor.

That is, $B \propto \frac{1}{d}$

so, graph in option (a) is the correct one.

61. A horizontal overhead powerline is at height of 4m from the ground and carries a current of 100A from east to west. The magnetic field directly below it on the ground is

$(\mu_0 = 4\pi \times 10^{-7}$ Tm $A^{-1})$

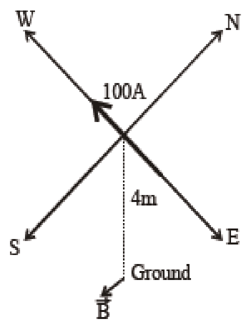
[2008]

(a) $2.5 \times 10^{-7} T$ southward **(b) 5×10^{-6} Tnorthward**

(c) $5 \times 10^{-6} T$ southward **(d) 2.5×10^{-7} Tnorthward**

SOL. (c) The magnetic field is

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = 10^{-7} \times \frac{2 \times 100}{4} = 5 \times 10^{-6} T$$



Current flows from east to west. Point is below the power line, using right hand thumb rule, the magnetic field is directed towards south.

62. A long straight wire of radius a carries a steady current i. The current is uniformly distributed across its cross section. The ratio of the magnetic field at $a/2$ and $2a$ is

[2007]

(a) 1/2 (b) 1/4 (c) 4 (d) 1

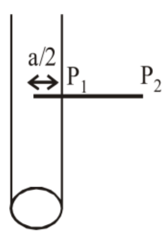
SOL. (d) Since uniform current is flowing through a straight wire, current enclosed in the amperian path formed at a distance $r_1 \left(= \frac{a}{2}\right)$ is

$$i = \left(\frac{\pi r_1^2}{\pi a^2}\right) \times I ,$$

where I is total current

Using Ampere circuital law,

$$\oint B \cdot \vec{dl} = \mu_0 i$$



$$\Rightarrow B_1 = \frac{\mu_0 \times \text{current enclosed}}{\text{Path}}$$

$$\Rightarrow B_1 = \frac{\mu_0 \times \left(\frac{\pi r_1^2}{\pi a^2}\right) \times I}{2\pi r_1} = \frac{\mu_0 \times I r_1}{2\pi a^2}$$

Now, magnetic field induction at point P_2,

$$B_2 = \frac{\mu_0}{2\pi} \cdot \frac{I}{(2a)} = \frac{\mu_0 I}{4\pi a}$$

$$\frac{B_1}{B_2} = \frac{\mu_0 I r_1}{2\pi a^2} \times \frac{4\pi a}{\mu_0 I}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{2r_1}{a} = \frac{2 \times \frac{a}{2}}{a} = 1.$$

63. A current *I* flows along the length of an infinitely long, straight, thin walled pipe. Then

[2007]

(a) the magnetic field at all points inside the pipe is the same, but not zero

(b) the magnetic field is zero only on the axis of the pipe

(c) the magnetic field is different at different points inside the pipe

(d) the magnetic field at any point inside the pipe is zero

SOL. (d) There is no current inside the pipe.

From Ampere's circuital law $\oint \vec{B} \cdot \overline{dl} = \mu_0 I$

$$I = 0$$

$$B = 0$$

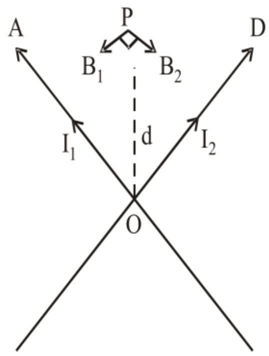
64. Two identical conducting wires *AOB* and *COD* are placed at right angles to each other. The wire *AOB* carries an electric current I_1 and *COD* carries a current I_2. The magnetic field on a point lying at a distance d from *O*, in a direction perpendicular to the plane ofthe wires *AOB* and *COD*, will be given by

[2007]

(a) $\frac{\mu_0}{2\pi d}\left(l_1^2 + l_2^2\right)$ **(b)** $\frac{\mu_0}{2\pi}\left(\frac{I_1 + I_2}{d}\right)^{\frac{1}{2}}$

(c) $\frac{\mu_0}{2\pi d}\left(I_1^2 + I_2^2\right)^{\frac{1}{2}}$ **(d)** $\frac{\mu_0}{2\pi d}\left(I_1 + I_2\right)$

SOL. (c) The direction of magnetic field induction due to current through AB and CD at P are indicated as B_1 and B_2. The magnetic fields at a point P, equidistant from AOB and COD will have directions perpendicular to each other, as they are placed normal to each other.



Magnetic field at P due to current through AB, $B_1 = \frac{\mu_0 I_1}{2\pi d}$

Magnetic field at P due to current through CD,

$$B_2 = \frac{\mu_0 I_2}{2\pi d}$$

Resultant field, $B = \sqrt{B_1^2 + B_2^2}$

$$B = \sqrt{\left(\frac{\mu_0}{2\pi d}\right)^2 (I_1^2 + I_2^2)}$$

or, $B = \frac{\mu_0}{2\pi d}(I_1^2 + I_2^2)^{1/2}$

65. A long solenoid has 200 turns per cm and carries a current i. The magnetic field at its centre is 6.28×10^{-2} Weber/m^2. Another long solenoid has 100 turns per cm and it carries a current $\frac{i}{3}$. The value of the magnetic field at its centre is

[2006]

(a) 1.05×10^{-2} Weber/m^2 **(b) 1.05×10^{-5} Weber/m^2**

(c) 1.05×10^{-3} Weber/m^2 **(d) 1.05×10^{-4} Weber/m^2**

SOL. (a) Magnetic field due to long solenoid is given by $B = \mu_0 n I$

In first case $B_1 = \mu_0 n_1 I_1$

In second case, $B_2 = \mu_0 n_2 I_2$

$$\frac{B_2}{B_1} = \frac{\mu_0 n_2 i_2}{\mu_0 n_1 i_1}$$

$$\Rightarrow \frac{B_2}{6.28 \times 10^{-2}} = \frac{100 \times \frac{i}{3}}{200 \times i}$$

$$\Rightarrow B_2 = \frac{6.28 \times 10^{-2}}{6} = 1.05 \times 10^{-2} \text{ Wb/m}^2$$

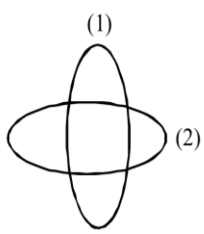
66. **Two concentric coils each of radius equal to 2 π cm are placed at right angles to each other. 3 ampere and 4 ampere are the currents flowing in each coil respectively. The magnetic induction in Weber/m^2 at the centre of the coils will be $(\mu_0 = 4\pi \times 10^{-7} \text{Wb/A.m})$**

[2005]

(a) 10^{-5} **(b) 12×10^{-5}** **(c) 7×10^{-5}** **(d) 5×10^{-5}**

SOL.

(d)



The magnetic field due to circular coil (1) is

$$B_1 = \frac{\mu_0 i_1}{2r} = \frac{\mu_0 i_1}{2(2\pi \times 10^{-2})} = \frac{\mu_0 \times 3 \times 10^2}{4\pi}$$

Magnetic field due to coil (2)

$$B_2 = \frac{\mu_0 i_2}{2(2\pi \times 10^{-2})} = \frac{\mu_0 \times 4 \times 10^2}{4\pi}$$

Total magnetic field, $B = \sqrt{B_1^2 + B_2^2}$

$$= \frac{\mu_0}{4\pi} \cdot 5 \times 10^2$$

$$\Rightarrow B = 10^{-7} \times 5 \times 10^2$$

$$\Rightarrow B = 5 \times 10^{-5} \backslash\text{Vb/m}^2$$

67. A current i ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at anypoint inside the tube is

[2004]

(a) $\frac{\mu_0}{4\pi}.\frac{2i}{r}$ tesla **(b) zero** **(c) infinite** **(d) $\frac{2i}{r}$ tesla**

SOL. (b) From Ampere's circuital law

$$\int \vec{B} \cdot \overline{dl} = \mu_0 i$$

$$\Rightarrow B \times 2\pi r = \mu_0 i$$

Here i is zero, for $r < R$, whereas R is the radius

$$B = 0$$

68. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be

[2004]

(a) $2nB$ **(b) n^2B** **(c) nB** **(d) $2\,n^2B$**

SOL. (b) Magnetic field at the centre of a circular coil of radius R carrying current i is $B = \frac{\mu_0 i}{2R}$

The circumference of the first loop $= 2\pi R$.

If it is bent into n circular coil ofradius r'.

$$n \times (2\pi r') = 2\pi R$$

$\Rightarrow nr' = R$ (1)

New magnetic field, $B' = \frac{n \cdot \mu_0 i}{2r'}$ (2)

From(1) and(2),

$$B' = \frac{n\mu_0 i \cdot n}{2\pi R} = n^2 B$$

69. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of loop?

[2004]

(a) 125 μT **(b) 150 μT** **(c) 250 μT** **(d) 75 μT**

SOL. (c) The magnetic field at a point on the axis of a circular loop at a distance x from centre is,

$$B = \frac{\mu_0 i a^2}{2(x^2 + a^2)^{3/2}}$$

Magnetic field at the centre of loop is

$$B' = \frac{\mu_0 i}{2a}$$

$$B' = \frac{B \cdot (x^2 + a^2)^{3/2}}{a^3}$$

Put $x = 4$ & $a = 3$

$$\Rightarrow B' = \frac{54(5^3)}{3 \times 3 \times 3} = 250\mu T$$

70. If in a circular coil A of radius R, current I is flowing and in another coil B ofradius $2R$ a current $2I$ is flowing, then the ratio ofthe magnetic fields B_A and B_B, produced by them willbe

[2002]

(a) 1 **(b) 2** **(c) 1/2** **(d) 4**

SOL. (a) Magnetic field induction at the centre of current carrying circular coil of radius r is

$$B = \frac{\mu_0 I}{4\pi R} \times 2\pi$$

Here $B_A = \frac{\mu_0 I}{4\pi R} \times 2\pi$ and $B_B = \frac{\mu_0 2I}{4\pi 2R} \times 2\pi$

$$\Rightarrow \frac{B_A}{B_B} = \frac{I/R}{2I/2R} = 1$$

TOPIC-3 Force and Torque on Current Carrying Conductor

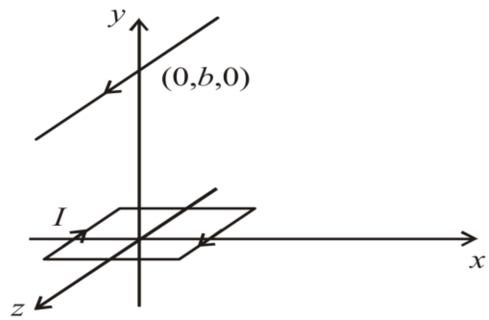
71. **A square loop of side $2a$ and carrying current I is kept *is xz* plane with its centre at origin. A long wire carrying the same current I is placed parallel to z-axis and passing through point $(0, b, 0)$, $(b >> a)$. The magnitude oftorque on the loop about z-axis will be :**

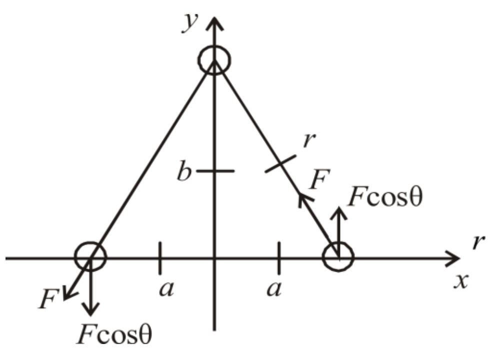
[Sep. 06, 2020 (II)]

(a) $\frac{2\mu_0 I^2 a^2}{\pi b}$ **(b)** $\frac{2\mu_0 I^2 a^2 b}{\pi\left(a^2+b^2\right)}$ **(c)** $\frac{\mu_0 I^2 a^2 b}{2\pi\left(a^2+b^2\right)}$ **(d)** $\frac{\mu_0 I^2 a^2}{2\pi b}$

SOL.

(b)





r= $\sqrt{b^2 + a^2}$

Force, $F = BI\,2a = \frac{\mu_0 I}{2\pi r} I \times 2a$

Force, $F = \frac{\mu_0 I^2 a}{\pi\sqrt{b^2+a^2}}$

Torque, $\tau = F_1 \times$ Perpendicular distance $= F\cos\theta \times 2a$

$$= \frac{\mu_0 I^2 a}{\pi\sqrt{b^2+a^2}} \times \frac{b}{\sqrt{b^2+a^2}} \times 2a$$

$$\Rightarrow \Gamma = \frac{2\mu_0 I^2 a^2 b}{\pi(a^2+b^2)}$$

If $b >> a$ then $\Gamma = \frac{2\mu_0 I^2 a^2}{\pi b}$

72. A square loop of side $2a$, and carrying current I, is kept in XZ plane with its centre at origin. Along wire carrying the same current I is placed parallel to the z-axis and passing through the point $(0, b, 0)$, $(b >> a)$. The magnitude of the torque on the loop about z-axis is given by:

[Sep. 05, 2020 (I)]

(a) $\frac{\mu_0 I^2 a^2}{2\pi b}$ **(b)** $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ **(c)** $\frac{2\mu_0 I^2 a^2}{\pi b}$ **(d)** $\frac{2\mu_0 I^2 a^3}{\pi b^2}$

SOL. (c) Torque on the loop,

$$'\overline{\tau} = \overline{M} \times \overline{B} = MB \sin\theta = MB \sin 90°$$

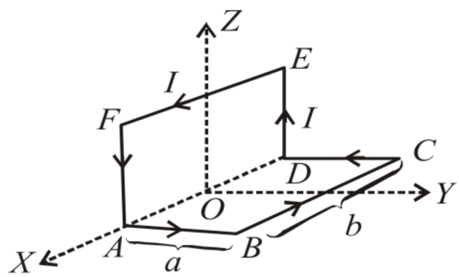
Magnetic field, $B = \frac{\mu_0 I}{2\pi d}$

$$\tau = I_1 (2a)^2 \left(\frac{\mu_0 I_2}{2\pi d}\right) \sin 90^\circ$$

$$= \frac{2\mu_0 I_1 I_2}{\pi d} \times a^2 = \frac{2\mu_0 I^2 a^2}{\pi d}$$

73. A wire carrying current I is bent in the shape ABCDEFA as shown, where rectangle *ABCDA* and *ADEFA* are perpendicular to each other. If the sides of the rectangles are of lengths a and b, then the magnitude and direction of magnetic moment of the loop *ABCDEFA* is:

[Sep. 02, 2020 (II)]



(a) abI, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$ **(b) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$**

(c) $\sqrt{2}abI$, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$ **(d) abI, along $\left(\frac{\hat{j}}{\sqrt{5}}+\frac{2\hat{k}}{\sqrt{5}}\right)$**

SOL. (b) Magnetic moment of loop *ABCD*,

M_1 = area of loop × current

$\vec{M}_1 = (abI)(\hat{j})$ (Here, ab = area of rectangle)

Magnetic moment of loop *DEFA*, $\vec{M}_2 = (abI)(\hat{\imath})$

Net magnetic moment,

$$\vec{M} = \vec{M}_1 + \vec{M}_2 \Rightarrow \vec{M} = abI(\hat{\imath} + \hat{j})$$

$$\Rightarrow |\vec{M}| = \sqrt{2}abI\left(\frac{\hat{j}}{\sqrt{2}}+\frac{\hat{k}}{\sqrt{2}}\right)$$

74. A small circular loop of conducting wire has radius a and carries current I. It is placed in a uniform magnetic field B perpendicular to its plane such that when rotated slightly about its diameter and released, it starts performing simple harmonic motion of time period T. If the mass of the loop is m then:

[9 Jan 2020, II]

(a) $T = \sqrt{\frac{2m}{IB}}$ **(b) $T = \sqrt{\frac{\pi m}{2IB}}$** **(c) $T = \sqrt{\frac{2\pi m}{IB}}$** **(c) $T = \sqrt{\frac{\pi m}{IB}}$**

SOL. (c) Torque on circular loop, $\tau = MB\sin\theta$

where, M=magnetic moment

B = magnetic field

Now, using $\tau = I\alpha$

$$\Gamma = MB \sin\theta = I\alpha$$

$$\Rightarrow \pi R^2 IB\theta = \frac{mR^2\alpha}{2}$$

($m = IA$ and moment of inertia of circular loop, $I = \frac{mR^2}{2}$)

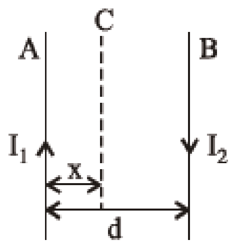
$$\Rightarrow \pi R^2 IB\theta = \frac{mR^2}{2} c0\theta$$

$$\Rightarrow (j) = \sqrt{\frac{2\pi IB}{m}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{2\pi IB}{m}}$$

$$\Rightarrow T = \sqrt{\frac{2\pi m}{IB}}$$

75. Two wires A&B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are:

[10 April 2019, I]



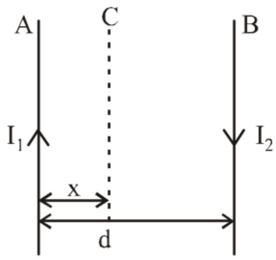
(a) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \frac{I_2}{(I_1 + I_2)}d$

(b) $x = \left(\frac{I_2}{(I_1 + I_2)}\right)d$ and $x = \left(\frac{I_2}{(I_1 - I_2)}\right)d$

(c) $x = \left(\frac{I_1}{(I_1 + I_2)}\right)d$ and $x = \left(\frac{I_2}{(I_1 - I_2)}\right)d$

(d) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

SOL.

(d) 

As net force on the third wire C is zero.

$$\Rightarrow \vec{\Gamma} = \frac{\mu_0 I_1}{2\pi x} + \frac{\mu_0 I_2}{2\pi(d-x)} = 0$$

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi(x-d)}$$

$$I_1 x - I_1 d = I_2 x$$

$$x = \frac{I_1 d}{I_1 - I_2}$$

Two cases maybepossib1e if $I_1 > I_2$ or $I_2 > I_1$

76. A rectangular coil (Dimension 5 cm $\times$ 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction, is kept centered at the origin and in the X-Z plane. A magnetic field of l T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is:

[9 April 2019 I]

(a) 0.38 Nm **(b) 0.55 Nm** **(c) 0.42Nm** **(d) 0.27Nm**

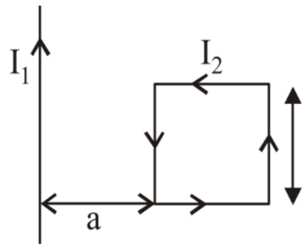
SOL. (d) $\tau = MB\sin 45^\circ = N(iA)B\sin 45^\circ$

$$= 100 \times 3(5 \times 2.5) \times 10^{\triangleleft} \times 1 \times \frac{1}{\sqrt{2}}$$

$$= 0.27 N - m$$

77. A rigid square of loop of side a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be:

[9 April 2019 I]



(a) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$ (b) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$

(c) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$ (d) Zero

SOL. (c) $F = \frac{\mu_0}{2\pi}\left(\frac{i_i i_2}{a} - \frac{i_1 i_2}{2a}\right) \times a = \frac{\mu_0 i_1 i_2}{4\pi}$

78. A circular coil having N turns and radius r carries a current *I*. It is held in the XZ plane in a magnetic field B. The torque on the coil due to the magnetic field is:

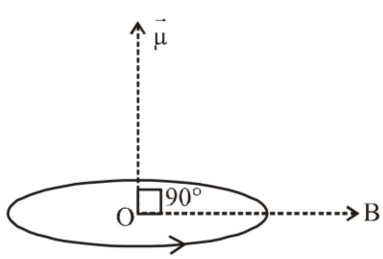
[8 April 2019 I]

(a) $\frac{Br^2 I}{\pi N}$ **(b)** $B\pi r^2 IN$ **(c)** $\frac{B\pi r^2 I}{N}$ **(d) Zero**

SOL (b) $|\vec{\Gamma}| = |\vec{\mu} \times \vec{B}|$ $[\mu = NIA]$

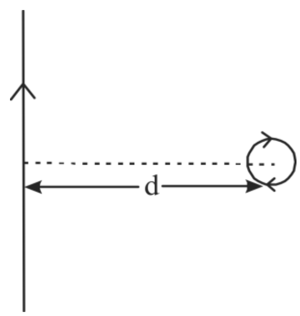
$$= NIA \times B \sin 90^o \ [A = \pi r^2]$$

$$\Rightarrow \tau = NI\pi r^2 B$$



79. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $(d >> a)$. If the loop applies a force *Γ* on the wire then:

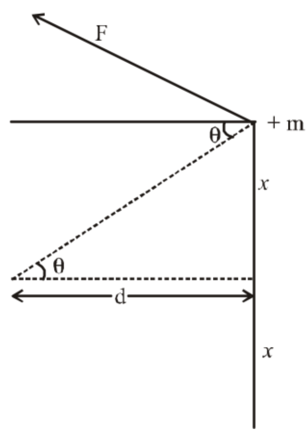
[9 Jan. 2019 I]



(a) $F = 0$ **(b)** $F \propto \left(\frac{a}{d}\right)$ **(c)** $F \propto \left(\frac{a^2}{d^3}\right)$ **(d)** $F \propto \left(\frac{a}{d}\right)^2$

SOL.

(d)



Force on one pole,

$$\Gamma = m \times \frac{\mu_0 I}{2\pi\sqrt{d^2 + x^2}}$$

Total force, $\Gamma_{\text{tota1}} = 2\Gamma \sin\theta$

$$= 2 \times \frac{\mu_0 \text{ Im}}{2\pi\sqrt{d^2 + a^2}} \times \frac{x}{\sqrt{d^2 + a^2}}$$

$$= \frac{\mu_0 \text{ Im x}}{\pi(d^2 + a^2)}$$

Magnetic moment, $M = I\pi a^2 = m \times 2$

or, Total force, $F_{\text{total}} = \frac{\mu_0 I a^2}{2(d^2 + a^2)}$

$$=\frac{\mu_0 Ia^2}{2d^2}[d >> a]$$

Clearly $F_{total} \propto \frac{a^2}{d^2}$

80. A charge q is spread uniformly over an insulated loop of radius r. If it is rotated with an angular velocity w with respect to normal axis then the magnetic moment of the loop is

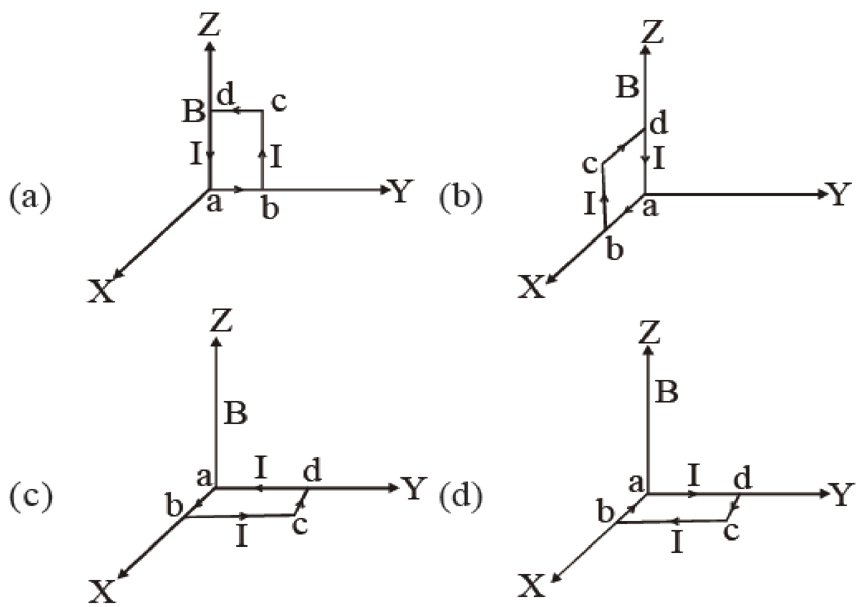
[Online April 16, 2018]

(a) $\frac{1}{2}qwr^2$ **(b) $\frac{4}{3}qwr^2$** **(c) $\frac{3}{2}qwr^2$** (d) qwr^2

SOL. (a) Magnetic moment, $\mu = IA = \frac{qv}{2\pi r}(\pi r^2)$

or, $\mu = \frac{qrw}{2\pi r}\left(\pi r^2\right) = \frac{1}{2}qr^2 w$

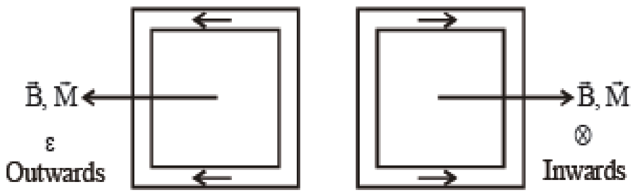
81. A uniform magnetic field B of 0.3T is along the positive Z direction. A rectangular loop (abcd) of sides 10 cm × 5 cm carries a current I of 12A. out of the following different orientations which one corresponds to stable equilibrium?



SOL. (c) Magnetic moment of current carrying rectangular loop of area A is given by M = NIA

Magnetic moment of current carrying coil is a vector and its direction is given by right hand thumb rule.

For rectangular loop, B1 at centre due to current in loop and M1 are always parallel.



Hence, (c) corresponds to stable equilibrium.

82. Two coaxial solenoids of different radius carry current I in the same direction. $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then:

[2015]

(a) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$

(b) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

(c) $\vec{F_1} = \vec{F_2} = 0$

(d) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards

SOL. (c) $\vec{F}_1 = \vec{F}_2 = 0$

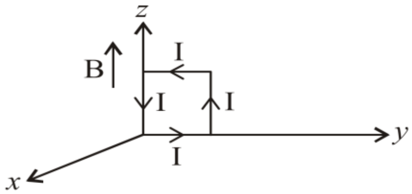
because of action and reaction pair

83. A rectangular loop of sides 10 cm and 5 cm carrying a current 1 of 12 A is placed in different orientations as shown in the figures below:

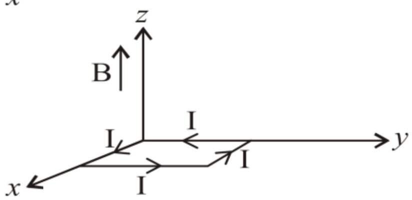
Out of the following different orientations which one corresponds to stable equilibrium?

[Online April 9, 2017]

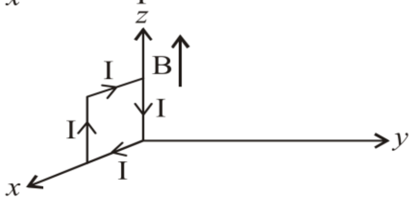
(A)



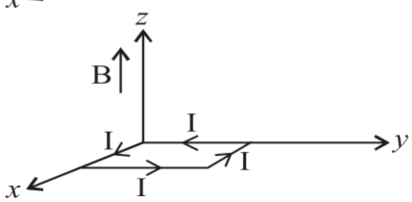
(B)



(C)



(D)



If there is a uniform magnetic field of $0.3T$ in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

[2015]

(a) (B) and (D), respectively **(b) (B) and (C), respectively**

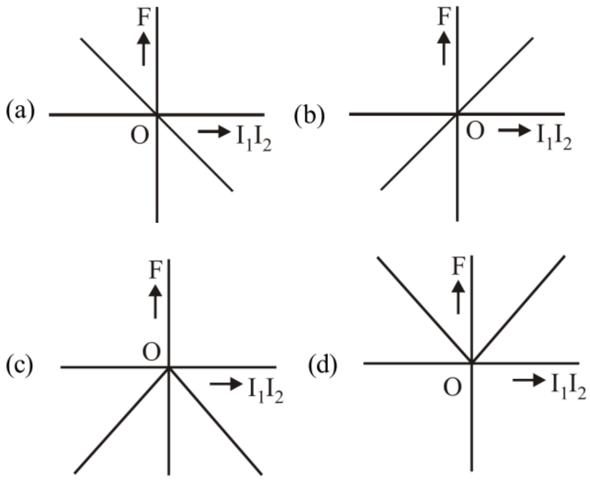
(c) (A) and (B), respectively **(d) (A) and (C), respectively**

SOL. (a) For stable equilibrium $\vec{M} || \vec{B}$

For unstable equilibrium $\vec{M} || (-\vec{B})$

84. Two long straight parallel wires, carrying (adjustable) current I_1 and I_2, are kept at a distance d apart. If the force ' Γ' between the two wires is taken as 'positive' when the wires repel each other and 'negative' when the wires attract each other, the graph showing the dependence of Γ', on the product $I_1 I_2$, would be :

[Online April 11, 2015]



SOL. (a) $I_1 I_2$ = Positive

(attract) F = Negative

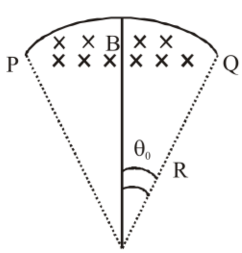
$I_1 I_2$ = Negative

(repel) F = Positive

Hence, option (a) is the correct answer.

85. A wire carrying current I is tied between points P and Q and is in the shape of a circular arc of radius R due to a uniform magnetic field B (perpendicular to the plane of the paper, shown by xxx) in the vicinity of the wire. If the wire subtends an angle $2\theta_0$ at the centre of the circle (of which it forms an arc) then the tension in the wire is:

[Online April 11, 2015]

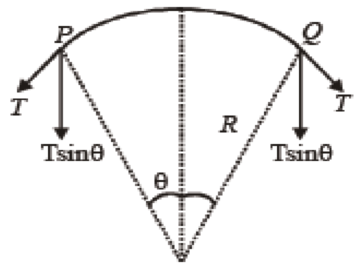


(a) $\frac{IBR}{2 \sin \theta_0}$ **(b)** $\frac{IBR\theta_0}{\sin \theta_0}$ **(c) IBR** **(d)** $\frac{IBR}{\sin \theta_0}$

SOL. (c) For small arc length

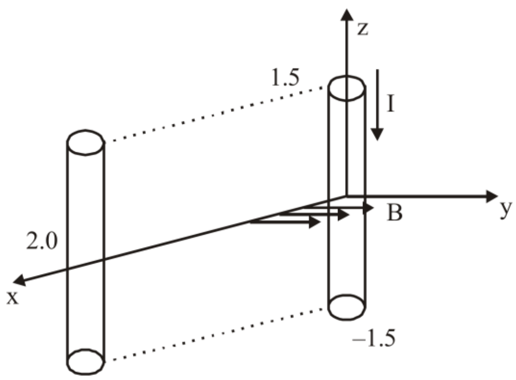
$2T \sin \theta = \text{BIR } 2\,\theta$ (As $F = \text{BIL}$ and $L = \text{R}Z\theta$)

$$T = \text{BIR}$$



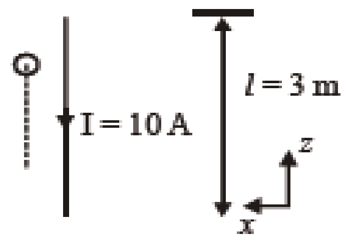
86. **A conductor lies along the z-axis at $-1.5 \le z < 1.5$m and cames a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0 \times 10^{-4} e^{-0.2x} \hat{a}_y$T, find the power required to move the conductor at constant speed to $x = 2.0$m, $y = 0$m in 5×10^{-3}s. Assume parallel motion along the x-axis.**

[2014]



(a) 1.57W **(b) 2.97W** **(c) 14.85W** **(d) 29.7W**

SOL. (b) Work done in moving the conductor is,



$$W = \int_0^2 F\,dx = \int_0^2 3.0 \times 10^{-4} e^{-0.2x} \times 10 \times 3 dx$$

$$= 9 \times 10^{-3} \int_0^2 e^{-0.2x}\,dx$$

$$= \frac{9 \times 1.0^{-3}}{02}\left[-e^{-0.2\times 2} + 1\right]$$

$$= \frac{9 \times 1.0^{-3}}{02} \times [1 - e^{-0.4}]$$

$$= \frac{9 \times 10^{-3} \times (0.33)}{2} = \frac{2.97 \times 10^{-3}}{2}$$

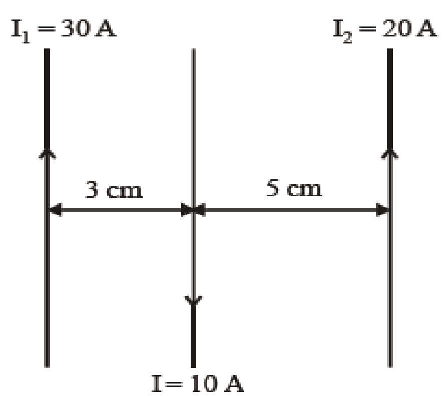
Power required to move the conductor is,

$$P = \frac{W}{t}$$

$$P = \frac{2.97 \times 10^{-3}}{(0.2) \times 5 \times 10^{-3}} = 2.97\text{W}$$

87. Three straight parallel current carrying conductors are shown in the figure. The force experienced by the middle conductor of length 25 cm is:

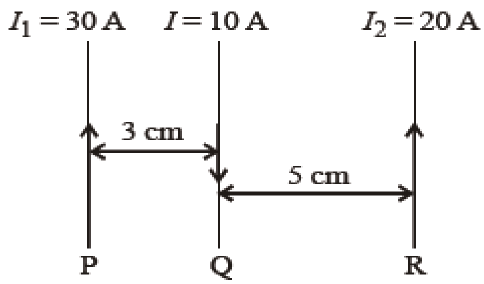
[Online April 11, 2014]



(a) 3×10^{-4}N toward right **(b) 6×10^{-4}N toward right**

(c) 9×10^{-4}N toward right **(d) Zero**

SOL. (a)



Also given; length of wire Q = 25cm = 0.25m

Force on wire Q due to wire R

$$F_{QR} = 10^{-7} \times \frac{2 \times 2.0 \times 10}{005} \times 0.25$$

$$= 20 \times 10^{-5}\text{N (Towards left)}$$

Force on wire Q due to wire P

$$F_{QP} = 10^{-7} \times \frac{2 \times 30 \times 10}{0.03} \times 0.25$$

$$= 50 \times 10^{-5}\text{N (Towards right)}$$

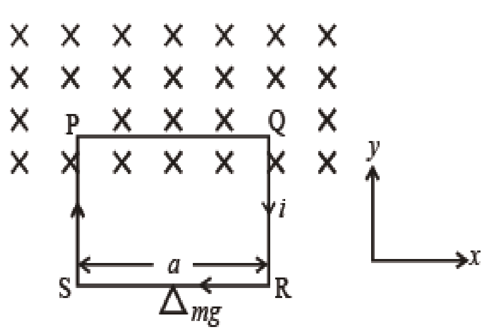
Hence, $F_{\text{net}} = F_{QP} - F_{QR}$

$$= 50 \times 10^{-5}\text{N} - 20 \times 10^{-5}\text{N}$$

$$= 3 \times 10^{-4}\text{N towards right}$$

88. A rectangular loop of wire, supporting a mass m, hangs with one end in a uniform magnetic field $\vec{B}$ pointing out of the plane of the paper. A clockwise current is set up such that $i > mglBa$, where a is the width of the loop. Then:

[Online April 23, 2013]



(a) The weight rises due to a vertical force caused by the magnetic field and work is done on the system.

(b) The weight do not rise due to vertical for caused by the magnetic field and work is done on the system.

(c) The weight rises due to a vertical force caused by the magnetic field but no work is done on the system.

(d) The weight rises due to a vertical force caused by the magnetic field and work is extracted from the magnetic field.

SOL. (c)

89. Currents of a 10 ampere and 2 ampere are passed through two parallel thin wires A and B respectively in opposite directions. Wire A is infinitely long and the length ofthe wire B is 2 m. The force acting on the conductor B, which is situated at 10 cm distance from A will be

[Online May 26, 2012]

(a) 8×10^{-5}N (b) 5×10^{-5}N (c) $8\pi \times 10^{-7}$N (d) $4\pi \times 10^{-7}$N

SOL. (a) Force acting on conductor B due to conductor A is given by relation

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

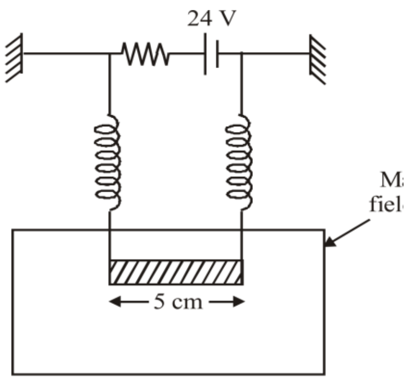
l-length of conductor B

r-distance between two conductors

$$F = \frac{4\pi \times 10^{-7} \times 10. \times 2 \times 2}{2 \times \pi \times 01} = 8 \times 10^{-5}\text{N}$$

90. The circuit in figure consists of wires at the top and bottom and identical springs as the left and right sides. The wire at the bottom has a mass of 10 g and is 5 cm long. The wire is hanging as shown in the figure. The springs stretch 0.5 cm under the weight of the wire and the circuit has a total resistance of 12 Ω. When the lower wire is subjected to a static magnetic field, the springs, stretch an additional 0.3 cm. The magnetic field is

[Online May 12, 2012]



(a) 0.6 T and directed out of page

(b) 1.2 T and directed into the plane of page

(c) 0.6 T and directed into the plane of page

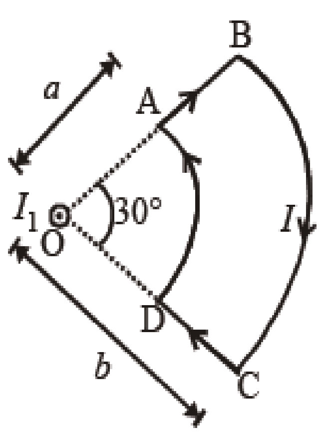
(d) 1.2 T and directed out of page

SOL. (a)

Directions : Question numbers 91 and 92 are based on the following paragraph

A current ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius=b) and DA (radius = *a*) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by *AB* and *CD* at the origin O is 30^0. Another straight thin wire with steady current I_1 flowing out of the paper is kept at the origin

[2009]



91. The magnitude of the magnetic field (*B*) due to the loop *ABCD* at the origin (*O*) is :

(a) $\frac{\mu_0 I(b-a)}{24ab}$ **(b)** $\frac{\mu_o I}{4\pi}\left[\frac{b-a}{ab}\right]$

(c) $\frac{m_o I}{4\pi}\left[2(b-a)+\pi/3(a+b)\right]$ **(d) zero**

SOL. (a) The magnetic field at O due to current in DA is

$B_1 = \frac{\mu_o I}{4\pi a} \times \frac{\pi}{6}$ (directed vertically upwards)

The magnetic field at O due to current in BC is

$B_2 = \frac{\mu_o I}{4\pi b} \times \frac{\pi}{6}$ (directed vertically downwards)

The magnetic field due to current AB and CD at O is zero.

Therefore the net magnetic field is

$B = B_1 - B_2$ (directed vertically upwards)

$$= \frac{\mu_0 I\pi}{4\pi a 6} - \frac{\mu_0 I}{4\pi b} \times \frac{\pi}{6}$$

$$= \frac{\mu_o I}{24}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{\mu_o I}{24ab}(b-a)$$

92. Due to the presence of the current I_1 at the origin:

(a) The forces on AD and BC are zero.

(b) The magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi} m_o[2(b-a)+\pi/3(a+b].$

(c) The magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$

(d) The forces on AB and DC are zero.

SOL. (d) $\vec{F} = I\left(\vec{l} \times \vec{B}\right)$

The force on AD and BC due to current I_1 is zero. This is because the directions of current element $I\overline{d\ell}$ and magnetic field B are parallel

93. Two long conductors, separated by a distance d carry current I_1 and I_2 in the same direction. They exert a force F on each other. Now the current in one ofthem is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value ofthe force between them is

[2004]

(a) $-\frac{2F}{3}$ (b) $\frac{F}{3}$ (c) $-2F$ (d) $-\frac{F}{3}$

SOL. (a) Force acting between two long conductor carrying current,

$$F=\frac{\mu_0}{4\pi}\frac{2I_1I_2}{d}\times\ell$$

Where d = distance between the conductors

ℓ = length of conductor

In second case, $F'=-\frac{\mu_0}{4\pi}\frac{2(2I_1)I_2}{3d}\ell$ -----(ii)

From equation (i) and (ii), we have

$$\frac{F'}{F}=\frac{-2}{3}$$

94. If a current is passed through a spring then the spring will

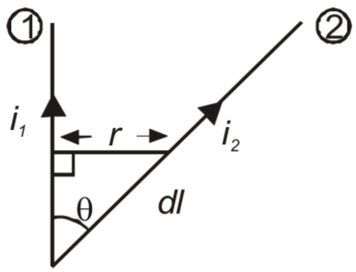
[2002]

(a) expand (b) compress (c) remains same (d) none of these

SOL. (b) When current is passed through a spring then current flows parallel in the adjacent turns in the same direction. As a result the various turn attract each other and spring get compress.

95. Wires 1 and 2 carrying currents i_1 and i_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl ofwire 2 at a distance of r from wire 1 (as shown in figure) due to the magnetic field of wire 1?

[2002]



(a) $\frac{\mu_0}{2\pi r} i_1 i_2 dl \tan\theta$

(b) $\frac{\mu_0}{2\pi r} i_1 i_2 dl \sin\theta$

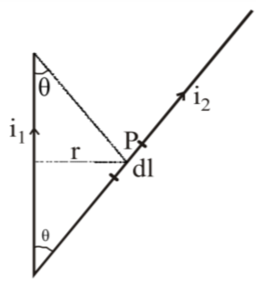
(c) $\frac{\mu_0}{2\pi r} i_1 i_2 dl \cos\theta$

(d) $\frac{\mu_0}{4\pi r} i_1 i_2 dl \sin\theta$

SOL. (c) Magnetic field due to current in wire 1 at point P distant r from the wire is

$$B = \frac{\mu_0}{4\pi}\frac{i_1}{r}[\cos\theta + \cos\theta]$$

$$B = \frac{\mu_0}{2\pi}\frac{i_1 \cos\theta}{r}$$



This magnetic field is directed perpendicular to the plane of paper, inwards.

The force exerted due to this magnetic field on current element $i_2 dl$ is

$$dF = i_2 dlB \sin 90^\circ$$

$$dF = i_2\ dl\ B$$

$$\Rightarrow dF = i_2 dl\left(\frac{\mu_0 i_1 \cos\theta}{4\pi r}\right)$$

$$= \frac{\mu_0}{2\pi r} i_1 i_2\ dl \cos\theta$$

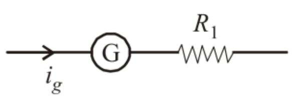
TOPIC-4 Galvanometer and its conversion into Ammeter and Voltmeter

96. A galvanometer of resistance G is converted into a voltmeter of range $0 - 1V$ by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to increase the range of the voltmeter to $0 - 2V$ will be:

[Sep. 05, 2020 (I)]

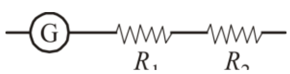
(a) G **(b) R_1** **(c) $R_1 - G$** **(d) $R_1 + G$**

SOL. (d) Galvanometer of resistance (G) converted into a voltmeter of range 0-1 V.



$V = 1 = i_g (G + R_1) - - - - - - (i)$

To increase the range of voltmeter 0-2V



$2 = i_g(R_1 + R_2 + G)$ -----(ii)

Dividing eq. (i) by(ii),

$$\Rightarrow \frac{1}{2} = \frac{G + R_1}{G + R_1 + R_2}$$

$$\Rightarrow G + R_1 + R_2 = 2G + 2R_1$$

$$R_2 = G + R_1$$

97. A galvanometer is used in laboratory for detecting the null point in electrical experiments. If on passing a current of 6 mA it produces a deflection of 2°, its figure of merit is close to :

[Sep. 05, 2020 (II)]

(a) 333°A/div. **(b) $6 \times 10^{-3} \text{A/div.}$** **(c) 666°A/div.** **(d) $3 \times 10^{-3} \text{A/div.}$**

SOL. (d) Given

Current passing through galvanometer, $I = 6\text{mA}$

Deflection, $\theta = 2°$

Figure of merit of galvanometer $= \frac{I}{\theta} = \frac{6 \times 10^{-3}}{2} = 3 \times 10^{-3} \text{A/div}$

98. A galvanometer coil has 500 turns and each turn has an average area of $3 \times 10^{-4} \mathrm{~m}^2$. If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is

[NA Sep. 03, 2020 (II)]

SOL. (20)

Given,

Area of galvanometer coil, $A=3 \times 10^{-4} \mathrm{~m}^2$

Number of turns in the coil, $N=500$

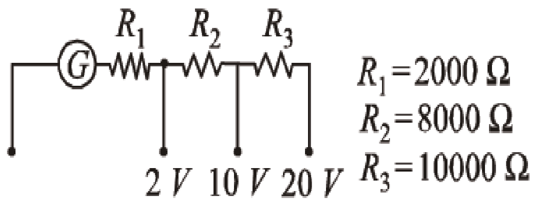
Current in the coil $I=0.5 \mathrm{A}$

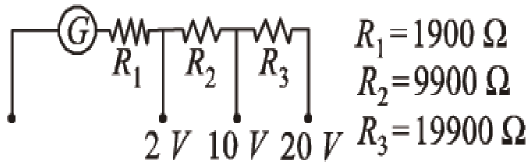
Torque $\Gamma=|\vec{M} \times \vec{B}|=N i A B \sin \left(90^{\circ}\right)=N i A B$

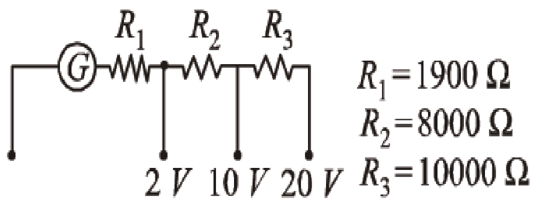
$$\Rightarrow B=\frac{\Gamma}{N i A}=\frac{1.5}{500 \times 0.5 \times 3 \times 10^{-4}}=20 T$$

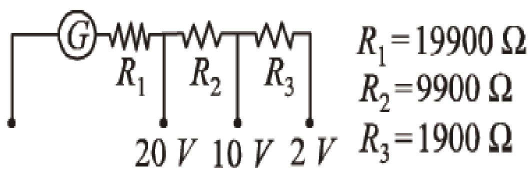
99. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of $0-2 \mathrm{V}$, $0-10$ V and 0-20 V. The appropriate circuit to do so is:

[12 April 2019, I]

(a) 

(b) 

(c) 

(d) 

SOL. (c) $i_g = 20 \times 50 = 1000\mu A = 1\text{mA}$

Using, $= i_g(G+R)$, we have

$$2 = 10^{-3}(100 + R_1)$$

$$R_1 = 1900\Omega$$

when, $V = 10$ volt

$$10 = 10^{-3}(100 + R + R)$$

$$10000 = (100 + R_2 + 1900)$$

$$R_2 = 8000\Omega$$

100. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter ofrange 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V $(V = GI_0)$ by connecting a series resistance R_v to it. Then,

[12 April 2019, II]

(a) $R_A R_V = G^2 \left(\frac{I_0 - I_g}{I_g}\right)$ **and** $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(b) $R_A R_V = G^2$ **and** $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(c) $R_A R_V = G^2 \left(\frac{I_g}{I_0 - I_g}\right)$ **and** $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(d) $R_A R_V = G^2$ **and** $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

SOL. (b) In an ammeter,

$$i_g = i_0 \frac{R_A}{R_A + G}$$

and for voltmeter,

$$V = i_g (G + R_V) = G i_0$$

On solving above equations, we get

$$R_A R_V = G^2$$

And $\frac{R_A}{R_V} = \left(\frac{i_g}{i_o - i_g}\right)^2$

101. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore the value of shunt resistance required to convert the above galvanometer into an ammeter of range $0 - 10$mA is:

[10April 2019, I]

(a) 500 Ω **(b) 100 Ω** **(c) 200 Ω** **(d) 10 Ω**

SOL. (Bonus) $v = i_g (R + G)$

$$\Rightarrow 5 = 10^{-4} (2 \times 10^6 + x)$$

$$x = -195 \times 10^4 \Omega$$

102. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to:

[9 April 2019 I]

(a) 40 V **(b) 15V** **(c) 20V** **(d) 10V**

SOL. (c) $V = i_g(G + R) = 4 \times 10^{-3}(50 + 5000) = 20V$

103. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately:

[9 April 2019, II]

(a) 10^{-4} **(b) 10^{-2}** **(c) 10^{-1}** **(d) 10^{-3}**

SOL. (d) $C\theta = NBiA \sin 90°$

or $10^{-6}\left(\frac{\pi}{180}\right) = 175B(10^{-3}) \times 10^{-4}$

$B = 10^{-3}$T

104. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it order to convert it into an ammeter of range $0-0.5$A?

[9 April 2019, II]

(a) 0.5 ohm **(b) 0.002 ohm** **(c) 0.02 ohm** **(d) 0.2 ohm**

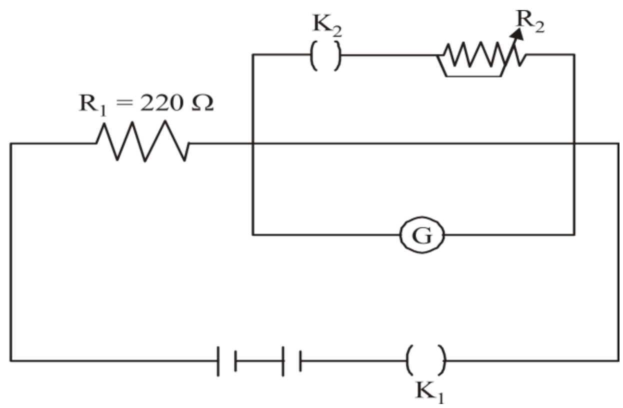
SOL. (d) Using, $i_g = i\frac{S}{S+G}$

$$0.002 = 0.5\frac{S}{S+50}$$

On solving, we get

$$S = \frac{100}{498} = 0.2\Omega$$

105. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance ofthe galvanometer is, then, given by [Neglect the internal resistance of battery]: [12 Jan 2019, I]



(a) 5Ω **(b) 22Ω** **(c) 25Ω** **(d) 12Ω**

SOL. (b) When key K_1 is closed and key K_2 is open

$$i_g = \frac{E}{220+R_g} = C\theta_0 \text{ ... (i)}$$

When both the keys are closed

$$i_g = \left(\frac{E}{220+\frac{5}{(R_g+5)}}\right) \times \frac{5}{(R_g+5)} = \frac{C\theta_0}{5}$$

$$\Rightarrow \frac{5E}{225R_g+1100} = \frac{C\theta_0}{5} \text{ ---------(ii)}$$

$$\frac{E}{220+R_g} = C\theta_0 \text{ ... (i)}$$

Dividing (i) by (ii), we get

$$\Rightarrow \frac{225R_g + 1100}{1100 + 5R_g} = 5$$

$$\Rightarrow 5500 + 25R_g = 225R_g + 1100$$

$$200R_g = 4400$$

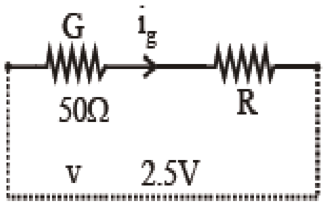
$$R_g = 22\Omega$$

106. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of:

[12 Jan 2019, II]

(a) 250 ohm **(b) 200 ohm** **(c) 6200 ohm** **(d) 6250 ohm**

SOL. (b) Galvanometer has 25 divisions $I_g = 4 \times 1\sigma^4 \times 25 = 10^{-2}$A



$V = I_g R_{net}$

$$v = I_g(G + R)$$

$$2.5 = (50 + R)10^{-2} \therefore R = 200\Omega$$

107. A galvanometer having a resistance of 20 Ω and 30 division on both sides has figure ofmerit 0.005 ampere/ division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is:

[11 Jan 2019, II]

(a) 100 Ω **(b) 120 Ω** **(c) 80 Ω** **(d) 125 Ω**

SOL. (c) Deflection current $= Ig_{\max} = nxk = 0.005 \times 30$

Where, n = Number of divisions = 30 and k = 0.005amp/ division

$$= 15 \times 10^{-2} = 0.15$$

$$v = I_g[20 + R]$$

$$15 = 0.15[20 + R]$$

$$100 = 20 + R$$

$$R = 80\Omega$$

108. **A galvanometer having a coil resistance 100 Ω gives a full scale deflection when a current of l mA is passed through it. What is the value of the resistance which can convert this galvanometer into a voltmeter giving full scale deflection for a potential difference of 10 V?**

[8 Jan 2019, II]

(a) $10\mathrm{k}\Omega$ **(b) $8.9\mathrm{k}\Omega$** **(c) $7.9\mathrm{k}\Omega$** **(d) $9.9\mathrm{k}\Omega$**

SOL. (d) Given,

Resistance of galvanometer, $G = 100\Omega$, Current, $i_g = 1mA$

A galvanometer can be converted into voltmeter by connecting a large resistance R in series with it.

Total resistance of the combination $= G + R$

According to Ohm's law, $V = i_g(G + R)$

$$10 = 1 \times 10^{-3}(100 + R_0)$$

$$\Rightarrow 10000 - 100 = 9900\Omega = \mathrm{R}_0$$

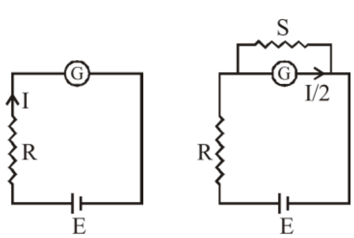
$$\Rightarrow \mathrm{R}_0 = 9.9\mathrm{k}\Omega$$

109. **In a circuit for finding the resistance of a galvanometer by half deflection method, a 6V battery and a high resistance of ll kΩ are used. The figure of merit of the galvanometer 60μA/division. In the absence of shunt resistance, the galvanometer produces a deflection of $\theta = 9$ divisions when current flows in the circuit. The value of the shunt resistance that can cause the deflection of $\theta/2$, is closest to**

[Online Apri116, 2018]

(a) 55Ω **(b) 110Ω** **(c) 220Ω** **(d) 550Ω**

SOL. (b) Figure of merit of a galvanometer is the current required to produce a deflection of one division in the galvanometer i. e., figure of merit $= \frac{\mathrm{I}}{\theta}$



$$I = \frac{\varepsilon}{R+G} \qquad G = \frac{1}{9} K\Omega$$

$$\frac{1}{2} = \frac{\varepsilon}{R + \frac{GS}{G+S}} \times \frac{S}{S+G} \Rightarrow \frac{1}{2} = \frac{\varepsilon S}{R(S+G)+GS}$$

$$S = \frac{RG \times \frac{1}{2}}{\varepsilon - \frac{(R+G)I}{2}}$$

$$S = \frac{11 \times 10^3 \times \frac{1}{2} \times 10^2 \times 270 \times 10^{-6}}{6 - \left(\frac{6}{2}\right)} = 110\Omega$$

110. A galvanometer with its coil resistance 25Ω requires a current of $1mA$ for its full deflection. In order to construct an ammeter to read up to a current of $2A$, the approximate value of the shunt resistance should be

[Online Apri116, 2018]

(a) $2.5 \times 10^{-2}\Omega$ (b) $1.25 \times 10^{-3}\Omega$ (c) $2.5 \times 10^{-3}\Omega$ (d) $1.25 \times 10^{-2}\Omega$

SOL. (d) According to question, current through galvanometer, $I_g = 1mA$

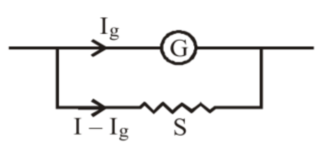
Current through shunt (I-Ig) $= 2A$

Galvanometer resistance $R_g = 25\Omega$

Resistance of shunt, S = ?

$I_0R_0 = (I - I_g)S$

$$\Rightarrow S = \frac{10^{-3} \times 25}{2}$$



$$S = 1.25 \times 10^{-2}\Omega$$

111. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 , it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range $0 - 10V$ is

[2017]

(a) $2.535 \times 10^3\Omega$ (b) $4.005 \times 10^3\Omega$ (c) $1.985 \times 10^3\Omega$ (d) $2.045 \times 10^3\Omega$

SOL. (c) Given : Current through the galvanometer,

$$i_g = 5 \times 10^{-3} A$$

Galvanometer resistance, $G = 15\Omega$

Let resistance R to be put in series with the galvanometer to convert it into a voltmeter.

$$V = i_g(R + G)$$

$$10 = 5 \times 10^{-3}(R + 15)$$

$$R = 2000 - 15 = 1985 = 1.985 \times 10^3 \Omega$$

112. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a currect of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is:

[2016]

(a) 0.1Ω (b) 3 Ω (c) 0.01Ω (d) 2 Ω

SOL. (c) $\text{Ig G} = (\text{I} - \text{Ig})\text{s}$

$$10^{-3} \times 100 = (10 - 10^{-3}) \times \text{S}$$

$$\text{S} \approx 0.01\Omega$$

113. A 50Ω resistance is connected to a battery of 5V. A galvanometer of resistance 100 Ω is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit?

[Online April 9, 2016]

(a) $r_s = 0.5\Omega$ in series with the galvanometer

(b) $r_s = 1\Omega$ in series with galvanometer

(c) $r_s = 1\Omega$ in parallel with galvanometer

(d) $r_s = 0.5\Omega$ in parallel with the galvanometer.

SOL. (d) As we know, $\text{I} = \frac{\text{V}}{\text{R}} = \frac{5}{50} = 0.1$

$$\text{I}' = 0.099$$

When Galvanometer is connected

$$R_{eq} = 50 + \frac{100S}{100 + S} = \frac{V}{I}$$

$$\Rightarrow \frac{100S}{100 + S} = \frac{5}{0.099} - 50$$

$$\Rightarrow \frac{100S}{100 + S} = 50.50 - 50 \Rightarrow \frac{100S}{100 + S} = 0.5$$

$$\Rightarrow 100S = 50 + 0.55 \Rightarrow 99.5S = 50$$

$$S = \frac{50}{99.05} = 0.5\Omega$$

So, shunt of resistance $= 0.5\Omega$ is connected in parallel with the galvanometer.

114. To know the resistance G of a galvanometer by half deflection method, a battery of emf V_E and resistance R is used to deflect the galvanometer by angle θ. If a shunt of resistance S is needed to get half deflection then G, R and S related by the equation:

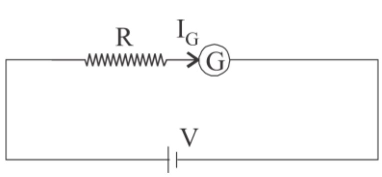
[Online April19, 2016]

(a) $S(R + G) = RG$ **(b) $2S(R + G) = RG$** **(c) $2G = S$** **(d) $2S = G$**

SOL. (a) According to Ohm's Law, $I = \frac{V}{R}$

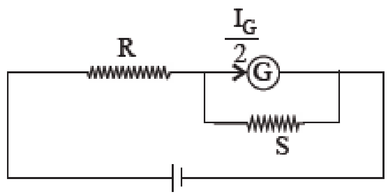
$$I_g = \frac{V}{R + G}$$

where, I_g-Galvanometer current, G-Galvanometer resistance



When shunt of resistance S is connected parallel to the Galvanometer then $G = \frac{GS}{G+S}$

$$I = \frac{V}{R + \frac{GS}{G + S}}$$



Equal potential difference is given by

$$I_g^{\uparrow}G = \left(I - I_g'\right)S$$

$$I_g^{\uparrow}(G + S) = IS$$

$$\Rightarrow \frac{I_g}{2} = \frac{IS}{G + S}$$

$$\Rightarrow \frac{V}{2(R + G)} = \frac{V}{R + \frac{GS}{G + S}} \times \frac{S}{G + S}$$

$$\Rightarrow \frac{1}{2(R + G)} = \frac{S}{R(G + S) + GS}$$

$$\Rightarrow R(G + S) + GS = 2S(R + G)$$

$$\Rightarrow RG + RS + GS = 2S(R + G)$$

$$\Rightarrow RG = 2S(R + G)\text{-}S(R+G)$$

$$RG = S(R + G)$$

115. The AC voltage across a resistance can be measured using a :

[Online April 11, 2015]

(a) hot wire voltmeter **(b) moving coil galvanometer**

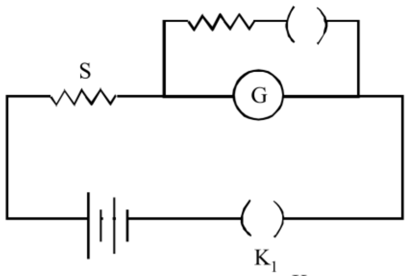
(c) potential coil galvanometer **(d) moving magnet galvanometer**

SOL. (b) To measure AC voltage across a resistance a moving coil galvanometer is used.

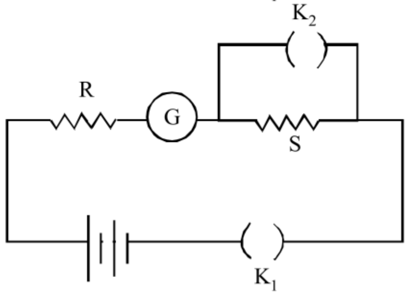
116. In the circuit diagrams (A, B, C and D) shown below, R is a high resistance and S is a resistance of the order of galvanometer resistance G. The correct circuit, corresponding to the half deflection method for finding the resistance and figure of merit of the galvanometer, is the circuit labelled as: R

[OnlineK_2 April 11, 2014]

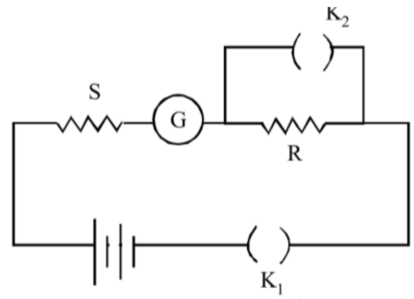
(A)



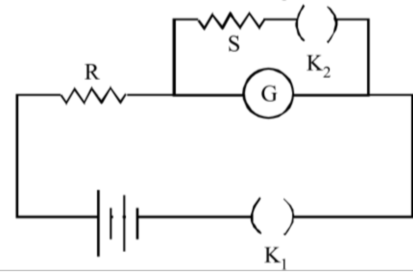
(B)



(C)



(D)



(a) Circuit A with $G = \frac{RS}{(R-S)}$ **(b) Circuit B with G = S**

(c) Circuit C with G = S **(d) Circuit D with** $G = \frac{RS}{(R-S)}$

SOL. (d) The correct circuit diagram is *D* with galvanometer resistance

$$G = \frac{RS}{R - S}$$

117. This questions has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes into two Statements.

Statement-I: Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.

[2013]

(a) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of Statement-I.

(b) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

(c) Statement-I is true, Statement-II is false.

(d) Statement-I is false, Statement-II is true.

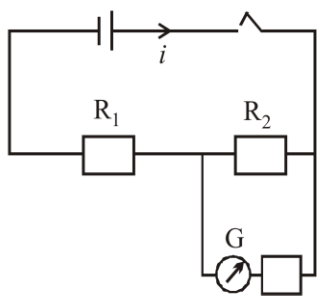
SOL. (d) Statements I is false and Statement II is true

For ammeter, shunt resistance, $S = \frac{IgG}{I - Ig}$

Therefore for I to increase, S should decrease, So additional S can be connected across it.

118. To find the resistance of a galvanometer by the half deflection method the following circuit is used with resistances $R_1 = 9970W$, $R_2 = 30W$ and $R_3 = 0$. The deflection in the galvanometer is d. With $R_3 = 107W$ the deflection changed to $\frac{d}{2}$. The galvanometer resistance is approximately:

[Online April 22, 2013]



R_3

(a) 107 Ω **(b) 137 Ω** **(c) 107/2 Ω** **(d) 77Ω**

SOL. (d)

119. A shunt of resistance 1 Ω is connected across a galvanometer of 120 Ω resistance. A current of 5.5 ampere gives full scale deflection in the galvanometer. The current that will give full scale deflection in the absence of the shunt is nearly:

[Online April 9, 2013]

(a) 5.5 ampere **(b) 0.5 ampere** **(c) 0.004 ampere** **(d) 0.045 ampere**

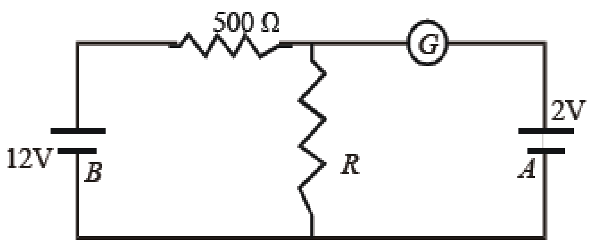
SOL. (d) The current that will given full scale deflection in the absence of the shunt is nearly equal to the current through the galvanometer when shunt is connected i.e. I_g

As $I_g = \frac{IS}{G+S}$

$= \frac{5.5 \times 1}{120+1} = 0.045$ ampere.

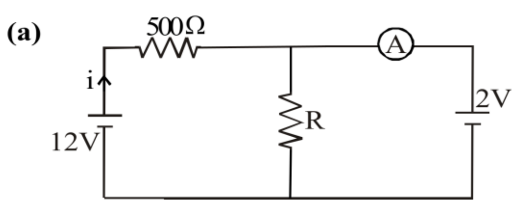
120. In the circuit , the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value ofthe resistor *R* will be-

[2005]



(a) 100Ω **(b) 200Ω** **(c) 1000Ω** **(d)500Ω**

SOL.



$$12 - 2 = (500\Omega)i \Rightarrow i = \frac{10}{500} = \frac{1}{50}$$

Again, $i = \frac{12}{500+R} = \frac{1}{50}$

$$\Rightarrow 500 + R = 600$$

$$\Rightarrow R = 100\Omega$$

121. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10-divisions per mill ampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 volt, the resistance in ohms needed to be connected in series with the coil will be-

[2005]

(a) 10^5 **(b) 10^3** **(c) 9995** **(d) 99995**

SOL. (c) Resistance of Galvanometer,

$$G=\frac{\text{Currentsensitivity}}{\text{Vo1tagesensitivity}}\Rightarrow G=\frac{10}{2}=5\Omega$$

Here $i_g=F$ull scale deflection current $=\frac{150}{10}=15\text{mA}$

$V=$ voltage to be measured $=150$ volts

(such that each division reads 1 volt)

$$\Rightarrow R=\frac{150}{15\times10^{-3}}-5=9995\Omega$$

MAGNETISM AND MATTER

Magnet: A body which attracts Iron,Cobalt, Nickel, like substances and which exhibits directive property is called Magnet.

Types of Magnet:

i) Natural magnets: a) The magnet which is found in nature is called a natural magnet

Eg: magnetite. (Fe_3O_4).

b) Generally they are weak magnets.

ii) Artificial magnets: The magnets which are artificially prepared are known as artificial magnets. These are generally made of iron, steel and nickel.

Properties of Magnets:

1) **Attractive property :** The property of attracting pieces of iron, steel , cobalt , nickel etc by a magnet is called attractive property. It was found that when a magnet is dipped into iron fillings the concentrations of iron fillings is maximum at ends and minimum at centre. The places in a magnet where the attracting power is maximum are called poles.

2. **Directive property :** If a magnet is suspended freely, its length becomes parallel to N-S direction. This is called directive property. The pole at the end pointing north is called north pole while the other pointing south is called south pole.

- Magnetic poles always exist in pairs If a magnet is broken into number of pieces, each piece becomes a magnet with two equal and opposite poles This implies that monopole do not exist.
- The two poles of a magnet are found to be equal in strength and opposite in nature.
- Unlike poles attract each other and like poles repel each other
- There can be magnets with no poles.

Eg: Solenoid and toroid has properties of magnet but no poles.

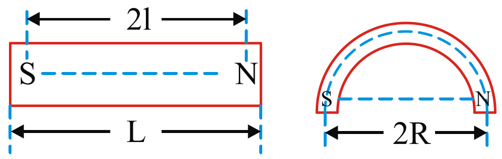
Magnetic axis and magnetic meridian

The line joining the poles of a magnet is called magnetic axis and the vertical plane passing through the axis of a freely suspended magnet is called magnetic meridian

Geometrical length (L) : The actual length of magnet is called geometric length

Magnetic length $(2l)$ The shortest distance between two poles of a magnet along the axis is called magnetic length or effective length. As the poles are not exactly at the ends the magnetic length is always lesser than geometric length of a magnet. Effective length depends only on the positions of the poles but not on the magnet

Examples :



Magnetic length = 2l   Magnetic length = 2R

Geometrical length = L Geometrical length $= \pi R$

Magnetic length is a vector quantity. its direction is from south pole to north pole along its axis

Magnetic length = $\frac{5}{6}$ Geometrical length

Pole Strength (m) : The ability of a pole to attract or repel another pole of a magnet is called pole strength S.I Unit : ampere - meter. Pole strength is a scalar It depends on the area of cross section of the pole. Its dimensional formula is $M^0LT^0A^1$

Inductive property: When a magnetic substance such as iron bar is kept very close to a magnet an opposite pole is induced at the nearer end and a similar pole is induced at the farther end of the magnetic substance.This property is known as inductive property.

A magnet attracts certain other magnetic substance through the phenomenon of magnetic induction. induction precedes attraction.

- Repulsion is a sure test of magnetism.A pole of a magnet attracts the opposite pole while repels similar pole.How ever a sure test of magnetism is repulsion but not attraction.Because attraction can takes place between opposite poles or between a pole and a piece of unmagnetized material due to induction.

Magnetic Moment

Magnetic dipole and magnetic dipole moment (M) : A configuration of two magnetic poles of opposite nature and equal strength separated by a finite distance is called as magnetic dipole.

The product of pole strength (either pole) and magnetic length of the magnet is called magnetic dipole moment or simply magnetic moment.

If **'m'** be the pole strength of each pole and '$2l$' be the magnetic length, then magnetic moment M is given by $M = m \times 2l$

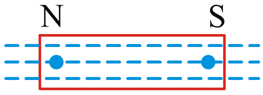
In vector form, $\overline{M} = \overline{2\ell}m$

Magnetic moment is a vector whose direction is along the axis of the magnet from south to north pole. The S.I. unit of magnetic moment is ampere-meter^2 (A-m^2) its dimensional formula [AL^2]

Variation of magnetic moment due to cutting of magnets :

Consider a bar magnet of length '$2l$', pole strength **'m'** and magnetic moment **'M'**

- **When the bar magnet is cut into ‘n’ equal parts parallel to its length, then**



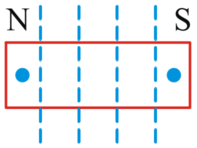
Pole strength of each part = m/n
($\because$ area of cross section becomes (1/n) times of original magnet)
Length of each part = $2l$ (remains same)

$\therefore$Magnetic moment of each part, $M^1 = 2l \times \frac{m}{n} = \frac{M}{n}$

Note: If it is cut ‘n’ times , parallel to its length then magnetic moment of each part is

$$M^1 = 2l \times \frac{m}{n+1} = \frac{M}{n+1}$$

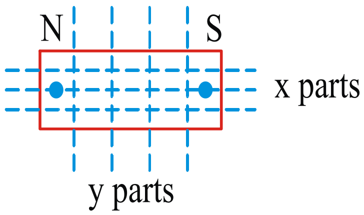
- **When the magnet is cut into ‘n’ equal parts perpendicular to its length then**



Pole strength of each part =m ($\because$ area of cross section remains same)
Length of each part $= 2l/n$

Magnetic moment of each part, $M^1 = \frac{2l}{n} \times m = \frac{M}{n}$

- **When the magnet is cut into ‘x’ equal parts parallel to its length and ‘y’ equal parts perpendicular to its length, then**

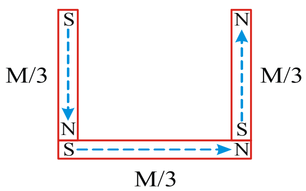


pole strength of each part = m/x
Length of each part = $2l/y$

Magnetic moment of each part, $M^1 = \frac{2l}{y} \times \frac{m}{x} = \frac{M}{xy}$

Variation of magnetic moment due to bending of magnets

- **When a bar magnet is bent, its pole strength remains same but magnetic length decreases. Therefore magnetic moment decreases.**
- **When a thin bar magnet of magnetic moment M is bent in the form of ⊔ -shape with the arms of equal length as shown in figure, then**

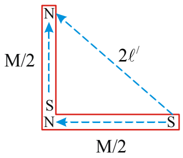


Magnetic moment of
each part = M / 3
Net magnetic moment of the combination,

$$\overline{M^1} = \frac{M}{3}\left(-\bar{j}\right) + \frac{M}{3}\left(\bar{i}\right) + \frac{M}{3}\left(\bar{j}\right) = \frac{M}{3}\left(\bar{i}\right)$$

$$\therefore M^1 = \frac{M}{3}$$

- **When a thin magnetic needle of magnetic moment M is bent at the middle, so that the two equal parts are perpendicular as shown in figure, then**

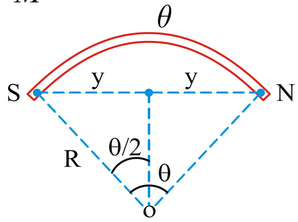


Magnetic moment of each part = $\frac{M}{2}$

Net magnetic moment of the combination, $\overline{M^1} = \frac{M}{2}\left(-\bar{i}\right) + \frac{M}{2}\left(\bar{j}\right) \quad \therefore M^1 = \sqrt{2} \times \frac{M}{2} = \frac{M}{\sqrt{2}}$

- **When a thin bar magnet of magnetic moment M is bent into an arc of a circle subtending an angle 'θ' radians at the centre of the circle**, then its new magnetic moment is given by $M^1 = \frac{2M\sin\left(\frac{\theta}{2}\right)}{\theta}$ (θ must be in radians)

$ie\ \theta = \frac{21}{R} \Rightarrow R = \frac{21}{\theta}$

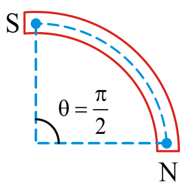


from the figure, Effective length = 2y= 2R $\sin\frac{\theta}{2}$ $\left(\because \sin\frac{\theta}{2} = \frac{y}{R} \Rightarrow y = R\sin\left(\frac{\theta}{2}\right)\right)$

$\therefore$ New Magnetic Moment, $M^1 = m \times 2y = m \times 2\left(\frac{21}{\theta}\right)\sin\frac{\theta}{2}$

$$\Rightarrow M^1 = \frac{2M\sin\left(\frac{\theta}{2}\right)}{\theta} \qquad \left(\because M = 21 \times m\right)$$

- If $\theta = \frac{\pi}{2}$ radians,

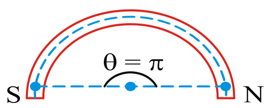


i.e., if the magnet is bent in the form of quadrant of a circle, then

$$M^1 = \frac{2M \sin\frac{\pi}{4}}{\left(\frac{\pi}{2}\right)} = \frac{2\sqrt{2}M}{\pi}$$

- If $\theta = \pi$ radians, i.e., if the magnet is bent in the form of a semi circle, then

$$M^1 = \frac{2M \sin\frac{\pi}{2}}{\pi} = \frac{2M}{\pi}$$



- If $\theta = 2\pi$ radians, i.e., if the magnet is bent in the form of a circle, then

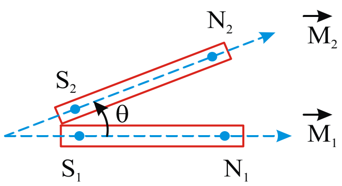
$$M^1 = \frac{2M \sin\pi}{2\pi} = 0$$

- **When a magnet in the form of an arc of a circle making an angle 'θ'at the centre having magnetic moment 'M' is straightened, then**

A Effective length of the magnet increases. Hence Magnetic moment increases

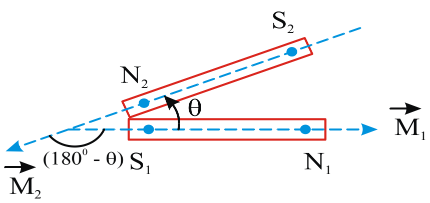
ANew magnetic moment is given by $M^1 = \frac{M\theta}{2\sin\left(\frac{\theta}{2}\right)}$ (θ must be in radians)

Resultant Magnetic Moment due to combination of Magnets :

- 

When two bar magnets of moments M_1 and M_2 are joined so that their like poles touch each other and their axes are inclined at an angle 'θ', then the resultant magnetic moment of the combination 'M^1' is given by $M^1 = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos\theta}$

($\theta =$ angle between the directions of magnetic moments)

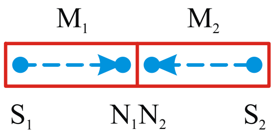
- 

When two bar magnets of moments M_1 and M_2 are joined so that their unlike poles touch each other and their axes are inclined at an angle 'θ', then the resultant magnetic moment

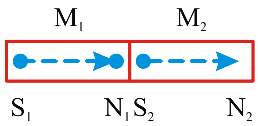
$$M^1 = \sqrt{M_1^2 + M_2^2 + 2M_1M_2\cos(180^0 - \theta)}$$

[$\because$ angle between directions of magnetic moments is $(180^0 - \theta)$]

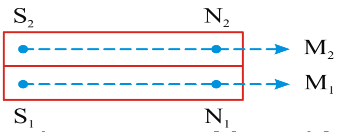
$$\therefore M^1 = \sqrt{M_1^2 + M_2^2 - 2M_1M_2\cos\theta}$$

- 

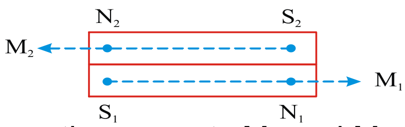
When two bar magnets of moments M_1 and M_2 ($M_1 > M_2$) are placed coaxially with like poles in contact then resultant magnetic moment, $M^1 = M_1 – M_2$
($\because$ angle between directions of magnetic moments, $\theta = 180^0$)

- 

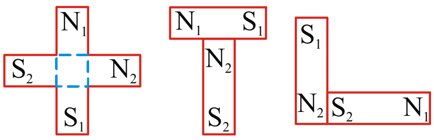
When two bar magnets of moments M_1 and M_2 ($M_1 > M_2$) are placed coaxially with unlike poles are in contact then resultant magnetic moment, $M^1 = M_1 + M_2$
($\because$ angle between directions of magnetic moments, $\theta = 0^0$)

- 

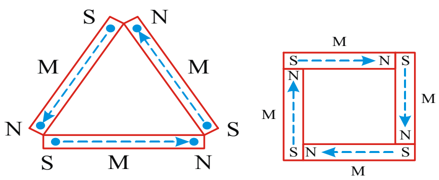
When two bar magnets of magnetic moments M_1 and M_2 are placed one over the other with like poles on the same side, then resultant magnetic moment, $M^1 = M_1 + M_2$ $\left(Q\ \theta = 0^0\right)$

- 

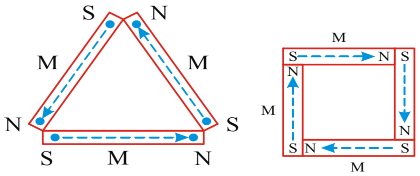
When two bar magnets of magnetic moments M_1 and M_2 are placed one over the other with unlike poles on the same side, then resultant magnetic moment, $M^1 = M_1 : M_2$. $\left(Q\ \theta = 180^0\right)$

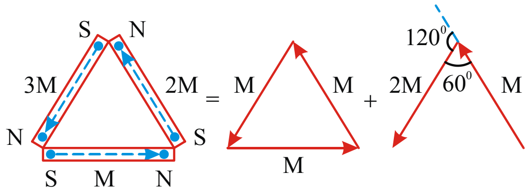
- 

When two bar magnets of magnetic moments M_1 and M_2 are placed at right angles to each other then resultant magnetic moment, $M^1 = \sqrt{M_1^2 + M_2^2}$ $\left(Q\ \theta = 90^0\right)$.

- 

When identical magnets each of magnetic moment M are arranged to form a closed polygon like a triangle (or) square with unlike poles at each corner, then resultant magnetic moment, $M^1 = 0$.

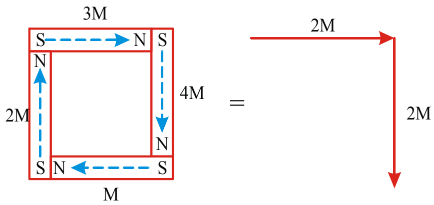
- 

  When identical magnets each of magnetic moment M are arranged to form a closed polygon like a triangle (or) square with unlike poles at each corner, then resultant magnetic moment, $M^1 = 0$.
- In the above point , if one of the magnets is reversed pole to pole then resultant magnetic moment, $M^1 = 2M$
- 

  When three bar magnets of equal length but moments M, 2M and 3M are arranged to form an equilateral triangle with unlike poles at each corner, resultant magnetic moment is given by

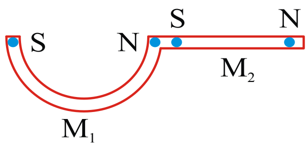
$$M^1 = \sqrt{(2M)^2 + M^2 + 2(2M)(M)\cos 120^0} = \sqrt{3}M$$

- When four bar magnets of moments M, 2M, 3M & 4M are arranged to form a square with unlike poles at each corner, then resultant magnetic moment is given by



$$M^1 = \sqrt{(2M)^2 + (2M)^2 + 2(2M)(2M)\cos 90^0} = 2\sqrt{2}M$$

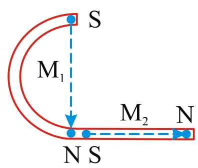
- When half of the length of a thin bar magnet of magnetic moment M is bent into a semi circle as shown in figure, then



resultant magnetic moment, $M^1 = M_1 + M_2$ $= \frac{2\left(\frac{M}{2}\right)}{\pi} + \frac{M}{2} = \frac{M}{\pi} + \frac{M}{2} = M\left(\frac{2+\pi}{2\pi}\right)$

- In the above case if the two parts are arranged perpendicular to each other, then resultant magnetic moment is

$$M^1 = \sqrt{M_1^2 + M_2^2} \quad = \sqrt{\left(\frac{M}{\pi}\right)^2 + \left(\frac{M}{2}\right)^2} = \frac{M}{2\pi}\sqrt{\left(4+\pi^2\right)}$$

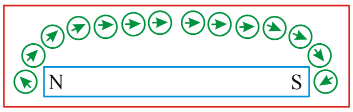


Magnetic field :

- Around a pole there exist a region called magnetic field in which the influence of the pole is felt.
- The space around the magnet is said to be associated with a field known as magnetic field, if another magnet is brought into the space, it is acted upon by a force due to this energy.
- Magnetic induction is the measure of magnetic field both in magnitude and direction.

Magnetic Field Lines :

- The imaginary path in which a free unit north pole would tend to move in a magnetic field is known as a magnetic line of force (or) simply magnetic "field line".



Magnetic line of force with magnetic needle

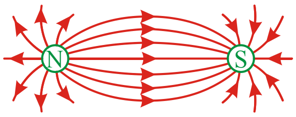
Characteristics of lines of force :

i) Magnetic lines of force are closed curves. Outside the magnet, their direction is from north to south pole, while inside the magnet they are from south to north pole. Hence they have neither origin nor end.

ii) Tangent, at any point to the line of force gives the direction of magnetic field at that point.

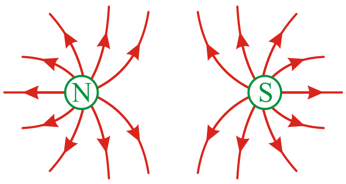
iii) Two lines of force never intersect each other. If the two lines of force intersect, at the intersecting point the field should have two directions, which is not possible.

iv) The lines of force tend to contract longitudinally or length wise . Due to this property the two unlike poles attract each other.



Magnetic lines of force between two unlike poles.

v) The lines of force tend to repel each other laterally. Due to this property the two similar poles repel each other.



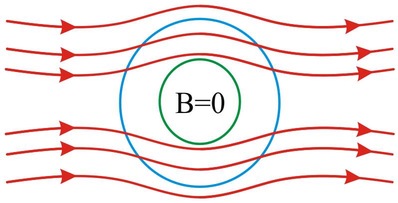
Magnetic lines of force between two like poles

vi) If in any point, in the combined field due to two magnets, there are no lines of force, it follows that the resultant field at that point is zero. Such points are called null or neutral points.

vii) Lines of force in a field represent the strength of the field at a point in the field. Lines of force are crowded themselves in regions where the field is strong and they spread themselves apart at places where the field is weak.

viii) Lines of force have a tendency to pass through magnetic substances. They show maximum tendency to pass through ferro magnetic materials.

ix) When a soft iron ring is placed in magnetic field, then most of lines of force pass through the ring and no lines of force pass through the space inside the ring as shown in figure. The phenomenon is known as magnetic screening or shielding.



x) If the magnetic lines of force are straight and parallel, and equally spaced the magnetic field is said to be uniform.

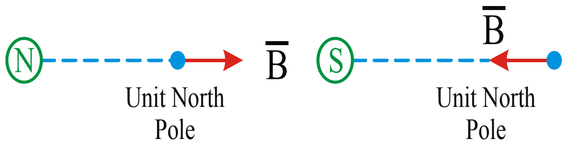
Magnetic Induction (or) Induction Field Strength (B)

Magnetic induction field strength at a point in the magnetic field is defined as the force experienced by unit north pole placed at that point. It is denoted by 'B'.

If a pole of strength 'm' placed at a point in a magnetic field experiences a force 'F', the magnetic induction (B) at that point is given by

$\overline{B} = \frac{\overline{F}}{m}$ i.e., $\overline{F} = m\overline{B}$

- B is a vector quantity directed away from N-pole or towards S-pole.



S.I. Unit of B :

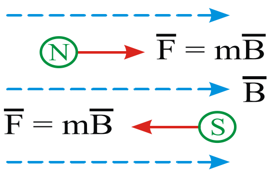
$$\frac{N}{A-m}(or)\frac{J}{A-m^2}(or)\frac{V-s}{m^2}(or)\frac{wb}{m^2}(or)tesla(T)$$

CGS Unit of B : gauss (G) 1G = 10^{-4} T

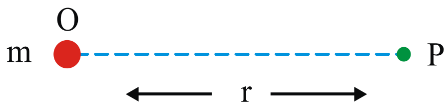
Dimensions of B :

$$B = \frac{F}{m} = \frac{[MLT^{-2}]}{[AL]} = [MT^{-2}A^{-1}]$$

When placed in an external magnetic field, all N-poles experience a force (F = mB) in the direction of the field and all S-poles experience the same force in the direction opposite to the field.



Magnetic induction at a point due to an isolated magnetic pole :
Consider a magnetic pole of strength 'm' kept at the point 'O'. Consider a point 'P' at a distance 'r' from 'O'. To find the magnetic induction at the point 'P', imagine a unit north pole at P.



Force on unit north pole at $P = \frac{\mu_o}{4\pi} \frac{m \times 1}{r^2}$ N

Force on unit north pole at 'P' gives the magnetic induction at that point.
∴ Magnetic induction at P is

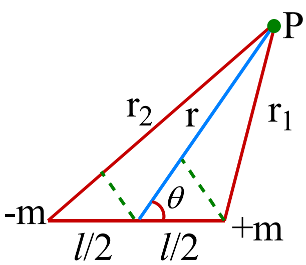
$B = \frac{\mu_o}{4\pi} \frac{m}{r^2}$ newton/amp–metre (or) tesla (T)

Magnetic potential : The amount of work done in bringing a unit north pole from infinity to a point in magnetic field is known as magnetic potential at the point.
It is a scalar
SI unit Joule/amp-m
For a pole strength m, the field at a distance r is

$B = \frac{\mu_0}{4\pi} \frac{m}{r^2}$, and radially away from the pole. The potential at a distance r is given by,

$V = -\int_{\infty}^{r} \vec{B} \cdot d\vec{r} = -\int_{\infty}^{r} \frac{\mu_0}{4\pi} \frac{m}{r^2} \times dr = \frac{\mu_0}{4\pi} \frac{m}{r}$ **Note :** $B = -\frac{\delta V}{\delta r}$

Magnetic potential due to a dipole : Consider a magnetic dipole of moment M. If m is the pole strength and *l* is the distance between the poles, then $M = ml$. If r_1 and r_2 are the distances of point P from the poles, then $r_1 = r - \frac{l}{2}\cos\theta$ and $r_2 = r + \frac{l}{2}\cos\theta$



Magnetic potential at P,

$$V = V_N + V_S = \frac{\mu_0}{4\pi}\left[\frac{m}{r_1} - \frac{m}{r_2}\right] = \frac{\mu_0 m}{4\pi}\left(\frac{l\cos\theta}{r^2 - \frac{l^2}{4}\cos^2\theta}\right)$$

Putting $ml = M$, and neglecting l^2 in comparison to r, we get

$$V = \frac{\mu_0}{4\pi}\frac{M\cos\theta}{r^2}$$

Case I : On the axial line of short Barmagnet

$$\theta = 0^0 \Rightarrow V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$$

Case II : On the equitorial line of short Barmagnet

$$\theta = 90^0 \Rightarrow V = 0$$

Types of Magnetic Field

- **Uniform magnetic field:** The magnetic field, in which the magnetic induction field strength is same both in magnitude and direction at all points, is known as uniform magnetic field.
- In such a magnetic field the magnetic lines of force are equidistant and parallel straight lines.
  Ex: Horizontal component of earth's magnetic field in a limited region.
- **Non uniform magnetic field:** The magnetic field, in which the magnetic induction or field strength differs either in magnitude, in direction or both is known as non uniform magnetic field.
- It is represented by non-parallel lines of force
  Ex: The magnetic field near the pole of any magnet

Magnetic flux (ϕ): It is equal to the total number of magnetic lines of force passing normal through a given area. Its S.I. unit is weber and C.G.S. unit is maxwell

1 weber = 10^8 maxwell

$$\phi = \vec{B}.\vec{A} = BA\cos\theta$$

Where 'θ' is the angle made by magnetic field $(\vec{B})$ with the area $(\hat{n})$

$\vec{A} = A\hat{n}$ A = area of the coil

It is a scalar. Dimensional formula is $\left[ML^2T^{-2}I^{-1}\right]$.

Magnetic Flux Density (B): The number of magnetic flux lines passing per unit area of cross section normal to the cross section is called magnetic flux density.

B= ϕ_B / A

SI unit is weber metre^{-2} or tesla or $NA^{-1}m^{-1}$.

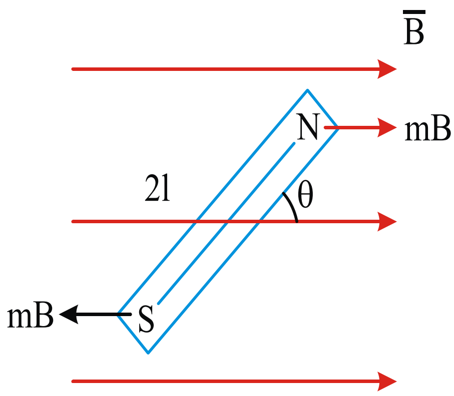
Its C.G.S. unit is gauss

1 gauss = 10^{-4} tesla

- Its dimensional formula is $[M^1L^0T^{-2}A^{-1}]$
- It is also known as magnetic induction and magnetic field.
- The relation between B and H is $B_0 = m_0H$ in vacuum and B = m H in a material medium Where m is the absolute permeability of the medium.
- The force experienced by a pole of strength 'm' ampere meter in a field of induction B is F = m B

Couple acting on the bar magnet (or) Torque on a Magnetic Dipole

- When a bar magnet of moment M and length 2l is placed in a uniform field of induction B, then each pole experiences a force mB in opposite directions.



As a result the bar magnet experiences a couple and moment of couple is developed.

- Moment of couple acting on the bar magnet is C = Force x perpendicular distance between two forces.

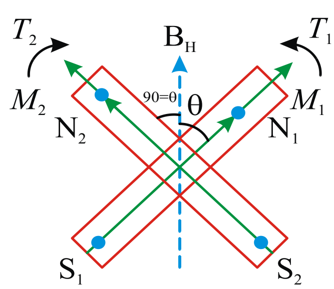
  $C = (m)(2l)\, B\sin\theta$ (or) C = M B sinq Where q is the angle between magnetic moment and magnetic field.

  In vector notation $\vec{C} = \vec{M} \times \vec{B}$

- When the bar magnet is either along or opposite to the direction of magnetic field then moment of couple=0.
- When the bar magnet is perpendicular to the direction of applied magnetic field, then the moment of couple is maximum. i.e. C_{max} = MB
- In a uniform magnetic field a bar magnet experiences only a couple but no net force. Therefore it undergoes only rotatory motion.
- In a non-uniform magnetic field a bar magnet experiences a couple and also a net force. So it undergoes both rotational and translational motion
- Two magnets of magnetic moments M_1 and M_2 are joined in the form of a (+) and this arrangement is pivoted so that it is free to rotate in a horizontal plane under the influence of earth's horizontal magnetic field. If 'θ' is the angle made by the magnetic meridian with M_1 in equilibrium position, then

  $\tau_1 = \tau_2$; i.e., $M_1 B_H \sin\theta$

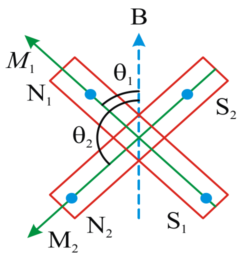
  $= M_2 B_H \sin(90-\theta)$; $\therefore \tan\theta = \frac{M_2}{M_1}$



- Two magnets of moments M_1 and M_2 are joined as shown in figure and the arrangement is pivoted so that it is free to rotate in a horizontal plane under the influence of magnetic field B. Then net torque acting on the system is given by

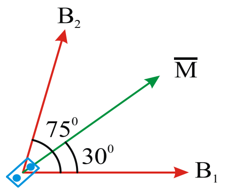
$$\tau = \tau_1 + \tau_2 \ ; \quad = M_1 B \sin\theta_1 + M_2 B \sin\theta_2$$

$$= B\left(M_1 \sin\theta_1 + M_2 \sin\theta_2\right)$$



Two uniform magnetic fields of strengths B_1 and B_2 acting at an angle 75^0 with each other in horizontal plane are applied on a magnetic needle of moment M, which is free to move in the horizontal plane. If the needle gets aligned at an angle 30^0 with B_1, then the ratio B_1/B_2 is

In equilibrium position,



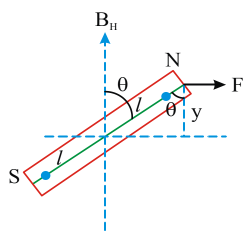
$\tau_1 = \tau_2$; i.e., $MB_1 \sin 30^0$

$= M B_2 \sin(75^0 - 30^0)$; $\therefore \dfrac{B_1}{B_2} = \dfrac{\sin 45^0}{\sin 30^0} = \dfrac{\sqrt{2}}{1}$

- A pivoted magnetic needle of length $2l$ and pole strength 'm' is at rest in magnetic meridian. It is held in equilibrium at an angle 'θ' with B_H by pulling its north pole towards east by a string. Then tension in the string is

from the figure ,

$\cos\theta = \dfrac{y}{l} \Rightarrow y = l\cos\theta$ In equlibrium

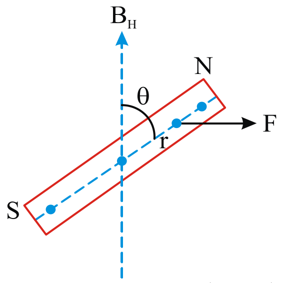


$\tau_{tension} = \tau_{B_H}$; i.e., $Fl\cos\theta = MB_H \sin\theta$

(or) $Fl\cos\theta = 2l\, m B_H \sin\theta$

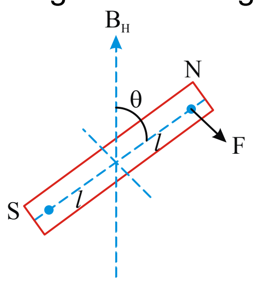
$\therefore F = 2mB_H \tan\theta$

b) In the above case, if the magnetic needle is held in equilibrium at an angle 'θ' to a uniform magnetic induction field B_H by applying a force F at a distance 'r' from the pivot along a direction perpendicular to the field, then



$Fr\cos\theta = MB_H\sin\theta$; $\therefore F = \frac{MB_H\tan\theta}{r}$ $= \frac{(2lm)B_H\tan\theta}{r}$

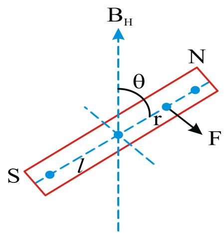
c) In the above case, if the force is applied at one end which is always perpendicular to length of the magnetic needle, then



$\tau_{tension} = \tau_{B_H}$

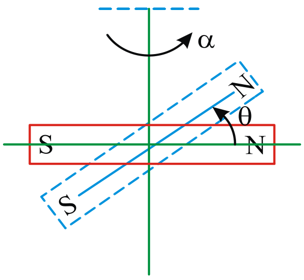
i.e., $Fl = (2lm)B_H\sin\theta$; $\therefore F = 2mB_H\sin\theta$

d) In the above case, if the force applied is always perpendicular to length of the magnetic needle but at a distance 'r' from the pivot, then



$Fr\sin 90^0 = MB_H\sin\theta$ $\therefore F = \frac{MB_H\sin\theta}{r}$; $= \frac{2lmB_H\sin\theta}{r}$

- A magnet of moment 'M' is suspended in the magnetic meridian with an untwisted wire. The upper end of the wire is rotated through an angle 'α' to deflect the magnet by an angle 'θ' from magnetic meridian. Then deflecting couple acting on the magnet = $MB_H\sin\theta$



Restoring couple developed in suspension wire = $C(\alpha-\theta)$ where C is couple per unit twist of suspension wire. $\therefore$ In equilibrium position, $MB_H\sin\theta = C(\alpha-\theta)$

EX. 1 : When a bar magnet is placed at 90° to a uniform magnetic field, it is acted upon by a couple which is maximum. For the couple to be half of the maximum value, at what angle should the magnet be inclined to the magnetic field (B) ?

Sol. We know that, $\tau = MB \sin\theta$

If $\theta = 90°$ then $\tau_{max} = MB$ ..... (1)

$\frac{\tau_{max}}{2} = MB\sin\theta$ ..... (2)

From equations (1) and (2)

$2 = \frac{1}{\sin\theta}$ or $\sin\theta = \frac{1}{2}$ or $\theta = 30°$

EX. 2 : A bar magnet of magnetic moment M_1 is suspended by a wire in a magnetic field. The upper end of the wire is rotated through 180^0, then the magnet rotated through 45^0. Under similar conditions another magnet of magnetic moment M_2 is rotated through 30^0. Then find the ratio of M_1 & M_2 .

Sol. $C(\alpha - \theta) = MB\sin\theta$

For first magnet, $C(180-45) = M_1 B \sin 45^0$ ----(1)

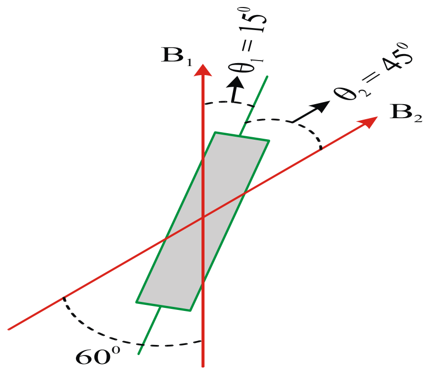
For second magnet, $C(180-30) = M_2 B \sin 30^0$ ----(2)

Diving equation (1) by equation (2)

$\frac{135}{150} = \frac{M_1}{M_2} \times \sqrt{2} \Rightarrow \frac{M_1}{M_2} = \frac{9}{10\sqrt{2}}$

EX. 3 :: A magnetic dipole is under the influence of two magnetic fields. The angle between the two field directions is 60^0 and one of the fields has a magnitudeof 1.2×10^{-2}T. If the dipole comes to stable equilibrium at an angle of 15^0 with this field, what is the magnitude of the other field?

Sol. Here $B_1 = 1.2 \times 10^{-2}$ T Inclination of dipole with B_1 is $\theta_1 = 15^0$ Therefore, inclination of dipole with B_2 is $\theta_2 = 60^0 - 15^0 = 45^0$ As the dipole is in equilibrium, therefore the torque on the dipole due to the two fields are equal and opposite. If M is magnetic dipole moment of the dipole, then



$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$ or $B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$

$= \frac{1.2 \times 10^{-2} \times \sin 15^0}{\sin 45^0}$; $= \frac{1.2\times10^{-2}\times0.2588}{0.707}$; $= 4.39\text{x}10^{-3}$ T

EX. 4 : A compass needle of magnetic moment $60 A\text{-}m^2$, pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is 40μ wb/m^2 experiences a torque of 1.2×10^{-3} Nm. Find the declination at that place.

Sol. If θ is the declination of the place, then the torque acting on the needle is $\tau = M B_H \sin\theta$

$$\Rightarrow \sin\theta = \frac{\tau}{M B_H} = \frac{1.2 \times 10^{-3}}{60 \times 40 \times 10^{-6}} = \frac{1}{2} \therefore \theta = 30^0$$

Work done in rotating a magnetic dipole in a magnetic field

- The work done in deflecting a magnet from angular position θ_1 to an angular position θ_2 with the field is change in PE given as $W = MB(\cos\theta_1 - \cos\theta_2)$
- The work done in deflecting a bar magnet through an angle θ from its state of equilibrium position in a uniform magnetic field is given by $W = MB(1-\cos\theta)$ $\left[\text{here } \theta_1 = 0^0, \theta_2 = \theta\right]$
  When it is released, this workdone converts into rotational KE
  $MB(1\text{-}Cos\theta) = \frac{1}{2} I\omega^2$
- When a bar magnet is held at an angle θ with the magnetic field, the potential energy possessed by the magnet is $U = -MB\cos\theta$
- When the bar magnet is parallel to the applied field, then $\theta = 0^0$ and potential energy is (-MB).It is said to be stable equilibrium.
- When the bar magnet is perpendicular to the applied field, then $\theta = 90^0$ and potential energy is zero
- When the bar magnet is anti-parallel to the applied field, then $\theta = 180^0$ and potential energy is maximum i.e. $U = +MB$.It is said to be unstable equilibrium.

EX. 5: A magnet is suspended at an angle 60^0 in an external magnetic field of 5×10^{-4} T. What is the work done by the magnetic field in bringing it in its direction ? [The magnetic moment = 20 A-m^2]

Sol. Work done by the magnetic field, $W = MB(\cos\theta_1 - \cos\theta_2)$ Here $\theta_1 = 60^0$ and $\theta_2 = 0^0$

$$\therefore W = 20 \times 5 \times 10^{-4}\left[\cos 60^0 - \cos 0^0\right] \qquad = 10^{-2}\left[\frac{1}{2} - 1\right] = -5 \times 10^{-3} \text{ J}.$$

EX. 6 : A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60^0. What is the torque needed to maintain the needle in this positon?

Sol. In case of a dipole in a magnetic field,

$W = MB(\cos\theta_1 - \cos\theta_2)$ and $C = MB\sin\theta$

Here, $\theta_1 = 0^0$ and $\theta_2 = 60^0$

So, $W = MB(1-\cos\theta) = 2MB\sin^2\frac{\theta}{2}$

and , $C = MB\sin\theta = 2MB\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

So, $\frac{C}{W} = \cot\left(\frac{\theta}{2}\right)$, i.e $C = W\cot 30^0 = \sqrt{3}W$

EX. 7 : A bar magnet has a magnetic moment 2.5 J T^{-1} and is placed in a magnetic field of 0.2 T. Calculate the work done in turning the magnet from parallel to antiparallel position relative to field direction.

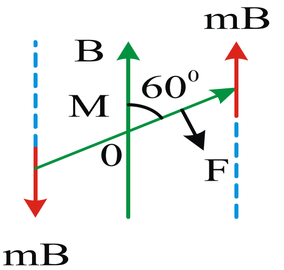
Sol. Work done in changing the orientation of a dipole of moment M in a field B from position $\theta_1 to \theta_2$ is given by $W = MB(\cos\theta_1 - \cos\theta_2)$

Here, $\theta_1 = 0^0$ and $\theta_2 = 180^0$

So, $W = 2MB = 2\times 2.5\times 0.2 = 1J$

EX. 8 : A bar magnet with poles 25 cm apart and pole-strength 14.4 A-m rests with its centre on a frictionless pivot. It is held in equilibrium at 60^0 to a uniform magnetic field of induction 0.25 T by applying a force F at right angles to its axis, 10cm from its pivot. Calculate F. What will happen if the force is removed?

Sol. The situation is shown in figure. In equilibrium the torque on M due to B is balanced by torque due to F, i.e., i.e., $\vec{M}\times\vec{B} = \vec{r}\times\vec{F}$



$MB\sin\theta = Fr\sin 90^0$ **or** $F = \dfrac{(m\times 2l)B\sin\theta}{r}$

(as M = m x 2l) ; So substituting the given data,

$F = \dfrac{14.4\times(25\times10^{-2})\times0.25(\sqrt{3}/2)}{10\times10^{-2}} = 7.8$ N

If the force $\vec{F}$ is removed, the torque $\vec{M}\times\vec{B}$ will become unbalanced and under its action the magnet will execute oscillatory motion about the direction of B on its pivot O which will not be simple harmonic as $\sin\theta \neq \theta$

Field of a Bar Magnet

Axial line: The magnetic induction at a point on the axial line is $B_a = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{2Md}{(d^2 - l^2)^2}$

For a short bar magnet i.e. $l <<<< d$

then $B_a = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{2M}{d^3}$

↳ The direction of magnetic induction on the axial line is along the direction of magnetic moment.

Equatorial line: The magnetic induction at a point on the equatorial line at a distance d from the centre is $B_e = \dfrac{\mu_0}{4\pi}\dfrac{M}{(d^2 + l^2)^{3/2}}$

For a short bar magnet i.e. $l << d$

then $B_e = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{M}{d^3}$

↳ The direction of magnetic induction on the equatorial line is in the direction opposite to magnetic moment.

At any point in the plane of axial and equatorial lines:

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{M\sqrt{(3\cos^2\theta+1)}}{d^3}$$

$\theta = 0^0$ for axial line ; $\theta = 90^0$ for equatorial line

↳ For a short bar magnet , at two equidistant points , one on the axial and the other on equitorial line $B_a = 2B_e$

Force between two magnets : When one magnet is placed in the field of another magnet it usually experiences a couple or force or both and has potential energy. Depending on the orientation of the magnets relative to each other, the following situations are discussed .

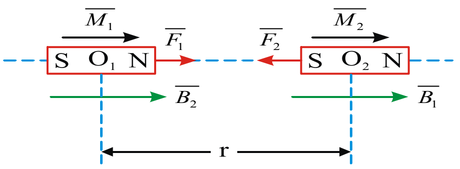
↳ **When magnets are along the line joining their centres**

If the opposite poles of two magnets face each other as shown in Fig.(A), the field due to $\vec{M}_1$ at the position of $\vec{M}_2$, i.e., at O_2, will be :

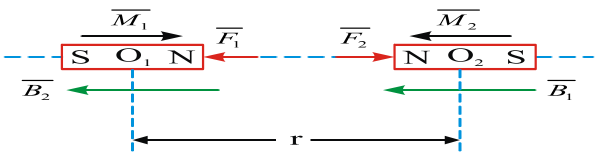
$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{2M_1}{r^3} \quad \text{with } \theta = 0^0 \qquad [\text{ as } O_2 \text{ lies on the axis of } \vec{M}_1]$$

So couple on $\vec{M}_2$ due to $\vec{M}_1$, i.e., $\vec{B}_1$ is

$$\vec{C}_2 = \vec{M}_2 \times \vec{B}_1 = 0 \qquad [\text{ as } \vec{M}_2 \text{ is parallel to } \vec{B}_1 \text{, i.e., } \theta = 0] \text{........(1)}$$



Attraction
(A)



Repulsion
(B)

Similarly, $\vec{C}_1 = \vec{M}_1 \times \vec{B}_2 = 0$

[as $\vec{M}_1$ is parallel to $\vec{B}_2$ i.e., $\theta = 0$] ....(2)

i.e., the magnets will not exert any couple on each other

And as $U = -\vec{M}\bullet\vec{B}$, the interaction energy of the system (i.e., P.E. of $\vec{M}_2$ in the field of $\vec{M}_1$ or P.E. of $\vec{M}_1$ in the field of $\vec{M}_2$) will be

$$U = -\vec{M}_2 \bullet \vec{B}_1 = -\vec{M}_1 \bullet \vec{B}_2 = -\frac{\mu_0}{4\pi}\frac{2M_1M_2}{r^3} \qquad [\text{as } \vec{M}_2 \text{ is parallel to } \vec{B}_1 \text{, i.e., } \theta = 0^0] \text{.........(3)}$$

Now as F = - (dU/dr) so force on $\vec{M}_1$ due to $\vec{M}_2$ or force on $\vec{M}_2$ due to $\vec{M}_1$ will be

$$F_1 = F_2 = -\frac{d}{dr}\left[-\frac{\mu_0}{4\pi}\frac{2M_1M_2}{r^3}\right]$$

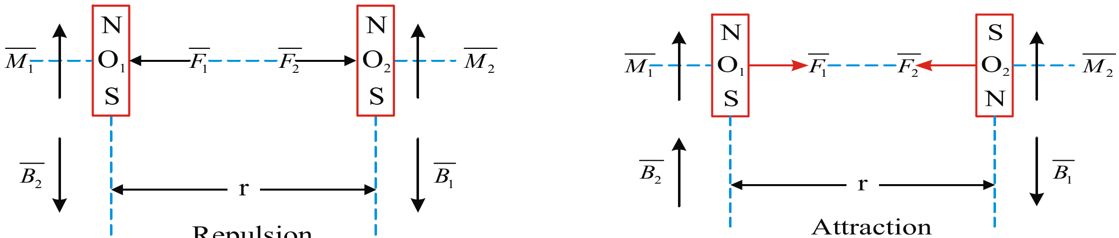
$$= -\frac{\mu_0}{4\pi}\frac{6M_1M_2}{r^4} \qquad \text{———(4)}$$

From equation (4) it is clear that **interaction**

↳ **force between the magnets varies as ($1/r^4$).**

When magnets are perpendicular to the line joining their centres

If the similar poles of two magnets face each other as shown in Fig. (A), the field due to $\vec{M}_1$ at the position of $\vec{M}_2$, i.e., at O_2 will be



Repulsion
(A)

Attraction
(B)

$$\vec{B}_1 = \frac{\mu_0}{4\pi}\frac{M_1}{r^3} \text{ with } \phi = 90^0 \text{ [as } O_2 \text{ lies on the equatorial line of } \vec{M}_1\text{]}$$

Now as $\vec{B}_1$ is antiparallel to $\vec{M}_2$ and $\vec{B}_2$ to $\vec{M}_1$, i.e., $\theta = 180^0$, so

$$\vec{C}_2 = \vec{M}_2 \times \vec{B}_1 = 0 \text{ and } \vec{C}_1 = \vec{M}_1 \times \vec{B}_2 = 0 \quad \text{........... (5)}$$

i.e., the magnets will not exert any couple on each other

And as $U = -\vec{M}.\vec{B}$, the interaction energy of the system (i.e.,P.E. of $\vec{M}_2$ in the field of $\vec{M}_1$ or P.E. of $\vec{M}_1$ in the field of $\vec{M}_2$) will be

$$U = -\vec{M}_2.\vec{B}_1 = -\vec{M}_1.\vec{B}_2 = \frac{\mu_0}{4\pi}\frac{M_1M_2}{r^3} \quad \text{........... (6)}$$

[as $\vec{M}_2$ is antiparallel to $\vec{B}_1$, i.e., $\theta = 180^0$]

Now as F = -(dU/dr), so force on $\vec{M}_1$ due to $\vec{M}_2$ or on $\vec{M}_2$ due to $\vec{M}_1$ will be

$$F_1 = F_2 = -\frac{d}{dr}\left(\frac{\mu_0}{4\pi}\frac{M_1M_2}{r^3}\right) = \frac{\mu_0}{4\pi}\frac{3M_1M_2}{r^4} \quad \text{.............. (7)}$$

From equation (7) it is clear that **interaction force varies as ($1/r^4$).**

Superposition of Magnetic fields

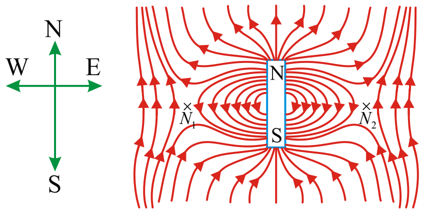
Neutral points and their location :

In the combined field due to bar magnet and horizontal component of earth's magnetic field (B_H) :

Earth's magnetic field is present every where and its horizontal component extends from south to north. When a magnet is placed any where, its field gets superimposed over the earth's field, giving rise to resultant magnetic field. In this resultant magnetic field, there are certain points where the resultant magnetic induction field becomes zero. At these points, the horizontal component of earth's magnetic field exactly balances the field due to the magnet. These points are called null points or neutral points.

"The points in the magnetic field where the resultant magnetic induction field becomes zero are called null points".

North pole of the magnet pointing towards geographical north : When a magnet is placed in the magnetic meridian, with its north pole facing geographic north, the combined magnetic field lines due to earth and the bar magnet are as shown in the figure.



Magnetic lines of force when northpole of the magnet pointing towards geographic north

Results:

- Along the axial line, on both sides, the two fields have same direction. The magnitude of resultant magnetic field is the sum of the magnitudes of two fields.
- As we deviate from axial line, the two fields differ in direction.
- On the equatorial line, the direction of the two fields are exactly opposite to each other.
- At N_1 and N_2 on the equatorial line, the magnetic induction field due to the magnet is exactly same as that of earth's horizontal component. These points are called null points. If the average distance of N_1 and N_2 from the centre of the magnet is 'd' then

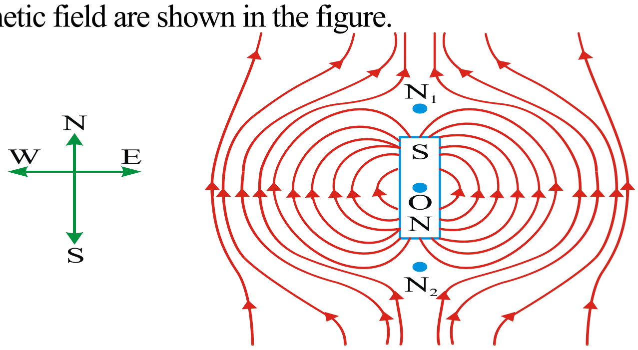
  $B_{magnet} = B_H$ (horizontal component of earth's magnetic field)

  $$\therefore \frac{\mu_0}{4\pi}\frac{M}{\left(d^2+l^2\right)} = B_H$$

  For short magnet $\frac{\mu_0}{4\pi}\frac{M}{d^3} = B_H$

North pole of the magnet pointing towards geographic south:

When a magnet is placed in the magnetic meridian with its north pole facing geographic south, the field lines of the resultant magnetic field are shown in the figure.



Magnetic lines of force when north pole of the magnet pointing towards geographic south

Results:

- The directions of the two fields (horizontal component of earth's magnetic field and the field due to the magnet) are exactly opposite to each other, on the axial line.
- As we deviate from the axial line, the two fields differ in direction.
- The directions of the two fields at all points on the equatorial line is the same.
- Along the axial line, the magnetic field due to magnet decreases in magnitude on moving away from the centre of the magnet. There will be points N_1 and N_2 situated at equal distances from the centre of the magnet where the fields are exactly balanced by the earth's horizontal component field. These points are called null points.

At null points, $B = \frac{\mu_0}{4\pi}\frac{2Md}{\left(d^2-1^2\right)^2} = B_H$ (where B_H is earth's horizontal magnetic induction field)

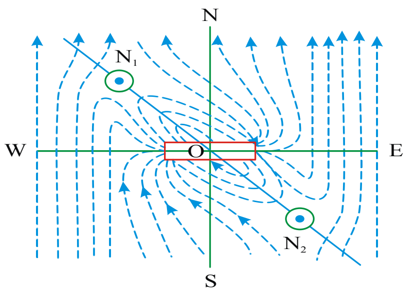
For short magnet, $\frac{\mu_0}{4\pi}\frac{2M}{d^3} = B_H$

If the horizontal component of earth's magnetic field B_H at the given place is known, the magnetic moment (M) of the magnet can be determined by locating the neutral points.

Magnet placed perpendicular to the magnetic meridian : When a bar magnet is placed with its axial line perpendicular to the magnetic meridian with its north pole facing east of earth, the resultant magnetic field is shown in the figure Along a line making an angle of $\tan^{-1}\left(\sqrt{2}\right)$ with east - west line, there are two points (N_1 and N_2) where the resultant magnetic induction field is zero. Thus N_1 (on the N-W line) and N_2 (on the S-E line) are the null points.

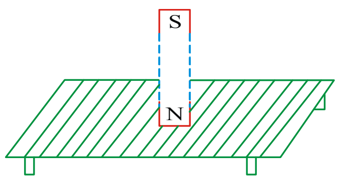
At the null point,

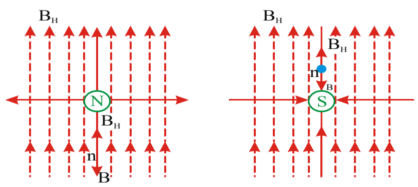
$$B_H = \frac{\mu_0}{4\pi}\frac{M}{d^3}\sqrt{1+3\cos^2\theta} \quad \text{Where } \tan\theta = \sqrt{2}$$



$$\therefore \cos\theta = \frac{1}{\sqrt{3}} \quad \Rightarrow B_H = \sqrt{2}\frac{\mu_0}{4\pi}\frac{M}{d^3}$$

If a very long magnet is placed vertically with its one pole on a horizontal wooden table (or) when an isolated magnetic pole is kept in the earth's magnetic field, then





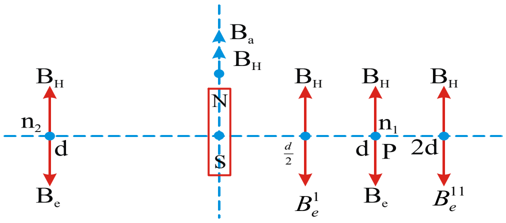
↳ A single neutral point will be formed in the combined field on the horizontal table.

↳ If 'm' is the polestrength and 'd' is the distance from the pole of the magnet where the neutral point is formed, then $\boxed{B_H = \frac{\mu_0}{4\pi}\frac{m}{d^2}}$

↳ If the north pole is on the table, then the neutral point is formed towards geographic south side of the pole.

iv) If the south pole is on the table, then the neutral point is formed towards geographic north side of the pole.

Note: A short bar magnet is kept along magnetic meridian with its north pole pointing north. A neutral point is formed at point 'P' at distance 'd' from the centre of the magnet then



↳ At a distance 'd' on equatorial line, net megnetic induction $B_{net} = 0$

ie $B_e = B_H \Rightarrow \frac{\mu_0}{4\pi}.\frac{M}{d^3} = B_H$

↳ At a distance d/2 from the centre of the magnet on equatorial line, the net magnetic induction is given by

$$B_{net} = B_e^{|} - B_H = \frac{\mu_0}{4\pi}\frac{M}{\left(\frac{d}{2}\right)^3} - B_H = 7B_H$$

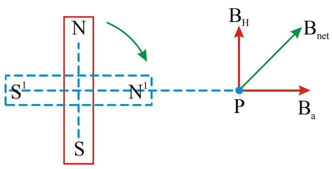
↳ At a distance '2d' on equatorial line, the net magnetic induction is given by

$$B_{net} = B_H - B_e^{||} = B_H - \frac{\mu_0}{4\pi}\frac{M}{(2d)^3} = B_H - \frac{B_H}{8} = \frac{7B_H}{8}$$

↳ At a distance 'd' on axial line of the bar magnet, the net magnetic induction is given by $B_{net} = B_a + B_H = 2B_e + B_H = 2B_H + B_H = 3B_H$.

↳ If the axis of the bar magent is rotated through 90^0 clockwise at the same position then the net magnetic magnetic induction at the same point 'P' is

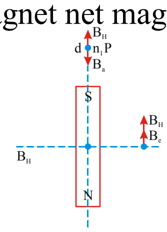
$$B_{net} = \sqrt{B_a^2 + B_H^2} = \sqrt{5}\ B_H \quad (\because B_a = 2B_e = 2B_H)$$



↳ If the axis of the magnet is rotated through 180^0 at the same position, then net magnetic induction at the same point 'P' is $B_{net} = B_e + B_H = 2B_H$

Note: A short bar magnet is kept along magnetic meridian with its south pole pointing north. A neutral point is formed at a point 'P' at a distance 'd' from the centre of the magnet then

↳ at a distance 'd' on axial line of the bar magnet net magnetic induction,



$$B_{net} = 0 \text{ i.e., } B_a = B_H \Rightarrow \frac{\mu_0}{4\pi}.\frac{2M}{d^3} = \mathbf{B}_H$$

- At a distance $\frac{d}{2}$ on axial line of bar magnet, netmagnetic induction is given by

$$B_{net} = B_a^1 - B_H = \frac{\mu_0}{4\pi}.\frac{2M}{\left(\frac{d}{2}\right)^3} - B_H = 7B_H$$

- At a distance '2d' on axial line of the bar magnet, net magnetic induction is given by

$$B_{net} = B_H - B_a^{||} = B_H - \frac{\mu_0}{4\pi}\frac{2M}{(2d)^3} \qquad = B_H - \frac{B_H}{8} = \frac{7B_H}{8}$$

- At a distance 'd' on equatorial line of the bar magnet, net magnetic induction is

$$B_{net} = B_e + B_H = \frac{B_a}{2} + B_H \qquad = \frac{B_H}{2} + B_H = \frac{3}{2}B_H$$

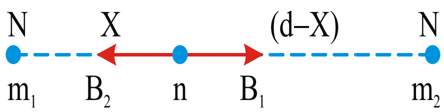
- If axis of the magnet is rotated through 90^0 clockwise at the same position, then net magnetic induction at the same point 'P' is given by

$$B_{net} = \sqrt{B_e^2 + B_H^2} = \frac{\sqrt{5}}{2}B_H \left(\because B_e = \frac{B_a}{2} = \frac{B_H}{2}\right)$$

- If axis of the magnet is rotated through 180^0 ,then magnetic induction at the point 'P' is $B_{net} = B_a + B_H = B_H + B_H = 2B_H$

Neutral points in the combined field due to isolated magnetic poles :

- When two like magnetic poles of pole strengths m_1 and m_2 ($m_1 < m_2$) are separated by a distance 'd', then neutral point is formed in between the poles and on the line joining them. Let 'x' be the distance of neutral point from weaker pole of strength m_1 .

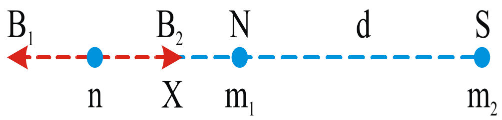


At neutral point, $B_1 = B_2$

$$\Rightarrow \frac{\mu_0}{4\pi}.\frac{m_1}{x^2} = \frac{\mu_0}{4\pi}.\frac{m_2}{(d-x)^2}$$

on solving, we get $x = \frac{d}{\sqrt{\frac{m_2}{m_1}}+1}$

- When two unlike magnetic poles of strengths $m_1$and m_2 ($m_1 < m_2$) are separated by a distance 'd', then neutral point is formed outside and on the line passing through the poles. It always lies closertoweakerpole.



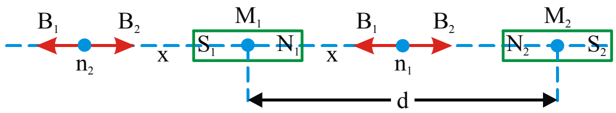
At neutral point, $B_1 = B_2$

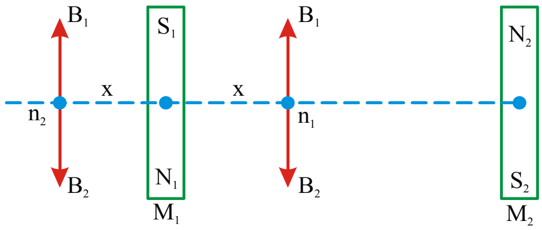
$$\Rightarrow \frac{\mu_0}{4\pi}.\frac{m_1}{x^2} = \frac{\mu_0}{4\pi}.\frac{m_2}{(d+x)^2}$$

$$\text{on solving, we get } x = \frac{d}{\sqrt{\frac{m_2}{m_1}} - 1}$$

Neutral points in the combined field due to short bar magnets :

Two short bar magnets of magnetic moments M_1 and M_2 ($M_1 < M_2$) are placed at a distance 'd' between their centres with their magnetic axes oriented as shown in the figure, Then two neutral points are formed (i) in between and (ii) outside and on the line passing through centres of the magnets. In either case, null point is always closer to magnet of weaker moment.





Case i) : If the neutral point is formed in between the magnets, then $B_1 = B_2$

$$\Rightarrow \frac{\mu_0}{4\pi}.\frac{2M_1}{x^3} = \frac{\mu_0}{4\pi}.\frac{2M_2}{(d-x)^3}$$

$$\text{on solving, we get } x = \frac{d}{\left(\frac{M_2}{M_1}\right)^{1/3} + 1}$$

Case ii) : If the neutral point is formed outside the combination, then

$$\Rightarrow \frac{\mu_0}{4\pi}.\frac{2M_1}{x^3} = \frac{\mu_0}{4\pi}.\frac{2M_2}{(d+x)^3}$$

$$\textit{on solving, we get } x = \frac{d}{\left(\frac{M_2}{M_1}\right)^{1/3} - 1}$$

Note: No null points are obtained when unlike poles of the magnets are placed closer to each other

↳ When two or more magnetic fields are superimposed in the same region, according to the resultant magnetic field the space in the region gets modified.

↳ The magnetic field of induction at any point is the resultant of all the fields superimposed at that point.

Null Point (or) Neutral Point : The point at which the resultant magnetic field is zero is called null point.

↳ If two poles of pole strengths m_1 and m_2 $(m_1 < m_2)$ are separated by a distance d, then the distance of the neutral point from the first pole m_1 is

$$x = \frac{d}{\sqrt{\frac{m_2}{m_1}} \pm 1} \quad \begin{pmatrix} +\text{for like poles} \\ -\text{for unlike poles} \end{pmatrix}$$

a) For like poles the neutral point is situated in between the poles

b) For unlike poles the neutral point is situated on line joining the poles. But not in between them.

c) In either case null point is always closer to the weaker pole.

↳ If two short bar magnets of magentic moments M_1 and $M_2\left(M_1 < M_2\right)$ are placed along the same line with like poles facing each other and 'd' is the distance between their centres, the distance of null point from M_1 is $x = \dfrac{d}{\left(\dfrac{M_2}{M_1}\right)^{1/3} \pm 1}$

a) + for null point formed between the magnets.

b) – for null point formed outside the magnets.

c) When unlike poles face each other, null point

Time period of Suspended Magnet in the Uniform Magnetic Field

↳ **Principle :** When a bar magnet is suspended freely in a uniform magnetic field and displaced from its equilibrium, it starts executing angular SHM.

↳ Time period of oscillation and frequency of magnet is

$$T = 2\pi\sqrt{\frac{I}{MB_H}} \quad \text{and} \quad n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{MB_H}{I}}$$

where M magnetic moment, B_H Horizontal component of earth magnetic induction and I moment of inertia, $I = m\dfrac{(l^2+b^2)}{12}$

for a thin bar magnet $I = \dfrac{ml^2}{12}$

where m is mass, l is length and b is breadth of the magnet.

↳ For small percentage changes in moment of inertia $\dfrac{\Delta T}{T} \times 100 = \dfrac{1}{2}\dfrac{\Delta I}{I} \times 100$

↳ As M increases , T increases

↳ For small percentage changes in magnetic moment $\dfrac{\Delta T}{T} \times 100 = \dfrac{-1}{2}\dfrac{\Delta M}{M} \times 100$

↳ As M increases , T decreases

Comparision of magnetic moments :

- If two magnets of moment M_1 and M_2 of same dimensions and same mass are oscillating in the same field separately, then

$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} \text{ (Bar magnets of equal size) } \left(\because T \propto \frac{1}{\sqrt{M}}\right)$$

- A magnet is oscillating in a magnetic field B and its time period is Tsec. If another identical magnet is placed over that magnet with similar poles together, then time period remain unchanged.

$(\because I^{|} = 2I$ and $M^{|} = 2M$,

$$T^{|} = 2\pi\sqrt{\frac{I^{|}}{M^{|}B}} = 2\pi\sqrt{\frac{2I}{2MB}} = 2\pi\sqrt{\frac{I}{MB}} = T\,)$$

- A magnet is oscillating in a magnetic field B and its time period is T sec. If another identical magnet is placed over that magnet with unlike poles together, then time period becomes infinite. i.e., it does not oscillate.

$$\left[M^{|} = M - M = 0; T = 2\pi\sqrt{\frac{I}{0 \times B}} = \propto\right]$$

- The time period of a thin bar magnet is T. It is cut into 'n' equal parts by cutting it normal to its length. The time period of each piece when oscillating in the same magnetic field will be $T^{|} = \frac{T}{n}$

$$\left(\because I^1 = \frac{\left(\frac{m}{n}\right)\left(\frac{1}{n}\right)^2}{12} = \frac{I}{n^3} \;\&\; M^1 = \frac{M}{n}\right) \therefore T^1 = 2\pi\sqrt{\frac{I^1}{M^1 B}} = \frac{T}{n}$$

- The time period of a thin bar magnet is T. It is cut into 'n' equal parts by cutting it along its length. The time period of each piece remains unchanged, when oscillating in the same field.

$$\because M^{|} = \frac{M}{n} \;\&\; I^{|} = \frac{\left(\frac{m}{n}\right)l^2}{12} = \frac{I}{n}$$

$$\Rightarrow T^{|} = 2\pi\sqrt{\frac{I^{|}}{M^{|}B}} = 2\pi\sqrt{\frac{I/n}{\frac{M}{n}B}} = T$$

- a) Two magnets of magnetic moments M_1 and M_2 (M_1>M_2) are placed one over the other. If T_1 is the time period when like poles touch each other and T_2 is the time peiod when unlike poles touch each other, then

$$\frac{T_2^2}{T_1^2} = \frac{M_1 + M_2}{M_1 - M_2} \left(\because T\alpha\frac{1}{\sqrt{M}}\right) \Rightarrow \frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2}$$

b) If n_1 and n_2 are the corresponding

frequencies, then $\frac{M_1}{M_2} = \frac{n_1^2 + n_2^2}{n_1^1 - n_2^2}$

↳ When same bar magnet used in the vibration magnetometer at two different places 1 and 2, then

$$\frac{B_{H_1}}{B_{H_2}} = \frac{T_2^2}{T_1^2} \qquad \left(\because T \alpha \frac{1}{\sqrt{B_H}}\right)$$

↳ When two bar magnets of moments M_1 and M_2 are placed one over the other such that (i) like poles together (ii) unlike poles together and (iii) their axes are perpendicular to each other. When vibrated in the same magnetic field, the ratio of their time periods respectively is

$$T = 2\pi\sqrt{\frac{I}{MB}} \Rightarrow T\alpha\frac{1}{\sqrt{M}}$$

$$\therefore T_1 : T_2 : T_3 = \frac{1}{\sqrt{M_1 + M_2}} : \frac{1}{\sqrt{M_1 - M_2}} : \frac{1}{\sqrt{\left(\sqrt{M_1^2 + M_2^2}\right)}}$$

↳ If T_0 is the time period of oscillation of the experimental magnet oscillating in B_H. An external field B is applied due to a bar magnet in addition to B_H at the point where the first magnet is oscillating. Then its new time period is T.

Then $\frac{T_0}{T} = \sqrt{\frac{B_r}{B_H}}$ where $\vec{B_r} = \vec{B} + \vec{B_H}$

a) If $\vec{B}$ and $\vec{B_H}$ are along the same direction, $B_r = B + B_H \Rightarrow T < T_0$

b) If $\vec{B}$ and $\vec{B_H}$ are in opposite directions, $B_r = B - B_H \Rightarrow T_0 < T$

c) If $\vec{B}$ and $\vec{B_H}$ are in opposite directions, and also if $|\vec{B}| = |\vec{B_H}|$ then $B_r = B \sim B_H = 0 \Rightarrow T = \propto$ (i.e., it does not oscillate)

d) If $\vec{B}$ and $\vec{B_H}$ are perpendicular to each other, $B_r = \sqrt{B^2 + B_H^2} \Rightarrow T < T_0$

Here $B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$ (if the point is on the axial line)

$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$ (if the point is on the equatorial line)

e) If a straight wire carries current vertically up or down placed on the east or west or north or south side, then $B = \frac{\mu_0}{2\pi}\frac{i}{r}$ (From ampere's law in electro magnetism)

↳ If n_1 and n_2 are frequencies of oscillation of the bar magnet in uniform magnetic field when B supports B_H and when B opposes B_H

then $\frac{n_1}{n_2} = \sqrt{\frac{B + B_H}{B - B_H}}$ (let $B > B_H$)

$$\Rightarrow \frac{B}{B_H} = \frac{n_1^2 + n_2^2}{n_1^2 - n_2^2}$$

↪ For a bar magnet , $T \propto \frac{1}{\sqrt{B_H}}$ (or) $n \propto \sqrt{B_H}$

If B_1 and B_2 be the earth's magnetic induction at two different places having angles of dip θ_1 and θ_2 then

$$\frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} = \sqrt{\frac{B_2 Cos\theta_2}{B_1 Cos\theta_1}}$$

or $\frac{n_1}{n_2} = \sqrt{\frac{B_{H1}}{B_{H2}}} = \sqrt{\frac{B_1 Cos\theta_1}{B_2 Cos\theta_2}}$

EX. 9: Two bar magnets placed together in a vibration magnetometer take 3 seconds for 1 vibration. If one magnet is reversed, the combination takes 4 seconds for 1 vibration. Find the ratio of their magnetic moments.

Sol. Given that, T_1 = 3s and T_2 = 4s

$$\frac{M_1}{M_2} = \frac{T_2^2 + T_1^2}{T_2^2 - T_1^2} = \frac{4^2 + 3^2}{4^2 - 3^2} = \frac{16+9}{16-9} = \frac{25}{7} \text{ or } \frac{M_1}{M_2} = 3.57$$

EX. 10 : A bar magnet makes 40 oscillations per minute in a vibration magnetometer. An identical magnet is demagnetised completely and is placed over the magnet in the magnetometer. Calculate the time taken for 40 oscillations by this combination. Ingore induced magnetism.

Sol. In the first case, frequency of oscillation,

$$n = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

In the second case, frequency of oscillation,

$$n^1 = \frac{1}{2\pi}\sqrt{\frac{MB}{2I}} \Rightarrow \frac{n^1}{n} = \frac{1}{\sqrt{2}} \Rightarrow \frac{T^1}{T} = \sqrt{2}$$

(or) $T^1 = \sqrt{2}T$ (or) $40T^1 = \sqrt{2} \times 40T$

(or) $t^1 = \sqrt{2}t = \sqrt{2}$ minute = 1.414 minute

EX. 11 : A short magnet oscillates in a vibration magnetometer with a time period of 0.1s where the horizontal component of earth's magnetic field is $24\mu T$. An upward current of 18A is established in the vertical wire placed 20 cm east of the magnet. Find the new time period ?

Sol. $\frac{T_2}{T_1} = \sqrt{\frac{B_1}{B_2}}$ Where $B_1 = B_H = 24 \times 10^{-6}$T

and $B_2 = B_H : B = B_H : \frac{\mu_0 i}{2\pi r}$

$$= 24 \times 10^{-6} : \frac{4\pi \times 10^{-7} \times 18}{2\pi \times 0.2} = 6 \times 10^{-6} T$$

$$\therefore \frac{T_2}{0.1} = \sqrt{\frac{24 \times 10^{-6}}{6 \times 10^{-6}}} = 2 \quad \Rightarrow T_2 = 0.2s$$

EX. 12: A magnet is suspended so as to swing horizontally makes 50 vibrations/min at a place where dip is 30^0, and 40 vibrations / min where dip is 45^0. Compare the earth's total fields at the two places.

Sol. $n \alpha \sqrt{B_H}$

$$\Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{B_1 Cos\,\theta_1}{B_2 Cos\,\theta_2}} \quad ie\, \frac{50}{40} = \sqrt{\frac{B_1}{B_2} \times \frac{Cos 30^0}{Cos 45^0}}$$

$$\Rightarrow \frac{25}{16} = \frac{B_1}{B_2} \times \frac{\sqrt{3}}{\sqrt{2}} \text{ (or) } \frac{B_1}{B_2} = \frac{25}{8\sqrt{6}}$$

EX. 13: When a short bar magnet is kept in tan A position on a deflection magnetometer, the magnetic needle oscillates with a frequency'f' and the deflection produced is 45^0. If the bar magnet is removed find the frequency of osciullation of that needle ?

Sol. $n\alpha\sqrt{B} \quad \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{B_1}{B_2}}$

Where $B_1 = \sqrt{B^2 + B_H^2} = \sqrt{\left(B_H \tan 45^0\right)^2 + B_H^2}$

$= \sqrt{2}\, B_H$ & $B_2 = B_H$

$$\therefore \frac{n_1}{n_2} = \sqrt{\frac{\sqrt{2} B_H}{B_H}} = 2^{1/4} \quad \Rightarrow n_2 = \frac{n_1}{2^{1/4}} = \frac{f}{2^{1/4}}$$

EX. 14: Two bar magnets of the same length and breadth but having magnetic moments M and 2M are joined with like poles together and suspended by a string. The time of oscillation of this assembly in a magnetic field of strength B is 3 sec. What will be the period of oscillation, if the polarity of one of the magnets is changed and the combination is again made to oscillate in the same field ?

Sol. As magnetic moment is a vector, so when magnets are joined with like poles together $M_1 = M + 2M = 3M$, so

$$T = 2\pi\sqrt{\frac{(I_1 + I_2)}{3MB}} \quad \text{........... (1)}$$

When the polarity of one of the magnets is reversed, $M_2 = M \sim 2M = M$;

$$\text{so } T' = 2\pi\sqrt{\frac{(I_1 + I_2)}{MB}} \quad \text{............ (2)}$$

Dividing Eq. (2) by (1),

$$\frac{T'}{T} = \sqrt{3}, \quad i.e., T' = \left(\sqrt{3}\right) T = 3\sqrt{3} \text{ sec}$$

Magnetic Materials

- Curie and Faraday discovered that all the materials in the universe are magnetic to some extent. These magnetic substances are categorized mainly into two groups.
- Weak magnetic materials come under diamagnetic and paramagnetic materials. Strong magnetic materials are Ferro-magnetic materials.
- According to the modern electron theory of magnetism, the magnetic response of any material is due to the circulating electrons in the atoms. Each circulating charge constitutes a magnetic moment in a direction perpendicular to the plane of circulation.
- In magnetic material all these magnetic moments due to the orbital and spin motion of all the electrons in the atoms of the material, vectorially add up to a resultant magnetic moment. The magnitude and direction of this resultant magnetic moment is responsible for the magnetic behaviour of the material.
- Magnetic material are studied interms of the following physical parameters

Intensity of Magnetising field($\vec{H}$) :

Any magnetic field in which a magnetic material is placed for its magnetization is called magnetising field.

In a magnetising field the ratio of magnetising field $\vec{B}_o$ to the permeability of free space is called intensity of magnetising field

In air . $\vec{H} = \frac{\vec{B}_o}{\mu_o}$ or $\vec{B}_o = \mu_o \vec{H}$

In a medium $H = \frac{B}{\mu}$

The value of H is independent of medium .
Intensity of magnetising field is a vector in the direction of magnetic field and has unit

$$\frac{Wb/m^2}{H/m} = \frac{V \times s}{\Omega \times s \times m} = \frac{A}{m}$$ Dimensions AL^{-1}

2) Intensity of magnetisation $\vec{I}$: When a magnetic material is magnetised by placing it in a magnetising field, the induced dipole-moment per unit volume in the specimen is called intensity of magnetisation.

i.e $\vec{I} = \frac{\vec{M}}{V}$ but as $\vec{M} = mL\hat{n}$ and V = SL $\vec{I} = \frac{m}{S}\hat{n}$ i.e , intensity of magnetisation is numerically equal to the induced pole-strength per unit area of cross -section.It is a vector quantity having direction of magnetising field or opposite to it as shown in figure. Its unit is (A/m) and dimensions $\left[AL^{-1}\right]$

Magnetic Susceptibility(χ_m)**:** The ratio of magnitude of intensity of magnetisation to that of magnetising field strength is called magnetic susceptibility $\chi_m = \frac{I}{H}$

It is a scalar with no units and dimensions. It physically represents the ease with which a magnetic material can be magnetised.i.e large value of χ_m implies that the material is more susceptible to the field and hence can be easily magnetised.

Magnetic permeability (μ) : When a magnetic material is placed in a magnetising field, the ratio of magnitude of total field inside the material to that of intensity of magnetising field is called magnetic permeability; i.e.,

$$\mu = \frac{B}{H}, i.e., B = \mu H$$

It measures the degree to which a magnetic material can be penetrated by the magnetising field or ability of the material to allow magnetic lines of force. It is a scalar having unit Hm^{-1} and dimensions $(MLT^{-2}A^{-2})$.

Relative permeability (μ_r) :

It is the ratio of magnitudes of total field inside the material to that of magnetising field or it is the ratio of permeability of a medium to that of free space.

$$\mu_r = \frac{B}{B_0} = \frac{\mu H}{\mu_0 H} = \frac{\mu}{\mu_0}$$

It has no units and dimensions

Relation between relative permeability and susceptibility :

We know $B = \mu_0 (H + I)$ or, $\frac{B}{H} = \mu_0 \left(1 + \frac{I}{H}\right)$

or, $\mu = \mu_0 (1 + \chi)$ [as $\frac{B}{H} = \mu$ and $\frac{I}{H} = \chi$]

(or) $\frac{\mu}{\mu_0} = 1 + \chi$ $\quad\therefore \mu_r = 1 + \chi$ This is the desired result.

EX. 15: A magnetising field of 1600Am^{-1} produces a magnetic flux of 2.4 × 10^{-5} weber in a bar of iron of cross section 0.2 cm^2. Calculate permeability and susceptibility of the bar.

Sol. Magnetic induction, $B = \frac{\phi}{A} = \frac{2.4 \times 10^{-5}}{0.2 \times 10^{-4}} = 1.2 \text{ Wb/m}^2$

i) Permeability, $\mu = \frac{B}{H} = \frac{1.2}{1600} = 7.5 \times 10^{-4} TA^{-1}m$

ii) As $\mu = \mu_0 (1 + \chi)$ then

Susceptibility, $\chi = \frac{\mu}{\mu_0} - 1 = \frac{7.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1 = 596.1$

EX. 16: The permeability of substance is 6.28×10^{-4} wb/A-m. Find its relative permebility and suscepibility ?

Sol. $\mu_\gamma = \frac{\mu}{\mu_0} = \frac{6.28 \times 10^{-4}}{4\pi \times 10^{-7}} = 500$ $\quad \mu_\gamma = 1 + \chi \quad \therefore \chi = \mu_\gamma - 1 = 500 - 1 = 499$

EX. 17 : The magnetic moment of a magnet of mass 75 gm is 9×10^{-7} A-m^2. If the density of the material of magnet is 7.5×10^3 kg m^{-3}, then find intensity of magnetisation is

Sol. $I=\frac{M}{V}$ Where volume, $V=\frac{mass(m)}{density(\rho)}$

$$=\frac{M\times\rho}{m}=\frac{9\times10^{-7}\times7.5\times10^{3}}{75\times10^{-3}}=0.09A/m$$

EX. 18 : A magnetic field strength (H) 3×10^3Am^{-1} produces a magnetic field of induction (B) of $12\pi T$ in an iron rod. Find the relative permeability of iron ?

Sol. $\mu=\frac{B}{H}=\frac{12\pi}{3\times10^{3}}=4\pi\times10^{-3}$

$$\therefore \mu_r=\frac{\mu}{\mu_0}=\frac{4\pi\times10^{-3}}{4\pi\times10^{-7}}=10^4$$

EX. 19 : An iron bar of length 10 cm and diameter 2 cm is placed in a magnetic field of intensity 1000Am^{-1} with its length parallel to the direction of the field. Determine the magnetic moment produced in the bar if permeability of its material is 6.3×10^{-4} TmA^{-1}.

Sol. we know that, $\mu=\mu_0(1+\chi)$

$$\Rightarrow \chi=\frac{\mu}{\mu_0}-1=\frac{6.3\times10^{-4}}{4\pi\times10^{-7}}-1=500.6$$

Intensity of magnetisation,

$I=\chi H=500.6\times1000=5\times10^5$ Am^{-1}

$\therefore$ magnetic moment, M = I x V $=I\times\pi r^2 1$

$=5\times10^5\times3.14\times(10^{-2})^2\times(10\times10^{-2})$=17.70 A-m^2

Electron Theory of Magnetism

- i) Molecular theory of magnetism was first given by Weber and was later developed by Ewing.
- ii) Electron theory of magnetism was proposed by Langevin.
- iii) The main reason for the magnetic property of a magnet is spin motion of electron. Most of the magnetic moment is produced due to electron spin. The contribution of the orbital revolution is very small.

A) Explanation of diamagnetism:

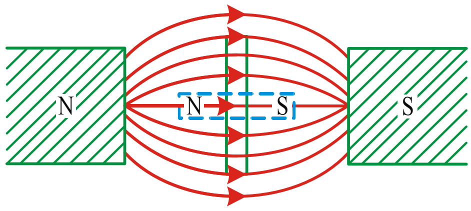
- i) Since diamagnetic substance have paired electrons, magnetic moments cancel each other and there is no net magnetic moment.
- ii) When a diamagnetic substance is placed in an external magnetic field each electron experiences radial force F = Bev either inwards or outwards. Due to this the angular velocity, current, and magnetic moment of one electron increases and of the other decreases. This results in a non-zero magnetic moment in the substances in a direction opposite to the field.
- iii) Since the orbital motion of electrons in atoms is an universal phenomenon, diamagnetism is present in all materials. Hence diamagnetism is a universal property.

Properties of Dia-magnetic substances

- The substances which when placed in a external magnetic field acquire feeble magnetism opposite to the direction of the magnetising field are known as dia-magnetic substances.

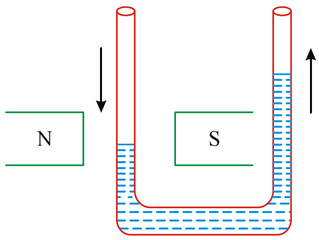
Ex: Bismuth (Bi), Zinc (Zn), Copper (Cu), Silver (Ag), Gold (Au). Salt (Nacl), Water (H_2O), Mercury (Hg), Hydrogen (H_2O) etc.

↳ When a bar of dia-magnetic substance is suspended freely between two magnetic poles [see figure] , then the axis of the bar becomes perpendicular to magnetic field.



↳ When a dia-magnetic material is placed inside a magnetic field, the magnetic field lines become less dense in the material.

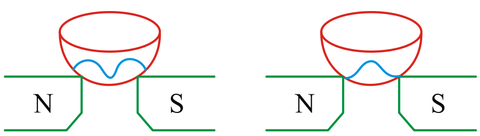
↳ If one limb of a narrow U-tube containing a dia-magnetic liquid is placed between the poles of an electromagnet, then on switching the field, the liquid shows a depression. This is shown in figure.



↳ When a dia-magnetic substance is placed in a non-uniform field, then it tends to move towards the weaker part from the stronger part of the field as shown in figure.

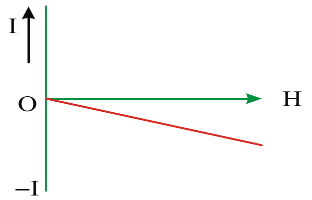
Properties of Dia, Para and Ferror Magnetic materials		
DIA	**PARA**	**FERRO**
1. They are feebly repelled by a magnet.	1. They are feebly attracted by a magnet	1. They are strongly attracted a magnet
2. The net magnet moment due to all the electrons in the atom is zero	2. The net magnetic moment atoms due to all electrons is not zero.	2. The net magnetic moment in atoms is very strong.
3. When subjected to the magnetising field they are feebly magnetised in opposite direction to the magnetising field	3. Magnetised feebly in the direction of magnetising field.	3. Magnetized strongly in the direction of magnetising field.
4. When suspended inside the magnetic field, they align their length perpendicular to the magnetic field.	4. They align with their length along the direction of magnetic field.	4. They align with their length along the direction of magnetic field.

5. Magnetic lines of force prefer to move out of the specimen.	5. Few lines pass through the specimen.	5. Almost all lines prefer to move through the specimen.
6. They move from stronger part of the magnetic field to the weaker part of the magnetic field	6. They move from weaker to stronger part of the magnetic field	6. They move from weaker to stronger part of the magnetic field.
7. $\mu_r < 1$	7. $\mu_r > 1$	7. $\mu_r >>> 1$
8. Intensity of magnetization (I) is small and negative.	8. I is small and positive	8. I is high and positive
9. χ_m is small and negative	9. χ_m is small and positive	9. χ_m is highly positive
10. χ_m is independent of temperature.	10. χ_m is dependent on temperature.	10. χ_m is dependent on temperature
11. Doesn't obey Curie law.	11. Obey Curie law	11. Obey Curie law and at Curie temperature they are turned to paramagnetic materials.
12. Substances following Diamagnetism are Bismuth, Copper, lead, silicon, water, glass etc.	12. Substances following paramagnetism are Aluminum, Platinum, Manganese, Chromium, Calcium, Oxygen, Nitrogen (at STP)	12. Substances following ferromagnetism are Iron, Cobalt, Nickel and alloys like alnico

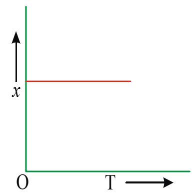


a) Magnet closely spaced   b) pole pieces moved apart

↳ Dia magnetic substances acquire feeble magnetism in a direction oppoite to magnetising field. The intensity of magnetisation I is very small, negative and is directly proportional to magnetising field H as shown in figure.



↪ The magnetic susceptibility χ (I/H) is small and negative(Because I is small and opposite in direction to H). This is independent of temperature as shown in figure.



↪ The relative permeability is less than unity because $\mu_r = (1+\chi)$ and χ is negative.

↪ The origin of diamagnetism is the induced dipole moment due to change in orbital motion of electrons in atoms by the applied field. Dia-magnetism is shown only by those substances which do not have any permanent magnetic moment.

B) Explanation of Paramagnetism:

↪ i) Paramagnetic materials have a permanent magnetic moment in them. The moments arise from both orbital motion of electrons and the spinning of electrons in certain axis.

↪ ii) In atoms whose inner shells are not completely filled, there is a net moment in them since more number of electrons spin in the same direction. This permanent magnet behaves like a tiny bar magnet called atomic magnet.

↪ iii) In absence of external magnetic field atomic magnets are randomly oriented due to the thermal agitation and the net magnetic moment of the substance is zero.

↪ iv) When it is placed in an external magnetic field the atomic magnets align in the direction of the field and thermal agitation oppose them to do so.

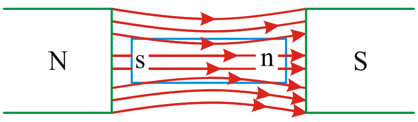
↪ v) At low fields the total magnetic moment would be directly proportional to the magnetic field B and inversely proportional to temperature T.

Properties of Paramagnetic substances

(i) The substances which when placed in a magnetic field, acquire feeble magnetism in the direction of magnetising field are known as paramagnetic substances.

Ex: Aluminium (Al), Platinum (Pt), Manganese (Mn), Copper chloride ($CuCl_2$), Oxygen (O_2), solutions of salts of iron etc. are examples of paramagnetic substances.

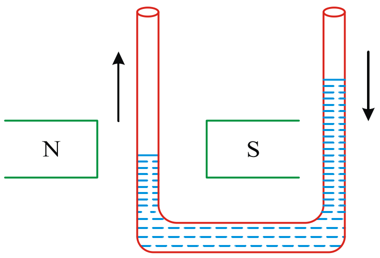
(ii) When a bar of paramagnetic substance is placed in a magnetic field, it tries to concentrate the lines of force into it as shown in figure



This shows that the magnetic induction **B** in it is numerically slightly greater than the applied field **H.** So the permeability μ is greater than one because $\mu = (B/H)$**.**

(iii) When the bar of paramagnetic material is suspended freely between two magnetic poles, its axis becomes parallel to magnetic field. Move over, the poles produced at the ends of the bar are opposite to nearer magnetic poles.

(iv) If a paramagnetic solution is poured in a U-tube and if one limb is placed between the poles of an electromagnet in such a way that liquid level is parallel to field, then on switching the field, the liquid rises. This is shown in figure.

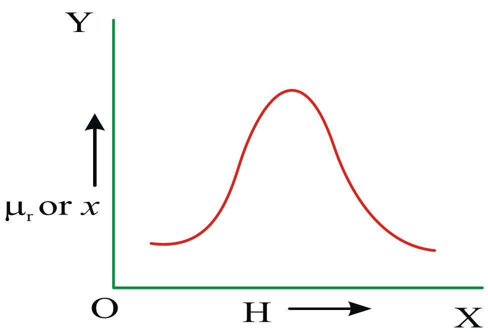


(v) In a non-uniform magnetic field, the paramagnetic substances are attracted towards the stronger parts of the magnetic field from the weaker parts of the field. The situation is shown in figure. In figure (a), the field is stronger in the middle as the poles

fig(a) fig(b)

are near to each other. In figure (b), the distance between the poles is increased. i.e., the field is stronger near the poles.

(vi) The intensity of magnetisation I is very small and compared to one. It follows from the relation $\mu_r = 1 + \chi_m$. The variation of μ_r or χ with **H** is shown in figure. As is clear from the figure, the variation is non-linear. The large value of μ_r is due to the fact that the field **B** inside the material is much stronger than the magnetising field due to 'pulling in' of a large number of lines of force by the material.



Curie's law : Curie law states that far away from saturation, the suceptibility χ (I/H) of paramagnetic substance is inversely proportional to absolute temperature, i.e.,

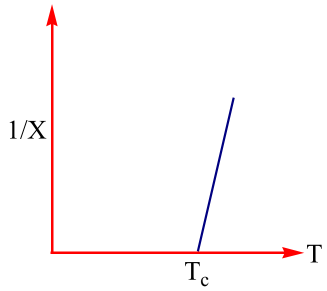
$$\chi \propto \frac{1}{T} \quad \text{or} \quad \chi = \frac{C}{T}$$

where C is constant and is called as Curie constant.

Curie's temperature

When a ferromagnetic material is heated, it becomes paramagnetic at a certain temperature. This temperature is called as Curie temperature and is denoted by T_C. After this temperature, the susceptibility varies with temperature as

$$\chi = \frac{C^|}{(T - T_c)}$$



where $C^|$ is another constant. For iron, $T_c = 1043\,K = 770^0\,C$

Ferromagnetic substances

These substances possess a very large resultant magnetic moment. According to Frenkel and Heisenberg domain theory.

(i) The spin magnetic moments of the electrons are responsible for the magnetic properties of ferromagnetics.

(ii) Under certain definite forces, these spin magnetic moments are lined up parallel to one another. This results in setting up of regions of spontaneous magnetisation which are called domains.

(iii) A domain contains from 10^{21} to 10^{17} atoms and has dimensions of the order of 10^{-8} to 10^{-12} m^3.

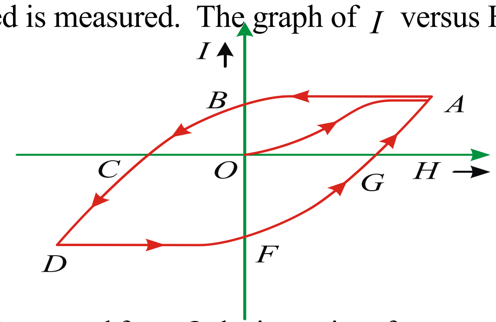
(iv) The magnetisation of the domains tends to align in the direction of the field and the piece of matter becomes a magnet.

Hysteresis: It is defined as the tendency of demagnetisation to lag behind the change in magnetic field applied to a ferromagnetic material.

The process of taking a ferromagnet through a cycle of magnetisation results in loss of energy. This is called hysterisis loss and it appears in the form of heat.

Area of hysteresis loop is equal to the energy loss per cycle per unit volume.

When a bar of ferromagnetic material is magnetized by a varying magnetic field H and the intensity of magnetization I induced is measured. The graph of I versus H is as shown is figure.

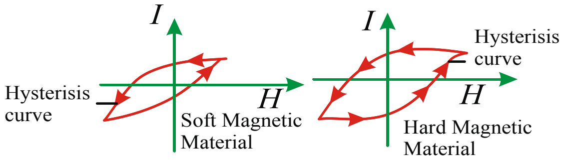


- When magnetising field is increased from O the intensity of magnetisation I increases and becomes maximum i.e at point (A). This maximum value is called the saturation value.
- When H is reduced, I reduces but is not zero when H = 0. The remainder value OB of magnetisation when H = 0 is called the residual magnetism or retentivity. OB is retentivity.
- When magnetic field H is reversed, I reduces and becomes zero i.e., for H = OC, I = 0. This value of H is called the coercivity.

- When field H is further increased in reverse direction, the intensity of magnetisation attains saturation value in reverse direction (i.e., point D). When H is decreased to zero and changed direction in steps, we get the part DFGA

- When field H is further increased in reverse direction, the intensity of magnetisation attains saturation value in reverse direction (i.e., point D). When H is decreased to zero and changed direction in steps, we get the part DFGA

Properties of soft iron and steel: For soft iron, the susceptibility, permeability and retentivity are greater while coercivity and hysteresis loss per cycle are smaller than those of steel.



- For soft iron area of hysteresis loop is less and thus low energy loss.
- For steel area of hysteresis loop is large and thus high energy loss.
- Magnetisation and demagnetisation of soft iron are easy, where as difficult for steel.
- Permanent magnets are made of steel and cobalt while electromagnets are made of soft iron.
- Diamagnetism is universal. It is present in all materials. But it is weak and hard to detect if substance is para or ferromagnetic

Shielding from magnetic fields: For shielding a certain region of space from magnetic field, we surround the region by soft iron rings. Magnetic field lines will be drawn into the rings and the space enclosed will be free of magnetic field.

Elements of Earth's Magnetism (Terristrial Magnetism): There are three elements of earth's magnetism

(i) Angle of declination (ii) Angle of dip

(iii) Horizontal component of earth's field.

- **Geographical Meridian:** A vertical plane passing through the axis of rotation of the earth is called the geographic meridian.
- **Magnetic Meridian :** A vertical plane passing through the axis of a freely suspended magnet is called the magnetic meridian.
- **Angle of Declination (α) :** The acute angle between the magnetic meridian and the geographical meridian is called the 'angle of declination' at any place.
- The value of declination at equator is 17^0
- **Earth's Magnetic Field:** The earth's magnetic field B_e in the magnetic meridian may be resolved into a horizontal component B_H and vertical component B_V at any place.

$$B_r = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3} \quad \text{and} \quad B_\theta = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{r^3}$$

and as $\tan\phi = \frac{B_V}{B_H} = -\frac{B_r}{B_\theta}$, so in the light of Eq. (1)

Apparent Dip: If the dip circle is not kept in the magnetic meridian, the needle will not show the correct direction of earth's magnetic field. The angle made by the needle with the horizontal is called the apparent dip for this plane. If the dip circle is at an angle θ to the meridian, the effective horizontal component in this place is $B'_H = B_H \cos\theta$. The vertical component is still B_V. If δ_1 is the apparent dip and δ is the true dip, we have

$$\tan\delta_1 = \frac{B_V}{B'_H} = \frac{B_V}{B_H \cos\theta}$$

or $\tan\delta_1 = \dfrac{\tan\delta}{\cos\theta}$ $\left(\because \tan\delta = \dfrac{B_V}{B_H}\right)$ ..... (1)

Now suppose, the dip circle is rotated through an angle of 90^0 from this position. It will now make an angle $(90° - \theta)$ with the meridian. The effective horizontal component in this plane is $B''_H = B_H \sin\theta$. if δ_2 be the apparent dip, we shall have

$$\tan\delta_2 = \frac{B_V}{B''_H} = \frac{B_V}{B_H \sin\theta} \text{ or } \tan\delta_2 = \frac{\tan\delta}{\sin\theta} \text{.........(2)}$$

From (1) and (2) $\cot^2\delta_1 + \cot^2\delta_2 = \cot^2\delta$

Thus, one can get the true dip δ without locating the magnetic meridian.

More about angle of dip (δ) :

(i) At a place on poles, earth's magnetic field is perpendicular to the surface of earth, i.e., $\delta = 90^0$

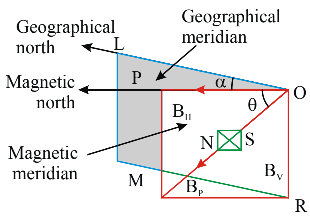
$\therefore B_V = B\sin 90^0 = B$

Further, $B_H = B \cos 90^0 = 0$

So, except at poles, the earth has a horizontal component of magnetic induction field.

(ii) At a place on equator, earth's magnetic field is parallel to the surface of earth, i.e., $\delta = 0^0$

$\therefore B_H = B\cos 0^0 = B$



θ=dip (or) inclination α = declination

↪ Horizontal component of earth's magnetic field

$B_H = B_e \cos\theta$ ....(1)

↪ Vertical component of earth's magnetic field

$B_V = B_e \sin\theta$ ....(2) $B_e = \sqrt{(B_H^2 + B_V^2)}$

↪ Dividing equation (2) by equation (1), we have $\dfrac{B_V}{B_H} = \dfrac{B_e \sin\theta}{B_e \cos\theta} = \tan\theta$

Magnetic Maps : Usually lines are drawn joining all places having same value of an element. Such maps are called magnetic maps. The alue of all the three magnetic elements (a) declination (b) dip and (c) Horizontal component are found to be different at different places on the surface of earth.

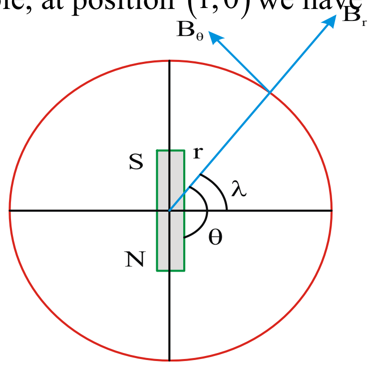
i) Isogonic lines : Lines passing through different places having the same declination are called isogonic lines.

ii) Isoclinic lines : These are lines passing through place of equal dip. The line joining places of zero dip is called aclinic line.

E.X: 20 Considering the earth as a short magnet with its centre coinciding with the centre of earth, show that the angle of dip ϕ is related to magnetic latitude λ through the relation

$\tan\phi = 2\tan\lambda$

Sol. Considering the situation for dipole, at position (r, θ) we have



$\tan\phi = -2\cot\theta$; But From figure $\theta = 90^0 + \lambda$

So, $\tan\phi = -2\cot(90^0 + \lambda)$; i.e., $\tan\phi = 2\tan\lambda$

Further $B_V = B_H \sin 0^0 = 0$

So, except at equator, the earth has a vertical component of magnetic induction field.

(iii) In a vertical plane at an angle θ to magnetic meridian

$B_H' = B_H \cos\theta$ and $B_V' = B_V$

So, the angle of dip δ' in a vertical plane making an angle θ to magnetic meridian is given by

$\tan\delta' = \dfrac{B_V'}{B_H'} = \dfrac{B_V}{B_H\cos\theta}$ or $\tan\delta' = \dfrac{\tan\delta}{\cos\theta}$ $\left(\because \dfrac{B_V}{B_H} = \tan\delta\right)$

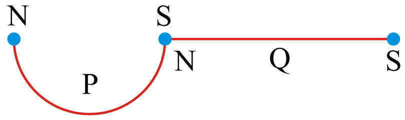
(a) For a vertical plane other than magnetic meridian, $\theta > 0^0$ and $\cos\theta < 1$, i.e., $\delta^1 > \delta$ (angle of dip increases)

(b) For a plane perpendicular to magnetic meridian, $\theta = 90^0$

$\therefore \tan\delta^1 = \dfrac{\tan\delta}{\cos 90} = \infty$ or $\delta^1 = 90^0$

This shows that in a plane perpendicular to magenetic meridian, the dip needle will become vertical.

EX. 21 : **A magnet of length 2L and moment 'M' is axially cut into two equal halves 'P' and 'Q'. The piece 'P' is bent in the form of semi circle and 'Q' is attached to it as shown. Its moment is**



1) $\frac{M}{\pi}$ 2) $\frac{M}{2\pi}$ 3) $\frac{M(2+\pi)}{2\pi}$ 4) $\frac{M\pi}{(2+\pi)}$

Sol..KEY(3) In the arrangement magnetic moment of P is

$M_1 = \frac{2\frac{M}{2}\sin\frac{\pi}{2}}{\pi} = \frac{M}{\pi}$ and magnetic moment of Q is $M_2 = \frac{M}{2}$ $\therefore$ Resultant magnetic moment

$$M_r = M_1 + M_2 = \frac{M}{\pi} + \frac{M}{2} = \frac{M(\pi+2)}{2\pi}$$

EX. 22: . **A magnet is suspended in the magnetic meridian with an untwisted wire. The upper end of the wire is rotated through 180^0 to deflect the magnet by 30^0 from magnetic meridian. Now this magnet is replaced by another magnet. Now the upper end of the wire is rotated through 270^0 to deflect the magnet 30^0 from the magnetic meridian. The ratio of the magnetic moments of the two magnets is**

1) 3 : 4 2) 1 : 2 3) 4 : 7 4) 5 : 8

Sol. KEY(4) $C(180-30) = M_1 B_H Sin30 - (1)$

$C(270-30) = M_2 B_H Sin30 - (2)$

Divide $\frac{M_1}{M_2} = \frac{5}{8}$

EX. 23 : .**A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal haves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T' then $\frac{T'}{T}$ is**

1) $\frac{1}{4}$ 2) $\frac{1}{2\sqrt{2}}$ 3) $\frac{1}{2}$ 4) 2

Sol..KEY(3) When magnet is divided into two equal halves, mass is reduced by a factor of 2 and length is also reduced by factor of 2. So new moment of inertia is $\frac{1}{8}$ th of the initial moment of inertia.

Pole strength is unchanged and the length is halved. So, new magnetic moment is one-half of the initial magnetic moment.

$T' = 2\pi\sqrt{\frac{I'}{M'B}}$ Now, $= 2\pi\sqrt{\frac{I/8}{\frac{M}{2}B}} = \frac{T}{\sqrt{4}} = \frac{T}{2}$ $\frac{T'}{T} = \frac{1}{2}$

EX. 24 A compass needle makes 10 oscillations per minute in the earths horizontal field.A bar magnet deflects the needle by 60^0 from the magnetic meridian. The frequency of oscillation in the deflected position in oscillations per minute is (field due to magnet is perpendicular to B_H)

1) $5\sqrt{2}$ 2) $20\sqrt{2}$ 3) $10\sqrt{2}$ 4) 10

Sol..KEY(3) $n_1 = \frac{1}{2\pi}\sqrt{\frac{M B_H}{I}}$ ________ (1)

$n_2 = \frac{1}{2\pi}\sqrt{\frac{M\sqrt{B_H^2 + B^2}}{I}}$ ______ (2)

$B = B_H \, Tan60$

$B = \sqrt{3}B_H$ ________ (3)

Solving $n_2 = \sqrt{2}\, n_1$ $n_2 = 10\sqrt{2}$

EX. 25 Two bar magnets are placed in vibration magnetometer and allowed to vibrate. They make 20 oscillations per minute when their similar poles are on the same side, while they make 15 osillations per minute when their opposite poles lie on the same side. The ratio of their magnetic moments is

(Eam (M) 2008, E(2009)

1) 7 : 25 2) 25 : 7 3) 25 : 16 4) 16 : 25

Sol..KEY(2)

$20 = 2\pi\sqrt{\frac{(M_1 + M_2)B_H}{2I}}$

$15 = 2\pi\sqrt{\frac{(M_1 - M_2)}{2I}}B_H$

$\frac{4}{3} = \sqrt{\frac{M_1 + M_2}{M_1 - M_2}}$ Solving $\frac{M_1}{M_2} = \frac{25}{7}$

Magnetism and Matter

(Jee main previous year questions)

Topic 1: Magnetism, Gauss's Law, Magnetic Moment, Properties of Magnet

1. A small bar magnet placed with its axis at 30^{o} with an external field of 0.06 T experiences a torque of 0.018 Nm. The minimum work required to rotate it from its stable to unstable equilibrium position is :

[Sep. 04, 2020 (I)]

(a) 6.4×10^{-2}J (b) 9.2×10^{-3}J (c) 7.2×10^{-2}J (d) 11.7×10^{-3}J

SOL. (c) Here, $\theta = 30°$, $\tau = 0.018$ N-m, $B = 0.06$T

Torque on a bar magnet :

$$T = MB \sin\theta$$

$0.018 = M \times 0.06 \times \sin 30°$

$$\Rightarrow 0.018 = M \times 0.06 \times \frac{1}{2} \qquad \Rightarrow M = 0.6\text{A} - \text{m}^2$$

Position of stable equilibrium ($\theta = 0°$)

Position of unstable equilibrium ($\theta = 180°$)

Minimum work required to rotate bar magnet from stable to unstable equilibrium

$\Delta U = U_f - U_j = -MB \cos 180° - (-MB \cos 0°)$

$$W = 2MB = 2 \times 0.6 \times 0.06$$

$$W = 7.2 \times 10^{-2} \text{J}$$

2. **A circular coil has moment of inertia 0.8 kg m^2 around any diameter and is carrying current to produce a magnetic moment of 20 Am^2. The coil is kept initially in a vertical position and it can rotate freely around a horizontal diameter. When a uniform magnetic field of 4 T is applied along the vertical, it starts rotating around its horizontal diameter. The angular speed the coil acquires after rotating by $60°$ will be:**

[Sep. 04, 2020 (II)]

(a) 10 rad s^{-1} **(b) 10π rad s^{-1}** **(c) 20π rad s^{-1}** **(d) 20 rad s^{-1}**

SOL. (a) Given,

Moment of inertia of circular coil, $I = 0.8$ kg m^2

Magnetic moment of circular coil, $M = 20$ Am^2

Rotational kinetic energy of circular coil, $\text{KE} = \frac{1}{2} Iw^2$

Here, w = angular speed of coil

Potential energy of bar magnet $= -MB \cos \varphi$

From energy conservation

$$\frac{1}{2} Iw^2 = U_{in} - U_f = -MB \cos 60^\circ - (-MB)$$

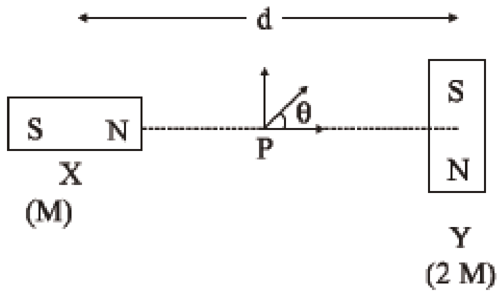
$$\Rightarrow \frac{MB}{2} = \frac{1}{2} Iw^2 \qquad \Rightarrow \frac{20 \times 4}{2} = \frac{1}{2}(0.8) w^2$$

$$\Rightarrow 100 = w^2 \Rightarrow w = 10 \text{ rad}$$

3. **Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their midpoint P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What**

would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)

[8 April 2019 II]



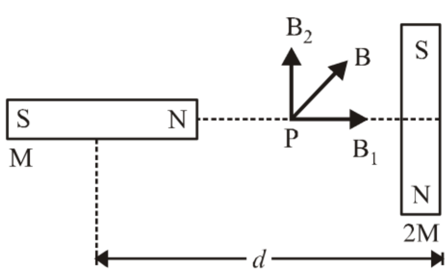
(a) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(b) 0

(c) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{(d/2)^3}\times qv$

(d) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{(d/2)^3}\times qv$

SOL. (b) $B_1=\frac{\mu_0 2M}{4\pi(d/2)^3}$



and $B_2=\frac{\mu_0 2M}{4\pi(d/2)^3}$

$$\tan\theta=\frac{B_2}{B_1}=\frac{\frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}}{\frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}}=1$$

or $\theta=45^{\circ}$

The resultant field is 45^{o} from B_1.

The angle between $\vec{B}$ and $\vec{v}$ zero, so force on the particle is zero.

4. **A magnet of total magnetic moment $10^{-2}\,\hat{i}\ A-m^2$ is placed in a time varying magnetic field, $B\ \hat{i}$ (cos wt) where $B = 1$ Tesla and $w = 0.125\ rad/s$. The work done for reversing the direction of the magnetic moment at $t = 1$ second, is:**

[10 Jan. 2019 I]

(a) 0.01 J **(b) 0.007 J** **(c) 0.028 J** **(d) 0.014 J**

SOL. (c)

Work done, $W = 2m \cdot B$

$= 2 \times 10^2 \times 1 \cos\ (0.125)$

$= 0.02J$

5. **A magnetic dipole in a constant magnetic field has:**

[Online April 8, 2017]

(a) maximum potential energy when the torque is maximum

(b) zero potential energy when the torque is minimum.

(c) zero potential energy when the torque is maximum.

(d) minimum potential energywhen the torque is maximum.

SOL. (c) Potential energy of dipole,

$$U = -pE\cos\theta$$

Torque experienced by dipole $\tau = pE\sin\theta$

Torque will be maximum (Γ_{max}) when $\theta = 90°$ then potential energy $U = 0$

6. A magnetic dipole is acted upon by two magnetic fields which are inclined to each other at an angle of $75°$. One of the fields has a magnitude of 15 mT. The dipole attains stable equilibrium at an angle of $30°$ with this field. The magnitude of the other field (in mT) is close to:

[Online April 9, 2016]

(a) 1 (b) 11 (c) 36 (d) 1060

SOL. (b) We know that, magnetic dipole moment

$M = NiA\cos\theta$ i.e., $M \propto \cos\theta$

When two magnetic fields are inclined at an angle of $75°$ the equilibrium will be at $30°$, so

$$\cos\theta = \cos(75° - 30°) = \cos 45° = \frac{1}{\sqrt{2}}$$

$$\frac{x}{\sqrt{2}} = \frac{15}{2} \ldots\ldots x \approx 11$$

7. A 25 cm long solenoid has radius 2cm and 500 total number of turns. It carries a current of 15 A. If it is equivalent to a magnet of the same size and magnetization $\vec{M}$ (magnetic moment/volume), then $|\vec{M}|$ is:

[Online April 10, 2015]

(a) $30000\pi Am^{-1}$ (b) $3\pi Am^{-1}$ (c) $30000\ Am^{-1}$ (d) $300Am^{-1}$

SOL. (c) $\vec{M}(\text{mag. moment/volume}) = \frac{NiA}{Al}$

$$= \frac{Ni}{\ell} = \frac{(500)15}{25 \times 10^{-2}} = 30000Am^{-1}$$

8. A bar magnet of length 6 cm has a magnetic moment of $4\ JT^{-1}$. Find the strength of magnetic field at a distance of 200 cm from the centre of the magnet along its equatorial line.

[Online May 7, 2012]

(a) 4×10^{-8} tesla **(b) 3.5×10^{-8} tesla** **(c) 5×10^{-8} tesla** **(d) 3×10^{-8} tesla**

SOL. (c) Along the equatorial line, magnetic field strength

$$B=\frac{\mu_0 M}{4\pi\left(r^2+l^2\right)^{3/2}}$$

Given: $M = 4\,J\Gamma^{-1}$ $r = 200\text{cm} = 2\text{m}$

$$\ell=\frac{6\text{cm}}{2}=3\text{cm}=3\times10^{-2}\text{m}$$

$$B=\frac{4\pi\times10^{-7}}{4\pi}\times\frac{4}{[2^2+(3\times10^{-2})^2]^{3/2}}$$

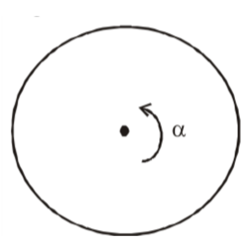
Solving we get, $B = 5\times10^{-8}$ tesla

9. A thin circular disc of radius R is uniformly charged with density $\sigma>0$ per unit area. The disc rotates about its axis with a uniform angular speed (w) . The magnetic moment of the disc is

[2011 RS]

(a) $\pi R^4\sigma w$ **(b)** $\frac{\pi R^4}{2}\sigma w$ **(c)** $\frac{\pi R^4}{4}\sigma w$ **(d)** $2\pi R^4\sigma w$

SOL. (c) $\frac{q}{2m}=\frac{\text{Magnetic dipole moment}}{\textit{Angular momentum}}$



Magnetic dipole moment(M)

$$M=\frac{q}{2m}\left(\frac{mR^2}{2}\right)w=\frac{1}{4}\sigma\pi R^4 w$$

10. A magnetic needle is kept in a non-uniform magnetic field. It experiences

[2005]

(a) neither a force nor a torque **(b) a torque but not a force**

(c) a force but not a torque **(d) a force and a torque**

SOL. (d) A magnetic needle kept in non uniform magnetic field experience a force and torque due to unequal forces acting on poles.

11. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along its length into three equal parts and these parts are then placed on each other with their like poles together. The time period of this combination will be

[2004]

(a) $2\sqrt{3}$s **(b) $\frac{2}{3}$s** **(c) 2 s** **(d) $\frac{2}{\sqrt{3}}$s**

SOL. (b) Initially, time period of magnet

$$T = 2\pi\sqrt{\frac{I}{MB}} = 25 \text{ where } I = \frac{1}{12}m\ell^2$$

When the magnet is cut into three pieces the pole strength will remain the same and Moment of inertia of each part,

$$(I^1) = \frac{1}{12}\left(\frac{m}{3}\right)\left(\frac{\ell}{3}\right)\times 3 = \frac{I}{9}$$

We have, Magnetic moment (M) = Pole strength $(m) \times \ell$

New magnetic moment,

$$M^{\uparrow} = m \times \left(\frac{\ell}{3}\right) \times 3 = m\ell = M$$

New time period, $T^1 = 2\pi\sqrt{\frac{I^1}{M^1 B}}$

$= 2\pi\sqrt{\frac{I}{9MB}} \Rightarrow T' = \frac{T}{\sqrt{9}} = \frac{2}{3}s.$

12. A magnetic needle lying parallel to a magnetic field requires W units of work to turn it through 60^0. The torque needed to maintain the needle in this position will be

[2003]

(a) $\sqrt{3}$W **(b) W** **(c) $\frac{\sqrt{3}}{2}$W** **(d) 2W**

SOL. (a) Workdone to turn a magnetic needle from angle θ_1 to θ_2 is given by

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

$$W = MB(\cos 0° - \cos 60°)$$

$= MB\left(1 - \frac{1}{2}\right) = \frac{MB}{2}$

Torque, $\tau = MB \sin\theta = MB \sin 60° = \sqrt{3}\frac{MB}{2} = \sqrt{3}W$

13. The magnetic lines of force inside a bar magnet

[2003]

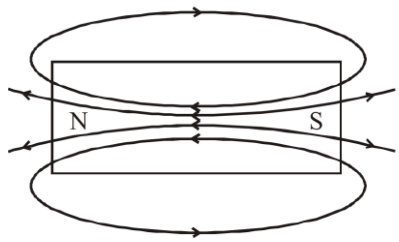
(a) are from north-pole to south-pole of the magnet

(b) do not exist

(c) depend upon the area of cross-section of the bar magnet

(d) are from south-pole to north-pole of the Magnet

SOL. (d) The magnetic field lines of bar magnet form closed lines. As shown in the figure, the magnetic lines of force are directed from south to north inside a bar magnet. Outside the bar magnet magnetic field lines directed from north to south pole.



Topic 2: The Earth Magnetism, Magnetic Materials and their properties

14. An iron rod of volume $10^{-3}\mathrm{m}^3$ and relative permeability 1000 is placed as core in a solenoid with 10 turns/cm. If a current of 0.5 A is passed through the solenoid, then the magnetic moment of the rod will be :

[Sep. 05, 2020 (ID]]

(a) $50 \times 10^2 \mathrm{Am}^2$ (b) $5 \times 10^2 \mathrm{Am}^2$ (c) $500 \times 10^2 \mathrm{Am}^2$ (d) $0.5 \times 10^2 \mathrm{Am}^2$

SOL. (b) Given,

Volume of iron rod, $V = 10^{-3}\mathrm{m}^3$

Relative permeability, $\mu_r = 1000$

Number of turns per unit length, $n = 10$

Magnetic moment of an iron core solenoid,

$$M = (\mu_r - 1) \times NiA$$

$$\Rightarrow M = (\mu_r - 1) \times Ni\frac{V}{l} \Rightarrow M = (\mu_r - 1) \times \frac{N}{l} iV$$

$$\Rightarrow M = 999 \times \frac{10}{10^{-2}} \times 0.5 \times 10^{-3} = 499.5 \approx 500.$$

15. A paramagnetic sample shows a net magnetization of 6A/m when it is placed in an external magnetic field of 0.4T at a temperature of 4 K. When the sample is placed in an external magnetic field of 0.3T at a temperature of 24 K, then the magnetisation will be:

[Sep. 04, 2020 (II)]

(a) 1A/m (b) 4A/m (c) 2.25A/m (d) 0.75A/m

SOL. (d) For paramagnetic material. According to curies law

$$\chi \propto \frac{1}{T}$$

For two temperatures T_1 and T_2

$$\chi_1 T_1 = \chi_2 T_2$$

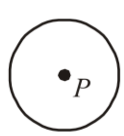
But $\chi = \frac{I}{B}$

$$\frac{I_1}{B_1} T_1 = \frac{I_2}{B_2} T_2$$

$$\Rightarrow \frac{6}{0.4} \times 4 = \frac{I_2}{0.3} \times 24 \Rightarrow l_2 = \frac{0.3}{0.4} = 0.75\text{A/m}$$

16. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which is filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is:

[Sep. 03, 2020 (II)]



(a) $\vec{B}$

(b) zero

(c) much large than $|\vec{B}|$ and parallel to $\vec{B}$

(d) much large than $|\vec{B}|$ but opposite to $\vec{B}$

SOL. (b) When magnetic field is applied to a diamagnetic substance, it produces magnetic field in opposite direction so net magnetic field inside the cavity of sphere will be zero. So, field inside the paramagnetic substance kept inside the cavity is zero.

17. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required?

[Sep. 02, 2020 (I)]

(a) T : Large retentivity, small coercivity

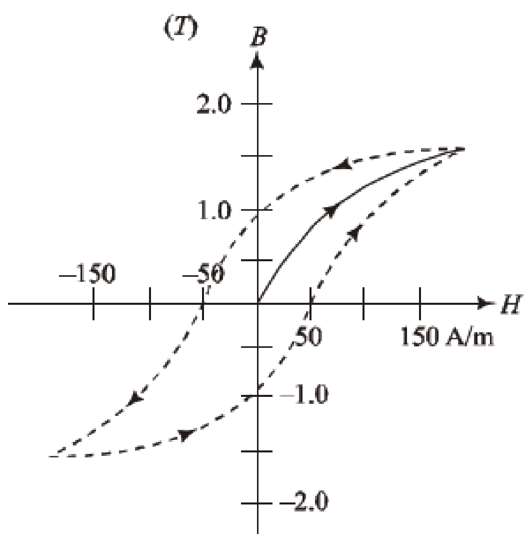
(b) P: Small retentivity, large coercivity

(c) T: Large retentivity, large coercivity

(d) P: Large retentivity, large coercivity

SOL. (d) Permanent magnets(P) are made of materials with large retentivity and large coercivity. Transformer cores (T) are made of materials with low retentivity and low coercivity.

18.



The figure gives experimentally measured *B* vs. *H* variation in a ferromagnetic material. The retentivity, co-ercivity and saturation, respectively, of the material are:

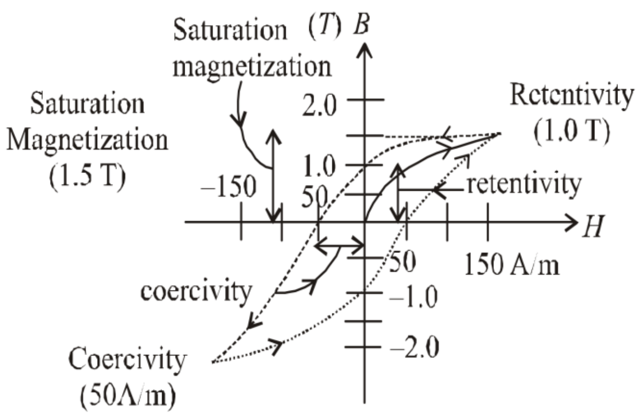
[7 Jan. 2020 II]

(a) $1.5T$, $50A/m$ and 1.0 T **(b) $1.5T$, $50A/m$ and $1.0T$**

(c) $150A/m$, $1.0T$ and $1.5T$ **(d) $1.0T$, $50A/m$ and $1.5T$**

SOL.

(d)



19. **A paramagnetic material has 10^{28} atoms$/m^3$. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is:**

[12 Jan. 2019 II]

(a) 3.267×10^{-4} **(b) 3.672×10^{-4}** **(c) 3.726×10^{-4}** **(d) 2.672×10^{-4}**

SOL. (a) According to Curie law for paramagnetic substance,

$$\chi \propto \frac{1}{T_C} \Rightarrow \frac{\chi_1}{\chi_2} = \frac{T_{C_2}}{T_{C_1}}$$

$$\frac{2.8 \times 10^{-4}}{x_2} = \frac{300}{350}$$

$$\chi_2 = \frac{2.8 \times 350 \times 10^{-4}}{300} = 3.266 \times 10^{-4}$$

20. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is:

[11 Jan. 2019 II]

(a) 3.3×10^{-2} (b) 4.3×10^{-2} (c) 2.3×10^{-2} (d) 3.3×10^{-4}

SOL. (d) Magnetic susceptibility,

$$\chi = \frac{I}{H}$$

where, $I = \frac{\text{Magneticmoment}}{\text{Vo1ume}} = \frac{20 \times 10^{-6}}{10^{-6}} = 20 N/m^2$

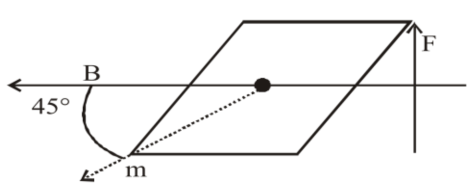
Now, $\chi = \frac{20}{60 \times 10^{3}} = \frac{1}{3} \times 10^{-3} = 3.3 \times 10^{-4}$

21. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle oflength 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is:

[10 Jan. 2019 II]

(a) 3.6×10^{-5}N (b) 1.8×10^{-5}N (c) 1.3×10^{-5}N (d) 6.5×10^{-5}N

SOL. (d) using, MB $\sin\theta = F\ell \text{Sin}\theta(\tau)$



MB $\sin 45° = F\frac{\ell}{2} \sin 45°$

$$F = 2MB = 2 \times 1.8 \times 18 \times 10^{-6} = 6.5 \times 10^{-5} N$$

22. **A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is:**

[9 Jan. 2019 I]

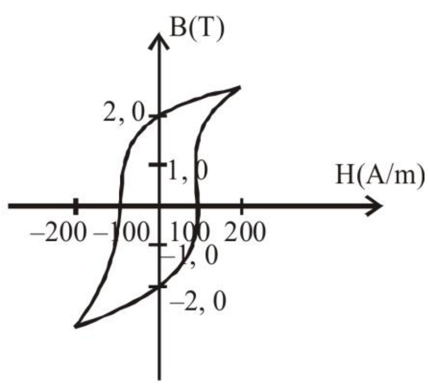
(a) 285A/m **(b) 2600A/m** **(c) 520A/m** **(d) 1200A/m**

SOL. (b) Coercivity, $H = \frac{B}{\mu_0}$ and $B = \mu_0 ni$ $\left(n = \frac{N}{\ell}\right)$

or, $H = \frac{N}{\ell} i = \frac{100}{0.2} \times 5.2 = 2600 \text{A/m}$

23. **The B-H curve for a ferromagnet is shown in the figure. The ferromagnet is placed inside a long solenoid with 1000 turns/cm.. The current that should be passed in the solenoid to demagnetize the ferromagnet completely is:**

[Online Apri115, 2018]



(a) 2 mA **(b) 1 mA** **(c) 40 μA** **(d) 20 μA**

SOL. (b) Given Number of turns,

$$n = 1000 \text{ tums/cm} = 1000 \times 100\text{mms/m}$$

Coercivity of ferromagnet, $H = 100\text{A/m}$

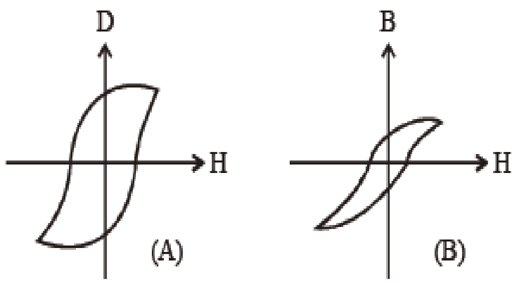
Current to demagnetise the ferromagnet, I =?

Using, $H = nI$

or, $100 = 10^5 \times I$

$$I = \frac{100}{10^5} = 1mA$$

24. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

[2016]

(a) A for transformers and B for electric generators.

(b) B for electromagnets and transformers.

(c) A for electric generators and transformers.

(d) A for electromagnets and B for electric generators.

SOL. (b) Graph [A] is for material used for making permanent magnets (high coercivity)

Graph [B] is for making electromagnets and transformers.

25. A fighter plane of length 20 m, wing span (distance from tip of one wing to the tip of the other wing) of l5m and height 5m is lying towards east over Delhi. Its speed is 240 ms^{-1}. The earth's magnetic field over Delhi is 5×10^{-5}T with the declination angle ~0° and dip of θ such

that $\sin\theta = \frac{2}{3}$. If the voltage developed is V_B between the lower and upper side of the plane and V_W between the tips of the wings then V_B and V_W are close to:

[Online April 10, 2016]

(a) $V_B = 40mV; V_W = 135mV$ with left side of pilot at higher voltage

(b) $V_B = 45mV; V_W = 120mV$ with right side of pilot at higher voltage

(c) $V_B = 40mV; V_W = 135mV$ with right side of pilot at higher voltage

(d) $V_B = 45mV; V_W = 120mV$ with left side of pilot at higher voltage

SOL. (d) $V_B = VB_H l = 240 \times 5 \times 10^5 \cos(\theta) \times 5 = 44.7$ mv

By right hand rule, the charge moves to the left of pilot.

26. A short bar magnet is placed in the magnetic meridian of the earth with north pole pointing north. Neutral points are found at a distance of 30 cm from the magnet on the East- West line, drawn through the middle point of the magnet. The magnetic moment of the magnet in Am^2 is close to: (Given $\frac{\mu_0}{4\pi} = 10^{-7}$ in SI units and B_H =Horizontal component of earth's magnetic field $= 3.6$

$\times 10^{-5}$ tesla)

[Online April 11, 2015]

(a) 14.6 (b) 19.4 (c) 9.7 (d) 4.9

SOL. (c) Here, $r = 30cm = 0.3cm$

we know $\frac{\mu_0 M}{4\pi r^3} = B_H = 3.6 \times 10^{-5}$

$\Rightarrow M = \frac{3.6 \times 10^{-5}}{10^{-7}} (0.3)^3$

Hence, $M = 9.7Am^2$

27. The coercivity of a small magnet where the ferromagnet gets demagnetized is $3 \times 10^3 Am^{-1}$ The current required to be passed in a solenoid oflength 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is:

[2014]

(a) 30mA (b) 60 mA (c)3A (d) 6A

SOL. (c) Magnetic field in solenoid $B = \mu_0 ni$

$$\Rightarrow \frac{B}{\mu_0} = ni$$

(Where n = number ofturns per unit length)

$$\Rightarrow \frac{B}{\mu_0} = \frac{Ni}{L} \Rightarrow 3 \times 10^3 = \frac{100i}{10 \times 10^{-2}}$$

$$\Rightarrow i = 3A$$

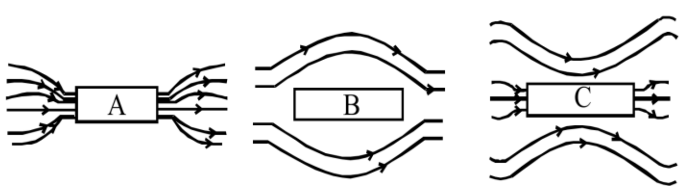
28. An example of a perfect diamagnet is a superconductor. This implies that when a superconductor is put in a magnetic field of intensity B, the magnetic field B_s inside the superconductor will be such that:

[Online Apri119, 2014]

(a) $B_s = -B$ (b) $B_s = 0$ (c) $B_s = B$ (d) $B_s < B$ but $Bs \neq 0$

SOL. (b) Magnetic field inside the superconductor is zero. Diamagnetic substances are repelled in external magnetic field.

29. Three identical bars A, B and C are made of different magnetic materials. When kept in a uniform magnetic field, the field lines around them look as follows:



Make the correspondence of these bars with their material being diamagnetic (D), ferromagnetic (*F*) and paramagnetic (P):

[Online April 11, 2014]

(a) $A \leftrightarrow D, B \leftrightarrow P, C \leftrightarrow F$ **(b) $A \leftrightarrow F, B \leftrightarrow D, C \leftrightarrow P$**

(c) $A \leftrightarrow P, B \leftrightarrow F, C \leftrightarrow D$ **(d) $A \leftrightarrow F, B \leftrightarrow P, C \leftrightarrow D$**

SOL. (b) Diamagnetic materials are repelled in an external magnetic field.

30. The magnetic field of earth at the equator is approximately 4×10^{-5} T. The radius of earth is 6.4×10^6m. Then the dipole moment of the earth will be nearly of the order of:

[Online April 9, 2014]

(a) 10^{23} A m^2 **(b) 10^{20} A m^2** **(c) 10^{16} A m^2** **(d) 10^{10} A m^2**

SOL. (a)Given, $B = 4\times10^{-5}$ T, $R_E = 6.4\times10^6 m$

Dipole moment of the earth M =?

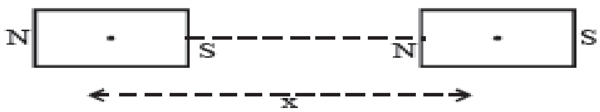
$$B = \frac{\mu_0}{4\pi}\frac{M}{d^3}$$

$$4 \times 10^{-5} = \frac{4\pi \times 10^{-7} \times M}{4\pi \times (6.4 \times 10^6)^3}$$

$$M \cong 10^{23} Am^2$$

31. The mid points of two small magnetic dipoles of length d in end-on positions are separated by a distance x, (x >> d). The force between them is proportional to x^{-n} where n is:

[Online April 9, 2014]



(a) 1 **(b) 2** **(c) 3** **(d) 4**

SOL. (d) In magnetic dipole

$$\text{Force} \propto \frac{1}{r^4}$$

In the given question, $\text{Force} \propto x^{-n}$ Hence, $n = 4$

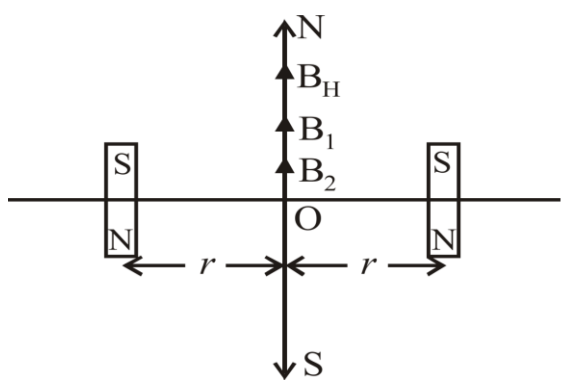
32. Two short bar magnets of length 1 cm each have magnetic moments $1.20\ Am^2$ and $1.00\ Am^2$ respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of earth's magnetic induction is $3.6 \times 10.5\ Wb/m^2$)

[2013]

(a) $3.6 \times 10^5\ Wb/m^2$ **(b)** $2.56 \times 10^4\ Wb/m^2$

(c) $3.50 \times 10^4\ Wb/m^2$ **(d)** $5.80 \times 10^4\ Wb/m^2$

SOL. (b) Given: $M_1 = 1.20Am^2$



$$M_2 = 1.00Am^2; r = \frac{20}{2}cm = 0.1\text{m}$$

$$B_{net} = B_1 + B_2 + B_H$$

$$B_{net} = \frac{\mu_0(M_1 + M_2)}{4\pi r^3} + B_H$$

$$=\frac{10^{-7}(1.2+1)}{(0.1)^3}+3.6\times10^{-5}=2.56\times10^{-4}\,wb/m^2$$

33. The earth's magnetic field lines resemble that of a dipole at the centre of the earth. If the magnetic moment of this dipole is close to $8\times10^{22}\text{Am}^2$, the value of earth's magnetic field near the equator is close to (radius of the earth $=6.4\times10^6\text{m}$)

[Online April 25, 2013]

(a) 0.6 Gauss (b) 1.2 Gauss (c) 1.8 Gauss (d) 0.32 Gauss

SOL. (a) Given $M=8\times10^{22}\text{Am}^2$

$$d=R_e=6.4\times10^6\text{m}$$

Earth's magnetic field, $B=\frac{\mu_0}{4\pi}\cdot\frac{2M}{d^3}$

$$=\frac{4\pi\times10^{-7}}{4\pi}\times\frac{2\times8\times10^{22}}{(6.4\times10^6)^3}\cong0.6\ \text{Gauss}$$

34. Relative permittivity and permeability of a material ε_r and μ_r , respectively. Which of the following values of these quantities are allowed for a diamagnetic material?

[2008]

(a) $\varepsilon_r=0.5,\mu_r=1.5$ (b) $\varepsilon_r=1.5,\mu_r=0.5$

(c) $\varepsilon_r=0.5,\mu_r=0.5$ (d) $\varepsilon_r=1.5,\mu_r=1.5$

SOL. (b) For a diamagnetic material, the value of μ_r is slightly less than one.

For any material, the value of $\in_r$ is always greater than 1.

35. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will

[2006]

(a) attract N_1 and N_2 strongly but repel N_3

(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly

(c) attract N_1 strongly, but repel N_2 and N_3 weakly

(d) attract all three of them

SOL. (b) Ferromagnetic substance has magnetic domains whereas paramagnetic substances have magnetic dipoles which get attracted to a magnetic field. Ferromagnetic material magnetised strongly in the direction of magnetism field, Hence, N_1 will be attracted paramagnetic substance attract weekly in the direction offield. Hence, N_2 will weakly attracted. Diamagnetic substances do not have magnetic dipole but in the presence ofexternal magnetic field due to their orbital motion of electrons these substances are repelled. Hence, N_3 will be repelled.

36. The materials suitable for making electromagnets should have

[2004]

(a) high retentivity and low coercivity **(b) low retentivity and low coercivity**

(c) high retentivity and high coercivity **(d) low retentivity and high coercivity**

SOL. (b) Electromagnet should be amenable to magnetisation & demagnetization.

Materials suitable for making electromagnets should have low retentivity and low coercivity should be low.

37. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate fieely in the same field. If its period of oscillation is T', the ratio $\frac{T'}{T}$ is

[2003]

(a) $\frac{1}{2\sqrt{2}}$ **(b)** $\frac{1}{2}$ **(c) 2** **(d)** $\frac{1}{4}$

SOL. (b) The time period of a rectangular magnet oscillating in earth's magnetic field is given by $T = 2\pi\sqrt{\frac{I}{MB_H}}$

where I = Moment of inertia of the rectangular magnet M = Magnetic moment

B_H = Horizontal component of the earth's magnetic field Initially,

the time period of the magnet $T = 2\pi\sqrt{\frac{I}{MB_H}}$ where I $= \frac{1}{12}M\ell^2$

Case 2

Magnet is cut into two identical pieces such that each piece has half the original length.

Then $T^{'} = 2\pi\sqrt{\frac{I^{'}}{MB_H}}$

Moment of inertia of each part

$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2 \times 8 \times 10^{22}}{(6.4 \times 10^6)^3} \cong 0.6$ and $M^{'} = \frac{M}{2}$

$$\frac{T'}{T} = \sqrt{\frac{I'}{M} \times \frac{M}{I}} = \sqrt{\frac{I/8}{M/2} \times \frac{M}{I}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

38. Curie temperature is the temperature above which

[2003]

(a) a ferromagnetic material becomes paramagnetic

(b) a paramagnetic material becomes diamagnetic

(c) a ferromagnetic material becomes diamagnetic

(d) a paramagnetic material becomes ferromagnetic

SOL. (a) The temperature above which a ferromagnetic substance becomes paramagnetic is called Curie's temperature.

Topic 3: Magnetic Equipment

39. A ring is hung on a nail. It can oscillate, without slipping or sliding (i) in its plane with a time period T_1 and, (ii) back and forth in a direction perpendicular to its plane, with a period T_2. The ratio $\frac{T_1}{T_2}$ will be:

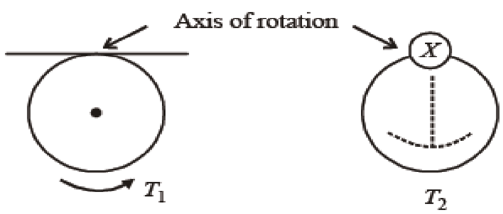
[Sep. 05, 2020 (II)]

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2}{3}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

SOL. (a) Let I_1 and I_2 be the moment ofinertia in first and second case respectively.

$$I_1 = 2MR^2$$

$$I_2 = MR^2 + \frac{MR^2}{2} = \frac{3}{2}MR^2$$



Time period, $T = 2\pi\sqrt{\frac{I}{mgd}}$

$$T \propto I$$

$$\therefore \frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

40. **A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45^o, and 40 times per minute where the dip is 30^o. If B_1 and B_2 are respectively the total magnetic field due to the earth and the two places, then the ratio B_1/B_2 is best given by:**

[12 April 2019 I]

(a) 1.8 **(b) 0.7** **(c) 3.6** **(d) 2.2**

SOL. (Bonus) We have, $T = 2\pi\sqrt{\frac{I}{MB_x}}$

$$\frac{T_1^2}{T_2^2} = \frac{Bx_2}{Bx_1}$$

or $\left(\frac{2}{1.5}\right)^2 = \frac{B_2 \cos 45^\circ}{B_1 \cos 30^\circ} = \frac{B_2 \times 2}{\sqrt{2} \times B_1 \times \sqrt{3}}$

$$\left(\frac{4}{3}\right)^2 = \frac{B_2}{B_1} \times \frac{2}{\sqrt{6}}$$

$$\frac{B_1}{B_2} = \frac{9}{8\sqrt{6}} = 0.46$$

41. **A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then:**

[10 Jan. 2019 II]

(a) $T_h = T_c$ **(b) $T_h = 2T_c$** **(c) $T_h = 1.5T_c$** **(d) $T_h = 0.5T_c$**

SOL. (a) Using, time /oscillation period,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Where, M = magnetic moment, I moment of inertia and B = magnetic field

$$T_h = 2\pi\sqrt{\frac{mR^2}{(2MB)}}$$

$$T_c = 2\pi\sqrt{\frac{1/2mR^2}{MB}}$$

Clearly, $T_h = T_c$

42. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is:

[2017]

(a) 6.98 s **(b) 8.76 s** **(c) 6.65 s** **(d) 8.89 s**

SOL. (c) Given: Magnetic moment, $M = 6.7 \times 10^2 Am^2$ Magnetic field, $B = 0.01T$

Moment of inertia, $I = 7.5 \times 10^{-6}$Kgm2

Using, $T = 2\pi\sqrt{\frac{I}{MB}}$

$$= 2\pi\sqrt{\frac{75 \times 10^{-6}}{67 \times 10^{-2} \times 0.01}} = \frac{2\pi}{10} \times 1.06s$$

Time taken for 10 complete oscillations

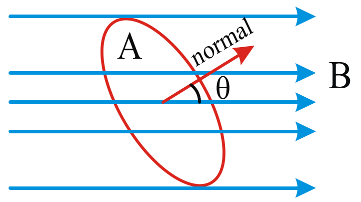
$$t = 10T = 2\pi \times 1.06$$

$$= 6.6568 \approx 6.65s$$

ELECTRO MAGNETIC INDUCTION

↳ The phenomenon in which electric current is induced by varying magnetic fields is called electromagnetic induction.

Magnetic Flux (ϕ) : The number of magnetic lines of force passing normally through given area is called magnetic flux.



When a surface of area A is placed in a uniform magnetic field of induction B, such that the unit vector along the normal ($\hat{n}$) makes an angle ' θ ' with direction of magnetic field then the flux passing through it is given by

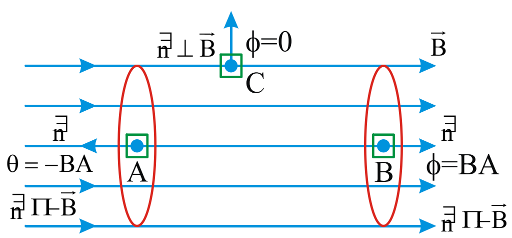
$\phi = \overline{B}.\overline{A} = BA\cos\theta$

↳ If magnetic field is non uniform then $\phi = \int \overline{B}.\overline{ds}$

↳ The SI Unit of flux is weber (Wb).
CGS unit of flux is maxwell (Mx)
1 weber = 1 tesla - meter^2
1 weber = 10^8 maxwell
Dimensional formula of the magnetic flux is $ML^2T^{-2}A^{-1}$
Magnetic flux is a scalar

↳ Magnetic flux can be positive, negative or zero depending upon the angle between area vector and field direction.

↳ When a cylinder is placed in a uniform magnetic field as shown in the below figure



i) When the plane of the surface is parallel to the direction of the magnetic field (or) normal drawn to the surface is perpendicular to the magnetic field $\left(\hat{n} \perp \vec{B}\right)$ then magnetic flux linked with the surface is zero i.e., $\phi = 0 \left[\because \theta = 90^0\right]$

ii) When the plane of the surface is perpendicular to magnetic field (or) normal drawn to the surface is parallel to the magnetic field $\left(\hat{n} \parallel \vec{B}\right)$, then magnetic flux linked with the surface is maximum. i.e., $\phi_{max} = BA\left(\because \theta = 0^0\right)$

iii) When the flux entering the surface is opposite to the area vector ($\hat{n}$) then $\phi = -BA\left(\because \theta = 180^0\right)$

↪ The magnetic flux linked with a coil $\left(\phi = NBA\cos\theta\right)$ can be changed by

a) Changining the no. of turns (N)
b) Varying the magnetic field (B)
c) Changing the area of the magnetic field bounded by the coil by moving the coil into or out of the magnetic field
d) Changing the angle made by the coil with the direction of the field

↪ **The change of flux due to rotation of the coil:** When the coil is rotated from an angle of θ_1 to an angle of θ_2 (both are measured w.r.t normal) in a uniform magnetic field then the initial flux through the coil is

$\phi_i = \text{NBA}\cos\theta_1$

The final flux through the coil after rotation is

$\phi_f = \text{NBA}\cos\theta_2$

The change in the flux associated with the coil is

$\Delta\phi = \phi_f - \phi_i$

$\Delta\phi = \text{NBA}\left(\cos\theta_2 - \cos\theta_1\right)$

if $\theta_1 = 0^0$ and $\theta_2 = 90^0$ then $\Delta\phi = -\text{NBA}$

if $\theta_1 = 90^0$ and $\theta_2 = 180^0$ then $\Delta\phi = -\text{NBA}$

if $\theta_1 = 0^0$ and $\theta_2 = 180^0$ then $\Delta\phi = -2\text{NBA}$

Ex:1 A rectangular loop of area 0.06 m^2 is placed in a uniform magnetic field of 0.3T with its plane (i) normal to the field (ii) inclined 30^0 to the field (iii) parallel to the field. Find the flux linked with the coil in each case.

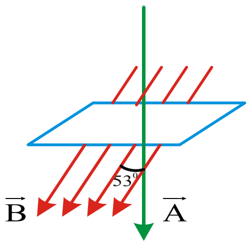
Sol. $\phi = \text{NBA}\cos\theta$

i) $\phi = 1 \times 0.06 \times 0.3 \times \cos 0^0 = 0.018$ weber

ii) $\phi = 1 \times 0.06 \times 0.3 \times \cos 60^0 = 0.009$ weber

iii) $\phi = 1 \times 0.06 \times 0.3 \times \cos 90^0 = 0$

Ex 1(a): **t a certain location in the northern hemisphere, the earth's magnetic field has magnitude of $42\mu T$ and points downwards at 53^0 to the vertical. Calculate the flux through a horizontal surface of area** $2.5m^2.\left[\sin 53^0 = 0.8\right]$



Sol. $\phi_B = BA\cos\theta = 42\times10^{-6}\times2.5\times\cos 53^0 = 63\mu Wb$

Faraday's laws of electro magnetic induction

First Law : Whenever the magnetic flux linked with an electric circuit (coil) changes, an emf is induced in the circuit (coil). The induced emf exists as long as the change in magnetic flux continues.

Second Law : The induced emf produced in the coil is equal to the negative rate of change of magnetic flux linked with it.

$$e = -\frac{d\phi}{dt}$$

where $\phi =$ flux through each turn

If the coil contains N turns, an emf appears in every turn all these emfs are to be added. Then, the induced emf is given by

$$e = -N.\frac{d\phi}{dt} = -\frac{d}{dt}(N\phi)$$

Where '$N\phi$' is total flux linked with the coil of N turns.

(or)

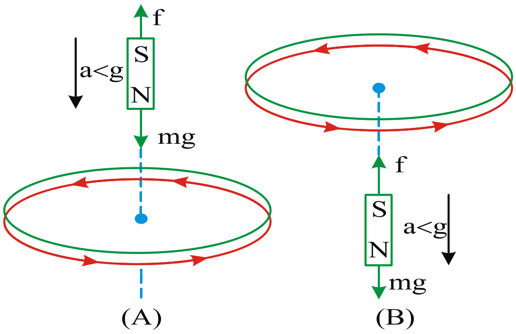
$$e = -\frac{d}{dt}(N\phi) = -\frac{d}{dt}(\text{NBA}\cos\theta)$$

Negative sign is in accordance with Lenz's law. The above law is also called **Neumann's law**.

Lenz's Law and Conservatin of Energy

"The direction of the induced emf is always such that it tends to produce a current which opposes the change in magnetic flux"

- Induced emf can exist whether the circuit is opened or closed. But induced current can exist only in the closed circuits.
- A metallic ring is held horizontally and a bar magnet is dropped through the ring with its length along the axis of the ring, as shown in figure.



In both the cases net force on the magnet is

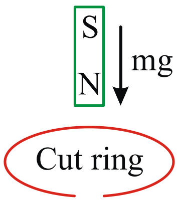
$F_{net} = mg - f$

Hence net acceleration of the fall is

$a_{net} = g - \frac{f}{m} \Rightarrow a_{net} < g$

where f=force exerted by the induced magnetic field of ring on the magnet.

↳ When the magnet is allowed to fall through an open ring (or) cut ring, then



a) an emf is induced

b) No current is induced (since the ring is not closed) and hence no induced magnetic field.

c) No opposition to the motion of the magnet.

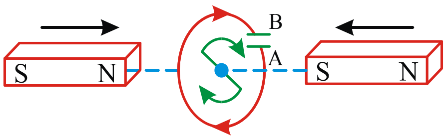
d) $F_{net} = mg$

e) $a_{net} = g$ Magnet falls with an acceleration = g

↳ When a magnet is allowed to fall through two identical metal coils at different temperatures then magnet falls slowly through the coil at low temperature as its resistance is less more induced current flows so more is the opposition.

↳ A magnet allowed to fall through a long cylindrical pipe then the acceleration of magnet is always less than 'g' and the acceleration continuously decreases due to induced currents. But the velocity increases until the magnet moves with acceleration. At a particular instant the acceleration becomes zero and the magnet moves downwards with uniform velocity, called terminal velocity.

↳ When the two magnets are moved perpendicular to plane of coil as shown, then



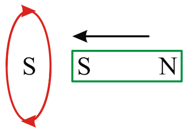
a) emf is induced

b) Induced current flows from A to Balong the coil when A and B are connected through resistor.

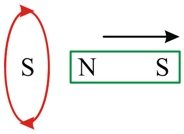
c) Electrons flow from B to A along the coil

d) Hence plate A will become negatively charged and plate B becomes positively charged.

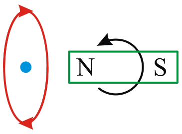
↳ The directions of iduced current in coil for different kinds of motion of magnets

(a) 

Clockwise induced current

(b) 

Clockwise induced current

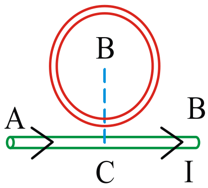
(c) 

No induced current

(because there is no change of flux linked with the coil)

↳ When a current carrying conductor is placed beside a closed loop in its plane then the induced current direction for the following are

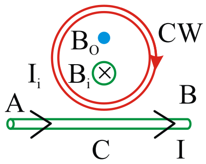
a) Current in conductor is constant.



I= Constant
ϕ= Constant
I_i= Zero
(A)

$\therefore$ No induced current

b) Current through the conductor increases as shown.

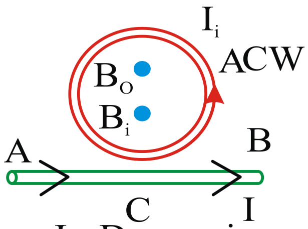


B_O - Original field
B_i - Induced field

I= Increasing
ϕ= Increasing
I_i= clock wise
(B)

↳ In this case, the flux through the loop due to current carrying wire is out of the plane of the coil.
As current is increasing, the outward flux through the coil also increases.
Hence to oppose this, an inward flux is created by the clock wise induced current.

c) Current through the conductor decreases as shown.



B_O - Original field
B_i - Induced field

I= Decreasing
ϕ= Decreasing
I_i= Anti clockwise

(C)

In this case, the flux through the loop due to current carrying wire is out of the plane of the coil. As current is decreasing, the outward flux through the coil also decreases.
Hence to oppose this, an outward flux is created by the anti-clock wise induced current.

Expressions for induced EMF, Induced current and Induced Change

↳ According to Faraday's second law and Lenz's law the induced emf is given by $e = -\frac{d\phi}{dt}$

If the coil has N turns then $e = -N\frac{d\phi}{dt}$

$$\Rightarrow e = -N\frac{(\phi_2 - \phi_1)}{dt}$$

↳ As $\phi = BAN\cos\theta$ and $e = -\frac{d\phi}{dt}$

The emf is induced (or) change in flux is caused by changing B (or) A (or) N (or) θ

↳ If 'B' is changed then

a) Average induced emf

$$e = -AN\cos\theta\frac{(B_2 - B_1)}{(t_2 - t_1)}$$

Here B_1 is magnetic field induction at an instant t_1 B_2 is magnetic field induction at an instant t_2

b) If the plane of the coil is perpendicular to magnetic field, then $\theta = 0^0 \Rightarrow \cos\theta = 1$

then $e = -AN\frac{(B_2 - B_1)}{(t_2 - t_1)}$

c) Instantaneous emf $e = -AN\cos\theta\frac{dB}{dt}$

↳ If 'A' is changed then

a) Average induced emf

$$e = -BN\cos\theta\frac{(A_2 - A_1)}{(t_2 - t_1)}$$

b) If the plane of the coil is perpendicular to magnetic field, then $\theta = 0^0 \Rightarrow \cos\theta = 1$

then $e = -BN\frac{(A_2 - A_1)}{(t_2 - t_1)}$

c) Instantaneous emf $e = -BN\cos\theta\frac{dA}{dt}$

↳ If 'θ' is changed (i.e., if coil is rotated)

a) Average induced emf

$$e = -BAN\frac{(\cos\theta_2 - \cos\theta_1)}{(t_2 - t_1)}$$

b) Instantaneous emf $e = -BAN\frac{d}{dt}(\cos\theta)$

If the coil is rotated with constant angular velocity 'ω' then $\theta = \omega t$ and

$$e = -BAN\frac{d}{dt}(\cos\omega t) = BAN\,\omega\,\sin\,\omega t$$

$\therefore e = BAN\omega\sin\omega t$

c) $\omega t = 90^0$, if the plane is parallel to the magnetic field then induced emf is maximum. Then Peak emf.

$e_0 = BAN\,\omega$ $\qquad\qquad \therefore e = e_0 \sin\omega t$

This is the principle of AC generator.

Induced Current

- If the magnetic flux in a coil of resistance R changes from ϕ_1 to ϕ_2 in a time 'dt', then a current 'i' is induced in the coil as $i = \frac{e}{R}$ $\qquad i = \frac{N(\phi_2 - \phi_1)}{Rdt}$ $\qquad \left(\because e = -N.\frac{d\phi}{dt}\right)$

$\therefore$ Induced current is given by $\qquad$ Magnitude of current

$$i = \frac{\text{Induced emf}}{\text{Resistance in the circuit}} = \frac{N}{R}\left(\frac{d\phi}{dt}\right)$$

Induced Charge

- The amount of charge induced in a conductor is given as follows

We know, $I = \frac{e}{R}$ (or) $I = \frac{1}{R}\left(-\frac{d\phi}{dt}\right)$ $\qquad \Rightarrow \frac{dq}{dt} = -\frac{1}{R}\frac{d\phi}{dt}$ (or) $dq = -\frac{1}{R}d\phi$

$\therefore$ Induced charge, $q = -\frac{1}{R}\int_{\phi_i}^{\phi_f} d\phi$

$q = -\frac{1}{R}\left[\phi_f - \phi_i\right]$ (or) $q = \frac{\phi_i - \phi_f}{R}$ (magnitude of charge)

$\therefore$ In general, induced charge is given by

$$q = \frac{\text{change of magnetic flux}}{\text{resistance}}$$

For N turns, the induced charge is $q = \frac{N}{R}(d\phi)$

- Induced emf is independent of total resistance of the circuit but depends on time of change of flux.
- Induced current depends on both time of change of flux and resistance of circuit
- Induced charge is independent of time but depends on the resistance of circuit.
- When a magnet is moved towards a stationary coil(i) slowly and (ii) quickly, then
  a) induced charge is same in both cases
  b) induced emf is more in second case
  c) induced current is more in second case

E.X: 1(b). 3:The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi_B = \left(5t^3 + 4t^2 + 2t - 5\right)$ weber. Calculate the induced current through the coil at $t = 2$ second. The resistance of the coil is 5Ω.

Sol. $\phi = 5t^3 + 4t^2 + 2t - 5$

$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$ at $t = 2\sec, e = 78V$

$i \times 5 = 15 \times 4 + 8 \times 2 + 2 \Rightarrow i = 15.6A$

E.X: 2EX. 4:A circular coil of 500 turns of wire has an enclosed area of $0.1m^2$ per turn. It is kept perpendicular to a magnetic field of induction 0.2T and rotated by 180^0 about a diameter perpendicular to the field in 0.1s. How much charge will pass when the coil is connected to a galvanometer with a combined resistance of 50Ω.

Sol. $q = \frac{\phi_i - \phi_f}{R} = \frac{NBA - (-NBA)}{R} = \frac{2NBA}{R}$

$q = \frac{2 \times 500 \times 0.2 \times 0.1}{50} = 0.4C$.

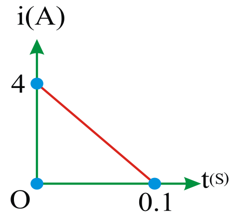
E.X: 3EX. 5: Some magnetic flux is changed from a coil of resistance 10Ω. As a result an induced current is developed in it, which varies with time as shown in figure. What is the magnitude of change in flux through the coil ?

Sol. The induced charge is $q = \frac{\Delta\phi}{R}$

But, Area of i-t curve gives charge

$\therefore \Delta\phi = R \times$ Area of $i - t$ curve ; $\Delta\phi = qR$

$= \frac{1}{2} \times 4 \times 0.1 \times 10$



$\therefore \Delta\phi = 2wb$

E.X: 4EX. 6: A long solenoid with 1.5 turns per cm has a small loop of area $2.0\,cm^2$ placed inside the solenoid normal to its axis. If the current in the solenoid changes steadily from 2.0 A to 4.0 A in 1.0s. The emf induced in the loop is

Sol. The magnetic field along the axis of solenoid is $B = \mu_0 ni$ where n is no. of turns per unit length. flux through the smaller loop placed in solenoid is $\phi = B \times A$ Since current in solenoid is changing,

emf induced in loop is $e = \frac{d\phi}{dt} = \frac{d}{dt}\left[\mu_0 niA\right]$; $e = \mu_0 nA\left(\frac{di}{dt}\right)$

$= 4\pi \times 10^{-7} \times 1.5 \times 10^2 \times 2 \times 10^{-4} \times \left(\frac{4-2}{1-0}\right)$

$= 0.75 \times 10^{-6} V$

E.X: 5EX. 7:: A sqaure loop of side 10cm and resistance $0.5\,\Omega$ is placed vertically in the east-west plane. A uniform magnetic field of 0.10T is set up across the plane in the north-east direction. The magnetic field is decreased to zero in 0.70s at a steady rate. The magnitude of current in this time-interval is.

Sol. The initial magnetic flux is given by

$\phi = BA\cos\theta$

Given, B=0.10 T, area of square loop $= 10\times 10 = 100 cm^2 \quad = 10^{-2} m^2$

$\therefore \qquad \phi = \frac{0.1\times 10^{-2}}{\sqrt{2}} Wb$

Final flux, $\phi_{min} = 0$

The change in flux is brought about in 0.70 s

The magnitude of the induced emf is

$$e = \frac{\Delta\phi}{\Delta t} = \frac{|\phi - 0|}{\Delta t} = \frac{10^{-3}}{\sqrt{2}\times 0.7} = 1\,mV$$

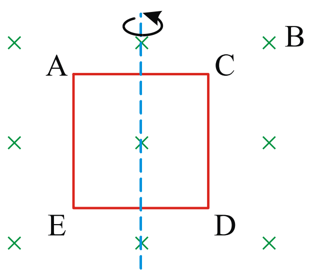
The magnitude of current is

$$I = \frac{e}{R} = \frac{10^{-3}}{0.5} = 2\,mA$$

E.X: 6EX. 8: **A square loop ACDE of area $20 cm^2$ and resistance 5Ω is rotated in a magnetic field B = 2T through 180^0**

a) in 0.01 s and b) in 0.02 s.

Find the magnetiude of e,i and Δq in both the cases.



Sol. Let us take the area vector S perpendicular to plane of loop inwards. So intially dS parallel to B and when it is rotated by 180^0, S is anti parallel to B. Hence, initial flux passing through the loop,

$\phi_i = BS\ \cos 0^0 = (2)(20\times 10^{-4})(1)$

$= 4\times 10^{-3}\, Wb$

Flux passing through the loop when it is rotated by 180^0, $\phi_f = BS\cos 180^0$

$= (2)(20\times 10^{-4})(-1) \quad = -4.0\times 10^{-3} Wb$

Therefore, change in flux,

$\Delta\phi_B = \phi_f - \phi_i \ ; \quad = -8\times 10^{-3} Wb$

(a) Given $\Delta t = 0.01s,\ R = 5\Omega$;∴ $|e| = \left|\frac{\Delta\phi_B}{\Delta t}\right| = \frac{8\times10^{-3}}{0.01} = 0.8V$; or $i = \frac{|e|}{R} = \frac{0.8}{5} = 0.16A$

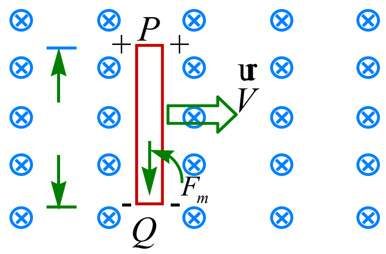
and $\Delta q = i\Delta t = 0.16\times0.01;\ = 1.6\times10^{-3}C$

b) $\Delta t = 0.02s;\ \therefore\ |e| = \left|-\frac{\Delta\phi_B}{\Delta t}\right|$

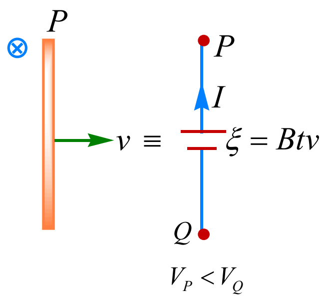
$= \frac{8\times10^{-3}}{0.02}$; = 0.4V ; $i = \frac{|e|}{R} = \frac{0.4}{5} = 0.08A$ and $\Delta q = i\Delta t = (0.08)(0.02);\ = 16\times10^{-3}C$

Motional EMF

Let a thin conducting rod PQ of length l move in a uniform magnetic field B directed perpendicular to plane of paper inwards. Let the velocity v of rod be in the plane of paper towards right



By Fleming's Let Hand Rule a positive charge (q) in the rod suffers magnetic force qvB directed from Q to P along the rod while an electron will experience a force evB directed from P to Q along the length of the rod. Due to this force the free electrons of rod move from P to Q, thus making end Q negative and end P positive. This causes a potential difference along the ends of rod. This potential difference developed is called induced emf ξ.



If E is electric field developed in the rod, then $E = \frac{\xi}{l}$ ξ being e.m.f. induced across the rod

For equilibrium of charges
Electrical force = Magnetic force

$\Rightarrow eE = evB$ $\Rightarrow E = vB$

So, induced e.m.f. $\xi = El = Blv$

If the rod moves across the magnetic field moving at an angle θ with it, then induced e.m.f.

$\xi + B_n vl = Bv_n l$ where B_n is component of magnetic field normal to v.

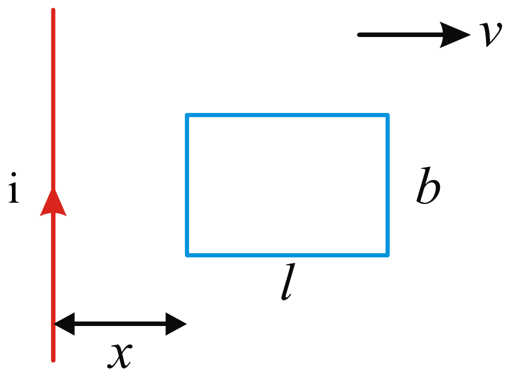
Hence, $B_n = B\sin\theta$ $\Rightarrow$ Induced e.m.f. $\xi = Bvl\sin\theta$

Please note that the equivalent replacement of motional emf by a battery is shown here. Do not confuse that the direction of current is shown right but $V_P < V_Q$ as induced current with go from higher potential to lower potential. Hes, of course this is true but for the external circuit (excluding battery) so, note that in the rod in motion the induced coventional current is going from lower potential to higher potential. Just think this away that in the external circuit current goes from positive terminal to the negative terminal and inside the battery it goes from negative terminal to positive terminal.

- The motional emf is the emf which results from relative motion between a conductor and the source of magnetic field.
- When a condutor of length l is moved with a velocity v perpendicular to its length in uniform magnetic field (B), which is perpendicular to both its length and as well as its velocity, the emf induced across its ends e=Blv
- If the rod moved making an angle θ with its length, then $e = Blv\sin\theta$
- In vector form $e = \vec{B}.(\vec{l} \times \vec{v})$ or $\vec{l}.(\vec{v} \times \vec{B})$
- among $\vec{B}, \vec{l}$ and $\vec{v}$, if any two are parallel the emf induced across the conductor is zero

E.X: 7 A rectangular loop of length 'l' and breadth 'b' is placed at a distance of x from an infinitely long wire carrying current 'i' such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity 'v', the magnitude of the e.m.f. in the loop is : (μ_0 = permeability of free space)

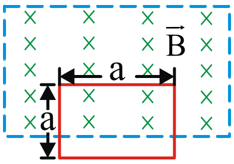
Sol.



emf = Blv =Bbv $= (B_1 - B_2)bv$

$$= \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+l)}\right]bv; \quad = \frac{\mu_0 ibv}{2\pi}\left[\frac{1}{x} - \frac{1}{l+x}\right]; \quad = \frac{\mu_0 ilbv}{2\pi x(x+l)}$$

E.X: 8 A horizontal magnetic field B is produced across a narrow gap between the two square iron pole pieces. A closed square loop of side a, mass m and resistance R is allowed to fall with the top of the loop in the field. The loop attains a terminal velocity equal to :



Sol. Induced emf in the loop, when it is falling with terminal velocity

e = BVa ; $i = \frac{e}{R} = \frac{Bva}{R}$

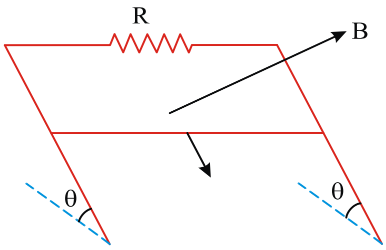
Vertically upward force experienced by loop due to this

F = Bia ; $= B\left(\frac{Bva}{R}\right)a$; $= \frac{B^2va^2}{r}$

When the loop attains terminal velocity 'v'

$mg = \frac{B^2va^2}{R}$; $V = \frac{mgR}{B^2a^2}$

E.X: 9 A conducting wire of mass m slides down two smooth conducting bars, set at an angle θ to the horizontal as shown in figure. The seperation between the bars is l. The system is located in the magnetic field B, perpendicular to the plane of the sliding wire and bars. The constant velocity of the wire is



Sol. Along inclined plane the force acting downwards $= mg\sin\theta$ ....(1)

magnetic force acting upwards

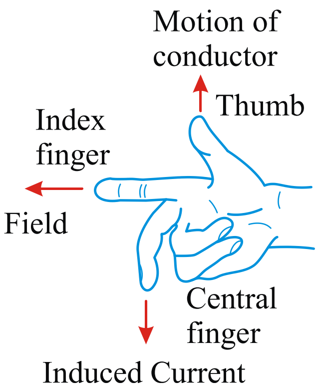
$\Rightarrow F = Bil \Rightarrow B\left(\frac{Blv}{R}\right)l$; $= \frac{B^2l^2v}{R}$ ......(2)

From (1) and (2)

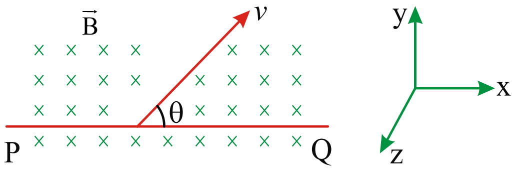
$\frac{B^2l^2v}{R} = mg\sin\theta$; $v = \frac{mgR\sin\theta}{B^2l^2}$

Flemings's Right Hand Rule

- Stretch the first three fingers of right hand such that they are mutually perpendicular to each other. If the fore finger represents the direction of magnetic field and the thumb represents the direction of the motion of the conductor, then the central finger indicates the direction of induced current



- A conductor of length 'l' measured from P to Q is moved with a speed of 'v' in a uniform magnetic field 'B' as shown in figure.

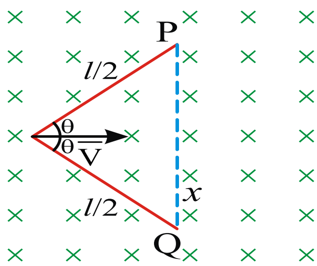


Here $\vec{B} = B(-\hat{k}), \vec{l} = l(\hat{i})$ and $\vec{v} = v\cos\theta\hat{i} + v\sin\theta\hat{j}$

Induced emf is $e = \vec{l}\cdot\left(\vec{v}\times\vec{B}\right) = l(\hat{i})\cdot\left(v\cos\theta\hat{i} + v\sin\theta\hat{j}\right)\times B(-\hat{k}) = -Blv\sin\theta$

The change in the flux in the time of 'Δt' is $\therefore \Delta\phi = e\Delta t = -Blv\sin\theta\Delta t$

- A conductor of length 'l' is bent at its midpoint and is moved along its perpendicular bisector with a constant speed of 'v' in a uniform magnetic field of strength 'B' as shown in figure



From the figure $\sin\theta = \dfrac{x}{l/2} \Rightarrow x = \dfrac{l}{2}\sin\theta$

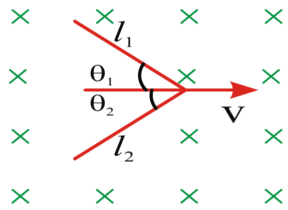
Here $\vec{B} = B(-\hat{k}), \vec{v} = v\hat{i}$ and effective length of the conductor $\vec{l} = 2x(-\hat{j}) = l\sin\theta(-\hat{j})$

Induced emf is $e = \vec{l}\cdot\left(\vec{v}\times\vec{B}\right) = l\sin\theta(-\hat{j})\cdot v\hat{i}\times B(-\hat{k}) = -Blv\sin\theta$

The change in the flux associated in time internal of 'Δt' is $\Delta\phi = e\Delta t = -Blv\sin\theta\Delta t$

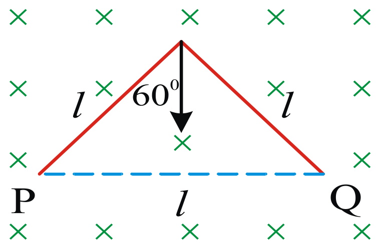
Here the effective length between free ends of conductor is $l\sin\theta$.

↳ The emf induced across the ends of the conductor shown in the figure is



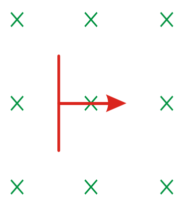
$e = BVl = BV(l_1 \sin\theta_1 + l_2 \sin\theta_2)$

E.X: 10 A wire of length 2l is bent at mid point so that the angle between two halves is 60^0. If it moves as shown with a velocity v in a magnetic field B find the induced emf.



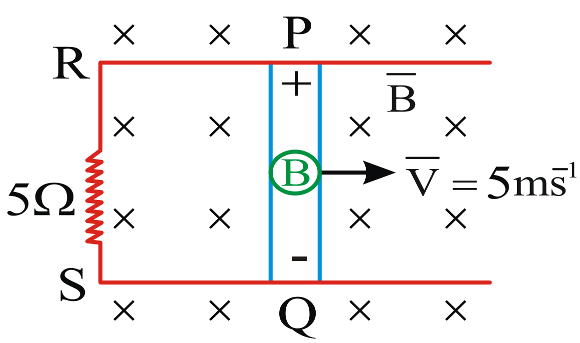
Sol. e=Blv. Here l = Effective length = PQ

E.X: 11 A conductor of length 0.1m is moving with a velocity of 4m/s in a uniform magnetic field of 2T as shown in the figure. Find the emf induced?



Sol. e=Blv sin 90^0 = (2) (0.1) (4) = 0.8 Volt

E.X: 12 Figure shows a conducting rod PQ in contact with metal rails RP and SQ, which are 0.25m apart in a uniform magnetic field of flux density 0.4T acting perpendicular to the plane of the paper. Ends R and S are connected through a 5Ω resistance. What is the emf when the rod moves to the right with a velocity of $5ms^{-1}$? What is the magnitude and direction of the current through the 5Ω resistance? If the rod PQ moves to the left with the same speed, what will be the new current and its direction ?



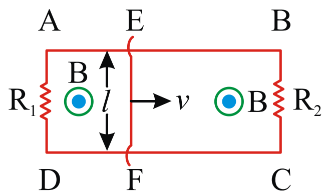
Sol. $|e| = Blv = 0.4 \times 0.25 \times 5 = 0.5V$ Current, $I = \frac{|e|}{R} = \frac{0.5V}{5\Omega} = 0.1A$

As the rod 'PQ' moves to right as shown, the free electrons in it experience a Lorentz force. According to F.L.H., the force is towards the end 'Q' of rod. $\therefore$ They move from P to Q, hence the end of the rod P becomes deficient of electrons $\Rightarrow V_P > V_Q$

Applying Flemming's right hand rule, the current in the rod shall flow from Q to P.

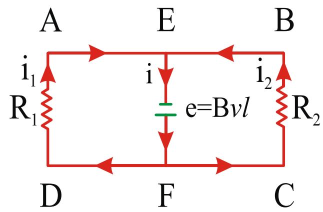
(b) : If the rod PQ moves to the left with the same speed, then the current of 0.1 A will flow inthe rod PQ from P to Q

E.X: 13 A loop ABCD containing two resistors as shown in figure is placed in a uniform magnetic field B directed outwards to the plane of page. A sliding conductor EF of length l and of negligible resistance moves to the right with a uniform velocity v as shown in Fig. Determine the current in each branch.



Sol. The magnetic field induction B, length l and the velocity v of the conductor EF are mutually perpendicular, hence the emf induced in it is e=Blv (with end F of the rod at higher potential)

$\therefore$ The effective electric circuit can be redrawn as shown in Fig.



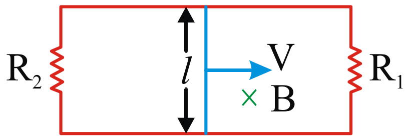
The resistance R_1 and R_2 are in parallel, so the equivalent resistance R is given by $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

From Ohms law, the total current is $i = \frac{e}{R}$

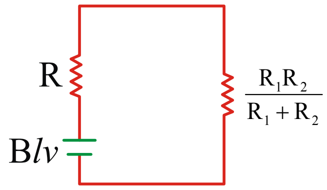
$$i = Blv\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

Current in AD is $i_1 = \frac{BlV}{R_1}$; Current in BC is $i_2 = \frac{BlV}{R_2}$

E.X: 14 A rectangular loop with a slide wire of length l is kept in a uniform magnetic field as shown in the figure. The resistance of slider is R. Neglecting self inductance of the loop find the current in the connector during its motion with a velocity v.



Sol. The equivalent circuit is

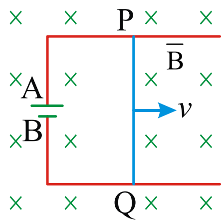


The equivalent resistance of the circuit is $R = \left(R + \frac{R_1 R_2}{R_1 + R_2}\right)$

Hence the current in the connector is $i = \frac{e}{R}$

$\therefore \qquad i = \frac{Blv(R_1 + R_2)}{(RR_1 + RR_2 + R_1 R_2)}$

E.X: 15 A conducting rod PQ of length L = 1.0m is moving with a uniform speed v=2.0m/s in a uniform magnetic field B=4.0T directed into the paper. A capacitor of capacity $C = 10\mu F$ is connected as shown in the figure. Then what are the charges on the plates A and B of the capacitor.



Sol. The motional emf is

$\therefore$ p.d across the capacitor $= Blv = 4 \times 1 \times 2 = 8V$

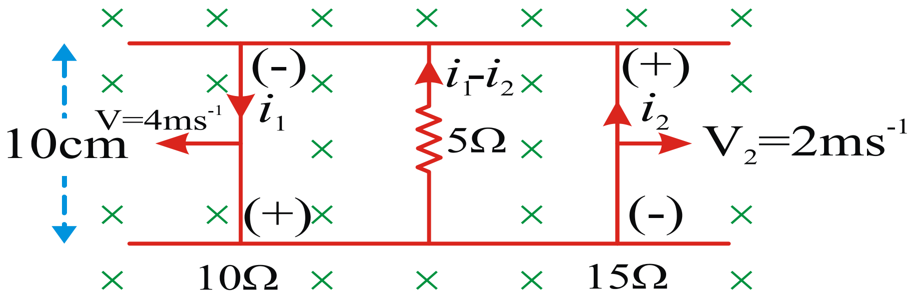
$q = CV = 10 \times 8 = 80\mu C$

A is +Ve w.r.t. B (from fleming right hand rule)

The charge on plate A is $q_A = 80\mu C$

The charge on plate B is $q_B = -80\mu C$

E.X: 16 **Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0Ω resistor. The circuit also contains two metal rods having resistances of 10.0Ω and 15.0Ω along the rails. The rods are pulled away from the resistor at constant speeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01T is applied perpendicular to the plane of the rails. Determine the current in the 5.0Ω resistor.**



Sol. In the figure $R = 5.0\Omega, r_1 = 10\Omega, r_2 = 15\Omega$,

$$e_1 = Blv_1 = 0.01\times 0.1\times 4 = 4\times 10^{-3} V$$

$$e_2 = Blv_2 = 0.01\times 0.1\times 2 = 2\times 10^{-3} V$$

Applying Kircoff's law to the left loop :

$$10i_1 + 5(i_1 - i_2) = 4\times 10^{-3}$$

$$\Rightarrow 15i_1 - 5i_2 = 4\times 10^{-3} \quad \rightarrow (1)$$

Right loop : $15i_2 - 5(i_1 - i_2) = 2\times 10^{-3}$

$$\Rightarrow 20i_2 - 5i_1 = 2\times 10^{-3} \quad \rightarrow (2)$$

Solving (1) and (2) gives :

$$i_1 = \frac{18}{55}\times 10^{-3} A \text{ and } i_2 = \frac{10}{55}\times 10^{-3} A$$

$\Rightarrow$ Current through $5\Omega = i_1 - i_2$

$$= \frac{8}{55}\times 10^{-3} A = \frac{8}{55} mA$$

E.X: 17 **conducting rod MN moves with a speed v parallel to a long straight wire which carries a constant current i, as shown in Fig. The length of the rod is normal to the wire. Find the emf induced in the total length of the rod. State which end will be at a lower potential.**

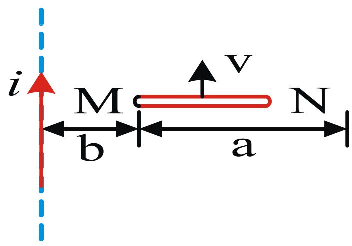


Figure (a)

Sol. The magnetic field induction due to current i is different at different sections of the rod, because they are at different distances from the wire.
Let us, first of all, subdivide the entire length of the conductor MN into elementary sections. Consider a section (shown shaded in the figure (b)) of thickness dx at a distance x from the wire. As all the three, v, B and (dX) are mutually normally to each other, so the emf induced in it is de=Bvdx.
(from N to M by Fleming's right hand rule)

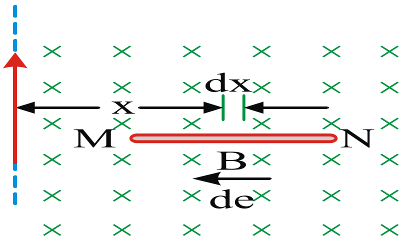


Figure (b)

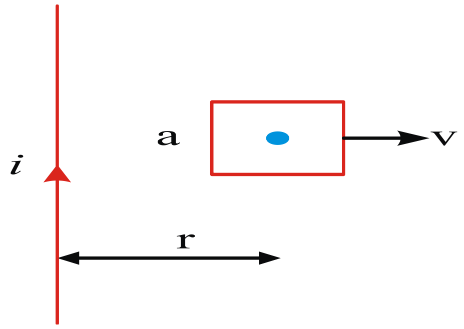
For the rest of sections, the induced emf is in the same sense, (i.e., from N to M)

$\therefore$ Total emf induced in the conductor is $e = \int de = \int_b^{b+a} Bv\,dx$

Substituting for $B = \frac{\mu_0 i}{2\pi x}$, the above equation gets changed to

$e = \int_b^{b+a} \frac{\mu_0 iv\,dx}{2\pi x}$ $e = \frac{\mu_0 iv}{2\pi}[\ln x]_b^{b+a}$ or, $e = \frac{\mu_0 iv}{2\pi}\ln(1 + a/b)$

E.X: 18 A square loop of side a is placed in the same plane as a long straight wire carrying a current i. The centre of the loop is at a distance r from the wire where $r >> a$. The loop is moved away from the wire with a constant velocity v. The induced e.m.f. in the loop is



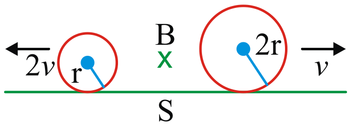
Sol. Magnetic field by the straight wire of current i at a distance r is $B = \frac{\mu_0 i}{2\pi r}$

flux associated with the loop is $\phi = BA = \frac{\mu_0 i}{2\pi r} a^2$

$\therefore e = \frac{-d\phi}{dt} = \frac{-\mu_0}{2\pi} ia^2 \frac{d}{dt}\left(\frac{1}{r}\right) = \frac{-\mu_0}{2\pi} ia^2 \left(\frac{-1}{r^2}\right)\frac{dr}{dt}$

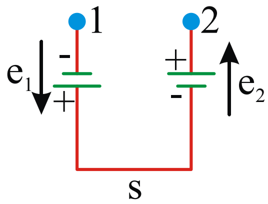
Hence the induced emf in the loop is $e = \frac{\mu_0}{2\pi} i \frac{a^2}{r^2} v \quad \left(\because \frac{dr}{dt} = v \right)$

E.X: 19 Two conducting rings of radii r and 2r move in apposite directions with velocities 2υ and υ respectively on a conducting surface S. There is a uniform magnetic field of magnitude B perpendicular to the plane of the rings. The potential difference between the highest points of the two rings is



Sol. Replace the induced emfs in the rings by cells emfs $e_1 = B2r(2v) = 4Brv$

$e_2 = B(4r)v = 4Brv$ The equivalent circuit is



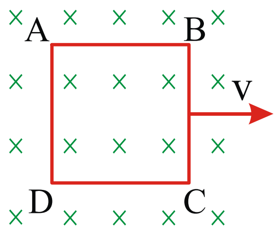
Hence the potential difference between the highest points of the two rings is $V_2 - V_1 = e_1 + e_2 = 8Brv$

E.X: 20 A metallic square loop ABCD is moving in its own plane with velocity v in a uniform magnetic field perpendicular to its plane as shown in the figure. Find

a) In which sides of the loop electric field is induced.

b) Net emf induced in the loop

c)If one 'BC' is outside the field with remaining loop in the field and is being pulled out with a costant velocity then induced current in the loop.



Sol. a) The metallic square loop moves in its own plane with velocity v.

A uniform magnetic field is imposed perpendicular to the plane of the square loop.

AD and BC are $\perp$ to the velocity as well as $\perp$ to field applied. Hence electric field is induced across the sides AD and BC only.

b) As there is no change of flux through the entire coil net emf induced in the coil is zero.

c)Induced current $i = \frac{e}{R}$ Where R is the resistance of the coil.

$\Rightarrow i = \frac{Blv}{R}$ (Only the side AD cuts the flux)

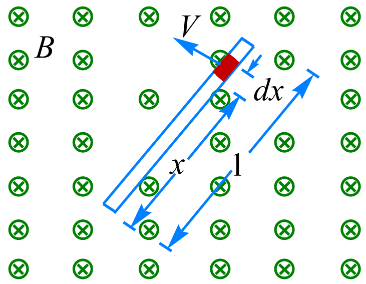
Motional EMF Induced in a Rotating bar

Method-1 : Consider a conducting rod of length l rotating about the point O(at one end of the rod) in a uniform magnetic field b. To find the e.m.f. induced across the ends of the rod, let us consider an infinitesimal element of length dx at a distance x from O, having a velocity v, as shown in figure. If $d\xi$ be the induced e.m.f. across the element, then

$d\xi = B(dx)v$, where v = v = x ω

$\Rightarrow d\xi = B\omega x dx$

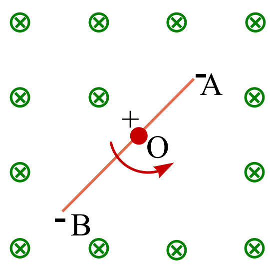
$$\Rightarrow \xi = B\omega \int_0^l x dx = B\omega \frac{x^2}{2}\Big|_0^l = \frac{1}{2}B\omega l^2$$



Method-2 : When the rod is rotating in the field with angular velocity ω, then the induced e.m.f. is

$$\xi = \frac{B(\text{Area swept by the Rod})}{\text{Time to complete one Revolution}} \Rightarrow \xi = \frac{B(\pi l^2)}{\frac{2\pi}{\omega}} \Rightarrow \xi = \frac{1}{2}B\omega l^2$$

↳ In the above case if the rod is rotated about an axis passing through its centre (O) and perpendicular to tis length them emf across its ends is zero



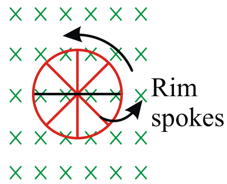
emf across OA is $e = +\frac{1}{8}Bl^2\omega$ emf across OB is $e = -\frac{1}{8}Bl^2\omega$

Net emf across AB is zero

end 'A' is -ve with respect to 'O'

end 'B' is -ve with respect to 'O'

↳ A spoked wheel of spoke length 'l' is rotated about its axis with an angular velocity 'ω' in a plane normal to uniform magnetic field B as shown.



The emf induced across the ends of each spoke is $e = \frac{1}{2}Bl^2\omega$, with axle (centre) at higher potential.

Since all the spokes are parallel between axle and rim, the emf induced between axle and rim is

$e = \frac{1}{2}Bl^2\omega$.

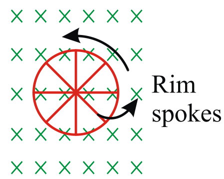
It is independent of number of spokes.

E.X: 21 A copper rod of length 2m is rotated with a speed of 10 rps, in a uniform magnetic field of 1 tesla about a pivot at one end. The magnetic field is perpendicular to the plane of rotation. Find the emf induced across its ends

Sol. $e = \frac{1}{2}B\omega l^2 = \frac{1}{2}B(2\pi n)l^2 = \pi Bnl^2$

$e = 3.14 \times 1 \times 10 \times 2 \times 2 = 125.6$ volt

E.X: 22 A wheel with 10 metallic spokes, each 0.5m long, is rotated with a speed of 120 rev/ minute in a plane normal to the earth's magnetic field at the place. If the magnitude of the field is 0.40 gauss, what is the induced emf between the axle and the rim of the wheel ?



Sol. Here each spoke of wheel act as a source of an induced emf (cell) and emf's of all spokes are parallel.

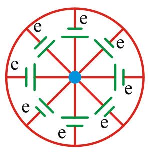
f=120 rev/min = 2 rev/second,

B=0.40 gauss= $0.4 \times 10^{-4} T$,

Area swept, by each spoke per second, $A = \pi r^2 f$

Magnetic flux cut by each spoke per second, $\frac{d\phi_B}{dt} = BA = B\pi r^2 f$

Induced emf, $e = B\pi r^2 f$ (numerically) $e = 0.4 \times 10^{-4} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2$

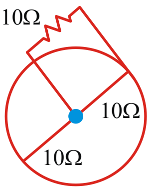


$e = 6.29 \times 10^{-5}$ volt

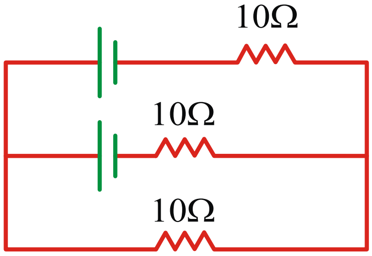
Induced emf in a wheel is independent of no. of spokes.

E.X: 23 A metal rod of resistance 20Ω is fixed along a diameter of a conducting ring of radius 0.1m and lies on x-y plane. There is a magnetic field $\vec{B} = (50T)\hat{k}$. The ring rotates with an angular velocity $\omega = 20\, rad/s$ about its axis. An external resistance of 10Ω is connected across the centre of the ring and rim. The current through external resistance is

Sol.



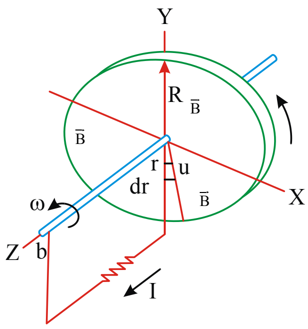
The equivalent circuit is



$e = \frac{1}{2}Bl^2\omega = \frac{1}{2}\times 50\times 0.1\times 0.1\times 20$;

$\therefore e = 5V$ Hence the current through the external resistance is $i = \frac{e}{R}$ $\therefore i = \frac{5}{15} = \frac{1}{3}A$

Motional EMF Induced in a Rotating Disc



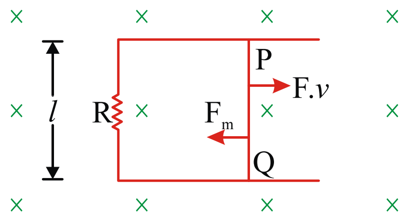
↳ A circular disc of radius 'R' is rotating with an angular velocity 'ω' about an axis passing through centre and plane of rotation is normal to an uniform magnetic field of induction B. It is equivalent to a spoked wheel with a large number of spokes each of length 'R' between centre and rim without any air gap. The emf induced between centre and rim is independent of number of spokes.

So, the emf induced between centre and rim is $e = \frac{1}{2}Bl^2\omega = \frac{1}{2}BR^2\omega$

E.X: 24 A copper disc of radius 1m is rotated about its natural axis with an angular velocity 2 rad/sec in a uniform magnetic field of 5 telsa with its plane perpendicular to the field. Find the emf induced between the centre of the disc and its rim.

Sol. $e = \frac{1}{2}B\omega r^2$; $e = \frac{1}{2}\times 5\times 2\times 1\times 1 = 5\, volt$

Energy consideration



↳ A conductor PQ is moved with a constant velocity v on parallel sides of a U shaped conductor in a magnetic field as shown in figure. Let R be the resistance of the closed loop.

The emf induced in the rod is e=Blv

The current in the circuit is $i = \frac{e}{R} = \frac{Blv}{R}$

As current flows in the conductor PQ from Q to P of the conductor. So, an equal and opposite force F has to be applied on the conductor to move the conductor with a constant velocity v.

Thus, $F = F_m = \frac{B^2 l^2 v}{R}$

The rate at which work is done by the applied force to move the rod is,

$P_{\text{applied}} = Fv = \frac{B^2 l^2 v^2}{R}$

The rate at which energy is dissipated in the circuit is,

$P_{\text{dissipated}} = i^2 R = \left(\frac{Bvl}{R}\right)^2 R = \frac{B^2 l^2 v^2}{R}$

This is just equal to the rate at which work is done by the applied force.

E.X: 25 A 0.1 m long conductor carrying a current of 50 A is perpendicular to a magnetic field of 1.25 mT. The mechanical power to move the conductor with a speed of $1 ms^{-1}$ is

Sol. Power P=Fv ; P=Bilv ; l=0.1m ; i=50

$B = 1.25\times10^{-3}$; v=1m/sec ; $\therefore p = Bilv$

$= 1.25\times10^{-3}\times50\times0.1\times1; = 6.25\times10^{-3}; = 6.25$ mW

E.X: 26 A short - circuited coil is placed in a time varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the radius of the wire is to be halved, then find the electrical power dissipated.

Sol. Current is induced in the short-circuited coil due to the imposed time - varying magnetic field.

Power $P = \frac{e^2}{R}$; Here $e = -\frac{d\phi}{dt}$ where $\phi = NBA$

and $R = \frac{\rho l}{\pi r^2}$ wher l and r are length and radius of the wire.

$\therefore P = \frac{\pi r^2}{\rho l}\left[\frac{d}{dt} NBA\right]^2$ or

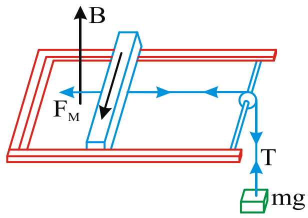
$$P = \frac{\pi r^2}{\rho l} N^2 A^2 \left(\frac{dB}{dt}\right)^2 \text{ or P=(constant)} \frac{N^2 r^2}{l}$$

when $r_2 = \frac{r_1}{2}$ then $t_2 = 4l_1$

$$\therefore \frac{P_2}{P_1} = \frac{(4N)^2}{N^2} \times \left(\frac{r}{2r}\right)^2 \times \left(\frac{l}{4l}\right) \qquad \therefore \frac{P_2}{P_1} = \frac{16N^2 \times r^2 \times l}{N^2 \times 4r^2 \times 4l} \text{ or } \frac{P_2}{P_1} = \frac{1}{1}$$

$\therefore$ Power dissipated is the same.

E.X: 27 A pair of parallel horizontal conducting rails of negligible resistance, shorted at one end is fixed on a table. The distance between R can slide on the rails frictionlessly. The rod is tied to a massless string which passes over a pulley fixed to the edge of the table. A mass m, tied to the other end of the string, hangs vertically. A constant magnetic field B exists perpendicular to the table. If the system is released from rest, calculate :



i) The terminal velocity achieved by the rod.

ii) The acceleration of the mass at the instant when the velocity of the rod, is half the terminal velocity.

Sol. i) the velocity of rod = V

Intensity of magnetic field = B $\qquad \therefore$ emf induced in rod (e)=BLV

$\therefore$ current induced in rod $(i) = \frac{BlV}{R}$

Force on the rod $F = BiL = \frac{B^2VL^2}{R}$

Net force on the system = mg - T

mg - T = ma

but $T = F = \frac{B^2VL^2}{R}$ Hence, $mg - \frac{B^2VL^2}{R} = ma$ or $a = g - \frac{B^2VL^2}{mR}$ ....(i)

For rod to achieve terminal velocity $V_T, a = 0 \quad \therefore 0 = g - \frac{B^2V_TL^2}{mR}$

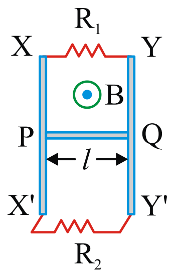
or Terminal velocity $(V_T) = \frac{mgR}{B^2L^2}$ ....(ii)

ii) Acceleration of mass when $V = \frac{V_T}{2}$

or $V = \frac{mgR}{2B^2L^2}$. Put this value of V in (i)

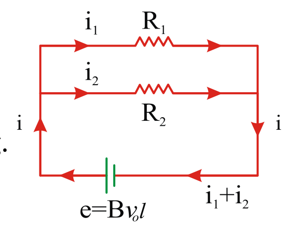
$\therefore a = g - \frac{B^2L^2}{mR} \times \left(\frac{mgR}{2B^2L^2}\right)$ or $a = g - \frac{g}{2}$ $\qquad$ or $a = \frac{g}{2}$ ....(iii)

E.X: 28 Two parallel vertical metallic bars XX^1 and YY^1, of negligible resistance and separated by a length 'l ', are as shown in Fig. The ends of the bars are joined by resistance R_1 and R_2. A uniform magnetic field of induction B exists in space normal to the plane of the bars. A horizontal metallic rod PQ of mass m starts falling vertically, making contact with the bars. It is observed that in the steady state the powers dissipated in the resistance R_1 and R_2 and the terminal velocity attained by the rod PQ.



Sol. Let V_0 be the terminal velocity attained by the rod PQ (in the steady state). If i_1 and i_2 be the currents flowing through R_1 and R_2 in this state, then current flowing through the rod PQ is

$i = i_1 + i_2$ (see the circuit diagram) as shown in Fig.



$\therefore$ Applying Kirchoff's loop rule, yields.

$i_1R_1 = BV_0l$ and $i_2R_2 = BV_0l$

$$\therefore i_1 + i_2 = BV_0l\left(\frac{1}{R_1}+\frac{1}{R_2}\right) \qquad ......(i)$$

Given that, $P_1 = i_1^2R_1 = \dfrac{B^2V_0^2l^2}{R_1}$ .....(ii)

and $P_2 = i_2^2R_2 = \dfrac{B^2V_0^2l^2}{R_2}$ .....(iii)

Also in the steady state, the acceleration of PQ=0

$\Rightarrow mg = B(i_1+i_2)l$ (or) $mg = B^2l^2V_0\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right) = P_1 + P_2$

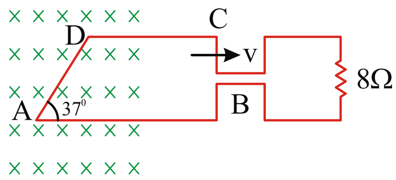
[From equation (ii) and (iii)] $\therefore$ The terminal velocity is $V_0 = \dfrac{P_1+P_2}{mg}$

Substituting for V_0 in equation (ii), $P_1 = \dfrac{B^2l^2}{R_1}\left(\dfrac{P_1+P_2}{mg}\right)^2 \Rightarrow R_1 = \left[\dfrac{Bl(P_1+P_2)}{mg}\right]^2 \times \dfrac{1}{P_1}$

Similarly from equation (iii)

$$R_2 = \left[\frac{Bl(P_1+P_2)}{mg}\right]^2 \times \frac{1}{P_2}$$

E.X: 29 The loop ABCD is moving with velocity 'v' towards right. The magnetic field is 4T. The loop is connected to a resistance of 8Ω. If steady current of 2A flows in the loop then value of 'v' if loop has a resistance of 4Ω, is : (Given AB=30cm, AD=30 cm)



Sol. The induced emf in the loop is $e = Blv$

$$e = B(AD)\sin 37^0 v = 4\times 0.3 \sin 37^0 v$$

Effective resistance of the circuit is

$R = (4+8) = 12\Omega$; Hence $i = \frac{e}{R} = \frac{Blv}{R}$

$$\Rightarrow 2 = \frac{4\times 0.3\times \sin 37^0 v}{(4+8)} \ ; \ \therefore v = \frac{100}{3} m/s$$

E.X: 30 A square loop of side 12cm with its sides parallel to x and y-axes is moved with a velocity 8 cm/s along positive x-direction in an environment containing magnetic field along +ve z-direction. The field has a gradient of 10^{-3} tesla/em along -ve x-direction (increasing along -ve x-axis) and also decreases with time at the rate of 10^{-3} tesla/s. The emf induced in the loop is

Sol. The magnetic field in loop varies with position 'x' of loop and also with time simultaneously.

The rate of change of flux due to variation of 'B' with time is $\frac{d\phi}{dt} = A\times\frac{dB}{dt}$

The rate of change of flux due to variation B with position 'x' is

$$\frac{d\phi}{dt} = A\times\frac{dB}{dt} = A\frac{dB}{dx}\times\frac{dx}{dt} = A\frac{dB}{dx}\times v$$

Since both cause decrease in flux, the two effects will add up

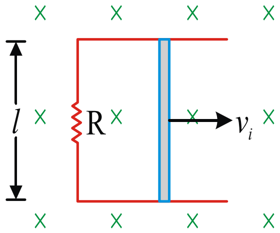
$\therefore$ The net emf induced

$$e = \frac{d\phi}{dt} = A\frac{dB}{dt} + A\frac{dB}{dx}\times v = A\left[\frac{dB}{dt} + v.\frac{dB}{dx}\right]$$

$$= 144\times 10^{-4}\left[10^{-3} + 8\times 10^{-3}\right]$$

$$= 144\times 9\times 10^{-7} = 129.6\times 10^{-6} V$$

E.X: 31 A bar of mass m and length l moves on two frictionless parallel rails in the presence of a uniform magnetic field directed into the plane of the paper. The bar is given an initial velocity v_i to the right and released. Find the velocity of bar, induced emf across the bar and the current in the circuit as a function of time



Sol. The induced current is in the counter clockwise direction and the magnetic force on the bar is given by $F_B = -ilB$. The negative sign indicates that the force is towards the left and retards motion.

F=ma

$$-ilB = m.\frac{dv}{dt}$$

Because the force depends on current and the current depends on the speed, the force is not constant and the acceleration of the bar is not constant. The induced current is given by $i = \frac{Blv}{R}$;

$$-ilB = m.\frac{dv}{dt}$$

$$-\left(\frac{Blv}{R}\right)lB = m.\frac{dv}{dt} \Rightarrow \frac{dv}{v} = -\frac{B^2l^2}{mR}dt$$

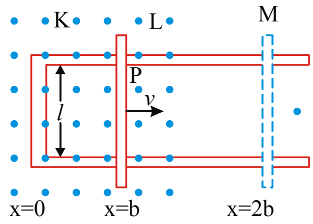
$$\int_{v_1}^{v}\frac{dv}{v} = -\frac{B^2l^2}{mR}\int_0^t dt \; ; \; \ln\left(\frac{v}{v_1}\right) = -\frac{B^2l^2}{mR}t = \frac{-t}{T}$$

where $T = \frac{mR}{B^2l^2} \Rightarrow v = v_i e^{\frac{t}{T}}$

The speed of the bar therefore decreases exponentially with time under the action of magnetic retarding force.

emf $= iR = Blv_i e^{\frac{t}{T}}$; current : $i = \frac{Blv}{R} = \frac{Bl}{R}v_1 e^{\frac{t}{T}}$

E.X: 32 The arm PQ of the rectangular conductor is moved from x=0, outwards in the uniform magnetic field which extends from x=0 to x=b and is zero for x>b as shown. Only the arm PQ possesses substantial resistance r. Consider the situation when the arm PQ is pulled outwards from x=0 to x=2b, and is then moved back to x=0 with constant speed v. Obtain expressions for the flux, the induced emf, the force necessary to pull the arm and the power dissipated as Joule heat. Sketch the variation of these quantities with distance.

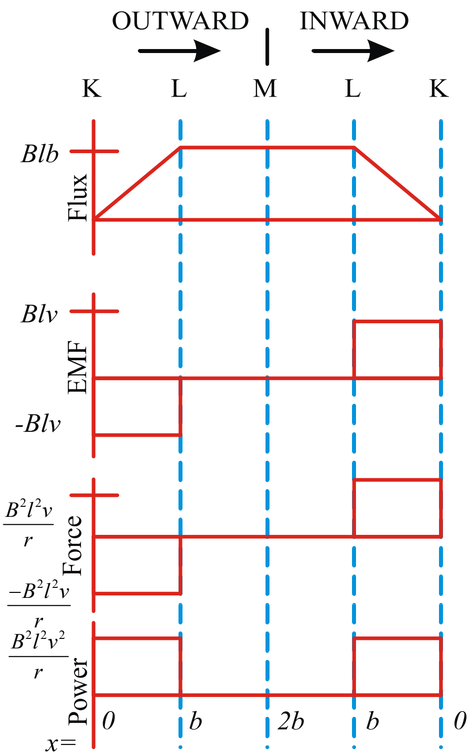


Sol. Let us first consider the forward motion from $x=0$ to $x=2b$. The flux ϕ_B linked with the circuit SPQR is

The induced emf is, $\varepsilon = -\dfrac{d\phi_B}{dt} = -Blv \qquad 0 \le x < b$

$\varepsilon = 0 \qquad b \le x < 2b$

When the induced emf is nonzero, the current I is $I = \dfrac{Blv}{r}$ (in magnitude)



The force required to keep the arm PQ in constant motion is IlB. Its direction is to the left.

$$F = \frac{B^2l^2v}{r} \quad 0 \le x < b : F = 0 \quad b \le x < 2b$$

The Joule heating loss is

$$P_J = I^2 r = \frac{B^2l^2v^2}{r} \qquad 0 \le x < b \qquad P_J = 0 \qquad b \le x < 2b$$

One obtains similar expressions for the inward motion from $x=2b$ to $x=0$. One can appreciate the whole process by examining the sketch of various quantities displayed in Fig

Eddy Currents

- When bulk pieces of conductors are subjected to changing magnetic flux, induced currents are produced in them.
- The flow patterns of induced currents resemble the whirling eddies in water. This effect was discovered by Foucault and these currents are called eddy currents (or) Foucault currents.
- A copper plate is allowed to swing like a simple pendulum between the pole pieces of a strong magnet, its motion is damped and the plate comes to rest inthe magnetic field due to eddy currents in the plate.
- If rectangular slots are made in the copper plate area available to the flow of eddy currents is less. So, electromagnetic damping is reduced and the plate swings more freely.
- The eddy currents heat up the metallic cores and dissipate electrical energy inthe form of heat in the devices like transformers, electric motors and other such devices.
- The eddy currents are minimized by using. laminations of metal to make a metal core. The laminations are separated by an insulating material like lacquer.
- The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths reduces the strength of the eddy current.
- **Advantages :**

  Eddy currents are used in

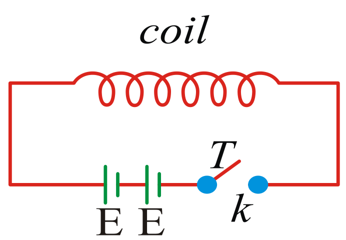
  a) Magnetic braking in trains.

  b) Electromagnetic damping.

  c) Induction furnace.

  d) Electric power meters.

Self induction :



- If current flowing in a coil changes, the magnetic flux linked with the coil changes. Then emf induced in the coil is called self induced emf and the phenomenon is called self induction.
- If 'i' is the current flowing through the coil and 'ϕ' is magnetic flux linked with the coil, then

  $$\phi \propto i \quad \Rightarrow \phi = L\,i, \qquad \therefore L = \frac{\phi}{i}$$

  Here 'L' is called coefficient of self induction of the coil or self inductance of the coil.

  $$\phi \propto i \quad \Rightarrow \phi = L\,i, \qquad \therefore L = \frac{\phi}{i}$$

- Self induced e.m.f is given by

  $$e = \frac{-d\phi}{dt} = -L\frac{di}{dt}$$

- Self inductance of a coil is magnetic flux linked with the coil when unit current flows through it (or) emf induced in the coil when current changes in it at the rate of 1 A/sec.

- S.I. Unit of self inductance : Henry.

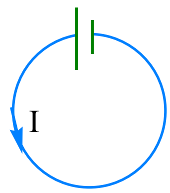
  Other Units : weber / ampere, volt-second/ampere, $J/amp^2, Wb^2/J, voltsec^2 coul^{-1}$.

  Dimensional formula of L is $\left[ML^2T^{-2}I^{-2}\right]$
- A coil having high self inductance is called inductor.
- Self induction is also known as inertia of electricity as it opposes the growth or decay of the current in the circuit.
- Inductance may be viewed as electrical inertia. It is analogous to inertia in mechanics. It does not oppose the current, but is opposes the change in current.

Self Inductance of a flat circular coil:

Let us consider a circular coil of radius r and containing N-turns. Suppose it carries a current 'i '.

The magnetic field at the centre due to this current $B = \frac{\mu_0 Ni}{2r}$

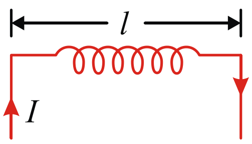


And total flux $= NBA = N\left(\frac{\mu_0 Ni}{2r}\right)\pi r^2 = \frac{\mu_0 \pi N^2 ri}{2}$

Now comparing with $N\phi_B = Li$ we get $L = \frac{\mu_0 \pi N^2 r}{2}$

Self inductiance of a solenoid:

Consider a long solenoid of length l, area of cross section A and number of turns per unit length n and length is very large when compared with radius of cross section.
Let I be the current flowing through the solenoid. The magnetic field inside the long solenoid is uniform and is given by $B = \mu_0 nI$
Total number of turns in the solenoid of length l is N=nl.
Now, the magnetic flux linked with each turn of the solenoid $B \times A = \mu_0 nIA$



$\therefore$ Total magnetic flux linked with the whole solenoid, ϕ=magnetic flux with each turn $\times$ number of turns in the solenoid.

$\phi = \mu_0 nIA \times nl = \mu_0 n^2 IAl$ ....(1)

But $\phi = LI \Rightarrow LI = \mu_0 n^2 IAl$ from (1) & (2)

$\therefore L = \mu_0 n^2 Al$ Since $n = \frac{N}{l}, L = \mu_0 \frac{N^2}{l} A$

↳ Self inductance of coil depends on

i) Geometry of the coil

i.e., a) Number of turns of the coil

b) The length (l) of the solenoid,

c) The area of cross-section (A) of the solenoid,

ii) Medium inside the coil (permeability)

iii) Nature of the material of the core of the solenoid.

↳ More is the permeability of the medium, more is the self inductance

↳ An inductor will have large inductance and low resistance.

↳ Resistor opposes the current, inductor opposes the change of current

↳ One can have resistance without inductance↓ One cannot have inductance without resistance.

↳ An ideal inductor has inductance and no resistance.

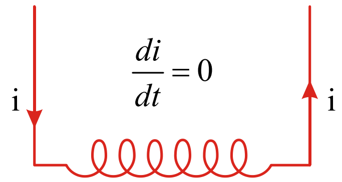
↳ When the current in the coil either increases or decreases at a rate, then the coil can be imagined to be a cell of emf $e = L.\frac{di}{dt}$

↳ One can have self inductance without mutual inductance.

↳ One cannot have mutual inductance without self inductance.

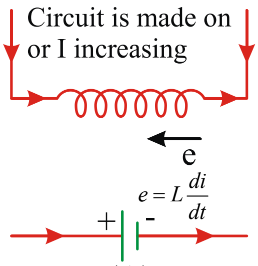
The direction of induced emf for different states of current in a coil :

a) Steady current



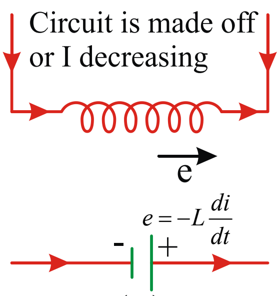
e = 0 no opposition

b) Make of circuit or increasing current



(A)

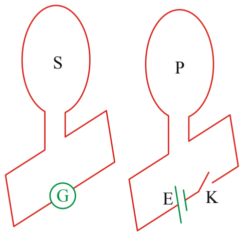
c) Breaking of circuit of decreasing of current



(B)

Mutual induction

↳ When current in one coil changes, magnetic flux linked with the second coil placed near by it also changes. The emf induced in secondary is called mutually induced emf and the phenomenon is called mutual induction.



↳ If 'i_p' is current flowing in the primary coil, 'ϕ_S' is magnetic flux linked with secondary coil, then

$\phi_S \propto i_p$

$\Rightarrow \phi_S = Mi_p, \qquad \therefore M = \frac{\phi_S}{i_p}$

Here 'M' is called coefficient of mutual induction or mutual inductance.

↳ Induced emf in secondary coil is

$$e = \frac{-d\phi}{dt} = -M\left(\frac{di_p}{dt}\right) \text{ (or) } M = \frac{e}{-di_p/dt}$$

↳ Mutual indictance between two coils is equal to the magnetic flux linked in the secondary coil when unit current passes through the primary coil (or) emf induced in one coil when current in the other coil changes at the rate of 1 Amp/second.

↳ S.I. unit : Herry

↳ Dimensional formula of self inductance or mutual inductance is $ML^2T^{-2}A^{-2}$

↳ The value of mutual inductance depends on

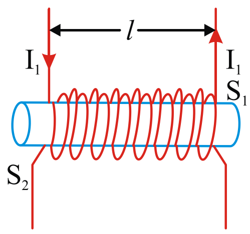
1) Distance between the two coils
2) Number of turns of coils
3) Geometrical shape of the coil
4) Material of the core medium between the coils
5) Orientation of the coils i.e., angle between the axes of the coils.

If the axes the parallel, then M is maximum

If the axes are perpendicular then M is minimum

Mutual inductance of two long coaxial solenoids:

↳ Consider two solenoids S_1 and S_2 such that the solenoid S_2 completely surrounds the solenoid S_1.



Let l be length of each solenoid (or length of primary coil) and of nearly same area of cross-section A. N_1 and N_2 are the total number of turns of solenoid S_1 and S_2 respectively.

∴ Number of turns per unit length of solenoid S_1 is, $n_1 = \frac{N_1}{l}$

Number of turns per unit length of solenoid S_2 is, $n_2 = \frac{N_2}{l}$

Magnetic field inside the solenoid S_1 is given by $B_1 = \mu_0 n_1 I_1 = \mu_0 \frac{N_1}{l} I_1$

∴ Magnetic flux linked with each turn of solenoid $S_2 = B_1 A = \mu_0 \frac{N_1}{l} I_1 A$

∴ Total magnetic flux linked with N_2 turns of the solenoid S_2 is

$$\phi_2 = N_2 (B_1 A) = \mu_0 \frac{N_1}{l} I_1 A \times N_2$$

$$\phi_2 = \frac{\mu_0 N_1 N_2 I_1 A}{l} \text{ .... (i)}$$

But $\phi_2 = M_{12} I_1$ .... (ii)

Where M_{12} is the mutual inductance when current varies in solenoid S_1 and makes magnetic flux linked with solenoid S_2,

from (i) and (ii) we get

$$M_{12} I_1 = \frac{\mu_0 N_1 N_2 I_1 A}{l} \quad \therefore \quad M_{12} = \frac{\mu_0 N_1 N_2 A}{l}$$

Similarly, $M_{21} = \frac{\mu_0 N_1 N_2 A}{l}$, where M_{21} is the mutual inductance when current varies in solenoid S_2 and makes magnetic flux linked with solenoid S_1.

It can be proved that $M_{12} = M_{21} = M$

The above equation is treated as a general result, if the two solenoids are wound on a magnetic substance of relative permeability μ_r, then the mutual inductance is given by

$$M = \frac{\mu_0 \mu_r N_1 N_2 A}{l} = \mu_0 \mu_r n_1 n_2 A l$$

E.X: 33**EX. 35: Two different coils have self inductance $L_1 = 8\text{mH}$ and $L_2 = 2\text{mH}$. The currents in both are increasing at the same constant rate. At a certain instant of time, the power given to the two coils is the same. At this moment the current, the induced voltage and energy stored in the first coil are i_1, V_1 and U_1 respectively. The corresponding values in the second coil are i_2, V_2 and U_2 respectively. Then the values of $\frac{i_1}{i_2}, \frac{V_1}{V_2}$ and $\frac{U_1}{U_2}$ are respectively**

Sol. $\frac{i_1}{i_2} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$; $\frac{v_1}{v_2} = \frac{L_1}{L_2} = \frac{8}{2} = 4$

$$\frac{U_1}{U_2} = \frac{L_2}{L_1} = \frac{2}{8} = \frac{1}{4}$$

E.X: 34 Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10cm^2$ and length = 20cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is $\left(\mu_0 = 4\pi \times 10^{-7} TmA^{-1}\right)$

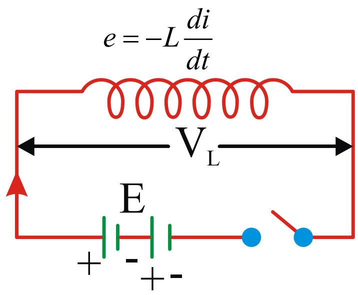
Sol. $M = \dfrac{\mu_0 N_1 N_2 A}{L}$

$$M = \frac{4\pi \times 10^{-7} \times 3 \times 10^2 \times 4 \times 10^2 \times 10^{-3}}{2 \times 10^{-1}}$$

$$= 2.4\pi \times 10^{-4} H$$

Energy stored in a inductor

↳ Consider an ideal inductor of inductance 'L' connected with a battery. Let I be the current inthe circuit at any instant 't '



This induced emf is given by $\qquad e = -L\dfrac{dI}{dt}$

-ve sign shows that 'e' opposes the change of current I in the inductor.
To drive the current through the inductor against the induced emf 'e', the external voltage is applied.
Here external voltage is emf of the battery = E
According to Kirchoff's voltage law, E+e=0

$$E = -e \;\; ; \;\; E = L\frac{dI}{dt}$$

Let an infinitesimal charge dq be driven through the inductor in time dt. So, the rate of work done by the external voltage is given by

$$\frac{dW}{dt} = EI = L\frac{dI}{dt} \times I = LI\frac{dI}{dt}$$

The total work done in establishing a current through the inductor from 0 to I is given by

$$W = \int dW = \int_0^I LI \; dI \; ; \quad W = L\left(\frac{I^2}{2}\right) = \frac{1}{2}LI^2$$

$$\boxed{W = \frac{1}{2}LI^2}$$

The work done in maintaining the current through the inductor is stored as the potential energy (U) in its magnetic field. Hence energy stored inthe inductor is given by $\boxed{U = \dfrac{1}{2}LI^2}$

- The equation $U=\frac{1}{2}LI^2$ is similar to the expression for kinetic energy $E=\frac{1}{2}mv^2$. It shows that L is analogues to mass 'm' and self inductance is called electrical inertia.
- The self inductance of a coil is numerically equal to twice the energy stored in it when unit current flows through it.
  i.e., When i=1A, L=2U
- Induced power $P=e\times i=Li\left(\frac{di}{dt}\right)$.
- In case of solenoid $L=\mu_0 n^2 Al$
- Magnetic energy stored per unit volume

  $$u_B=\frac{\frac{1}{2}Li^2}{Al}\Rightarrow u_B=\frac{1}{2}u_0n^2i^2 \quad \text{Hence } u_B=\frac{B^2}{2\mu_0}$$

- The magnetic energy stored per unit volume similar to electrostatic energy stored per unit volume in a parallel plate capacitor $u_B=\frac{1}{2}\varepsilon_0E^2$

  In both cases the energy is proportional to the square of field stregth

E.X: 35 The self-inductance of a coil having 200 turns is 10 milli henry. Calculate the magnetic flux through the cross-section of the coil corresponding to current of 4 milliampere. Also determine the total flux linked with each turn.

Sol. Total magnetic flux linked with the coil,

$$N\phi=LI=10^{-2}\times4\times10^{-3}=4\times10^{-5}Wb$$

$$\therefore \text{Flux per turn, } \phi=\frac{4\times10^{-5}}{200}=2\times10^{-7}Wb$$

E.X: 36 A coil of inductance 0.2 henry is connected to 600 volt battery. At what rate, will the current in the coil grow when circuit is completed ?

Sol. As the battery and inductor are in parallel, at any instant, emf of the battery and self emf in the inductor are equal

$$|e|=L\frac{dI}{dt} \text{ or } \frac{dI}{dt}=\frac{|e|}{L}=\frac{600V}{0.2H}=3000A\ s^{-1}$$

E.X: 37 An inductor of 5H inductance carries a steady current of 2A. How can a 50V self-induced emf be made to appear in the inductor

Sol. $L=5H;|e|=50V$; Let us produce the required emf by reducing current to zero

$$\text{Now, } |e|=L\frac{dI}{dt} \text{ or } dt=\frac{LdI}{|e|}=\frac{5\times2}{50}s$$

$$\frac{10}{50}s=\frac{1}{8}s=0.2s$$

So, the desired emf can be produced by reducing the given current to zero in 0.2 second

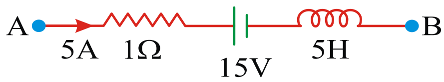
E.X: 38 Two different coils have self-inductances $L_1 = 16\,mH$ and $L_2 = 12\,mH$. At a certain instant, the current in the two coils is increasing at the same rate of power supplied to the two coils is the same. Find the ratio of i) induced voltage ii) current iii) energy stored in the two coils at that instant.

Sol. i) $V_1 = L_1 \dfrac{dI}{dt}; V_2 = L_2 \dfrac{dI}{dt}; \dfrac{V_1}{V_2} = \dfrac{L_1}{L_2} = \dfrac{16}{12} = \dfrac{4}{3}$

ii) $P = V_1 I_1 = V_2 I_2 \Rightarrow \dfrac{I_1}{I_2} = \dfrac{V_1}{V_2} = \dfrac{3}{4}$

iii) $\dfrac{U_1}{U_2} = \dfrac{\frac{1}{2} L_1 I_1^2}{\frac{1}{2} L_2 I_2^2} = \left(\dfrac{L_1}{L_2}\right)\left(\dfrac{I_1}{I_2}\right)^2 = \dfrac{4}{3}\left(\dfrac{3}{4}\right)^2 = \dfrac{3}{4}$

E.X: 39 The network shown is a part of the closed circuit in which the current is changing. At an instant, current in it is 5A. Potential difference between the points A and B if the current is

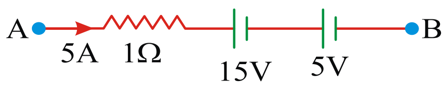


1) Increasing at 1A/sec

2) Decreasing at 1A/sec

Sol. 1) The coil can be imagined as a cell of emf

$e = L\left(\dfrac{di}{dt}\right) = 5 \times 1 = 5V$; $\therefore$ Equivalent circuit is

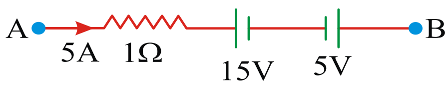


$V_A - 5(1) - 15 - 5 = V_B$

Hence $V_A - V_B = 5 + 15 + 5 = 25V$

2) The coil can be imagined as a cell of emf

$e = L\left(\dfrac{di}{dt}\right) = 5 \times 1 = 5V$; $\therefore$ Equivalent circuit is



$V_A = 5(1) - 15 + 5 = V_B$

Hence $V_A - V_B = 5 + 15 - 5 = 15V$

Relation between, L_1, L_2 and M :

The flux linked with coil 1 is $N_1\phi_1 = L_1 i_1 \Rightarrow L_1 = \frac{N_1\phi_1}{i_1}$

The flux linked with coil 2 is $N_2\phi_2 = L_2 i_2 \Rightarrow L_2 = \frac{N_2\phi_2}{i_2}$

M on 1 because of 2 ; $M_{12} = \frac{N_1\phi_1}{i_2}$

M on 2 because of 1 ; $M_{21} = \frac{N_2\phi_2}{i_1}$

↪ If the flux in linkage is maximum, then $M_{12} = M_{21} = M$; $M_{12} \times M_{21} = \frac{N_2\phi_2}{i_1} \times \frac{N_1\phi_1}{i_2}$

$M^2 = L_1 L_2$; $\therefore \quad M = \sqrt{L_1 L_2}$

This is the maximum mutual inductance when all the flux linked with one coil is also completely linked with the other.

In general, only a fraction of the total flux will be linked with the coil due to the flux leakage.

$\therefore \quad M = K\sqrt{L_1 L_2}$

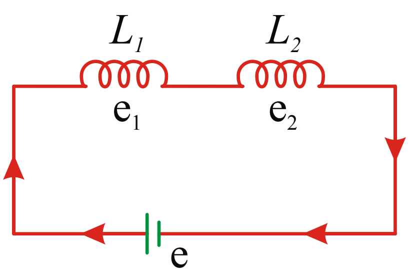
Where K-coefficient of coupling $(K \le 1)$

For tight coupling (or) if the coils are closely wound, then K=1.

$\therefore \quad M_{\max} = \sqrt{L_1 L_2}$

Inductors in Series:

If two coils of inductances L_1 and L_2 are connected in series then the potential divides.



i.e., $e = e_1 + e_2$ (or) $L_S \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$

Since in series, $\frac{di}{dt}$ is same for all coils

$\therefore L_S = L_1 + L_2$

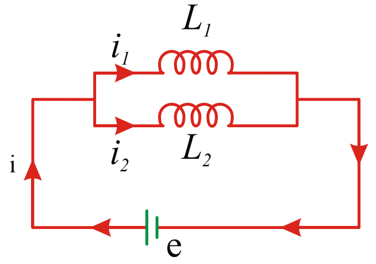
If n coils of inductances $L_1, L_2, L_3 ....L_n$ are connected in series then effective inductance of the arrangement,

$L = L_1 + L_2 + L_3 + ... + L_n$

(when coils are far away)

Inductors in parallel :

If two coils of inductances L_1 and L_2 are connected in parallel then the current divides.



i.e., $i = i_1 + i_2$ (or) $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} \Rightarrow \frac{e}{L_P} = \frac{e_1}{L_1} + \frac{e_2}{L_2}$

However in parallel as potential difference remains same i.e, $e = e_1 = e_2$, so

$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2}$ (or) $L_P = \frac{L_1 L_2}{(L_1 + L_2)}$

If n coil of inductances $L_1, L_2, L_3 ....L_n$ are connected in parallel then effetive inductance of the arrangement,

$\frac{1}{L_P} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_n}$

(when coils are far away)

↳ Let two coils of inductances L_1 and L_2 are connected in series and M is their mutual inductance. The flux linked with one coil will be the sum of two fluxes which exist independently. When the flux in the two coils support each other

$N_1\phi_1 = L_1 i_1 + M_{12} i_2$

From Faraday's law, $e_1 = -L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt}$

Similarly $N_2\phi_2 = L_2 i_2 + M_{21} i_1$

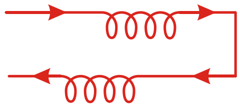
$e_2 = -L_2 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt}$

$e = e_1 + e_2 = -L_1 \frac{di_1}{dt} - M_{12} \frac{di_2}{dt} - L_2 \frac{di_2}{dt} - M_{21} \frac{di_1}{dt}$

In series the current i and the change in current di is same $e = -(L_1 + M_{21} + L_2 + M_{12}) \frac{di}{dt}$

$L = (L_1 + M_{21} + L_2 + M_{12}) = L_1 + L_2 + 2M$

If the two coils oppose each other, then



$L = (L_1 - M) + (L_2 - M) = L_1 + L_2 - 2M$

E.X: 40 Calculate the mutual inductance between two coils when a current of 2A changes to 6A in 2 seconds and induces an emf of 20 mV in the secondary coil

Sol. $|e| = M\frac{dI}{dt}$

$20\times10^{-3} = M\frac{(6-2)}{2}$ (or) M = 10mH

E.X: 41 If the coefficient of mutual induction of the primary and secondary coils of an induction coil is 6H and a current of 5A is cut off in 1/5000 second, calculate the emf induced in the secondary coil.

Sol. $|e| = M\frac{dI}{dt}; e = 6\times\frac{5}{1/5000}V = 15\times10^{4}V$

E.X: 42 A solenoid is of length 50 cm and has a radius of 2cm. It has 500 turns. Around its central section a coil of 50 turns is wound. Calculate the mutual inductance of the system.

Sol. $N_P = 500, N_S = 50$; $A = \pi\times0.02\times0.02m^2$

$\mu_0 = 4\pi\times10^{-7}Hm^{-1}, 1 = 50\,cm = 0.5\,m$

Now, $M = \frac{\mu_0 N_P N_S A}{l}$

$= \frac{4\pi\times10^{-7}\times500\times50\times\pi\times(0.02)^2}{0.5}H$

$= 789.8\times10^{-7}H = 78.98\,\mu H$

E.X: 43 A solenoidal coil has 50 turns per centimetre along its length and a cross-sectional area of $4\times10^{-4}m^2$. 200 turns of another wire is wound round the first solenoid co-axially. The two coils are electrically insulated from each other. Calculate the mutual inductance between the two coils.

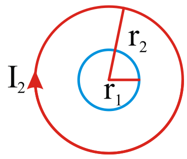
Sol. $n_1 = 50$ turns per cm ; = 5000 turns per metre

$n_2 l = 200, A = 4\times10^{-4}m^2$; $M = \mu_0 n_1 (n_2 l) A$

$= 4\pi\times10^{-7}\times5000\times200\times4\times10^{-4}H = 5.03\times10^{-4}H$

E.X: 44 Two circular coils, one of smaller radius r_1 and the other of very large radius r_2 are placed co-axially with centres coinciding. Obtain the mutual inductance of the arrangement.

Sol. Suppose a current I_2 flows through the outer circular coil. The field at the centre of the coil is



$$B_2 = \frac{\mu_0 I_2}{2r_2}$$

The second co-axially placed coil has very small radius. So B_2 may be considered constant over its cross-sectional area.

Now, $\phi_1 = \pi r_1^2 B_2 = \pi r_1^2\left(\frac{\mu_0 I_2}{2r_2}\right)$ or $\phi_1 = \frac{\mu_0 \pi r_1^2}{2r_2} I_2$

Comparing with $\phi_1 = M_{12} I_2$, we get ; $M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2}$

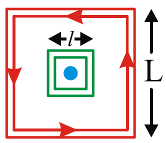
Also, $M_{21} = M_{12} = \frac{\mu_0 \pi r_1^2}{2r_2} \Rightarrow M \propto \frac{r_1^2}{r_2}$

It would have been difficult to calculate the flux through the bigger coil of the nouniform field due to the current in the smaller coil and hence the mutual inductance M_{12}. The equality $M_{12} = M_{21}$ is helpful. Note also that mutual inductance depends solely on the geometry.

E.X: 45 A small square loop of wire of side l is placed inside a large square loop of wire of side $L(>>l)$. The loops are coplanar and their centres coincide. What is the mutual inductance of the system ?

Sol. Considering the large loop to be made up of four rod each of length L, the field at the centre, i.e., at a distance (L/2) from each rod, will be

$B = 4\times\frac{\mu_0}{4\pi}\frac{I}{d}[\sin\alpha + \sin\beta]$ i.e., $B = 4\times\frac{\mu_0}{4\pi}\frac{I}{(L/2)}\times 2\sin 45$



i.e., $B_1 = \frac{\mu_0}{4\pi}\frac{8\sqrt{2}}{L} I$

So the flux linked with smaller loop

$$\phi_2 = B_1 S_2 = \frac{\mu_0}{4\pi}\frac{8\sqrt{21}}{L} l^2$$

and hence, $M = \frac{\phi_2}{I} = 2\sqrt{2}\frac{\mu_0}{\pi}\frac{l^2}{L} \Rightarrow M \propto \frac{l^2}{L}$

E.X: 46 Derive an expression for thetotal magnetic energy stored in two coils with inductances L_1 and L_2 and mutual inductance M when the currents in the coils are I_1 and I_2 respectively.

Sol. When the currents are increasing in the circuit, we have for emf's

$$\varepsilon_1 = -L_1 \frac{dI_1}{dt} \pm M \frac{dI_2}{dt}$$

$$\varepsilon_2 = -L_2 \frac{dI_2}{dt} \pm M \frac{dI_1}{dt}$$

$$dW = -\varepsilon_1 dq_1 - \varepsilon_2 dq_2$$

$$= L_1 \frac{dI_1}{dt} dq_1 \, \text{m}M \frac{dI_2}{dt} dq_1 + L_2 \frac{dI_2}{dt} dq_2 \, \text{m}M \frac{dI_1}{dt} dq_2$$

$$U = \int dW = \int_0^{I_1} I_1 dI_1 + L_2 \int_0^{I_2} I_2 dI_2 \, \text{m}M \int_0^{I_1 I_2} d(I_1 I_2)$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + M I_1 I_2$$

Ac Generator:

- An ac generator converts mechanical energy into electrical energy. The device used for the purpose is called ac generator.
- When the coil having N turns is rotated with a constant angular speed ω, the angle between the area vector A and the magnetic field vector B is at any instant t is $\theta = \omega t$ (assuming $\theta = 0^0$ at t=0).

  The flux linked with the coil at any instant t is $\phi_B = NBA \cos\theta = NBA \cos\omega t$

  From Faraday's law, the induced emf for the rotating coil of N turns is,

$$\varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(NBA \cos\omega t) = NBA \ \omega \sin\omega t$$

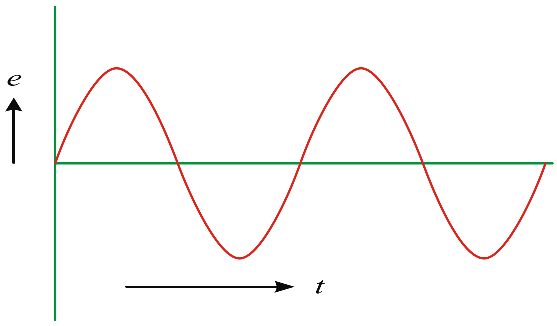
- The magnitude of induced emf is

  $\varepsilon = NBA \ \omega \sin\omega t = \varepsilon_0 \sin\omega t$

  where $e_0 = NBA\omega$ is the maximum value of the emf.

  ε_0 is called the amplitude or peak value of emf.
- The induced emf depends upon (i) strength of the magnetic field, (ii) area of the coil, (iii) speed of rotation, and (iv) the number of turns of the coil.

  If f be the frequency of rotation of coil, then $\varepsilon = \varepsilon_0 \sin 2\pi f t$
- A graph plotted between ε and ωt, is a sine curve as shown in Fig.



E.X: 47 A boy pedals a stationary bicycle at one revolution per second. The pedals are attached to 100 turns coil of are 0.1m² and placed in a uniform magnetic field of 0.1T. What is the maximum voltage generated in the coil ?

Sol. $\varepsilon_0 = NBA\omega = NBA(2\pi f)(\because f = 1)$

$\varepsilon_0 = 100\times0.1\times0.1(2\times3.14\times1)V = 6.28V$

E.X: 48 A coil of 800 turns and 50 cm^2 area makes 10 rps about an axis in its own plane in a magnetic field of 100 gauss perpendicular to this axis. What is the instantaneous induced emf in the coil?

Sol. $A = 50cm^2 = 50\times10^{-4}m^2$

$n = 10\,rps,\ N = 800$

$B = 100\,gauss = 100\times10^{-4}T = 10^{-2}T$

Now, $\varepsilon = \varepsilon_0 \sin\omega t = NBA\omega\sin\omega t$

$= 800\times10^{-2}\times50\times10^{-4}\times2\pi\times10\sin(20\pi t)$

or $\varepsilon = 2.5\sin(20\pi t)\,volt$

E.X: 48 A person peddles a stationary bicyle the pedals of the bicycle are attached to a 100 turn coil of area 0.10 m². The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil, What is the maximum voltage generated in the coil ?

Sol. Here $f = 0.5Hz : N = 100,\ A = 0.1m^2$ and $B = 0.01T$ from the equation

$\varepsilon = \varepsilon_0 \sin\omega t = NBA\omega\sin\omega t$ maximum emf $\varepsilon_0 = NBA\omega = NBA(2\pi f)$

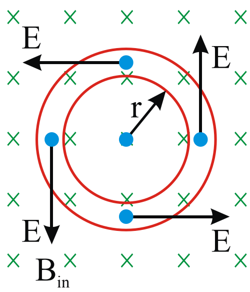
$\varepsilon_0 = 100\times0.01\times0.1\times2\times3.14\times0.5 = 0.314V$

Inducedele electric fields:

When a conducting loop is placed in a varying magnetic field, a varying electric field produced in the loop, is called induced electric field.

An electric field is always generated by a changing magnetic field, even in free space where no charges are present.

Consider a conducting loop of radius R, situated in a uniform magnetic field $\overline{B}$ that is perpendicular to the plane of the loop as shown in the figure



If the magnetic field changes with time, then an emf $e = \frac{-d\phi}{dt}$ is induced in the loop. The induced current thus produced implies the presence of an induced electric field E that must be tangential to the loop in order to provide an electric force on the
charge around the loop.
The work done by the electric field on the loop in moving a test charge q once around the loop=qe. Because the magnitude of electric force onthe charge is qE, the work done by the electric field can also be expressed as $qE(2\pi r)$, where $2\pi r$ is the circumference of the loop. These two expressions for the work must be equal; therefore, we see that

$$qe = qE(2\pi r) \ ; \ E = \frac{e}{2\pi r}$$

Using this result along with Faraday's law and the fact that $\phi_B = BA = B\pi r^2$ for a circular loop, the induced electric field can be expressed as

$$E = \frac{1}{2\pi r}\left(-\frac{d\phi_B}{dt}\right) = -\frac{1}{2\pi r}\frac{d}{dt}\left(B\pi r^2\right) = -\frac{r}{2}\frac{dB}{dt}$$

The emf for any closed path can be expressed as the line integral of $\vec{E}.\vec{dl}$ over that path. Hence, the general form of Faraday's law of induction is

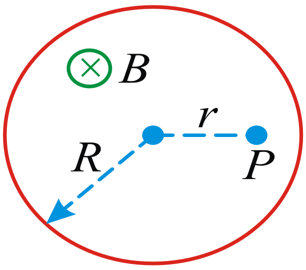
$$e = \oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$$

It is important to recognize that the induced electric field E that appears in the equation is a non-convervative field that is generated by a changing magnetic field.

↳ Points to remember about induced electric field.

1) The induced electric field is produced only by changing magnetic field and not by charged particles.
2) One cannot define potentials w.r.t this induced field
3) The lines of induced electric field are closed curves and have no starting and terminating points.
4) As long as the magnetic field keeps on changing, the induced electric field will be present because this electric field is produced only by variable magnetic field.

E.X: 50 A uniform magnetic field of induction B is confined in a cylinderical region of radius R. If the field is increasing at a constant rate of $\frac{dB}{dt} = \alpha T/s$, then the intensity of the electric field induced at point P, distant r from the axis as shown in the figure is proportional to :



Sol. For r < R ; $e=\int E.ds \; ;=\frac{d\phi_B}{dt}$

$$E.2\pi r=-A\left(\frac{dB}{dt}\right);\; E.2\pi r=-\pi r^2\left(\frac{dB}{dt}\right)$$

$$E=-\frac{r}{2}\left(\frac{dB}{dt}\right); E=-\frac{r}{2}\alpha \;;\; \left(\overline{E}\right)=\frac{r}{2}\alpha \;;\; E\alpha r$$

E.X: 51 Magnetic flux linked with a stationary loop of resistance R varies with respect to time during the time period T as follows: $\phi=at\left(T-t\right)$ the amount of heat generated in the loop during that time (inductance of the coil is negligible) is

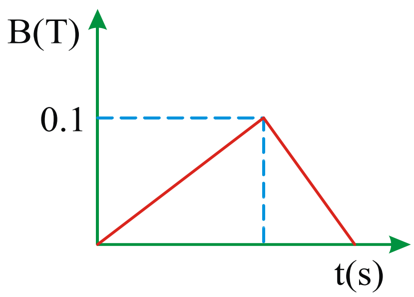
Sol. Give that $\phi=at\left(T-t\right)$; induced emf, $E=\frac{d\phi}{dt}$

$$=\frac{d}{dt}\left[at\left(T-t\right)\right];=at\left(0-1\right)+a\left(T-1\right);\;=a\left(T-2t\right)$$

So, induced emf is also a function of time Heat generated

$$H=\int_0^T\frac{E^2}{R}dt\;;\;=\frac{a^2}{R}\int_0^T\left(T-2t\right)dt\;;\;=\frac{a^2T^3}{3R}$$

E.X: 52 A closed loop of cross-sectional area $10^{-2}\,m^2$ which has inductance L=10 mH and negligible resistance is placed in a time-varying magnetic field. Figure shows the variation of B with time for the interval 4 s. The field is perpendicular to the plane of the loop (given at $t=0, B=0, I=0$). The value of the maximum current induced in the loop is



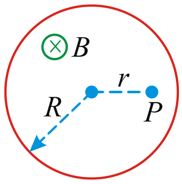
Sol. Induced emf (e) $=L\frac{di}{dt}$

$$\Rightarrow A\left(\frac{dB}{dt}\right)=L\frac{di}{dt}\Rightarrow di=\frac{A}{L}\left(\frac{dB}{dt}\right)\times dt$$

$$\int_0^I di=\int_0^B\left(\frac{A}{L}\right)dB\;;I=\frac{A}{L}B\quad;\qquad\Rightarrow I_{max}=\frac{A}{L}B_{max}$$

$$=\frac{10^{-2}}{10\times10^{-3}}\times0.1\;;\;=0.1A=100mA$$

E.X: 53 **A magnetic field directed into the page changes with time according to the expression $B=(0.03t^2+1.4)T$, where t is in seconds. The field has a circular cross - section of radius R - 2.5cm. What is the magnitude and direction of electric field at P, when t = 3.0s and r = 0.02 m.**

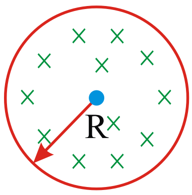


Sol. $e=\oint E.dl=\frac{+d\phi}{dt}$

$$E(2\pi r)=A.\frac{dB}{dt}=\pi r^2\times\frac{d}{dt}(0.03t^2+1.4)\quad E=\frac{\pi r^2}{2\pi r}\times(0.06t)=\frac{r}{2}(0.06t)$$

$$|E|=\frac{0.02}{2}\times0.06\times3=18\times10^{-4}N/C$$

E.X: 54 **The magnetic field at all points within the cylindrical region whose cross- section is indicated in the accompanying figure starts increasing at a constant rate 'α'. Find the magnitude of electric field as a function of r, the distance from the geometric centre of the region.**



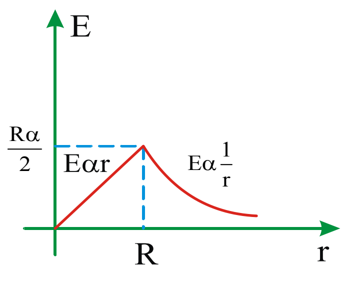
Sol. Case -1 : For r < R
Case -2 : r = R

$$E.2\pi r=-A\frac{dB}{dt};\ E.2\pi R=-\pi R^2\frac{dB}{dt}$$

$$E.2\pi r=-\pi r^2\frac{dB}{dt}\ ;\ E=\frac{R}{2}\frac{dB}{dt}$$

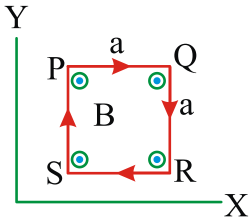
$$E=-\frac{r}{2}\frac{dB}{dt}=-\frac{r}{2}\alpha\,;E=-\frac{R\alpha}{2}\,;\ E\ \alpha\ r$$

Case - 3 r > R ; $E.2\pi r=-\pi R^2\frac{dB}{dt}$



$E=-\frac{R^2}{2r}\frac{dB}{dt}$; $E=-\frac{R^2}{2r}\alpha$; $E_{out}\alpha\frac{1}{r}$

E.X: 55 A wire is bent in the form of a square of side 'a' in a varying magnetic field $\overline{B}=\alpha\ B_0 t\ \hat{k}$. If the resistance per unit length is λ, then find the following.



i)The direction of induced current

ii) The current in the loop

iii) Potential difference between P and Q

Sol. i) Direction of current is closewise.

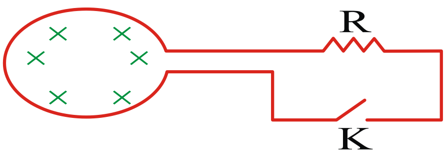
ii) $|e|=\frac{d\phi}{dt}=\frac{d}{dt}(BA)=a^2(\alpha B_0.t)=a^2\alpha B$

Current : $i=\frac{e}{R}=\frac{a^2\alpha B_0}{4a\lambda}(\because R=4a\lambda)=\frac{a\,\alpha\,B_0}{4\lambda}$

iii) $v_p+\frac{e}{4}-i.a\lambda=V_Q$,where 'e' is the total emf induced or $V_P-V_Q=ia\lambda-\frac{e}{4}$

or $V_P-V_Q=\frac{e}{4a\lambda}.a\lambda-\frac{e}{4}$; $V_P-V_Q=\frac{e}{4}-\frac{e}{4}=0$

E.X: 56 Shown in the figure is a circular loop of radius r connected to a resistance R. A variable magnetic field of induction $B=e^{-t}$ is established inside the coil. If the key(k) is closed, find the electric power developed ?



Sol. $E=\frac{-d\phi}{dt}=-A.\frac{dB}{dt}$ or $E=-\pi r^2\frac{d}{dt}\left(e^{-t}\right)$

$\Rightarrow E=\pi r^2\ e^{-t}$; $P=\frac{E^2}{R}=\frac{\pi^2 r^4 e^{-2t}}{R}$;

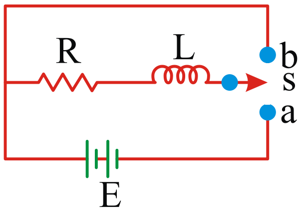
at t = 0 ; $P=\frac{\pi^2 r^4}{R}$

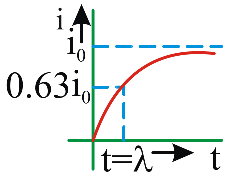
D.C. Circuits

Growth and decay of current in an inductor Resistor (L - R) circuit

I. Growth of current

Consider a circuit shown in the diagram





a) When a switch S is connected to 'a' , the current in the circuit beings to increase from zero to a maximum value 'i_0'. The Inductor opposes the growth of the current.

$$\therefore \quad E - L\frac{di}{dt} = Ri$$

Where 'i ' is the current in the circuit at any instant 't ' and $i = i_0\left\{1 - e^{\frac{-t}{\lambda}}\right\}$

Where i_0 is the maximum current. Here $\lambda = \frac{L}{R}$ called Inductive time constant

b) At $t = \lambda, i = i_0\left(1 - \frac{1}{e}\right) = 0.63\ i_0$

c) Thus the inductive time constant of a circuit is defined as the time in which the current rises from zero to 63% of its final value.

d) Greater the value of 'λ' smaller will be the rate of growth of current.

e) Current reaches i_0 after infinite time.

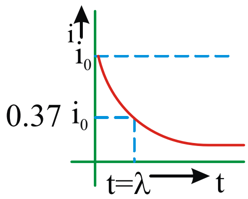
f) When current attains maximum value, Inductor doesn't work. $\therefore \quad i_0 = \frac{E}{R}$

II. Decay of Current

a) When circuit is disconnected from the battery and switch 's' is connected to point 'b', the current now beings to fall. But inductor opposes decay of current $\therefore -L\frac{di}{dt} = Ri$

Where i is the current at any instant and $i = i_0 e^{\frac{-t}{\lambda}}$

where $t = \lambda = \frac{L}{R}$



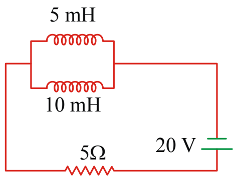
b) At $t = \lambda, i = \frac{i_0}{e} = 0.37\, i_0$

c) The inductive time constant (λ) can also be defined as the time interval during which the current decays to 37% of the maximum current.

d) For small value of 'L', rate of decay of current will be large.

e) Current becomes zero after infinite time.

E.X: 57 In the given circuit, current through the 5 mH inductor in steady state is



Sol. 5mH, 10mH are connected in parallel

$\therefore$ Equivalent inductance

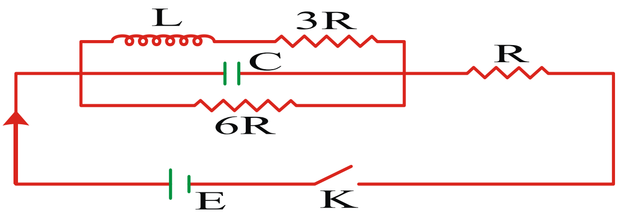
$$L_{eq} = \frac{5 \times 10}{5+10} = \frac{50}{15}; \quad = \frac{10}{3} mH$$

Current at steady state ; $I = \frac{20}{5} = 4A$

As L_1 and L_2 are in parallel

$$I_1 = \left(\frac{L_2}{L_1 + L_2}\right) I; \; = \left(\frac{10}{10+5}\right) \times 4 \qquad ; = \frac{10}{15} \times 4; \qquad = \frac{8}{3} Amp$$

E.X: 58 In the given circuit diagram, key K is switched on at $t = 0$. The ratio of current i through the cell at $t = 0$ to that at $t = \infty$ will be



Sol. At t = 0, the branch containing L will offer infinite resistance while the branch containing the capacitor will be effectively a short circuit.

Hence, $(i)_{t=0} = \frac{E}{R}$

Similarly, at $t = \infty$, L will offer zero, resistance, where as 'c' will be an open circuit.

Hence, effective resistance $= R + \frac{6R \times 3R}{6R + 3R}$; $(i)_{t=\infty} = \frac{e}{3R}$

The required ratio ; $= \frac{e}{R} \times \frac{3R}{e}$; $= 3 : 1$

E.X: 59 An inductor of inductance $L = 400\,mH$ and resistors of resistance $R_1 = 4\Omega$ and $R_2 = 2\Omega$ are connected to battery of emf 12V as shown in the figure. The internal resistance of the battery is negligible. The swich S is closed at $t = 0$. The potential drop across L as a function of time is

Sol. $I_1 = \frac{E}{R_1} = \frac{12}{2} = 6A$; $E = L\frac{dl_2}{dt} + R_2 \times l_2$

$I_2 = I_0\left(1 - e^{-t/t_c}\right)$; $\Rightarrow I_0 = \frac{E}{R_2} = \frac{12}{2} = 6A$

$t_c = \frac{L}{R} = \frac{400 \times 10^{-3}}{2} = 0.2$; $I_2 = 6\left(1 - e^{-t/0.2}\right)$

Potential drop across L

$V_L = E - R_2 I_2 = 12 - 2 \times 6\left(1 - e^{-bt}\right)$; $= 12e^{-5t}$

E.X: 60 An inductor of 3H is connected to a battery of emf 6V through a resistance of 100Ω. Calculate the time constant. What will be the maximum value of current in the circuit ?

Sol. Give that $L = 3H, E = 6V, R = 100\Omega$

Time constant $\tau_L = \frac{L}{R} = \frac{3}{100} = 0.03\,\text{sec}$

Maximum Current $I_0 = \frac{E}{R} = \frac{6}{100}\text{amp} = 0.06\,\text{amp}$

E.X: 61 A cell of 1.5V is connected across an inductor of 2mH in series with a 2Ω resistor. What is the rate of growth of current immediately after the cell is switched on.

Sol. $E = L\frac{dI}{dt} + IR$, therefore, $\frac{dI}{dt} = \frac{E - = IR}{L}$

$E = 1.5Volt, R = 2\Omega, L = 2mH = 2 \times 10^{-3} H$

When the cell is switched on, I = 0

Hence $\frac{dI}{dt} = \frac{E}{L} = \frac{1.5}{2 \times 10^{-3}} As^{-1} = 750\ As^{-1}$

E.X: 62 A coil having resistance 15Ω and inductance 10H is connected across a 90 Volt dc supply. Determine the value of current after 2sec. What is the energy stored in the magnetic field at that instant.

Sol. Give that ; $R = 15\Omega, L = 10H, E = 90Volt$

Peak value of current

$$I_0 = \frac{E}{R} = \frac{90}{15}A = 6A \text{ also, } \tau_L = \frac{L}{R} = \frac{10}{15} = 0.67\sec$$

Now, $I = I_0\left(1 - e^{\frac{-Rt}{L}}\right)$, After 2sec,

$$I = 6\left[1 - e^{-2/0.67}\right] = 6[1 - 0.05] = 5.7A$$

Energy stored in the magnetic field

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\times 10\times(5.7)^2 J = 162.45J.$$

E.X: 63 Calculate the back e.m.f of a 10H, 200Ω coil 100 ms after a 100V d.c supply is connected to it.

Sol. The value of current at 100ms after the switch is closed is

$$I - I_0\left[1 - e^{\frac{-t}{T_0}}\right], \text{ Here, } I_0 = \frac{100}{200} = 0.5\,\text{amp};$$

$$\tau_0 = \frac{L}{R} = \frac{10}{200} = 0.05\sec;\ t = 0.1\sec$$

$$I = 0.5\left(1 - e^{-0.1/0.05}\right) = 0.5\left(1 - e^{-2}\right) = 0.4325A$$

Now, $E = IR + L\frac{dI}{dt}$, or

$$100 = 0.4325\times 200 + L\frac{dI}{dt}$$

Back e.m.f $= L\frac{dI}{dt} = 100 - 0.4325\times 200 = 13.5V$

E.X: 64 A coil of resistance 20Ω and inductance 0.5 henry is switched to dc 200 volt supply. Calculate the rate of increase of current:

a) At the instant of closing the switch and

b) After one time constant

c) Find the steady state current in the circuit.

Sol. a) This is the case of growth of current in an L - R circuit. Hence, current at time t is given by $i = i_0\left(1 - e^{\frac{-t}{\tau_L}}\right)$.

Rate of increase of current, $\frac{di}{dt} = \frac{i_0}{\tau_L} e^{\frac{-t}{\tau_L}}$; At $t = 0 \frac{di}{dt} = \frac{i_0}{\tau_L} = \frac{E/R}{L/R} = \frac{E}{L}$

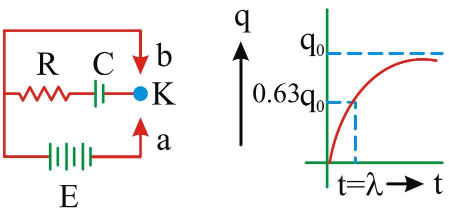
$\frac{di}{dt} = \frac{200}{0.5} = 400\,A/s$

b) At $t = \tau_L$, $\frac{di}{dt}(400)e^{-1} = (0.37)(400) = 148\,A/s$

c) The steady state current in the circuit, is $i_0 = \frac{E}{R} = \frac{200}{20} 10\,A$

Growth and decay of charge in a capacitor - Resistor (C - R) circuit

I. Growth of Charge : Consider a circuit shown in the diagram



a) When the key's is connected to point 'a', the charging of capacitor takes place until the potential difference across the plates of the condenser becomes E.

b) But charge attained already on the plates opposes further introduction of charge

$E - \frac{q}{c} = Ri$ (or) $E - \frac{q}{c} = R\frac{dq}{dt}$

Where 'q' is the instantaneous charge, i is the instaneous current in the circuit.

and $q = q_0\left(1 - e^{\frac{-t}{\lambda}}\right)$

where q_0 is the maximum charge.

Where $\lambda = CR$, called **capacitive time constant**

c) When $t = \lambda$. $q = q_0\left(1 - \frac{1}{e}\right) = 0.63\,q_0$

d) Thus the capacitive time constant is the time in which the charge on the plates of the capacitor becomes $0.63\,q_0$

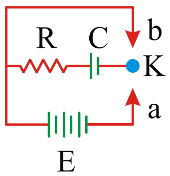
e) Smaller the value of CR, more rapid is the growth of charge on the condenser.

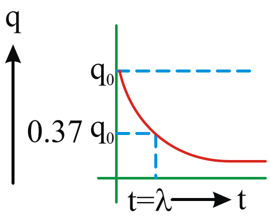
f) Charge on the capacitor becomes maximum after infinite time and it is $q_0 = EC$. Then current in the circuit becomes zero.

II. Decay of charge :

a) When the capacitor is fully charged the key is connected to point 'b'.
b) Charge slowly reduces to zero after infinite time.

$$\therefore -\frac{q}{c} = Ri \text{ (or) } \frac{-q}{c} = R\frac{dq}{dt} \text{ and } q = q_0 e^{\frac{-t}{\lambda}}$$



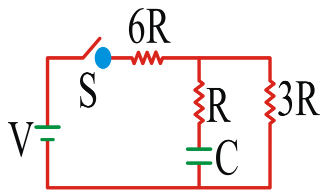


c) At $t = \lambda, q = \frac{q_0}{e} = 0.37\ q_0$

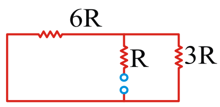
d) Thus capacitive time constant can also be defined as the time interval in which the charge decreases to 37% of the maximum charge

e) Smaller the time constant, quicker is the discharge of the condenser.

E.X: 65 In the circuit shown in figure switch S is closed at time t = 0. Find the current through different wires and charge stored on the capacitor at any time t.



Sol. Calculation of equivalent time constant



In the circuit shown in figure, after short circuiting the battery 3R and 6R are parallel, so their combined resistance is $\frac{(6R)(3R)}{6R+3R} = 2R$. Now this 2R is in series with the remaining R.

Hence, $R_{net} = 2R + R = 3R$; $\tau_c = (R_{net})C = 3RC$

Calculation of steady state charge q_0:

At $t = \infty$, capacitor is fully charged and no current flows through it.

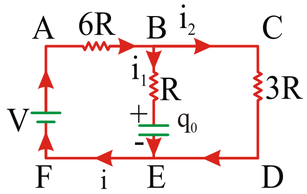
P.D across capacitor = P.D across 3R

$$= \left(\frac{V}{9R}\right)(3R) = \frac{V}{3}, \qquad q_0 = \frac{CV}{3}$$

Now, let charge on the capacitor at any time t be q and current through it is i_1. Then

$$q = q_0\left(1 - e^{-t/\tau_c}\right) \text{ i.e., } q = q_0\left(1 - e^{\frac{t}{3RC}}\right)$$

$$\text{and } i_1 = \frac{dq}{dt} = \frac{q_0}{\tau_c}e^{-t/\tau_c} = \frac{q_0}{3RC}e^{\frac{1}{3RC}} \qquad ...(1)$$



Applying Kirchhoff's second law in loop

ACDFA, we have $-6iR-3i_2R+V=0$ $\qquad 2i+i_2=\dfrac{V}{3R}$ ...(ii)

Applying Kirchoff's junction law at B, we have $\qquad i=i_1+i_2$ ...(iii)

Solving Eqs. (i), (ii) and (iii), we have

$$i_2=\frac{V}{9R}-\frac{2}{3}i_1=\frac{V}{9R}-\frac{2q_0}{3t_c}e^{-t/t_c}$$

where $q_0=\dfrac{CV}{3}$ i.e., $i_2=\dfrac{V}{9R}-\dfrac{2q_0}{3RC}e^{\frac{t}{3RC}}$

$$i=\frac{V}{9R}+\frac{q_0}{3t_c}e^{-t/t_c}=\frac{V}{9R}+\frac{q_0}{3RC}e^{\frac{t}{3RC}}$$

E.X: 66 A parallel - plate capacitor, filled with a dielectric of dielectric constant k, is charged to a potential V_0. It is now disconnected fromthe cell and the slab is removed. If it now discharges, with time constant τ, through a resistance, then find time after which the potential difference across it will be V_0?

Sol. When slab is removed, the potential difference across capacitor increases to kV_0

$$CV_0=kCV_0\ e^{-\frac{t}{\tau}} \text{ as } q_0=KCV_0$$

$$\frac{1}{k}=e^{-\frac{t}{\tau}}\Rightarrow k=e^{\frac{t}{\tau}}\ ;\quad \therefore ln=\frac{t}{\tau}\Rightarrow t=\tau lnk$$

E.X: 67 $4\mu F$ capacitor and a resistance $2.5\,M\Omega$ are in series with 12V battery. Find the time after which the potential difference across the capacitor is 3 times the potential difference across the resistor. [Given ln(2)=0.693]

Sol. a) Charging current $i=\dfrac{V_0}{R}e^{\frac{t}{RC}}$

$\therefore$ Potential difference across R is $\qquad V_R-iR=V_0e^{-\frac{t}{RC}}$

$\therefore$ Potential difference across 'C' is $V_C=V_0-V_R$; $\quad =V_0-V_0e^{-\frac{t}{RC}}=V_0\left(1-e^{-\frac{t}{RC}}\right)$

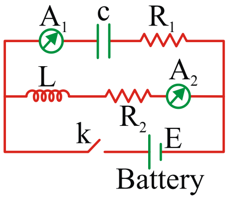
but given $V_C=3V_R$, we get $\qquad 1-e^{-t/RC}=3e^{-t/RC}$ or $4e^{-t/RC}$

$$e^{\frac{-t}{RC}} = 4 \Rightarrow \frac{t}{RC} = ln\ 4 \Rightarrow t = 2RC\ ln\ 2$$

$t = 2.5 \times 10^{6} \times 4 \times 10^{-6} \times 2 \times 0.693$

or t = 13.86 sec

E.X: 68 In a circuit inductance L and capacitance C are connected as shown in figure and A_1 and A_2 are ammeters. When key k is pressed to complete the circuit, then just after closing key k, the reading of A_1 and A_2 will be :

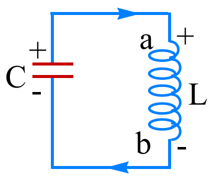


Sol. At t = 0 capacitor offers zero resistance and acts like a short circuit. While inductor offers infinite resistance and it acts like an open circuit. Therefore no current flow through inductor branch and maximum current flows through capacitor branch.

Hence reading of A_2 is zero and reading A_1 is given by $\frac{E}{R_1}$

LC Oscillations

A capacitor (C) and an inductor (L) are connected as shown in the figure. Initially the charge on the capacitor is Q



$\therefore$ Energy stored in the capacitor $U_E = \frac{Q^2}{2C}$

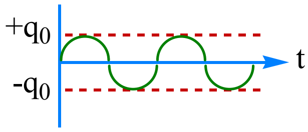
The energy stored in the inductor, $U_B = 0$.

The capacitor now begins to discharge through the inductor and current begins to flow in the circuit.

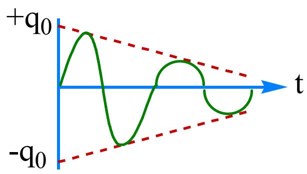
As the charge on the capacitor decreases, U_E decreases but the energy $U_B = \frac{1}{2}LI^2$ in the magnetic field of the inductor increases. Energy is thus transferred from capacitor to inductor. When the whole of the charge on the capacitor disappears, the total energy stored in the electric field in the capacitor gets converted into magnetic field energy in the inductor. At this stage, there is maximum current in the inductor.

Energy now flows from inductor to the capacitor except that the capacitor is charged oppositely. This process of energy transfer continues at a definite frequency (ν). Energy is continuously shuttled back and forth between the electric field in the capacitor and the magnetic field in the inductor.

If no resistance is present in the LC circuit, the LC oscillation will continue infinitely as shown.



However in an actual LC circuit, some resistance is always present due to which energy is dissipated in the form of heat. So LC oscillation will not continue infinitely with same amplitude as shown.



Let q be the charge on the capacitor at any time t and $\frac{di}{dt}$ be the rate of change of current.

Since no battery is connected in the circuit, $\frac{q}{c} - L.\frac{di}{dt} = 0$ but $i = -\frac{dq}{dt}$

from the above equations, we get $\frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

The above equation is analogus to $\frac{d^2x}{dt^2} + \omega^2 x = 0$ (differential equation of S.H.M)

Hence on comparing $\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

$$2\pi f = \frac{12\pi}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

The charge therefore oscillates with a frequency $f = \frac{1}{2\pi\sqrt{LC}}$ and varies sinusoidally with time.

Comparison of L-C oscillations with SHM :

The L-C oscillations can be compared to S.H.M of a block attached to a spring

- In L-C oscillations $\omega_0 = \frac{1}{\sqrt{LC}}$
- In Mechanical oscillations $\omega_0 = \sqrt{\frac{K}{m}}$ where K is the spring constant
- In L-C oscillations $\frac{1}{C}\left(=\frac{V}{q}\right)$ tells us the potential difference required to store a unit charge
- In a mechanical oscillation $K\left(=\frac{F}{x}\right)$ tells us the external force required to produce a unit displacement of mass

↪ In L-C oscillations current is the analogous quanitty for velocity of the mass in mechanical oscillations

↪ In L-C oscillations energy stored in capacitor is analagous to potential energy in mechanical oscillations.

↪ In L-C oscillations energy stored in inductor is analogous to kinetic energy of the mass in mechanical oscillations.

↪ In L-C oscillations maximum charge on capacitor q_0 is analogous to amplitude in mechanical oscillations

↪ $\therefore$ As $V_{max} = A\omega$ in mechanical oscillations,

$I_0 = q_0\omega_0$ in L-C oscillations

Analogies between Electrical and Mechanical Systems

One dimensional Mechanical system	Electric Circuit	Analogy
Position	Charge	$Q \leftrightarrow x$
Velocity	Current	$Q \leftrightarrow x_x$
Force	Potential difference	$Q \leftrightarrow x$
Viscous damping coefficient	Resistance	$Q \leftrightarrow x$
(k-spring constant)	Capacitance	$Q \leftrightarrow x$
Mass	Inductance	$L \leftrightarrow m$
Velocity = time derivation of position	Current = time derivative of change	$I = \frac{dQ}{dt} \leftrightarrow u_x = \frac{dx}{dt}$
Acceleration = second time derivative of position	Rate of change of current = second time derivative of charge	$\frac{dI}{dt} = \frac{d^2Q}{dt^1} \leftrightarrow a_x = \frac{du_x}{dt} = \frac{d^2x}{dt^2}$
Kinetic energy of moving object	Energy in inductor	$U_C = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$
Potential energy stored in a spring	Energy in capacitor	$U_C = \frac{1}{2}LI^2 \leftrightarrow K = \frac{1}{2}mv^2$
Rate of energy loss due to friction	Rate of energy loss due to resistance	$I^2R \leftrightarrow bv^2$
Damped object on a spring	RLC circuit	$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0 \leftrightarrow$ $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

Energy of LC oscillations : Let q_0 be the initial charge on a capacitor. Let the charged capacitor be connected to an inductor of inductance L. LC circuit will sustain an oscillations with frequency $\left(\omega = 2\pi f = \frac{1}{\sqrt{LC}}\right)$ At an instant t, charge q on the capacitor and the current i are given by;

$q(t) = q_0 \cos \omega t; i = -q_0 \omega \sin \omega t$

Energy stored in the capacitor at time is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{q_0^2}{2C}\cos^2(\omega t)$$

Energy stored in the inductor at time t is $U_M = \frac{1}{2}Li^2$

$$= \frac{1}{2}Lq_0^2\omega^2 \sin^2(\omega t) = \frac{q_0^2}{2C}\sin^2(\omega t)\left(\because \omega^2 = \frac{1}{\sqrt{LC}}\right)$$

Sum of energies

$$U_E + U_M = \frac{q_0^2}{2C}\left(\cos^2 \omega t + \sin^2 \omega t\right) = \frac{q_0^2}{2C}$$

As q_0 and C, both are time independent, this sum of energies stored in capacitor and inductor is constant in time. Note that it is equal to the initial energy of the capacitor.

E.X: 69 A capacitor of capacitance $25\mu F$ is charged to 300V. It is then connected across a 10 mH inductor. The resistance in the circuit is negligible.

a) Find the frequency of oscillation of the circuit.

b) Find the potential difference across capacitor and magnitude of circuit current 1.2 ms after the inductor and capacitor are connected.

c) Find the magnetic energy and electric energy at t=0 and t=1.2 ms.

Sol. a) $f = \frac{1}{2\pi\sqrt{L.C}} \Rightarrow f = 3183.3Hz$

b) $q = q_0 \cos(\omega t)$

$$\Rightarrow I = \frac{dq}{dt} = -q_0\omega \sin(\omega t)$$

Now, charge in the capacitor after $t = 1.2 \times 10^{-4} s$ is

$$q = (7.5 \times 10^{-3})\cos\left((2\pi \times 318.3)(1.2 \times 10^{-3})\right)C$$

$$\Rightarrow q = -5.53 \times 10^{-3} C$$

$$V = \frac{|q|}{C} = \frac{5.53 \times 10^{-3}}{25 \times 10^{-6}} = 221.2V$$

The magnitude of current in the circuit at t=1.2 x 10^{-3}s is,

$$|I| = q_0\omega \sin(\omega t) \Rightarrow |I| = 10.13A$$

c) At t=0 the current in the circuit is zero. Hence, U_L=0. So, charge in the capacitor in maximum

$$\Rightarrow U_C = \frac{1}{2}\frac{q_0^2}{C}$$

$$\Rightarrow U_C = \frac{1}{2} \times \frac{\left(7.5\times10^{-3}\right)^2}{\left(25\times10^{-4}\right)} = 1.125J$$

$\Rightarrow$ Total energy $E = U_L + U_C = 1.125J$

At t=1.2 ms, we have

$$U_L = \frac{1}{2}U^2 = \frac{1}{2}\left(10\times10^{-3}\right)\left(10.13\right)^2$$

$$\Rightarrow U_l = 0.513J$$

$$\Rightarrow U_C = E - U_L = 1.125 - 0.513 = 0.612J$$

Else, U_C can also be calculated as,

$$U_C = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\times\frac{\left(5.53\times10^{-3}\right)^2}{\left(25\times10^{-6}\right)} = 0.612J$$

E.X: 70 An inductor of inductance 2 mH is connected across a charged capacitor of capacitance $5\mu F$ and the resulting LC circuit is set oscillating at its natural frequency. Let Q de-note the instantaneous charge an the capacitor adn I the current in the circuit. It is found that the maximum value of Q is $200\mu C$

a) When Q=$100\mu C$, what is the value of $\left|\frac{dT}{dt}\right|$?

b) When Q=$20\mu C$, what is the value of I?

c) Find the maximum value of I?

d) When I is equal to one-half its maximum values, what is the value of |Q| ?

Sol. This problem is dealing with LC oscillations. The charge stored in teh capacitor oscillates simple harmonically as,

$$Q = Q_0 \sin\left(\omega t \pm \phi\right)$$

Here Q_0 = maximum value of

$$Q = 20\mu C = 2\times10^{-4}C$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(2\times10^{-3}H\right)\left(5\times10^{-6}F\right)}} = 10^4 s^{-1}$$

Let at t=0, Q = Q_0

$$Q(t) = Q_0\cos\left(\omega t\right) \qquad \text{------1}$$

$$\Rightarrow I(t) = \frac{dQ}{dt} = -Q_0\omega\sin(\omega t) \qquad \text{--------2}$$

$$\Rightarrow \frac{dI(t)}{dt} = -Q_0\omega^2\cos(\omega t)$$

a) $Q = 100\mu C = \frac{Q_0}{2}$

At $\cos(\omega t) = \frac{1}{2}$, from equation (3), we get

$$\left|\frac{dI}{dt}\right| = \left(2\times10^{-4}C\right)\left(10^4 s^{-1}\right)^2\left(\frac{1}{2}\right)$$

$$\Rightarrow \left|\frac{dI}{dt}\right| - 10^4 As^{-1}$$

b) $Q = 200\mu C = Q_0$ when

$\cos(\omega t) = 1, i.e., \omega t = 0, 2\pi.,$

$$\Rightarrow I(t) = 0.\left(\sin 0^0 = 2\sin 2\pi = 0\right)$$

c) $I(t) = -Q_0\omega\sin(\omega t)$

the maximum value of I is $Q_0\omega$

$$\Rightarrow I_{\max} = Q_0\omega = 2A$$

d) From energy conservation,

$$\frac{1}{2}LI_{\max}^2 = \frac{1}{2}LI^2 + \frac{1}{2}\frac{Q^2}{C}$$

$$\Rightarrow Q = \sqrt{LC\left(I_{\max}^2 - I^2\right)}$$

For, $I = \frac{I_{\max}}{2} = 1A$, we get

$$Q = \sqrt{\left(2\times10^{-3}\right)\left(5\times10^{-6}\right)\left(2^2 - 1^2\right)}$$

$$\Rightarrow Q = \sqrt{3}\times10^{-4}C \Rightarrow Q = 1.732\times10^{-4}C$$

Electro Magnetic Induction

(Jee main previous year questions)

Topic 1: Magnetic Flux, Faraday's and Lenz's Law

1. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$A. Where t is in sec. The emf induced in C_1 (in mV), at the instant $t = 1$s is $\frac{4}{x}$. The value of x is

[NA Sep. 05, 2020 (I)]

SOL. (5)

For coil C_1, No. of turns $N_1 = 500$ and radius, $r = 1$ cm.

For coil C_2, No. of turns $N_2 = 200$ and radius, $R = 20$ cm

$$I = (5t^2 - 2t + 3) \Rightarrow \frac{dI}{dt} = (10t - 2)$$

$$\varphi_{\text{small}} = BA = \left(\frac{\mu_0 I N_2}{2R}\right)(\pi r^2)$$

Induced emf in small coil, $e = \frac{d\varphi}{dt} = \left(\frac{\mu_0 N_2}{2r}\right)\pi r^2 N_1 \frac{di}{dt} = \left(\frac{\mu_0 N_1 N_2 \pi r^2}{2R}\right)(10t - 2)$

At $t = 1$s

$$e = \left(\frac{\mu_0 N_1 N_2 \pi r^2}{2R}\right) 8 = 4\frac{\mu_0 N_1 N_2 \pi r^2}{R}$$

$$= \frac{4(4\pi)10^{-7} \times 200}{20} \times 500 \times \frac{10^{-4}}{10^{-2}}\pi$$

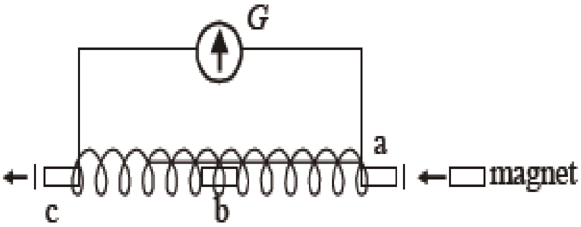
$$= 80 \times \pi^2 \times 10^{-7} \times 10 \times 10^2 \times 10^{-2}$$

$$= 8 \times 10^{-4} \text{ volt}$$

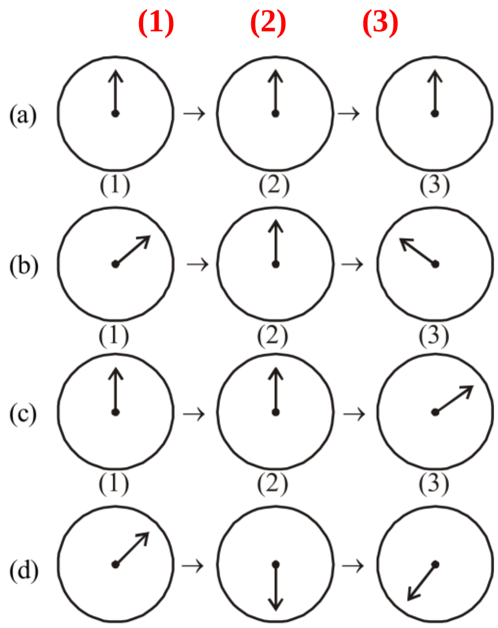
$$= 0.8\text{mV} = \frac{4}{x} \Rightarrow x = 5.$$

2. **A small bar magnet is moved through a coil at constant speed from one end to the other. Which of the following series of observations will be seen on the galvanometer *G* attached across the coil?**

[Sep. 04, 2020 (I)]



Three positions shown describe: (1) the magnet's entry (2) magnet is completely inside and (3) magnet's exit.



SOL. (b)

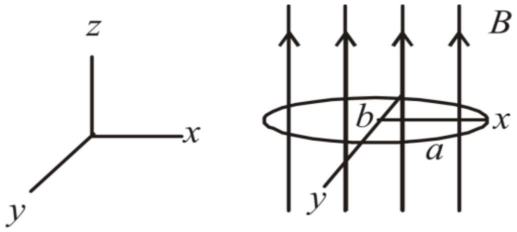
Case (a): When bar magnet is entering with constant speed, flux (φ) will change and an e.m.f. is induced, so galvanometer will deflect in positive direction.

Case (b): When magnet is completely inside, flux (φ) will not change, so galvanometer will show null deflection.

Case (c) : When bar magnet is making on exit, again flux (φ) will change and an e.m.f. is induced in opposite direction so galvanometer will deflect in negative direction i.e. reverse direction.

3. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency w, the average power loss in the loop due to Joule heating is :

[Sep. 03, 2020 (I)]



(a) $\frac{\pi^2 a^2 b^2 B^2 w^2}{2R}$ **(b) zero** **(c)** $\frac{\pi abBw}{R}$ **(d)** $\frac{\pi^2 a^2 b^2 B^2 w^2}{R}$

SOL. (a) As we know, emf $\varepsilon = NAB\text{w} \quad \cos \text{w}t$, Here $N = 1$

Average power,

$$P = \frac{\varepsilon^2}{R} = \frac{A^2 B^2 \text{w}^2 \cos^2 wt}{R} = \frac{A^2 B^2 w^2}{R}\left(\frac{1}{2}\right)$$

Therefore average power loss in the loop due to Joule heating

$$\langle P \rangle = \frac{\pi^2 a^2 b^2 B^2}{2R} \text{w}^2$$

4. A uniform magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. The wire has a diameter of 4 mm and a total length of 30 cm. The magnetic field changes with time at a steady rate $dB/dt = 0.032 \text{Ts}^{-1}$. The induced current in the loop is close to (Resistivity of the metal wire is $1.23 \times 10^{-8} \Omega\text{m}$)

[Sep. 03, 2020 (II)]

(a) 0.43A **(b) 0.61A** **(c) 0.34A** **(d) 0.53A**

SOL. (b) Given,

Length of wire, $l = 30 \text{ cm}$

Radius of wire, $r = 2\text{mm} = 2 \times 10^{-3}\text{m}$

Resistivity of metal wire, $\text{p} = 1.23 \times 10^{-8} \Omega\text{m}$

Emf generated, $|e| = \frac{d\varphi}{dt} = \frac{dB}{dt}(A) \quad (\varphi = \text{B.A.})$

Current, $i=\frac{e}{R}$

But, resistance of wire, $R=\text{p}\frac{l}{A}$

$$j=|\frac{dB}{dt}|\frac{(A)^2}{\text{pl}}=\frac{0.032\times\{\pi\times 2\times 10^{-3}\}^2}{1.23\times 10^{-8}\times 0.3}=0.61\text{A}.$$

5. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5}T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes halfofrotation in $0.2s$. The maximum value of EMF induced (in μV) in the coil will be close to the integer .

[NA Sep. 02, 2020 (I)]

SOL. (15)

Here, $B=3.0\times 10^{-5}\text{T},\ R=10\text{cm}=0.\text{lm}$

$$\text{w}=\frac{2\pi}{2T}=\frac{\pi}{0.2}$$

Flux as a function of time $\varphi=\vec{B}\cdot\vec{A}=AB\cos(\text{w}t)$

Emf induced, $e=\frac{-d\varphi}{dt}=ABw\ \ \sin(wt)$

Max. value of Emf $=AB_{\text{tiJ}}=\pi R^2B\text{w}$

$$=3.14\times 0.1\times 0.1\times 3\times 10^{-5}\times\frac{\pi}{0.2}$$

$$=15\times 10^{-6}\text{V}=15\mu\text{V}$$

6. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self-inductance of the choke (in mH) is estimated to be .

[NA 9 Jan. 2020 I]

SOL. (10) Given $dI=0.25-0=0.25\text{A}$

$dt=0.025$ ms

Induced voltage $E_{ind}=100\text{v}$

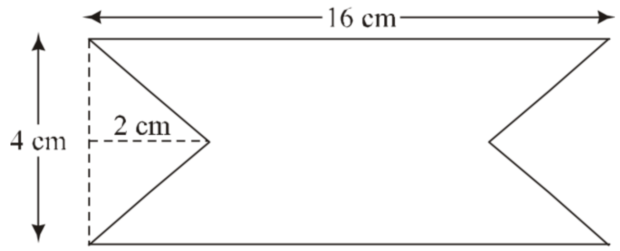
Self-inductance, L =?

Using, $E_{\text{ind}} = \frac{\Delta\varphi}{\Delta t} \quad \Rightarrow 100 = \frac{L(0.25-0)}{025 \times 10^{-3}}$

$$\Rightarrow \mathrm{L} = 10^{-3}\mathrm{H} = 10\mathrm{mH}$$

7. At time $t = 0$ magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is:

[NA 8 Jan. 2020 I]



(a) 56μV **(b) 28 μV** **(c) 48 μV** **(d) 36μV**

SOL. (a) According to question, $dB = 1000 - 500 = 500$ gauss

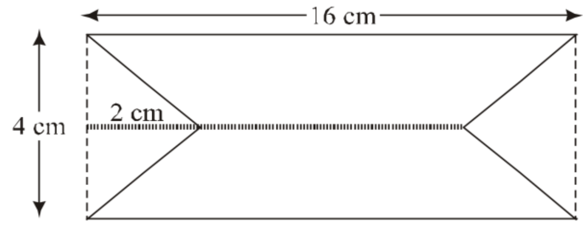
$$= 500 \times 10^{-4}T$$

Time $dt = 5$s

Using faraday law

Induced EMF, $e = \left|-\frac{d\varphi}{dt}\right| = \left|A\frac{dB}{dt}\right|$

$$\frac{dB}{dt} = \frac{1000 - 500}{5} \times 10^{-4} = 10^{-2}\mathrm{T/sec}$$



Area, A = area of □ − 2 area ofΔ =(16 × 4 − 2 × Area of triangle) cm^2

$$= \left(64 - 2 \times \frac{1}{2} \times 2 \times 4\right) \mathrm{cm}^2$$

$$= 56 \times 10^{-4}\mathrm{m}^2$$

$$\varepsilon_{\text{induced}} = |A\frac{dB}{dt}| = 56 \times 10^{-4} \times 10^{-2} = 56 \times 10^{-6}V = 56\mu V$$

8. Consider a circular coil ofwire carrying constant current I, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by φ_j .The magnetic flux through the area ofthe circular coil area is given by φ_0. Which of the following option is correct?

[7 Jan. 2020 I]

(a) $\varphi_j = \varphi_0$ (b) $\varphi_j > \varphi_0$ (c) $\varphi_j < \varphi_0$ (d) $\varphi_i = -\varphi_0$

SOL. (d) As magnetic field lines form close loop, hence every magnetic field line creating magnetic flux through the inner region (φ_i) must be passing through the outer region. Since flux in two regions are in opposite region.

$\therefore \quad \phi_i = -\phi_0$

9. A long solenoid of radius R carries a time (t)- dependent current $I(t) = I_0 t(1-t)$. A ring of radius $2R$ is placed coaxially near its middle. During the time interval $0 \leq t \leq 1$, the induced current (I_R) and the induced $EMF(V_R)$ in the ring change as:

[7 Jan. 2020 I]

(a) Direction of I_R remains unchanged and V_R is maximum at $t = 0.5$

(b) At $t = 0.25$ direction of I Reverses and V_R is maximum

(c) Direction of I_R remains unchanged and V_R is zero at $t = 0.25$

(d) At $t = 0.5$ direction of I_R reverses and V_R is zero

SOL. (d) According to question,

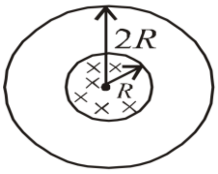
$$I(t) = I_0 t(1-t)$$

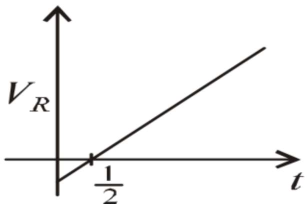
$$I = \mathrm{I}_0 \mathrm{t} - I_0 t^2$$

$$\varphi = B.A$$

$$\varphi = (\mu_0 nl) \times (\pi \mathrm{R}^2)$$

$(B = \mu_0 \text{nI} \;\; and \;\; A = \pi R^2)$





$$V_R = \frac{-d\varphi}{dt}$$

$$V_R = \mu_0 n \pi R^2 (I_0 - 2I_0 t)$$

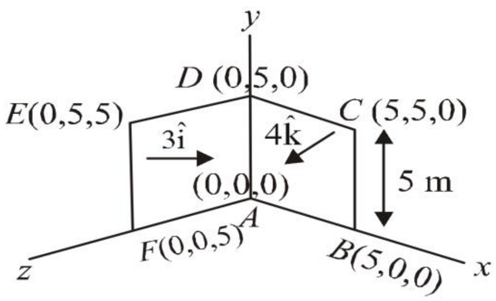
$\Rightarrow V_R = 0$ at $t = \frac{1}{2} s$

10. A loop ABCDEFA of straight edges has six corner points $A(0,0,0)$, $B\{5,0,0)$, $C(5,5,0)$, $D(0,5,0)$, $E(0,5,5)$and $F(0,0,5)$. The magnetic field in this region is $B = (3\hat{i} + 4\hat{k})$T. The quantity of flux through the loop *ABCDEFA* (in Wb) is

[NA 7 Jan. 2020 I]

SOL.

(175.00)



Flux through the loop ABCDEFA,

$$\varphi = \vec{B}.\vec{A} = (3\hat{i} + 4\hat{k}).(25\hat{i} + 25\hat{k})$$

$\Rightarrow \varphi = (3 \times 25) + (4 \times 25) = 175$ weber

11. A planar loop of wire rotates in a uniform magnetic field. Initially, at $t = 0$, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at:

[7 Jan. 2020 II]

(a) $2.5s$ and $7.5s$ **(b) $2.5s$ and $5.0s$**

(c) 5.0s and 7.5s **(d) 5.0s and 10.0s**

SOL. (b) We have given, time period, $T = 10\text{s}$

Angular velocity, $\text{w} = \frac{2\pi}{10} = \frac{\pi}{5}$

Magnetic flux, $\varphi(\text{t}) = BA\cos\ wt$

Emf induced, $E = \frac{-d\varphi}{dt} = BA\text{w}\ \ \sin \text{w}t = BA\text{w}\ \ \sin(\text{w}t)$

Induced emf, $|\varepsilon|$ is maximum when w$t = \frac{\pi}{2}$

$$\Rightarrow t = \frac{\pi}{2} / \frac{\pi}{5} = 2.5\text{s}$$

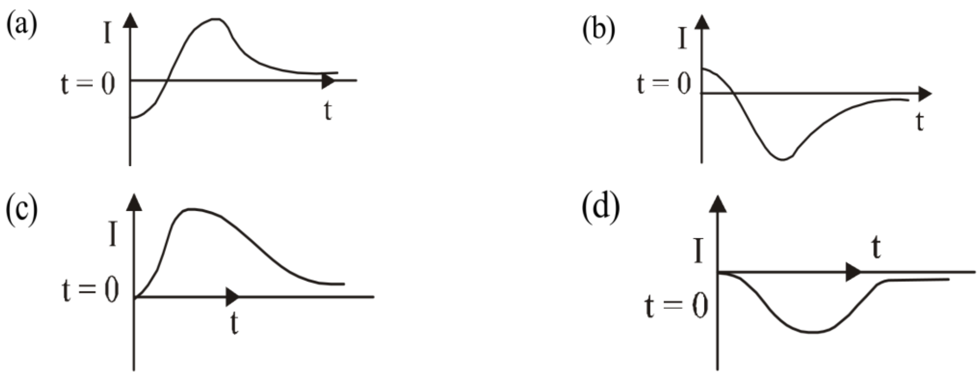
For induced emf to be minimum i.e zero

$$t = \frac{\pi}{\pi/5}$$

Induced emf is zero at $\text{t} = 5\text{s}$

12. A very long solenoid of radius R is carrying current $\mathbf{I(t) = kte^{\alpha t}} (k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by:

[9 Apr. 2019 II]



SOL. (a) $Q = BA$

$$= (\mu_0 \text{ni})A$$

$= \mu_0 n\ \ (kt\ \ e^{-\alpha t})A$

$$e = -\frac{dQ}{dt} = -\mu_0 nAk\frac{d}{dt}(te^{-\alpha t})$$

$$= -\mu_0 nAk[t(-1)e^{-\alpha t} + e^{-\alpha t} \times 1]$$

$$= -\mu_0 nAk[e^{-\alpha t}(1-t)]$$

$$i = \frac{e}{R} = \frac{-\mu_0 nAk}{R}[e^{-\alpha t}(1-t)]$$

At $t = 0,\ i \Rightarrow$-ve

13. Two coils P′ and Q′ are separated by some distance. When a current of 3A flows through coil P′, a magnetic flux of 10^{-3} Wb passes through Q′. No current is passed through ‘ Q′. When no current passes through ‘P’ and a current of 2A passes through ‘Q’, the flux through‘P’ is:

[9 Apr. 2019 II]

(a) 6.67×10^{-4}Wb **(b) 3.67×10^{3}Wb**

(c) 6.67×10^{3}Wb **(d) 3.67×10^{-4}Wb**

SOL. (a) $Q_{\text{coi1}} = (NQ) \propto i$

So, $\frac{Q_1}{Q_2} = \frac{i_1}{i_2} = \frac{3}{2}$

or $Q_2 = \frac{2}{3}Q_1 = \frac{2}{3} \times 10^{-3} = 6.67 \times 10^{-4}$Wb

14. The self-induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in ls, the change in the energy of the inductance is:

[9 Jan. 2019 II]

(a) 740 J **(b) 437.5J** **(c) 540J** **(d) 637.5J**

SOL. (b) According to faraday’s law of electromagnetic induction, $e = \frac{-d\varphi}{dt}$

$L \times \frac{di}{dt} = 25 \Rightarrow L \times \frac{15}{1} = 25$ or $L = \frac{5}{3}$H

Change in the energy of the inductance,

$$\Delta E = \frac{1}{2}L(i_1^2 - i_2^2) = \frac{1}{2} \times \frac{5}{3} \times (25^2 - 10^2)$$

$$= \frac{5}{6} \times 525 = 437.5J$$

15. A conducting circular loop made of a thin wire, has area $3.5 \times 10^3 m^2$ and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4T)\sin(50\pi t)$. The the net charge flowing through the loop during $t = 0s$ and $t = 10$ ms is close to:

[9 Jan. 2019 I]

(a) 14mC **(b) 7mC** **(c) 21mC** **(d) 6mC**

SOL. [Bonus]

Net charge $Q = \frac{\Delta\varphi}{R} = \frac{1}{10} A(B_f - B_i) = \frac{1}{10} \times 3.5 \times 10^{-3}$

$$\left(0.4 \sin \frac{\pi}{2} - 0\right)$$

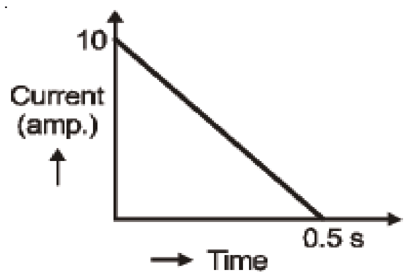
$$= \frac{1}{10}(3.5 \times 10^{-3})(0.4 - 0)$$

$$= 1.4 \times 10^{-4}$$

No option matches, So it should be a bonus.

16. In a coil of resistance 100Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is

[2017]



(a) 250 Wb **(b) 275 Wb** **(c) 200 Wb** **(d) 225 Wb**

SOL. (a) According to Faraday's law of electromagnetic induction, $e = \frac{-d\varphi}{dt}$

Also, $\varepsilon = iR$

$$iR = \frac{d\varphi}{dt} \Rightarrow \int d\varphi = R\int idt$$

Magnitude of change in flux $(d\varphi) = R \times$ area under current vs time graph or, $d\varphi = 100 \times \frac{1}{2} \times \frac{1}{2} \times 10 = 250$ Wb

17. A conducting metal circular–wire–loop of radius r is placed perpendicular to a magnetic field which varies with time as $B = B_0 e^{-t/\tau}$, where B_0 and τ are constants, at time $t = 0$. If the resistance ofthe loop is R then the heat generated in the loop after a long time $(t \to \infty)$ is;

[Online Apri110, 2016]

(a) $\frac{\pi^2 r^4 B_0^4}{2\tau R}$ **(b)** $\frac{\pi^2 r^4 B_0^2}{2\tau R}$ **(c)** $\frac{\pi^2 r^4 B_0^2 R}{\tau}$ **(d)** $\frac{\pi^2 r^4 B_0^2}{\tau R}$

SOL. (b) Electric flux is given by

$$\varphi = B.A$$

$$\varphi = B_0 \pi r^2 e^{-t/\tau} \qquad (B = B_0 e^{-t/\tau})$$

Induced E.m. f. $\varepsilon = \frac{d\varphi}{dt} = \frac{B_0 \pi r^2}{\Gamma^2} e^{-t/\tau}$

Heat $= \int_0^\infty \frac{\varepsilon^2}{R} = \frac{\pi^2 r^4 B_0^2}{2\tau R}$

18. When current in a coil changes from 5 A to 2 A in 0.1 s, average voltage of 50 V is produced. The self- inductance of the coil is:

[Online Apri110, 2015]

(a) 6 H **(b) 0.67 H** **(c) 3 H** **(d) 1.67H**

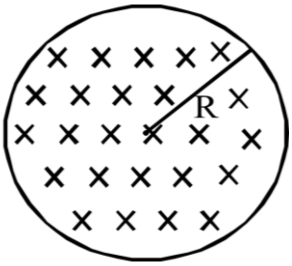
SOL. (d) According to Faraday's law of electromagnetic induction,

Induced emf, $e = \frac{Ldi}{dt}$

$$50 = L\left(\frac{5-2}{0.1 \text{ sec}}\right)$$

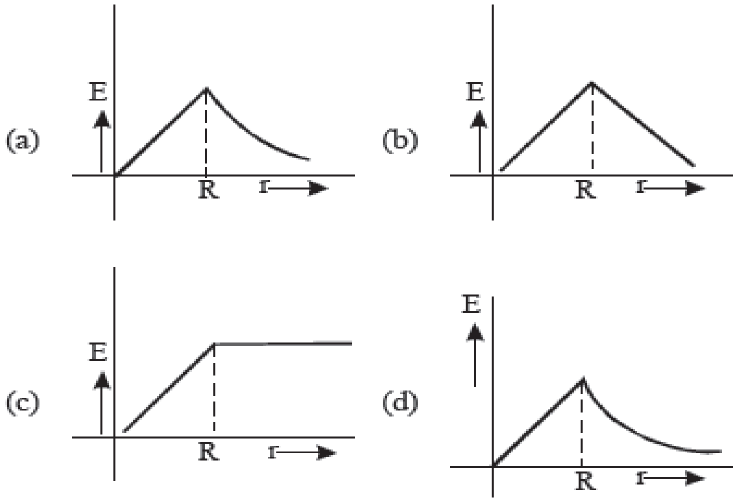
$$\Rightarrow L = \frac{50 \times 0.1}{3} = \frac{5}{3} = 1.67\text{H}$$

19. Figure shows a circular area of radius-R where a uniform magnetic field $\vec{B}$ is going into the plane of paper an increasing in magnitude at a constant rate.



In that case, which of the following graphs, drawn schematically, correctly shows the variation of the induced electric field $E(r)$?

[Online April 19, 2014]



SOL. (a) Inside the sphere field varies linearly i. e., $E \propto r$ with distance

and outside varies according to $E \propto \frac{1}{r^2}$

Hence the variation is shown by curve (a)

20. A coil of circular cross-section having 1000 turns and 4 cm^2 face area is placed with its axis parallel to a magnetic field which decreases by $10^{-2} Wbm^{-2}$ in $0.01s$. The e.m.f. induced in the coil is:

[Online April 11, 2014]

(a) 400 mV (b) 200 mV (c) 4 mV (d) 0.4 mV

SOL. (a) Given: No. of turns $N = 1000$

Face area, $A = 4cm^2 = 4 \times 10^{-4} m^2$

Change in magnetic field,

$$\Delta B = 10^{-2} \text{ wbm}^{-2}$$

Time taken, $t = 0.01\text{s} = 10^{-2}$ sec

Emf induced in the coil $e = ?$

Applying formula,

Induced emf, $e = \dfrac{-d\varphi}{dt} = N\left(\dfrac{\Delta B}{\Delta t}\right)A\cos\theta$

$$= \frac{1000 \times 10^{-2} \times 4 \times 10^{\triangleleft}}{10^{-2}} = 400\text{mV}$$

21. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is

[2013]

(a) 9.1×10^{-11} weber **(b) 6×10^{-11} weber**

(c) 3.3×10^{-11} weber **(d) 6.6×10^{-9} weber**

SOL. (a) As we know, Magnetic flux, $\varphi = B.A$

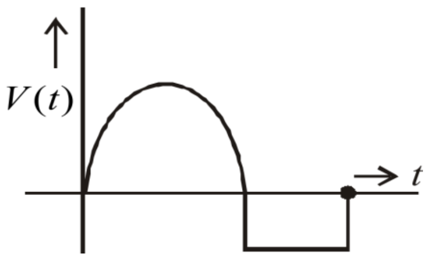
$$\frac{\mu_0(2)(20 \times 10^{-2})^2}{2[(0.2)^2 + (0.15)^2]} \times \pi(0.3 \times 10^{-2})^2$$

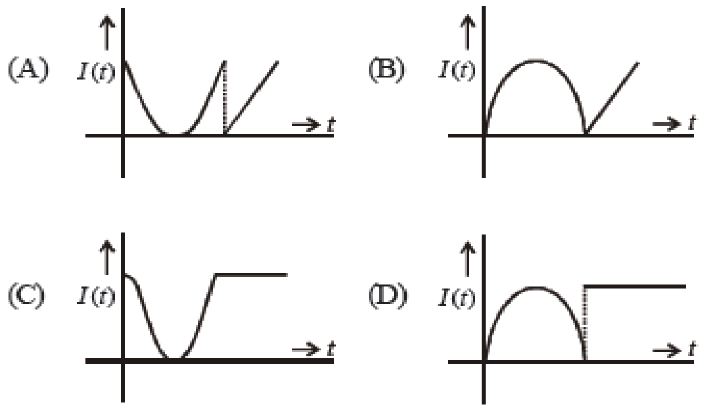
On solving

$= 9.216 \times 10^{-11} = 9.2 \times 10^{-11}$ weber

22. Two coils, X and Y, are kept in close vicinity of each other. When a varying current, $I(t)$, flows through coil X, the induced emf $(V(t))$ in coil Y, varies in the manner shown here. The variation of $I(t)$, with time, can then be represented by the graph labelled as graph :

[Online April 9, 2013]





(a) A **(b) C** **(c) B** **(d) D**

SOL. (a) Induced emf

$$\varepsilon \propto \frac{-di}{dt}$$

23. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; It is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to:

[2012]

(a) development of air current when the plate is placed

(b) induction of electrical charge on the plate

(c) shielding of magnetic lines of force as aluminium is a paramagnetic material.

(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

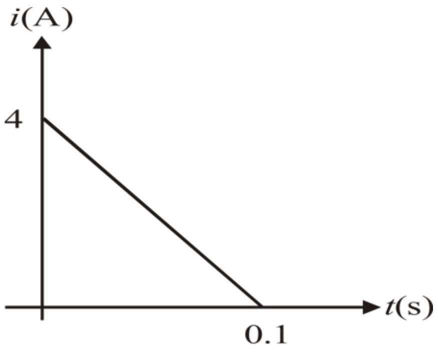
SOL. (d) Because of the Lenz's law of conservation of energy.

Length of straight wire, $\ell = 20\text{m}$, Earth's Magnetic field,

$$B = 0.30 \times 10^{-4}\ \text{Wb/m}^2.$$

24. Magnetic flux through a coil of resistance 10 Ω is changed by $\Delta\varphi$ in 0.1 s. The resulting current in the coil varies with time as shown in the figure. Then $|\Delta\varphi|$ is equal to (in weber)

[Online May 12, 2012]



(a) 6 **(b) 4** **(c) 2** **(d) 8**

SOL. (c) As $e = \frac{\Delta\varphi}{\Delta t}$ or $Ri = \frac{\Delta\varphi}{\Delta t}$ $(\because e = Ri)$

$\Rightarrow \Delta\varphi = R(i.\Delta t)$

$= R \times$ area under $i - t$ graph

$= 10 \times \frac{1}{2} \times 4 \times 0.1 = 2$ weber

25. The flux linked with a coil at any instant t' is given by $\varphi = 10t^2 - 50t + 250$. The induced emf at $t = 3s$ is

[2006]

(a) $-190V$ **(b) $-10V$** **(c) $10V$** **(d) $190V$**

SOL. (b) Electric flux, $\varphi = 10t^2 - 50t + 250$

Induced emf, $e = -\frac{d\varphi}{dt} = -(20t - 50)$

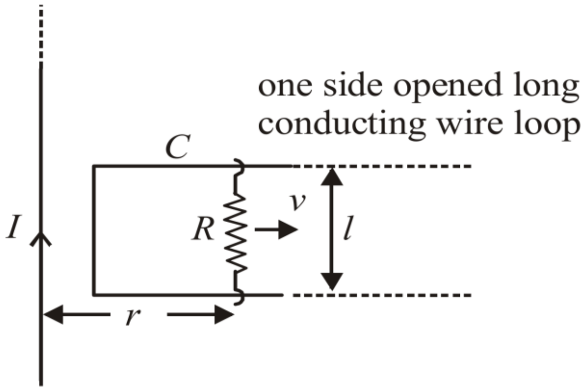
$$e_{t=3} = -10V$$

Topic 2: Motional and Static EMI and Application of EMI

26. An infinitely long straight wire carrying current I, one side opened rectangular loop and a conductor C with a sliding connector are located in the same plane, as shown in the figure. The connector has length l and resistance R. It slides to the right with a velocity v. The resistance of the conductor and the self inductance of the loop are negligible. The induced current in the loop, as a function of separation r, between the connector and the straight

wire is:

[Sep. 05, 2020 (II)]



(a) $\frac{\mu_0 Ivl}{4\pi Rr}$ **(b)** $\frac{\mu_0 Ivl}{\pi Rr}$ **(c)** $\frac{2\mu_0 Ivl}{\pi Rr}$ **(d)** $\frac{\mu_0 Ivl}{2\pi Rr}$

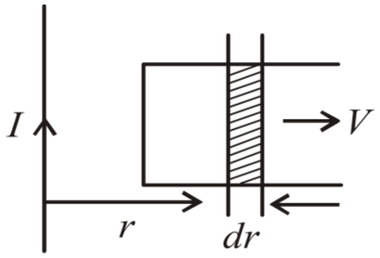
SOL. (d) Magnetic field at a distance r from the wire

$$B = \frac{\mu_0 l}{2\pi r}$$

Magnetic flux for small displacement dr

$\varphi = B \cdot A = Bldr$ $[A = ldr$ and $B.A = BA\cos 0°]$

$\Rightarrow \varphi = \frac{\mu_0 I}{2\pi r} ldr$

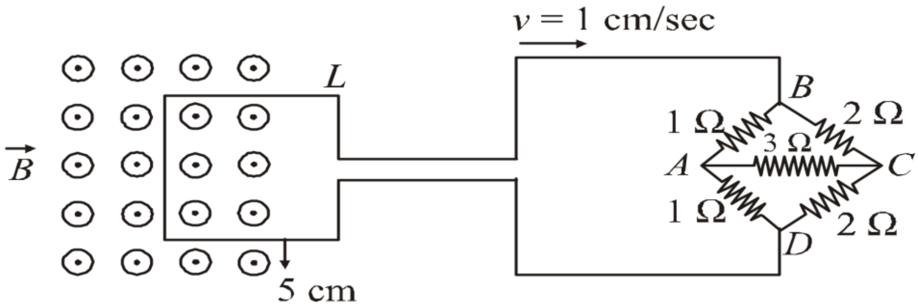


Emf, $e = \frac{d\varphi}{dt} = \frac{\mu_0 Il}{2\pi r} \cdot \frac{dr}{dt} \Rightarrow e = \frac{\mu_0}{2\pi} \cdot \frac{Ivl}{r}$

Induce current in the loop, $i = \frac{e}{R} = \frac{\mu_0}{2\pi} \cdot \frac{Ivl}{Rr}$

27. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of l cm s^{1}. At some instant, a part of L is in a uniform magnetic field of l T, perpendicular to the plane of the loop. If the resistance of L is 1. 7 &!, the current in the loop at that instant will be close to:

[12 Apr. 2019 I]



(a) $60\mu A$ **(b) 170 μA** **(c) 150 μA** **(d) 115 μA**

SOL. (b) Induced emf,

$$e = Bv\ell = 1 \times 10^{-2} \times 0.05 = 5 \times 10^{-4} \text{V}$$

Equivalent resistance,

$$R = \frac{4 \times 2}{4 + 2} + 1.7 = \frac{4}{3} + 1.7 = 3\Omega$$

Current, $i = \frac{e}{R} = \frac{5 \times 10^{-4}}{3} = 170 \mu \text{A}$

28. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to:

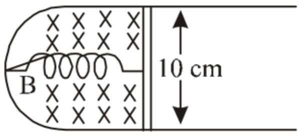
[9 April 2019 I]

(a) L **(b) L^2** **(c) $1/L^2$** **(d) $1/L$**

SOL. (d) Inductance $= \frac{\mu_0 N^2 A}{L}$

29. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant $0.5 Nm^{-1}$ (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of strip is 50 grams, its resistance is 10Ω and air drag negligible, N will be close to:

[8 April 2019 I]

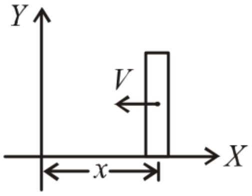


(a) 1000 **(b) 50000** **(c) 5000** **(d) 10000**

SOL. (c) Force on the strip when it is at stretched position x from mean position is

$$F = -kx - iIB = -kx - \frac{BIv}{R} \times IB$$

$$F = -kx - \frac{B^2 I^2}{R} \times v$$



Above expression shows that it is case of damped oscillation,

so its amplitude can be given by

$$\Rightarrow A = A_0 e^{\frac{bt}{2m}}$$

$$\Rightarrow \frac{A_0}{e} = A_0 e^{-\frac{bt}{2m}}$$ [as per question $A = \frac{A_0}{e}$]

$$\Rightarrow t = \frac{2m}{\left(\frac{B^2 I^2}{R}\right)} = \frac{2 \times 50 \times 10^{-3} \times 10}{0.01 \times 0.01}$$

Given, $m = 50 \times 10^{-3}$ kg

$B = 0.1\text{T}$ $\quad l = 0.1\text{m}$ $\quad R = 10\Omega$ $\quad k = 0.5\text{N}$

Time period, $T = 2\pi\sqrt{\frac{m}{k}} = 2\text{s}$

so, required number of oscillations,

$$N = \frac{10000}{2} = 5000$$

30. A 10m long horizontal wire extends from North East to South West. It is falling with a speed of $5.0\ \text{ms}^{-1}$, at right angles to the horizontal component of the earth's magnetic field, of $0.3 \times 10^{-4}\text{Wb/m}^2$. The value of the induced emf in wire is:

[12 Jan. 2019 II]

(a) $1.5 \times 10^{-3}\text{V}$ (b) $1.1 \times 10^{-3}\text{V}$ (c) $2.5 \times 10^{-3}\text{V}$ (d) $0.3 \times 10^{-3}\text{V}$

SOL. (a) Induced emf, $\varepsilon = \text{Bv}\ell$

$$= 0.3 \times 10^{-4} \times 5 \times 10$$

$$= 1.5 \times 10^{-3}\text{V}$$

31. There are two long co-axial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number ofturns per unit lengh n_1 and n_2, respectively. The ratio of mutual

inductance to the self-inductance of the inner-coil is:

[11 Jan. 2019 I]

(a) $\frac{n_1}{n_2}$ **(b)** $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ **(c)** $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ **(d)** $\frac{n_2}{n_1}$

SOL. (d) The rate of mutual inductance is given by

$M = \mu_0 n_1 n_2 \pi r_1^2$ (i)

The rate of self inductance is given by

$L = \mu_0 n_1^2 \pi r_1^2$ (ii)

Dividing (i) by (ii) $\Rightarrow \frac{M}{L} = \frac{n_2}{n_1}$

32. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil:

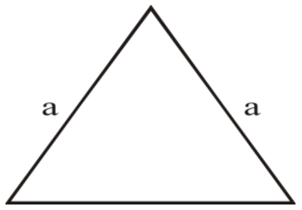
[11 Jan. 2019 II]

(a) decreases by a factor of 9 **(b) increases by a factor of 27**

(c) increases by a factor of 3 **(d) decreases by a factor of $9\sqrt{3}$**

SOL. (c) As total length L of the wire will remain constant $L = (3a)N$ ($N =$ total turns)

and length of winding $= (d)N$



($d =$ diameter ofwire)

self inductance $= \mu_0 n^2 A\ell$

$= \mu_0 n^2 \left(\frac{\sqrt{3}a^2}{4}\right) dN$

$\propto a^2 N \propto a$ [as $N = L/3a \Rightarrow N \propto \frac{1}{a}$]

Now a' increased to 3a' So self inductance will become 3 times

33. A solid metal cube of edge length 2 cm is moving in a positive y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is:

[10 Jan. 2019 I]

(a) 12 mV (b) 6mV (c) 1 mV (d) 2 mV

SOL. (a) Potential difference between two faces perpendicular to x-axis

$$= lV.B = 2 \times 10^{-2}(6 \times 0.1) = 12\text{mV}$$

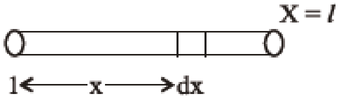
34. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin $(x = 0)$ and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is:

[10 Jan. 2019 I]

(a) $\pi n \rho l^3$ **(b)** $\frac{\pi}{3} n \rho l^3$ **(c)** $\frac{\pi}{4} n \rho l^3$ **(d)** $n \rho l^3$

SOL.

(c)



Magnetic moment, M = NIA

$$dQ = pdx$$

$$dI = \frac{dQ}{2\pi}.w$$

$$dM = dI \times A$$

$$= \frac{w}{2\pi}.\frac{p_0}{\ell}.x\pi x^2 dx \Rightarrow M = \frac{p_0}{\ell} n\pi \int_0^{\ell} x^3\, dx$$

$$= \frac{\pi}{4}.np\ell^3$$

35. A coil of cross-sectional area A having n turns is placed in a uniform magnetic field B. When it is rotated with an angular velocity w, the maximum e.m.f. induced in the coil will be

[Online April 16, 2018]

(a) nBAw **(b) $\frac{3}{2}$nBAw** **(c) 3nBAw** **(d) $\frac{1}{2}$nBAw**

SOL. (a) Induced emf in a coil, $e = -\frac{d\varphi}{dt} =$ NBAsin w t

Also, $e = e_0 \sin$ oot

Maximum emf induced, e_0 =nBAw

36. An ideal capacitor of capacitance $0.2\ \mu F$ is charged to a potential difference of 10V. The charging battery is then disconnected. The capacitor is then connected to an ideal inductor of self-inductance 0.5mH. The current at a time when the potential difference across the capacitor is 5V, is:

[Online April 15, 2018]

(a) 0.17A **(b) 0.15A** **(c) 0.34A** **(d) 0.25A**

SOL. (a) Given: Capacitance, $C = 0.2\mu F = 0.2 \times 10^{-6} F$

Inductance $L = 0.5\text{mH} = 0.5 \times 10^{-3}\text{H}$

Current $I = ?$

Using energy conservation

$$\frac{1}{2}CV^2 = \frac{1}{2}CV_1^2 + \frac{1}{2}LI^2$$

$$\frac{1}{2} \times 0.2 \times 10^{-6} \times 10^2 + 0$$

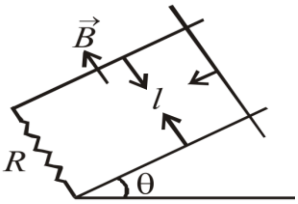
$$= \frac{1}{2} \times 0.2 \times 10^{-6} \times 5^2 + \frac{1}{2} \times 0.5 \times 10^{-3} I^2$$

$$I = \sqrt{3} \times 10^{-1}\text{A} = 0.17\text{A}$$

37. A copper rod of mass m slides under gravity on two smooth parallel rails, with separation 1 and set at an angle of θ with the horizontal. At the bottom, rails are joined by a resistance R. There is a uniform magnetic field B normal to the plane of the rails, as shown in the figure. The terminal

speed of the copper rod is:

[OnlineApril l5, 2018]



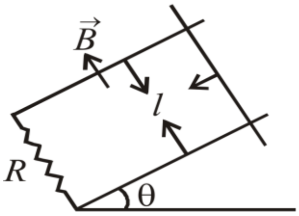
(a) $\frac{\text{mgR}\cos\theta}{\text{B}^2 l^2}$ **(b)** $\frac{\text{mgR}\sin\theta}{\text{B}^2 l^2}$ **(c)** $\frac{\text{mgR}\tan\theta}{\text{B}^2 l^2}$ **(d)** $\frac{\text{mgR}\cot\theta}{\text{B}^2 l^2}$

SOL. (b) From Faraday's law of electro magnetic induction,

$$e = \frac{d\varphi}{dt} = \frac{d(BA)}{lt} = \frac{d(Bll)}{dt}$$

$$= \frac{Bdl \times l}{dt} = BVl$$

Also, $F = ilB = \left(\frac{BV}{R}\right)(l^2 B) = \frac{B^2 l^2 V}{R}$



At equilibrium

$$mg\sin\theta = \frac{B^2 lV}{R} \Rightarrow V = \frac{mgR\sin\theta}{B^2 l^2}$$

38. At the centre of a fixed large circular coil of radius R, amuch smaller circular coil of radius r is placed. The two coils are concentric and are in the same plane. The larger coil carries a current I. The smaller coil is set to rotate with a constant angular velocity w about an axis along their common diameter. Calculate the emf induced in the smaller coil after a time t of its start of rotation.

[Online Apri115, 2018]

(a) $\frac{\mu_0 \text{I}}{2\text{R}} \text{wr}^2 \sin wt$ **(b)** $\frac{\mu_0 \text{I}}{4\text{R}} \text{w}\pi\text{r}^2 \sin wt$

(c) $\frac{\mu_0 \text{I}}{2\text{R}} \text{w}\pi\text{r}^2 \sin wt$ **(d)** $\frac{\mu_0 \text{I}}{4\text{R}} \text{wr}^2 \sin wt$

SOL. (c) According to Faraday's law of electromagnetic induction,

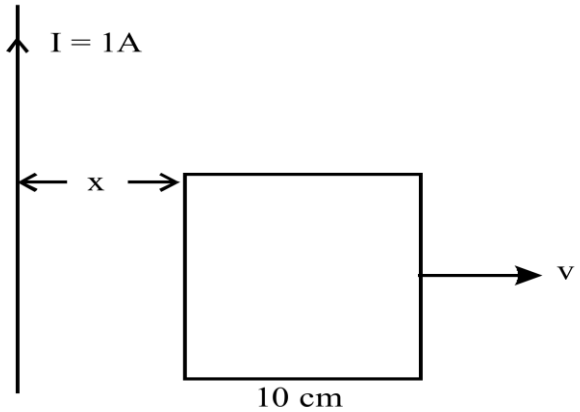
$e = -\frac{d\varphi}{dt}$ and $\varphi = BA\cos\text{wt} = B\pi r^2 \cos\text{wt}$

$\Rightarrow\ e = -\frac{d}{dt}(\pi r^2 B \cos wt) = \pi r^2 B \sin wt\ (\text{w})$

$$e = \frac{\mu_0 I}{2R}\pi \text{w} r^2 \sin \text{w}t \left(\because B = \frac{\mu_0 I}{2R}\right)$$

39. A square frame of side 10 cm and a long straight wire carrying current 1 A are in the plate of the paper. Starting from close to the wire, the frame moves towards the right with a constant speed of 10 ms^{-1} (see figure). The emf induced at the time the left arm of the frame is at $x = 10$ cm from the wire is:

[Online April 19, 2014]



(a) 2 μV (b) 1 μV (c) 0.75μV (d) 0.5μV

SOL. (b) In the given question,

Current flowing through the wire, $\text{I} = 1\text{A}$

Speed of the frame, $\text{v} = 10\text{ms}^{-1}$

Side of square loop, $l = 10$ cm

Distance of square frame from current carrying wires $\text{x} = 10\text{cm}$.

We have to find, e.m.f induced $\text{e} = ?$

According to Biot-Savart's law

$$\text{B} = \frac{\mu_0 \text{I dl} \sin\theta}{4\pi \text{x}^2}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{1 \times 10^{-1}}{(10^{-1})^2}$$

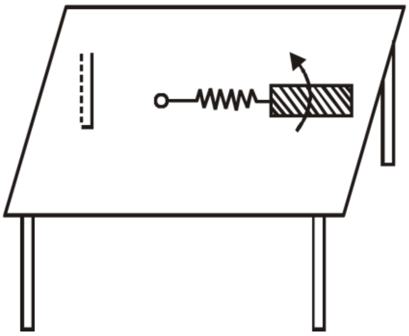
$$= 10^{-6}$$

$$\text{Induced e.m.f. } e = Blv$$

$$= 10^{-6} \times 10^{-1} \times 10 = 1\mu v$$

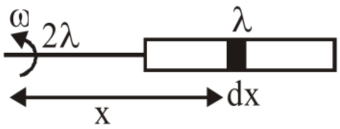
40. A metallic rod of length ℓ' is tied to a string of length 2ℓ and made to rotate with angular speed w on a horizontal table with one end ofthe string fixed. If there is a vertical magnetic field B' in the region, the e.m.f. induced across the ends of the rod is

[2013]



(a) $\frac{2Bw\ell^2}{2}$ **(b)** $\frac{3Bw\ell^2}{2}$ **(c)** $\frac{4Bw\ell^2}{2}$ **(d)** $\frac{5Bw\ell^2}{2}$

SOL. (d) Here, induced e.m.f.

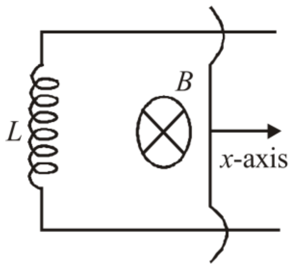


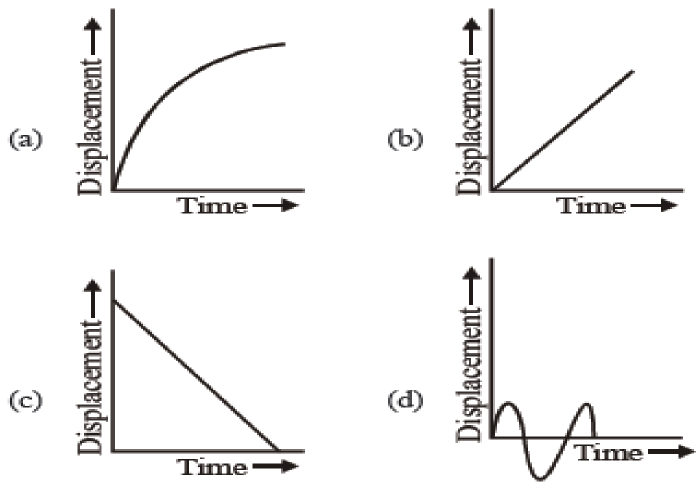
$$e = \int_{2p}^{3p} (wx) Bdx = Bw \frac{\left[(3\ell)^2 - (2\ell)^2\right]}{2}$$

$$= \frac{5B\ell^2 w}{2}$$

41. A coil of self-inductance L is connected at one end of two rails as shown in figure. A connector of length l, mass m can slide freely over the two parallel rails. The entire set up is placed in a magnetic field ofinduction B going into the page. At an instant $t = 0$ an initial velocity v_0 is imparted to it and as a result of that it starts moving along x-axis. The displacement of the connector is represented by the figure.

[Online May 19, 2012]





SOL. (d)

42. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: Self inductance of a long solenoid of length *L*, total number of turns N and radius r is less than $\frac{\pi\mu_0 N^2 r^2}{L}$.

Statement 2: The magnetic induction in the solenoid in Statement 1 carrying current *I* is $\frac{\mu_0 Nl}{L}$ in the middle ofthe solenoid but becomes less as we move towards its ends.

[Online May 19, 2012]

(a) Statement 1 is true, Statement 2 is false.

(b) Statement 1 is true, Statement2 is true, Statement 2 is the correct explanation of Statement 1.

(c) Statement 1 is false, Statement2 is true.

(d) Statement 1 is true, Statement2 is true, Statement 2 is not the correct explanation of Statement 1.

SOL. (b) Self inductance of a long solenoid is given by

$$L = \frac{\mu_0 N^2 A}{l}$$

Magnetic field at the centre of solenoid

$$B = \frac{\mu_0 N I}{l}$$

So both the statements are correct and statement 2 is correct explanation of statement 1

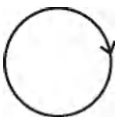
43. A boat is moving due east in a region where the earth's magnetic field is $5.0 \times 10^{-5} \mathrm{NA^{-1} m^{-1}}$ due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the Mat is $1.50 \mathrm{~ms^{-1}}$, the magnitude of the induced emf in the wire of aerial is:

[2011]

(a) $0.75 \mathrm{mV}$ (b) $0.50 \mathrm{mV}$ (c) $0.15 \mathrm{mV}$ (d) $1 \mathrm{mV}$

SOL. (d) As magnetic field lines form close loop,

hence every magnetic field line creating magnetic flux through the inner region (φ_i) must be passing through the outer region. Since flux in two regions are in opposite region.



$$\varphi_i = -\varphi_0$$

44. A horizontal straight wire 20 m long extending from east to west falling with a speed of $5.0 \mathrm{~m/s}$, at right angles to the horizontal component of the earth's magnetic field $0.30 \times 10^{-4} \mathrm{Wb/m^2}$. The instantaneous value of the e.m.f. induced in the wire will be

[2011 RS]

(a) $3 \mathrm{mV}$ (b) $4.5 \mathrm{mV}$ (c) $1.5 \mathrm{mV}$ (d) $6.0 \mathrm{mV}$

SOL. (a) Induced, emF, $\varepsilon = Bv\ell$

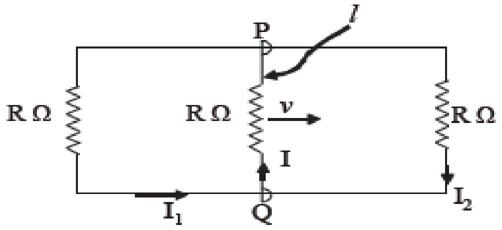
$$= 0.3 \times 10^{-4} \times 5 \times 20$$

$$= 3 \times 10^{-3} \mathrm{V} = 3 \mathrm{mV}.$$

45. A rectangular loop has a sliding connector PQ of length l and resistance $R\Omega$ and it is

moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1 , I_2 and I are

[2010]



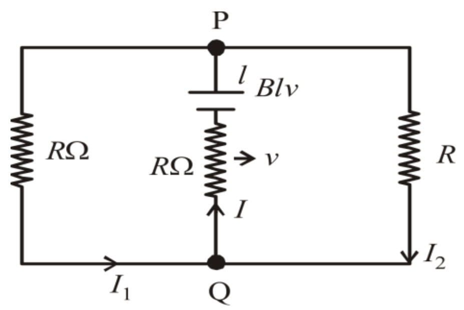
(a) $I_1 = -I_2 = \frac{Blv}{6R}, I = \frac{2Blv}{6R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

SOL. (b) Due to the movement of resistor R, an emf equal to Blv will be induced in it as shown in figure clearly,



$I = I_1 + I_2$ Also, $I_1 = I_2$ Solving the circuit,

we get $I_1 = I_2 = \frac{Blv}{3R}$ and $I = 2I_1 = \frac{2Blv}{3R}$

46. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\text{cm}^2$ and length $= 20$ cm. If one of the solenoid has 300 tums and the other 400 turns, their mutual inductance is $(\mu_0 = 4\pi \times 10^{-7}\text{TmA}^{-1})$

[2008]

(a) $2.4\pi \times 10^{-5}\text{H}$ **(b)** $4.8\pi \times 10^{-4}\text{H}$ **(c)** $4.8\pi \times 10^{-5}\text{H}$ **(d)** $2.4\pi \times 10^{-4}\text{H}$

SOL. (d) Given, Area of cross-section of pipe, $A = 10\text{cm}^2$

Length of pipe, $\ell = 20$ cm

$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

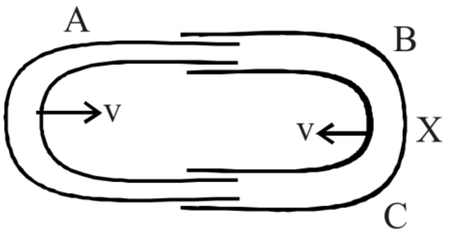
$$= \frac{4\pi \times 10^{-7} \times 300 \times 400 \times 100 \times 10^{-4}}{02}$$

$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

$$= 2.4\pi \times 10^{-4}\text{H}$$

47. One conducting U tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v where l is the width of each tube, will be

[2005]



(a) $-Blv$ **(b) *Blv*** **(c) 2 *Blv*** **(d) zero**

SOL. (c) Relative velocity of the tube of width l,

$$= v - (-v)v = 2v$$

Induced emf. $= B.l(2v)$

48. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 radians per second. If the horizontal component of earth's magnetic field is 0.2×10^{-4}T, then the e.m.f. developed between the two ends of the conductor is

[2004]

(a) 5mV **(b) 50μV** **(c) 5μV** **(d) 50mV**

SOL. (b) Given, length of conductor $\ell = 1$m,

Angular speed, $w = 5\text{rad/s}$,

Magnetic field, $B = 0.2 \times 10^{-4} T$

EmF generated between two ends of conductor

$$\varepsilon = \frac{Bwl^2}{2} = \frac{0.2 \times 10^{-4} \times 5 \times 1}{2} = 50\mu V$$

49. A coil having n tums and resistance $R\Omega$ is connected with a galvanometer of resistance $4R\Omega$. This combination is moved in time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is

[2004]

(a) $-\frac{(W_2 - W_1)}{Rnt}$ **(b)** $-\frac{n(W_2 - W_1)}{5Rt}$ **(c)** $-\frac{(W_2 - W_1)}{5Rnt}$ **(d)** $-\frac{n(W_2 - W_1)}{Rt}$

SOL. (b) $\frac{\Delta\varphi}{\Delta t} = \frac{(W_2 - W_1)}{t}$

$$R_{tot} = (R + 4R)\Omega = 5R\Omega$$

$$i = \frac{nd\varphi}{R_{tot}dt} = \frac{-n(W_2 - W_1)}{5Rt}$$

(W_2& W_1 are magnetic flux)

50. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon [2003]

(a) the rates at which currents are changing in the two coils

(b) relative position and orientation of the two coils

(c) the materials of the wires of the coils

(d) the currents in the two coils

SOL. (b) Mutual inductance depends on the relative position and orientation of the two coils.

51. When the current changes from $+2A$ to $-2A$ in 0.05 second, an e.m.f. of 8 V is induced in a coil. The coefficient of self -induction of the coil is

[2003]

(a) $0.2H$ **(b) $0.4H$** **(c) $0.8H$** **(d) $0.1H$**

SOL. (d) Induced emf,

$$e = -\frac{\Delta\varphi}{\Delta t} = \frac{-\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

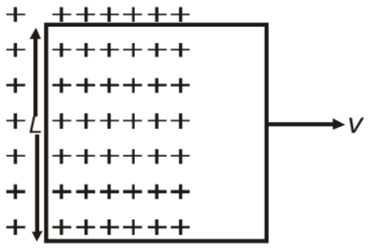
$$|e| = L\frac{\Delta I}{\Delta t}$$

$$\Rightarrow 8 = L \times \frac{[2-(-2)]}{0.05}$$

$$\Rightarrow L = \frac{8 \times 0.05}{4} = 0.1\text{H}$$

52. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is

[2002]



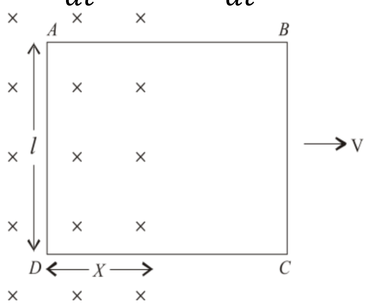
(a) zero (b) RvB (c) vBL/R (d) vBL

52. (d) As the side BC is outside the field, no emf is induced across BC. Further, sides AB and CD are not cutting any flux. So, they will not contribute in flux.

Only side AD is cutting the flux, so emf will be induced due to AD only.

The induced emf is

$$e = \frac{-d\varphi}{dt} = -\frac{d(\vec{B}\cdot\vec{A})}{dt} = \frac{-d(BA\cos 0^o)}{dt}$$



$$e = -B\frac{dA}{dt} = -B\frac{d(\ell \times x)}{dt}$$ $$e = -B\ell\frac{dx}{dt} = -B\ell v$$

Alternating Current

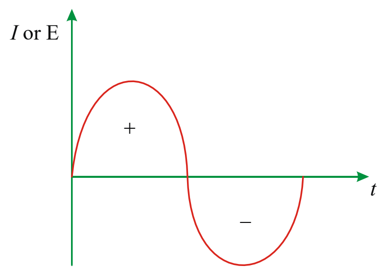
When a resistor is connected across the terminals of a battery, a current is established in the circuit. The current has a unique direction, it goes from the positive terminal to the negative terminal via the external resistor. The magnitude of the current also remains almost constant. This is called direct current (**dc**). If the direction of the current in a resistor or in any other element changes alternately, the current is called an alternating current (**ac**). In this chapter, we shall study the alternating current that varies sinusoidally with time.

Alternating Current(A.C.)

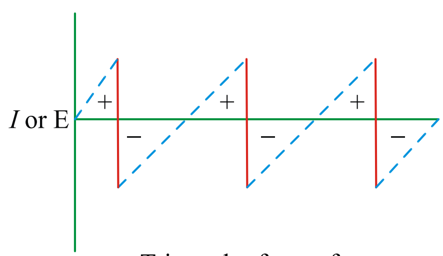
- Electric current, which keeps on changing in magnitude and direction periodically is defined as alternating current.
- It obeys Ohm's law and Joule's heating law.
- It is produced using the principle of electromagnetic induction.
- Graphical representations for alternating quantities can be represented in the form of the following graphs.

Alternating Voltage (A.V)

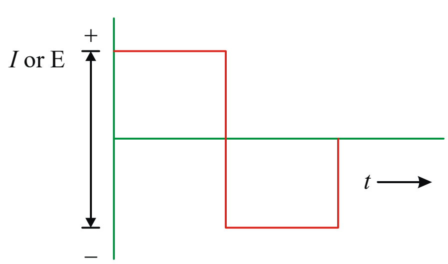
- The voltage, which changes in magnitude and direction with respect to time is defined as alternating voltage.
- The alternating voltage in general use is sinusoidal voltage. It is produced by rotating a coil in a uniform magnetic field with uniform angular velocity.



sinusoidal form of ac



Triangular form of ac



Square form of ac

Advantages of Alternating current over direct current

- The cost of generation of **ac** is less than that of **dc**.
- **ac** can be conveniently converted into **dc** with the help of rectifiers.
- By supplying **ac** at high voltages, we can minimise transmission losses or line losses.
- **ac** is available in a wide range of voltages. These volatages can be easily stepped up or stepped down with the help of transformers.

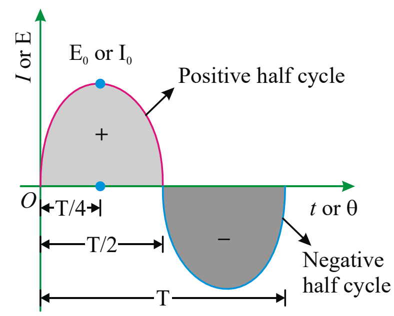
Disadvantages of alternating current over direct current

- **ac** is more dangerous than **dc**.
- **ac** is transmitted more by the surface of the conductor. This is called skin effect. Due to this reason that several strands of thin insulated wire, instead of a single thick wire, need be used.
- For electrorefining,electro - typing electroplating, only dc can be used but not ac.

Instantaneous Value Of Current Or Voltage (I Or E)

- The value of current or voltage in an **ac** circuit at any instant of time is called its instantaneous value.
- Instantaneous current, $I = I_0 \sin\omega t$ (or) $I = I_0 \sin(\omega t + \phi)$
- Instantaneous voltage, $E = E_0 \sin\omega t$ (or) $E = E_0 \sin(\omega t + \phi)$

Where $(\omega t + \phi)$ is called phase



Amplitude Of A.C. (Peak Value) (I_0) Or (I_m):

It is the maximum value of A.C. The value of A.C. becomes maximum twice in one cycle.

Note: Average value of a function $F(t)$ over a period of T is given by

$$<F(t)> = F_{avg} = \frac{\int_0^T F(t)\,dt}{\int_0^T dt} = \frac{1}{T}\int_0^T F(t)\,dt$$

Eg:-

$$<\sin^2 \omega t> = \frac{1}{2} : <\cos^2 \omega t> = \frac{1}{2}$$

$$<\sin 2\omega t> = 0 ; <\cos 2\omega t> = 0$$

Average Value Of A.C. < I >

- The value of current at any instant 't' is given by $I = I_0 \sin\omega t$.
- The average value of a sinusoidal wave over one complete cycle is given by

$$I_{avg} = \frac{\int_0^T I.dt}{\int_0^T dt} = \frac{\int_0^T I_0 \sin \omega t.dt}{\int_0^T dt} = 0$$

For half cycle:

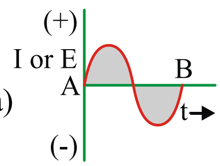
$$< I > = \frac{\int_0^{T/2} Idt}{\int_0^{T/2} dt} = \frac{\int_0^{T/2} I_0 \sin \omega t \, dt}{\int_0^{T/2} dt} = \frac{2I_0}{\pi} = 0.636 I_0$$

$$I_{avg} = 63.7\% \text{ of } I_0$$

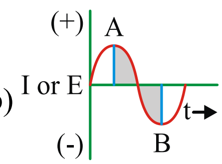
Similarly

$$E_{avg} = \frac{2E_o}{\pi} = 0.637 E_0 = 63.7\% E_0$$

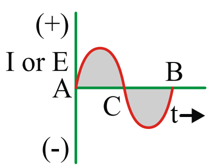
Note:

a) 

Between A and B,
$I_{avg} = 0$
$E_{avg} = 0$

b) 

Between A and B,
$I_{avg} = 0$
$E_{avg} = 0$

c) 

Between A and C
or
C and B
$I_{avg} \neq 0$
$E_{avg} \neq 0$

Frequence of A.C (F)

It is the number of cycles completed by A.C. in one second.

Time Period Of A.C. (T)

It is the time taken by A.C. to complete one cycle.

$$f = 1/T$$

Mean Square Value Of A.C. $< I^2 >$

$$< I^2 > = \frac{I_0^2}{2}$$

R.M.S. Value (I_{rms}) Or Effective Value (I) Or Virtual Value Of A.C.

It is the square root of the average of squares of all the instantaneous values of current over one complete cycle.

$$I_{rms}^2 = \frac{\int_0^T I^2.dt}{\int_0^T dt} = \frac{\int_0^T I_0^2.\sin^2 \omega t.dt}{T}$$

$$= \frac{I_0^2}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right]dt = \frac{I_0^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T = \frac{I_0^2}{2} \;;$$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = 0.707\ I_0$$

It is equal to that direct current which produces same heating in a resistance as is produced by the A.C. in same resistance during same time.

Mean Square Value Of A.C. $< I^2 >$

$$< I^2 > = \frac{I_0^2}{2}$$

Form Factor

$$\textbf{Form factor} = \frac{\text{rms value}}{\text{average value over half cycle}}$$

$$Form\ factor = \frac{I_{rms}}{I_{avg}} = \frac{E_{rms}}{E_{avg}}$$

$$\text{We know that } I_{rms} = \frac{I_0}{\sqrt{2}} \text{ and } I_{ave} = \frac{2I_0}{\pi}$$

$$\therefore \text{Form factor} = \frac{I_0}{\sqrt{2}} \times \frac{\pi}{2I_0} = \frac{\pi}{2\sqrt{2}} = 1.11$$

Note :

- **ac** ammeter and voltmeter read the r.m.s value i.e., effective value of alternating current and voltage respectively.
- **ac** can be measured by using hot wire ammeters or hot wire voltmeters because the heat generated is independent of the direction of current.
- **ac produces the same heating effects as that of dc of magnitude $i = i_{rms}$**
- **ac** is more dangerous than **dc** of same voltage.
- 100V **ac means** $E_{rms} = 100V$,

  $E_0 = 100\sqrt{2}\,V$

  100V **dc is equivalent to** E_{rms}
- **ac** can be produced by the principle of electromagnetic induction.

Power in ac Circuits:

In **dc** circuits power is given by P = VI. But in **ac** circuits, since there is some phase angle between voltage and current, therefore power is defined as the product of voltage and that component of the current which is in phase with the voltage.

Thus $\mathbf{P = EI}\cos\phi$, where E and I are r.m.s. values of voltage and current.

Power factor:

The quantity $\cos\phi$ is called power factor.

a) In stantaneous power :

Suppose in a circuit $E = E_0 \sin \omega t$

and $I = I_0 \sin(\omega t + \phi)$

then $P_{\text{instantaneous}} = EI = E_0 I_0 \sin \omega t \sin(\omega t + \phi)$

b) Average power (True power) :

The average of instantaneous power in an **ac** circuit over a full cycle is called average power. Its unit is watt i.e.

$$P_{avg} = \frac{W}{t} = \frac{\int_0^T P.dt}{\int_0^T dt} = \frac{\int_0^T P.dt}{T} ; \; W = \int_0^T P.dt$$

$$W = E_0 I_0 \cos\phi \int_0^T \sin^2 \omega t dt + \frac{E_0 I_0}{2} \sin\phi \int_0^T \sin 2\omega t dt$$

$$W = E_0 I_0 \cos\phi X \frac{T}{2}$$

Average power over complete cycle,

$$P_{avg} = \frac{W}{T} = \frac{E_0 I_0}{2} \cos\phi$$

$$= \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\phi = E_{rms} I_{rms} \cos\phi$$

c) Apparent or virtual power :

The product of apparent voltage and apparent current in an electric circuit is called apparent power. This is always positive.

$$P_{app} = E_{rms} I_{rms} = \frac{E_0 I_0}{2}$$

Resistance(R)

It is the opposition offered by a conductor to the flow of direct current.

Impedance(Z)

It is the opposition offered by a conductor to the flow of alternating current.

$$Z = \frac{|\text{alternating emf}|}{|\text{alternating current}|}$$

$$= \frac{\text{peak value of alternating voltage}}{\text{peak value of AC}}$$

$$= \frac{\text{RMS value of alternating voltage}}{\text{RMS value of AC}}$$

Admittance(Y):

Reciprocal of impedance of a circuit is called admittance of the circuit.

$$\text{admittance (Y)} = \frac{1}{Z}$$

S.I. Unit:ohm^{-1} i.e. mho or siemen.

Phase:

The physical quantity which represents both the instantaneous value and direction of A.C. at any instant is called its phase.

It is dimensionless quantity and its unit is Radian

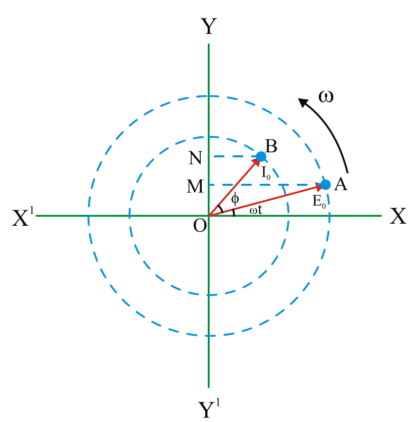
Phase Difference:

The diffrerence between the phases of current and voltage is called Phase difference.

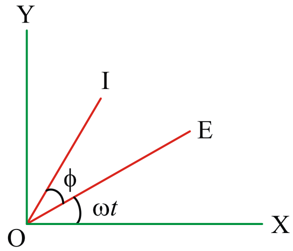
If alternating emf and current are $E = E_0 \sin(\omega t + \phi_1)$ and $i = i_0 \sin(\omega t + \phi_2)$

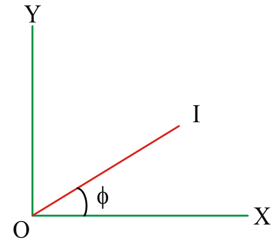
then phase difference is $\phi = \phi_1 \sim \phi_2$

- **The quantity varies sinusoidally with time and can be represented as projection of a rotating vector, is called as phasor.**
- **A diagram, representing alternating emf and current (of same frequency) as rotating vectors (Phasors) with phase angle between them is called as phasor diagram.**



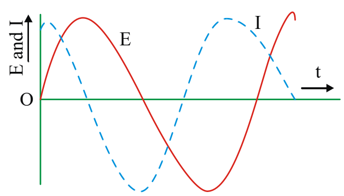
- In the above figure , $\overline{OA}$ and $\overline{OB}$ represent two rotating vectors having magnitudes E_0 and I_0 in anti clock wise direction with same angular velocity 'ω'.
- OM and ON are the projections of $\overline{OA}$ and $\overline{OB}$ on Y-axis respectively.
- OM = E and ON = I, represent the instantaneous values of alternating emf and current.
- $\underline{|BOA} = \phi$ represents the phase angle by which current I_0 leads the alternating emf E_0.
- The phasor diagram, in a simple representation is





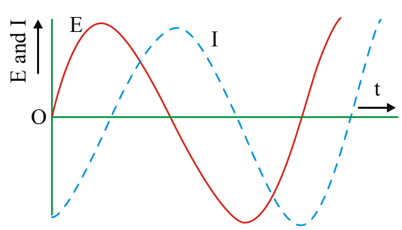
Note:

If e.m.f (or voltage) in A.C. is $E = E_0 \sin \omega t$ and the current $I = I_0 \sin(\omega t + \varphi)$ Where phase difference φ is Positive if current leads, Negative if current lags and zero if current is inphase with the emf (or voltage).



- instantaneous emf is $E = E_0 \sin \omega t$
- instantaneous current $I = I_0 \sin(\omega t + \phi)$

where $\left[\phi = \frac{\pi}{2}\right]$; Current leads emf by $\frac{\pi}{2}$



$E = E_0 \sin \omega t$; $I = I_0 \sin(\omega t - \phi)$ where $\phi = \frac{\pi}{2}$

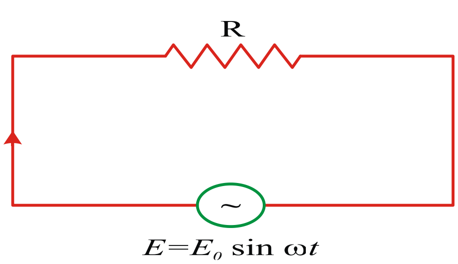
- Current lags emf by $\pi/2$

or

emf leads current by $\pi/2$

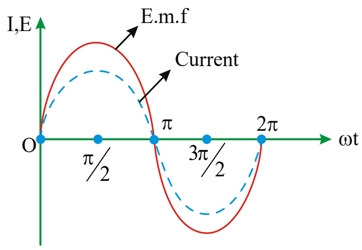
A.C Through a resistor

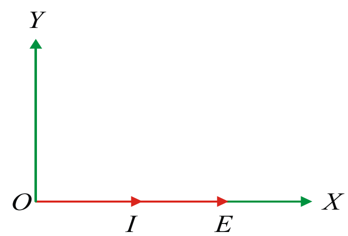
A pure resistor of resistance R is connected across an alternating source of emf



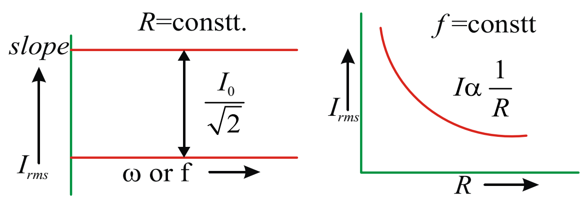
- The instantaneous value of alternating emf is $E = E_0 \sin \omega t$
- The instantaneous value of alternating current is $I = \frac{E}{R} = \frac{E_0}{R}(\sin \omega t) = I_0 \sin \omega t$
- Peak value of current, $I_0 = \frac{E_0}{R}$

Phasor diagrams:





- emf and current will be in phase $(\Delta\phi = 0°)$
- emf and current have same frequency
- Peak emf is more than peak current
- The value of impedance (Z) is equal to R and reactance (X) is zero
- Apart from instantaneous value, current in the circuit is independent of frequency and decreases with increase in R (similar to that in dc circuits).



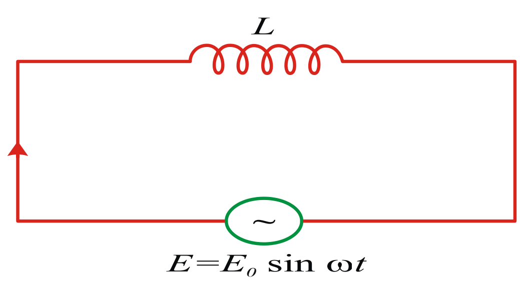
Power

- power factor $\cos\phi = \cos 0° = 1$
- Instantaneous power $P_i = E_o I_o \sin^2 \omega t$
- Average power over time $'T'$ sec =

$$P_{avg} = E_{rms} I_{rms} \cos\phi = E_{rms} . I_{rms} = \frac{E_{rms}^2}{R}$$

A.C Through an inductor

A pure inductor of inductance L is connected across an alternating source of emf E



- The instantaneous value of alternating emf is $E = E_0 \sin \omega t$ ....(1)

- The induced emf across the inductor $= -L.\frac{dI}{dt}$

which opposes the growth of current in the circuit. As there is no potential drop across the circuit, so

$$E + \left(-L.\frac{dI}{dt}\right) = 0 \quad \text{or} \quad L.\frac{dI}{dt} = E$$

$$\frac{dI}{dt} = \frac{E_0}{L}\sin \omega t \text{ ; On integrating}$$

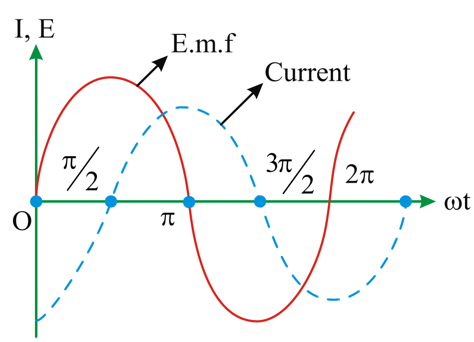
$$I = -\frac{E_0}{L\omega}\cos \omega t = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \quad ....(2)$$

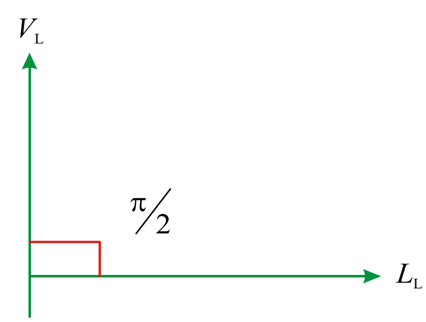
- The instantaneous value of alternating current is $\Rightarrow I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

Where Peak value of current, $I_0 = \frac{E_0}{\omega L} = \frac{E_0}{X_L}$

- From equation 1 & 2 **Phase difference between alternating voltage and current is** $\frac{\pi}{2}$
- **The alternating current lags behind the emf** by a phase angle of $\frac{\pi}{2}$

Phasor diagram





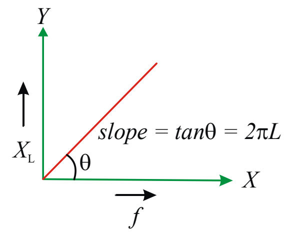
Inductive Reactance (X_l)

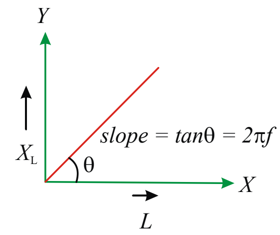
- The opposition offered by an inductor to the flow of **ac** is called an inductive reactance.
- The quantity ωL is analogous to resistance and is called reactance of Inductor represented by X_L.
- It allows D.C. but offers finite impedance to the flow of A.C.
- Its value depends on L and f.
- Inductance not only causes the current to lag behind emf but it also limits the magnitude of current in the circuit.
- $$I_0 = \frac{E_0}{\omega L} \Rightarrow \omega L = \frac{E_0}{I_0} = X_L ,$$

$$\therefore X_L = \omega L = 2\pi f L \Rightarrow X_L \alpha f ;$$

$X_L - f$ curve

$X_L - L$ curve





- For dc, $f = 0 \quad \therefore X_L = 0$
- For ac, high frequencies, $X_L = \infty$

  $\therefore$ dc can flow easily through inductor.
- Inductive reactance in terms of RMS value is $X_L = \omega L = \dfrac{E_{rms}}{I_{rms}}$

Power supplied to inductor

The instantaneous power supplied to the inductor is

$$P_L = iv = i_0 \sin\left(\omega t - \frac{\pi}{2}\right) \times v_0 \sin(\omega t)$$

$$= -i_0 v_0 \cos(\omega t)\sin(\omega t) \quad = -\frac{i_0 v_0}{2}\sin(2\omega t)$$

So, the average power over a complete cycle is

$$P_{avg} = E_{rms} \, . \, I_{rms} \cos\phi = 0 \qquad (\because \Delta\phi = 90°)$$

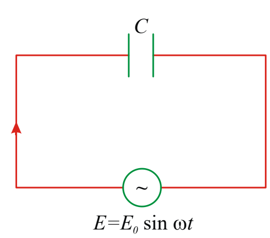
$$= -\frac{i_0 v_0}{2}\langle \sin(2\omega t)\rangle = 0$$

Since the average of $\sin(2\omega t)$ over a complete cycle is zero.

Thus, the **average power supplied to an inductor over one complete cycle is zero.**

A.C Throgh a Capacitor

- When an alternating emf is applied to a capacitor, then alternating current is constituted in the circuit. Due to this, charge on the plates and electric field between the plates of capacitor vary sinusoidally with time.
- At any instant the potential difference between the plates of a capacitor is equal to applied emf at that time.



- A capacitor of capacity C is connected across an alternating source of emf
- The instantaneous value of alternating emf is $E = E_0 \sin\omega t$ ....(1)
- Let q be the charge on the capacitor at any instant.

  Accoding to kirchhoff's loop rule

$$E - \frac{q}{C} = 0 \Rightarrow q = CE_0 \sin\omega t$$

$$q = CE = CE_0 \sin \omega t$$

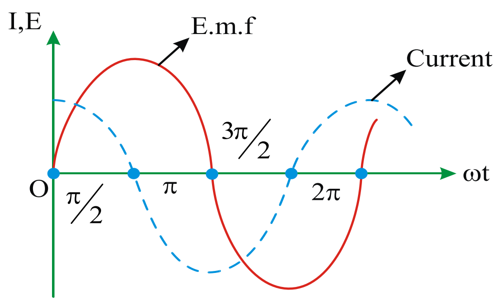
- $I = \frac{dq}{dt} = C\omega E_0 \cos\omega t = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$..............(2)

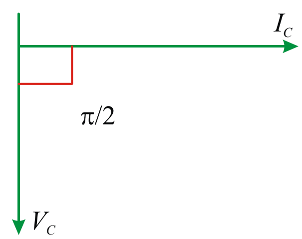
- The instantaneous value of alternating current is $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$..............(2)

where peak value of current, $I_0 = \frac{E_0}{\left(\frac{1}{\omega C}\right)}$

- From equation 1 & 2 **current leads the emf by an angle $\frac{\pi}{2}$.**

Phasor diagram



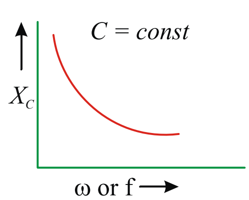


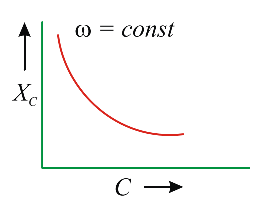
Capacitive Reactance (X_C)

- The resistance offered by a capacitor to the flow of **ac** is called capacitive reactence.
- The quantity $\frac{1}{\omega C}$ is analogous to resistance and is called reactance of capacitor represented by X_C

$$I_0 = \frac{E_0}{\left(\frac{1}{\omega C}\right)} \Rightarrow X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{E_0}{I_0} = \frac{E_{rms}}{I_{rms}}$$

- It is the part of impedance in which A.C. leads the A.V. by a phase angle of $\frac{\pi}{2}$.
- Its value is $X_c = \frac{1}{\omega C} = \frac{1}{2\pi fC}$.
- Its value depends on C and f.
- It bypasses A.C. but blocks D.C.
- It is produced due to pure capacitor or induced charge.
- $X_C - f$ curve $X_C - C$ curve





Note:

Resistance, Impedance and Reactance have the same units and Dimensional Formulae.

i.e. SI unit is ohm; Dimensional Formula is $\left(ML^2T^{-3}A^{-2}\right)$

Power supplied to capacitor:

The instantaneous power supplied to the capacitor is $P_c = iv = i_0\cos(\omega t)v_0\sin(\omega t)$

$$= i_0v_0\cos(\omega t)\sin(\omega t)\,; \quad = \frac{i_0v_0}{2}\sin(2\omega t)$$

So, the average power over a complete cycle is zero

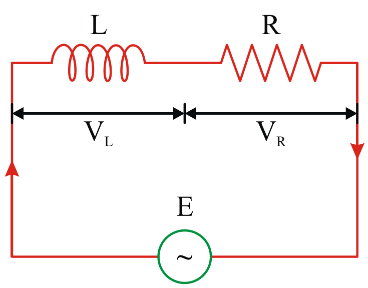
since $<\sin(2\omega t)>= 0$ over a complete cycle.

$$P_{avg} = V_{rms}.I_{rms}\cos\phi = V_{rms}.I_{rms}\cos 90° = 0$$

$\therefore$ **no power is consumed in a purely capacitive circuit.**

A.C Through LR Series Circuit

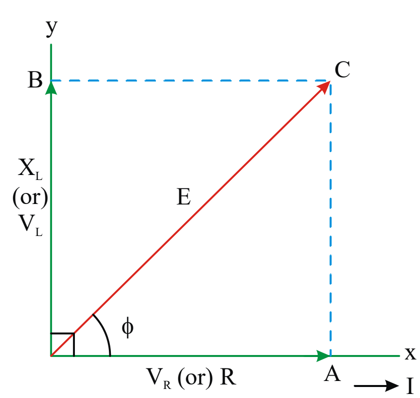
- LR circuit consists of a resistor of resistance R and an inductor of inductance L in series with a source of alternating emf
- The instantaneous value of alternating emf is $E = E_0 \sin \omega t$



- The potential difference across the inductor is given by, $V_L = IX_L$ .....(1)
- The potential difference across the resistor, $V_R = IR$ .....(2)

current I lags the Voltage V_L by an angle of $\frac{\pi}{2}$,

Therefore, the resultant of V_L and V_R is $OC = \sqrt{OA^2 + OB^2}$ or $E = \sqrt{V_R^{\,2} + V_L^{\,2}}$



Using equations (1) and (2), we get

$$E = \sqrt{I^2R^2 + I^2X_L^{\,2}} = I\sqrt{R^2 + X_L^{\,2}}$$

where $X_L = \omega L$ is the inductive reactance.

or $I = \dfrac{E}{\sqrt{R^2 + X_L^{\,2}}}$ .....(3)

$$I = \frac{E}{Z_{LR}};$$

$$Z_{LR} = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + L^2\omega^2}$$

The effective opposition offered by LR circuit to **ac** is called the **impedance** of LR circuit.

Let ϕ be the angle made by the resultant of V_L and V_R with the X-axis, then from figure, we get

$$\tan\phi = \frac{AC}{OA} = \frac{OB}{OA} = \frac{V_L}{V_R} = \frac{IX_L}{IR}$$

$$\text{or } \tan\phi = \frac{X_L}{R} = \frac{\omega L}{R}$$

Note:

In series LR circuit, emf leads the current or the current is said to lag behind the emf by an angle ϕ

$\therefore$ Current in L-R series circuit is given by $I = \dfrac{E}{Z_{LR}} = \dfrac{E_0}{Z_{LR}}\sin(\omega t - \phi)$

(or) $I = I_0 \sin(\omega t - \phi)$

Note:

$$Z_{LR} = \sqrt{R^2 + L^2\omega^2} = \sqrt{R^2 + L^2 \times 4\pi^2 f^2}\,.$$

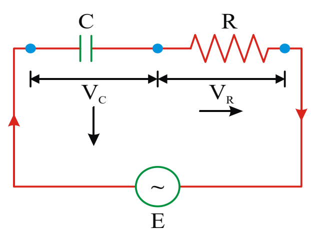
Thus Z_{LR} increases with the frequency of **ac**,

so Z_{LR} is low for lower freqeuncy of **ac** and high for higher frequency of **ac**

The phase angle between voltage and current increases with the increase in the frequency of **ac**

C-R Series Circuit with alternating Voltage

- Let an alternating source of emf $E = E_0 \sin\omega t$ is connected to a series combination of a pure capacitor of capacitance (C) and a resistor of resistance (R) as shown in figure (a)



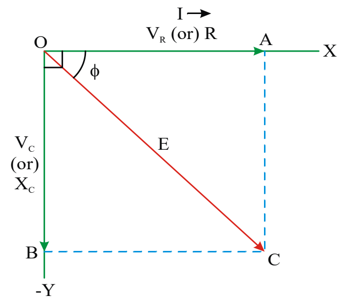
- Let I be the r.m.s value of current flowing through the circuit. The potential difference across the capacitor,

$V_C = IX_C$ **.....(i)**

- The current leads emf by an angle $\frac{\pi}{2}$ when ac flows through capacitor.

- The potential difference across the resistor, $V_R = IR$ .....(ii)
- The emf and current are in phase when ac flows through resistor.

Phasor diagram.



- In figure V_C is represented by OB along negative Y - axis and the current I is represented along X - axis.
- V_R is represented by OA along X - axis.
- The resultant potential difference of V_C and V_R is represented by OC.
- Also, the emf and current are in phase when **ac** flows through the resistor. So, V_R is represented by OA along X-axis.
- Therefore, the resultant potential difference of V_C and V_R is represented by OC and is given by

$$OC = \sqrt{OA^2 + OB^2} \quad or\ E = \sqrt{V_R^{\ 2} + V_C^{\ 2}}$$

Using equations (i) and (ii), we get

$$E = \sqrt{I^2R^2 + I^2X_C^{\ 2}} = I\sqrt{R^2 + X_C^{\ 2}}$$

$$\text{or}\quad I = \frac{E}{\sqrt{R^2 + X_C^{\ 2}}} = \frac{E}{Z_{CR}}$$

From the above equations of I and E, we have $Z_{CR} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{C\omega}\right)^2}$

Where Z_{CR} is the effective opposition offered by the CR circuit to **ac**, which is the **impedance** of CR circuit.

Let ϕ be the angle made by E with X-axis $\tan\phi = \frac{AC}{OA} = \frac{V_C}{V_R} = \frac{IX_C}{IR}$

$$\text{or}\ \tan\phi = \frac{X_C}{R} = \frac{I}{C\omega R}$$

In series CR circuit, emf lags behind the current or in other words, the current is said to lead the emf by an angle ϕ given by the above equation.

$\therefore$ Current in C-R series circuit is given by $I = \frac{E}{Z_{CR}} = \frac{E_0}{Z_{CR}}\sin(\omega t + \phi)$

(or) $I = I_0.\sin(\omega t + \phi)$

Note:

- The resultant potential difference of V_C and V_R is represented by OC Impedance of CR circuit.

$$Z_{CR} = \sqrt{R^2 + {X_C}^2} = \sqrt{R^2 + \frac{1}{C^2\omega^2}} = \sqrt{R^2 + \frac{1}{4\pi^2 f^2 C^2}}$$

$$\text{Thus } Z_{CR} \propto \frac{1}{f}$$

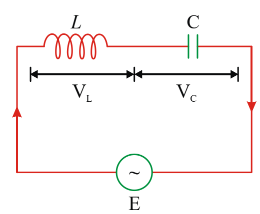
- The resultant potential difference of V_C and V_R is represented by OC For very high frequency (f) of **ac**. $Z \to R$ and for very low frequency of **ac**, $Z \to \infty$
- Phase angle between voltage and current is given by

$$\tan\phi = \frac{1}{C\omega R} = \frac{1}{2\pi fCR}$$

As f increases, phase angle ϕ decreases.

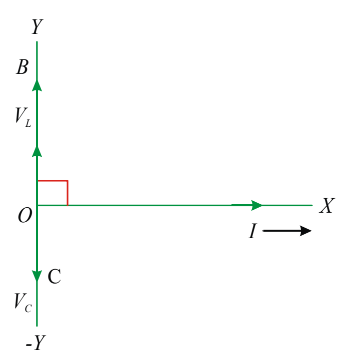
L - C Series Circuit With Alternating Voltage.

- Let an alternating source of emf $E = E_0 \sin\omega t$ is connected to the series combination of a pure capacitor of capacitance (C) and an inductor of inductance (L) is shown in fig.



- Let I be the rms value of current flowing in the circut
- The P.D across 'L' is $V_L = I.X_L$
- The current I lags V_L by an angle $\pi/2$.
- The P.D across capacitance is $V_C = I.X_C$.
- The current I leads V_C by an angle $\pi/2$.

The voltage V_L and V_C are represented by OB and OC respecitvely.



The resultant P.D of V_L and V_C is

$V = V_L \sim V_C = I(X_L \sim X_C)$ $\quad = \left[\omega L \sim \frac{1}{\omega C}\right] = IZ_{LC}$

From the above equations, **Impedance of L -C** circuit is

$$Z_{LC} = \left[(\omega L) \sim \frac{1}{\omega C}\right]$$

- If $\omega L > \frac{1}{\omega C}$ i.e, $X_L > X_C$ then $V_L > V_C$ potential difference $V = V_L - V_C$.
- Now current lags behind voltage by $\pi/2$.
- If $\omega L < \frac{1}{\omega C}$ then $V_L < V_C$ resultant potential difference $(V) = V_C - V_L$

Now current leads emf by $\pi/2$.

If $\omega L = \frac{1}{\omega C}$ then $Z = \omega L - \frac{1}{\omega C} = 0$

Current $I = \frac{E}{Z} = \alpha$

In L - C, circuit, the phase difference between voltage and current is always $\pi/2$.

Power factor $\cos\phi = \cos\pi/2 = 0$.

So, power consumed in L - C circuit is

$$P = V_{rms} \times I_{rms} \times \cos\phi = 0$$

$\therefore$ **In L - C circuit no power is consumed.**

Note:

- In L - C, circuit, the impendence $Z = \left|\omega L - \frac{1}{\omega C}\right|$

Current $I = \frac{E}{Z}$.

So, the impedence and current varies with frequency.

- At a particular angular frequency, $\omega L = \frac{1}{\omega C}$

and current $I = \frac{E}{Z}$ becomes maximum (I_0) and resonance occurs.

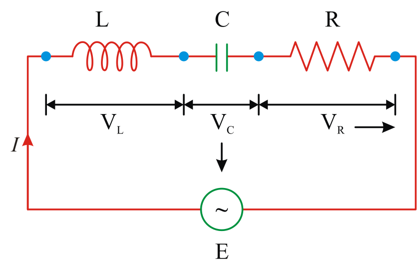
At resonance $Z = 0$ and $I_0 = \frac{E_0}{Z} = \infty$.

Resonant angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

Resonant frequecny $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

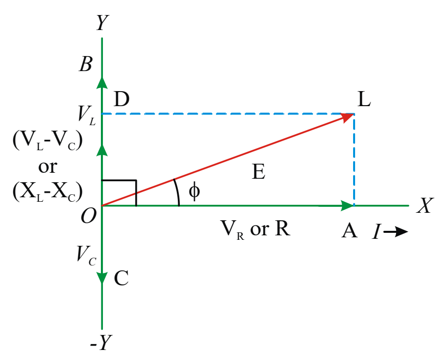
A.C Through LCR Series Circuit

- A circuit containing pure inductor of inductance (L), pure capacitor of capacitance (C) and resistor of resistance (R), all joined in series, is shown in figure.
- Let E be the r.m.s value of the applied alternating emf to the LCR circuit.



- The potential difference across L, $V_L = IX_L$ .....(i)
- The potential difference across C, $V_C = IX_C$ .....(ii)
- The potential difference across R, $V_R = IR$ .....(iii)

PHASOR DIAGRAM



- Since V_L and V_C are in opposite phase, so their resultant $(V_L - V_C)$ is represented by OD (Here $V_L > V_C$)
- The resultant of V_R and $(V_L - V_C)$ is given by OL.

The magnitude of OL is given by $OL = \sqrt{(OA)^2 + (OD)^2}$; $= \sqrt{V_R^{\,2} + (V_L - V_C)^2}$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \frac{E}{I} = \sqrt{R^2 + (X_L - X_C)^2}$$

$\therefore$ Impedance (Z) of LCR circuit is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}} \; ; \; = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}}$$

- Let ϕ be the phase angle between E and I, then from Phasor diagram

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_L - IX_C}{IR} = \frac{X_L - X_C}{R}$$

$$\tan\phi = \frac{\left(L\omega - \frac{1}{C\omega}\right)}{R}$$

$\therefore$ Current in L-C- R series circuit is given by $I = \frac{E}{Z} = \frac{E_0}{Z}\sin(\omega t \pm \phi)$

(or) $I = I_0.\sin(\omega t \pm \phi)$

- If X_L and X_C are equal then Z = R i.e., expression for pure resistance circuit.

  If $X_L = 0$ then $Z = \sqrt{R^2 + X_C{}^2}$ i.e., expression for series RC circuit.

- Similarly if $X_C = 0$ then $Z = \sqrt{R^2 + X_L{}^2}$ i.e. expression for series RL circuit.

  Also, $\cos\phi = \frac{R}{Z}$

Case (i) :

If $X_L > X_C$ then ϕ is +ve.

In this case the current lags behind the emf by a phase angle $\phi = Tan^{-1}\left(\frac{X_L - X_C}{R}\right)$

Case (ii) :

If $X_L < X_C$ then ϕ is -ve.

In this case the current leads the emf by a phase angle $\phi = Tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

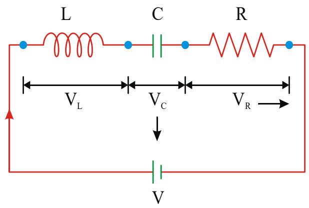
Case (iii):

If $X_L = X_C$ then ϕ is 0.

In this case the current and emf are in phase.

- If $X_L > X_C$, then the circuit will be inductive
- If $X_L < X_C$, then the circuit will be capacitive
- If $X_L = X_C$, then the circuit will be purely resistive.
- The LCR circuit can be inductive or capacitive or purely resistive depending on the value of frequency of alternating source of emf.
- At some frequency of alternating source, $X_L > X_C$ and for some other frequency, $X_L < X_C$. There exists a particular value of frequency where $X_L = X_C$ (This situation is explained under resonance of LCR series circuit)

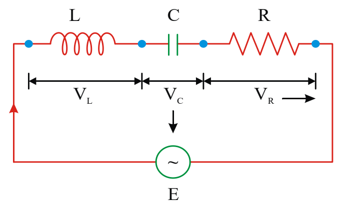
Note: Relation between applied pd & pd's across the components in L - C - R circuit



For 'dc'

$$\boxed{\mathbf{V = V_R + V_L + V_C}}$$

(only before steady state)



For 'ac'

$$V = IZ$$

$$= I\sqrt{R^2 + \left(X_L - X_C\right)^2}$$

$$= \sqrt{(IR)^2 + \left(IX_L - IX_C\right)^2}$$

$$\mathbf{V^2 = V_R{}^2 + \left(V_L - V_C\right)^2}$$

where $V_L = IX_L = I\omega L$

$$V_C = IX_C = \frac{I}{\omega C}$$

$$V_R = IR$$

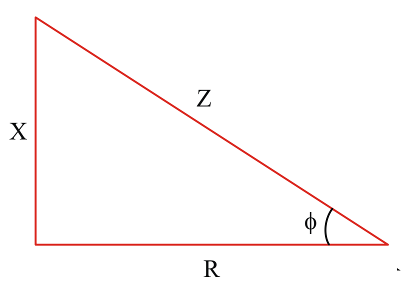
Note: Rules to be followed for various combinations of ac circuits

- Compute effective resistance of the circuit as R
- Calculate the net reactance of the circuit as $X = X_L - X_C$ where $X_L = \omega L$, $X_C = \dfrac{1}{\omega C}$.
- Resistance offered by all the circuited elements to the flow of **ac** is impedance (Z)

$$\therefore\ Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

- Calculate the peak value of current as $I_0 = \dfrac{E_0}{Z}$

The phase difference between emf & current can be known by constructing an **ac** triangle as



$$\tan\phi = \frac{X}{R}$$

$$\sin\phi = \frac{X}{Z}$$

$$\cos\phi = \frac{R}{Z}$$

Resonant Frequency

Electrical Resonance Series L-C-R Circuit

Electrical resonance is said to take place in a series LCR circuit, when the circuit allows maximum current for a given frequency of alternating supply, at which capacitive reactance becomes equal to the inductive reactance.

The current (I) in a series LCR circuit is given by

$$I = \frac{E}{Z} = \frac{E}{\sqrt{R^2 + \left(L\omega - \frac{1}{C\omega}\right)^2}} \quad .....(i)$$

From the above equation (i), it is clear that current I will be maximum if the impedance (Z) of the circuit is minimum.

At low frequencies, $L\omega = L \times 2\pi f$ is very small and $\frac{1}{C\omega} = \frac{1}{C \times 2\pi f}$ is very large.

At high frequencies, $L\omega$ is very large and $\frac{1}{C\omega}$ is very small.

For a particular frequency (f_0), $L\omega = \frac{1}{C\omega}$ i.e. $X_L = X_C$ and the impedance (Z) of LCR circuit is minimum and is given by Z = R.

Therefore, at the particular frequency (f_0), the current in LCR circuit becomes maximum. The frequency (f_0) is known as the **resonant frequency** and the phenomenon is called electrical resonance.

Again, for electrical resonance $(X_L - X_c) = 0$.

i.e. $X_L = X_C$

or $L\omega = \frac{1}{C\omega} \Rightarrow \omega^2 = \frac{1}{LC}$

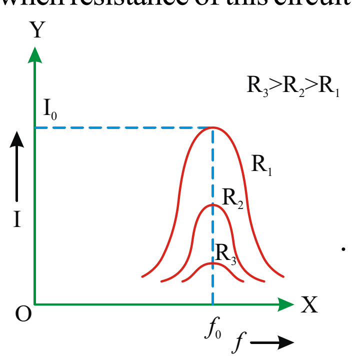
or $\omega = \frac{1}{\sqrt{LC}} \Rightarrow (2\pi f_0) = \frac{1}{\sqrt{LC}}$

or $f_0 = \frac{1}{2\pi\sqrt{LC}}$ .....(ii)

This is the value of resonant frequency.

The resonant frequency is independent of the resistanace R in the circuit. However, the sharpness of resonance decreases with the increase in R.

Series LCR circuit is more selective when resistance of this circuit is small.



Note: Series LCR circuit at resonance admit maximum current at particular frequencies, so they can be used to tune the desired frequency or filter unwanted frequencies. They are used in transmitters and receivers of radio, television and telephone carrier equipment etc.

Resonance in L -C Circuit:

At resonance ,

a) Net reactance X = 0

b) $X_L = X_C$

c) Impedance Z = 0

d) peak value of current $I_0 = \frac{E_0}{Z} = \infty$

e) Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

f) Voltage and current differ in phase by $\frac{\pi}{2}$

g) Power factor $\cos\phi = 0$

Resonance in L-C -R Circuit:

At resonance,

a) Net reactance X = 0

b) $X_L = X_C$

c) Impedance Z = R (minimum)

d) peak value of current $I_0 = \frac{E_0}{Z} = \frac{E_0}{R}$ (maximum but not infinity)

e) Resonant frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

f) Voltage and current will be in phase

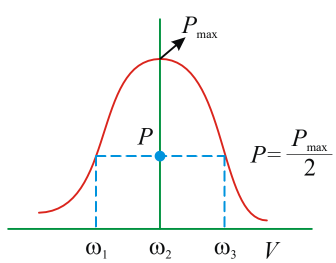
g) power factor $\cos\phi = 1$

h) Resonant frequency is independent of value of R.

i) **A series L - C - R circuit behaves like a pure resistive circuit at resonance.**

Half power frequencies and band width.

- The frequencies at which the power in the circuit is half of the maximum power (The power at resonance) are called half power frequencies.



- The current in the circuit at half power frequecies (HPF) is $1\sqrt{2}$ or 0.707 or 70.7% of maximum current (current at resonance).
- There are two half power frequencies

$\omega_1 \rightarrow$ called lower half power frequency. At this frequency the circuit is capacitive.

$\omega_3 \rightarrow$ called upper half power frequency. It is greater than ω_2. At this frequency the circuit is inducitve.

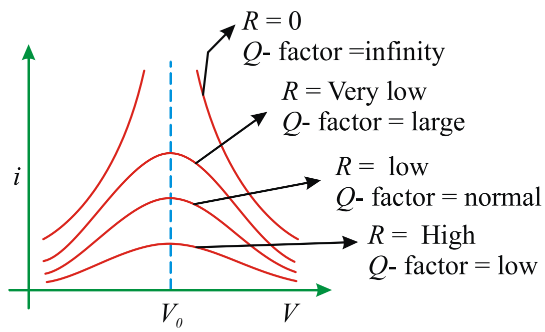
Band width $(\Delta\omega)$:

The difference of half power frequencies ω_1 and ω_2 is called band width $(\Delta\omega)$ and $\Delta\omega = \omega_3 - \omega_1$.

For series resonant circuit it can be proved $(\Delta\omega = R/L)$

Quality factor (A - Factor) of Series Resonant Circuit.

- The characteristic of a series resonant circuit is determined by the quality factor $(Q-\text{factor})$ of the circuit.
- It defines sharpness of $i-v$ curve at resonance when $Q-$factor is large, the sharpness of resonance curve is more and vice - versa.



Resonance curve

- $Q-$factor also defined as follows

$$Q-\text{factor} = 2\pi \times \frac{\text{Maximum energy stored}}{\text{energy dissipation}}$$

$$= \frac{2\pi}{T} \times \frac{\text{Maximum energy stored}}{\text{Mean power dissipated}}$$

$$= \frac{\text{Resonant frequency}}{\text{Band width}} = \frac{\omega_0}{\Delta\omega}$$

$$Q-\text{factor} = \frac{V_L}{V_R} \text{ or } \frac{V_C}{V_R} = \frac{\omega_0 L}{R} \text{ or } \frac{1}{\omega_0 CR}$$

$$\Rightarrow Q-\text{factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Wattless Current:

In an ac circuit, $R = 0 \Rightarrow \cos\phi = 0$

so $P_{av} = 0$,

i.e.., **in resistanceless circuit the power consumed is zero, Such a circuit is called the wattless circuit and the current flowing is called the wattless current.**

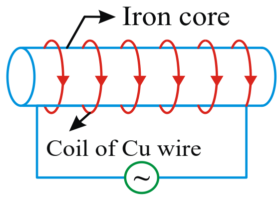
Or

The component of current which does not contribute to the average power dissipation is called wattless current.

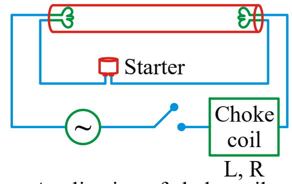
wattless current $= I_{rms} \sin\phi$

Choke Coil:

- Choke coil (or ballast) is a device having high inductance and negligible resistance.
- It is used to control current in ac circuits and is used in fluorescent tubes.
- The power loss in a circuit containing choke coil is least.
- In a dc circuit current is reduced by means of a rheostat.This resutls in a loss of electrical energy I^2R per sec.



Choke coil



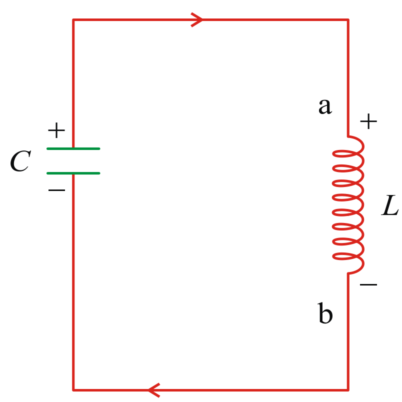
Application of choke coil

- It consists of a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced.
- Soft iron is used to improve inductance (L) of the circuit.
- The inductive reactance or effective opposition of the choke coil is given by $X_L = \omega L = 2\pi \nu L$
- For an ideal choke coil $r = 0$, no electric cnergy is wasted, i.e., average power P = 0.
- In actual practice choke coil is equivalent to a $R - L$ circuit.
- Choke coil for different frequencies are made by using different substances in their core.
- For low frequency L should be large thus iron core choke coil is used. For high frequency ac circuit, L Should be small, so air cored choke coil is used.
- The choke coil can be used only in ac circuits not in dc circuits, because for dc frequency $\nu = 0$. Hence $X_L = 2\pi \nu L = 0$.
- Choke coil is based on the principle of wattless current.
- The current in the circuit $I = \dfrac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$.
- The power loss in the choke $p_{av} = V_{rms} I_{rms} \cos\phi \to 0$

  as $\cos\phi = \dfrac{r}{Z} = \dfrac{r}{\sqrt{r^2 + \omega^2 L^2}} = \dfrac{r}{\omega L} \to 0$

LC OSCILLATIONS

A capacitor (C) and an inductor (L) are connected as shown in the figure. Initially the charge on the capacitor is Q



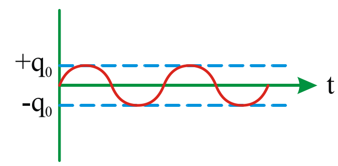
$\therefore$ Energy stored in the capacitor $U_E = \dfrac{Q^2}{2C}$

The energy stored in the inductor, $U_B = 0$.

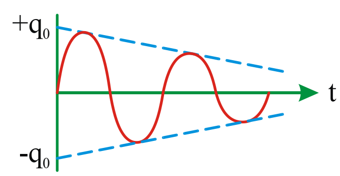
The capacitor now begins to discharge through the inductor and current begins to flow in the circuit. As the charge on the capacitor decreases, U_E decreases but the energy $U_B = \frac{1}{2}LI^2$ in the magnetic field of the inductor increases. Energy is thus transferred from capacitor to inductor. When the whole of the charge on the capacitor disappears, the total energy stored in the electric field in the capacitor gets converted into magnetic field energy in the inductor. At this stage, there is maximum current in the inductor.

Energy now flows from inductor to the capacitor except that the capacitor is charged oppositely. This process of energy transfer continues at a definite frequency (v). Energy is continuosly shuttled back and forth between the electric field in the capacitor and the magnetic field in the inductor.

If no resistance is present in the LC circuit, the LC oscillation will continue infinitely as shown.



However in an actual LC circuit, some resistance is always present due to which energy is dissipated in the form of heat. So LC oscillation will not continue infinitely with same amplitude as shown.



Let q be the charge on the capacitor at any time t and $\frac{di}{dt}$ be the rate of change of current. Since no battery is connected in the circuit,

$$\frac{q}{c} - L.\frac{di}{dt} = 0 \quad \text{but } i = -\frac{dq}{dt}$$

from the above equations, we get

$$\frac{q}{C} + L\frac{d^2q}{dt^2} = 0 \Rightarrow \frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$$

The above equation is analogus to $\frac{d^2x}{dt^2} + \omega^2 x = 0$ (differential equation of S.H.M)

Hence on comparing $\omega^2 = \frac{1}{LC} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$

$$2\pi f = \frac{1}{\sqrt{LC}} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

The charge therefore oscillates with a frequency

$f = \frac{1}{2\pi\sqrt{LC}}$ and varies sinusoidally with time.

Comparison of L-C Oscilations with SHM:

The L - C oscillations can be compared to S.H.M of a block attached to a spring

* In L - C oscillations $\omega_0 = \frac{1}{\sqrt{LC}}$
* In Mechanical oscillations $\omega_0 = \sqrt{\frac{K}{m}}$ where K is the spring constant
* In L - C oscilations $\frac{1}{C}\left(=\frac{V}{q}\right)$ tells us the potential difference required to store a unit charge
* In a mechanical oscillation $K\left(=\frac{F}{x}\right)$ tells us the external force requred to produce a unit displacement of mass
* In L - C oscillations current is the analogous quantity for velocity of the mass in mechanical oscillations
* In L - C oscillations energy stored in capacitor is analagous to potential energy in mechanical oscillations
* In L - C oscillations energy stored in inductor is analogous to kinetic energy of the mass in mechanical oscillations
* In L - C oscillations maximum charge on capacitor q_0 is analogous to amplitude in mechanical oscillations
* $\therefore$ As $V_{max} = A\omega$ in mechanical oscillations,
  $I_0 = q_0\omega_0$ in L- C oscillations

Analogies between Mechanical and Electrical Quantities	
Mechanical System	Electrical System
Mass m	Inductance L
Force constant k	Reciprocal capacitance 1/C
Displacement x	Charge q
Velocity v = dx/dt	Current I = dq/dt
Mechnical energy	Electromagnetic energy

Energy of LC Oscillations:

Let q_0 be the initial charge on a capacitor. Let the charged capacitor be connected to an inductor of inductance L. LC ciruit will sustain an oscillations with frequency $(\omega = 2\pi f = \frac{1}{\sqrt{LC}})$ At an instant t, charge q on the capacitor and the current i are given by; $q(t) = q_0 \cos \omega t$; $i = -q_0\omega \sin \omega t$

Energy stored in the capacitor at time t is

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{q^2}{C} = \frac{q_0^2}{2C}\cos^2(\omega t)$$

Energy stored in the inductor at time t is $U_M = \frac{1}{2}Li^2$

$$= \frac{1}{2}Lq_0^2\omega^2 \sin^2(\omega t) = \frac{q_0^2}{2C}\sin^2(\omega t)(\because \omega^2 = \frac{1}{\sqrt{LC}})$$

Sum of energies

$$U_E + U_M = \frac{q_0^2}{2C}(\cos^2 \omega t + \sin^2 \omega t) = \frac{q_0^2}{2C}$$

As q_0 and C, both are time independent, this sum of energies stored in capacitor and inductor is constant in time. Note that it is equal to the initial energy of the capacitor.

Transformer

- A transformer works on the principle of mutual induction.
- It is a static device that is used to increase or decrease the voltage in an AC circuit.
- On a laminated iron core two insulated copper coils called primary and secondary are wound.
- Primary is connected to an alternating source of emf, By mutual induction, an emf is induced in the secondary.

Voltage Ratio:

- If V_1 and V_2 are the primary and secondary voltages in a transformer, N_1 and N_2 are the number of turns in the primary and secondary coils of the transformer, then $\frac{V_1}{V_2} = \frac{N_1}{N_2}$.
- In a transformer the voltage per turn is the same in primary and secondary coils.
- The ratio N_2/N_1 is called transformation ratio.
- The voltage ratio is the same as the ratio of the number of turns on the two coils.

Current Ratio:

- If the primary and secondary currents are I_1 and I_2 respectively, then for ideal transformer $\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$.
- In an ideal transformer the ampere turns are the same in primary and secondary coils.
- If $N_s > N_P$ voltage is stepped up, then the transformer is called step - up transformer.
- If $N_s < N_P$ voltage is stepped down, then the transformer is called step - down transformer.
- In step - up transformer, $V_S > V_P$ and $I_S < I_P$
- In step - down transformer, $V_S < V_P$ and $I_S > I_P$
- Frequency of input a.c is equal to frequency of output a.c
- Transformation of voltage, is not possible with d.c

Efficiency of transformer (η)

Effeiciency is defined as the ratio of output power and input power.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}}$$

$$\text{i.e., } \eta\% = \frac{P_{out}}{P_{in}} \times 100 = \frac{V_s i_s}{V_P i_P} \times 100$$

- For an ideal transformer $P_{out} = P_{in}$ so $\eta = 100\%$ (But efficiency of practical transformer lies between 70% - 90 %)

For practical transformer $P_{in} = P_{out} + P_{losses}$

$$\text{So } \eta = \frac{P_{out}}{P_{in}} \times 100$$

$$= \frac{P_{out}}{(P_{out} + P_L)} \times 100 = \frac{(P_{in} - P_L)}{P_{in}} \times 100$$

- In an ideal transformer the input power is equal to the output power. $V_1 I_1 = V_2 I_2$
  The efficiency of an ideal transformer is 100%.

Losses in a Tranformer:

- The losses in a transformer are divided in to two types. They are copper losses and iron losses.
- The loss of energy that occurs in the copper coils of the transformer (i.e. primary and secondary coils) is called 'copper losses'. These are nothing but joule heating losses where electrical energy is converted in to heat energy.
  The loss of energy that occurs in the iron core of the transformer (i.e. hysteresis loss and eddy current loss) is called 'iron losses'.

Minimizing the Losses in a Transformer:

- The core of a transformer is laminated and each lamination is coated with a paint of insulation to reduce the 'eddy current' losses.
- By choosing a material with narrow 'hysteresis loop' for the core, the hysteresis losses are minimized.

Uses of transformer:

- A transformer is used in almost all ac operations, e.g
- In voltage regulators for TV, refrigerator, computer, air conditioner etc.
- In the induction furnaces.
- Step down transformer is used for welding purposes.
- In the transmission of ac over long distnace.
- Step down and step up transformers are used in electical power distribution.
- Audio Frequency transformers are used in radiography, television, radio, telephone etc.
- Radio frequency transformers are used in radio communication.

Skin Effect::

- A direct current flows uniformly throughout the cross section of the conductor.
- An alternaitng current, on the other hand, flows mainly along the surface of the conductor. This effect is known as skin effect.
- When alternating current flows through a conductor, the flux changes in the inner part of the conductor are higher.
- Therefore, the inductance of the inner part is higher than that of the outer part. Higher the frequency of alternating current, more is the skin effect.
- The depth upto which ac current flows through a wire is called skin depth (δ).

$$R = \frac{V_R^2}{P_R} \Rightarrow R \alpha V_R^2$$

$(V_R = \text{rated voltage, } P_R = \text{rated power})$

PROBLEMS

1. You have two copper cables of equal length for carrying current. One of them has a single wire of area of across section A, the other has ten wires each of cross section area A/10. Judge their suitability for transporting ac and dc.

SOLUTION:

For transporting d.c.., both the wires are equally suitable, but for transporting a.c., we prefer wire of multiple strands.**ac** is transmitted more by the surface of the conductor. This is called skin effect .Due to this

2. If the voltage in an ac circuit is represented by the equation.

$V = 220\sqrt{2}\sin(314t-\phi)$ **volt calculate (a) peak and rms value of the voltage, (b) average voltage, (c) frequency of ac.**

SOLUTION:

(a) As in case of ac,

$V = V_0 \sin(\omega t - \phi)$; The peak value

$V_0 = 220\sqrt{2} = 311V$ and as in case of ac.

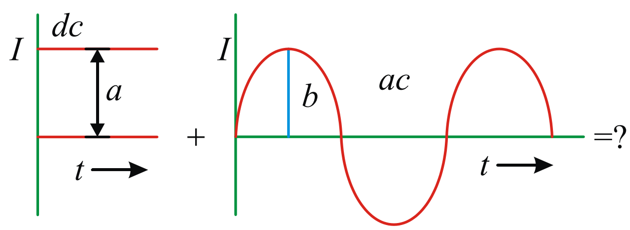
$V_{rms} = \frac{V_0}{\sqrt{2}}; V_{rms} = 220V$; (b) In case of ac

$$V_{avg} = \frac{2}{\pi}V_0 = \frac{2}{\pi}\times 311 = 198.17V$$

(c) As $\omega = 2\pi f, 2\pi f = 314$

i.e, $f = \frac{314}{2\times\pi} = 50Hz$

3. If a direct current of value a ampere is superimposed on an alternating current $I = b\sin\omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



SOLUTION:

As current at any instant in the circuit will be, $I = I_{dc} + I_{ac} = a + b\sin\omega t$

$$\text{So, } I_{eff} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt}\right]^{\frac{1}{2}} = \left[\frac{1}{T}\int_0^T (a + b\sin\omega t)^2\, dt\right]^{\frac{1}{2}}$$

$$\text{i.e, } I_{eff} = \left[\frac{1}{T}\int_0^T \left(a^2 + 2b\sin\omega t + b^2 + \sin^2\omega t\right)dt\right]^{\frac{1}{2}}$$

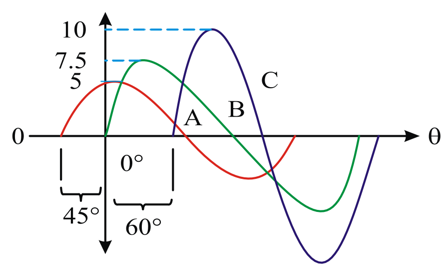
But as

$$\frac{1}{T}\int_0^T \sin\omega t\ dt = 0 \quad \text{and} \frac{1}{T}\int_0^T \sin^2\omega t\, dt = \frac{1}{2}$$

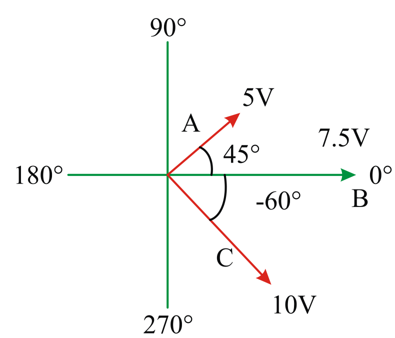
$$\text{So, } I_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$$

4. Use a phasor diagram to represent the sine waves in the following Figure.

SOLUTION:



The phasor diagram representing the sine waves is shown in figure.The length of each phasor represents the peak value of the sine wave.



5. An alternating voltage $E = 200\sqrt{2}\sin(100t)$ volt is connected to a $1\mu F$ capacitor through an ac ammeter. What will be the reading of the ammeter?

SOLUTION:

$$\text{Comparing } E = 200\sqrt{2}\sin(100t) \text{with}$$

$$E = E_0\sin\omega t\,;\ E_0 = 200\sqrt{2}V \text{ and } \omega = 100(rad/s)$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\times10^{-6}} = 10^4\Omega$$

$$I_{rms} = \frac{E_{rms}}{Z} = \frac{E_0}{\sqrt{2}X_C} = \frac{200\sqrt{2}}{\sqrt{2}\times10^4} = 20\,mA$$

6. A 0.21 H inductor and a 12 ohm resistance are connected in series to a 220 V. 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage

SOLUTION: .

Here

$$X_L = \omega L = 2\pi fL = 2\pi \times 50 \times 0.21 = 21\pi\,\Omega$$

$$Z = \sqrt{R^2 + X_L{}^2} = \sqrt{12^2 + (21\pi)^2} = \sqrt{144 + 4348}$$

$$Z = \sqrt{4492} \simeq 67.01\Omega \;\; ; \;\; I = \frac{V}{Z} = \frac{220}{67.02} = 3.28A$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{21\pi}{12}\right)$$

The current will lag the applied voltage by an angle $\tan^{-1}\left(\frac{21\pi}{12}\right)$.

7. A 10µF capacitor is in series with a 50Ω resistance and the combination is connected to a 220V, 50 Hz line. Calculate (i) the capacitive reactance, (ii) the impedance of the circuit and (iii) the current in the circuit.

SOLUTION:

$$\text{Here, } C = 10\mu F = 10 \times 10^{-6} = 10^{-5} F$$

$$R = 50 \text{ ohm}, E_{rms} = 220V, \nu = 50Hz,$$

(i) Capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 10^{-5}} = 318.5\ \Omega$$

(ii) Impedance of CR circuit.

$$Z_{CR} = \sqrt{R^2 + X_C{}^2} = \sqrt{(50)^2 + (318.5)^2} = 322.4\Omega$$

$$\text{(iii) Current, } I_{rms} = \frac{E_{rms}}{Z_{CR}} = \frac{220}{322.4} = 0.68A$$

8. A coil has an inductance of 0.7 H and is joined in series with a resistance of 220 Ω. When an alternating e.m.f of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is

SOLUTION:

$$\tan\phi = \frac{X_L}{R} = \frac{\omega L}{R} = \frac{2\pi \times 50 \times 0.7}{220} = 1$$

$$\therefore \phi = 45°, Z = \sqrt{R^2 + X_L^2} = \sqrt{220^2 + 220^2}$$

$$= 220\sqrt{2}\Omega$$

$$\text{Wattless component of curnet} = I_v \sin\phi$$

$$= \frac{E_v}{Z}\sin 45° = \frac{220}{220\sqrt{2}} \times \frac{1}{\sqrt{2}} = 0.5A$$

9. In a circuit L, C and R are connected in series with an alternating voltage source of frequency f. The current leads the voltage by $45°$. The value of C is :

SOLUTION:

As current leads the voltage by $45°$,

$$\therefore \tan\theta = \frac{X_C - X_L}{R} = \tan 45° = 1$$

$$\therefore X_C - X_L = R \text{ or } X_C = X_L + R$$

$$or \ \frac{1}{\omega C} = \omega L + R \Rightarrow C = \frac{1}{\omega(\omega L + R)}$$

$$C = \frac{1}{2\pi f(2\pi fL + R)}$$

10. In an A.C circuit the instantaneous values of current and voltage are $I = 120 \sin \omega t$ ampere and $E = 300\sin(\omega t + \pi/3)$ volt respectively. What will be the inductive reactance of series LCR circuit if the resistance and capacitive reactance are 2 ohm and 1 ohm respectively?

1) 4.5 ohms 2) 2 ohms 3) 2.5 ohms 4) 3 ohms

SOLUTION:

$$I = 120\sin\omega t, E = 300\sin(\omega t + \pi/3)$$

Clearly, $\phi = \pi/3$,

Now, $\cos\phi = \frac{R}{Z} = \cos 60° = \frac{1}{2} \therefore Z = 2R$

As $R = 2\Omega, \therefore Z = 2 \times 2 = 4\Omega; X_C = 1\Omega$

Now $(X_L - X_C)^2 = Z^2 - R^2 = 4^2 - 2^2 = 12$

$$X_L - X_C = \pm\sqrt{12} = \pm 2\sqrt{3}$$

$$X_L = X_C \pm 2\sqrt{3} = 1 \pm 3.464$$

Taking + value, $X_L = 1 + 3.464 = 4.465\,\Omega$

11. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10V each. If the capacitance is short circuited then the voltage across the inductance will be

SOLUTION:

As $V_R = V_L = V_C$; $R = X_L = X_C$

$Z = R$; $V = IR = 10$ volt

When capacitor is short circuited,

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + R^2} = R\sqrt{2}$$

New current, $I' = V/Z = \frac{V}{R\sqrt{2}} = \frac{10}{R\sqrt{2}}$

Potential drop across inductance

$$= I'X_L = I'R = \frac{10 \times R}{R\sqrt{2}} = \frac{10}{\sqrt{2}} \text{ volt}$$

12. An inductance of $\frac{200}{\pi} mH$. a capacitance of $\frac{10^{-3}}{\pi} F$ and a resistance of 10Ω are connected in series with an AC source of 220 V, 50 Hz. The phase angle of the circuit is

SOLUTION:

$$\text{Here, } L = \frac{200}{\pi} mH = \frac{200 \times 10^{-3}}{\pi} H = \frac{0.2}{\pi} H$$

$$C = \frac{10^{-3}}{\pi} F, R = 10\Omega \; ; E_v = 220V, n = 50Hz$$

$$X_L = \omega L = 2\pi n L = 2\pi \times 50 \times \frac{0.2}{\pi} = 20\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi n C} = \frac{\pi}{2\pi \times 50 \times 10^{-3}} = 10\Omega$$

$$\tan\phi = \frac{(X_L - X_C)}{R} = \frac{20-10}{10} = 1 \; ; \; \phi = \frac{\pi}{4}$$

13. In a series LCR circuit, $R = 200\Omega$, thevoltage and the frequency of the main supply is 220 V and 50 Hz. respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by $30°$.On taking out the inductor from the circuit, the current leads the voltage by 30^0. The power dissipated in the LCR circuit is

SOLUTION:

$$\text{Here, } R = 200\Omega, E_v = 220V$$

$$\text{In L-R circuit, } \tan 30° = \frac{X_L}{R}$$

$$\text{In C - R circuit, } \tan 30° = \frac{X_C}{R}$$

$$\therefore \frac{X_L}{R} = \frac{X_C}{R} \text{ or } X_L = X_C$$

In L - C - R circuit, if θ is the phase difference between voltage and current, then

$$\tan\theta = \frac{X_L - X_C}{R} = \frac{0}{200} = 0 \Rightarrow \theta = 0°$$

i.e., current and voltage are in the same phase.

$$\therefore \text{Average power} = E_v \times I_v \cos\theta = \frac{E_v^2}{R} (\because \theta = 0)$$

$$= \frac{(220)^2}{200} = 242W$$

14. An LCR circuit has L = 10 mH. R = 3 ohm and $C = 1\mu F$ connected in series to a source of 15 cos ωt volt. What is average power dissipated per cycle at a frequecny that is 10% lower than the resonant requency?

SOLUTION:

$$\text{Here, } L = 10^{-2} H, R = 3\Omega, C = 10^{-6} F$$

Resonant frequency,

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \times 10^{-6}}} = 10^4 \text{ rad/s}$$

$$\text{Actual frequency, } \omega = (90\%)\,\omega_0$$

$$= 9 \times 10^3 \text{ rad/s}$$

$$X_L = \omega L = 9 \times 10^3 \times 10^{-2} = 90\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{9 \times 10^3 \times 10^{-6}} = \frac{1000}{9}\Omega$$

$$Z = \sqrt{R^2 + (X_C - X_L)^2} = \sqrt{3^2 + \left(\frac{100}{9} - 90\right)^2} = 21.3\Omega$$

$$\text{Power dissipated / cycle} = E_v I_v \cos\phi$$

$$= E_0 \left(\frac{E_v}{Z}\right)\frac{R}{Z} = \left(\frac{E_v}{Z}\right)^2 \times R$$

$$= \left(\frac{15}{\sqrt{2} \times 21.3}\right)^3 \times 3 = 0.744\,W$$

15. A current is made of two components a dc component $i_1 = 3$A and an ac component $i_2 = 4\sqrt{2}\sin\omega t$. Find the reading of hot wire ammeter?

SOLUTION:

$$i = i_1 + i_2 = 3 + 4\sqrt{2}\sin\omega t$$

$$i_{rms}^2 = \frac{\int_0^T i^2 dt}{\int_0^T dt} = \frac{\int_0^T \left(3 + 4\sqrt{2}\sin\omega t\right)^2 dt}{T}$$

$$i_{rms}^2 = \frac{1}{T}\int_0^T \left(9 + 24\sqrt{2}\sin\omega t + 32\sin^2\omega t\right)dt$$

$$\therefore i_{rms} = 5\text{A}$$

16. A 750 Hertz - 20 volt source is connected to a resistance of 100 ohm, an inductance of 0.1803 henry and a cpacitance of $10\mu F$, all in series. What is the time in which the resistance (Thermal capacity $= 2$ joule/°C) will get heated by $10°C$?

SOLUTION:

$$\text{Here, } v = 750Hz,\ E_v = 20V,\ R = 100\Omega$$

$$L = 0.1803H, C = 10\mu F = 10^{-5}F, t = ?$$

$$\Delta\theta = 10°C, \text{thermal capacity} = 2\text{J/°C}$$

$$X_L = \omega L = 2\pi vL = 2\times 3.14\times 750\times 0.1803$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times 750\times 10^{-5}} = 21.2\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{100^2 + (850 - 21.2)^2} = 835\Omega$$

$$\text{Power dissipated} = E_v I_v \cos\phi$$

$$= E_v\left(\frac{E_v}{Z}\right)\left(\frac{R}{Z}\right) = \frac{20^2\times 100}{(835)^2} = 0.0574\text{W}$$

$$\text{Heat produced in resistance} = 2\times 10 = 20J$$

If t is the required time, then

$$P\times t = 20 \Rightarrow t = \frac{20}{P} = \frac{20}{0.0574} = 348\text{ s}$$

17. A pure resistive circuit element 'x' when connected to an A.C. supply of peak voltage 100 V gives a peak current of 4 A which is in phase with the voltage. A second circuit element 'y' when connected to the same AC supply also gives the same value of peak current but the current lags behind by 90^0. If the series combination of 'x' and 'y' is connected to the same supply. R.M.S. value of current is

1) $\frac{5}{\sqrt{2}}A$ 2) 2A 3) 1/2 A 4) $\frac{\sqrt{2}}{5}A$

SOLUTION:

$$X_L = \frac{\varepsilon_o}{I_o} = 25\Omega\ ; R = \frac{\varepsilon_o}{I_o} = 25\Omega;\ Z = \sqrt{R^2 + X_C{}^2}\ ;$$

$$I_0^1 = \varepsilon_o / Z = 4/\sqrt{2}A;\ I_{r.m.s.} = I_0^1/\sqrt{2} = \frac{4/\sqrt{2}}{\sqrt{2}} = 2A$$

18. An ideal choke coil takes a current of 8 ampere when connected to an AC supply of 100 volt and 50 Hz. A pure resistor under the same conditions takes a current of 10 ampere. If the two are connected to an AC supply of 150 volts and 40 Hz. then the current in a series combination of the above resistor and inductor is

SOLUTION:

For pure inductor,

$$X_L = \frac{E_0}{I_v} = \frac{100}{8} = \frac{25}{2}\Omega$$

$$\omega L = \frac{25}{2}; L = \frac{25}{2\omega} = \frac{25}{2\times 2\pi \times 50} = \frac{1}{8\pi}H$$

$$R = \frac{V}{I} = \frac{100}{10} = 10\Omega$$

For the combination, the supply is 150 v, 40 Hz

$$\therefore X_L = \omega L = 2\pi \times 40 \times \frac{1}{8\pi} = 10\Omega$$

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2}\text{ohm}$$

$$I_v = \frac{E_v}{Z} = \frac{150}{10\sqrt{2}}A = \frac{15}{\sqrt{2}}A$$

19, Find the maximum value of current when a coil of inductance 2H is connected to 150V, 50 cycles / sec supply.

SOLUTION:

Here $L = 2H,\ E_{rms} = 150\ V,\ f = 50\ Hz$

$$X_L = L\omega = L\times 2\pi f = 2\times 2\times 3.14\times 50 = 628\text{ ohm}$$

RMS value of current through the inductor ,

$$I_{rms} = \frac{E_{rms}}{X_L} = \frac{150}{628} = 0.24A$$

Maximum value (or peak value) of current is given by $I_{rms} = \frac{I_0}{\sqrt{2}}$

or $I_0 = \sqrt{2}\ I_{rms} = 1.414\times 0.24 = 0.339A$

20. An inductor of 1 henry is connected across a 220 v, 50 Hz supply. The peak value of the current is approximately.

SOLUTION:

Peak value of current

$$i_0 = \frac{E_0}{X_L} = \frac{\sqrt{2}E_{rms}}{\omega L} = \frac{\sqrt{2}E_{rms}}{2\pi fL} = \frac{\sqrt{2}(220)}{2\pi \times 50\times 1} = 0.99A$$

21. A capacitor of $2\mu F$ is connected in a radio circuit. The source frequency is 1000 Hz. If t h e current through the capacitor branch is
2 mA then the voltage across the capacitor is

SOLUTION:

$$V_C = IX_C = I \times \frac{1}{\omega C} = \frac{I}{2\pi fC}$$

$$= \frac{2\times10^{-3}}{2\pi\times10^3\times2\times10^{-6}} = 0.16V$$

22. An electric bulb has a rated power of 50 W at 100 V. If it is used on an AC source of 200 V, 50 Hz, a choke has to be used in series with it. This choke should have an inductance of

SOLUTION:

Here, $P = 50W, V = 100\text{volt}$

$$I = \frac{P}{V} = \frac{50}{100} = 0.5A,\ R = \frac{V}{I} = \frac{100}{0.5} = 200\Omega$$

Let L be the inductance of the choke coil

$$\therefore I_v = \frac{E_v}{Z} = \text{ or } Z = \frac{E_v}{I_v} = \frac{200}{0.5} = 400\,\Omega$$

Now $X_L = \sqrt{Z^2 - R^2} = \sqrt{400^2 - 200^2}$

$$\omega L = 100\times2\sqrt{3}$$

$$L = \frac{200\sqrt{3}}{\omega} = \frac{200\sqrt{3}}{2\pi v} = \frac{200\sqrt{3}}{100\pi} = \frac{2\times1.732}{3.14} = 1.1H$$

23. A 100 V a.c source of frequency 50 Hz is connected to a LCR circuit with L = 8.1 millihenry, $C = 12.5\mu F$ and $R = 10ohm$, all connected in series. What is the potential difference across the resistance?
1) 100 V 2) 200 V 3) 300 V 4) 450 V

SOLUTION:

. Here, $E_v = 100V, v = 500Hz$

$$L = 8.1\times10^{-3}H, C = 12.5\times10^{-6}F, R = 10\Omega$$

$$X_L = \omega L = 2\pi vL = 1000\pi\times8.1\times10^{-3} = 25.4\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{1000\pi\times12.5\times10^{-6}}$$

$$= \frac{10^3}{12.5\pi}\Omega = 25.4\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{10^2 + (25.4 - 25.4)^2} = 10\Omega$$

$$I_v = \frac{E_v}{Z} = \frac{100}{10} = 10A$$

Potential difference across $R = I_v R = 10 \times 10 = 100V$

24. A transformer having efficiency 90% is working on 100 V and at 2.0 kW power. If the current in the secondary coil is 5A, calculate (i) the current in the primary coil and (ii) voltage across the secondary coil.

SOLUTION:

Here $\eta = 90\% = \frac{9}{10}, I_s = 5A$, $E_p = 100V$,

$$\text{(i)} \; E_p I_p = 2kW = 2000W$$

$$I_p = \frac{2000}{E_p} \text{ or } I_p = \frac{2000}{100} = 20A$$

$$\text{(ii)} \; \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{E_s I_s}{E_p I_p} \text{ or } E_s I_s = \eta \times E_p I_p$$

$$= \frac{9}{10} \times 2000 = 1800W$$

$$\therefore E_s = \frac{1800}{I_s} = \frac{1800}{5} = 360 \text{ volt}$$

25. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

1) The bulb glows dimmer
2) The bulb glows brignther
3) Total impedance of the circuit is unchanged
4) Total impedance of the circuit increases

SOLUTION:

In $R - C$ circuit, the impedance is

$$Z = \sqrt{R^2 + \frac{1}{\omega^2 + C^2}};$$

As ω increases, Z decreases.

Since, Power $\alpha \frac{1}{\text{impedance}}$, therefore the bulb glows brighter.

26. When 100 volt dc is applied across a coil, a current of 1 amp flows through it; when 100 V ac of 50 Hz is applied to the same coil, only 0.5 amp flows. Calculate the resistance and inductance of the coil.

SOLUTION:

In case of a coil, i.e, L - R circuit,

$$I = \frac{V}{Z} \text{ with } Z = \sqrt{R^2 + X_L^{\,2}} = \sqrt{R^2 + (\omega L)^2}$$

So when dc is applied, $\omega = 0$, so $Z = R$

and hence $I = \frac{V}{R}, i.e, R = \frac{V}{I} = \frac{100}{1} = 100\Omega$

and when ac of 50 Hz is applied.

$$I = \frac{V}{Z}, \text{ i,e. } Z = \frac{V}{I} = \frac{100}{0.5} = 200\Omega$$

but $Z = \sqrt{R^2 + \omega^2 L^2}$, i.e, $\omega^2 L^2 = Z^2 - R^2$

i.e, $(2\pi fL)^2 = 200^2 - 100^2 = 3 \times 10^4$

$$L = \frac{\sqrt{3} \times 10^2}{2\pi \times 50} = \frac{\sqrt{3}}{\pi} H = 0.55H$$

27. A circuit containing resistance R_1, Inductance L_1 and capacitance C_1 connected in series resonates at the same frequency 'n' as a second combination of R_2, L_2 and C_2. If the two are connected in series. Then the circuit will resonates at

1) n 2) 2n 3) $\sqrt{\frac{L_2C_2}{L_1C_1}}$ 4) $\sqrt{\frac{L_1C_1}{L_2C_2}}$

SOLUTION:

$$n = \frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}}$$

$$L_1C_1 = L_2C_2 \; ; \; L_{net} = L_1 + L_2; C_{net} = \frac{C_1C_2}{C_1 + C_2}$$

$$L_{net}C_{net} = (L_1 + L_2)\left(\frac{C_1C_2}{C_1 + C_2}\right) ; \; L_{net}C_{net} = L_2C_2$$

28. An AC source of variable frequency is applied across a series L-C-R circuit. At a frequency double the resonance frequency. The impedance is $\sqrt{10}$ times the minimum impedance. The inductive reactance is

1) R 2) 2R 3) 3R 4) 4R

SOLUTION:

$$Z^2 = R^2 + (\omega L - 1/\omega C)^2$$

$$10R^2 = R^2 + (2\omega_o L - 1/2\omega_o C)^2$$

minimum impedance Zmin = R

$$\omega_o^2 LC = 1 \quad \text{-------} \quad (1)$$

$$2\omega_o L - \frac{1}{2\omega_o C} = 3R \quad \text{-------} \quad (2)$$

from(1) $\frac{1}{2\omega_o C} = R \quad \therefore \; X_C = R$

from(2) $X_C = 2\omega_o L = 3R + R = 4R$

29. An AC source of angular frequency ω is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. The ratio of reactance to resistance at the original frequency ω is

1) $\sqrt{\frac{3}{5}}$ 2) $\sqrt{\frac{5}{3}}$ 3) $\frac{3}{5}$ 4) $\frac{5}{3}$

SOLUTION:

$$\text{at frequency } \omega,\ X_C = 1/\omega C$$

$$\text{at frequency } \omega/3,\ X_C^{'} = \frac{3}{\omega C} = 3X_C$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}};\ \frac{I}{2} = \frac{V}{\sqrt{R^2 + 9X_C^2}};\ \frac{X_C}{R} = \sqrt{\frac{3}{5}}$$

30. An LCR circuit has L = 10 mH, R = 3Ω, and C = 1 μF connected in series to a source of $15\cos\omega t$ volt. The current amplitude at a frequency that is 10% lower than the resonant frequency is

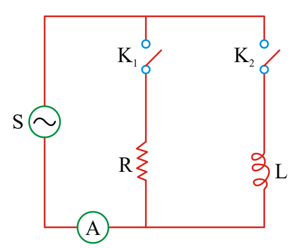
1) 0.5 A 2) 0.7 A 3) 0.9 A 4) 1.1 A

SOLUTION:

$$c_v = \frac{90}{100}c_{v_0} = \frac{90}{100}x\frac{1}{\sqrt{LC}} = 9000 \text{ rad/s}$$

$$i_0 = \frac{E_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

31. In the given circuit, R is a pure resistor, L is a pure inductor, S is a 100V, 50 Hz AC source, and A is an AC ammeter. With either K_1 or K_2 alone closed, the ammeter reading is I. If the source is changed to 100 V, 100 Hz, the ammeter reading with K_1 alone closed and with K_2 alone closed will be respectively.



1) $I, I/2$ 2) $I, 2I$ 3) $2I, I$ 4) $2I, I/2$

SOLUTION:

In the second case induction reactance becomes 2 times thus current through L when K_2 is closed becomes $\frac{i}{2}$. But current through R when K_1 is closed does not change

32. A capacitor has a resistance of $1200\ M\Omega$ and capacitance of $22\ \mu F$. When connected to an a.c. supply of frequency 80 hertz, then the alternating voltage supply required to drive a current of 10 virtual ampere is

1) $904\sqrt{2}$V 2) 904V 3) $904/\sqrt{2}$V 4) 452V

SOLUTION:

$$f = 80Hz,\ I_V = 10A$$

$$\text{Current through R, } I_R = \frac{E_V}{R} = \frac{E_V}{12\times10^8}$$

$$\text{Current through C } I_C = \frac{E_V}{X_C} = 2\pi fC \times E_V$$

$$= 2\pi\times80\times22\times10^{-6}\times E_V$$

$$= 352\pi\times10^{-5}\times E_V \qquad I_V^2 = I_R^2 + I_C^2$$

$$(10^2)\frac{E_V^2}{(12\times10^8)^2} + (352\times10^{-5}\times E_V)^2$$

$$= E_V^2\left(\frac{1}{144\times10^{16}} + 1.2\times10^{-4}\right)$$

$$E_V^2 = \frac{100\times10^4}{1.2} \quad E_V \approx 904 \text{ volt}$$

33. A 120V, 60Hz a.c. power is connected 800Ω non-inductive resistance and unknown capcitance in series. The voltage drop across the resistance is found to be 102V, then voltage drop across capacitor is

1) 8V 2) 102V 3) 63V 4) 55V

SOLUTION:

$$V^2 = V_R^2 + V_C^2;$$

$$V_C^2 = V^2 - V_R^2$$

$$V_C^2 = (120)^2 - (102)^2$$

$$V_C = 63V$$

34. A series combination of R, L, C is connected to an a.ac. source. If the resistance is 3Ω and the resistance is 4Ω, the power factor of the circuit is

1) 0.4 2) 0.6 3) 0.8 4) 1.0

SOLUTION:

$$x = 4\Omega, R = 3\Omega$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{3^2 + 4^2} = 5$$

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{3}{5} = 0.6$$

35. Two alternating voltage generators produce emfs of the same amplitude E_0 **but with a phase difference of $\frac{\pi}{3}$. The resultant e.m.f.is**

1) $E_0 \sin\left(\omega t + \frac{\pi}{3}\right)$ 2) $E_0 \sin\left(\omega t + \frac{\pi}{6}\right)$

3) $\sqrt{3}E_0 \sin\left(\omega t + \frac{\pi}{6}\right)$ 4) $\sqrt{3}E_0 \sin\left(\omega t + \frac{\pi}{2}\right)$

SOLUTION:

$$E_1 = E_0 \sin \omega t;\ E_2 = E_0 \sin(\omega t + \pi/3)$$

$$E = E_2 + E_1$$

$$= E_0 \sin(\omega t + \pi/3) + E_0 \sin \omega t$$

$$= 2E_0 \sin(\omega t + \pi/6)\cos(\pi/6)$$

$$= \sqrt{3}E_0 \sin(\omega t + \pi/6)$$

36. A lamp consumes only 50% of peak power in an a.c. circuit. What is the phase difference between the applied voltage and the circuit current

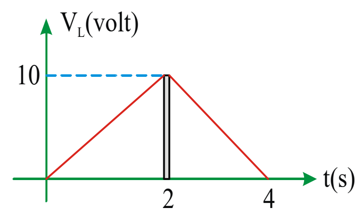
1) $\frac{\pi}{6}$ 2) $\frac{\pi}{3}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{2}$

SOLUTION:

$$P = \frac{1}{2} \times V_0 i_0 \cos\phi \Rightarrow P = P_{peak} \cos\phi$$

$$\Rightarrow \frac{1}{2}\left(P_{peak}\right) = P_{peak} \cos\phi \Rightarrow \cos\phi \frac{1}{2} \Rightarrow \phi = \frac{\pi}{3}$$

37. The potential difference across a 2H inductor as a function of time is shown in figure. At time t = 0, current is zero.
Current t = 2 second is



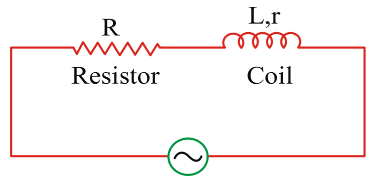
1) 1A 2) 3A 3) 4A 4) 5A

SOLUTION:

$$|e| = L\frac{di}{dt} \Rightarrow |e|dt = L(i_2 - i_1)$$

$$|e|dt = \text{area of } \Delta \text{le for } t = 0 \text{ to } 2 \text{ sec.}$$

38. For the circuit shown in the figure the rms value of voltages across R and coil are E_1 and E_2, respectively.



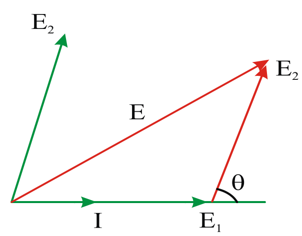
The power (thermal) developed across the coil is

1) $\frac{E-E_1^2}{2R}$ 2) $\frac{E-E_1^2-E_2^2}{2R}$ 3) $\frac{E^2}{2R}$ 4) $\frac{(E-E_1)^2}{2R}$

SOLUTION:

Draw the phasor diagram.

$$E^2 = E_1^2 + E_2^2 + 2E_1E_2\cos\theta.$$



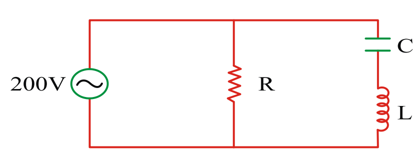
Thermal power developed in coil is

$$P = E_2\cos\theta \times I$$

$$\text{and } I = \frac{E_1}{R}$$

$$\Rightarrow P = \frac{E_1E_2}{R}\cos\theta = \frac{E^2-E_1^2-E_2^2}{2R}$$

39. In the circuit diagram shown, $X_C = 100\,\Omega, X_L = 200\,\Omega \,\&\, R = 100\,\Omega$. The effective current through the source is



1) 2 A 2) $2\sqrt{2}\,A$ 3) 0.5 A 4) $\sqrt{0.4}\,A$

SOLUTION:

$$I_R = \frac{V}{R} = \frac{200}{100} = 2A;\quad I_{LC} = \frac{200}{X_L \times X_C} = 2A$$

$I=\sqrt{2^2+2^2}=2\sqrt{2}\ A$ as I_R inphase with V

I_{LC} lages behind V by $\pi/2$

40. If a current I given by $I_0 \sin(\omega t-\pi/2)$ flows in an ac circuit across which an ac potential of $E_0 \sin(\omega t)$ has been applied, then the power consumption P in the circuit will be

1) $E_0 I_0/\sqrt{2}$ 2) $E_0 I_0/2$ 3) $EI/\sqrt{2}$ 4) Zero

SOLUTION:

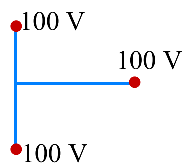
$$P=E_{rms}I_{rms}\cos\phi=\frac{E_0}{\sqrt{2}}\times\frac{I_0}{\sqrt{2}}\cos\frac{\pi}{2}=0$$

41. A high impedance AC voltmeter is connected in turn across the inductor, the capacitor and the resistor in a series circuit having an AC source of 100 V(rms) and gives the same reading in volts in each case. This reading is :

1) 100 V 2) 141 V 3) 150 V 4) 200 V

SOLUTION:

It is condition of resonance then only potential or each are equal and 100V.



42. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC sources is 100 V, the VR is close to

1) 50V 2) 70 V 3) 90 V 4) 100 V

SOLUTION:

Given $V_L : V_C : V_R = 1:2:3$

$V = 100$ V

$V_R = ?$

As we know,

$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$

Solving we get, $V_R = 90$ V

43. A sinusoidal voltage V (t) = 100 sin (500t) is applied across a pure inductance of L=0.02H. The current through the coil is

1) 10 cos (500t) 2) -10 cos (500t) 3) 10 sin (500t) 4) -10 sin (500t)

SOLUTION:

$$i = i_0 \sin\left(\omega t - \frac{\pi}{2}\right), i_0 = \frac{V_0}{X_L}$$

44. In an a.c.circuit, V & I are given by V=100sin(100 t)volt.

$$I = 100\sin\left(100t + \frac{\pi}{2}\right)mA$$

The power dissipated in the circuit is :

1) 1 watt 2) 10 watt 3) zero 4) 5 watt

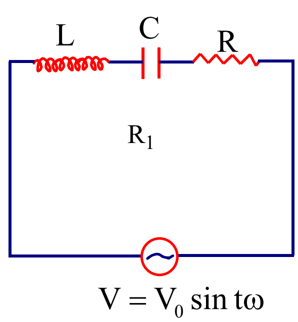
SOLUTION:

$$P = V_{rms} I_{rms} \cos\phi$$

$$= \frac{100}{\sqrt{2}}\left(\frac{100}{\sqrt{2}} \times 10^{-3}\right)\cos 90^0 = 0$$

45. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C', when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C', must have been connected in

(JEE Mains online April 11, 2014)



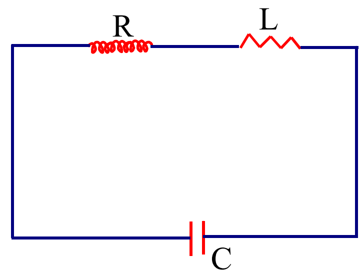
1) series with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$ 2) series with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

3) parallel with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$ 4) parallel with C and has a magnitude $\frac{C}{(\omega^2 LC - 1)}$

SOLUTION:

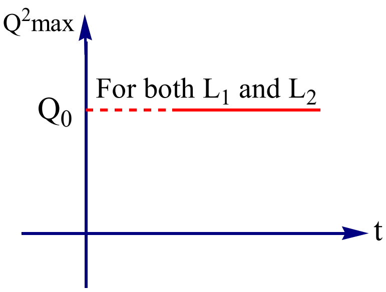
$$X_L = X_C \text{ for } \cos\phi = 1$$

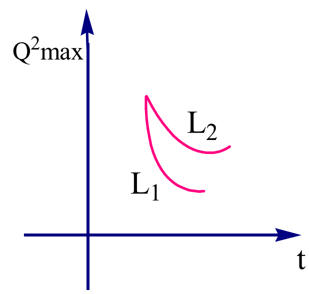
46. An LCR circuit is equivalent to a damped pendulum. In an L-C-R circuit, the capacitor in charged to Q_0 and then connected to L and R as shown.

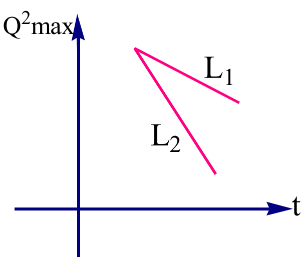


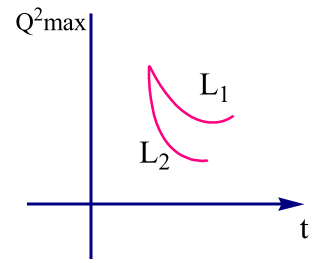
If a student plots graphs of the square of maximum charge $\left(Q^2_{Max}\right)$ on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly ? (JEE Main 2015)

(Plots are schematic and not drawn to scale)

1) 

2) 

3) 

4) 

SOLUTION:

If q is charge on capacitor at any time 't' and current is 'i', then applying Kirchoff's law

$$\frac{q}{C} - ie - \frac{Ldi}{dt} = 0$$

Putting $i = \frac{dq}{dt}$ and $\frac{di}{dt} = \frac{-d^2q}{dt^2}$

In the above equation $\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{q}{LC} = 0$

which has general solution of amplitude

$$q_{max} = Q_0 e^{-\frac{RT}{2L}}$$

$$Q^2_{max} = Q^2_0 e^{-\frac{RT}{L}}$$

47. **When an A.C. voltage of 220 V is applied to the capacitor C**

1) the maximum voltage between plates is 220 V

2) the current is in phase with the applied voltage

3) the charge on the plates is in phase with the applied voltage

4) power delivered to the capacitor is zero

ANSWER : 3 , 4

SOLUTION:

$$E = E_0 \sin \omega t$$

$Q = CE = CE_0 \sin \omega t$, therefore E and Q are in same phase.

In case of capacitor, $\phi = \frac{\pi}{2}$

$$\therefore P = E_{rms} I_{rms} \cos \phi = 0.$$

48. **An arc lamp requires a direct current of 10 A and 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is closed to (JEE Main 2016)**

1) 80 H 2) 0.08 H 3) 0.044 H 4) 0.065 H

SOLUTION:

.
$$I_{rms} = \frac{E_{rms}}{z}$$

49. **The r.m.s. value of an ac of 50 Hz is 10 amp. The time taken by the alternating current in reaching from zero to maximum value and the peak value of current will be**

1) 2×10^{-2} sec and 14.14 amp 2) 1×10^{-2} sec and 7.07 amp

3) 5×10^{-3} sec and 7.07 amp 4) 5×10^{-3} sec and 14.14 amp

SOLUTION:

$$I_{rms} = 10A \Rightarrow I_0 = 10\sqrt{2}A = 14.14Amp$$

Let I=0 when t=0.

$$\therefore I = I_2 \sin \omega t$$

$$I_0 = I_0 \sin \omega t \Rightarrow \omega t = \frac{\pi}{2}$$

$$\Rightarrow t = \frac{\pi}{2\omega} = \frac{\pi}{2 \times 2\pi \times f} = \frac{1}{200} = 5 \times 10^{-3} \text{ sec}$$

50. An electric bulb is designed to operate at 12 volts DC. If this bulb is connected to an AC source and gives normal brightness, what would be the peak voltage of the source?

1) 37 V 2) 17 V 3) 18 V 4) 10 V

SOLUTION:

$$i_{rms} = \frac{i_0}{\sqrt{2}}; V_{DC} = \frac{V_0}{\sqrt{2}}$$

$$12(1.4) = 16.8 \approx 17$$

51. An inductor of reactance 1Ω and a resistor of 2Ω are connected in series to the terminals of a 6V(rms) a.c. source. The power dissipated in the circuit is

1) 8 W 2) 12 W 3) 14.4 W 4) 18 W

SOLUTION:

Here, $X_L = 1\Omega, R = 2\Omega, V_{rms} = 6V$

Impedance of the circuit

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{6}{\sqrt{5}} A$$

Power dissipated

$$P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \frac{R}{Z}$$

$$= 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{72}{5} = 14.4W$$

52. A coil having an inductance of $1/\pi$ henry is connected in series with a resistance of 300Ω. If 20 volt from a 200 cycle source are impressed across the combination, the value of the phase angle between the voltage and the current is :

1) $\tan^{-1}\frac{5}{4}$ 2) $\tan^{-1}\frac{4}{5}$ 3) $\tan^{-1}\frac{3}{4}$ 4) $\tan^{-1}\frac{4}{3}$

SOLUTION:

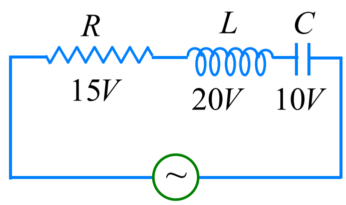
$$X_L = 2\pi fL = 2\pi \times 200 \times \frac{1}{\pi} = 400\Omega$$

$$R = 300\Omega$$

$$\tan\phi = \frac{X_L}{R} = \frac{400}{300} = \frac{4}{3}$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{4}{3}\right)$$

53. In the circuit as shown in the figure, if value of $R = 60\Omega$, then the current flowing through the condenser will be



1) 0.5 A 2) 0.25 A 3) 0.75 A 4) 1.0 A

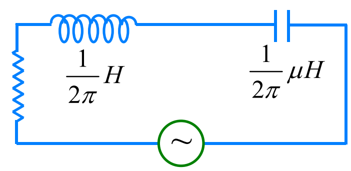
SOLUTION:

I = current flowing through condenser/capacitor.

R,L,C are connected in series. So same current flows through R.

$$V_R = IR \Rightarrow 15 = I \times 60 \Rightarrow I = \frac{1}{4} = 0.25A$$

54. In the a.c. circuit shown in figure, the supply voltage has a constant r.m.s. value but variable frequency f. Resonance frequency is



1) 10 Hz 2) 100 Hz 3) 1000 Hz 4) 200 Hz

SOLUTION:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{2\pi}.\frac{1}{2\pi}\times 10^{-6}}} = 1000Hz$$

55. In an a.c. circuit V and I are given by $V = 100\sin(100t)\, volts$; $I = 100\sin\left(100t + \frac{\pi}{3}\right)mA$.

The power dissipated in teh circuit is

1) 10^4 watt 2) 10 watt 3) 2.5 watt 4) 5 watt

SOLUTION:

$$E_{rms} = \frac{100}{\sqrt{2}} volt.$$

$$I_{rms} = \frac{100}{\sqrt{2}} mA = \frac{100}{\sqrt{2}} \times 10^{-3} A, \phi = \frac{\pi}{3}$$

$$E_{rms}I_{rms}\cos\phi = \frac{100}{\sqrt{2}}\times\frac{100}{\sqrt{2}}\times 10^{-3}\times\cos\frac{\pi}{3}$$

$$= \frac{100\times 100}{2}\times 10^{-3}\times\frac{1}{2} = 2.5watt$$

56. In R-L series circuit, we have same current at angular frequencies ω_1 and ω_2. The resonant frequency fo circuit is

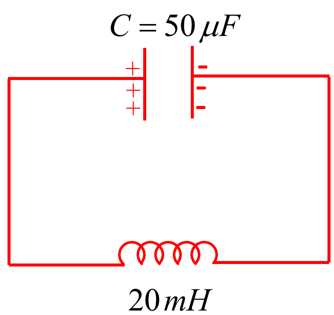
1) $\frac{\omega_1^2}{\omega_2}$ 2) $\frac{\omega_2^2}{\omega_1}$ 3) $\sqrt{\omega_1\omega_2}$ 4) $\omega_1+\omega_2$

SOLUTION:

$$\omega_1 L - \frac{1}{\omega_1 C} = \omega_2 L - \frac{1}{\omega_2 C}$$

$$\text{Solving } \omega_1\omega_2 = \frac{1}{LC} = \omega_0^2$$

57. An LC circuit contains a 20 mH inductor and a $50\,\mu F$ capacitor with initial change of 10 mC. The resistance of the circuit in negligible. Let the instant the circuit is closed be $t = 0$.



Chose the correct option

1) Energy stored in the circuit in completely electrical at $t = \frac{n\pi}{2000}$

2) Energy stored in the circuit in completely magentic at $t = \frac{(2n+1)\pi}{2000}$

3) Energy stored in the circuit in shared equally between the inductor and capacitor at $t = \frac{(2n+1)\pi}{4000}$

4) Energy stored in the circuit is shared equally between the inductor and capacitor at $t = \frac{n\pi}{2000}$

ANSWER : 1, 2, 3

SOLUTION:

Instantaneous electrical energy

$$U_E = \frac{q_0^2\cos^2\omega t}{2C}$$

At $\omega t = 0, \pi, 2\pi, 3\pi, ...$ the energy is completely electrical.

$$t = \frac{n\pi}{2\pi f} = \frac{n}{2f} = \frac{n\pi}{1000}\sec; n = 0,1,2,3,4$$

or $t = 0, T/2, 3T/2, ...$

Instantaneous magnetic energy

$$U_B = \frac{1}{2}Lq_0^2\omega^2 \sin^2 \omega t \text{ or } U_B = \frac{q_0^2}{2C}\sin^2 \omega t$$

So, at $\omega t = \pi/2, 3\pi/2, 5\pi/2, ...$

The energy is completely magnetic

$$t = \frac{(2n+1)\pi}{2(2\pi f)} = \frac{(2n+1)}{4f} = \frac{(2n+1)\pi}{2000}\sec$$

Where $n = 0,1,2,3,4,.....$or t =T/4,3T/4,5T/ 4,.....

Timings for energy shared equally between inductor and capacitor.

$$U_B = U_E$$

$$\frac{q_0^2}{2C}\sin^2 \omega t = \frac{q_0^2}{2C}\cos^2 \omega t$$

$$\tan^2 \omega t = 1 \text{ or } \tan \omega t = \tan \pi/4$$

$$t = \frac{\pi}{4\omega}, \frac{3\pi}{4\omega}, \frac{5\pi}{4\omega}, ... \text{ or } t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...$$

58. If the rms current in a 50 Hz a.c. circuit is 5A, the value of the current 1/300 seconds after its value becomes zero is

1) $5\sqrt{2}A$ **2)** $5\sqrt{\frac{3}{2}}A$ **3)** $\frac{5}{6}A$ **4)** $\frac{5}{\sqrt{2}}A$

SOLUTION:

$$\text{Here, } I_{rms} = 5A, \upsilon = 50\,Hz, t = \frac{1}{300}s$$

$$I_0 = \sqrt{2}I_{rms} = 5\sqrt{2}A.$$

Form $I = I_0 \sin \omega t = I_0 \sin 2\pi\upsilon t$

$$I = 5\sqrt{2}\sin\left(2\pi \times 50 \times \frac{1}{300}\right)$$

$$= 5\sqrt{2}\sin\frac{\pi}{3} = 5\sqrt{2}\frac{\sqrt{3}}{2} = 5\sqrt{\frac{3}{2}}A$$

59. An alternating current generator has an internal resistance R_g and an internal reactance X_g , it is used to supply power to a passive load consisting of a resistance R_g and a rectance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to

1) zero 2) X_g 3) $-X_g$ 4) R_g

SOLUTION:

For maximum power to be delivered from the generator to the load, the total reactance must vanist.

i.e., $X_L + X_g = 0$ or $X_L = -Xg$

60. To reduce the resonant frequency in an LCR series circuit with a generator

1) the generator frequency should be reduced.

2) another capacitor should be added in parallel to the first.

3) the iron core of the inductor should be removed.

4) dielectric in the capacitor should be removed.

SOLUTION:

Resonant frequency in a series LCR circuit is

$$\upsilon_r = \frac{1}{2\pi\sqrt{LC}}$$

If capacitance C increases, the resonant frequency will reduce, which can be achieved by adding another capacitor in parallel to the first.

61. Which of the following combinations should be selected for better tuning of an LCR circuit used for communication?

1) $R = 20\Omega, L = 1.5H, C = 35\mu F$ **2)** $R = 25\Omega, L = 2.5H, C = 45\mu F$

3) $R = 15\Omega, L = 3.5H, C = 30\mu F$ **4)** $R = 25\Omega, L = 1.5H, C = 45\mu F$

SOLUTION:

For better tunning of an LCR circuit used for communication the circuit should possess high quality factor of resonance.

i.e. $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ should be high.

For it R should be low, L should be high and C should below, therefore combination in option (c) is correct.

62. A 20V, 750 HZ source is connected to a series combination of R = 100Ω **, C =** $10\,\mu$ F **and L = 0.1803 H. Calculate the time in which resistance will get heated by** $10^o C$ **. (If thermal capacity of the material = 2 J /** ^oC **)**

1) 328 sec 2) 348 sec 3) 3.48 sec 4) 4.32 sec

SOLUTION:

$$X_C = \frac{1}{2\pi nc} = 21.2\Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 835\Omega$$

$$I_V = Ev/z = 0.0239A$$

$$I_V{}^2Rt = (ms)\Delta\theta \Rightarrow t = \frac{(ms)\Delta\theta}{I_V{}^2R}$$

63. The output of a step-down transformer is measured to be 24V when connected to a 12 watt light bulb. The value of the peak current is

1) $\frac{1}{\sqrt{2}}A$ **2)** $\sqrt{2}A$ **3)** $2A$ **4)** $2\sqrt{2}A$ **More than One Option Correct**

SOLUTION:

Here, $V_s = 24V, P_s = 12W$

$$I_s = \frac{P_s}{V_s} = \frac{12}{24} = 0.5\,A$$

$$I_m = \sqrt{2}I_s = \sqrt{2} \times 0.5 = \frac{1}{\sqrt{2}}A$$

64. A step up transformer operates on a 230 V line and a load current of 2 ampere. The ratio of the primary and secondary windings is 1 : 25. What is the current in the primary ?

SOLUTION:

Using the relation

$$\frac{N_P}{N_S} = \frac{I_S}{I_P}; I_P = \frac{N_S I_S}{N_P}$$

Here $N_p / N_s = 1/25$ (or) $N_s / N_p = 25/1 = 25$

and $I_S = 2A$

Current in primary, $I_P = 25 \times 2 = 50A$

65. As the frequency of an a.c. circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?

1) Inductor and capacitor **2) Resistor and inductor**

3) Resistor and capacitor **4) Resistor, inductor and capacitor**

ANSWER : 1 , 4

SOLUTION:

We know that,

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = 2\pi\upsilon L \text{ and } x_C = \frac{1}{2\pi\upsilon C}$$

So with increase in frequency, R remains constant, inductive reactance increases and capacitive reactance decreases.

66. **Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is(are) correct?**

1) For a given power level, there is a lower current

2) Lower current implies less power less.

3) Transmission lines can be made thinner.

4) It is easy to reduce the voltage at the receiving end using step-down transformers.

ANSWER : 1 , 2 , 4

SOLUTION:

According to relation, $P = EI$, when I is low, power loss $(= I^2R)$ is also low. A step down trans- former lowers voltage by increasing current.

67. **For an LCR circuit, the power transferred from the driving source to the driven oscillator is** $P = I^2 Z \cos\phi$.

1) Here, the power factor $\cos\phi \geq 0, P \geq 0$

2) The driving force can give no energy to the oscillator $(P = 0)$ **in some cases**

3) The driving force cannot syphon out $(P < 0)$ **the energy out of oscillator**

4) The driving force take away energy out ofthe oscillator

ANSWER : 1 , 2 , 3

SOLUTION:

Power is transferred from driving source to driven oscillator.

$\therefore P \geq 0$ and power factor $\cos\phi \geq 0$.

$P = 0$, when $\phi = \frac{\pi}{2}$ for L and C and $P < 0$ is not possible.

68. **If the phase difference between voltage and current is** $\pi/6$ **and the resistance in the circuit is** $\sqrt{300}\Omega$**, then the impedance of teh circuit will be**

1) 40Ω 2) 20Ω 3) 50Ω 4) 13Ω

SOLUTION:

$$\cos\phi = \frac{R}{|Z|} or \frac{\sqrt{3}}{2} = \frac{\sqrt{300}}{|Z|} or \ Z = 20\Omega$$

69. **The line that draws power supply to your hosue from street has**

1) zero average current

2) 220 V average-voltage

3) voltage and current out ofphasc by 90^0

4) voltage and current possibly differing in phase ϕ **such that** $|\phi| < \frac{\pi}{2}$

ANSWER : 1 ,4

SOLUTION:

As the supply current is alternating, so average current over one cycle is zero. The line that draws power supply has some resistance, inductance and capacitance, hence voltage and current differ in phase ϕ such that $|\phi| < \pi/2$

70. A coil has an inductance of 0.7H and is joined in series with a resistance of 220 Ω. When an alternating e.m.f. of 220V at 50 c.p.s. is applied to it, then the wattless component of the current in the circuit is

1) 5 ampere 2) 0.5 ampere
3) 0.7 ampere 4) 7 ampere

SOLUTION:

Watt less component of

$$\text{A.C.} = I_V \sin\theta = \frac{E_V}{Z}\sin\theta$$

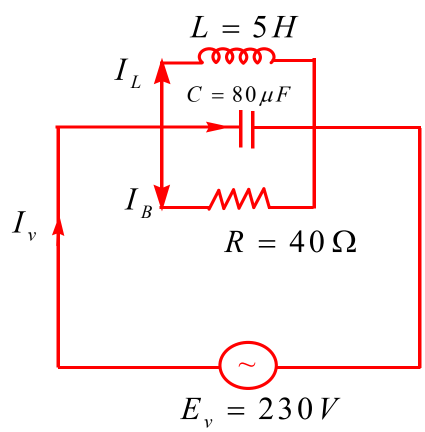
$$= \frac{220}{\sqrt{R^2 + L^2\omega^2}} \times \frac{L\omega}{\sqrt{R^2 L^2 \omega^2}} \quad \therefore L0.7 \times 2\pi \times 50$$

$$= \frac{220 \times L\omega}{(R^2 + L^2\omega^2)} = 0.7 \times 2 \times \frac{22}{7} \times 50$$

$$= \frac{220 \times (0.7 \times 2\pi \times 50)}{(220^2 + 220^2)} = 220\Omega$$

$$= \frac{220 \times 220}{220^2(2)} = \frac{1}{2} = 0.5$$

71. If the three elements, L, C and R are arranged in parallel. Source has emf 230 V and L = 5. H, $C = 80\,\mu F$ and $R = 40\Omega$.



1) The minimum impedance in the circuit is 40Ω

2) The maximum impedance in the circuit is 40Ω

3) The impedance is minimum at $\omega = 50\,rads^{-1}$ **of the source**

4) The impedance is maximum at $\omega = 50\,rads^{-1}$ **of the source**

ANSWER : 2, 3

SOLUTION:

Resonating angular frequency $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{5\times80\times10^{-6}} = 50 rad\ s^{-1}$

$\therefore$ Resonance of L and C in parallel can be calculated.

$$\frac{1}{X} = \frac{1}{X_L} - \frac{1}{X_C} = \frac{1}{\omega L} - \omega C$$

Impedence of R and X in parallel is given by $\frac{1}{Z} = \sqrt{\frac{1}{R^2} + \frac{1}{X^2}}$

At resonating frequency of series LCR, $X_L = X_C$

So, $\frac{1}{X} = \frac{1}{X_L} - \frac{1}{X_C} = 0$

Thus, impedances Z=R and will be maximum, Hence, in parallel resonant circuit, current is minimum at resonant frequency.

72. When a voltage measuring device is connected to a.c. mains, the meter shows the steady input voltage of 220V. This means

1) input voltage cannot be a.c. voltage, but a d.c. voltage.

2) maximum input voltage is 220V.

3) the meter reads not v but $< v^2 >$ and is calibrated to read $\sqrt{< v^2 >}$.

4) the pointer of the meter is stuck by some mechanical defect.

SOLUTION:

The voltmeter connected to a.c. mains is calibrated to read root mean square value or virtual value of a.c. voltage.

73. An ideal inductor takes a current of 10 A when connected to a 125 V, 50 Hz AC supply. A pure resistor across the same source takes 12.5 A. if the two are connected in series across a $100\sqrt{2}V$, 40 Hz supply, the current through the circuit will be

1) 10 A 2) 12.5 A 3) 20 A 4) 25 A

SOLUTION:

For 50 Hz and 125 V supply

$$X_L = \omega L = \frac{V}{i_L} \Rightarrow L = \frac{1}{8\pi},\ R = \frac{V}{i_R} = 10\Omega$$

For 40 Hz, $100\sqrt{2}V$ supply

$$i = \frac{V}{\sqrt{R^2 + X_L^2}} = \frac{V}{\sqrt{R^2 + 4\pi^2 f^2 L^2}}$$

74. A bulb is rated at 100 V, 100 W, it can be treated as a resistor. Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz

1) $\frac{\pi}{\sqrt{3}}H$ 2) 100 H 3) $\frac{\sqrt{2}}{\pi}H$ 4) $\frac{\sqrt{3}}{\pi}H$

SOLUTION:

$$\text{Resistance of bulb is } R=\frac{100\times 100}{100}=100\Omega$$

$$\text{Rated current is } \frac{100}{100}=1A$$

$$\text{In ac, } I_{rms}=\frac{V_{rms}}{Z}\ ;\ Z=200\Omega$$

$$\sqrt{100^2+(\omega L)^2}=200 \Rightarrow \omega^2L^2=30000 \text{ and}$$

$$L=\sqrt{\frac{30000}{(100\pi)^2}}=\frac{\sqrt{3}}{\pi}\text{ henry.}$$

PRACTICE BITS

1. The r.m.s. value of an a.c. of 50 Hz is 10 A. The time taken by the alternating current in reaching from zero to maximum value and the peak value of current will be

1) 2×10^{-2} sec and 14.14 A
2) 1×10^{-2} sec and 7.07 A
3) 5×10^{-3} sec and 7.07 A
4) 5×10^{-3} sec and 14.14 A

KEY:4

HINT::

$$i_0=\sqrt{2}i_{rms}\quad ,T=\frac{1}{f},t=\frac{T}{4}$$

2. An inductor has a resistance R and inductance L. It is connected to an A.C. source of e.m.f E_V **and angular frequency ω, then the current I_v in the circuit is**

1) $\frac{E_V}{\omega L}$ 2) $\frac{E_V}{R}$ 3) $\frac{E_V}{\sqrt{R^2+\omega^2L^2}}$ 4) $\sqrt{\left(\frac{E_V}{R}\right)^2+\left(\frac{E_V}{\omega L}\right)^2}$

KEY:3

HINT::

$$i=\frac{E_0}{\sqrt{R^2+X_L^2}},\ X_L=L\omega$$

3. The peak voltage of 220 Volt AC mains (in Volt) is

1) 155.6 2) 220.0 3) 311 4) 440.0

KEY:3

HINT::

$$V_0=\sqrt{2}.V_{r.m.s.}=\sqrt{2}\times200=311\,volt$$

4. The peak value of A.C. is $2\sqrt{2}A$. It's apparent value will be

1) 1A 2) 2A 3) 4A 4) zero

KEY:2

HINT::

$$I_{rms} = \frac{I_0}{\sqrt{2}}$$

5. Alternating current in circuit is given by $I = I_0 \sin 2\pi nt$. Then the time taken by the current to rise from zero to r.m.s. value is equal to

1) 1/2n 2) 1/n 3) 1/4n 4) 1/8n

KEY:4

HINT::

$$t = \frac{T}{4} = \frac{1}{4f}$$

6. Using an A.C. voltmeter the potential difference in the electrical line in a house is read to be 234 volt. If the line frequency is known to be 50 cycles/second, the equation for the line voltage is

1) V = 165 $\sin(100\pi\ t)$ 2) V = 331 $\sin(100\pi\ t)$

3) V = 220 $\sin(100\pi\ t)$ 4) V = 440 $\sin(100\pi\ t)$

KEY:2

HINT::

$E = E_0 \sin \omega$ t ; voltage read is r.m.s. value

$E_0 = \sqrt{2} \times 234V = 331$ volt

and ω t $= 2\pi\, n\, t = 2\pi \times 50 \times t = 100\,\pi\, t$

Thus, the eqn of line voltage is given by

V = 331 $\sin(100\pi\ t)$

7. A mixer of 100Ω resistance is connected to an A.C. source of 200V and 50 cycles/sec. The value of average potential difference across the mixer will be

1) 308V 2) 264V 3) 220V 4) zero

KEY:4

HINT::

For one complete rotation, average voltage is zero

8. The equation of an alternating voltage is E=220 $\sin(\omega t + \pi/6)$ and the equation of the current in the circuit is I=10 $\sin(\omega t - \pi/6)$. Then the impedance of the circuit is

1) 10 ohm 2) 22 ohm 3) 11 ohm 4) 17 ohm

KEY:2

HINT::

$$Z = \frac{E_0}{I_0}$$

9. A steady P.D. of 10V produces heat at a rate 'x' in resistor. The peak value of A.C. voltage which will produce heat at rate of x/2 in same resistor is

1) 5 V 2) $5\sqrt{2}$ V 3) 10 V 4) $10\sqrt{2}$ V

KEY:3

HINT::

$$\frac{v^2}{R} = x, \frac{v_1^2}{R} = \frac{x}{2} \Rightarrow v_1 = \frac{v}{\sqrt{2}}$$

$\therefore$ in the second case $V_{rms} = V_1$ $\therefore$ $V_0 = \sqrt{2}V_1$

10. An alternating voltage of $E = 200\sqrt{2}\sin(100t)V$ is connected to a condenser of 1 μF through an A.C. ammeter. The reading of the ammeter will be

1) 10 mA 2) 40 mA 3) 80 mA 4) 20 mA

KEY:4

HINT::

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_0 \omega C}{\sqrt{2}}$$

11. The inductance of a coil is 0.70 henry. An A.C. source of 120 volt is connected in parallel with it. If the frequency of A.C. is 60Hz, then the current which is flowing in inductance will be

1) 4.55 A 2) 0.355 A 3) 0.455 A 4) 3.55 A

KEY:3

HINT::

$$X_L = 2\pi fl = 6.28 \times 60 \times 0.70 = 263.76\Omega$$

$$I = \frac{V}{X_L} = \frac{120}{263.76} = 0.455A$$

12. A transformer steps up an A.C. voltage from 230 V to 2300 V. If the number of turns in the secondary coil is 1000, the number of turns in the primary coil will be

1) 100 2) 10,000 3) 500 4) 1000

KEY:1

HINT::

$$\frac{n_s}{n_p} = \frac{V_s}{V_p}$$

13. The transformer ratio of a transformer is 5. If the primary voltage of the transformer is 400 V, 50 Hz, the secondary voltage will be

1) 2000 V, 250 Hz 2) 80 V, 50 Hz

3) 80 V, 10 Hz 4) 2000 V, 50 Hz

KEY:4

HINT::

Frequency remains same. $\frac{V_s}{V_p} = 5$

14. A step-up transformer works on 220V and gives 2 A to an external resistor. The turn ratio between the primary and secondary coils is 2:25. Assuming 100% efficiency, find the secondary voltage, primary current and power delivered respectively

1) 2750 V, 25 A, 5500 W

2) 2750 V, 20 A, 5000 W

3) 2570 V, 25 A, 550 W

4) 2750 V, 20 A, 55 W

KEY:1

HINT::

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{i_p}{i_s}, \quad P = E_s i_s$$

15. A coil of self - inductance $\left(\frac{1}{\pi}\right)$H is connected in series with a 300 Ω resistance. A voltage of 200V at frequency 200Hz is applied to this combination. The phase difference between the voltage and the current will be

1) $\tan^{-1}\left(\frac{4}{3}\right)$ 2) $\tan^{-1}\left(\frac{3}{4}\right)$ 3) $\tan^{-1}\left(\frac{1}{4}\right)$ 4) $\tan^{-1}\left(\frac{5}{4}\right)$

KEY:1

HINT::

$$\tan\theta = \frac{2\pi \mathrm{fL}}{\mathrm{R}}, \quad \mathrm{f} = \frac{1}{2\pi\sqrt{\mathrm{LC}}}$$

16. A condenser of 10μF and an inductor of 1H are connected in series with an A.C. source of frequency 50Hz. The impedance of the combination will be (take $\pi^2 = 10$)

1) zero 2) Infinity 3) 44.7 Ω 4) 5.67 Ω

KEY:1

HINT::

$$\mathrm{Z} = \left(2\pi \mathrm{fL} - \frac{1}{2\pi \mathrm{fC}}\right)$$

17. A 100 km telegraph wire has capacity of 0.02 $\mu F/km$, if it carries an alternating current of frequency 5 kHZ. The value of an inductance required to be connected in series so that the impedence is minimum.

1) 50.7mH 2) 5.07mH 3) 0.507mH 4) 507mH

KEY:3

HINT::

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow L = \frac{1}{\omega^2 C} = \frac{1}{(2\pi n)^2 C}$$

18. In an LCR series circuit the rms voltages across R, L and C are found to be 10 V, 10 V and 20 V respectively. The rms voltage across the entire combination is

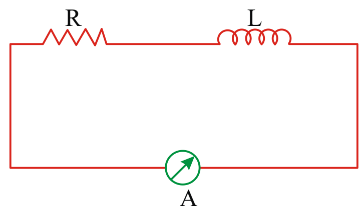
1) 30 V   2) 1 V   3) $20V$ 4) $10\sqrt{2}\,V$

KEY:4

HINT::

$$V=\sqrt{V_R^2+(V_L-V_C)^2}$$

19. In the circuit shown, a 30V d.c. source gives a current 2.0 A as recorded in the ammeter A and 30V a.c. source of frequency 100Hz gives a current 1.2A. The inductive reactance is



1) 10 ohm   2) 20 ohm   3) $5\sqrt{34}$ ohm   4) 40 ohm

KEY:2

HINT::

When d.c. source, $R=\frac{V}{I}=\frac{30}{2}=15\Omega$

When a.c. source, $Z=\frac{30}{1.2}=25\Omega$

$X_L=\sqrt{(25)^2-(15)^2}=\sqrt{625-225}=20\Omega$

20. A choke coil has negligible resistance. The alternating potential drop across it is 220 volt and the current is 5mA. The power consumed is

1) $220\times\frac{5}{1000}$W   2) $\frac{220}{5}$W

3) zero   4) 2.20 x 5W

KEY:3

HINT::

Average power is zero

21. In an A.C. circuit, the instantaneous values of e.m.f. and current are E = 200 sin 314t volt and $I=\sin(314t+\pi/3)$ **ampere then the average power consumed in watts is**

1) 200   2) 100   3) 0   4) 50

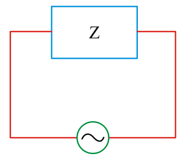
KEY:4

HINT::

$$P_{avg}=I_{rms}E_{rms}\cos\phi=\frac{1}{\sqrt{2}}\times\frac{200}{\sqrt{2}}\cos 60^\circ$$

50W

22. In a black box of unkown elements (L, C or R or any other combination) an AC voltage $E = E_0 \sin(\omega t + \phi)$ is applied and current in the circuit was found to be $i = i_0 \sin(\omega t + \phi + \pi/4)$. Then the unknown elements in the box may be



1) only capacitor 2) both inductor and resistor
3) either capacitor, resistor and inductor or only capacitor and resistor
4) only resistor

KEY:3

HINT::

Here current leads the voltage. So, there is reactance which is capacitive

$\Rightarrow X = X_C - X_L \quad or \quad X = X_C$ alone besides R

23. The instantaneous value of current and emf in an AC circuit are $I = \frac{1}{\sqrt{2}} \sin 314t$ amp and $E = \sqrt{2} \sin\left(314t - \frac{\pi}{6}\right) V$, respectively. The phase difference between E and I (with respect to I) will be

1) $-\frac{\pi}{6}$ rad 2) $-\frac{\pi}{3}$ rad 3) $\frac{\pi}{6}$ rad 4) $\frac{\pi}{3}$ rad

KEY:1

HINT::

Ans : (a)

$$V = \frac{V_0 t}{T/4} = \frac{4V_0 t}{T}$$

$$V_{rms} = \sqrt{\langle V^2 \rangle} = \frac{4V_0}{T} \left\{ \frac{\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt} \right\} = \frac{V_0}{\sqrt{3}}$$

24. The power in ac circuit is given by $P = E_{rms} I_{rms} \cos\phi$. The value of $\cos\phi$ in series LCR circuit at resonance is :

1) zero 2) 1 3) $\frac{1}{2}$ 4) $\frac{1}{\sqrt{2}}$

KEY:2

HINT::

$$\cos\phi = \frac{R}{Z} = \frac{R}{R} = 1$$

25. The secondary coil of an ideal step down transformer is delivering 500 watt power at 12.5 A current. if the ratio of turns in the primary to the secondary is 5:1; then the current flowing in the primary coil will be :

1) 62.5 A 2) 2.5 A 3) 6 A 4) 0.4 A

KEY:2

HINT::

For an ideal transformer (100% efficient)

$$\Rightarrow P_{input} = P_{output} \qquad V_1 I_1 = V_2 I_2$$

$$\Rightarrow I_1 = \frac{V_2 I_2}{V_1} = \frac{40(12.5)}{40 \times 5} = 2.5A$$

$$\left[\because \frac{n_1}{n_2} = \frac{V_1}{V_2} \Rightarrow \frac{5}{1} = \frac{V_1}{40}\right]$$

26. In a step-up transformer the turn's ratio is 10. If the frequency of the curernt in the primary coil is 50 Hz then the frequency of the current in the second ary coil will be

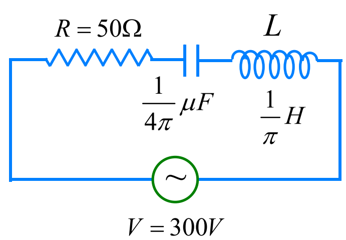
1) 500 Hz 2) 5 Hz 3) 60 Hz 4) 50 Hz

KEY:4

HINT::

Frequency of the current remains same, only magnitudes of current changes in a transform

27. In the a.c. circuit shown in the figure. The supply voltage has a constant r.m.s value V, but variable frequency f. Resonance frequency in hertz is



1) 10 2) 100 3) 1000 4) 200

KEY:3

HINT::

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi \times \sqrt{\frac{1}{\pi} \times \frac{1}{4\pi} \times 10^{-6}}} = 1000 Hz$$

28. **The average current of a sinusoidally varrying alternating current of peak value 5A with initial phase zero, between the instants t = T/8 to t = T/4 is (Where 'T' is time period)**

1) $\frac{10}{\pi}\sqrt{2}A$ 2) $\frac{5}{\pi}\sqrt{2}A$ 3) $\frac{20\sqrt{2}}{\pi}A$ 4) $\frac{10}{\pi}A$

KEY:1

HINT: $<i>=\frac{\int_{T/8}^{T/4} i\,dt}{\int_{T/8}^{T/4} dt}$

29. **A 100Ω resistance is connected in series with a 4H inductor. The voltage across the resistor is $V_R = 2\sin(1000t)V$. The voltage across the inductor is**

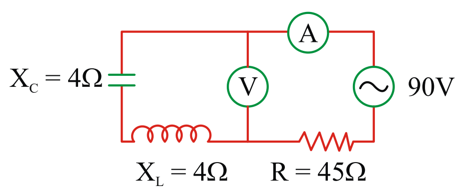
1) $80\sin\left(1000t+\frac{\pi}{2}\right)$ 2) $40\sin\left(1000t+\frac{\pi}{2}\right)$

3) $80\sin\left(1000t-\frac{\pi}{2}\right)$ 4) $40\sin\left(1000t-\frac{\pi}{2}\right)$

KEY:1

HINT: $i=\frac{(V_0)_R}{R}, V_L=(V_0)_L\sin\left(\omega t+\frac{\pi}{2}\right) and (V_0)_L = X_L i$

30. **The reading of voltmeter and ammeter in the following figure will respectively be**



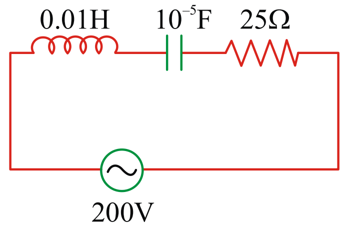
1) 0 and 2A 2) 2A and 0V
3) 2V and 2A 4) 0V and 0A

KEY:1

HINT $: I_{rms} = \frac{E_{rms}}{R} = 2A$; $V_{rms} = I_{rms}(X_L - X_C) = 0$

$\therefore$ circuit is at resonance

31. In the following circuit, the values of current flowing in the circuit at f = 0 and f = ∞ will respectively be

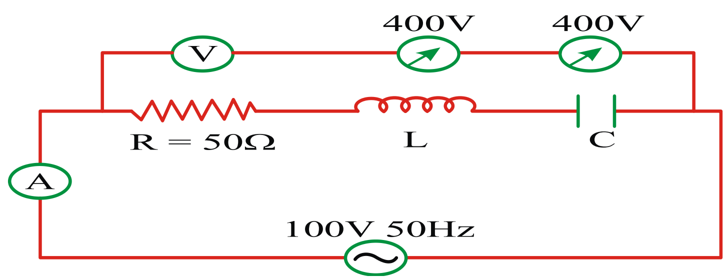


1) 8A and 0A 2) 0A and 0A 3) 8A and 8A 4) 0A and 8A

KEY:2

HINT: $$I=\frac{E}{Z}=\frac{E}{\sqrt{R^2+\left[2\pi f L-\frac{1}{2\pi f C}\right]^2}}$$

32. In the series L-C-R circuit figure the voltmeter and ammeter readings are



1) V=100 volt, I=2A 2) V=100 volt, I = 5 A 3) V=1000 volt, I=2A 4) V=300 volt, I = 1 A

KEY:1

HINT: $$I_{r.m.s.}=\frac{V_{r.m.s.}}{Z}=\frac{V_{r.m.s.}}{R}=\frac{100}{50}=2A$$

$$V=\sqrt{V_R^2+\left(V_L-V_C\right)^2}$$

33. The potential difference between the ends of a resistance R is V_R, between the ends of capacitor is $V_C = 2V_R$ and between the ends of inductance is $V_L = 3V_R$. Then the alternating potential of the source in terms of V_R will be

1) $\sqrt{2}V_R$ 2) V_R 3) $\frac{V_R}{\sqrt{2}}$ 4) $5V_R$

KEY:1

HINT: $$\overline{V_S}=\overline{V_B}+\overline{V_C}+\overline{V_L}=V_R\hat{i}-2V_R\hat{j}+3V_R\hat{j}$$

$$=V_R\hat{i}+V_R\hat{j}\ ,\ \left|\overline{V}\right|=\sqrt{2}V_R$$

34. **A 220V, 50Hz a.c. generator is connected to an inductor and a 50Ω resistance in series. The current in the circuit is 1.0A. The P.D. across inductor is**

1) 102.2V 2) 186.4V 3) 213.6V 4) 302V

KEY:3

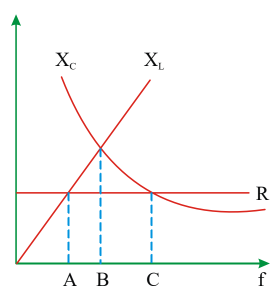
HINT $:I=\frac{E}{Z}, \therefore I=\frac{220}{Z}, Z=220\Omega$

$Z^2=R^2+X_L^2 \quad \therefore \quad X_L=\sqrt{Z^2-R^2}$

$L=\frac{1}{\omega}\sqrt{Z^2-R^2} \quad \therefore L=\frac{1}{2\pi f}\sqrt{Z^2-R^2}=0.68H$

$\therefore V_L=\omega LI=2\pi\times0.5\times0.68\times1=213.6\text{V}$

35. The figure shows variation of R, X_L **and X_C with frequenc f in a series L, C, R circiut. Then for what frequency point, the circiut is inductive**

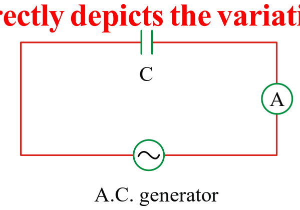


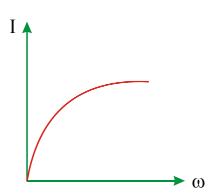
1) A 2) B 3) C 4) All points

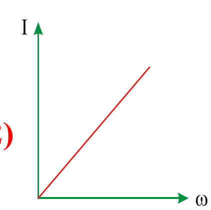
KEY:3

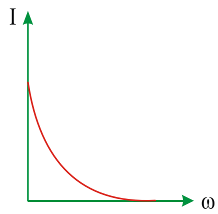
HINT: At A : $X_C > X_L$; At B : $X_C = X_L$; At C : $X_C < X_L$

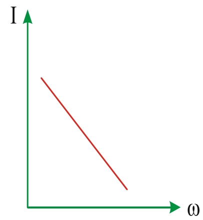
36. **A constant voltage at different frequencies is applied across a capacitance C as shown in the figure. Which of the following graphs correctly depicts the variation of current with frequency**



1) 

2) 

3) 

4) 

KEY:2

HINT: For capacitive circuits $X_C = \frac{1}{\omega C}$

$\therefore i = \frac{V}{X_C} V\omega C \Rightarrow i \propto \omega$

37. In a series $L-C-R$ circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220 V and 50Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30^0. On taking out the inductor from the circuit the current leads the voltage by 30^0. The power dissipated in the L-C-R circuit is

1) 305 W 2) 210 W 3) zero 4) 242 W

KEY:1

HINT :The given circuit is under resonance as $X_L = X_C$

Hence, power dissipated in the circuit is $P = \frac{V^2}{R} = 242W$

38. In a series resonant LCR circuit, the voltage across R is 100V and $R = 1k\Omega$ with $C = 2\mu F$. The resonant frequency ω is 200 rad/s. At resonance the voltage across L is

1) $2.5\times10^{-2}V$ 2) 40 V 3) 250 V 4) $4\times10^{-3}V$

KEY:2

HINT :At resonance, $\omega L = \frac{1}{\omega C}$

current flowing through the circuit $I = \frac{V_R}{R} = \frac{100}{1000} = 0.1A$

So, voltage across L is given by

$V_L = I X_L = I\omega L$ but $\omega L = \frac{1}{\omega C}$

$V_L = \frac{1}{\omega C} = \frac{0.1}{200\times2\times10^{-6}} = 250V$

39. The series RLC circuit in resonance is called :

1) Selector circuit 2) rejector circuit
3) amplifier circuit 4) oscillator circuit

KEY:1

HINT The series RLC circuit at resistance selects that current out of many currents whose frequency is

equal to its

:natural frequency, hence called as 'acceptor' or 'selector' circuit.

40. In a series R-L-C circuit, the frequency of the source is half of the resonance frequency. The nature of the circuit will be

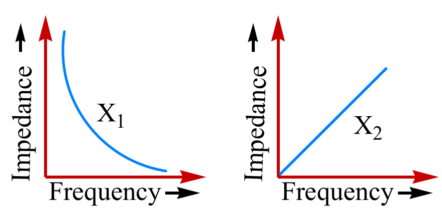
1) capacitive 2) inductive
3) purely resistive 4) selective

KEY:1

HINT: $X_c = \frac{1}{\omega C}$ and $X_L = \omega L$

At $\omega < \omega_{res}, X_C > X_L$ $\therefore$ The circuit capacitive.

41. The graphs given below depict the dependence of two reactive impedances X_1 and X_2 on the frequency of the alternating e.m.f. applied individually to them. We can then say that



1) X_1 is an inductor and X_2 is a capacitor
2) X_1 is a resistor and X_2 is a capacitor
3) X_1 is a capacitor and X_2 is a inductor
4) X_1 is an inductor and X_2 is a resistor

KEY:3

HINT: $X_L = wL$ and $X_C = \frac{1}{wC}$

so X_1 capacitive reactance and X_2 is incuctive reactance

THEORY BITS

1. In non-resonant circuit, the nature of circuit for frequencies greater than the resonant frequency is
   1) resistive 2) capacitive
   3) inductive 4) both 1 and 2

KEY:3

2. The average e.m.f during the positive half cycle of an a.c. supply of peak value E_0 is
   1) E_0/π 2) $E_0/\sqrt{2}$ 3) $E_0/2\pi$ 4) $2E_0/\pi$

KEY:4

3. The phase difference between voltage and current in an LCR series circuit is
   1) zero always 2) $\pi/4$ always
   3) π 4) between 0 and $\pi/2$

KEY:4

4. Alternating current is transmitted to distant places at
   1) high voltage and low current
   2) high voltage and high current
   3) low voltage and low current
   4) low voltage and high current

KEY:1

5. For an ideal transformer ratio of output to the input power is always
   1) greater than one 2) equal to one
   3) less than one 4) zero

KEY:2

6. In case of a.c circuit, Ohm's law holds good for
   a) Peak values of voltage and current
   b) Effective values of voltage and current
   c) Instantaneous values of voltage and current
   1) only a is true 2) only a and b are true3) only c is true 4) a, b and c are true

KEY:2

7. The unit of impedence is
   1) ohm 2) mho 3) ampere 4) volt

KEY:1

8. In case of AC circuits the relation V = i Z, where Z is impedance, can directly applied to
   1) peak values of voltage and current only
   2) rms values of voltage and current only
   3) instantaneous values of voltage and current only
   4) both 1 and 2 are true

KEY:4

9. If in a series L - C - R ac circuit, the voltages across R, L, C are V_1, V_2, V_3 respectively. Then the voltage of applied AC source is always equal to
   1) $V_1+V_2+V_3$ 2) $\sqrt{V_1^2+(V_2+V_3)^2}$
   3) $V_1-V_2-V_3$ 4) $\sqrt{V_1^2+(V_2-V_3)^2}$

KEY:4

10. Alternating current can not be measured by direct current meters, because
1) alternating current can not pass through an ammeter
2) the average value of current for complete cycle is zero
3) some amount of alternating current is destroyed in the ammeter
4) peak value of current is zero
KEY:2

11. If the instantaneous values of current is $I = 2\cos(\omega t + \theta)$ A in a circuit, the r.m.s. value of current in ampere will be
1) 2 2) $\sqrt{2}$ 3) $2\sqrt{2}$ 4) zero
KEY:2

12. The ratio of primary voltage to secondary voltage in a transformer is 'n'. The ratio of the primary current to secondary current in the transformer is
1) n 2) 1/n 3) n^2 4) $1/n^2$
KEY:2

13. If a capacitor is connected to two different A.C. generators, then the value of capacitive reactance is
1) directly proportional to frequency
2) inversely proportional to frequency
3) independent of frequency
4) inversely proportional to the square of frequency
KEY:2

14. In general in an alternating current circuit
1) the average value of current is zero
2) the average value of square of the current is zero
3) average power dissipation is zero
4) the phase difference between voltage and current is zero
KEY:1

15. A stepup transformer develops 400V in secondary coil for an input of 200V A.C. Then the type of transformer is
1) Steped down 2) Steped up 3) Same
4) Same but with reversed direction
KEY:2

16. The magnitude of induced e.m.f in an LR circuit at break of circuit as compared to its value at make of circuit will be
1) less 2) more
3) some times less and some times more 4) nothing can be said
KEY:2

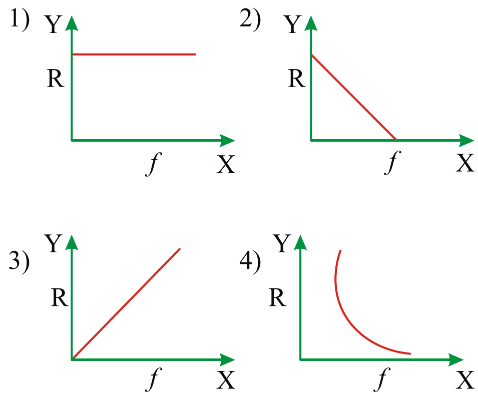
17. If the frequency of alternating e.m.f. is f in L-C-R circuit, then the value of impedance Z will change with log (frequency) as
1) increases
2) increases and then becomes equal to resistance, then it will start decreasing
3) decreases and when it becomes minimum equal to the resistance then it will start increasing
4) go on decreasing
KEY:3

18. The emf and current in a circuit are such that $E = E_0 \sin \omega t$ and $I = I_0 \sin (\omega t - \theta)$. This AC circuit contains.

1) R and L 2) R and C 3) only R 4) only C

KEY:1

19. The correct graph between the resistance of a conductor with frequency is



KEY:1

20. Same current is flowing in two alternating circuits. The first circuit contains only inductance and the other contains only a capacitor. If the frequency of the e.m.f. is increased, the current will

1) increase in first circuit and decrease in the other
2) increase in both circuits
3) decrease in both circuits
4) decrease in first circuit and increase in the other

KEY:4

21. For an ideal transformer ratio of output to the input power is always

1) greater than one 2) equal to one
3) less than one 4) zero

KEY:2

22. Ratio of impedence to capacitive reactance has

1) no units 2) ohm 3) ampere 4) tesla

KEY:1

23. An inductor coil having some resistance is connected to an AC source. Which of the following have zero average value over a cycle

1) induced emf in the inductor only
2) current only 3) both 1 and 2 4) neither 1 nor 2

KEY:3

24. In an AC circuit containing only capacitance the current

1) leads the voltage by 180^o
2) lags the voltage by 90^o
3) leads the voltage by 90^o
4) remains in phase with the voltage

KEY:3

25. A bulb is connected first with dc and then ac of same voltage.Then it will shine brightly with
1) AC 2) DC 3) Equally with both 4) Brightness will be in ratio 1/14
KEY:3
26. A capacitor of capacity C is connected in A.C. circuit. If the applied emf is $V = V_0 \sin \omega t$, then the current is
1) $I = \frac{V_0}{L\omega} \sin \omega t$ 2) $I = \frac{V_0}{\omega C} \sin\left(\omega t + \frac{\pi}{2}\right)$
3) $I = V_0 C\omega \sin \omega t$ 4) $I = V_0 C\omega \sin\left(\omega t + \frac{\pi}{2}\right)$
KEY:4

27. Statement (A) : The reactance offered by an inductance in A.C. circuit decreases with increase of AC frequency.
Statement (B) : The reactance offered by a capacitor in AC circuit increases with increase of AC frequency.
1) A is true but B is false 2) Both A and B are true
3) A is false but B is true 4) Both A and B are false
KEY:4
28. Statement (A) : With increase in frequency of AC supply inductive reactance increases.
Statement (B) : With increase in frequency of AC supply capacitive reactance increase
1) A is true but B is false 2) Both A and B are true
3) A is false but B is true 4) Both A and B are false
KEY:1
29. In an A.C circuit having resistance and capacitance
1) emf leads the current
2) current lags behind the emf
3) both the current and emf are in phase
4) current leads the emf.
KEY:4
30. Select the correct options among the following: In an R-C circuit
a) instantaneous A.C is given by $I = I_0 \sin(wt + \phi)$
b) the alternating current in the circuit leads the emf by a phase angle ϕ.
c) Its impedance is $\sqrt{R^2 + (\omega c)^2}$
d) Its capacitive reactance is ω c
1) a, b are ture 2) b, c, d are true
3) c, d are true 4) a, c are true
KEY:1
31. If the frequency of alternating e.m.f. is f in L-C-R circuit, then the value of impedance Z will change with log (frequency) as
1) increases
2) increases and then becomes equal to resistance, then it will start decreasing
3) decreases and when it becomes minimum equal to the resistance then it will start increasing
4) go on decreasing
KEY:3

32. An inductance and resistance are connected in series with an A.C circuit. In this circuit

1) the current and P.d across the resistance lead P.d across the inductance by $\pi/2$

2) the current and P.d across the resistance lags behind the P.d across the inductance by angle $\pi/2$

3) The current across resistance leads and the P.d across resistance lags behind the P.d across the inductance by $\pi/2$

4) the current across resistance lags behind and the P.d across the resistance leads the P.d across the inductance by $\pi/2$

KEY:2

33. An LCR circuit is connected to a source of alternating current. At resonance, the applied voltage and the current flowing through the circuit will have a phase difference of

1) $\pi/4$ 2) zero 3) π 4) $\pi/2$

KEY:2

34. The incorrect statement for L-R-C series circuit is

1) The potential difference across the resistance and the appleid e.m.f. are always in same phase

2) The phase difference across inductive coil is 90^0

3) The phase difference between the potential difference across capacitor and potential difference across inductance is 90^0

4) The phase difference between potential difference across capacitor and potential difference across resistance is 90^0

KEY:3

35 In series L - C - R resonant circuit, to increase the resonant frequency

1) L will have to be increased

2) C will have to be increased

3) LC will have to be decreased

4) LC will have to be increased

KEY:3

36. If in a series L - C - R ac circuit, the voltages across R, L, C are V_1, V_2, V_3 respectively. Then the voltage of applied AC source is always equal to

1) $V_1+V_2+V_3$ 2) $\sqrt{V_1^2+(V_2+V_3)^2}$

3) $V_1-V_2-V_3$ 4) $\sqrt{V_1^2+(V_2-V_3)^2}$

KEY:4

37. In non-resonant circuit, the nature of circuit for frequencies greater than the resonant frequency is

1) resistive 2) capacitive

3) inductive 4) both 1 and 2

KEY:3

38. The phase difference between voltage and current in an LCR series circuit is

1) zero always 2) $\pi/4$ always

3) π 4) between 0 and $\pi/2$

KEY:4

39. In an LCR a.c circuit at resonance, the current

1) Is always in phase with the voltage 2) Always leads the voltage

3) Always lags behind the voltage 4) May lead or lag behind the voltage

KEY:1

40. An inductance L and capacitance C and resistance R are connected in series across an AC source of angular frequency ω. If $\omega^2 > \frac{1}{LC}$ then

1) emf leads the current 2) both the emf and the current are in phase

3) current leads the emf 4) emf lags behind the current

KEY:1

41. Consider the following two statements A and B and identify the correct answer.

A) At resonance of L - C - R series circuit, the reactance of circuit is minimum.

B) The reactance of a capacitor in an A.C circuit is similar to the resistance of a capacitor in a D.C. circuit

1) A is true but B is false 2) Both A and B are true

3) A is false but B is true 4) Both A and B are false

KEY:1

42. Choose the wrong statement of the following.

1) The peak voltage across the inductor can be less than the peak voltage of the source in an LCR circuit

2) In a circuit containing a capacitor and an ac source the current is zero at the instant source voltage is maximum

3) When an AC source is connected to a capacitor,then the rms current in the circuit gets increased if a dielectric slab is inserted into the capacitor.

4) In a pure inductive circuit emf will be in phase with the current.

KEY:4

43. The essential difference between a d.c. dynamo and an a.c. dynamo is that

1) a.c. has an electromagnet but d.c. has a permanent magnet

2) a.c. will generate a higher voltage

3) a.c.has slip rings but the d.c. has a commutator

4) a.c. dynamo has a coil wound on soft iron, but the d.c. dynamo has a coil wound on copper

KEY:3

44. The power factor of a.c. circuit having L and R connected in series to an a.c. source of angular frequency ω is given by

1) $\frac{\sqrt{R^2+\omega^2L^2}}{R}$ 2) $\frac{R}{\sqrt{R^2+\omega^2L^2}}$ 3) $\frac{\omega L}{R}$ 4) $\frac{R}{\omega L}$

KEY:2

45. The capacitor offers zero resistance to

1) D.C. only 2) A.C. & D.C. 3) A.C. only 4) neither A.C. nor D.C.

KEY:4

46. In an ac circuit the current

1) is in phase with the voltage 2) leads the voltage 3) lags the voltage

4) any of the above depending on the circumstances

KEY:4

47. Power factor is defined as

1) apparent power/true power 2) true power/apparent power

3) true power (apparent power)^2 4) true power x apparent power

KEY:2

48. At low frequency a condenser offers

1) high impedance 2) low impedance

3) zero impedance 4) impedance of condenser is independent of frequency

KEY:1

Transformer

49. The core of a transformer is laminated so that

1) energy loss due to eddy currents may be reduced

2) rusting of the core may be prevented

3) change in flux may be increased

4) ratio of voltage in the primary to that in the secondary may be increased

KEY:1

50. A step up transformer is used to

1) increase the current and increase the voltage 2) decrease the current and increase the voltage

3) increase the current and decrease the voltage 4) decrease the current and decrease the voltage

KEY:2

51. A transformer changes the voltage

1) without changing the current and frequency

2) without changing the current but changes the frequency

3) without changing the frequency but changes the current

4) without changing the frequency as well as the current

KEY:3

52. In a step down transformer, the number of turns in the primary is always

1) greater than the number of turns in the secondary

2) less than the number of turns in the secondary

3) equal to the number of turns in the secondary

4) either greater than or less than the number of turns in the secondary

KEY:1

53. The phase angle between current and voltage in a purely inductive circuit is

1) zero 2) π 3) $\pi/4$ 4) $\pi/2$

KEY:4

54. The transformer ratio of a step up transformer is

1) greater than one 2) less than one

3) less than one and some times greater than one

4) greater than one and some times less than one

KEY:1

55. Assertion(A) : If changing current is flowing through a machine with iron parts, results in loss of energy.

Reason(R): Changing magnetic flux through an area of the iron parts causes eddy currents.

1)Both A and R are individually true and R is the correct explanation of A

2)Both A and R are individually true but R is not the correct explanation of A

3) A is true but R is false 4) Both A and R are false

KEY:1

56. Transformers are used in
1) d.c circuits only 2) a.c. circuits only
3) Both a.c and d.c circuits 4) Integrated circuits.
KEY:2
57. The magnitude of the e.m.f. across the secondary of a transformer does not depend on
1) The number of the turns in the primary
2) The number of the turns in the secondary
3) The magnitude of the e.m.f applied across the primary
4)The resistance of the primary and the secondary
KEY:4
58. For an ideal transformer ratio of output to the input power is always
1) greater than one 2) equal to one
3) less than one 4) zero
KEY:2
59. Consider the following two statements A and B and identify the correct answer.
A) In a transformer a large alternating current at low voltage can be transformed into a small alternating current at high voltage
B) Energy in current carrying coil is stored in the form of magnetic field.
1) A is true but B is false 2) Both A and B are true
3) A is false but B is true 4) Both A and B are false
KEY:2
60. Statement (A) : Flux leakage in a transformer can be minimized by winding the primary and secondary coils one over the other.
Statement (B) : Core of the transformer is made of soft iron
KEY:4
61. Statement (A) : In high current low voltage windings of a transformer thick wire is used to minimize energy loss due to heat produced
Statement (B) : The core of any transformer is laminated so as to reduce the energy loss due to eddy currents
KEY:2
62. Statement (A) : Step up transformer converts low voltage, high current to high voltage, low current
Statement (B) : Transformer works on both ac and dc
KEY:1
63. To reduce the iron losses in a transformer, the core must be made of a material having
1) low permeability and high resistivity
2) high permeability and high resistivity
3) low permeability and low resistivity
4) high permeability and low resistivity
KEY:2
64. At low frequency a condenser offers
1) high impedance 2) low impedance
3) zero impedance
4) impedance of condenser is independent of frequency
KEY:1

65. Maximum efficiency of a transformer depends on
1) the working conditions of technicians.
2) weather copper loss =1/2 x iron loss
3) weather copper loss = iron loss
4) weather copper loss =2 x iron loss

KEY:3

66. For a LCR series circuit with an A.C. source of angular frequency ω
1) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$
2) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$
3) power factor of circuit will be unity if capacitive reactance equals inductive reactance
4) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

KEY:3

67. The value of current in two series L C R circuits at resonance is same when connected across a sinusoidal voltage source. Then
1) both circuits must be having same value of capacitance and inductance
2) in both circuits ratio of L and C will be same
3) for both the circuits X_L / X_C must be same at that frequency
4) both circuits must have same impedance at all frequencies

KEY:3

68. When an AC source of emf $e = E_0 \sin(100t)$ is connected across a circuit, the phase difference betwen the emf e and the current i in the circuit is observed to be $\frac{\pi}{4}$ ahead,If the circuit consists possibly of $R-C$ or $R-L$ or $L-C$ in series, find the relationship between the two elements:
1) $R = 1k\Omega,\ C = 10\mu F$ 2) $R = 1k\Omega,\ C = 1\mu F$
3) $R = 1k\Omega,\ L = 10H$ 4) $R = 1k\Omega,\ L = 1H$

KEY:1

69. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased
1) the bulb glows dimmer 2) the bulb glows brighter
3) total impendance of the circuit is unchanged
4) total impendance of the circuit increases

KEY:2

ASSERTION & REASON

1) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
2) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
3) Assertion is true but Reason is false
4) Assertion is false but Reason is true

70. Assertion (A): The average value of $<\sin^2 \omega t>$ is zero.

Reason (R): The average value of function $F(t)$ over a period T is $<F(t)>=\frac{1}{T}\int_0^T F(t)dt$

KEY:4

71. Assertion (A): If current varies sinusoidally the average power consumed in a cycle is zero.
    Reason (R): If current varies sinusoidally the average power consumed is zero

KEY:4

72. Assertion (A) : The power consumed in an electric circuit is never negative
    Reason (R) : The average power consumed in an electric circuit is $P=\frac{V^2}{R}=I^2R$

KEY:1

73. Assertion (A): The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance circuit.
    Reason (R): The inductive reactance is directly proportional to the inductance and to the frequency of the varying current.

KEY:2

74. Assertion (A) : An ac emf which oscillates symmetrically about zero, the current it sustains also oscillates symmetrically about zero.
    Reason (R): In any circuit element, current is always in the phase with voltage

KEY:4

75. Assertion (A): A lamp is connected in series with a capacitor and ac source connected across their terminals consequently current flow in the circuit and the lamp will shine.
    Reaosn(R): capacitor block dc current and allow ac current

KEY:1

76. Assertion (A): An electric lamp is connected in series with a long solenoid of copper with air core and then connected to AC source. If an iron rod is inserted in solenoid the lamp will become dim.
    Reason (R): If iron rod is inserted in solenoid, the induction of solenoid increases.

KEY:1

77. An inductor, capacitor and resistance connected in series. The combination is connecte across AC source.
    Assertion (A): Peak current through each remains same
    Reason (R) : Average power delivered by source is equal to average power consumed by resistance

KEY:.2

78. Assertion (A): when frequency is greater than resonance frequency in a series LCR circuit, it will be an inductive circuit.
    Reason (R): Resultant voltage will lead the current

KEY:1

79. Assertion (A): Maximum power is dessipated in a circuit (through R) in resonance
    Reason (R) : At resonance in a series LCR circuit, the voltage across indcutor and capacitor are out of phase.

KEY:1

80. Assertion (A): The D.C and A.C both can be measured by a hot wire instrument.
    Reason (R) : The hot wire instrument is based on the principle of magnetic effect of current

KEY:3

78. Assertion (A): The electrostatic energy stored in capacitor plus magnetic energy stored in inductor will always be zero in a series LCR circuit driven by ac voltage source under condition of resonance.
Reason (R) : The complete voltage of ac source appears across the resistor in a series LCR circuit driven by ac voltage source under condition of resonance.

KEY:4

79. Assertion (A): The r.m.s. value of alternating current is defined as the square root of the average of I^2 during a complete cycle.
Reason (R) : For sinusoidal a.c.

$$\left(I = I_0 \sin wt\right) I_{rms} = \frac{I_0}{\sqrt{2}}$$

KEY:2

80. Assertion (A): In series LCR circuit resonance can take place.
Reason (R) : Resonance takes if inductive reactance and capacitive reactance are equal with phase difference 180^0.

KEY:1

81. The r.m.s. value of potential due to superposition of given two alternating potentials $E_1 = E_0 \sin \omega t$ and $E_2 = E_0 \cos \omega t$ will be

1) E_0 2) $2E_0$ 3) $E_0\sqrt{2}$ 4) Zero

KEY:1

82. The current does not rise immediately in a circuit containing inductance

1) because of induced emf 2) because of high voltage drop
3) both 1 and 2 4) because of joule heating

KEY:3

83. A stepup transformer develops 400V in secondary coil for an input of 200V A.C. Then the type of transformer is

1) Steped down 2) Steped up 3) Same 4) Same but with reversed direction

KEY:2

84. In an AC circuit containing only capacitance the current

1) leads the voltage by 180^o 2) lags the voltage by 90^o
3) leads the voltage by 90^o 4) remains in phase with the voltage

KEY:3

85. A step up transformer is connected on the primary side to a rechargable battery which can deliver a large current. If a bulb is connected in the secondary, then

1) the bulb will glow very bright 2) the bulb will get fused
3) the bulb will glow, but with less brightness 4) the bulb will not glow

KEY:4

86. When an a.c source is connected across a resistor

1) The current leads the voltage in phase 2) The current lags behind the voltage in phase
3) The current and voltage are in same phase 4) The current and voltage are out of phase

KEY:3

ALERNATING CURRENT

JEE MAIN PREVIOUS YEARS QUESTIONS :

1. An alternating voltage $v(t) = 220 \sin\ 100$Àt volt is applied to a purely resistive load of 50Ω. The time taken for the current to rise from half of the peak value to the peak value is: [8 April 2019 I]

(a) 5 ms (b) 2.2 ms (c) 7.2 ms (d) 3.3 ms

SOLUTION: (d)

As $V(t) = 220 \sin 100\pi t$

so, $I(t) = \frac{220}{50} \sin 100\pi t$

i.e., $I = I_m = \sin(100\pi t)$

For $I = I_m$

$$t_1 = \frac{\pi}{2} \times \frac{1}{100\pi} = \frac{1}{200} \text{sec.}$$

and for $I = \frac{I_m}{2}$

$$\Rightarrow \frac{I_m}{2} = I_m \sin(100\pi t_2) \Rightarrow \frac{\pi}{6} = 100\pi t_2$$

$$\Rightarrow t_2 = \frac{1}{600} s$$

$$t_{req} = \frac{1}{200} - \frac{1}{600} = \frac{2}{600} = \frac{1}{300} s = 3.3 \text{ ms}$$

2. A small circular loop ofwire ofradius a is located at the centre ofa much larger circular wire loop ofradius b. The two loops are in the same plane. The outer loop ofradius b carries an alternating current $I = I_o \cos$ (oot). The emf induced in the smaller inner loop is nearly: [Online April 8, 2017]

(a) $\frac{\pi\mu_0 I_o}{2} . \frac{a^2}{b} 0o \sin(oot)$ (b) $\frac{\pi\mu_0 I_o}{2} . \frac{a^2}{b} 00 \cos(00t)$

(c) $\pi\mu_0 I_o \frac{a^2}{b} 0) \sin(0)t)$ (d) $\frac{\pi\mu_0 I_o b^2}{a}$(ocos $(c0t)$

SOLUTION: (a)

For two concentric circular coil,

Mutual Inductance $M = \frac{\mu_0 \pi N_1 N_2 a^2}{2b}$

here, $N_1 = N_2 = 1$

Hence, $M = \underline{\mu_0 \pi a^2}$ ..... (i)

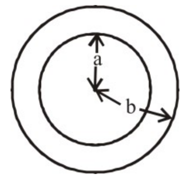
$2b$

and given $I = I_0 \cos$ oot (ii)

Now according to Faraday^{1}s second law induced emf

$$e = -M\frac{dI}{dt}$$

Fromeq. (ii),



$$e = \frac{-\mu_0 \pi a^2}{2b}\frac{d}{dt}(I_0 \cos o)t)$$

$$e = \frac{\mu_0 \pi a^2}{2b} I_0 \sin o)t(0))$$

$$e = \frac{\pi \mu_0 I_0}{2}.\frac{a^2}{b} 0) \sin o)t$$

3. Asinusoidal voltage V(t) $= 100 \sin(500t)$ is applied acrossa pure inductance of$L = 0.02$ H. The current through the coil is: [Online Apri112, 2014]

(a) $10 \cos(500t)$ (b) $-10 \cos(500t)$ (c) $10 \sin(500t)$ (d) $-10 \sin(500t)$

SOLUTION: $(.b)$

In a pure inductive circuit current always lags behindthe emf by $\frac{\pi}{2}$.

If $v(t) = v_0$ sinoot

then $I = I_0 \sin(0)t - \frac{\pi}{2})$

Now, given $v(t) = 100 \sin(500t)$

and $I_0 = \frac{E_0}{coL} = \frac{100}{500\times0.02} [\because L = 0.02H]$

$$I_0 = 10 \sin\left(500t - \frac{\pi}{2}\right)$$

$$I_0 = -10 \cos(500t)$$

4. In an a. c. circuit the voltage applied is $E = E_0 \sin o)\ t$. The resulting current in the circuit is $I = {}_{I_0} \sin\left((j)t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by [2007]

(a) $P = \sqrt{2}E_0 I_0$ (b) $P = \frac{E_0 I_0}{\sqrt{2}}$ (c) $P =$ zero (d) $P = \frac{E_0 I_0}{2}$

SOLUTION: (c)

We know that power consumed in a. c. circuit is given

by,

$$P = E_{rms}.I_{rms} \cos\varphi$$

Here, $E = E_0 \sin\ ciJt$

$$I = I_0 \sin\left(tiJt - \frac{\pi}{2}\right)$$

This means the phase difference, is $\varphi = \frac{\pi}{2}$

$$\cos\varphi = \cos\frac{\pi}{2} = 2$$

$$P = E_{rms}.I_{rms}.\cos\frac{\pi}{2} = 0$$

5. In a uniform magnetic field of induction B a wire in the form of a semicircle ofradius r rotates about the diameter of the circle with an angular frequency 00. The axis of rotation is perpendicular to the field. Ifthe total resistance ofthe circuit is R, the mean power generated per period of rotation is [2004]

(a) $\frac{(B\pi r o))^2}{2R}$ (b) $\frac{(B\pi r^2 o))^2}{8R}$ (c) $\frac{B\pi r^2 0)}{2R}$ (d) $\frac{(B\pi r o)^2)^2}{8R}$

SOLUTION: .(b)

$$\varphi = \vec{B}.\vec{A};\ \varphi = BA\cos cj)t$$

$$\varepsilon = -\frac{d\varphi}{dt} = 0)BA\sin o)t\ ;\ i = \frac{coBA}{R}\sin o)t$$

$$P_{inst} = i^2R = \left(\frac{\sigma)BA}{R}\right)^2 \times R\sin^2\sigma)t$$

$$P_{avg} = \frac{\int_0^T P_{Inst} \times dt}{\int_0^T dt} = \frac{((iJBA)^2 \int_0^T \sin^2(iJtdt}{R\int_0^T dt} = \frac{1}{2}\frac{((oBA)^2}{R}$$

$$P_{avg} = \frac{(0)B\pi r^2)^2}{8R}\left[A = \frac{\pi r^2}{2}\right]$$

6. Alternating current can not be measured by D.C. ammeter because [2004]

(a) Average value of current for complete cycle is zero (b) A.C. Changes direction

(c) A.C. can not pass through D.C. Ammeter (d) D.C. Ammeter will get damaged.

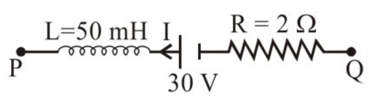
SOLUTION: . (a)

D.C. ammeter measure average value ofcurrent. In AC

current, average value ofcurrent in complete cycle is zero.

Hence reading will be zero.

7. Apart of a complete circuit is shown in the figure. At some instant, the value ofcurrent I is 1 A and it is decreasing at a rate of $10^2 As^{-1}$. The value ofthe potential difference $V_P - V_Q$,(in volts) at that instant, is . [NA Sep. 06, 2020 (I)]



SOLUTION: . (33)

$$\text{Here, } L = 50\text{mH} = 50 \times 10^{-3}\text{H};\ I = 1\text{A}, R = 2\Omega$$

$$V_P - L\frac{dl}{dt} - 30 + RI = V_Q$$

$$\Rightarrow V_P - V_Q = 50 \times 10^{-3} \times 10^2 + 30 - 1 \times 2$$

$$= 5 + 30 - 2 = 33 \text{ V.}$$

8. An AC circuit has $R = 100\Omega$, $C = 2\mu\text{F}$ and $L = 80\text{mH}$, connected in series. The quality factor ofthe circuit is: [Sep. 06, 2020 (I)]

(a) 2 (b) 0.5 (c) 20 (d) $\lhd\ \omega$

SOLUTION: . (a)

Quality factor,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{100}\sqrt{\frac{80 \times 10^{-3}}{2 \times 10^{-6}}}$$

$$= \frac{1}{100}\sqrt{40 \times 10^3} = \frac{200}{100} = 2$$

9. In a series LR circuit, power of 400 W is dissipated from a source of250 V, 50 Hz. The power factor ofthe circuit is 0.8. In order to bring the power factor to unity, a capacitor ofvalue C is added in series to the L and R. Taking the value C as $\left(\frac{\text{n}}{3\pi}\right)\mu\text{F}$, then value ofn is_. [NA Sep. 06, 2020 (ID]

SOLUTION: . (400)

Given: Power $P = 400\text{W}$, Voltage $V = 250\text{V}$

$$P = V_m \cdot I_{\text{rms}} \cdot \cos\varphi$$

$$\Rightarrow 400 = 250 \times I_{\text{rms}} \times 0.8 \Rightarrow I_{\text{rms}} = 2\text{A}$$

Using $P = I_{\text{rms}}^2 R$

$$(I_{\text{nns}})^2 \cdot R = P \Rightarrow 4 \times R = 400$$

$$\Rightarrow R = 100\Omega$$

Power factor is, $\cos\varphi = \frac{R}{\sqrt{R^2 + X_L^2}}$

$$\Rightarrow 0.8 = \frac{100}{\sqrt{100^2 + X_L^2}} \Rightarrow 100^2 + X_L^2 = \left(\frac{100}{0.8}\right)^2$$

$$\Rightarrow X_L = \sqrt{-100^2 + \left(\frac{100}{08}\right)^2} \Rightarrow X_L = 75\Omega$$

When power factor is unity, $X_C = X_L = 75 \Rightarrow \frac{1}{\text{co}C} = 75$

$$\Rightarrow C=\frac{1}{75\times 2\pi\times 50}=\frac{1}{7500\pi}F$$

$$(10^6\ 1)$$

$$=\left(\frac{-\times-}{25003\pi}\right)=\frac{400}{3\pi}\mu F$$

$$N=400$$

10. A series L - R circuit is connected to a battery ofemf V. If the circuit is switched on at $t=0$, then the time at which the energy stored in the inductor reaches $\left(\frac{1}{n}\right)$ times of its maximumvalue, is: [Sep. 04, 2020 (II)]

(a) $\frac{L}{R}\ \ln\left(\frac{\sqrt{n}}{\sqrt{n}-1}\right)$ (b) $\frac{L}{R}\ \ln\left(\frac{\sqrt{n}+1}{\sqrt{n}-1}\right)$ (c) $\frac{L}{R}\ \ln\left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)$ (d) $\frac{L}{R}\ \ln\left(\frac{\sqrt{n}-1}{\sqrt{n}}\right)$

SOLUTION: . (a)

Potential energy stored in the inductor$U=\frac{1}{2}LI^2$

During growth ofcurrent,$i=I_{\max}\left(1-e^{-Rt/L}\right)$

For U to be $\frac{U_{\max}}{n}$; i has to be $\frac{I_{\max}}{\sqrt{n}}$

$$\frac{I_{\max}}{\sqrt{n}}=I_{\max}\left(1-e^{-Rt/L}\right)$$

$$\Rightarrow e^{-Rt/L}=1-\frac{1}{\sqrt{n}}=\frac{\sqrt{n}-1}{\sqrt{n}}$$

$$\Rightarrow -\frac{Rt}{L}=\ \ln\left(\sqrt{n}\right)$$

$$\left(\sqrt{n}-1\right)$$

$$\Rightarrow t=\frac{L}{R}\ln(\quad)(\quad)$$

11. A 750 Hz, 20 V(rms) source is connected to a resistance of 100 Ω, an inductance of0.1803 H and a capacitance of10 μF all in series. The time in which the resistance (heat capacity2 J/^{0}C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to: [Sep. 03, 2020 (I)]

(a) 418 s (b) 245 s (c) 365 s d) 348 s

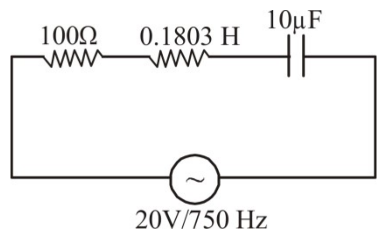
SOLUTION: . (d)

$$\text{Here, } R=100, X_L=L_{0)}=0.1803\times 750\times 2\pi=850\Omega,$$

$$X_c=\frac{1}{Cc0}=\frac{1}{10^{-5}\times 2\pi\times 750}=21.23\Omega$$

$$\text{Impedance } Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$=\sqrt{100^2+(850-21.23)^2}=834.77=835$$



$$H = i_{\text{rms}}^2 Rt = \left(\frac{V_{\text{rms}}}{|Z|}\right)^2 RT = (ms)\Delta t$$

$$\Rightarrow \frac{20}{835} \times \frac{20}{835} \times 100t = (2) \times 10$$

$$V_{\text{rms}} = 20\text{V and } \Delta\text{t} = 10^\circ\text{C}$$

$$\text{Time, } t = 348.61\text{s.}$$

12. **An inductance coil has a reactance of100 Ω. When an AC signal offrequency 1000 Hz is applied to the coil, the applied voltage leads the current by45°. The self - inductance of the coil is: [Sep. 02, 2020 (II)]**

(a) 1.1×10^{-2}H **(b) 1.1×10^{-1}H** **(c) 5.5×10^{-5}H** **(d) 6.7×10^{-7}H**

SOLUTION: . (a)

Given,

Reactance of inductance coil, $Z = 100\Omega$

Frequency ofAC signal, $v = 1000$ Hz

Phase angle, $\varphi = 45^\circ$

$$\tan\varphi = \frac{X_L}{R} = \tan 45^\circ = 1$$

$$\Rightarrow X_L = R$$

$$\text{Reactance, } Z = 100 = \sqrt{X_L^2 + R^2}$$

$$\Rightarrow 100 = \sqrt{R^2 + R^2}$$

$$\Rightarrow \sqrt{2}R = 100 \Rightarrow R = 50\sqrt{2}$$

$$X_L = 50\sqrt{2}$$

$$\Rightarrow Lo) = 50\sqrt{2}\left(\because X_{L=0)}L\right)$$

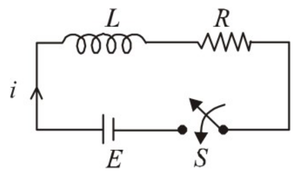
$$\Rightarrow L = \frac{50\sqrt{2}}{2\pi \times 1000}(\because 0) = 2\pi v)$$

$$= \text{mH}\underline{25\sqrt{2}}$$

$$\pi$$

$$= 1.1 \times 10^{-2}\text{H}$$

13. Consider the LR circuit shown in the figure. Ifthe switch S is closed at $t = 0$ then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is : [12 April 2019 II]



(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$ (c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

SOLUTION: . (b)

We have, $i = i_0(1 - e^{-t/c}) = \frac{\varepsilon}{R}(1 - e^{-t/c})$

Charge, $q = \int_0^\tau i\, dt$

$$= \frac{\mathcal{E}}{R}\int_0^\tau (1 - e^{-t/\tau})\, dt = \frac{E\,\tau}{R\,e} = \frac{E}{R} \times \frac{(L/R)}{e} = \frac{EL}{2.7R^2}$$

14. A coil of selfinductance 10 mH and resistance $0.1\ \Omega$ is connected through a switch to a battery of internal resistance 0.9Ω. After the switch is closed, the time taken for the current to attain 80% ofthe saturation value is [take $ln5 = 1.6$] [10 April 2019 II]

(a) $0.324s$ (b) $0.103s$ (c) $0.002s$ (d) $0.016s$

SOLUTION: . (d)

$$I = I_0\left(1 - e^{-\frac{Rt}{L}}\right)(\quad) \text{ Here } R = R_L + r = 1\Omega$$

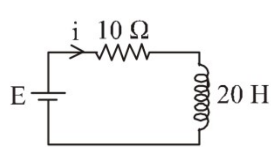
$$0.8I_0 = I_0\left(1\left(-e^{\frac{t}{01}}\right)\right)$$

$$\Rightarrow 0.8 = 1 - e^{-100t}$$

$$\Rightarrow e^{-100t} = 0.2 = \left(\frac{1}{5}\right)$$

$$\Rightarrow 100t = \ln 5 \Rightarrow t = \frac{1}{100}\ln 5 = 0.016s$$

15. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule' s heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is : [8 April 2019 I]



(a) $\frac{2}{ln2}$ (b) $\frac{1}{2}ln2$ (c) $2\ ln2$ (d) $ln2$

SOLUTION: . (c)

$$i^2 R = \left(\tau \frac{di}{dt}\right) i$$

$$\Rightarrow \frac{di}{dt} = \frac{i}{\tau}$$

$$\Rightarrow \mathrm{t} = \tau ln2 = 2ln2 \left[as\tau = \frac{L}{R} = \frac{20}{10} = 2\right]$$

16. A circuit connected to an ac source $ofemfe = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the $emfe$ and current i. Which ofthe following circuits will exhibit this? [8 April 2019 II]

(a) RL circuit with $\mathrm{R} = 1\mathrm{k}\Omega$ and $\mathrm{L} = 10\mathrm{mH}$
(b) RL circuit with $\mathrm{R} = 1\mathrm{k}\Omega$ and $\mathrm{L} = 1\mathrm{mH}$
(C) RC circuit with $\mathrm{R} = 1\mathrm{k}\Omega$ and $\mathrm{C} = 1\mu\mathrm{F}$
(d) RC circuit with $\mathrm{R} = 1\mathrm{k}\Omega$ and $\mathrm{C} = 10\mu\mathrm{F}$.

SOLUTION: . (d)

$$0) = 100\mathrm{rad/s}$$

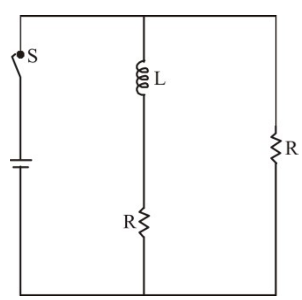
We know that

$$\tan\varphi = \frac{X_C}{R} = \frac{1}{\mathrm{co}CR}$$

$$\text{or } \tan 45\mathrm{o} = \frac{1}{\mathrm{co}CR} \text{ or } 0)\mathrm{CR} = 1$$

$$\text{LHS: } 0)\mathrm{CR} = 10 \times 10 \times 10^{\triangleleft} \times 10^3 = 1$$

17. In the figure shown, a circuit contains two identicalresistors with resistance $\mathrm{R} = 5\Omega$ and an inductance with $\mathrm{L} = 2\mathrm{mH}$. An ideal battery of15 V is connected in the circuit. What will be the current through the battery long after the switch is closed? [12 Jan. 2019 I]

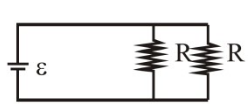


(a) $5.5\mathrm{A}$ (b) $7.5\mathrm{A}$ (c) 3 A (d) 6 A

SOLUTION: . (d)

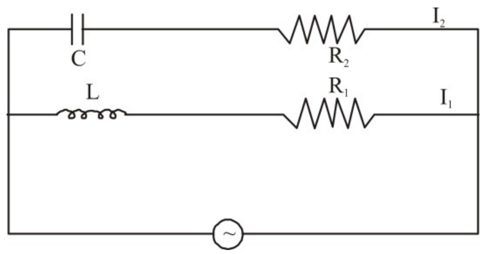
Long time after switch is closed, the inductor will be

idle so, the equivalent diagram will be as below



$$I = \frac{\varepsilon}{\left(\frac{R \times R}{R + R}\right)} = \frac{2\varepsilon}{R} = \frac{2 \times 15}{5} = 6A$$

18.



In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\Omega$, $L = \frac{\sqrt{3}}{10}H$ and $R_1 = 10\Omega$. Current in $L - R_1$ path is I_1 and in $C - R_2$ path it is I_2. The voltage ofA.C source is given by, $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is:

[12 Jan. 2019 II]

(a) 60° (b) 30° (c) 90° (d) 0

SOLUTION: . (Bonus)

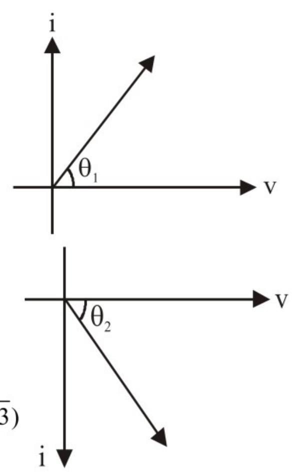
Capacitive reactance,

$$X_c = \frac{1}{\omega C} = \frac{4}{10^{-6} \times \sqrt{3} \times 100} = \frac{2 \times 10^4}{\sqrt{3}}$$

$\tan\theta_1 = \frac{X_C}{R_2} = \frac{10^3}{\sqrt{3}}$

θ_1 is close to 90°

For L-R circuit



$X_L = \omega L = 100 \times \frac{\sqrt{3}}{10} = 10\sqrt{3}$

$R_1 = 10$

$\tan\theta_2 = \frac{X_L}{R_1}$

$\tan\theta_2 = \sqrt{3} \Rightarrow \theta_2 = \tan^{-1}(\sqrt{3})$

$\theta_2 = 60°$

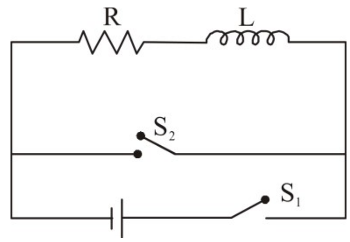
So, phase difference comes out $90° + 60° = 150°$

IfR_2 is 20 $K\Omega$

then phase difference comes out to be $60 + 30 = 90°$.

Therefore Ans. is Bonus

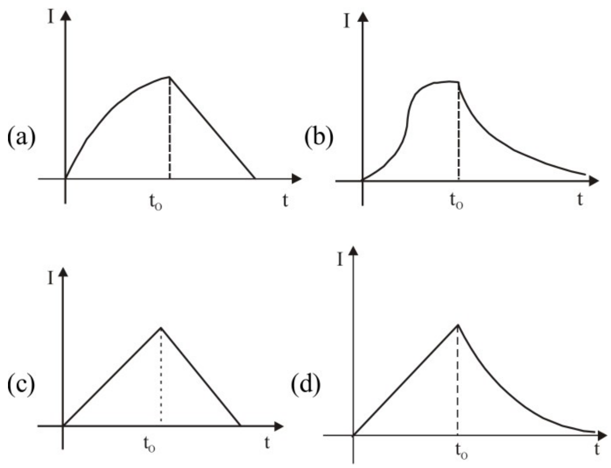
19. In the circuit shown,



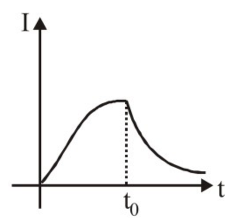
ε

the switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0) , the switch S_1 is opened and S_2 is closed. the behaviour ofthe current I as a function

oftime t’ is given by: [11 Jan. 2019 II]



SOLUTION:(b)



The current will grow for the time $t = 0$ to $t = t_0$ and after that decay of current takes place.

20. A series AC circuit containing an inductor (20mH), a capacitor $(120\mu\text{F})$ and a resistor (60Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is: [9 Jan. 2019 I]

(a) $5.65 \times 10^2\text{J}$ (b) $2.26 \times 10^3\text{J}$ (c) $5.17 \times 10^2\text{J}$ (d) $3.39 \times 10^3\text{J}$

SOLUTION: . (c)

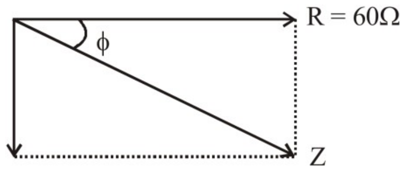
Given: $R = 60\Omega$, $f = 50$ Hz, $0)^{=2\pi f=}100\pi$ and $v = 24v$

$$C = 120\mu f = 120 \times 10^{-6} f$$

$$x_C = \frac{1}{\text{coC}} = \frac{1}{100\pi \times 120 \times 10^{-6}} = 26.52\Omega$$

$$x_L = 0)L = 100\pi \times 20 \times 10^{-3} = 2\pi\Omega$$

$$x_C - x_L = 20.24 \approx 20$$



$$z = \sqrt{R^2 + (x_C - x_L)^2}$$

$$z = 20\sqrt{10}\Omega$$

$$\cos\varphi = \frac{R}{z} = \frac{60}{20\sqrt{10}} = \frac{3}{\sqrt{10}}$$

$P_{avg} = VI\cos\varphi, I = \frac{v}{z} = \frac{v^2}{z}\cos\varphi = 8.64$ watt Energy dissipated(Q) in time $t = 60s$ is

$$Q = P.t = 8.64 \times 60 = 5.17 \times 10^2 J$$

21. In LC circuit the inductance $L = 40mH$ and capacitance $C = 100\mu F$. Ifa voltage $V(t) = 10\sin(314t)$ is applied to the circuit, the current in the circuit is given as: [9 Jan. 2019 II]

(a) $0.52\ \cos 314t$ (b) $10\ \cos 314t$ (c) $5.2\ \cos 314t$ (d) $0.52\ \sin 314t$

SOLUTION: (a)

Given, Inductance, $L = 40mH$

Capacitance, $C = 100\mu F$

Impedance, $Z = X_C - X_L$

$$\Rightarrow Z = \frac{1}{coC} - tiJL\ (\because X_c = \frac{1}{coC} \text{ and } X_L = (iJL)$$

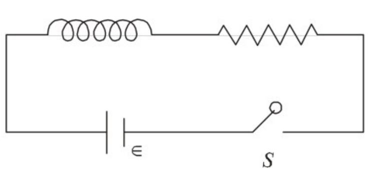
$$= \frac{1}{314 \times 100 \times 10^{-6}} - 314 \times 40 \times 10^{-3}$$

$$= 19.28\Omega$$

$$\text{Current, } i = \frac{V_0}{Z}\sin(0)t + \pi/2)$$

$$\Rightarrow j = \frac{10}{19.28}\cos c0t = 0.52\cos(314t)$$

22.



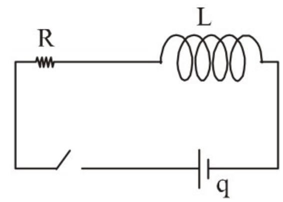
As shown in the figure, abattery ofemf $\in$ is connected to an inductor L and resistance R in series. The switch is closed at $t = 0$. The total charge that flows from the battery, between $t = 0$ and $t = t_c$ (t_c is the time constant ofthe circuit) is: [8 Jan. 2020 II]

(a) $\frac{\in R}{eL^2}$ (b) $\frac{\in L}{R^2}\left(1-\frac{1}{e}\right)$ (c) $\frac{\in L}{R^2}$ (d) $\frac{\in R}{eL^2}$

SOLUTION: (a)

For series connection ofa resistor and inductor, time

variation ofcurrent is $I = I_0\left(1 - e^{-t/T_c}\right)$



Here, $T_C = \frac{L}{R}$

$$q = \int_0^{T_c} i\,dt$$

$$\Rightarrow \int dq = \int \frac{E}{R}\left(1 - e^{-t/t_c}\right)dt$$

$$\Rightarrow q = R{-}\in \left\{ {}^{-t/t}_{c^{e}} \right\}$$

$$\Rightarrow q = R{-}\in \left[t_C + \frac{t_C}{e} - t_C\right]$$

$$\Rightarrow q = \frac{L}{\text{Re}}\overline{R} \in$$

$$q = \frac{\in L}{R^2 e}$$

23. ALCR circuit behaves like adamped harmonic oscillator. Comparing it with a physical spring - mass damped oscillator having damping constant 'b', the correct equivalence would be: [7 Jan. 2020 I]

(a) $L \leftrightarrow m, C \leftrightarrow k, R \leftrightarrow b$ (b) $L \leftrightarrow \frac{1}{b}, C \leftrightarrow \frac{1}{m}, R \leftrightarrow \frac{1}{k}$

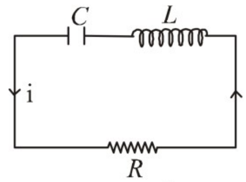
(c) $L \leftrightarrow k, C \leftrightarrow b, R \leftrightarrow m$ (d) $L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$

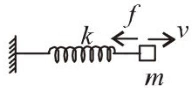
SOLUTION: (d)

In damped harmonic oscillation,

$$\frac{md^{2x}}{dt^2} = -kx - bv$$

$$\Rightarrow \frac{md^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \text{ (i)}$$





In LCR circuit, $\frac{-q}{C} - iR - \frac{Ldi}{dt} = 0$

$$L\frac{d^2}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0 \text{ (ii)}$$

Comparing equations (i) & (ii)

$$L \leftrightarrow m, C \leftrightarrow \frac{1}{k}, R \leftrightarrow b$$

24. An emfof20V is applied at time $t = 0$to acircuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time $t = \infty$ and at $t = 40$s is close to:(Take $e^2 = 7.389$) [7 Jan. 2020 II]

(a) 1.06 (b) 1.15 (c) 1.46 (d) 0.84

SOLUTION: . (a)

The current (I) in LR series circuit is given by

$$I = \frac{V}{R}\left(1 - e^{\frac{tR}{L}}\right)(\quad)$$

At $t = \infty$,

$$I_\infty = \frac{20}{5}\left(I - e^{\frac{-\infty}{L/R}}\right) = 4(\quad) \text{ (i)}$$

At $t = 40$s,

$$\left(1 - e^{\frac{-40\times5}{10\times10^{-3}}}\right) = 4(1 - e^{-20,000}) \text{ (ii)}$$

Dividing (i) by (ii) we get

$$\Rightarrow \frac{I_\infty}{I_{40}} = \frac{1}{1-e^{-20,000}},$$

25. In an a. c. circuit, the instantaneous e. m. f. and current aregiven by $e = 100 \sin 30t i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a. c., the average power consumed by the circuit and the wattless current are, respectively: [2018]

(a) 50W, 10A (b) $\frac{1000}{\sqrt{2}}$W, 10A (c) $\frac{50}{\sqrt{2}}$W, 0 (d) 50W, 0

SOLUTION:(b)

As we know, average power $P_{avg} = V_{nns}I_{nns}\cos\theta$

$$= \left(\frac{V_0}{\sqrt{2}}\right)\left(\frac{I_0}{\sqrt{2}}\right)\cos\theta = \left(\frac{100}{\sqrt{2}}\right)\left(\frac{20}{\sqrt{2}}\right)\cos 45^\circ (\because \theta = 45^\circ)$$

$$P_{avg} = \frac{1000}{\sqrt{2}} \text{ watt}$$

$$\text{Wattless current } I = I_{rms} \sin \theta$$

$$= \frac{I_0}{\sqrt{2}} \sin \theta = \frac{20}{\sqrt{2}} \sin 45^\circ = 10A$$

26 . For an RLC circuit driven with voltage ofamplitude $v_m$and frequency $0)0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by: [2018]

(a) $\frac{c0_0 L}{R}$ (b) $\frac{c0_0 R}{L}$ (c) $\frac{R}{(c0_0 C)}$ (d) $\frac{CR}{c0_0}$

SOLUTION: . (a)

$$\text{Quality factor } Q = \frac{00_0}{2\Delta c0} = \frac{c0_0 L}{R}$$

27. A sinusoidal voltage of peak value 283 V and angular frequency $320/s$ is applied to a series LCR circuit. Given that $R = 5\Omega, L = 25mH$ and $C = 1000\mu F$. The total impedance,and phase difference between the voltage across the source and the current will respectively be: [Online April 9, 2017]

(a) 10 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$ (b) 7 Ω and 45° (c) 10 Ω and $\tan^{-1}\left(\frac{8}{3}\right)$ (d) 7 Ω and $\tan^{-1}\left(\frac{5}{3}\right)$

SOLUTION: (b)

Given,

$$V_0 = 283 \text{ volt}, co = 320, R = 5\Omega, L = 25mH, C = 1000\mu F$$

$$x_L = 0)L = 320 \times 25 \times 10^3 = 8\Omega$$

$$x_c = \frac{1}{coC} = \frac{1}{320 \times 1000 \times 10^{-6}} = 3.1\Omega$$

Total impedance ofthe circuit:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{25 + (49)^2} = 7\Omega$$

Phase difference between the voltage and current

$$\tan (|) = \frac{x_L - x_C}{R}$$

$$\tan \varphi = \frac{4.9}{5} \approx 1 \Rightarrow \varphi = 45^\circ$$

28. An arc lamp requires a direct current of10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to: [2016]

(a) $0.044H$ (b) $0.065H$ (c) 80 H (d) $0.08H$

SOLUTION: . (b)

$$i = \frac{e}{\sqrt{R^2 + X_L^2}} = \frac{e}{\sqrt{R^{2+0)^2}L^2}} = \frac{e}{\sqrt{R^2 + 4\pi^2 v^2 L^2}}$$

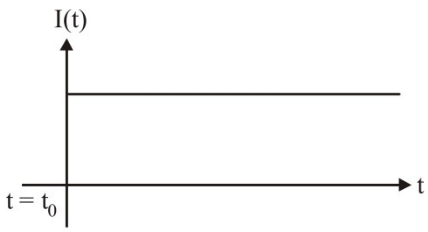
$$10 = \frac{220}{\sqrt{64 + 4\pi^2 (50)^2 L}}$$

$$\left[\because R = \frac{V}{I} = \frac{80}{10} = 8\right]$$

On solving we get

$$L = 0.065H$$

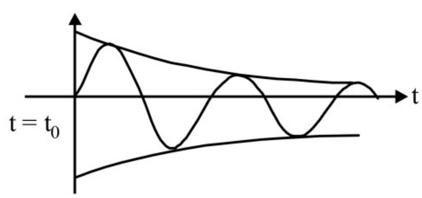
29. **A series LR circuit is connected to a voltage source with $V(t) = V_0$ sinoot. After very large time, current I(t) behaves as $(t_0 >> \frac{L}{R})$:** **[Online April 9, 2016]**



(a)

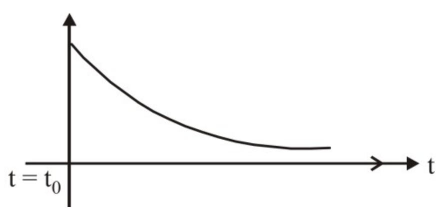
I(t)

(b)



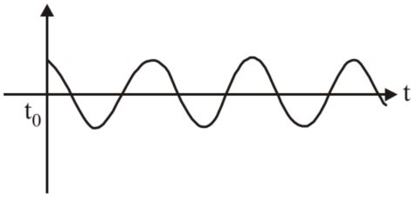
I(t)

(c)



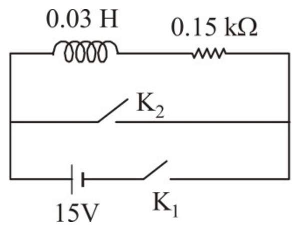
I(t)

(d)



SOLUTION: . (d)

30. An inductor $(L = 0.03H)$ and a resistor $(R = 0.15k\Omega)$ are connected in series to a battery of $15V$ emfin a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1$ ms, the current in the circuit willbe: $(\cong 150)$ [2015]



(a) $6.7mA$ (b) $0.67mA$ (c) $100mA$ (d) 67 mA

SOLUTION:(b)

$$I(0) = \frac{15 \times 100}{0.15 \times 10^3} = 0.1A$$

$$I(\infty) = 0$$

$$I(t) = [I(0) - 1(\infty)]e^{\frac{-t}{L/R}} + i(\infty)$$

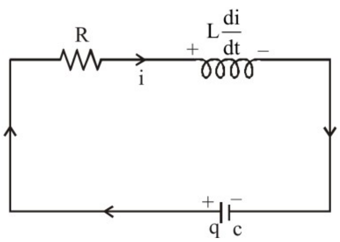
$$I(t) = 0.1e^{\frac{-t}{L/R}} = 0.1e^{\frac{R}{L}}$$

$$I(t) = 0.1e^{\frac{0.15 \times 1000}{0.03}} = 0.67mA$$

31. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

SOLUTION: . (c)

From KVL at anytime t



$$\frac{q}{c} - iR - L\frac{di}{dt} = 0$$

$$j = -\frac{dq}{dt} \Rightarrow \frac{q}{c} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{Lc} = 0$$

From damped harmonic oscillator, the amplitude is given by $A = A_o e - \frac{dt}{2m}$

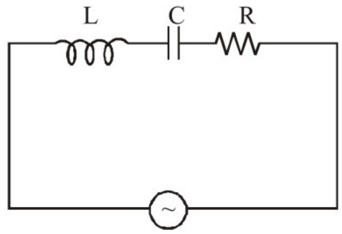
Double differential equation

$$\frac{d^{2x}}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

$$Q_{\max} = Q_0 e^{\frac{Rt}{2L}} \Rightarrow Q^2_{\max} = Q_0^2 e^{\frac{Rt}{L}}$$

Hence damping will be faster for lesser selfinductance.

32. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C′, when joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C′, must have been connected in : [Online April 11, 2015]



$\mathrm{V} = \mathrm{V_0 sinteo}$

(a) series with C and has a magnitude $\frac{\mathrm{C}}{(0)^2\mathrm{LC}-1)}$ (b) series with C and has a magnitude $\frac{1-\mathrm{c}0^2\mathrm{LC}}{0)^2\mathrm{L}}$

(c) parallel with C and has a magnitude $\frac{1-\mathrm{c}0^2\mathrm{LC}}{0)^2\mathrm{L}}$ (d) parallel with C and has a magnitude $\frac{\mathrm{C}}{(0)^2\mathrm{LC}-1)}$

SOLUTION: (c)

Power factor

$$\cos\varphi = \frac{R}{\sqrt{R^2 + \left[\sigma \mathrm{J} L - \frac{1}{\sigma \mathrm{J}(C + C^1)}\right]^2}} = 1$$

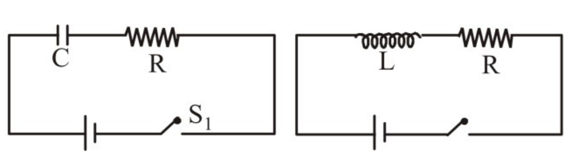
On solving we get,

$$0)L = \frac{1}{0)(C + C')}$$

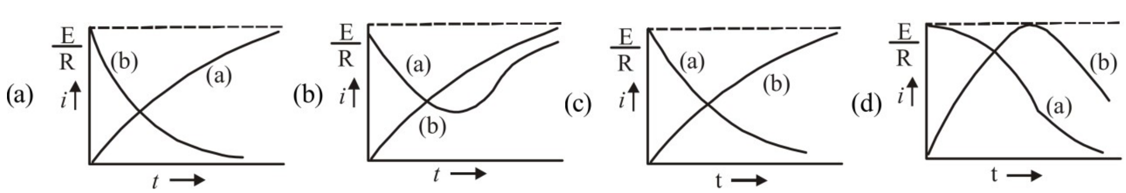
$$C^{\dagger} = \frac{1 - \mathrm{Q})^{2LC}}{\mathrm{Q})^{2L}}$$

Hence option (c) is the correct answer.

33. In the circuits (a) and (b) switches S_1 and S_2 are closed at $t = 0$ and are kept closed for a long time. The variation of current in the two circuits for $t \geq 0$ are roughly shown by figure (figures are schematic and not drawn to scale): [Online Apri110, 2015]



E



SOLUTION: (c)

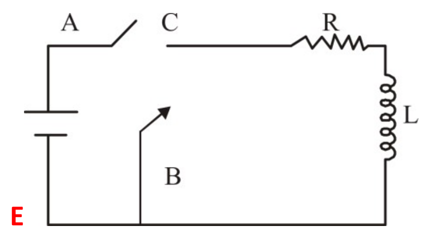
For capacitor circuit, $i = i_0 e^{-t/RC}$

For inductor circuit, $i = i_0\,(1 - e|-|RtL) -$

Hence graph (c) correctly depicts i versus t graph.

34. In the circuit shown here, the point C' is kept connectedto point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point A' and connected to point B' at time $t = 0$. Ratio of the voltage across resistance and the

inductor at $t = L/R$ will be equal to: [2014]

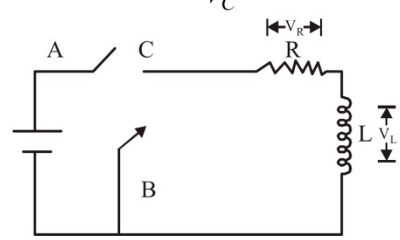


(a) $\frac{e}{1-e}$ (b) 1 (c) -1 (d) $\frac{1-e}{e}$

SOLUTION: . (c)

Applying Kirchhoffs law ofvoltage in closed loop

$$-V_R - V_C = 0 \Rightarrow \frac{V_R}{V_C} = -1$$



35 When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. Ifthe rms voltage ofthe AC sources is 100 V, the V_R is close to: [Online April 9, 2014]

(a) 50V (b) 70V (c) 90V (d) 100V

SOLUTION: . (c)

Given, $V_L : V_C : V_R = 1 : 2 : 3$

$$V = 100V$$

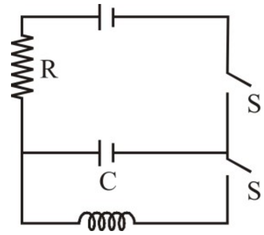
$$V_R = ?$$

As we know,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Solving we get, $V_R = 90V$

36. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which ofthe following statement is correct? [2013]



(a) Work done by the battery is half of the energy dissipated in the resistor (b) At, $t = \tau, q = CV/2$

(c) At, $t = 2\tau, q = CV(1 - e^{-2})$ (d) At, $t = 2\tau, q = CV(1 - e^{-1})$

SOLUTION: . (c)

Charge on he capacitor at any time t is given

by $q = CV(1 - e^{t/\tau})$

at $t = 2\tau$

$q = CV(1 - e^{-2})$

37. A series LR circuit is connected to an ac source of frequency 00 and the inductive reactance is equal to$2R$. Acapacitance ofcapacitive reactance equal to R is added in series with L and R. The ratio ofthe new power factor to the old one is: [Online April 25, 2013]

(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{2}{5}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $\sqrt{\frac{5}{2}}$

SOLUTION: . (d)

Power factor (old)

$$= \frac{R}{\sqrt{R^2 + X_L^2}} = \frac{R}{\sqrt{R^2 + (2R)^2}} = \frac{R}{\sqrt{5}R}$$

Power factor$_{(new)}$

$$= \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + (2R - R)^2}} = \frac{R}{\sqrt{2}R}$$

$$\frac{\text{Newpowerfactor}}{\text{O1dpowerfactor}} = -\frac{R}{\sqrt{2}R} = \sqrt{\frac{5}{2}} \frac{R}{\sqrt{5}R}$$

38. When resonance is produced in a series LCR circuit, then which ofthe following is not correct? [Online April 25, 2013]

(a) Current in the circuit is in phase with the applied voltage. (b) Inductive and capacitive reactances are equal.

(c) If R is reduced, the voltage across capacitor willincrease. (d) Impedance ofthe circuit is maximum.

SOLUTION: (d)

Impedance (Z) ofthe series LCR circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

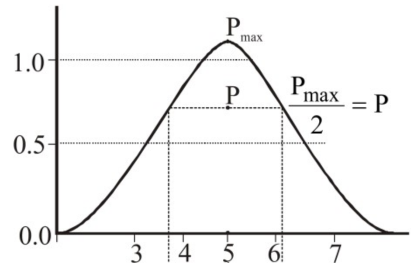
At resonance, $X_L = X_C$

Therefore, $Z_{mmimum} = R$

39. The plot given below is ofthe average power delivered to an LRC circuit versus frequency The quality factor of the circuit is: [Online April 23, 2013]

(a) 5.0 (b) 2.0 (c) 2.5 (d) 0.4

SOLUTION: . (b)



$0)_1\ 0)_0\ 0)_2$

Quality factor of the circuit

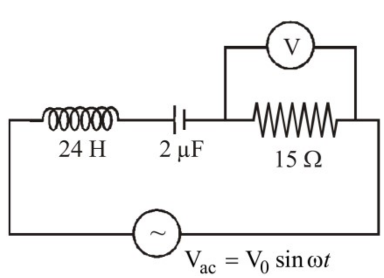
$$= \frac{0)0}{0_{2-0)1}}) = \frac{5}{2.5} = 2.0$$

40. In a series L - C - R circuit, $C = 10^{-11}$ Farad, $L = 10^{-5}$ Henryand $R = 100$ Ohm, when a constant D.C. voltage E is applied to the circuit, the capacitor acquires a charge 10^{-9} C. The D.C. source is replaced by a sinusoidal voltage source in which the peak voltage E_0 is equal to the constant D.C.voltage E. At resonance the peak value of the charge acquired by the capacitor will be : Online April 22, 2013I

(a) 10^{-15}C (b) 10^{-6}C (c) 10^{-10}C (d) 10^{-8}C

SOLUTION: . (d)

41. An LCR circuit as shown in the figure is connected to a voltage frequency can be varied.



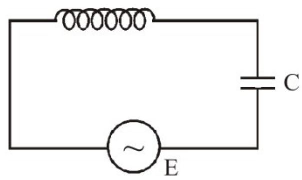
The frequency, at which the voltage across the resistor is maximum, is: [Online April 22, 2013]

(a) 902 Hz (b) 143 Hi (c) 23 Hz (d) 345 Hz

SOLUTION: . (c)

$$\text{Frequency } f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.4\sqrt{24\times 2\times 10^{-6}}} = 23\text{Hz}$$

42. In the circuit shown here, the voltage across E and C are respectively 300 V and 400 V. The voltage E ofthe ac source is: [Online April 9, 2013]



(a) 400Volt (b) 500Volt (c) 100Volt (d) 700Volt

SOLUTION: . (c)

Voltage E ofthe ac source

$$E = V_C - V_L = 400\text{ V} - 300\text{ V} = 100\text{V}$$

43. A resistance R and a capacitance C are connected in series to a battery ofnegligible internal resistance through a key. The key is closed at $t = 0$. If after t sec the voltage across the capacitance was seven times the voltage across R, the value oft is [Online May 12, 2012]

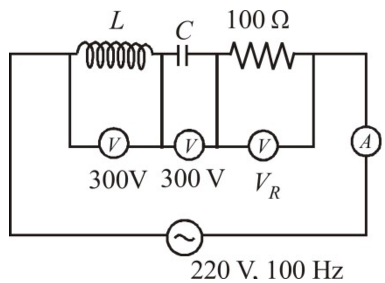
(a) $3RCln2$ (b) 2 $RCln2$ (c) 2 $RC\ell n7$ (d) 3 $RC\ell n7$

SOLUTION: . (a)

$$t = 3RCln2$$

44. In an LCR circuit shown in the following figure, what will be the readings of the voltmeter across the resistor and ammeter ifan $a.c.$ source of220V and 100 Hz is connected to it as shown?

[Online May 7, 2012]



(a) $800\text{V}, 8\text{A}$ (b) $110\text{V}, 1.1\text{A}$

(c) 300 V, 3 A (d) $220\text{V}, 2.2\text{A}$

SOLUTION: . (d)

In case ofseries RLC circuit,

Equation ofvoltage is given by

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$\text{Here, } V = 220V; V_L = V_C = 300\text{V}$$

$$V_R = \sqrt{V^2} = 220\text{V}$$

$$\text{Current } i = \frac{V}{R} = \frac{220}{100} = 2.2\text{A}$$

45. A fully charged capacitor C with initial charge q_0 is connected to a coil ofselfinductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is: [2011]

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$ (c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

SOLUTION: . (a)

$$\text{Energy stored in magnetic field} = \frac{1}{2}\text{Li}^2$$

$$\text{Energy stored in electric field} = \frac{1}{2}\frac{q^2}{C}$$

Energy will be equal when

$$\frac{1}{2}Li^2 = \frac{1}{2}\frac{q^2}{C}$$

$$\tan \text{o})t = 1$$

$$q = q_0 \cos \text{o})t$$

$$\Rightarrow \frac{1}{2}L(0)q_0 \sin \text{o})t)^2 = \frac{(q_0 \cos \text{oo}t)^2}{2C}$$

$$\Rightarrow 0) = \frac{1}{\sqrt{LC}} \Rightarrow 0)t = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{\pi}{4}\sqrt{LC}$$

46. A resistor 'R' and $2\mu F$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. $(\log_{10}2.5 = 0.4)$ [2011]

(a) $1.7 \times 10^5\Omega$ (b) $2.7 \times 10^6\Omega$ (c) $3.3 \times 10^7\Omega$ (d) $1.3 \times 10^4\Omega$

SOLUTION: . (b)

$$\text{We have, V} = \text{V}_0\left(1 - e^{-t/RC}\right)$$

$$\Rightarrow 120 = 200\left(1 - e^{-t/RC}\right)$$

$$e^{-t/r} = \frac{200 - 120}{200} = \frac{80}{200}$$

$$t = \log_e(2.5)$$

$$\Rightarrow t = RC \text{ in}(2.5)\ [r = RC]$$

$$\Rightarrow R = 2.71 \times 10^6\Omega$$

47. Combination oftwo identical capacitors, a resistor R and a dc voltage source ofvoltage $6\mathrm{V}$ is used in an experiment on a (C - R) circuit. It is found that for a parallel combination ofthe capacitor the time in which the voltage of the fully charged combination reduces to halfits original voltage is 10 second. For series combination the time for needed for reducing the voltage of the fully charged seriescombination byhalfis [2011 RS]

(a) 10 second (b) 5 second (c) 2.5 second (d) 20 second

SOLUTION: (c)

Time constant for parallel combination$= 2RC$

Time constant for series combination $= \frac{RC}{2}$

In first case :$V = V_0\left(\frac{t}{CR}\right) \Rightarrow \frac{V_0}{2} = V_0 - V_0 e^{\frac{t}{CR}}$

$V = V_0 e = \frac{V_0}{2}\frac{t_1}{2RC}$ (1)

In second case :

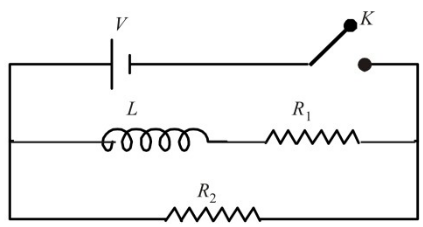
In series grouping, equivalent capacitance $= \frac{C}{2}$

$V = V_0 e^{\frac{t_2}{(RC/2)}} = \frac{V_0}{2}$ (2) From (1) and (2)

$$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)}$$

$\Rightarrow t_2 === 2.5\underline{t_1}\underline{10}$ sec.

48. In the circuit shown below, the keyK is closed at $t = 0$. The current through the battery is [2010]



(a) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(b) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = \infty$

(c) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t = \infty$

(d) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

SOLUTION: . (c)

At $t = 0$, no current will flow through L and R_1 as

inductor will offer infinite resistance.

Current through battery, $i = \frac{V}{R_2}$

At $t = \infty$, inductor behave as conducting wire

Effective resistance, $R_{eff} = \frac{R_1R_2}{R_1+R_2}$

$$\text{Current through battery} = \frac{V}{R_{eff}} = \frac{V(R_1+R_2)}{R_1R_2}$$

49. In a series LCR circuit $R = 200\Omega$ and the voltage and the frequency of the main supply is 220V and 50 Hz

respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is [2010]

(a) 305 W (b) 210 W (c) Zero W (d) 242 W

SOLUTION: . (d)

When only the capacitance is removed phase

difference between current and voltage is

$$\tan\varphi = \frac{X_L}{R} \Rightarrow \tan\varphi = \frac{(iJL}{R}$$

$$\Rightarrow 0)L = R\tan\varphi = 200 \times \frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

When only inductor is removed, phase difference betweencurrent and voltage is

$$\tan\varphi = \frac{1}{0)CR}$$

$$\Rightarrow \frac{1}{0)C} = R\tan\varphi = 200 \times \frac{1}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

$$\text{Impedance ofthe circuit,} Z = \sqrt{R^2 + \left(\frac{1}{0)C} - 0JL\right)^2}$$

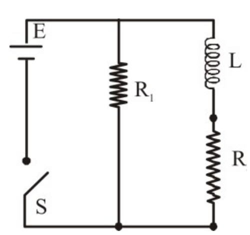
$$= \sqrt{(200)^2 + \left(\frac{200}{\sqrt{3}} - \frac{200}{\sqrt{3}}\right)^2} = 200\Omega$$

$$\text{Power dissipated in the circuit} = V_{\text{nns}} I_{\text{nns}} \cos\varphi$$

$$= V_{\text{rms}} \cdot \frac{V_{\text{rms}}}{Z} \cdot \frac{R}{Z}\left(\because \cos\varphi = \frac{R}{Z}\right) = \frac{V_{\text{rms}}^2 R}{Z^2}$$

$$= \frac{(220)^2 \times 200}{(200)^2} = \frac{220 \times 220}{200} = 242\text{W}$$

50.



An inductor of inductance $L = 400\text{mH}$ and resistors of resistance $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery ofemf 12 V as shown in the figure. The internal resistance ofthe battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function oftime is [2009]

(a) $\frac{12}{t}e^{-3t}\mathrm{V}$ (b) $6(1-e^{-t/02})\mathrm{V}$ (c) $12\mathrm{e}^{-5t}\mathrm{V}$ (d) $6\mathrm{e}^{-5t}\mathrm{V}$

SOLUTION: (c)

Growth in current in branch containing L and R_2 whenswitch is closed is given by

$$i=\frac{E}{R_2}\left[1-e^{-R_2 t/L}\right]$$

$$\Rightarrow \frac{di}{dt}=\frac{E}{R_2}\cdot\frac{R_2}{L}\cdot e^{-R_2 t/L}=\frac{E}{L}e^{\frac{R_2 t}{L}}$$

Hence, potential drop across $LV_L=\frac{Ldi}{dt}=\left(\frac{E}{L}e^{-R_2 t/L}\right)L$

$2t$

$$=Ee^{-R_2 t/L=}12e^{\overline{400\times 10^{-3}}}=12\mathrm{e}^{-5t}\mathrm{V}-$$

51. In a series resonant LCR circuit, the voltage across R is 100 volts and $R=1\mathrm{k}\Omega$ with $\mathrm{C}=2\mu\mathrm{F}$. The resonant frequency 00 is 200 $\mathrm{rad/s}$. At resonance the voltage across L is [2006]

(a) $2.5\times 10^{-2}\mathrm{V}$ (b) $40\mathrm{V}$ (c) $250\mathrm{V}$ (d) $4\times 10^{-3}\mathrm{V}$

SOLUTION: . (c)

Across resistor, $I=\frac{V}{R}=\frac{100}{1000}=0.1A$

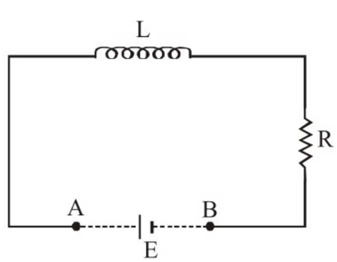
At resonance,

$$X_L=X_C=\frac{1}{\mathrm{co}C}=\frac{1}{200\times 2\times 10^{-6}}=2500$$

Voltage across L is

$$IX_L=0.1\times 2500=250\mathrm{V}$$

52. An inductor $(L=100\mathrm{mH})$, a resistor $(R=100\Omega)$ and abattery $(E=100\mathrm{V})$ are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is [2006]



(a) $1/\mathrm{eA}$ (b) eA (c) $0.1\mathrm{A}$ (d) 1A

SOLUTION: (a)

Initially, when steady state is achieved, $j=\frac{E}{R}$

Let E is short circuited at $t=0$. Then

At $t=0$

Maximum current, $i_0=\frac{E}{R}=\frac{100}{100}=1A$

Let during decay ofcurrent at any time the current flowing

is $-L\frac{di}{dt}-iR=0\Rightarrow\frac{di}{i}=-\frac{R}{L}di$

$$\Rightarrow\int_{i_0}^{i}\frac{di}{i}=\int_0^t-\frac{R}{L}dt$$

$$\Rightarrow\log_e\frac{i}{i_0}=-\frac{R}{L}t$$

$$\Rightarrow i=i_0e^{\frac{R}{L}t}$$

$$\Rightarrow i=\frac{E}{R}e^{\frac{R}{L}t}=1\times e^{\frac{-100\times10^{-3}}{100\times10^{-3}}}=\frac{1}{e}$$

53. In an AC generator, a coil with Nmms, all ofthe same area A and total resistance R, rotates with frequency 00 in amagnetic field B. The maximum value ofemfgenerated in the coil is [2006]

(a) N.A.B.R. (*j*) (b) N.A. B (c) N.AB. R (d) N.A.B. (*j*)

SOLUTION: (d)

$$e=-\frac{d\varphi}{dt}=-\frac{d(N\overline{B}.\overline{A})}{dt}$$

$$=-N\frac{d}{dt}(BA\cos\text{o})t)=NBA(0\sin\sigma)t$$

$\Rightarrow$ e $_{\text{max}}$ =NBAo)

54. The phase difference between the alternating current and emfis $\frac{\pi}{2}$. Which ofthe following cannot be the constituent ofthe circuit? [2005]

(a) R, L (b) C alone (c) L alone (d) L, C

SOLUTION: . (a)

Phase difference for $R-L$ circuit lies between

$\left(0,\frac{\pi}{2}\right)$ but 0 or $\pi/2$

55. A circuit has a resistance of12 ohm and an impedance of 15 ohm. The power factor ofthe circuit will be [2005]

(a) 0.4 (b) 0.8 (c) 0.125 (d) 1.25

SOLUTION: . (b!))

Given, Resistance ofcircuit, $R=12\Omega$

Imedance of circuit, $Z = 15\Omega$

Power factor $=\ \cos\varphi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$

56. A coil of inductance 300 mH and resistance 2 Ω is connected to a source ofvoltage $2\mathrm{V}$. The current reaches half of its steady state value in [2005]

(a) 0.1 s (b) 0.05s (c) 0.3s (d) 0.15s

SOLUTION: (a)

Current in inductor circuit is given by,

$$i = i_0\ (1 - |e| - |RtL) -$$

$$\frac{i_0}{2} = i_0\left(1 - e^{\frac{Rt}{L}}\right) \Rightarrow e^{\frac{Rt}{L}} = \frac{1}{2}$$

Taking $\log$ on both the sides, $-\frac{Rt}{L} = \log 1 - \log 2$

$$\Rightarrow t = \frac{L}{R}\log 2 = \frac{300\times 10^{-3}}{2} \times 0.69 \Rightarrow t = 0.1 \text{ sec.}$$

57. The selfinductance ofthe motor ofan electric fan is 10H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of [2005]

(a) 8 μF (b) 4μF (c) 2 μF (d) 1 μF

SOLUTION: (d)

For maximum power, $X_L = X_C$, which yields

$$C = \frac{1}{(2\pi n)^2 L} = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$$

$$C = 0.1 \times 10^{-5} F = 1\mu F$$

58. In an LCR series a. c. circuit, the voltage across each of the components, L, C and R is $50\mathrm{V}$. The voltage across the LC combination will be [2004]

(a) $100\mathrm{V}$ (b) $50\sqrt{2}\mathrm{v}$ (c) $50\mathrm{V}$ (d) OV(zero)

SOLUTION: . (d)

In a series LCR circuit voltage across the inductor

and capacitor are in opposite phase

Net voltage difference across

$$LC = 50 - 50 = 0$$

59 . In a LCR circuit capacitance is changed from C to 2 C. For the resonant frequency to remain unchanged, the inductance should be changed from L to [2004]

(a) $Ll2$ (b) $2L$ (c) $4L$ (d) $Ll4$

SOLUTION: . (a)

Resonant frequency, $F_r = \frac{1}{2\pi\sqrt{LC}}$

For resonant frequency to remain same

$LC =$ constant

$LC = L'C'$

$\Rightarrow LC = L' \times 2C$

$\Rightarrow L' = \frac{L}{2}$

60. The power factor of an AC circuit having resistance (R) and inductance (L) connected in series and an angular velocity (*j*) is [2002]

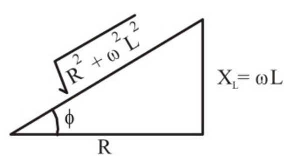
(a) $RlCiJL$ (b) $Rl(R^2 + CiJ^2L^2)^{1/2}$ (c) *CiJLIR* (d) $Rl(R^2 - CiJ^2L^2)^{1/2}$

SOLUTION: . $(\mathbf{b})$

Resistance ofthe inductor, $X_L = 0)L$

The impedance triangle for resistance (R) and inductor (L)

connected in series is shown in the figure.

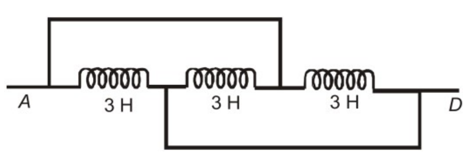


Net impedance ofcircuit $Z = \sqrt{X_L^2 + R^2}$

Power factor, $\cos\varphi = \frac{R}{Z}$

$\Rightarrow \cos\varphi = \frac{R}{\sqrt{R^2 + c0^2L^2}}$

61. The inductance between A and D is [2002]



(a) $3.66\mathrm{H}$ (b) $9\ \mathrm{H}$ (c) $0.66\ \mathrm{H}$ (d) $1\ \mathrm{H}$

SOLUTION: (d)

All three inductors are connected in parallel. The

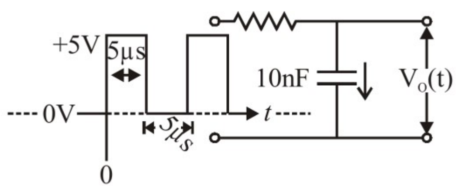
equivalent inductance L_p is given by

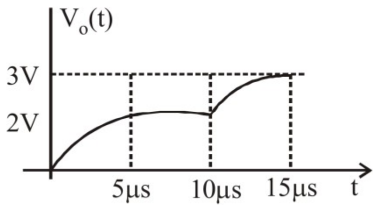
$$\frac{1}{L_p}=\frac{1}{L_1}+\frac{1}{L_2}+\frac{1}{L_3}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=\frac{3}{3}=1$$

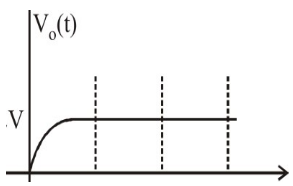
$$L_p=1$$

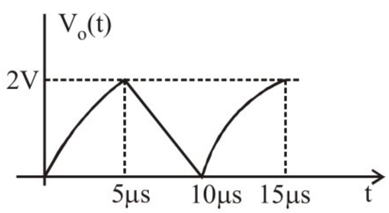
62. **For the given input voltage waveform $V_{in}(t)$, the output voltage waveform $V_o(t)$, across the capacitor is correctly depicted by:** **[Sep. 06, 2020 (I)]**

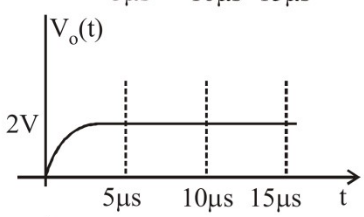
1kΩ

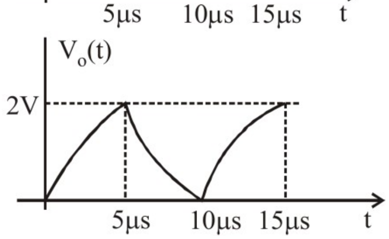


(a)




(b)


(c)


(d)


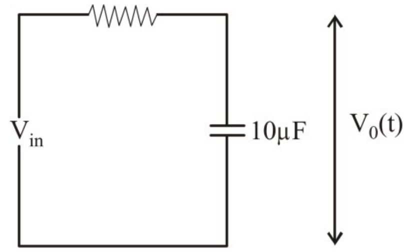
SOLUTION: . (a)

When first pulse is applied, the potential across

capacitor

$$V_0(t) = V_{in(\ )}(\ )$$

At $t = 5\mu s = 5 \times 10^{-6} s$



$$V_0(t) = 5\left(1\left(-e^{\frac{5\times10^{-6}}{10^3\times10\times10^{-9}}}\right)\right) = 5(1 - e^{-0.5}) = 2V$$

When no pulse is applied, capacitor will discharge. Now, $V_{in} = 0$ means discharging.

$$V_0(t) = 2e^{\frac{1}{RC}} = 2e^{-0.5} = 1.21\text{V}$$

Now for next 5 μs

$$V_0(t) = 5 - 3.79e^{\frac{1}{RC}}$$

After 5 μs again, $V_0(t) = 2.79$ Volt $\approx 3\text{V}$ Hence, graph (a) correctly depicts.

63. A transformer consisting of300 turns in the primary and 150 turns in the secondary gives output power of2.2kW. Ifthe current in the secondary coil is 10 A, then the input voltage and current in the primary coil are: [10 April 2019 I]

(a) 220 V and 20 A (b) 440 V and 20 A (c) 440 V and 5 A (d) 220 V and 10 A

SOLUTION: . (c)

Power output $(V_2I_2) = 2.2\text{kW}$

$$V_2 = \frac{2.2\text{kW}}{(10\text{A})} = 220 \text{ volts}$$

Input voltage for step - down transformer

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = 2$$

$$V_{input} = 2 \times V_{output} = 2 \times 220$$

$$= 440\text{V}$$

Also $\frac{I_1}{I_2} = \frac{N_2}{N_1}$

$$I_1 = \frac{1}{2} \times 10 = 5\text{A}$$

64. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. Ifthe current in the primary ofthe transformer is 5A and its efficiency is 90%, the output current would be: [9 Jan. 2019 II]

(a) 50 A (b) 45 A (c) 35 A (d) 25 A

SOLUTION: (b)

$$\text{Efficiency}, \eta = \frac{P_{out}}{P_{in}} = \frac{V_s I_s}{V_p I_p}$$

$$\Rightarrow 0.9 = \frac{230 \times I_s}{2300 \times 5}$$

$$\Rightarrow I_s = 0.9 \times 50 = 45A$$

$$\text{Output current} = 45A$$

65. A power transmission line feeds input power at2300 Vto a step down transformer with its primary windings having 4000 turns, giving the output power at 230 V. Ifthe current in the primary of the transformer is 5 A, and its eificiency is 90%, the output current would be: [Online Apri116, 2018]

(a) 20A (b) 40A (c) 45A (d) 25A

SOLUTION: (c)

$$\text{Given: } V_P = 2300V, V_s = 230V, I_P = 5A, \text{n}=90\% = 0.9$$

$$\text{Efficiencyn} = 0.9 = \frac{P_s}{P_P} \Rightarrow P_s = 0.9P_p$$

$$V_S I_S = 0.9 \times V_P I_P \ (P = VI)$$

$$I_s = \frac{0.9 \times 2300 \times 5}{230} = 45A$$

66. In an oscillating LC circuit the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic field is [2003]

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

SOLUTION: . (c)

When the capacitor is completely charged, the total

energy in the LC circuit is with the capacitor and that

energy is given by

$$U_{\max} = \frac{1}{2}\frac{Q^2}{C}$$

When halfenergyis with the capacitor in the form ofelectric

field between the plates ofthe capacitor we get

$$\frac{U_{\max}}{2} = \frac{1}{2}\frac{q'^2}{C}$$

Here q' is the charge on the plate ofcapacitor when energy

is shared equally.

$$\frac{1}{2} \times \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{q'^2}{C} \Rightarrow q' = \frac{Q}{\sqrt{2}}$$

67. The core ofany transformer is laminated so as to [2003]

(a) reduce the energy loss due to eddy currents (b) make it light weight

(c) make it robust and strong (d) increase the secondary voltage

SOLUTION: (a)

Laminated core provide less area ofcross - section for

the current to flow. Because ofthis, resistance of the core

increases and current decreases there by decreasing the

energy loss due to eddy current.

68. In atransformer, number ofmrns in theprimarycoil are 140 and that in the secondary coil are 280. Ifcurrent in primary coil is 4 A, then that in the secondary coil is [2002]

(a) 4A (b) 2A (c) 6A (d) 10A.

SOLUTION: (b)

Number ofturns in primary $N_p = 140$

Number ofturns in secondary $N_s = 280, I_p = 4A, I_s = ?$

Using transformation ratio for a transformer $\frac{I_s}{I_p} = \frac{N_p}{N_s}$

$$\Rightarrow \frac{I_s}{4} = \frac{140}{280}$$

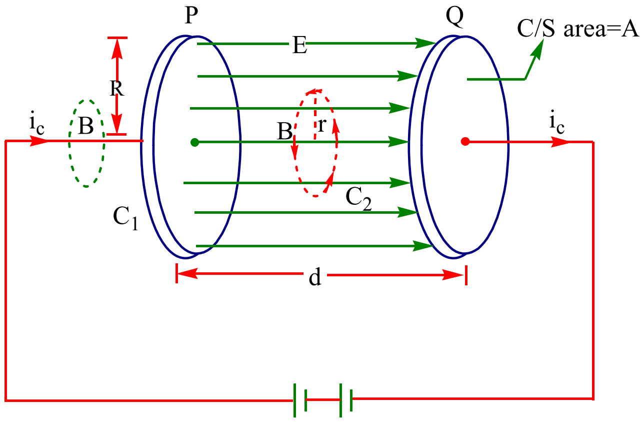
$$\Rightarrow I_s = 2A$$

EM WAVES

A According to Maxwell, an accelerated charge produces a sinusoidal time-varying magnetic field, which in turn produces a sinusoidal time varying electric field. The two fields so produced are mutually perpendicular. They constitute electro magnetic waves which can propagate through empty space.

A **Displacement current:-** According to Ampere's circuital Law, the magnetic field B is related to steady current I as $\oint \vec{B}.\vec{dl} = \mu_0 I$ ....(i) where I is the current travelling through the surface bounded by closed loop.

In 1864, Maxwell showed that relation (i) is logically inconsistent. He accounted for this inconsistency as follows: Consider a parallel plate capacitor having plates P and Q being charged with battery B.



A During charging, a current I flows through the connecting wires which changes with time. This current will produce magnetic field around the wires which can be detected using a magnetic compass needle. Consider two loops c_1 and c_2 parallel to the plates P and Q of the capacitor. c_1 is enclosing only the connecting wire attached to the plate P of the capacitor and c_2 lies in the region between the two plates of capacitor. For the loop c_1, a current I is flowing through it, hence Ampere's circuital law for loop c_1 gives

$$\oint_{C_1} \vec{B}.\vec{dl} = \mu_0 I \quad ....(ii)$$

A Since the loop c_2 lies in the region between the plates of the capacitor, no current flows in this region. Hence Ampere's circuital law for loop c_2 gives $\oint_{C_2} \vec{B}.\vec{dl} = 0 \quad ....(iii)$

A The relations (ii) and (iii) continue to be true even if two loops c_1 and c_2 are infinitesimally close to the plate P of the capacitor. In the other hand, as the loops c_1 & c_2 are infinitesimally close, it is expected that

$$\oint_{C_1} \vec{B}.\vec{dl} = \oint_{C_2} \vec{B}.\vec{dl} \quad ...(iv)$$

Thus, relation (iv) is in contradiction with relations (ii) and (iii). This led Maxwell to point out that Ampere's circuital law as given by (i) is logically inconsistent.

A **Idea of Displacement Current :** Maxwell predicted that not only a current flowing in a conductor produces magnetic field but also a time-varying electric field (i.e., changing electric field) in a vacuum/free space (or in a dielectric) produces a magnetic field. It means a changing electric field gives rise to a current which flows through a region so long as the electric field is changing there. Maxwell also predicted that this current produces the same magnetic field as a conduction current can produce. This current is known as 'displacement current'.

A Thus, displacement current is that current which comes into play in the region in which the electric field and hence the electric flux is changing with time.

Maxwell defined this displacement current in space where electric field is changing with time as

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt} \qquad ...(v)$$

where ϕ_E is the electric flux.

A Maxwell also found that conduction current (I) and displacement current (I_D) together have the property of continuity, although, individually, they may not be continuous.

A This idea led Maxwell to modify Ampere's circuital law in order to make the same logically consistent. He states Ampere circuital law in the form, $\oint_C \vec{B}.\vec{dl} = \mu_0 (I + I_D)$

$$= \mu_0 \left(I + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

It is now called as Ampere-Maxwell's law.

A This means that out side the capacitor plates, we have only conduction current $i_c = i$ and no displacement current $i_d = 0$. On the other hand, inside the capacitor, there is no conduction current $i_c = 0$ and there is only displacement current $i_d = i$

Note : (i) Between the capacitor plates the displacement current can be treated as the output of the constant current density j given by j=

Thus, i_d, corresponding to r will be

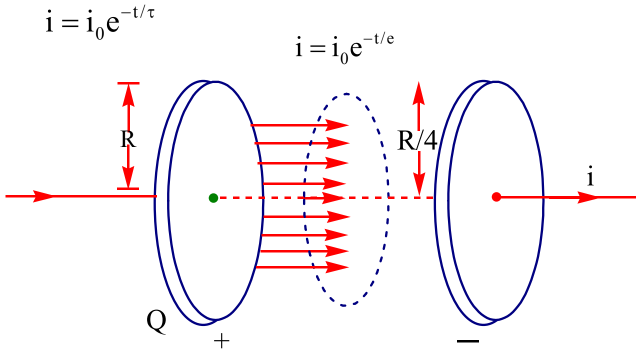
$$i_d = j(\pi r^2) = i_c \left(\frac{r}{R} \right)^2$$

(ii) Eq. Reflects that the magnetic field inductio B varies linearly with r, so that it is zero at the axis (r=0) and maximum at the periphery of the cylindrical volume enclosing the plates (i.e., r=R)

EX.1 : A circular parallel plate capacitor with plate radius R is charged by means of a cell, at time t=0. The initial conduction current is i_0. Consider a circular area of radius R/4 coplanar with the capacitor plates and located symmetrically between them. Find the time rate of elecric flux change through this area after one time constant.

Sol. The conduction current at the end of one time constant can be obtained by substituting t= in the expression

$i = i_0 e^{-t/\tau}$



$$i = \frac{i_0}{e} = i'$$

(Where e is the base of natural logarithm).

If Q be the charge at the mentioned isnatant then, the electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 \left(\pi R^2\right)}$$

$\therefore$ The electric flux through the specified area is

$$\phi_E' = E\pi(R/4)^2 = \frac{Q}{\varepsilon_0\left(\pi R^2\right)}\left(\frac{\pi R^2}{16}\right) = \frac{Q}{16\varepsilon_0}$$

Rate of electric flux change is

$$\frac{d\phi_E'}{dt} = \frac{1}{16\varepsilon_0}\left(\frac{dQ}{dt}\right) = \frac{1(i')}{16\varepsilon_0} = \frac{i_0}{16e\varepsilon_0}$$

Maxwell's Equations

- Maxwell, in 1862, gave the basic Laws of electricity and magnetism in the form of four fundamental equations which are known as Maxwell's equations. In the absence of any dielectric and magnetic material may be stated in the integral form as below.

1. Gauss's Law for electrostatics :-

This Law gives the total electric flux in terms of charge enclosed by the closed surface.

In the usual notations $\oint \vec{E}.\vec{dS} = \frac{q_{in}}{\varepsilon_0}$

This Law states that electric lines of force start from positive charge and end at negative charge i.e., electric lines force do not form closed paths.

2. Gauss's Law for magnetism :-

Mathematically $\oint \vec{B}.\vec{dS} = 0$

- This Law shows that the no. of magnetic lines of force entering a closed surface is equal to no. of magnetic lines of force leaving that closed surface.
- This law tells that the magnetic lines of force form a continuous closed path.
- This Law also predicts that the isolated magnetic monopoles does not exist.

3. Faraday's Law of electro magnetic induction :-

Mathematic cally $\oint \vec{E}.\vec{dl} = \frac{-d\phi_B}{dt}$ = induced emf.

- This law gives a relation between electric field and changing magnetic flux.
- This law tells that changing magnetic field is a source of electric field.

4. Ampere's-Maxwell's Law :-

Mathematically $\oint \vec{B}.\vec{dl} = \mu_0\left(i_c + i_d\right)$

$$= \mu_0\left(i_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

- This law states that magnetic field can be produced by a conduction current as well as by displacement current.
- At any instant in a circuit, conduction current is equal to displacement current.

5. Lorentz Force :- Force acting on a charge 'q' moving in a region where electric and magnetic fields similar to EM waves are existing simultaneously is $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

EX. 2: What is the instantanuous displacement current in space between plates of parallel plate capacitor of capacitor $1\mu F$ which is charging at rate of $10^6 V / S$

Sol. As $I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 A \frac{d}{dt}(E) = \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$

$= \frac{\varepsilon_0 A}{d} \frac{dV}{dt} = C \frac{dV}{dt}$; $I_d = 10^{-6} \times 10^6 = 1A$

EX. 3: Electro magnetic waves travel in a medium with speed of $2 \times 10^8 m/\sec$. The relative permeability of the medium is 1 find relative permittivity.

Sol. Give $C = 2 \times 10^8 m/\sec, \mu_r = 1$

Speed of EM waves in medium

$$C_{med} = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}} = \frac{C_0}{\sqrt{\mu_r \varepsilon_r}} \; ; \; \varepsilon_r = \frac{c_0^2}{c^2 \mu_r} = \frac{\left(3 \times 10^8\right)^2}{\left(2 \times 10^8\right)^2 \times 1} = 2.25$$

EX. 4: Suppose that the electric field amplitude of an EM wave is $E_0 = 120 N/C$ and that its frequency $\nu = 50 \mu HZ$.Determine (a) B_0, ω, λ and K (b) Find expressions for E and B

Sol. a) i) Using $C = \frac{E_0}{B_0}$ we get

$$B_0 = \frac{E_0}{C} = \frac{120}{3 \times 10^8} = 4 \times 10^{-7} T = 400 nT$$

ii) $\omega = 2\pi\nu = 2 \times \pi \times 50 \times 10^6 = 3.14 \times 10^8 rad / Sec$ iii) $c = \upsilon\lambda \Rightarrow \lambda = \frac{C}{\upsilon} = \frac{3 \times 10^8}{50 \times 10^6} = 6m$

iv) $K = \frac{2\pi}{\lambda} = \frac{2\pi}{6} = \frac{2 \times 3.14}{6} = 1.05 m^{-1}$

b) $\vec{E} = E_0 \sin(kx - \omega t)$

$= 120 \sin\left(1.05x - 3.14 \times 10^8 t\right)$

$B = B_0 \sin(kx - \omega t)$

$= 400 \times 10^{-9} \sin\left(1.05x - 3.14 \times 10^8 t\right)$

Energy density of EM waves:

A Consider a plane electro magnetic wave propagating along x-axis. The electric and magnetic fields in a plane EM wave can be given by
$E = E_0 \sin(kx - \omega t)$ and $B = B_0 \sin(kx - \omega t)$

A In any small volume 'dV', the energy of electric field is $U_E = \frac{1}{2}\varepsilon_0 E^2 dV$ and energy of the magnetic field in volume 'dV' is $U_B = \frac{B^2}{2\mu_0} dV$

A Thus total energy of EM wave is $u = \frac{1}{2}\varepsilon_0 E^2 dV + \frac{B^2}{2\mu_0} dV$

A Energy density of EM wave is $U = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0}$

$= \frac{1}{2}\varepsilon_0 E_0^2 \sin^2(kx - wt) + \frac{B_0^2}{2\mu_0}\sin^2(kx - \omega t)$

If we take average over a long time, the $\sin^2$ terms have an average value of $\frac{1}{2}$

Thus $u_{aV} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{B_0^2}{4\mu_0}$

Now $E_0 = CB_0$ and $\mu_0\varepsilon_0 = \frac{1}{C^2}$

$\therefore u_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0\left(C^2 B_0^2\right) = \frac{1}{4}\varepsilon_0 \frac{1}{\mu_0\varepsilon_0} B_0^2 = \frac{B_0^2}{4\mu_0} = u_B$

Hence is an EM wave, average energy density of electric field is equal to average energy density of magnetic field.

The units of u_E & u_B are Jm^{-3}

$\therefore$ average energy density of EM wave

$u = u_E + u_B = 2u_E = 2u_B = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$

Intensity of electro magnetic wave:

Intensity of EM wave is defined as the energy crossing per second per unit area of a source perpendicular to the direction of propagation of the wave. It is denoted by I.

i.e. Intensity $I = \frac{total\ EM\ wave\ energy}{Surface\ area \times time}$

$\frac{u_{aV} \times \Delta V}{A\Delta t} = \frac{u_{aV} \times Ac\Delta t}{A\Delta t}$

Interms of electric fields $I = \frac{1}{2}\varepsilon_0 E_0^2 C$ ---------(1)

Interms of magnetic field $I = \frac{B_0^2}{2\mu_0} C$ ---------(2)

Either eq (1) or (2) may be used to find intensity of EM waves

ii) The intensity of EM radiation from an isotropic point source at a distance r is $I = \frac{P}{4\pi r^2}$ where P is power of source

Note: The rate of flow of energy crossing a unit area in an EM wave is described by the vector 'S' called poynting vector which is described by the expression.

$\vec{S} = \frac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$

Since $\vec{E}$ and $\vec{B}$ are mutually perpendicular

$\left|\vec{E}\times\vec{B}\right| = EB$

Thus magnitude of Poynting vector

$$S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 C}$$

SI unit of S is $J\sec^{-1}m^{-2}\ (or) Wm^{-2}$

This relation shows that the value of electric vector at any instant in the EM wave is about 377 times the value of magnetic vector. It is because of this reason, optical properties of light is due to electric field.

Average of Poynting vector is given by

$$I = S_{aV} = \frac{E_0 B_0}{2\mu_0} = \frac{1}{2}\varepsilon E_0^2 C = \frac{CB_0^2}{2\mu_0}$$

EX. 5 : The electric field of an electro magnetic wave is given by $E = 50\sin\omega\left(t - \frac{x}{c}\right)N/C$. Find energy contained in a cylinder of cross-section $10cm^2$ and length $50cm$ along x-axis

Sol: Average volume of energy density $u_{aV} = \frac{1}{2}\varepsilon_0 E_0^2$

Total volume of cylinder $V = Al$

Total energy of contained in cylinder

$$U = (U_{aV})V = \left(\frac{1}{2}\varepsilon_0 E_0^2\right)(Al) = 5.5\times10^{-12}J$$

Momentum and Radiation Pressure

i) Electro magnetic waves have linear momentum as well as energy. When EM waves strike a surface, pressure is exerted on it, called radiation pressure.

ii) When EM waves are incident on a surface and the total energy transferred to the surface in a time t is U then magnitude of momentum transferred to surface is $p = \frac{U}{C}$ (total absorption)

$$\left[E = mC^2 = (mC)C \Rightarrow mC = \frac{E}{C} \Rightarrow P = \frac{E}{C}\right]$$

iii) According to quantum theory of radiation, linear momentum associated with a photon is

$\mathrm{P} = \frac{\mathrm{E}}{\mathrm{C}} = \frac{\mathrm{h}\nu}{\mathrm{C}} = \frac{\mathrm{h}}{\mathrm{C}}\times\frac{\mathrm{C}}{\lambda} = \frac{\mathrm{h}}{\lambda}$ where λ = wave length, v = frequency, C = velocity of light

iv) When radiation incident on a surface is entirely reflected back along its original path, magnitude of momentum delivered to the surface is $p = \frac{2U}{C}$ where 'C' is velocity of light.

v) When the radiation incident on a surface (Perfect absorber) radiaton pressure

$$P_r = \frac{F}{A} = \frac{1}{A}\frac{dp}{dt} = \frac{1}{A}\frac{d}{dt}\left(\frac{U}{C}\right) = \frac{1}{AC}\frac{dU}{dt} = \frac{S}{C}$$

$\frac{dU/dt}{A}$ is called average value of Poynting vector.

If the surface is perfect reflector radiation pressure $P_r = \frac{2S}{C}$

vi) Consider a beam of electro magnetic radiation of intentisyt I, and of cross sectional area A which falls on a surface of a body normally

Case (i) :

a) If the surface absorbs the radiation falling on it completely, force excerted by the radiation on the surface = Rate of change of linear momentum

$$F = \frac{dp}{dt} = \frac{dE}{C} \times \frac{1}{dt} = \frac{1}{C}\left(\frac{dE}{dt}\right) = \frac{IA}{C}$$

b) Pressure excerted on the surface $P' = \frac{F}{A} = \frac{I}{C}$

Case (ii)

If the surface reflects the radiation completely (falling on it normally), force excerted on the surface

$F = \frac{dp}{dt} = 2\left(\frac{dE}{C}\right) \times \frac{1}{dt} = \frac{2}{C}\left(\frac{dE}{dt}\right) = \frac{2IA}{C}$ and pressure on the surface $P' = \frac{F}{A} = \frac{2I}{C}$

Case (iii) :

a) If the radiation falls normally and the surface is partially reflecting and absorbing the remaining with reflection and absorption coefficients r and a respectively, then force on the surface (no force acts on the surface due to transmission)

$$F = \left[r \times 2\left(\frac{dE}{C}\right) \times \frac{1}{dt}\right] + \left[a\left(\frac{dE}{C}\right) \times \frac{1}{dt}\right]$$

$$= \frac{1}{C}\left(\frac{dE}{C}\right)(2r + a) = \frac{IA}{c}(2r + 1 - r) = \frac{IA}{c}(1 + r)$$

$(Q\ a + r = 1)$

Pressure $P' = \frac{F}{A} = \frac{I}{C}(1 + r)$

b) In this case if surface is partially transmitting with reflection , absorption and transmission coefficients r,a and t repectively, then force on the surface

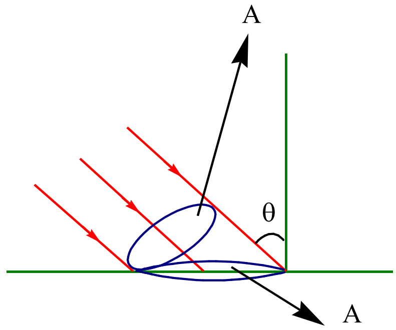
$$F = r \times 2\left(\frac{dE}{C}\right) \times \frac{1}{dt} + a\left(\frac{dE}{C}\right) \times \frac{1}{dt}$$

$$= \frac{1}{C}\left(\frac{dE}{C}\right)(2r + a) = \frac{IA}{c}(2r + a)$$

and $r + a + t = 1$

Case(iv) :

Let a parallel beam of radiation falls on a plane surface at an angle with normal to the surface and A be the cross-sectional area of the beam



a) If the radiation is absorbed by the surface completely, force on the surface normal to it is

$$F_n = \left(\frac{dE\cos\theta}{C}\right)\times\frac{1}{dt} = \frac{1}{C}\left(\frac{dE}{dt}\right)\cos\theta = \frac{IA}{C}\cos\theta$$

Force on the surface parallel to the surface is

$$F_t = \left(\frac{dE\sin\theta}{C}\right)\times\frac{1}{dt} = \frac{1}{C}\left(\frac{dE}{dt}\right)\sin\theta = \frac{IA}{C}\sin\theta$$

Resultant force on the surface $F = \sqrt{F_n^2 + F_t^2} = \frac{IA}{C}$ at an angle with normal to the surface

$$\text{Pressure} = \frac{\text{normal force}}{\text{area}} = \frac{F_n}{\left(\frac{A}{\cos\theta}\right)} = \frac{IA\cos\theta}{C\left(\frac{A}{\cos\theta}\right)} = \frac{I}{C}\cos^2\theta$$

b) In this case if radiation is completely reflected at the same angle, then force on the surface

$$F = 2\left(\frac{dE\cos\theta}{C}\right)\times\frac{1}{dt} = \frac{2}{C}\left(\frac{dE}{dt}\right)\cos\theta = \frac{2IA}{C}\cos\theta$$

and force parallel to the surface=0($\because$ no change in linear momentum parallel to the surface)

$$\text{Pressure } P' = \frac{F}{\left(\frac{A}{\cos\theta}\right)} = \frac{2IA\cos^2\theta}{C}$$

c) In this case if the surface partially reflects (at same angle) and absorbs the remaining with reflection and absorption coefficients r and a respectively (r+a=1), then force on surface normal to it due to the reflected and absorbed parts of the radiation

$$F_n = \left[r\times\frac{dE\cos\theta}{C}\times\frac{1}{dt}\right] + \left[a\left(\frac{dE\cos\theta}{C}\right)\frac{1}{dt}\right]$$

$$= \frac{1}{C}\left(\frac{dE}{dt}\right)\cos\theta(2r+a) = \frac{IA\cos\theta}{C}(2r+1-r)$$

$$= \frac{IA\cos\theta}{C}(1+r)$$

Force on the surface parallel to it (this is due to the absorbed portion of the radiation only) is

$$F_t = a\frac{dE\sin\theta}{C}\times\frac{1}{dt} = \frac{a}{C}\left(\frac{dE}{dt}\right)\sin\theta$$

$$= \frac{IA}{C}\sin\theta(1-r)$$

Resultant force on the surface is

$$F=\sqrt{F_n^2+F_t^2}=\frac{IA}{C}=\frac{IA}{C}\sqrt{(1+r)^2\cos^2\theta+(1-r)^2\sin^2\theta}$$

This force acts at an angle $\alpha=\tan^{-1}\left(\frac{F_t}{F_n}\right)$ with normal to the surface

ie $\alpha=\tan^{-1}\left[\tan\theta\left(\frac{1-r}{1+r}\right)\right]$

Pressure $P'=\frac{\text{normal force}}{\text{area}}=\frac{F_n}{\frac{A}{\cos\theta}}$

EX. 6: Light with an energy flux of $18 W/cm^2$ falls on a non reflecting surface at normal incidence. If the surface has an area of 20 cm^2 then find average force exerted on the surface during a 30 minute time span.

Sol. Total energy falling on the surface is

$$U=(18\times10^4)(20\times10^{-4})(30\times60)=6.48\times10^5 J$$

Total momentum delivered (complete absorption)

$$p=\frac{U}{C}=\frac{6.48\times10^5}{3\times10^8}=2.16\times10^{-3}\,kg\,m/\sec$$

average force exerted

$$F=\frac{p}{t}=\frac{2.16\times10^{-3}}{30\times60}=1.2\times10^{-6}N$$

EX. 7: The rms value of electric field of light coming from sun is 720N/C. Find average energy density of em wave.

Sol: Total average energy density $=\frac{1}{2}\varepsilon_0E_0^2=\varepsilon_0E_{rms}^2$

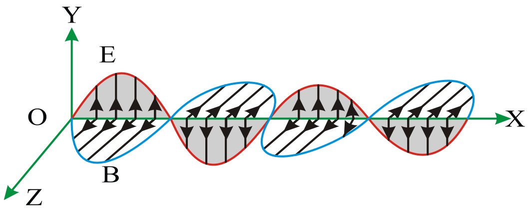
$$\left(\therefore E_{rms}=\frac{E_0}{\sqrt{2}}\right)=8.85\times10^{-12}\times(720)^2=4.58\times10^{-6}Jm^{-3}$$

Source of EM waves :-

- Accelerated charges radiate energy in the form of EM waves. So it is source of EM waves
- An oscillating charge produces an oscillating electric field inturn which produces an oscillating magnetic field.
- The oscillating electric and magnetic fields regenerate each other and propagate through space as waves called EM waves.
- An electric charge oscillating harmonically with frequency 'υ' produces EM waves of same frequency.

Characteristics of EM waves :

1) EM waves are transverse in nature whose speed is same as that of speed of light
2) The two fields $\vec{E}$ and $\vec{B}$ have same frequency of oscillation and they are in phase with each other.
3) Keeping these features in mind, we can assume that if EM wave is travelling along positive direction along x-axis, the electric field is oscillating parallel to the y-axis and that magnetic field is parallel to z-axis, then we can write the electric and magnetic fields as sinusoidal functions of position 'x' and time 't'



$E = E_0 Sin(kx - wt)$; $B = B_0 \sin(kx - wt)$

In this, E_0 & B_0 are the amplitudes of the fields

4) EM waves can be polarised.

5) EM waves are self-sustaining oscillations of electric and magnetic fields in free space or vaccum. EM waves travel through vaccum with speed of light 'C' where. $C = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 m / S\text{ec}$

6) The speed of EM waves in any other medium of permittivity ε and permeability μ is

$$C_{med} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_r \mu_0 \varepsilon_r \varepsilon_0}} = \frac{C_0}{\sqrt{\mu_r \varepsilon_r}}$$

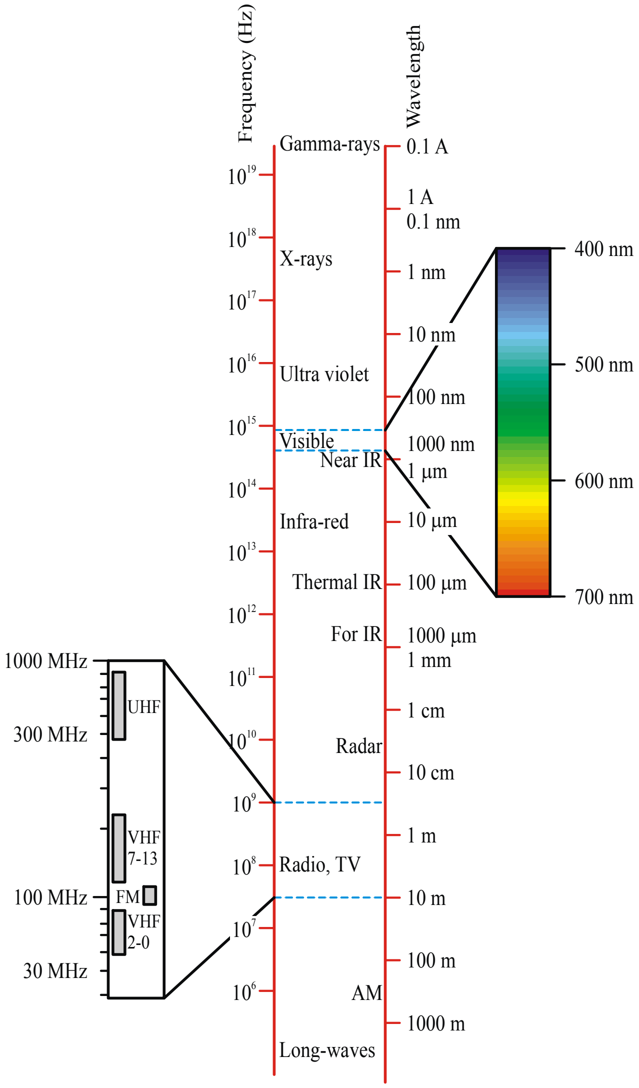
$$\sqrt{\mu_r \varepsilon_r} = \frac{C_0}{C_{med}} = n \text{ R.I. of medium}$$

7) In vaccum, EM waves are of different wavelengths, but velocity is same.

Electro magnetic spectrum :

A The array obtained on arranging all the electromagnetic waves in an order on the basis of their wavelength is called the electromagnetic spectrum

In the order of increasing wavelength, these waves are (i) Gamma rays, (ii) X-rays, (iii) Ultraviolet rays, (iv) Visible light, (v) infrared waves, (vi) Microwaves and (vii) Radio waves.



The figure illustrates the general spectrum of the electromagnetic radiations, in which the wavelength is expressed in metre.

1) **Gamma rays :** They were discovered by Becquerel and Curie in 1896. Their wavelength is of the order of 10^{-14} to $10^{-10}\,m$. The main sources are the natural and artificial radioactive substances. These rays affect the photographic plate. These rays are mainly used in the treatment of cancer disease.

2) **X-rays :** They were discovered by Roentgen in 1895. Their wavelength is of the order of $10^{-12}\,m$ to $10^{-18}\,m$. X-rays are produced when highly energetic cathode rays are stopped by a metal target of high melting point. They affect the photographic plate and can penetrate through the transparent materials. They are mainly used in detecting the fracture of bones, hidden bullet, needle, costly material, etc., inside the body and also used in the study of crystal structure.

3) **Ultraviolet rays :** They were discovered by Ritter in 1801. Their wavelength is of the order $10^{-9}\,m$ to $4\times10^{-7}\,m$. In the radiations received from sun, major part is that of the ultraviolet radiation. Its other sources are the electric discharge tube, carbon arc etc. These radiations are mainly used in excitation of photoelectric effect and to kill the bacteria of many diseases.

4) **Visible light :** This was first studied in 1666 by Newton. The radiations in the range of wavelength from $4\times10^{-7}\,m$ to $7\times10^{-7}\,m$ fall in the visible region. The wavelength of the light of violet colour is the shortest

and that of red colour is the longest. Visible light is obtained from the glowing bodies, while they are white hot. The light obtained from the electric bulbs, sodium lamp, fluorescent tube is the visible light.

5) **Thermal or infrared waves:** They were discovered by Herchell in 1800. Their wavelength is of the order of $7\times10^{-7}m$ to $10^{-3}m$. A body on being heated, emits out the infrared waves. These radiations have the maximum heating effect. The glass absorbs these radiations, therefore for the study of these radiations rock salt prism is used instead of a glass prism. These waves are mainly used for therapeutic purpose by the doctors because of their heating effect.

6) **Microwaves:** They were discovered by Hertz in 1888. Their wavelength is in the range of nearly $10^{-4}m$ to 1m. These waves are produced by the spark discharge or magnetron valve. They are detected by the crystal or semiconductor detector. These waves are used mainly in radar and long distance communication.

7) **Radiowaves:** They were first discovered in 1895 by Marconi. Their wavelength is in the range of 0.1m to $10^{5}m$. They can be obtained by the flow of high frequency alternating current in an electric conductor. These waves are detected by the tank circuit in a radio receiver or transmitter.

Application of EM waves

1) Radio and microwave radiations are used in radio and TV communication system. Microwave radiations are mainly used in radar and TV communication.

2) **Infrared radiations are used**
i) in green houses to keep the plants warm
ii) in revealing the secret writings on the ancient walls
iii) for looking through haze, fog and mist during war time, as these radiations can pass through them.

3) **Ultraviolet radiations are used**
i) in preserving the food stuffs.
ii) in the detection of invisible writing, forged documents, finger prints in forensic laboratory.
iii) Ultraviolet radiations are also used for knowing the structure of the molecules and arrangement of electrons in the external shells.

4) **X-rays many applications** these rays provide us valuable information
i) about the structure of atomic nuclei
ii) in the study of crystal structure
iii) in the fracture of bones etc.

5) $\gamma-$ **rays were used**
i) in treatment of cancer and tumours
ii) to produce nuclear reactions.

Electromagnetic spectrum

S No.	Name	Frequency range (Hz)	Wavelength range (m)	Production
1	Gamma (γ) rays	$5x10^{22}$ to $5x10^{18}$	$0.6x10^{-14}$ to 10^{-10}	Nuclear origin
2	X-rays	$3x10^{21}$ to $1x10^{16}$	10^{-13} to $3x10^{-8}$	Bombardment of high Z target by electrons
3	Ultraviolet rays (UV)	$8x10^{14}$ to $8x10^{16}$	$4x10^{-9}$ to $4x10^{-7}$	Excitation of atoms and spark
4	Visible light	$4x10^{14}$ to $8x10^{14}$	$4x10^{-7}$ to $8x10^{-7}$	Excitation of atoms, spark and arc flame
5	Thermal of infrared rays (IR)	$3x10^{11}$ to $4x10^{14}$	$8x10^{-9}$ to $3x10^{-3}$	Excitation of atoms and molecules
6	Microwaves	$3x10^{8}$ to $3x10^{11}$	10^{-3} to 1	Klystron value or magnetron value
7	Radiowaves	$3x10^{3}$ to $3x10^{11}$	10^{-3} to 10^{5}	Oscillating circuits

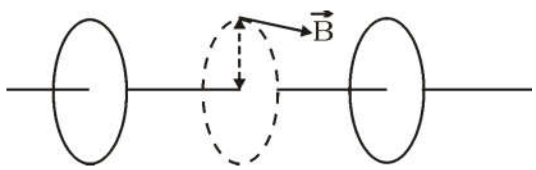
EX8:A plane electromagnetic wave is incident on a material surface. If the wave delivers momentum p and energy E, then

(a) p = 0, E = 0 (b) p ¹ 0, E ¹ 0

(c) p ¹ 0, E = 0 (d) p = 0, E ¹ 0.

SOL;An electromagnetic wave has both energy and momentum.

EX9:An expression for the magnetic field strength B at the point between the capacitor plates indicates in Fig. Express B in terms of the rate of change of the electric field strength i.e. dE/dt between the plates



(a) $\dfrac{\mu_0 I}{2\pi r}$ (b) $\dfrac{\in_0 \mu_0 r}{2} dE/dt$ (c) Zero (d) $\dfrac{\mu_0 I}{2r}$.

SOL: $B = \dfrac{\mu_0}{4\pi}\dfrac{2i_D}{r} = \dfrac{\mu_0}{4\pi}\dfrac{2}{r} \times \in_0 \dfrac{d\phi_E}{dt} = \dfrac{\mu_0}{2\pi r} \in_0 \dfrac{d}{dt}\left(E\pi r^2\right)$

EX10:The electric field (in NC^{-1}) in an electromagnetic wave is given by E = 50 sin w (t-*x*/c) The energy stored in a cylinder of crossection 10 cm^2 and length 100 cm along the *x*-axis will be

(a) $5.5\times 60^{-12} J$ (b) $1.1\times 10^{-11} J$ (c) $2.2\times 10^{-11} J$ (d) $1.65\times 10^{-11} J$

SOL:Energy contained in a cylinder

$$U = \text{average energy density} \ ′\ \text{Volume} = \frac{1}{2} \in_0 E_0^2 \times Al$$

$$= \frac{1}{2} \times \left(8.85 \times 10^{-12}\right) \times (50)^2 \times \left(10 \times 10^{-4}\right) \times 1 = 1.1 ' 10^{-11} \text{ J}$$

EX11: If c is the speed of electromagnetic waves in vacuum, its speed v in a medium of dielectric constant K and relative permeability m_r is

(a) $\nu = \frac{1}{\sqrt{\mu_r K}}$ (b) $\nu = c\sqrt{\mu_r K}$ (c) $\nu = \frac{c}{\sqrt{\mu_r K}}$ (d) $\nu = \frac{K}{\sqrt{\mu_r c}}$

SOL: $c = \frac{1}{\sqrt{\mu_0 \in_0}}$ $\upsilon = \frac{1}{\sqrt{\mu_0 \mu_r \in_0 K}} = \frac{c}{\sqrt{\mu_r K}}$

EX12:If a source is transmitting electromagnetic wave of frequency $8.2 ' 10^6$ Hz, then wavelength of the electro-magnetic waves transmitted from the source will be

(a) 36.6 *m* (b) 40.5 *m* (c) 42.3 *m* (d) 50.9 *m*

SOL: Here, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{8.2 \times 10^6} = 36.6\text{m}.$

EX13: Television signals reach us only through the ground waves. The range R related with the transmitter height h is in proportion to

(a) *h* (b) $h^{1/2}$ (c) $h^{-1/2}$ (d) h^{-1}

SOL: Range, $R = \sqrt{2hr}$ where r is the radius of earth so $R \propto h^{1/2}$

EX14: The minimum frequency $\nu_{\min}$ of continuous X-rays is related to the applied pot. diff. V as

(a) $v_{min} \mu$ V (b) $v_{min} \mu V^{1/2}$ (c) $v_{min} \mu V^{-3}$ (d) $v_{min} \mu V^4$.

SOL: $h\nu_{\min} = eV$ or $\nu_{\min} \propto V$.

EX15 **An electron is constrained to move along they axis with aspeed of $0.1\ c$ (is the speed oflight) in the of electromagnetic wave, whose electric field is . The maximum magnetic force experienced by the electron will be :(given & electron charge)**

[Sep. 05, 2020 (I)]

(a) 3.2×10^{-18}N **(b)** 2.4×10^{-18}N **(c)** $4.8 X 10^{-19} N$ **(d)** $8 X 10^{-19} N$

SOLUTION: **(c) In electromagnetic wave,** $\frac{E_0}{B_0} = C$

Maximum value ofmagnetic field, $B_0 = \frac{E_0}{c}$

$$F_{\max} = qVB_{\max} \sin 90^\circ = \frac{qV_0E_0}{C}$$

(Given $V_0 = 0.1C$ **and** $E_0 = 30$)

$$= \frac{1.6 \times 10^{-19} \times 0.1 \times 3 \times 10^8 \times 30}{3 \times 10^8} = 4.8 \times 10^{-19}\text{N}$$

$)\ B_0$

EX16: **The electric field ofa plane electromagnetic wave is given by** $\vec{E} = E_0(\hat{x}+\hat{y})\sin(kz-(j)t)$

Its magnetic field will be given by: [Sep. 04, 2020 (II)]

(a) $\frac{E_0}{c}(-\hat{x}+\hat{y})\sin(kz-(j)t)$

(b) $\frac{E_0}{c}(\hat{x}+\hat{y})\sin(kz-0)t)$

(c) $\frac{E_0}{c}(\hat{x}-\hat{y})\sin(kz-(j)t)$

(d) $\frac{E_0}{c}(\hat{x}-\hat{y})\cos(kz-(j)t)$

SOLUTION: $\vec{E} = E_0(\hat{x}+\hat{y})\sin(kz-(j)t)$

Direction ofpropagation ofem wave $= +\hat{k}$

Unit vector in the direction ofelectric field, $\hat{E} = \frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$

The direction ofelectromagnetic wave is perpendicular to both electric and magnetic field. $\hat{k} = \hat{E} \times \hat{B}$

$$\Rightarrow \hat{k} = (\quad)\times^{\hat{B}}(\quad) \Rightarrow \hat{B} = \frac{-\hat{\imath}+\hat{\jmath}}{\sqrt{2}}$$

$$\vec{B} = \frac{E_0}{c}(-\hat{x}+\hat{y})\sin(kz-00t)$$

EX17: **The magnetic field ofa plane electromagnetic wave is**

$\cos((w)t\left(-k\frac{\hat{\imath}+\hat{\jmath}}{\sqrt{2}}\right))$

$\vec{B} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{\imath}T$

where $c = 3\times10^{8}\text{ms}^{-1}$ **is the speed oflight.**

The corresponding electric field is: [Sep. 03, 2020 (I)] (a) $\vec{E} = 9\sin[200\pi(y+ct)]\hat{k}\text{V/m}$

(b) $\vec{E} = -10^{-6}\sin[200\pi(y+ct)]\hat{k}\text{V/m}$

(c) $\vec{E} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{k}\text{V/m}$

(d) $\vec{E} = -9\sin[200\pi(y+ct)]\hat{k}\text{V/m}$

SOLUTION:4. (d) Given: $\overline{B} = 3\times10^{-8}\sin[200\pi(y+ct)]\hat{\imath}T$

$B_0 = 3\times10^{-8}$ $E_0 = CB_0 \Rightarrow E_0 = 3\times10^{8}\times3\times10^{-8} = 9\text{V/m}$

Directiono fwave propagation

$(\overline{E}\times\overline{B})\|\overline{C}$ $\hat{B} = \hat{\imath}$ **and** $\hat{C} = -\hat{\jmath}$ $\hat{E} = -\hat{k}$

$\overline{E} = E_0\sin[200\pi(y+ct)](-\hat{k})\text{V/m}$

or, $\overline{E} = -9\sin[200\pi(y+ct)]\hat{k}\text{V/m}$

EX18: **The electric field of a plane electromagnetic wavepropagating along the** x **direction in vacuum is** $\vec{E} = E_0\hat{j}\cos(0)t-kx)$**. The magnetic field** $\vec{B}$**, at the moment** $t = 0$ **is: [Sep. 03, 2020 (If)]**

(a) $\vec{B} = \frac{E_0}{\sqrt{\mu_0\varepsilon_0}}\cos(kx)\hat{k}$ (b) $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{j}$

(C) $\vec{B} = E_0\sqrt{\mu_0\varepsilon_0}\cos(kx)\hat{k}$ (d) $\vec{B} = \frac{E_0}{\sqrt{\mu_0\varepsilon_0}}\cos(kx)\hat{j}$

SOLUTION: **(c) Relation between electric field and magnetic field for an electromagnetic wave in vacuum is** $B_0 = \frac{E_0}{c}$

In free space, its speed $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$

Here, μ_0 **=absolute permeability,** ε_0 **=absolute permittivity**

$$B_0 = \frac{E_0}{c} = \frac{E_0}{1/\sqrt{\mu_0 \varepsilon_0}} = E_0\sqrt{\mu_0 \varepsilon_0}$$

As the electromagnetic wave is propagating along x direction and electric field is along y direction.

$\hat{E} \times \hat{B} \parallel \hat{C}$ (Here, $\hat{C}$ = direction of propagation of wave)

$\vec{B}$ should be in $\hat{k}$ direction.

$B = E_0\sqrt{\mu_0 \varepsilon_0}\cos(\omega t - kx)\hat{k}$

At $t = 0$

$B = E_0\sqrt{\mu_0 \varepsilon_0}\cos(kx)\hat{k}$

EX19: A plane electromagnetic wave, has frequency of 2.0×10^{10} Hz and its energy density is $1.02 \times 10^{-8} J/m^3$ in vacuum. The amplitude of the magnetic field of the wave is close to ($\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$ and $c = 3 \times 10^8 ms^{-1}$): [Sep. 02, 2020 (I)]

(a) 150nT **(b)** 160nT **(c)** 180nT **(d)** 190nT

solution: (b) Energy density $= \frac{1}{2}\frac{B^2}{\mu_0} \Rightarrow B = \sqrt{2 \times \mu_0 \times \text{Energy density}}$

$$\mu_0 = \frac{1}{C^2 \varepsilon_0} = 4\pi \times 10^{-7}$$

$$B = \sqrt{2 \times 4\pi \times 10^{-7} \times 102 \times 10^{-8}} = 160 \times 10^{-9}$$

$$= 160 nT$$

EX20: In a plane electromagnetic wave, the directions of electric field and magnetic field are represented by $\hat{k}$ and $2\hat{i}\ 2\hat{j}$, respectively. What is the unit vector along direction of propagation of the wave. [Sep. 02, 2020 (II)] $\vec{E} = E_0\frac{\hat{i}+\hat{j}}{\sqrt{2}}\cos(kz + \omega t)$

(a) $\frac{1}{\sqrt{2}}(\hat{i}+\hat{j})$ **(b)** $\frac{1}{\sqrt{2}}(\hat{j}+\hat{k})$ **(c)** $\frac{1}{\sqrt{5}}(\hat{i}+2\hat{j})$ **(d)** $(2\hat{i}+\hat{j})$

solution: (a) Electromagnetic wave will propagate perpendicular to the direction of Electric and Magnetic fields

$\hat{C} = \hat{E} \times \hat{B}$

Here unit vector $\hat{C}$ is perpendicular to both $\hat{E}$ and $\hat{B}$ Given, $\vec{E} = \hat{k}, \vec{B} = 2\hat{i} - 2\hat{j}$

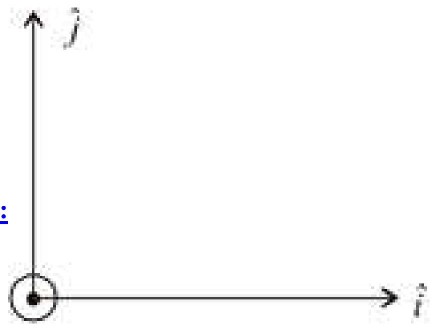
$$\hat{C} = \hat{E} \times \hat{B} = \frac{1}{\sqrt{2}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \frac{\hat{i}+\hat{j}}{\sqrt{2}} \qquad \Rightarrow \hat{C} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$$

EX21: The electric fields of two plane electromagnetic plane waves in vacuum are given by $\vec{E}_1 = E_0\hat{j}\cos(\omega t - kx)$ **and** $\vec{E}_2 = E_0\hat{k}\cos(\omega t - ky)$.
At $t = 0$, a particle of charge q is at origin with a velocity $\vec{v} = 0.8c\hat{j}$ (c is the speed of light in vacuum). The instantaneous force experienced by the particle is: [9 Jan 2020, I]

(a) $E_0 q(0.8\hat{i} - \hat{j} + 0.4\hat{k})$ **(b)** $E_0 q(0.4\hat{i} - 3\hat{j} + 0.8\hat{k})$

(c) $E_0 q(-0.8\hat{i} + \hat{j} + \hat{k})$ **(d)** $E_0 q(0.8\hat{i} + \hat{j} + 0.2\hat{k})$

solution:



Given: $\vec{E}_1 = E_0 j \cos((j)t - kx)$ i.e., **Travelling in** $+vex$ **direction** $\vec{E} \times \vec{B}$ **should be in** x **direction**

$\vec{B}$ **is in** $\hat{K}$

$\vec{B}_1 = \frac{E_0}{C} \cos((j)t - kx)\hat{k} \left(\because B_0 = \frac{E_0}{C}\right)$

$\vec{E}_2 = E_0\hat{k} \cos(wt - ky)\hat{k} \quad \vec{B}_2 = \frac{E_c}{c}\hat{\underline{\imath}} \cos(wt - ky)$

Travelling in $+vey$ **axis** $\vec{E} \times \vec{B}$ **should be in** y **axis**

Net force $\vec{F} = q\vec{E} + q(\vec{v} \times \vec{B}) = q(\vec{E}_1 + \vec{E}_2) + q(0.8c\hat{j} \times (\vec{B}_1 + \vec{B}_2))$

If t = 0 **and** x = y = 0

$\vec{E}_1 = E_0 j\, \vec{E}_2 = E_0\hat{k}\vec{B}_1 = \frac{E_0}{c}\hat{k}\, \vec{B}_2 = \frac{E_0}{c}\hat{\imath}$

$\vec{F}_{net} = qE_0(\hat{j} + \hat{k}) + q \times 0.8c \times \frac{E_0}{c} j \times (k + \hat{\imath})$

$= qE_0(j + \hat{k}) + 0.8qE_0 (\hat{\imath} - k)$

$= qE_0$ (0.8î+j +0.2k)

EX22: **A plane electromagnetic wave is propagating along the direction** $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ **with its polarization along the direction** $\hat{k}$**. The correct form of the magnetic field of the wave would be (here** B_0 **is an appropriate constant): [9 Jan 2020, II]**

(a) $B_0 \frac{\hat{i}-\hat{j}}{\sqrt{2}} \cos\left((w)t\left(-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\right)$

(b) $B_0 \frac{\hat{j}-\hat{i}}{\sqrt{2}} \cos\left((w)t\left(+k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\right)$

(C) $B_0\hat{k} \cos\left((w)t\left(-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\right)$

(d) $B_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}} \cos\left((w)t\left(-k\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)\right)$

soluttion: (a) Direction ofpolarisation $= \hat{E} = \hat{k}$

Direction ofpropagation $= \hat{E} \times \hat{B} = \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

But $\vec{E}.\vec{B} = 0\ \hat{B} = \frac{\hat{i}-\hat{j}}{\sqrt{2}}$

EX23:A plane electromagnetic wave of frequency 25 GHz ispropagating in vacuum along the z direction. At aparticular point in space and time, the magnetic field isgiven by $\vec{B} = 5 \times 10^{-8} \hat{j} T$. The corresponding electricfield $\vec{E}$ is (speed oflight $c = 3 \times 10^{8}$ ms 1)[8 Jan 2020, II]

(a) $1.66 \times 1t^{16} \hat{i}$ V/m (b) $-1.66 \times 10^{16} \hat{i}$ (c) $\hat{i}$ (d) $15\hat{i}$

solution:(d) Amplitude ofelectric field(E) and Magnetic field(B) ofan electromagnetic wave are related by the relation

$\frac{E}{B} = c \quad \Rightarrow E = Bc \qquad \Rightarrow E = 5 \times 10^{8} \times 3 \times 10^{8} = 15 N/C$

EX24:. Ifthe magnetic field in a plane electromagnetic wave isgiven by $\vec{B} = 3 \times 10^{8} \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t) \hat{j}$ T, thenwhat will be expression for electric field?[7 Jan 2020, I]

(a) $\vec{E} = \left(60 \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)\hat{k}\,{}^{V}/m\right)$

(b) $\vec{E} = \left(9 \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)\hat{k}\,{}^{V}/m\right)$

(c) $\vec{E} = \left(3 \times 10^{8} \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)\hat{k}\,{}^{V}/m\right)$

(d) **none**

solution:(b) Given, $\vec{B} = 3 \times 10^{-8} \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)$

Using, $E_0 = B_0 \times C = 3 \times 10^{-8} \times 3 \times 10^{8} = 9 V/m$

Electric field,

$\vec{E} = 9 \sin(1.6 \times 10^{3} x + 48 \times 10^{10} t)\hat{k} V/m$

EX25The electric field of a plane electromagnetic wave isgiven byAt $t = 0$, a positively charged particle is at the point$(x, y, z) = \left(0, 0, \frac{\pi}{k}\right)$. Ifits instantaneous velocity at $(t = 0)$is $v_0\hat{k}$, the force acting on it due to the wave is:[7 Jan 2020, II]

(a) parallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (b) zero (c) antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (d) parallel to $\hat{k}$

solution: (c) At $t = 0$, $z = \frac{\pi}{k}$ $\vec{E} = \frac{E_0}{\sqrt{2}}(\hat{i}+\hat{j}) \cos[\pi] = -\frac{E_0}{\sqrt{2}}(\hat{i})$

$\vec{F}_E = q\vec{E}$

Force due to electric field will be in the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Force due to magnetic field is in direction $q(\vec{v} \times \vec{B})$ and $\vec{v} \| \hat{k}$. Therefore, it is parallel to $\vec{E}$.

$\Rightarrow \vec{F}_{net} = \vec{F}_E + \vec{F}_B$ is antiparallel to $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$

EX26:An electromagnetic wave is represented by the electricfield $\vec{E} = E_0 \hat{n} \sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z directions to be $\hat{i}, \hat{j}, \hat{k}$, the direction ofpropogation $\hat{s}$ is: [12 April 2019, I]

(a) $\hat{s} = \frac{3\hat{i} - 4\hat{j}}{5}$ (b) $\hat{s} = \frac{-4\hat{k} + 3\hat{j}}{5}$ (c) $\hat{s} = \left(\frac{-3\hat{j} + 4\hat{k}}{5}\right)$ (d) $\hat{s} = \frac{3\hat{j} - 3\hat{k}}{5}$

solution:((c) $\hat{S} = \frac{6\hat{j}+8\hat{k}}{\sqrt{6^2+8^2}} = \frac{-3\hat{j}+4\hat{k}}{5}$

EX27. A plane electromagnetic wave having a frequency $v = 23.9$ GHz propagates along the positive z direction infree space. The peak value ofthe Electric Field is 60 V/m Which among the following is the acceptable magneticfield component in the electromagnetic wave?
[12 April 2019, II]

(a) $\overline{B} = 2 \times 10^{7} \sin(0.5 \times 10^{3} z + 1.5 \times 10^{11} t)\hat{i}$

(b) $\overline{B} = 2 \times 10^{-7} \sin(0.5 \times 10^{3} z - 1.5 \times 10^{11} t)\hat{i}$

(c) $\overline{B} = 60 \sin(0.5 \times 10^{3} x + 1.5 \times 10^{11} t)\hat{k}$

(d) $\overline{B} = 2 \times 10^{-7} \sin(1.5 \times 10^{2} x + 0.5 \times 10^{11} t)\hat{j}$

solution:ti))$B_0 = \frac{E_0}{c} = \frac{60}{3\times10^8} = 20 \times 10^{8}\,\mathrm{T} = 2 \times 10^{7}\,\mathrm{T}$ $\quad K = \frac{(j)}{v} = \frac{2\pi f}{v} = \frac{2\pi \times 23.9 \times 10^{9}}{3 \times 10^{8}} = 500$

$\rightarrow$

Therefore, $B = B_0 \sin(kz - CiJt)$
$= 2 \times 10^{7} \sin(0.5 \times 10^{3} z - 1.5 \times 10^{11} t)\hat{i}$

EX28: The electric field ofa plane electromagnetic wave is given by $\overline{E} = E_0 \hat{i} \cos(kz) \cos(cot)$ The corresponding magnetic field is then given by:
[10 April 2019, I]

(a) $\overline{B} = \frac{E_0}{C}\hat{j} \sin(kz) \sin(wt)$ (b) $\overline{B} = \frac{E_0}{C}\hat{j} \sin(kz) \cos(wt)$
(c) ($\overline{B} = \frac{E_0}{C}\hat{j} \cos(kz) \sin(wt)$ (d) $\overline{B} = \frac{E_0}{C}\hat{k} \sin(kz) \cos(wt)$

solution: (a) $\frac{E_0}{B_0} = C \Rightarrow B_0 = \frac{E_0}{C}$

Given that $\vec{E} = E_0 \cos(kz) \cos(wt)\,\hat{i}$

$\vec{E} = \frac{E_0}{2}[\cos(kz\ wt)\,\hat{i}\ \ \cos(kz + wt)\,\hat{i}]$

Correspondingly $\vec{B} = \frac{B_0}{2}[\cos(kz - wt)\hat{j} - \cos(kz + wt)\hat{j}]$

$\vec{B} = \frac{B_0}{2} \times 2 \sin\ kz\ \sin\ wt$

$\vec{B} = (\frac{E_0}{C} \sin\ kz\ \sin wt)\,\hat{j}$

PREVIOUS MAINS QUESTIONS

29. Light is incident normally on a completely absorbing surface with an energy flux of25 Wcm 2. Ifthe surface has an area of25 cm^2, the momentum transferred to the surfacein 40 min time duration will be: [10 April 2019, II]

(a) $6.3 \times 10^{\rightarrow\downarrow}$Ns (b) 1.4×10^{4}Ns
(c) 5.0×10^{3}Ns (d) $3.5 \times 10^{\triangleleft}$Ns

solution (c) Pressure, $P = \frac{I}{C} \Rightarrow \frac{F}{A} = \frac{I}{C} \Rightarrow F = \frac{IA}{C} = \frac{\Delta p}{\Delta t} \Rightarrow \Delta p = \frac{I}{C} A \Delta t$

$= \frac{(25\times25)\times10^{4}\times10^{-4}\times40\times60}{3\times10^{8}}$ N-s

$= 5 \times 10^{3}$ N-s

30. The magnetic field of a plane electromagnetic wave isgiven

by:$\vec{B} = B_0\ \hat{\imath}\ [\cos(kz - \omega t)] + B_1 \hat{\jmath} \cos(kz + \omega t)$Where $B_0 = 3 \times 10^{5}$T and

$B_1 = 2 \times 10^{6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{\rightarrow\downarrow}$C at $z = 0$ is closest to: [9 April 2019 I]

(a) 0.6N (b) 0.1N(c) 0.9N (d) 3×10^{-2}N

Solution. (a) $B_0 = \sqrt{B_0^2 + B_1^2} = \sqrt{30^2 + 2^2} \times 10^{-6} = 30 \times 10^{4}$T

$E_0 = CB = 3 \times 10^{8} \times 30 \times 10^{-6} = 9 \times 10^{3}$V/m

$\frac{E_0}{\sqrt{2}} = \frac{9}{\sqrt{2}} \times 10^{3} Vlm$

Force on the charge,$F = EQ = \frac{9}{\sqrt{2}} \times 10^{3} \times 10^{-4} \simeq 0.64N$

31. A plane electromagnetic wave offiiequency50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}$V/m. The corresponding magnetic field $\vec{B}$, at that point will be:

[9 April 2019 I]

(a) $18.9 \times 10^{8}\hat{k}$T (b) $2.1 \times 10^{8}\hat{k}$T (c) $6.3 \times 10^{8}\hat{k}$T (d) $18.9 \times 10^{8}\hat{k}$T

Solution. b)) As we know,

$$|\vec{B}| = \frac{|\vec{E}|}{C} = \frac{6.3}{3 \times 10^{8}} = 2.1 \times 10^{-8}\text{T}$$

and $\hat{E} \times \hat{B} = \hat{C}$

$\hat{J} \times \hat{B} = \hat{\imath}$ [EM wave travels along $+$(ve)x-direction.]

$\widehat{B} = \hat{k}$ or $\vec{B} = 2.1 \times 10^{-8}\hat{k}T$

32. 50 $Wm2$ energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy(25%) is reflected from the surface and the rest is absorbed. The force exerted on $1m^2$ surface area will BeClose to
$(c = 3 \times 10^8 m/s)$: [9 April 2019, II]
(a) $15 \times 10^8 N$ (b) $20 \times 10^8 N$
(c) $10 \times 10^8 N$ (d) $35 \times 10^8 N$

Solution. b)$F = (1 + r)\frac{IA}{C} = \frac{(1+0.25)\times 50\times 1}{3\times 10^8} = 20 \times 10^{-8} N$

33. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6$ V/m 1 along$y$-direction. Its corresponding magnetic field component,B would be : [8 April 2019 I]
(a) $2 \times 10^8 T$ along z-direction
(b) $6 \times 10^8 T$ along x-direction
(c) $6 \times 10^8 T$ along z-direction
(d) $2 \times 10^8 T$ along y-direction
Solution. (a) The relation between amplitudes of electric and magnetic field in fi¡ee space is given by

$$B_0 = \frac{E_0}{c} = \frac{6}{3 \times 10^8} = 2 \times 10^{-8} T$$

Propagation direction $= \hat{E} \times \hat{B}$
$\hat{\imath} = \hat{\jmath} \times B$

$$\Rightarrow \hat{B} = \hat{k}$$

The magnetic field component will be along z direction.
34. The magnetic field of an electromagnetic wave is given by.

$\overline{B} = 1.6 \times 10^{-6} \cos(2 \times 10^7 z + 6 \times 10^{15} t)\ (2\hat{\imath}+\hat{\jmath})\ \frac{Wb}{m^2}$

The associated electric field will be: [8 April 2019, II]

(a) $\overline{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t)\ (2\hat{\imath}+\hat{\jmath})\ \frac{V}{m}$

(b) $\overline{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z - 6 \times 10^{15} t)(-2\hat{\jmath} + \hat{\imath})\frac{V}{m}$

(c) $\overline{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)\ (-\hat{\imath}+2\hat{\jmath})\ \frac{V}{m}$

(d) $\overline{E} = 4.8 \times 10^2 \cos(2 \times 10^7 z + 6 \times 10^{15} t)\ (\hat{\imath}-2\hat{\jmath})\ \frac{V}{m}$

Solution. (c) $E_0 = cB_0 = 3 \times 10^8 \times 1.6 \times 10^{-6} = 4.8 \times 10^2 V/m$

Also $\vec{S} \Rightarrow \vec{E} \times \vec{B}$

or $-\vec{K} \Rightarrow \overline{E} \times (2\hat{\imath}+\hat{\jmath})$

Therefore direction of $\overline{E}$ $is(-\hat{\imath}+2\hat{\jmath})$

35. The mean intensity of radiation on the surface of the Sun is about $10^8 W/m^2$. The rms value ofthe correspondingmagnetic field is closest to: [12 Jan 2019, II]

(a) 1 T (b) 10^2T (c) 10^{-2}T (d) 10^{-4}T

Solution. $\left(I = \frac{B_0^2}{2\mu_0} \cdot C\right.$

$$\Rightarrow \frac{B_0^2}{2} = \frac{I\mu_0}{C}$$

$$\Rightarrow B_{rms} = \sqrt{\frac{I\mu_0}{C}}$$

$$= \sqrt{\frac{10^8 \times 4\pi \times 10^{-7}}{3 \times 10^8}}$$

$$= 6 \times 10^{-4} T$$

Which is closest to 10^{-4}

36. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields, and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by:

[11 Jan 2019, I]

(a) $\left(\frac{1}{\sqrt{n}} \frac{1}{\sqrt{n}}\right)$ (b) $\left(\sqrt{n}, \sqrt{n}\right)$ (c) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\frac{1}{\sqrt{n}} \sqrt{n}\right)$

Solution. (c) The speed of electromagnetic wave in free space is given by

$C = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (i)

In medium, $v = \frac{1}{\sqrt{k\epsilon_0 \mu_0}}$ ... (ii)

Dividing equation (i) by(ii), we get$\frac{C}{V} = \sqrt{k} = n$

$\frac{1}{2} \in_{\cap} E_{\cap}^{2}C =$ intensity $= \frac{1}{2} \in_{\cap} kE^{2}vE_{0}^{2}C = kE^{2}v$

$$\Rightarrow \frac{E_0^2}{E^2} = \frac{kV}{C} = \frac{n^2}{n} \Rightarrow \frac{E_0}{E} = \sqrt{n}$$

similarly

$$\frac{B_0^2 C}{2\mu_0} = \frac{B^2 v}{2\mu_0} \supset \frac{B_0}{B} = \frac{l}{\sqrt{n}}$$

37. A27mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by:
[Given permittivity of space $\in 0 = 9 \times 10^{-12}$ SI units, Speed Of light $c = 3 \times 10^{8} m/s$] [11 Jan2019, II]

(a) $2kV/m$ (c) $0.7kV/m$

(b) lkV/m (d) $1.4kV/m$

38. If the magnetic field of a plane electromagnetic wave is given by (The speed of light $= 3 \times 10^{8} m/s$)

$$B = 100 \times 10^{-6} \sin \left[2\pi \times 2 \times 10^{15} \left(t - \frac{x}{c}\right)\right]$$

then the maximum electric field associated with it is:
[10 Jan. 2019 I]

(a) $6 \times 10^{4} N/C$ (b) $3 \times 10^{4} N/C$

(c) $4 \times 10^{4} N/C$ (d) $4.510^{4} N/C$

39. The electric field of a plane polarized electromagnetic wave in fi$_{\text{i}}$ee space at time $t = 0$ is given by an expression

$$\vec{E}(x, y) = 10\hat{j} \cos [(6x + 8z)]$$

The magnetic field $\vec{B}(x, z, t)$ is given by: (c is the velocity of light) [10 Jan 2019, II]

(a) $\frac{1}{c}$ (6k+8î) $\cos [(6x - 8z + 10ct)]$

(b) $\frac{1}{c}$ (6k̂-8î) $\cos [(6x + 8z - 10ct)]$

(c) $\frac{1}{c}$ (6k+8î) $\cos [(6x + 8z - 10ct)]$

(d) $\frac{1}{c}$ (6k-8î) $\cos [(6x + 8z + 10ct)]$

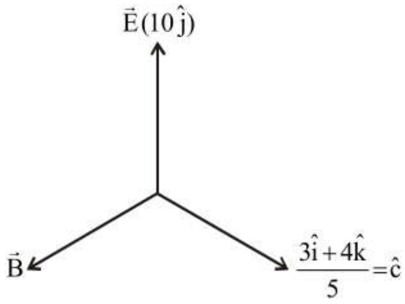
Solution. $\hat{C} \times \hat{E} = \frac{-4\hat{i}+3\hat{k}}{5}$

$$\vec{B} = \frac{E}{C} = \frac{10}{C}$$

$$\vec{B}\frac{10}{C}\left(\frac{-4\hat{i}+3\hat{k}}{5}\right) = \left(\frac{-8\hat{i}+6\hat{k}}{C}\right)$$

or, magnetic field $\vec{B}(x, z, t) = \frac{1}{C}$

(6k-8î)cos $(6x + 8z - 10ct)$



40. An EM wave from air enters a medium. The electric fields are

$\vec{E}_1 = E_{01}\hat{x} \cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and

$\vec{E}_2 = E_{02}\hat{x} \cos[k(2z - ct)]$ in medium, where the wavenumber k and frequency v

refer to their values in air. The medium is nonmagnetic. If $\in_1$ and $\in_{r_2}$ refer to

relative permittivity's ofair and medium respectively, which of thefollowing options is correct? [9 Jan 2019, I]

(a) $\in_{r_2}\underline{\in_{\gamma_1}} = 4$ (b) $\in\underline{\in_{r_1}}\ r_2 = 2$

(c) $\in r_1 \underline{\in\ }\ r_2 = \frac{1}{4}$ (d) $\in_{r_2}\underline{\in_{r_1}} = \frac{1}{2}$

Solution. (c) Velocity ofEM wave is given by $v = \frac{1}{\sqrt{\mu\in}}$

Velocity in air $= \frac{(j)}{k} = C$

Velocity in medium $= \frac{C}{2}$

Here, $\mu_1 = \mu_2 = 1$ as medium is non-magnetic$-\frac{1}{\sqrt{\in_{r_2}}}\frac{1}{\sqrt{\in,1}} = \frac{C}{\left(\frac{C}{2}\right)} = 2 \Rightarrow \in\in_{r_1} r_2 = \frac{1}{4}$

41. The energy associated with electric field is (U_E) and withmagnetic fields is (U_B) for an electromagnetic wave infree space. Then : [9 Jan 2019, II]

(a) $U_E = \frac{U_B}{2}$ (b) $U_E > U_B$

(c) $U_E < U_B$ (d) $U_E = U_B$

Solution. (d) Average energy density ofmagnetic field,

$$u_B = \frac{B_0^2}{4\mu_0}$$

Average energy density of electric field,

$$u_E = \frac{\varepsilon_0 E_0^2}{4}$$

Now, $E_0 = CB_0$ and $C^2 = \frac{1}{\mu_0 \in_0}$

$$u_E = \frac{\varepsilon_0}{4} \times C^2 B_0^2 = \frac{\varepsilon_0}{4} \times \frac{1}{\mu_0 \varepsilon_0} \times B_0^2 = \frac{B_0^2}{4\mu_0} = u_B$$

$$u_E = u_B$$

Since energy density of electric and magnetic field is same, so energy associated with equal volume will be equal i. e.,

$$u_E = u_B$$

42. A plane electromagnetic wave of wavelength λ has anintensity I. It is propagating along the positive Y −direction. The allowed expressions for the electric andmagnetic fields are given by [Online Apri116, 2018]

(a) $\bar{E} = \sqrt{\frac{I}{\varepsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y\text{-}ct)\right]\hat{\imath}$; $\vec{B} = \frac{1}{c} E\hat{k}$

(b) $\bar{E} = \sqrt{\frac{I}{\varepsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}; \vec{B} = -\frac{1}{c} E\hat{\imath}$

(c) $\bar{E} = \sqrt{\frac{2I}{\varepsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y - ct)\right]\hat{k}; \vec{B} = +\frac{1}{c} E\hat{\imath}$

(d) $\bar{E} = \sqrt{\frac{2I}{\varepsilon_0 C}} \cos\left[\frac{2\pi}{\lambda}(y + ct)\right]\hat{k}; \vec{B} = \frac{1}{c} E\hat{\imath}$

Solution. (c) IfE_0 is magnitude of electric field then

$$\frac{1}{2}\varepsilon_0 E^2 \times C = 1 \Rightarrow E_0 = \sqrt{\frac{2I}{C\varepsilon_0}}$$

$$E_0 = \frac{E_0}{C}$$

Direction of $\vec{E} \times \vec{B}$ will be along $+\hat{j}$.

43. A monochromatic beam of light has a frequency $v = \frac{3}{2\pi} \times 10^{12}$ Hz and is propagating along the direction $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$. It is polarized along the $\hat{k}$ direction. The acceptableform for the magnetic field is: [Online April 15, 2018]

(a) $k\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right) \cos\left[10^4\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right).\vec{r} - (3\times10^{12})t\right]$

(b) $\frac{E_0}{C}\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right) \cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r} - (3\times10^{12})t\right]$

(c) $\frac{E_0}{C}\hat{k} \cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r} + (3\times10^{12})t\right]$

(d) $\frac{E_0}{C}\frac{(\hat{i}+\hat{j}+\hat{k})}{\sqrt{3}} \cos\left[10^4\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right).\vec{r} + (3\times10^{12})t\right]$

Solution. (c) $\hat{E} \times \hat{B}$ should give the direction ofwave propagation

$\Rightarrow \hat{K} \times \hat{B} || \frac{\hat{i}\times\hat{j}}{\sqrt{2}} \Rightarrow \hat{K} \times (\quad)(\quad) = \frac{\hat{j}-(-\hat{i})}{\sqrt{2}} = \frac{\hat{i}+\hat{j}}{\sqrt{2}} || \frac{\hat{i}+\hat{j}}{\sqrt{2}}$

Option (a), option (b) and option (d) does not satisfy

Wave propagation vector $\hat{K}$ should along $\frac{\hat{i}+\hat{j}}{\sqrt{2}}$.

44. The electric field component of monochromatic radiation is given by $\vec{E} = 2E_0\hat{i}\cos\ kz\ \ coswt$ Its magnetic field $\vec{B}$ is then given by: [Online April 9, 2017]

(a) $\frac{2E_0}{c}\hat{j}\sin\ kz\ \cos\ oot$ (b) $\frac{2E_0}{c}\hat{j}\sin\ kz\ \sin\ oot$

(c) $\frac{2E_0}{c}\hat{j}\sin\ kz\ \sin\ oot$ (d) $\frac{2E_0}{c}\hat{j}\cos\ kz\ \cos\ oot$

Solution. (c) Given, Electric field component of monochromatic radiation, $(\vec{E}) = 2E_0\hat{i}\ coskz\ \ \cos wt$

We know that, $\frac{dE}{dz} = -\frac{dB}{dt}$

$$\frac{dE}{dz} = -2E_0 k \sin\ kz\ \ \cos\ o)t = -\frac{dB}{dt}$$

$dB = +2E_0 k \sin \ kz \ \cos \ oot\, dt$ (i)
Integrating eq^n(i) , we have

$B = +2E_0 k \sin \ kz \int \ \cos \ oot\, dt$

Magnetic field is given by,

$= +2E_0 \frac{k}{0} \sin)\ kz \ \sin wt$

We also know that, $\frac{E_0}{B_0} = \frac{t0}{k} = c$

Magnetic field vector, $\vec{B} = \frac{2E_0}{c} \hat{j} \sin \ kz \ \sin \ wt$

45. Magnetic field in a plane electromagnetic wave is given by

$\vec{B} = B_0 \sin(kx + wt)\hat{j}$ T.Expression for corresponding electric field will be:

Where c is speed oflight. [Online April 8, 2017]

(a) $\vec{E} = B_0 c \sin\ (kx + 0)t)\hat{k} V/m$

(b) $\vec{E} = \frac{B_0}{c} \sin\ (kx + 0lt)\hat{k} V/m$

(c) $\vec{E} = -B_0 c \sin\ (kx + (ot)\hat{k} V/m$

(d) $\vec{E} = B_0 c s\dot{m}(kx - 0)t)\hat{k} V/m$

Solution. (a) Speed ofEM wave in force space (c) $= \frac{E_0}{B_0}$

or $\vec{E} = cB_0 \sin\ (kx + 0)t)\hat{k}$

46. Consider an electromagnetic wave propagating in vacuum.
Choose the correct statement: [Online Apri110, 2016]

(a) For an electromagnetic wave propagating in +ydirection the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(x, t)\hat{z}$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_z(x, t)\hat{y}$

(b) For an electromagnetic wave propagating in +ydirection the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(x, t)\hat{y}$ andthe magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}(x, t)\hat{z}$

(c) For an electromagnetic wave propagating in+x direction the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(y, z, t)(\hat{y} + \hat{z})$ and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}(y, z, t)(\hat{y} + \hat{z})$

(d) For an electromagnetic wave propagating in +xdirection the electric field is $\vec{E} = \frac{1}{\sqrt{2}} E_{yz}(x,t)(\hat{y} - \hat{z})$and the magnetic field is $\vec{B} = \frac{1}{\sqrt{2}} B_{yz}(x,t)(\hat{y} + \hat{z})$

Solution. (d) Wave in X-direction means E and B should be function ofx and t.

$$\hat{y} - \hat{z} \perp \hat{y} + \hat{z}$$

47. For plane electromagnetic waves propagating in thez-direction, which one of the following combination gives the correct possible direction for $\overline{E}$ and $\overline{B}$ field respectively? [Online April 11, 2015]

(a) $(2\hat{\imath}+3\hat{\jmath})$ and $(\hat{\imath}+2\hat{\jmath})$ (b) $(-2\hat{\imath}-3\hat{\jmath})$ and $(3\hat{\imath}-2\hat{\jmath})$

(c) $(3\hat{\imath}+4\hat{\jmath})$ and $(4\hat{\imath}-3\hat{\jmath})$ (d) $(\hat{\imath}+2\hat{\jmath})$ and $(2\hat{\imath}-\hat{\jmath})$

Solution. b) As we know, $\vec{E}.\overline{B} = 0$ $[\vec{E} \perp \overline{B}]$and $\vec{E} \times \overline{B}$ should be along Z direction

As $(-2\hat{\imath} - 3\hat{\jmath}) \times (3\hat{\imath} - 2\hat{\jmath}) = 5\hat{k}$Hence option (b) is the correct answer.

48. An electromagnetic wave travelling in the x-direction has frequency of2×10^{14} Hz and electric field amplitude of 27Vm^{-1}. From the options given below, which one describesthe magnetic field for this wave? [Online Apri110, 2015]

(a) $\vec{B}(x,t) = (3 \times 10^{-8}\text{T})\hat{j} \sin[2\pi(1.5 \times 10^{-8}x - 2 \times 10^{14}t)]$

(b) $\vec{B}(x,t) = (9 \times 10^{-8}T)\hat{\imath} \sin[2\pi(1.5 \times 10^{-8}x - 2 \times 1014t)]$

(c) $\overline{B}(x,t) = (9 \times 10^{-8}T)\hat{\jmath} \sin[1.5 \times 10^{-6}x - 2 \times 10^{14}t)]$

(d) $\vec{B}(x,t) = (9 \times 10^{-8}T)\hat{k} \sin[2\pi(1.5 \times 10^{-6}x - 2 \times 10^{14}t)]$

Solution. (d) As we know,$B_0 = \frac{E_0}{C} = \frac{27}{3\times10^8} = 9 \times 10^{-8}$ tesla

Oscillation of B can be only along $\hat{j}$ or $\hat{k}$ direction.

$(w) = 2\pi f = 2\pi \times 2 \times 10^{14}$ Hz $\overline{B}(x,t) = (9 \times 10^{-8}T)\hat{k} \sin[2\pi(1.5 \times 10^{-6} \times -2 \times 10^4 t)]$

49. During the propagation of electromagnetic waves in a medium: [2014]

(a) Electric energy density is double of the magnetic energy density.

(b) Electric energy density is half of the magnetic energy density.
(c) Electric energy density is equal to the magnetic energy density.
(d)Both electric and magnetic energy densities are zero.

Solution. (c) $E_0 = CB_0$ and $C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Electric energy density $= \frac{1}{2}\varepsilon_0 E_0^2 = \mu_E$

Magnetic energy density $= \frac{1}{2}\frac{Bo^2}{\mu_0} = \mu_B$

Thus, $\mu_E = \mu_B$ Energy is equally divided between electric and magneticfield.

50. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5m from the lamp will be nearly: [Online Apri112, 2014]
(a) $1.34\mathrm{V/m}$ (b) $2.68\mathrm{V/m}$
(c) $4.02\mathrm{V/m}$ (d) $5.36\mathrm{V/m}$

Solution b) Wavelength ofmonochromatic green light
$= 5.5 \times 10^{-5}$ cm

Intensity $\mathrm{I} = \frac{\text{Power}}{\text{Area}}$

$$= \sqrt{\frac{2\times\left(\frac{3}{100}\pi\right)}{\left(\frac{1}{4\pi\times9\times10^9}\right)\times\left(3\times10^8\right)}} = \frac{100\times(3/100)}{4\pi(5)^2} = \frac{3}{100\pi}\mathrm{Wm}^{-2}$$

Now, half of this intensity(I) belongs to electric field and
Halfof that to magnetic field, therefore,

$$\frac{\mathrm{I}}{2} = \frac{1}{4}\varepsilon_0 \mathrm{E}_0^2 \mathrm{C} \text{or } \mathrm{E}_0 = \sqrt{\frac{2\mathrm{I}}{\varepsilon_0 \mathrm{C}}} = \sqrt{\frac{6}{25}\times 30} = \sqrt{72}$$
$$\mathrm{E}_0 = 2.68\mathrm{V/m}$$

51. An electromagnetic wave of frequency 1×10^{14} hertz is propagating along z-axis. The amplitude of electric field is4 $\mathrm{V/m}$. If $\varepsilon_0 = 8.8 \times 10^{-12}\mathrm{C}^2/\mathrm{N}-\mathrm{m}^2$, then average energy density of electric field will be: [Online April, 2014]
(a) $35.2 \times 10^{-10}\mathrm{J/m}^3$ (b) $35.2 \times 10^{-11}\mathrm{J/m}^3$
(c) $35.2 \times 10^{-12}\mathrm{J/m}^3$ (d) $35.2 \times 10^{-13}\mathrm{J/m}^3$

Solution (c) Given: Amplitude ofelectric field,$E_0 = 4\mathrm{v/m}$
Absolute permitivity,$\varepsilon_0 = 8.8 \times 10^{-12}\mathrm{c}^2/\mathrm{N}-\mathrm{m}^2$

Average energy density u_E =?

Applying formula, Average energy density $u_E = \frac{1}{4}\varepsilon_0 E^2$

$$\Rightarrow u_E = \frac{1}{4} \times 8.8 \times 10^{-12} \times (4)^2$$
$$= 35.2 \times 10^{-12} \mathrm{J/m^3}$$

52. The magnetic field in a travelling electromagnetic wave has a peak value of20 nT. The peak value ofelectric fieldstrength is : [2013]

(a)$3\mathrm{V/m}$ (b)$6\mathrm{V/m}$ (c) $9\mathrm{V/m}$ (d) $12\mathrm{V/m}$

Solution. From question,$B_0 = 20nT = 20 \times 10^{-9} T$

(velocity of light in vacuum $C = 3 \times 10^8 \mathrm{ms^{-1}}$)$\vec{E}_0 = \vec{B}_0 \times \vec{C}$

$|\vec{E}_0| = |\vec{B}| \cdot |\vec{C}| = 20 \times 10^{-9} \times 3 \times 10^8 = 6\mathrm{V/m}.$

53. A plane electromagnetic wave in anon-magnetic dielectric medium is given by $\overline{E} = \overline{E}_0(4 \times 10^{-7} x - 50t)$ with distance being in meter and time in seconds. The dielectric constant of the medium is: [Online April 22, 2013]

(a) 2.4 (b)5.8 (c) 8.2 (d) 4.8

54. Select the correct statement from the following:
[Online April 9, 2013]

(a) Electromagnetic waves cannot travel in vacuum.
(b) Electromagnetic waves are longitudinal waves.
(c) Electromagnetic waves are produced by charges moving with uniform velocity.
(d) Electromagnetic waves carry both energy and momentum as they propagate through space.

Solution (d) Electromagnetic waves do not required any medium to propagate. They can travel in vacuum. They are transverse in nature like light. They carry both energy and momentum. A changing electric field produces a changing magnetic field and vice-versa. Which gives rise to a transverse wave known as electromagnetic wave.

55. An electromagnetic wave in vacuum has the electric and magnetic field $\vec{E}$ and $\vec{B}$, which are always perpendicular to each other. The direction of polarization is given by $\vec{X}$and that of wave propagation by $\vec{k}$. Then [2012]

(a) $\vec{X}||\vec{B}$ and $\vec{k}||\vec{B}\times\vec{E}$

(b) $\vec{X}||\vec{E}$ and $\vec{k}||\vec{E}\times\vec{B}$

(c) $\vec{X}||\vec{B}$ and $\vec{k}||\vec{E}\times\vec{B}$

(d) $\vec{X}||\vec{E}$and$\vec{k}||\vec{B}\times\vec{E}$

Solution b) The E.M. wave are transverse in nature *i. e.*,

$=\frac{\vec{k}\times\vec{E}}{\mu}=\vec{H}$ ... (i)

where $\vec{H}=\frac{\vec{B}}{\mu}$

and $\frac{\vec{k}\times\vec{H}}{\text{tiJ8}}=-\vec{E}$ ... (ii)

$\vec{k}$ is $\perp\vec{H}$ and $\vec{k}$ is also $\perp$ to $\vec{E}$The direction of wave propagation is parallel to

$\vec{E}\times\vec{B}$.The direction ofpolarization is parallel to electric field

56. An electromagnetic wave with frequency w and wavelength λ travels in the $+y$ direction. Its magnetic fieldis along $+x$-axis. The vector equation for the associated electric field (of amplitude E_0) is [Online May 19, 2012]

(a) $\to E=-E_0\cos\left((w)t+\frac{2\pi}{\lambda}y\right)\hat{x}$

(b) $\to E=E_0\cos\left((w)t-\frac{2\pi}{\lambda}y\right)\hat{x}$

(c) $\to E=E_0\cos(cotw\text{-}\ \frac{2\pi}{\lambda}y)\ \hat{z}$

(d) $\to E=-E_0\cos\left(wt+\frac{2\pi}{\lambda}y\right)\hat{z}$

Solution (c) In an electromagnetic wave electric field and magnetic field are perpendicular to the direction of propagation of wave. The vector equation for the electric field is $\vec{E}=E_0\cos(\left(\text{wt}-\frac{2\pi}{\lambda}y\right)\hat{z}$

57. An electromagnetic wave of frequency $v=3.0$ MHzpasses from vacuum into a dielectric medium withpermittivity $\in=4.0$. Then [2004]

(a) wave length is halved and frequency remains unchanged

(b) wave length is doubled and frequency becomes half

(c) wave length is doubled and the frequency remains Unchanged wave length and fiiequency both remain unchanged.

Solution. (a) Frequency remains unchanged during refraction

Velocity ofEM wave in vacuum

$$V_{\text{vacuum}} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = C$$

$$v_{\text{med}} = \frac{1}{\sqrt{\mu_0 \epsilon_0 \times 4}} = \frac{c}{2}$$

$$\frac{\lambda_{\text{med}}}{\lambda_{\text{vacuum}}} = \frac{v_{\text{med}}}{v_{\text{vacuum}}} = \frac{c/2}{c} = \frac{1}{2}$$

Wavelength is halved and frequency remains unchanged

58. *Electromagnetic waves are transverse in nature* is evident by [2002]

(a) polarization (b) interference(c) reflection (d) diffraction

Solution The phenomenon of polarization is shown only by transverse waves. The vibration of electromagnetic wave are restricted through polarization in a direction perpendicular to wave propagation.

59. The correct match between the entries in column I and column II are: [Sep. 05, 2020 (II)]

I	II
Radiation	Wavelength
(A) Microwave	(i) 100m
(B) Gamma rays	(ii) $1\sigma_{\text{m}}^{15}$
(C) A.M. radio waves	$(iii) 10^{-10}$m
(D) X-rays	(iv) $1\sigma^{-3}$m

(a) (A) – (ii), (B) – (i), (C) – (iv), (D) – (iii)

(b) (A) – (i), (B) – (iii), (C) – (iv), (D) – (ii)

(c) (A) – (iii), (B) – (ii), (C) – (i), (D) – (iv)

(d) (A) – (iv), (B) – (ii), (C) – (i), (D) – (iii)

Solution (d) Energy sequence of radiations is

E_{Y}-Rays $> E_{\text{X}}$-Rays $>$ Emicrowave $> E_{\text{AM}}$ Radiowaves

λ_{Y} -Rays $< \lambda_{\text{X}}$-Rays $< \lambda_{\text{microwave}} < \lambda_{\text{AM}}$ Radiowaves

From the above sequence, we have(a) Microwave $\rightarrow 10^{-3}$m

(iv) Gamma Rays $\rightarrow 10^{-15}$m

(ii)(c) AM Radio wave $\rightarrow 100$m

(i)(d) X –Rays

60. Chose the correct option relating wavelengths of different parts of electromagnetic wave spectrum:

[Sep. 04, 2020 (I)]

(a) $\lambda_{\text{visible}} < \lambda_{\text{micro}}$ waves $< \lambda_{\text{radio}}$ waves $< \lambda_{x}$-rays

(b) λ_{radio} waves $> \lambda_{\text{micro}}$ waves $> \lambda_{\text{visible}} > \lambda_{x}$-rays

(c) λ_{x}-rays $< \lambda_{\text{micro}}$ waves $< \lambda_{\text{radio}}$ waves $< \lambda_{\text{visible}}$

(d) $\lambda_{\text{visible}} > \lambda_{x}$-rays $> \lambda_{\text{radio}}$ waves $> \lambda_{\text{micro}}$ waves

Solution b) The orderly arrangement ofdifferent parts ofEM wave in decreasing order of wavelength is as follows:

$\lambda_{\text{radiowaves}} > \lambda_{\text{microwaves}} > \lambda_{\text{visible}} > \lambda_{\text{X}}$-rays

61. Given below in the left column are different modes of communication using the kinds of waves given in the right column. [10 April 2019, I]

A. Optical Fiber	P. Ultrasound Communication
B. Radar	Q. InfiaredLight
C. Sonar	R Microwaves
D. Mobile Phones	S. Radio Waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(a) A-Q, B – S, C-R, D-P

(b) A-S, B-Q, C-R, D-P

(c) A-Q, B-S, C-P, D-R

(d) A-R, B-P, C-S, D-Q

Solution (c) Optical Fibre Communication- Infiared Light

Radar- Radio Waves

Sonar- Ultrasound

Mobile Phones- Microwaves

E, Decreases

62. Arrange the following electromagnetic radiations per quantum in the order of increasing energy: [2016]

A: Blue light B: Yellow lightC: X-ray D: Radiowave.

(a) C, A, B, D (b) B, A, D, C

(c) D, B, A, C (d) A, B, D, C

Solution (c) y-rays X-rays uv-rays Visible rays IR rays Radio VIBGYOR Microwaves waves

Radio wave $<$ yellow light $<$ blue light $<$ X-rays

(Increasing order of energy

63.Microwave oven acts on the principle of:

[Online April 9, 2016]

(a) giving rotational energy to water molecules

(b) giving translational energy to water molecules
(c) giving vibrational energy to water molecules
(d) transferring electrons $from$ lower to higher energy levels in water molecule

Solution (c) Microwave oven acts on the principle of giving vibrational energyto water molecules.

64. If microwaves, X rays, infrared, gamma rays, ultra-violet, radio waves and visible parts of the electromagnetic spectrum are denoted by$\mathrm{M}, \mathrm{X}, \mathrm{I}, \mathrm{G}, \mathrm{U}, \mathrm{R}$ and V then which of the following is the arrangement in ascending order of wavelength? [Online Apri119, 2014]

(a) $\mathrm{R}, \mathrm{M}, \mathrm{I}, \mathrm{V}, \mathrm{U}, \mathrm{X}$ and G
(b) $\mathrm{M}, \mathrm{R}, \mathrm{V}, \mathrm{X}, \mathrm{U}, \mathrm{G}$ and I
(c) $\mathrm{G}, \mathrm{X}, \mathrm{U}, \mathrm{V}, \mathrm{I}, \mathrm{M}$ and R
(d) $\mathrm{I}, \mathrm{M}, \mathrm{R}, \mathrm{U}, \mathrm{V}, \mathrm{X}$ and G

Solution (c) Gamma rays $<$ X $-$rays $<$ Ultra violet $<$ Visible rays $<$ Infiared rays $<$ Microwaves $<$ Radio waves.

DUAL NATURE OF RADIATION AND MATTER WAVES

Photon:

According to Eienstein's quantum theory light propagates in the bundles (packets or quanta) of energy, each bundle being called a photon and possessing energy.

Energy of photon :

Energy of photon is given by $E = h\nu = \dfrac{hc}{\lambda}$;

where c = Speed of light,

h = Plank's constant = 6.6 ´ 10^{-34} *J-sec*,

n = Frequency in *Hz*,

l = Wavelength of light.

In electron volt $E(eV) = \dfrac{hc}{e\lambda} = \dfrac{12375}{\lambda(\mathring{A})}$ » $\dfrac{12400}{\lambda(\mathring{A})}$

Mass of photon :

Actually rest mass of the photon is zero. But it's effective mass is given as

$$E = mc^2 = h\nu$$

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{c\lambda}.$$

This mass is also known as kinetic mass of the photon

Momentum of the photon

Momentum $p = m \times c = \dfrac{E}{c} = \dfrac{h\nu}{c} = \dfrac{h}{\lambda}$

Number of emitted photons :

The number of photons emitted per second from a source of monochromatic radiation of wavelength l and power P is given as

$$(n) = \frac{P}{E} = \frac{P}{h\nu} = \frac{P\lambda}{hc};$$

where E = energy of each photon

Intensity of light (I) :

Energy crossing per unit area normally per second is called intensity or energy flux

i.e. $I = \dfrac{E}{At} = \dfrac{P}{A}$

$$\left(\frac{E}{t} = P = \text{radiation power}\right)$$

At a distance r from a point source of power P intensity is given by

$$I = \frac{P}{4\pi r^2}$$

$$I \propto \frac{1}{r^2}$$

Number of photons falling per second (n) :

If P is the power of radiation and E is the energy of a photon then

$$n = \frac{P}{E}$$

Electron Emission :

♦ Metals have free electrons and these normally cannot escape out of the metal surface.

♦ The free electron is held inside the metal surface by the attractive forces of the ions.

♦ A certain minimum amount of energy is requried to be given to an electron to pull it out from the surface of the metal and this energy is known as "Work Function".

♦ Work function (ϕ) = 5.65 eV, highest (for platinum) ϕ=1.88 eV, lowest (for ceasium)

♦ This minimum energy required for the electron emission can be supplied by any one of the following processes.

a) Thermionic emission :

"Sufficient thermal energy can be imported to free electrons" by suitably heating

b) Field emission:

"By applying a very strong electric field ($\approx 10^8 V/m$)".

c) Photo electric emission:

"By irradiating the metal surface with suitable E.M radiaton".

Photo electric effect :

- The photo-electric effect is the emission of electrons (called photo-electrons when light strikes a surface. To escape from the surface, the electron must absorb enough energy from the incident radiation to overcome the attraction of positive ions in the material of the surface.

♦ The emission of electrons from a metal plate when illuminated by electromagnetic radiation of suitable wavelength is called Photoelectric effect.

The photoelectric effect is based on the principle of conservation of energy.

♦ Photo electric effect was discovered by Hertz in 1887. In his experiments, Hertz observed that high voltage spark passes across the metal electrodes more easily when cathode is illuminated with ultra violet rays from an arc lamp.

♦ In 1888 Hallwachs under took the study further. He connected zinc plate to an electroscope. He found that when zinc plate is illuminated with ultra violet light it became positively charged. A positively charged zinc plate became more positively charged when it is further illuminated with ultra violet light .

♦ From these observations he concluded that negatively charged particles were emitted by the zinc plate under the action of ultra violet light. After the discovery of electron these particles were called as photo electrons.

Work function (or threshold energy) (W_0) :

The minimum energy of incident radiation, required to eject the electrons from metallic surface is defined as work function of that surface.

$$W_0 = h\nu_0 = \frac{hc}{\lambda_0} Joules ;$$

n_0 = Threshold frequency;
l_0 = Threshold wavelength

Work function in electron volt $W_0(eV) = \frac{hc}{e\lambda_0} = \frac{12375}{\lambda_0(\AA)}$

Threshold frequency (n_0) :

The minimum frequency of incident radiations required to eject the electron from metal surface is defined as threshold frequency.

If incident frequency $n < n_0$ No photoelectron emission

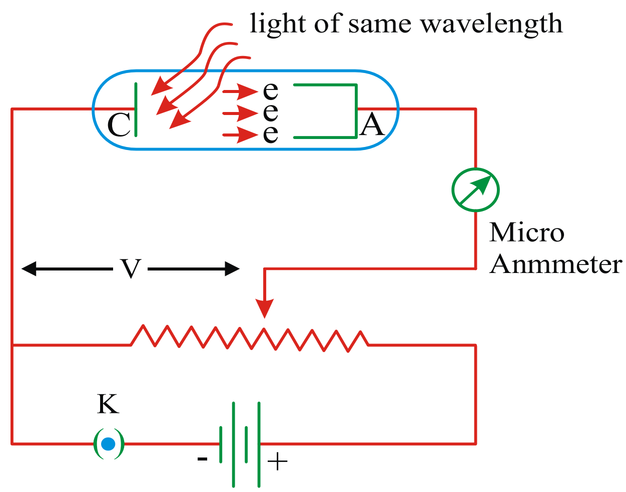
For most metals the threshold frequency is in the ultraviolet (corresponding to wavelengths between 200 and 300 *nm*), but for potassium and cesium oxides it is in the visible spectrum (l between 400 and 700 *nm*)

Threshold wavelength (l_0) :

The maximum wavelength of incident radiations required to eject the electrons from a metallic surface is defined as threshold wavelength.

If incident wavelength $l > l_0$ No photoelectron emission

Lenard's Experimental Study of Photoelectric effect :



♦ The apparatus used for experimental study of photoelectric effect. A metal plate C called cathode (emitter) and a metal cup A called anode (collector) are sealed in a vacuum chamber.

♦ A beam of monochromatic light enters the window of a vacuum chamber and falls on cathode C. The photoelectrons emitted are collected by the anode A.

♦ When key K is open and monochromatic light is made incident on the cathode, then current is measured by the ammeter. i.e., even though applied voltage is zero current flows in the circuit. These photoelectrons emitted from the cathode C moves towards anode A. But less energetic electrons comes to rest before reaching the anode.

♦ When anode is given positive potential w.r.t the cathode, electrons in the space charge are attracted towards the anode so photocurrent increases. If potential of the anode is increased gradually the effect of space charge becomes negligible at some potential and then every electron that is emitted from the cathode will be able to reach the anode. The current then becomes constant even though voltage is increased and this current is called saturation photocurrent.

♦ When anode is given negative potential w.r.t the cathode, the photo electrons will be repelled by the anode and some electrons will go back to cathode so current decreases. At some negative potential anode current becomes zero. This potential is called stopping potential.

♦ The minimum negative potential (V_0) given to the collector with respect to the emitter for which 'photocurrent' becomes zero is called 'stopping potential'.

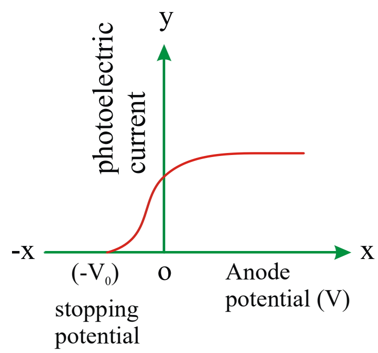
♦ Stopping potential is related to maximum kinetic energy of photoelectrons, because at this potential even the most energetic electron just fails to reach the anode.

So work done by the stopping potential is equal to the maximum kinetic energy of the electrons.

$$(-e)(-V_0) = \frac{1}{2}mv_{max}^2 - 0;$$

$$\therefore\ eV_0 = \frac{1}{2}mv_{max}^2$$

♦ A graph is plotted with current on y-axis and applied voltage on x-axis. It is as shown in below graph

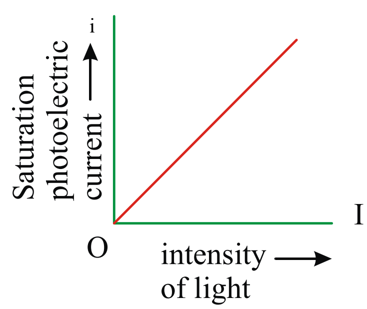


Experimental Results

1. Variation of Photo current with intensity of incident light :

Keeping the frequency of incident light and nature of the cathode constant, for different intensities of incident light saturation photo current is measured.

When a graph is plotted with saturation photocurrent on y-axis and intensity of incident light on x-axis, it is as shown in figure.

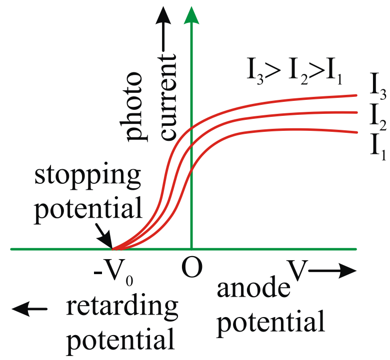


It is observed that saturation photocurrent (i) is proportional to the intensity (I) of incident light at a given frequency

2. Variation of saturation photo current with stopping potential at constant intensity :

Keeping the frequency of incident light and nature of the cathode constant, for different intensities of incident light photo current is measured.

When a graph is plotted with photocurrent on y-axis and applied voltage on x-axis. It is as shown in figure.



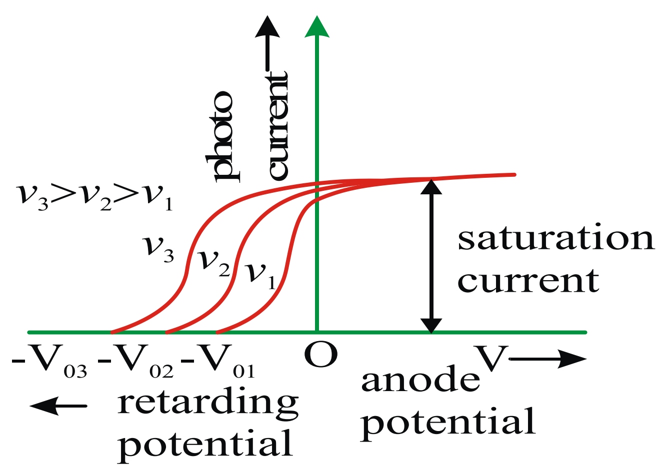
The value of stopping potential is independent of the intensity of incident light, if frequency is constant.

The magnitude of saturation current depends on the intensity of light. Higher the intensity, larger the saturation current.

3. Variation of frequency of incident light on stopping potential :

Keeping the intensity of incident light and nature of the cathode constant, for different frequencies of incident light, photo current is measured.

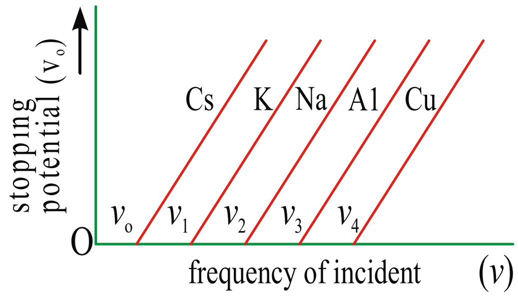
When a graph is plotted with photocurrent on y-axis and applied voltage on x-axis. It is as shown in figure.



- ♦ Larger the frequency of incident radiation, larger is the stopping potential.
- ♦ So The maximum kinetic energy of the emitted electrons depends on the frequency of incident light and nature of the metal plate. Maximum kinetic energy of photo electrons is independent of the intensity of incident light.
- ♦ The saturation photo current is independent of the frequency of incident radiation.

4. Variation of Stopping potential with frequency of incident light :

When a graph is plotted with stopping potential on y-axis and frequency of incident radiation on x-axis, keeping the metal constant, then it is as shown in figure.



- Threshold frequency (ν_0) is a characteristic of the metal plate and at this frequency, kinetic energy of the photo electrons is zero.
- Above threshold frequency, kinetic energy of photo electrons range from zero to a maximum value.
- Maximum kinetic energy and Stopping potential increases linearly with increasing frequency as shown in the above figure.

Laws of Photoelectric Effect:

- If the frequency of incident radiation is less than a certain value called threshold frequency, electrons are not emitted from a given metal surface, whatever be the intensity of the incident radiation.
- The maximum kinetic energy of photoelectrons depends on the frequency of the incident radiation, but it is independent of the intensity of the radiation. The maximum kinetic energy of photoelectrons is a linear function of the frequency of the incident radiation.
- The photocurrent increases with intensity of incident radiation, but it is independent of the frequency of incident radiation.
- There is no time lag between the incidence of the incident radiation and the emission of photo electrons.

Einsten's Photo Electric Equation:

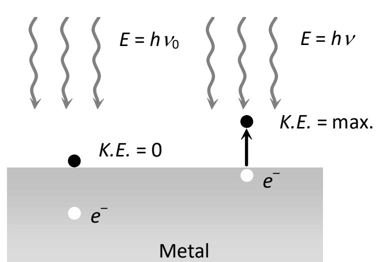


Fig. 25.17

- For explaining photoelectric effect, Einstein postulated that light consists of particles called photons. Energy of a photon of frequency ν is $h\nu$.
- According to this theory the emission of a photoelectron was the result of the interaction of a single photon with an electron, in which the photon is completely absorbed by the electron.
- The minimum amount of energy required to eject an electron from a metal surface is called work function (W) of that metal. It is also called threshold energy.
- The minimum frequency of radiation required to eject an electron from a metal surface is called threshold frequency(ν_0) for that metal. $\therefore \quad W = h\nu_0$
- Work function of a metal depends on nature of the metal, it will not depend on frequency and intensity of the radiation.
- When a photon of energy $h\nu$ is absorbed by an electron, an amount of energy atleast equal to work

function W (provided $h\nu$ >W) is used up in liberating the electron from the surface and the difference ($h\nu$ - W) is equal to the maximum kinetic energy.of that electron.

$$\therefore \frac{1}{2}mV_{max}^2 = h\nu - W \quad \rightarrow (1)$$

$$\therefore h\nu = W + \frac{1}{2}mV_{max}^2 \quad \rightarrow (2)$$

$$\therefore h\nu = h\nu_0 + \frac{1}{2}mV_{max}^2 \quad \rightarrow (3)$$

The above relation is called the Einstein's Photoelectric equation. Here 'm' is the mass of the electron and V_{max} is the maximum velocity of the photoelectrons. Infact, most of the electrons possess kinetic energy less than the maximum value, as they lose a part of their kinetic energy due to collisions before escaping from the metal.

Thus from the above discussion the laws of photoelectric effect from Einstein's Photoelectric equations are deduced.

■From equation (1) maximum kinetic energy of photoelectrons is

$$KE_{max} = h\nu - h\nu_0 .$$

For photoelectric emission to take place kinetic energy of electrons must be positive.

It follows that $h\nu > h\nu_0 \quad \Rightarrow \quad \nu > \nu_0$.

It proves that for photoelectric emission to take place, from a given metal the frequency f the incident radiation must be greater than threshold frequency for that metal.

If frequency of the incident radiation is less than threshold frequency then no photoelectric emission will take place, whatever be the intensity of the incident radiation, or how long it falls on the metal surface.

■From equation (1) it follows that maximum kinetic energy of photoelectrons depends linearly on the frequency.

It proves that the maximum kinetic energy of photoelectrons increases as frequency of incident radiation increases.

Since Einstein's equation does not involve a factor representing intensity, it proves that the maximum kinetic energy of emitted electrons is independent of the intensity of incident radiation.

■According to Einstein, the photoelectric effect arises , when a single photon is absorbed by a single electron.

So number of photoelectrons ejected will be large if intense radiation is incident. This is because intensity of radiation is proportional to number of photons per unit area per unit time. Hence if intensity of incident radiation is larger, then number photons incident is larger and number of electrons ejected is larger.

It proves that number of photoelectrons ejected from a metal surface depends on intensity of incident radiation. Further, there is no effect of frequency of incident radiation on number of photoelectrons emitted. It is because one photon is capable of ejecting only one electron, provided , $\nu > \nu_0$

■According to Einstein, the basic process in photoelectric emission is absorption of a photon of light by an electron. So as the photon is absorbed, emission of electron takes place instantaneously irrespective of intensity.

Note :

♦ Alkali metals can cause photoelectric effect with visible light.

♦ Work function of Alkali metals is around 2eV.

♦ Among all metals work function is least for Cesium(2.14eV)

- Work function $W = h\nu_0 = \frac{hC}{\lambda_0}$

where ν_0 =threshold frequency,

λ_0 =threshold wavelength

- Einstein's equation can be written as follows: $KE_{\max} = E - W$

(or) $KE_{\max} = h\nu - h\nu_0$

(or) $KE_{\max} = \frac{hC}{\lambda} - \frac{hC}{\lambda_0}$

- $\frac{1}{2}mV_{\max}^2 = E - W$ (or) $\frac{1}{2}mV_{\max}^2 = h\nu - h\nu_0$

$\frac{1}{2}mV_{\max}^2 = \frac{hC}{\lambda} - \frac{hC}{\lambda_0}$

- $eV_0 = E - W$ (or) $eV_0 = h\nu - h\nu_0$

(or) $eV_0 = \frac{hC}{\lambda} - \frac{hC}{\lambda_0}$

::PROBLEMS::

1. A photon of energy 2.5 eV and wavelength λ falls on a metal surface and the ejected electrons have velocity 'v'. If the λ of the incident light is decreased by 20%, the maximum velocity of the emitted electrons is doubled. The work function of the metal is

1) 2.6 eV 2) 2.23 eV 3) 2.5 eV 4) 2.29 eV

SOLUTION :

$$\frac{v_1^2}{v_2^2} = \frac{\frac{hc}{\lambda_1} - \omega}{\frac{hc}{\lambda_2} - \omega}$$

use $E = \frac{12400}{\lambda(A^0)}$

2. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly

1) 1.2 nm 2) 1.2 x 10^{-3} 3) 1.2 x 10^{-6} nm 4) 1.2 x 10 nm

SOLUTION :

Given in the quesiton.

Energy of a photon, $E = 1MeV \Rightarrow = 10^6\, eV$

Now, hc =1240 eVm ; Now, $E = \frac{hc}{\lambda}$

3. While working with light and X-rays, there is a useful relation between the energy of a photon in electron volts (eV) and the wavelength of the photon in angstom (A^0). Suppose the wavelength of aphoton is λA^0. Then energy of the photon is

SOLUTION :

$$E = hv = \frac{hc}{\lambda}$$

Here wavelength =

$$\lambda \times 10^{-10} m; h = 6.62 \times 10^{-34} Js, c = 3 \times 10^8 ms^{-1}$$

$$\therefore E = \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{\lambda \times 10^{-10}}$$

$$= \frac{(6.62 \times 10^{-34}) \times (3 \times 10^8)}{(\lambda \times 10^{-10}) \times (1.6 \times 10^{-19})} eV = \frac{12400}{\lambda} eV$$

$$\therefore E = \frac{12400}{\lambda} eV$$

Note : (λ is taken in A^0 and 12400 in A^0 eV)

4. When a metal surface is illuminated by light of wavelengths 400 nm and 250 nm, the maximum velocities of the photoelectrons ejected are V and 2V respectively. The work function of the metal is

1) 2hc x 10^6 J 2) 1.5hc x 10^6 J 3) hc x 10^6 J 4) 0.5hc x 10^6 J

SOLUTION :

$$\frac{hc}{\lambda} = w + \frac{1}{2}mv^2$$

5. If wavelength of radiation is 4000 A^0 = 400nm then the energy of the photon is

SOLUTION :

$$E = \frac{hC}{\lambda} = \frac{12400eVA^0}{4000A^0} = \frac{1240eVnm}{400nm} = 3.1eV$$

6. Light described at a place by the equation $E = (100\, V/M) \times \left[\sin(5 \times 10^{15} s^{-1})t + \sin(8 \times 10^{15} s^{-1})t\right]$ **falls on a metal surface having work fucntion 2.0 eV. Calculate the maximum kinetic energy of the photoelectrons**

1) 3.27 eV 2) 5 eV 3) 1.27 eV 4) 2.5 eV

SOLUTION :

$$E = 100\sin 5\times10^{15}t + 100\sin 8\times10^{15}t$$

$$\nu_{max} = \frac{8\times10^{15}}{2\pi};$$

$$K.E_{max} = h\nu_{max} - w; K.E_{max} = 3.27eV$$

7. The electric field associated with a light wave is given by $E = E_0 \times \sin\left[\left(1.57\times10^7 m^{-1}\right)(x-ct)\right]$. Find the stopping potential when this light is used in an experiment on photoelectric effect with the similar having work function 1.9 eV

1) 1.2 V 2) 1.1 V 3) 2 V 4) 2.1 V

SOLUTION :

$$\nu = \frac{1.57\times10^7\times3\times10^8}{2\pi} = 0.75\times10^{15}\,Hz$$

$$E = \frac{6.62\times10^{-34}\times0.75\times10^{15}}{1.6\times10^{-19}}\quad eV = 3.1eV$$

$$eV_0 = E - w\ ;$$

$$V_0 = 1.2V$$

8. A monochromatic source of light operating at 200 W emits 4×10^{20} photons per second. Find the wagelength of the light.

SOLUTION :

$$\text{Power} = P = \frac{N}{t}h\upsilon$$

$$\text{Energy of photon} = E = \frac{P}{\left(\frac{N}{t}\right)} = \frac{200}{4\times10^{20}} = 5\times10^{-19}$$

$$\lambda = \frac{\left(6.62\times10^{-34}\right)\times\left(3\times10^8\right)}{5\times10^{-19}}\text{m} = 3.972\overset{0}{\text{A}}$$

9. Radiation of wavelength 200 nm propagating in the form of a parallel beam, fall normally on a plane metallic surface. The intensity of the beam is 5mW and its cross sectional area 1.0 mm^2. Find the pressure exerted by the radiation on the metallic surface, if the radiation is completely reflected. (Roorkee 2001)

SOLUTION :

$$\text{E} = \frac{12400}{\lambda} = \frac{12400}{200} = 6.2\text{eV} \approx 10^{-18}\text{J}$$

Number of photons passing a point per second is $\text{n} = \frac{\text{P}}{\text{E}} = \frac{5\times10^{-19}}{10^{-18}} = 5\times10^9$. momentum of each photon

$\text{P} = \frac{\text{E}}{\text{C}} = 3.3\times10^{-27}\text{J/s}$. Charge in momentum after each strike = 2p =6.6 x 10^{-27} J/s

Total momentum change per second is

$$F = \frac{dp}{dt} = \frac{n \times 2p}{t} = 5\times10^{9}\times6.6\times10^{-27} = 33\times10^{-18}\,N$$

$$\therefore \text{pressure}\,\frac{F}{A} = 33\times10^{-12}\,N/m^{2}$$

10. When a surface 1 cm thick is illuminated with light of wave lenght λ the stopping potential is V_0, but when the same surface is illuminated by light of wavelength 3 λ, the stopping potential is $\frac{V_0}{6}$. The threshold wavelength for metallic surface is:

1) 4λ 2) 5λ 3) 3λ 4) 2λ

SOLUTION :

$$eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

$$\frac{eV_0}{6} = hc\left(\frac{1}{3\lambda} - \frac{1}{\lambda_0}\right)$$

11. The work function of a metal is 3.0eV. It is illuminated by a light of wave length 3 x 10^{7}m. Calculate i) threshold frequency, ii) the maximum energy of photoelectrons, iii) the stopping potential. (h =6.63 x 10^{-34} Js and c = 3 x 10^{8}ms^{-1}).

SOLUTION :

i) $W = h\nu_0 = 3.0eV = 3 \times 1.6 \times 10^{-19}J$

Threshold frequency

$$\nu_0 = \frac{W}{h} = \frac{3\times1.6\times10^{-19}}{6.63\times10^{-34}} = 0.72\times10^{15}\,Hz\,.$$

ii) Maximum kinetic energy $(E_{max}) = h(\nu - \nu_0)$

$$\lambda = 3\times10^{-7}\,m,\ \nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{3\times10^{-7}} = 1\times10^{15}\,Hz$$

$$K_{max} = h(\nu - \nu_0) = 6.63 \times 10^{-34}(1 - 0.72) \times 10^{15}\,J = 1.86 \times 10^{-19}\,J.$$

iii) $K_{max} = e\,V_0$ where V_0 is stopping potential in volt and e is the charge of electron

$V_0 = \frac{K_{max}}{e}$. Here $K_{max} = 1.86 \times 10^{-19}$ J and

$e = 1.6 \times 10^{-19}$ C;

$$V_0 = \frac{1.86\times10^{-19}\,J}{1.6\times10^{-19}\,C} = 1.16V$$

12. The work function of a photosensitive element is 2eV. Calculate the velocity of a photoelectron when the element is exposed to a light of wavelength $4\times10^{3}\,\overset{0}{A}$.

SOLUTION :

Einstein's photoelectric equation is

$$\frac{1}{2}mv^2 = \frac{hc}{\lambda} - W_0$$

$$\frac{1}{2}mv^2 = \frac{6.62\times 3}{4\times 10^3\times 10^{-10}}\times 10^{-26} - 2\times 1.6\times 10^{-19}$$

$$v^2 = \frac{1.765\times 2}{9.1}\times 10^{12}$$

$$v = \sqrt{\frac{1.765\times 2}{9.1}}\times 10^6 = 6.228 \text{ x } 10^5 \text{ ms}^{-1}$$

13. A metal of work function 4eV is exposed to a radiation of wavelength 140 x 10^{-9}m. Find the stopping potential developed by it.

SOLUTION :

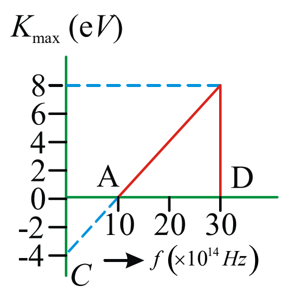
$$E = \frac{hc}{\lambda} \quad E = \frac{6.62\times 10^{-34}\times 3\times 10^8}{140\times 10^{-9}\times 1.6\times 10^{-19}}eV = 8.86\text{eV}$$

work function $W_0 = 4$eV

$eV_0 = E - W_0 = 8.86 - 4 = 4.86$eV

$\therefore$ Stopping potential $V_0 = 4.86V$

14. A graph regarding photoeletric effect is shown between the maximum kinetic energy of electrons and the frequency of the incident light. On the basis of data as shown in the graph, calculate the work fucntion



1) 2 eV 2) 4 eV 3) 4.2 eV 4) 2.5 eV

SOLUTION :

From the graph

threshold frquency $(f_0) = 10\times 10^{14}\,Hz$

$$h = \frac{8\times 1.6\times 10^{-19}}{20\times 10^{14}} = 6.4\times 10^{-34}\,J$$

work function $= hf_0 = 4eV$

15. In a photocell bi chromatic light of wave length 2480 A^0 and 6000A^0 are incident on a cathode whose workfunction is 4.8eV. If a uniform magnetic field of 3x10^{-5} T exists parallel to the plate, find the radius of the circular path described by the photoelectron. (mass of electron is $9x10^{-31}kg$)

SOLUTION :

$$E_1 = \frac{12400}{\lambda_1} = \frac{12400}{2480} = 5eV\,;$$

$$E_2 = \frac{12400}{\lambda_2} = \frac{12400}{6000} = 2.06eV$$

As $E_2 < W_0$ and $E_1 > W_0$, photo electric emission is possible only with λ_1.

Maximum K.E of emitted photo electrons

$K = E_1 - W_0 = 0.2$ eV.

Photo electrons experience magnetic force and move along a circular path of radius

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq} \Rightarrow r = 5cm.$$

16. A monochromatic light of wavelength λ is incident on an isolated metalic sphere of radius a. The threshold wavelength is λ_0 which is larger than λ. Find the number of photoelectrons emitted before the emission of photo electrons stops.

SOLUTION :

As the metallic sphere is isolated, it becomes positively charged when electrons are ejected from it. There is an extra attractive force on the photoelectrons. If the potential of the sphere is raised to V, the electron should have a minimum energy W + eV to be able to come out. Thus, emission of photoelectrons will stop when

$$\frac{hc}{\lambda} = W + eV = \frac{hc}{\lambda_0} + eV \quad \text{or,} \quad V = \frac{hc}{e}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right).$$

The charge on the sphere needed to take its potential to V is $Q = (4\pi\varepsilon_0 a)V$

The number of electrons emitted is, therefore,

$$n = \frac{Q}{e} = \frac{4\pi\varepsilon_0 aV}{e} = \frac{4\pi\varepsilon_0 ahc}{e^2}\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

17. From the above figure the values of stopping potentials for M_1 and M_2 for a frequency $\nu_3 (> \nu_{02})$ of the incident radiatioins are V_1 and V_2 respectively. Then the slope of the line is equal to

1) $\frac{V_2 - V_1}{\nu_{02} - \nu_{01}}$ 2) $\frac{V_1 - V_2}{\nu_{02} - \nu_{01}}$ 3) $\frac{V_2}{\nu_{02} - \nu_{01}}$ 4) $\frac{V_1}{\nu_{02} - \nu_{01}}$

SOLUTION :

$$h\nu_{01} + eV_1 = h\nu_{02} + eV_2$$

$$e(V_1 - V_2) = h(V_{02} - V_{01});$$

$$\frac{h}{e} = \frac{(V_1 - V_2)}{(V_{02} - V_{01})}$$

18. A small metal plate (work function W) is kept at a distance d from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.

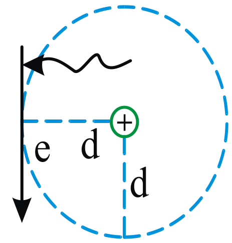
SOLUTION :

Electron is moving around the ion in a Circle of radius 'd'. $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{d^2} = \frac{mV^2}{d}$, $\therefore mV^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{d}$

$$\therefore K.E = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{d} \quad \text{----------(1)}$$

But $$K.E_{max} = \frac{hc}{\lambda} - W \quad \text{----------(2)}$$

$$\therefore \lambda = \frac{hc\ 8\pi\varepsilon_0 d}{e^2 + 8\pi\varepsilon_0 dW}$$



19. A source of light is placed above a sphere of radius 10cm. How many photoelectrons must be emitted by the sphere before emission of photoelectrons stops? The energy of incident photon is 4.2 eV and the work function of the metal is 1.5 eV.

1) 2.08×10^{18} 2) 1.875×10^{8} 3) 2.88×10^{18} 4) 4×10^{19}

SOLUTION :

Stopping potential energy $= eV_0 = E - \omega$

$$V_0 = \frac{E-\omega}{e} = \frac{9\times10^9 ne}{r};$$

n = no of electrons

Dual nature of matter -(de-Broglie hypothesis)

♦ Photoelectric effect and Compton effect proves that radiation behaves like particles (photons), where as Interference and Diffraction proves that radiation behaves like waves.

So 'radiation has dual nature' i.e., radiation behaves like particles when interacting with matter and radiation behaves like waves when propagating in a medium.

♦ **de Broglie Hypothesis**

- ♦ The universe consists of matter and radiation only.
- ♦ Nature loves symmetry
- ♦ If radiation has dual nature then matter also should have dual nature.

♦ According to de Broglie particles like electron, proton and neutron, also have both wave and particle properties. The waves associated with moving particle are called matter waves and the wavelength is called the de Broglie wavelength of a particle.

For a photon Energy, $E = \dfrac{hC}{\lambda} = mC^2$

where m = effective mass then wavelength $\lambda = \dfrac{h}{mC} = \dfrac{h}{p}$

where p = momentum of the photon

de Broglie extended the same for particles also.

So if a particle of mass 'm' is moving with velocity 'v' then its momentum p = mv, hence

de Broglie wave length of the matter wave associated with is given by $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$

Davisson and Germer studied the scattering of electrons by a nikel target. The wavelength

λ of diffracted electrons was determined by Davisson and Germer. The experimental values of wavelength λ were found to agree with the theoretical value $\lambda = \frac{h}{m\,v}$

Hence it is concluded that electrons behaves like waves and undergo diffraction.

- For definite sized objects like a car the corresponding wavelength is very small to detect the wave properties. But the de-Broglie wavelength of the electron is large enough to be observed.

Because of their small mass, electrons have a small momentum and hence large wavelength $\lambda = h/p$.

Note :

- deBroglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

  Where momentum p = mv ; m=mass, v= velocity

- deBroglie wavelength $\lambda = \frac{h}{\sqrt{2mK}}$

  where kinetic energy, $K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$

- If a particle having charge q starting from rest is accelerated through a potential difference V then gain in kinetic energy, K=qV

  so, deBroglie wavelength $\lambda = \frac{h}{\sqrt{2mqV}}$

- For electron $\lambda = \frac{12.27}{\sqrt{V}}\overset{o}{A} = \sqrt{\frac{150}{V}}\overset{o}{A}$

- For proton $\lambda = \frac{0.286}{\sqrt{V}}\overset{o}{A} = \sqrt{\frac{0.082}{V}}\overset{o}{A}$

- For dueteron $\lambda = \frac{0.202}{\sqrt{E}}\mathring{A}$

- For α particle $\lambda = \frac{0.101}{\sqrt{V}}\mathring{A}$

- for neutron $\lambda = \frac{0.286}{\sqrt{E}}\mathring{A}$

  where E = kinetic energy in electron volts

- The de-Broglie wavelength of a particle is independent of nature of the particle and these waves are not electromagnetic. Diffraction effects have been obtained with streams of electrons, protons, neutrons and alpha particles.

- de-Broglie explains Bohr's criterion to select the allowed orbits in which angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$. According to his hypothesis, an electron revolving round the nucleus is associated with certain wavelength 'λ' which depends on its momentum mv. It is given by $\lambda = \frac{h}{mv} = \frac{h}{p}$

In an allowed orbit, an electron can have an integral multiple of this wavelength.

That is the n^{th} orbit consists of n complete de-Broglie wavelengths $2\pi r_n = n\lambda_n$,

where n is the principle quantum number.

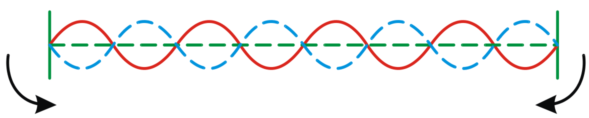
where r_n is the radius of n^{th} orbit and λ_n is the wavelength of electron in n^{th} orbit

$$\lambda_n = \frac{2\pi r_n}{n}$$

$$\lambda_n = \frac{2\pi}{n}(0.53\times n^2)\ \AA.$$

$$\lambda_n = 2\pi\times 0.53 n A^\circ$$

(a)



(b)

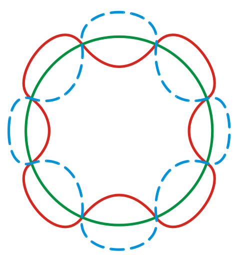


Figure (a) shows the waves on a string have a wavelength related to the length of the string allowing them to interfere constructively as shown If we imagine the string bent into a closed circle we get an idea of how electrons in circular orbits can interfere constructively as shown in figure(b). If the wavelength does not fit in to the circumference the electron interferes destructively, electron can not exist in such an orbit.

Heisenberg uncertainity Principle

- The matter-wave picture elegantly incorporated the Heisenberg's uncertainty principle. According to the principle, it is not possible to measure both the position and momentum of an electron (or any other particle) at the same time exactly. There is always some uncertainity (Δx) in the specification of position and some uncertainity (Δp) in the specification of momentum. The product of Δx and Δp is of the order of h (with $\lambda = \frac{h}{2\pi}$)

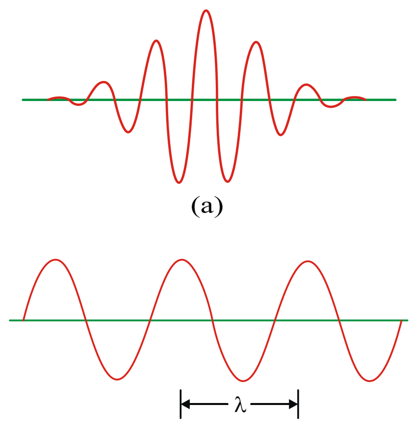
  i.e., $\Delta x \Delta p = h$.
- Equation allows the possibility that Δx is zero, but then Δp must be infinite in order that the product is nonzero. Similarly, if Δp is zero, Δx must be infinite. Ordinarily, both Δx and Δp are nonzero such that their product is of the order of h.
- Now, if an electron has a definite momentum p, (i.e., Δp=0), by the de Broglie relation, it has a definite wavelength λ. A wave of definite (single) wavelength extends all over space. By Born's probability interpretation this mens that the electron is not localizedin any finite region of space. That is, its position uncertainity is infinite $(\Delta x \to \infty)$, which is consistent with the uncertainity principle.
- In general, the matter wave associated with the electron is not extended all over space. It is a wve packet extending over some finite region of space. In that case Δx is not infinite but has some

finite value depending on the extension of the wave packet. Also, you must appreciate that a wave packet of finite extension does not have a single wavelength. It is built up wavelengths spread around some central wavelength.

♦ By de Broglie's relation, then, the momentum of the electron will also have a spread - an uncertainity Δp. This is as expected from the uncertainity principle. It can be shown that the wave packet description together with de Broglie relation and Born's probability interpretation reproduce the Heisenberg's uncertainity princile exactly.

♦ The de Broglie relation will be seen to justify bohr's postulate on quantisation of angular momentum of electron in an atom.

Figure shows a schematic diagram of (a) a localised wave packet, and (b) an extended wave with fixed wavelength.

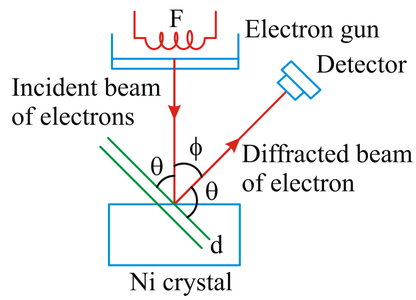


(a) (b)

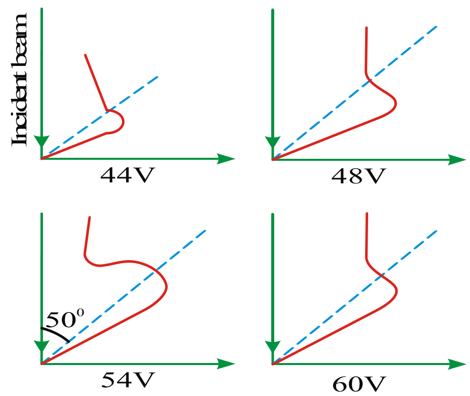
Figure (a) the wave packet description of an electron. The wave packet corresponds to a spread of wavelength around some central wavelength (and hence by de Broglie relation, a spread in momentum). Consequently, it is associated with an uncertainity in position (Δx) and an uncertainity in momentum (Δp). (b) the matter wave corresponiding to a definite momentum of an electron extends all over space. In this case, $\Delta p = 0$ and $\Delta x \to \infty$.

Davisson and Germer's electron diffraction experiment

- The first experimental evidence of matter wave was given by two American physicists, Davisson and Germer in 1927. They also succeeded in measuring the de - Broglie wave length associated with slow electrons.
- A beam of electron emitted by electron gun is made to fall on nikel crystal cut along cubical axis at a particular angle.
- Ni crystal behaves like a three dimensional diffraction grating and it diffracts the electron beam obtained from electron gun.
- The diffracted beam of electrons received by the detector which can be positioned at any angle by rotating about the point of incidence.



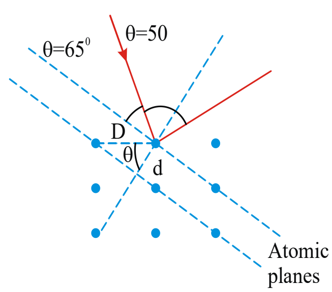
- The energy of the incident beam of electron can also be varied by changing the applied voltage to the electron gun.
- According to classical physics, the intensity of scattered beam of electrons was not the same but different at different angles of scattering. It is maximum for diffracting angle 50^0 at 54 volt P.D.
- It is seen that a bump begins to appear in the curve for 44 volt electrons. With increasing potential in the bump moves upwards and becomes most prominent in the curve for 54 volt electrons at $\phi = 50^0$. At higher potential the bump gradually disappears.



- If the de Broglie waves are associated with electron, then these should be diffracted like x - rays. using the Bragg's formula $2d\sin\theta = n\lambda$, we can determine the wavelength of these waves.

  Where 'd' is the distance between the diffracting planes. $\theta = \left[\frac{180-\phi}{2}\right]$ = glancing angle for incident beam = Bragg's angle.
- The distance between diffracting planes in Ni - crystal for this experiment is $d = 0.91A^0$ and for n = 1; $\lambda = 2\times 0.91\times 10^{-10}\ \sin 65 = 1.65A^0$

  Now debroglie wave length can also be determined using the formula ; $\lambda = \frac{12.27}{\sqrt{V}} = \frac{12.27}{\sqrt{54}} = 1.67A^0$



  Thus the deBroglie hypothesis is verified.
- The Bragg's formula can be rewritten in the form containing inter atomic distance D and scattering angle 'ϕ'.

$\therefore \theta = 90 - \frac{\phi}{2}$ and $d = D\cos\theta = D\sin\frac{\phi}{2}$

using $\sin\theta = \cos\frac{\phi}{2}$

$\lambda = 2d\sin\theta = 2d\left(\sin\frac{\phi}{2}\right)\cos\frac{\phi}{2} = d\sin\phi$

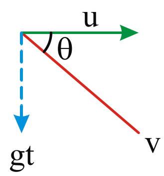
$\boxed{\lambda = d\sin\phi}$

::PROBLEMS ::

1. A particle of mass 'm' projected horizontally with velocity u. If it makes an angle θ, with the horizontal after some time, then at that instant, its de Broglie wavelength is

SOLUTION :

For a projectile horizontal component of velocity is constant.



$\therefore v_x = u_x ; V\cos\theta = u$

$\therefore$ de Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h\cos\theta}{mu}$

2. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV.

1) 0.5 eV 2) 1.8 eV 3) 1.02 eV 4) 2.5 eV

SOLUTION :

Given,

For the first condition, Wavelength of light = 600 nm and for the second condition wavelength of light X' = 400 nm

Also, maximum kinetic energy for the second condition is equal to the twice of the kinetic energy in first condition. i.e., $K_{max} = 2K_{max}$

Here, $K_{max} = \frac{hc}{\lambda} - \phi$

$$\Rightarrow 2K_{max} = \frac{hc}{\lambda'} - \phi_0$$

$$\Rightarrow 2\left(\frac{1230}{600} - \phi\right) = \left(\frac{1230}{400} - \phi\right)$$

$$[\because hc \simeq 1240\, eVnm]$$

$$\Rightarrow \phi = \frac{1230}{1200} = 1.02\, eV$$

3. Assuming an electron is confined to a lnm wide region, find the uncertainty in moment using Heisenberg uncertainty principle $(\Delta x \times \Delta p \approx h)$. You can assume the uncertainty in position Δx as 1 nm.Assuming $p \approx \Delta p$, find the energy of the electron in electronvolts.

1) 1.6 meV **2) 3.8 meV**

3) 0.16 meV **4) 0.38 meV**

SOLUTION :

19. **Here,** $\Delta x = 1nm = 10^{-9}\,m, \Delta p = ?$

As $\Delta x \Delta p \approx h$

$$\therefore \Delta p = \frac{h}{\Delta x} = \frac{h}{2\pi \Delta x}$$

$$= \frac{6.62 \times 10^{-34}\,Js}{2 \times (22/7)(10^{-9})m}$$

$$= 1.05 \times 10^{-25}\,kg\ m/s$$

Energy, $E = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} \qquad [\because \rho \approx \Delta p]$

$$= \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} J$$

$$= \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}} eV$$

$$= 3.8 \times 10^{-2}\,eV$$

4. Electrons are accelerated through a potential difference of 150V. Calculate the de Broglie wavelength.

SOLUTION :

V = 150V; h=6.62 x 10^{-34}Js, m=9.1 x 10^{-31}kg,
e = 1.6 x 10^{-19}C

$$\therefore \lambda = \frac{h}{\sqrt{2Vem}} = \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 150}} = 1\overset{0}{A}$$

5. A proton, a neutron, an electron and an α-particle have same energy. Then their de-Broglie wavelengths compare as

1) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ **2)** $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$ **3)** $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ **4)** $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

SOLUTION :

We know that the relation between λ and K is given by $\lambda = \frac{h}{\sqrt{2mk}}$

Here, for the given value of energy K, $\frac{h}{\sqrt{2k}}$ **is a constant.**

$$\text{Thus, } \lambda \propto \frac{1}{\sqrt{m}}$$

$$\therefore \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha = \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$$

Since, $m_p = m_n$, hence $\lambda_p = \lambda_n$

As, $m_\alpha > m_p$**, therefore** $\lambda_\alpha < \lambda_p$

As, $m_e < m_n$, therefore $\lambda_e < \lambda_p$

Hence, $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$

6. Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively

SOLUTION :

$$\text{Since, } \lambda = \frac{h}{m\upsilon} = \frac{h}{\sqrt{3mkT}};$$

$$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{8}{3}}$$

7. The de-Broglie wavelength of a photo is twice, the de-Broglie wavelength of an electron. The speed of the electron is $v_c = \frac{c}{100}$**. Then,**

1) $\frac{E_e}{E_p} = 10^{-4}$ **2)** $\frac{E_e}{E_p} = 10^{-2}$ **3)** $\frac{P_e}{m_e C} = 10^{-2}$ **4)** $\frac{P_e}{m_e C} = 10^{-4}$

SOLUTION :

Suppose, Mass of electron = m_e,

Mass of photon = m_p

Velocityofelectron=v_e

Velocity of photon = v_p

Thus, for electron, de-Broglie wavelenth

$$\lambda_e = \frac{h}{m_e v_e} = \frac{h}{m_e(C/100)} = \frac{100h}{m_e C} \text{ (given)}$$

Kinetic enegy, $E_0 = \frac{1}{2} m_e v_e^2$

$$\Rightarrow m_e v_e = \sqrt{2E_e m_e}$$

so, $\lambda_e = \frac{h}{m_e v_e} = \frac{h}{\sqrt{2m_e E_e}}$

$$\Rightarrow E_e = \frac{h^2}{2\lambda_e^2 m_e}$$

For photon of wavelength λ_p**, energy**

$$E_p = \frac{hc}{\lambda_p} = \frac{hc}{2\lambda_e}$$

$$\therefore \frac{E_p}{E_e} = \frac{hc}{2\lambda_e} \times \frac{2\lambda_e^2 m_e}{h^2}$$

$$= \frac{\lambda_e m_e C}{h} = \frac{100h}{m_e c} \times \frac{m_e C}{h} = 100$$

So, $\frac{E_e}{E_p} = \frac{1}{100} = 10^{-2}$

For electron, $p_e = m_e v_e = m_e \times c/100$

So, $\frac{p_e}{m_e c} = \frac{1}{100} = 10^{-2}$

8. With what velocity must an electron travel so that its momentum is equal to that of a photon with a wavelength of $5000\overset{0}{A}$ **(** $h = 6.6x10^{-34} Js$ **,** $m_e = 9.1x10^{-31} Kg$ **)**

SOLUTION :

$$mv = \frac{h}{\lambda} \Rightarrow v = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 5000 \times 10^{-10}} = 1450 m/s$$

9 . An electron is moving with an initial velocity $v = v_0 \hat{i}$ **and is in a magnetic field** $B = B_0 \hat{j}$**. Then it's de Broglie wavelength**

1) Remains constant **2) Increases with time**

3) Decreases with time **4) Increases and decreases perdiocally.**

SOLUTION :

Here, $\vec{v} = v_0 \hat{i}, \vec{B} = B_0 \hat{j}$ **Force on moving electron due to magnetic field is**

$$\vec{F} = -e\left(\vec{v} \times \vec{B}\right) = -e\left(v_0 \hat{i} \times B_0 \hat{j}\right) = -e v_0 B_0 \hat{k}$$

As this force is perpendicular to $\vec{v}$ and $\vec{B}$, so the magnitude of $\vec{v}$ will not change. i.e. momentum (= *mv*) will remain constant in magnitude. Therefore, de Broglie wavelength, $\lambda\left(=\frac{h}{mv}\right)$ remains constant.

10. If 10,000V applied across an X-ray tube, what will be the ratio of deBroglie wavelength of the incident electrons to the shortest wavelength of X-ray produced (e/m of electron is $1.7x10^{11}C/Kg$)

SOLUTION :

Debroglie wave length of incident electron is $\lambda_1 = \frac{h}{\sqrt{2meV}}$ 1

Shortest wavelength of x ray photon is $\lambda_2 = \frac{hc}{Ve}$ 2

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{1}{c}\sqrt{\left(\frac{V}{2}\right)\left(\frac{e}{m}\right)} = 0.1$$

11. Photons of energies 4.25eV and 4.7eV are incident on two metal surfaces A and B respectively. The maximum KE of emitted electrons are respectively T_A eV and T_B=(T_A-1.5)eV. The ratio de-Broglie wavelengths of photo electrons from them is $\lambda_A : \lambda_B$=1:2, then find the work function of Aand B

SOLUTION :

Debroglie wavelength

$$\lambda = \frac{h}{\sqrt{2km}} \Rightarrow \lambda \propto \frac{1}{\sqrt{k}} \text{ (k = k.E = T)}; \frac{\lambda_B}{\lambda_A} = \sqrt{\frac{T_A}{T_B}}$$

$$2 = \sqrt{\frac{T_A}{T_A - 1.5}} \Rightarrow T_A = 2eV$$

$$\Rightarrow W_A = 4.25 - T_A = 2.25\,eV$$

$$\Rightarrow T_B = T_A - 1.5 = 2 - 1.5 = 0.5eV$$

$$\Rightarrow W_B = 4.7 - T_B = 4.7 - 0.5 = 4.2\,eV$$

12, If the uncertainity in the position of proton is $6\times10^{8}\,m$, then the minimum uncertainity in its speed is

SOLUTION :

$$\Delta p = m\Delta v = \frac{h}{\Delta x}$$

$$\text{or } \Delta v = \frac{h}{m\Delta x} = \frac{1.034\times10^{-34}}{1.67\times10^{-27}\times6\times10^{-8}} = 1\ ms^{-1}$$

13. The correctness of velocity of an electron movign with velocity 50 ms^{-1} is 0.005%. The accuracy with which its position can be measured will be

SOLUTION :

Here, $\Delta v = \frac{0.005 \times 50}{100} = 0.0025 ms^{-1}$

$$\Delta x = \frac{h}{m\Delta v} = \frac{1.034 \times 10^{-34}}{9.1 \times 10^{-31} \times 0.0025}$$
$$= 4634 \times 10^{-5} m$$

14. A particle is dropped from a height H. The de-Broglie wavelength of the particle as a function of height is proportional to

1) H 2) $H^{1/2}$ 3) H^0 4) $H^{-1/2}$

SOLUTION :

Velocity of a body falling from a height H is given by $v = \sqrt{2gH}$

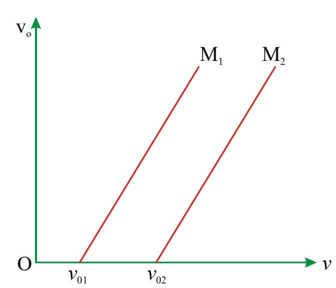
We know that de-broglie wavelength

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}} \Rightarrow = \frac{h}{m\sqrt{2g}\sqrt{H}}$$

Here, $\frac{h}{m\sqrt{2g}}$ **is a sonstant** ϕ **say 'K'**

So, $\lambda = K\frac{1}{\sqrt{H}} \Rightarrow \lambda \propto \frac{1}{\sqrt{H}} \Rightarrow \lambda \propto H^{-1/2}$

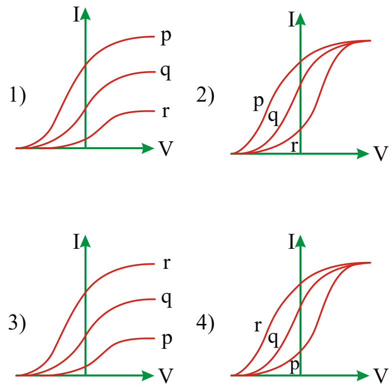
15. Figure shows the variation of the stopping potential (V_0) with the frequency (v) of the incident radiations for two different photosensitive material M_1 and M_2.What are the values of work functions for M_1 and M_2 respectively



1) hv_{01}, hv_{02} 2) hv_{02}, hv_{01} 3) hv_{01}, hv_{01} 4) hv_{02}, hv_{02}

SOLUTION :. $W = hv$

16. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0eV$, $\phi_q = 2.5eV$ and $\phi_r = 3.0eV$ respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is : [Take *hc* = 1240 eV nm]



SOLUTION :.

Explain based on graph between V & I for different metals and light of different wave lengths.

Passage

Photoelectric threhold of silver is $\lambda = 3800\,\overset{o}{A}$. ultraviolet light of $\lambda = 2600\,\overset{o}{A}$ is incident on silver surface. (Mass of the electron $9.11\times10^{-31}kg$)

17. Calculate the value of work function in eV.

1) 1.77 2) 3.27

3) 5.69 4) 2.32

SOLUTION :.

$$E = hv \qquad \frac{hc}{\lambda}$$

18. Calculate the maximum kinetic energy (in eV) of the emitted photoelectrons.

1) 1.51 2) 2.36

3) 3.85 4) 4.27

SOLUTION :. $E = W.E. + K.E.$

19. Calculate the maximum velocity of the photoelectrons.

1) 72.89×10^{8} 2) 57.89×10^{8}

3) 42.93×10^{8} 4) 68.26×10^{8}

SOLUTION :. $K = \frac{1}{2}mv^2$

20. An electron (mass m) with an initial velocity v = v_oi (v_o >0) is in an electric field E = -$E_o\hat{i}$ (E_0 = constan t > 0). It's de-Broglie wavelength at time t is given b

1) $\dfrac{\lambda_0}{\left(1+\dfrac{eE_0}{m}\dfrac{t}{v_0}\right)}$ 2) $\lambda_0\left(1+\dfrac{eE_0t}{mv_0}\right)$ 3) λ_0 4) $\lambda_0 t$

SOLUTION :.

Initial de-Broglie wavelength of electron,

$$\lambda_0 = \frac{h}{mv_0} \qquad \text{.....(i)}$$

Force on electron in electrid field,

$$F = -eE = -e\left[-E_0\hat{i}\right] = eE_0\hat{i}$$

Acceleration of electron $a = \frac{F}{m} = \frac{eE_0\hat{i}}{m}$

Velocity of electron after time t, $v = v_0\hat{i} + \left(\frac{eE_0}{mv_0}t\right)\hat{i}$

de-Broglie wavelength associated with electron at time t is

$$\lambda = \frac{h}{mv} = \frac{h}{m\left[v_0\left(1+\frac{eE_0}{mv_0}t\right)\right]} = \frac{\lambda_0}{\left[1+\frac{eE_0}{mv_0}t\right]} \qquad \left[\because \lambda_0 = \frac{h}{mv_0}\right]$$

CONCEPTUAL BITS

1. The process of photo electric emission depends on
   1) Temperature of incident light
   2) Nature of surface
   3) Speed of emitted photo electrons
   4) Speed of the incident light

KEY :2

2. Which of the following statement is wrong?
   1) Einstein explained photo electric effect with the help of quantum theory
   2) Millikan determined the value of planck's constant depending upon the property of photo electric effect
   3) The maximum K.E. of the photo electrons depends upon the intensity of incident radiation
   4) As the frequency of incident photon increases the corresponding stopping potential also increases

KEY :3

3. .The stopping potential of the photocell is independent of
   1) wavelength of incident light
   2) nature of the metal of photo cathode
   3) time for which light is incident
   4) frequency of incident light

KEY :3

4. In photoelectric emission, the energy of the emitted electron is
   1) larger than that of the incident photons
   2) smaller than that of the incident photons
   3) same at that of the incident photons
   4) proportional to the intensity of the incident light

KEY :2

5. A laser beam of output power 'P' consists only of wavelength λ. If Planck's constant is h and the speed of light is c, then the number of photons emitted per second is
   1) $P\lambda/hc$ 2) $P\lambda/h$ 3) $hc/P\lambda$ `4) hc/P

KEY :1

6. In photoelectric effect, which of the following property of incident light will not affect the stopping potential
   1) Frequency 2) Wavelength 3) Energy 4) Intensity

KEY :4

7. The best suitable metal for photoelectric effect is
   1) Iron 2) Steel 3) Aluminium 4) Cesium

KEY :4

8. If Planck's constant is denoted by h and electronic charge by e, then photoelectric

effect allows determination of:
1) Only h 2) Only e
3) Both h and e 4) Only h/e

KEY :4

9. Photo electric effect can be explained only by assuming that light
1) is a form of transverse waves
2) is a form of longitudinal waves
3) can be polarized 4) consists of quanta

KEY :4

10. If the energy and momentum of a photon are E and P respectively, then the velocity of photon will be
1) E/P 2) $(E/P)^2$ 3) EP 4) $3\text{x}10^7$ m/s

KEY :1

11. The photo electric effect proves that light consists of
1) Photons 2) Electrons
3) Electromagnetic waves 4) Mechanical waves

KEY :1

12. Intensity of light incident on a photo sensitive surface is doubled. Then
1) the number of emitted electrons is tripuled
2) the number of emitted electrons is doubled
3) the K.E of emitted electrons is doubled
4) the momentum of emitted electrons is doubled

KEY :2

13. The deBroglie wavelength associated with a particle of mass m, moving with a velocity v and energy E is given by
1) h/mv^2 2) mv/h^2
3)$h/\sqrt{2mE}$ 4)$\sqrt{2mE}/h$

KEY :3

14. A point source of light is used in a photoelectric effect. If the source is moved farther from the emitting metal, the stopping potential
1) will increase
2) will decrease
3) will remain constant
4) will either increase or decrease

KEY :3

15. With the decrease in the wave length of the incident radiation the velocity of the photoelectrons emitted from a given metal
1) remains same
2) increases
3) decreases

4) increases first and then decreases

KEY :2

16. Consider the following statements A and B, identify the correct choice in the given answers.

A) Tightly bound electrons of target material scattered X-ray photon,resulting in the Compton effect.

B) Photoelectric effect takes place with free electrons.

KEY :4

17. In photo electric effect, the slope of the straight line graph between stopping potential and frequency of the incident light gives the ratio of Planck's constant to

1) charge of electron 2) work function

3) photo electric current 4) K.E. of electron

KEY :1

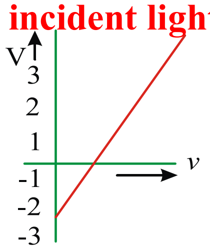
18. When ultraviolet radiation is incident on a surface, no photoelectrons are emitted. If a second beam causes emission of photoelectrons, it may consist of :

1) radio waves 2) infrared rays

3) visible light rays 4) X-rays

KEY :4

19. From the graph shown, the value of Work function if the stopping potential (V), and frequency of the incident light, ν, are on y and x- axes respectively is



1) 1eV

2) 2eV

3) 4eV

4) 3eV

KEY :4

20. Moving with the same velocity ,one of the following has the longest deBroglie wavelength

1) β-particle 2) α-particle

3) proton 4) neutron

KEY :1

21. In an experiment of photo electric emission for incident light of 4000 A^0, the stopping potential is 2V. If the wavelength of incident light is made 3000 A^0 , then the stopping potential will be

1) Less than 2 volt 2) More than 2 volt

3) 2 volt 4) Zero

KEY :2

22. The energy of a photon of frequency n is E=hn and the momentum of a photon of wavelength λ is $p = h / \lambda$. From this statement one may conclude that the wave

velocity of light is equal to :

1) $3 \times 10^8\ ms^{-1}$ 2) $\frac{E}{P}$ 3) EP 4) $\left(\frac{E}{P}\right)^2$

KEY :2

23. Light of wavelength λ falls on a metal having work function hc / λ_o Photoelectric effect will take place only if

1) $\lambda \geq \lambda_0$ 2) $\lambda \geq 2\lambda_0$ 3) $\lambda \leq \lambda_0$ 4) $\lambda < \lambda_0 / 2$

KEY :3

24. The work function for aluminium surface is 4.2 eV and that for sodium surface is 2.0 ev. The two metals were illuminated with appropriate radiations so as to cause photo emission. Then :

1) Both aluminium and sodium will have the same threshold frequency
2) The threshold frequency of aluminium will be more than that of sodium
3) The threshold frequencyof aluminium will be less than that of sodium
4) The threshold wavelength of aluminium will be more than that of sodium

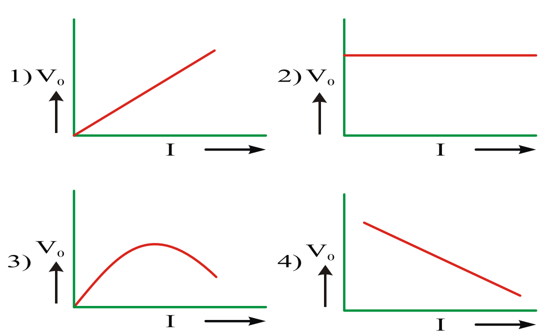
KEY :2

25. The threshold wavelength of lithium is 8000 A^0. When light of wavelength 9000 A^0 is made to be incident on it, then the photo electrons

1) Will not be emitted
2) Will be emitted
3) Will sometimes be emitted and sometimes not 4) Data insufficient

KEY :1

26. The correct curve between the stopping potential (V_o) and intensity of incident light (I) is



KEY :2

27. Debroglie wavelength of protons accelerated by an electric field at a potential difference v is

1) $\frac{0.108}{\sqrt{V}}$ 2) $\frac{0.202}{\sqrt{V}}$ 3) $\frac{0.286}{\sqrt{V}}$ 4) $\frac{0.101}{\sqrt{V}}$

KEY :3

28. The necessary condition for photo electric emission is

1) $h\upsilon \leq h\upsilon_0$ 2) $h\upsilon \geq h\upsilon_0$

3) $E_k > h\upsilon_0$ 4) $E_k < h\upsilon_0$

KEY :2

29. Stopping potential depends on

1) Frequency of incident light
2) Intensity of incident light
3) Number of emitted electrons
4) Number of incident photons

KEY :1

30. Work function is the energy required

1) to excite an atom
2) to produce X-rays
3) to eject an electron just out of the surface 4) to explode the atom

KEY :3

31. Threshold wavelength depends on

1) frequency of incident radiation
2) work function of the substance
3) velocity of electrons
4) energy of electrons

KEY :2

32. When monochromatic light falls on a photosensitive material, the number of photoelectrons emitted per second is n and their maximum kinetic energy is K_{max}. If the intensity of the incident light is doubled keeping the frequency same, then :

1) both n and K_{max} are doubled
2) both n and K_{max} are halved
3) n is doubled but K_{max} remains the same
4) K_{max} is doubled but n remains the same

KEY :3

33. The work function of a metal is X eV When light of energy 2X eV is made to be incident on it then the maximum kinetic energy of emitted photo electron will be

1) 2 eV 2) 2X eV 3) X eV 4) 3X eV

KEY :3

34. If the distance of 100W lamp is increased from a photocell, the saturation current i in the photo cell varies with distance d as

1) $i \propto d^2$ 2) $i \propto d$ 3) $i \propto \frac{1}{d}$ 4) $i \propto \frac{1}{d^2}$

KEY :4

35. A source of light is placed at a distance 4m from a photocell and the stopping potential is then 7.7 volt. If the distance is halved, the stopping potential now will be

1) 7.7 volt 2) 15.4 volt

3) 3.85 volt 4) 1.925 volt

KEY :1

36. A milliammeter in the circuit of a photocell measures

1) number of electrons released per second

2) energy of photon

3) velocity of photoelectrons

4) momentum of the photo electrons

KEY :1

37. The Einstein's photoelectric equation is based upon the conservation of

1) Mass 2) momentum

3) angular momentum 4) energy

KEY :4

38. The maximum energy of emitted photo electrons is measured by

1) the current they produce

2) the potential difference they produce

3) the largest potential difference they can transverse

4) the speed with which they emerge

KEY :3

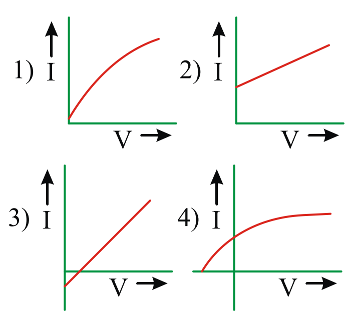
39. Three metals have work functions in the ratio 2:3:4. Graphs are drawn for all between the stopping potential and the incident frequency. The graphs have slopes in the ratio

1) 2: 3: 4 2) 4: 3: 2

3)6: 4: 3 4)1: 1: 1

KEY :4

40. The curve between current (I) and potential difference (v) for a photo cell will be



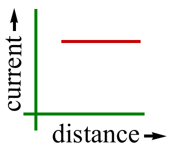
KEY :4

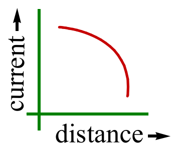
41. Matter waves are:

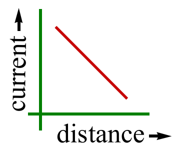
1) electromagnetic waves
2) mechanical waves
3) either mechanical or electromagnetic waves
4) neither mechanical nor electromagnetic waves

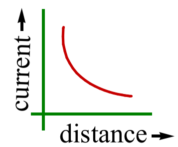
KEY :4

42. A point source causes photoelectric effect from a small metal plate. Which of the following curves may represent the saturation photocurrent as a function of the distance between the source and the metal?

1) 

2) 

3) 

4) 

KEY :4

43. In photo electric effect, the photo electric current

1) increases when the frequency of incident photon increases
2) decreases when the frequency of incident photon decreases
3) does not depend upon the photon frequency but depends on the intensity of incident beam
4) depends both on the intensity and frequency of the incident beam.

KEY :3

44. The photoelectric current can be increased by

1) increasing frequency 2) increasing intensity
3) decreasing intensity 4) decreasing wavelength

KEY :2

45. The threshold wavelength for sodium is 5×10^{-7} m. Photoemission occurs for light of

1) Wavelength of 6×10^{-7} m and above
2) Wavelength of 5×10^{-7} m and below
3) Any wavelength
4) All frequencies below 5×10^{14} Hz

KEY :2

46. The electron behaves as waves because they can

1) be diffracted by a crystal
2) ionise a gas
3) be deflected by magnetic fields
4) be deflected by electric fields

KEY :1

47. The mass of a photon in motion is (given its frequency = x)

1) $\frac{hx}{c^2}$ 2) hx^3 3) $\frac{hx^3}{c^2}$ 4) zero

KEY :1

48. A nonmonochromic light is used in an experiment on photoelectric effect. The stopping potential

1) is related to the mean wavelength
2) is related to the longest wavelength
3) is related to the shortest wavelength
4) is not related to the wavelength

KEY :3

49. The incident photon involved in the photoelectriceffect experiment

1) completely disappears
2) comes out with increased frequency
3) comes out with a decreased frequency
4) comes out with out change in frequency

KEY :1

50. A proton and an electron both have energy 50 eV.
Statement-I: Both have different wavelengths
Statement-II: Wavelength depends on energy and not on mass.

KEY :3

51. In a photoelectric experiment, the maximum velocity of photoelectrons emitted

1) depends on intensity of incident radiation
2) does not depend on cathode material
3) depends on frequency of incident radiation
4) does not depend on wavelength of incident radiation

KEY :3

52. The number of electrons emitted by a surface exposed to light is directly proportional to

1) Frequency of light 2) Work function
3) Thereshold wavelength 4) Intensity of light

KEY :4

53. Emission of electrons in photo electric effect is possible, if

1) metal surface is highly polished
2) the incident light is of sufficiently high intensity
3) the light is incident at right angles to the surface
4) the incident light is of sufficiently low wavelength

KEY :4

54. When orange light falls on a photo sensitive surface the photocurrent begins to flow. The velocity of emitted electrons will be more when surface is hit by

1) red light 2) violet light

3) thermal radiations 4) radio waves

KEY :2

55. When the amplitude of the light wave incident on a photometal sheet is increased then

1) the photoelectric current increases

2) the photoelectric current remains unchanged

3) the stopping potential increases

4) the stopping potential decreases

KEY :1

56. Which of the following is dependent on the intensity of incident radiation in a photoelectric experiment

1) work function of the surface

2) amount of photoelectric current

3) stopping potential

4) maximum kinetic energy

KEY :2

57. When stopping potential is applied in an experiemtn on photoelectric effect, no photocurrent is observed. This means that

1) the emission of photoelectrons is stopped

2) the photoelectrons are emitted but are reabsorbed by the emitter metal

3) the photoelectrons are ccumulated near the collector plate

4) the photoelectrons are dispersed from the sides of the apparatus.

KEY :2

58. Which one of the following is true in photoelectric emission

1) photoelectric current is directly proportional to the amplitude of light of given frequency

2) photoelectric current is directly proportional to the intensity of light of given frequency at moderate intensities

3) above the threshold frequency the maximum kinetic energy of photoelectrons is inversely proportional to the frequency of incident light

4) the threshold frequency depends on the intensity of incident light

KEY :2

59. If the work function of the metal is W and the frequency of the incident light is υ, then there is no emission of photoelectrons if

1) ν < W/h 2) ν > W/h 3) $\nu \geq$ W/h 4) $\nu \leq$ W/h

KEY :1

60. The total energy E of a sub-atomic particle of rest mass m_0 moving at non-relativistic speed v is

1) $E = m_0c^2$ **2)** $E = \frac{1}{2}m_0v^2$

3) $E = m_0c^2 + \frac{1}{2}m_0v^2$ **4)** $E = m_0c^2 - \frac{1}{2}m_0v^2$

KEY :3

61. A desktop illuminates a desk top with light of wavelength λ. The amplitude of this electromagnetic wave is E_0. Assuming illumination to be normally on the surface, the number of photons striking the desk per second per unit area N is

1) $N = \frac{\lambda\varepsilon_0E_0^2}{h}$ **2)** $N = \frac{2\lambda\varepsilon_0E_0^2}{h}$ **3)** $N = \frac{\lambda\varepsilon_0E_0^2}{2h}$ **4) Data insufficient**

KEY :3

62. The function of photoelecrtic cell is

1) to convert electrical energy into light energy.

2) to convert light energy into electrical energy

3) to convert mechanical energy into electrical energy

4) to convert DC into AC.

KEY :2

63. The rest mass of a photon is

1) zero **2)** $1.6\times10^{-19}kg$ **3)** $3.1\times10^{-30}kg$ **4)** $9.1\times10^{-31}kg$

KEY :1

64. When light falls on a photosensitive surface, electrons are emitted from the surface .The kineticenergy of these electrons does not depend on the:

1) Wave length of light

2) thickness of the surface layer

3) type of material used for the layer

4) intensity of light.

KEY :4

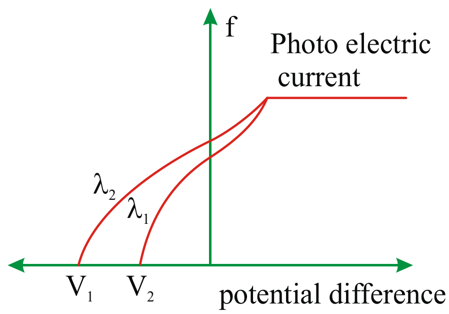
65. Though quantum theory of light can explain a number of phenomena observed with light, it is necessary to retain the wave-nature of light to explain the phenomena of:

1) photoelectric effect 2) diffraction

3) compton effect 4) black body radiation

KEY :2

66. In the following diagram if $V_2 > V_1$ then



1) $\lambda_1 = \sqrt{\lambda_2}$ 2) $\lambda_1 < \lambda_2$ 3) $\lambda_1 = \lambda_2$ 4) $\lambda_1 > \lambda_2$

KEY :4

67. When an X-ray photon collides with an electron and bounces off, its new frequency

1) is lower than its original frequency
2) is same as its original frequency
3) is higher than its original frequency
4) depends upon the electron's frequency

KEY :1

68. De-Broglie wavelength depends on

1) mass of the particle 2) size of the particle
3) material of the particle 4) shape of the particle

KEY :1

69. The photo electrons emitted from the surface of sodium metal are

1) Of speeds from 0 to a certain maximum
2) Of same de Broglie wavelength
3) Of same kinetic energy
4) Of same frequency

KEY :3

70. Choose the correct statement

1) Any charged particle in rest is accompanied by matter waves
2) Any uncharged particle in rest is accompanied by matter waves
3) The matter waves are waves of zero amplitude
4) The matter waves are waves of probability amplitude

KEY :4

71. Two separate monochromatic light beams A and B of the same intensity (energy per unit area per unit time) are falling normally on a unit area of a metallic surface. Their wavelength are λ_A and λ_B respectively. Assuming that all the incident light is used in ejecting the photoelectrons, the ratio of the number of photoelectrons from beam A to that from B is

1) $\left(\frac{\lambda_A}{\lambda_B}\right)$ 2) $\left(\frac{\lambda_B}{\lambda_A}\right)$ 3) $\left(\frac{\lambda_A}{\lambda_B}\right)^2$ 4) $\left(\frac{\lambda_B}{\lambda_A}\right)^2$

KEY :1\

72. Statement I: Davisson-Germer experiment established the wave nature of electrons
Statement II: If electrons have wave nature, they can interface and show diffraction. [AIEEE-2012]

KEY :1

73. Which of the following particles - neutron, proton, electron and deuteron has the lowest energy if all have the same de Broglie wavelength

1) neutron 2) proton3)electron 4) deuteron

KEY :4

74. A wave is associated with matter when it is

1) stationary
2) in motion with a velocity
3) in motion with speed of light
4) in motion with speed greater than that of light

KEY :2

75. An electron of mass 9.1 x 10^{-31}kg and charge 1.6 x 10^{-19} C is accelerated through a potential difference of V volt. The de Broglie wavelength (λ) associated with the electron is

1) $\frac{12.27}{\sqrt{V}}$ A^0 2) $\frac{12.27}{V}$ A^0 3) $12.27\sqrt{V}$ A^0 4) $\frac{1}{12.27\sqrt{V}}$ A^0

KEY :1

76. The de Broglie wavelength of a molecule of thermal energy KT (K is Boltzmann constant and T is absolute temperature) is given by

1) $\frac{h}{\sqrt{2mKT}}$ 2) $\frac{h}{2mKT}$ 3) $h\sqrt{2mKT}$ 4) $\frac{1}{h\sqrt{2mKT}}$

KEY :1

77. The wavelengths of a proton and a photon are same. Then

1) Their velocities are same
2) Their momenta are equal
3) Their energies are same
4) Their speeds are same

KEY :2

78. A particle of mass M at rest decays into two particles of masses m_1 and m_2, having non zero velocities. The ratio of the de Broglie wavelengths of the particles, $\frac{\lambda_1}{\lambda_2}$ is :

1) $\frac{m_1}{m_2}$ 2) $\frac{m_2}{m_1}$ 3) 1:1 4) $\sqrt{\frac{m_2}{m_1}}$

KEY :3

79. The wavelength of matter waves does not depend on

1) Momentum 2) Velocity 3) Mass 4) Charge

KEY :4

80. If the frequency of light in a photoelectric experiment is doubled, the stopping potenital will

1) be doubled 2) be halved

3) become more than double

4) become less than double

KEY :3

81. The wave nature of matter is not observed in daily life because their wave length is

1) Less 2) More

3) In infrared region 4) In ultraviolet region

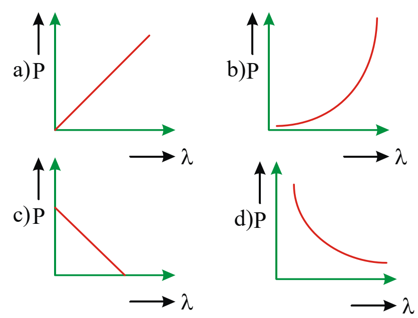
KEY :1

82. The ratio of the wavelengths of a photon and that of an electron of same energy E will be [m is mass of electron]

1) $\sqrt{\frac{2m}{E}}$ 2) $\sqrt{\frac{E}{2m}}$ 3) $C\sqrt{\frac{2m}{E}}$ 4) $\sqrt{\frac{EC}{2m}}$

KEY :3

83. One of the following figures respesents the variation of particle momentum with associated de Broglie wavelength



1) a 2) b 3) c 4) d

KEY :4

84. The work function of a metal

1) is different for different metals

2) is the same for all the metals

3) depends on the frequency of the light

4) depends on the intensity of the incident light

KEY :1

85. Let p and E denote the linear momentum and the energy of a photon. If the wavelength is decreased,

1) both p and E increase

2) p increases and E decreases

3) p decreases and E increases

4) both p and E decreases

KEY :1

86. Photoelectric effect supports the quantum nature of light because

1) There is minimum frequency of light above which no photo electrons are emitted

2) The maximum kinetic energy of photo electrons depends on both frequency and intensity of light

3) Even when a metal surface is faintly illuminated, the photoelectrons do not leave the surface immediately

4) The maximum K.E. of photo electrons depends only on the frequency of light and not on intensity

KEY :4

87. If the work function of a metal is ϕ_0, then its threshold wavelength will be

1) hcϕ_0 2) $\frac{c\phi_0}{h}$ 3) $\frac{h\phi_0}{c}$ 4) $\frac{hc}{\phi_0}$

KEY :4

88. The incorrect statement is

1) Material wave (de-Broglie wave) can travel in vacuum

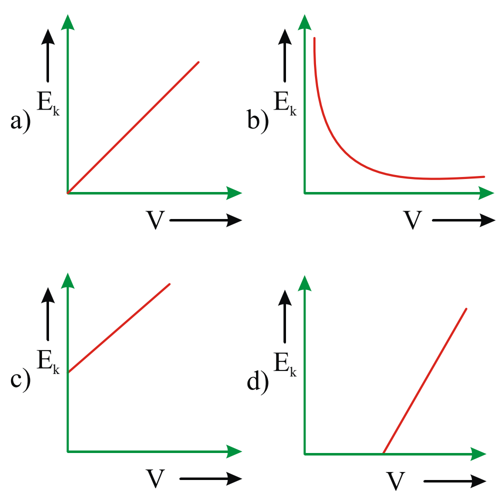
2) Electromagnetic wave can travel through vacuum

3) The velocity of photon is the same as light passes through any medium

4) Wavelength of de-Broglie wave depends upon velocity

KEY :3

89. Maximum kinetic energy (E_k) of a photoelectron varies with the frequency (ν) of the incident radiation as



1) a 2) b 3) c 4) d

KEY :4

90. The magnitude of the de-Broglie wavelength (λ) of an electron (e),proton(p),neutron (n) and α - particle (α) all having the same energy of MeV, in the increasing order will follow the sequence:

1) $\lambda_e, \lambda_p, \lambda_n, \lambda_\alpha$ 2) $\lambda_\alpha, \lambda_n, \lambda_p, \lambda_e$

3) $\lambda_e, \lambda_n, \lambda_p, \lambda_\alpha$ 4) $\lambda_p, \lambda_e, \lambda_\alpha, \lambda_n$

KEY :2

91. Debroglie wavelength of a particle at rest position is

1) zero 2) finite

3) infinity 4) cannot be calculated

KEY :3

92. When green light is incident on a metal, photo electrons are emitted by it but no photo electrons are obtained by yellow light. If red light is incident on that metal then

1) No electron will be emitted

2) Less electrons will be emitted

3) More electrons will be emitted

4) we can not predict

KEY :1

93. Debroglie wavelength of uncharged particles depends on

1) mass of particle

2) kinetic energy of particle

3) nature of particle

4) All above

KEY :4

94. Debroglie wavelength of a moving gas molecule is

1) proportional to temperature

2) inversely proportional to temperature

3) independent of temperature

4) inversely proportional to square root of temperature

KEY :4

95. Sodium surface is illuminated with ultraviolet light and visible radiation successively and the stopping potentials are determined. Then the potential

1) is equal in both the cases

2) greater for ultraviolet light

3) more for visible light

4) varies randomly

KEY :2

96. The wavelength λ of de Broglie waves associated with an electron (mass m, charge e) accelerated through a potential difference of V is given by (h is Planck's constant) :

1) $\lambda = h/mV$ 2) $\lambda = h/2\ meV$ 3) $\lambda = h/\sqrt{meV}$ 4) $\lambda = h/\sqrt{2meV}$

KEY :4

97. If a proton and an electron are confined to the same region, then uncertainity in momentum

1) for proton is more, as compared to the electron

2) for electron is more, as compared to the proton

3) same for both the particles

4) directly proportional to their masses

KEY :3

98. At stopping potential, the photo electric current becomes

1) Minimum 2) Maximum 3) Zero 4) Infinity

KEY :3

99. Which phenomenon best supports the theory that matter has a wave nature ?

1) electron momentum 2) electron diffraction

3) photon momentum 4) photon diffraction

KEY :2

100. Einstein's photoelectric equation states that $E_k = h\nu - W$, In this equation E_k refers to :

1) kinetic energy of all ejected electrons

2) mean kinetic energy of emitted electrons

3) minimum kinetic energy of emitted electrons 4) maximum kinetic energy of emitted electrons

KEY :4

101. The wavelength of de-Broglie wave associated with a thermal neutron of mass m at absolute temperature T is given by (Here, k is the Boltzmann constant)

1) $\frac{h}{\sqrt{2mkT}}$ 2) $\frac{h}{\sqrt{mkT}}$ 3) $\frac{h}{\sqrt{3mkT}}$ 4) $\frac{h}{2\sqrt{mkT}}$

KEY :3

In each of the following questions, a statement is given and a corresponding statement or reason is given just below it. In the statements, mark the correct answer as

1) If both Assertion and Reason are true and Reason is correct explanation of Assertion.

2) If both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

3) If Assertion is true but Reason is false.

4) If both Assertion and Reason are false.

102. Assertion (A) : For a fixed incident photon energy, photoelectrons have a wide range of energies ranging from zero to the maximum value $K_{\max}$

Reason (R) : Initially, the electrons in the metal are at different energy level.

KEY :1

103. Photoelectric effect is described as the ejection of electrons form the surface of a metal when:

1) it is heated to a high temparature

2) light of a suitable wave lenght is incident on it

3) electrons of a suitable velocity impinge on it

4) it is placed in a strong electric field

KEY :2

104. Photoelectric effect can be explained only by assuming that light:

1) is a form of transverse waves

2) is a form of longitudinal waves

3) can be polarised 4) consists of quanta

KEY :4

In each of the following questions, a statement is given and a corresponding statement or reason is given just below it. In the statements, mark the correct answer as

1) Statement I is true, Statement II is true; statement II is a correct explanation of statement I.

2) Statement I is true, Statement II is true, Statement II is NOT a correct explanation for statement I.

3) Statement I is true, Statement II is false

4) Statement I is false, Statemenet II is true.

105. The frequency and intensity of a light source are both doubled. Consider the following statements.

(A) The saturation photocurrent remains almost the same.

(B) The maximum kinetic energy of the photoelectrons is doubled.

1) Both A and B are true 2) A is true but B is false

3) A is false but B is true

4) Both A and B are false

KEY :2

106. Statement I: Though light of a single frequency (monochromic light) is incident ona metal, the energies of emitted photoelectrons are different.

Statement II: The energy of electrons just after they absorb photons incident on the metal surface may be lost in collision with other atoms in the metal before the electron is ejected out of the metal.

KEY :1

107. Statement I: The de Broglie wavelength of a molecule (in a sample of ideal gas) varies inversely as the square root of absolute temperature.

Statement II:The deBroglie wavelength of a molecule (in sample of ideal gas) depends on temperature

KEY :.2

108. Statement-I: A metallic surface is irradiated by a monochromatic light of frquency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are $K_{\max}$ and v_0 are also doubled.

Statement-II: The maximum kinetic energy and he stopping potential of photoelectrons emitted from a surface are linearly dependent on the frquency of incident light. [AIEEE-2011]

KEY :3

PREVIOUS MAINS QUESTIONS

Matter waves , Cathode and Positive Rays

1. An electron, a doubly ionized helium ion (He^{++}) and a proton are having the same kinetic energy. The relation between their respective de-Broglie wavelengths $\lambda_e, \lambda_{He++}$ andλ_p is: [Sep. 06, 2020 (I)]

(a) $\lambda_e > \lambda_{He++} > \lambda_p$ (b) $\lambda_e < \lambda_{He++} = \lambda_p$

(c) $\lambda_e > \lambda_p > \lambda_{He++}$ (d) $\lambda_e < \lambda_p < \lambda_{He++}$

SOLUTION : (c)

$$\text{de - Broglie wavelength, } \lambda = \frac{h}{P} = \frac{h}{\sqrt{2m(\mathrm{KE})}}$$

$$\lambda \propto \frac{1}{\sqrt{m}}$$

$$\text{As } m_{He^{H}} > m_P > m_e$$

$$\lambda_{He^{++}} > \lambda_P > \lambda_e \text{ or } \lambda_e > \lambda_P > \lambda_{He^{++}}$$

2. Assuming the nitrogen molecule is moving with r.m.s.velocity at 400 K, the de-Broglie wavelength of nitrongen molecule is close to:

(Given: nitrogen molecule weight: 4.64×10^{26}kg, Boltzman constant: 1.38×10^{23}J/K Planck constant: 6.63×10^{34} J. s) [Sep. 06, 2020 (II)]

(a) 0.24 Â (b) 0.20A (c) 0.34A (d) 0.44A

SOLUTION : (a)

$$\text{Rms speed ofgas molecule, } V_{rms} = \sqrt{\frac{3kT}{m}}$$

$$\text{de Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mk}}$$

$$\lambda = \frac{h}{\sqrt{2m \times \frac{1}{2} m V_{rms}^2}} = \frac{h}{\sqrt{m \times \frac{3}{2} kT}} = \frac{h}{\sqrt{3mkT}}$$

Substituting the respective values we get

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{3 \times 464 \times 10^{-26} \times 138 \times 10^{-13} \times 400}} = 0.24\text{A}$$

3. **Particle A ofmass $m_A = \frac{m}{2}$ moving along the x-axis with velocity v_0 collides elastically with another particle B at rest having mass $m_B = \frac{m}{3}$. Ifboth particles move along the x-axis after the collision, the change $\Delta\lambda$ in de-Broglie wavelength of particle A, in terms of its de-Broglie wavelength (λ_0) before collision is : [Sep. 04, 2020 (I)]**

(a) $\Delta\lambda = \frac{3}{2}\lambda_0$ **(b) $\Delta\lambda = \frac{5}{2}\lambda_0$** **(c) $\Delta\lambda = 2\lambda_0$** **(d) $\Delta\lambda = 4\lambda_0$**

SOLUTION : . (d)

$$(\mathrm{m}/2)\ (\mathrm{m}/3)\ (\mathrm{m}/2)$$

$$-\underline{V_B}$$

$$-V_0$$

A B (rest) (A) V_A (B) $(\mathrm{m}/3)$ Before collision After collision

Applying momentum conservation

$$\frac{m}{2} \times V_0 + \frac{m}{3} \times (0) = \frac{m}{2} V_A + \frac{m}{3} V_B$$

$$= \frac{V_0}{2} = \frac{V_A}{2} + \frac{V_B}{3} \text{ (i)}$$

Since, collision is elastic

$$e = 1 = \frac{V_B - V_A}{V_0} \Rightarrow V_0 = V_B - V_A$$

On solving equations (i) and (ii) : $V_A = \frac{V_0}{5}$

Now, de - Broglie wavelength of A before collision:

$$\lambda_0 = \frac{h}{m_A V_0} = \frac{h}{\left(\frac{m}{2}\right) V_0} \Rightarrow \lambda_0 = \frac{2h}{mV_0}$$

Final de - Broglie wavelength:

$$\lambda_f = \frac{h}{m_A V_0} = \frac{h}{\frac{m}{2} \times \frac{V_0}{5}} \Rightarrow \lambda_f = \frac{10h}{mV_0}$$

$$\Delta\lambda = \lambda_f - \lambda_0 = \frac{10h}{mV_0} - \frac{2h}{mV_0}$$

$$\Rightarrow \Delta\lambda = \underline{8h} \Rightarrow \Delta\lambda = 4 \times \underline{2h}$$

$$mv_0 \; mv_0$$

$$\Delta\lambda = 4\lambda_0$$

4. A particle is moving 5 times as fast as an electron. The ratio ofthe de-Broglie wavelength ofthe particle to that of the electron is 1.878×10^{-4}. The mass of the particle is close to : [Sep. 02, 2020 (II)]

(a) 4.8×10^{-27} kg (b) 9.1×10^{-31}kg

(c) 1.2×10^{-28} kg (d) 9.7×10^{-28} kg

SOLUTION : . (d)

de Broglie wavelength

$$\lambda = \frac{h}{mv} \Rightarrow m = \frac{h}{\lambda v}$$

Clearly, $m \propto \frac{1}{\lambda v}$

If λ and v be the wavelength and velocity of electron and λ' and v' be the wavelength and velocity of the particle then

$$\Rightarrow \frac{m'}{m} = \frac{v\lambda}{v'\lambda'} = \frac{1}{5} \times \frac{1}{1.878} \times 10^{-4}$$

$$\Rightarrow m = 9.7 \times 10^{-28}\text{kg}$$

5. A particle moving with kinetic energy E has de Broglie wavelength λ. If energy ΔE is a dded to its energy, the wavelength become $\frac{\lambda}{2}$. Value ofΔE, is: [9 Jan. 2020 I]

(a) E (b) ffl (c) 3E (d) 2E

SOLUTION : . (c)

As per question, when KE ofparticle E, wavelength λ and when KE becomes $E + \Delta E$ wavelength becomes $\lambda/2$

Using, $\lambda = \frac{h}{\sqrt{2mKE}}$

$$\frac{\lambda}{2} = \frac{h}{\sqrt{2m(KE + \Delta E)}}$$

$$\Rightarrow \frac{\lambda}{\lambda/2} = \sqrt{\frac{KE + \Delta E}{KE}}$$

$$\Rightarrow 4 = \frac{KE + \Delta E}{KE}$$

$$\Rightarrow 4KE - KE = \Delta E$$

$$\Delta E = 3KE = 3E$$

6. An electron ofmass m and magnitude ofcharge $|e|$initially at rest gets accelerated by a constant electric field E. The rate ofchange ofde-Broglie wavelength ofthis electron at time t ignoringrelativistic effects is: [9 Jan. 2020 II]

(a) $-\frac{h}{|e|E\sqrt{t}}$ (b) $\frac{|e|Et}{h}$ (c) $-\frac{h}{|e|Et}$ (d) $\underline{-h}$d) $|e|Et^2$

SOLUTION : . (d)

Acceleration ofelectron in electric field, $a = \frac{eE}{m}$ Using equation

$$v = u + at$$

$$\Rightarrow v = 0 + \frac{eE}{m}t$$

$$\Rightarrow v = \underline{eEt}$$

(i)

m

De - broglie wavelength λ is given by

$\lambda = \frac{h}{mv} = \frac{h}{m\left(\frac{eEt}{m}\right)}$ [using (i)]

$$\Rightarrow \lambda = \frac{h}{eEt}$$

Differentiating w.r.t. t

$$\frac{d\lambda}{dt} = \frac{d\left(\frac{h}{eEt}\right)}{dt} \Rightarrow \frac{d\lambda}{dt} = \frac{-h}{eEt^2}$$

7. An electron (mass m) with initial velocity $\vec{v} = v_0\hat{\imath} + v_0 j$ isin an electric field $\vec{E} = -E_0\hat{k}$. If λ_0 is initial de-Broglie wavelength ofelectron, its de-Broglie wave length at time t is given by: **[8 Jan. 2020 II]**

(a) $\frac{\lambda_0\sqrt{2}}{\sqrt{1+\frac{e^2E^2t^2}{m^2v_0^2}}}$

(b) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E_0^2t^2}{m^2v_0^2}}}$

(c) $\frac{\lambda_0}{\sqrt{1+\frac{e^2E^2t^2}{2m^2v_0^2}}}$

(d) $\frac{\lambda_0}{\sqrt{2+\frac{e^2E^2t^2}{m^2v_0^2}}}$

SOLUTION; (C)

Given, Initial velocity, $u = v_0\hat{\imath} + v_0\hat{\jmath}$

Acceleration, $a = \frac{qE_0}{m} = \frac{eE_0}{m}($

Using $v = u + at$

$$v = v_0\hat{\imath} + v_0\hat{\jmath} + \frac{eE_0}{m}t\hat{k}$$

$$|\vec{v}| = \sqrt{2v_0^2 + \left(\frac{eE_0t}{m}\right)^2}$$

de - Broglie wavelength, $\lambda = \frac{h}{p}$

$$\Rightarrow \lambda = \frac{h}{mv} (\because p = mv)$$

Initial wavelength, $\lambda_0 = \frac{h}{mv_{0\sqrt{2}}}$

Final wavelength,

$$\lambda = \frac{h}{m\sqrt{2v_0^2 + \left(\frac{eE_0t}{m}\right)^2}}$$

$$\frac{\lambda}{\lambda_0} = \frac{1}{\sqrt{1+\left(\frac{eE_0 t}{\sqrt{2} m v_0}\right)^2}}$$

$$\Rightarrow \lambda = \frac{\lambda_0}{\sqrt{1+\frac{e^2 E_0^2 t^2}{2m^2 v_0^2}}}$$

8. A particle P' is formed due to a completely inelastic collision ofparticles 'x' and 'y' having de-Broglie wavelengths γ_X' and γ_y' respectively. Ifx and y were moving oppositedirections, then the de-Broglie wavelength of P' is:

[9 Apr. 2019 II]

(a) $\frac{\gamma_x \gamma_y}{\gamma_x + \gamma_y}$ (b) $\frac{\gamma_x \gamma_y}{|\gamma_x - \gamma_y|}$ (c) $\gamma_x - \gamma_y$ (d) $\gamma_x + \gamma_y$

SOLUTION : . (b)

$$P_1 - P_2 = (P_1 + P_2) = P \text{ As } P \propto \frac{1}{\lambda}$$

$$\text{or } \frac{1}{\lambda_{\lambda i}} - \frac{1}{\lambda_y} = \frac{1}{\lambda}$$

$$\text{or } \frac{\lambda_y - \lambda_x}{\lambda_x \lambda_y} = \frac{1}{\lambda}$$

9. Two particles move at right angle to each other. Their de Broglie wavelengths are λ_1 and $\lambda_2$respectively. The particles suffer perfectly inelastic collision. The de Broglie wavelength λ, ofthe fmal particle, is given by: [8 April 2019 I]

(a) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$ (b) $\lambda = \sqrt{\lambda_1 \lambda_2}$

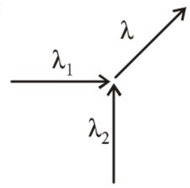
(c) $\lambda = \frac{\lambda_2 + \lambda_2}{2}$ (d) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

SOLUTION : (a)

(a) From the de-Broglie relation,

$$p_1 = \frac{h}{\lambda_1}$$

$$p_2 = \frac{h}{\lambda_2}$$



Momentum ofthe fmal particle (p_f) is given by

$$p_f = \sqrt{p_1^2 + p_2^2}$$

$$\Rightarrow \frac{h}{\lambda} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}}$$

$$\Rightarrow \frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

10. **Aparticle A ofmass m' and charge q' is acceleratedbya potential difference of50v Another particle B ofmass '4m' and charge' q' is accelerated by a potential differnce of 2500V. The ratio ofde-Broglie wavelength $\frac{\lambda_A}{\lambda_B}$ is [12 Jan. 2019 I]**

(a) $10.\alpha)$ (b) 0.07 (c) 14.14 (d) 4.47

SOLUTION : . (c)

de Broglie wavelength (λ) is given by

$$K = qV$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}} (\because p = \sqrt{2mK})$$

Substituting the values we get

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{2m_B q_B V_B}}{\sqrt{2m_A q_A V_A}} = \sqrt{\frac{4mq2500}{mq50}}$$

$$= 2\sqrt{50} = 2 \times 7.07 = 14.14$$

11. **Ifthe deBroglie wavelength ofan electron is equal to 10^-3 times the wavelength ofa photon offiiequency 6×10^{14} Hz, then the speed ofelectron is equal to:**

(Speed oflight $= 3 \times 10^8 m/s$) Planck's constant $= 6.63 \times 10^{-34} J.s$

Mass ofelectron $= 9.1 \times 10^{-31} kg$) [11 Jan. 2019 I]

(a) $1.1 \times 10^6 m/s$ (b) $1.7 \times 10^6 m/s$

(c) $1.8 \times 10^6 m/s$ (d) $1.45 \times 10^6 m/s$

SOLUTION : (d)

de - Broglie wavelength,

$$\lambda = \frac{h}{mv} = 10^{-3}(\ \ )(\ \ )\left[\because \lambda = \frac{c}{v}\right]$$

$$v = \frac{6.63 \times 10^{-34} \times 6 \times 10^{14}}{9.1 \times 10^{-31} \times 3 \times 10^{5}}$$

$$v = 1.45 \times 10^6 m/s$$

12 . In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12}m, the minimum electron energy required is close to:[10 Jan. 2019 I]

(a) 500 keV (b) 100keV (c) 1keV (d) 25 keV

SOLUTION : (d)

Using, $\lambda = \frac{h}{p}$ {given: $\lambda = 7.5 \times 10^{12}$}

$$\Rightarrow P = \frac{h}{\lambda}$$

Minimum energy required,

$$KE = \frac{P^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{\left\{\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right\}^2}{2 \times 9.1 \times 10^{-31}} J = 25keV$$

13. Two electrons are moving with non-relativistic speeds perpendicular to each other. If corresponding de Broglie wavelengths are λ_1 and λ_2, their de Broglie wavelength in the frame ofreference attached to their centre ofmass is:

[Online Apri115, 2018]

(a) $\lambda_{CM} = \lambda_1 = \lambda_2$ (b) $\frac{1}{\lambda_1} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

(c) $\lambda_{CM} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2}}$ (d) $\lambda_{CM} = \left(\frac{\lambda_1 + \lambda_2}{2}\right)$

SOLUTION ; (c)

Momentum (p) ofeach electron $\frac{h}{\lambda_1}\hat{i}$ and $\frac{h}{\lambda_2}\hat{j}$

Velocity ofcentre ofmass

$$V_{cm} = \frac{h}{2m\lambda_1}\hat{i} + \frac{h}{2m\lambda_2}\hat{j}\ (p = mv)$$

Velocity oflst particle about centre ofmass

$$V_{lcm} = \frac{h}{2m\lambda_1}\hat{i} - \frac{h}{2m\lambda_2}\hat{j}$$

$$\lambda_{cm} = \frac{h}{\sqrt{\frac{h^2}{4\lambda_1^2} + \frac{h^2}{4\lambda_2^2}}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} \left(\because \lambda = \frac{h}{p}\right)$$

14. **Ifthe de Broglie wavelengths associated with aproton and an α-particle are equal, then the ratio of velocities of the proton and the α-particle will be:[Online Apri115, 2018]**

(a) 1: 4 **(b) 1: 2** **(c) 4: 1** **(d) 2: 1**

SOLUTION : . (c)

According to question, $\lambda = \lambda$

$p\ \alpha$

Using, $\lambda = \frac{h}{p} = \frac{h}{mv}$

So, $\frac{h}{m_p v_p} \times = \frac{h}{m_\alpha v_\alpha} \times$

$\Rightarrow v_{p==} m_\alpha 4m_p$

$v_\alpha\ m_p\ m_p$

(mass of α - particle is 4 times of mass ofproton)

So, $\frac{v_p}{v_\alpha} = \frac{4}{1}$; i.e., 4: 1

15. Aparticle A ofmass m and initial velocityv collides witha particle B ofmass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is [2017]

(a) $\frac{\lambda_A}{\lambda_B}=\frac{2}{3}$ (b) $\frac{\lambda_A}{\lambda_B}=\frac{1}{2}$

(c) $\frac{\lambda_A}{\lambda_B}=\frac{1}{3}$ (d) $\frac{\lambda_A}{\lambda_B}=2$

SOLUTION : (d)

From question, $m_A=M; m_B=\frac{m}{2}$

$u_A=V\ u_B=0$

Let after collision velocity of $A=V_1$ and

velocity of $B=V_2$

Applying law of conservation ofmomentum,

$$mu=m\text{v}_1+\left(\frac{m}{2}\right)\text{v}_2$$

$$\text{or, } 24=2\text{v}_1+\text{v}_2 \ldots.(i)$$

By law ofcollision

$$e=\frac{v_2-v_1}{u-0}$$

$$\text{or, } u=\text{v}_2-\text{v}_1\ (ii)$$

[collision is elastic, $e=1$] using eqns (*i*) and (*ii*)

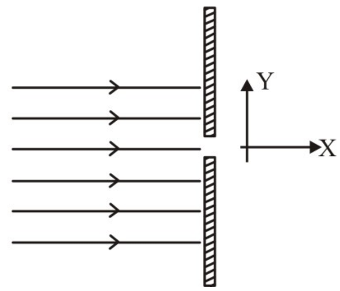
$$\text{v}_1=\frac{4}{3} \text{ and } \text{v}_2=\frac{4}{3}u$$

$$\text{de - Broglie wavelength } \lambda=\frac{h}{p}$$

$$\frac{\lambda_A}{\lambda_B}=\frac{P_B}{P_A}=\frac{\frac{m}{2}\times\frac{4}{3}u}{m\times\frac{4}{3}}=2$$

16. A parallel beam of electrons travelling in x-direction falls on a slit ofwidth $\mathbf{d}$ (see figure). If after passing the slit, an electron acquires momentum $\mathbf{p_y}$ in the $\mathbf{y}$-direction then for a majority ofelectrons passing through

the slit ($\mathbf{h}$ is Planck's constant): [Online Apri110, 2015]



(a) $|\mathbf{P_y}|\mathbf{d} > h$ (b) $|\mathbf{P_y}|\mathbf{d} < h$

(c) $|\mathbf{P_y}|\mathbf{d} = \mathbf{h}$ (d) $|\mathbf{P_y}|\mathbf{d} >> h$

SOLUTION : . (a)

From Bragg's equation

$$d \sin \theta = \lambda$$

$$\sin \theta = \frac{\lambda}{d} < 1 \; \lambda < d$$

$$\frac{h}{|p_y|} < d \left[\because \lambda = \frac{h}{|p_y|}\right]$$

$$h < |p_y| d$$

17. de-Broglie wavelengh ofan electron acceleratedbyavoltage of 50 V is close to ($|\mathbf{e}| = 1.6 \times 10^{-19}$C, $\mathbf{m_e} = 9.1 \times 10^{-31}$kg, $\mathbf{h} = 6.6 \times 10^{-34}$ Js) : [Online Apri110, 2015]

(a) 2.4 A (b) 0.5A (c) 1.7A d) 1.2A

SOLUTION : . (c)

de - Broglie wavelength,

$$\lambda = \frac{h}{P} = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$$

$$\text{or, } \lambda = \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 91 \times 10^{-31} \times 16 \times 10^{-19} \times 50}}$$

$$= 1.7\text{A}$$

18. For which ofthe following particles will it be most difficult to experimentally veri5γ the de-Broglie relationship? [Online April 9, 2014]

(a) an electron (b) a proton

(c) an α-particle (d) a dust particle

SOLUTION : (d)

Among the given particles most difficult to experimentally veriђγ the de - broglie relationship is for a dust particle.

19. Electrons are accelerated through apotential difference V and protons are accelerated through a potential difference 4 V. The de-Broglie wavelengths are λ_e and λ_p for electrons and protons respectively. The ratio of $\frac{\lambda_e}{\lambda_p}$ is given by : (given m_e is mass of electron and m_p is mass ofproton).

[Online April 23, 2013]

(a) $\frac{\lambda_e}{\lambda_p}=\sqrt{\frac{m_p}{m_e}}$ (b) $\frac{\lambda_e}{\lambda_p}=\sqrt{\frac{m_e}{m_p}}$ (c) $\frac{\lambda_e}{\lambda_p}=\frac{1}{2}\sqrt{\frac{m_e}{m_p}}$ (d) $\frac{\lambda_e}{\lambda_p}=2\sqrt{\frac{m_p}{m_e}}$

SOLUTION : . (d)

Energy injoule (E)

=charge $\times$ potential dilf. in volt

$E_{electron}=q_eV$ and $E_{proton}=q_p4V$

de - Broglie wavelength $\lambda=\frac{h}{P}=\frac{h}{\sqrt{2mE}}$

$\lambda_e=\frac{h}{\sqrt{2m_eeV}}$ and $\lambda_P=\frac{h}{\sqrt{2m_Pe4V}}(q_e=q_p)$

$$\frac{\lambda_e}{\lambda_P}=-=\frac{h}{\sqrt{2m_Pe4V}}\frac{h}{\sqrt{2m_eeV}}\sqrt{\frac{2m_Pe4V}{2m_eeV}}=2\sqrt{\frac{m_P}{m_e}}$$

20. If the kinetic energy of a free electron doubles, it's deBroglie wavelength changes by the factor [2005]

(a) 2 (b) $\frac{1}{2}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

SOLUTION : . (d)

de - Broglie wavelength,

$$\lambda = \underline{h} = \underline{h}$$

(i)

$$p \; mv$$

but $K.E = \frac{1}{2}mv^2$

$$\Rightarrow K.E = \frac{(mv)^2}{2m}$$

$$\Rightarrow mv = \sqrt{2m}K.E$$

$$\lambda = \frac{h}{\sqrt{2mK.E}}$$

21. Formation ofcovalentbonds in compounds exhibits [2002]

(a) wave nature of electron (b) particle nature ofelectron

(c) both wave and particle nature of electron (d) none of these

SOLUTION : . (a)

Covalent bonds are formed by sharing of electrons with different compounds. Formation ofcovalent bond is best explained by molecular orbital theory.

Photon , Photoelectric Effect X – rays and Davisson – Germer Experiment

22. A beam of electrons of energy E scatters from a target having atomic spacing oflA. The first maximum intensity occurs at $\theta = 60^{o}$. Then E (in eV) is .

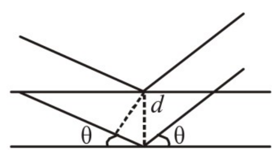
(Plank constant $h = 6.64 \times 10^{-34}$ Js, 1 eV $= 1.6 \times 10^{-19}$J,

electron mass $m = 9.1 \times 10^{-31}$kg) [NA Sep. 05, 2020 (I)]

SOLUTION : *(50)*

From Bragg' s equation $2d \sin\theta = \lambda$ and de - Broglie

wavelength, $\lambda = \frac{h}{P} = \frac{h}{\sqrt{2mE}}$



$$2d \sin\theta = \lambda = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow 2 \times 10^{-10} \times \frac{\sqrt{3}}{2} = \frac{6.6 \times 10^{-34}}{\sqrt{2mE}}$$

$[\because \theta = 60^{\circ}$ and $d = 1A = 1 \times 10^{-10}\text{m}]$

$$E = \frac{1}{2} \times \frac{6.64^{2} \times 10^{-48}}{9.1 \times 10^{-31} \times 3 \times 1.6 \times 10^{-19}} = 50\text{eV}$$

23. The surface of a metal is illuminated alternately with photons ofenergies $E_1 = 4$eV and $E_2 = 2.5$ eV respectively The ratio ofmaximum speeds ofthe photoelectrons emitted in the two cases is 2. The work function ofthe metal in (eV)

is [NA Sep. 05, 2020 (II)]

SOLUTION : . 2

From the Einstein' s photoelectric equation

Energy of photon

= Kinetic energy ofphotoelectrons + Work function

$\Rightarrow$ Kinetic energy = Energy of Photon - Work Function Let φ_0 be the work function ofmetal and v_1 and v_2 be the velocity ofphotoelectrons. Using Einstein' s photoelectric equation we have

$$\frac{1}{2}mv_1^2 = 4 - \varphi_0 \text{ (i)}$$

$$\frac{1}{2}mv_2^2 = 2.5 - (|)0 \text{ (ii)}$$

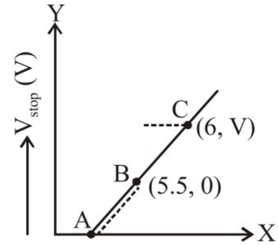
$$\Rightarrow \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{4-\varphi_0}{2.5-\varphi_0}$$

$$\Rightarrow (2)^2 = \frac{4-\varphi_0}{2.5-\varphi_0} \Rightarrow 10 - 4(|)0 = 4 - \varphi_0$$

$$\varphi_0 = 2eV$$

24. **Given figure shows few data points in a photo electric effect experiment for a certain metal. The minimum energy for ejection ofelectron from its surface is: (Plancks constant**

$h = 6.62 \times 10^{-34} \mathrm{J.s})$ **[Sep. 04, 2020 (I)]**



$\rightarrow^{5}$ $f(10^{14}$ **Hz)**

(a) $2.27\mathrm{eV}$ **(b)** $2.59\mathrm{eV}$ **(c)** $1.93\mathrm{eV}$ **(d)** $2.10\mathrm{eV}$

SOLUTION : . (a)

Graph of V_s and f given at $B(5.5, 0)$

Minimum energy for ejection ofelectron

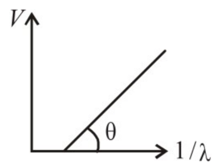
$=$ Work function (φ) .

$\varphi = hV$ joule or $\varphi = \frac{hV}{e}$ eV (for $V = 0$)

$$\varphi = \frac{6.62 \times 10^{-34} \times 5.5 \times 10^{14}}{1.6 \times 10^{-19}} \mathrm{eV} = 2.27\mathrm{eV}$$

25 **. In aphotoelectric effect experiment, the graph ofstopping potential V versus reciprocal of wavelength obtained is shown in the figure. As the intensity ofincident radiation**

is increased : [Sep. 04, 2020 (II)]



(a) Straight line shifts to right (b) Slope of the straight line get more steep

(c) Straight line shifts to left (d) Graph does not change

SOLUTION : (d)

According to Einstein's photoelectric equation

$$K_{\max} = hv - \varphi_0$$

$$\Rightarrow eV_s = \frac{hc}{\lambda} - \varphi_0$$

$$\Rightarrow V_s = \frac{hc}{\lambda e} - \frac{\varphi_0}{e}$$

where $\lambda =$ wavelength ofincident light

$\varphi_0 =$ work function

$V_s =$ stopping potential

Comparing the above equation with $y = mx + c$, we get slope $= \frac{hc}{e}$

Increasing the fiiequency ofincident radiation has no effect on work function and frequency So, graph will not change.

26. When the wavelength of radiation falling on a metal is changed fi: om500 nm to 200 mn, the maximum kinetic energy ofthe photoelectrons becomes three times larger. The work function ofthe metal is close to: [Sep. 03, 2020 (I)]

(a) $0.81\mathrm{eV}$ (b) $1.02\mathrm{eV}$ (c) $0.52\mathrm{eV}$ (d) $0.61\mathrm{eV}$

SOLUTION : . (d)

Using equation, $= \frac{hc}{\lambda} - (|)$

$$KE_{\max} = \frac{hc}{\lambda} - (|) = \frac{hc}{500} - \varphi \quad (1)$$

Again, $3KE_{\max} = \frac{hc}{200} - (|)$ (2)

Dividing equation (2) by(1),

$$\frac{3KE_{\max}}{KE_{\max}} = \frac{3}{1} = \frac{\frac{hc}{200} - \varphi}{\frac{hc}{500} - \varphi}$$

Putting the value of $hc = 1237.5$ and solving we get, work function, $\varphi = 0.61\text{eV}$.

27 . Two sources oflight emit X-rays ofwavelength 1 nm and visible light ofwavelength 500 nm, respectively. Both the sources emit light ofthe same power 200 W. The ratio of the number density of photons of X-rays to the number density of photons of the visible light of the given wavelengths is: [Sep. 03, 2020 (II)]

(a) $\frac{1}{500}$ (b) 250 (c) $\frac{1}{250}$ (d) $5\alpha)$

SOLUTION : . (a)

Given,

Wavelength of X - rays, $\lambda_1 = 1\text{nm} = 1 \times 10^{-9}\text{m}$

Wavelength ofvisible light, $\lambda_2 = 500 \times 10^{-9}\text{m}$

The number ofphotons emitted per second from a source ofmonochromatic radiation of wavelength λ and power P is given as

$$n = \frac{P}{E} = \frac{P}{hv} = \frac{P\lambda}{hc} (\because \text{E} = \text{hv and v} = \frac{c}{\lambda})$$

$\Rightarrow$Clearly $n \propto \lambda$

$$\Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{500}$$

28. When radiation of wavelength λ is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation ofwavelength 3λ, the stopping potential is $\frac{V}{4}$. If the theshold wavelength for the metallic surface is $n\lambda$ then value of $n$will be

[NA Sep. 02, 2020 (I)]

SOLUTION : . (9)

When radiation ofwavelength A, λ_A is used to illuminate, stopping potential $V_A = V$

$\frac{hc}{\lambda} = \varphi + eV$ (i)

When radiation ofwavelength B, λ_B is used to illuminate, stopping potential, $V_B = \frac{V}{4}$

$\frac{hc}{3\lambda} = (|) + \frac{eV}{4}$ (ii)

From eq. $(\mathrm{i}) - (\mathrm{ii})$,

$$\frac{hc}{\lambda}\left(1 - \frac{1}{3}\right) = \frac{3}{4}eV$$

$$\Rightarrow \frac{hc2}{\lambda 3} = \frac{3}{4}eV \Rightarrow eV = \frac{8}{9}\frac{hc}{\lambda}$$

$$\frac{hc}{\lambda} = (|) + \frac{8}{9}\frac{hc}{\lambda}$$

$\varphi = \frac{hc}{9\lambda} = \frac{hc}{n\lambda}$, so, $n = 9$.

29. Radiation, with wavelength 6561 A falls on a metal surface to produce photoelectrons. The electrons are made to enter a uniform magnetic field of$3 \times 10^{\rightarrow\downarrow}$ T. Ifthe radius ofthe largest circular path followed by the electrons is 10 mm, the work function ofthe metal is close to:

[9 Jan. 2020 I]

(a) 1.1 ev (b) 0.8 ev (c) 1.6 ev (d) 1.8 ev

SOLUTION : . (a)

Using Einstein’s photoelectric equation,

$$E = 0)_0 + KE_{\mathrm{mx}}$$

$$\Rightarrow 0)_0 = KE_{\max} - E$$

$$p = \sqrt{2mKE} \Rightarrow KE = \frac{p^2}{2m}$$

$$r = \frac{p}{eB} \Rightarrow p = reB$$

$$K_{\max} = \frac{r^2e^2B^2}{2m} KE_{\max} = \frac{12420}{\lambda} - 0)0$$

$$\Rightarrow c0_0 = \frac{12420}{6561} - \frac{r^2eB^2}{2m} (In\ \mathrm{eV})$$

$$= 1.89(eV) - \frac{(10^{-4})(1.6 \times 10^{-19})9 \times 10^5}{2 \times 9.07 \times 10^{-31}}$$

$$= 1.89(eV) - \frac{(10^{-4})(1.6 \times 10^{-19})9 \times 10^5}{2 \times 9.07 \times 10^{-31}}$$

$$= (1.89 - 0.79)eV = 1.1eV$$

30. When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_AeV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon ofenergy 4.50eV is $T_B = (T_A - 1.5)$eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function ofmetalB is:[8 Jan. 2020 I]

(a) 4 eV **(b) 2 eV** **(c) 1.5eV** **(d) 3 eV**

SOLUTION : . (a)

de - Broglie wavelength (λ) ,

Momentum, $mv = \frac{h}{\lambda} = p = \sqrt{2m(KE)}$

$$\lambda = \frac{h}{\sqrt{2mKE}} \Rightarrow \lambda \propto \frac{1}{\sqrt{KE}}$$

$\frac{\lambda_A}{\lambda_B} = \sqrt{\frac{K_B}{K_A}} = \sqrt{\frac{T_A - 15}{T_A}}$ (as given) Also, $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

On solving we get, $T_A = 2eV$

$$\mathfrak{M}_B = T_A - 1.5 = 2 - 1.5 = 0.5eV$$

Work function ofmetal B is

$$\varphi_B = E_B - \mathfrak{M}_B = 4.5 - 0.5 = 4eV$$

31. A beam of electromagnetic radiation of intensity $6.4 \times 10^5 \mathrm{W/cm^2}$ is comprised ofwavelength, $\lambda = 310\mathrm{nm}$. It falls normally on a metal (work function $\phi = 2$eV) ofsurface area ofl cm^2. Ifone in 10^3 photons ejects an electron, total number ofelectrons ejected in 1 s is 10^X. ($hc = 1240$eVnm, $= 1.6 \times 10^{19}$J) , then x is . [NA7Jan. 2020I]

SOLUTION : . (11.00)

Energy of proton

$$E = \frac{hc}{\lambda} = \frac{1240}{310} = 4eV > 2eV F\varphi]$$

(so emission ofphotoelectron will take place)

$= 4 \times 1.6 \times 10^{19} = 6.4 \times 10$ 19joule

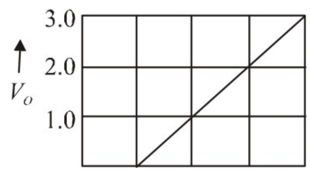
$$N = \frac{6.4 \times 10^{-5} \times 1}{4 \times 6.4 \times 10^{-19}} = 10^{14}$$

No. ofphotoelectrons emitted per second

$= \frac{10^{14}}{10^3} = 10^{11}$ (1 in 10^3 photons ejects an electron) Value of $X = 11.00$

32. he stopping potential V_0(in volt) as a function offiiequency (v) for a sodium emitter, is shown in the figure. The work function of sαlium, from the data plotted in the figure, will : (Given: Planck's constant $(h) = 6.63 \times 10$ 34Js, electron

charge $e = 1.6 \times 10^{19}$C) [12 Apr. 2019 I]



2 4 6 8 10

$v(10^{14} \rightarrow$ Hz)

(a) 1.82eV (b) 1.66eV(c)1.95eV (d) 2.12eV

SOLUTION : (b)

$$f_0 = 4 \times 10^{14} \text{ Hz}$$

$$W_0 = hf_0 = 6.63 \times 10^{34} \times (4 \times 10^{14})\text{J}$$

$$= \frac{(6.63 \times 10^{-34}) \times (4 \times 10^{14})}{1.6 \times 10^{-19}}$$

$$= 1.66\text{eV}$$

33. In a photoelectric effect experiment the threshold wavelength oflight is 380 nm. Ifthe wavelength ofincident light is 260 nm, the maximum kinetic energy of emitted electrons will be:

Given E(in eV) $=$ [10 Apr. 2019 I]

(a) 1.5eV (b) 3.0eV (c) 4.5e (d) 15.1eV

SOLUTION : . (a)

$$KE_{max} = E - \varphi_0$$

(where E = energy ofincident light φ_0 = work function)

$$= \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$$

$$= 1237\left[\frac{1}{260} - \frac{1}{380}\right]$$

$$= \frac{1237 \times 120}{380 \times 260} = 1.5eV$$

34. A $2mW$ laser operates at a wavelength of 500 nm. The number ofphotons that will be emitted per second is:

[Given Planck's constant $h = 6.6 \times 10^{34}$ Js, speed oflight

$c = 3.0 \times 10^8 m/s$] [10 Apr. 2019 II]

(a) 5×10^{15} (b) 1.5×10^{16} (c) 2×10^{16} (d) 1×10^{16}

SOLUTION : . (a)

Energy ofphoton (E) is given by

$$E = \frac{hc}{\lambda}$$

Number ofphotons ofwavelength λ emitted in t second from laser ofpower P is given by

$$n = \frac{Pt\lambda}{hc}$$

$$\Rightarrow n = \frac{2 \times \lambda}{hc} = \frac{2 \times 10^{-3} \times 5 \times 10^{-7}}{2 \times 10^{-25}} (t = 1S)$$

$$\Rightarrow n = 5 \times 10^{15}$$

35. The electric field oflight wave is given as

$$\vec{E} = 10^3 \cos\left(\frac{2\pi x}{5 \times 10^{-7}} - 2\pi \times 6 \times 10^{14} t\right)^{\wedge} x \frac{N}{C}$$

This light falls on a metal plate ofwork function $2eV$. The stopping potential of the photo-electrons is: Given, $E(\text{in eV}) = \frac{12375}{\lambda(\text{inA})}$ [9 April 2019 I]

(a) $2.0V$ (b) $0.72V$ (c) $0.48V$ (d) $2.48V$

SOLUTION : . (c)

$$\text{Here} w = 2\pi \times 6 \times 10^{14} \text{or} f = 6 \times 10^{14}\text{Hz}$$

$$\text{Wavelength } \lambda = \frac{C}{f} = \frac{3\times10^{8}}{6\times10^{14}} = 05 \times 10^{-6} m = 5000\text{A}^{o} \text{ Now } E = \frac{12374}{5000} = 2.48eV$$

$$\text{Using } \mathrm{E} = \mathrm{w} + \mathrm{eV_s}$$

$$2.48 = 2 + \mathrm{eV_s} \text{ or } \mathrm{V_s} = 0.48\mathrm{V}$$

36. When a certain photosensistive surface is illuminated with monochromatic light offrequency ν, the stopping potential for the photo current is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency *v/2*, the stoppoing potential is $-V_0$. The threshold frequency for photoelectric emission is: [12 Jan. 2019 II]

(a) $\frac{5v}{3}$ (b) $\frac{4}{3}v$ (c) 2 v (d) $\frac{3v}{2}$

. (BONUS)

37. In aFrank-Hertz experiment, an electron ofenergy $5.6eV$ passes through mercury vapour and emerges with an energy $0.7eV$. The minimum wavelength ofphotons emitted by mercury atoms is close to: [12 Jan. 2019 II]

(a) $17\alpha)$nm (b) 2020nm (c) 220nm (d) 250nm

SOLUTION : (d)

$$\text{Using, wavelength, } \lambda = \frac{12375}{\Delta\mathrm{E}}$$

$$\text{or, } \lambda = \frac{12375}{4.9} = 250\mathrm{nm}$$

38. In aphotoelectric experiment, the wavelength ofthe light incident on a metal is changed from 300 nm to 400 nm. The decrease in the stopping potential is close to: [1lJan. 2019 II]

($\frac{hc}{e} = 1240$ nm-V)

(a) $0.5V$ (b) $1.5V$ (c) $1.0V$ (d) $2.0V$

SOLUTION : (c)

$$\text{Let } \varphi = \text{work function ofthe metal, } \frac{hc}{\lambda_1} = (|) + eV_1 \text{ (i)}$$

$$\frac{hc}{\lambda_2} = (1) + eV_2 \text{ (ii)}$$

Sutracting (ii) $\mathrm{fi_iom}$ (i) we get

(1 1)

$$h_C(\quad)^{=e(V_1-V_2)}$$

$$\Rightarrow V_1 - V_2 = \frac{hc}{e}(\quad)(\quad)\begin{cases}\lambda_1 = 300nm \\ \lambda_2 = 400nm \\ \frac{hc}{e} = 1240nm - V\end{cases}$$

$$= (1240nm - v)(\quad)(\quad)$$

$$= 1.03V \approx 1V$$

39. A metal plate of area $1 \times 10^{-4} m^2$ is illuminated by a radiation ofintensity 16 mW/m^2. The work function ofthe metal is 5 eV. The energy ofthe incident photons is 10 eV and only 10% of it produces photo electrons. The number ofemitted photo electrons per second and their maximum energy, respectively, will be: $[1eV = 1.6 \times 10^{-19} J]$ [10 Jan. 2019 II]

(a) 10^{14} and 10 eV (b) 10^{12} and 5 eV

(c) 10^{11} and5 eV (d) 10^{10} and5 eV

SOLUTION : . (c)

using, intensity $I = \frac{nE}{At}$

$n =$ no. of photoelectrons

$$\Rightarrow 16 \times 10^3 = \left(\frac{n}{t}\right) \times \frac{10 \times 1.6 \times 10^{19}}{10^{-l}} \text{ or, } \frac{n}{t} = 10^{12}$$

So, effective number of photoelectrons ejected per unit time $= 10^{12} \times 10/100 = 10^{11}$

40. Surface ofcertain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, bylight ofwavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of(2) The work function ofthe metal (in eV) is close to:

(Energy ofphoton $= \frac{1240}{\lambda(innm)} eV$) [9 Jan. 2019 I]

(a) 1.8 (b) 2.5 (c) 5.6 (d) 1.4

SOLUTION : . (a)

From Einstein’s photoelectric equation,

$$\frac{hc}{\lambda_1} = \varphi + \frac{1}{2}m(2v)^2 \text{ (i)}$$

$$\text{and } \frac{hc}{\lambda_2} = \varphi + \frac{1}{2}mv^2 \text{ (ii)}$$

As per question, maximum speed of photoelectrons in two cases differ by a factor 2

From eqn. (i) & (ii)

$$\Rightarrow \frac{\frac{hc}{\lambda_1} - \varphi}{\frac{hc}{\lambda_2} - \varphi} = 4 \Rightarrow \frac{hc}{\lambda_1} - \varphi = \frac{4hc}{\lambda_2} - 4\varphi$$

$$\Rightarrow \frac{4hc}{\lambda_2} - \frac{hc}{\lambda_1} = 3\varphi \Rightarrow \varphi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4\times350-540}{350\times540}\right) = 1.8\text{eV}$$

41. The magnetic field associated with a light wave is given at the origin by

$$B = B_0[\sin(3.14 \times 10^7)ct + \sin(6.28 \times 10^7)ct].$$

Ifthis light falls on a silver plate having a work function of 4.7eV, what will be the maximum kinetic energy of the photoelectrons? [9 Jan. 2019 II]

$$(c = 3 \times 10^8 ms^{-1}, h = 6.6 \times 10^{-34} J - s)$$

(a) 6.82eV (b) 12.5eV (c) 8.52eV (d) 7.72eV

SOLUTION : . (d)

According to question, there are two EM waves with different frequency,

$$B_1 = B_0 \sin(\pi \times 10^7 c)t$$

$$\text{and } B_2 = B_0 \sin(2\pi \times 10^7 c)t$$

To get maximum kinetic energy we take the photon with higher frequency

$$\text{using, } B = B_0 \sin \text{ oot and } (j)^{=}2\pi v \Rightarrow v = \frac{0)}{2\pi}$$

$$10^7$$

$$B_1 = \text{BOsin}(\pi \times 10^7 c)t \Rightarrow v_1 =$$

$$\overline{2} \times c$$

$$B_2 = B_0 \sin(2\pi \times 10^7 c)t \Rightarrow v_2 = 10^7 c$$

where c is speed oflight $c = 3 \times 10^8 m/s$

Clearly, $v_2 > v_1$

so KE of photoelectron will be maximum for photon of higher energy.

$v_2 = 10^{7c}$ Hz

$$h\nu = \varphi + KE_{max}$$

energy of photon

$$E_{ph} = h\nu = 6.6 \times 10^{-34} \times 10^7 \times 3 \times 10^9$$

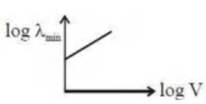
$$E_{ph} = 6.6 \times 3 \times 10^{-19} J$$

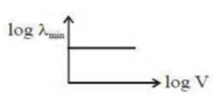
$$= \frac{6.6. \times 3 \times 10^{-19}}{16 \times 10^{-19}} eV = 12.375 eV$$

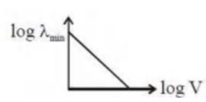
$$KE_{max} = E_{ph} - \varphi$$

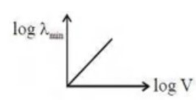
$$= 12.375 - 4.7 = 7.675 eV \approx 7.7 eV$$

42. **An electron beam is acceleratedbya potential differenceV to hit a metallic target to prαluce X-rays. It produces continuous as well as characteristic X-rays. If λ_{mm} is the smallest possible wavelength ofX-rayin the spectrum, the variation oflog λ_{mm} with log V is correctlyrepresented in:** **[2017]**

(a) 

(b) 

(c) 

(d) 

SOLUTION : (c)

In X - ray tube, $\lambda_{min} = \frac{hc}{eV}$

$$\ln \lambda_{min} = In\left(\frac{hc}{e}\right) - InV$$

Clearly, $\log \lambda_{m\,ln}$ versus $\log V$ graph

slope is negative hence option (c) correctly depicts.

43. ALaser light ofwavelength 660 nm is used to weld Retina detachment. Ifa Laser pulse ofwidth 60 ms and power $0.5\mathrm{kW}$ is used the approximate number of photons in the pulse are : [Take Planck's constant $\mathrm{h}=6.62\times 10^{-4}$ Js] [Online April 9, 2017]

(a) 10^{20} (b) 10^{18} (c) 10^{22} (d) 10^{19}

SOLUTION : . (a)

Given, $\lambda=660$ nm, Power $=0.5\mathrm{kW}$, $\mathrm{t}=60$ ms

Power $\mathrm{P}=\frac{\mathrm{nhc}}{\lambda\mathrm{t}}\Rightarrow\mathrm{n}=\frac{\mathrm{p}\lambda\mathrm{t}}{\mathrm{hc}}$

$$=0.5\times 10^{3}\times\frac{660\times 10^{-9}\times 60\times 10^{-3}}{6.6\times 10^{-34}\times 3\times 10^{8}}$$

$$=100\times 10^{18}=10^{20}$$

44. The maximum velocity ofthe photoelectrons emitted from the surface is v when light offiiequencyn falls on a metal surface. Ifthe incident frequency is increased to $3\mathrm{n}$, the maximum velocity of the ejected photoelectrons will be : [Online April 8, 2017]

(a) less than $\sqrt{3}\mathrm{v}$ (b) v

(c) more than $\sqrt{3}\mathrm{v}$ (d) equal to $\sqrt{3}\mathrm{v}$

SOLUTION : . (c)

As the metal surface is same, work function (φ) is same for both the case.

Initially $\mathrm{KE}_{\max}=\mathrm{nh}-\varphi$ (i)

After increase

$\mathrm{KE}'_{\max}=3\mathrm{nh}-\varphi$ (ii)

For work function $\varphi-$ not to be - ve or zero, $\mathrm{v}'>\sqrt{3}\mathrm{v}$

45. Radiation ofwavelength λ, is incident on a photocell. The fastest emitted electron has speed v. Ifthe wavelength is changed to $\frac{3\lambda}{4}$, the speed ofthe fastest emitted electron willbe: [2016]

(a) $=\mathrm{v}\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (b) $=\mathrm{v}\left(\frac{3}{4}\right)^{\frac{1}{2}}$

(c) $>v\left(\frac{4}{3}\right)^{\frac{1}{2}}$ (d) $<v\left(\frac{4}{3}\right)^{\frac{1}{2}}$

SOLUTION : (c)

$$h\frac{C}{\lambda} - h\nu_0 = \frac{1}{2}mv^2$$

$$\frac{4}{3}\frac{hc}{\lambda} - h\nu_0 = \frac{1}{2}mv'^2$$

$$\therefore \frac{v'^2}{v^2} = \frac{\frac{4}{3}\nu - \nu_0}{\nu - \nu_0} \quad \therefore \quad v' = v\sqrt{\frac{\frac{4}{3}\nu - \nu_0}{\nu - \nu_0}}$$

$$v' > v\sqrt{\frac{4}{3}}$$

46. A photoelectric surface is illuminated successively by monochromatic light of wavelengths λ and $\frac{\lambda}{2}$. If the maximum kinetic energy ofthe emitted photoelectrons in the second case is 3 times that in the first case, the work ffinction ofthe surface is: [Online Apri110, 2016]

(a) $\frac{hc}{2\lambda}$ (b) $\frac{hc}{\lambda}$ (c) $\frac{hc}{3\lambda}$ (d) $\frac{3hc}{\lambda}$

SOLUTION : . (a)

From Einstein's photoelectric equation

$$K.E_{\lambda} = \frac{hc}{\lambda} - (|) \text{ (i)}$$

(for monochromatic light ofwavelength λ) where φ is work function

$$K.E_{\lambda/2} = \frac{hc}{\lambda/2} - (|) \text{ (ii)}$$

(for monochromatic light ofwavelength $\lambda/2$) From question,

$$K.E_{\lambda/2} = 3(K.E_{\lambda}) \Rightarrow \frac{hc}{\lambda/2} - \varphi = 3\left(\frac{hc}{\lambda} - \varphi\right)$$

$$\frac{2hc}{\lambda} - (|) = 3\frac{hc}{\lambda} - 3_{t}|)$$

$$\Rightarrow 2\varphi = \frac{hc}{\lambda}\varphi = \frac{hc}{2\lambda}$$

47. When photons ofwavelength λ_1 are incident on an isolated sphere, the corresponding stopping potential is found to be V. When photons of wavelength λ_2 are used, the corresponding stopping potential was thrice that of the above value. Iflight ofwavelength λ_3 is used then find the stopping potential for this case: [Online April 9, 2016]

(a) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(b) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{1}{\lambda_1}\right]$

(c) $\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{1}{\lambda_2}-\frac{1}{\lambda_1}\right]$

(d) $\frac{hc}{e}\left[\frac{1}{\lambda_3}+\frac{1}{2\lambda_2}-\frac{3}{2\lambda_1}\right]$

SOLUTION : (None)

From Einstein' s photoelectric equation, we have $\frac{hc}{\lambda_1}=\frac{hc}{\lambda_0}+eV$ (1)

$\frac{hc}{\lambda_2}=\frac{hc}{\lambda_0}+eV$ (2)

$\underline{hc} = +3eV'\underline{hc}$

(3)

$\lambda_3\ \lambda_0$

From equation (1)& (2)

$$\frac{3}{2\lambda_1}-\frac{2}{2\lambda_2}=\frac{1}{\lambda_0}$$

$$\frac{hc}{\lambda_1}-hc\left[\frac{3}{2\lambda_1}-\frac{1}{2\lambda_2}\right]=eV'$$

$$\frac{hc}{e}\left[\frac{1}{\lambda_3}-\frac{3}{2\lambda_1}+\frac{1}{2\lambda_2}\right]=V'$$

48. Match List-I (Fundamental Experiment) with List- II (its conclusion) and select the correct option from the choices given below the list: [2015]

Lis t-I	List-II
A. Franck-Hertz Exp eriment	(i) Particle nature of light
B. Photo-electric experiment	(ii) Discrete energy levels ofatom

C. Davison-Germer experiment	$(Reject)$ Wave nature of electron
	(iv) Structure ofatom

(a) (A)-(ii); (B)-(i); (C)-(iii)

(b) $(\mathbf{A})-(\mathbf{iv});(\mathbf{B})-(\mathbf{iii});(\mathbf{C})-(\mathbf{ii})$

(c) $(\mathbf{A})-(\mathbf{i});(\mathbf{B})-(\mathbf{iv});(\mathbf{C})-(\mathbf{iii})$

(d) $(\mathbf{A})-(\mathbf{ii});(\mathbf{B})-(\mathbf{iv});(\mathbf{C})-(\mathbf{iii})$

SOLUTION : (a)

Frank - Hertz experiment - Discrete energy levels of atom, Photoelectric effect - Particle nature oflight.

Davison - Germer experiment - wave nature ofelectron.

49. Abeam oflight has two wavelengths of4972Â and $6216\mathrm{A}$ with a total intensity of $3.6 \times 10^{-3}\mathrm{Wm}^{-2}$ equally distributed among the two wavelengths. The beam falls normally on an area of $1\ \mathrm{cm}^2$ of a clean metallic surface ofwork function $.3\ \mathrm{eV}$. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number ofphotoelectrons liberated in $2s$ is approximately: [Online April 12, 2014]

(a) 6×10^{11} (b) 9×10^{11} (c) 11×10^{11} (d) 15×10^{11}

SOLUTION : (b)

Given, $\lambda_1 = 4972\mathrm{A}$

and $\lambda_2 = 6216\mathrm{A}$

and $\mathrm{I} = 3.6 \times 10^{-3}\mathrm{Wm}^{-2}$

Intensity associated with each wavelength

$$= \frac{3.6 \times 10^{-3}}{2}$$

$$= 1.8 \times 10^{-3}\mathrm{Wm}^{-2}$$

work function $\varphi = \mathrm{hv}$

$$= \frac{\mathrm{hc}}{\lambda}$$

$$= \frac{(6.62 \times 10^{-34})(3 \times 10^8)}{\lambda}$$

$$= \frac{12.4 \times 10^3}{\lambda} \text{ev}$$

for different wavelengths

$$\varphi_1 = \frac{12.4 \times 10^3}{\lambda_1} = \frac{12.4 \times 10^3}{4972} = 2.493\text{eV} = 3.984 \times 10^{-19}\text{J}$$

$$\varphi_2 = \frac{12.4 \times 10^3}{\lambda_2} = \frac{12.4 \times 10^3}{6216} = 1.994\text{eV} = 3.184 \times 10^{-19}\text{J}$$

Work function for metallic surface $\varphi = 2.3\text{eV}$ (given)

$$\varphi_2 < \varphi$$

Therefore, φ_2 will not contribute in this process.

Now, no. of electrons per $\mathrm{m}^2 - \mathrm{s} = \mathrm{no.\ ofphotons\ per\ m}^2 - \mathrm{s}$ no. of electrons per $\mathrm{m}^2 - \mathrm{s} = \frac{1.8 \times 10^{-3}}{3.984 \times 10^{-19}} \times 10^{-4}$

$$(\because 1^{2\triangleleft}_{\text{Cm}=10\text{m}^2)=0.45 \times 10^{12}}$$

So, the number ofphoto electrons liberated in 2 sec.

$$= 0.45 \times 10^{12} \times 2$$

$$= 9 \times 10^{11}$$

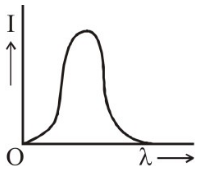
50. A photon ofwavelength λ is scattered from an electron, which was at rest. The wavelength shift $\Delta\lambda$ is three times of λ and the angle ofscattering θ is $60°$. The angle at which the electron recoiled is φ. The value oftan φ is: (electron speed is much smaller than the speed oflight)

[Online April 11, 2014]

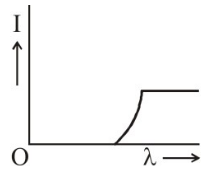
(a) 0.16 (b) 0.22 (c) 0.25 (d) 0.28

SOLUTION ;(b)

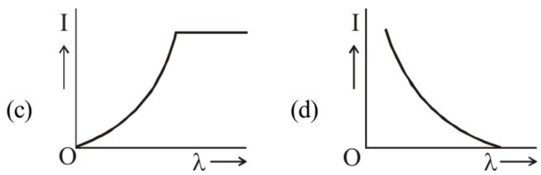
51. The anode voltage of a photocell is kept fixed. The wavelength λ ofthe light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows: [2013]



(a)



(b)

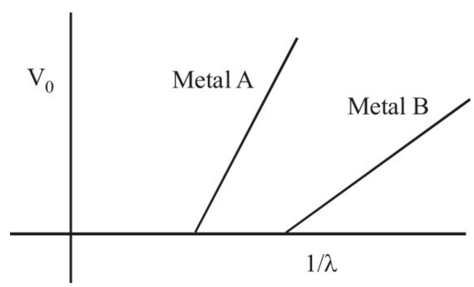


SOLUTION : . (d)

As λ is increased, there will be a value of λ above which photoelectrons will be cease to come out so photocurrent will become zero. Hence(d) is correct answer.

52. **In an experiment on photoelectric effect, a student plots stopping potential V_0 against reciprocal ofthe wavelength λ ofthe incident light for two different metals A and B.**

These are shown in the figure. **[Onhne April 25, 2013]**



Looking at the graphs, you can most appropriately say that:

(a) Work function ofmetal B is greater than that ofmetalA

(b) For light ofcertain wavelength falling on both metal, maximum kinetic energy ofelectrons emitted fi$_i$omA will be greater than those emitted fi$_i$om B.

(c) Work function ofmetalA is greater than that ofmetal B

(d) Students data is not correct

SOLUTION : (d)

$$\frac{hc}{\lambda} - \varphi = eV_0$$

$$v_0 = \frac{hc}{e\lambda} - \frac{\varphi}{e}$$

For metal A For metal B

$$\frac{\varphi A}{hc}=\frac{1}{\lambda}\frac{\varphi B}{hc}=\frac{1}{\lambda}$$

As the value of $\frac{1}{\lambda}$ (increasing and decreasing) is not specified hence we cannot say that which metal has comparatively greater or lesser work function (φ) .

53. A copper ball ofradius 1 cm and work function $4.47\mathrm{eV}$ is irradiated with ultraviolet radiation ofwavelength 2500 A. The effect ofirradiation results in the emission ofelectrons from the ball. Further the ball will acquire charge and due to this there will be a finite value ofthe potential on the ball. The charge acquired by the ball is: [Online April 25, 2013]

(a) $5.5\times 10^{-13}\mathrm{C}$

(b) $7.5\times 10^{-13}\mathrm{C}$

(c) $4.5\times 10^{-12}\mathrm{C}$

(d) $2.5\times 10^{-11}\mathrm{C}$

SOLUTION : . (a)

54. This equation has statement 1 and statement2. Ofthe four choices given after the statements, choose the one that describes the two statements.

Statement 1: Davisson-Germer experiment established the wave nature of electrons.

Statement 2 : If electrons have wave nature, they can interfere and show diffraction. [2012]

(a) Statement 1 is false, Statement 2 is true.

(b) Statement 1 is true, Statement 2 is false

(c) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of statement 1

(d) Statement 1 is true, Statement2 is true, Statement 2 is not the correct explanation ofStatement 1

SOLUTION : (a)

Davisson Germer experiment showed that electron beams can undergo diffraction when passed through atomic crystal. This established wave nature of electron as waves can eAibit interference and diffiiaction.

55. Photoelectrons are ejected from a metal when light of frequency u falls on it. Pick out the wrong statement from the following. [Online May 26, 2012]

(a) No electrons are emitted if u is less than Wlh, where W is the work function ofthe metal

(b) The ejection ofthe photoelectrons is instantaneous.

(c) The maximum energy of the photoelectrons is hu.

(d) The maximum energy of the photoelectrons is independent ofthe intensity ofthe light.

SOLUTION : (c)

According to photo – electric equation :

$K.E_{max} = h\nu - h\nu_0$ (Work function)

Some sort of energy is used in ejecting the photoelectrons.

56. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: A metallic surface is irradiated by a monochromatic light of fiiequency $u > u_0$ (the threshold fiiequency). Ifthe incident fiiequency is now doubled, the photocurrent and the maximum kinetic energy are also doubled.

Statement 2: The maximum kinetic energy of photoelectrons emitted from a surface is linearly dependent on the fiiequency of the incident light. The photocurrent depends only on the intensity ofthe incident light. [Online May 19, 2012]

(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of

Statement 1.

(b) Statement 1 is false, Statement 2 is true.

(c) Statement 1 is true, Statement 2 is false.

(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of Statement 1.

SOLUTION : (b)

The maximum kinetic energy of photoelectrons depends upon frequency on incident light and photo current depends upon intensity of incident light.

57. This question has Statement-l and Statement-2. Ofthe four choices given after the statements, choose the one that best describes the two statements. [2011] Statement— 1: A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold fiiequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement—2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency ofincident light.

(a) Statement-l is true, Statement-2 is true, Statement2 is the correct explanation

ofStatement -1.

(b) Statement-l is true, Statement-2 is true, Statement -2 is not the correct explanation of Statement—l.

(c) Statement—l is false, Statement-2 is true.

(d) Statement-l is true, Statement-2 is false.

SOLUTION : . (c)

By Einstein photoelectric equation,

$$K_{\max} = eV_0 = hv - hv_0$$

When v is doubled, $K_{\max}$ and V_0 become more than double.

58. Statement -1 : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is $K_{\max}$ .When the ultraviolet light is replaced by X-rays, both V_0 and $K_{\max}$ increase.

Statement -2 : Photoelectrons are emitted with speeds ranging $\mathrm{fi_{i}om}$ zero to a maximum value because of the range of frequencies present in the incident light. [2010]

(a) Statement-l is true, Statement -2 is true; Statement -2 is the correct explanation of Statement-l.

(b) Statement-l is true, Statement -2 is true; Statement -2 is not the correct explanation of Statement -1

(c) Statement-l is false, Statement -2 is true.

(d) Statement-l is true, Statement -2 is false.

SOLUTION : . (d)

We know that

$$eV_0 = K_{\max} = h\mathrm{v} - \varphi$$

where, φ is the work function.

X - rays have higher frequency (v) than ultraviolet rays. Therefore as v increases $K.E$ and V_0 both increases.

The kinetic energy ranges from zero to maximum because of loss of energy due to subsequent collisions before getting ejected.

59. The surface ofa metal is illuminted with the light of400 nm. The kinetic energy ofthe ejected photoelectrons was found to be $.68$ eV. The work function ofthe metal is:

$(hc = 1240 \text{eV}.\text{nm})$ [2009]

(a) 1.41 eV (b) 1.51 eV (c) 1.68 eV (d) 3.09 eV

SOLUTION : . (a)

Wavelength ofincident light, $\lambda = 400$ nm $hc = 1240$

eV. nm

$$K.E = 1.68 \text{eV}$$

Using Einstein' s photoelectric equation

$$\frac{hc}{\lambda} - W = K.E$$

$$\Rightarrow W = \frac{hc}{\lambda} - K.E$$

$$\Rightarrow W = \frac{1240}{400} - 1.68$$

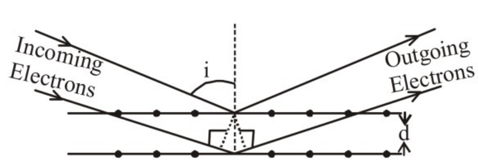
$$= 3.1 - 1.68$$

$$= 1.41 \text{eV}$$

Directions: Question No. 60 and 61 are based on the following paragraph.

Wave property ofelectrons implies that they will show diffiaction effects. Davisson and Germer demonstrated this by diffiacting electrons from crystals. The law governing the diffiiaction $\mathrm{fi_{i}om}$ a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure).

''''''1''''



Crystal plane

60. Electrons accelerated bypotential V are diffiacted from a crystal. Ifd $= 1\text{A}$ and i $= 30°$, V should be about [2008]

$$(h = 6.6 \times 10^{-34}\text{Js}, m_e = 9.1 \times 10^{-31}\text{kg}, e = 1.6 \times 10^{-19}\text{C})$$

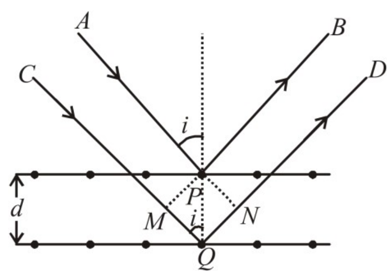
(a) $20\alpha)\text{V}$ (b) 50V (c) 500V (d) $10\alpha)\text{V}$

SOLUTION : . (b)

The path difference between the rays APB and CQD is

$$\Delta x = \text{MQ} + \text{QN} = d \cos i + d \cos i$$

$$\Delta x = 2d \cos i$$



For constructive interference the path difference is integral multiple ofwavelength

$$n\lambda = 2d \cos i$$

From de - broglie concept

Wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\text{mK.E}}} = \frac{h}{\sqrt{2\text{meV}}}$$

$$\frac{nh}{\sqrt{2\text{meV}}} = 2d \cos i$$

Squaring both side

$$\frac{n^2h^2}{2meV} = 4d^2\cos^2 i$$

For first order interference $n = 1$

$$V = \frac{h^2}{8med^2\cos^2 i}$$

$$= \frac{(.6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times 16 \times 10^{-19} \times (10^{-10})^2 \times \cos^2 30}$$

$= 50V$

61. Ifa strong diffiaction peak is observed when electrons are incident at an angle i' from the normal to the crystal planes with distance '*d*' between them (see figure), de Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer) [2008]

(a) $d \sin i = n\lambda_{dB}$ (b) $2d \cos i = n\lambda_{dB}$

(c) $2d \sin i = n\lambda_{dB}$ (d) $d \cos i = n\lambda_{dB}$

SOLUTION : . (b)

For constructive interference,

$$2d \cos i = n\lambda_{dB}$$

62. Photon of frequency v has a momentum associated with it. Ifc is the velocity oflight, the momentum is [2007]

(a) $\frac{hv}{c}$ (b) $\frac{v}{c}$ (c) hvc (d) $\frac{hv}{c^2}$

SOLUTION : . (a)

Energy of a photon of frequency v is given by

$$E = hv.$$

Also, $E = mc^2, mc^2 = hv$

$$\Rightarrow mc = \underline{hv} \Rightarrow p = \underline{hv}$$

63. The threshold fiiequency for ametallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in

[2006]

(a) ultra-violet region (b) infia-red region

(c) visible region (d) X-rayregion

SOLUTION : (a)

Work function, $\varphi = 6.2\text{eV} = 6.2 \times 1.6 \times 10^{-19}\text{J}$

Stopping potential, $V = 5$ volt

From the Einstein's photoelectric equation

$$\frac{hc}{\lambda} - \varphi = eV_0$$

$$\Rightarrow \lambda = \frac{hc}{\varphi + eV_0}$$

$$= \frac{6..6 \times 10^{-34} \times 3 \times 10^{8}}{16 \times 10^{-19}(6.2+5)} \approx 10^{-7}\mathrm{m}$$

This range lies in ultra violet range.

64. The time taken by a photoelectron to come out after the photon strikes is approximately [2006]

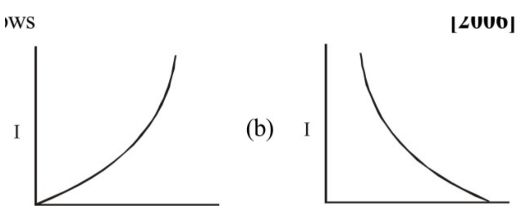
(a) 10^{-4}s (b) 10^{-10}s (c) 10^{-16}s (d) 10^{-1}s

SOLUTION : (b)

The photoelectric emission is an instantaneous process without any apparent time lag. It is known that emission starts in the time ofthe order of 10^{-9} second. So, the approximate time taken by a photoelectron to come out after the photon strikes is 10^{-10} second.

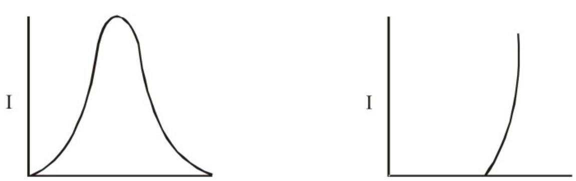
65. The anode voltage of a photocell is kept fixed. The wavelength λ ofthe light falling on the cathode is gradually changed. The plate current I of the photocell varies as rsnn′τ

follc



(a)

0 λ 0 λ



(c) **(d)**

0 λ 0 λ

SOLUTION : (b)

ti)) As λ decreases, y increases and hence the speed of photoelectron increases. The chances of photo electron to meet the anode increases and hence photo electric current increases.

66 . Aphotocell is illuminated bya small bright source placed 1 m away. When the same source oflight is placed $\frac{1}{2}$ m away, the number of electrons emitted by photocathode would [2005]

(a) increase by a factor of 4

(b) decrease by a factor of 4

(c) increase by a factor of2

(d) decrease by a factor of 2

SOLUTION : . (a)

$$I \propto \frac{I}{r^2}; \frac{I_1}{I_2} = (\quad)(\quad)^2 = \frac{1}{4}$$

$$I_2 \rightarrow 4 \text{ times } I_1$$

When intensity becomes 4 times, no. of photoelectrons emitted would increase by4 times, since number ofelectrons emitted per second is directly proportional to intensity.

67. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is [2004]

(a) Ec

(b) $2Elc$

(c) Elc

(d) Elc^2

SOLUTION : (b)

) Momentum ofphoton of energy E is $= \frac{E}{c}$

When a photon hits a perfectlyreflecting surface, it reflects black in opposite direction with same energy and momentum.

Change in momentum $= \frac{E}{C} - \left(\frac{-E}{C}\right) = \frac{2E}{C}$

This is equal to momentum transferred to the surface.

68. According to Einstein's photoelectric equation, the plot ofthe kinetic energy ofthe emitted photo electrons fi: om a metal Versus the frequency ofthe incident radiation gives a straight line whose slope [2004]

(a) depends both on the intensity of the radiation and the metal used

(b) depends on the intensity ofthe radiation

(c) depends on the nature of the metal used

(d) is the same for the all metals and independent of the intensity ofthe radiation

SOLUTION : . (d)

From the Einstein photoelectric equation $K.E. = h\text{v} - \varphi$ Here, $\varphi =$ work function ofmetal

$h =$ Plank's constant

slope ofgraph $of K.E.$ &v is h (Plank's constant) which is same for all metals.

69. The work function of a substance is . 0 eV. The longest wavelength oflight that can cause photoelectron emission from this substance is approximately [2004]

(a) 310nm (b) 400nm (c) 540nm (d) 220nm

SOLUTION : . (a)

Work function ofmetal (φ) is given by

$$\varphi = \frac{hc}{\lambda}$$

$$\Rightarrow \lambda = \frac{hc}{\varphi}$$

$$\Rightarrow \lambda = \frac{6.63\times10^{-34}\times3\times10^{8}}{4\times1.6\times10^{-19}} = 310 \text{ nm}$$

70. Two identical photocathodes receive light of fiequencies f_1 and f_2. If the velocites of the photo electrons (of mass m) coming out are respectively v_1 and v_2, then [2003]

(a) $v_1{}^2 - v_2{}^2 = \frac{2h}{m}(f_1 - f_2)$

(b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$

(c) $v_1{}^2 + v_2{}^2 = \frac{2h}{m}(f_1 + f_2)$

(d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

SOLUTION : . . (a)

Let work function be W and v_1 and v_2 be the velocity of electrons for frequencies f_1 and f_2.

Using Einstein's photo electric equation for one photodiode, we get

$$hf_1 - W = \frac{1}{2}mv_1^2 \text{ (i)}$$

Using Einstein's photo electric equation for another photodiode we get,

$$hf_2 - W = \frac{1}{2}mv_2^2 \text{ (ii)}$$

Subtracting (ii) fi;om (i) we get

$$(hf_1 - W) - (hf_2 - W) = \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2$$

$$h(f_1 - f_2) = \frac{m}{2}(v^2 1 - v_2^2)$$

$$v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$$

71. Sodium and copper have work ffinctions 2.3eV and 4.5eV respectively. Then the ratio ofthe wavelengths is nearest to [2002]

(a) 1: 2 (b) 4: 1 (c) 2: 1 (d) 1: 4

SOLUTION : . (c)

We know that work function,

$$E = h\text{u} = \frac{hC}{\lambda}$$

where

$h = \text{Planck'}$ s constant

$C =$ velocity of light

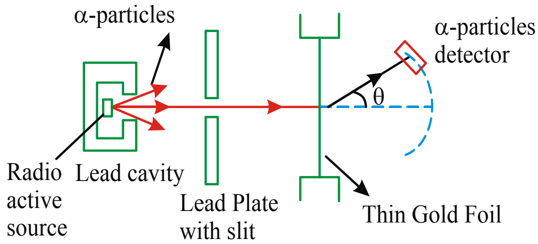
$\lambda =$ wavelength oflight

$$\frac{E_{Na}}{E_{Cu}} = \frac{\lambda_{Cu}}{\lambda_{Na}}$$

$$\Rightarrow \frac{\lambda_{Na}}{\lambda_{Cu}} = \frac{E_{Cu}}{E_{Na}} = \frac{4.5}{2.3} \approx \frac{2}{1}$$

ATOMS

Rutherford's α -particle Scattering Experiment:

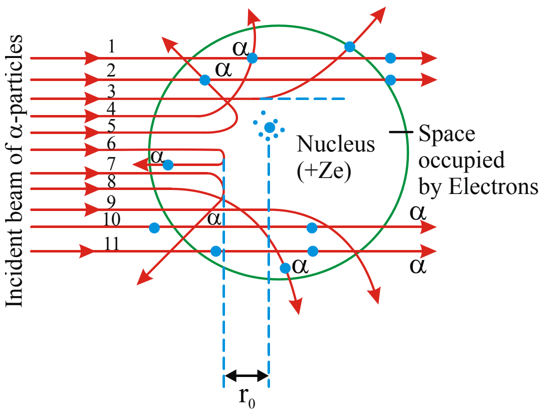


Experimental Observations:

a) Most of the α-particles were found to pass through the gold- foil without being deviated from their paths.

b) Some α-particles were found to be deflected through small angles $\theta < 90°$.

c) Few α-particles were found to be scattered at fairly large angles from their initial path $\theta > 90°$



d) A very small number of α-particles about 1 in 8000 practically retracted their paths or suffered deflections of nearly 180° .

- The observation (a) indicates that most of the portion of the atom is hollow inside.
- Because α-particle is positively charged, from the observations (b), (c) and (d) atom also have positive charge and the whole positive charge of the atom must be concentrated in small space which is at the centre of the atom is called nucleus. The remaining part of the atom and electrons are revolving around the nucleus in circular objects of all possible radii. The positive charge present in the nuclei of different metals is different . Higher the positive charge in the nucleus, larger will be the angle of scattering of α-particle.

Distance of Closest Approach :

- An α-particle which moves straight towards the nucleus in head on direction reaches the nucleus i.e, it moves close to a distance r_0 as shown the figure.
- As the α-particle approaches the nucleus, the electrostatic repulsive force due to the nucleus increases and kinetic energy of the alpha particle goes on converting into the electrostatic potential energy. When whole of the kinetic energy is converted into electrostatic potential energy, the α-particle cannot further move towards the nucleus but returns back on its initial path i.e α-particle is scattered through an angle of 180°. The distance of α-particle from the nucleus in this stage is called as the distance of closest approach and is represented by r_0.
- Let m_α and v_α be the mass and velocity of the α-particle directed towards the centre of the nucleus. Then kinetic energy of the α-particle $K = \left(\frac{1}{2}\right) m_\alpha v^2{}_\alpha$

  Because the positive charge on the nucleus is Ze and that on the α-particle 2e, hence the electrostatic potential energy of the α-particle, when at a distance r_0 from the centre of the nucleus, is given by $U = \frac{1}{4\pi \in_0} \cdot \frac{(2e)(Ze)}{r_0}$

  Because at r = r_0 kinetic energy of the α-particle appears as its potential energy, hence, K=U

$$\frac{1}{4\pi \in_0} \cdot \frac{(2e)(Ze)}{r_0} = \frac{1}{2} m_\alpha v_\alpha^2$$

$$r_0 = \frac{1}{4\pi \in_0} \frac{4Ze^2}{m_\alpha {v_\alpha}^2}$$

Alpha partical scattering (additional)

- When a mono energitic beam of α particles is projected towards a thin metal foil, some of the particles are found to deviate from their original path. This phenomenon is called α ray scattering
- It is caused by coulomb repulsive force between α particles and positive charges in atom.

- The number of α-particles scattered at an angle θ is given by $N = \frac{Q\,n\,t\,z^2\,e^4}{(8\pi\varepsilon_0)^2\, r^2\, E^2 \sin^4\left(\frac{\theta}{2}\right)}$

where

Q$\rightarrow$Total number of α particles striking the foil

n$\rightarrow$number of atoms per unit volume of the foil

r$\rightarrow$distance of screen from the foil

t$\rightarrow$thickness of the foil

z$\rightarrow$Atomic number of the foil atoms

$\theta \rightarrow$angle of scettering

E$\rightarrow$kinetic energy of α particles

$N \alpha t$; $N \alpha z^2$; $N\alpha \frac{1}{\sin^4 \frac{\theta}{2}}$

$N \alpha \frac{1}{E^2}$ or $N \alpha \frac{1}{\upsilon^4}$

where ϑ is the velocity of α particles falling on the foil.

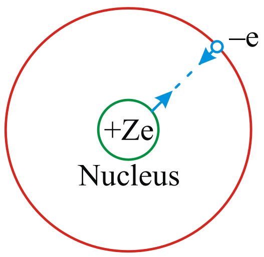
- **Impact Parameter(b):**The perpendicular distance of the initial velocity vector of the α- particle from centre of the nucleus is called "impact parameter".

$$b = \frac{Ze^2 \cot\left(\frac{\theta}{2}\right)}{4\pi \in_0 \times \frac{1}{2} mv^2}$$

Bohr's model of atom :

- Electron can revolve round the nucleus only in certain allowed orbits called stationary orbits and the Coulomb's force of attraction between electron and the positively charged nucleus provides necessary centripetal force.

$$\frac{k(Ze)e}{r^2} = \frac{mV^2}{r}$$



- Suppose m is the mass of electron, V is the velocity and 'r' is the radius of the orbit, then in stationary orbits the angular momentum of the electron is an integral multiple of $\frac{h}{2\pi}$, where h is the Planck's constant. The angular momentum $L = I\omega = mVr = n\frac{h}{2\pi}$ where n is called principal quantum number.

- An electron in a stationary orbit has a definite amount of energy. It posses kinetic energy because of its motion and potential energy on account of the attraction of the nucleus. Each allowed orbit is therefore associated with a certain quantity of energy called the energy of the orbit, which equals the total energy of the electron in it. In these allowed orbits electrons revolve without radiating energy .
- Energy is radiated or absorbed when an electron jumps from one stationary orbit to another stationary orbit. This energy is equal to the energy difference between these two orbits and emitted or absorbed as one quantum of radiation of frequency ν given by Planck's equation $E_2 - E_1 = h\nu = \frac{hc}{\lambda}$. This is called Bohr's frequency condition.

Conclusion

- **(i) Radius of Bohr's orbit :** When mass of the nucleus is large compared to revolving electron, then electron revolves around the nucleus in circular orbit. According to first postulate

$$\frac{k(Ze)e}{r^2} = \frac{mV^2}{r} \left(where \;\; k = \frac{1}{4\pi \in_0} \right) ......(1)$$

According to second postulate

$$mVr = n\frac{h}{2\pi} \text{ where n = 1,2,3,4............} \qquad \text{(or) } V = \frac{nh}{2\pi mr} \text{ .... (2)}$$

After solving the equations, radius of the orbit $r = \frac{n^2h^2}{4\pi^2 kZme^2}$

For nth orbit $r_n = \frac{h^2}{4\pi^2 ke^2} \cdot \left(\frac{n^2}{mZ} \right)$ ....(3)

For hydrogen atom Z = 1, radius of the first orbit (n = 1) is given by $r_1 = 0.529 \times 10^{-10} m \;;\; 0.53 \mathring{A}$

This value is called as Bohr's radius and the orbit is called Bohr's orbit. In general, the radius of the nth orbit of a hydrogen like atom is given by

$$r_n = 0.53 \left(\frac{n^2}{Z} \right) \mathring{A} \quad where\, n = 1,2,3,....... (4)$$

(ii) Velocity of the Electron in the orbit :

The velocity of an electron in nth orbit $V_n = \frac{nh}{2\pi m r_n}$ hence

$$V_n = \frac{2\pi ke^2}{h} \cdot \left(\frac{Z}{n} \right) \left(\because r_n = \frac{n^2h^2}{4\pi^2 kZme^2} \right) ..(5)$$

i.e the velocity of electron in any orbit is independent of the mass of electron. The above equation can also be written as

$$\therefore V_n = \left(\frac{c}{137}\right).\frac{Z}{n} \; m/s$$ .... (6)

Where 'c' is the speed of light in vacuum.

(iii) Time period of electron in the orbit :

Angular velocity of electron in n^{th} orbit

$\omega_n = \frac{V_n}{r_n} = \frac{\omega_0 Z^2}{n^3}$ where $\omega_0 = \frac{8\pi^3 k^2 e^4 m}{h^3}$ ......(7) is the angular velocity of electron in first Bohr's orbit. The time period of rotation of electron in n^{th} orbit $T = \frac{2\pi}{\omega_n} = \frac{n^3}{2\pi\omega_0 Z^2}$ .... (8) i.e $T \propto \frac{n^3}{Z^2}$.

The time period of rotation increases as n increases and is independent on the mass of the electron.

(iv) Kinetic Energy of the electron in the orbit

- The kinetic energy of the electron revolving round the nucleus in n^{th} orbit is given by

$$K_n = \frac{1}{2}mV^2 = \frac{1}{2}m\left[\frac{2\pi ke^2}{h}.\frac{Z}{n}\right]^2$$

$$K_n = \frac{2\pi^2 k^2 e^4}{h^2}.\left(\frac{mZ^2}{n^2}\right)$$ ....(9); $K_n \,\alpha\, \frac{mZ^2}{n^2}$

(v) Potential Energy of the electron in the orbit :

$$U_n = -\frac{k(Ze)e}{r_n} = -kZe^2\left[\frac{4\pi^2 kmZe^2}{n^2h^2}\right]$$

$$U_n = -\frac{4\pi^2 k^2 e^4}{h^2}\left(\frac{mZ^2}{n^2}\right)$$ .... (10)

(vi)Total energy of the electron in n^{th} orbit

Total energy of the electron in n^{th} orbit

$$E_n = K_n + U_n = -\frac{2\pi^2 k^2 mZ^2 e^4}{n^2 h^2}$$

$$E_n = -\frac{2\pi^2 k^2 e^4}{h^2}\left(\frac{mZ^2}{n^2}\right)$$ .... (11)

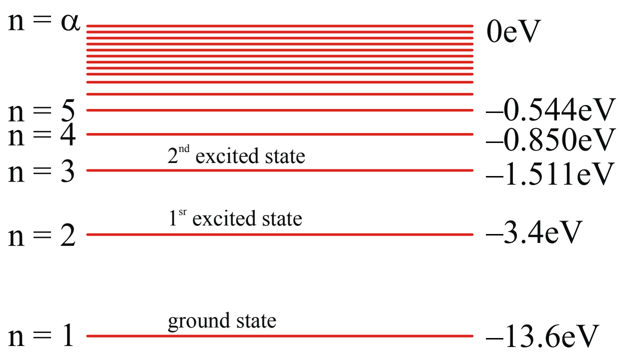
The expression of total energy for hydrogen like atom may be simplified as

$$E_n = -13.6\frac{Z^2}{n^2} \quad eV, n = 1,2,3..... \quad (12)$$

Where -13.6 eV is the total energy of the electron in the ground state of an hydrogen atom.
From the equations (9),(10)&(11) it is clear that
PE : K.E : T.E = -2 : 1 : -1

$$\text{i.e} \frac{PE}{-2} = \frac{KE}{1} = \frac{TE}{-1}$$

A The state n = 1 is called ground state and n > 1 states are called excited states. When electron go from lower orbit to higher orbit speed and hence kinetic energy decrease, but both potential energy and total energy increases. $E \alpha \frac{1}{n^2}$ tells us that the energy gap between the two successive levels decreases as the value of n increases. At infinity level the total energy of the atom becomes zero. Energy level diagram of hydrogen atom (Z = 1) for normal and excited states as shown the figure. The energy level diagram of hydrogen like atom with atomic number Z for normal and excited states as shown in Figure.



A The total energy of the electron is negative implies the atomic electron is bound to the nucleus. To remove the electron from its orbit beyond the attraction of the nucleus , energy must be required.

A The minimum energy required to remove an electron from the ground state of an atom is called its ionization energy and it is 13.6 Z^2 eV.

A In hydrogen atom the ground state energy of electron is –13.6 eV, so 13.6 eV is the ionization energy of the Hydrogen atom.

Emission of radiation:

A When an electron jumps from higher energy level

n_2 to a lower energy level n_1 in stationary atom, the difference in energy is radiated as a photon whose frequency ν is given by Planck's formula.

$$E_{n_2} - E_{n_1} = h\nu$$

(or) $h\nu = E_2 - E_1 = 13.6Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \; e.V$

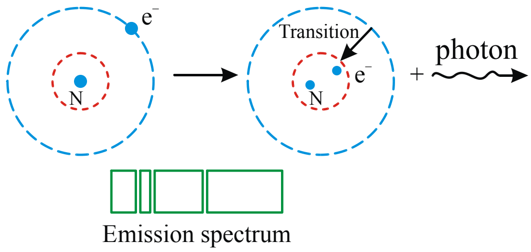
$\left(\therefore E_n = -\frac{13.6Z^2}{n^2} \; e.V\right)$ since $i.e., V = 1.6\times10^{-19} J$ hence $h\frac{c}{\lambda} = (12.8\times10^{-18})Z^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \; J$

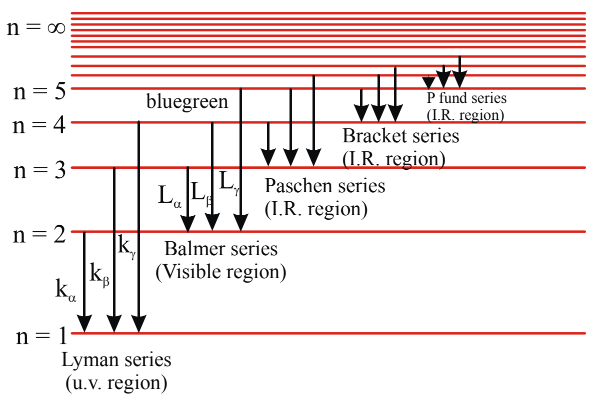
(or) wave number $\bar{v} = \frac{1}{\lambda} = RZ^2 . \left[\frac{1}{n_1^2} - \frac{1}{n_2^2} \right] \ m^{-1}$

where R is called for "Rydberg constant", when the nucleus is infinitely massive as compared to the revolving electron. In other words the nucleus is considered to be stationary. The numerical value of R is 1.097×10^7 m^{-1}.

Emission Spectrum of Hydrogen atom :

Electron in hydrogen atom, can be in excited state for very small time of the order of 10^{-8} second. This is because in the presence of conservative force system particles always try to occupy stable equilibrium position and hence minimum potential energy, which is least in ground state. Because of instability, when an electron in excited state makes a transition to lower energy state, a photon is emitted. Collection of such emitted photon frequencies is called an emission spectrum. This is as showing in figure.





The Spectral Series of Hydrogen Atom as shown in figure, are explained below.

a) Lyman Series : Lines corresponding to transition from outer energy levels $n_2 = 2, 3, 4, ...\infty$ to first orbit ($n_1 = 1$) constitute Lyman series. The wave numbers of different lines are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2} \right]$$

↳ Line corresponding to transition from $n_2 = 2$ to $n_1 = 1$ is first line; its wavelength is maximum.

$\frac{1}{\lambda_{max}} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 1.1 \times 10^7 \left[\frac{1}{1} - \frac{1}{4}\right] \therefore \lambda_{max} = 1212 Å$ Similarly transition from $n_2 = \infty$ to $n_1 = 1$ gives line of minimum wavelength. $\frac{1}{\lambda_{min}} = R\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] = 1.1 \times 10^7 \therefore \lambda_{min} = 912 Å$

↳ Lyman series lies in ultraviolet region of electro magnetic spectrum.

↳ Lyman series is obtained in emission as well as in absorption spectrum.

b) Balmer Series: Lines corresponding to $n_2 = 3, 4, 5, ...\infty$ to $n_1 = 2$ constitute Balmer series. The wave numbers of different lines are given by, $\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]$

↳ Line corresponding to transition $n_2=3$ to $n_1=2$ is first line, wavelength corresponding to this transition is maximum. Line corresponding to transition $n_2 = \infty$ to $n_1 = 2$ is last line; wavelength of last line is minimum.

$\frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] \quad \therefore \lambda_{max} = 6568 Å \quad \frac{1}{\lambda_{min}} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] \therefore \lambda_{min} = 3636 Å$

↳ Balmer series lies in the visible region of electromagnetic spectrum. The wavelength of L_α line is 656.8 nm (red). The wavelength of L_β line is 486 nm (blue green). The wavelength of L_γ line is 434 nm (violet). The remaining lines of Balmer series closest to violet light wavelength. The speciality of these lines is that in going from one end to other, the brightness and the separation between them decreases regularly.

↳ This series is obtained only in emission spectrum. Absorption lines corresponding to Balmer series do not exist, except extremely weakly, because very few electrons are normally in the state n = 2 and only a very few atoms are capable of having an electron knocked from the state n = 2 to higher states. Hence photons that correspond to these energies will not be strongly absorbed. In highly excited hydrogen gas there is possibility for detecting absorption at Balmer-line wavelengths.

c) Paschen Series: Lines corresponding to $n_2 = 4,5,6,.......\infty$ to $n_1 = 3$ constitute Paschen series. The wave number of different lines are given by $\bar{v} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right]$

↳ Line corresponding to transition $n_2 = 4$ to $n_1 = 3$ is first line, having maximum wavelength. Line corresponding to transition $n_2 = \infty$ to $n_1 = 3$ is last line, having minimum wavelength

$\frac{1}{\lambda_{max}} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] \therefore \lambda_{max} = 18747 Å$

$\frac{1}{\lambda_{min}} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = 1.1 \times 10^7 \times \left[\frac{1}{9} - 0\right]$

$\therefore \lambda_{min} = 8202 Å$

↳ Paschen series lies in the infrared region of electromagnetic spectrum.
↳ This series is obtained only in the emission spectrum.

d) Bracket Series: The series corresponds to transitions from $n_2 = 5,6,7......., \infty$ to $n_1 = 4$. The wave number are given by,

$$\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right]$$

↳ Line corresponding to transition from n_2=5 to n_1=4 has maximum wavelength and $n_2 = \infty$ to n_1 = 4 has minimum wavelength.

$$\frac{1}{\lambda_{max}} = R\left[\frac{1}{4^2} - \frac{1}{5^2}\right] \therefore \lambda_{max} = 40477Å$$

$$\frac{1}{\lambda_{min}} = R\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right] \therefore \lambda_{min} = 14572Å$$

↳ This series lies in the infrared region of electromagnetic spectrum.

e) Pfund Series: This series corresponds to transitions from $n_2 = 6,7,8,....,\infty$ to $n_1 = 5$. The wave numbers are given by $\bar{v} = \frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right]$

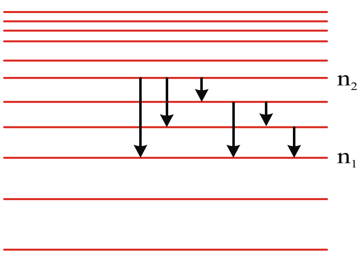
↳ Line corresponding to transition from $n_2 = 6$ to $n_1 = 5$ has maximum wavelength and $n_2 = \infty$ to n_1 = 4 has minimum wavelength. $\frac{1}{\lambda_{max}} = R\left[\frac{1}{5^2} - \frac{1}{6^2}\right]$

$\therefore \lambda_{max} = 74563Å$ $\frac{1}{\lambda_{min}} = R\left[\frac{1}{5^2} - \frac{1}{\infty^2}\right]$

$\therefore \lambda_{min} = 22768Å$

↳ This series lies in infrared region of electromagnetic spectrum.

Note : In an atom emission transition may start from any higher energy level and end at any energy level below of it. Hence in emission spectrum the total possible number of emission lines from some excited state n_2 to another energy state $n_1(< n_2)$ is $\frac{(n_2 - n_1)(n_2 - n_1 + 1)}{2}$



Note 1: for $n_2 = 4$, and $n_1 = 1$, the number of possible lines are 6.

Note 2 : If ΔE is the energy difference between two given energy states, then due to transition between these two states wavelength of emitted photon is $\lambda(\AA) = \dfrac{12400}{\Delta E(eV)}$

Limitation of Bohr's model :

Despite its considerable achievements, the Bohr's model has certain short coming.

- It could not interpret the details of optical spectra of atoms containing more than one electron.
- It involves the concept of orbit which could not be checked experimentally
- It could be successfully applied only to single-electron atoms (e.g., H, He^+ , Li^{2+}, etc.)
- Bohr's model could not explain the binding of atoms into molecules.
- No justification was given for the "principle of quantization of angular momentum".
- Bohr's model could not explain the reason why atoms should combine to form chemical bonds and why do the molecules become more stable on such combinations.
- Bohr had assumed that an electron in the atom is located at definite distance from the nucleus and is revolving with a definite velocity around it. This is against the Heisenberg uncertainty principle. With the advancements in quantum mechanics, it be came clear that there are no well defined orbits; rather there are clouds of negative charge.

S.No	Name of the series	Final State (n_1)	Initial State (n_2)	Formula	Series limit	Maximum wavelength	Region
1.	Lyman	$n_1 = 1$	$2, 3, 4, ...\infty$	$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_2^2}\right)$	$\lambda = \frac{1}{R} = 911 A°$	$\lambda = \frac{4}{3R}$	UV
2.	Balmer	$n_1 = 2$	$3, 4, 5 ....\infty$	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right)$	$\lambda = \frac{4}{R}$	$\lambda = \frac{36}{5R}$	Visible
3.	Paschen	$n_1 = 3$	$4, 5, 6 ....\infty$	$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_2^2}\right)$	$\lambda = \frac{9}{R}$	$\lambda = \frac{144}{7R}$	Near IR
4.	Brackett	$n_1 = 4$	$5, 6, 7 ....\infty$	$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_2^2}\right)$	$\lambda = \frac{16}{R}$	$\lambda = \frac{400}{9R}$	Middle IR
5.	Pfund	$n_1 = 5$	$6, 7, 8 ....\infty$	$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_2^2}\right)$	$\lambda = \frac{25}{R}$	$\lambda = \frac{9000}{11R}$	Far IR

Ex-1 The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. What are the possible values of n_1 and n_2 ?

Sol. Since, $T \propto n^3$; $\quad \frac{T_1}{T_2} = \frac{n_1^3}{n_2^3}$, As $T_1 = 8T_2$, $\left(\frac{n_1}{n_2}\right)^3 = 8$ (or) $n_1 = 2n_2$.

Thus the possible values of n_1 and n_2 are $n_1 = 2, n_2 = 1, n_1 = 4, n_2 = 2, n_1 = 6, n_2 = 3;$ and so on.

Ex-2 Find the kinetic energy, potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Sol. E_1 = -13.60eV; $K_1 = -E_1$ = 13.60eV

$U_1 = 2E_1$ = -27.20eV $E_2 = -3.40eV$ K_2 = 3.40eV and U_2 = -6.80eV

Now, U_1 = 0, i.e., potential energy has been increased by 27.20eV. So, we will increase U and E in all energy states by 27.20eV while kinetic energy will remain unchanged.

Hence K(eV), U(eV),E(eV)

First orbit are 13.6, 0, 13.6

in Second orbit 3.40, 20.40, 23.80

Ex-3 A small particle of mass m moves in such a way that the potential energy U = ar² where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of nth allowed orbit.

Sol. The force at a distance r is, $F = -\frac{dU}{dr} = -2ar$ Suppose r be the radius of n^{th} orbit. Then the necessary centripetal force is provided by the above force.Thus, $\frac{mv^2}{r} = 2ar$ -------- (i)

Further, the quantization of angular momentum gives, $mvr = \frac{nh}{2\pi}$ -------- (ii)

Solving Eqs. (i) and (ii) for r, we get $r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$

Ex-4 Consider a hydrogen-like atom whose energy in nth excited state is given by $E_n = -\frac{13.6Z^2}{n^2}$ when this excited atom makes transition from excited state to ground state most energetic photons have energy $E_{max} = 52.224eV$ and least energetic photons have energy $E_{min} = 1.224eV$. Find the atomic number of atom and the state of excitation.

Sol. Maximum energy is liberated for transition $E_n \to 1$ and minimum energy for $E_n \to E_{n-1}$

Hence, $\frac{E_1}{n^2} - E_1 = 52.224eV$ ... (1)

and $\frac{E_1}{n^2} - \frac{E_1}{(n-1)^2} = 1.224\,eV$ ...... (2)

Solving above equations simultaneously, we get

$E_1 = -54.4eV$ and n = 5 Now $E_1 = -\frac{13.6Z^2}{1^2} = -54.4eV$. Hence, Z = 2 i.e, gas is helium originally excited to n = 5 energy state.

Ex-5 A hydrogen-like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV and 17.00 eV respectively. Alternatively the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n and Z (ionization energy of hydrogen atom = 13.6 eV)

Sol. The electronic transitions in a hydrogen-like atom from a state n_2 to a lower state n_1 are given by

$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$. For the transition from a higher state n to the first excited state n_1 = 2, the total energy released is (10.2 + 17.0) eV or 27.2eV. Thus ΔE = 27.2 eV, $n_1 = 2$ and $n_2 = n$.

We have $27.2 = 13.6Z^2\left[\frac{1}{4}-\frac{1}{n^2}\right]$ ...... (1)

For the eventual transition to the second excited state $n_1 = 3$,
the total energy released is (4.25 + 5.95) eV or 10.2eV.

Thus $10.2 = 13.6Z^2\left[\frac{1}{9}-\frac{1}{n^2}\right]$.....(2)

Dividing the Eq. (1) by Eq. (2) we get $\frac{27.2}{10.2}=\frac{9n^2-36}{4n^2-36}$.

Solving we get $n^2 = 36$ or n = 6
Substituting n = 6 in any one of the above equations, we obtain $Z^2 = 9$ (or) Z = 3,
Thus n=6 and Z=3.

Ex-6 A doubly ionized lithium atom is hydrogen like with atomic number Z = 3. Find the wavelength of the radiation required to excite the electron in Li^{2+} from the first to the third Bohr orbit. Given the ionization energy of hydrogen atom as 13.6 eV.

Sol. The energy of n^{th} orbit of a hydrogen-like atom is given as $E_n = -\frac{13.6Z^2}{n^2}$

Thus for Li^{2+} atom, as Z = 3, the electron energies of the first and third Bohr orbits are
For n = 1, $E_1 = -122.4$eV, for n = 3, $E_3 = -13.6$eV.
Thus the energy required to transfer an electron from E_1 level to E_3 level is, $E = E_3 - E_1 = -13.6-(-122.4) = 108.8\text{eV}$.
Therefore, the radiation needed to cause this transition should have photons of this energy.
hv = 108.8 eV.

The wavelength of this radiation is or $\lambda = \frac{hc}{108.8\text{eV}}$ = 114.25 Å

Ex-7 A hydrogen atom in a state of binding energy 0.85 eV makes a transition to a state of excitation energy of 10.2 eV.

(i) What is the initial state of hydrogen atom?

(ii) What is the final state of hydrogen atom ?

(iii) What is the wavelength of the photon emitted ?

Sol. (i) Let n_1 be initial state of electron. Then $E_1 = -\frac{13.6}{n_1^2} eV$ Here E_1 = - 0.85 eV, therefore $-0.85 = -\frac{13.6}{n_1^2}$

or $n_1 = 4$

(ii) Let n_2 be the final excitation state of the electron. Since excitation energy is always measured with respect to the ground state, therefore

$\Delta E = 13.6\left[1 - \frac{1}{n_2^2}\right]$ here $\Delta E = 10.2 eV$, therefore,

$10.2 = 13.6\left[1 - \frac{1}{n_2^2}\right]$ or $n_2 = 2$ Thus, the electron jumps from $n_1 = 4$ to $n_2 = 2$.

(iii) The wavelength of the photon emitted for a transition between $n_1 = 4$ to $n_2 = 2$, is given by

$\frac{1}{\lambda} = R_\infty\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right]$ (or) $\frac{1}{\lambda} = 1.09 \times 10^7\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$ =4860Å .

Ex-8 A hydrogen atom initially in the ground level absorbs a photon, which excites it to the n = 4 level. Determine the wavelength and frequency of photon. To find the wavelength and frequency of photon use the relation of energy of electron in hydrogen atom is $E_n = -\frac{13.6}{n^2} eV$.

Sol. For ground state $n_1 = 1$ to $n_2 = 4$.

Energy absorbed by photon, $E = E_2 - E_1$

$= +13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \times 1.6 \times 10^{-19}$ J

$= 13.6\left(\frac{1}{1} - \frac{1}{4^2}\right) \times 1.6 \times 10^{-19}$

$= 13.6 \times 1.6 \times 10^{-19}\left(\frac{15}{16}\right) = 20.4 \times 10^{-19}$.

or $E = h\nu = 20.4 \times 10^{-19}$

Frequency $\nu = \frac{20.4 \times 10^{-19}}{h} = \frac{20.4 \times 10^{-19}}{6.63 \times 10^{-34}}$

$= 3.076 \times 10^{15} = 3.1 \times 10^{15}$ Hz.

Wavelength of photon $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.076 \times 10^{15}} = 9.74 \times 10^{-8}$ m. Thus, the wavelength is 9.7×10^{-8} m and frequency is 3.1×10^{15} Hz.

Ex-9 (a) Using the Bohr's model calculate the speed of the electron in a hydrogen atom in the n = 1, 2 and 3 levels.

(b) Calculate the orbital period in each of these levels.

Sol. (a) Speed of the electron in Bohr's nth orbit

$$\nu = \frac{c}{v}\alpha \quad \text{where,} \quad \alpha = \frac{2\pi Ke^2}{ch}$$

$$\alpha = 0.0073 \qquad \therefore \quad v = \frac{c}{n} \times 0.0073$$

For n = 1, $v_1 = \frac{c}{1} \times 0.0073 = 3 \times 10^8 \times 0.0073 = 2.19 \times 10^6$ m/s

For n = 2 $v_2 = \frac{c}{2} \times 0.0073 = \frac{3 \times 10^8 \times 0.0073}{2} = 1.095 \times 10^6$ m/s

For n = 3 $v_3 = \frac{c}{3} \times 0.0073 = \frac{3 \times 10^8 \times 0.0073}{3} = 7.3 \times 10^5$ m/s.

(b) Orbital period of electron is given by $T = \frac{2\pi r}{v}$

Radius of nth orbit $r_n = \left(\frac{n^2}{m}\right)\left(\frac{h}{2\pi}\right)^2 \frac{4\pi\varepsilon_0}{e^2}$

$$\therefore \ r_1 = \frac{(1)^2 \times (6.63 \times 10^{-34})^2}{4 \times 9.87 \times (9 \times 10)^9 \times 9 \times 10^{-31} \times 6.6 \times 10^{-19})} = 0.53 \times 10^{-10} \text{ m.}$$

For n = 1 $T_1 = \frac{2\pi r_1}{v_1} = \frac{2 \times 3.14 \times 0.53 \times 10^{-10}}{2.19 \times 10^6} = 1.52 \times 10^{-16}$ s

For n = 2, radius $r_n = n^2 r_1$

$r_2 = 2^2 . r_1 = 4 \times 0.53 \times 10^{-10}$ m and

velocity $v_n = \frac{v_1}{n}$ $\quad \therefore \ v_2 = \frac{v_1}{2} = \frac{2.19 \times 10^6}{2}$

Time period $T_2 = \frac{2 \times 3.14 \times 4 \times 0.53 \times 10^{-10} \times 2}{2.19 \times 10^6} = 1.216 \times 10^{-15}$ s.

For n = 3, radius $r_3 = 3^2$

$r_1 = 9 r_1 = 9 \times 0.53 \times 10^{-10}$ m

and velocity $v_3 = \frac{v_1}{3} = \frac{2.19 \times 10^6}{3}$ m/s

Time period $T_3 = \frac{2\pi r_3}{v_3} = \frac{2 \times 3.14 \times 9 \times 0.53 \times 10^{-10} \times 3}{2.19 \times 10^6} = 4.1 \times 10^{-15}$ s.

Ex-10 The radius of the innermost electron orbit of a hydrogen atom is 5.3 × 10^{-11} m. What are the radii of the n = 2 and n = 3 orbits?

Sol. Given, the radius of the innermost electron orbit of a hydrogen $r_1 = 5.3 \times 10^{-11}$ m.

As we know that $r_n = n^2 r_1$

For n = 2, radius $r_2 = 2^2$

$r_1 = 4 \times 5.3 \times 10^{-11} = 2.12 \times 10^{-10}$ m.

For n = 3, radius $r_3 = 3^2$

$r_1 = 9 \times 5.3 \times 10^{-11} = 4.77 \times 10^{-10}$ m.

Ex-11 A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. What series of wavelength will be emitted?

Sol. Energy of electron beam E=12.5 eV

$= 12.5 \times 1.6 \times 10^{-10}$ J

Planck's constant $h=6.63 \times 10^{-34}$ J-s

Velocity of light $c=3\times 10^8$ m/s

Using the relation $E=\frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{12.5\times1.6\times10^{-19}} = 0.993 \times 10^{-7}\ m = 993 \times 10^{-10}\ m = 993\ A^0$

This wavelength falls in the range of Lyman series (912 A^0 to 1216A^0)

thus, we conclude that Lyman series of wavelength 993 A^0 is emitted.

Ex-12 In accordance with the Bohr's model, find the quantum number that characterises the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of earth = 6.0 × 10^{24} kg).

A. Given, radius of orbit $r = 1.5 \times 10^{11}$ m

Orbital speed $v=3\times 10^4$m/s;

Mass of earth $M=6\times 10^{24}$kg

Angulalr momentum, $mvr = \frac{nh}{2\pi}$ or $n = \frac{2\pi vrm}{h}$

[where, n is the quantum number of the orbit]

$$= \frac{2\times3.14\times3\times10^{4}\times1.5\times10^{11}\times6\times10^{24}}{6.63\times10^{-34}} = 2.57 \times 10^{74} \text{ or } n = 2.6 \times 10^{74}.$$

Thus, the quantum number is 2.6 × 10^{74} which is too large. The electron would jump from n=1 to n=3

$$E_3 = \frac{-13.6}{3^2} = -1.5\ eV.$$

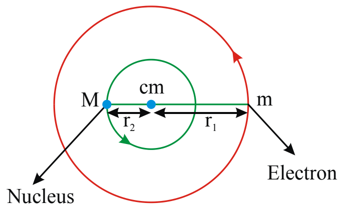
So, they belong to Lyman series.

EFFECT OF FINITE MASS OF NUCLEUS ON BOHR'S MODEL OF AN ATOM

i) In the atomic spectra of hydrogen and hydrogen like atoms a very small deviation with Bohr's model results

ii) This is in the assuption that the nucleus is infinitely massive when compared to mass of eletron so that it remains stationary during the rotation of eletron around it

iii) Infact the nucleus is not infinitely massive and hence both the nucleus and eletron revolve around their centre of mass with same angular velocity ω

v) 

v) let m be the mass of eletron, M be the mass of the nucleus , Z be its atomic number and r be the separation between them. If r_1, r_2 are distances of centre of mass from electron and nucleus respectively then $r_1 + r_2 = r$ and

$$r_1 = \frac{Mr}{M+m}, r_2 = \frac{mr}{M+m}$$

vi) For both the eletron and the nucleus the necessary centripetal force to revolve in circular orbits is provided by the electrostaic force between them.

i.e., $mr_1\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$; $m\frac{Mr}{M+m}\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$

i.e., $\mu r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}$ ----- (1)

where $\mu = \frac{Mm}{M+m}$ called reduced mass

vii) From Bohr's theory of quantization of angular moentum, total angular momentum of the system

$$L = I_1\omega + I_2\omega = \frac{nh}{2\pi};\quad mr_1^2\omega + Mr_2^2\omega = \frac{nh}{2\pi}$$

$$m\frac{M^2r^2}{(M+m)^2}\omega + \frac{Mm^2r^2}{(M+m)^2}\omega = \frac{nh}{2\pi}$$

$\frac{mMr^2\omega}{(M+m)^{2}}(M+m) = \frac{nh}{2\pi}$ ie $\mu r^2\omega = \frac{nh}{2\pi}$ --- (2)

viii) A system of this type is equalent a single particle of mass μ revolving around the position of the heavier particle(nucleus) in an orbit of radius r.

From (1) and (2) $r = \frac{\varepsilon_0 n^2h^2}{\pi z\mu e^2}$

ix) Radius of orbit of such a particle in a quantum state n is $r_n = \frac{\varepsilon_0 n^2h^2}{\pi z\mu e^2} \Rightarrow r_n \propto \frac{n^2}{\mu z}$

x) Potential energy of the system $PE = \frac{-Ze^2}{4\pi\varepsilon_0 r}$ and kinetic energy of the system

$$KE = \frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2 = \frac{1}{2}(I_1 + I_2)\omega^2$$

$$= \frac{1}{2}(mr_1^2 + Mr_2^2)\omega^2$$

$$= \frac{1}{2}\left[m\frac{M^2r^2}{(M+m)^2} + M\frac{m^2r^2}{(M+m)^2}\right]\omega^2$$

$$= \frac{1}{2}\frac{mMr^2\omega^2}{(M+m)^{\not 2}}\cancel{(M+m)} = \frac{1}{2}\left(\frac{mM}{M+m}\right)r^2\omega^2$$

ie $KE = \frac{1}{2}\mu r^2\omega^2 = \frac{1}{2}\frac{Ze^2}{4\pi\varepsilon_0 r}\left(\because \mu r^3\omega^2 = \frac{Ze^2}{4\pi\varepsilon_0}\right)$

$\therefore$ Total energy of the system (or equivalent particle of mass μ) $E = PE + KE$

$$E = -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0} \times \frac{\pi Z\mu e^2}{n^2h^2\varepsilon_0} = -\frac{Z^2\mu e^4}{8\varepsilon_0^2 n^2 h^2}$$

ie Energy of n[th] quantum state

$$E_n = \frac{-Z^2\mu e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{-mZ^2\mu e^4}{8\varepsilon_0^2 n^2 h^2 \times m} = \frac{-me^4}{8\varepsilon_0^2 h^2} \times \left(\frac{Z^2\mu}{n^2 m}\right)$$

$$E_n = -13.6\left(\frac{z^2}{n^2}\right) \times \frac{\mu}{m}eV = -13.6\left(\frac{Z^2}{n^2}\right) \times \frac{M}{M+m}eV$$

$$= \frac{-13.6}{1+\frac{m}{M}}\left(\frac{Z^2}{n^2}\right)eV$$

The formulae for r_n and E_n can be obtained simply by replacing m by μ in the formulae for stationary nucleus, If λ is wave length of photon emitted due to transition from a quantum state n_2 to a quantum state n_1, then $\frac{hc}{\lambda} = \frac{me^4z^2}{8\varepsilon_0^2 h^2} \times \frac{1}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$$\frac{1}{\lambda} = \frac{me^4z^2}{8\varepsilon_0^2 h^3 c} \times \frac{1}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$\frac{1}{\lambda}=\frac{R_0Z^2}{\left(1+\frac{m}{M}\right)}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ where R_0 is Rydberg's constant when the nucleus is stationary

EXCITATION BY COLLISION

i) When an atom is bombarded by particles like electron, proton, neutron, $\alpha-$ particle etc, the loss in KE of the system during collision may be used in excitation of the atom

ii) If loss in KE of the system during collision (during deformation phase)is less than the energy required to excite the eletron to next higher energy state, eletron can't be excited and the loss of KE of the system during deformation phase again converts into KE of the system and totally there will be no loss of KE of of the system and hence the collision is elastic

iii) If loss in KE of the system during deformation phase is more than or equal to the energy required to excite the electron to next higher state, excitation of the electron may take place and hence kinetic energy of the system may not be conserved hence the collision may be inelastic or even perfectly inelastic.

iv) If loss in KE is sufficient even ionization may take place.Even though the possible loss in KE is greater than or equal to excitation energy of electron, excitation may not take place necessarily and hence collision may be elastic.

v) Consider a particle of mass m moving with velocity u which strikes a stationary hydrogen like atom of mass M which is in ground state.

vi) Loss in KE will be maximum in perfectly inelastic collision. In this case if V is common velocity after collision, from conservation of linear momentum mu+0=(M+m)V $\Rightarrow V=\frac{mu}{M+m}$

$\therefore$ Maximum possible loss in KE is

$$\Delta K=\frac{1}{2}mu^2-\frac{1}{2}(M+m)V^2$$

i.e $\Delta K=\frac{1}{2}\left(\frac{Mm}{M+m}\right)u^2$

vii) If ΔE is minimum excitation energy (ex: n=1 to n=2 in ground state) and if $\Delta K<\Delta E$ eletron can't be excited hence there will be no loss of KE of the system hence the collision is elastic.

viii) If $\Delta K=\Delta E$ the electron may get excited and the collision may be perfectly in elastic.

ix) If $\Delta K>\Delta E$ the electron may get excited to higher energy states or even removed from the atom and may have some kinetic energy. In this case the collision may be inelastic or may be perfectly inelastic as there is loss in KE of the system, or even elastic if excitation dose not take place.

x) If $KE_{\min}$ is the minimum KE that should bepocessed by the colliding particle to excite the electron, $\Delta K\geq\Delta E$ for excitation

$$\frac{1}{2}\frac{mM}{M+m}u^2\geq\Delta E\Rightarrow\frac{1}{2}mu^2\geq\Delta E\left(\frac{M+m}{M}\right)$$

$$\Rightarrow KE_{\min}=\Delta E\left(\frac{M+m}{M}\right)=\Delta E\left(1+\frac{m}{M}\right)$$

In the same way we can calculate $KE_{\min}$ in other cases where atom is also moving using the conservation of linear momentum.

X-Rays :

Roentgen discovered the X-rays.

i) Most commonly x-rays are produced by the deceleration of high energy electrons bombarding a hard metal target.

ii) The target should have
a) high atomic weight
b) high melting point
c) high thermal conductivity

iii) They are electromagnetic waves of very short wavelength. i.e., order of wavelength 0.1A° to 100A°, order of frequency 10^{16}Hz to 10^{19} Hz, order of energy 124eV to 124keV

iv) Most of the kinetic energy of electrons is converted into heat and only a fraction is used in producing x-rays (less than 1% x - rays and more than 99% heat).

v) Intensity of x-rays depends on the number of electrons striking the target which inturn depends on filament current.

vi) Quality of x - rays (hard /soft) depends on P.D applied to x - rays tube.

vii) high frequency x-rays are called hard x-rays

viii) low frequency x-rays are called soft x-rays

ix) Penetrating power of x-rays is a function of potential difference between cathode and target.

x) Interatomic distance in crystals is of the order of the wavelength of x-rays hence crystals diffract x-rays.

xi) Production of x-rays is converse of photoelectric effect.

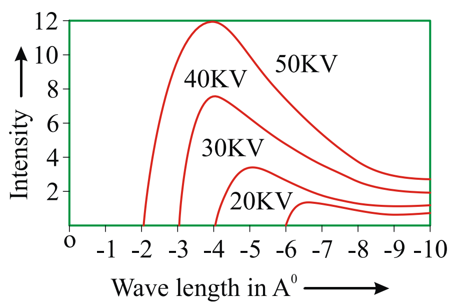
X-Ray spectrum

i) Continuous X-ray spectrum:

a) It is produced when high speed electrons are suddenly stopped by a metal target.

b) It contains all wave lengths above a minimum wavelength λ_m. ($\therefore$ continuous spectrum) For a given accelerating potential, λ_m is called cut off wave length.

c) Properties of continuous x - rays spectra are independent of nature of target metal and they depend only on accelerating potential.



d) $$\boxed{\lambda_{min} = \frac{hc}{eV} = \frac{12400}{V} A^0}$$

$\therefore \lambda_{min} \alpha \frac{1}{V}$ it is Duane and Hunt's law

e) Maximum frequency of emitted x - ray photon is $\boxed{\nu_{\max} = \frac{ev}{h}}$

f) In this spectrum intensity first increases, reaches a maximum value I_{max} and then decreases.

g) Every spetrum starts with certian minimum wave length called limiting wave length or cut off wave length $\lambda_{\min}$.

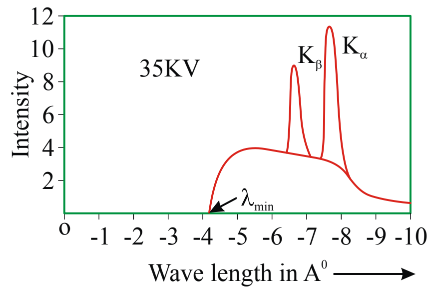
h) With the increase in target potential, $\lambda_{\min}$ and wavelength corresponding to maximum intensity λ_0 shifts towards minimum wavelength side.

i) At a given potential the range of wave length of continous x - rays produced is $\lambda_{\min}$ to ∞.

j) Efficiency of x - ray tube $\eta = \frac{out\ put\ power}{input\ power} x100$

input power P = VI. Where V is P.D applied to x - ray tube I = anode current

ii) Characterstic X-ray spectrum:

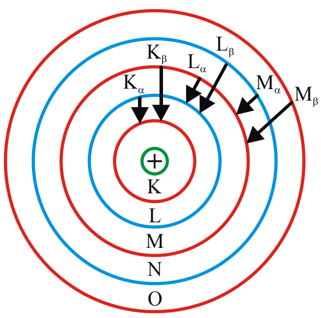


a) Produced due to transition of electrons from higher energy level to lower energy level in target atoms

b) Wavelengths of these x-rays depend only on atomic number of the target element and independent of target potential.

c) Characteristic x-rays of an element consists of K, L,M and N series.

d) K-series of lines are obtained when transition takes place from higher levels to k shell



e) This spectrum is useful in identifying the elements by which they are produced.

f) Relation among the energies $E_{k\alpha} < E_{k\beta} < E_{k\gamma}$, $E_{k\alpha} > E_{L\alpha}$

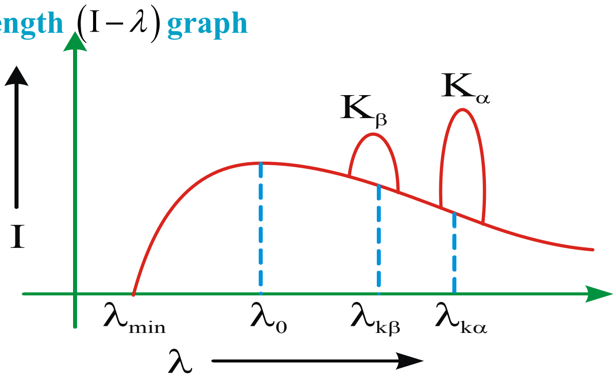
g) Intensity of x - rays $I_{k\alpha} > I_{k\beta} > I_{k\gamma}$

h) Relation among frequences $\nu_{k\alpha}, \nu_{k\beta}$ and $\nu_{L\alpha}$ is $\nu_{k\beta} = \nu_{k\alpha} + \nu_{L\alpha} \Rightarrow \frac{1}{\lambda_{K\beta}} = \frac{1}{\lambda_{K\alpha}} + \frac{1}{\lambda_{L\alpha}}$

h) $E_K - E_L = h\nu_{K\alpha} = \dfrac{hc}{\lambda_{K\alpha}}$ $E_K - E_M = h\nu_{K\beta} = \dfrac{he}{\lambda_{K\beta}}$

$E_L - E_M = h\nu_{L\alpha} = \dfrac{he}{\lambda_{K\alpha}}$

iii) Intensity and wavelength $(I-\lambda)$ graph



As target potential V is increased

a) $(\lambda_0 - \lambda_{min})$ decreases

b) Wavelength of k_α remains constant.

c) diffrence between λ_{k_α} and λ_{min} increases

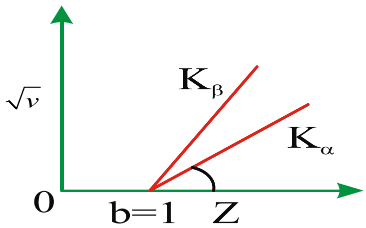
d) diffrence between λ_{k_α} line and λ_{k_β} line remains constant.

e) Difference between $\lambda_{k\alpha} - \lambda_0$ increases.

Moseley's Law

i) "The square root of frequency (ν) of the spectral line of the characteristic x-rays spectrum is directly proportional to the atomic number(z) of the target element.

$\sqrt{\nu} \propto Z$ or $\sqrt{\nu} = a(Z-b)$



ii) The slope(a) of $\sqrt{\nu}$ -Z curve varies from series to series and also from line to line of a given series.

For K series $\sqrt{\dfrac{\nu_1}{\nu_2}} = \left(\dfrac{Z_1 - 1}{Z_2 - 1}\right) \Rightarrow \sqrt{\dfrac{\lambda_2}{\lambda_1}} = \left(\dfrac{Z_1 - 1}{Z_2 - 1}\right)$

iii) $a_{k\gamma} > a_{k\beta} > a_{k\alpha}$

iv) The intercept on 'Z' axis gives the screening constant 'b' and it is constant for all spectral lines in given series but varies with the series.

b = 1 for k series $(k_\alpha, k_\beta, k_\gamma)$

b = 7.4 for L series

v) The wavelength of characteristic X-rays is given by $\frac{1}{\lambda}$=R(Z-b)^2 $\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

vi) Ratio of k_α and k_β lines from a given target is $\frac{\lambda_{k\alpha}}{\lambda_{k\beta}}=\frac{32}{27}$

vii) **Significance :**

a) The elements must be arranged in the periodic table as per their atomic numbers but not on their atomic weights.

b) Helped to discover new elements like masurium (43) and illinium (61) etc.

c) Decided the positions and atomic numbers of rare earth metals.

Ex-13 Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of emitted X-rays is

A) $\lambda_0=\frac{2mc\lambda^2}{h}$ **B)** $\lambda_0=\frac{2h}{mc}$

C) $\lambda_0=\frac{2m^2c^2\lambda^3}{h^2}$ **D)** $\lambda_0=\lambda$

Sol. $\frac{hc}{\lambda_0}=KE_e=\frac{P_e^2}{2m_e}=\frac{(h/\lambda)^2}{2m_e}=\frac{h^2}{2m\lambda^2}$

$\Rightarrow \lambda_0=\frac{2mC\lambda^2}{h}$

Ex-14 In coolidge tube the potential difference of electron gun is increased from 12.4 KV to 24.8 KV. As a result the value of $\left|\lambda_{K_\alpha}-\lambda_C\right|$ increase two fold. The wave length of K_α line is (λ_C = cut off wave length)

A) 1A^0 B) 0.5A^0 C) 1.5A^0 D) 1.25A^0

Sol. conceptual

Ex-15 Wavelength the K_α X-ray of on element A is λ_1 and wavelength of K_α X-ray element B is $\lambda_2 . \frac{\lambda_1}{\lambda_2}$ is equal to $\frac{1}{4}$ and Z_1 & Z_2 are atoms number of A and B respectively then

A) $2Z_2-Z_1=1$ **B)** $Z_2-2Z_1=1$

C) $\frac{Z_2}{Z_1}=4$ **D)** $\frac{Z_1}{Z_2}=4$

Sol. $\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}}=\frac{Z_1-1}{Z_2-1}$

Ex-16 An X-ray tube produces a continuous spectrum of radiation with its short-wavelength end at 0.45A^0. What is the maximum energy of a photon in the radiation? (b) From your answer to (a), guess what order of accelerating voltage (for electrons) is required in such a tube?

Sol. a) $\lambda_{\min} = 0.45A^0$

$$E_{\max} = h\nu_{\max} = \frac{hc}{\lambda_{\min}} \qquad = \frac{12431}{0.45} = 27624.44eV = 27.624KeV$$

b) The minimum accelerating voltage for electrons is $\dfrac{27.6keV}{e} = 27.6kV$

i.e. of the order of 30 kV

Ex-17 The wavelength of the characteristics X-ray K_α line emitted from zinc (Z=30) is 1.415A^0. Find the wavelength of the K_α line emitted from molybdenum (Z=42).

Sol. According to Moseley's law, the frequency for K series is given by

$$\nu \propto (Z-1)^2$$

or $\dfrac{c}{\lambda} \propto (Z-1)^2$ or $\dfrac{1}{\lambda} = k(Z-1)^2$ ...(3.6)

Where k is a constant. Let λ' be the wavelength of K_α line emitted from molybdenum, then

$$\frac{1}{\lambda'} = k(Z-1)^2 \qquad ...(3.7)$$

Dividing (3.6) and (3.7) we get

$$\lambda' = \left(\frac{Z-1}{Z'-1}\right)^2 \lambda = \left(\frac{30-1}{42-1}\right)^2 \times 1.415A^0 = 0.708A^0$$

Ex-18 If the short series limit of the Balmer series for hydrogen is 3644A^0, find the atomic number of the element which give X-ray wavelengths down to 1A^0. Identify the element

Sol. If short series limit of the Balmer series is corresponding to transition $n=\infty$ to $n=2$ which is given by

$\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{\infty^2}\right) = \dfrac{R}{4}$ or $R = \dfrac{4}{\lambda} = \dfrac{4}{3644}\left(A^0\right)^{-1}$

The shortest wavelength corresponds to $n=\infty$ to n=1. Therefore λ_c is given as

$$\frac{1}{\lambda_c} = R(Z-1)^2\left[\frac{1}{1^2} - \frac{1}{\infty^2}\right] \text{ or}$$

$$(Z-1)^2 = \frac{1}{\lambda_c R} = \frac{1}{1A^0 \times \frac{4}{3644}\left(A^0\right)^{-1}} = \frac{3644}{4} = 911$$

or Z-1 = 30.2 or Z=31.2; 31

Thus the atomic number of the element is 31 which is gallium.

Ex-19 A material whose K absorption edge is 0.2A⁰ is irradiated by X-rays of wavelength 0.15A⁰. Find the maximum energy of the photoelectrons that are emitted from the K shell.

Sol. The binding energy for k shell in eV is

$$E_k = \frac{hc}{\lambda_k} = \frac{12431}{0.2} eV = 62.155 KeV$$

The energy of the incident photon in eV is

$$E = \frac{hc}{\lambda} = \frac{12431}{0.15} = 82.873 KeV$$

Therefore, the maximum energy of the photoelectrons emitted from the K shell is

$$E_{max} = E - E_k = 82.873 - 62.155 KeV$$

= 20.718 KeV

Ex-20 The wavelength of K_α X-rays produced by a X-ray tube is 0.76A⁰. What is the atomic number of the anode material of the tube?

Sol. K_α X-rays are produced when an electron makes a transition from n=2 to n=1 to fill a vacancy in K-shell. The wavelength of X-ray lines is given by

$$\frac{1}{\lambda_{K_\alpha}} = (Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4}R(Z-1)^2$$

or $(Z-1)^2 = \frac{4}{3R\lambda_{K_\alpha}} \quad = \frac{4}{3\times(1.097\times10^7)\times(0.76\times10^{-10})} = 1599.25$

or $(Z-1)^2=1600$ or Z-1=40 or Z=41

Ex-21 The K-absorption edge of an unknown element is 0.171A⁰

a) Identify the element

b) Find the average wavelengths of the K_α, K_β & K_λ lines.

c) If a 100 eV electron strike the target of this element, what is the minimum wavelength of the X-ray emitted?

Sol. From Moseley's law, the wavelength of k series of X-rays is given by taking $\sigma = 1$ in modified in rydberg's formula given as

$$\frac{1}{\lambda} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right)$$ for K lines where, n=2,3,4,...

a) For K-absorption edge, we put $n = \infty$, in above expression gives

$$(Z-1) = \sqrt{\frac{1}{\lambda R}} \qquad \text{or } Z = \sqrt{\frac{1}{(0.171\times10^{-10})(1.097\times10^7)}} + 1 = 74$$

The element is Tungsten

b) For K_α line : $\frac{1}{\lambda_{K_\alpha}} = R(74-1)^2\left[1-\frac{1}{2^2}\right]$

$\lambda_{K_\alpha} = 0.228\overset{0}{A}$

For K_β line : $\frac{1}{\lambda_{K_\beta}} = R(74-1)^2\left[1-\frac{1}{3^2}\right]$

$\lambda_{K_\beta} = 0.192\overset{0}{A}$

For K_γ line : $\frac{1}{\lambda_{K\gamma}} = R(74-1)^2\left[1-\frac{1}{4^2}\right]$

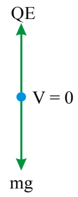
$\lambda_{K_\gamma} = 0.182\overset{0}{A}$

c) The shortest wavelength corresponding to an electron with kinetic energy 100 eV is given by

$\lambda_c = \frac{hc}{E} = \frac{12431}{100}\overset{0}{A} = 124.31\overset{0}{A}$

Ex-22 IIn Millikan's oil drop experiment an oil drop of radius r and charge Q is held in equilibrium between the plates of a charged parallel plate capacitor when the potential difference is V. To keep another drop of same oil whose radius is 2r and carrying charge 2Q in equilibrium between the plates, find the new potential difference required.

Sol. Since drop is at rest QE = mg



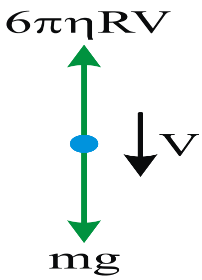
$Q\frac{V}{d} = \frac{4\pi}{3}r^3\rho g$; $V \propto \frac{r^3}{Q}$; $\frac{V_2}{V_1} = \left(\frac{r_2}{r_1}\right)^3 \frac{Q_1}{Q_2}$

$\frac{V_2}{V} = 8\times\frac{1}{2}$; $\therefore V_2 = 4V$

Ex-23 A charged oil drop of charge q is falling under gravity with terminal velocity v in the absence of electric field. A electric field can keep the oil drop stationary. If the drop acquires an additional charge, it moves up with velocity 3v in that field. Find the new charge on the drop.

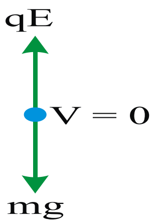
Sol. In the absence of electric field

$mg = 6\pi\eta RV \rightarrow \quad (1)$



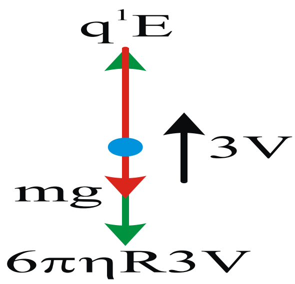
when electric field is applied

$mg = qE \rightarrow \quad (2)$



If q^1 = new charge and drop is moving up then

$q^1E = mg + 3 \times 6\pi\eta RV \rightarrow \quad (3)$



$q^1E = 4mg = 4qE \qquad \therefore q^1 = 4q$

Ex-24 In Millikan's method of determining the charge of an electron, the terminal velocities of oil drop in the presence and in the absence of an electric field are xcm/s upwards and y cm/s downwards respectively. Find the ratio of electric force to gravitational force on the oil drop. (Neglect Buoyancy)

Sol. In Gravitational field, weight = viscous force

$W = 6\pi\eta ry........(1)$

In electric field,

Electric force = weight + viscous force

$Eq = W + 6\pi\eta rx \text{ ....}(2)$

Substitute (1) in (2) then $Eq = 6\pi\eta r(y + x) \text{ ....}(3)$

$$\frac{\text{Electric force}}{\text{Gravitational force}} = \frac{x+y}{y}$$

Ex-25 **In a Millikan's experiment an oil drop of radius 1.5 x 10^{-6} m and density 890 kg/m^3 is held stationary between two condenser plates 1.2 cm apart and kept at a p.d of 2.3 kV. If upthrust due to air is ignored, then the number of excess electrons carries by the drop will be**

Sol Eq = mg; $\frac{V}{d}.ne = \frac{4}{3}\pi r^3.\rho g$; $n = \frac{4}{3}\pi \times r^3 \rho g \frac{d}{Ve}$

$$n = \frac{4}{3} \times \frac{\pi \times 3.375 \times 10^{-18} \times 890 \times 9.8 \times 1.2 \times 10^{-2}}{2.3 \times 10^3 \times 1.6 \times 10^{-19}}$$

$n ;\ 4$

Ex-25 **A charged oil drop is of charge q is falling freely under gravity in the absence of electric field with a velocity 'v'. It is held stationary in an electric field, as it acquires a charge it moves up with a velocity '3v. Now the charge on the drop is**

Sol $mg = 6\pi\eta r\upsilon$, Eq = mg

$Eq^1 = mg + 6\pi\eta r(3\upsilon)$, $Eq^1 = Eq + 3Eq$

$Eq^1 = 4Eq \Rightarrow q^1 = 4q$

Ex-26 **A charged oil drop falls with a terminal velocity V in the absence of electric field. An electric field E keeps the oil drop stationary in it. When the drop acquires a charge 'q' it moves up with same velocity. Find the initial charge on the drop.**

Sol $mg = 6\pi\eta r\upsilon$,Eq = mg

$E(Q+q) = mg + 6\pi\eta r\upsilon$,$E(Q+q) = 2EQ$

$Q + q = 2Q \Rightarrow q = Q$

Ex-27 **Two oil drops in Millikan's experiment are falling with terminal velocities in the ratio 1:4. The ratio of their de-Broglie wave length is**

Sol $\upsilon_T \propto r^2$ $\qquad r \propto \sqrt{V_T}$

$\frac{r_1}{r_2} = \sqrt{\frac{V_1}{V_2}} = \frac{1}{2}$ $\qquad \frac{m_1}{m_2} = \frac{1}{8}$

$\therefore \frac{\lambda_1}{\lambda_2} = \frac{m_2}{m_1}.\frac{\upsilon_2}{\upsilon_1}$ $\qquad = \frac{8}{1}.\frac{4}{1} = 32 : 1$

Ex-28 $\alpha -$**particles are projected towards the nuclei of the different metals, with the same kinetic energy. The distance of closest approach is minimum for**

1) Cu(Z=29) 2) Ag(Z=47)
3) Au(Z=79) 4) Pd(Z=46)

Sol.. $r = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{(KE)} \Rightarrow r \alpha q_2$

Ex-29: In Rutherford experments on $\alpha-$ray scattering the number of particles scattered at 90^0 be 28 per minute. Then the number of particles scattered per minute by the same foil but at 60^0 are

1) 56 2) 112 3) 60 4) 120

Sol..No. of particles scattered at angle θ is $N\alpha \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$

Ex-30 For a given impact parameter (b), if the energy increase then the scattering angle (θ) will

1) Decrease 2) increase

3) become zero 4) become

Sol.. $N\alpha \frac{1}{\sin^{-4}\left(\frac{\theta}{2}\right)}$

Ex-31 Find the frequency of revolution of the electron in the first stationary orbit of H-atom

1) $6\times10^{14}Hz$ 2) $6.6\times10^{10}Hz$ 3) $6.6\times10^{-10}Hz$ 4) $6.6\times10^{15}Hz$

Sol.. $T=\frac{2\pi r}{V}$ where $V=\frac{3\times10^8}{137}$; $r=0.53\times10^{-10}m$

Ex-32 Let the potential energy of a hydrogen atom in the ground state be zero. Then its energy in the first excited state will be

1) 10.2eV 2) 13.6eV 3) 23.8eV 4) 27.2eV

Sol.. $P.E_2=-\frac{27.2}{4}+27.2=20.4$ eV

$K.E_1=13.6eV,\quad K.E_2=3.4eV$

$\therefore T.e_2=20.4+3.4\quad=23.8eV$

Ex-33 According to bohr model, the diameter of first orbit of hydrogen atom will be

1) $1.A^0$ 2) $0.529A^0$ 3) $2.25A^0$ 4) $0.725A^0$

Sol..Diameter $=2r_0$ =2 x 0.529 =1.058 $\overset{0}{A}$

Ex-34 The energy required to excite an electron from n=2 to n=3 energy state is 47.2 e V. The charge number of the nucleus, around which the electron revolving will be

1) 5 2) 10 3)15 4) 20

Sol. . $\Delta E = E_3 - E_2 = \frac{13.6Z^2}{4} - \frac{13.6Z^2}{9} = 47.2$

$\Rightarrow Z = ?$

Ex-35 An orbital electron in the ground state of hydrogen has the magnetic moment μ_1. This orbital electron is excited to 3rd excited state by some energy transfer to the hydrogen atom. The new magnetic moment of the electron is μ_2, then

1) $\mu_1 = 2\mu_2$ 2) $2\mu_1 = \mu_2$ 3) $16\mu_1 = \mu_2$ 4) $4\mu_1 = \mu_2$

Sol. . $\mu = niA = iA$, $= \frac{qv}{2\pi r} \times \pi r^2$

$M \alpha vr$ where $v \alpha \frac{z}{n}$ & $r \alpha \frac{n^2}{z}$ $\therefore \mu \alpha n$

Ex-36 When an electron in the hydrogen atom in ground state absorbs a photon of energy 12.1 eV, its angular momenturm

1) decreases by $2.11 \times 10^{-34} J-s$

2) decreases by $1.055 \times 10^{-34} J-s$

3) increases by $2.11 \times 10^{-34} J-s$

4) increases by $1.055 \times 10^{-34} J-s$

Sol. . After absorbing a photon of energy 12.1 eV electron jumps from ground state (n=1) to second excited state (n=3).Therefore change in angular momentum $\Delta L = L_3 - L_1$

$= 3\left(\frac{h}{2\pi}\right) - \frac{h}{2\pi} = \frac{h}{\pi}$

$= \frac{6.6 \times 10^{-34}}{3.14} J-s = 2.11 \times 10^{-34} J-s$

Ex-37 A neutron moving with a speed v makes a head on collision with a hydrogen atom in ground state kept at rest. The minimum kinetic energy of neutron for which inelastic collision will take place is (assume that mass of proton is nearly equal to the mass of neutron)

1) 10.2 eV 2) 20.4eV 3) 12.1eV 4) 16.8ev

Sol. . Let v=speed of neutron before collision

v_1= speed of neutron after collision

v_2= speed of proton or hydrogen atom after collision and ΔE=energy of exitation.

From conservation of linear momentum

$mv = mv_1 + mv_2$ ....(1)

From conservation of energy

$$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \qquad ....(2)$$

from 1 and 2 As $v_1 - v_2$ must be real

$$v^2 - 4\frac{\Delta E}{m} \geq 0$$

$$\Rightarrow \frac{1}{2}mv^2 = 2\Delta E = 2\times 10.2 = 20.4\,eV$$

Ex-38. An electron in hydrogen atom after absorbing an energy photon jumps from energy state n_1 to n_2. Then it returns to ground state after emitting six different wavelengths in emission spectrum. The energy of emitted photons is either equal to, less than or greater than the absorbed photons. Then n_1 and n_2 are

1) $n_2 = 4, n_1 = 3$ 2) $n_2 = 5, n_2 = 3$

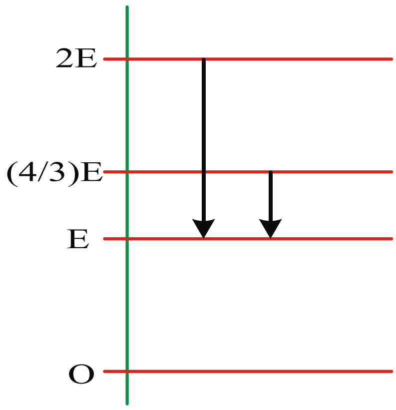
3) $n_2 = 4, n_1 = 2$ 4) $n_2 = 4, n_1 = 1$

Sol. . From $n_2 = 4$, six lines are obtained in emission spectrum. Now: $E_{4-2} = E_{absorbed}$

$E_{4\to 3} < E_{absorbed}$ and $E_{4-1}E_{3-1}, E_{2-1} > E_{absorbed}$

Hence, $n_1 = 2$ and $n_2 = 4$

Ex-39. The figure shows energy levels of a certain atom, when the system moves from level 2E to E, a photon of wavelength λ is emitted. The wavelength of photon produced during its transition from level 4/3 E to E level is:



1) 3λ 2) $3/4\,\lambda$ 3) $\lambda/4$ 4) 2λ

Sol. . $\frac{hc}{\lambda_1} = 2E - E = E \to 1$; $\frac{hc}{\lambda_2} = \frac{4}{3}E - E = \frac{E}{3} \to 2$

$$\frac{\lambda_2}{\lambda_1} = \frac{E}{\frac{E}{3}} = \frac{3}{1}$$

Ex-40 **When the electron in hydrogen atom jumps from the second orbit to the first orbit, the wavelength of the radiation emitted is λ. When the electron jumps from the third to the first orbit, the wavelength of the radiation emitted is**

1) $\frac{9}{4}\lambda$ 2) $\frac{4}{9}\lambda$ 3) $\frac{27}{32}\lambda$ 4) $\frac{32}{27}\lambda$

Sol. . $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

Ex-**41. An electron in a hydrogen atom makes a transition $n_1 \rightarrow n_2$, where n_1 and n_2 are principal quantum numbers of the states. Assume the Bohr's model to be valid. The time period of the elctron in the initial states is eight times to that of final state. What is ratio of $\frac{n_1}{n_2}$**

1) 8:1 2) 4:1 3) 2:1 4) 1:2

Sol. . Time period of revolution of electron in the nth orbit is $T \alpha \ n^3$

Ex-42. Let ϑ_1 be the frequency of the series limit of the Lyman series and ϑ_2 be the frequency of the first line of the Lyman series and ϑ_3 be the frequency of the series limit of Balmar series, then

1) $\vartheta_1 - \vartheta_2 = \vartheta_3$ 2) $\vartheta_2 - \vartheta_1 = \vartheta_3$
3) $2\vartheta_3 = \vartheta_1 + \vartheta_2$ 4) $\vartheta_1 + \vartheta_2 = \vartheta_3$

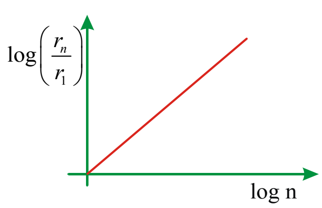
Sol. .$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\alpha^2}\right)$ series limit for lyman series

$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ series for lyman first line

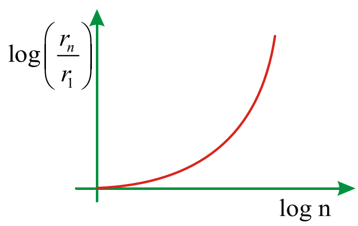
$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\alpha^2}\right)$ series limit for balmer series

Ex-43. In hydrogen atom, the radius of n^{th} Bohr orbit is V_n. The graph between $\log\left(\frac{r_n}{r_1}\right)$ and $\log n$ will be

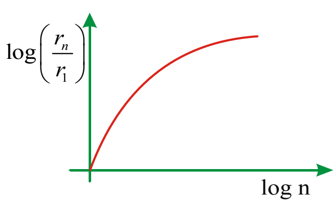
1)



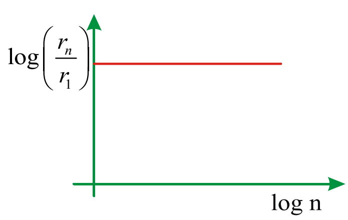
2)



3)



4)



Sol. . $r_n \propto n^2$

Ex-44. An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (In eV) required to remove both the electrons from a neutral helium atom is : [JEE'95, 01]

1) 38.2 2) 49.2 3) 51.8 4) 79.0

Sol. . After removing one electron He^+ becomes hydrogen like and energy required to remove the second electron

$$= 0 - \left(\frac{-13.6 \times Z^2}{n^2}\right) = 13.6 \times 4 = 54.4 eV$$

$\therefore$ energy required to remove both the electrons $= 24.6 + 54.4 = 79.0 eV$

Ex-45. The frequency of the first line in Lyman series in the hydrogen spectrum is n. What is the frequency of the corresponding line in the spectrum of doubly ionized Lithium ?

1) n 2) 3 n 3) 9 n 4) 27 n

Sol. . $E = 13.6\left(\frac{Z^2}{n^2}\right)$

$$\Delta E_H = \frac{13.6(1)^2}{(1)^2} - \frac{13.6(1)^2}{(2)^2} = 10.2 \text{ eV} = h\nu$$

$$\Delta E_{Li} = \frac{13.6(3)^2}{(1)^2} - \frac{13.6(3)^2}{(2)^2} = 91.80\text{eV} = h(9\nu)$$

Ex-46. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which following statements is true?

1) Its kinetic energy increases and its potential and total energies decrease

2) Its kinetic energy decreases, potential energy increases and its total energy remains the same

3) Its kinetic and total energies decrease and its potential energy increases

4) Its kinetic, potential and total energies decrease

Sol. . Kinetic energy (K.E.) = $\frac{13.6\, z^2}{n^2}$ eV

Potential (P.E.) = $\frac{-2(13.6)\, z^2}{n^2}$ eV

Total energy (T.E.) = $\frac{-13.6\, z^2}{n^2}$ eV

When an electron in H–atom makes a transition from an excited state to the ground state, value of n decreases, hence K.E. increases and its P.E. & T.E. decrease.

Ex-47. Three photons coming from excited atomic-hydrogen sample are picked up. Their energies are 12.1eV, 10.2eV and 1.9eV. These photons must come from

1) a single atom
2) two atoms
3) three atoms
4) either two atoms or three atoms

Sol. $12.1 = E(n = 3) - E(n = 1)$

$10.2 = E(n = 2) - E(n = 1)$

$1.9 = E(n = 3) - E(n = 2)$

At least two atoms must be involved as there can not be two transitions from same level from same atom.

Ex-48. The ionization energy of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by electromagnetic radiation of energy 12.1 eV. How many spectral lines will be emitted by the hydrogen atoms?

1) one 2) two 3) three 4) four

Sol. . 12.1 eV radiation will excite a hydrogen atom in ground state to n = 3 state.

$\therefore$ number of possible transitions = ${}^nC_1 = {}^3C_1 = 3$.

Ex-49. The wavelength of the first line in balmer series in the hydrogen spectrum is λ. What is the wavelength of the second line :

1) $\frac{20\lambda}{27}$ 2) $\frac{3\lambda}{16}$ 3) $\frac{5\lambda}{36}$ 4) $\frac{3\lambda}{4}$

Sol. . $\frac{1}{\lambda_1} = R\left(\frac{1}{4} - \frac{1}{9}\right) \Rightarrow \lambda_1 = \frac{4 \times 9}{5R}$

similarly $\frac{1}{\lambda_2} = R\left(\frac{1}{4} - \frac{1}{4^2}\right)$

$\Rightarrow \lambda_2 = \frac{16}{3R} = \frac{16}{3} \times \frac{5\lambda}{4 \times 9} = \frac{20}{27}\lambda$

Ex-50. An electron with kinetic energy 5 eV is incident on a hydrogen atom in its ground state. The collision

1) must be elastic
2) may be partially elastic
3) must be completely inelastic
4) may be completely inelastic

Sol. . 5eV electron cannot excite a hydrogen atom and hence there will be no loss of K.E.and hence the collision is elastic.

Atoms

(Jee main previous year questions)

Topic 1: Atomic Structure and Rutherford's Nuclear Model

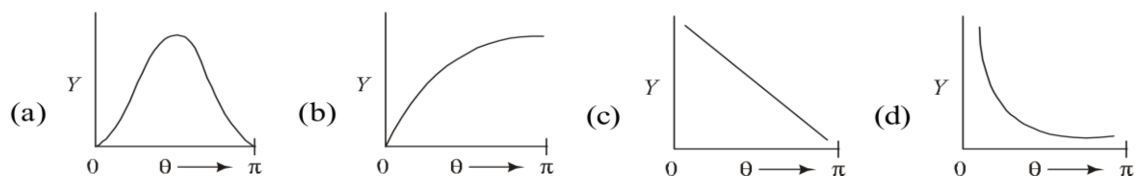
1. The graph which depicts the results of Rutherford gold foil experiment with α-particles is:

θ: Scattering angle

Y: Number of scattered α-particles detected

(Plots are schematic and not to scale)

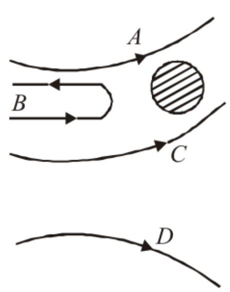
[8 Jan. 2020 I]



SOL. (c) conceptual

2. In the Rutherford experiment, α-particles are scattered from a nucleus as shown. Out of the four paths, which path is not possible?

[Online May 7, 2012]



(a) *D* **(b) *B*** **(c) *C*** **(d) *A***

SOL. (c) As α-particles are doubly ionized helium He^{++} i.e. positively charged and nucleus is also positively charged and we know that like charges repel each other.

3. An alpha nucleus of energy $\frac{1}{2}mv^2$ bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to

[2006]

(a) v^2 **(b) $\frac{1}{m}$** **(c) $\frac{1}{v^2}$** **(d) $\frac{1}{Ze}$**

SOL. (b) Work done to stop the α particle is equal to K.E.

$$qV = \frac{1}{2}mv^2 \Rightarrow q \times \frac{K(Ze)}{r} = \frac{1}{2}mv^2$$

$$\Rightarrow r = \frac{2(2e)K(Ze)}{mv^2} = \frac{4KZe^2}{mv^2}$$

$$\Rightarrow r \propto \frac{1}{v^2} \text{ and } r \propto \frac{1}{m}.$$

4. An α-particle of energy 5 MeV is scattered through 180^o by a fixed uranium nucleus. The distance of closest approach is of the order of

[2004]

(a) 10^{-12} cm **(b) 10^{-10} cm** **(c) 10^{-14} cm** **(d) 10^{-15} cm**

SOL. (a) Distance of closest approach

$$r_0 = \frac{Ze(2e)}{4\pi\varepsilon_0\left(\frac{1}{2}mv^2\right)}$$

Energy, $E = 5 \times 10^6 \times 1.6 \times 10^{-19}$J

$$r_0 = \frac{9 \times 10^9 \times (92 \times 1.6 \times 10^{-19})(2 \times 1.6 \times 10^{-19})}{5 \times 10^6 \times 1.6 \times 10^{-19}}$$

$\Rightarrow r_h = 5.2 \times 10^{-14} m = 5.3 \times 10^{-12}$ cm

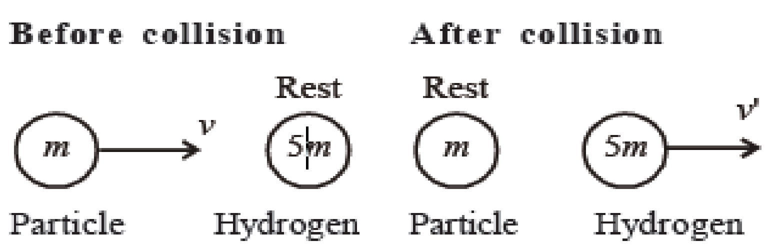
Topic 2: Bohr's Model and the Spectra of the Hydrogen Atom

5. A particle of mass 200 $\mathrm{MeV/c^2}$ collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first excited state. The initial kinetic energy of the particle (in eV) is $\frac{N}{4}$. The value of N is:

[NA Sep. 05, 2020 (I)]

(Given the mass of the hydrogen atom to be 1 $\mathrm{GeV/c^2}$)

SOL. (51)



From linear momentum conservation, $L_i = L_f$

$mV + 0 = 0 + 5mV^{\uparrow} \Rightarrow V' = \frac{v}{5}$

Loss of KE $= KE_i - KE_f = \frac{1}{2}mv^2 - \frac{1}{2}(5m)\left(\frac{v}{5}\right)^2$

$$= \frac{1}{2}mv^2\left(1 - \frac{1}{5}\right) = \frac{4}{5}\left(\frac{mv^2}{2}\right)$$

$$= \frac{4}{5}KE_i = 10.2\text{eV}$$

[Energy in first excited state of atom $= 10.2$eV]

$$KE_i = 12.75\text{eV} = \frac{\text{N}}{4} \Rightarrow N = 51$$

The value of $N = 51$.

6. In the line spectra of hydrogen atom, difference between the largest and the shortest wavelengths of the Lyman series is 304 A^0. The corresponding difference for the Paschan series in A^0 is:

[NA Sep. 04, 2020 (I)]

SOL. (10553.14)

From Bohr's formula for hydrogen atom,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$R = 1.097 \times 10^7 \text{m}^{-1}$$

For Lyman series:

$$\frac{1}{\lambda_{\min}} = R(1) = R \qquad \text{since, } n_2 = \infty \text{ and } n_1 = 1$$

$$\frac{1}{\lambda_{\max}} = R(1 - \frac{1}{4}) = \frac{3R}{4} \qquad \text{since, } n_1 = 2, n_1 = 1$$

$$\lambda_{\max.} - \lambda_{\min.} = \frac{4}{3R} - \frac{1}{R} = \frac{1}{3R} = 304 \text{ (Given)}$$

For Paschen series:

$$\lambda_{\min.} = R\left(\frac{1}{9}\right) \qquad \text{and } \lambda_{\max.} = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{16\times 9}$$

$$\lambda_{\max.} - \lambda_{\min.} = \frac{16\times 9}{7R} - \frac{9}{R} = \frac{81}{7R}$$

$$\text{or, } \lambda_{\max.} - \lambda_{\min.} = \frac{81}{7R} = \frac{81\times 3}{7\times 3R} = \frac{81\times 3}{7} \times 304$$

$$\left(\because \frac{1}{3R} = 304\text{A}\right)$$

For Paschen series, $\lambda_{max} - \lambda_{min} = 10553.14$

7. In a hydrogen atom the electron makes a transition from $(n+1)^{\text{th}}$ level to the n^{th} level.

If >> 1 , the frequency of radiation emitted is proportional to:

[Sep. 02, 2020 (II)]

(a) $\frac{1}{n}$ **(b)** $\frac{1}{n^3}$ **(c)** $\frac{1}{n^2}$ **(d)** $\frac{1}{n^4}$

SOL. (b) Total energy of electron in n^{th} orbit of hydrogen atom

$$E_n = -\frac{Rhc}{n^2}$$

Total energy of electron in $(n+1)^{\text{th}}$ level of hydrogen atom

$$E_{n+1} = -\frac{Rhc}{(n+1)^2}$$

When electron makes a transition from $(n+1)^{\text{th}}$ level to n^{th} level

Change in energy,

$$\Delta E = E_{n+1} - E_n$$

$h\text{v} = Rhc \cdot \left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$ $(\because E = h\text{v})$

$$\text{v} = R \cdot c\left[\frac{(n+1)^2 - n^2}{n^2(n+1)^2}\right]$$

$$\text{v} = R \cdot c\left[\frac{1+2n}{n^2(n+1)^2}\right]$$

For $n >> 1$

$$\Rightarrow \text{v} = R \cdot c\left[\frac{2n}{n^2 \times n^2}\right] = \frac{2RC}{n^3}$$

$$\Rightarrow \text{v} \propto \frac{1}{n^3}$$

8. The energy required to ionize a hydrogen like ion in its ground state is 9 Rydberg. What is the wavelength of the radiation emitted when the electron in this ion jumps from the second excited state to the ground state?

[9 Jan. 2020 II]

(a) 24.2 nm **(b) 11.4 nm** **(c) 35.8nm** **(d) 8.6nm**

SOL. (b) According to Bohr's Theory the wavelength of the radiation emitted from hydrogen atom is given by

$$\frac{1}{\lambda} = RZ^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$Z = 3$$

$$\frac{1}{\lambda} = 9R\left[1 - \frac{1}{9}\right]$$

$$\Rightarrow \lambda = \frac{1}{8R} = \frac{1}{8\times 10973731.6} \ (R = 10973731.6m - 1)$$

$$\Rightarrow \lambda = 11.39 \text{ nm}$$

9. The first member of the Balmer series of hydrogen atom has a wavelength of 6561 A^0. The wavelength of the second member of the Balmer series (in nm) is .

` **[NA 8 Jan. 2020 II]**

SOL. (486.00)

The wavelength of the spectral line of hydrogen spectrum is given by formula

$$\frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

Where, R = Rydberg constant

For the first member of Balmer series $n_f = 2, n_i = 3$

$\frac{1}{\lambda} = \mathrm{R}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$ --------(i)

For last member of Balmer series, $n_f = 2, n_i = 4$

So, $\frac{1}{\lambda^1} = \mathrm{R}\left[\frac{1}{4} - \frac{1}{16}\right]$ --------(ii)

Dividing (i) by (ii), we get

$$\Rightarrow \frac{\lambda'}{\lambda} = \frac{5 \times 16}{9 \times 4 \times 3}$$

$$\Rightarrow \lambda' = \frac{5 \times 4 \times 656.1}{9 \times 3}(nm) = 486nm$$

10. The time period of revolution of electron in its ground state orbit in a hydrogen atom is 1.6×10^{-16}s. The frequency of revolution of the electron in its first excited state (in s^1) is:

[7 Jan. 2020 I]

(a) 1.6×10^{14} **(b) 7.8×10^{14}** **(c) 6.2×10^{15}** **(d) 5.6×10^{12}**

SOL. (b) For first excited state $\mathrm{n}' = 3$

Time period $T \propto \frac{n^3}{z^2}$

$$\Rightarrow \frac{T_2}{T_1} = \frac{n'^3}{n^3}$$

$\mathrm{T}_2 = 8\mathrm{T}_1 = 8 \times 1.6 \times 10^{-16}\mathrm{s}$

Frequency, $v = \frac{1}{T_2} = \frac{1}{8 \times 1.6 \times 10^{-16}} \approx 7.8 \times 10^{14}$ Hz

11. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n, corresponding to its initial excited state is (for photon of wavelength)

Energy E $= \frac{1240eV}{\lambda(innm)}$

[12 April 2019 II]

(a) $n = 4$ (b) $n = 5$ (c) $n = 7$ (d) $n = 6$

SOL. (b) $E = E_1 + E_2$

$$13.6 \frac{z^2}{n^2} = \frac{1240}{\lambda_1} + \frac{1240}{\lambda_2}$$

or $\frac{13.6(2)^2}{n^2} = 1240\left(\frac{1}{108.5} + \frac{1}{30.4}\right) \times \frac{1}{10^{-9}}$

On solving, $n = 5$

12. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths, λ_1/λ_2, of the photons emitted in this process is:

[12 Apri12019 II]

(a) 20/7 (b) 27/5 (c) 7/5 (d) 9/7

SOL. (a) $\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{16\times 9}$

And $\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$

Now $\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{5R}{36}\right)}{\frac{7R}{(16\times 9)}} = \frac{20}{7}$

13. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius $4.65\ A^0$). The de-Broglie wavelength of this electron is:

[12 April 2019 II]

(a) $3.5\ A^0$ (b) $6.6\ A^0$ (c) $12.9\ A^0$ (d) $9.7\ A^0$

SOL. (d) $v = \frac{c}{137n} = \frac{c}{137\times 3}$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\left(\frac{m \times c}{3 \times 137}\right)} = \frac{h}{mc} \times (3 \times 137) \quad = 9.7 A^0$$

14. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of ?

[10 April 2019 II]

(Given: $h = 6.63 \times 10^{34}$ Js; $c = 3 \times 10^{8} ms^{1}$)

(a) 11.4nm (b) 9.4nm (c) 12.3 nm (d) 10.8nm

SOL. (d) Spectral lines obtained on account of transition from n th orbit to various lower orbits is $\frac{n(n-1)}{2}$ $\Rightarrow 6 = \frac{n(n-1)}{2}$

$$\Rightarrow \text{n} = 4$$

$$\Delta E = \frac{hc}{\lambda} = \frac{-Z^2}{n^2}(13.6eV)$$

$$\Rightarrow \frac{1}{\lambda} = Z^2\left(\frac{13.6eV}{hc}\right)\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

$$= (13.4)(3)^2\left[1 - \frac{1}{16}\right]eV$$

$$\Rightarrow \lambda = \frac{1.242 \times 16}{(134) \times (9)(15)}\text{nm} = 10.8\text{nm}$$

15. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2^{nd} Balmer line(n = 4 to n = 2) will be;

[9 April 2019 I]

(a) 889.2nm (b) 488.9nm (c) 642.7nm (d) 388.9nm

SOL. (b)

$$\frac{1}{\lambda_1} = -R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{3R}{16}$$

$$\frac{\lambda_2}{\lambda_1} = \frac{80}{108}$$

$\lambda_2 = \frac{80}{108}\lambda_1 = \frac{80}{108} \times 660 = 488.9$ nm.

16. A He^+ ion is in its first excited state. Its ionization energy is:

[9 April 2019 II]

(a) 48.36eV (b) 54.40eV (c) 13.60eV (d) 6.04eV

16. (c) $E_n = -13.6\frac{z^2}{n^2}$

For He^+, $E_2 = \frac{-13.6(2)^2}{2^2} = -13.60\text{eV}$

Ionization energy $= 0 - \text{E2} = 13.60\text{eV}$

17. Radiation coming from transitions $n = 2$ to $n = 1$ of hydrogen atoms fall on He^+ ions in $n = 1$ and $n = 2$ states. The possible transition of helium ions as they absorb energy from the radiation is:

[8 April 2019 I]

(a) $n = 2 \rightarrow n = 3$ (b) $n = 1 \rightarrow n = 4$

(c) $n = 2 \rightarrow n = 5$ (d) $n = 2 \rightarrow n = 4$

SOL. (d) Energy released by hydrogen atom for transition $n = 2$ to $n = 1$

$$\Delta E_1 = 13.6 \times \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3}{4} \times 13.6\text{eV}$$

$$= 10.2\text{eV}$$

This energy is absorbed by He + ion in transition from n = 2 to n = nl (say)

$$\Delta E_2 = 13.6 \times 4 \times \left(\frac{1}{4} - \frac{1}{n_1^2}\right) = 10.2\text{eV}$$

⇒ nl = 4

So, possible transition is n = 2 → n = 4

18. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 A^0. The radius of the atom in the excited state, in terms of Bohr radius a_0, will be:

[11 Jan 2019 I]

(a) $25a_0$ (b) $9a_0$ (c) $16a_0$ (d) $4a_0$

SOL. (3) Energy of photon $= \frac{hc}{\lambda} = \frac{12500}{980} = 12.75\text{eV}$

Energy of electron in n^{th} orbit is given by

$$En = \frac{-13.6}{n^2} \Rightarrow E_n - E_1 = -13.6\left[\frac{1}{n^2} - \frac{1}{1^2}\right]$$

$$\Rightarrow 12.75 = 13.6\left[\frac{1}{1^2} - \frac{1}{n^2}\right] \Rightarrow n = 4$$

Electron will excite to n = 4

We know that R’ ∝ n^2

Radius of atom will be $16a_0$

19. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be:

[11 Jan 2019 II]

(a) $\frac{27}{20}\lambda$ (b) $\frac{16}{25}\lambda$ (c) $\frac{25}{16}\lambda$ (d) $\frac{20}{27}\lambda$

SOL. (d) When electron jumps from M $\rightarrow$ L shell

$$\frac{1}{\lambda}=K\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{K\times 5}{36} \text{ ..... (i)}$$

When electron jumps from N $\rightarrow$ L shell

$$\frac{1}{\lambda'}=K\left(\frac{1}{2^2}-\frac{1}{4^2}\right)=\frac{K\times 3}{16} \text{ -------(ii)}$$

solving equation (i) and (ii) we get

$$\lambda^{\uparrow}=\frac{20}{27}\lambda$$

20. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength ofthe electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

[2018]

(a) $\Lambda_n \approx A+\frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A+B\lambda_n$

(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ **(d) $\Lambda_n^2 \approx \lambda$**

SOL. (a) Wavelength of emitted photon from n^{th} state to the ground state,

$$\frac{1}{\Lambda_n} = RZ^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2}\left(1 - \frac{1}{n^2}\right)^{-1}$$

Since n is very large, using binomial theorem

$$\Lambda_n = \frac{1}{RZ^2}\left(1 + \frac{1}{n^2}\right)$$

$$\Lambda_n = \frac{1}{RZ^2} + \frac{1}{RZ^2}\left(\frac{1}{n^2}\right)$$

As we know, $\lambda_n = \frac{2\pi r}{n} = 2\pi\left(\frac{n^2h^2}{4\pi^2 mZe^2}\right)\frac{1}{n} \propto n$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

21. If the series limit frequency of the Lyman series is v_1 , then the series limit frequency of the P-fund series is:

[2018]

(a) 25 v_L **(b) 16 v_L** **(c) $v_L/16$** **(d) $v_L/25$**

SOL. (d) $hv_L = E_\infty - E_1$ --------(i)

$hv_f = E_\infty - E_5$ ------------(ii)

$$E \propto \frac{z^2}{n^2} \Rightarrow \frac{E_5}{E_1} = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$$

$$\text{Eqn }(i)/(ii) \Rightarrow \frac{hv_L}{hv_f} = \frac{E_1}{E_5}$$

$$\Rightarrow \frac{v_L}{v_f} = \frac{25}{1} \Rightarrow v_f = \frac{v_L}{25}$$

22. The de-Broglie wavelength (λ_B) associated with the electron orbiting in the second excited state of hydrogen atom is related to that in the ground state (λ_G) by

[Online April 16, 2018]

(a) $\lambda_B = \lambda_G / 3$ **(b)** $\lambda_B = \lambda_G / 2$ **(c)** $\lambda_B = 2\lambda_G$ **(d)** $\lambda_B = 3\lambda_G$

SOL. (d) de-Broglie wavelength, $\lambda = \frac{h}{P}$

$$\frac{\lambda_B}{\lambda_G} = \frac{P_a}{P_B} = \frac{mv_G}{mv_B}$$

Speed of electron $V \propto \frac{z}{n}$

so $\frac{\lambda_B}{\lambda_G} = \frac{n_B}{n_G} = \frac{3}{1} \Rightarrow \lambda_B = 3\lambda_G$

23. The energy required to remove the electron from a singly ionized Helium atom is 2.2 times the energy required to remove an electron from Helium atom. The total energy required to ionize the Helium atom completely is:

[Online April 15, 2018]

(a) 20 eV **(b) 79 eV** **(c) 109 eV** **(d) 34 eV**

SOL. (b) Energy required to remove e^- from singly ionized helium atom

$$=\frac{(13.6)Z^2}{1^2}=54.4\text{eV} \ (\because Z=2)$$

Energy required to remove $e^-$from helium atom $=$ xeV

According to question, $54.4\text{eV} = 2.2x \Rightarrow x = 24.73\text{eV}$

Therefore, energy required to ionize helium atom

$$= (54.4 + 24.73)\text{eV} = 79.12\text{eV}$$

24. Muon (μ^{-1}) is negatively charged $(|q| = |e|)$ with a mass $m_\mu = 200m_e$, where m_e is the mass of the electron and e is the electronic charge. If μ^{-1} is bound to a proton to form a hydrogen like atom, identify the correct statements

[Online Apri115, 2018]

(A) Radius of the muonic orbit is 200 times smaller than that of the electron

(B) the speed of the μ^{-1} in the nth orbit is $\frac{1}{200}$ times that of the election in the nth orbit

(C) The lionization energy of muonic atom is 200 times more than that of an hydrogen atom

(D) The momentum of the muon in the nth orbit is 200 times more than that of the electron

(a) (A), (B), (D) **(b) (B), (D)** **(c) (C),(D)** **(d) (A), (C), (D)**

SOL. (d)

(A) Radius of muon $= \frac{\text{Radius of hydrogen}}{200}$

Radius of H atom $= r = \frac{\epsilon_0 n^2 h^2}{\pi m e^2}$

Radius of muon $= r_\mu = \frac{\epsilon_0 n^2 h^2}{\pi \times 200 m e^2}$

$$r_\mu = \frac{r}{200}$$

(B) Velocity relation given is wrong

(C) Ionization energy in e^-H atom

$$E = \frac{+me^4}{8 \in^2 0n^2h^2}$$

$$E_\mu = \frac{200me^4}{8 \in^2 0n^2h^2} = 200E$$

(D) Momentum of H-atom

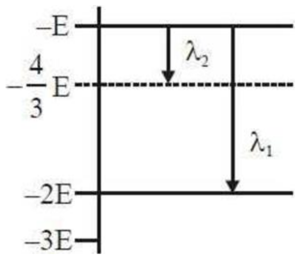
$$mvr = \frac{nh}{2\pi}$$

Hence (A), (C), (D) are correct.

25. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths

$r = \lambda_1/\lambda_2$, is given by

[2017]



(a) $r = \frac{3}{4}$ **(b) $r = \frac{1}{3}$** **(c) $r = \frac{4}{3}$** **(d) $r = \frac{2}{3}$**

SOL. (b) From energy level diagram, using $\Delta E = \frac{hc}{\lambda}$

For wavelength λ_1, $\Delta E = -E - (-2E) = \frac{hc}{\lambda_1}$

$$\lambda_1 = \frac{hc}{E}$$

For wavelength λ_2, $\Delta E = -E - \left(-\frac{4E}{3}\right) = \frac{hc}{\lambda_2}$

$$\lambda_2 = \frac{hc}{\left(\frac{E}{3}\right)} \; r = \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

26. The acceleration of an electron in the first orbit of the hydrogen atom $(z = 1)$ is:

[Online April 9, 2017]

(a) $\frac{h^2}{\pi^2 m^2 r^3}$ **(b) $\frac{h^2}{8\pi^2 m^2 r^3}$** **(c) $\frac{h^2}{4\pi^2 m^2 r^3}$** **(d) $\frac{h^2}{4\pi m^2 r^3}$**

SOL. (c) Speed of electron in first orbit $(n = 1)$ of hydrogen atom $(z = 1)$,

$$v = \frac{e^2}{2\varepsilon_0 h}$$

radius of Bohr's first orbit,

$$r=\frac{h^2\varepsilon_0}{\pi me^2}\Rightarrow \varepsilon_0=\frac{r\pi me^2}{h^2} \text{ .... (i)}$$

Acceleration of electron,

$$\frac{v^2}{r}=\frac{e^4}{4\varepsilon_0^2h^2}\times\frac{\pi me^2}{h^2\varepsilon_0}$$

$$=\frac{e^4\times\pi me^2}{4h^4\varepsilon_0^3} \text{ .... (ii)}$$

Eliminating ε_0 from eq(ii),

$$=\frac{e^4\pi me^2h^6}{4h^4r^3\pi^3m^3e^6} \text{ from eqn(i)}$$

$$=\frac{h^2}{4\pi^2m^2r^3}$$

27. According to Bohr's theory, the time averaged magnetic field at the centre (i.e. nucleus) of a hydrogen atom due to the motion of electrons in the n^{th} orbit is proportional to:

(n = principal quantum number)

[Online April 8, 2017]

(a) n^{-4} **(b) n^{-5}** **(c) n^{-3}** **(d) n^{-2}**

SOL. (d) Magnetic field at the centre of nucleus of H-atom, $B=\frac{\mu_0 I}{2r}$ ..... (i)

According to Bohr's model, radius of orbit $r\propto n^2$

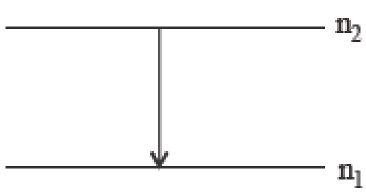
from eq. (i) we can also write as $B\propto n^{-2}$

28. A hydrogen atom makes a transition from n = 2 to n = 1 and emits a photon. This photon strikes a doubly ionized lithium atom (z = 3) in excited state and completely removes the orbiting electron. The least quantum number for the excited state of the ion for the process is:

[Online April 9, 2016]

(a) 2 **(b) 4** **(c) 5** **(d) 3**

SOL. (b) A hydrogen atom makes a transition from n = 2 to n = 1



Then wavelength $= Rcz^2\left|\frac{1}{n_1^2}-\frac{1}{n_2^2}\right| = Rc(1)^2\left[1-\frac{1}{4}\right]$

$\lambda = Rc\left|\frac{3}{4}\right|$ -------(1)

For ionized lithium

$\lambda = Rc(3)^2\left|\frac{1}{n^2}\right| = Rc9\left|\frac{1}{n^2}\right|$ --------(2)

$$Rc\left[\frac{3}{4}\right] = Rc9\left[\frac{1}{n^2}\right]$$

$$\Rightarrow \frac{3}{4} = \frac{9}{n^2} \Rightarrow n = \sqrt{12} = 2\sqrt{3}$$

The least quantum number must be 4.

29. **As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion:**

[2015]

(a) kinetic energy decreases, potential energy increases but total energy remains same

(b) kinetic energy and total energy decrease but potential energy increases

(c) its kinetic energy increases but potential energy and total energy decrease

(d) kinetic energy, potential energy and total energy decrease

SOL. (c) Kinetic energy of electron is

K.E. $\propto \left(\frac{Z}{N}\right)^2$

When the electron makes transition from excited state to ground state, then n increases and kinetic energy increases.

Total energy $= -$KE

Total energy also decreases.

Potential energy is lowest for ground state.

30. **The de-Broglie wavelength associated with the electron in the $n = 4$ level is :**

[Online April 11, 2015]

(a) $\frac{1}{4}$ th of the de-Broglie wavelength of the electron in the ground state.

(b) four times the de-Broglie wavelength of the electron in the ground state

(c) two times the de-Broglie wavelength of the electron in the ground state

(d) half of the de-Broglie wavelength of the electron in the ground state

SOL. (b) De-Broglie wavelength of electron $\lambda = \frac{h}{mV}$

As we know, $V \propto \frac{1}{n}$

So, $\lambda \propto n$

$$\lambda_4 = 4\lambda_1$$

λ_1 is the de-Broglie wavelength of the electron in the ground state.

31. If one were to apply Bohr model to a particle of mass m' and charge q' moving in a plane under the influence of a magnetic field B′, the energy of the charged particle in the n^{th} level will be: [Online Apri110, 2015]

(a) $n\left(\frac{hqB}{2\pi m}\right)$ (b) $n\left(\frac{hqB}{8\pi m}\right)$ (c) $n\left(\frac{hqB}{4\pi m}\right)$ (d) $n\left(\frac{hqB}{\pi m}\right)$

SOL. (c) $qVB = \frac{mv^2}{r}$ -----(i)

$\frac{nh}{2\pi} = mvr$ ....(ii)

Multiplying equation (i) and (ii),

$$\frac{qBnh}{2\pi} = m^2 \mathrm{v}^2$$

Now multiplying both sides by $\frac{1}{2m}$,

$$n\frac{qBh}{4\pi m} = \frac{1}{2}m\mathrm{v}^2$$

i.e. KE $= n\left[\frac{qBh}{4\pi m}\right]$

32. The radiation corresponding to $3 \to 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius ofthe largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to:

[2014]

(a) 1.8eV **(b) 1.1eV** **(c) 0.8eV** **(d) 1.6eV**

SOL. (b) Radius of circular path followed by electron is given by,

$$r = \frac{mv}{qB} = \frac{\sqrt{2meV}}{\mathrm{e}B} = \frac{1}{B}\sqrt{\frac{2m}{\mathrm{e}}V}$$

$\Rightarrow V = \frac{B^2r^2\mathrm{e}}{2m} = 0.8V$

For transition between 3 to 2.

$$E = 13.6\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{13.6 \times 5}{36} = 1.88eV$$

Work function $= 1.88\text{eV} - 0.8\text{eV} = 1.08\text{eV} \approx 1.\text{leV}$

33. **Hydrogen $({}_1H^1)$, Deuterium $({}_1H^2)$, singly ionised Helium $(He^4)^+$ and doubly ionised lithium $(Li^6)^{++}$ all have one electron around the nucleus. Consider an electron transition from $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are λ_1 , λ_2 , λ_3 and λ_4 respectively then approximately which one of the following is correct?**

[2014]

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ **(b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$**

(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ **(d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$**

SOL. (c) Wave number $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\Rightarrow \lambda \propto \frac{1}{Z^2}$$

$\lambda Z^2 = \text{constant}$

By question $n = 1$ and $n_1 = 2$ Then, $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

34. **Match List - I (Experiment performed) with List-II (Phenomena discovered/associated) and select the correct option from the options given below the lists:**

[Online Apri119, 2014]

List - I		List - II	
(1)	Davisson and Germer experiment	(i)	Wave nature of electrons
(2)	Millikan's oil drop experiment	(ii)	Charge of an electron
(3)	Rutherford experiment	(iii)	Quantisation of energy levels
(4)	Franck-Hertz experiment	(iv)	Existence of nucleus

(a) (1) -(i), (2) -(ii), (3) -(iii), (4) -(iv) **(b) (1) -(i), (2) -(ii), (3) -(iv), (4) -(iii)**

(c) (1) -(iii), (2) – (iv), (3) -(i), (4) – (ii) **(d) (1) -(iv), (2) -(iii), (3) -(ii), (4) -(i)**

SOL. (b)

(1) Davisson and Germer experiment-wave nature of electrons.

(2) Millikan's oil drop experiment- charge of an electron.

(3) Rutherford experiment- Existence of nucleus.

(4) Frank-Hertz experiment - Quantization of energy levels.

35. The binding energy of the electron in a hydrogen atom is $13.6 \mathrm{eV}$, the energy required to remove the electron from the first excited state of Li^{++} is:

[Online April 9, 2014]

(a) $122.4 \mathrm{eV}$ **(b) $30.6 \mathrm{eV}$** **(c) $13.6 \mathrm{eV}$** **(d) $3.4 \mathrm{eV}$**

SOL. (b) For first excited state, $n = 2$ and for Li^{++} $Z = 3$

$$E_n = \frac{13.6}{n^2} \times Z^2 = \frac{13.6}{4} \times 9 = 30.6 eV$$

36. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to:

[2013]

(a) $\frac{1}{n}$ **(b)** $\frac{1}{n^2}$ **(c)** $\frac{1}{n^{3/2}}$ **(d)** $\frac{1}{n^3}$

SOL. (d) $\Delta E = h\nu$

$$v = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k(2n-1)}{n^2(n-1)^2}$$

$\approx \frac{2k}{n^3}$ or $v \propto \frac{1}{n^3}$

37. A 12.5eV electron beam is used to bombard gaseous hydrogen at room temperature. It will emit:

[Online April 25, 2013]

(a) 2 lines in the Lyman series and 1 line in the Balmer series

(b) 3 lines in the Lyman series

(c) 1 line in the $L_{\mathcal{W}}$an series and 2 lines in the Balmer series

(d) 3 lines in the Balmer series

SOL. (a) $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{6.62\times10^{-34}\times3\times10^{8}}{12.5\times1.6\times10^{-19}} = 993A^{\circ}$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_{1^2}} - \frac{1}{n_{2^2}}\right)$$

(where Rydberg constant , $R = 1.097 \times 10^7$)

or, $\frac{1}{993\times10^{-10}} = 1.097\times10^{7}\left(\frac{1}{1^2} - \frac{1}{n_{2^2}}\right)$

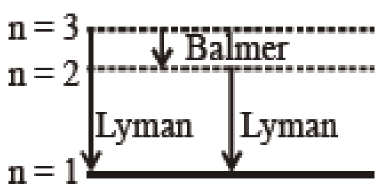
Solving we get $n_2 = 3$

Spectral lines

Total number of spectral lines = 3

Two lines in Lyman series for $n_1 = 1$, $n_2 = 2$ and $n_1 = 1, n_2 = 3$

and one in Balmer series for $n_1 = 2$, $n_2 = 3$



38. In the Bohr's model of hydrogen-like atom the force between the nucleus and the electron is modified as $F = \frac{e^2}{4\pi\varepsilon_0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$, where β is a constant. For this atom, the radius of the n^{th} orbit in terms of the Bohr radius is: $\left(a_0 = \frac{\varepsilon_0 h^2}{m\pi e^2}\right)$

[Online April 23, 2013]

(a) $r_n = a_0 n - \beta$ **(b) $r_n = a_0 n^2 + \beta$** **(c) $r_n = a_0 n^2 - \beta$** **(d) $r_n = a_0 n + \beta$**

SOL. (c) As $F = \frac{mv^2}{r} = \frac{e^2}{4\pi \in 0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$

and $mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$

$$m\left(\frac{nh}{2\pi mr}\right)^2 \times \frac{1}{r} = \frac{e^2}{4\pi \in 0}\left(\frac{1}{r^2} + \frac{\beta}{r^3}\right)$$

or, $\frac{1}{r^2} + \frac{\beta}{r^3} = \frac{mn^2h^2 4\pi \in 0}{4\pi^2 m^2 e^2 r^3}$

or, $\frac{a_0 n^2}{r^3} = \frac{1}{r^2} + \frac{\beta}{r^3}$ $\left(\because a_0 = \frac{\in 0 h^2}{m\pi e^2} \text{ Given}\right)$

For n^{th} atom

$$r_n = a_0 n^2 - \beta$$

39. Orbits of a particle moving in a circle are such that the perimeter of the orbit equals an integer number of de-Broglie wavelengths of the particle. For a charged particle moving in a plane perpendicular to a magnetic field, the radius of the n^{th} orbital will therefore be proportional to : [Online April 22, 2013]

(a) n^2 **(b) n** **(c) $n^{1/2}$** **(d) $n^{1/4}$**

SOL. (c) According to the question,

$$2\pi r = n\lambda = \frac{nh}{p} = \frac{nh}{mv}$$

Or $mvr = \frac{nh}{2\pi}$ or $mv = \frac{nh}{2\pi r}$

$$F = qvB = \frac{mv^2}{r} \qquad \text{or, } qB = \frac{mv}{r} = \frac{nh}{2\pi r.r}$$

or, $r^2 = \frac{nh}{2\pi qB}$ or, $r = \sqrt{\frac{nh}{2\pi qB}}$

i.e., $r \propto n^{1/2}$

40. In the Bohr model an electron moves in a circular orbit around the proton. Considering the orbiting electron to be a circular current loop, the magnetic moment of the hydrogen atom, when the electron is in n^{th} excited state, is:

[Online April 9, 2013]

(a) $\left(\frac{en^2h}{2m2\pi}\right)$ **(b)** $\left(\frac{e}{m}\right)\frac{nh}{2\pi}$ **(c)** $\left(\frac{e}{2m}\right)\frac{nh}{2\pi}$ **(d)** $\left(\frac{e}{m}\right)\frac{n^2h}{2\pi}$

SOL. (c) Magnetic moment of the hydrogen atom, when the electron is in n^{th} excited state,

i. e., $n' = (n+1)$

As magnetic moment $M_n = I_nA = I_n(\pi r_n^2)$

$$i_n = eV_n = \frac{mz^2e^5}{4\varepsilon_0^2 n^3 h^3}$$

$$r_n = \frac{n^2h^2}{4\pi^2 kzme^2}\left(k = \frac{1}{4\pi \in 0}\right)$$

Solving we get magnetic moment of the hydrogen atom for n^{th} excited state

$$M_{n^t} = \left(\frac{e}{2m}\right)\frac{nh}{2\pi}$$

41. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be:

[2012]

(a) 2 **(b) 3** **(c) 5** **(d) 6**

SOL. (d) For ground state, the principal quantum no. $(n) = 1$. Principal quantum number 4 belongs to 3rd excited state. The possible number of the spectral lines from a state n to ground state is

$$= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

42. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by: (n is an integer)

[2012]

(a) $\frac{(m_1+m_2)^2 n^2h^2}{2m_1^2m_2^2r^2}$ **(b)** $\frac{n^2h^2}{2(m_1+m_2)r^2}$

(c) $\frac{2n^2h^2}{(m_1+m_2)r^2}$ **(d)** $\frac{(m_1+m_2)n^2h^2}{2m_1m_2r^2}$

SOL. (d) The energy of the system of two atoms of diatomic molecule $E = \frac{1}{2} Iw^2$

where I = moment of inertia

w = Angular velocity $= \frac{L}{I}$,

L = Angular momentum

$$I = \frac{1}{2}(m_1 r_{1^2} + m^2 r_2^2)$$

Thus, $E = \frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2\right) w^2$ ... (i)

$$E = \frac{1}{2}(m_1 r_{1^2} + m_2 r_{2^2}) \frac{L^2}{I^2}$$

$$L = nh$$

(According to Bohr's Hypothesis)

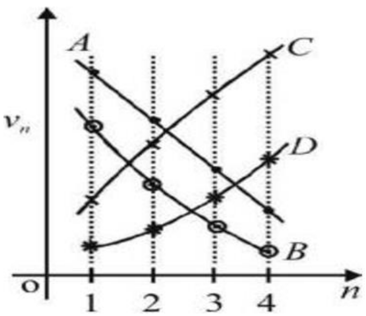
$$E = \frac{1}{2}(m_1 r_{1^2} + m_2 r_2^2) \frac{L^2}{(m_1 r_1^2 + m_2 r_2^2)^2}$$

$$E = \frac{1}{2} \frac{L^2}{(m_1 r_1^2 + m_2 r_2^2)} = \frac{n^2 h^2}{8\pi^2 (m_1 r_1^2 + m_2 r_2^2)}$$

$$E = \frac{(m_1 + m_2) n^2 h^2}{8\pi^2 r^2 m_1 m_2}$$

43. Which of the plots shown in the figure represents speed (v_n) of the electron in a hydrogen atom as a function of the principal quantum number (n) ?

[Online May 26, 2012]



(a) *B* **(b) *D*** **(c) *C*** **(d) *A***

SOL. (a) Velocity of electron in n^{th} orbit of hydrogen atom is given by :

$$V_n = \frac{2\pi KZe^2}{nh}$$

Substituting the values we get,

$V_n = \frac{2.2\times10^6}{n}$ m/s or $V_n \propto \frac{1}{n}$

As principal quantum number increases, velocity decreases.

(1 1)

44. A doubly ionized Li atom is excited fiom its ground state $(n = 1)$ to $n = 3$ state. The wavelengths of the spectral lines are given by λ_{32}, λ_{31} and λ_{21}. The ratio $\lambda_{32}/\lambda_{31}$ and $\lambda_{21}/\lambda_{31}$ are, respectively

[Online May 12, 2012]

(a) $8.1, 0.67$ **(b) $8.1, 1.2$** **(c) $6.4, 1.2$** **(d) $6.4, 0.67$**

SOL. (c) $\frac{1}{\lambda}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ where R = Rydberg constant

$\frac{1}{\lambda_{32}}=\left(\frac{1}{4}-\frac{1}{9}\right)=\frac{5}{36} \Rightarrow \lambda_{32}=\frac{36}{5}$

Similarly solving for λ_{31} and λ_{21}

$\lambda_{31}=\frac{9}{8}$ and $\lambda_{21}=\frac{4}{3}$

$\frac{\lambda_{32}}{\lambda_{31}}=6.4$ and $\frac{\lambda_{21}}{\lambda_{31}}=1.2$

45. A hypothetical atom has only three energy levels. The ground level has energy, $E_1=-8\text{eV}$. The two excited states have energies, $E_2=-6\text{eV}$ and $E_3=-2\text{eV}$. Then which ofthe following wavelengths will not be present in the emission spectrum of this atom?

[Online May 12, 2012]

(a) 207 nm **(b) 465 nm** **(c) 310nm** **(d) 620nm**

SOL. (b) $E=\frac{hc}{\lambda}$

46. The electron of a hydrogen atom makes a transition from the $(n+1)^{\text{th}}$ orbit to the n^{th} orbit. For large n the wavelength of the emitted radiation is proportional to

[Online May 7, 2012]

(a) n **(b) n^3** **(c) n^4** **(d) n^2**

SOL. (b) If $n_1=n$ and $n_2=n+1$

Maximum wavelength $\lambda_{max} = \dfrac{n^2(n+1)^2}{(2n+1)R}$

Therefore, for large n, $\lambda_{max} \propto n^3$

47. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is:

[2011]

(a) $36.3eV$ (b) $108.8eV$ (c) $122.4eV$ (d) $12.1eV$

SOL. (b) Energy of excitation (ΔE) is

$$\Delta E = 13.6z^2\left(\frac{1}{n_1} - \frac{1}{n_2}\right)eV$$

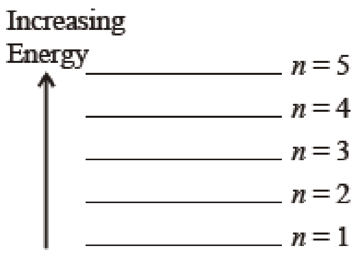
$$\Rightarrow \Delta E = 13.6(3)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 108.8eV$$

48. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from:

[2009]

(a) $3 \to 2$ (b) $4 \to 2$ (c) $5 \to 4$ (d) $2 \to 1$

SOL. (c) It is given that transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom result in ultraviolet radiation. For infrared radiation $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ should be less. The only option is $5 \to 4$.



49. **Suppose an electron is attracted towards the origin by a force $\frac{k}{r}$ where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the n^{th} orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be T_n'. Then which of the following is true?**

[2008]

(a) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$

(b) T_n **independent of** n, $r_n \propto n$

(c) $T_n \propto \frac{1}{n}, r_n \propto n$

(d) $T_n \propto \frac{1}{n^3}, r_n \propto n^2$

SOL. (b) Given,

Centripetal force $= \frac{k}{r}$ Then

$$\frac{k}{r} = \frac{mv^2}{r}$$

$\Rightarrow k = mv^2 \Rightarrow Tn = \frac{1}{2}mv^2 = \frac{1}{2}k$

Tn is independent of n

Also,

Angular momentum, $L = \frac{nh}{2\pi}$

$\Rightarrow mvr_n = \frac{nh}{2\pi} (\because L = mvr)$

$\Rightarrow r_n = \frac{nh}{2\pi\sqrt{km}}$ $(m^2v^2 = km)$

Clearly, $r_n \propto n$

50. Which of the following transitions in hydrogen atoms emit photons of highest frequency?

[2007]

(a) $n = 1$ to $n = 2$ **(b) $n = 2$ to $n = 6$**

(c) $n = 6$ to $n = 2$ **(d) $n = 2$ to $n = 1$**

SOL. (d) We have to find the frequency of emitted photons. For emission of photons electron should makes a transition from higher energy level to lower energy level.

so, option (a) and (b) are incorrect.

Frequency of emitted photon is given by

$$h\text{v} = -13.6\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$$

For transition from $n = 6$ to $n = 2$,

$$\text{v}_1 = \frac{-13.6}{h}\left(\frac{1}{6^2} - \frac{1}{2^2}\right) = \frac{2}{9} \times \left(\frac{13.6}{h}\right)$$

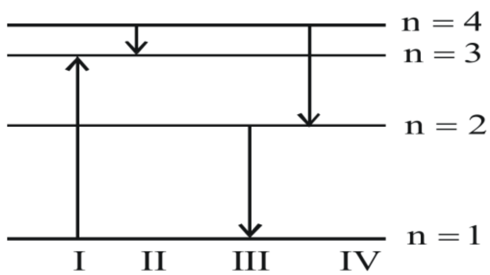
For transition from $n = 2$ to $n = 1$,

$$v_2 = \frac{-13.6}{h}\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = \frac{3}{4} \times \left(\frac{13.6}{h}\right)$$

$$v_1 < v_2$$

51. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy?

[2005]



(a) IV **(b) III** **(c) II** **(d) I**

SOL. (b) Energy of radiation that corresponds to energy difference between two energy levels n_1 and n_2 is given as

$$E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

E will be maximum for the transition for which $\left[\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right]$ is maximum.

Here n_2 is the higher energy level.

Clearly, $\left[\frac{1}{n_{1^2}} - \frac{1}{n_{2^2}}\right]$ is maximum for the third transition, $i.e.$, $2 \rightarrow 1$.

I transition is showing the absorption of energy

52. The wavelengths involved in the spectrum of deuterium $_1^2D$ are slightly different from that of hydrogen spectrum, because

[2003]

(a) the size of the two nuclei are different

(b) the nuclear forces are different in the two cases

(c) the masses of the two nuclei are different

(d) the attraction between the electron and the nucleus is different in the two cases

SOL. (c) The wavelength of spectrum is given by

$$\frac{1}{\lambda} = Rz^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

where $R = \frac{1.097 \times 10^7}{1 + \frac{m}{M}}$

where m = mass ofelectron

M = mass of nucleus.

Thus, wavelength involved in the spectrum of hydrogen like atom depends upon masses of nucleus. The mass number of hydrogen and deuterium is 1 and 2 respectively, so spectrum of deuterium will be different from hydrogen.

53. **If 13.6eV energy is required to ionize the hydrogen atom, then the energy required to remove an electron from $n = 2$ is**

[2002]

(a) 10.2 eV **(b) 0eV** **(c) 3.4 eV** **(d) 6.8 eV**

SOL. (c) The energy required to remove the electron from the n^{th} orbit of hydrogen is given by

$E_n = \frac{13.6}{n^2}$ eV/atom

For $n = 2$, $E_n = \frac{13.6}{4} = 3.4eV$

Therefore the energy required to remove electron from $n = 2$ is $+3.4$eV.

NUCLEI

Nucleus:

- The nucleus of an atom is at the centre. Most of the mass of an atom is at the centre. The entire positive charge of an atom lies in the nucleus.
  All atomic nuclei are made up of elementary particles called protons and neutrons. Proton is the nucleus of the hydrogen atom. It has a positive charge of 1.6×10^{-19} C having a mass of 1.6726×10^{-27} kg. This is nearly equal to 1836 times the electron mass. Neutron is electrically neutral (i.e. neutron carries no charge). Mass of neutron is slightly greater than that of the proton (1.6750×10^{-27} kg). Both the proton and neutron together constitute the nucleus. They are called nucleons.
- Generally, atomic number is denoted by Z and mass number is denoted by A and (A-Z) gives number of neutrons (N) in the nucleus.
  $\therefore N = A - Z$; $A = Z + N$
- Nucleus is positively charged and its shape is considered as spherical.

Types of Nuclei:

- **Isotopes:** Atomic nuclei having same atomic number but different mass numbers are known as isotopes. They occupy same position in the periodic table and possess identical chemical properties. They have same proton number.
  Ex: 1) ${}_3Li^6, {}_3Li^7$ 2) ${}_1H^1, {}_1H^2, {}_1H^3$
- **Isotones :** Atomic nuclei having same number of neutrons are called isotones.
  Ex: 1) ${}_{17}Cl^{37}, {}_{19}K^{39}$, 2) ${}_7N^{17}, {}_8O^{18}, {}_9F^{19}$
- **Isobars:** Atomic nuclei having same mass number but different atomic numbers are called Isobars. They have same number of nucleons.
  Ex:–1) ${}_{18}Ar^{40}, {}_{20}Ca^{40}$, 2) ${}_{32}Ge^{76}, {}_{34}Se^{76}$
- **Isomers:** Atomic nuclei having same mass number and same atomic number but different nulear properties are called isomers.
  Ex:- $m\,{}_{35}Br^{80}$ metastable Bromine and $g_{35}Br^{80}$ ground state Bromine are two isomers with different half lives
- **Isodiaphers :** Nuclei having different Atomic number (Z) and mass number (A) but with same excess number of neutrons over protons (A-2Z) are called isodiaphers.
  Ex:- ${}_{11}Na^{23}$, ${}_{13}Al^{27}$

Size of the Nucleus:

- Nuclear sizes are very small and are measured in fermi (or) femtometer. 1 fermi=10^{-15}m
- Radius of the nucleus depends on number of nucleons. $R = R_0 A^{1/3}$

  above equation does not apply to heavy nucleides
  Value of R_o = 1.4 x 10^{-15}m
- Radius of the nucleus is in the order of 10^{-15} m.
- Size of an atom is in the order of 10^{-10}m.
- If an α-particle with an initial kinetic energy E approaches a target of atomic number Z, if the distance of closest approach is "d" then

  $\frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{d} = E$ (Where 'e' is charge of an electron) If "v" represents the initial velocity of α particle, (m is mass of "α" particle) then $\frac{1}{4\pi\epsilon_0}\frac{2Ze^2}{d} = \frac{1}{2}mv^2$

Note : If a particle of charge q, mass m is projected towards a nucleus of charge Q with velocity v from infinity then the distance of closest approach d is give by $\frac{1}{4\pi\epsilon_0}\frac{qQ}{d} = \frac{1}{2}mv^2$

Note : If R, S and V be the Radius, surface area and volume of a nucleus with mass number A then

$$R \propto A^{\frac{1}{3}} \Rightarrow \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}}; S \propto R^2 \propto A^{\frac{2}{3}} \Rightarrow \frac{S_1}{S_2} = \left(\frac{A_1}{A_2}\right)^{\frac{2}{3}}$$

$$V \propto R^3 \propto A \Rightarrow \frac{V_1}{V_2} = \frac{A_1}{A_2}$$

Note : If a stationary nucleus splits in to two lighter nuclei with mass numbers A_1 and A_2 then according to law of conservation of linear momentum, the two lighter nuclei move in opposite directions with equal momenta hence $\mathbf{m_1v_1 = m_2v_2}$

Ratio of velocities of the two nuclei

$$\frac{v_1}{v_2} = \frac{m_2}{m_1} = \frac{A_2}{A_1} = \left(\frac{R_2}{R_1}\right)^3 \quad (\because m \propto A \propto R^3)$$

Ratio of kinetic energy of the two nuclei

$$\frac{KE_1}{KE_2} = \frac{m_2}{m_1} = \frac{A_2}{A_1} = \left(\frac{R_2}{R_1}\right)^3$$

$$\left(\because KE = \frac{p^2}{2m} \;\&\; KE \propto \frac{1}{m} \text{ when p is constant}\right)$$

Density of the Nucleus:

- Density of nucleus is independent of mass number of the atom.
- Density of the nucleus is 1.45×10^{17} Kgm^{-3}.
- The density is maximum at the centre and gradually falls to zero as we move radially out wards.
- Radius of the nucleus is taken as the distance between the centre and the point where the density falls to half of its value at the centre.
- Density of nucleus is of the order of $10^{14} gm/cc = 10^{17} kg/m^3$

EX. 1: Compare the radii of the nuclei of mass numbers 27 and 64.

Sol. The ratio of the radii of the nuclei is

$$\frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{\frac{1}{3}} = \left(\frac{27}{64}\right)^{\frac{1}{3}} (\because R = R_0 A^{1/3}) = \frac{3}{4}$$

EX. 2: The radius of the oxygen nucleus ${}^{16}_{8}O$ is 2.8×10^{-15}m. Find the radius of lead nucleus ${}^{205}_{82}Pb$.

Sol. $R_0 = 2.8 \times 10^{-15}$ m, $A_0 = 16$, $A_{Pb} = 205$ $R \propto A^{1/3}$

$$\frac{R_0}{R_{Pb}} = \left(\frac{A_0}{A_{Pb}}\right)^{1/3} = \frac{2.8 \times 10^{-15}}{R_{Pb}} = \left(\frac{16}{205}\right)^{1/3}$$

$R_{Pb} = 6.55 \times 10^{-15}$m.

Atomic Mass Unit (A.M.U):

i) The masses of atoms, nuclei, sub atomic particles are very small. Hence, a small unit is used to express these masses. This unit is called as atomic mass unit (amu). **1 amu is equal to one twelth part of the mass of carbon ($_6C^{12}$) isotope.**
Mass of $_6C^{12}$ is exactly 12 amu

ii) Now, the mass of 1 gm -mole of carbon is 12 gm and according to Avogadro's Hypothesis it has N (Avogadro's Number) atoms. Thus, the mass of one atom of carbon is (12/N) gm. According to the definition.

$$1amu = 1u = \frac{1}{12} \times (mass\ of\ one\ carbon\ atom)$$

$$= \frac{1}{12} \times \frac{12}{N} = \frac{1}{N} gm = \frac{1}{6.023 \times 10^{23}} gm$$

$$= 1.660565 \times 10^{-24} gm = 1.660565 \times 10^{-27} Kg$$

Mass - Energy Equivalence : According to Einstein's mass-energy equivalence principle, mass is another form of energy. Mass can be converted into energy & energy can be converted into mass according to the equation $E = mC^2$

Here m is the mass that disappears and E is the energy liberated. C is the velocity of light in vacuum.

When 1 amu of mass is converted in to energy

Energy liberated is given by

$E = (1.660565 \times 10^{-27}) \times 9 \times 10^{16}$ J= 931.5 MeV

hence 1 amu of mass is equivalent to 931.5 MeV of energy $\therefore \boxed{1\text{ amu} = 931.5\text{ MeV/C}^2}$

The masses of electron, proton and neutron in terms of various units are :

Mass of the electron = $m_e = 9.1095 \times 10^{-31}$ kg

= 0.000549 u = 0.511 MeV/C^2

Mass of the proton = $m_p = 1.6726 \times 10^{-27}$ kg

= 1.007276 u = 938.28 MeV/C^2

Mass of the neutron = $m_n = 1.6750 \times 10^{-27}$ kg

= 1.008665 u = 939.573 MeV/C^2 .

Nuclear Forces:

The attractive force which holds the nucleons together in the nucleus is called nuclear force.

Properties of nuclear forces :

1) Nuclear forces are strongest forces in nature. Nuclear forces are about 10^{38} times as strong as gravitational forces.The relative strengths of the gravitational, Coulomb's and nuclear forces are

$$F_g : F_e : F_n = 1 : 10^{36} : 10^{38}$$

2) **Nuclear forces are short range forces .**
3) **Nuclear forces are basically strong attractive forces, but contain a small component of repulsive forces.**
4) **Nuclear forces are saturated forces.**
5) **Nuclear forces are charge independent.** 6) **Nuclear forces are spin-dependent.**
7) **Nuclear forces are exchange forces.** 8) **Nuclear forces are non-central forces.**

Mass defect, binding enegry, Einstein's Mass energy Relation

- When matter is completely annihilated, energy released is $E = mc^2$
- The energy equivalent to 1 amu is 931.5 M eV = $1.4925 \times 10^{-10} J$.

Mass Defect: Atomic mass is always less than the sum of the masses of constituent particles. The difference between the total mass of the nucleons and mass of the nucleus of an atom gives mass defect.

$$\Delta m = \left[\left[ZM_p + (A - Z)M_n\right] - M_{nucleus}\right]$$

Z = Atomic number; M_p = Mass of proton

M_n = Mass of neutron; A=Mass number

$M_{nucleus}$ = Mass of nucleus.

Binding Energy: The energy required to bring the nucleons from infinity to form the nucleus is called binding energy or it is the energy required to split a nuclens into nucleons. It is energy equivalent of mass defect BE= $[\Delta m]C^2$

NOTE: BE = mass defect x 931.5 MeV if mass is expressed in a.m.u.

B.E. per nucleon= Binding fraction

$$\frac{\text{Binding Energy}}{\text{Mass Number}} = \frac{\Delta m \text{ x } 931 \text{ Mev}}{A}$$

Averge Binding energy or Binding energy fraction: It is the Binding energy per nucleon (or)

the average energy needed to separate a nuclei in to its individual nucleons.

- Binding energy is not a measure of stability of a nucleus.

Packing fraction of a Nucleus :

Packing fraction : It is defined as the mass defect per nucleon. Packing fraction $= \frac{\Delta m}{A} = \frac{M - A}{A}$

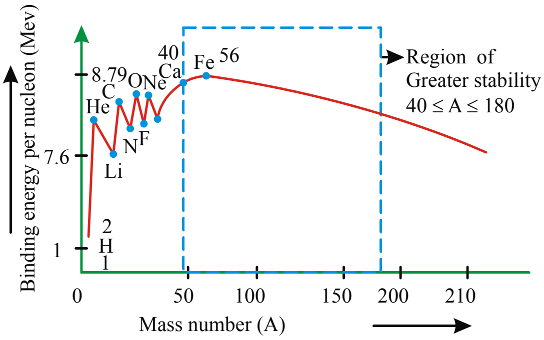
If the packing fraction is negative then the nucleus is more stable.

If the packing fraction is positive then the nucleus is unstable.

Packing fraction is zero for ${}_6C^{12}$

- Packing fraction measures the stability of a nucleus. Smaller the value of packing fraction, large is the stability of the nucleus.

Variation of B.E. per nucleon With Mass Number



The main features of binding energy curve shown in figure are :

1) The minimum value of binding energy per nucleon is in the case of deuteron (1.11MeV).

2) The maximum value of $\frac{BE}{A}$ is 8.7MeV for the nucleus ${}_{28}Fe^{56}$ (iron) which is the most stable.

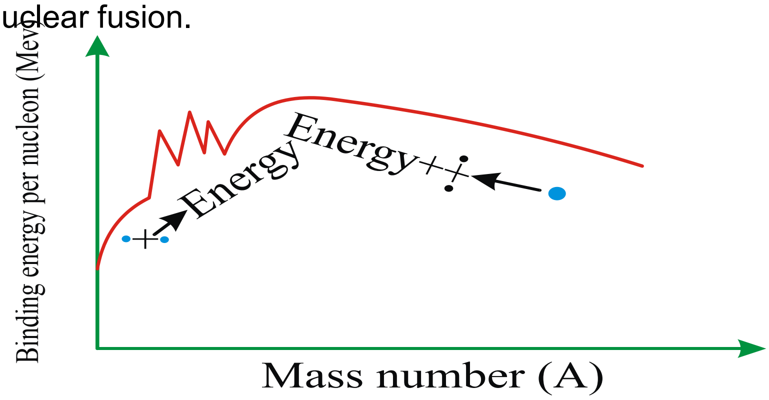
3) Binding energy is high in the range 28<A<138. The binding energy of these nuclei is very close to 8.7 MeV.

4) Further increase in the mass number, binding energy per nucleon decreases and consequently for the heavy nuclei like uranium it is 7.6 MeV.

5) In the region of smaller mass numbers, the binding energy per nucleon curve shows the characteristic minima and maxima. Minima are associated with nuclei containing an odd number of protons and neutrons such as ${}_{3}^{6}Li, {}_{5}^{10}B, {}_{7}^{14}N$ and the maxima are associated with nuclei having an even number of protons and neutrons such as ${}_{2}^{4}He, {}_{6}^{12}C, {}_{8}^{16}O$.

6) Nuclei with A > 220 are distinctly unstable. That means from A > 220 single heavy nucleus breaks into two nearly equal nuclei with mass number A < 150 and so which are most stable. This process takes at right of the BE curve as shown in figure . This process explains the nuclear fission.

7) Light nuclei such as hydrogen combine to form heavy nucleus to form helium for greater stability. This process takes at left of the BE curve as shown in figure. This process explains the nuclear fusion.



Note : Iron $\left({}_{28}Fe^{56}\right)$ whose binding energy per nucleon stands maximum at 8.7 MeV is most stable and will undergo neither fission nor fusion.

Exo-ergic Reaction : The reaction in which energy will be released is called exo-ergic Reaction. A + B $\rightarrow$ C + D + Q

Here A and B are called Reactants

C and D are called Products

Q is the amount of energy released

In an Exo - ergic Reaction

Mass of reactants > Mass of products

$\Delta m = M_R - M_P = (M_A + M_B) - (M_C + M_D)$

Energy Released $Q = \Delta m \times C^2$ joule [Δm is in kg]

$= \Delta m \times 931.5$ MeV (Δm is in amu)

If Binding energies are given then for Exo-ergic reactions.

(B.E) Products > (B.E) Reactants

Energy released Q = $(B.E)_P - (B.E)_R$

$= [(B.E)_C + (B.E)_D] - [(B.E)_A + (B.E)_B]$

Endo-ergic Reaction : The reaction in which energy will be absorbed is called Endo-ergic Reaction. A + B $\rightarrow$ C + D – Q

Here A and B are called Reactants

C and D are called Products

Q is the amount of energy absorbed

In an Endo - ergic Reaction

mass of reactants < Mass of products

$$\Delta m = M_P - M_R = (M_C + M_D) - (M_A + M_B)$$

Energy absorbed $Q = \Delta m \times C^2$ joule [Δm is in kg]

$\Rightarrow \Delta m \times 931.5$ MeV (Δm is in amu)

If Binding energies are given then for

Endo-ergic reaction.

(B.E) Products < (B.E) Reactants

Energy absorbed Q = $(B.E)_R - (B.E)_P$

= $[(B.E)_A + (B.E)_B] - [(B.E)_C + (B.E)_D]$

Note: A nuclear reaction can occur only if certain conservation laws are followed. These are :

1. Conservation of mass number A.
2. Conservation of charge.
3. Conservation of energy, linear momentum and angular momentum.

EX. 3 : Find the binding energy of $^{56}_{26}Fe$. Atomic mass of Fe is 55.9349u and that of Hydrogen is 1.00783u and mass of neutron is 1.00876u

Sol. Mass of the hydrogen atom m_H = 1.00783u; Mass of neutron m_n = 1.00867 u; Atomic number of iron Z = 26; mass number of iron A = 56; Mass of iron atom M_a = 55.9349u

Mass defect Δm = $[Zm_H+(A-Z)m_n] - M_a$

= [26 x 1.00783+(56-20)1.00867]-55.93493

u = 0.5287 u.

$\therefore$ Binding energy = $(\Delta m)c^2$ = (0.52878)

c^2 = (0.52878)(931.5MeV) = 492.55 MeV

EX. 4 : Find the energy required to split $^{16}_{8}O$ nucleus into four α - particles. The mass of an α - particle is 4.002603u and that of oxygen is 15.994915u.

Sol. Mass of a-particle = 4.002603 u

Mass of oxygen = 15.994915u

B.E= [Mass of 4 particles-Mass of oxygen] x 931.5MeV

B.E = [4 x 4.002603 - 15.994915] x 931.5 MeV =(16.010412-15.994915)x931.5MeV

= 0.015497 x 931.5 ; B.E = 14.43 MeV

EX. 5 : Calculate the binding energy per nucleon of $^{40}_{20}Ca$. Given that mass of $^{40}_{20}Ca$ nucleus = 39.962589 u, mass of proton = 1.007825 u. mass of Neutron = 1.008665 u and 1 u is equivalent to 931 MeV.

Sol. A =40, Z = 20, A -Z = 20

$\Delta m = \{Zm_p + (A-Z)m_n\} - M_n$

= { (20 x 1.007825+(20x1.008665)} - 39.962589

= 40.329800 - 39.962589 ; $\Delta m = 0.367211$

Binding energy per nucleon =

$$\frac{\Delta m \times 931}{A} = \frac{0.367211 \times 931}{40} = 8.547 MeV.$$

EX. 6 : The binding energies per nucleon for deuterium and helium are 1.1 MeV and 7.0 MeV respectively. What energy in joules will be liberated when 2 deuterons take part in the reaction.

Sol. $^2_1H + ^2_1H \rightarrow ^4_2He + Q$

Binding energy per nucleon of helium (^4_2He) =7 MeV

Binding energy = 4 x7 = 28 MeV

Binding energy per nucleon of deuterium (^2_1H)=1.1MeV

Binding energy = 2 x 1.1 = 2.2 MeV

Energy liberated (Q) = (28 -(2.2)2] = 23.6 Mev.

i.e. $Q=23.6\times10^6\times1.6\times10^{-19}$; $Q = 37.76\times10^{-13} J$

EX. 7: The kinetic energy of α-particles emitted in the decay of $_{88}Ra^{226}$ into $_{86}Rn^{222}$ is measured to be 4.78 MeV. What is the total disintegration energy or the 'Q-value of this process' ?

Sol. The standard relation between the kinetic energy of the α-particle (KE_a) and the Q-value (or total disintegration energy) is

$KE_a = \left(\frac{A-4}{A}\right).Q \qquad Q = \left(\frac{A}{A-4}\right).KE_a$

$= \left(\frac{226}{226-4}\right)\times 4.78MeV \quad = \frac{226}{222}\times 4.78MeV$

$Q = 4.865MeV \approx 4.87MeV$

EX. 8: A nucleus X-initially at rest, undergoes alpha-decay, according to the equation.

$$^A_{92}X \rightarrow ^{228}_{z}Y + \alpha$$

The α-particle in the above process is found to move in a circular track of radius $1.1\times10^2 m$ in a uniform magnetic field of $3.0\times10^3 T$.

The energy (in MeV) released during the process and binding energy of the parent nucleus X, respectively.

Given: $m_y = 228.03 amu \quad m_\alpha = 4.003 amu^\dagger \quad m(^1_0n) = 1.009 amu \quad m(^1_0H) = 1.008 amu^\dagger$

$1 amu = 1.66\times10^{-27} \quad kg \equiv 931.5 MeV/c^2$

Sol: The given equation is ${}^{A}_{92}X \to {}^{228}_{z}Y + {}^{4}_{2}He$

$A = 228 + 4 = 232$; $92 = z + 2 \quad \therefore z = 90$

$\frac{m_\alpha v_\alpha^2}{r} = qv_\alpha B$; $v_\alpha = \sqrt{\frac{rqB}{m_\alpha}}$

$= \sqrt{\frac{1.1\times10^2 \times 2\times1.6\times10^{-19}\times3\times10^3}{4.003\times1.66\times10^{-27}}}$

$= 4.0\times10^6 m/s$

From conservation of linear momentum, $m_\alpha v_\alpha = m_y v_y$

$v_y = \frac{m_\alpha v_\alpha}{m_y} = \frac{(4.003)(4.0\times10^6)}{(228.03)} = 7.0\times10^4 m/s$

There fore, energy released during the process $= \frac{1}{2}\left[m_\alpha v_\alpha^2 + m_y v_y^2\right] = \frac{(1.66\times10^{-27})}{(2\times1.6\times10^{-13})}$

$\left[(4.003)(4.0\times10^6)^2 + (228.03)(7.0\times10^4)^2\right]MeV$

$= 0.34\,MeV = \frac{0.34}{931.5}amu = 0.000365amu$

Therefore, mass of ${}^{232}_{92}X = m_y + m_\alpha + 0.000365$

$= 232.033365\ u$

Mass defect $\qquad \Delta m = 92(1.008) + (232 - 92)$

$(1.009) - 232.033365$.

$\therefore$ Binding energy $= 1.962635\times931.5\ MeV$

$= 1828.2\,MeV$

Natural radio activity :

Spontaneous decay of naturally occurring unstable nuclei by emission of certain sub particles (like α, β, and γ radiation) is called natural radio activity.

The emission of these rays takes place because of the instability of the nucleus. In the process of emitting these rays a nucleus tries to attain the stability.

In general natural radioactivity takes place in heavy nuclei beyond lead in the periodic table. There are also naturally radioactive light nuclei, such as potassium isotope ${}_{19}K^{40}$, the carbon isotope ${}_{6}C^{14}$ and the rubidium isotope ${}_{37}Rb^{87}$.

Regarding radioactivity.

i) It is completely uneffected by the physical and chemical conditions to which the nucleus is subjected i.e we cannot change the radio activity by applying high temperature, high pressure and strong electric field etc.

ii) The nucleus can disintegrate immediately (or) it may take infinite time.

iii) The energy liberated during the radioactive decay comes from individual nuclei.

Modes of Decay:

The radioactive nucleus before decay is called a parent nucleus, the nucleus resulting from its decay by particles (Radiation) emission is called daughter nuclei.

This daughter nuclei may be stable (or) unstable.

$$\underset{\text{Parent}}{X} \longrightarrow \underset{\text{Daughter}}{Y} + \underset{\text{Radiation}}{R} + \underset{\text{Energy}}{Q}$$

Here R may be either α particle (or) β particle (or) γ radiation. Q is the energy of the emitted particles (or radiation).

α **-decay : When a nucleus disintegrates by radiating α -rays, it is said to undergo α -decay.** An α - particle is a helium nucleus. Thus a nucleus emitting an α particle losses two protons and two neutrons, as a result its atomic number Z decreases by 2, the mass number A decreases by 4 and the neutron number N decreases by 2.

$${}_{z}X^{A} \rightarrow {}_{z-2}Y^{A-4} + {}_{2}He^{4} + Q$$

Ex: ${}_{88}Ra^{226} \rightarrow {}_{86}Rn^{222} + {}_{2}He^{4} + 4.87MeV$

Both electric charge and nucleon number are conserved in the process of α decay.

Application : When a stationary Radio active nucleus x decays into another nucleus y by emitting an α -particle. $x \rightarrow y + \alpha$ particle $+Q$

Applying LCLM if α particle moves forward with a momentum 'P' then daughter nucleus y recoils with same momentum 'P' so that total momentum of the system is zero. Hence $\boxed{P_y = P_\alpha}$

The energy released 'Q' is in the form of K.E of daughter nucleus 'y' and 'α' particle.

$$\boxed{Q = KE_y + KE_\alpha}$$

Ratio of kinetic energies $\boxed{\dfrac{KE_y}{KE_\alpha} = \dfrac{M_\alpha}{M_y}}$

$\left(\because KE = \dfrac{P^2}{2m}\right.$ and $KE \ \alpha \ \dfrac{1}{m}$ when 'P' is same)

$$1 + \frac{KE_y}{KE_\alpha} = 1 + \frac{M_\alpha}{M_y}\ ; \quad \frac{KE_\alpha + KE_y}{KE_\alpha} = \frac{M_y + M_\alpha}{M_y}$$

$$\boxed{KE_\alpha = Q\left(\frac{M_y}{M_\alpha + M_y}\right)}\ ; \quad \boxed{KE_y = Q\left(\frac{M_\alpha}{M_\alpha + M_y}\right)}$$

Notice that KE_α is very close to (but smallerthan) Q.

β **-Decay : When a nucleus disintegrates by radiating $\beta -$ rays, it is said to undergo $\beta -$ decay.**

i) β particles are nothing but electrons. Hence when a nucleus emits a β particle, the atomic number (Z) increases by 1 unit, but the mass number does not change.

The general form of $\beta -$ decay can be written as

$${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + {}_{-1}e^{0}.$$

Ex: ${}_{90}Th^{234} \rightarrow {}_{91}Pa^{234} + {}_{-1}e^{0}$

Both electric charge and nucleon number are conserved in β decay also.

γ **-Decay: When a nucleus disintegrates by radiating $\gamma -$ rays, it is said to undergo $\gamma -$ decay.**

Gamma rays are nothing but electromagnetic radiations of short wavelengths (not exceeding 10^{-10}m.)

The emission of $\gamma -$ rays from the nucleus does not alter either atomic number Z or mass number A. It just results in the change of the energy state of a nucleus.

When a parent nucleus emits an α or a β particle, the daughter nucleus may be formed in one of excited states. Such a nucleus will eventually comes to the ground state. In this process $\gamma -$ radiation will be emitted.

$${}_{Z}X^{A} \rightarrow {}_{Z}X^{A} + \gamma \ Photon(s)$$

Example: ${}_{38}{}^{*}Sr^{87} \rightarrow {}_{38}Sr^{87} + \gamma$.

${}_{38}Sr$ is isomer of ${}_{38}Sr$.

Note : When a Radio active nucleus emits an α - particle followed by two β - particles, its isotope is formed.

$${}_{Z}X^{A} \xrightarrow{\alpha} {}_{Z-2}Y^{A-4} \xrightarrow{2\beta^{-}} {}_{Z}X^{A-4}$$

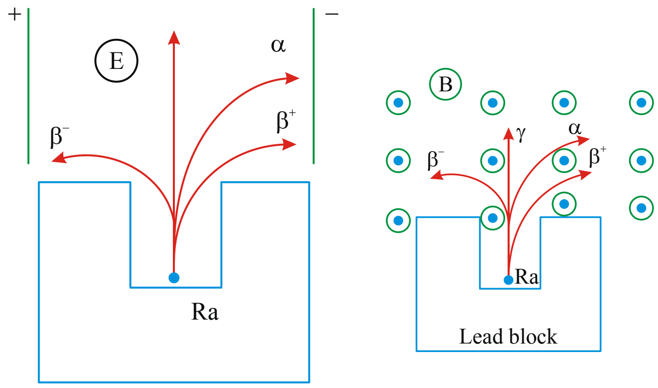
Note : When a Radio active nucleus emits a β - particle its isobar is formed.

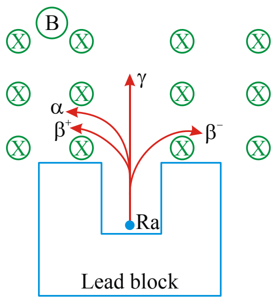
$${}_{Z}X^{A} \xrightarrow{\beta^{-}} {}_{Z+1}Y^{A}$$

Note: When a Radio active nucleus emits a γ - particle its isomer is formed

$${}_{Z}X^{A} \xrightarrow{\gamma} {}_{Z}X^{A}$$

Deflection of Radioactive radiations in electric and magnetic fields :





EX. 9: The nucleus $^{23}_{10}Ne$ decays by β^- emission. Write down the β-decay equation and determine the maximum kinetic energy of the electrons emitted. Given that :
m ($^{23}_{10}Ne$) = 22.994466u; m($^{23}_{11}Na$)=22.989770 u

Sol. $^{23}_{10}Ne \rightarrow ^{23}_{11}Na + \bar{e} + \bar{v} + Q$

For β^- - decay, Q = [M(x) - M(y)]C^2 = [22.994466-22.989770]931.5

= 0.004696 x 931.5 = 4.37 MeV

EX. 10: Calculate the binding energy of an α -particle. Given that mass of proton = 1.0073u, mass of neutron = 1.0087u. and mass of α -particle = 4.0015u.

Sol. m_p = 1.0073u, m_N = 1.0087u, M = 4.0015u

N = A -Z =4 -2 =2 $\quad \left(Q_2\ He^4 = _Z X^A\right)$

B.E = Δm x 931.5 MeV

$= \left\{\left[Zm_p + (A-Z)m_n\right] - M\right\} \times 931.5$

$\left[\left[(2\times1.0073)+(2\times1.0087)-4.0015\right]\right]\times 931.5MeV$

= 0.0305 $\times$ 931.5 MeV ; B.E = 28.4 MeV

EX. 11: How many α and β - particles are emitted when uranium nucleus $({}_{92}U^{238})$ decay to ${}_{82}Pb^{214}$?

Sol. Let n be the number of α - particles and m be the number of β - particles emitted.

${}_{92}U^{238} \rightarrow {}_{82}Pb^{214} + n_2He^4 + m\ {}_{-1}e^0.$

As mass is conserved, 238 = 214 + 4n + m (0)

= 214 + 4n ; 4n = 24; n = 6

As charge is conserved , 92 = 82 + 2n + m (-1)

10 = 2(6) - m $(\because n = 6)$; m = 2.

$\therefore 6\alpha$ - particles and 2 β - particles are emitted

Radioactive Decay Law:

Based on their experimental observations and analysis of certain radioactive materials Rutherford and Soddy formulated a theory of radioactive decay. According to them

After decay of a nucleus the new product (daughter) of nucleus has totally different physical as well as chemical properties.

The rate of radioactive decay (or) the number of nuclei decaying per unit time at any instant is directly proportional to the number of nuclei (N) present at that instant and is independent of the external physical conditions like temperature, pressure etc.

Let 'N' be the number of radioactive atoms present at a time 't' and N_0 is the initial number of radio active nuclei. Let dN atoms disintegrate in time 'dt'. According to the law of radioactive decay

$\left(\frac{dN}{dt}\right) \propto N$; $\left(\frac{dN}{dt}\right) = -\lambda N$ .... (1)

The proportionality constant λ is called decay constant (or) disintegration constant. The negative sign indicates that as time increases N decreases.

From eqn (1) $\frac{dN}{N} = -\lambda\, dt$ ....(2)

Integrating eq (2) on both sides $\int \frac{dN}{N} = -\lambda \int dt$

$\log_e N = -\lambda t + C$ .....(3)

Here C is the constant of integration

At t = O, N= N_0 Substituting in eqn (3),

we get, $\log_e N_0 = C$

$\therefore \log_e N = -\lambda t + \log_e N_0$;

$\therefore \log_e N - \log_e N_0 = -\lambda t$

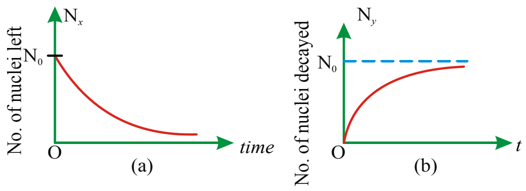
$\therefore \log_e \left(\frac{N}{N_0}\right) = -\lambda t$; $\frac{N}{N_0} = e^{-\lambda t}$; $\boxed{N = N_0 e^{-\lambda t}}$ ... (4)

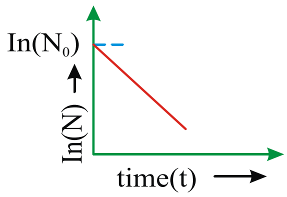
This shows that the number of radioactive nuclei decreases exponentially with time.

Above equation is known as the decay law (or) the law of radio-active decay. It is an exponential law.

taking logarithm on both sides for the above equation. $\log_e N = \log_e N_o - \lambda t$; $\lambda t = \log_e \frac{N_0}{N}$

$$\therefore \boxed{t = \frac{1}{\lambda} \ln\left(\frac{N_0}{N}\right)}$$





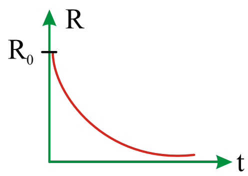
(C)

Activity (R) :

The number of decays per unit time (or) decay rate is called activity (R)

$$|R| = \left|\frac{dN}{dt}\right| = \frac{d}{dt}\left(N_0 e^{-\lambda t}\right) \text{(or)} \ R = \lambda N = \lambda N_0 e^{-\lambda t} \ \text{(or)}$$

$R = R_0 e^{-\lambda t}$, where $R_0 = \lambda N_0$ is the decay rate at $t = 0$, called initial activity.



If a nucleus can decay simultaneously by n processes, which have activities R_1, R_2, ... and R_n. Then the resultant activity $R = R_1 + R_2 +$............... $+ R_n$. **If nucleus decays simultaneously more than one process is called parallel decay.**

The S.I unit of activity is **Becquerel (Bq)** and other units are **curie (Ci) and Rutherford (Rd).**

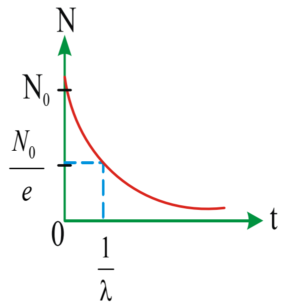
1 Bq = 1 decay per second,

1 Rd = 10^6 decays per second.

1 Ci = 3.7×10^{10} decays per second.

Note : Curie is approximately equal to the activity of one gram of pure radium.

Decay Constant (λ) : It gives the ability of a nucleus to decay. The decay constant λ for a given radio active sample is defined as the reciprocal of the time during which the number of nuclei decreases to $\frac{1}{e}$ times their original value.



1) **Larger value of λ corresponding to decay in smaller time and vice versa.**
2) $\lambda = 0$ for stable nuclei.
3) Decay constant is the characteristic of the sample taken and does not vary with time.
4) If a nucleus can decay simultaneously by more than one process (say n), which have decay constants $\lambda_1, \lambda_2 ....and\ \lambda_n$, then the effective decay constant is $\lambda = \lambda_1 + \lambda_2 + ... + \lambda_n$. This is called parallel decay.

Half life (T) : As the name suggests, the half life of a radioactive sample is defined as "The time interval during which the activity of a radio active sample falls to half of its value, (or) The time interval during which the number of radio active nuclei of a sample disintegrate to half of its original number of nuclei" Half lives vary from isotope to isotope. While T may be as small as 10^{-16} s, its largest value may be as big as 10^9 years.

Eg: Half-life of uranium $\left({}^{238}_{92}U\right)$ is 4.47 x 10^9 years

half-life of krypton $\left({}^{89}_{36}Kr\right)$ is 3.16 minutes.

Relation between decay constant (λ) and half life period (T).

From Law of Radioactive decay $\frac{N}{N_0} = e^{-\lambda t}$

when $N = \frac{N_0}{2}$, $t = T$ $\therefore \frac{1}{2} = e^{-\lambda T}$ or $2 = e^{\lambda T}$

taking logirthms on both sides $\ln 2 = \lambda T$

(or) $\log_e 2 = \lambda T$ $\therefore T = \frac{2.303 \log_{10} 2}{\lambda} = \frac{0.693}{\lambda}$

$$\therefore \boxed{T = \frac{\ln 2}{\lambda} = \frac{2.303 \log 2}{\lambda} = \frac{0.693}{\lambda}}$$

The above relation establishes that the half - life (T) depends upon the decay constant λ of the radioactive substance. The value of λ is different for different radioactive substances.

Note :

i) Half life is the characteristic property of the sample and T cannot be changed by any known method.

ii) At any given instant whatever be the amount of the undecayed sample, it will be reduced to exactly half its value after a time equal to the half life of the sample.

iii) In parallel decay $\lambda = \lambda_1 + \lambda_2 + ...\lambda_n$ hence $\frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2} + ...\frac{1}{T_n}$, where T is the equivalent half-life and T_1, T_2 ....T_n are the half-lives in individual decay.

Application :

In a radioactive sample the number of nuclides undecayed after n-half lives (i.e., t = nT) is

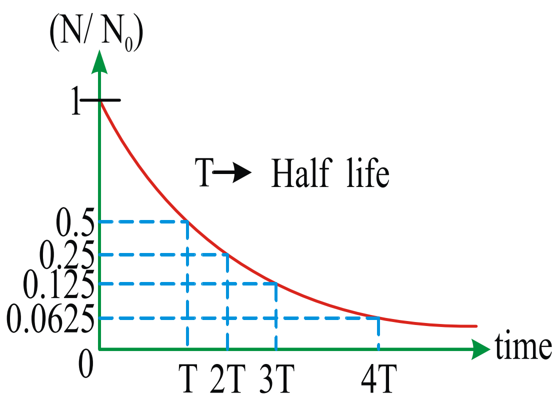
$$t = nT = \frac{1}{\lambda} \ln\left(\frac{N_0}{N}\right) \text{ or } \frac{n(\ln 2)}{\lambda} = \frac{1}{\lambda}\ln\left(\frac{N_0}{N}\right)$$

or $2^n = \frac{N_0}{N}$; or $\boxed{N = N_0\left(\frac{1}{2}\right)^n}$

Note: The number of nuclei remain in the sample after **half of half life period (t=1/2T) is** given by $N = N_0\left(\frac{1}{2}\right)^n$ here $n = \frac{1}{2}$ then $N = N_0\left(\frac{1}{2}\right)^{\frac{1}{2}}$

$\therefore \boxed{N = \frac{N_0}{\sqrt{2}}}$ taking $N_0 = 100$, $N = 50\sqrt{2} = 70.7$

70.7% of nuclei remain and 29.3% of nuclei decayed.



Average life (or) Mean life :

The phenomenon of radioactivity is random because we just can't predict which of the atoms in a given sample will decay first and when. Hence radioactivity process totally depends on chance. In decay process some of the atoms of the given sample may have very short life span, and others may not decay even after a very large span of time. So to determine the ability of the nucleus to decay it would be useful to calculate the average life. **Hence average life is defined as the total life time of all the nuclei divided by the total number of original nuclei.**

i.e $\tau = \dfrac{\sum \text{life span of individual nucleus}}{\text{Total number of original nuclei}} = \dfrac{\sum t}{N_0}$

Let N_0 be the radio active nuclei that are present at t = 0 in the radioactive sample.

The number of nuclei which decay between t and (t + dt) is dN i.e the life time of these nuclei is 't'.

The total life time of these dN nuclei is (t dN)

$\therefore$ The total life time of all the nuclei present initially in the sample = $\int_{t=0}^{t=\infty} t\,dN$ [$\because$ N = 0 at infinity]

Average life time $\tau = \dfrac{\int t\,dN}{N_0}$ But $\dfrac{-dN}{dt} = \lambda N$

$dN = -\lambda N dt = -\lambda N_0 e^{-\lambda t} dt \quad (\because N = N_0 e^{-\lambda t})$

$\tau = \int_0^{\infty} t \dfrac{\lambda N_0 e^{-\lambda t}}{N_0} dt$; $\boxed{\tau = \dfrac{1}{\lambda}}$

The mean life (or) average life of a radio active sample is reciprocal to decay constant.

We know that $N = N_0 e^{-\lambda t}$; When $t = \tau$, $\quad N = N_0 e^{-\frac{1}{\tau} x \tau} = \dfrac{N_0}{e} = 0.37 N_0 = 37\%$ of N_0

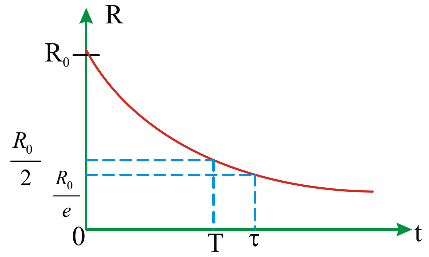
Hence average life period of a radio active sample can also be defined as **"The time interval during which 63% of sample decays or sample reduces to 37% of its original amount"**.

Relation Between Half Life Period and Average Life Period

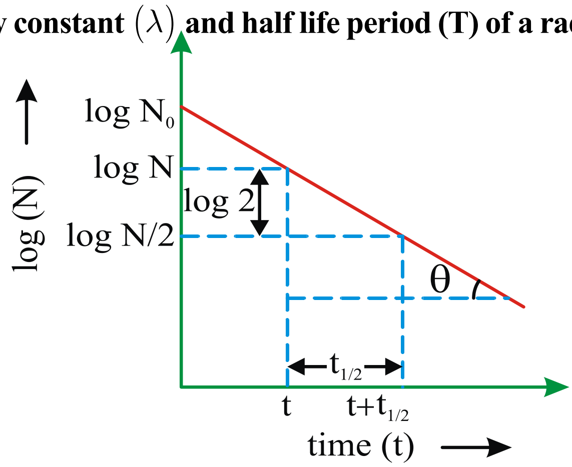
We know that $T = \dfrac{0.693}{\lambda}$ & $\tau = \dfrac{1}{\lambda}$

Hence $T = 0.693\tau$ (or) $\boxed{\tau = \dfrac{T}{0.693} = 1.443T}$

From the above equation it is clear that average life period is 44.3% greater than half life period.



Determination of decay constant (λ) and half life period (T) of a radioactive sample graphically



If N_0 and N be the number of atoms present undecayed initially and after a time t, then

We know that $N=N_0\ e^{-\lambda t}$ taking log on both sides

$$\log_e N = \log_e N_0 - \lambda t \Rightarrow \log N = \log N_0 - \frac{\lambda t}{2.303}$$

$$\log N = \left(\frac{-\lambda}{2.303}\right)t + \log N_0$$

Slope of the graph $m = -\tan\theta = \frac{-\lambda}{2.303}$

$$\Rightarrow \boxed{\lambda = 2.303\tan\theta}$$

Half life period $T = \frac{2.303\log 2}{\lambda}$

$$T = \frac{2.303\log 2}{2.303\tan\theta} \qquad \therefore \boxed{T = (\log 2)\cot\theta}$$

Note : In radioactive sample decay

1) The probability survival of nucleus after time $P_s = \frac{N}{N_0} = e^{-\lambda t}$.

2) The probability of nucleus to disintegrate in time t is $P_d = 1 - P_s = 1 - e^{-\lambda t}$.

EX. 12: A radioactive sample has an activity of 5.13 x10^7 Ci. Express its activity in 'becqueral' and 'rutherford'.

Sol. Since 1 Ci = 3.7 x 10^{10} decays per second,

activity = 5.13 x 10^7 Ci

=5.13 x 10^7 x 3.7 x 10^{10} Bq = 1.9 x 10^{18} Bq

Since , 1 x 10^6 decay per second = 1Rd

Activity =1.9 x 10^{18} Bq $= \frac{1.9\times10^{18}}{1\times10^{6}} Rd$ = 1.9 x 10^{12} Rd.

EX. 13:A radioactive substance has 6.0×10^{18} active nuclei initially. What time is required for the active nuclei of the same substance to become 1.0×10^{18} if its half-life is 40 s.

Sol. The number of active nuclei at any instant of time t,

$$\frac{N_0}{N}=e^{\lambda t};\quad \log_e\left(\frac{N_0}{N}\right)=\lambda t$$

$$\therefore t=\frac{\log_e\left(\frac{N_0}{N}\right)}{\lambda}=\frac{2.303\log_{10}\left(\frac{N_0}{N}\right)}{\lambda}$$

In this problem, the initial number of active nuclei,

$N_0=6.0\times10^{18}$; $N=1.0\times10^{18}, T=40s,$

$$\lambda=\frac{0.693}{T}=\frac{0.693}{40}=1.733\times10^{-2}\,s^{-1}.$$

$$t=\frac{2.303\log_{10}\left(\frac{6.0\times10^{18}}{1.0\times10^{18}}\right)}{1.733\times10^{-2}}$$

$$=\frac{2.303\log_{10}(6)}{1.733\times10^{-2}}=\frac{2.303\times0.7782}{1.733\times10^{-2}}=103.4s.$$

EX. 14: A radioactive sample can decay by two different processes. The half-life for the first process is T_1 and that for the second process is T_2. Find the effective half-life T of the radioactive sample.

Sol. Let N be the total number of atoms of the radioactive sample initially. Let $\frac{dN_1}{dt}$ and $\frac{dN_2}{dt}$ be the initial rates of disintegrations of the radioactive sample by the two processes respectively.

Then $\frac{dN_1}{dt}=\lambda_1 N$ and $\frac{dN_2}{dt}=\lambda_2 N$

Where λ_1 and λ_2 are the decay constants for the first and second processes respectively.

The initial rate of disintegrations of the radioactive sample by both the processes

$$=\frac{dN_1}{dt}+\frac{dN_2}{dt}=\lambda_1 N+\lambda_2 N=(\lambda_1+\lambda_2)N.$$

If λ is the effective decay constant of the radioactive sample , its initial rate of disintegration.

$$\frac{dN}{dt}=\lambda N$$

But $\frac{dN}{dt}=\frac{dN_1}{dt}+\frac{dN_2}{dt}$

$\lambda N=(\lambda_1+\lambda_2)N$

$\lambda=\lambda_1+\lambda_2$

$\frac{0.693}{T_1}+\frac{0.693}{T_2}=\frac{0.693}{T}$

$\frac{1}{T}=\frac{1}{T_1}+\frac{1}{T_2};T=\frac{T_1T_2}{T_1+T_2}.$

EX. 15: Plutonium decays with a half life of 24,000 years. If plutonium is stored for 72,000 years, what fraction of it remains?

Sol. $T_{1/2}$ = 24,000 years

Duration of time (t) = 72,000 years

Number of half lifes (n) = $\frac{t}{T_{1/2}} = \frac{72000}{24000} = 3$

$\therefore\ 1g\xrightarrow{1}\frac{1}{2}g\xrightarrow{2}\frac{1}{4}g\xrightarrow{3}\frac{1}{8}g$

$\therefore$ Fraction of plutonium remains = $\frac{1}{8}g$

EX. 16: A certain substance decays to 1/32 of its initial activity in 25 days. Calculate its half -life.

Sol. $1\text{g}\xrightarrow{1}\frac{1}{2}\text{g}\xrightarrow{2}\frac{1}{4}\text{g}\xrightarrow{3}\frac{1}{8}\text{g}\xrightarrow{4}\frac{1}{16}\text{g}\xrightarrow{5}\frac{1}{32}\text{g} \qquad \therefore n=5$

$(n)=\frac{t}{t_{1/2}}\Rightarrow t_{1/2}=\frac{t}{n}=\frac{25}{5};\ t_{1/2}=5\,\text{days}$

EX. 17: The half -life period of a radioactive substance is 20 days. What is the time taken for 7/8th of its original mass to disintegrate?

Sol. Let the initial mass be one unit.

Mass reamaining = $1-\frac{7}{8}=\frac{1}{8}$

A mass of 1 unit becomes $\frac{1}{2}$ unit in 1 half life

$\frac{1}{2}$ unit becomes $\frac{1}{4}$ unit in 2nd half life

$\frac{1}{4}$ unit becomes $\frac{1}{8}$ unit in 3rd half life

$\therefore$ Time taken = 3 half lifes = 3 x 20 = 60 days

EX. 18: How many disintegrations per second will Occur in one gram of $^{238}_{92}U$, if its half-life against α -decay is 1.42 x 10^{17}s?

Sol. Given Half -life period (T) $= \dfrac{0.693}{\lambda} =$

1.42×10^{17} s

$$\lambda = \frac{0.693}{1.42 \times 10^{17}} = 4.88 \times 10^{-18}$$

Avagadro number (N) = 6.023 $\times$ 10^{23} atoms

n = Number of atoms present in 1 g of $^{238}_{92}U = \dfrac{N}{A}$

$$= \frac{0.623 \times 10^{23}}{238} = 25.30 \times 10^{20}$$

Number of disintegrations $= \dfrac{dN}{dt} = \lambda n$

= 4.88 x10^{-18} x 25.30 x 10^{20} = 1.2346 x 10^{4} disintegrates/sec

EX. 19: One gram of radium is reduced by 2 milligram in 5 years by α -decay. Calculate the half-life of radium.

Sol: Initial mass = 1 g, t = 5 years

Reduced mass = 2mg = 2 x 10^{-3} g $= \dfrac{2}{1000}$ g

Remaining mass = 1 - $\dfrac{2}{1000} = \dfrac{998}{1000}$

$$\frac{N}{N_0} = \frac{998}{1000} (\because Mass \propto Number\ of\ atoms)$$

$$\frac{N}{N_0} = e^{-\lambda t} \ ; \ \frac{998}{1000} = e^{-\lambda t}$$

$$\frac{1000}{998} = e^{\lambda t} = e^{5\lambda} \Rightarrow \log_e\left(\frac{1000}{998}\right) = 5\lambda$$

2.303 (3.0000 -2.9991) = 5 λ

$$\lambda = \frac{2.303 \times 1 \times 0.0009}{5}$$

$$(T_{1/2}) = \frac{0.693}{\lambda} = \frac{0.693 \times 5}{2.303 \times 0.0009} = 1671.7 \text{ years}$$

EX. 20: The half-life of a radioactive substance is 5000 years . In how many years, its activity will decay to 0.2 times of its initial value? Given $\log_{10}5 = 0.6990$.

Sol. T = 5000 years,

$$\frac{N}{N_0} = 0.2 = \frac{2}{10} = \frac{1}{5}$$

$$\lambda = \frac{0.693}{T} = \frac{0.693}{5000}$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$\frac{1}{5} = \frac{1}{e^{\lambda t}} \Rightarrow 5 = e^{\lambda t}$$

$\log_e 5 = \lambda t$

$2.303 \times 0.6990 = \lambda t$

$$t = \frac{2.303 \times 0.6990 \times 5000}{0.693}$$

t = 11614.6years $= 1.1615 \times 10^4$ years

EX. 21: Obtain the amount of ${}^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half -life of ${}^{60}_{27}Co$ is 5.3 years.

Sol. Half - life of ${}^{60}{}_{27}Co$ = 5.3 years

= 5.3x365x24 x 60 x 60 S = 5.3 x 3.15 x 10^7 s

Now `x` gm of ${}^{60}{}_{27}Co$ contains $\frac{10^{-3}x}{60}$ k - mole = $\frac{x \times 10^{-3}}{60} \times 6.025 \times 10^{26}$

N = 0.1004x x 10^{23} = 1.004 x x 10^{22} atoms

Required strength of ${}^{60}{}_{27}Co$ = 8 m Ci

= 8 x 10^{-3} x 3.7 x10^{10} dis/s

We know that, decay rate (R) = λN

$$N = \frac{R}{\lambda} = \frac{8 \times 3.7 \times 10^7}{0.693 / T_{1/2}}$$

$$= \frac{29.6 \times 10^7}{0.693} \text{x5.3x3.15x}10^7 = 713.0909 \times 10^{14}$$

1.004x x 10^{22} = 713.0909 x 10^{14}

x = 710.2499 x 10^{-8} kg = 7.1 x 10^{-6} g

Artificial Transmutation of Elements

The conversion of one element into another by artificial means is called artificial transmutation of the element. Rutherford performed number of experiments in which the atoms of different stable elements, such as nitrogen, aluminium, phosphorus, etc, were bombarded by high speed $\alpha -$particles from natural radioactive substances. Finally in 1919, he discovered the phenomenon of artificial transmutation.

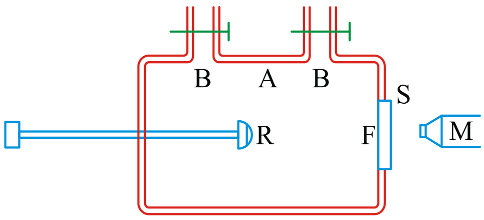


Fig.

The apparatus used by Rutherford is as shown in Fig.

i. It consists of a chamber A provided with an adjustable rod, carrying a radio - active substance R (Radium C).
ii. The side of the glass tube facing 'R' is covered by metal plate with a central hole which is closed by a thin silver foil 'F'.
iii. A screen 'S', coated with a fluorescent material like zinc sulphide is arranged infront of the silver foil and the scintillations produced on it can be observed through the microscope 'M'.
iv. The side tubes B, B were used to fill various gases in the chamber.
v. The source of α - particles, Ra was placed on a small disc at R. Its distance from F was adjustable.
vi. The radio-active substance emits α -particles whose range in air was found to be about 7cm.
vii. When the glass tube is filled with nitrogen gas, scintillations are observed, even when 'R' is at a distance of 40cm from the foil.
viii. These particles producing scintillations can not be α -particles as they can not have such a long range.
ix. Rutherford concluded that nitrogen nucleus hit by an α $({}_2He^4)$-particle transmutes into oxygen nucleus along with a proton $({}_1H^1)$.
x. The nuclear reaction causing artificial transmutation can be represented as ${}_7N^{14} + {}_2He^4 \rightarrow {}_8O^{17} + {}_1H^1$

Thus an atom of nitrogen is transformed into an isotope of oxygen. This process is called transmutation of elements.

* High energy α - particles were used in the discovery of artificial transmutation and neutron because α - particles produce intense ionisation of the medium through which they pass and can be stopped after travelling a few mm in air.

Significance :

i) It leads to the discovery of proton and neutron.
ii) It helps to produce radio isotopes.
iii) It helps to produce transuranic elements.

Radio isotopes and their uses:

Radio isotopes have very short half lives and hence used for various purposes.

1) Medical applications :

a) $_{13}Al^{27} + _0n^1 \rightarrow _{11}Na^{24*} + _2He^4$

Radio – sodium is used to find out how a given medicine is circulated in the body. It is also used to find out circulatory disorders in blood vessels.

b) $_{15}P^{31} + _1H^2 \rightarrow _{15}P^{32*} + _1H^1$

Radio – phosphorus is used in the treatment of skin diseases. It is also used for the treatment of blood disorders.

c) $_{53}I^{127} + _0n^1 \rightarrow _{53}I^{128*} + \gamma - \text{rays}$

Radio – iodine is used in the treatment of thyroid glands. Radio - iodine (I^{131}) is used for diagnosis and treatment of brain tumor and for the study of pumping condition of heart.

d) $_{27}Co^{59} + _0n^1 \rightarrow _{27}Co^{60*} + \gamma - \text{rays}$ Radio – cobalt is used in the detection and treatment of cancer

e) Radio-iron is used to detect anemia and treat anemia.

2) In Geology:

a) Radio carbon(C^{14}) is used to determine the age of fossils by radio - carbon dating

b) Radio isotopes are used to determine the age of rocks by the ratio of U^{238} to Pb^{206}

3) In industry :

a) Radio – isotopes are used to find the wear and tear of machine parts

b) Radio isotopes are used to detect flaws in metal structures

c) Radio isotopes are used for treatment of alloys such as quenching , annealing and hardening.

d) Radio isotopes are used in the selection of appropriate lubricants.

4) In research: Radio - isotopes are used in the study of nuclear disintegrations of elements.

5) In food preservation: By exposing vegetables and other food stuffs to radiations from radio - active isotopes, their shelf life can be increased.

6) In agriculture:

a) Radio phosphorus (P^{32}) is used to study the uptake of phosphorus by plants using.

b) Radio sulphur (S^{34}) is used to study the transport ofminerals in plants .

c) Radio zinc is used to develop new species of plants by causing genetic mutation.

d) Irradiation by γ - radiations of seeds to improve yields.

7) In Chemistry :

a) Radio oxygen (O^{18}) is used to study the mechanisms of photosynthesis and hydrolysis of ester

b) Radio isotopes are used in the chemical analysis of solubility of sparingly soluble salts such as $PbSO_4$ and AgCl and determination of trace amounts of elements in industrial raw materials and products.

Neutron

- It is electrically neutral and its mass is slightly greater than that of proton. It was discovered by chadwick
- Bothe – Becker equation:

$$_{4}Be^{9} + {}_{2}He^{4} \rightarrow \left[{}_{6}C^{13}\right] \rightarrow {}_{6}C^{12} + {}_{0}n^{1}$$

- Neutron is unstable outside the nucleus.

$_{0}n^{1} \rightarrow {}_{1}H^{1} + {}_{-1}e^{0} + \bar{\nu}$ (anti neutrino)

- It has high penetrating power and low ionizing power.
- Slow moving neutrons are called thermal neutrons. Fast moving neutrons convert into thermal neutrons when they pass through a substance called moderator.
- Thermal neutrons have an average energy of nearly 0.025 eV. Fast moving neutrons have an average energy of 2 MeV.

Nuclear Fission

- Nuclear Fission is a nuclear reaction in which a heavy atomic nucleus like U^{235} splits into two approximately equal parts, emitting neutrons and liberating large amount of energy.
- Bohr and Wheeler proposed liquid drop model to explain this fission process.
- Nucleus of U^{235} undergoes fission when it is struck by slow neutrons. This fission is not due to the impact of neutron.
- Energy of about 200 MeV is released during one fission reaction of $_{92}U^{235}$. The most probable nuclear fission reaction is

$${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3\,{}_{0}n^{1} + energy$$

There is no guarantee that U^{235} always breaks into Barium and Krypton.

- On an average,in the fission of U^{235}, 2.5 neutrons are emitted per fission when fission occurs due to slow neutrons. U^{235} undergoes fission with fast neutrons also. But this probability is minimum.
- Fission fragments are unstable and emit neutrons some time after fission reaction which are called "delayed neutrons"
- 99% of neutrons emitted during fission process are prompt.
- Delayed neutrons play an important role in chain reaction

Chain Reaction:

- If the mass of fissionable material exceeds a critical value, chain reaction or self propagating fission reaction takes place.
- The rate of reaction increases in geometric progression during uncontrolled chain reaction.
- Chain-reaction : The process of continuation of nuclear fission which when once started continues spontaneously without the supply of additional neutrons from outside is defined as chain reaction.

Reproduction factor (K): "It is the ratio of number of neutrons in any particular generation to the number of neutrons in the preceeding generation.

Case(i): K<1 ; Chain reaction is not maintained.(sub-critical state)

Case (ii): K=1 : Chain reaction is maintained at steady rate. (critical state). In the state electricity is produced in the reactors at steady rate

Case (iii): K>1 : Chain reaction becomes self sustained and lead to atomic explosion(super critical state)

- Uncontrolled chain reaction takes place in atom bomb.

Nuclear Reactor or atomic pile:

- Nuclear reactor is a device in which nuclear fission is produced by controlled self sustaining chain reaction. And is used for the production of nuclear power (energy).
- The essential parts of a nuclear reactor are (i) the fuel, (ii) moderator, (iii) control rods, (iv) coolant, (v) radiation shields.
- **THE FUEL:** The common fuels used are uranium $\left(U^{238}\right)$, enriched uranium (U^{235}) and plutonium $\left(Pu^{236}\right)$ and Th^{232}.

Moderator:

- The function of a moderator is to slow down the fast moving neutrons to increase the rate of fission.
- The commonly used moderators in the order of efficiency are (i) Heavy water (ii) graphite, (iii) Berillium and Berillium Oxide
- Heavy water is a best moderator
- A good moderator should have

  1) low atomic mass 2) poor absorption of neutrons

  3) good scattering property.

  4) The size of moderator atom should be nearly of same size as that of the size of a prompt neutron.

Control Rods:

- The function of a control rod is to absorb (capture) the neutrons.
- Cadmium, Boron and steel rods are used as control rods in a nuclear reactor.
- Cadmium rods are best control rods
- They regulate the net rate of neutron production and hence they control the intensity of fission process.

Coolant:

- The function of a coolant is to keep the reactor temperature at a low value so that there may not be any danger of heat damage to the reactor.
- Air and CO_2 are used as gaseous coolants. Water, Organic liquids, Helium, Liquid Sodium are used as liquid coolants. Liquid sodium is best coolant.
- **Protective shield:** The process of preventing radioactive effect around nuclear reactor is called Protective Shield.
- During the working of a nuclear reactor dangerous radiations such as high energy neutrons, gamma rays and thermal radiations are produced. To protect the persons working there, the reactor is thoroughly shielded with concrete wall of several feet thick and lined with metals like lead.

Power of A Nuclear Reactor

In the nuclear reactor, large amount of heat will be generated in the core. These reactors have elaborate cooling systems that use water. This water absorbs the heat and produces stea m. This steam in turn is used to run the steam turbines which ultimately generate electric power. Such reactors are called power reactors.

The power generated by a nuclear reactor is $\boxed{P=\frac{nE}{t}}$ here $\frac{n}{t}$ be the number of fissions per second and E be the energy released in each fission

E = 200 MeV $=200\times10^{6}\times1.6\times10^{-19}J$

= 3.2 x 10^{-11}J

Note : Number of fissions per sec in a reactor of power 1 W is given by $\frac{n}{t}=\frac{P}{E}$

$=\frac{1}{3.2\times10^{-11}}$ = 3.125 x 10^{10} fissions per sec

Note : If only x% of energy released in fission is converted into electrical energy then out put power of reactor is $P=\frac{x}{100}\left(\frac{nE}{t}\right)$

Note: If ‘x’gm of fuel with mass number ‘A’ completely undergo nuclear fission in time t sec in a reactor then its power is given by

Number of moles in x gm of fuel = $\frac{x}{A}$

Number of atoms (nuclei) present in x gm of fuel $n=\left(\frac{x}{A}\right)N_A$. Where N_A is Avogadro number

$\therefore$ power $P=\frac{nE}{t}$ $\Rightarrow$ $\boxed{P=\frac{xN_AE}{At}}$

Uses of Nuclear Reactors:

1) To generate electric power.

2) To produce nuclear fuel plutonium -239 and other radioactive materials which have a wide variety of applications in the fields of medicine, industry and research.

Uses of atomic energy :

1. **Generation of electric power :** The coolant in a nuclear reactor absorbs the heat generated as a result of the chain reaction and it releases the heat to the water which is converted into high pressure steam. This steam is used to drive turbine and operate the electric generator.

2. **Production of radio isotopes** : A small amount of the pure element is placed in an aluminium container and the container is placed in the reactor for a few days. The element absorbs neutrons and the element becomes radioactive isotope.

$${}_{53}I^{127} + {}_{0}n^{1} \rightarrow {}_{53}I^{128*} + \gamma .$$

Radioiodine obtained in this way can be used to treat the thyroid gland. These radio isotopes have a number of applications in the field of medicine , agriculture, industry and basic research.

3. **Source of neutrons :** A large number of neutrons are produced in a reactor. They are used in research . The effect of neutrons on biological tissues is studied. A new branch of physics called Neutron Physics has come up.
4. Atomic energy is used to create artificial lakes, to divert the course of a river , to make tunnels for laying new railway tracks etc.
5. Atomic energy is used for driving automobiles, submarines and war - planes.
6. Atomic energy is used in war - fare for creating destructive atom bombs and hydrogen bombs.

Nuclear Fusion:

- The phenomenon in which two lighter nuclei combine to form a heavier nucleus of mass less than the total mass of the combining nuclei is called nuclear fusion. This mass defect appears as energy.
- At temperatures of about 10^7K, light nuclei combine to give heavier nuclei. Hence, fusion reactions are called thermo nuclear reactions.
- Nuclear fusion takes place in the sun and other stars.
- Energy produced in a single fission of ${}_{92}U^{235}$ is larger than that in a single fusion of Hydrogen into Helium.
- But fusion produces more energy than fission per nucleon.
- In fission, 0.09% of mass is converted into energy. In fusion 0.66% of mass is converted into energy.
- Hydrogen bomb is a fission – fusion bomb.

Stellar and solar energy:

Stellar and solar energy is due to fusion.

The cycles that occur are. Proton - Proton Cycle & Carbon - Nitrogen Cycle

Proton - Proton Cycle:

The Thermonuclear reactions involved are:

$$2({}_{1}H^{1}) + 2({}_{1}H^{1}) \rightarrow 2({}_{1}H^{2}) + 2({}_{+1}e^{0}) + Q_1$$

$$2({}_{1}H^{2}) + 2({}_{1}H^{1}) \rightarrow 2({}_{2}He^{3}) + Q_2$$

$${}_{2}He^{3} + {}_{2}He^{3} \rightarrow ({}_{2}He^{4}) + 2({}_{1}H^{2}) + Q_3$$

On adding up these reactions, we obtain.

$$4({}_{1}H^{1}) \rightarrow {}_{2}He^{4} + 2\,{}_{+1}e^{0} + Q$$

Where $Q = Q_1 + Q_2 + Q_3$ is the total energy evolved in the fusion of 4 hydrogen nuclei (protons) to form Helium nucleus. The value of Q as calculated from mass defect comes out to be 26.7MeV

Carbon - Nitrogen Cycle:

Proposed by bethe. It consists of following reactions.

$_{6}C^{12} + _{1}H^{1} \rightarrow _{7}N^{13} + Q_1$; $_{7}N^{13} \rightarrow _{6}C^{13} + (_{1}e^{0}) + Q_2$

$_{6}C^{13} + _{1}H^{1} \rightarrow _{7}N^{14} + Q_3$; $_{7}N^{14} + _{1}H^{1} \rightarrow _{8}O^{15} + Q_4$

$_{8}O^{15} \rightarrow _{7}N^{15} + (_{+1}e^{0}) + Q_5$;

$_{7}N^{15} + _{1}H^{1} \rightarrow _{6}C^{12} + _{2}He^{4} + Q_6$

On adding all these 6 equations, we get

$4\,_{1}H^{1} \rightarrow _{2}He^{4} + 2(_{+1}e^{0}) + Q$

Where $Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6$ The value of Q as calculated from mass defect is 26.7 Mev.

- In the sun both proton - proton cycle and carbon- nitrogen cycles occur with equal probabilities. In stars, whose interior temperatures are less than that of the sun, proton -proton cycle dominates the energy generation. Again in stars, whose interior temperatures are more than that of the sun, the energy generation is mainly due to carbon- nitrogen cycle.
- The core temperature of heavier stars may be larger than that of the sun and much larger nuclei may be formed.

NOTE: Due to enormous energy released in Sun and Stars the atmosphere of them will be in ionised state which is called Plasma (Which contains fast moving neutrons and electrons). Nuclear fusion can not be controlled.

Nuclear Fission	Nuclear Fusion
1) Neutrons are required for it	1) Protons are required for it
2) It is possible at normal pressure and temperature	2) It is possible at high pressure and temperature
3) Energy released per nucleon $\cong 0.9 Mev$	3) Energy released per nucleon $\cong 6 Mev$
4) % of mass getting converted into energy =0.1%	4) % of m ass getting converted into energy =0.7%
5) Fissionable materials are expensive	5) Fusion materials are cheap
6) Harmful reactions are produced	6) Harmful reactions are not produced

EX. 22: An explosion of atomic bomb releases an energy of 7.6 x 10^{13} J. If 200 MeV energy is released on fission of one ^{235}U atom calculate (i) the number of uranium atoms undergoing fission. (ii) the mass of uranium used in the atom bomb

Sol: E=7.6x10^3J;Energy released per fission = 200 MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11}$ J

$$\text{Number of uranium atoms (n)} = \frac{\text{Total energy}}{\text{Energy per fission}}$$

$$n = \frac{7.6 \times 10^{13}}{3.2 \times 10^{-11}} = 2.375 \times 10^{24} \text{ atoms}$$

Avagadro number (N) = 6.023×10^{23}atoms

Mass of uranium =

$$\frac{n \times 235}{N} = \frac{2.375 \times 10^{24} \times 235}{6.023 \times 10^{23}} = 92.66\text{g}$$

EX. 23: Calculate the energy released by fission from 2 g of $^{235}_{92}$U in kWh. Given that the energy released per fission is 200 MeV.

Sol. Mass of uranium = 2g

Energy released per fission = 200 MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11}$ J

Number of atoms in 2 gram of uranium is

$$n = \frac{2 \times 6.023 \times 10^{23}}{235} = 5.125 \times 10^{21} \text{ atoms}$$

Total energy released = No. of atoms x energ released per fission

$= 5.125 \times 10^{21} \times 3.2 \times 10^{-11} = 16.4 \times 10^{10}$J

$$\therefore \text{Energy in Kwh} = \frac{16.4 \times 10^{10}}{36 \times 10^5} \text{Kwh}$$

$= 0.455 \times 10^5$ Kwh $= 4.55 \times 10^4$ Kwh

EX. 24: 200 Mev energy is released when one nucleus of ^{235}U undergoes fission. Find the number of fissions per second required for producing a power of 1 megawatt.

Sol. Energy released = 200MeV

$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11}$J

P = 1 mega watt = 10^6 watts.

$$\text{No.of fissions per second (n)} = \frac{\text{Total energy}}{\text{Energy per fission}}$$

$$n = \frac{10^6}{3.2 \times 10^{-11}} = 3.125 \times 10^{16} \text{ Fissions}$$

EX. 25: How much ^{235}U is consumed in a day in an atomic power house operating at 400 MW, provided the whole of mass ^{235}U is converted into energy?

Sol. Power = 400 MW = 400 x 10^6 W;

time = 1 day = 86, 400 s.

Energy produced , E = power $\times$ time = 400 $\times$ 10^6 $\times$ 86,400 = 3.456 $\times$ 10^{13} J.

As the whole of mass is converted into energy , by Einstein's mass -energy relation.

E = Mc^2

$$\frac{E}{c^2} = \frac{3.456 \times 10^{13}}{(3\times 10^8)^2} = 3.84 \times 10^{-4}\ kg = 0.384\ g.$$

EX. 26: How long can an electric lamp of 100W be kept glowing by fusion of 2.0 kg of deuterium ? Take the fusion reaction as $^2_1H + ^2_1H \rightarrow ^3_2He + n + 3.27$ MeV

Sol. $_1H^2 +_1 H^2 \rightarrow^3_2 He + n + 3.27 MeV$

No. of atoms in 2 kg of $_1H^2$ = 2/2 x 6.023 x 10^{26}

= 6.023 x 10^{26} atoms

In the above reaction two deuterium nuclei are combined

Power (p) = w x rate of fusion.

$$= 3.27 \text{ MeV x } \frac{Number\ of\ atoms}{Time\ expended}$$

$$100 = 3.27 \text{ x } 10^6 \text{ x } 1.6 \text{ x } 10^{-19} \text{ x } \frac{6.023\times10^{26}}{2x}$$

$$\therefore x = \frac{3.27\times1.6\times6.023\times10^{+11}}{2} = 15.756 \text{ x } 10^{11}\text{S}$$

$$= \frac{15.756\times10^{11}}{365\times24\times60\times60} = \frac{15.756\times10^{11}}{3.15\times10^7} = 5\text{x}10^4 \text{ years}$$

EX. 27: Suppose India had a target of producing by 2020 AD, 200,000 MW of electric power, ten percent of which was to be obtained from nuclear power plants. Suppose we are given that, on an avedrage, the efficiency of utilization (i.e conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020 ? Take the heat energy per fission of ^{235}U to be about 200 MeV.

Sol. Required power from nuclear plants

= 10% of 2,00,000 Mw = 2 x 10^{10}w

Required electric energy from nuclear plants in one year = 2 x 10^{10} x 365 x24 x 60 x 60

= 2 x 10^{10} x 3.15 x 10^7 = 6.30 x10^7 J

Available electric energy per fission = 25% of 200 MeV = 50 MeV = 8 x 10^{-12}J

$$\text{Req. no. of fissions per year} = \frac{6.30\times10^{17}}{8\times10^{-12}} = 0.7875\text{x}10^{29}$$

$$\text{Req. no. of moles of } U^{238} = \frac{0.7875\times10^{29}}{6.023\times10^{23}} = 0.1307 \text{ x}10^6$$

Required mass of U^{238} = 0.1307 x 235 x10^6 g

= 30.71 x10^6 gm = 30.71 x10^6 x 10^{-3} kg

= 0.03071 x 10^6 kg = 3.071 x 10^4 kg

EX. 28: Calculate the energy released by the fission 1 g of ^{235}U in joule, given that the energy released per fission is 200 MeV.
(Avogadro's number = 6.023 $\times$ 10^{23})

Sol. The number of atoms in 1 g of ^{235}U

$$= \frac{Avogadro's\ number}{Mass\ \ number} = \frac{6.023 \times 10^{23}}{235} = 2.563 \times 10^{21}$$

Energy released per fission = 200 MeV

$$= 200 \times 10^6 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-11} J.$$

Energy released by 1 g of ^{235}U

= Number of atoms $\times$ energy released per fission

$$= 2.563 \times 10^{21} \times 3.2 \times 10^{-11} J = 8.202 \times 10^{10} J$$

EX. 29: In the process of nuclear fission of 1 gram uranium, the mass lost is 0.92 milligram. The efficiency of power house run by it is 10%. To obtain 400 megawatt power from the power house, how much uranium will be required per hour? (c = 3 x 10^8 ms^{-1})

Sol. Power to be obtained from power house = 400 mega watt\ Energy obtained per hour = 400 meagwatt x 1hour = (400 x 10^6 watt) x 3600 second = 144 x 10^{10} joule

Here only 10% of input is utilised. In order to obtain 144 x 10^{10} joule of useful energy, the output energy from the power house $\frac{10E}{100}$ = 144 x 10^{10} J

E = 144 x 10^{11} joule

Let, this energy is obtained from a mass–loss of Δm kg.

Then $(\Delta m)c^2 = 144 \times 10^{11}$ joule

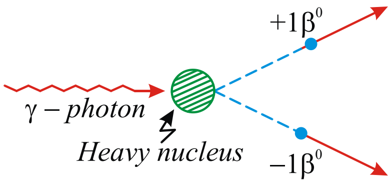
$$\Delta m = \frac{144 \times 10^{11}}{\left(3 \times 10^8\right)^2} = 16 \times 10^{-5} kg \quad = 0.16 \text{ g}$$

Since 0.92 milli gram (= 0.92 x 10^{-3} g) mass is lost in 1 g uranium, hence for a mass loss of 0.16g, the uranium required is $= \frac{1 \times 0.16}{0.92 \times 10^{-3}} = 174\ g$

Thus to run the power house, 174 gm uranium is required per hour.

PAIR AND PRODUCTION AND PAIR ANNIHILATION : When an energetic γ- photon falls on a heavy nucleus, it is absorbed by the nucleus and a pair of electron and positon is produced. This phenomenon is called as pair production and can be represented by the following equation:

$$\underset{(\gamma-photon)}{hv} = \underset{(Positron)}{_{+1}\beta^0} + \underset{(electron)}{_{-1}\beta^0}$$



The rest mass energy of electron or positron is:

$$E_0 = m_0c^2 = \left(9.1\times10^{-31}\right)\times\left(3\times10^8\right)^2$$

$$= 8.2\times10^{-14} J \; ; \; 0.51 MeV.$$

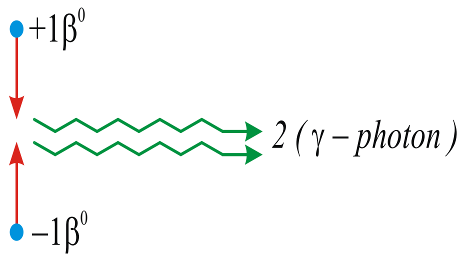
Hence for pair production, the minimum energy of γ-photon must be $2\times0.51 = 1.02 MeV$. If the energy of γ-photon is less than this, there may be Compton's effect. If energy of γ-photon is greater than E_0, then extra energy will become kinetic energy of the particles. If E is the energy of γ-photon, then kineric energy of each particle will be,

$$K_{electron} = K_{positron} = \frac{E - 2E_0}{2}$$

The inverse process of pair production is called pair annihilation. According to it when electron and a positron come close to each other, annihilate each other and produces minimum two γ-photons.

Thus

$$\underset{(Positron)}{_{+1}\beta^0} + \underset{(electron)}{_{-1}\beta^0} = \underset{(\gamma-photon)}{2hf}$$



Additional information

Elementary particles :

We have realized, so far that there are only four fundamental constituents of matter. We can describe various physical processes involving atoms, molecules and nuclei in terms of electrons, protons, neutrons and photons. The first three are the building blocks of atoms and hence matter. The fourth one (i.e photon) is the quantized energy which is exchanged whenever electronic or nucleonic transition is involved.

Subsequently many more elementary particles and antiparticles have been discovered, using giant and modern accelerating machines.

The particles which are not constituted by any other particles are called Elementary particles. A brief discussion of important fundamental particles is as follows.

i) **Electron :** It was discovered in 1897 by Thomson. Its charge is –e and mass is 9.1 X 10^{-31} kg. Its symbol is e^- (or $_{-1}\beta^0$). It is a stable particle having spin = 1/2

ii) **Proton :** It was discovered in 1919 by Rutherford in artificial nuclear disintegration. It has a positive charge +e and its mass is 1836 times (1.673 X 10^{-27}kg) the mass of electron. In free state, the proton is a stable particle. Its symbol is P^+. It is also written as $_1H^1$. It is a stable particle having spin = 1/2.

iii) **Neutron :** It was discovered in 1932 by Chadwick. Electrically it is a neutral particle. Its mass is 1839 times (1.675 X 10^{-27} kg) the mass of electron. In free state the neutron is unstable. Inside the nucleus the neutron is stable. Its symbol is n (or) $_0n^1$.

iv) **Positron :** It was discovered by Anderson in 1932. It is the antiparticle of electron, i.e., its charge is +e and its mass is equal to that of electron. Its symbol is e^+ (or $_{+1}\beta^0$)

v) **Antiproton :** It is the antiparticle of proton. It was discovered in 1955. Its charge is –e and its mass is equal to that of proton. Its symbol is P^-.

vi) **Antineutron :** It was discovered in 1956. It has no charge and its mass is equal to the mass of neutron. **The only difference between neutron and antineutron is that their magnetic momenta will be equal in magnitude and opposite in direction**. The symbol for antineutron is $\bar{n}$.

vii) **Neutrino and antineutrino :** The existence of these particles was predicted in 1930 by Pauli while explaining the emission of β- particles from radioactive nuclei, but these particles were actually observed experimentally in 1956. Their rest mass and charge are both zero but they have energy and momentum. These are mutually antiparticles of each other. They have the symbol ν and $\bar{\nu}$

viii) **Pi - Mesons :** The existence of pi - mesons was predicted by Yukawa in 1935, but they were actually discovered in 1947 in cosmic rays. Nuclear forces are explained by the exchange of pi-mesons between the nucleons. pi - mesons are of three types, positive π - mesons (π^+), negative pi-mesons (π^-) and neutral π - mesons (π^0). Charge on $\pi^\pm$ is $\pm$e. Whereas mass of $\pi^\pm$ is 274 times the mass of electron. π^0 has mass nearly 264 times the electronic mass. These are unstable having half life 10^{-8} sec and spin = 0

ix) Mu-Mesons : These were discovered in 1936 by Anderson and Neddermeyer. These are found in abundance in the cosmic rays at the ground level. There are two types of mu-mesons. Positive mu-meson (μ^+) and negative mu-meason (μ^-). There is no neutral mu-meson. Both the mu-mesons have the same rest mass 207 times the rest mass of the electron. These are unstable having half life 10^{-6} sec and spin = 1/2.

x) Photon : These are bundles of electromagnetic energy and travel with the speed of light. Energy and momentum of a photon of frequency ν are $h\nu$ and $\frac{h\nu}{c}$ respectively. They posses no charge. no mass and spin = 1 and are stable.

xi) Gravitons : Hypothetical particles that carry gravitational energy are called Gravitons. They possess no mass, no charge and spin = 2 as proposed by Dirac.

Antiparticles : An antiparticle is a form of matter that has the same mass as the particle but carries an opposite charge and / or a magnetic moment that is oriented in an opposite direction relative to the spin.

Name of the particle	Antiparticle
Electron (e^{-1})	Positron (e^{+1})
Proton(P^+)	Anti proton (P^-)
Neutron (n)	Anti Neutron ($\bar{n}$)
Neutrino (v)	Anti Neutrino ($\bar{\nu}$)
Positive Pi-Meson (π^+)	Negative Pi-Meson(π^-)
Positive Mu-Meson(μ^+)	Negative Mu-Meson(μ^-)

Note: A few electrically neutral particles, like the photon and neutral π meson are their own antiparticles. A collision between a particle and an antiparticle results in annihilation of matter.

Classification of particles based on spin

1) **Bosons :** These particles have spin in the integral multiples of unity
2) **Fermions :** These particles have spin in the integral multiples of 1/2.

Classification of particles based on rest mass

1) **Photons :** Particles with zero rest mass
2) **Leptons :** Lighter particles
3) **Mesons :** Particles with intermediate mass
4) **Baryons :** Heavier particles

Classification of particles based on interaction

1) **Photons:** representing electromagnetic interactions.
2) **Leptons:** representing weak interactions.
3) **Hadrons:** representing strong interactions.
4) **Gravitons:** representing gravitational interactions.

EX. 30: An electron-positron pair is produced when a γ -ray photon of energy 2.36MeV passes close to a heavy nucleus. Find the kinetic energy carried by each particle produced, as well as the total energy with each.

Sol. The reaction is represented by

$\gamma \to ({}_{-1}e^0) + ({}_{+1}e^0)$, sothat

$E = m_0C^2 + K.E_{electron} + m_0C^2 + K.E_{positron}$ 2.36MeV = $2m_0.C^2$ + K.E $_{(electron)}$ + K.E $_{(positron)}$

= 1.02 MeV + $K.E_{(e^-)} + K.E_{(e^+)}$

K.E. of $(e^-) = K.E_{(e^+)} = \frac{1}{2}(2.36 - 1.02)MeV$,

(K.E. carried each) = 0.67 MeV (motional energy)

Total energy shared by each particle is obviously $m_0C^2 + K.E = 0.51MeV + 0.67MeV$ = 1.18MeV.

EX. 31: A gamma ray photon of energy 1896 MeV annihilates to produce a proton-antiproton pair. If the rest mass of each of the particles involved be 1.007276 a.m.u approximately, find how much K.E these will carry?

Sol. Working on the same lines as an electron-positron pair production, we notice that the reaction. $\gamma \to$ proton + antiproton, has the energy balance

E = $m_{0\ (proton)}\ C^2$ + K.E $_{(proton)}$ +

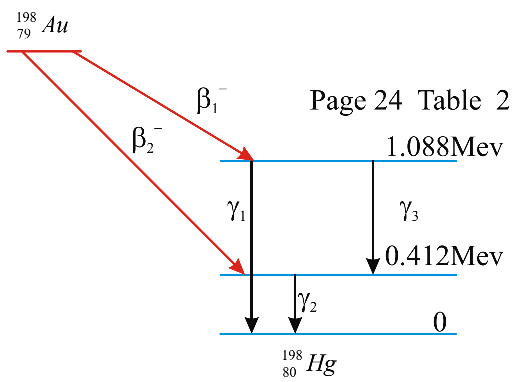
$m_{0\ (antiproton)}C^2$ + K.E $_{(antiproton)}$

But m_0C^2 = energy equivalent of 1.007276 a.m.u $\approx$ 938 MeV. [Q 1.007276 x 931 $\approx$ 938 MeV]

Thus K.E of each particle

$= \frac{1}{2}$ [1896 MeV - 2 $\times$ 938 MeV] = 10MeV.

EX. 32: Obtain the maximum kinetic energy of β -particles, and the radiation frequencies of γ decays in the decay scheme shown in Fig. 14.6. You are given that m(^{198}Au) = 197.968233 u; m(^{198}Hg) = 197.966760 u



Sol. γ-rays are electro magnetic radiations having energy E = hυ $\Rightarrow \upsilon = \frac{E}{h}$ where h = plank`s constant = 6.625 x 10^{-34} J.S

1) Frequencies of γ_1, γ_2 and γ_3 are calculated as follows

$$\gamma_1 = \frac{\Delta E}{h} = \frac{(1.088-0)MeV}{6.625\times10^{-34}J.s} = \frac{1.088\times10^6\times1.6\times10^{-19}}{6.625\times10^{-34}}$$

= 0.2627 x 10^{21} = 2.627 x 10^{20} Hz

$$\gamma_2 = \frac{\Delta E}{h} = \frac{(0.412-0)MeV}{6.625\times10^{-34}J.s} = \frac{0.412\times1.6\times10^{-19}\times10^6}{6.625\times10^{-34}}$$

= 0.0995 x10^{21} = 9.95 x 10^{19}Hz

$$\gamma_3 = \frac{\Delta E}{h} = \frac{(1.088-0.412)\times10^6\times1.6\times10^{-19}}{6.625\times10^{-34}}$$

= 0.1632 x 10^{21} = 1.632 x 10^{20} Hz

2) Now maximum K.E of β_1^- = [M($^{198}_{79}$Au)

M($^{198}_{80}$Hg)-$\frac{1.088}{931.5}$] c^2

($\because$ 1 amu = 931.5 MeV$\Rightarrow$ 1MeV = $\frac{1}{931.5}U$)

= [197.968233-197.966760-0.001168]931.5 MeV

= 0.000305 x 931.5 = 0.284 MeV

Maximum K.E of β_2^- =

[M($^{198}_{79}$Au) - M($^{198}_{80}$Hg) - $\frac{0.412}{931.5}$] c^2

= [197.968233-197.966760-0.000442]931.5

= 0.001031 x 931.5 =0.9603 MeV

EX. 33 A radioactive isotope is being produced at a constant rate A. The isotope has a half -life T initially there are no nuclei, after a time t >>T, the number of nuclei becomes constant. The value of this constant is

1) AT 2) $\frac{A}{T}\ln(2)$ 3) $AT\ln(2)$ 4) $\frac{AT}{\ln(2)}$

Sol. key(4) $A = N\lambda$; $\therefore N = \frac{A}{\lambda} = \frac{AT}{\ln 2}$

EX. 34 The probability of survival of a radioactive nucleus for one mean life is

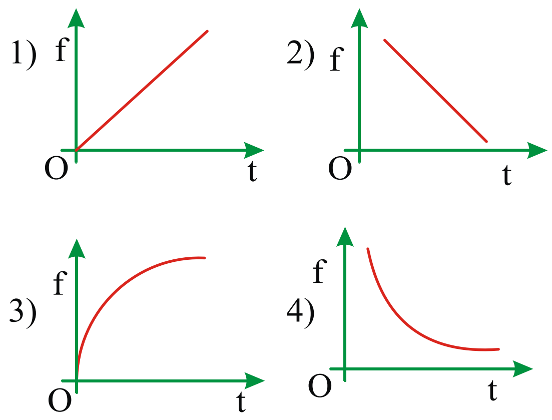
1) $\frac{1}{e}$ 2) $1-\frac{1}{e}$ 3) $\frac{\ln 2}{e}$ 4) $1-\frac{\ln 2}{e}$

Sol. key(1) Probability of survival for any nucleus at time t is

$$P = \frac{N}{N_0} = \frac{N_0 e^{-\lambda t}}{N_0} = e^{-\lambda t}$$

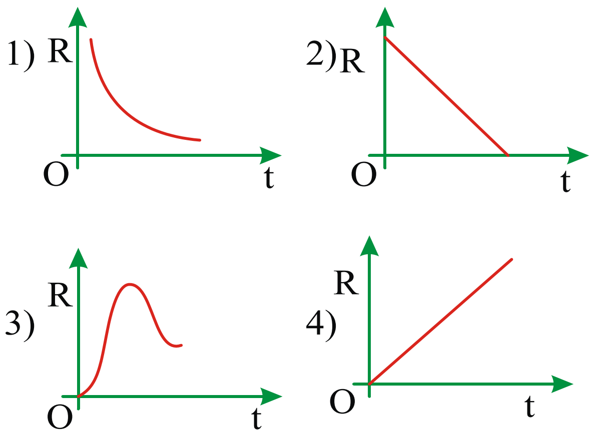
So, in one mean life, required probability is $e^{-\lambda \times \frac{1}{\lambda}} = \frac{1}{e}$;

EX. 35 The fraction f of radioactive element decayed change with respect to time (t). The curve representing the correct variation is



Sol. key(3)

EX. 36 The rate of decay (R) of nuclei in a radiocative sample is plotted against time (t). Which of the following best represents the resulting curve?



Sol. key(1)

EX. 37 A sample of uranium is a mixture of three isotopes ${}_{92}U^{234}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$ present in the ratio 0.006%, 0.71% and 99.284% respectively. The half lives of then isotopes are 2.5 x 10^5 years, 7.1 x 10^8 years and 4.5 x 10^9 years respectively. The contribution to activity (in %) of each isotope in the sample respectively

1) 51.41%, 2.13%, 46.46%

2) 51.41%, 46.46%, 2.13%

3) 2.13%, 51.41%, 46.46%

4) 46.46%, 2.13%, 51.41%

Sol. key(1) Let m is the total mass of the uranium mixture. The masses of the isotopes ${}_{92}U^{234}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$ in the mixture are $m_1 = \frac{0.006}{100}m$;

$m_2 = \frac{0.71}{100}m$, and $m_3 = \frac{99.284}{100}m$.

If N_A is the Avogadro number, then number of atoms of threeisotopes are; $N_1 = \frac{m_1 N_A}{M_1}$,

$N_2 = \frac{m_2 N_A}{M_2}$, and $N_3 = \frac{m_3 N_A}{M_3}$

Activity of radioactive sample $A = \lambda N$.

As $\lambda = \frac{0.693}{t_{1/2}}$, $\therefore$ $A = \frac{0.693}{t_{1/2}}N$

If t_1, t_2 and t_3 be the half lives, then

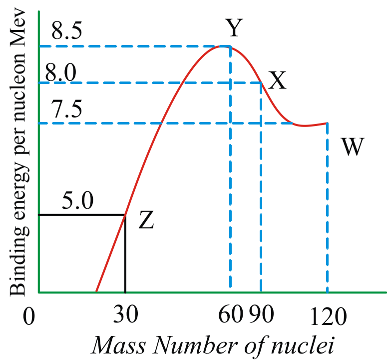
$$A_1 : A_2 : A_3 = \frac{N_1}{t_1} : \frac{N_2}{t_2} : \frac{N_3}{t_3}$$

or $A_1 : A_2 : A_3 = \dfrac{m_1}{M_1 t_1} : \dfrac{m_2}{M_2 t_2} : \dfrac{m_3}{M_3 t_3}$

$$= \frac{0.006}{234\left(2.5\times10^5\right)} : \frac{0.71}{235\left(7.5\times10^8\right)} : \frac{99.284}{238\left(4.5\times10^9\right)}$$

$$= 51.41\% : 2.13\% : 46.46\%$$

EX. 38 Binding energy per nucleon versas mass number curve for nuclei is shown in the fig. W,X,Y and Z are four nuclei indicated on the curve. The process that would release energy is



1) $Y \to 2Z$ 2) $W \to X + Z$

3) $X \to Y + Z$ 4) $W \to 2Y$

Sol. .key(4) Energy of products must be more then the reactants to release energy.

EX. 39 A small quantity of a solution containing N^{24} radio - nuclide of half - life T and activity R_0 is injected into blood of a person. $1\,cm^3$ of sample of blood taken from the blood of the person shows activity R_1. If the total volume of the blood in the body of the person is V, find the timer after which sample is taken.

1) $\dfrac{T}{\ln(2)}\left[1n\dfrac{R_0}{VR_1}\right]$ 2) $\dfrac{T}{\ln(2)}\left[1n\dfrac{VR_0}{R_1}\right]$

3) $\dfrac{T}{\ln(2)}\left[1n\dfrac{VR_1}{R_0}\right]$ 4) $\dfrac{T}{\ln(2)}\left[1n\dfrac{R_1}{VR_0}\right]$

Sol. . key(1) Total volume of blood,

$$V = \frac{\text{Total activity R}}{\text{Activity per cm}^3 (R_1)}$$

$$= \frac{R_0 e^{-\lambda t}}{R_1}, \text{ (or) } \frac{VR_1}{R_0} = e^{-\lambda t}$$

$$-\ln\left(\frac{VR_1}{R_0}\right) = \lambda t = \frac{\ln(2)t}{T};\ t = \frac{T}{\ln(2)}\ln\left(\frac{R_0}{VR_1}\right)$$

EX. 40 A nucleus with mass number 220 initially at rest emits an α - particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of α -particle.

1) 4.4 MeV 2) 5.4 MeV 3) 5.6MeV 4) 6.5 MeV

Sol. .key(2) $K_1 + K_2 = 5.5\,MeV$

From conservation of linear momentum

$$[2K_1(216m)]^{1/2} = [2K_2(4m)]^{1/2};\ K_2 = 54K_1$$

From the above $K_2 = 5.4\,MeV$

EX. 41 Some amount of radioactive substance (half-life=10 days) is spread inside a room and consequently the level of radiation becomes 50times the permissible level for normal occupancy of the room. The room be safe for occupation after

1) 20days 2) 34.8 days 3) 56.4 days 4) 62.9 days

Sol. .key(3) Since the intial activity is 50 times the activity for safe occupancy, therefore, $R_0 = 50R$ where $R = \lambda N$ since,

$R \propto N$

$$\frac{R}{R_0} = \frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{t/T} \quad or$$

$$\left(\frac{1}{2}\right)^{t/10} = \frac{1}{50}$$

EX. 42 The fraction of a radiactive sample will decay during half of its half-life period is

1) $\frac{1}{\sqrt{2}}$ 2) $\frac{1}{\sqrt{2}-1}$ 3) $\frac{\sqrt{2}-1}{\sqrt{2}}$ 4) $\frac{1}{2}$

Sol. .key(3) $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{\frac{t}{T}}$ *Here,* $t = \frac{T}{2}$ *or* $\frac{t}{T} = \frac{1}{2}$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}};\ \frac{N_0 - N}{N_0} = 1 - \frac{1}{\sqrt{2}} = \frac{\sqrt{2}-1}{\sqrt{2}}$$

EX. 43 In moon rock sample the ratio of the number of stable argon-40 atoms present to the number of radioactive potassium-40 atoms is 7:1. Assume that all the argon atoms were produced by the decay of potassium atoms, with a half-life of $2.5\times10^9\,yr$. The age of the rock is

1) $2.5\times10^9\,yr$ 2) $5.0\times10^9\,yr$

3) $7.5\times10^9\,yr$ 4) $10^{10}\,yr$

Sol. .key(3) Let the number of radioactive Potassium atoms present initially (t=0) is N_0 and the number of stable argon atoms at t=0 is zero. After time t the number of stable argon atoms is m and the radioactive potassium atoms is $N_0 - m$ given that

$$\frac{N_0 - m}{m} = \frac{1}{7}, m = \frac{7}{8}N_0 \ and \ N_0 - m = \frac{1}{8}N_0$$

since after one half-life N_0 reduces to $N_0/2$ after 2 half-lives $N_0/4$ and after 3 half-lives it reduces to $N_0/8$

Thus, $t = nT = 3\times2.5\times10^9$ years

$= 7.5\times10^9$ year

EX. 44 The half-life of a radioactive sample is T. If the activities of the sample at time t_1 adn t_2 ($t_1 < t_2$) are R_1 and R_2 respec tively, then the number of atoms disintegrated in time $t_2 - t_1$ is proportional to

1) $(R_1 - R_2)T$ 2) $(R_1 + R_2)T$

3) $\frac{R_1R_2}{R_1 + R_2}T$ 4) $\frac{R_1 + R_2}{T}$

Sol. .key(1) Activity $R = \lambda N$

so that

$$R_1 = \lambda N_1 \ and \ R_2 = \lambda N_2$$

$$R_1 - R_2 = \lambda(N_1 - N_2) = \frac{0.6931}{T}(N_1 - N_2)$$

$$N_1 - N_2 = \frac{(R_1 - R_2)T}{0.6931}$$

$$N_1 - N_2 \propto (R_1 - R_2)T\ ; \ \therefore t_2 - t_1 \alpha (R_1 - R_2)T$$

EX. 45 **The half life of a radioactive substance is 20 minutes.The approximate time interval $(t_2 - t_1)$ between the time t_2, when 2/3 of it has decayed and time t_1 and 1/3 of it had decayed is**

1) 14 minutes 2) 20 minutes

3) 28 minutes 4) 7 minutes

Sol. . key(2) At t_1 $\frac{2}{3} = \frac{1}{2^{t_1/20}}$; At t_2 $\frac{1}{3} = \frac{1}{2^{t_2/20}}$

$t_2 - t_1 = 20$ mins.

EX. 46 A charged capacitor of capacitance C is discharged through a resistance R. A radio active sample decays with an average life t. Find R interms of C and t in order that the ratio of the electrostatic energy stored in the capacitor to the activity of the radio active sample remains constant with time

1) $\frac{2t}{C}$ 2) $\frac{C}{2t}$ 3) 2t C 4) t C

Sol. key(1) $\frac{U_c}{A}=$constant; $\frac{\frac{1}{2}CV_o^2}{\lambda N_o}=\frac{\frac{1}{2}CV^2}{\lambda N}$

Where $V=V_o e^{-t/CR}$, $N=N_o e^{-\lambda t}$ and $\lambda=\frac{1}{t}$

EX. 47 A radioactive sample can decay by two differentprocesses. The half-life for the first process is T_1 and that for the second process is T_2. The effective half-life T of the radioactive sample is

1) $T=T_1+T_2$ 2) $\frac{1}{T}=\frac{1}{T_1}+\frac{1}{T_2}$

3) $T=\frac{T_1+T_2}{T_1T_2}$ 4) $T=\frac{T_1-T_2}{T_1T_2}$

Sol. . key(2) $\frac{dN}{dt}\propto\lambda N$; $\lambda=\lambda_1+\lambda_2$, $\lambda\propto\frac{1}{T}$

EX. 48 In nuclear fusion,One gram hydrogen is converted into 0.993gm.If the efficiency of the generator be 5%,then the energy obtained in KWH is

1) 8.75×10^3 2) 4.75×10^3 3) 5.75×10^3 4) 3.73×10^3

Sol. key(1). Efficency $=\frac{output}{\Delta mc^2}$

EX. 49 A photon of energy 1.12 Mev splits into electron positron pair. The velocity of electron is (Neglect relativistic correction)

1) 3 x 10^8 ms^{-1} 2) 1.33 x 10^8 ms^{-1}

3) 6 x $10^8$$ms^{-1}$ 4) 9 x $10^8$$ms^{-1}$

.**Sol.** key(2) $E_\gamma=\left(\frac{1}{2}mv^2\right)2+2E_0$

EX. 50. A sample of radioactive material has mass m, decay constant λ and molecular weight M. Avagadro constant = N_A . The activity of the sample after time t will be

1) $\left(\frac{mN_A}{M}\right)e^{-\lambda t}$ 2) $\left(\frac{mN_A\lambda}{M}\right)e^{-\lambda t}$

3) $\left(\frac{mN_A}{M\lambda}\right)e^{-\lambda t}$ 4) $\frac{m}{\lambda}\left(1-e^{-\lambda t}\right)$

Sol. key(2). activity = $\lambda N_o e^{-\lambda t}$ where $N_o = \frac{N_A m}{M}$

EX. 51 **Samples of two radioactive nuclides A and B are taken. λ_A and λ_B are the disintegration the following cases, the two samples can simultaneously have the same decay rate at any time?**

1) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A = \lambda_B$
2) Initial rate of decay of A is twice the initial rate of decay of B and $\lambda_A > \lambda_B$
3) Initial rate of decay of B is twice the initial rate of decay of A and $\lambda_A > \lambda_B$
4) Initial rate of decay of B is same as the rate of decay of A at t = 2h and $\lambda_B = \lambda_A$

Sol. key(2). $\left|\frac{dN}{dt}\right| \to$ rate of decay $= \lambda N$

(1) $t = 0$

$$\frac{\lambda}{A} N_{0A} = 2\frac{\lambda}{B} N_{0B} \Rightarrow N_{0A} = 2N_{0B}$$

$$\left|\frac{dN}{dt}\right|_A = \lambda_A N_A = \lambda A_A N_{0A} e^{-\lambda_A t} = 2\lambda_A N_{0A} e^{-\lambda A t}$$

$$\left|\frac{dN}{dt}\right|_B = \lambda_B N_B = \lambda_B N_{0B} e^{-\lambda_A t}$$

Q $\lambda_A = \lambda_B$

$$\left|\frac{dN}{dt}\right|_A \neq \left|\frac{dN}{dt}\right|_B$$

(2) $\lambda_A N_{0A} = 2\lambda_B N_{0B}\left(\lambda_A > \lambda_B\right)$

at $t = t$

$$\left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t}$$

$\Rightarrow$ If equal : $\lambda_A N_{0A} e^{-\lambda_A t} = \lambda_B N_{0B} e^{-\lambda_B t}$

$2 = e^{(\lambda_A - \lambda_B)t} \left(\lambda_A > \lambda_B\right) \Rightarrow t$

can have real solution.

(3) Proceed similar to (2) twill have no real soltion.

(4) $\lambda_B = N_{0B} = \lambda_A N_{0A} e^{-2\lambda A}$

$N_{0B} = N_{0A} e^{-2\lambda A}$

at some $t = t,$ if equal rates occur

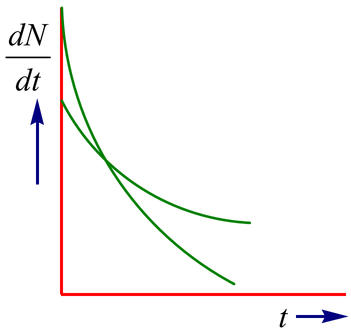
$\lambda_B N_{0B} e^{-\lambda_B t} = \lambda_A N_{0A} e^{-\lambda A t}$

$N_{0B} e^{-\lambda Bt} = \lambda_A N_{0A} e^{-\lambda At} \left(\lambda_A = \lambda_B\right)$

$N_{0B} e^{-\lambda At} = N_{0B} e^{2\lambda A} e^{-\lambda_{At}}$

$\boxed{\Rightarrow e^{2\lambda A} = 1}$ not possible.

EX. 52. **The variation of decay rate of two radioactive samples A and B with time is shown in figure. Which of the following statements are true?**



1) Decay constant of A is greater than that of B, hence A always decays faster than B.
2) Decay constant of B is greater than that of A but its decay rate is always smaller than that of A.
3) Decay constant of A is greater than that of B but it does not always decay faster than B.
4) Decay constant of B is smaller than that of A, but still its decay rate becomes equal to that of A at a later instant.

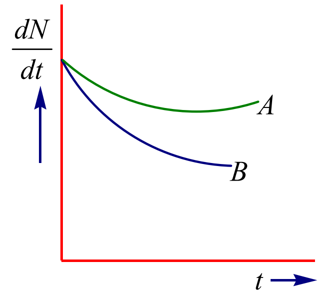
Sol. key (3, 4). $\left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} \Rightarrow \left|\frac{d^2N}{dt^2}\right| = \lambda^2 N_0 e^{-\lambda t}$

at instant when

$\left|\frac{dN}{dt}\right|_A = \left|\frac{dN}{dt}\right|_B, \left(slope\right)_A > \left(slope\right)_B$

$\Rightarrow \lambda_A > \lambda_B.$

EX. 53. **Which sample A or B shown in figure has shorter mean-life?**



1) $\tau_B < \tau_A$ 2) $\tau_B > \tau_A$

3) $\tau_B = \tau_A$ 4) Nothing can be concluded

Sol. key(1). at $t = 0$; $\left|\frac{dN}{dt}\right|_A = \left|\frac{dN}{dt}\right|_B$

but $\left|\frac{d^2N}{dt^2}\right|_B > \left|\frac{d^2N}{dt^2}\right|_A \Rightarrow \lambda_B > \lambda_A \Rightarrow \tau_A < \tau_A$

ADVANCED MAIN POINTS

Q-Value of Energy of a Reaction :

Consider the nuclear reaction $a + x \rightarrow y + b$; where a - bombarding particle x - Target nucleus y - daughter nucleus b - emitted particle. Let m_1, m_2, m_3 and m_4 be the masses of a, x, y & b respectively and k_1, k_2, k_3 and k_4 be their KEs. From principle of conservation of energy

$m_1c^2 + k_1 + m_2c^2 + k_2 = m_3c^2 + k_3 + m_3c^2 + k_4$ or

$\left[(m_1 + m_2) - (m_3 + m_4)\right]c^2 = (k_3 + k_4) - (k_1 + k_2)$

The Q-Values is

$(k_3 + k_4) - (k_1 + k_2) = \left[(m_1 + m_2) - (m_3 + m_4)\right]c^2$

Q-Values of various decays:-

a) For α-decay; ${}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4He$

$Q = \left[m\left({}_Z^AX\right) - m\left({}_{Z-2}^{A-4}Y\right) - m\left({}_2He^4\right)\right]c^2$

b) For β^{-1}-decay; ${}_Z^AX \rightarrow {}_{Z+1}^AY + {}_{-1}^0e + \overline{v}$

$Q = \left[m\left({}_ZX^A\right) - m\left({}_{Z+1}Y^A\right)\right]c^2$

c) For β^+-decay; ${}_Z^AX \rightarrow {}_{Z-1}^AY + {}_{+1}^0e + v$

$Q = \left[m\left({}_ZX^A\right) - m\left({}_{Z-1}Y^A\right) - 2m_e\right]C^2$

d) For K-capture; ${}_{Z}^{A}X + {}_{-1}^{0}e \rightarrow {}_{Z-1}^{A}Y + v$

$$Q = \left[m\left({}_{Z}X^{A}\right) - m\left({}_{Z-1}Y^{A}\right)\right]c^{2}$$

Types of nuclear collisions:-

Exoergic reaction / collision :

i) If Q - value is positive, rest mass energy is converted into kinetic mass energy, radiation or both.

ii) In α-emision, kinetic energy of the emitted

α - particle $= \left(\frac{A-4}{A}\right)Q$

Where A is the mass number of parent nucleus and Q is the Q- value of the reaction.

iii) In β^{-} - decay process, the energy Q is shared by the anti-neutirinos and the beta particle. The Kinetic Energy of the beta particle can be anything between zero and a maximum value Q.

Endoergic Collision / Reaction:

i) If Q is negative, the reaction is endoergic.

ii) Some minimum energy called threshold energy is required to intitiate the nuclear reaction.

$E_{th} = |Q|\left[\frac{m_1}{m_2}+1\right]$. Where m_1 is the mass of the bombarding particle and m_2 is the mass of the target nucleus. The threshold energy is some what greater than $|Q|$ because the outgoing particles must have some kinetic energy to conserve momentum.

Radioactivity law for different types of disintegration.

a. Only disintegration:- $\frac{-dN}{dt} = \lambda N \Rightarrow N = N_0 e^{-\lambda t}$

b. Disintegration with continuous production:-

$$\xrightarrow[q]{\text{formation rate}} A \xrightarrow[(\lambda)]{\text{decay}} B(\text{stable})$$

$$\frac{dN}{dt} = \lambda N - q \Rightarrow N = \frac{1}{\lambda}\left[q + \left(\lambda N_0 - q\right)e^{-\lambda t}\right]$$

Successive Disintegration

A parent nucleus may decays into a daughter nucleus, which may decay into another daughter nucleus and so on. Such decay is called successive disintegration or series decay. The chain stops only when the end product is stable.

c. For successive disintegration of the products:

$$A \xrightarrow[\lambda_1]{decay} B \xrightarrow[\lambda_2]{decay} C(stable)$$

For A $-\frac{dN_A}{dt} = \lambda_1 N_A \Rightarrow N_A = N_0 e^{-\lambda_1 t}$

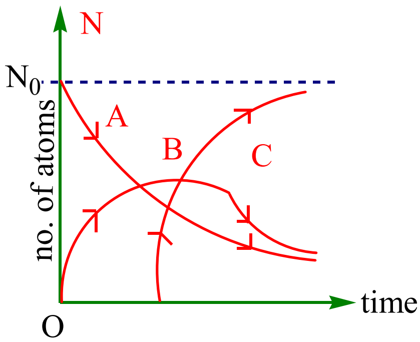
For B $+\frac{dN_B}{dt} = \lambda_1 N_A - \lambda_2 N_B$

$$\Rightarrow N_B = \frac{N_0 \lambda_1}{\lambda_2 - \lambda_1}\left[e^{-\lambda_1 t} - e^{-\lambda_2 t}\right]$$

For C $\frac{dN_C}{dt} = \lambda_2 N_B$.

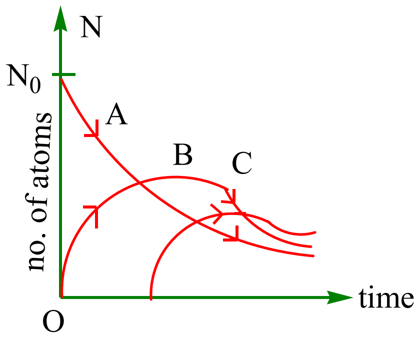
$$N_c = N_0\left[1 - \frac{\lambda_2 e^{-\lambda_1 t}}{(\lambda_2 - \lambda_1)} + \frac{\lambda_1 e^{-\lambda_2 t}}{(\lambda_2 - \lambda_1)}\right]$$

If 'C' is the final stable product, then the decay and recovery curves for the substances A, B and C are as shown



At peak of B, the rate of formation of B = rate of disintegration of B. Hence $\lambda_1 N_A = \lambda_2 N_B$.

If 'C' is also an unstable product, then the decay and recovery curves for the substances A,B and C are as shown



Also, $N_0 = N_A + N_B + N_C$; at t = 0, $N_B = 0$;

at $0 < t < \tau; N_B \to$ increases

at $t = \tau = \left(\frac{1}{\lambda_2 - \lambda_1}\right)\ln\left(\frac{\lambda_2}{\lambda_1}\right); N_B$ = maximum

at $\tau < t < \infty; N_B \to$ decreases to zero

Permanent or Secular Equlibrium

If half lives T_A, T_B of the species A and B are such that $T_A >> T_B$ i.e parent nuclei has longer half life, then their decay constants obey $\lambda_1 << \lambda_2$.

Let us choose, $T_A \to \infty$ and $T_B \to 0$

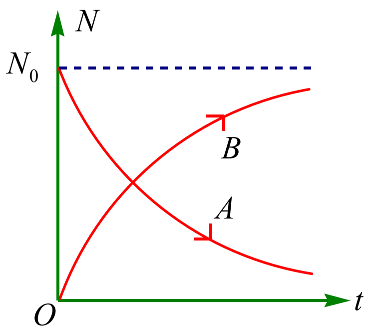
$\lambda_1 = 0$ and so $\lambda_2 - \lambda_1 \approx \lambda_2$

then $N_B = \frac{\lambda_1 N_0}{\lambda_2}\left[1 - e^{-\lambda_2 t}\right]$

For λ_2 be large $e^{-\lambda_2 t} = 0$

hence $N_B \lambda_2 = N_0 \lambda_1$

i.e Thus B is in permanent of secular equlibrium with A (parent)



Transient Equlibrium

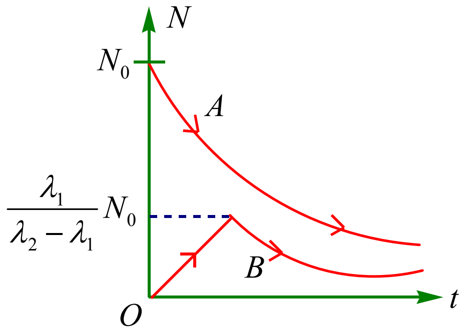
If the parent is long-lifed compared to daughter but half-life of the parent not very large i.e $T_A > T_B$ Then $\lambda_1 < \lambda_2$, but $\lambda_1 \neq 0$.

For $\frac{N_B}{N_A} = \frac{\lambda_1}{\lambda_2 - \lambda_1}\left[1 - \frac{e^{-\lambda_2 t}}{e^{-\lambda_1 t}}\right]$

After sufficiently long time $e^{-\lambda_2 t}$ becomes negligible compared to $e^{-\lambda_2 t}$.

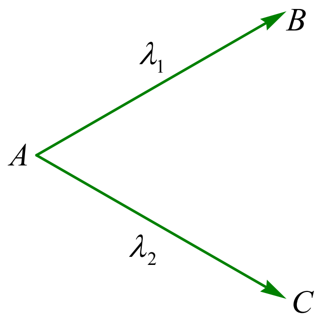
So that $\frac{N_B}{N_A} = \frac{\lambda_1}{\lambda_2 - \lambda_1} = \text{constant}$

$\therefore$ After sufficient time, the ratio of parent atom to daughter atom become constant and both eventually decay with same half life. This is known as transient equilibrium



Note : It is to be noted that if the parent has shorter half-life than that of the daughter $(\lambda_1 > \lambda_2)$, no state of equilibrium is attained

d. Simultaneous disintegration of parent nuclei:-



$\lambda_{eff} = \lambda_1 + \lambda_2$ or

$$\frac{1}{T_{eff}} = \frac{1}{T_1} + \frac{1}{T_2} \Rightarrow T_{eff} = \frac{T_1 T_2}{T_1 + T_2}$$

Where T_1 and T_2 are the respective half lifes

e. Radioactive equilibrium:

After a period of time, successive daughter nucleus decays at the same rate as it is formed. The situation is called radioactive equilibrium.

$$\lambda_A N_A = \lambda_B N_B = \lambda_C N_C = ...$$

Nuclei

(Jee main previous year questions)

Topic 1: Composition and Size of the Nuclei

1. **The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})A^{1/3}$m. It follows that the mass density of a nucleus is of the order of:**

   $(M_{\text{prot.}} \cong M_{\text{neut.}} = 1.67 \times 10^{-27}\text{kg})$

   [Sep. 03, 2020 (II)]

   (a) 10^{3} kg m^{-3}   **(b) 10^{10} kg m^{-3}**

   (c) 10^{24}kgm^{-3}   **(d) 10^{17} kg m^{-3}**

SOL. (d) Density of nucleus, $\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{mA}{\frac{4}{3}\pi R^3}$

$$\Rightarrow \text{p} = \frac{mA}{\frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3} \qquad (R = R_0 A^{1/3})$$

Here m = mass of a nucleon

$$\rho = \frac{3\times 1.67\times 10^{-27}}{4\times 3.14\times\left(1.3\times 10^{-15}\right)^3} \text{ (Given, } R_0 = 1.3 \times 10^{-15})$$

$$\Rightarrow \text{p} = 2.38 \times 10^{17}\text{kg/m}^3$$

2. **The ratio of the mass densities of nuclei of ^{40}Ca and O^{16} is close to :**

   [8 April 2019 II]

   (a) 1   **(b) 0.1**   **(c) 5**   **(d) 2**

SOL. (a) Nuclear density is independent of atomic number.

3. **An unstable heavy nucleus at rest breaks into two nuclei which move away with velocities in the ratio of 8:27. The ratio of the radii of the nuclei (assumed to be spherical) is:**

[Online Apri1 15, 2018]

(a) 8: 27 **(b) 2: 3** **(c) 3: 2** **(d) 4: 9**

SOL. (c) Let heavy nucleus breaks into two nuclei of mass m_1 and m_2 and move away with velocities V_1 and V_2 respectively.

According to question, $\frac{V_1}{V_2} = \frac{8}{27}$

$m_1 V_1 = m_2 V_2$(Law of momentum conservation)

$$\Rightarrow \frac{m_1}{m_2} = \frac{V_2}{V_1} = \frac{27}{8}$$

$$\frac{\mathrm{p} \times \frac{4}{3}\pi R_1^3}{\mathrm{p} \times \frac{4}{3}\pi R_2^3} \quad (\because \text{ density } \mathrm{p} = \frac{\text{mass}}{\text{volume}})$$

$$\Rightarrow \left(\frac{R_1}{R_2}\right) = \left(\frac{27}{8}\right)^{\frac{1}{3}} = \left(\frac{3}{2}\right)^{3 \times \frac{1}{3}} \qquad \frac{\mathrm{R}_1}{\mathrm{R}_2} = \frac{3}{2}$$

4. Which of the following are the constituents of the nucleus?

[2007]

(a) Electrons and protons **(b) Neutrons and protons**

(c) Electrons and neutrons **(d) Neutrons and positrons**

SOL. (b)

5. If radius of the Al_{13}^{27} nucleus is estimated to be 3.6 fermi then the radius of Te_{52}^{125} nucleus be nearly

[2005]

(a) 8 fermi **(b) 6 fermi** **(c) 5 fermi** **(d) 4 fermi**

SOL. (b) Radius of a nucleus,

$$R = R_0 (A)^{1/3}$$

Here, R_0 is a constant

$A =$ atomic mass number

$$\frac{R_1}{R_2}=\left(\frac{A_1}{A_2}\right)^{1/3}=\left(\frac{27}{125}\right)^{1/3}=\frac{3}{5}$$

$\Rightarrow R_2=\frac{5}{3}\times 3.6=6$ fermi

Topic 2: Mass-Energy Equivalence and Nuclear Reactions

6. You are given that mass of $Li_3^7=7.0160\text{u}$, Mass of $He_2^4=4.0026\text{u}$ and Mass of $H_1^1=1.0079\text{u}$. When 20 g of Li_3^7 is converted into He_2^4 by proton capture, the energy liberated, (in kWh), is: [Mass of nucleon $=1\text{GeV}/c^2$]

[Sep. 06, 2020 (D)]

(a) 4.5×10^5 (b) 8×10^6 (c) 6.82×10^5 (d) 1.33×10^6

SOL. (d) ${}_3^7\text{Li}+\text{H}_1^1\rightarrow 2\left(\text{He}_2^4\right)$

$$\Delta m\rightarrow[m_{\text{Li}}+m_{\text{H}}]-2[M_{\text{He}}]$$

Energy released $=\Delta mc^2$

In use of 1 g Li energy released $=\dfrac{\Delta mc^2}{m_{\text{Li}}}$

In use of 20 g energy released $=\dfrac{\Delta mc^2}{m_{Li}}\times 20\text{g}$

$$=\frac{[(7.016+1.0079)-2\times 4.0026]u\times c^2}{7.016\times 1.6\times 10^{-24}}\times 20\text{g}$$

$$=480\times 10^{10}\text{J}$$

$$1\text{J}=2.778\times 10^{-7}\text{kWh}$$

Energy released $=480\times 10^{10}\times 2.778\times 10^{-7}$

$$=1.33\times 10^6\text{kWh}$$

7. Given the masses of various atomic particles $m_p=1.0072\text{u}$, $m_n=1.0087\text{u}$, $m_e=0.000548\text{u}$, $m_{\bar{v}}=0$, $m_d=2.0141\text{u}$, where p $\equiv$ proton, n $\equiv$ neutron, e $\equiv$ electron, $\bar{v}\equiv$ antineutrino and d $\equiv$ deuteron. Which of the following process is allowed by

momentum and energy conservation?

[Sep. 06, 2020 (II)]

(a) $n+n \rightarrow$ deuterium atom (electron bound to the nucleus)

(b) $p \rightarrow n+e^{+}+v$

(c) $n+p \rightarrow d+\gamma$

(d) $e^{+}+e \rightarrow \gamma$

SOL. (c) For the momentum and energy conservation, mass defect (Δm) should be positive. Since some energy is lost in every process.

$$(m_p+m_n)>m_d$$

8. Find the Binding energy per nucleon for Sn_{50}^{120}. Mass of proton $m_p=1.00783\text{U}$, mass of neutron $m_n=1.00867$ U and mass of tin nucleus $m_{\text{Sn}}=119.902199$ U. (take $1\text{U}=931\text{MeV}$)

[Sep. 04, 2020 (II)]

(a) 7.5MeV (b) 9.0MeV (c) 8.0MeV (d) 8.5MeV

SOL. (d) Mass defect,

$$\Delta m=(50m_p+70m_n)-(m_{sn})$$

$$=(50\times 1.00783+70\times 1.008)-(119.902199)$$

$$=1.096$$

Binding energy $=(\Delta m)C^2=(\Delta m)\times 931=1020.56$

$$\frac{\text{Bindingenergy}}{\text{Nuc1eon}}=\frac{1020.5631}{120}=8.5\text{MeV}$$

9. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 MeV. Given that the Avogadro number, $N=6.023\times 10^{26}$ per kilo mole and 1 eV $=1.6\times 10^{-19}$ J. The power output of the reactor is close to:

[Sep. 02, 2020 (I)]

(a) 35 MW (b) 60 MW (c) 125 MW (d) 54 MW

SOL. (b) Power output of the reactor,

$$P=\frac{\text{energy}}{\text{time}}$$

$$=\frac{2}{235}\times\frac{6.023\times10^{26}\times200\times1.6\times10^{-19}}{30\times24\times60\times60}=60\text{MW}$$

10. Consider the nuclear fission

$$\mathbf{Ne^{20}\rightarrow 2He^{4}+C^{12}}$$

Given that the binding energy/nucleon of Ne^{20}, $\mathbf{He^4}$ and $\mathbf{C^{12}}$ are, respectively, $\mathbf{8.03MeV}$, $\mathbf{7.07MeV}$ and $\mathbf{7.86MeV}$, identify the correct statement:

[10 Jan. 2019 II]

(a) energy of $\mathbf{12.4}$ MeV will be supplied

(b) $\mathbf{8.3}$ MeV energy will be released

(c) energy of $\mathbf{3.6MeV}$ will be released

(d) energy of $\mathbf{11.9MeV}$ has to be supplied

SOL. (d)

11. Imagine that a reactor converts all given mass into energy and that it operates at a power level of 10^9 watt. The mass of the fuel consumed per hour in the reactor will be : (velocity of light, c is $\mathbf{3\times10^8 m/s}$)

[Online April 9, 2017]

(a) $\mathbf{0.96}$ *gm*   (b) $\mathbf{0.8}$ gm   (c) $\mathbf{4\times10^2}$ gm   (d) $\mathbf{6.6\times10^5}$ gm

SOL. (c) Power level of reactor, $P=\frac{E}{\Delta t}=\frac{\Delta mc^2}{\Delta t}$

mass of the fuel consumed per hour in the reactor,

$$\frac{\Delta m}{\Delta t}=\frac{P}{c^2}=\frac{10^9}{(3\times10^8)^2}=4\times10^{-2}\text{gm}$$

12. Two deuterons undergo nuclear fusion to form a Helium nucleus. Energy released in this process is : (given binding energy per nucleon for deuteron $\mathbf{=1.1MeV}$ and for helium= $\mathbf{7.0MeV}$)

[Online April 8, 2017]

(a) $\mathbf{30.2MeV}$ (b) $\mathbf{32.4MeV}$ (c) $\mathbf{23.6MeV}$ (d) $\mathbf{25.8MeV}$

SOL. (c) $H^2 + H^2 \rightarrow He^4$

Total binding energy of two deuterium nuclei $= 1.1 \times 4 = 4.4\text{MeV}$

Binding energy of a $({}_2He^4)$ nuclei $= 4 \times 7 = 28\text{MeV}$

Energy released in this process $= 28 - 4.4 = 23.6\text{MeV}$

13. When Uranium is bombarded with neutrons, it undergoes fission. The fission reaction can be written as:

$U_{92}^{235} + n_0^1 \rightarrow Ba_{56}^{141} + \text{Kr}_{36}^{92} + 3x + \text{Q}$ **(energy)**

where three particles named x are produced and energy Q is released. What is the name of the particle x ?

[Online April 9, 2013]

(a) electron **(b) α-particle** **(c) neutron** **(d) neutrino**

SOL. (c) Nuclear fission equation

$U_{92}^{235} + n_0^1 \rightarrow Ba_{56}^{141} + \text{Kr}_{36}^{92} + 3n_0^1 + \text{Q}$ (energy)

Hence particle x is neutron.

14. Assume that a neutron breaks into a proton and an electron. The energy released during this process is: (mass of neutron $= 1.6725 \times 10^{-27}$kg, mass of proton $= 1.6725 \times 10^{-27}$ kg, mass of electron $= 9 \times 10^{-31}$kg).

[2012]

(a) 0.51MeV **(b) 7.10MeV** **(c) 6.30MeV** **(d) 5.4MeV**

SOL. (a) $n_0^1 \rightarrow H_1^1 + {}_{-1}\text{e}^0 + \overline{\nu} + Q$

The mass defect during the process

$$\Delta m = m_n - m_H - m_e = 1.6725 \times 10^{-27}$$

$$-(1.6725 \times 10^{-27} + 9 \times 10^{-31}\text{kg})$$

$$= -9 \times 10^{-31}\text{kg}$$

The energy released during the process $E = \Delta m c^2$

$E = 9 \times 10^{-31} \times 9 \times 10^{16} = 81 \times 10^{-15}$ Joules

$$E = \frac{8.1 \times 10^{-15}}{16 \times 10^{-19}} = 0.511\text{MeV}$$

15. Ionisation energy of Li (Lithium) atom in ground state is 5.4eV. Binding energy of an electron in Li^+ ion in ground state is 75.6 eV. Energy required to remove all three electrons of Lithium (Li) atom is

[Online May 19, 2012]

(a) 81.0eV (b) 135.4eV (c) 203.4eV (d) 156.6eV

SOL. (d)

16. After absorbing a slowly moving neutron of mass m_N (momentum ≈ 0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1 (6m_1 = M + m_N)$ respectively. If the de Broglie wavelength of the nucleus with mass m_1 is λ, the de Broglie wavelength of the nucleus will be [2011]

(a) 5λ (b) $\lambda/5$ (c) λ (d) 25λ

16. (c) Initial momentum of system, $p_j = 0$

Let p_1 and p_2 be the momentum ofbroken nuclei ofmasses m_1 and $5m_1$ respectively.

$$p_f = p_1 + p_2$$

From the conservation of momentum

$$p_i = p_f$$

$$0 = p_1 + p_2$$

$$p_1 = -p_2$$

From de Broglie relation, wavelength

$\lambda_1 = \frac{h}{p_1}$ and $\lambda_2 = \frac{h}{p_2}$

$$|\lambda_1| = |\lambda_2|$$

$$\lambda_1 = \lambda_2 = \lambda.$$

DIRECTIONS: Questions number 17-18 are based on the following paragraph.

A nucleus of mass $M+\Delta m$ is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed oflight is c.

[2010]

17. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(a) $E_2 = 2E_1$ **(b)** $E_1 > E_2$ **(c)** $E_2 > E_1$ **(d)** $E_1 = 2E_2$

SOL. (c) In nuclear fission, the binding energy per nucleon of daughter nuclei is always greater than the parent nucleus.

18. The speed of daughter nuclei is

(a) $c\frac{\Delta m}{M+\Delta m}$ **(b)** $c\sqrt{\frac{2\Delta m}{M}}$ **(c)** $c\sqrt{\frac{\Delta m}{M}}$ **(d)** $c\sqrt{\frac{\Delta m}{M+\Delta m}}$

SOL. (b) Mass defect, $\Delta M = \left[(M+\Delta m) - \left(\frac{M}{2}+\frac{M}{2}\right)\right]$

$$= [M + \Delta m - M] = \Delta m$$

Energy released, $Q = \Delta M c^2 = \Delta m c^2$ -----(i)

From the law of conservation of momentum

$$(M+\Delta m)\times 0 = \frac{M}{2}v_1 - \frac{M}{2}\times v_2$$

$$\Rightarrow v_1 = v_2$$

Now,

$$Q = \frac{1}{2}\left(\frac{M}{2}\right)v_1^2 + \frac{1}{2}\left(\frac{M}{2}\right)v_2^2 - \frac{1}{2}(M+\Delta m)\times(0)^2$$

$= \frac{M}{2}v_1^2 \quad (\because v_1 = v_2)$ -------(ii)

From equation (i) and (ii), we get

$$\left(\frac{M}{2}\right)v_1^2 = \Delta m c^2$$

$$\Rightarrow v_1^2 = \frac{2\Delta m c^2}{M} \quad \Rightarrow V_1 = c\sqrt{\frac{2\Delta m}{M}}$$

19. Statement-l: Energy is released when heavy nuclei undergo fission or light nuclei undergo

fusion and

Statement-2 : For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei it decreases with increasing Z.

[2008]

(a) Statement-l is false, Statement-2 is true

(b) Statement- 1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement- 1

(c) Statement-l is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-l

(d) Statement-l is true, Statement-2 is false

SOL. (d) We know that energy is released when heavy nuclei undergo fission or light nuclei undergo fusion. Therefore statement (1) is correct.

The second statement is false because for heavy nuclei the binding energy per nucleon decreases with increasing Z and for light nuclei, B. $\mathrm{E/nucleon}$ increases with increasing Z.

20. If M_O is the mass of an oxygen isotope $8^{O^{17}}$, ^{M}P and M_N are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is

[2007]

(a) $(M_O - 17M_N)c^2$ (b) $(M_O - 8M_P)c^2$ (c) $(M_O - 8M_P - 9M_N)c^2$ (d) $M_O c^2$

SOL. (c) Number of protons in oxygen isotope, $Z = 8$

Number of neutrons $= 17 - 8 = 9$

Binding energy

$$= [\mathrm{ZM_P} + (\mathrm{A} - \mathrm{Z})\mathrm{M_N} - \mathrm{M}]\mathrm{c}^2$$

$$= [8\mathrm{M_P} + (17 - 8)\mathrm{M_N} - \mathrm{M}]\mathrm{c}^2$$

$$= [8\mathrm{M_P} + 9\mathrm{M_N} - \mathrm{M}]\mathrm{c}^2$$

$$= [8\mathrm{M_P} + 9\mathrm{M_N} - \mathrm{M_o}]\mathrm{c}^2$$

21. When Li_3^7 nuclei are bombarded by protons, and the resultant nuclei are Be_4^8, the emitted

particles will be

[2006]

(a) alpha particles **(b) beta particles** **(c) gamma photons** **(d) neutrons**

SOL. (c) $Li_3^7 + p_1^1 \rightarrow Be_4^8 + \gamma$

We see that both proton number and mass number are equal in both sides, so emitted particle should be massless gamma photons.

22. If the binding energy per nucleon in Li_3^7 and He_2^4 nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction $p + Li_3^7 \rightarrow 2\,He_2^4$. energy of proton must be

[2006]

(a) 28.24MeV **(b) 17.28MeV** **(c) 1.46MeV** **(d) 39.2MeV**

SOL. (b) Given,

Binding energy per nucleon of $Li_3^7 = 5.60\text{MeV}$

Binding energy per nucleon of $He_2^4 = 7.06\text{MeV}$

Let E be the energy ofproton, then

$$E + 7 \times 5.6 = 2 \times [4 \times 7.06]$$

$$\Rightarrow E = 56.48 - 39.2 = 17.28\text{MeV}$$

23. A nuclear transformation is denoted by $X(n,\alpha)\,Li_3^7$ Which of the following is the nucleus of element X ?

[2005]

(a) B_5^{10} **(b)** C_6^{12} **(c)** Be_4^{11} **(d)** B_5^9

SOL. (a) $z^{X^A} + n^1 \rightarrow 3^{Li^7} + 2He^4$

Using conservation of mass number

$$A + 1 = 4 + 7$$

$$\Rightarrow A = 10$$

Using conservation of charge number

$$Z + 0 = 2 + 3 \Rightarrow Z = 5$$

It is boron $5^{B^{10}}$

24. A nucleus disintegrated into two nuclear parts which have their velocities in the ratio of 2: 1. The ratio of their nuclear sizes will be

[2004]

(a) $3^{1/2}:1$ **(b)** $1:2^{1/3}$ **(c)** $2^{1/3}:1$ **(d)** $1:3^{1/2}$

SOL. (b) Given:

$$\frac{v_1}{v_2} = \frac{2}{1}$$

From conservation of momentum $m_1v_1 = m_2v_2$

$$\Rightarrow \left(\frac{m_1}{m_2}\right) = \left(\frac{v_2}{v_1}\right) = \frac{1}{2}$$

We know that mass of nucleus, $m \propto A$

Nuclear size $R \propto A^{1/3} \propto m^{1/3}$

$$\frac{R_1}{R_2} = \left(\frac{m_1}{m_2}\right)^{1/3} \Rightarrow \frac{R_1^3}{R_2^3} = \frac{1}{2} \quad \Rightarrow \left(\frac{R_1}{R_2}\right) = \left(\frac{1}{2}\right)^{1/3}$$

25. The binding energy per nucleon of deuteron H_1^2 and helium nucleus He_2^4 is 1.1MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is

[2004]

(a) 23.6MeV **(b)** 26.9MeV **(c)** 13.9MeV **(d)** 19.2MeV

SOL. (a) The chemical reaction of process is $2{}_1^2\mathrm{H} \rightarrow 2^{\mathrm{He}}4$

Binding energy of two deuterons,

$$4 \times 1.1 = 4.4\text{MeV}$$

Binding energy of helium nucleus $= 4 \times 7 = 28\text{MeV}$

Energy released $= 28 - 4.4 = 23.6\text{MeV}$

26. **When a U^{238} nucleus originally at rest, decays byemitting an alpha particle having a speed 'u', the recoil speed of the residual nucleus is [2003]**

(a) $\frac{4u}{238}$ **(b)** $-\frac{4u}{234}$ **(c)** $\frac{4u}{234}$ **(d)** $-\frac{4u}{238}$

SOL. (c) Mass of α particle, $m_\alpha = 4u$

Mass of nucleus after fission, $m_n = 234u$

From conservation of linear momentum we have

$$238 \times 0 = 4u + 234v$$

$$v = -\frac{4}{234}u$$

Speed $= |\vec{v}| = \frac{4}{234}u$

27. **In the nuclear fusion reaction** $H_1^2 + H_1^3 \rightarrow He_2^4 + n$

given that the repulsive potential energy between the two nuclei is $\sim 7.7 \times 10^{-14}\text{J}$, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's Constant $k = 1.38 \times 10^{-23}\text{J/K}$]

[2003]

(a) 10^7K **(b)** 10^5K **(c)** 10^3K **(d)** 10^9K

SOL. (d) The average kinetic energy per molecule at temperature T is $= \frac{3}{2}kT$

Where $k =$ Boltzmann's constant

This kinetic energy should be able to provide the repulsive potential energy

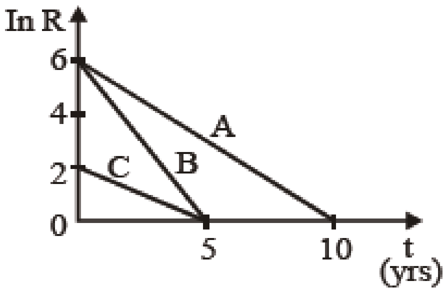
$$\frac{3}{2}kT = 7.7 \times 10^{-14}$$

$$\Rightarrow T = \frac{2 \times 7.7 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^9 K$$

Topic 3: Radioactivity

28. **Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{1/2}(A):T_{1/2}(B):T_{1/2}(C)$ are in the ratio:**

[Sep. 05, 2020 (I)]



(a) 2: 1 : 1 **(b) 3: 2: 1** **(c) 2: 1 : 3** **(d) 4: 3 : 1**

SOL. (c) Since, $R = R_0 e^{-\lambda t}$

$$\ln R = \ln R_0 + (-\lambda \ln t)$$

$\lambda = \dfrac{\ln 2}{t_{1/2}} =$ Slope

$\lambda_A = \dfrac{6}{10} \Rightarrow T_A = \dfrac{10}{6}\ln 2$

$$\lambda_B = \frac{6}{5} \Rightarrow T_B = \frac{5\ln 2}{6}$$

$$\lambda_c = \frac{2}{5} \Rightarrow T_C = \frac{5\ln 2}{6}$$

$$T_{1/2}(A):T_{1/2}(B):T_{1/2}(C) = \frac{10}{6}:\frac{5}{6}:\frac{15}{6} = 2:1:3$$

29. **A radioactive nucleus decays by two different processes. The half life for the first process is 10 s and that for the second is 100 s. The effective halflife of the nucleus is close to:**

[Sep. 05, 2020 (II)]

(a) 9 sec. **(b) 6 sec.** **(c) 55 sec.** **(d) 12 sec.**

SOL. (a) Let λ_1 and λ_2 be the decay constants of two process. N be the number of nuclei left undecayed after two process. From the law of radioactive decay we have

$$-\frac{dN}{dt} = \lambda_1 N + \lambda_2 N \qquad \left[\because -\frac{dN}{dt} = \lambda N\right]$$

$$\Rightarrow -\frac{dN}{dt} = (\lambda_1 + \lambda_2)N$$

$$\Rightarrow \lambda_{eq.} = (\lambda_1 + \lambda_2)$$

$$\Rightarrow \frac{\ln 2}{T} = \frac{\ln 2}{T_1} + \frac{\ln 2}{T_2} \quad \left(\because \lambda = \frac{\ln 2}{T}\right)$$

$$\Rightarrow \frac{1}{T} = \frac{1}{T_1} + \frac{1}{T_2}$$

$\Rightarrow \frac{1}{T} = \frac{1}{10} + \frac{1}{100} = \frac{11}{100}$ [Given: $T_1 = 10$ s & T2=100 s]

$\Rightarrow T = \frac{100}{11} = 9$ sec.

30. In a radioactive material, fraction of active material remaining after time *t* is 9/16. The fraction that was remaining after $tl2$ is:

[Sep. 03, 2020 (I)]

(a) $\frac{4}{5}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{3}{4}$ **(d)** $\frac{7}{8}$

SOL. (c) As we know, for first order decay, $N(t) = N_0 e^{-\lambda t}$

According to question,

$$\frac{N(t)}{N_0} = \frac{9}{16} = e^{-\lambda t}$$

After time, *t/2*;

$$N(t/2) = N_0 e^{-\lambda(t/2)}$$

$$\frac{N(t/2)}{N_0} = \sqrt{e^{-\lambda t}} = \sqrt{\frac{9}{16}}$$

$$N(tl2) = \frac{3}{4} N_0$$

31. The activity of a radioactive sample falls from 700 s^{-1} to 500 s^{-1} in 30 minutes. Its halflife is close to:

[7 Jan. 2020, II]

(a) 72 min **(b) 62 min** **(c) 66 min** **(d) 52 min**

SOL. (b) We know that

Activity, $A = A_0 e^{-\lambda t}$

$$A = A_0 e^{-t\ln 2/T_{1/2}} \left(\left(\because \lambda = \frac{In_2}{T_{1/2}}\right)\right)$$

$$\Rightarrow 500 = 700 e^{-tIn2/T_{1/2}}$$

$\Rightarrow In\frac{7}{5} = \frac{30In2}{T_{1/2}}$ (t = 30 minute)

$\Rightarrow T_{1/2} = 30\frac{In2}{In1.4} = 61.8$ minute

$$(\ln 2 = 0.693 \text{ and } \ln.1.4 = 0.336)$$

$\Rightarrow T_{1/2} \approx 62$ minute

32. Two radioactive materials A and B have decay constants 10 λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $1/e$ after a time:

[10 April 2019, I]

(a) $\frac{1}{9\lambda}$ **(b)** $\frac{1}{11\lambda}$ **(c)** $\frac{11}{10\lambda}$ **(d)** $\frac{1}{10\lambda}$

SOL. (a) As, $N = N_0 e^{-\lambda t}$

so, $\frac{N_A}{N_B} = e^{(\lambda_B - \lambda_A)t} = \frac{1}{e} \Rightarrow (\lambda_B - \lambda_A)t = -1$

$$\Rightarrow (\lambda_A - \lambda_B).t = 1$$

$$\Rightarrow t = \frac{1}{(\lambda_B - \lambda_A)} \qquad t = \frac{1}{10\lambda - \lambda} = \frac{1}{9\lambda}$$

33. Two radioactive substances A and B have decay constants 5λ and λ respectively. At $t = 0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be:

[10 April 2019, II]

(a) $1/2\lambda$ **(b)** $1/4\lambda$ **(c)** $1/\lambda$ **(d)** $2/\lambda$

SOL. (a) Let N_1 and N_2 be the number ofradioactive nuclei of substance at anytime t.

$N_1(\text{at } t) = N_0 e^{-5\lambda t}$ (i)

$N_2(\text{at } t) = N_0 e^{-\lambda t}$ (ii)

Dividing equation (i) by (ii), we get

$$\frac{N_1}{N_2} = \frac{1}{e^2} = e^{-4\lambda t} \Rightarrow 4\lambda t = 2$$

$$\Rightarrow t = \frac{2}{4\lambda} = \left(\frac{1}{2\lambda}\right)$$

34. In a radioactive decay chain, the initial nucleus is Th_{90}^{232}.

At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $_Z^X A$, A and Z are given by:

[12 Jan. 2019, II]

(a) $A = 208; Z = 80$ **(b) $A = 202; Z = 80$**

(c) $A = 208; Z = 82$ **(d) $A = 200; Z = 81$**

SOL. (c) When one α- particle emitted then daughter nuclei has 4 unit less mass number (A) and 2 unit less atomic number (z)

$$\text{Th}_{90}^{232} \rightarrow \text{Y}_{78}^{208} + 6 He_2^4$$

$$\text{Y}_{78}^{208} \rightarrow X_{82}^{208} + 4\beta \text{ particle}$$

35. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At $t = 0$ it was 1600 counts per second and $t = 8$ seconds it was 100 counts per second. The count rate observed, as counts per second, at $t = 6$ seconds is close to:

[10 Jan. 2019 I]

(a) 200 **(b) 150** **(c) 400** **(d) 360**

SOL. (a) According to question,

at $t = 0$, $A_0 = \frac{dN}{dt} = 1600\,C/s$ and

at $t = 8s, A = 100C/s$

$$\therefore \frac{A}{A_0} = \frac{1}{16} \text{ in 8 sec}$$

Therefore half life period, $t1/2 = 2s$

Activity at $t = 6s = 1600\left(\frac{1}{2}\right)^3 = 200C/s$

36. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci $= 3.7 \times 10^{10}$decays/s), has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The coniect choices for half-lives of A and B would then be respectively: [9 Jan. 2019 I]

(a) 5 days and 10 days **(b) 10 days and 40 days**

(c) 20 days and 5 days **(d) 20 days and 10 days**

SOL. (c) Activity A $= 1N$

For material, A $10 = (2N0)1A$

For material, B $20 = N01B$

$$\Rightarrow \lambda_B = 4\lambda_A \quad \therefore T_{/\frac{1}{2}A} = 4T_{/\frac{1}{2}B} \left[\because T_{/\frac{1}{2}} = \frac{0.693}{\lambda}\right]$$

i.e. 20 days half-lives for A and 5 days $\left(T_{\sqrt{2}}\right)_B$ For material B.

37. At a given instant, say $t = 0$, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is ln2, the half-life of B is:

[9 Jan. 2019, II]

(a) $4ln2$ **(b)** $\frac{ln2}{2}$ **(c)** $\frac{ln2}{4}$ **(d)** $2ln2$

SOL. (c) Half life of A $= \ell n\ 2$

$$(t_{1/2})A = \frac{\ell n2}{\lambda}$$

$$\lambda_A = 1$$

at $t = 0$ $R_A = R_B$

$$N_A e^{-\lambda AT} = N_B e^{-\lambda BT}$$

$N_A = N_B$ at $t = 0$

At $t = t$, $\frac{R_B}{R_A} = \frac{N_0 e^{-\lambda_B t}}{N_0 e^{-\lambda_A t}}$

$$e^{-(\lambda_B - \lambda_A)t} = e^{-3t}$$

$$\Rightarrow \lambda_B - \lambda_A = 3$$

$$\lambda_B = 3 + \lambda_A = 4$$

$$(t_{1/2})_B = \frac{\ell n 2}{\lambda_B} = \frac{\ell n 2}{4}$$

38. At some instant, a radioactive sample S_1 having an activity $5\mu Ci$ has twice the number of nuclei as another sample S_2 which has an activity of $10\mu Ci$. The half-lives of S_1 and S_2 are [Online Apri116, 2018]

(a) 10 years and 20 years, respectively

(b) 5 years and 20 years, respectively

(c) 20 years and 10 years, respectively

(d) 20 years and 5 years, respectively

SOL. (b) Given: $N_1 = 2N_2$

Activity of radioactive substance $= \lambda N$

Half life period $t = \frac{\ln 2}{\lambda}$ or, $\lambda = \frac{\ln 2}{T}$

$\lambda_1 N_1 = \frac{\ln 2}{t_1} \times N_1 = 5\mu Ci$ ..... (i)

$\lambda_2 N_2 = \frac{\ln 2}{t_2} \times N_2 = 10\mu Ci$ ..... (ii)

Dividing equation (ii) by (i)

$$\frac{t_2}{t_1} \times \frac{N_1}{N_2} = \frac{1}{2}$$

$$\frac{t_2}{t_1} = \frac{1}{4} \Rightarrow t_1 = 4t_2$$

i. e., Half life of S_1 is four times of sample S_2. Hence 5 years and 20 years.

39. A solution containing active cobalt Co_{27}^{60} having activity of $0.8\,\mu Ci$ and decay constant λ is injected in an animal's body. If $1 cm^3$ of blood is drawn from the animal's body after 10 hrs of injection, the activity found was 300 decays per minute. What is the volume of blood

that is flowing in the body? (1Ci $= 3.7 \times 10^{10}$ decay per second and at $t = 10$ hrs, $e^{-\lambda t} = 0.84$) [Online Apri115, 2018]

(a) 6 litres **(b) 7 litres** **(c) 4 litres** **(d) 5 litres**

SOL. (d) Let initial activity $= N_0 = 0.8\mu$ ci

$=0.8 \times 3.7 \times 10^4$ dps

Activity in 1 cm^3 of blood at $t = 10$hr,

$$n = \frac{300}{60} \text{dps} = 5\,\text{dps}$$

N = Activity of whole blood at time $t = 10$hr.

Total volume of the blood in the person, $V = \frac{N}{n} = \frac{N_0 e - \lambda t}{n} = \frac{0.8 \times 3.7 \times 10^4 \times 0.7927}{5} \cong 5$ litres

40. A radioactive nucleus A with a half life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

[2017]

(a) $t = T \log(1.3)$ **(b)** $t = \frac{T}{\log(1.3)}$

(c) $t = T\frac{\log 2}{\log 1.3}$ **(d)** $t = \frac{\log 1.3}{\log 2}$

SOL. (d) Let initially there are total N_0 number of nuclei

At time t

$$\frac{N_B}{N_A} = 0.3(\text{given})$$

$$\Rightarrow N_B = 0.3N_A$$

$$N_0 = N_A + N_B = N_A + 0.3N_A$$

$$N_A = \frac{N_0}{1.3}$$

As we know $N_t = N_0 e^{-\lambda t}$

or, $\frac{N_0}{1.3} = N_0 e^{-\lambda t}$

$$\frac{1}{1.3} = e^{-\lambda t} \qquad \Rightarrow ln(1.3) = \lambda t$$

or, $t = \frac{ln(1.3)}{\lambda} \quad \Rightarrow t = \frac{ln(1.3)}{\frac{ln(2)}{T}} = \frac{ln(1.3)}{ln(2)} T$

41. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed number of A and B nuclei will be:

[2016]

(a) 1:4 (b) 5:4 (c) 1:16 (d) 4: 1

SOL. (b) For $A_{t^1/_2} = 20$ min , t = 80 min , number of half-lives $n = 4$

Nuclei remaining $= \frac{N_0}{2^4}$.

Therefore nuclei decayed

$$= N_0 - \frac{N_0}{2^4}$$

For B $t^1/_2 = 40$ min ., $t = 80$ min , number of half-lives $n = 2$

Nuclei remaining $= \frac{N_0}{2^2}$.

Therefore nuclei decayed

$$= N_0 - \frac{N_0}{2^2}$$

Required ratio $= \frac{N_0 - \frac{N_0}{2^4}}{N_0 - \frac{N_0}{2^2}} = \frac{1 - \frac{1}{16}}{1 - \frac{1}{4}} = \frac{15}{16} \times \frac{4}{3} = \frac{5}{4}$

42. Let N_β be the number of β particles emitted by 1 gram of Na^{24} radioactive nuclei (half life $= 15$ hrs) in 7.5 hours, N_β is close to (Avogadro number $= 6.023 \times 10^{23}$/g. mole):

[Online April 11, 2015]

(a) 6.2×10^{21} (b) 7.5×10^{21} (c) 1.25×10^{22} (d) 1.75×10^{22}

SOL. (b) We know that $N_\beta = N_0(1 - e^{-\lambda t})$

$$N_\beta = \frac{6.023 \times 10^{23}}{24}\left[1 - e\frac{\ell n2}{15} \times 7.5\right]$$

on solving we get,

$$N_\beta = 7.4 \times 10^{21}$$

43. A piece of wood from a recently cut tree shows 20 decays per minute. A wooden piece of same size placed in a museum (obtained from a tree cut many years back) shows 2 decays per minute. If half life of C^{14} is 5730 years, then age of the wooden piece placed in the museum is approximately:

[Online Apri1 19, 2014]

(a) 10439 years (b) 13094 years (c) 19039 years (d) 39049 years

SOL. (c) Given: $\frac{dN_0}{dt} = 20 decays/\min$

$$\frac{dN}{dt} = 2 decays/\min$$

$T_{1/2} = 5730$ years

As we know,

$$N = N_0 e^{-\lambda t}$$

$$\text{Log}\,\frac{N_0}{N} = \lambda t$$

$$t = \frac{1}{\lambda}\text{Log}\,\frac{N_0}{N}$$

$$= \frac{2.303 \times T_{1/2}}{0.693} \times \text{Log}_{10}\frac{N_0}{N}$$

But $\dfrac{\frac{dN_0}{dt}}{\frac{dN}{dt}} = \frac{N_0}{N} = \frac{20}{2} = 10$

$$t = \frac{2.303 \times 5730}{0.693} \times 1$$

$= 19039$ years

44. A piece of bone of an animal from a ruin is found to have C^{14} activity of 12 disintegrations per minute per gm of its carbon content. The C^{14} activity of a living animal is 16

disintegrations per minute per gm. How long ago nearly did the animal die? (Given half life of ^{14}C is $t_{1/2} = 5760$ years)

[Online April 12, 2014]

(a) 1672 years (b) 2391 years (c) 3291 years (d) 4453 years

SOL. (b) Given, for C^{14}

$A_0 = 16$ dis $min^{-1}g^{-1}$ $\quad$ $A = 12$ dis $min^{-1}g^{-1}$ $\quad$ $t_{1/2} = 5760$ years

Now, $\lambda = \frac{0.693}{t_{1/2}}$

$\lambda = \frac{0.693}{5760}$ per year

Then, from, $t = \frac{2.303}{\lambda}\log_{10}\frac{A_0}{A}$

$$= \frac{2.303 \times 5760}{0.693}\log_{10}\frac{16}{12}$$

$$= \frac{2.303 \times 5760}{0.693}\log_{10}1.333$$

$= \frac{2.303 \times 5760 \times 0.1249}{0.693} = 2390.81 \approx 2391$ years.

45. A radioactive nuclei with decay constant $0.5/s$ is being produced at a constant rate of 100 nuclei/s. If at $t = 0$ there were no nuclei, the time when there are 50 nuclei is:

[Online April 11, 2014]

(a) 1s (b) $2\ln\left(\frac{4}{3}\right)s$ **(c) $ln\ 2$ s** (d) $\ln\left(\frac{4}{3}\right)s$

SOL. (b) Let N be the number ofnuclei at any time t then,

$\frac{dN}{dt} = 100 - \lambda N$ $\quad$ or $\quad$ $\int_0^N \frac{dN}{(100 - \lambda N)} = \int_0^t dt$

$-\frac{1}{\lambda}\left[\log(100 - \lambda N)\right]_0^N = t$

$$\log(100 - \lambda N) - \log 100 = -\lambda t$$

$$\log\frac{100 - \lambda N}{100} = -\lambda t$$

$$\frac{100-\lambda N}{100}=e^{-\lambda t} \qquad 1-\frac{\lambda N}{100}=e^{-\lambda t}$$

$$N=\frac{100}{\lambda}\left(1-e^{-\lambda}t\right)$$

As, $N=50 \text{and} \lambda=0.5/\sec$

$$50=\frac{100}{0.5}\left(1-e^{-0.5t}\right)$$

Solving we get,

$$t=2\ln\left(\frac{4}{3}\right)\sec$$

46. The half-life of a radioactive element A is the same as the mean-life of another radioactive element B. Initially both substances have the same number of atoms, then :

[Online April 22, 2013]

(a) A and B decay at the same rate always.

(b) A and B decay at the same rate initially.

(c) A will decay at a faster rate than B.

(d) B will decay at a faster rate than A.

SOL. (d) $\left(T_{1/2}\right)_A=(t_{mean})_B$

$$\Rightarrow \frac{0.693}{\lambda_A}=\frac{1}{\lambda_B} \Rightarrow \lambda_A=0.693\lambda_B$$

or $\lambda_A<\lambda_B$

Also rate of decay $=\lambda N$

Initially number of atoms (N) of both are equal but since $\lambda_B>\lambda_A$,

therefore B will decay at a faster rate than A

47. The counting rate observed from a radioactive source at $t=0$ was 1600 counts s^{-1} , and $t=8s$, it was 100 counts s^{-1}. The counting rate observed as counts s^{-1} at $t=6s$ will be

[Online May 26, 2012]

(a) 250 (b) 400 (c) 300 (d) 200

SOL. (d) As we know,

$$\left[\frac{N}{N_0}\right]=\left[\frac{1}{2}\right]^n \text{ --(i)}$$

n = no. of half life   N- no. of atoms left N_0- initial no. of atoms

By radioactive decay law,

$$\frac{dN}{dt} = kN \qquad k\text{- disintegration constant}$$

$$\frac{\frac{dN}{dt}}{\frac{dN_0}{dt}}=\frac{N}{N_0} \text{ ---(ii)}$$

From (i) and (ii) we get

$$\frac{\frac{dN}{dt}}{\frac{dN_0}{dt}}=\left[\frac{1}{2}\right]^n$$

or, $\left[\frac{100}{1600}\right]=\left[\frac{1}{2}\right]^n \Rightarrow \left[\frac{1}{2}\right]^4=\left[\frac{1}{2}\right]^n$

therefore $n = 4$,

Therefore, in 8 seconds 4 half life had occurred in which counting rate reduces to 100 counts s^{-1}.

Half life, $T_{\frac{1}{2}} = 2$ sec

In 6 sec, 3 half life will occur

$$\left[\frac{\frac{dN}{dt}}{1600}\right]=\left[\frac{1}{2}\right]^3 \qquad \Rightarrow \frac{dN}{dt}=200 \text{ counts } s^{-1}$$

48. The decay constants of a radioactive substance for α and β emission are λ_α and λ_β respectively. If the substance emits α and β simultaneously, then the average halflife of the material will be

[Online May 19, 2012]

(a) $\frac{2T_\alpha T_\beta}{T_\alpha + T_\beta}$ **(b)** $T_\alpha + T_\beta$ **(c)** $\frac{T_\alpha T_\beta}{T_\alpha + T_\beta}$ **(d)** $\frac{1}{2}(T_\alpha + T_\beta)$

SOL. (c) $T_{av} = \frac{T_\alpha T_\beta}{T_\alpha + T_\beta}$

If α and B are emitted simultaneously.

49. Which of the following Statements is correct?

[Online May 12, 2012]

(a) The rate of radioactive decay cannot be controlled but that of nuclear fission can be controlled.

(b) Nuclear forces are short range, attractive and charge dependent.

(c) Nuclei of atoms having same number of neutrons are known as isobars.

(d) Wavelength of matter waves is given by de Broglie formula but that of photons is not given by the same formula

SOL. (a) Radioactive decay is a continuous process. Rate of radioactive decay cannot be controlled.

Nuclear fission can be controlled but not of nuclear fusion.

50. A sample originally contained 10^{20} radioactive atoms, which emit α-particles. The ratio of α-particles emitted in the third year to that emitted during the second year is 0.3. How many α-particles were emitted in the first year?

[Online May 7, 2012]

(a) 3×10^{18} **(b)** 3×10^{19} **(c)** 5×10^{18} **(d)** 7×10^{19}

SOL. (b)

51. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it had decayed and time t_1 when $\frac{1}{3}$ of it had decayed is:

[2011]

(a) 14 min **(b) 20 min** **(c) 28 min** **(d) 7 min**

SOL. (b) Number of undecayed atom after time t_2;

$$\frac{N_0}{3} = N_0 e^{-\lambda t_2} \qquad \text{(i)}$$

Number of undecayed atom after time t_1; $\frac{2N_0}{3} = N_0 e^{-\lambda t_1}$ (ii)

Dividing (ii) by (i), we get

$$2 = e^{\lambda(t_2 - t_1)}$$

$$\Rightarrow \text{In}2 = \lambda(t_2 - t_1)$$

$$\Rightarrow t_2 - t_1 = \text{In}2/\lambda$$

52. Statement-l : A nucleus having energy E_1 decays by β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement-2 : To conserve energy and momentum in β^- decay at least three particles must take part in the transformation.

[2011 RS]

(a) Statement-l is correct but statement-2 is not correct.

(b) Statement-l and statement-2 both are correct and statement-2 is the correct explanation of statement-l.

(c) Statement-l is correct, statement-2 is correct and statement-2 is not the correct explanation of Statement-l

(d) Statement-l is incorrect, statement-2 is correct.

SOL. (b) Statement-l: A nucleus having energy E_1 decays by β- emission to daughter nucleus having energy E_2 then β- rays are emitted with continuous energy spectrum with energy $E_1 - E_2$.

Statement-2: For energy conservation and momentum conservation at least three particles, daughter nucleus, β particle and antineutrino are required.

53. A radioactive nucleus (initial mass number A and atomic number Z emits 3α- particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

[2010]

(a) $\frac{A-Z-8}{Z-4}$ **(b)** $\frac{A-Z-4}{Z-8}$ **(c)** $\frac{A-Z-12}{Z-4}$ **(d)** $\frac{A-Z-4}{Z-2}$

SOL. (b) When a radioactive nucleus emits 1 α-particle, the mass number decreases by 4 units and atomic number decreases by 2 units. When a radioactive nucleus emits 1 positron the atomic number decreases by 1 unit but mass number remains constant.

Mass number of final nucleus $= A - 12$

Atomic number of final nucleus $= Z - 8$

Number of neutrons, $N_n = (A-12) - (Z-8) = A - Z - 4$

Number of protons, $N_p = Z - 8$

Required ratio $= \frac{N_n}{N_p} = \frac{A-Z-4}{Z-8}$

54. The half-life period of a radio-active element X is same as the mean life time of another radio-active element Y. Initially they have the same number of atoms. Then

[2007]

(a) X and Y decay at same rate always

(b) X will decay faster than Y

(c) Y will decay faster than X

(d) X and Y have same decay rate initially

SOL. (c) Let λ_X and λ_Y be the decay constant of X and Y.

Half life of X, $=$ average life of Y

$$T_{1/2} = T_{av}$$

$$\Rightarrow \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$$

$$\Rightarrow \lambda_X = (0.693).\lambda_Y$$

$$\lambda_X < \lambda_Y.$$

Now, the rate of decay is given by

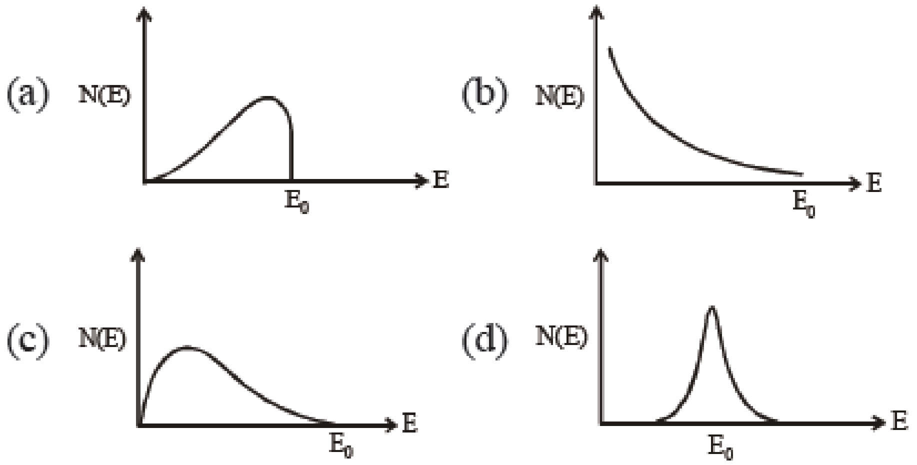
$$-\left(\frac{dN}{dt}\right)_x = \lambda_X N_0$$

$$-\left(\frac{dN}{dt}\right)_y = \lambda_y N_0$$

As the rate of decay is directly proportional to decay constant, Y will decay faster than X.

55. The energy spectrum of β-particles [number N(E) as a function of β-energy E emitted from a radioactive source is

[2006]



SOL. (c) The range of energy of β-particles is from zero to some maximum value.

56. Starting with a sample of pure 66_{Cu}, $\frac{7}{8}$ of it decays into Zn in 15 minutes. The corresponding halflife is

[2005]

(a) 15 minutes (b) 10 minutes (c) $7\frac{1}{2}$ minutes (d) 5 minutes

SOL. (d) It is given that

$\frac{7}{8}$ of Cu decays in 15 minutes.

Cu left undecayed is

$$N = 1 - \frac{7}{8} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$$

No. of half-lives $= 3$

$$n = \frac{t}{T} \Rightarrow 3 = \frac{15}{T}$$

$\Rightarrow T =$ halflife period $= \frac{15}{3} = 5$ minutes

57. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $\frac{1}{8}$. The thickness of lead which will reduce the intensityto $\frac{I}{2}$ will be [2005]

(a) 9mm (b) 6mm (c) 12mm (d) 18mm

SOL. (c) Let intensity of gamma radiation from source be I_0.

Intensity $I = I_0 . e^{-\mu d}$

Where d is the thickness of lead.

Applying logarithm on both sides,

$$-\mu d = \log\left(\frac{I}{I_0}\right)$$

For $d = 36$ mm, intensity $= \frac{I}{8}$

$-\mu \times 36 = \log\left(\frac{I/8}{I}\right)$ -----(i)

For intensity $I/2$, thickness $= d$

$$-\mu \times d = \log\left(\frac{I/2}{I}\right) \text{ -----}(ii)$$

Dividing (i) by (ii),

$$\frac{36}{d} = \frac{\log\left(\frac{1}{8}\right)}{\log\left(\frac{1}{2}\right)} = \frac{3\log\left(\frac{1}{2}\right)}{\log\left(\frac{1}{2}\right)} = 3 \quad \text{or } d = \frac{36}{3} = 12\text{mm}$$

58. Which of the following cannot be emitted by radioactive substances during their decay? [2003]

(a) Protons (b) Neutrinos (c) Helium nuclei (d) Electrons

SOL. (a) The radioactive substances emit α -particles (Helium nucleus), β-particles (electrons) and neutrinos. Protons cannot be emitted by radioactive substances during their decay.

59. A nucleus with $Z = 92$ emits the following in a sequence:

$$\alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$$

Then Z ofthe resulting nucleus is

[2003]

(a) 76 (b) 78 (c) 82 (d) 74

SOL. (b) The number of α-particles released $= 8$

Decrease in atomic number $= 8 \times 2 = 16$

The number of β^--particles released $= 4$

Increase in atomic number $= 4 \times 1 = 4$

Also the number of β^+ particles released is 2, which should decrease the atomic number by 2.

Therefore the final atomic number of resulting nucleus

$$= Z - 16 + 4 - 2 = Z - 14$$

$$= 92 - 14 = 78$$

60. A radioactive sample at any instant has its disintegration rate 5000 disintegrations per minute. After 5 minutes, the rate is 1250 disintegrations per minute. Then, the decay constant (per minute) is

[2003]

(a) $0.4 \ln 2$ (b) $0.2 \ln 2$ (c) $0.1 \ln 2$ (d) $0.8 \ln 2$

SOL. (a) Initial activity, $A_o = 5000$ disintegration per minute.

Activity after 5 min, $A = 1250$ disintegration per minute

$$A = A_o e^{-\lambda t}$$

$$\Rightarrow e^{-\lambda t} = \frac{A_o}{A}$$

$$\Rightarrow \lambda = \frac{1}{t} \log_e \frac{A_o}{A} = \frac{1}{5} \log_e \frac{5000}{1250}$$

$$= \frac{2}{5} \log_e 2 = 0.4 \log_e 2$$

61. At a specific instant emission of radioactive compound is deflected in a magnetic field. The compound can emit

(i) electrons (ii) protons (iii) He^{2+} (iv) neutrons

The emission at instant can be

[2002]

(a) i, ii,iii (b) i, ii, iii, iv (c) iv (d) ii, *iii*

SOL. (a) Charged particles are deflected in magnetic field. Electrons, protons and He^{2+} all are charged species. Hence, correct option is (a).

62. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ years, then the amount of substance left after 15 years is

[2002]

(a) $N_0/8$ (b) $N_0/16$ (c) $N_0/2$ (d) $N_0/4$

SOL. (a) After every half-life, the mass of the substance reduces to half its initial value.

$$N_0 \xrightarrow{5\text{ years}} \frac{N_0}{2} \xrightarrow{5\text{ years}} \frac{N_0/2}{2}$$

$$= \frac{N_0}{4} \xrightarrow{5\text{years}} \frac{N_0/4}{2} = \frac{N_0}{8}$$

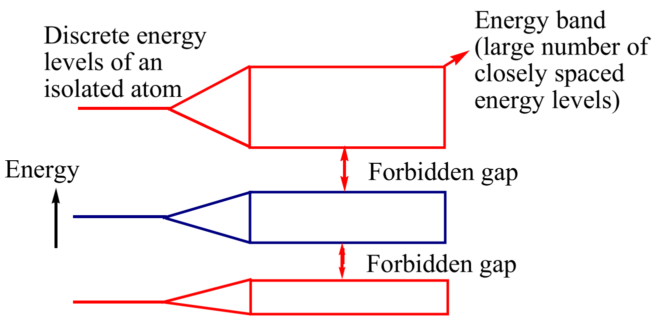
SEMICONDUCTORS

Classification of solids:

Solids are classified into two categories : amorphous solids and crystalline solids
In crystalline solids there is a long - range order in the arrangements of atoms. Crystalline solids have sharp melting points and are aniostropic. Sodium chloride, germanium, silicon etc., are crystalline solids
In amorphous solids there is no long - range order in the arrangement of atoms. Amorphous solids do not have sharp melting points and are isotropic. Glass is an example of an amorphous solids

Band theory in solids:

A The electrical conduction in solids is due to the prese nce of free electrons which are very loosely bound to the atom
A An isolated atom has well defined energy levels and energy of an electron depends on its orbit (Principal quantum number)
A But in solids atoms are so close such that the energies of their outer orbit electrons are influenced by neighbouring atoms. Hence it is not possible to talk about discrete energy levels for each atom.

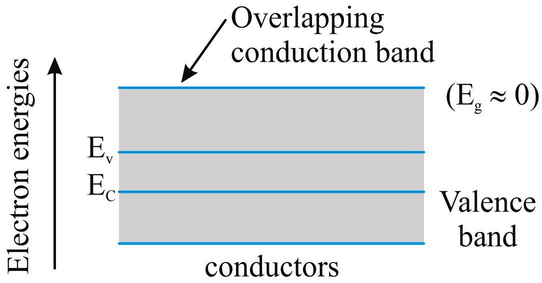


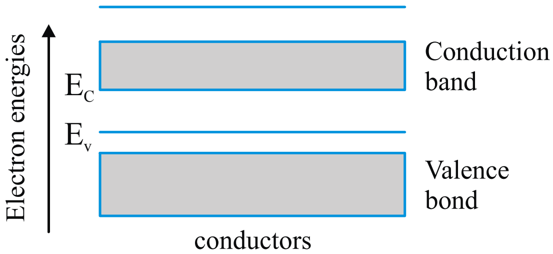
A Inside the crystal each electron has a unique position and no two electrons see exactly same pattern of surrounding charges and each electron has different energy level.
A Different energy levels are spread into bands called energy bands. Each band consists of closely spaced energy levels
A The highest filled energy band formed by a series of energy bands containing valance electrons is **valance band**.
A At 0 K, electrons start filling energy level in valance band starting from the lowest one. The highest energy level, occupied by an electron in the valance band at 0K is called **Fermi level**.
A The unfilled or partially filled energy band formed just above valance band is called **conduction band**. The conduction band is always occupied by free electrons which are responsible for conduction of a solid
A The energy difference between the top of the valence band and the bottom of conduction band is called energy gap (E_g). Energy gap also refered as forbidden energy gap is expressed in electron volt (eV)
A Resistivity (ρ), temparature coefficient of Resistivity (α) and number density (n) [number of charge carries per units volume] are three important electrical properties of solids. Basing

on the resistivites at room temparature, solids are classified into three categories : conductors, insulators and semiconductors.

1) Conductors : The energy band structure in conductors have two possibilities

A The valance band may be completely filled and the conduction band partially filled with an extremely small energy gap between them $E_g = 0$

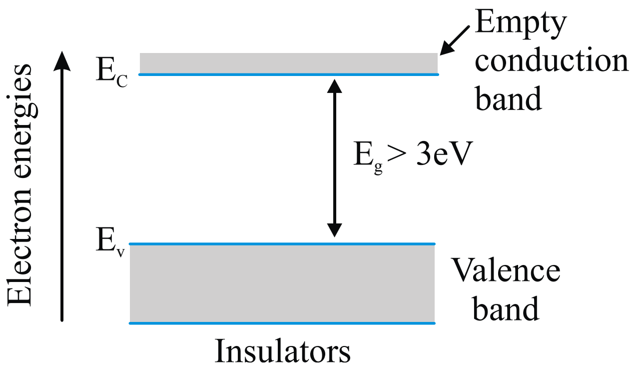




2) Insulators: In insulators forbidden energy gap is quite large. $E_g > 3ev$

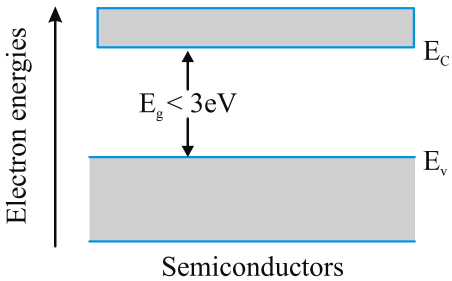
Eg. Energy gap for diamond is 5.5 eV.

Insulators have low conductivity($10^{-19}$to$10^{-11}$$Sm^{-1}$) or high resistivity ($10^{+11}$ to 10^{+19} Ω m)



3) Semi conductors: Semi conductors are the basic materials used in the present solid state devices like diode, transistor, Ic's.

A The energy band structure of the semiconductors is similar to insulators but in their case, the size of forbidden energy gap is much smaller than that of insulators. $E_g = 0.2eV\ to\ 3eV$



Eg: Forbidden energy gap for Ge is 0.67eV, for Si is 1.1 eV and for GaAs is 1.41eV
Germanium, silicon are natural semiconducting crystals while gallium arsenide, indium antimonide, cadmium sulphide, etc also exhibit semiconducting properties.
Semiconductors have conductivity ($10^5$to$10^{-6}$$Sm^{-1}$) or resistivity (10-^{5}to$10^6\,\Omega$ m), intermediate to metals and insulators
As the temperature increases more number of electrons jump into conduction band and hence the conductivity increases i.e, resistivity decreases Hence semiconductors have negative temperature coefficient of resistance.

Intrinsic semi conductor:

Semiconductors in the purest form are called as intrinsic semi conductors.

A At 0K all the valance electrons are involved in covalent bonding and so the crystal is a perfect insulator as there are no electrons available for conduction.

A At higher temperature due to thermal agitation, some of electrons gain sufficient energy to break away from covalent bonds and jump into conduction band thus becoming free electrons. (conductivity of the crystal increases)

A The electron that breaks away from covalent bond leaves behind a vacancy in the lattice. This vacancy is a site in the crystal in which there is an excess of positive charge. It is called as hole. Holes are always formed in valance band and their mobility is very less compared to that of free electrons.

A In intrinsic semiconductor electrons (free from covalent bonds) and holes are called intrinsic carriers and hence the name intrinsic semiconductor.

A When intrinsic semiconductor is connected to the terminals of a battery electrons drift towards positive terminal and holes towards negative terminal.

A In an intrinsic semiconductor if n_e denotes the electron number density in conduction band, n_h the hole number density in valence band and n_i the number density (or) concentration of intrinsic carriers, then $n_e = n_h = n_i$

A The intrinsic concentration n_i varies with T as $n_i^2 = A_0 T^3 e^{-Eg/KT}$ (A_0 is constant)

A The fraction of electrons of valence band present in conduction band is given by

$f \propto e^{Eg/KT}$

Where K is Boltzman's constant, E_g is forbidden energy gap and T is temperature

The Energy Gap : Experimentallly it has been found that the forbidden energy region E_g depends on temperature.

For silicon E_g (T) = 1.21 - 3.60 x 10^{-4} T

For germanium E_g (T) = 0.785 - 2.23 x 10^{-4} T

At room temperature (300K) for silicon E_g = 1.1 eV and for germanium E_g = 0.72 eV

Extrinsic semi conductor :

Doping: The process of adding small amounts (1 part in 1 million) of suitable material to intrinsic semi conductor so as to increase its conductivity enormously without distorting the basic crystal structure is called doping.

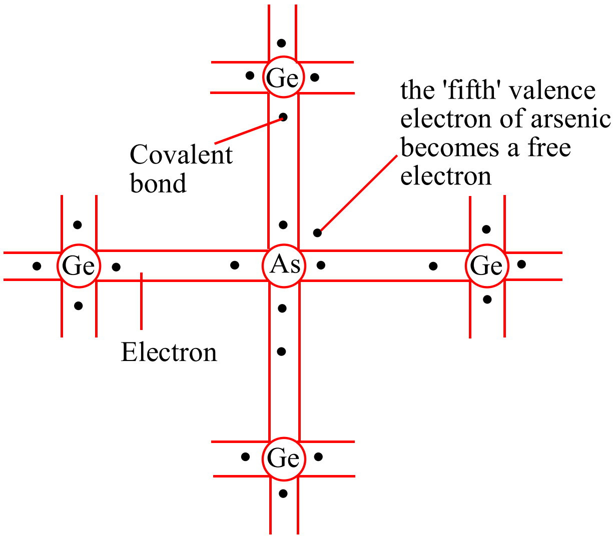
A The doping elements are generally referred as impurities. In doping process the impurity atoms occupy the empty sites of the crystal and hence the basic crystal structure is not distorted.

A The suitable impurities for semiconductors are i) pentavalent elements viz., antimony, arsenic etc., ii) trivalent impurities viz., indium, thallium etc.,

A An intrinsic semicondutor doped with a suitable impurity is called an extrinsic semiconductor. Two types of extrinsic semiconductors can be obtained i) n-type ii) p-type

n-type

A When instrinsic semiconductor is doped with a pentavalent impurity like arsenic, it occupies the empty site of the crystal and forms covalent bonds with four adjacent germanium atoms. The fifth electron of the arsenic atom requires least energy to jump to conduction band. The conduction band is thus filled with abundant electrons increasing the conductivity enormously.

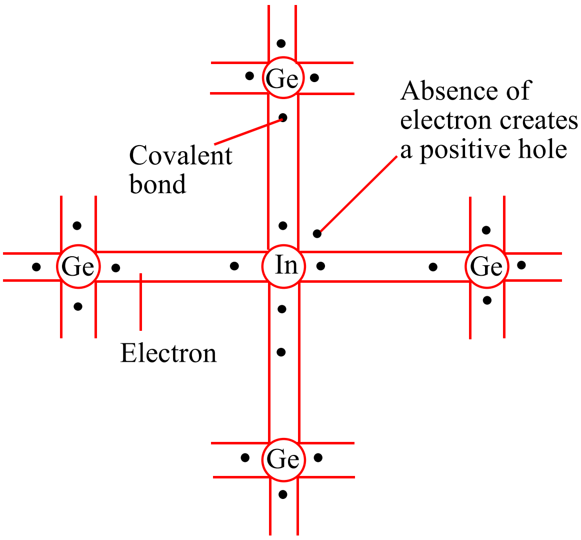


A The electrons are majority carriers and holes become minority carriers. Since electrons are negatively charged the extrinsic semiconductor is called n-type.

A Fermi level in n-type semi conductors (also known as donor energy level) lies in forbidden energy gap and is very close to conduction band ($\approx$0.01eV below conduction band)

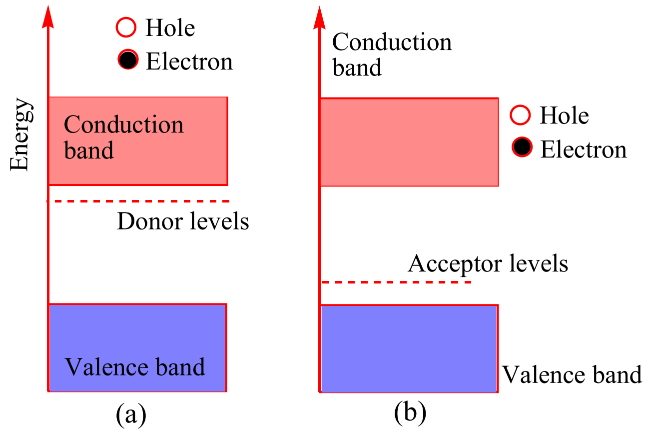
p-type

When instrinsic semiconductor is doped with a trivalent impurity like thallium, it occupies the empty site of the crystal and forms covalent bonds with three adjacent germanium atoms. The fourth electron of the thallium atom is unpaired which results in the formation of hole in valance band. The valance band is thus filled with abundant holes increasing the conductivity enormously.



A The holes are majority carriers and electrons become minority carriers. Since holes are positively charge the extrinsic semiconductor is called p-type.

A Fermi level in p-type semi conductor (also known as acceptor energy level) lies in forbidden energy gap and is very close to valence band ($\approx$0.01 to 0.05 eV above valence band)



A In a doped (or) extrinsic semi conductor number density of electrons in conduction band (n_e), number density of holes in the valence band (n_h) and number density of electrons in conduction band (or) holes in valence band in a pure semi conductor (n_i) then they are related as $n_e n_h = n_i^2$.

A Adding equal concentration of donor and acceptor or atoms to P type and N type semiconductor respectively results in an intrinsic semiconductor.

A When concentration of donor atoms exceeds the acceptor concentration in P type semiconductor, it changes from P type to N type semiconductor.

A In semi conductors the total current I is the sum of electron current I_e and holes current I_h
$I = I_e + I_h$

A Electrical conductivity $\sigma = \sigma_e + \sigma_h$

$$\sigma = n_e \mu_e e + n_h \mu_h e$$

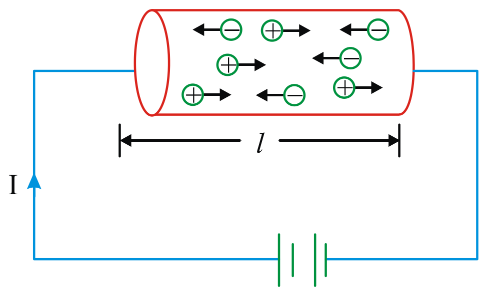
For intrinsic semi conductor $n_e = n_h = n_i$ so that $\sigma = n_i e(\mu_e + \mu_h)$

Electrical conductivity of Semiconductors: (Formulae derivation)

A Consider a block of semiconductor of length l, area of cross-section A and having number density of electrons and holes as n_e and n_h respectively .

A By that on applying a potential difference, say V, a current I flows through it as shown in figure.

A The electron current (I_e) and the hole current (I_h) constitute the total current I flowing through the semiconductor i.e ., $I = I_e + I_h$............(1)



A If n_e is the density of conduction band electrons in the semiconductor and V_e is the drift velocity of electrons, then electron current is given by $I_e = en_e A v_e$........(2)

Also, the hole current $I_h = en_h A v_h$......(3)
Using the equation (2) and (3) , the equation (1) becomes

$I = en_e A v_e + en_h A v_h$ or $I = eA(n_e v_e + n_h v_h)$........(4)
If ρ is the resistivity of the material of the semiconductor, then the resistance offered by the semiconductor to the flow of current is given by $R = \rho \frac{1}{A}$.........(5)

Since V=RI, from the equations (4) and (5),
we have

$$V = RI = \rho \frac{1}{A} \times eA(n_e v_e + n_h v_h)$$

or $V = \rho l\, e(n_e v_e + n_h v_h)$.............(6)

If E is the electric field set up across the semiconductor, then $E = \frac{V}{1}$........(7)

From the equations (6) and (7) , we have

$$E = \rho e(n_e v_e + n_h v_h)$$

or $\frac{1}{\rho} = e\left(n_e \frac{v_e}{E} + n_h \frac{v_h}{E}\right)$.........(8)
On applying electric field, the drift velocity acquired by the electrons (or holes) per unit strength of electric field is called mobility of electrons (or holes)

$\mu_e = \frac{v_e}{E}$ and $\mu_h = \frac{v_h}{E}$
Therefore, the equation (8) becomes

$$\frac{1}{\rho} = e(n_e \mu_e + n_h \mu_h)\text{.........}(9)$$

Also, $\sigma = \frac{1}{\rho}$ is called the conductivity of the material of semiconductor.

$\therefore \sigma = e(n_e \mu_e + n_h \mu_h)$ ....(10)

A Electrons mobility is greater than the hole mobility

A Mobility is a property of the semiconductor itself. It does not depends on the doping concentration.

A The mobility of an electron or hole generally decreases with increase temperature.

A Resistance of semi conductors decrease with the increase in temperature so semi conductors are insulators at low temperature but becomes slightly conducting at room tempeture.

A P- type (or) n- type semi conductor material is elctrically neutral.

EX. 1:The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneosuly with 5×10^{22} atoms per m^3 of Arsenic and 5×10^{20} per m^3 atoms of Indium. Calculate the number of electrons and holes. Given that $n_t = 1.5\times10^{16}\ m^{-3}$.Is the material - n type or p-type?

Sol. Arsenic is donor impurity No. of donor atoms added, $N_D = 5\times10^{22} m^{-3}$,

Indium is acceptor impurity, no.of accepter atoms added $N_A = 5\times10^{20} m^{-3}$

Therefore, no.of free electrons created $n_e = N_D = 5\times10^{22}$

Now, $n_e > n_h$, therefore, net no.of free electrons created,

$$n_e^{\ 1} = n_e - n_h = 5\times10^{22} - 5\times10^{20} = 4.95\times10^{22} m^{-3}$$

Also net no. of holes created $$n_h^1 = \frac{n_1^2}{n_e^1} = \frac{\left(1.5\times10^{16}\right)^2}{4.95\times10^{22}} = 4.55\times10^{9} m^{-3}$$

As $n_e^1 > n_h^1$, the resulting material is n-type semiconductor.

EX. 2: A semiconductor has an electron concentration of $0.45\times10^{12} m^{-3}$ and a hole concentration of $5.0\times10^{20} m^{-3}$. Calculate its conductivity . Given electron mobility $= 0.135 m^2V^{-1}s^{-1}$; hole mobility $= 0.048 m^2V^{-1}s^{-1}$,

Sol. The conductivity of a semicon ductor is the sum of the conductivities due to electrons and holes and is given by

$$\sigma = \sigma_e + \sigma_h = n_e e\mu_e + n_h e\mu_h = e\left(n_e\mu_e + n_h\mu_h\right)$$

As per given date , n_e is negligible as compared to n_h, so that we can write

$$\sigma = en_h\mu_h = \left(1.6\times10^{-19} C\right)\left(5.0\times10^{20} m^{-3}\right)\left(0.048 m^2V^{-1}s^{-1}\right)$$

$$= 3.84\Omega^{-1}m^{-1} = 3.84 Sm^{-1}$$

EX. 3: An N-type silicon sample of width $4\times10^{-3} m$ thickness and length $6\times10^{-2} m$ carries a current of 4.8mA when the voltage is applied across the length of the sample. What is the current density? If the free electron density is $10^{22} m^{-3}$, then find how much time it takes for the electrons to travel the full length of the sample .

Sol. The current density J is given by

$$J = \frac{I}{A} = \frac{4.8\times10^{-3}}{\left(4\times10^{-3}\right)\left(25\times10^{-5}\right)} = \frac{4.8\times10^{-3}}{10^{-6}}$$

The drift velocity v_d given by $$v_d = \frac{J}{ne} = \frac{4800}{10^{22}\times1.6\times10^{-19}} = 3m/s$$

The time taken ‘t’ is given by $t = \frac{L}{v_d} = \frac{6\times10^{-2}}{3} = 0.02\sec$

EX. 4:The energy gap of pure Si is 1.1 eV. The mobilities of electrons and holes are respectively $0.135m^2V^{-1}s^{-1}$ and $0.048m^2V^{-1}s^{-1}$ and can be taken as independent of temperature. The intrinsic carrier concentration is given by $n_i = n_0e^{-Eg/2kT}$. Where n_0 is a constant, E_g The gap width and k The Boltzmann's constant whose value is $1.38\times10^{-23}JK^{-1.}$ The ratio of the electrical conductivities of Si at 600K and 300K is.

Sol. The total electrical conductivity of a semiconductor is given by $\sigma = e(n_e\mu_e + n_h\mu_h)$

For an intrinsic semiconductor, $n_e = n_h = n_i$

We can thus write for the conductivity $\sigma = e(\mu_e + \mu_h)n_i$ or $\sigma = e(\mu_e + \mu_h)n_0e^{E_g/2kT}$

As the mobilities μ_e, μ_h are independent of temperature, they can be regarded as constant.

The ratio of the conductivities at 600 K and 300 K is then, $\frac{\sigma_{600}}{\sigma_{300}} = \frac{e(\mu_e + \mu_h)n_0e^{-E_g/2k\times600}}{e(\mu_e + \mu_h)n_0e^{-E_g/2k\times300}} = e^{-E_g/1200k}$

As per given data $E_g = 1.1eV$

$$k = 1.38\times10^{-23}JK^{-1}or\left(\frac{1.38\times10^{-23}}{1.6\times10^{-19}}\right)eVK^{-1}$$

$$\therefore k = 8.625\times10^{-5}eVK^{-1}$$

Solving we get the ratio of electrical conductivities is 4×10^4

EX. 5:In a p-n junction diode, the current I can expressed as $I = I_0\exp\left(\frac{eV}{2k_BT} - 1\right)$ where I_0 is called the reverse saturation current,V is the voltage across the diode and is positive for forward bias and negative for reverse bias, and I is the current through the diode, K_B is the Boltzmann constant $(8.6\times10^{-5}eV/K)$ and T is the absolute temperature. If for a given diode $I_o = 5\times10^{-12}$ A and T= 300K, then

(a) What will be the forward current at a forward voltage of 0.6V?

(b) What will be the increase in the current if the voltage across the diode is increased to 0.7V?

(c) What is the dynamic resistance?

(d) What will be current if reverse bias voltage changes from 1Vto 2V?

Sol. $I_o = 5\times10^{-12}A, k = 8.6\times10^{-5}eVk^{-1}$

$$= 8.6\times10^{-5}\times1.6\times10^{-19}Jk^{-1}$$

a) $I = I_0\left(e^{ev/2kT} - 1\right)$,

For V=0.6V,

$$I = 5\times10^{-12}\left(e^{\frac{1.6\times10^{-19}\times0.6}{2\times8.6\times10^{-5}\times1.6\times10^{-19}\times300}} - 1\right)$$

$$= 5\times10^{-12}\left(e^{23.52} - 1\right)$$

$=5\times10^{-12}\left(1.256\times10^{10}-1\right)=0.0628.A$

b) For v=0.7v, we have

$$I=5\times10^{-12}\left(e^{\frac{1.6\times10^{-19}\times0.7}{2\times8.6\times10^{-5}\times1.6\times10^{-19}\times300}}-1\right)$$

$$I=5\times10^{-12}\left(e^{27.32}-1\right)$$

$$=5\times10^{-12}\left(6.054\times10^{11}-1\right)=3.0271A$$

$\therefore \Delta I=3.271-0.0628=2.9643A$

c) $\Delta I=2.9643, \Delta v=0.7-0.6=0.1V$

dynamic resistance $=\frac{\Delta v}{\Delta I}=\frac{0.1}{2.9643}=0.0337\Omega$

d) For change in voltage from 1 to 2v, the current will remain equal to $I_0=5\times10^{-12}A$. It shows that the diode possesses practically infinite resistance
in reverse biasing

EX. 6:The energy of a photon of sodium light $(\lambda=589nm)$ equal to the band gap of a semiconducting material (a) Find the minimum energy E required to create a hole-electron pair (b) Find the value of E/kT at a temperature of 300K

Sol. **(a)** The energy of the photon in

$$eV=\frac{12400}{\lambda}=\frac{12400}{5890}=2.1eV$$

(Wavelength of photon =589nm=5890 A^0)

Thus the band gap is 2.1ev . This is also the minimum energy E required to push a n electron from the valence band into the conduction band. Hence the minimum energy required to create a hole-electron pair is 2.1eV

(b) At T=300K,kT$=\left(8.62\times10^{-5}eV/K\right)\left(300K\right)$

$=25.86\times10^{-3}eV$ Thus , $\frac{E}{kT}=\frac{2.1eV}{25.86\times10^{-3}eV}=81$

The availale thermal energy is nearly 81 times less than that of the required energy to create electron hole pair. So it is difficult for the thermal energy to create the hole-electron pair but a photon of light can do it easily

P-n junction: A two terminal device made out of a single semiconducting crystal, doped such that one side is p-type is other side is n-type is called a p-n junction diode. The plane that divides p-region and n-region is called p-n junction.

A At p-n junction migration of majority charge carriers i.e. holes form p side to n side and electrons from n side to p-side due to concentration difference is called diffussion

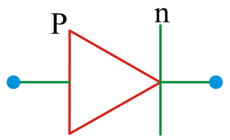
A These immobile ions at junction develop a potential difference and as it prevents further diffusion of charges across junction it is called potential barrier.

A The physical region of potential barrier is viod of charge carriers and hence called depletion layer. The width of the depletion layer is of the order of few micrometer.

A The large electric field of intensity (E) is directed from n-type to p-type at the junction and if potential barrier is V_b, width of depletion layer is d then $E = \frac{V_b}{d}$.

A The size potential barrier depends on nature of semi conductor crystal, temperature and amount of doping

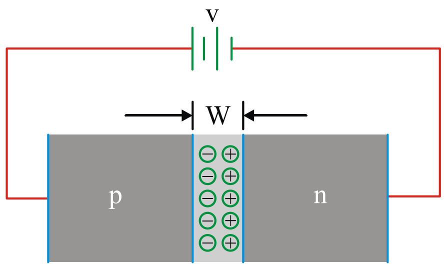
A The symbol of p-n junction diode is given below



Biasing pn-junction diode :

A Connecting the terminals of pn-junction diode to the terminals of a battery is called biasing. A diode can be biased in two ways, viz., i) forward bias ii) reverse bias.

1) Forward bias :



A When an external voltage V is applied across a semiconductor diode such that p-type is connected to +ve terminal and n-type to -ve terminal of battery (in general p - type to high voltage and n-type to low voltage), the diode is said to be forward biased.

A External voltage V is greater and opposite to barrier potential V_b. So width of depletion layer and resistance decrease.

A Effective barrier voltage under forward bias is V_b-V.

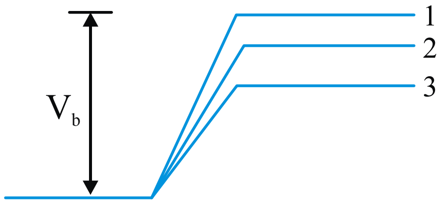


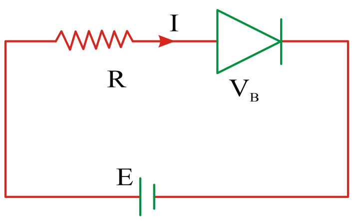
Fig. p-n junction diode under forward bias. Barrier potential (1) without battery, (2) low battery voltage, and (3) high voltage battery.

A Resistance of ideal diode in forward bias is zero

A If external voltage (V) is greater than barrier voltage then majority charge carriers diffuse across the junction and constitute diffusion current ($I = I_e + I_h$)

A Direction of diffusion current is from p- type to n-type

A The current flows through the diode in the below circuit is



$$I = \frac{E - V_B}{R + r_f}$$

Where R= external resistance,

r_f = resistance of diode in forward bias,

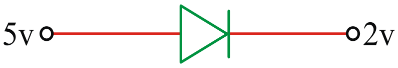
V_B =barrier potential.

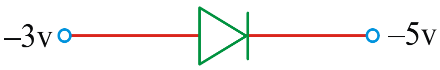
i) Power developed across the diode $= V_B I$

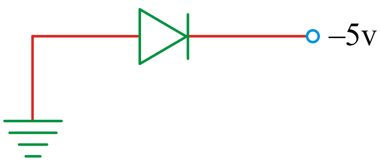
ii) Power developed across the resistor $= (E - V_B)I$,

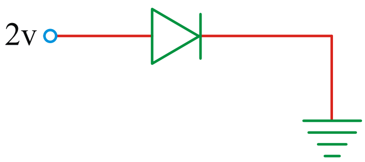
A The external voltage beyond which diode current start increasing rapidly is called knee voltage (V_{knee})

A The below diagram show forward bias of junction diode.

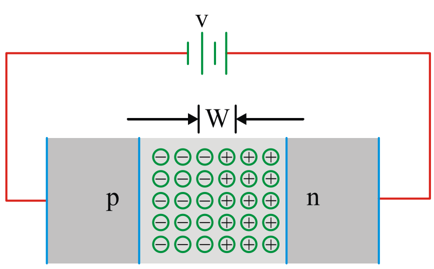
a) 

b) 

c) 

d) 

2) Reverse bias :



A When an external voltage V is applied across a semiconductor diode such that p-type is connected to -ve terminal and n-type to +ve terminal of battery (in general p - type to low voltage and n-type to high voltage), the diode is said to be reverse biased.

A External voltage V is greater and in same direction to barrier potential V_b, so width of depletion layer and resistance increase.

A Effective barrier voltage under reverse bias is $V_b + V$.

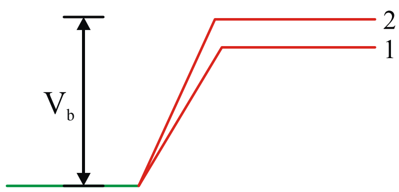


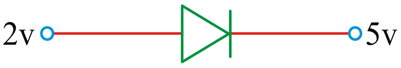
Fig : Diode under reverse bias, Barrier potential under reverse bias.

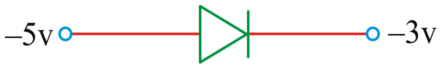
A Resistance of ideal diode in reverse bias is infinity upto a large external voltage (V).

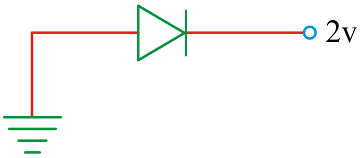
A At lower external voltage few covalent bonds are broken to liberate electrons and holes and these constitute reverse saturation current

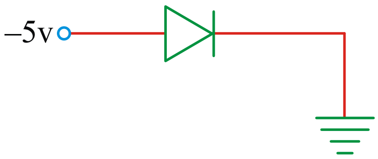
A At very high reverse bias voltage all the covalent bonds are broken to liberate large number of electrons called as breakdown voltage.

A The below diagram show reverse bias of p n junction diode.

a) 

b) 

c) 

d) 

A Thus there is an unexpected release (avalanche) of large number of electrons and holes there by sharp increase in current takes place at a voltage called avalanche breakdown voltage.

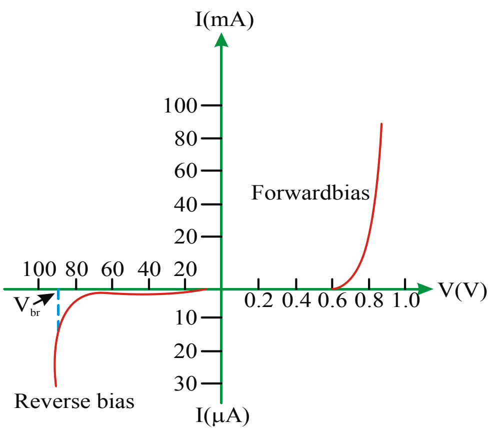


Fig : Avalanche breakdown

Note: The potential barrier existing across an unbiased p-n junction is V_B volt

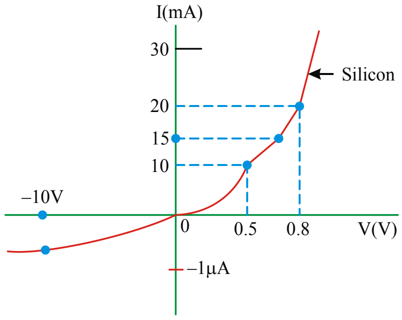
i) The minimum kinetic energy required by a hole to diffuse from the p-side to the n-side is 'eV_B'

ii) If the junction is forward biased at V volt, then the minimum kinetic energy required by a hole to diffuse from the p-side to the n-side is $e\left(V_B - V\right)$

iii) If the junction is reverse biased at V volt, then the minimum kinetic energy required by a hole to diffuse from the p-side to the n-side is $e\left(V_B + V\right)$

EX. 7: The V-I characteristic of a silicon diode is shown in the Fig. Calculate the

resistance of the diode at a) $I_D = 15mA \quad and \quad (b) V_D = -10V$



Sol. Considering the diode characteristics as a straight line between I=10mA to I=20mA passing through the origin, we can calculate the resistance using Ohm's law

a) From the curve at I=20mA,V=0.8V

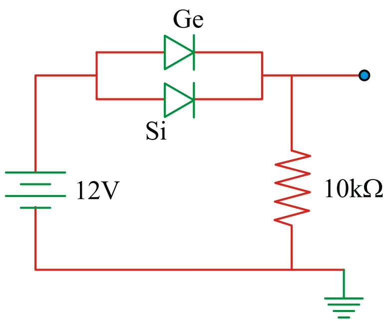
I=10mA,V=0.7V

$r_{fb} = \Delta V / \Delta I = 0.1V / 10mA = 10\Omega$

b) From the curve at $V = -10V,\ I = -1\mu A$

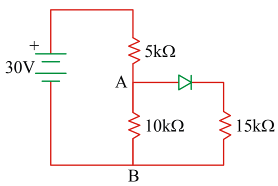
Therefore, $r_{rb} = 10V / 1\mu A = 1.0 \times 10^7 \Omega$

EX. 8: Two junction diodes, one of germanium (Ge) and other of silicon (Si) are connected as shown in fig to a battery of 12V and a load resistance $10k\Omega$. The germanium diode conducts at 0.3V and silicon diode at 0.7V. When current flows in the circuit, the potential of terminal Y will be



Sol. The Ge diode conducts for a p.d of 0.3V, therefore the current passes through it and the Si diode do not conduct. Hence the potential of terminal $Y = 12 - 0.3 = 11.7V$

EX. 9: Find maximum voltage across AB in the circuit shown in Fig. Assume that diode is ideal



Sol. As the diode is treated ideal, its forward resistance $R_f = zero$. It acts as short circuit. So $10\,k\Omega$ is in parallel with $15\,k\Omega$ and the effective resistance across AB is

$$R_{AB} = \frac{10 \times 15}{10 + 15} = \frac{10 \times 15}{25} = 6k\Omega$$

$6k\Omega$ is in series with $5k\Omega$

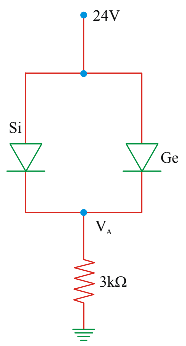
$\therefore$ total resistance $= R_T = 6k\Omega + 5k\Omega = 11k\Omega$,

V=30V. Current drawn from the battery is

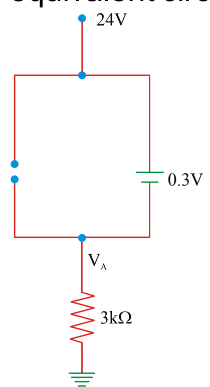
$I = \frac{V}{R_T} = \frac{30V}{11k\Omega} = 2.72mA$

$V_{AB} = IR_{AB} = 2.72mA \times 6k\Omega = 16.32V$

EX. 10:Find the voltage V_A in the circuit shown in figure . The potential barrier for Ge is 0.3 V and for Si is 0.7 V

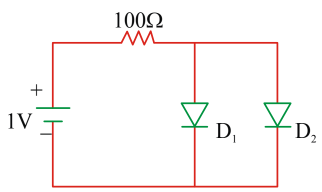


Sol. In the situation given, germanium diode will turn on first because potential barrier for germanium is smaller. The silicon diode will not get the opportunity to flow the current and so remains in open circuit. The equivalent circuit is as in figure

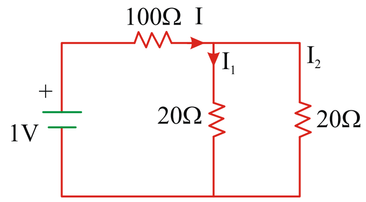


$V_A = 24 - 0.3 = 23.7V$

EX. 11:Considering the circuit and data given in the diagram, calculate the currents flowing in the diodes D_1 and D_2 Forward resistance of D_1 and D_2 is 20Ω



Sol. Since the positive terminal of battery is connected to P-type of both diodes D_1 and D_2, they are forward biased. These diodes are replaced by with their forward resis tance as shwon is Fig:

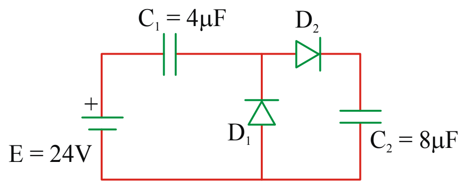


The resistance of 20Ω and 20Ω in parallel, $\frac{1}{R}=\frac{1}{20}+\frac{1}{20}(or)R=\frac{20}{2}=10\Omega$

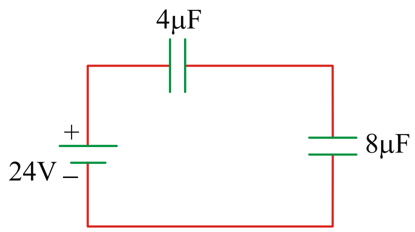
Therefore, total current I in the circuit

$I=\frac{1}{100+10}=\frac{1}{110}$ amp and $I_1=I_2=\frac{1}{2}\times\frac{1}{110}=\frac{1}{220}$ amp

EX. 12:In the circuit shown, the potential drop across each capacitor is (assuming the two diodes are ideal)



Sol. The diode D_1 is reverse biased (open circuit), but the diode D_2 is forward (short circuit).



$\therefore$ the potential of the battery divides across the two capacitors in the inverse ratio of their capacities.

$$i.e,\ \frac{V_1}{V_2}=\frac{C_2}{C_1}=\frac{8}{4}=\frac{2}{1}$$

$$V_1=\frac{2}{3}E=\frac{2}{3}\times 24=16V$$

$$V_2=\frac{1}{3}E=\frac{1}{3}\times 24=8V$$

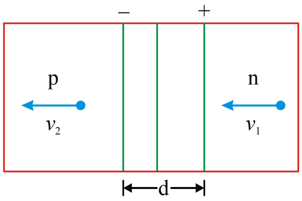
EX. 13: A potential barrier of 0.50V exists across a p-n junction

a) If the depletion region is $5.0\times10^{-7}m$ wide,what is the intensity of the electric field in this region?

b) An electron with a speed of $5.0\times10^{5}m/s$ approaches the p-n junction from the n-side. With what speed will it enter the p-side ?

Sol. a) The electric field is E=V/d

$$=\frac{0.50V}{5.0\times10^{-7}m}=1.0\times10^{6}V/m$$

b) 

Let the electron has a speed v_1 when it enters the depletion layer and v_2 when it comes out of it. As the potential energy increases by $e \times 0.50V$. From the principle of conservation of energy $\frac{1}{2}m\upsilon_1^2 = e \times 0.50 + \frac{1}{2}m\upsilon_2^2$

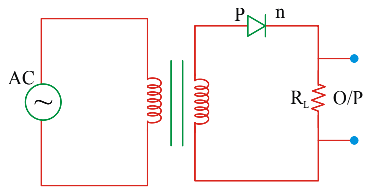
$$\frac{1}{2}\left(9.1\times10^{-31}\right)\left(5\times10^5\right)^2$$

$$= 1.6\times10^{-19}\times0.5+\frac{1}{2}\times9.1\times10^{-31}\times\upsilon_2^2$$

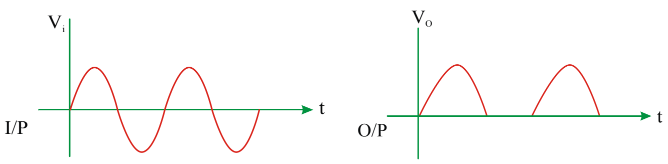
Solving this. $\upsilon_2 = 2.7\times10^5 m/s$

Application of junction diode as a Rectifier: The process of conversion of ac to dc is called rectification, tha arrangement is called rectfier. They are

A **Half wave rectifier**



Half Wave Rectifier



A In halfwave rectification we need atleast one semicondcutor diode.

A In half wave rectifier the efficiency is $\eta = 0.406\frac{R_L}{r_f + R_L}$ (where $r_f \rightarrow$ forward resistance of diode and $R_L \rightarrow$ load resistance.)
It's maximum value is 40.6%.

A Maximum current

$$I_m = \frac{V_m}{R_L + r_f}\left(V_m \text{ is maximum voltage}\right)$$

A Mean dc current $I_{dc} = \frac{I_m}{\pi}$

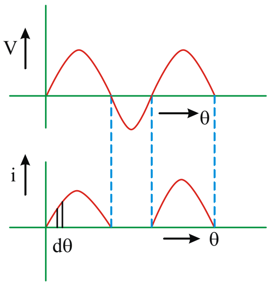
A $I_{rms} = \frac{I_m}{2}$

A Value of ripple factor = 1.21
A The ripple frequency is equal to the frequency of applied emf.
A The value of dc component in out put voltage is less than the ac.

A Input ac power = $I_{rms}^2 \left(R_L + r_f\right)$

A Out put dc power = $I_{dc}^2 R_L$

Efficiency of a half-wave rectifier (Formula derivation)



From fig

$$I_{average} = \frac{Area\ \ under\ \ the\, curve\ \ for\ \ a\ \ cycle}{base}$$

$$= \frac{\int_0^{\pi} i d\theta}{2\pi} \qquad = \frac{1}{2\pi}\int_0^{\pi} \frac{V_m \sin\theta}{r_f + R_L} d\theta$$

$$= \frac{V_m}{2\pi\left(r_1 + R_L\right)}\left[-\cos\theta\right]_0^{\pi} \qquad = \frac{V_m}{r_1 + R_L} \times \frac{1}{\pi} = \frac{I_m}{\pi}$$

$$\left[where\, I_m = \frac{V_m}{r_f + R_L}\right]$$

Where r_f and R_L denote diode resistance and load resistance respectively

Hence d.c. power $= I_{d.c}^2 \times R_L = \left(\frac{I_m}{\pi}\right)^2 \times R_L$

a.c. power input= $I_{r.m.s}^2 \left(r_f + R_L\right)$

For a half-wave rectifier $I_{r.m.s} = \frac{I_m}{2}$

Hence $P_{a.c} = \frac{I_m^2}{2} \times \left(r_f + R_L\right)$

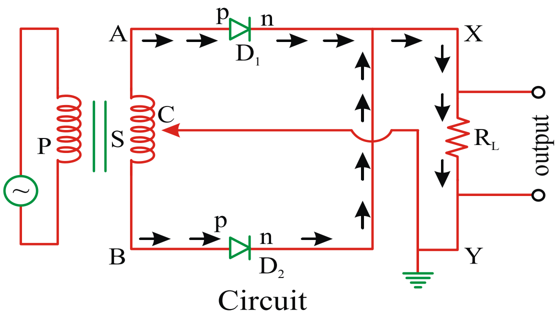
$\therefore$ Rectifier efficiency $= \frac{P_{dc}}{P_{ac}} \qquad = \frac{\left(I_m / \pi\right)^2 \times R_L}{\left(I_m / 2\right)^2 \left(r_f + R_L\right)} = \frac{0.406 R_L}{r_f + R_L}$

$$\eta = \frac{0.406}{1+\frac{r_f}{R_L}}$$

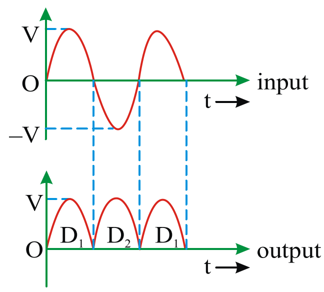
The efficiency is maximum when r_f is neglible.

$$\eta_{\max} = 40.6\%$$

A **Full wave rectifier:**



Circuit



A In fullfwave rectification we need at least two semicondcutor diodes.

A In full wave rectifier the efficiency is $\eta = 0.812\frac{R_L}{r_f + R_L}$. Its maximum value is 81.2% (where $r_f \to$ forward resistance of diode and $R_L \to$ load resistance.)

A Maximum current

$$I_m = \frac{V_m}{R_L + r_f}(V_m \text{ is maximum voltage})$$

A Mean dc current $I_{dc} = \frac{2I_m}{\pi}$

A $I_{rms} = \frac{I_m}{\sqrt{2}}$

A Value of ripple factor = 0.482

A The ripple frequency is twice to the frequency of applied emf.

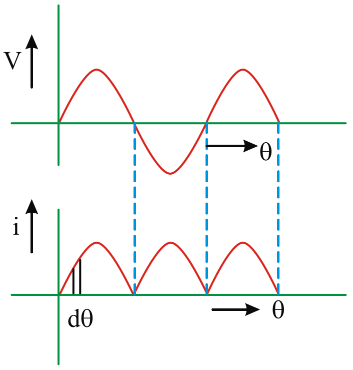
A The value of dc component in out put voltage is more than the ac.

A Input ac power = $I_{rms}^2 (R_L + r_f)$

A Out put dc power = $I_{dc}^2 R_L$

Efficiency of full-wave rectifier: (Formula derivation)

Efficiency of full-wave rectifier is given by



$$\eta = \frac{P_{d.c}}{P_{a.c}}$$

Instantaneous current is $i = \dfrac{\upsilon}{r_f + R_L}$

Average d.c. current is $I_{d.c} = \dfrac{2I_m}{\pi}$

$\therefore$ d.c . power output $= I^2{}_{d.c} \times R_L = \left(\dfrac{2I_m}{\pi}\right)^2 \times R_L$

a.c. input power is given by $P_{d.c} = I^2_{r.m.s}\left(r_f + R_L\right)$

For a full-wave rectified cycle, $I_{r.m.s} = I_m / \sqrt{2}$

$$P_{a.c} = \left(\frac{I_m}{\sqrt{2}}\right)^2 \left(r_f + R_L\right)$$

$\therefore$ Full-wave rectification efficiency,

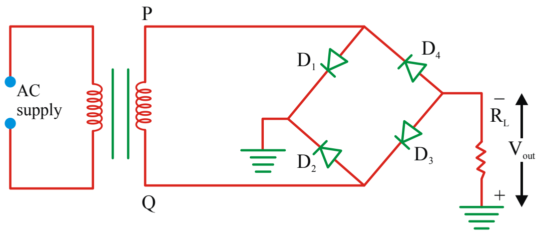
$$\eta = \frac{P_{d.c.}}{P_{a.c.}} = \frac{\left(2I_m / \pi\right)^2 R_L}{\left(\dfrac{I_m}{\sqrt{2}}\right)^2 \left(r_f + R_L\right)} = \frac{0.812 R_L}{r_f + R_L} = \frac{0.812}{1 + \dfrac{r_f}{R_L}}$$

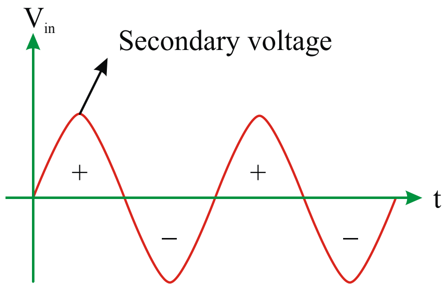
The efficiency will be maximum if r_f is zero

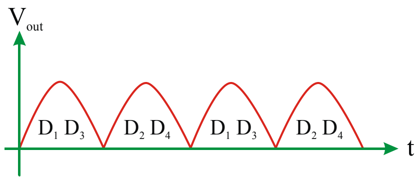
$\therefore$ maximum efficiency = 81.2%

The efficiency of a full-wave rectifier is double the efficiency of a half-wave rectifier

Full-wave bridge rectifier: In a centre tap full-wave rectifier, it is difficult to locate the centre tap on the secondary winding, which can be overcome in bridge rectifier. The circuit is shown in Figure Four diodes D_1, D_2, D_3 and D_4 are used in the circuit.







Ripple factor: The ratio of r.m.s value of a.c component to d.c. component in the rectifier output is called ripple factor.

$$\text{Ripple factor} = \frac{I_{a.c.}}{I_{d.c.}} = \sqrt{\left(\frac{I_{r.m.s}}{I_{d.c.}}\right)^2 - 1}$$

Ripple factor decides the effectiveness of a rectifier. The smaller value of ripple factor shows lesser a.c. component; hence more effectiveness of rectifier.

i) For half-wave rectification,

$$I_{r.m.s.} = \frac{I_m}{2}; \quad I_{d.c.} = \frac{I_m}{\pi}$$

$$\text{Ripple factor} = \sqrt{\left(\frac{I_m/2}{I_m/\pi}\right)^2 - 1} = 1.21$$

ii) For full-wave rectification

In full-wave rectification ,

$$I_{r.m.s.} = \frac{I_m}{\sqrt{2}}; \qquad I_{d.c.} = \frac{2I_m}{\pi}$$

$$\text{Ripple factor} = \sqrt{\left(\frac{I_m/\sqrt{2}}{2I_m/\pi}\right)^2 - 1} = 0.48$$

A **Form factor:** It is ratio of $I_{r.m.s.}$ and $I_{d.c.}$

i) For half-wave rectification:

$$F=\frac{I_{r.m.s}}{I_{d.c.}}=\frac{\pi}{2}=1.57$$

$$\left[as\ I_{r.m.s}=\frac{I_m}{2}\ and\ I_{d.c}=\frac{I_m}{\pi}\right]$$

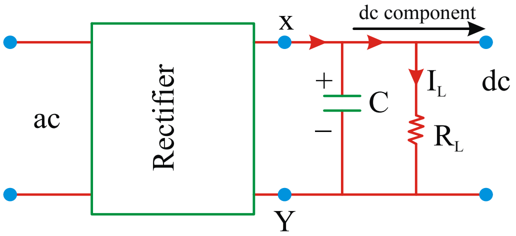
ii) For Full-wave rectification:

$$F=\frac{I_{r.m.s.}}{I_{d.c.}}=\frac{\pi}{2\sqrt{2}}$$

$$\left[as\ I_{r.m.s}=\frac{I_m}{\sqrt{2}}\ and\ I_{d.c}=\frac{2I_m}{\pi}\right]$$

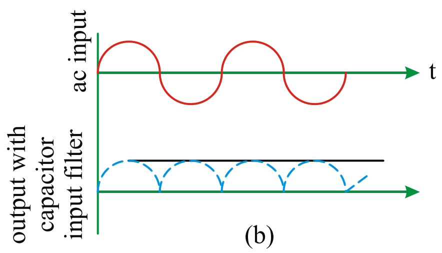
The role of capacitor in filtering:

- When the voltage across the capacitor is rising, it gets charged. If there is no external load,it remains charged to the peak voltage of the rectified output.
- When there is a load,it gets discharged through the load and the voltage across it begins to fall.In the next half-cycle of rectified output it again gets charged to the peak value in figure.
- The rate of fall of the voltage across the capacitor depends upon the inverse product of capacitor C and the effective resistance R_L used in the circuit and is called the time cosntant.
- To make the time constant large value of C should be large.So capacitor input filters use large capacitors .The output voltage obtained by using capacitor input filter is nearer to the peak volatge of the rectified voltage.This type of filter is most widely used in power supplies.



(a)

a) Full wave rectifier with capacitor filter



(b)

b) Input and out put voltage of rectifier in (a)

EX. 14:A p-n diode is used in a half wave rectifier with a load resistance of 1000Ω. If the forward resistance (r_f) of diode is 10Ω,calculate the efficiency of this half wave rectifier.

Sol. Load resistance $R_L=1000\Omega$

Forward resistance of the diode $=r_f=10\Omega$

Efficiency of half wave rectifier

$$\left[\frac{0.406R_L}{r_f + R_L}\right] = \frac{0.406 \times 1000}{1010} = 0.4019$$

The percentage efficiency of the half wave rectifier $\eta = 40.19\%$

EX. 15:Afull wave rectifier uses two diodes with a load resistance of 100Ω. Each diode is having negligible forward resistance . Find the efficiency of this full wave rectifier.

Sol. Forward resistance of the diode $r_f = 0$,

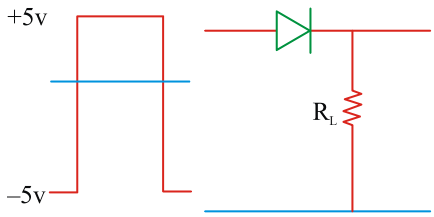
; Load resistance, $R_L = 100\Omega; \eta = ?$

efficiency of full wave rectifier

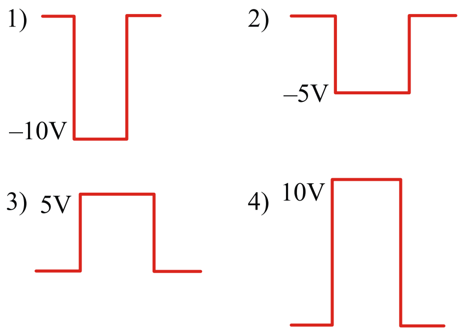
$$= \frac{0.812 \times 100}{100} = 0.812$$

The percentage efficiency of the full wave rectifier = 81.2%

EX. 16: If a p-n junction diode, a square input signal of 10V is applied as shwon



Then the out put signal across R_L will be



Sol. The junction diode will conduct when it is forward biased. Therefore, the output voltage will be obtainded during positive half cycle only . So option is (3)

p-n junction diodes are specially used as :

1) Zener diode
2) Opto electronic junction devices

1) Zener diode: A heavily doped p-n junction diode used to operate in reverse bias is called zener diode.

A The breakdown voltage in which the zener diode operates is the critical value of reverse potential difference at which the reverse current increases suddenly. The breakdown voltage is also referred as zener voltage.

A The breakdown in zener diode can occur by two distinct process depending on the level of doping of the diode. They are

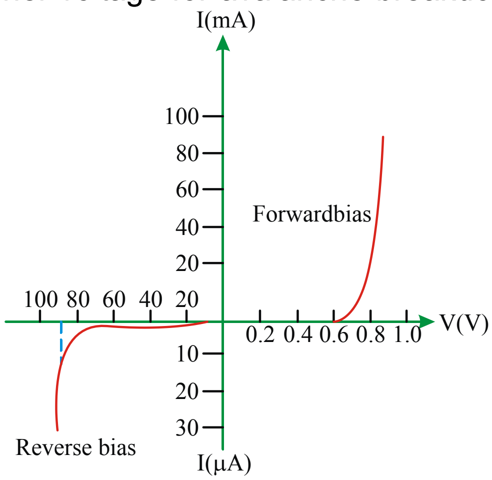
i) Zener breakdown (high level doping)
ii) Avalanche breakdown (low level doping)

A For zener breakdown, when reverse potential difference is increased the electric field across the depletion layer also increases and at a critical value of potential difference the electric field becomes so strong that it tears apart the covalent bonds releasing large number of electrons. This leads to massive increase in reverse current.

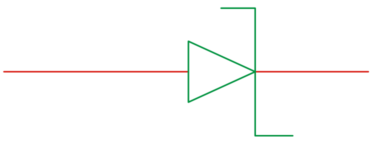
A Zener breakdown is predominent when the level of doping is high in the diode and is reversible. The zener voltage for zener breakdown is usually less than 5V.

A For avalanche breakdown, when reverse potential difference is increased to a very high value the electric field across the depletion layer accelerates the minority charges which gain sufficient energy and eject other electrons from the bonds. These ejected electrons cause further ionization by collisions with other electrons (avalanche) which finally leads to massive increase in reverse current.

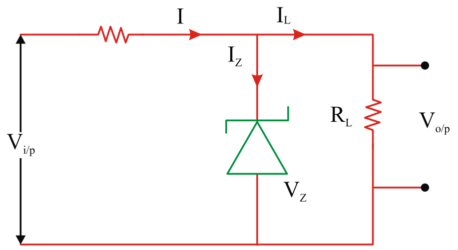
A Avalanche breakdown occurs in diodes that have low level doping and is irreversible. The zener voltage for avalanche breakdown is as high as 200V.



A Symbol of zener diode is



A Zener diode is used as voltage regulator its circuit diagram is



1) $I = I_Z + I_L$ 2) $V_{O/P} = V_Z = I_L R_L$

3) $V_{I/P} = IR + V_Z$

$V_{I/P} = IR + V_Z \Rightarrow V_{O/P} = V_{I/P} - (I_Z + I_L)R$

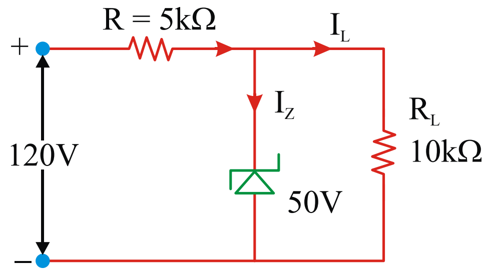
As $V_{I/P}$ changes current through zener diode change so that $V_{O/P}$ (or) V_Z remain constant.

EX. 17:For the circuit shown in figure, Find

1) the output voltage;

2) the voltage drop across series resistance;

3) the current through Zener diode.



Sol. From the figure $R = 5k\Omega = 5\times10^3\Omega;$

input voltage $V_{in} = 120V$; zener voltage , $V_z = 50V$

1) Output voltage $V_z = 50V$

2) Voltage drop across series tesistance R=

$V_{in} - V_z = 120 - 50 = 70V$

3) Load current $I_L = \frac{V_z}{R_L} = \frac{50}{10\times10^3} = 5\times10^{-3}A$

Current through R= $i = \frac{V_{in} - V_z}{R}$

$= \frac{70}{5\times10^3} = 14\times10^{-3}A$

According to Kirchoff's first law $I = I_L + I_z$

$\therefore$ Zener current

$I_Z = I - I_L = 14\times10^{-3} - 5\times10^{-3} = 9\times10^{-3} = 9mA$

2) Opto electronic junction devices Photodiode

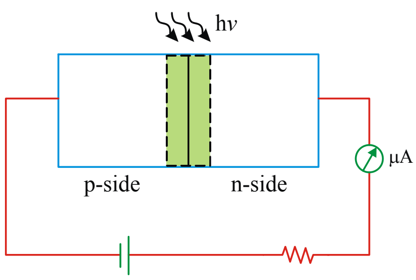


Fig : An illuminated photodiode under reverse bias

A Photocurrent is proportional to incident light intensity.

A Photodiode can be used as a photodetector to detect optical signals.

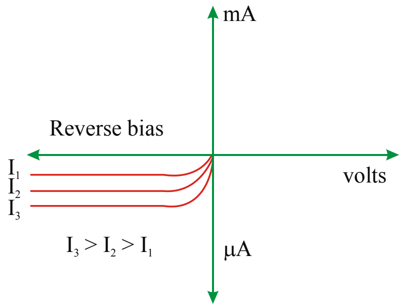


Fig : I-V characteristics of a photodiode for different illumination intensity $I_3 > I_2 > I_1$.

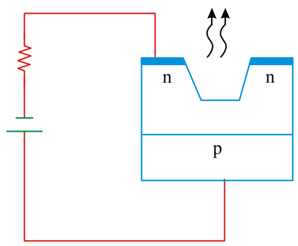
EX. 18: The current in the forward bias is known to be more (mA) than the current in the reverse bias (μA). What is the reason then to operate the photodiodes in reverse bias ?

Sol. Consider the case of an n-type semiconductor. The majority carrier density (n) is considerably larger than the minority hole density p(n>>p) On illumination, let the excess electrons and holes generated be $\Delta n \ \ and \ \ \Delta p$, respectively

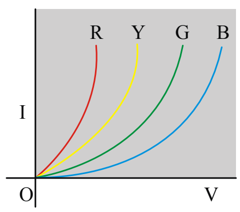
$$n^1 = n + \Delta n \quad ; \quad p^1 = p + \Delta p$$

Here n^1 and p^1 are the electron and hole concentrations at any particular illumination and n and p are carriers concentration when there is no illumination. Remember $\Delta n = \Delta p \quad and \quad n >> p$. Hence, the fractional change in the majority carriers (i.e., $\Delta n / n$)would be much less than that in the minority carrier dominated reverse bias current is more easily measurable than the fractional change in the forward bias current. Hence,photodiodes are preferably used in the reverse bias condition for measuring light intensity

Light Emitting Light (LED)



- A Light-emitting diode (LED) is a forward-biased p-n junction diode which emits visible light when energised .
- A The energy of radiation emitted by LED is equal to or less than the band gap of the semiconductor.
- A The band width of emitted light is $100 A^0$ to $500 A^0$ or in other words it is nearly (but not exactly) monochromatic.



The I-V characterstics of L.E.D:

Solar Cell:

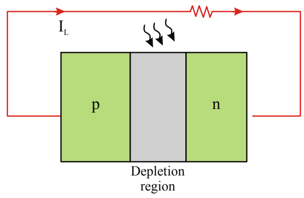


Fig : A typical illuminated p-n junction solar cell

- A Unlike a photodiode, a solar cell is not given any biasing. It supplies emf like an ordinary cell.
- A Sunlight is not always required for a solar cell.
- A Semiconductors with band gap close to 1.5ev. are ideal materials for solar cell fabrication
- A Si and GaAs are preferred material for solar cells.

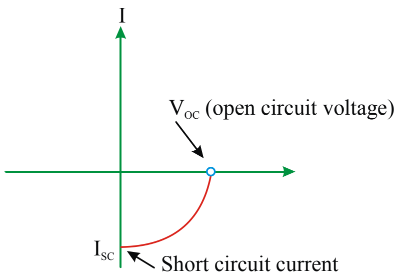


Fig : I-V characteristics of a solar cell

Selection of Solar material:

i) band gap (1.0 to 1.8eV)

ii) high optical absorption ($10^4 cm^{-1}$)

iii) good electrical conductivity

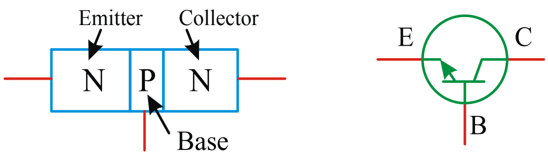
iv) availability of the raw material and

v) cost.

Transistors: Transfer + resistor = Transistor

- A Transistors are current operated solid-state devices.
- A Silicon is the element from which most of the transistors and other semiconductor components are made today.
- A Transistor has three regions known as the emitter (E), base (B) and collector (C)
  1) Emitter is heavily doped and medium in size.
  2) Base of a transistor is lightly doped and very thin.
  3) Collector of a transistor is moderately doped and large in size.
- A Transistor has two junctions
  1) emitter-base junction
  2) Collector-base junction
- A In a transistor the emitter base junction is forward biased and the collector base junction is reverse biased.

In an n-p-n Transistor:



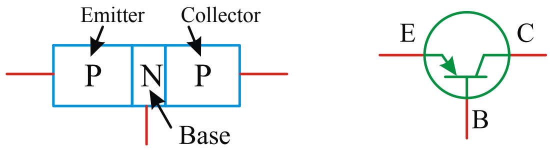
- A The current is due to electrons inside and outside the n-p-n transistor and they are the majority charge carriers
- A The conventional current flows from base to emitter.

A The emitter junction is forward biased and the collector junction is reverse biased.

A The emitter current (I_E) is the sum of base current (I_B) and collector current (I_C), i.e., $I_E = I_B + I_C$

A I_C is 97 to 98% of I_E and I_B is 2 to 3% of I_E.

In a p-n-p Transistor:



A The current is due to holes inside as they are the majority charge carriers and due to electons outside the p-n-p transistor.

A The conventional current is from emitter to base.

A The emitter junction is forward biased and the collector junction is reverse biased.

A Here also $I_E = I_C + I_B$

A I_C is 97 to 98% of I_E and I_B is 2 to 3% of I_E.

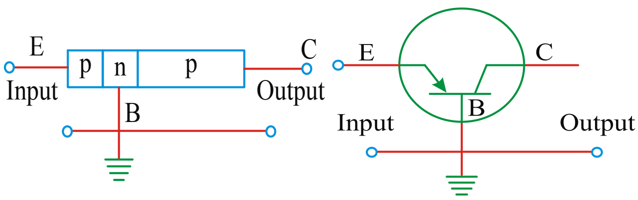
Thus the collector current is less than the emitter current ($I_C < I_E$)

Transistor Configurations: In electronic circuits transistors are connected in three ways. They are

1) Common base configuration ,
2) Common emitter configuration,
3) Common collector configuration

Common Base Configuration: In this configuration base is common to both input and output.

A Base terminal is earthed and input is given across base - emitter and output is taken across base - collector as shown in figure • This mode is called grounded base configuration.



A Current amplification factor of common base configuration for ac $\alpha = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{constant\ V_{CB}}$

A Current amplification factor of common base configuration for dc $\alpha = \frac{I_c}{I_E}$

A Values of α range from 0.95 to 0.99.

A Input resistance of transistor in CB configuration is $R_{in} = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CB}}$

A Output resistance of transistor in CB configuration is $R_{out} = \left(\frac{\Delta V_{CB}}{\Delta I_C}\right)_{I_E}$

A Voltage gain = current gain x resistance gain.

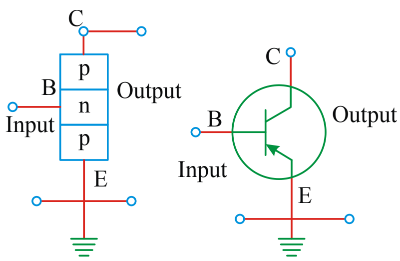
$$A_v = \alpha \times \frac{R_{out}}{R_{in}}$$

A Power gain = Voltage gain x current gain. $A_p = A_v \times \alpha = \alpha^2 \times \frac{R_{out}}{R_{in}}$

Common Emitter configuration: In this configuration emitter is common to both input and output.

A The emitter is earthed and input is given across base - emitter and output is taken across collecter - emitter as shown in fig

A This mode is called grounded emitter configuration.



A Current amplification factor of common emitter configuration for ac $\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{\text{constant } V_{CE}}$

A Current amplification factor of common emitter configuration for dc $\beta = \frac{I_c}{I_B}$

A Values of β range from 20 to 500.

A Input resistance of transistor in CE configuration is $R_{in} = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$

A Output resistance of transistor in CE configuration is $R_{out} = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B}$

A Voltage gain = current gain x resistance gain.

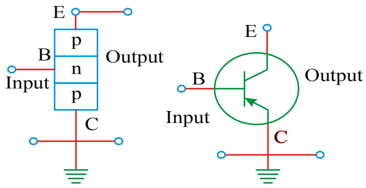
$$A_v = \beta \times \frac{R_{out}}{R_{in}}$$

A Power gain = Voltage gain x current gain. $A_p = A_v \times \beta = \beta^2 \times \frac{R_{out}}{R_{in}}$

Common collector configuration: In this configuration collector is common to both input and output.

A The collector is earthed and input is given across base - collector and output is taken across emitter -collector as shown in figure

A This mode is called grounded collector configuration.



A Current amplification factor of common collector configuration for ac $\gamma = \left(\dfrac{\Delta I_E}{\Delta I_B}\right)_{\text{constant } V_{CE}}$

A Current amplification factor of common collector configuration for dc $\gamma = \dfrac{I_E}{I_B}$

A Input resistance of transistor in CC configuration is $R_{in} = \left(\dfrac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}}$

A Output resistance of transistor in CCconfiguration is $R_{out} = \left(\dfrac{\Delta V_{CE}}{\Delta I_E}\right)_{I_B}$

Relation between α & β :-

ac current gain in C.B , $\alpha = \dfrac{\Delta I_C}{\Delta I_E}$

ac current gain in C.E , $\beta = \dfrac{\Delta I_C}{\Delta I_B}$

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{\Delta I_C}{\Delta I_E - \Delta I_C} = \frac{\dfrac{\Delta I_C}{\Delta I_E}}{1 - \dfrac{\Delta I_C}{\Delta I_E}} = \frac{\alpha}{1-\alpha}$$

$\Rightarrow \beta = \dfrac{\alpha}{1-\alpha}$ (or) $\alpha = \dfrac{\beta}{1+\beta}$

Relation between α , β and γ :

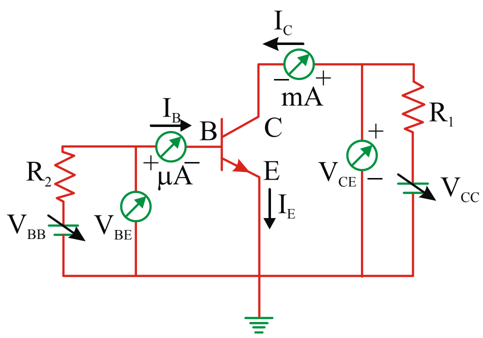
$$\gamma = \frac{\Delta I_E}{\Delta I_B} = \frac{1}{1-\alpha} = \frac{\beta}{\alpha} = \beta + 1$$

Comparative study of CB, CE, CC configurations

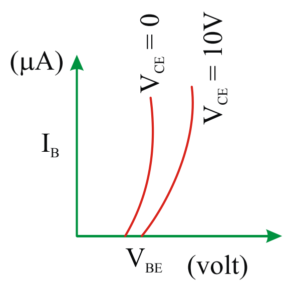
S.No	Parameter	CB configuration	CE configuration	CC configuration
1.	Input resistance	Minimum (50–20 Ω)	Medium (1–2K Ω)	Maximum (150–180K Ω)
2.	Output resistance	Maximum (1–2M Ω)	More (≈50K Ω)	Minimum (≈1K Ω)
3.	Current gain	Minimum $\alpha = 0.95 - 0.99$	More $\beta = 20 - 500$	Maximum $\gamma = 20 - 500$
4.	Voltage gain	Medium	Maximum	Minimum
5.	Power gain	Medium (20–30)	Maximum (30–40)	Minimum (≈10)
6.	Useful in application of	Current	Power	Impedance matching
7.	Phase difference	0	π rad (or) 180°	0

Characterestics of a transistor:

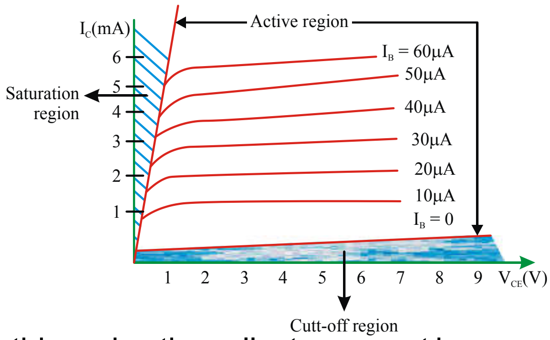
n–p–n Transistor (C.E)



1) **Input characterestics:** Input characterstics are drawn between V_{BE} verses I_B at constant V_{CE}. The output voltage V_{ce} is fixed (say at zero volts). The input voltage V_{be} is changed in steps (say 0.1V) upto 1 volt and the corresponding base current I_b is noted down. This process is repeated for different values (say 10V, 20V etc.) of V_{ce}.

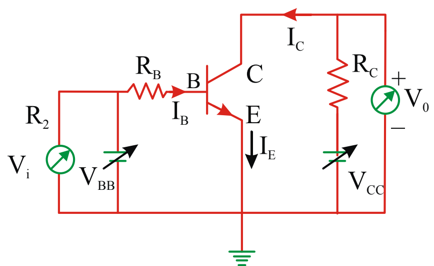


2) **Output characterestics:** Out put characterstics are drawn between V_{CE} verses I_C at constant I_B



- **Saturation region :-** In this region the collector current becomes almost independent of the base current. This happens when both junctions are forward biased.
- **Cut off region:-** In this region the collector curent is almost zero. This happens when both the junctions are reverse biased.
- **Active (or) Linear region:** In this region collector current (I_c) is many times greater than base current (I_b). A small change in input current (ΔI_b) produces a large change in the output current (ΔI_c). This happens when emitter junction is forward biased and collector junctions is reverse biased.
- The transistor works as an amplifier when operated in the active region.
- When the transistor is used in the cut off (or) saturation state it acts as a switch

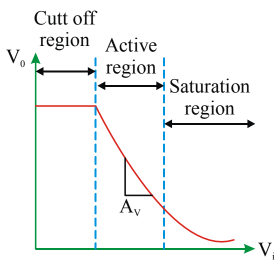
Transistor as a switch



- Appling KVL to the input and output sides of the circuit, We get

  $V_i = I_B R_B + V_{BE}$ and $V_O = V_{CC} - I_C R_C$
- In the case of Si transistor, as long as input Vi is less than 0.6 V, the transistor will be in cut off state and current I_C will be zero. Hence $V_o = V_{CC}$ When Vi becomes greater than 0.6 V the transistor is in active state ,so V_o decreases linearly till its value becomes less than about 1.0 V.

  Graph:



- If we plot the V_0 vs V_i curve [also called the transfer characterstics of the base biased transistor in figure,we see that between cut off state and active state and also between active state and saturation state there are regions of non-linearty showing that the transition from cut off state to active state and from active state to saturation state are not sharply defined.

Transistor as an Amplifier

(CE configuration)

The process of raising the strength of a weak input signal to a strong output signal is called 'amplification'.

Amplifier : It is a device which increases the weak input signal into strong output signal
Amplifier has wide applications in industries, T.V, radio and communication systems.
Amplifiers are of two types
1) Power amplifiers 2) Voltage amplifiers

1) **Power amplifier:** Amplifier which is used to raise the power level is known as "Power amplifier."

2) **Voltage amplifier:** The amplifier which is used to raise voltage level is known as voltage amplifier.

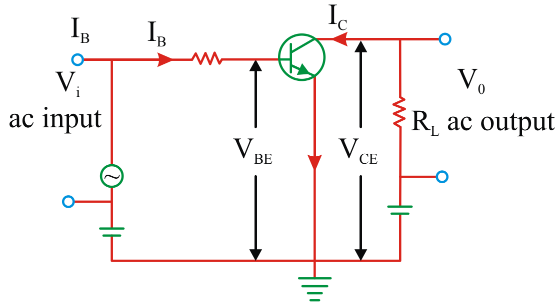


Fig:N-P-N TRANSISTOR AS AMPLIFIER

i) **Voltage gain:** It is defined as the ratio of change in output voltage to the change in input voltage.

$$A_v = \frac{\Delta V_{CE}}{\Delta V_{BE}} = -\frac{R_L(\Delta I_c)}{R_i(\Delta I_b)} = -\beta\frac{R_L}{R_i}$$

Negative sign indicates input and output voltages are in opposite phase.

ii) **Power gain:** It is defined as the ratio of output power to the input power.

power gain = $\frac{output\ \ power}{input\ \ power} = \frac{I_{out}V_{out}}{I_{in}V_{in}}$

Power gain= current gain × voltage gain

$$\Rightarrow A_p = \beta \times A_v = \beta^2 \times \frac{R_{out}}{R_{in}}$$

Note-1:

In common base amplifier, the phase difference between the input and output signals is zero

Note-2:

In common emitter amplifier, the phase difference between input and output signals is π

EX. 19:Current amplification factor of a common base configuration is 0.88. Find the value of base current when the emitter current is 1 m A.

Sol. In a common -base arragement, the current amplification factor $\alpha = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB}} = \frac{I_C}{I_E}$

Given $\alpha = 0.88, I_E = 1mA$

$\therefore$ Collector current

$I_C = \alpha I_E = 0.88 \times 1 = 0.88 mA$

Now since $I_E = I_B + I_C$

$\therefore$ Base current $I_B = I_E - I_C = 1 - 0.88 = 0.12 mA$

EX. 20: In a transistor, the emitter circuit resistance is 100Ω and the collector resistance is 100Ω. The power gain, if the emitter and collector currents are as sumed to be equal, will be

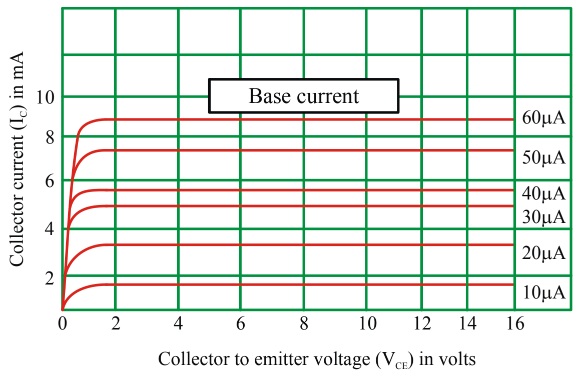
Sol. If $I_C \approx I_B \Rightarrow \beta \approx 1$

$$\therefore \; A_P = \beta^2 \left(\frac{R_L}{R_i}\right) = \left(\frac{R_L}{R_i}\right) = \frac{100 \times 10^3}{100} = 1000$$

EX:21: From the output characteristics shown in Fig .Calculate the values of β_{ac} *and* β_{dc} of the transistor when V_{CE} is 10V and $I_c = 4.0 mA$

Sol. Consider any two characteristics for two values of I_B which are above and below the given value of I_C, Here $I_C = 4.0 mA.$

(Choose characteristics for $I_B = 30$ and $20 \mu A$) At $V_{CE} = 10V$ we read the two values of I_C from the graph Then



$\Delta I_B = (30 - 20) \mu A$

$\Delta I_C = (4.5 - 3.0) mA = 1.5 mA$

Therefore

$$\beta_{ac} = \left(\frac{\Delta i_c}{\Delta I_B}\right)_{V_{CE}} = 1.5 mA / 10 \mu A = 150$$

For determining β_{dc} calculate the two values of β_{dc} for the two characteristics chosen and find their mean . Therefore for

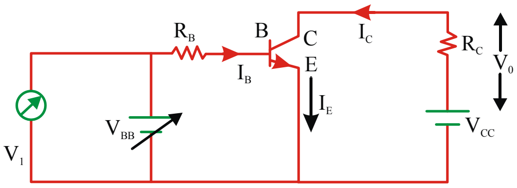
$I_C = 4.5 mA \;\; and \;\; I_B = 30 \mu A$ $\qquad \beta_{dc} = \frac{I_C}{I_B} = 4.5 mA \;/ 30 \mu A = 150$ and for

$I_C = 3.0 mA \;\; and \;\; I_B = 20 \mu A$

$\beta_{dc} = 3.0 mA / 20 \mu A = 150$

Hence $\beta_{dc} = (150+150)/2 = 150$

EX. 22: In Figure the V_{BB} supply can be varied form 0V to 5.0V. The Si transistor has $\beta_{dc} = 250$ and $R_c = 1K\Omega, V_{CC} = 5.0V$. Assume that when the transistor is saturated, $V_{CE} = 0V$ and $V_{BE} = 0.8V$. Calculate (a) the minumum base current, for which the transistor will reach saturation. Hence, (b) determine V_1 when the transistor is switched on(c) find the ranges of V_1 for which the transistor is switched off and switched on.



Sol. Given at saturation $V_{CE} = 0V,\ V_{BE} = 0.8V$

$V_{CE} = V_{CC} - I_C R_C \Rightarrow I_C = V_{CC}/R_C$

$= 5.0V / 1.0K\Omega = 5.0mA$

Therefore

$I_B = I_C/\beta = 5.0mA/250 = 20\mu A$

The input voltage at which the transistor will go into saturation is given by

$V_{IH} = V_{BB} = I_B R_B + V_{BC}$

$= 20\mu A \times 100K\Omega + 0.8V = 2.8V$

The value of input voltage below which the transistor remains cutoff is given by $V_{IL} = 0.6V, V_{IH} = 2.8V$.

Between 0.0V and 0.6V, the transistor will be in the 'switched off state. Between 2.8V and 5.0V,it will be in 'switched on' state

Note that the transistor is in active state when I_B varies from 0.0mA to 0.20 mA.

In this range, $I_C = \beta I_B$ is valid. In the

In this range, $I_C = \beta I_B$ is valid.

In the saturation range, $I_C \le \beta I_B$

EX. 23: Two amplifiers are connected one after the other in series (cascaded). The first amplifier has a voltage gain of 10 and the second has a voltage gain of 20. If the input signal is 0.01 volt, calculate the out put ac signal.

Sol. When the amplifiers are Connected in series, the net voltage gain is equal to the product of the gains of the individual amplifiers.

$\therefore Av = Av^1 \times Av^{11}$, hear $Av^1 = 10$, and $Av^{11} = 20$ also $Av = \dfrac{V_{output}}{V_{input}}$ $\therefore$ we can write $\dfrac{V_{output}}{V_{input}} = Av^1 \times Av^{11} : V_{in} = 0.01V$

$V_{out} = V_{in} \times Av^1 \times Av^{11} = 0.01 \times 10 \times 20 = 2V$

EX. 24: In a single state transistor amplifier, when the signal changes by 0.02V, the

base current by $10\mu A$ and collector current by 1mA. If collector load $R_C = 2k\Omega$ and $R_L = 10k\Omega$, Calculate: (i) Current Gain (ii)Input impedance,(iii) Effective AC load, (iv)Voltage gain and (v) Power gain.

Sol. i) Current Gain $\beta = \frac{\Delta i_c}{\Delta i_b} = \frac{1mA}{10\mu A} = 100$

ii) Input impedance

$$R_i = \frac{\Delta V_{BE}}{\Delta i_b} = \frac{0.02}{10\mu A} = 2000\Omega = 2k\Omega$$

iii) Effective (a,c) load

$$R_{AC} = R_C \,||\, R_L \quad \therefore R_{AC} = \frac{2\times 10}{2+10} = 1.66k\Omega$$

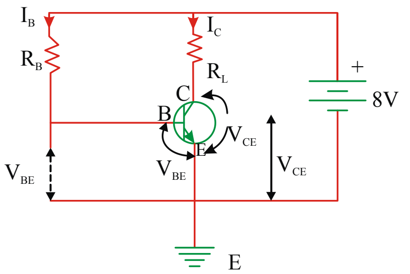
iv) Voltage gain $A_V = \beta \times \frac{R_{AC}}{R_{in}} = \frac{100\times 1.66}{2} = 83$

v) Power gain, A_p = Current gain $\times$ Voltage gain

$= 100\times 83 = 8300$

EX. 25: An n-p-n transistor in a common emitter mode is used as a simple voltage amplifier with a collector current of 4mA. The terminal of a 8V battery is connected to the collector through a load resistance R_L and to the base through a resistance R_B. The collector-emitter voltage $V_{CE} = 4V$, base-emitter voltage $V_{BE} = 0.6V$ and base current amplification factor $\beta_{d.c} = 100$.Calculate the values of R_L and R_B

Sol.



Potential difference across R_L

$= 8V - V_{CE} \quad = 8V - 4V \quad = 4V$

Now $I_C R_L = 4V$

$$R_L = \frac{4}{4\times 10^{-3}} = 10^3\Omega = 1k\Omega$$

Further for base emitter equation

$V_{CC} = I_B R_B + V_{BE}$

or $I_B R_B$ = Potential difference across R_B

$= V_{CC} - V_{BE} = 8.-0.6 = 7.4V$

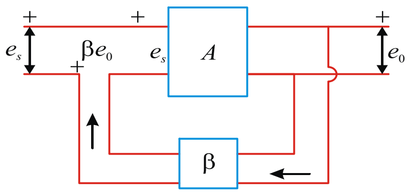
Again , $I_B = \frac{I_C}{\beta} = \frac{4\times 10^{-3}}{100} = 4\times 10^{-5} A$

$\therefore R_B = \dfrac{7.4}{4\times10^{-5}} = 1.85\times10^5\,\Omega = 185\,k\Omega$

Concept of feedback: When a part of the output voltage (or current) of an amplifier is injected back into the input circuit, feedback is said to exist.

- If the voltage feedback is in phase with the applied voltage, the feedback is said to be positive or regenerative;
- If the voltage feedback is in opposite phase to the incoming signal, the feedback is said to be negative or degenerative.

Fig. illustrates the principles of feedback.



A The gain of the amplifier without feedback is A. If a signal e_S is applied at the input terminals of the amplifier, then let the output voltage be e_0.

A If a fraction β ($\beta = \dfrac{e_f}{e_0}$) of this output voltage is fedback into the input in phase with the applied signal, then the actual input voltage of the amplifier,

$e_i = e_s + e_f$ $e_i = e_s + \beta e_0$ (for positive feed back)

A This total input voltage multiplied by the gain A of the amplifier must be equal to the output voltage i.e. $e_0 = Ae_i \Rightarrow e_0 = A(e_s + \beta e_0) = Ae_s + A\beta e_0$

$\Rightarrow e_0 - A\beta e_0 = Ae_s \Rightarrow e_0(1-\beta A) = Ae_s$ from which the gain A_f with feedback is,

$$A_f = \frac{e_0}{e_s} = \frac{A}{1-\beta A}$$

A The positive feedback thus, increases the gain of the amplifier. If too much positive feedback is applied so that $1-\beta A = 0$, the gain of the amplifier becomes infinite.

A For stable oscillation $\beta A = 1$ (Barkhausen's criteria)

A In this case the amplifier becomes unstable and the output can be obtained with no external input signal, i.e., the amplifier becomes an oscillator.

A In the case of negative feedback $e_i = e_s - e_f$

A In the case of negative feedback the voltage feedback βe_0 is in opposite phase to the applied voltage e_S, so that gain with negative feedback becomes, $A_f = \dfrac{e_0}{e_s} = \dfrac{A}{1+\beta A}$

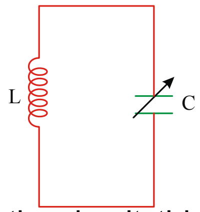
A The negative feed back also reduces noise & distortion in an amplifier.

Note:

i) When $|1+A\beta|>1, |A_f|<|A|$, feed back is negative

ii) When $|1+A\beta|<1, |A_f|>|A|$, feed back is positive

Transistor as an oscillator: The simplest eletrical oscillating system consists of an inductance L and capacitor C connected in parallel.



A Once an eletrical energy is given to the circuit, this energy oscillates between capacitance (in the form of eletrical energy) and inductance (in the form of magnetic energy) with a frequency $\nu = \dfrac{1}{2\pi\sqrt{LC}}$

A The amplitude of oscillations is damped due the presence of inherent resistance in the circuit

A In order to obtain oscillations of constant amplitide, an arrangement of regenerative or positive feedback from an output circuit to the input circuit is made so that the circuit losses may be compensated.

EX. 26: In a negative feedback amplifier, the gain without feedback in 100, feed back ratio is 1/25 and input voltage is 50m V. Calculate
(i)gain with feedback (ii) feedback factor
(iii) output voltage (iv) feedback voltage
(v) new input voltage so that output voltage with feedback equals the output voltage without feedback

Sol. i) Gain with feedback

$$A_f = \frac{A}{1+\beta A} = \frac{100}{1+(1/25)\times 100} = 20$$

ii) $\beta = \dfrac{1}{25}$

iii) Out put voltage

$$V_0' = A_f V_i = 20\times 50mV = 1volt$$

iv) Feedback voltage $\beta V_0' = \dfrac{1}{25}\times 1 = 0.04\ volt$

v) New increased input voltage $V_i^1 = V_i(1+\beta A)$

$$= 50\left(1+\frac{1}{25}\times 100\right) = 250mV$$

Digital Eletronics

1) **Binary number system:** It is a two-valued system developed by George Boole. Only two digits 0 and 1, called bits, are used in binary system. But binary addition (in this 1+1=10) is different from addition in Boolean algebra (in this 1+1=1 and remaining will be same i.e., 0+0=0, 1+0=1, 0+1=1).

A A number in a decimal system can be converted into binary by the successive division of 2 until the quotient is zero. The remainders obtained in the successive divisions, taken in the reverse order (from bottom to top shown below) give the binary representation of that number.

EX. 27: Convert the decimal number 23 into binary number.

Sol.

$$\begin{array}{l|l} 2 & 23 \\ 2 & 11-1 \\ 2 & 5-1 \\ 2 & 2-1 \\ 2 & 1-0 \\ 2 & 0-1 \end{array}$$

Arranging the remainders in the reverse order(from bottom to top shown below) the binary equivalent of 23 is found to be 10111. i.e., $23_{(10)} = 10111_{(2)}$

A In binary representation of any number, the first bit is the 'most significant bit' (MSB) and the last bit is the 'least significant bit' (LSB).

A Binary number can be converted into decimal number by multiplying each bit with 2^n and adding them as mention below. where n is positon of the bit from right side (LSB) to leftside (MSB).

EX. 28: Convert the binary number 10111 into decimal number.

Sol. $\overset{4\,3\,2\,1\,0}{10111}_{(2)} = 1\times2^4 + 0\times2^3 + 1\times2^2 + 1\times2^1 + 1\times2^0 \quad = 16 + 0 + 4 + 2 + 1 = 23_{(10)}$

$10111_{(2)} = 23_{(10)}$

2) Boolean Algebra: Only two states or values of a varible are allowed in Boolean algebra. In logic these two states correspond to 'on' and 'off' and 'saturation' of electronic devices.

A The two allowed states in Boolean alegbra are represented by the digits 0 and 1. 0 can also represent Off, low, false,No, 0 V 1 can also represent ON, high, true, Yes, 5 V

A The variables of Boolean algebra are subjected to three operations.

A The OR addition indicated by a plus (+) sign.

A The AND multiplication indicated by a cross (X) or a dot (.) sign.

A The NOT operation indicated by a bar over a variable.

A **OR Addition:** The + sign in Boolean algebra represents OR addition. The equation Y = A+B is read as 'Y equals A OR B'. The

A **AND multiplication:** In Boolean algebra the equation Y=AXB or Y=A.B or Y=AB is read as 'Y equals A AND B'

A **NOT operation:** The NOT operation on a variable A is represented by $\overline{A}$.

The equation $Y = \overline{A}$ is read as 'Y equals NOT A'.

A Rule for OR, AND and NOT functions in Boolean algebra

OR	AND	NOT
0+0=0	0.0=0	$\overline{0} = 1$
0+1=1	0.1=0	$\overline{1} = 0$
1+0=1	1.0=0	
1+1=1	1.1=1	

A Some useful laws of Boolean algebra

A **Commulative laws:** A+B=B+A; A.B=B.A

A **Associative laws:**

A+(B+C) = (A+B) +C ;A.(B.C) = (A.B).C

A **Distributive laws:**A.(B+C) = A.B +A.C

Basic OR and AND Relations:

OR	AND
A+0=A	A.0=0
A+1=1	A.1=A
A+A=A	A.A=A
A+ $\overline{A}$ =1	A. $\overline{A}$ =0

A **Double complement function:** $\overline{\overline{A}} = A$

A **De Morgan's theorems:**DeMorgan's theorems (or rules) are very useful in simplifying complicated logical expressions and can be stated as under :

Theorem 1: The complement of the sum of two (or more) variables is equal to the product of complements of the variables i.e. $\overline{A+B} = \overline{A} \bullet \overline{B}$ $\Rightarrow A+B = \overline{\bar{A}.\bar{B}}$

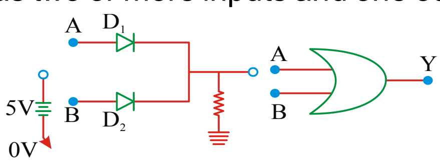
Theorem 2 : The complement of the product of two (or more) variables is equal to the sum of complements of the variables i.e $\overline{A \bullet B} = \overline{A} + \overline{B}$. $\Rightarrow A.B = \overline{\bar{A}+\bar{B}}$

Logic gates: A digital circuit having a certain logical relationship between the input and the output voltages is called a logic gate.

There are three basic logic gates (1) OR gate (2) AND gate (3) NOT gate

A **The OR gate**

1) It is a logic gate which has two or more inputs and one output.



OR gate Symbol OR gate

A If its one or more inputs are high, then iis output will be high. Therefore it has a logic of OR.

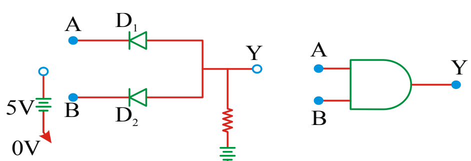
Boolean expression for OR gate:A + B = Y

which reads as A OR B is equal to Y.

A Truth table for OR gate -

A	B	Y = A +B
0	0	0
0	1	1
1	0	1
1	1	1

A **The AND gate:** It is logic gate which has two or more inputs and one output

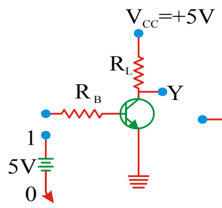


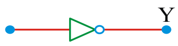
Circuit of AND gate Symbol of AND gate

- If its all inputs are high, then it is output will be high (1). Therefore it has a logic of AND.
- **Boolean expression for AND gate:**
  A. B = Y reads as A AND B is equal to Y.
- Truth table for AND gate

A	B	Y = A . B
0	0	0
0	1	0
1	0	0
1	1	1

The NOT gate: It is the logic gate which has one input and one output



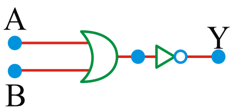


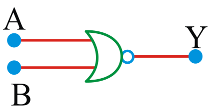
Circuit of NOT gate   Symbol of NOT gate

- If its input is high (1), then its output will be low(0). Therefore it has a logic of NOT.
- **Boolean expression for NOT gate:** $\overline{A} = Y$ reads as 'A NOT is equal to Y'.
- Truth table for NOT gate

A	$Y = \overline{A}$
0	1
1	0

- **Combination of gates:** In complicated digital circuits used in calculators, computers etc. the different types of combination of three basic logic gates are used.
- **NOR gate:** This logic gate is the combination of OR gate and NOT gate. OR + NOT = NOR
- In this logic gate the output of OR gate is given to the input of NOT gate as shown in the below figure.



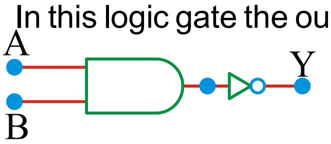


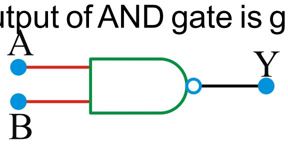
OR   NOT   NOR gate

- **Boolean expression for NOR gate:** $Y = \overline{A+B}$. Which reads as A OR B negated.
- Truth table for NOR gate

A	B	A+ B	$Y = \overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

- **NAND gate:** This logic gate is the combination of AND gate and NOT gate.
  AND+NOT $\rightarrow$ NAND
- In this logic gate the output of AND gate is given to the input of NOT gate as shown below





AND   NOT   NAND gate

A Boolean expression for NAND gate $Y = \overline{A.B}$

A Truth table for NAND gate

A B	A.B		$\overline{A.B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

A **Uses of NOR gate and NAND gate:** The NAND gate and NOR gates are the building blocks of digital circuits. All the basic gates (OR, AND and NOT) can be obtained by the repeated use of NAND or NOR gates.

NOT gate from NAND gate:

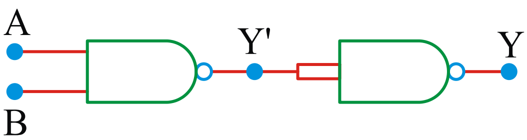
1) Diagram

2) Truth table

A B	$Y = \overline{A.A} = \overline{A}$	
0	0	1
1	1	0

AND gate from NAND gate

1) Diagram

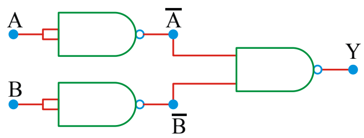


2) Truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR gate from NAND gate

1) Diagram



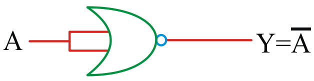
2) Truth table

A	B	$\overline{A}$	$\overline{B}$	$Y = (A+B)$
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1

1	1	0	0	1

NOT gate from NOR gate

1) Diagram

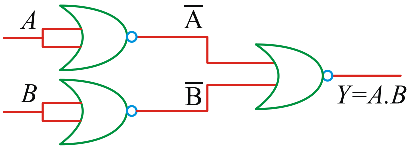


2) Truth table

A	B	$Y=\overline{A.A}=\overline{A}$
0	0	1
1	1	0

AND gate from NOR gate

1) Diagram

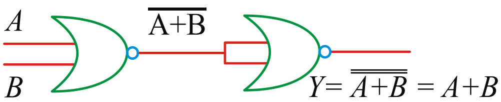


2) Truth table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR gate from NOR gate

1) Diagram



2) Truth table

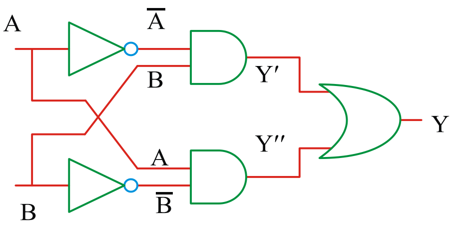
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

XOR GATE: XOR gate is obtained by using OR, AND and NOT gates. It is also called exlusive OR gate.

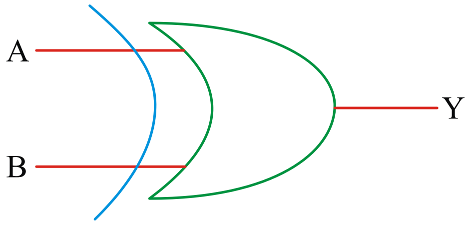
A The output of a two input XOR gate is 1 onl y when the two inputs are different.

A The Boolean equation is $Y=A.\overline{B}+B.\overline{A}$

1) two input XOR gate



2) circuit symbol



3) truth table

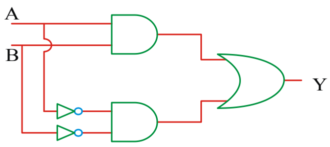
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

XNOR GATE: XNOR gate is obtained by using OR, AND and NOT gates.

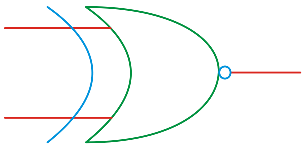
- It is also called exlusive NOR gate.
- The output of a two input XNOR gate is 1 only when both the inputs are same.
- The Boolean equation is $Y = A.B + \overline{A}.\overline{B}$

XNOR gate is inverse of XOR gate.

1) Diagram



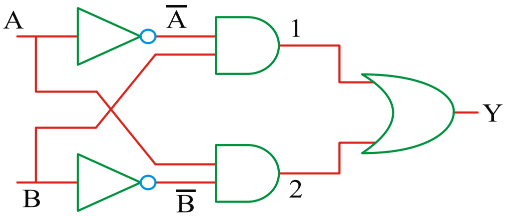
2) circuit symbol



3) truth table

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

EX. 29:The Boolean expression of the output Y of the inputs A and B for the circuit shown in the fig:

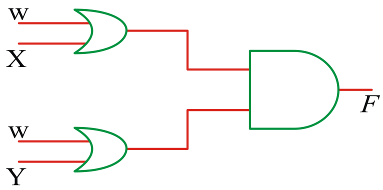


Sol. The output of AND gate 1 is $\overline{A}B$

The output of AND gate 2 is $A\overline{B}$

$\therefore$ the output of OR gate is Y $= \overline{A}B + A\overline{B}$

EX. 30: The diagram of a logic circuit is given below. The output of the circuit is represented by



Sol. $F = (W + X).(W + Y)$

$= W.W + W.Y + X.W + X.Y$

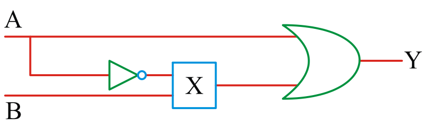
$= W + W.Y + X.W + X.Y$

$= W(1+Y) + X.W + X.Y = W + XW + X.Y$

$= W(1+X) + X.Y = W + X.Y$

EX. 31: The logic circuit and its truth table are given, what is the gate X in the diagram

A	B	Y
1	1	1
1	0	1
0	1	1
0	0	0



Sol. From the truth table we note that $Y = A + B$ **i.e., it is for OR gate (or)**

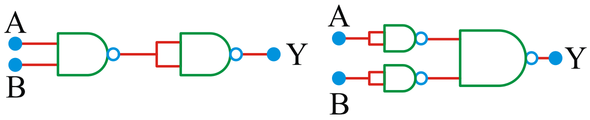
$A + X = A + B = A + B.(A + \overline{A})(\because A + \overline{A} = 1)$

then, $A + X = A + B.A + B.\overline{A}$

$= A.(1+B) + \overline{A}.B = A + \overline{A}.B$

So $X = A + \overline{A}.B$**, which is AND gate with inputs as** $\overline{A}$ **and B**

EX. 32: You are given two circuits as shown in figure which consist of NAND gates. Identify the logic operation carried out by the two circuits.



Sol. From fig(a). The output of NAND gate is connected to NOT gate (obtained from NAND gate) Let Y^1 be the output of NAND gate and the final output of the combination of two gates is Y. The output of a NAND gate is O only when both the inputs are zero, while in NOT gate, the input gets inverted. Truth table for the arrangement

A	B	Y'	Y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

=

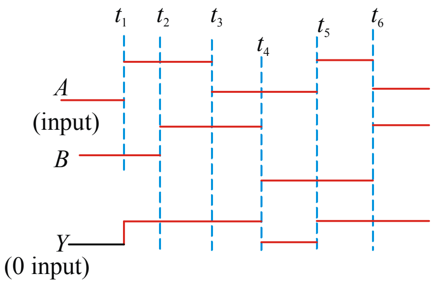
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

It is the truth tables of AND gate. Therefore the given circuit acts as AND gate.

b) The output of two NOT gates are connected to NAND gate Let Y_1 and Y_2 be the outputs of the two NOT gates and the final output of the combination of three gates be Y. In a NOT gate. the input gets inverted, while the output of a NAND gate is 'O' only when both the inputs are zero. Truth table for given arrangement.

A	B	Y
0	0	0
1	0	1
0	1	1
1	1	1

EX. 33:Justify the output waveform (Y) of the OR gate for the following inputs A and B given is
Fig



Sol. Note the following:

At $t < t_1$; $A = 0, B = 0$; $Hence Y = 0$

For $t_1\, to\, t_2$; $A = 1, B = 0$; $Hence\, Y = 1$

For $t_2\, to\, t_3$; $A = 1, B = 1$; $Hence\, Y = 1$

For $t_3\, to\, t_4$; $A = 0, B = 1$; $Hence\, Y = 1$

For $t_4\, to\, t_5$; $A = 0, B = 0$; $Hence\, Y = 0$

For $t_5\, to\, t_6$; $A = 1, B = 0$; $Hence\, Y = 1$

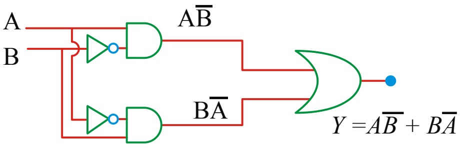
For $t > t_6$; $A = 0, B = 1$; $Hence\, Y = 1$

Therefore the wave form Y will be as shown in the Fig

EX. 34: Draw logic diagrams for the Boolean expressions given below.

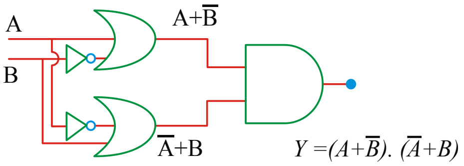
i) $A \bullet \overline{B} + \overline{A} \bullet B = Y$ **ii)** $\left(A + \overline{B}\right) \bullet \left(\overline{A} + B\right) = Y$

Sol. (i) The required logic diagram for the given Boolean expression is given in figure. Here the input B before applying to first AND gate and input A before applying to second AND gate have been inverted. The output of these gates are, therefore, $A\overline{B}$ and $\overline{A}B$ respectively. These outputs are fed to OR gate which gives $Y = A\overline{B} + \overline{A}B$ as shown in figure.



ii) The required logic diagram for the given Boolean expression is shown in figure. The input B to first OR gate and input A to second OR gate have been inverted. The output of these gates are, therefore, $A + \overline{B}$ and $\overline{A} + B$ as shown. These inputes when applied to AND gate give the required output.

$Y = \left(A + \overline{B}\right).\left(\overline{A} + B\right)$



INTEGRATED CIRCUITS: An entire circuit (consisting of many passive components like R and C active devices like diode and transistor) on a small single block (or chip) of a semicon ductor is known as Integrated Circuit (IC). The most widely used technology is the Monolithic Integrated Circuit.

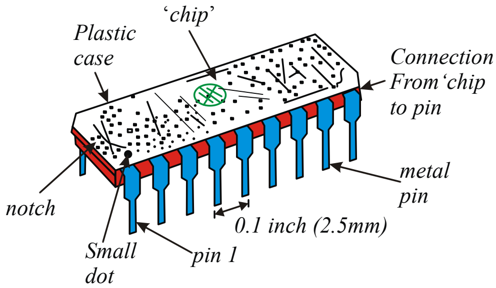
A The chip dimensions are as small as $1mm \times 1mm$ or it could even be smaller.

A Depending on nature of input signals, IC's can be grouped in two categories:
(a) linear or analogue IC's and
(b) digital IC's. The linear IC's process analogue signals varies linearly with the input.

A One of the most useful linear IC's is the operational amplifier.

A The digital IC's process signals that have only two value.

A They contain circuits such as logic gates. Depending upon the level of integration (i.e,. the number of circuit components or logic gates), the IC's are termed as
i) Small Scale Integration, SSI (logic gates $\leq$ 10)
ii) Medium Scale Integration, MSI
(logic gates < 100)
iii) Large Scale Integration, LSI
(logic gates <1000)
iv) very large scale integration VLSI
(logic gates > 1000).



The casing and connection of a 'chip'

PREVIOUS MAINS QUESTIONS

1.With increasing biasing voltage of a photodiode, the photocurrent magnitude:

[Sep. 05, 2020 (D)]

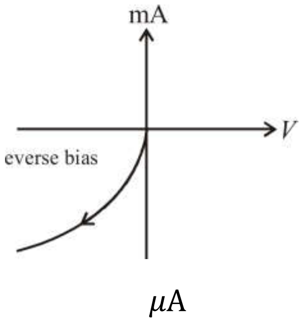
(a) remains constant

(b) increases initially and after attaining certain value, it decreases

(c) Increases linearly

(d) increases initially and saturates finally

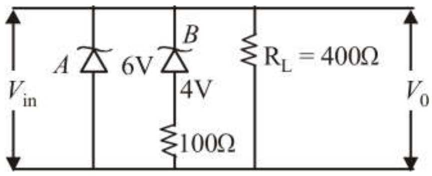
SOLUTION. (d) I-V characteristic of a photodiode is as follows:

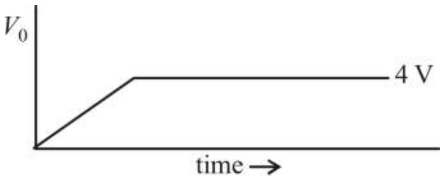


On increasing the biasing voltage of a photodiode, the magnitude of photocurrent first increases and then attains a saturation.

2. Two Zener diodes (A and B) having breakdown voltages of 6 V and 4V respectively, are Connected as shown in the circuit below. The output voltage V_0 variation with input voltage linearly increasing with time, is given by:($V_{\text{input}} = 0\text{V}$ at $t = 0$)(figures are qualitative)

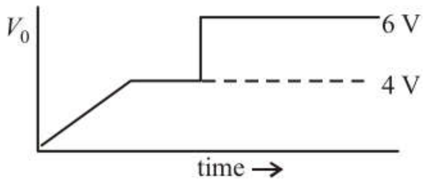
[Sep. 05, 2020 (II)]

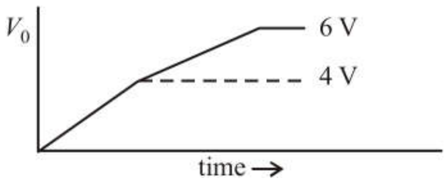




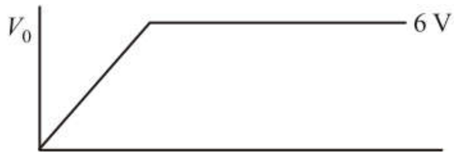
(a)

(b)



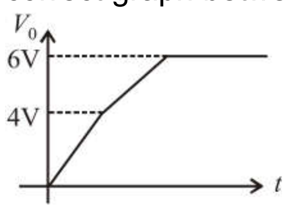


(c)

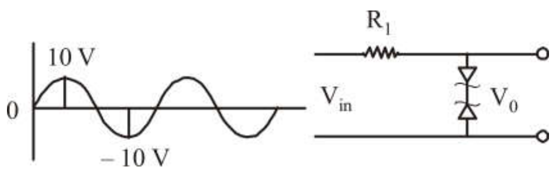
(d) 

time →

SOLUTION. (c) Till input voltage reaches 4 V. No Zener is in breakdown region so $V_0 = V_i$. Then now when V_i changes between 4Vto 6 V one Zener with 4 V will breakdown and P.D. across this Zener will become constant and remaining potential will drop across resistance in series with 4 V Zener. Now current in circuit increases abruptly and source must have an internal resistance due to which some potential will get drop across the source also so correct graph between V_0 and t will Be

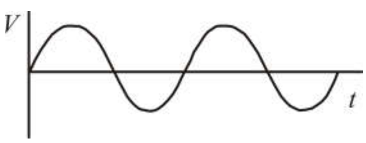


3. Take the breakdown voltage of the zener diode used in the given circuit as $6\mathrm{V}$. For the input voltage shown in figure below, the time variation of the output voltage is :(Graphs drawn are schematic and not to scale) [Sep. 04, 2020 (I)]

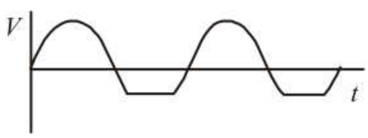


$V =$

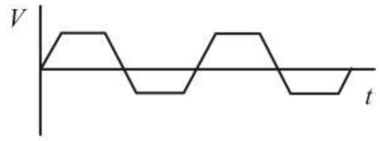
(a)



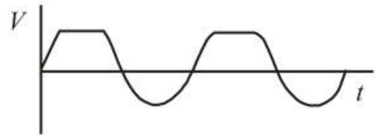
(b)



(c)



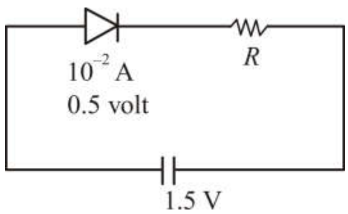
(d)



SOLUTION. (c) Here two Zener diodes are in reverse polarity so if one is in forward bias the other will be in reverse bias and above $6\mathrm{V}$ the reverse bias will too be in conduction mode. Hence when $\mathrm{V} > 6\mathrm{V}$ the output will be constant. And when$\mathrm{V} < 6\mathrm{V}$ it will follow the inut voltage.

4. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit ofcurrent through the diode is 10 mA.Ifa battery ofemf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is:
[Sep. 03, 2020 (I)]

(a) 300 Ω (b) 50 Ω (c) 100 Ω (d) 200 Ω



SOLUTION. (c) According to question, when diode is forward biased,$V_{\mathrm{DIODE}} = 0.5\mathrm{V}$
Safe limit of current, $I = 10\mathrm{mA} = 10^{-2}\mathrm{A}$ $R_{\min} = ?$
Voltage through resistance $V_R = 1.5 - 0.5 = 1$ volt $iR = 1$

$$R_{\min} = \frac{1}{i} = \frac{1}{10^{-2}} = 100\Omega$$

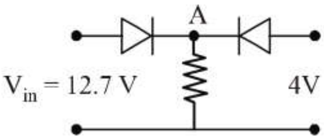
5. If a semiconductor photodiode can detect a photon with a maximum wavelength of400 nm, then its band gap energy is:$\mathrm{Plancks}$ constant, $h = 6.63 \times 10^{-34}$ J. s.
Speed of light, $c = 3 \times 10^8 \mathrm{m/s}$ [Sep. 03, 2020(II)]

(a) $1.1\mathrm{eV}$ (b) $2.0\mathrm{eV}$ (c) $1.5\mathrm{eV}$ (d) $3.1\mathrm{Ev}$

SOLUTION. (d) Given, Wavelength of photon, $\lambda = 400$ nm
A photodiode can detect a wavelength corresponding to the energy of band gap. If the signal is having wavelength greater than this value, photodiode cannot detect it.

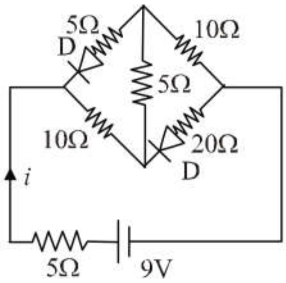
Band gap $E_g = \frac{hc}{\lambda} = \frac{1237.5}{400} = 3.09\mathrm{eV}$

6. Both the diodes used in the circuit shown are assumed to be ideal and have negligible resistance when these are forward biased. Built in potential in each diode is 0.7 V. Forthe input voltages shown in the figure, the voltage (inVolts) at point A is [NA 9 Jan. 2020 I]



SOLUTION. (12) Right hand diode is reversed biased and left-hand diode is forward biased. Hence Voltage at 'A' $V_A = 12.7 - 0.7 = 12$volt

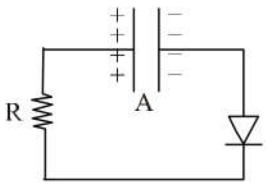
7. The current i in the network is: [9 Jan. 2020 II]

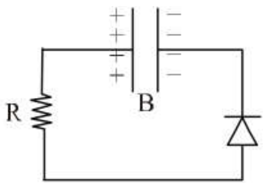


(a) $0.2\mathrm{A}$ (b) $0.6\mathrm{A}$ (c) $0.3\mathrm{A}$ (d) OA

SOLUTION. (c) Both the diodes are reverse biased, so, there is no flow of current through 5Ω and 20Ω resistances. Now, two resistors of 10Ω and two resistors of 5Ω are in series. Hence current I through the network $= 0.3\mathrm{A}$

8. Two identical capacitors A and B, charged to the same potential $5\mathrm{V}$ are connected in two different circuits asshown below at time $t = 0$. If the charge on capacitors $\mathrm{A}$and B at time $t = \mathrm{CR}$ is QAand $\mathrm{Q_B}$ respectively, then (Here$\mathrm{e}$ is the base ofnatural logarithm) [9 Jan. 2020 II]





(a) $\mathrm{Q_A} = \frac{\mathrm{VC}}{e}$, $\mathrm{Q_B} = \frac{\mathrm{CV}}{2}$ (b) $\mathrm{Q_A} = \mathrm{VC}$, $\mathrm{Q_B} = \mathrm{CV}$

(c) $\mathrm{Q_A} = \mathrm{VC}$, $\mathrm{Q_B} = \frac{\mathrm{VC}}{e}$ (d) $\mathrm{Q_A} = \frac{\mathrm{CV}}{2}$, $\mathrm{Q_B} = \frac{\mathrm{VC}}{e}$

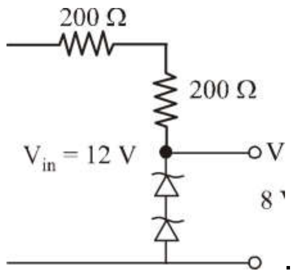
SOLUTION. (c) In case I diode is reverse biased, so no current flows $Q_A = CV$

In case II, current will flow as diode is forward biased. So, it offers negligible resistance to the flow of current and thus be replaced by short circuit. Now, the charge of capacitor will leak through the resistance and decay exponentially with time. During discharging of capacitor Potential difference across the capacitor at any instant

$$V' = Ve^{\frac{t}{CR}} \text{But } t = CR \quad V' = Ve^{-1} = \frac{V}{e}$$

$$\text{Charge } Q_B = CV' = \frac{CV}{e}$$

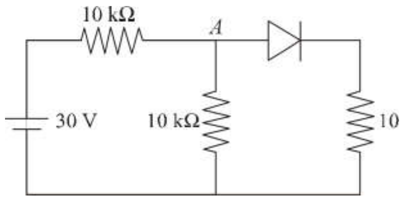
9. The circuit shown below is working as an 8 V dc regulated voltage source. When 12 V is used as input, the power dissipated (in mW) in each diode is; (considering both Zener diodes are identical). [9 Jan. 2020 II]



SOLUTION. (40) Current in the circuit, $I=\frac{12-8}{400}=10^{-2} A$Power dissipited in each diode,

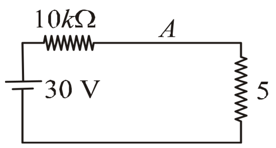
$P=V I \Rightarrow \qquad P=4 \times 10^{2}=40 mW$

10. In the figure, potential difference between A and B is: [7 Jan. 2020 II]



(a) $10 \mathrm{V}$ (b) $5 \mathrm{V}$ (c) $15 \mathrm{V}$ (d) zero

SOLUTION. 10. (a) The given circuit has two $10 k\Omega$ resistances in parallel, so we can reduce this parallel combination to a single equivalent resistance of $5 k\Omega$.

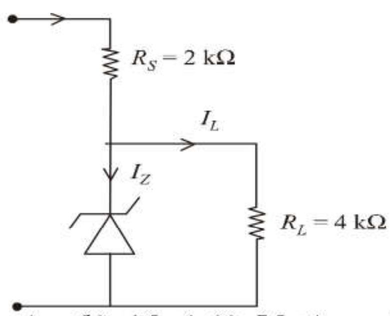


B

Diode is in forward bias. So it will behave like a conducting wire.

$$V_A-V_B=\frac{30}{5+10} \times 5=10 V$$

11. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage $=6 \mathrm{V}$. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current? [12 Apr. 2019 II]

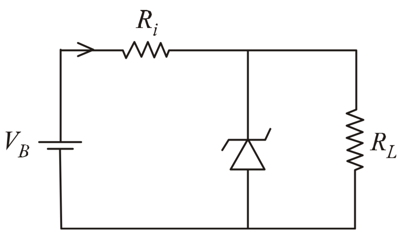


(a) 2.5 mA (b) 1.5 mA (c) 7.5 mA (d) 3.5 mA

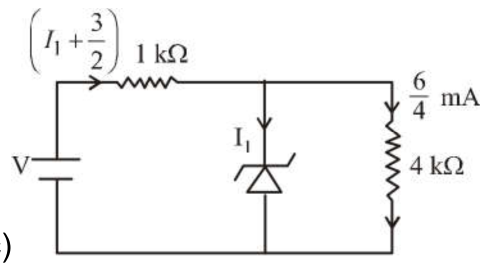
SOLUTION. (d) Current in load resistance, $i_{\mathrm{i}} = \frac{6}{4\times10^3} = 1.5\times10^3 \mathrm{A} = 1.5\mathrm{mA}$

For $V = 16$ volt, $i_s = \frac{(16-6)}{2\times10^3} = 5\mathrm{mA}$ $i_2 = i_s - i_1 = 5 - 1.5 = 3.5\mathrm{mA}$

12. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is6 V and the load resistance is $\mathrm{R_L} = 4\mathrm{k}$. The series resistanceofthe circuit is $\mathrm{R_i} = 1\mathrm{k}$. Ifthe batteryvoltage $\mathrm{V_B}$ varies fi$_{\mathrm{i}}$om8 V to 16V, what are the minimum and maximum values of the current through Zener diode? [10 Apr. 2019 II]



(a) $0.5\mathrm{mA}; 6\mathrm{mA}$ (b) $1\mathrm{mA}; 8.5\mathrm{mA}$ (c) $0.5\mathrm{mA}; 8.5\mathrm{mA}$ (d) $1.5\mathrm{mA}; 8.5\mathrm{mA}$

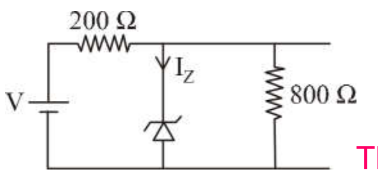


SOLUTION. (c)

For voltage, $\mathrm{V} = 8\mathrm{V}$ Current, $I_{\mathrm{i}} = \left(\frac{8-6-3}{2}\right) = \frac{1}{2} = 0.5\mathrm{mA}$

For voltage, $\mathrm{V} = 16\mathrm{V}$ Current, $I_2 = \left(\frac{16-6-3}{2}\right) = 8.5\mathrm{mA}$

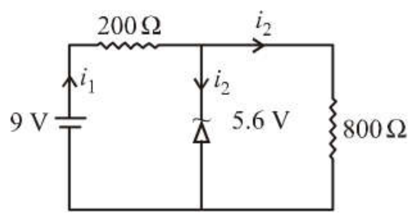
13. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.



The current I_Z through the Zener is: [8 April 2019 I]

(a) $10\mathrm{mA}$ (b) 17 mA (c) 15 mA (d) 7 mA

SOLUTION.

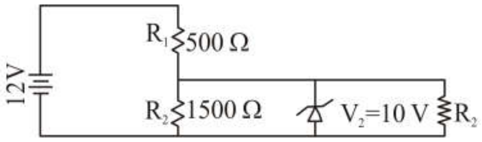


P.D. across 800Ω resistors $= 5.6\mathrm{V}$so, $I_{800\Omega} = \frac{5.6}{800} A = 7\mathrm{mA}$Now, P.D. across 200Ω

resistors $= 9 - 5.6\mathrm{V} = 3.4\mathrm{V}$so, $I_{200\Omega} = \frac{9-5.6}{200} = 17\mathrm{m}s$

so current through Zener diode $= \mathrm{I}_2 = 17 - 7 = 10\mathrm{mA}$

14. In the given circuit the current through Zener Diode is close to: [11 Jan. 2019 I]

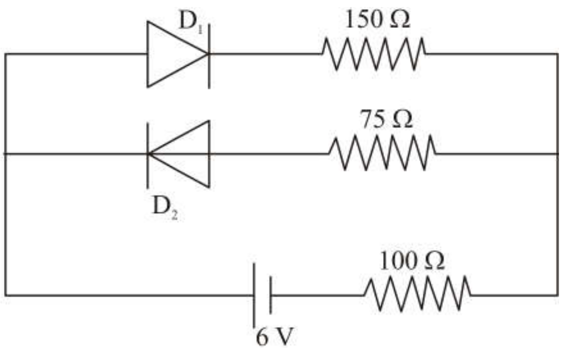


(a) $0.02\mathrm{mA}$ (b) $6.7\mathrm{mA}$ (c) $4.0\mathrm{mA}$ (d) $6.0\mathrm{mA}$

SOLUTION. now $\mathrm{R_{eq}} = 150 + 50 + 100 = 300\Omega$

So, required current $\mathrm{I} = \frac{\mathrm{BatteryVoltage}}{300}$ $\mathrm{I} = \frac{6}{300} = 0.02$

15. The circuit shown below contains two ideal diodes, each with a forward resistance of50 Ω. If the battery voltage is$6\mathrm{V}$, the current through the 100 Ω resistance (in Amperes) is: [11 jan. 2019 II]

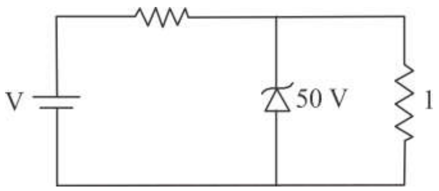


(a) 0.036 (b) 0.020(c) 0.027 (d) 0.030

SOLUTION. now $\mathrm{R_{eq}} = 150 + 50 + 100 = 300\Omega$

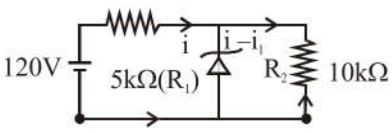
So, required current $\mathrm{I} = \frac{\mathrm{BatteryVoltage}}{300}\mathrm{I} = \frac{6}{300} = 0.02$

16. For the circuit shown below, the current through theZener diode is: [10 Jan. 2019 II]



(a) 9 mA (b) 5 mA(c) Zero (d) $14\mathrm{mA}$

SOLUTION. (a) The voltage across Zener diode is constant



$\mathrm{i_{(R_2)}} = \frac{\mathrm{V}}{\mathrm{R}} = \frac{50}{10\times10^3} = 5\times10^{-3}\mathrm{A}$

$$i_{(R_1)} = \frac{V}{R} = \frac{120-50}{5\times10^3} = \frac{70}{5\times10^3} 14\times10^{-3}A$$

$$i_{zenerdia1e} = 14\times10^{-3} - 5\times10^{-3} = 9\times10^{-3}A = 9mA$$

17. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is $10^{19}m3$ and their mobility is $1.6m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to:[9 Jan. 2019 I]

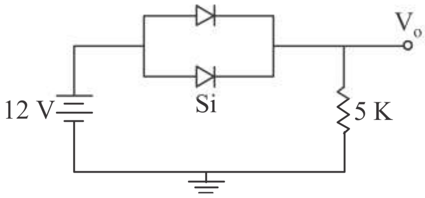
(a) 2 Ωm (b) 4 Ωm (c) $0.4\Omega m$ (d) $0.2\Omega m$

SOLUTION. (c) As we know, current density,j $= oE = nev_d$

$$o = ne\frac{v_d}{E} = ne\mu \quad \frac{1}{o} = P = \frac{1}{n_e e\mu_e} = \text{Resistivity}$$

$$= \frac{1}{10^{19}\times1.6\times10^{19-19}\times1.6} \text{or} \qquad P = 0.4\Omega m$$

18. Ge and Si diodes start conducting at 0.3 V and 0.7 Vrespectively. In the following figure if Ge diode connection are reversed, the value of V_o changes by:(assume that the Ge diode has large breakdown voltage) [9 Jan. 2019 II]



(a) 0.8 V (b) 0.6 V (c) 0.2 V (d) 0.4 V

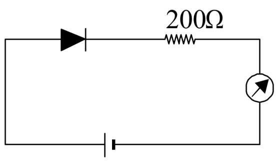
SOLUTION**(d)** Initially and Si are both forward biased so current will effectively pass through Ge diode

□ V∘= 12 – 0.3 =11.7 V And if "Ge" is reversed then current will flow through "Si" diode

□ V∘= 12 – 0.7 =11.3 V

Clearly, V∘ changes by 11.7 – 11.3 = 0.4 V

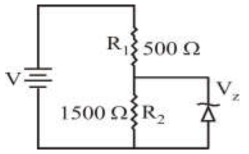
19.The reading of the ammeter for a silicon diode in the given circuit is :



(a) 0mA (b) 15 mA (c) 11.5mA (d) 2 mA

SOLUTION.c) Clearly from fig. given in question, Silicon diode is in forward bias. Potential barrier across diode$\Delta V = 0.7$ volts Current, $I = \frac{V-\Delta V}{R} = \frac{3-0.7}{200} = \frac{2.3}{200} = 11.5mA$

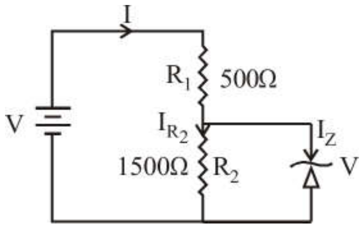
20. In the given circuit, the current through Zener diode is:[Online Apri116, 2018]



$Vz = 10V$ AND V=15volts

(a) $2.5mA$ **(b) $3.3mA$** **(c) $5.5mA$** **(d) $6.7mA$**

SOLUTION. b)



The voltage drops across R_2 is $V_{R_2} = V_Z = 10V$

The current through R_2 is$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{10V}{1500\Omega} = 0.667 \times 10^{-2}A = 6.67 \times 10^{-3}A = 6.67mA$

The voltage drops across R_1 is$V_{R_1} = 15V - V_{R_2} = 15V - 10V = 5V$

The current through R_1 is$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{5V}{500\Omega} = 10^{-2}A = 10 \times 10^{-3}A = 10mA$

The current through the Zener diode is$I_Z = I_{R_1} - I_{R_2} = (10 - 6.67)mA = 3.3mA$

21. What is the conductivity semiconductor sample having electron concentration of$5 \times 10^{18}m^{-3}$, hole concentrationof $5 \times 10^{19}m^{-3}$, electron mobility of $2.0m^2V^{-1}s^{-1}$ andhole mobility of $0.01m^2V^{-1}s^{-1}$?[Online April 8, 2017]

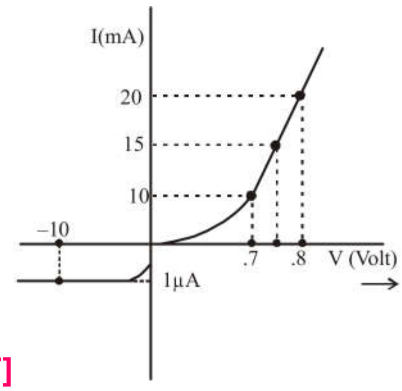
(Take charge of electron as $1.6 \times 10^{-19}C$)

(a) $1.68 \ (\Omega - m)^{-1}$ (b) $1.83 \ (\Omega - m)^{-1}$ (c) $0.59 \ (\Omega - m)^{-1}$ (d) $1.20 \ (\Omega - m)^{-1}$

SOLUTION. (a) The conductivity of semiconductor= $e(\eta_e\mu_e + \eta_h\mu_h)$

$$= 1.6 \times 10^{19}(5 \times 10^{18} \times 2 + 5 \times 10^{19} \times 0.01) = 1.6 \times 1.05 = 1.68$$

22. The V-I characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is:[Online April 8, 2017]



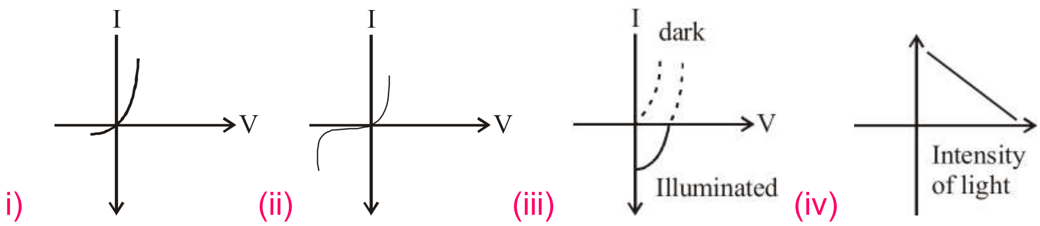
(a) 10 (b) 10^6 (c) 10^{-6} (d) 0

SOLUTION :(b) Forward bias resistance $=\frac{\Delta V}{\Delta I}=\frac{0.1}{10\times10^{-3}}=10\Omega$

Reverse bias resistance $=\frac{10}{10^{-6}}=10^{7}\Omega$

Ratio of resistances $=\frac{Forwardbiasresistance}{\text{Reversebiasresistance}}=10^{6}$

23. Identity the semiconductor devices whose characteristicsare given below, in the order (i), (ii), (iii), (iv) : [2016]



(a) Solar cell, Light dependent resistance, Zener diode, simple diode
(b) Zener diode, Solar cell, simple diode, Light dependent resistance
(c) Simple diode, Zener diode, Solar cell, Light dependent resistance
(d) Zener diode, Simple diode, Light dependent resistance, Solar cell

SOLUTION: (c) Graph (p) is for a simple diode.
Graph (q) is showing the V Break down used for Zener diode.
Graph (r) is for solar cell which shows cut-off voltage and open circuit current.
Graph (s) shows the variation of resistance h and hence current with intensity of light.

24. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400K, is best described by: [2016]
(a) Linear increase for Cu, exponential decrease of Si.
(b) Linear decrease for Cu, linear decrease for Si.
(c) Linear increase for Cu, linear increase for Si.
(d) Linear increase for Cu, exponential increase for Si.

SOLUTION

Metal (for limited range of temperature) Semiconductor

25. An experiment is performed to determine the $I-V$characteristics of a Zener diode, which has a protective resistance of $R=100\Omega$, and a maximum power ofdissipationrating of 1W. The minimum voltage range of the DC source in the circuit is: [Online April 9, 2016]
(a) $0-5V$ (b) $0-24V$(c) $0-12V$ (d) $0-8V$

25. **SOLUTION:** **c)** The minimum voltage range of DC source is given by V^2 = PR

P = 1 watt, R = 100 Ω V^2= 1×100 hence V = 10 volt.

26. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of lmfrom the diode is: [2015]

(a) $5.48 V/m$ (b) $\frac{7.75V}{m}$ (c) $1.73 V/m$ (d) $2.45 V/m$

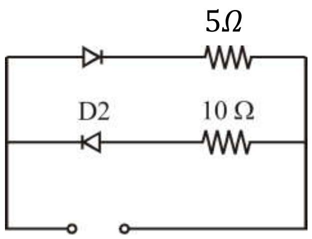
SOLUTION: (d) Using $U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$

But $U_{av} = \frac{P}{4\pi r^2 \times c}$ $hence \frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 \times c$

$$E_0^2 = \frac{2P}{4\pi r^2 \varepsilon_0 c} = \frac{2 \times 0.1 \times 9 \times 10^9}{1 \times 3 \times 10^8}$$

$E_0 = \sqrt{6} = 2.45 V/m$

26. A $2V$ battery is connected across AB as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to A and in other case when positive terminal of battery is connected to B will respectively be: [Online April 11, 2015]



(a) 0.4 A and 0.2A (b) 0.2 A and 0.4A (c) 0.1 A and 0.2A (d) 0.2 A and 0.1 A

SOLUTION: (a) When positive terminal connected to A then diode

D_1 is forward biased, current, $I = \frac{2}{5} = 0.4A$

When positive terminal connected to B then diode D_2

is forward biased, current, $I = \frac{2}{10} = 0.2A$

$$E_0^2 = \frac{2P}{4\pi r^2 \varepsilon_0 c} = \frac{2 \times 0.1 \times 9 \times 10^9}{1 \times 3 \times 10^8}$$

$$E_0 = \sqrt{6} = 2.45 V/m$$

28. In an unbiased n-p junction electron diffuse from n-region to p-region because: [Online Apri110, 2015]

(a) holes in p-region attract them

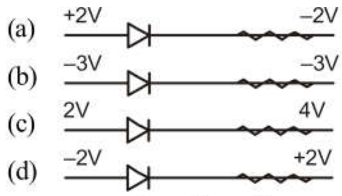
(b) electrons travel across the junction due to potential difference

(c) only electrons move from n to p region and not the vice-versa

(d) electron concentration in n-region is more compared to that in p-region

SOLUTION(d) Electrons in an unbiased *p-n* junction, diffuse from n-region i.e. higher electron concentration to p-region i.e. low electron concentration region.

29. The forward biased diode connection is: [2014]



SOLUTION. (a) 

For forward bias, p-side must be at higher potential than n-side. $\Delta V = (+)Ve$

30. For LED's to emit light in visible region of electromagnetic light, it should have energy band gap in the range of: [Online Apri112, 2014]

(a) 0.1eV to 0.4eV (b) 0.5eV to 0.8eV (c) $0.9\text{eV}$to 1.6eV (d) $1.7\text{eV}$to 3.0eV

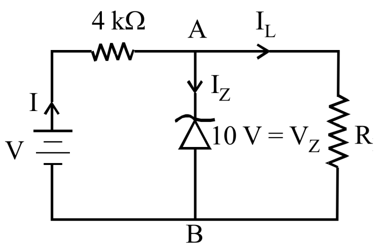
SOLUTION: (d) Energy band gap range is given by,$E_g = \frac{hc}{\lambda}$

For visible region $\lambda = (4 \times 10^{-7} - 7 \times 10^{-7})\text{m}$

$$E_g = \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{7 \times 10^{-7}} = \frac{19.8 \times 10^{-26}}{7 \times 10^{-7}} = \frac{2.\cdot 8 \times 10^{-19}}{16 \times 10^{-19}} \quad hence\ E_g = 1.75\text{eV}$$

31.A Zener diode is connected to a battery and a load as shown below:

[Online April II, 2014]



If v=60 volts the currents, I, I_Z and I_L are respectively.

(a) 15 mA, 5 mA, 10 mA (b) 15 mA, 7.5 mA, 7.5 mA
(c) 12.5 mA, 5 mA, 7.5 mA (d) 12.5 mA, 7.5 mA, 5 mA

SOLUTION(d) Here, $R = 4\text{k}\Omega = 4 \times 10^3 \Omega V_i = 60\text{V}$

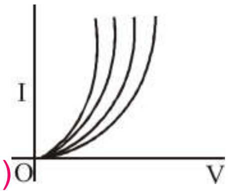
Zener voltage $V_Z = 10\text{V}$ $R_L = 2\text{k}\Omega = 2 \times 10^3 \Omega$

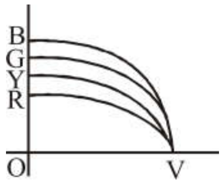
Load current, $I_L = \frac{V_Z}{R_L} = \frac{10}{2\times 10^3} = 5\text{mA}$

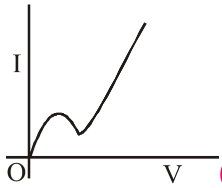
Current through R, $I = \frac{V_i - V_Z}{R} = \frac{60-10}{4\times 10^3} = \frac{50}{4\times 10^3} = 12.5\text{mA}$

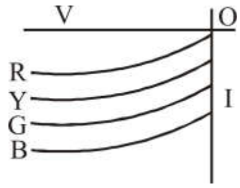
From circuit diagram,$I = I_Z + I_L \Rightarrow 12.5 = I_Z + 5 \Rightarrow I_Z = 12.5 - 5 = 7.5\text{mA}$

32. The I-V characteristic of an LED is [2013]

(a) 

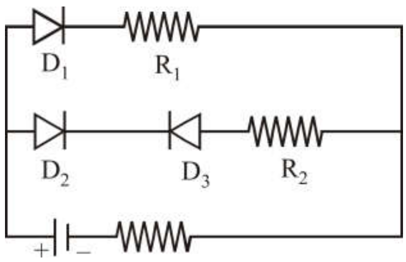
(b) 

(c) 

(d) 

SOLUTION(a) For same value of current higher value of voltage is required for higher frequency hence (a) is correct answer.

33. Figure shows a circuit in which three identical diodes are used. Each diode has forward resistance of20Ω and infinite backward resistance. Resistors $R_1 = R_2 = R_3 = 50\Omega$. Battery voltage is 6 V. The current through R_3 is: [Online April 22, 2013]



6V R3

(a) 50mA (b) 100mA (c) 60mA (d) 25 mA

SOLUTION: (a) Here, diodes D_1 and D_2 are forward biased and D_3 is reverse biased. Therefore, current through R_3 $i = \frac{V}{R'} = \frac{6}{120} = \frac{1}{20}A = 50\text{mA}$

34. This question has Statement 1 and Statement 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement 1: A pure semiconductor has negative temperature coefficient of resistance.

Statement 2: On raising the temperature, more charge carriers are released into the conduction band. [Online May 12, 2012]

(a) Statement 1 is false, Statement2 is true.

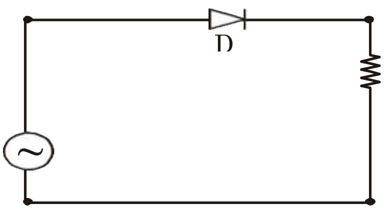
(b) Statement 1 is true; Statement 2 is false.

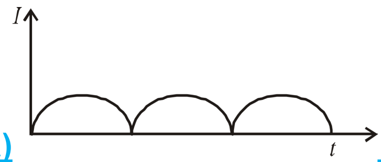
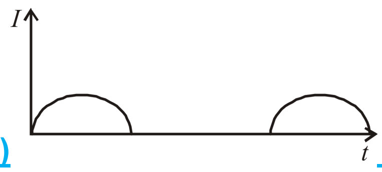
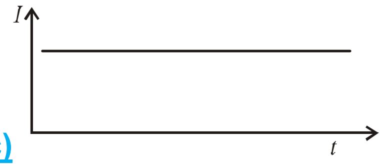
(c) Statement 1 is true, Statement2 is true, Statement 2 isnot a correct explanation of Statement 1.

(d) Statement 1 is true, Statement2 is true, Statement 2 isthe correct explanation of Statement 1.

SOLUTION: (d) Temperature coefficient of resistance is negative for pure semiconductor. And no. of charge carriers in conduction band increases with increase in temperature.

32. A *p-n* junction (*D*) shown in the figure can act as a rectifier. An alternating current source (*V*) is connected in the circuit.

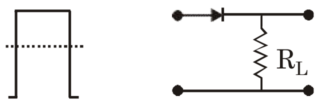


a)  b)  c) 

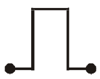
d)none

SOLUTION: The given circuit will work as halfwave rectifier as it conducts during the positive half cycle of input AC. Forward biased in one half cycle and reverse biased in the other half cycle].

36. If in a *p*-*n* junction diode, a square input signal of 10 V is applied as shown**[2007]**



Then the output signal across R_L will be

a) 

b) 

c) no current   d0data in sufficient

SOLUTION: (a) The current will flow through R_L when the diode is forward biased

37. Carbon, silicon and germanium have four valence electrons each. At room temperature which one of the following statements is most appropriate? **[2007]**

(a) The number of free electrons for conduction is significant only in Si and Ge but small in C.

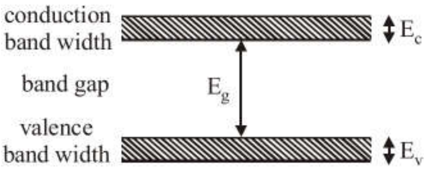
(b) The number of free conduction electrons is significant in C but small in Si and Ge.

(c) The number of free conduction electrons is negligibly small in all the three.

(d) The number of free electrons for conduction is significant in all the three.

SOLUTION: (a) Si and Ge are semiconductors but C is an insulator. InSi and Ge at room temperature, the energy band gap is low due to which electrons in the covalent bonds gains kinetic energy and break the bond and move to conduction band. As a result, hole is created in valence band. So, the number of free electrons is significant in Si and Ge.

38.Ifthe lattice constant of this semiconductor is decreased, then which of the following is correct? [2006]



(a) All E_c, E_g, E_v increase

(b) E_c and E_v increase, but E_g decreases

(c) E_c and E_v decrease, but E_g increases

(d) All E_c, E_g, E_v decrease

SOLUTION: (c) A crystal structure is made up of a unit cell arranged in a particular way; which is periodically repeated in three dimensions on a lattice. The spacing between unit cells in various directions is called its lattice constants. As lattice constants increases the band - gap (*Eg*), also increases which means more energy would be required by electrons to reach the conduction band from the valence band. Automatically E_c and E_v decreases.

39. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by [2006]

(a) Ionic bonding(b) Covalent bonding(c) Vander Waals bonding(d) Metallic bonding

SOLUTION: b) Van der Waal' s bonding is attributed to the attractive forces between molecules of a liquid. The conductivity of semiconductors (covalent bonding) and insulators (ionic bonding) increases with increase in temperature. Solid which is formed by covalent bond is not transparent to visible light and its conductivity increase with temperature.

40. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities? [2006]

(a) $\frac{5}{8}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{4}{7}$

SOLUTION: (c) Relation between drift velocity and current is

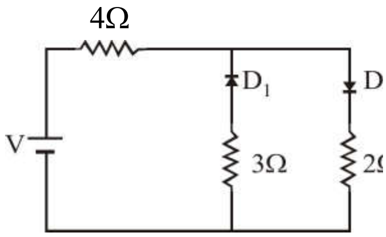
$$\mathrm{I = nAeV_d}$$

$$\frac{I_e}{I_h} = \frac{n_e eAv_e}{n_h eAv_h}$$

$$\Rightarrow \frac{7}{4} = \frac{7}{5} \times \frac{v_e}{v_h}$$

$$\Rightarrow \frac{v_e}{v_h} = \frac{5}{4}$$

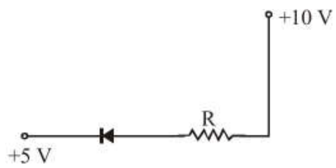
41. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit? [2006)



(a) 1.71A (b) 2.00A (c) 2.31A (d) 1.33A

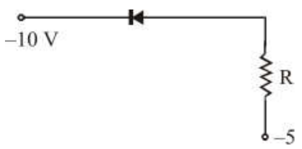
SOLUTION: b) D_2 is forward biased.D_1 is reversed biased. So, it will act like an open circuit. So effective resistance of the circuit $R=4+2=6\Omega$ $j=\frac{E}{R}=\frac{12}{6}=2A$

42. In the following, which one of the diodes reverse biased? [2006]



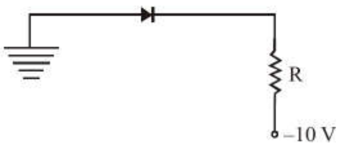
(a)

(b)

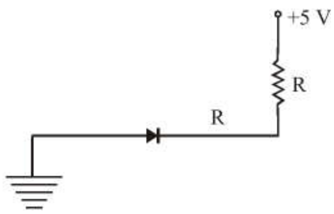


v

(c)



(d)



SOLUTION: (d) p-side connected to low potential and n-side is connected to high potential.

43. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm is incident on it. The band gap in (eV) for the semiconductor is [2005]

(a) $2.5 \mathrm{eV}$ (b) $1.1 \mathrm{eV}$ (c) $0.7 \mathrm{eV}$ (d) $0.5 \mathrm{eV}$

SOLUTION: (d) Band gap $=$ energy of photon of wavelength 2480 nm. So, Band gap,

$$\mathrm{E_g} = \frac{hc}{\lambda} = \left(\frac{6.63\times10^{-34}\times3\times10^{8}}{2480\times10^{-9}}\right)\times\frac{1}{1.6\times10^{-19}}eV = 0.5\mathrm{eV}$$

44. When p-n junction diode is forward biased then [2004]

(a) both the depletion region and barrier height are reduced

(b) the depletion region is widened and barrier height is reduced

(c) the depletion region is reduced and barrier height is increased

(d) Both the depletion region and barrier height are increased

SOLUTION: (a) In forward biasing, the p type is connected to positive terminal and n type is connected with negative terminal. So, holes from p region and electron from n region are pushed towards the Junction which reduces the width of depletion layer. Also, distance between diffused holes and electrons decrease, which decrease electric field hence barrier potential.

45. A strip of copper and another of germanium are cooled from room temperature to $80 \mathrm{K}$. The resistance of [2003]

(a) each of these decreases

(b) copper strip increases and that of germanium decreases

(c) copper strip decreases and that of germanium increases

(d) each of these increases

SOLUTION: (c) Copper is a conductor and in conductor resistance decreases with decrease in temperature. Germanium is a semiconductor. In semi-conductor resistance increases with decrease in temperature.

46. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the [2003]

(a) crystal structure

(b) variation of the number of charge carriers with temperature

(c) type of bonding

(d) variation of scattering mechanism with temperature

SOLUTION: b) When the temperature increases, certain bounded electrons become free which tend to promote conductivity. Simultaneously number of collisions between electrons and positive kernels increases which decrease the relaxation time.

47. In the middle of the depletion layer of a reverse- biased *p-n* junction, the [2003]

(a) electric field is zero

(b) potential is maximum

(c) electric field is maximum

(d) potential is zero

SOLUTION: (a) In reverse biasing the width of depletion region increases, and current flowing through diode is zero. Thus, electric field is zero at middle of depletion region.

48. At absolute zero, Si acts as [2002]

(a) non-metal (b) metal (c) insulator (d) none of these

SOLUTION: (c) Pure silicon, at OK, will contain all the electrons in bounded state. The conduction band will be empty. So, there will be no free electrons (in conduction band) and holes (in valence band). Therefore, no electrons from valence band are able to shift to conduction band due to thermal agitation. Pure silicon will act as insulator.

49. By increasing the temperature, the specific resistance of a conductor and a semiconductor [2002]

(a) increases for both (b) decreases for both

(c) increases, decreases (d) decreases, increases

SOLUTION: . (c) Specific resistance (resistivity) is given by $\Rightarrow$

where $n=\mathrm{no}$. offree electrons per unit volume

and $\tau=$ average relaxation time

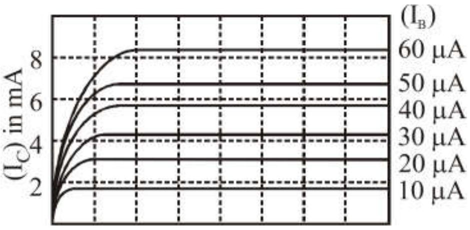
For a conductor with rise in temperature n increases. Increase in temperature results increase in number of collisions between free electrons due to which relaxation time T decreases. But the decrease in τ is more dominant than increase in n resulting an increase in the value of ρ. For a semiconductor with rise in temperature, n increases and τ decreases. But the increase in n is more dominant than decrease in τ resulting in a decrease in the value of ρ

50. The energy band gap is maximum in [2002]

(a) metals (b) superconductors (c) insulators (d) semiconductors.

SOLUTION: (c) In insulators, valence band is completely filled while conduction band is empty. The energy band gap is maximum in insulators.

51. The output characteristics of a transistor is shown in the figure. When $\mathrm{V_{CE}}$ is 10 V and $\mathrm{I_C}=4.0\mathrm{mA}$, then value of β_{ac} is. [Sep. 06, 2020 (II)]



2 4 6 8 10 12 14 16 (V_{CE}) in volts

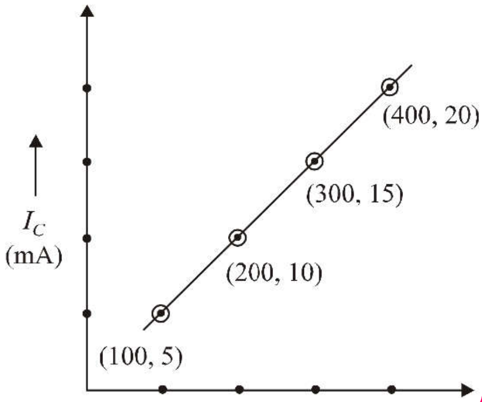
SOLUTION: (150) At $V_{CE} = 10\text{V} \text{and} I_c = 4\text{mA}$

Change in base current, $\Delta I_B = (30 - 20) = 10\mu A$

Change in collector current, $\Delta I_c = (4.5 - 3) = 1.5\text{mA}$

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right) = \frac{1.5mA}{10\mu A} = 150$$

52. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 $k\Omega$ respectively, is shown in the figure. The Voltage and Power gain, are respectively: [12 Apr. 2019 I]



(a) $2.5 \times 10^4, 2.5 \times 10^6$ (b) $5 \times 10^4, 5 \times 10^6$

(c) $5 \times 10^4, 5 \times 10^5$ (d) $5 \times 10^4, 2.5 \times 10^6$

SOLUTION: (Bonus) $\beta = \frac{\Delta i_c}{\Delta i_b} = \frac{200-100}{10-5} = 20$

Voltage gain $= \beta \frac{R_2}{R_1} = \frac{20 \times 100 \times 10^3}{100} = 20 \times 10^3$

Power gain $= \beta^2 \frac{R_2}{R_1} = 20^2 \left(\frac{100 \times 10^3}{100}\right) = 4 \times 10^5$

53. A npn transistor operates as a common emitter amplifier, with a power gain of60 dB. The input circuit resistance is100Ω and the output load resistance is 10 $k\Omega$. The common emitter current gain β is: [10 Apr. 2019 I]

(a) 10^2 (b) α) (c) 6×10^2 (d) 10^4

SOLUTION: (a) Power gain $= 60 = 10 \log \left(\frac{P_0}{p_i}\right) \Rightarrow 6 = \log \left(\frac{P_0}{p_i}\right)$

$\frac{p_o}{p_i} = 10^6 = \beta^2 \left(\frac{R_{out}}{R_{in}}\right) \Rightarrow \beta = 100$

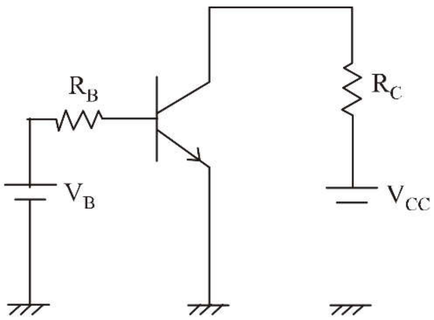
54. An NPN transistor is used in common emitter configurations as an amplifier with 1 k&! load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a3 mA change in the collector current and 15 $1/_4\text{A}$ change in the base current of the amplifier. The input resistance and voltage gain are: [9 April 2019 I]

(a) $0.33\text{k}\Omega 1.5$ (b) $0.67\text{k}\Omega 300$ (c) $0.67\text{k}\Omega 200$ (d) $0.33\text{k}\Omega 300$

SOLUTION: b) $\beta = \frac{\Delta Ic}{\Delta Ib} = \frac{3\times10^{-3}}{15\times10^{-6}} = 200$ We have $\frac{V_0}{V_i} = \beta\frac{R^2}{R_1}$

or $\frac{V_0}{V_i} = 200\left(\frac{1000}{R_1}\right)$ If $R_1 = 0.67\text{k}\Omega \Rightarrow \frac{V_0}{V_\text{i}} = 300$

55. A common emitter amplifier circuit, built using a npn transistor, is shown in the figure. Its dc current gain is 250,$\text{R}_\text{c} = 1$ k&! and $\text{V}_\text{cc} = 10\text{V}$. What is the minimum base current? for V_CE to reach saturation [8 Apr. 2019 II]



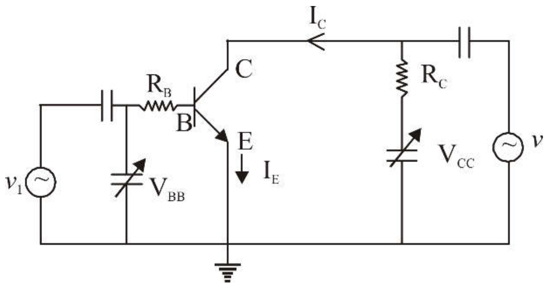
(a) 40 μA (b) 100 μA (c) 7 μA (d) 10 μA

SOLUTION: (a) Given, $\beta = 250$ Voltage gain, $\frac{V_{CC}}{V_B} = \beta\frac{R_C}{R_B}$

$$\frac{10}{V_B} = 250 \times \frac{10^3}{R_B}$$

$$\frac{V_B}{R_B} = \frac{1}{25\times10^3} = 40\mu\text{A}$$

56.In the figure, given that V_BB supply can vary from 0 to 5.0V, $\text{V}_\text{CC} = 5\text{V}$, $\beta_\text{dc} = 200$, $\text{R}_\text{B} = 100\text{k}\Omega$, $\text{R}_\text{C} = 1\text{K}\Omega$ and $\text{V}_\text{BE} = 1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively: [12 Jan. 2019 II]



(a) 25 μA and 3.5 V (b) 20 μA and 3.5 V (c) 25 μA and 2.8V (d) 20 μA and 2.8V

SOLUTION: (a) At saturation, $V_{CE} = 0$ $V_{CE} = V_{CC} - I_C R_c$

$$I_c = \frac{V_{cc}}{R_c} = 5\times10^{-3}$$

Current gain $\beta_{dc} = \frac{I_C}{I_B}$ $\Rightarrow I_B = \frac{5\times10^{-3}}{200} = 25\mu A$

At input side $V_{BB} = I_B R_B + V_{BE} = (25mA)(100k\Omega) + 1V = 3.5V$

57. In a common emitter configuration with suitable bias, it is given than R_L is the load resistance and R_{BE} is small signal dynamic resistance (input side). Then, voltage gain, current gain and power gain are given, respectively, by:(β is current gain, I_B, I_C, I_E are respectively base, collector and emitter currents:) [Online Apri115, 2018]

(a) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_E}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$

(b) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta\frac{R_L}{R_{BE}}$

(c) $\beta^2\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_E}, \beta^2\frac{R_L}{R_{BE}}$

(d) $\beta\frac{R_L}{R_{BE}}, \frac{\Delta I_C}{\Delta I_B}, \beta^2\frac{R_L}{R_{BE}}$

SOLUTION: (d) Current gain $\beta_{dc} = \frac{I_C}{I_B}$

Voltage gain $\wedge=$ Current gain $\times$ Resistance gain $= \beta\frac{R_L}{R_{BE}}$

Power gain $A_p = (\text{Current gain})^2 \times$ Resistance gain

$$= \beta^2\frac{R_L}{R_{BE}}$$

58. The current gain of a common emitter amplifier is 69. If the emitter current is 7.0mA, collector current is: [Online April 9, 2017]

(a) 9.6mA (b) 6.9mA (c) 0.69mA (d) 69 mA

SOLUTION: b) Given, current gain ofCE amplifier $\beta = 69$, $I_E = 7mA$ or $\frac{I_C}{I_B} = 69$

We know that, $\alpha = \frac{\beta}{1+\beta} = \frac{69}{70} = \frac{I_C}{I_E}$ $\quad I_C = I_E \times \frac{69}{70} = \frac{69}{70} \times 7$

Collector current, $I_c = 6.9\text{mA}$

59. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be: [Online April 2, 2017]

(a) $135°$ (b) $180°$ (c) $45°$ (d) $90°$

SOLUTION:. b) In common emitter configuration for *n-p-n* transistor input and output signals are $180°$ out of phase *i. e.*, phase difference between output and input voltage is $180°$.

60. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is: [2016]

(a) $\frac{1}{\beta} = \frac{1}{\alpha} + 1$ (b) $\alpha = \frac{\beta}{1+\beta}$ (c) $\beta = \alpha$ (d) None of these

SOLUTION: (b)We know that $\alpha = \frac{I_c}{I_e}$ and $\beta = \frac{I_c}{I_b}$ Also $I_e = I_b + I_c$

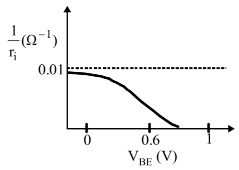
$$\alpha = \frac{Ic}{I_b + I_c} = \frac{\frac{I_c}{I_b}}{1 + \frac{I_c}{I_b}} = \frac{\beta}{1+\beta}$$

Option (b) and (d) are therefore incorrect.

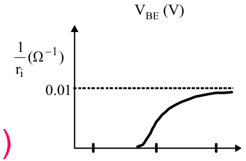
61. A realistic graph depicting the variation of the reciprocal of input resistance in an input characteristics measurement in a common emitter transistor configuration is

On x axis take $v_{BE}(v)$:[Online Apri10, 2016]

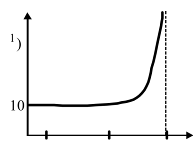
(a)

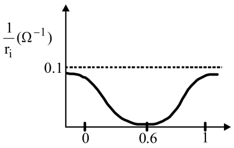


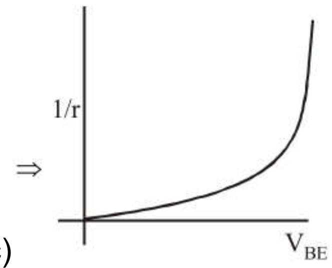
(b)



(c)



(d) 

SOLUTION: (c) 

62. The ratio (R) of output resistance r_0, and the input resistance r_i in measurements of input and output characteristics of a transistor is typically in the range:

[Online Apri110, 2016]

(a) $R \sim 10^2 - 10^3$ (b) $R \sim 1 - 10$ (c) $R \sim 0.1 - 1.0$ (d) $R \sim 0.1 - 0.01$

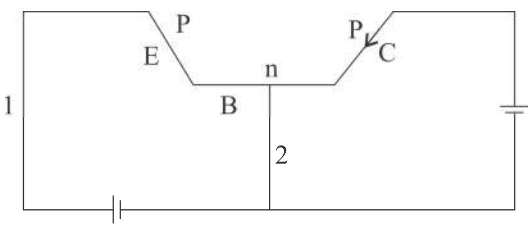
SOLUTION: (c) For C.B. configuration $\frac{r_i}{r_o} = 0.1\Omega$

For CE and CC-configuration $\frac{r_i}{r_0} \approx 1\Omega$.

63. An unknown transistor needs to be identified as a npn or pnp type. A multimeter, with $+$ve and-ve terminals, is used to measure resistance between different terminals of transistor. If terminal 2 is the base of the transistor then which of the following is correct for a pnp transistor? [Online April 9, 2016]

(a) $+$ve terminal 2, -ve terminal 3, resistance low
(b) $+$ve terminal 2, -ve terminal 1, resistance high
(c) $+$ve terminal 1, -ve terminal 2, resistance high
(d) $+$ve terminal 3, -ve terminal 2, resistance high

SOLUTION: (c) Connecting circuit according to question, it is clear



$+$ve terminal 1, -ve terminal 2, resistance high.

64. An n-p-n transistor has three leads A, B and C. Connecting B and C by moist fingers,

A to the positive lead of an ammeter, and C to the negative lead ofthe ammeter, one finds large deflection. Then, A, B and C refer respectively to: [Online April 9, 2014]

(a) Emitter, base and collector (b) Base, emitter and collector

(c) Base, collector and emitter (d) Collector, emitter and base.

SOLUTION: (c) In the given question, A, B and C refer base, collector and emitter respectively.

65. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some resistance is seen on the multimeter. Which of the following is true for the transistor? [2008]

(a) It is a npn transistor with R as base

(b) It is a pnp transistor with R as base

(c) It is a pnp transistor with R as emitter

(d) It is a npn transistor with R as collector

SOLUTION: b)) It is a *p-n-p* transistor with R as base.

66. In a common base mode of a transistor, the collector current is $5.488\mathrm{mA}$ for an emitter current of $5.60\mathrm{mA}$. The value of the base current amplification factor (β) will be [2006]

(a) 49 (b) 50 (c) 51 (d) 48

SOLUTION:(a) *Collector current,* $I_C = 5.488mA$,

Emitter $current I_e = 5.6mA$

$$\alpha = \frac{I_c}{I_e} = \frac{5.488}{5.6},$$

$$\beta = \frac{\alpha}{1-\alpha} = 49$$

67. In a common base amplifier, the phase difference between the input signal voltage and output voltage is [2005]

(a) π (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) 0

SOLUTION: (d) In common base amplifier circuit, input and output voltage are in the same phase. So, the phase difference between input voltage signal and output voltage signal is zero.

68. When npn transistor is used as an amplifier [2004]

(a) electrons move from collector to base

(b) holes move from emitter to base

(c) electrons move from base to collector

(d) holes move from base to emitter

SOLUTION: (c) In npn transistor, electrons moves from emitter to base.

69. For a transistor amplifier in common emitter configuration for load impedance of $1k\Omega$ ($h_{fe} = 50$ and $h_{oe} = 25$) the current gain is [2004]

(a) -24.8 (b) -15.7 (c) -5.2 (d) -48.78

SOLUTION: (d) In common emitter configuration for transistor amplifier current gain

$$A_j = \frac{-h_{fe}}{1 + b_{oe}R_L}$$

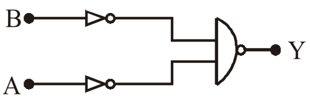
Where h_{fe} and h_{oe} are hybrid parameters. $A_i = \frac{-50}{1+25\times10^{-6}\times1\times10^{3}} = -48.78$

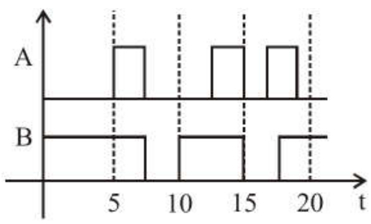
70. The part of a transistor which is most heavily doped to produce large number of majority carriers is [2002]

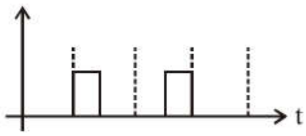
(a) emitter (b) base (c) collector (d) can be any of the above three.

SOLUTION:(a) Emitter main function is to supply the majority charge carriers towards the collector. Therefore, emitter is most heavily doped.

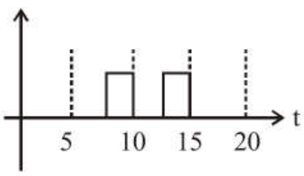
71. Identify the correct output signal Y in the given combination of gates (as shown) for the given inputs A and B. [Sep. 06, 2020 (D)]





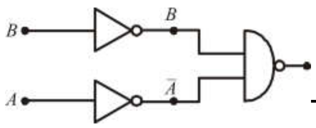
(a)


(b)


(c)


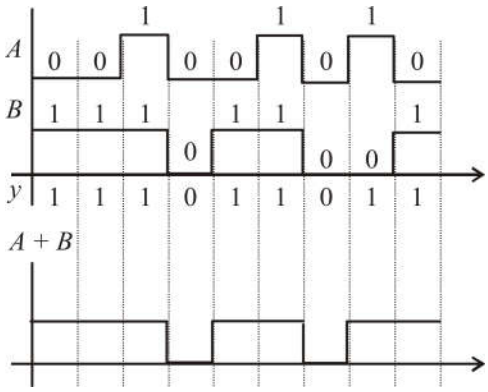
(d)


SOLUTION: (a) Boolean expression, $y = \overline{\overline{A} \cdot \overline{B}} = \overline{\overline{A}} + \overline{\overline{B}} = A + B$



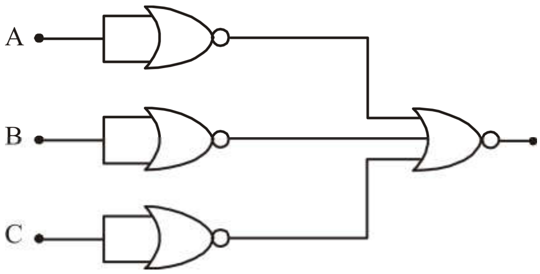
Truth table :

A	B	Y
0	1	1
1	0	1
0	0	0
1	1	1



y =A+B

72. identify the operation performed by the circuit given below: [Sep. 04, 2020 (II)]



(a) NAND (b) OR (c) AND (d) NOT

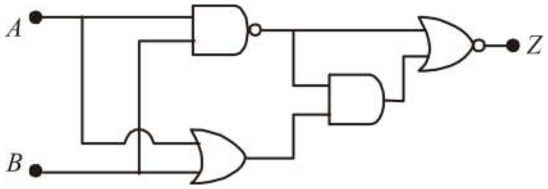
SOLUTION: (c) When two inputs of NAND gate are shorted, it behaves like a NOT gate so Boolean equation will be

$$y = \overline{\overline{A} + \overline{B} + \overline{C}} = A \cdot B \cdot C$$

Thus, whole arrangement behaves like a AND gate.

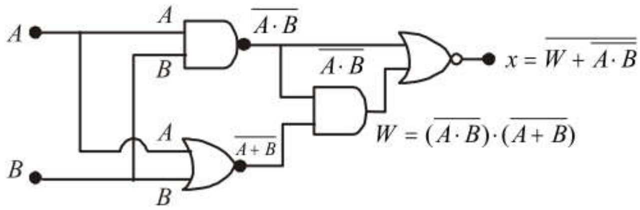
A	B	C	Y
0	0	0	0
1	0	0	0
0	1	0	0
0	0	1	0
1	1	0	0
1	0	1	0
0	1	1	0
1	1	1	1

73. In the following digital circuit, what will be the output at 'Z', when the input (A, B) are $(1,0)$, $(0,0)$, $(1,1)$, $(0,1)$:[Sep. 02, 2020 (II)]



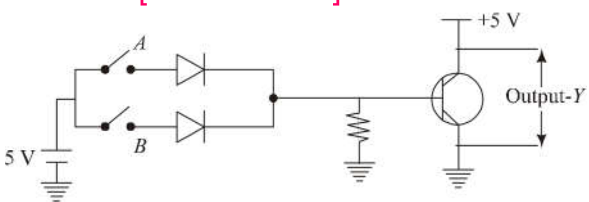
(a) 0,0,1,0 (b) 1, 0,1,1 (c) 1, 1, 0,1 (d) 0,1,0,0

SOLUTION: (a)



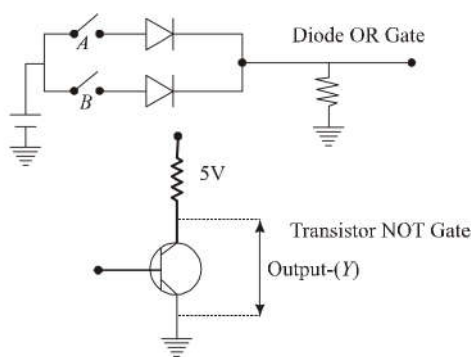
A	B	$\overline{A \cdot B}$	$\overline{A + B}$	$W = (\overline{A \cdot B}) \cdot (\overline{A + B})$	$Q = W + \overline{A \cdot B}$	$\overline{Q} = X$
1	0	1	0	0	1	0
0	1	1	0	0	1	0
1	1	0	0	0	0	1
0	0	1	1	1	0	0

74. Boolean relation at the output stage-Y for the following circuit is: [8 Jan. 2020 I]



(a) $\overline{A} + \overline{B}$ (b) A+B (c) A.B (d) $\overline{A}.\overline{B}$

(d)

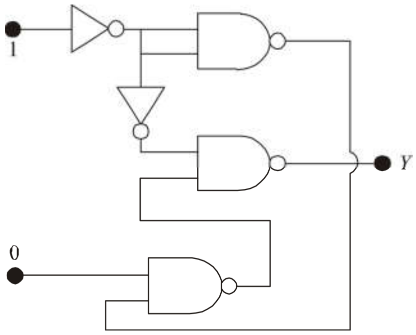


SOLUTION:

OR + NOT →NORGate

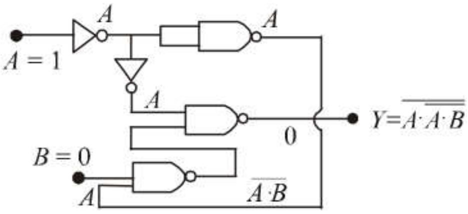
Hence Boolean relation at the output stage $-Y$ for thecircuit, $Y = \overline{A + B} = \overline{A}.\overline{B}$

75. In the given circuit, value of Y is:



(a) 0 (b) toggles between 0 and 1 (c) will not execute (d) 1

SOLUTION: (a)



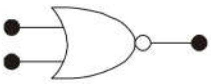
$$Y = AB.A = AB + A = AB + \overline{A}$$

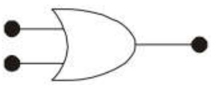
For $A = 1$, $B = 0$

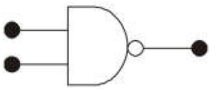
$$Y = (1) \times 0 + 0$$

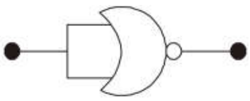
$$\Rightarrow Y = 0 + 0 = 0$$

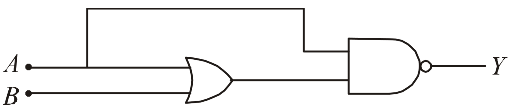
76. Which of the following gives a reversible operation? [7 Jan. 2020 I]

(a) 

(b) 

(c) 

(d) 



(a)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(b)

$$\begin{bmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

(c)

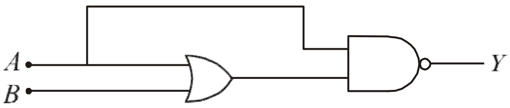
$$\begin{bmatrix} A & B & Y \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

(d)

$$\begin{bmatrix} A & B & Y \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

SOLUTION: (d) A logic gate is reversible if we can recover input data from the output. Hence NOT gate.

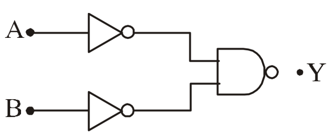
77. The truth table for the circuit given in the fig. is: [9 April 2019 I]



SOLUTION: (c)

A	B	(A + B)	(A + B). A	$\overline{(A+B).A}$
0	0	0	0	1
0	1	1	0	1
1	0	1	1	0
1	1	1	1	0

78. The logic gate equivalent to the given logic circuit is: [9 Apr. 2019 II]



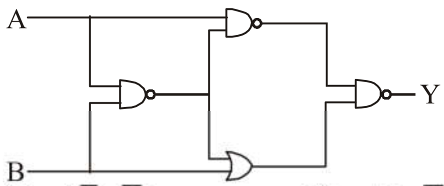
(a) NAND (b) OR (c) NOR (d) AND

SOLUTION: 78. b) Truth table → The output is of OR-gate

A	B	$\overline{A}$	$\overline{B}$	$\overline{\overline{A}.\overline{B}}$
0	0	1	1	0

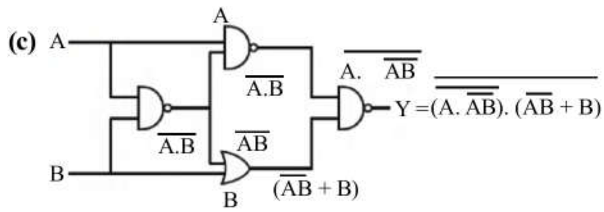
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

79. The output of the given logic circuit is: [12 Jan. 2019 I]



(a) $A\bar{B}+\bar{A}B$ (b) $AB+\bar{A}B$ (c) $A\bar{B}$ (d) $\bar{A}B$

SOLUTION:



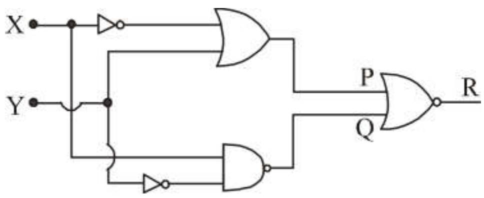
$$Y=\left(\overline{\bar{A}\cdot\overline{AB}}\right).\left(\overline{AB}+B\right)$$

$$=A\cdot\overline{AB}+AB\cdot\bar{B}$$

$$=A\cdot\left(\bar{A}+\bar{B}\right)+AB\cdot\bar{B}$$

$$=A\bar{B}$$

80. To get output 1 at R, for the given logic gate circuit the input values must be: [10 Jan. 2019 I]



(a) $X=0,\ Y=1$ (b) $X=1\ ,\ Y=1$ (c) $X=1,\ Y=0$ (d) $X=0,\ Y=0$

SOLUTION: (c) From the given logic circuit,

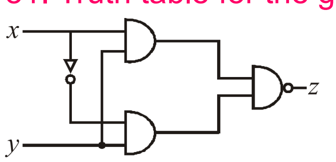
$$p=\bar{x}+y$$

$$Q=\bar{y}.x=y+\bar{x}$$

Output, $R=\overline{P+Q}$ To make output 1 $P+Q$ must be 0’

So, $x=1,\ y=0$

81. Truth table for the given circuit will be [Online Apri115, 2018]



x	y	z
0	0	1
0	1	1
1	0	1
1	1	0

(a)

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

(b)

x	y	z
0	0	1
0	1	1
1	0	1
1	1	1

(c)

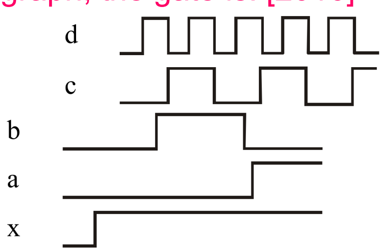
x	y	z
0	0	0
0	1	1
1	0	1
1	1	1

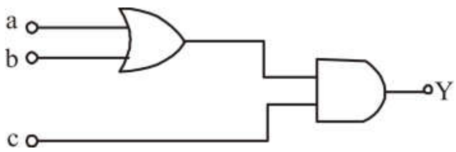
(d)

SOLUTION: (c) Truth table of the circuit is as follows

x	y	$\overline{x}$	$a = x.y$	$b = \overline{x}.y$	$z = \overline{a.b}$
0	0	1	0	0	1
0	1	1	0	1	1
1	0	0	0	0	1
1	1	0	1	0	1

82. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is: [2016]





(a) $a = 0,\ b = 0,\ c = 1$ (b) $a = 1\ ,\ b = 0,\ c = 0$

(c) $a = 1\ ,\ b = 0,\ c = 1$ (d) $a = 0,\ b = 1\ ,\ c = 0$

SOLUTION: . (a) In case of an 'OR' gate the input is zero when all inputs are zero. If anyone input is' 1', then the output is' 1'.

83. To get an output of 1 from the circuit shown in figure the input must be: [Online Apri11, 2016]

SOLUTION: . (c) Truth table for given logical circuit

a	b	$(a + b)$	c	$Y = (a + b).c$
0	0	0	0	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

Output of OR gate must be 1 and $c = 1$

So, $a = 1\ ,\ b = 0$ or $a = 0,\ b = 1$.

84. The truth table given in fig. represents: [Online April 9, 2016]

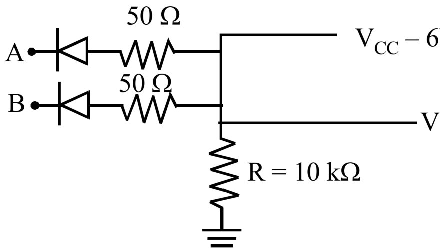
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

(a) OR- Gate (b) NAND- Gate (c) AND- Gate (d) NOR- Gate

SOLUTION: (a) It represents OR-Gate.

A	B	A+B=Y
0	0	0
0	1	1
1	0	1
1	1	1

85.Given: A and B are input terminals. Logic $1 => 5V$ Logic $0 =< 1V$

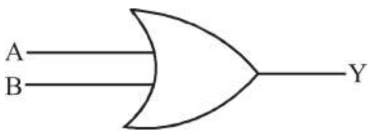


Which logic gate operation, the above circuit does?

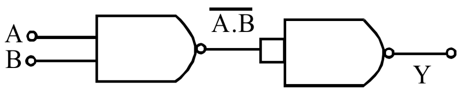
[Online Apri119, 2014]

(a) AND Gate (b) OR Gate (c) XOR Gate (d) NOR Gate

SOLUTION: (a) AND Gate



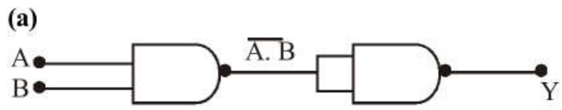
86.Identify the gate and match A, B, Y in bracket to check. [Online April 9, 2014]



(a) AND $(A = 1, B = 1, Y = 1)$
(b) OR $(A = 1, B = 1, Y = 0)$
(c) NOT $(A = 1, B = 1, Y = 1)$
(d) XOR $(A = 0, B = 0, Y = 0)$

(a) OR (b) NAND (c) NOT (d) AND

SOLUTION:

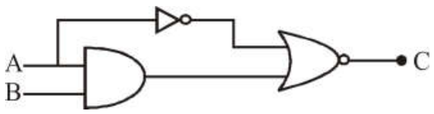


$Y = \overline{\overline{AB}.\overline{AB}} = AB + AB = AB$ In this case output Y is equivalent to AND gate.

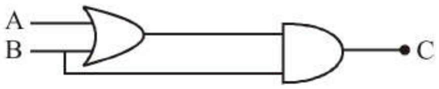
87. Which of the following circuits correctly represents the following truth table?

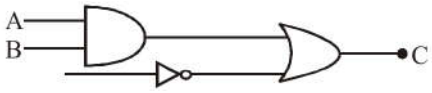
[Online April 25, 2013]

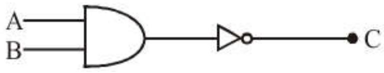
A	B	c
0	0	0
0	1	0
1	0	1
1	1	0



(a)

(b) 

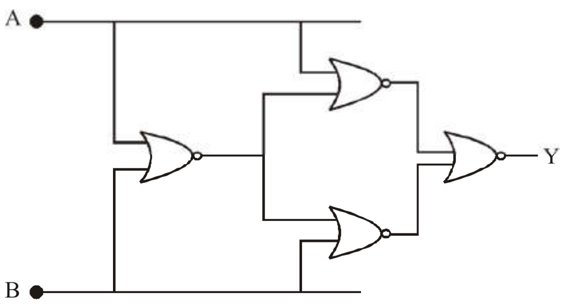
(c) 

(d) 

SOLUTION: (a) For circuit $A.B = \overline{Y + \overline{A}} = C$

A	B	y	$\overline{A}$	$\overline{Y + \overline{A}} = C$
0	0	0	1	0
0	1	0	1	0
1	0	0	0	1
1	1	1	0	0

88. A system of four gates is set up as shown. The 'truth table' corresponding to this system is : [Online April 23, 2013]



(a)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	0

(c)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(d)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

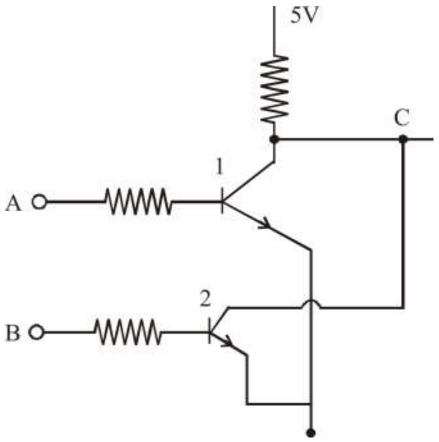
SOLUTION: (a) In the given system all four gates are NOR gate Truth Table

A	B	$\left(y' = \overline{A+B}\right)$	$y'' = \left(\overline{A+y^1}\right)$	$y''' = (A + y“)$	$y = y + y'''$
0	0	1	0	0	**1**
0	1	0	1	0	**0**
1	0	0	0	1	**0**
1	1	0	0	0	**1**

i.e.,

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

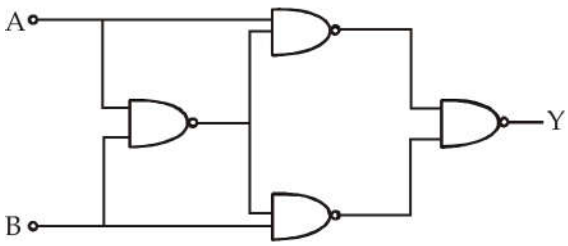
89. Consider two npn transistors as shown in figure. $If\ 0\ volts$ Volts corresponds to false and 5 Volts correspond to true then the output at C corresponds to: [Online Apri19, 2013]



(a) A NAND B (b) A OR B AAND B (d) A NOR B

SOLUTION: (a) The output at C corresponds to ANAND B or $\overline{A \cdot B} = C$

90. Truth table for system of four NAND gates as shown in figure is: **[2012]**



(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

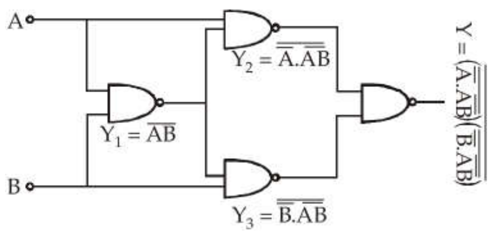
(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	1

SOLUTION: (a)

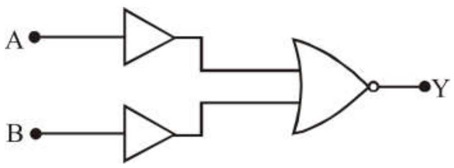


By expanding this Boolean expression $\mathrm{Y} = A.\overline{B} + B.\overline{A}$

Thus, the truth table for this expression should be (a).

91.The figure shows a combination of two NOT gates and a NOR gate.

[Online May 26, 2012]



The combination is equivalent to a

(a) NAND gate (b) NOR gate (c) AND gate (d) OR gate

SOLUTION: (c) Truth table is as shown:

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} + \overline{B}$	$\overline{\overline{A} + \overline{B}}$
0	0	1	1	1	0
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	0	1

Thus the combination of two NOT gates and one NOR gate is equivalent to a AND gate.

92. Which one of the following is the Boolean expression for NOR gate?

[Online May 19, 2012]

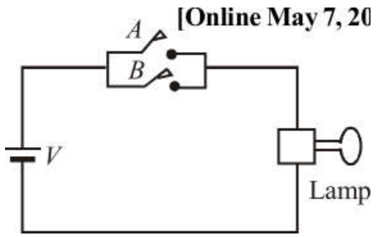
(a) $Y = \overline{A + B}$ (b) $Y = \overline{A.B}$ (c) $Y = A.B$ (d) $Y = \overline{A}$

SOLUTION: (a) NOR gate is the combination of NOT and OR gate.

Boolean expression for NOR gate is $Y = A + B$

93. Which logic gate with inputs A and B performs the same operation as that performed by the following circuit?

(a) NAND gate (b) OR gate (c) NOR gate (d) AND gate



SOLUTION: (b)When either of A or B is 1 i.e. closed then lamp will glow.

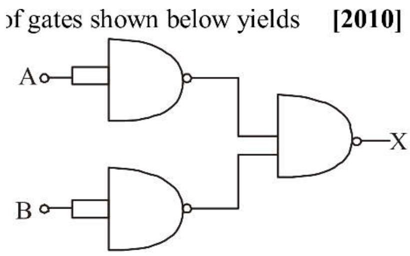
In this case, Truth table

Inputs		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

This represents OR gate.

94. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a: [2011 RS]

(a) NOT gate (b) NOR gate (c) AND gate (d) OR gate

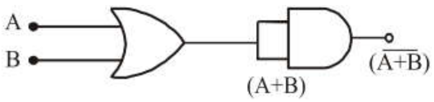


SOLUTION: . (b) When both inputs of NAND gate are jointed to form a single input, it behaves as NOT gate $\text{OR} +$ NOT $= \text{NOR}.$ $\overline{(\text{A} + \text{B})} =$ NOR gate

95. The combination of gates shown below yields

(a) OR gate (b) NOT gate (c) XOR gate (d) NAND gate

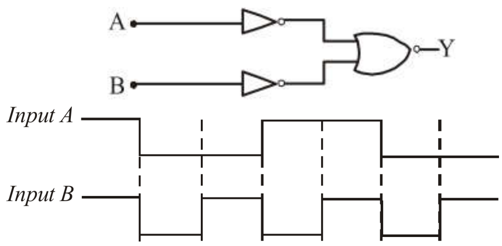
SOLUTION: . (a) The final Boolean expression of these gates is,



$$X = \overline{(\bar{A}.\bar{B})} = \left[\bar{\bar{A}} + \bar{\bar{B}}\right] = A + B \Rightarrow \text{OR gate}$$

It means OR gate is formed.

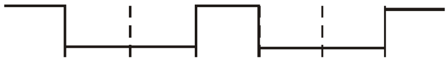
96. The logic circuit shown below has the input waveforms 'A' and 'B' as shown. Pick out the correct output waveform. [2009]

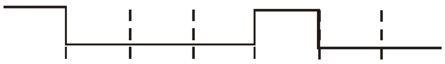


Output is

(a) 

(b) 

(c) 

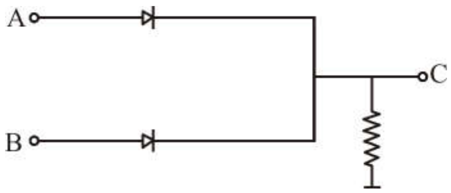
(d) 

SOLUTION: (d) The final Boolean expression

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$Y = (A + B) = A.B = A \times B$.Thus, it is an AND gate for which truth table is

97. In the circuit below, A and B represent two inputs and Crepresents the output. [2008]

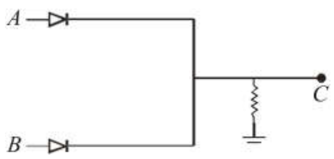


≡

The circuit represents

(a) NOR gate (b) AND gate (c) NAND gate (d) OR gate

SOLUTION: (d)



The truth table for the above circuit is:

A	B	C
1	1	1
1	0	1

0	1	1
0	0	0

when either A or B conducts, the gate conducts. It means $C = A + B$ which is for OR gate.

Communication System

Introduction:

Communication is an act of exchange of information between the sender and the receiver. Over decades, methods have been evolved t o develop languages, codes, signals etc to make communication effective. Communication through electrical signals has made things much simpler because they can be transmitted over extremely large distances in extremely short time as their speed is $3 \times 10^8 m/s$.

Modern communicaton has its roots in the 19th and 20th century in the work of scientists like J.C. Bose, F.B. Morse, G.Macroni and Alexander Graham Bell. The pace of development seems to have increased dramatically after the first half of the 20th century. We can hope to see many more accomplishments in the coming decades. The aim of this chapter is to introduce the concepts of communication, namely the mode of communication, the need for modulation, production and deduction of amplitude modulation.

Communication is basically of two types:

a) Point to point :- This takes place between a transmitter and a receiver. Telephonic conversion between two persons is a good example of it.

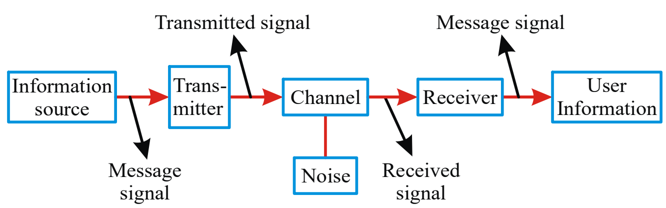
b) Broad cast mode :- Here, a large number of receivers receive the information from a single transmitter. Radio and television are good examples of broadcast mode.

Elements of Communication System

Basic units of a communication system.

a) Transmitter:

The part of the communication system, which sends out the information is called transmitter.



b) Transmission channel:

The medium or the link, which transfers message signal from the transmitter to the receiver of a communication system is called channel.

c) Receiver:

The part of the communication system, which picks up the information sent out by the transmitter is called **receiver.** The receiver consists of

Basic Terminology Used in Electronic Communication System

Some important terms needed to understand the basic elements of communication

a) Information : It is nothing but, the message to be conveyed. The message may be a symbol, code, group of words etc. Amount of information in message is measured in "bits"

b) Communication Channel : Physical medium through which signals propagate between transmitting and receiving stations is called communication channel.

Transmitter: Essential components of transmitter are as follows.

a)Transducer : Converts sound signals into electric signal. The device which converts a physical quantity (information) into electrical signal is known as **transducer**.

b)Modulator : Mixing of audio electric signal with high frequency radio wave.

c)Amplifier: Boosting the power of modulated signal.

d)Antenna : Signal is radiated in the space with the aid of an antenna.

Receiver: Basic componenets of receiver.

a) Pickup antenna: To pick the signal

b) **Demodulator:** To separate out the audio signal from the modulated signal

c) Amplifier: To boost up the weak audio signal

d) Transducer: To convert back audio signal in the form of electrical pulses into sound waves.

Message Signal:

Information converted in electrical form and suitable for transmission is called **signal**.

A signal is defined as a single-valued function of time (that conveys the information) and which, at every instant of time has a unique value.

Types of message signals

a) **Analog signal:** A signal, which is a continuous function of time (usually a sinusoidal function) is called analog signal**.**

b) **Digital signal:**A discrete signal (discontinuous function of time) which has only two levels is called digital signal.

Noise : This refers to undesired signals which disturb the transmission and processing of signals.

Attenuation :

It is the loss of strength of a signal during propagation in a medium.

Amplification :

It is the process of increasing the strength of the signal (amplitude) using an amplifier.

Range :

It is the maximum distance from a source upto which the signal is received with sufficient strength.

Repeaters :

These are the devices used to increase the range of communication system

Band width of signals (speech, T.V and digital data) :

Band width is the frequency range over which an equipment operates.

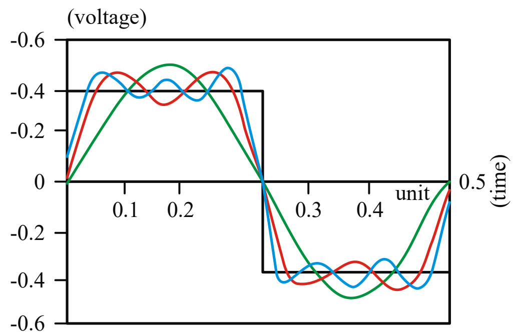
(or)

It is the portion of the spectrum occupied by the signal.

Bandwidth of signals :

In a communication system, the message signal may be voice, music, picture or data etc. Each of these signals has a spread of different range of frequencies. Hence, the type of communication system needed depends upon the band of frequencies involved. Speech signal requires the band width of 2800 Hz (3100 Hz to 300 Hz). For music, a bandwidth of about 20KHz is required (due to high frequency produced by musical instruments). The audible range of frequencies extends from 20Hz to 20KHz. Video signals require band width of 4.2 MHz for picture transmission. However, a band width of 6MHz is needed for T.V signals. (as it contains both voice and picture)

Digital signals are in the form of rectangular waves as shown in Fig. One can show that this rectangular wave can be decomposed into a superposition of sinuosidal waves of frequencies $f_0.\ 2f_0, 3f_0, 4f_0....nf_0$ where n is an integer extending to infinity and $f_0 = 1/T_0$. The fundamental (f_0), fundamental $(f_0)+$ second harmonic $(2f_0)$ and fundamental $(f_0)+$ second harmonic $(2f_0)+$ third harmonic $(3f_0)$, are shown in the same figure to illustrate this fact. It is clear that to reproduce the rectangular wave shape exactly we need to superimpose all the harmonics $f_0, 2f_0, 3f_0, 4f_0.....$ which implies an infinite bandwidth. However, for practical purposes, the contribution from higher harmonics can be neglected this limiting the bandwidth. As a result, received waves are a distorted version of the transmitted one. If the bandwidth is large enough to accommodate a few harmonics, the information is not lost and the rectangular signal is more or less recovered. This is so because the higher the harmonic. Less is its contribution to the wave form.



a) Rectangular wave

b) Fundamental (f_0)

c) Fundamental (f_0) + second harmonic $(2f_0)$

d) Fundamental (f_0) Second harmonic $(2f_0)$ + third harmonic $(3f_0)$

Bandwidth of transmission medium :

The most used transmission media are wire, free space, and fibre optic cable. Different transmission media offer different band width. Coaxial cable offers a band width of about 750 MHz. Radio wave communication through free space takes place over a wide range of frequencies from 100kHz-GHz.

Service	Frequency bands	Comments
Standard AM broadcast	540-1600 kHz	
FM broadcast	88-108 MHz	
Television	54-72 MHz	VHF(Very high frequencies)
	76-88 MHz	TV
	174-216 MHz	UHF(ultra high frequencies)
	420-890 MHz	TV
Cellular Mobile	896-901 MHz	Mobile to base station
	840-935 MHz	Base station to mobile
Satellite Communication	5.925-6.425 GHz	Uplink
	3.7 - 4.2 GHz	Downlink

↳ Optical communication using fibres is performed in the frequency range of 1 THz to 1000 THz (microwaves to ultraviolet).

↳ An optical fibre can offer a transmission band width in excess of 100 GHz.

Communication Channels:

The medium or the link, which transfers message signal from the transmitter to the receiver of a communication system is called **Communication channel**.

i) Space communication

i) Ground wave propagation

ii) Space wave propagation. (Tropospheric wave propagation. Surface wave propagation.)

iii) Sky wave propagation: A new dimension recently added to space communication is satellite communication.

2) Line communication

i) Two wire transmission line

ii) Coaxial cable

iii) Optical fibre cable

1) Space Communication

Propagation of EM waves in the atmosphere

The communication process utilizing the physical space around the earth is termed as space communication. Electromagnetic waves which are used in Radio, Television and other communication system are radio waves and microwaves.

The velocity of electromagnetic waves of different frequency in a medium is different. It is more for red light and less for violet light. Electromagnetic waves are of transverse nautre.

Earth's atmosphere

i) Earth's atmosphere is a gaseous envelope which surrounds the earth.

ii) Earth atmosphere mainly consists of nitrogen 78% , oxygen 21% along with a little protion of argon, carbon dioxide, water vapour, hydrocarbons, sulphur compounds and dust particles.

iii) The density of the atmospheric air goes on decreasing as we go up.

iv) The electrical conductivity of the atmospheric air increases as we go up.

v) The various regions of earth's atmosphere are:

Troposphere.

It extends upto a height of 12km

Stratosphere.

It extends from 12km to 50km. There is an ozone layer in stratosphere which mostly absorbs high energy radiations like ultraviolet radiations. etc. coming from outer space.

Mesosphere

. It extends from 50km to 80km.

Ionosphere.

i) It extends from 80 km to 400km.

ii) In this region, the temperature rises to some extent with height, hence it is called Thermosphere.

iii) The ionosphere which is composed of ionised matter (i.e. electrons and positive ions) plays an important role in space communication.

iv) The ionosphere is subdivided into four main layers as D, E, F_1 and F_2.

v) D- layer is at a virtual height of 80km from surface of earth and having electron density $\approx 10^9 m^{-3}$.

vi) The extent of ionisation of D layer depends upon the altitude of sun. This layer disappears at night. It reflects very low frequency (VLF) and low frequency (LF) electromagnetic waves, but absorbs medium frequency (MF) and high frequency (HF) electromagnetic waves to a certain degree,

(vii) E-layer is at a virtual height of 110km, from the surface of earth, having electron density $\approx 10^{11} m^{-3}$. The critical frequency* of this layer is about 4MHz. This layer helps to MF surface- w a v e propagation a little but reflects some high frequency waves in day time. It exists in day as well as in night time.

(viii) $F_1 -$ layer is at a virtual height of 180km from the surface of earth, having electron density $\approx 5 \times 10^{11} m^{-3}$. The critical frequency for this layer is 5 MHz. It reflects some of the high frequency waves but most of the high frequency waves pass through it and they get reflected from layer F_2 at night time.

(ix) F_2 layer is at a virtual height of about 300km in day time and about 350km in night time. The electron density of this layer is $\approx 8 \times 10^{11} m^{-3}$. The cirtical frequency of this layer is 8MHz in day time and 6MHz in night time. It reflects back the electromagnetic waves of frequency upto 30 MHz but cannot reflect back the electromagnetic waves of frequency 40MHz or more. It exists in day as well as night time

3. The electromagnetic waves of frequency ranging from a few kilo hertz to a few hundered mega hertz are called radio waves.

The various frequency ranges used in radio waves or micro wave communication system are as follows:

(a) Medium frequency band (M.F) 300 to 3000kHz.

(b) High frequency band (H.F) 3 to 30 MHz

(c) Very high frequency band (V.H.F) 30 to 300 MHz.

(d) Ultra high frequency band (U.H.F) 300 to 3000MHz

(e) Super high frequency band (S.H.F) 3000 to 30,000 MHz.

(f) Extra high frequency band (E.H.F) 30 to 300 GHz

The radio waves emitted from a transmitter antenna can reach the receiver antenna by the following mode of operation.

Kennely Heaviside Layer:

i) At 110 km above the surface of earth the concentration of electrons is very large. This layer is called Kennely Heaviside layer.

ii) The thickness of this layers is about of few km.

iii) Beyond this layer the electron concentration decreases upto 250 km

iv) From 250 km to 400 km, a layer of large concentration of electrons called Apple ton layer exists.

v) Above appleton layer, ie above Ionosphere the temperature is 927.6^{C}.

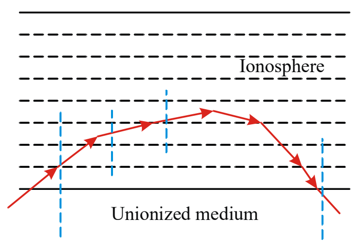
Ground wave propagation :

In this method, the radio waves are guided along the surface. The wave induces charges on the earth. These charges travel with the wave and this forms a current. Now the earth behaves like a leaky capacitor in carrying the induced current. The wave loses some energy, as energy is spent due to flow of charge through the earth's resistance. The wave also looses energy due to diffraction as it glides along the ground. The loss of energy increases as the frequency increases. Thus ground propagation is suitable upto 2MHz. As they loose energy they cannot go to long distances on the ground. Maximum range of the ground wave can be increased by increasing the power of the transmitter.

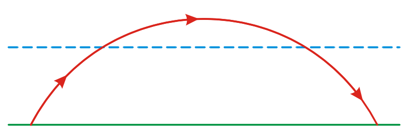
Sky wave propagation :

Above 2MHz and upto 30MHz, long distance communication takes place through ionosphere. The ionosphere reflects the radio waves back to the earth. This method is called sky wave propagation. It is used for shortwave broad casting services. Ionosphere is a thick blanket of 65 km to 400 km above the earth's surface. UV rays and other higher energy radiation coming from space results in the ionization of air molecules. The ionosphere is further divided into serval layers as shown in table below. It should be understood that degree of ionization changes with height. This is because the density of atmosphere decreases with height. At great heights, the radiation is intense, but the molecules available are few. On the other hand, near the earth's surface the molecular concentration is high but the intensity of radiation is low and thus again the ionization is low. Logically, the peak of ionization density occurs at some intermediate heights. The ionosphere acts as a mirror (reflector) for frequencies of 3-30 MHz. Electromagnetic waves of frequences greater than 30 MHz pass through the atmosphere and skip.

The process of bending of EM waves is similar to total internal reflection in optics. The bending of waves can be easily explained on the basis of variation of refractive index of the ionosphere with change in electron density. Suppose that a radio wave enters the ionosphere from the underlying unionized medium. Since the refractive index of ionosphere decreases from D layer to F_2 layer, consequently, the incident ray will move away from the normal drawn at the point of incidence following the ordinary laws of refraction



During the propagation in ionosphere the angle of refraction gradually increases and the ray goes on bending more and more till at some point, the angle of refraction becomes 90^0 and the wave travels parallel to the earth surface. This point is called point of reflection. Then the ray tends to move in the down ward direction and comes back to earth because of symmetry. Super high frequency (SHF) waves propagate as sky waves taking reflection at satellite.



The sky wave propagation can cover a very long distance and so round the globe communication is possible.
(c) The sky waves being electromagnetic in nature, changes the dielectric constant and refractive index of the ionosphere. The effective refractive index of the ionosphere is

$$n_{eff} = n_0\left[1 - \frac{Ne^2}{\varepsilon_0 m\,\omega^2}\right]^{1/2} = n_0\left[1 - \frac{80.5\,N}{f^2}\right]^{1/2}$$

Where n_0 = refractive index of free space, N = electron density of ionosphere, ε_0 = permittivity of free space, e = charge on electron, m = mass of electron w = angular frequency of EM wave.
(d) As we go deep into the ionosphere, N increases so n_{eff} decreases. The refractions or bending of the beam will continue and finally it reflects back.
(e) The highest frequency of radio wave, which gets reflected to earth by the ionosphere after having been sent straight to it is

Critical frequency (f_c)

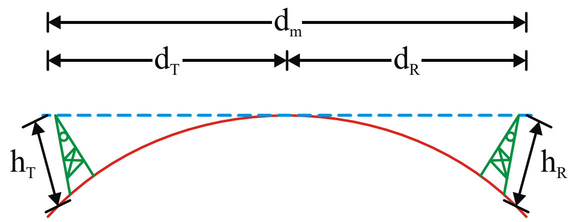
If maximum electron density of the ionosphere is N_{max} per m^3, then $f_c \approx 9(N_{max})^{1/2}$. Above f_c, a wave will penetrate the ionosphere and is not reflected by it.
(f): The highest frequency of radio waves which when sent at some angle of incidence, towards the ionosphere, get reflected and return to the earth is **Maximum usable**

frequency (MUF) $MUF = \frac{f_c}{\cos\theta}$

(g) The smallest distance from a transmitter along the earth's surface at which a sky wave of a fixed frequency but more than f_c is sent back to the earth is **Skip distance.**

(h) The fluctuation in the strength of a signal at a receiver due to interference of two waves is **fading**. Fading is more at high frequencies. It results into errors in data transmission and retrieval.

Space wave propagation : This method is used for line-of-sight [LOS] communication and also for satellite communication. At frequencies above 40MHz, communication is mainly by LOS method. At such frequencies, relatively smaller antenna can be erected above the ground. Because of LOS propagation, the direct waves get blocked, at some point due to the curvature of the earth as shown in the figure.



For the signal to be received beyond the horizon, the receiving antenna must be high enough to intercept the LOS waves. If the transmitting antenna is at a height h_T then it can be shown that the distance to the horizon d_T is given by $d_T = \sqrt{2Rh_T}$ where 'R' is the radius of earth. Similarly if the receiving antenna is at a height h_R, the distance to the horizon d_R is $d_R = \sqrt{2Rh_R}$

$\therefore$ The maximum distance d_M between the two antennas is $d_M = \sqrt{2Rh_T} + \sqrt{2RH_R}$ where

R = Radius of the earth.

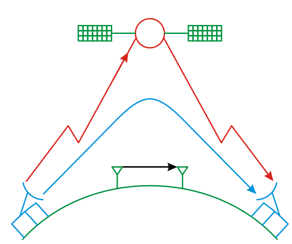
h_T = height of the transmitting antenna and

h_R = height of the receiving antenna.

If the Population density around the tower is given, the number of persons covered by the transmitting tower = (Area covered by the tower) $\times$ Population density.

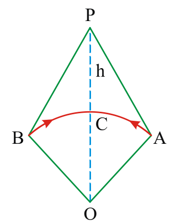
$\therefore$ No. of persons $= \pi d^2 \times$ covered population density (Here d = radius of the area covered by single transmitting tower of height h_T)

Television broadcast, microwave and satellite communications are a few examples of communication systems that use space wave propagation. The figure below illustrates the various modes of wave propagation.



Range of TV transmission :

As the frequency range of TV signals is 100-200 MHz, such signal transmission via ground waves is not possible. In such situations, we use line of sight transmission.



Let CP be the TV tower on the earth's surface. It's antenna is at P. Let PC = h. When TV broadcast is made, the signal can reach the earth upto A to B. There will be no reception of the signal beyond A and B. Arc length CA and CB is the range of TV transmission. If O is the centre of the earth, OA = OB = R is the radius of the earth, from right angled triangle OAP

$$OP^2 = OA^2 + PA^2$$

$$(h+R)^2 = R^2 + PA^2$$

$$PA = PB = d$$

$$(h+R)^2 = R^2 + d^2$$

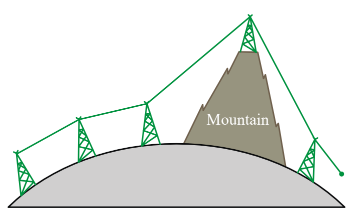
$$h^2 + R^2 + 2Rh = R^2 + d^2$$

As $h << R$ we can ignore h^2

$d^2 = 2Rh$ and $d = \sqrt{2Rh}$

Range of TV transmission depends upon the height of the transmission antenna. Broadcasts are made from tall transmitting antenna.

↳ A **repeater** is a combination of a receiver and a transmitter. A repeater, picks up the signal from the transmitter, amplifies and retransmits it to the receiver sometimes with a change in carrier frequency. Repeaters are used to extend the range of a communication system as shown in figure. A communication satellite is essentially a repeater station in space. Use of repeater station to increase the range of communication



These problems are solved by using geostationary satellite as a communication satellite.

Satellite Communication: Long distance communication beyond 10 to 20 MHz was not possible before 1960 because all the three modes of communication discussed above failed (ground waves due to conduction losses, space wave due to limited line of sight and sky wave due to the penetration of the ionosphere by the high frequencies beyond f_c).

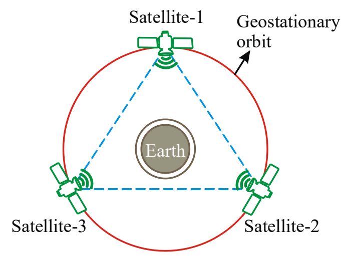
Ionosphere behaves as a rarer medium by which carrier wave is reflected back if its frequency $f\left(\le f_c\right)$

where f_c is called a "critical frequency" and is given by $f_c = f_0\left(N_{max}\right)^{\frac{1}{2}}$

[N_{max} = maximum electron density.]

Satellite communication made this possible. The basic principle of satellite communication is shown in figure.

A communication satellite is a spacecraft placed in an orbit around the earth. The frequencies used in satellite communication lie in UHF/ microwave regions. These waves can cross the ionosphere and reach the satellite.



- A geostationary satellite has the same time period of revolution of earth. It locates at the height of 36000 km above the earth's surface (well above the ionosphere).
- A communication satellite is a spacecraft placed in an orbit around the earth which carriers a transmitting and receiving equipment called radio transponder. It amplifies the microwave signals emitted by the transmitter from the surface of earth and send to the receiving station on earth.
- The transmitted signal is UP-LINKED and received by the satellite station which DOWN- LINKS it with the ground station through its transmitter.
- The up-link and down-link frequencies are kept different (both frequencies being in the regions of UHF/ microwave).
- At least three geo-stationary satellites are required which are 120° apart from each other to have the communication link over the entire globe of earth.
- Satellite technology is very useful in collecting information about various factors of the atmosphere which governs the weather and climatic conditions.
- The satellite communication can be used for establishing mobile communication with great use the communication satellites are now being used in Global Positioning System (GPS). The ordinary users can find their positions within accuracy of 100m.
- There are two types of satellites used for long distance transmission.
- (i) Passive satellite: It acts as reflector only for the signals transmitted from earth. Moon the natural satellite of earth is a passive satellite.
- (ii) Active satellite: It carries all the equipment used for receiving signals sent from the earth, processing them and then re-transmitting them to the earth. Now a days active satellites are in use.
- The Indian communication satellites INSAT-2B and INSAT-2C are positioned in such away in the outer space that they are accessible from any place in India.

Remote Sensing and Application of Satellite Communication.

Remote sensing is the technique to obtain information about an object by observing it from a distance and without coming to actual contact with it.

- There are two types of remote sensing instruments: active and passive. Active instruments provide their own energy to illuminate the object of interest, as radar does. They send an energy pulse to the object and then receive and process the pulse reflected from the object. Passive instruments sense only radiations emitted by the object or solar radiation reflected from the object.
- The remote sensing is done through a satellite. The satellite used in remote sensing should move in an orbit around the earth in such a way that it always passes over the particular area of the earth at the same local time.

  The orbit of such a satellite is known as sun-synchronous orbit. A remote sensing orbit can be circular polar orbit or in highly inclined elliptical orbit.
- A remote sensing satellite takes, photographs of a particular region which nearly the same illumination every time it passes through that region.

- The most useful remote sensing technology is that it makes possible the repetitive surveys of vast areas in a very short time, even if the areas are inaccessible.
- Space based remote sensing is a new technology. It has high potential for applications in nearly all aspects of resource management.
- The Indian remote sensing satellites are
  IRS-1A, IRS-1B, and IRS-1C.
- Remote sensing is applied in (i) Meteorology
  (ii) Climatology (iii) Oceanography
  (iv) Archaeology, geological surveys. (v) Water resource surveys, (vi) Urban land use surveys. (vii) Agriculture and forestry and natural disaster.
  (viii) To detect movements of enemy army. (ix)To locate the place where underground nuclear explosion has carried out.

2.Line Communication

Line communication means interconnection of two points with the help of wires for exchange of information. There are three principal types

(i)Two Wire Transmission Line

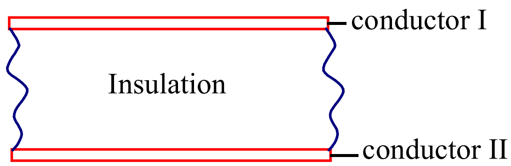
(ii) Coaxial wire lines (coaxial cables)

(iii) optical fibers

Two Wire Transmission Line

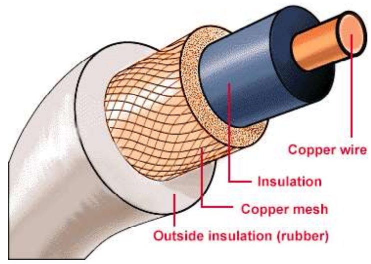
The most commonly used two wire lines are: Parallel wire, twisted pair wires and co-axial cable.

1) **Parallel wire line:** In a two wire transmission line, two metallic wires (may be hard or flexible) are arranged parallel to each other inside a protective insulation coating . Commonly used to connect an antenna with TV receiver. Such wires can suffer from interferences and losses.



2) **Twisted pair wire:** It consists of two insulated copper wires twisted around each other at regular intervals to minimize electrical interference (to connect telephone systems). Used to connect telephone systems. It works well up to small distances. They cannot transmit signals over very large distances. They transmit both, the analog and digital signals. They are easy to install and cost effective.

Coaxial wire lines: It consists of a central copper wire (which transmits surrounded by a PVC insulation over which a sleeve of copper mesh (outer conductor) is placed. The outer conductor is normally connected to ground and thus it provides an electrical shield to the signals carried by the central conductor. The outer conductor is externally covered with a polymer jacket for protection.



a) Co-axial line wires can be used for microwaves and ultra-high frequency waves.
b) The communication through co-axial lines is more efficient than through a twisted pair wire lines.
c) Co-axial cables can be gas filled also. To reduce flash over between the conductor handling high power, N_2-gas is used in the cable.

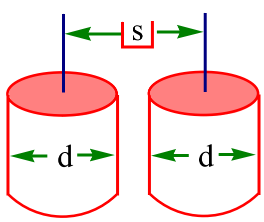
Impedance of Line :

1) Each portion of the transmission line can be considered as a small inductor, resistor and capacitor. As a result each length of transmission line has characteristic impedance.
2) In case of co-axial cable, the dielectric can be r epresented by a shunt resistance G.
3) When co-axial cable is used to transmit a radio frequency signal, X_L and X_C are large as compared to R and G respectively. Hence R and G can be neglected.
4) In co-axial cable, R is zero, so no loss of energy and hence no attenuation of frequency signal occurs when transmitted along it. That's why co-axial cables are specially used in cable TV network.

Characteristic impedance (Z_0) :

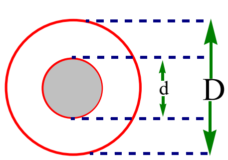
It is defined as the impedance measured at the input of a line of infinite length.

a) For parallel line $Z_0 = \frac{276}{\sqrt{k}} \log \frac{2s}{d}$



d = Diameter of each wire
s = Separation between the two wires
k = Dielectric constant of the insulating medium

b) For co-axial line wire $Z_0 = \frac{138}{\sqrt{k}} \log \frac{D}{d}$



d = Diameter of inner conductor
D = Diameter of outer conductor

c) At radio frequency $Z_0 = \sqrt{\frac{L}{C}}$

The usual range of characteristic impedance for parallel wire lines is 150W to 600 W and for co-axial wire it is 40 W to 150 W.

Velocity factor of a line (v. f.) : It is the ratio of reduction of speed of light in the dielectric of the cable

$$v.f. = \frac{v}{c} = \frac{\text{Speed of light in medium}}{\text{Speed of light in vacuum}} = \frac{1}{\sqrt{K}}$$

For a line v.f. is generally of the order of 0.6 to 0.9.

Telephone Links

1) A telephone (the most common means of communication) link can be established with the help of co-axial cables, ground waves, sky waves, microwaves or optical fiber cables.
2) Simultaneous transmission of a number of messages over a single channel without their interfering with one another is called multiplexing.
3) Twisted pair wire lines provide a band width of 2 MHz, while co-axial cable provides a band width of 20 MHz. For further increase in band width, we use (i) microwave link (ii) communication satellite link.

Optical Communication

The use of optical carrier waves for transmission of information from one place to another is called optical communication.

The information carrying capacity $\propto$ bandwidth $\propto$ frequency of carrier wave. Because of high frequency (10^{12} Hz to 10^{16} Hz) optical communication is better than others. (radio and microwave frequencies, 10^{6} Hz – 10^{11} Hz).

Basic optical communication link is a point to point link having transmitter at one end, receiver at the other end and consists of three components namely

1) Optical source and modulator
2) Optical signal detector or photodetector
3) Optical fibre cable through which optical signal is transmitted.

Optical sources for communication links

Light emitting diodes (LED) and diode lasers are preferred for optical source. LEDs are used for small distance transmission while diode laser is used for very large distance transmission.

For optical communication, light is to be **modulated** with the information signal. The frequency and intensity of light is sensitive to temparature changes,which is to be avoided. So suitable arrangement isrequired to obtain **thermal stability.**

Optical signal detector or photo detector:

The optical signal reaching the receiving end has to be detected by a detector which converts light intoelectrical signals, So that the transmitted information may be decoded.

The optical detector should have

i) size compatible with the fibre

ii) High sensitivity at the desired optical wavelength

iii) High response for fast speed data transmission/reception.

Semiconductor based photo-electors are used because they fullfill the above criteria

Modulation and Its Necessity:

Message signals are also called base band signals. Which essentially designate the band of frequencies representing the original signal, as delivered by the source of information. No signal, is a single frequency sinusoid, but it spreads over a range of frequencies called the signal bandwidth to transmit an electric signal frequency less than 20 kHz) over a long distance directly. It is clear that low frequency waves, can not travel long distances. Hence, to transmit low frequency wave over long distance, we take the help of high frequency waves called carrier wave. The low frequency wave is superposed over high frequency carrier wave. This process is called the modulation. The low frequency wave is called the modulating wave and the high frequency wave is called the carrier wave, and the resultant wave is called modulated wave. In this section we will discuss in detail about modulation. What is it? What is the need of modulation or how modulation is done etc.

No signal in general, is a single frequency but it spreads over a range of frequencies called the signal bandwidth. Suppose we with to transmit an electronic signal in the audio-frequency (20Hz-20kHz) range over a long distance. Can we do it? No it cannot because of the following problems.

Size of antenna : For transmitting a signal we need an antenna. This antenna should have a size comparable to the wavelength of the signal. For an electromagnetic wave of frequency 20kHz, wave length is 15km. Obviously such a long antenna is not possible and hence direct transmission of such signal is not practical.

The linear size of the antenna must be the order of the wave length and for effective transmission its length

must be $h = \frac{\lambda}{4}$

so that antenna properly senses the time variation of the signal.

Example1: For an electromagnetic wave of

f = 20 kHz, λ = 15 km Obviously, such a long antenna is not possible to construct and operate.

Hence direct transmission of such baseband signals is not practical.

Example2: If f = 1 MHz, then λ = 300 m

h = 75 m

Therefore, there is a need of translating the information contained in our original low frequency baseband signal into high or radio frequencies before transmission.

Additional Information:

a) The distance between transmitting antenna and the horizon, $D_t = \sqrt{2Rh_t}$.

Where h_t = height of transmitting antenna R = Radius of the earth

b) The distance between receiving antenna and the horizon, $D_r = \sqrt{2Rh_r}$.

Where h_r = height of receiving antenna

c) The maximum distance between the transmitting antenna and receiving antenna D_m.

$$D_m = D_r + D_t$$

$$D_m = \sqrt{2\,Rh_r} + \sqrt{2\,Rh_t}.$$

Where R is the radius of earth.

$h_r > h_t$ so then the receiving antenna intercepts the line of sight waves.

Single antenna

d) The radius "d" of the area covered by a single transmitting tower of height h is given by $d = \sqrt{2R_e h}$.

Where R_e is the radius of the Earth.

e) If the Population density around the tower is given, the number of persons covered by the tower is

= (Area covered by the tower) x Population density No. of persons covered = πd^2 x Population- density.

Effective power radiated by an antenna

Power radiated by an antenna is proportional to $\left(\frac{l}{\lambda^2}\right)$. Where l is length of the antenna .

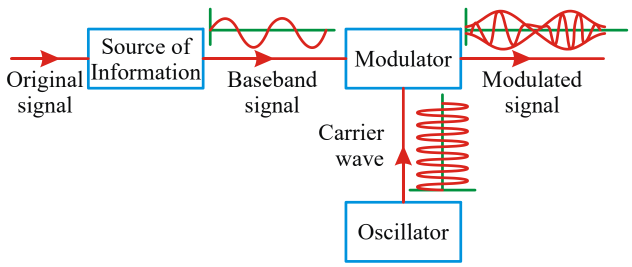
For a good transmissmission,high powers are required, hence low wavelength i.e high frequency transmissions are needed.

Mixing up of signals from different trans-mitters

Suppose many people are talking at the same time or many transmitters are transmitting baseband information signals simultaneously. All these signals will get mixed up and there is no simple way to distinguish between them. This points out towards a possible solution by using communication at high frequencies and alloting a

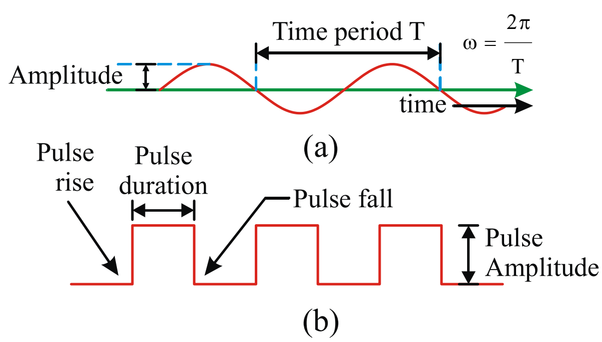
band of frequencies to each mesage signal for its transmission.
In doing so, we take the help of a high frequency signal, known as the carrier wave, and a process known as **modulation** which attaches information to it.

Modulation: The process of superimposing information contained in the low frequency , message signal on a high frequency carrier wave, near transmitter is known as **modulation**.



Types of Modulation:

The carrier wave may be continuous (sinusoidal) or in the form of pulses as shown in figure(2).



Therefore depending upon the specific characteristicof carrier wave which is being varied in accordance with the message signal, modulation can basically be differentiated as
i) continuous wave modulation; and
ii) pulse wave modulation.

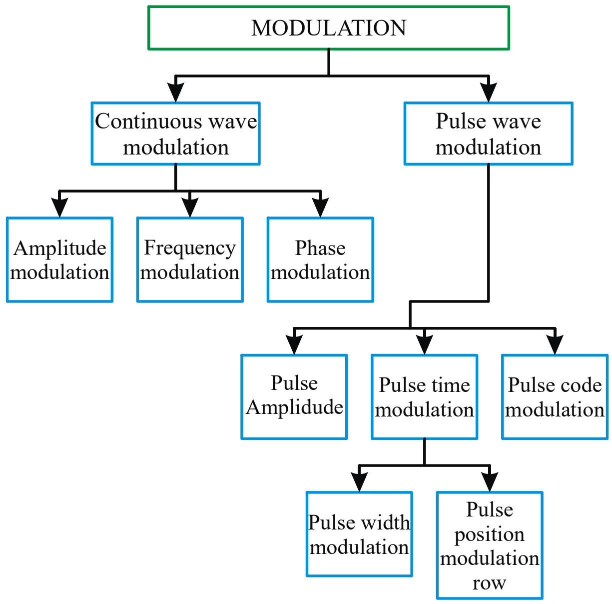
According to the type of modulation

For sinusodial continuous carrier waves

i) Amplitude Modulation (AM)

ii) Frequency Modulation (FM)

iii) Phase Modulation



For pulsed carrier waves

i) Pulse Amplitude Modulation (PAM)

ii) Pulse Time Modulation (PTM)

a) Pulse Position Modulation (PPM)

b) Pulse Width Modulation (PWM) or Pulse Duration Modulation (PDM)

iii) Pulse Code Modulation (PCM)

I) Continuous Wave Modulation

Eqution representing sinusoidal carrier wave can

$c(t) = A_c \sin(\omega_c t + \phi)$ ----(1)

where c(t) is the signal strength (voltage or current),

A_c is the amplitude

$(\omega_c t + \phi)$ is called argument of Phase angle of the carrier wave

$\omega_c (= 2\pi f_c)$ is the angular frequency

ϕ is the initial phase of the carrier wave. During the process of modulation, any of the two parameters, viz amplitude or phase angle, of the carrier wave can be controlled by the message or information signal. This results in two **types of modulations :**

i) Amplitude modulation (AM)

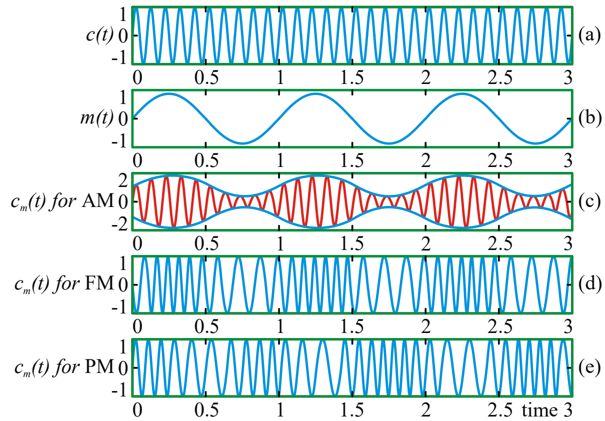
ii) Angle modulation

Angle modulation again can be of two types. They are

i) Frequency modulation (FM)

ii) Phase modulation (PM)

As shown in figure.



II) Pulse Wave Modulation.

The significant characteristics of a pulse are: i) Pulse amplitude
ii) Pulse duration or pulse width
iii) pulse position (denoting the time of rise or fall of the pulse amplitude) as shown in figure (3).

Types of pulse modulation:

a) pulse amplitude modulation (PAM),
b) pulse duration modulation (PDM) or pulse width modulation (PWM)
c) pulse position modulation (PPM).

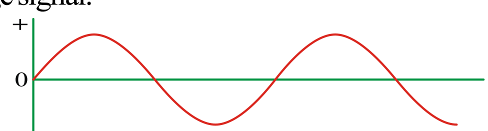
I) Continuous Wave Modulation:

1) Amplitude Modulation:

The method in which the amplitude of carrier is varied in accordance with the modulating signal keeping the frequency and phase of carrier wave constant is called amplitude modulation (AM).

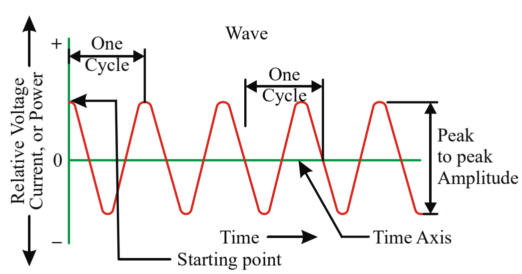
Here we explain amplitude modulation process using a sinusoidal signal as the modulating signal.

Let $m(t) = E_m \sin \omega_m t$ represent the message or the modulating or base band signal. Here $\omega_m = 2\pi f_m$ is the angular frequency of the message signal.



A-F Signal

$c(t) = E_c \sin \omega_c t$ represent carrier wave. Here $\omega_c = 2\pi f_c$ is the angular frequncy of the carrier signal



The modulated signal $c_m(t)$ can be written as

$$c_m(t) = (E_c + E_m \sin \omega_m t) \sin \omega_c t \text{ ----(1)}$$

$$c_m(t) = A_c\left(1 + \frac{E_m}{E_c} \sin \omega_m t\right) \sin \omega_c t \text{ -----(2)}$$

$$c_m(t) = E_c \sin\omega_c t + \mu E_c \sin\omega_m t \sin\omega_c t \text{ --(3)}$$

Using the trignometric relation

$\sin A \sin B = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$, we can write $c_m(t)$ of equation (3) as

$$c_m(t) = A_c \sin\omega_c t + \frac{\mu A_c}{2}\cos(\omega_c - \omega_m)t - \frac{\mu A_c}{2}\cos(\omega_c + \omega_m)t \text{ -----(4)}$$ Here

$\omega_c - \omega_m = 2\pi(f_c - f_m)$ = Lower side band frequency (LSB)

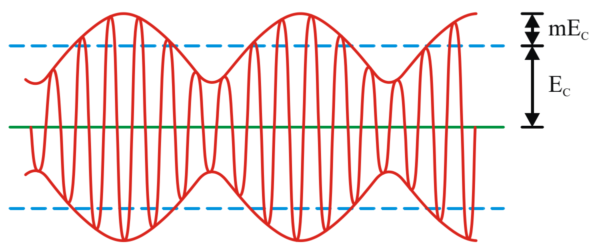
$\omega_c + \omega_m = 2\pi(f_c + f_m)$ =Upper side band frequency (USB)

Here (m or $\mu = E_m / E_c$) is the modulation index; (or) modulating factor.

In practice, μ is kept ≤ 1 to avoid distorition.

Depth of modulation $= \frac{A_m}{A_C} \times 100 = \mu \times 100$

Depth of modulation interms of E_{max} and E_{min}



A.M. Wave

$$E_{max} = E_c + E_m = E_c(1+m)$$

$$E_{min} = E_c - E_m = E_c(1-m)$$

$$\frac{E_{max}}{E_{min}} = \frac{E_c + E_m}{E_c - E_m} = \frac{E_c(1+m)}{E_c(1-m)}$$

$$\boxed{\frac{E_{max} - E_{min}}{E_{max} + E_{min}} = m_a}$$

The Band width of AM wave is " $2f_m$ "

The modulated signal now consists of the carrier wave of frequency ω_c plus two sinusoidal waves each with a frequency slightly different from ω_c, known as side bands. The frequency spectrum of the amplitude modulated signal is shown in

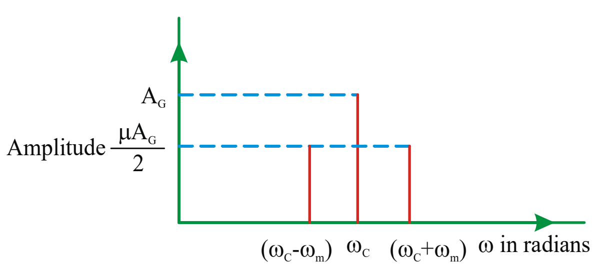
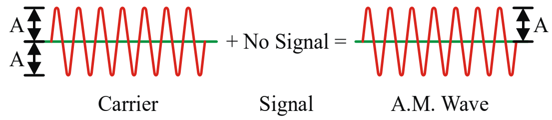


figure (5) A plot or amplitude versus ω for an amplitude modulated signal

As long as the broadcast frequencies (carrier waves) are sufficiently spaced out so that sidebands do not overlap, different stations can operate without interfering with each other.

Special cases of Amplitude modulation:

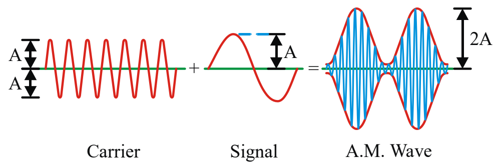
CaseI: In the absence of signal.



Modulation factor $m_a = \frac{O}{A} \times 100 = 0\%$

Case-II: When the signal amplitude is equal to CW wave.

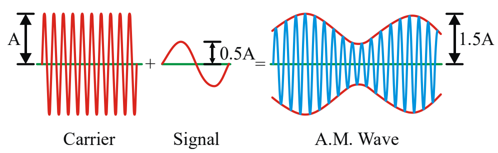
Amplitude varies from 2A to zero.



$$\frac{\text{Amplitude change in carrier wave}}{\text{Amplitude of CW}} = \frac{2A - A}{A} = 100\%$$

Case-III: When the amplitude of the signal is half of that of CW.

Amplitude of CW changes from A to $\left(A + \frac{A}{2}\right) = 1.5\,A$

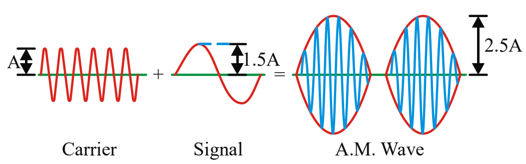


Modulation factor $= \frac{0.5\,A}{A} = 0.5$

$= 50\ \%$

Case-IV: When the amplitude of signal is 1.5 times that of the CW.

Amplitude of the modulated wave changes from 2.5 A to A



Modulation factor $m_a = \frac{2.5A - A}{A} = 1.5 = 150\ \%.$

In this case the quality of signal is lost

Note: A carrier wave is modulated by a number of sine waves with modulation indices m_1, m_2 and m_3. The total modulation index of the wave is

$$m = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

Power out put in AM wave

$P_t = P_c + P_s$

where P_t is power transmitted

P_c is power of carrier wave

P_s is total power of side bands

The equation of a carrier wave $Y_c = A_c \sin(w_c t + \phi)$

Power of carrier wave $P_c = \frac{[A_{rms}]^2}{R} = \frac{\left(\frac{A_c}{\sqrt{2}}\right)^2}{R} = \frac{A_c^2}{2R}$

The power of side bands = The power of lower side band + the power of upper side band

$$P_s = \frac{\left[\frac{(\mu A_c)}{2}/\sqrt{2}\right]^2}{R} + \frac{\left(\mu\frac{A_c}{2}/\sqrt{2}\right)^2}{R}$$

$$= \frac{\mu^2 A_c^2}{4R} = \frac{\mu^2}{2} P_c$$

$$P_t = P_c + P_s \quad = P_c + \frac{\mu^2}{2} P_c$$

$$= P_c\left(1 + \frac{\mu^2}{2}\right)$$

$$\therefore P_t = P_c\left(1 + \frac{\mu^2}{2}\right) ----(1)$$

$$\Rightarrow \frac{P_t}{P_c} = 1 + \frac{\mu^2}{2} ----(2)$$

$$\Rightarrow \left(\frac{i_t}{i_c}\right)^2 = 1 + \frac{\mu^2}{2} ----(3)\left[\because P \,\alpha\, i^2\right]$$

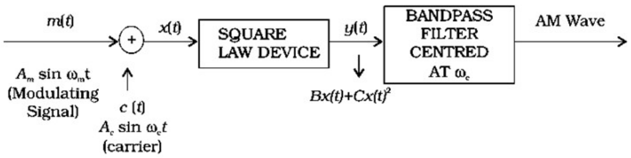
Example: If the modulation factor is 1 ie 100 % modulation then the useful power is $\frac{1}{3}$ of the total power radiated. The remaining 2/3 power is contained by carrier wave

$$\boxed{\frac{P_S}{P_T} = \frac{m_a^2}{2 + m_a^2} = \frac{1}{3}} \text{ and } \boxed{\frac{P_c}{P_T} = \frac{2}{2 + m_a^2} = \frac{2}{3}}$$

Transmission Efficiency $\eta = \frac{m^2}{2 + m^2}$

Production of Amplitude Modulated Wave

Amplitude modulation can be produced by a variety of methods. A conceptually simple method is shown in the block diagram of Fig.



Here the modulating signal $A_m \sin \omega_m t$ is added to the carrier signal $A_c \sin \omega_c t$ to produce signal $x(t) = A_m \sin \omega_m t + A_c \sin \omega_c t$

This signal is passed through a square law device which is a non linear device that can give tha output $y(t) = Bx(t) + Cx^2(t)$ where B and C are constants

Thus, $y(t) = BA_m \sin \omega_m t + BA_c \sin \omega_c t$

$+C\left[A_m^2 \sin^2 \omega_m t + A_c^2 \sin^2 \omega_c t + 2A_m A_c \sin \omega_m t \sin \omega_c t\right]$ -(6)

$= BA_m \sin \omega_m t + BA_c \sin \omega_c t + \frac{CA_m^2}{2} + A_c^2 - \frac{CA_m^2}{2}\cos 2\omega_m t - \frac{CA_c^2}{2}\cos 2\omega_c t$

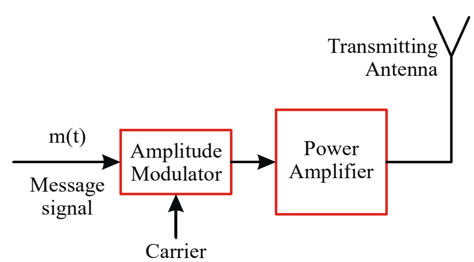
$+CA_m A_c \cos(\omega_c - \omega_m)t - CA_m A_c \cos(\omega_c + \omega_m)t$ -(7)

Where the trignometric relations $\sin^2 A = (1 - \cos 2A)/2$ and the relation for sinA sinB mentioned earilier are used.

In equation, there is a dc term $c/2\left(A_m^2 + A_c^2\right)$ and sinusoids of frequencies

$\omega_m, 2\omega_m, \omega_c, 2\omega_c, \omega_c - \omega_m$ and $\omega_c + \omega_m$. As shown in figure this signal is passed through a band pass filter which rejects dc and the sinusoids of frequencies $\omega_m, 2\omega_m$ and $2\omega_c$ and retains the frequencies $\omega_c, \omega_c - \omega_m$ and $\omega_c + \omega_m$. The output of the band pass filter therefore is of the same form as equation(4) and is therefore an AM wave.

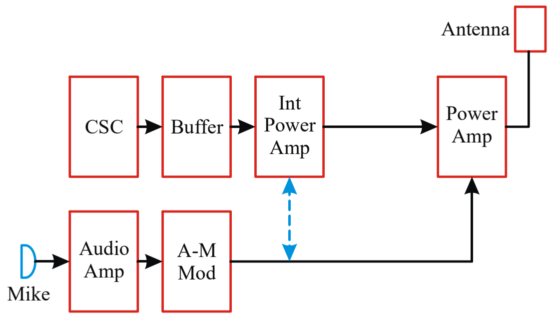
It is to be mentioned that the modulated signal cannot be transmitted as such. The modulator is to be followed by a power amplifier which provides the necessary power and then the modulated signal is fed to an antenna of appropriate size for radiation as shown in figure(7).



Figure(7)

A-M Transmitter.

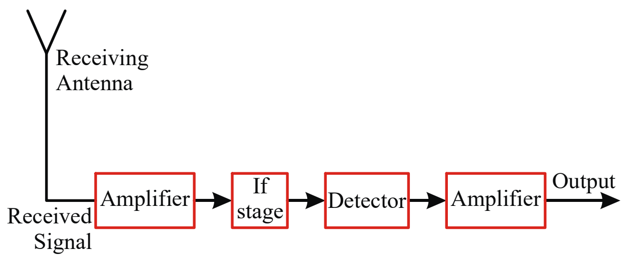
In the block diagram of the AM transmitter the r-f section consists of an oscillator feeding a buffer, which in turn feeds a system of frequency multipliers and/or intermediate power amplifiers. If frequency multiplication is unneccessary, the buffer feeds directly into the intermediate power amplifiers which, in turn, drive the final power amplifier. The input to the antenna is taken from the final power amplifier.



Amplitude-Modulated Transmitter

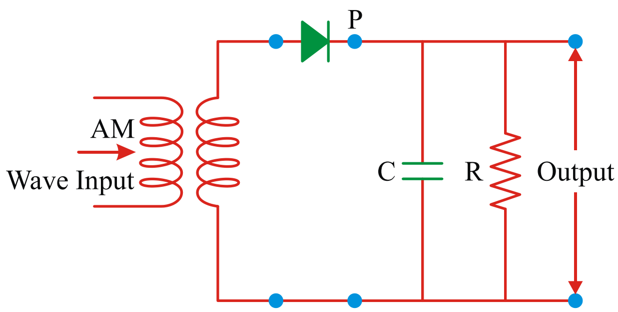
Detection of amplitude modulated wave

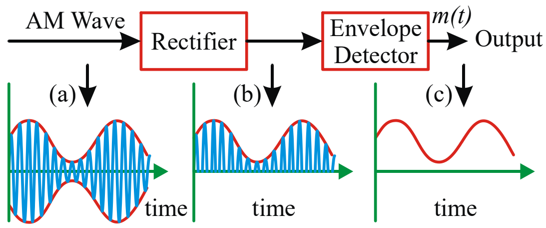
The transmitted message gets attenuated in propagating through the channel. The receiving antenna is therefore to be followed by an amplifier and a detector. In addition, to facilitate further processing, the carrier frequency is usually changed to a lower frequency by what is called an intermediate frequency (IF) stage preceding the detection. The detected signal may not be strong enough to be made use of and hence is required to be amplified. A block diagram of a typical receiver is shown in figure.



Simple demodulator circuit :

1) A diode can be used to detect or demodulate an amplitude modulated (AM) wave.
2) A diode basically acts as a rectifier i.e. it reduces the modulated carrier wave into positive envelope only.
3) The AM wave input is shown in figure. It appears at the output of the diode across PQ as a rectified wave (since a diode conducts only in the positive half cycle). This rectified wave after passing through the RC network does not contain the radio frequency carrier component. Instead, it has only the envelope of the modulated wave.





AM input wave | Rectified wave | Output (without RF component)

The capacitor connected in parallel with resistance R provides very low impedance at the carrier frequency and a much higher impedance at the modulating frequency. As a result capacitor effectively shorts or filters out the carrier, there by leaving the original modulating signal

In the actual circuit the value of RC (The time constant, t=RC) is chosen such that $\frac{1}{f_c} << RC$; where f_C = frequency of carrier signal.

Distortion in diode detectors: There are two types of distortions in diode detectors. Namely

a) Negative peak clipping

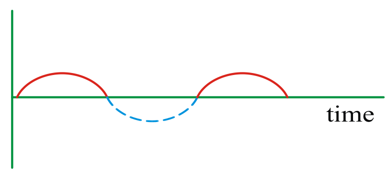


Figure shows the negative peak missing in the output message We know that

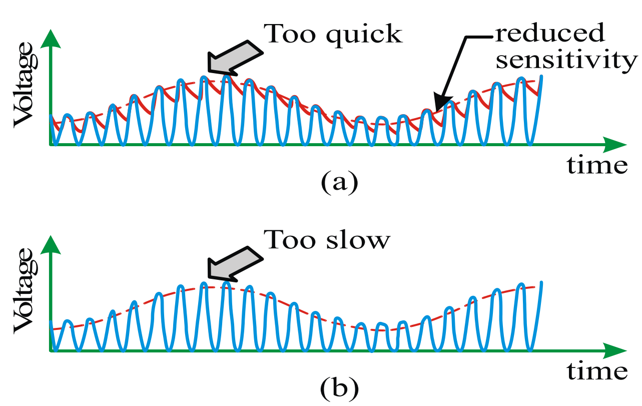
Modulation Index(m)=$\frac{E_m}{E_C} = \frac{I_m}{I_c}$ where $I_m = \frac{E_m}{Z_m}$ and $I_c = \frac{E_c}{R_c}$

Here z_m is audio load resistance of diode and R_c is dc diode resistance while $z_m < R_c$

hence $I_m > I_c$. This makes the modulation index in the demodulated wave higher than it was in modulated wave applied at the detector. In turn there is an increase in the chance of over modulation for modulation index nearer to 100% Due to this over modulation there is a negative clipping of the detector wave.

b) Diagonal clipping:

1. Diode ac load may no longer be purely resistive, it can contain reactive component.
2. At high modulation depths current will be changing so fast that the time constant of the load may be too slow to follow the changes. As a result current will decay exponentially. Hence output voltage follows the discharge law of the CR circuit



3. Condition necessary for avoiding distortion of this type is as follows.

$$I_m = \frac{E_m}{Z_m} = \frac{mE_c}{Z_m} \text{ and } I_c = \frac{E_c}{R_c}$$

$$m_d = \frac{I_m}{I_c} = \frac{mE_c/Z_m}{E_c/R_c} = \frac{mR_c}{Z_m}$$

Maximum value of $m_{dMaximum} = 1$

So maximum permissible transmitted modulation index will be $M_{\max imum} = m_d \frac{Z_m}{R_c} = 1 \times \frac{Z_m}{R_c}$

Limitation of amplitude modulation

i) Noisy reception ii) Low efficiency
iii) Small operating range iv) Poor audio quality

2) Angle modulation:

The angle of the carrier wave is varied according to the base band signal while the amplitude is maintained constant. This method provides better discrimination against NOISE and INTERFERENCE then the Amplitude modulation

There are two ways of varying the angle of the carrier.

$\theta_i(t)$ is the angle of moduilated sinusoidal carrier assumed to be a function of the message signal.

Equation of the angle modulated wave is $S(t) = A_c \cos(\theta_i(t))$

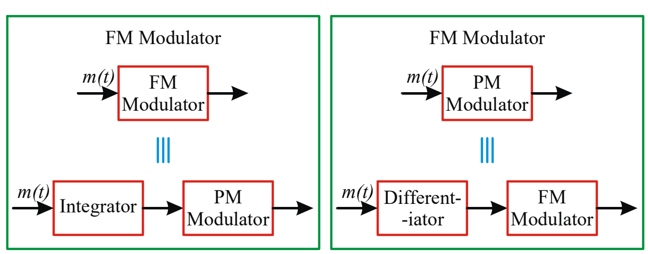
where A_c is amplitude of the carrier.

Instantaneous frequency of the angle modulated wave is $f_i(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$

here $\frac{d\theta_i(t)}{dt}$ is the angular velocity of the PHASOR of length A_c

a) By varying the frequency ω_c , Frequency Modulation.

b) By varying the phase ϕ_c , Phase Modulation.



A comparison of FM and PM modulators.

i) Frequency modulation (FM)

The method in which the frequency of carrier is varied in accordance to the modulating signal, keeping the amplitude and phase of the carrier the same is called **Frequency modulation** (FM)

a) Audio quality of AM transmission is poor. There is need to eliminate amplitude sensitive noise. This is possible if we **eliminate amplitude variation.**

b) In FM the overall **amplitude** of FM wave remains constant at all times.

c) In FM, the total transmitted **power** remains constant.

d) **Frequency deviation:** The maximum change in frequency from mean value (f_c) is known as frequency deviation. This is also the change or shift either above or below the frequency f_c and is called as frequency deviation.

$$\delta = (f_{\max} - f_c) = f_c - f_{\min} = k_f \cdot \frac{E_m}{2\pi}$$

k_f = Constant of proportionality.

It determines the maximum variation in frequency of the modulated wave for a given modulating signal.

e) Carrier swing (CS): The total variation in frequency from the lowest to the highest is called the carrier swing i.e. CS = 2 δ f

f) Frequency modulation index (m_f-) :

The ratio of maximum frequency deviation to the modulating frequency is called modulation index.

$$m_f = \frac{\delta}{f_m} = \frac{f_{\max} - f_c}{f_m} = \frac{f_c - f_{\min}}{f_m} = \frac{k_f E_m}{f_m}$$

g) Frequency spectrum: FM side band modulated signal consist of infinite number of side bands whose frequencies are

$$(f_c \pm f_m), (f_c \pm 2f_m), (f_c \pm 3f_m) \ldots\ldots$$

The number of side bands depends on the modulation index m_f.

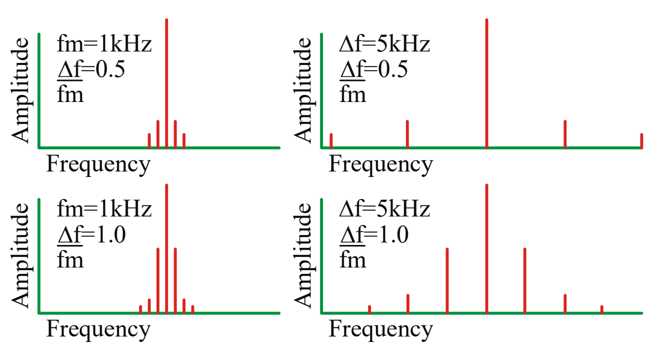
Band width = $2n \times f_m$; where n = number of significant side band pairs

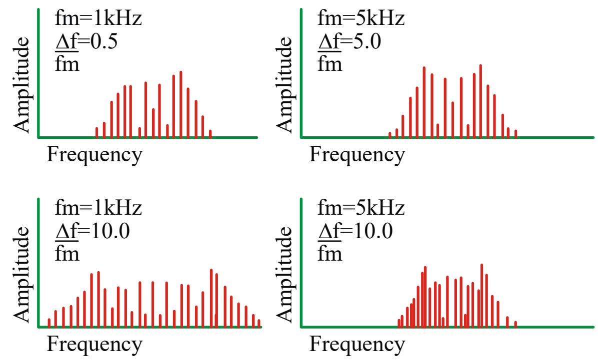
h) **Deviation ratio:** The ratio of maximum permitted frequency deviation to the maximum permitted audio frequency is known as deviation ratio. ($= \frac{(\Delta f)_{\max}}{(f_m)_{\max}}$)

i) Percent modulation: The ratio of actual frequency deviation to the maximum followed frequency deviation

$$m = \frac{(\Delta f)_{\text{actual}}}{(\Delta f)_{\max}}$$

j) Frequency spectra of fm waves under various conditions.



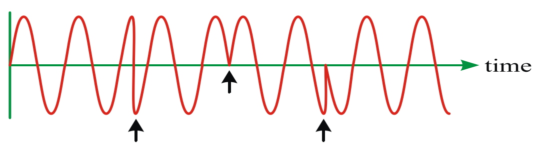


(A) Constant Modulating Frequency

(B) Constant Frequency Deviation

Δf Frequency Deviation   fm Modulation Frequency $\frac{\Delta f}{fm}$ Modulation Index

ii) **Phase Modulation** (PM), phase of carrier is varied in accordance with modulating signal keeping namplitude and frequency constant. We use the term phase shift to characterize such changes. If phase changes after cycle k, the next sinusoidal wave will start slightly later than the time at which cycle k completes.



II) **pulse wave modulation.**

Here the carrier wave is in the form of pulses. Pulse modulation is an analog process as the modulating signal is analog.

The common pulse modulating systems are:

i) **Pulse amplitude modulation(PAM):**

The amplitude of the pulse varies in accordance with the modulating signal.

ii) **Pulse time modulation (PTM):**

Or

Pulse duration modulation(PDM)

a) **Pulse width modulation (PWM):**

The pulse duration varies in accordance with the modulating signal, or the width of the unmodulated signal is constant.

b) **Pulse position modulation (PPM):**

In PPM, the position of the pulses of the carrier wave train is varied in accordance with the instantaneous value of the modulating signal.

iii) **Pulse code modulation (PCM):**

The pulse amplitude, pulse width and pulse position modulations are not completely digital. A completely digital modulation is obtained by pulse code modulation (PCM) by following three operations.

a) **Sampling:** It is the process of generating pulses of zero width and of amplitude equal to the instantaneous amplitude of the analog signal. The number of samples taken per second is called sampling rate.

b) **Quantization:** The process of dividing the maximum amplitude of the analog voltage signal into a fixed number of levels is called quantization.

c) **Coding:** The process of digitizing the quantised pulses according to some code is called coding.

ADDITIONAL INFORMATION

Digital Communication And Quantisation Of Message Signal

(Data Transmission and Retrieval)

Data means facts, concepts or instructions suitable for communication, interpretation or processing by human beings or by automatic means. Data in most cases consists of pulse type of signals.

In digital communication the modulating signals are discrete and are coded as represen tation of message signals to be transimitted. There are a number of encoding steps in digital communication, which makes its circutary complicated. Digital communication is error free and noise free.

The source encoder converts the information into binery code. Encoder first digitise the analog waveform. Some times an additional encoding called channel encoding is carried out. In the final step, before transmission,the channel codes modulate a continuous wave form.

The pulse code modulated (PCM) signal is a series of 1's and 0's. The following three modulation techniques are used to transmit a PCM signal.

1) Amplitude shift keying (ASK):

Two different amplitudes of the carrier represent the two binary values of the PCM signal. This method is also known as on-off keying (OOK)

1: Presence of carrier of same constant amplitude.

0: Carrier of zero amplitude.

2) Frequency shift keying (FSK): The binary values of the PCM signal are represented by two frequencies.

1: Increase in frequency

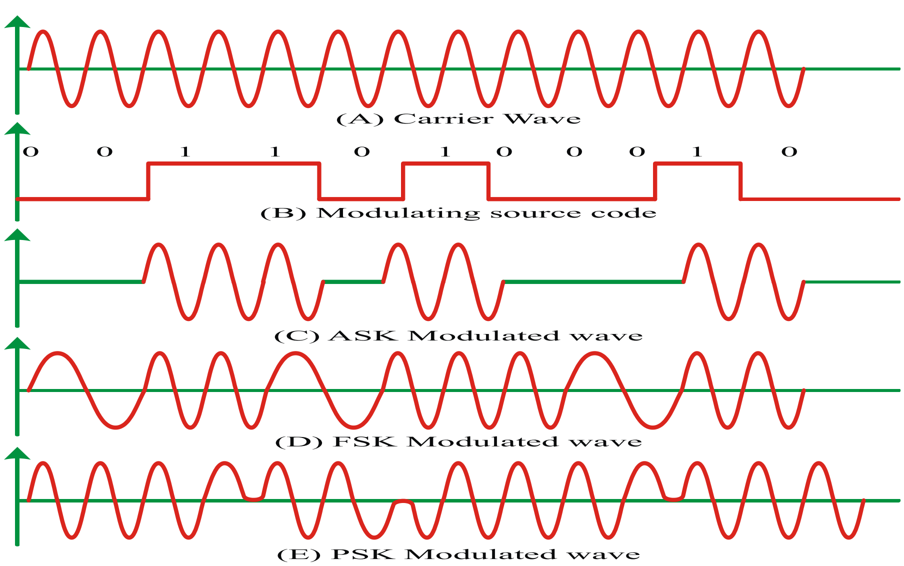
0: Frequency unaffected

3) Phase shift keying (PSK): The phase of the carrier wave is changed in accordance with modulating data signal.

1: Phase changed by p

0: Phase remains unchanged.

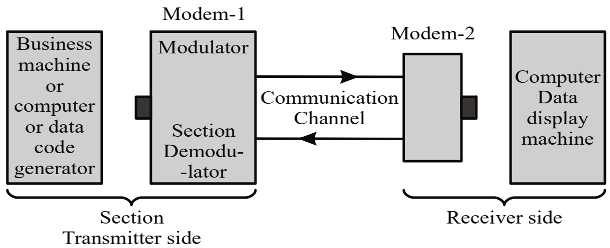
The analog signal is sampled by the sampler. The sampled pulses are then quantized. The encoder codes the quantized pulses according to the binary codes. After modulating the PCM signal (by ASK, FSK or PSK method)the modulated signal is, then transmitted into free space in the form of bits.



Modem and Fax

1) **Modem:** Modems are used to interface two digital sources/receivers.

i) Word modem implies MODulator and DEModulator. Both the functions (modulation and demodulation) are included in a signal unit.



ii) Modems are placed at both ends of the communication circuit.

iii) The modem at the transmitting station changes the digital output from a computer to an analog signal, which can be easily sent via communication channel. While the receiving modem reverses the process.

iv)There are three modes of operation of a modem.

a) Simplex mode: Data is transmitted in only one direction.

b) Half duplex: Data is transmitted between the transmitter and the receiver in both direction, but only in one direction at a time.

c) Full duplex: Data are transmitted between the transmitter and receiver in both directions at the same time.

2) **Fax (Facsimile transmission):** The electronic reproduction of a document at a distance place is known as facsimile transmission (FAX).

The original written document is converted into transmittable codes and is converted in to electrical signals, which are then modulated and transmitted on to the receiving end.

The Internet : It is a system with billions of users worldwide. It permits communication and sharing of all types of information between any two or more computers connected through a large and complex network. It was started in 1960's and opened for public use in 1990's. With the passage of time it has witnessed tremendous growth and it is still expanding its reach. Its applications include.

Email: It permits exchange of text/ graphic material using email software. We can write a letter and send it to the recipient through ISP's (internet Service Providers) who work like the dispatching and receiving post offices.

File transfer: A FTP (File Transfer Programmes) allows transfer of files/software from one computer to another connected to the internet.

World Wide Web (WWW): Computers that store specific information for sharing with others provide websites either directly or through web service providers. Government departments, companies, NGO's (Non-Government Organizations) and individuals can post information about their activities for resticted or free use on their websites. This information becomes accessible to the users. Several search engines like Google, Yahoo!etc. help us in finding information by listing the related websites. Hypertext is a powerful feature of the web that automatically links relevant information from one page on the web to another using HTML (hypertext markup language)

E-commerce: Use of the internet to promote business using electronic means such as using credit cards is called E-commerce. Customers view images and receive all the information about various products or services of companies through their websites. They can do on-line shopping from home/office. Goods are dispatched or services are provided by the company through mail/courier.

Chat: Real time conversation among people with common interests through typed messages is called chat. Everyone belonging to the chat group gets the message instantaneously and can respond rapidly.

D.Mobile telephony : The concept of mobile telephony was developed first in 1970's and it was fully implemented in the following decade. The central concept of this sytem is to divide the service area into a suitable number of cells centered on an office called MTSO (Mobile Telephone Switching Office). Each cell contains a low-power transmitter called a base station and caters to a large number of mobile receivers (popularly called cell phones). Each cell could have a service area of a few square kilometers or even less depending upon the number of customers. When a mobile receiver crosses the coverage area of one base station, it is necessary for the mobile user to be transferred to another base staion. This process is called handover or handoff. This process is carried out very rapidly, to the extent that the consumer does not even notice it. Mobile telephones operate typically in the UHF range of frequencies (about 800-950 MHz)

PROBLEMS

1. **How many AM broadcast stations can be accommodated in a 100 kHZ bandwidth if the highest modulating frequency of carrier is 5 kHZ?**

SOLUTION :

Any station being modulated by a 5 kHz singal will produce an upper side frequency 5 kHz above its carrier and a lower side frequency 5 kHz below its carrier, thereby requiring a bandwidth of 10 kHz. Thus,

Number of stations accommodated

$$\frac{\text{Total bandwidth}}{\text{Bandwidth per station}} = \frac{100}{10} = 10$$

2. **How many 500 kHz waves can be on a 10km transmission line simultaneously?**

SOLUTION :

Let λ be the wavelength of 500 kHz signal. Then, $\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{5.0 \times 10^5} m = 600m$

The number of waves on the line can be found from,

$$n = \frac{d}{\lambda} = \frac{10 \times 10^3}{600} = 16.67$$

3. **A two wire transmission line has a capacitance of 20 pF/m and a characteristic impedance of** 50Ω

a) What is the inductance per metre of this cable?

b) Determine the impedance of an infinitely long section of such cable.

SOLUTION :

a) The characteristic impedance. $Z = \sqrt{L/C}$ $L = (Z^2)(C) = (50)^2 (20 \times 10^{-12}) H = 0.05 H/m$

b) The characteristic impedance of a transmission line is the impedance that an infinite length of line would present to a power supply at the input end of the line. Thus, $Z_\infty = Z_0 = 50\Omega$

4. **An audio signal given by** $e_s = 15 \sin 2\pi (200t)$ **modulates a carrier wave given by**

$e_s = 60 \sin 2\pi (100,000t)$. **If calculate**

a) Percent modulation

b) Frequency spectrum of the modulated wave.

SOLUTION :

a) Signal Amplitude, $B = 15$

Carrier amplitude, $A = 60$

$$m = \frac{B}{A} = \frac{15}{60} = 0.25$$

$\therefore$ Percentage modulation $= 0.25 \times 100 = 25\%$

b) By comparing the given equations of signal and carrier with their standard form

$$e_s = E_s \sin \omega_s t = E_s \sin 2\pi f_s t \text{ and}$$

$$e_c = E_c \sin \omega_c t = E_c \sin 2\pi f_c t$$

we have signal frequency $f_s = 2000Hz$ and carrier frequency $f_c = 100,000Hz$

The frequencies present in modulated wave

i) $f_c = 100,000Hz = 100kHz$

ii) $f_c - f_s = 100,000 - 2000 = 98kHz$

iii) $f_c + f_s = 100kHz + 2kHz = 102kHz$

Therefore, frequency spectrum of modulated wave extends from 98kHz to 102 kHz is called band width.

5. The antenna current of an AM transmitter is 8A when only the carrier is sent but it increases to 8.93A when the carrier is modulated. Find percent modulation.

SOLUTION :

The modulated or total power carried by AM wave $P_T = P_C\left(1+\frac{m^2}{2}\right)$. If R is load resistance. I_m is the current when carrier is modulated and I_c the current when unmodulated, then

$$\frac{P_T}{P_C} = \frac{I_m^2 R}{I_c^2 R};$$

$$\therefore I + \frac{m^2}{2} = \frac{I_m^2 R}{I_c^2 R}$$

Given $I_m = 8.93A, I_c = 8A$

$$\therefore m^2 = 2\left[\left(\frac{8.93}{8.0}\right)^2 - 1\right] \therefore m = 0.7$$

Therefore, percentage modulation = 70%

6. The audio signal voltage is given by $V_m = 2\sin 12\pi \times 10^3 t$. The band width and LSB if carrier wave has a frequency $3.14\times10^6 rad/s$

1) 12 KHz; 494 KHz 2) 6 KHz; 313 KHz 3) 6 KHz; 494 KHz 4) 18 KHz; 494 KHz

SOLUTION :

band width = $2f_m$;

LSB= $f_c - f_m$

7. A sinusoidal carrier voltage of 80 volts amplitude and 1 MHz frequency is amplitude modulated by a sinusoidal voltage of frequency 5kHz producing 50% modulation. Calculate the amplitude and frequency of lower and upper side bands.

SOLUTION :

Amplitude of both LSB and USB are equal and given by

$$= \frac{mE_c}{2} = \frac{0.5\times80}{2} = 20volts$$

Now frequency of LSB = $f_c - f_s$

$$= (1000-5)kHz = 995kHz$$

Frequency of USB = $f_c + f_s$

$$= (1000+5)kHz = 1005kHz$$

8. The load current in the transmitting antenna of an unmodulated AM transmitter is 6 Amp. What will be the antenna current when modulation is 60%.

SOLUTION :

Total power carried by AM wave

$$P_T = P_C\left(1+\frac{m^2}{2}\right)......(1)$$

Where P_c is the power of carrier component and m is the modulation factor. If R is the resistance, I_m the antenna load current when modulation is 60% and I_c is the antenna load current when un modulated, then

$$\frac{P_T}{P_C} = \frac{I_m^2 R}{I_c^2 R}, \therefore 1+\frac{m^2}{2} = \frac{I_m^2}{I_c^2} \quad \text{using (1)}$$

$$\text{or } I_m = I_c\sqrt{\left\{\left(1+\frac{m^2}{2}\right)\right\}}$$

$$\text{Given } I_c = 6Amp, m = 0.6$$

$$I_m = 6\left[1+\frac{(0.6)^2}{2}\right]^{1/2} = 6[1.086] = 6.52Amp$$

9. An amplitude modulated wave is modulated to 50%. What is the saving in power, if carrier as well as one of the side bands are suppressed ?

SOLUTION :

$$\text{as total power } P_t = P_c + P_{SB}$$

$$\text{Here } P_C = \frac{A_C^2}{2R}$$

$$\text{and } P_{SB} = P_{LSB} + P_{USB} \text{ as } P_{LSB} = P_{USB} = \frac{\mu^2 A_C^2}{8R}$$

$$P_{SB} = \frac{\mu^2 A_C^2}{4R}$$

$$\text{\% saving } \frac{P_C + P_{LSB}}{P_t} \times 100$$

$$= \frac{\frac{A_C^2}{2R} + \frac{\mu^2 A_C^2}{8R}}{\frac{A_C^2}{2R} + \frac{\mu^2 A_C^2}{4R}} \times 100 = \frac{\frac{1}{2}+\frac{\mu^2}{8}}{\frac{1}{2}+\frac{\mu^2}{4}} \times 100$$

$$\text{Given that } \mu = \frac{50}{100} = \frac{1}{2}$$

substituting; we get % saving = 94.4 %

10. **The load on an Am diode detector consists of a resistance of 50 KΩ in parallel with a capacitor of 0.001 μF. Determine the maximum modulation index that the detector can handle without distortion when modulation frequency is (i) 1 kHz (ii) 5 kHz.**

SOLUTION :

$$Z_m = R_c \parallel C = \frac{1}{\sqrt{\frac{1}{(R_c)^2} + \frac{1}{(X_c)^2}}}$$

$$M_{\max} = \frac{Z_m}{R_c}$$

$$= \frac{1}{R_c\sqrt{\frac{1}{(R_c)^2} + \frac{1}{(X_c)^2}}} = \frac{1}{\sqrt{1+(2\pi fCR_c)^2}}$$

For f=1 kHz

$$M_{max} = \frac{1}{\sqrt{1 + 0.098696}} = 0.945$$

for f= 5 kHz

$$M_{max} = \frac{1}{\sqrt{1+2.4674}} = 0.537$$

11. **The tuned circuit of an oscillator in a simple AM transimitter employs a 250 micro henry coil and 1nF condenser. If the oscillator output is modulated by audio frequency upto 10KHz, the frequency range occupied by the side bands in KHz is**

1) 210 to 230 2) 258 to 278 3) 308 to 328 4) 118 to 128

SOLUTION :

$$f_c = \frac{1}{2\pi\sqrt{LC}} = 318\text{khz},\ f_m = 10\text{khz}$$

$$\text{LSB} = f_c - f_m = 308; \quad \text{USB} = f_c + f_m = 328$$

12. **A TV tower has a height of 70m. If the average population density around the tower is 1000km^{-2}, the population covered by the TV tower**

1) 2.816×10^{6} **2)** 2.86×10^{9} **3)** 2.816×10^{3} **4)** 2.816×10^{12}

SOLUTION :

$$\text{Population} = \text{Population density} \times \text{Area}$$
$$= \text{Population density} \times \pi\times 2Rh$$

13. A basic communication system consists of

A) transmitter B) information source

C) channel D) receiver

Choose the correct sequence in which these are arranged in a basic communication system:

1) ABCDE 2) BADEC

3) BDACE 4) BEADC

SOLUTION :

From the block diagram of basic commnication system, the sequence can be arranged.

14. A carrier wave is modulated by a number of sine waves with modulation indices 0.1, 0.2, 0.3. The total modulation index (m) of the wave is

1) 0.6 2) 0.2 3) $\sqrt{0.14}$ 4) $\sqrt{0.07}$

SOLUTION :

$$m = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

15. An audio signal of 15kHz frequency cannot be transmitted over long distances without modulation because

A) the size of the required antenna would be at least 5 km which is not convenient.

B) the audio signal can not be transmitted through sky waves.

C) the size of the required antenna would be at least 20 km, which is not convenient

D) effective power transmitted would be very low, if the size of the antenna is less than 5km.

ANSWER : 1 , 2 , 4

SOLUTION :

Here $f = 15\,kHz = 15\times10^3\,Hz$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{15\times10^3} = \frac{1}{5}\times10^5$$

Size of the antenna should be at least $l = \frac{\lambda}{4} = 5\,km$

Audio signal cannot be transmitted through the sky waves, because in sky wave propagation the frequency range is 2 MHz to 40 MHz. But frequency of this wave is 15 KHz.

For the effective power radiation by the antenna

$$p \propto \left(\frac{L}{\lambda}\right)^2$$

if 'L' decreases 'p' also decreases.

16. The maximum peak-to-peak voltage of an AM wave is 16 mV and the minimum peak-to-peak voltage is 4mV. The modulation factor is equal to

1) 0.6 2) 0.3 3) 0.8 4) 0.25

SOLUTION :

$$\mu = \frac{(16-4)/2}{(16+4)/2}$$

17. A carrier wave of 1000 W is subjected to 100% modulation. Calculate (i) Power of modulated wave, (ii) power is USB, (iii) power is LSB

SOLUTION :

i) Total power of modulated wave

$$P_T = P_C\left(1+\frac{m^2}{2}\right)\ ;\ =1000\left(1+\frac{1^2}{2}\right)=1500\,watt$$

ii) Power in USB $= \frac{1}{2}P_{SB}$

Where power carried by side bands is given by amplitude modulation and detection

$$P_{SB} = P_C\left(\frac{m^2}{2}\right)\ ;\ =1000\left(\frac{1^2}{2}\right)=500\,watt$$

$$P_{USB} = \frac{1}{2}P_{SB} = \frac{1}{2}\times 500 = 250\,watt$$

iii) Since power in LSB = Power in USB

$$P_{LSB} = P_{USB} = 250\,watt$$

18. A transmitting antenna at the top of a tower has a height 32m and the height of the receiving antenna is 50m. What is the maximum distance between them for satisfactory communication in LOS mode? Given radius of earth $6.4\times10^6\,m$.

SOLUTION :

$$d_m = \sqrt{2\times64\times10^5\times32}+\sqrt{2\times64\times10^5\times50m}$$

$$= 64\times10^2\times\sqrt{10}+8\times10^3\times\sqrt{10}m$$

$$= 144\times10^2\times\sqrt{10}m = 45.5km$$

19. A message signal of frequency 10 kHz and peak voltage of 10 volts is used to modulate a carrier of frequency 1 MHz and peak voltage of 20 volts. Determine

a) modulation index

ii) the side bands produced.

SOLUTION :

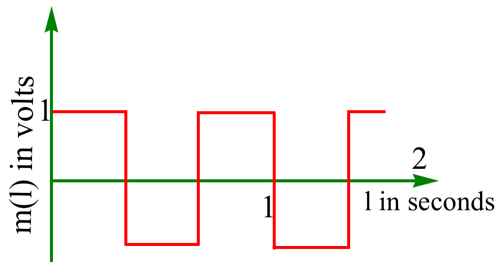
a) modulation index $= 10/20 = 0.5$

b) The side bands are at

(1000+10)kHz = 1010

kHz & (1000-10 kHz)= 990 kHz, are frequency.

20. A modulating signal is a square wave as shown in figure.



The carrier wave is given by
c(t)=2 sin (8 π t) volt
i) Sketch the amplitude modulated wave from
ii) What is the modulation index ? (NCERT)

SOLUTION :

i) Amplitude of modulating signal from fig is

$A_m = 1$

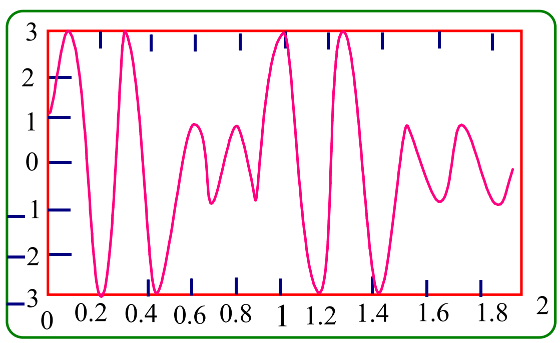
Amplitude of carrier wave from the equation is

$A_C = 2$

Then maximum amplitude of modulated wave is $A_{max} = A_C + A_m$

$A_{max} = 2 + 1 = 3$ and minimum amplitude of modulated wave is $A_{min} = A_C - A_m$

Then the sketch for amplitude modulated wave is as below.



ii) Modulation index $\mu = \dfrac{A_m}{A_C} = \dfrac{1}{2} = 0.5$

21. An AM wave is exressed as $e = 10(1 + 0.6\cos 2000\pi t)\cos 2\times10^8 \pi t$ volts, the minimum and maximum values of modulated carrier wave are

1) 10 V, 20 V 2) 4 V, 8 V 3) 16 V, 4 V 4) 8 V, 20 V

SOLUTION :

$$E_{max} = (1+\mu)E_c; \quad E_{min} = (1-\mu)E_c$$

22. A TV transmission tower at a particular station has a height of 160m. Radius of earth is 6400km

i) The range it covers is 45255 m

ii) The population that it covers is77.42 lakhs. When population density is $1200 km^2$

iii) The height of antenna should be increased by 480 m to double the coverage range

1) i and ii are true 2) ii and iii are true

3) i and iii are true 4) i, ii and iii are true

SOLUTION :

$$d = \sqrt{2Rh}$$

$$\text{Population} = \text{Population density} \times \pi d^2$$

$$d \alpha \sqrt{h}$$

23. The tuned circuit of the oscillator in a simple AM transmitter employs a 40 μH coil and 12 nanofarad(nF) capacitor. If the oscillator output is modulated by audio frequency of 5 kHz,Which of the following frequencies doesn't appear in the out put AM?

1) $f_{USB} = 225$ KHz 2) $f_{USB} = 235$ kHz **3) f_c=230 kHz 4) $f_c = 235$ kHz**

SOLUTION :

$$f_c = \frac{1}{2\pi\sqrt{LC}} = 230 KHz$$

$$\text{LSB} = f_c - f_m = 225 KHz$$

$$\text{USB} = f_c + f_m = 235 KHz$$

24. A 400 watt carrier is modulated to a depth of 80%. Caluculate the total power in the modulated wave

1) 528 W 2) 128 W 3) 256 W 4) 400 W

SOLUTION :

$$P_{Total} = P_C\left(1 + \frac{m^2}{2}\right) = 528 \text{ watts}$$

25. Caluculate modulation index if carrier waves is modulated by three signals with modulation indices as 0.6, 0.3 and 0.4

1) 1.0 2) 0.70 3) 0.78 4) 1.3

SOLUTION :

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2} = 0.78$$

26. A 1000 KHz carrier is simoultaneously modulated with f_{m1}= 300 Hz, f_{m2}= 800 Hz and f_{m3}= 1 KHz audio sine waves. What will be the frquencies present in the output?

a) $f_{LSB1} = 999.7 KHz$, $f_{USB1} = 1000.3 KHz$ **b)** $f_{LSB2} = 999.2 KHz$, $f_{USB2} = 1000.8 KHz$ **c)** $f_{LSB3} = 999 KHz$, $f_{USB3} = 1001 KHz$ **d)** $f_{LSB3} = 990 KHz$, $f_{USB3} = 1010 KHz$

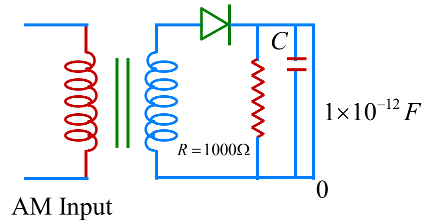
1) a only 2) b, c and d

3) a, b and c 4) a, c only

SOLUTION :

Frequencies present in the out put are $f_c - f_m$ and $f_c + f_m$

27. In the given detector circuit, the suitable value of carrier frequency is



1) << 10^9 Hz 2) << 10^5 Hz3) >> 10^9 Hz 4) << 10^3 Hz

SOLUTION :

$$\text{Using } \frac{1}{f_{carrier}} << RC$$

We get time constant,

RC = 1000 x 10^{-2}=10^{-9}s

$$\text{Now } \nu = \frac{1}{T} = \frac{1}{10^{-9}} = 10^9 \, Hz$$

Thus the value of carrier frequency should be much less than 10^9 Hz, say 100 KHz.

28. T.V. transmission tower at a particular station has a height of 160m.

a) What is the coverage range?

b) How much population is covered by transmission, if the average population density around the tower is 1200 per km^2 ?

c) What should be the height of tower to double the coverage range

SOLUTION :

$$\text{a) Coverage range } d = \sqrt{2Rh}$$

$$= \sqrt{2\times6400\times10^3\times160m}$$

$$= 45.254km$$

b) Population covered

$$= (population\, density)\times(area \text{ covered})$$

$$= (1200)\times(\pi d^2)$$

$$= (2400\pi Rh) = 2400\times3.14\times6.4\times10^3\times0.16$$

$$= 77.17\, lac$$

$$\text{c) Coverage range } \propto \sqrt{h}$$

Therefore coverage range can be doubled by making height of the tower four times to 640m. So, height of the tower should be increased by 480 m.

29. A carrier is simultaneously modulated by two sine waves with modulation indices of 0.4 and 0.3. The resultant modulation index will be

1) 1.0 2) 0.7 3) 0.5 4) 0.35

SOLUTION :

$$m = \sqrt{m_1^2 + m_2^2} = \sqrt{(0.16)+(0.09)} = 0.5$$

30. Mean optical power launched into an 8km fibre is 120mW and mean output power is 4mW, then the overall attenuation is (Given log 30 = 1.477)

1) 14.77 dB 2) 16.77 dB 3) 3.01 dB 4) 10.5 dB

SOLUTION :

$$\text{Attenuation} = 10\log\frac{120}{4} = 10\log 30$$

$$= 10\times 1.4771 = 14.77dB \cdot$$

31. Three waves A, B and C of frequencies 1600kHz, 5MHz and 60 MHz, respectively are to be transmitted from one place to another. Which of the following is the most appropriate mode of communications

1) A is transmitted via space wave while B and C are transmitted via sky wave.

2) A is transimitted via ground wave, B via sky wave and C via space wave.

3) B and C are transmitted via ground wave while A is transmitted via sky wave.

4) B is transmitted via ground wave while A and C are transmitted via space wave.

SOLUTION :

Ground wave propagation is suitable for the frequencies upto 2 MHz (less than few mHz)

Sky wave propagation is suitable for the sequencies upto 30 to 40 MHz

The space wave propagation is suitable for the freequencies above 40 MHz.

32. A 100m long antenna is mounted on a 500m tall building. The complex ca become a transmission tower of waves with λ

1) ~ 400 m 2) ~ 25 m 3) ~ 150 m 4) ~ 2400 m

SOLUTION :

$$\text{Length of the building } (l) = 500\,m$$

$$\text{length of antenna } (L) = 100\,m$$

wave length of the wave which can be tansmitted is λ

$$\text{as } L \approx \frac{L}{4} \Rightarrow \lambda = 4L$$

$$\lambda \approx 4(100) \approx 400\,m$$

33. A 1 KW signal is transmitted using a communication channel which provides attenuation at the rate of ~2dB per km. If the communication channel has a total length of 5km, the power of the signal received is

[gain in dB $= 10\log\left(\frac{P_0}{P_1}\right)$ **]**

1) 900 W 2) 100 W 3) 990 W 4) 1010 W

SOLUTION :

Power of the singal transmitted is

P_i = 1 KW = 1000W

rate of attentuation of signal is $= -2\,{}^{dB}\!/_{km}$

Length or total path = 5 km

thus lass suffered in the communication channel

$= (5)(-2) = -10\ dB$ and gain in

$$dB = 10\log\left(\frac{P_0}{P_i}\right)$$

$$dB = -10\log\left(\frac{P_i}{P_0}\right)$$

$$-10 = -10\log\left(\frac{P_i}{P_0}\right)$$

$$\log\frac{P_i}{P_0} = 1 \qquad \frac{P_i}{P_0} = 10$$

$$P_0 = \frac{P_i}{P_0} = \frac{1000}{10} = 100\,W$$

34. A speech signal of 3 kHz is used to modulate a carrier signal of frequency 1 MHz, using amplitude modulation. The frequencies of the side bands will be

1) 1.003 MHz and 0.997 MHz

2) 3001 kHz and 2997 kHz

3) 1003 kHz and 1000 kHz

4) 1 MHz and 0.997 MHz

SOLUTION :

the frequencies of side bands are

$LSB = f_c - f_m$ (Lower side Band)

$USB = f_c + f_m$ (Upper side Band)

35. A message signal frequency ω_m is superpiosed on a carrier wave of frequency ω_c to gent an amplitude modulated wave (AM). The frequency of the AM wave will be

1) ω_m 2) ω_c 3) $\frac{\omega_c + \omega_m}{2}$ 4) $\frac{\omega_c - \omega_m}{2}$

SOLUTION :

In amplitude modulation, frequency of carrier wave is equal to the frequency of modulated wave. Because in AM, Amplitude or carrier wave changes in accordance to the modulating signal

36. I-V characteristics of four devices are shown in Fig. 15.1

Identify devices that can be used for modulation :

1) 'i' and 'iii' 2) only 'iii'

3) 'ii' and some regions of 'iv'

4) All the devices can be used

SOLUTION :

A square law modulator is the device which can produce modulated waves by the apllication of message signal and the carrier wave.

Square law modulator is used for modulation purpose.

Charactersitics shown by (i) and (iii) correspond to linear devices.

And by (ii) and same part of (iv) corresponds to square law device which shows non-linear relations. Hence (ii) and (iv) can be used for modulation.

37. A male voice after modulation-transmission sounds like that of a female to the receiver. The problem is due to

1) poor selection of modulation index (selected $0 < m < 1$)

2) poor bandwidth selection of amplifiers

3) poor selection of carrier frequency

4) loss of energy in transmission

SOLUTION :

The frequency of modulated signal received becomes more due to more improper selection or band width and

$$\text{band width} = 2f_m$$

But frequency ofmale voice is less than that of a female.

38. Identify the mathematical expression for amplitude modulated wave :

1) $A_c \sin\left[\left\{\omega_c + KV_m(t)\right\}t + \phi\right]$

2) $A_c \sin\left[\left\{\omega_c + \phi + KV_m(t)\right\}t\right]$

3) $\left\{A_c + KV_m(t)\right\}\sin(\omega_c t + \phi)$

4) $A_c V_m(t)\sin(\omega_c t + \phi)$

SOLUTION :

$$\text{modulating signal } m(t) = A_m \sin\omega_m t$$

$$\text{carrier signal } c(t) = A_c \sin\omega_c t$$

modulated signal

$$c_m(t) = (A_c + A_m \sin\omega_m t)\sin\omega t$$

$$c_m(t) = A_c\left[1 + \frac{A_m}{A_c}\sin\omega_m t\right]\sin\omega_c t$$

$$c_m(t) = (A_c + \mu A_c \sin\omega_m t)\sin\omega_c t \quad \text{as } k = \mu A_c$$

$$c_m(t) = (A_c + k\sin\omega_m t)\sin\omega_m t$$

39. Compute LC product of a tuned amplifier circuit required to generate a carrier wave of 1 mHz for amplitude modulation

1) 52×10^{-15} **2)** 25×10^{-15}

3) 2.5×10^{-16} **4)** 2.0×10^{-15}

Multiple correct answer questions

SOLUTION :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

40. Audio sine waves of 3 kHz frequency are used to amplitude modulate a carrier signal of 0.5 MHz. Which of the following statements are true ?

1) The side band frequencies are 1506 kHz and 1494 kHz.

2) The bandwidth required for amplitude modulation is 6 kHz.

3) The bandwidth requried for amplitude modulation is 3 MHz.

4) The side band frequencies are 1503 kHz and 1497 kHz.

ANSWER : 2 , 4
SOLUTION :

$$LSB = f_c - f_m$$
$$USB = f_c + f_m$$
$$\text{Band width } = 2 f_m$$

41. A TV transmission tower has a height of 240m. Signals broadcast from this tower will be received by LOS communication at a distance of (assume the radius of earth to be 6.4 x 10^6m)
1) 100 km 2) 24 km 3) 55 km 4) 50 km

ANSWER : 2 , 3 , 4
SOLUTION :

Distance or Range or transmission of a tower

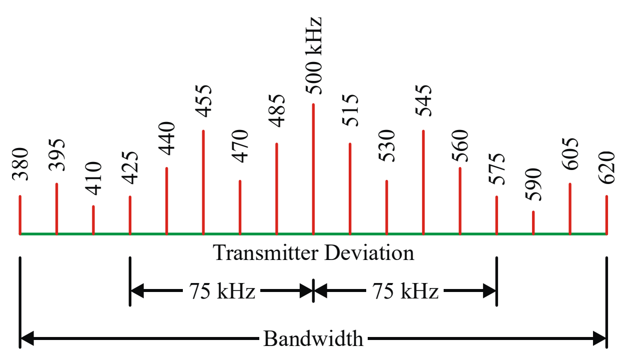
$$d = \sqrt{2Rh_t}$$
$$d = \sqrt{2\left(6.4\times10^6\right)240}$$
$$d = 55.4\,km$$

42. The carrier frequency is 500 kHz. The modulating frequency is 15 kilohertz and the deviation frequency is 75 kilohertz. Find
a) modulation index
b) Number of side bands
c) Band width

SOLUTION :

$$MI = \frac{\Delta f}{f_m} = 5$$

We can have that there are 16 significant sidebands for a modulation index of 5.



To determine total bandwidth for this case,

we use: $bW = f_m \times (\text{number of sidebands})$

bW = 15 x 16=240 kHz

43. The frequency response curve (Fig. 15.2) for the filter cirucit used for production of AM wave should be

1) (i) followed by (ii) **2) (ii) followed by (i)**

3) (iii) **4) (iv)**

ANSWER : 1 , 2 , 3

SOLUTION :

To produce an amplitude modulated wave, band width $= \omega_{USB} - \omega_{LSB} = 2\omega_m$

44. In amplitude modulation, the modulation index *m*, is kept less than or equal to 1 because

1) m > 1 , will result in interference between carrier frequency and message frequency, resulting into distortion.

2) m > 1 will result in overlapping of both side bands resulting into loss of information.

3) m >1 will result in change in phase between carrier signal and message signal.

4) m >1 indicates amplitude of message signal greater than amplitude of carrier signal resulting distortion.

ANSWER : 1, 4

SOLUTION :

Here $\mu = \frac{A_m}{A_c}$

when m >1 overlapping of both side bands takes place and it results loss of information and alos amplitude of message signal will be greater than amplitude or carrier signal which results destortion.

45. Choose correct statements in the following

1) A vibrating tuning fork produce analog signal

2) A musical sound due to vibrating sitar string produce analog signal

3) Light pulse produce digital signal

4) Out put of NAND Gate produce digital signal

ANSWER : 1 , 2 , 3, 4

SOLUTION :

Analog signals are continous signals but digiatal signals are in the form ofpulses

THEORY BITS

1. For transmitting audio signal properly
   1) it is first superimposed on high frequency carrier wave
   2) it is first superimposed on low frequency carrier wave
   3) It is sent directly without superimposing on any wave
   4) it is superposed with carrier wave of high velocity

KEY:1

2. The part of communication system that extracts the signal at the output of the channel is
   1) transducer 2) transmitter
   3) receiver 4) receiver or transmitter

KEY:3

3. Radio waves of constant amplitude can be generated with
   1) filter 2) rectifier 3) FET 4) oscillator

KEY:4

4. The attenuation of a signal is compensated by
   1) rectifier 2) oscillator
   3) modulator 4) amplifier

KEY:4

5. Modern communication systems use
   1) analog circuits
   2) digital circuits
   3) combination of analog & digital circuits
   4) radio circuits

KEY:2

6. Optical fibre communication is generally preferred over general communication system because
   1) it is more efficient 2) of signal security
   3) both (1) & (2) 4) it is easily available

KEY:3

7. A digital signal possess
   1) continuously varying values
   2) only two discrete values
   3) only four discrete values
   4) constant values

KEY:2

8. For TV transmission the frequency range employed (Karnataka 1990, 1989)
   1) 30 - 300 MHz 2) 30 - 300 GHz
   3) 30 - 300 KHz 4) 30 - 300 Hz

KEY:1

9. Digital signals
   1) provide continuous set of values
   2) represent values as randomly
   3) Utilise Decimal code system
   4) Utilise binary code system

KEY:4

10. Digital signals
    i) do not provide a continuous set of values.

ii) represents values as descrete steps.
iii) can utilize binary system
iv) can utilize decimal as well as binary system.
The true option is.
1) (i) & (ii) only 2) (ii) & (iii) only
3) (i), (ii) & (iii) only 4) (i),(ii),(iii)& (iv)

KEY:3

11 A digital signal
1) is less reliable than analog signal 2) is more reliable than analog signal
3) is equally reliable as the analog signal 4) Not at all reliable

KEY:2

12. The band width required for transmiting video signal is
1) 50 KHz 2) 1 MHz 3) 4.2 MHz 4) 6 MHz

KEY:3

13. A laser is a coherent source because it contains
1) Many wavelengths
2) Uncoordinated wave of a particular wavelength
3) Coordinated wave of many wavelengths
4) Coordinated waves of a particular wavelength

KEY:4

14. The short wave Radio broadcasting band is
1) 7 MHz to 22 MHz 2) 88 MHz to 108 MHz
3) 30 KHz to 300 KHz 4) 3 GHz to 30 GHz

KEY:1

15. The FM Radio broad casting band is
1) 5 MHz to 30 MHz 2) 88 MHz to 108 MHz
3) 30 KHz to 300 KHz 4) 3 GHz to 30 GHz

KEY:2

16. Modulation is required to
a) distinguish different transmissions
b) ensure that the information may be trans mitted over long distances
c) allow the information accessible for different people
1) a & b are true 2) b & c are true
3) c & a are true 4) a, b & c are true

KEY:4

17. A: Satellite communication uses different frequency bands for uplink and downlink
B: Bandwidth of video signals is 4.2 MHz
1) A is true but B is false 2) A is false but B is true
3) A and B are false 4) A and B are true

KEY:4

18. The higher frequency TV broad casting bands range is
1) 54 - 72 MHz and 76 to 88 MHz
2) 174 - 216 MHz and 420 to 890 MHz
3) 896 to 901 MHz and 840 to 935 MHz
4) 5.925 to 6.425 GHz and 3.7 to 4.2 GHz

KEY:2

19. Frequency ranges for micro waves are :

1) 3×10^{9} to 3×10^{4} Hz 2) 3×10^{13} to 3×10^{9} Hz

3) 3×10^{14} to 3×10^{9} Hz 4) 3×10^{11} to 3×10^{9} Hz

KEY:2

20. The frequency which is not part of AM broadcast

1) 100 kHz 2) 700 kHz 3) 600 kHz 4) 1500 kHz

KEY:1

21. The examples of broadcast are

A) radio B) television C) telephony D) internet

1) A & B 2) A, B & D 3) A,B & C 4) B & D

KEY:2

22. Cellular Mobile works in the frequency range of

1) 840 to 935 MHz 2) 3.7 to 4.2 GHz

3) 420 to 890 MHz 4) 30 to 300 GHz

KEY:1

23. Frequency range used in down linking in satellite communication is

1) 0.896 to 0.901 GHz 2) 0.420 to 0.890 GHz

3) 5.925 to 6.425 GHz 4) 3.7 to 4.2 GHz

KEY:4

24. The intensity of the ground waves decrease with increase of distance due to

1) Interference 2) Diffraction

3) Polarization 4) Due to unknown reason

KEY:2

25. In the satellite communication, the uplinking frequency range is

1) 0.896 to 0.901 GHz 2) 0.420 to 0.890 GHz

3) 5.925 to 6.425 GHz 4) 3.7 to 4.2 GHz

KEY:3

26. The frequency of a FM transmitter without signal input is called

1) Lower side band frequency

2) Upper side band frequency

3) Resting frequency

4) None of these

KEY:3

27. Television signals on earth cannot be received at distances greater than 100 km from the transmission station. The reason behind this is that

1) The receiver antenna is unable to detect the signal at a distance greater than 100 km

2) The TV programme consists of both audio and video signals

3) The TV signals are less powerful than radio signals

4) The surface of earth is curved like a sphere

KEY:4

28. Audio signal cannot be transmitted because

1) The signal has more noise

2) The signal cannot be amplified for distance communication

3) The transmitting antenna length is very small to design

4) The transmitting antenna length is very large and impracticable

KEY:4

28a. Because of tilting which waves finally disappear
1) Microwaves 2) Surface waves
3) Sky waves 4) Space waves
KEY:2
29. An antenna is a device
1) That converts electromagnetic energy into radio frequency signal
2) That converts radio frequency signal into electromagnetic energy
3) That converts guided electromagnetic waves into free space electromagnetic waves and vice-versa
4) None of these
KEY:3
30. An antenna
1) Converts AF wave to RF wave 2) RF signal into electromagnetic energy

3) Converts the guided EM waves into free space EM waves and vice versa
4) Super imposes AF wave on RF wave
KEY:3
31. The frequency at which communication will not be reliable is
1) 100 KHz 2) 1 MHz 3) 10 GHz 4) 100 GHz
KEY:1
32. An antenna behaves as a resonant circuit only when the length is
1) equal to $\lambda/4$ 2) equal to $\lambda/2$
3) equal to the integral multiples of $\lambda/2$
4) equal to $3\lambda/4$
KEY:1
33. A: At great heights from surface of earth and close to earth ionisation of air molecules is low.
B: EM waves of frequencies beyond 30 MHz penetrate ionosphere and escape.
1) both A and B are correct
2) both A and B are wrong
3) only A is correct 4) only B is correct
KEY:1
34. If audio signal is transmitted directly into space, the length of the transmitting antenna required will be
1) extremely small 2) extremely large
3) infinitely large 4) moderate
KEY:2
35. The height of the antenna
a) limits the population covered by the transmission
b) limits the ground wave propagation
c) effectively used in line of sight communication
1) a & b are true 2) b & c are true 3) c & a are true 4) a, b,c are true
KEY:4
36. Statement A: If the antenna is vertical the vertically polarised EM wave is radiated
Statement B: The vertically polarised EM wave has electrical variations in the vertical plane
1) A is true but B is false 2) A is false but B is true
3) A and B are false 4) A and B are true
KEY:4

37. Broadcasting antennas are generally
1) Omnidirectional type 2) Vertical type
3) Horizontal type 4) None of these
KEY:2
37a. Refractive index of ionosphere is
1) zero 2) more than one
3) less than one 4) one
KEY:3
37b. Electromagnetic waves with frequencies greater than the critical frequency of ionosphere cannot be used for communication using sky wave propagation because
1) the refractive index of the ionosphere becomes very high for $f>f_c$
2) the refractive index of the ionosphere becomes very low for $f>f_c$
3) the refractive index of ionosphere becomes very high for $f>f_c$
4) the refractive index of the ionosphere becomes very high for $f = f_c$
KEY:1
37c. In the night, ionosphere consists of
1) E, F_1 and F_2 layers 2) D, E, F_1 and F_2 layers
3) E and F_2 layers 4) D, E and F_2 layers
KEY:3
38. In frequency modulation
1) The amplitude of modulated wave varies as frequency of carrier wave
2) The frequency of modulated wave varies as amplitude of modulating wave
3) The amplitude of modulated wave varies as amplitude of carrier wave
4) The frequency of modulated wave varies as frequency of modulating wave
KEY:2
39. The process of recovering the audio signal from the modulated wave is known as
1) amplification 2) rectification
3) modulation 4) demodulation
KEY:4
40. The most commonly employed analog modulation technique in satellite communi-cation is the
1) amplitude modulation 2) frequency modulation
3) phase modulation 4) amplitude & phase modulation
KEY:2
41. The type of modulation is employed in India for radio transmission is
1) pulse modulation 2) frequency modulation
3) amplitude modulation 4) phase modulation
KEY:3
42. Match List-1 with List-2

List-1	List-2
Name of device	use
a) Antenna	e) sends out information
b) Transmitter	f) picksup information
c) Receiver	g) converts energy in one form to another form
d) Transducer	h) radiates signal

i) recieves signal

1) a-e; b-g,h;c-f;d-h 2) a-h,i;b-e;c-f;d-g

3) a-f; b-e;c-f,i;d-h 4) a-h;b-g,i;c-f;d-e

KEY:2

43. Modulation is used to

1) reduce the bandwidth

2) to seperate the transmission of different users

3) to ensure that intelligence may be transmitted to long distances

4) to allow the uses of practical antenna

KEY:1

44. The process of translating the information contained by the low base band signal to high frequencies is called

1) Detection 2) Modulation

3) Amplification 4) Demodulation

KEY:2

45. During the process of modulation the RF wave is called

1) Modulating wave 2) Modulated wave

3) Carrier wave 4) Audio wave

KEY:3

46. The physical quantities of the wave used for modulation

1) Amplitude only

2) Amplitude and frequency

3) Amplitude, frequency and phase

4) Only frequency

KEY:3

47. In Amplitude modulation

1) The amplitude of the carrier wave varies in accordance with the amplitude of the modulating signal

2) The amplitude of carrier wave remains constant, frequency changes in accordance with the modulating signal

3) The amplitude of carrier wave varies in accordance with the frequency of the modulating signal

4) The amplitude changes in accordance with the wave length of the modulating signal

KEY:1

48. In amplitude modulation, carrier wave frequencies are .... than that compared to those in frequency modulation

1) lower 2) higher

3) same 4) lower or higher

KEY:1

49. Draw backs of Amplitude modulation

1) During transmission extreneous noise creeps in.

2) Most of the transmitting power is wasted, as it does not contain useful information.

3) The reception is not clear in the case of weak signals due to noise

4) The receiver set is complex

KEY:2

50. The limitation of amplitude modulation is

1) clear reception 2) high efficiency

3) small operating range 4) good audio quality

KEY:3

51. In frequency modulation

1) Frequency of CW remains constant but amplitude changes in accordance with modulating wave frequency

2) Frequency of CW changes in accordance with the modulating wave frequency but the amplitude also changes.

3) Frequency of CW changes in accordance with the frequency of modulating wave frequency but the amplitude remains constant.

4) Frequency of CW changes in accordance with the amplitude of modulating wave amplitude

KEY:3

52. Device that converts one form of energy into another is called

1) transmitter 2) transducer

3) receiver 4) channel

KEY:2

53. In T.V. broadcasting both picture and sound are transmitted simultaneously. In this

1) audio signal is frequency modulated and video signal is amplitude modulated

2) both audio and video signals are frequency modulated

3) audio signal is amplitude modulated and video signal is frequency modulated

4) both audio and video signals are amplitude modulated

KEY:1

54. Effective power radiated by an antenna is

1) Proportional to the square at the length of the antenna

2) inversely proportional to the wavelength

3) inversely proportional to the square of the wavelength

4) proportional to the wavelength

KEY:3

55. Band width of an optical fiber is

1) more than 100 GHz 2) few kHz

3) less than 1MHz 4) less than 1GHz

KEY:1

56. The concepts of communication are

a) mode of communication

b) need for modulation

c) types of modulation

d) detection of modulated wave

1) a, b, c are true 2) b, c, d are true

3) c, d, a are true 4) a, b, c & d are true

KEY:4

57. Basically, the product modulator is

1) An amplifier 2) A mixer

3) A frequency separator 4) A phase separator

KEY:2

58. Which of the following is the disadvantage of FM over AM
1) Larger band width requirement
2) Larger noise
3) Higher modulation power
4) Low efficiency
KEY:1
59. Audio signal cannot be transmitted as such because
1) the signal has more noise
2) the signal cannot be amplified for distance communication
3) the transmitting antenna length is very small to design
4) the transmiting antenna length is very large and impracticable
KEY:4

60. The waves relavent to telecommunications are
1) visible light 2) infrared
3) ultraviolet 4) microwave
KEY:4
61. While tuning in a certain broad cast station with a receiver, we are actually
1) varying the local oscillator
2) varying the resonant frequency of the circuit for the radio signal to be picked up
3) tuning the antenna
4) varying the current of receiver set
KEY:3
62. Long distance short-wave radio broadcasting uses
1) Ground wave 2) Ionospheric wave
3) Direct wave 4) Sky wave
KEY:3
63. Advantage of HF transmission is
A) that the length of antenna is small
B) that the antenna can be mounted at larger heights
C) that the power radiated is more for a given length of antenna
1)a & b 2) b& c 3) a & c 4) a,b & c
KEY:4
64. A transducer used at the transmitting end, serves the purpose of converting
1) electrical signal to sound form
2) sound signal to electrical form
3) electrical signal to magnetic form
4) sound signal to magnetic form
KEY:2
65. High frequency waves are
1) absorbed by F layer
2) reflected by the E layer
3) capable of use for long distance transmission
4) affected by the solar cycle
KEY:2

66. As the e.m. waves travel in free space
1) absorption takes place 2) attennuation takes place
3) refraction takes place 4) reflection takes place
KEY:2
67. A: The frequency band of VHF is greater than UHF of TV transmission
B: Optical fiber transmission has frequency band of 1 THz to 1000 THz
1) A is true but B is false
2) A is false but B is true
3) A and B are false 4) A and B are true
KEY:2
68. The electromagnetic waves of frequency 80 MHz and 200 MHz
1) can be reflected by troposphere 2) can be reflected by ionosphere
3) can be reflected by mesosphere
4) cannot be reflected by any layer of earth's atmosphere
KEY:4
69. Micro wave link repeaters are typically 50 km apart
1) because of atmospheric attenuation
2) because of the earths curvature
3) to ensure that signal voltage may not harm the repeater
4) to reduce the interference of microwaves
KEY:2
70. Attenuation of ground waves is due to
1) Diffraction effect
2) Radio waves induce currents in the ground because of the polarisation
1) a & b are true 2) Only a is true
3) Only b is true 4) Both a & b false.
KEY:1
71. In a communication system, noise is most likely to affect the signal
1) At the transmitter
2) In the channel or in the transmission line
3) In the information source
4) At the receiver
KEY:2
72. The ground wave eventually disappears, as one moves away from the transmitter, because of
1) interference from the sky wave
2) loss of line of signal condition
3) maximum single - hop distance limitation
4) diffraction effect causing tilting of the wave
KEY:4
73. The range of ground wave transmission can be increased by
1) increasing the power of transmitter with the use of HF
2) increasing the power of transmitter with the use of VLF
3) decreasing the power and increasing the frequency of radio waves
4) decreasing both power and frequency of radio waves
KEY:2

74. In amplitude modulation

1) only amplitude is changed but frequency remains same

2) both amplitude & frequency changes equally

3) both amplitude & frequency changes unequally

4) only frequency changes but amplitude remains constant.

KEY:1

75. Space wave propagation is used in

a) microwave communication

b) satellite communication

c) TV transmission

1) Only a 2) Both a & b

3) Both b & c 4) a ,b & c

KEY:4

76. Frequencies in the UHF range normally propagate by means of:

1) Ground waves 2) Sky waves.

3) Surface waves 4) Space waves.

KEY:4

77. The TV broad casting bands are

1) MF and HF bands 2) VHF and UHF bands

3) UHF and SHF bands 4) SHF and EHF band

KEY:2

78. When a sky wave is reflected onto the ground

1) frequency of the reflected wave is different to that of incident wave

2) there is a phase difference introduced to the reflected wave

3) the reflected wave is out of phase with incident wave and reach the receving antenna along with the direct wave from transmitting antenna causing interference.

4) the waves are not reflected by the ground.

KEY:3

79. The electromagnetic waves of frequency 2 MHz to 30 MHz are

1) In ground wave propagation 2) In sky wave propagation

3) In microwave propagation 4) In satellite communication

KEY:2

80. The audio signal

1) can be sent directly over the air for large distance

2) can not be sent directly over the air for large distance

3) possesses very high frequency

4) possesses very low frequency

KEY:2

81. Among the following frequencies one will be suitable for beyond-the horizon communication using sky waves is

1) 10 kHz 2) 10 MHz 3) 1GHz 4)1000GHz

KEY:2

82. Among the following, the waves which can penetrate the ionosphere are

1)10GHz 2)10MHz 3)20MHz 4) 25 MHz

KEY:1

83. Through which mode of propagation, the radio waves can be sent from one place to another
1) Ground wave propagation   2) Sky wave propagation
3) Space wave propagation   4) All of them
KEY:4

84. The frequency above which radiation of electrical energy is practical is
1) 0.2 kHz 2) 2 kHz 3) 20 kHz 4) 2Hz
KEY:3
85. In a communication system, noise is most likely to affect the signal
1) at the transmitter   2) in the medium of transmission
3) information source signal   4) at the destination
KEY: 2
86. The radio waves of frequency 300 MHz to 3000 MHz belong to
1) High frequency band   2) Very high frequency band
3) Ultra high frequency band   4) Super high frequency band
KEY:3
87. Coaxial cable is an example of
1) Optical fibre   2) Free space
3) Wire medium   4) Sea medium
KEY:3
88. The attenuation in optical fibre is mainly due to 1) Absorption 2) Scattering
3) Neither absorption nor scattering
4) Both 1 and 2
KEY:4
89. Indicate which one of the following system is digital
1) Pulse position modulation   2) Pulse code modulation
3) Pulse width modulation   4) Pulse amplitude modulation
KEY:2
90. Consider telecommunication through optical fibres. Which of the following statements is not true [AIEEE 2003]
1) Optical fibres may have homogeneous core with a suitable cladding
2) Optical fibres can be of graded refractive index
3) Optical fibres are subject to electromagnetic interference from outside
4) Optical fibres have extremely low transmission loss
KEY:3
91. The phenomenon by which light travels in an optical fibres is
1) Reflection   2) Refraction
3)Total internal reflection 4) Transmission
KEY:3
92.. In which of the following remote sensing technique is not used
1)Forest density   2)Pollution
3)Wetland mapping   4)Medical treatment
KEY:4

93. Choose the one that best describes the two statements

A. Sky wave signals are used for long distance radio communication these signals are in general less stable than ground wave signals.

B. The state of ionosphere varies from hour, day to day and season to season.

1) both A and B are true

2) both A and B are false

3) A is true and B is false

4) A is false and B is true (JEE Mains-2011)

KEY:1

94. Choose the correct statement.

1) In the frequency modulation, the amplitude of high frequency carrier wave is mode to vary in proportion to the frequency of audio signal

2) In amplitude modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

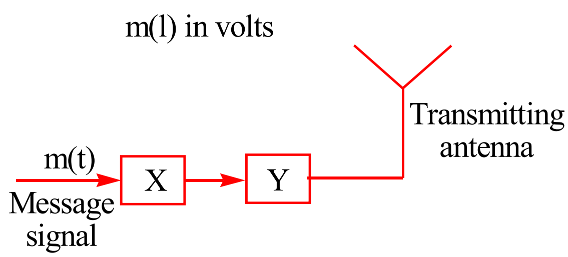
3) In amplitude modulation, the frequency or the high frequency carrier wave is made to vary in proportion to the amplitude of audio signal

4) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of audio signal.

(JEE Main -2016)

KEY:2

95. Figure shows a block diagram of a transmitter. Indentify the boxes X and Y ?



1) Amplitude modulator, detector

2) Detector, power amplifier

3) Amplitude modulator, power amplifier

4) Capacitor, Detector

KEY:3

96. Match the frequency band with the type of use

Frequency Band	Type of use
a) LF	e) Radio Broad casting
b) HF	f) Marine and navigational aid
c) VHF	g) Satellite communication
d) SHF	h) TV Broad casting

1) a - e; b - f; c - h; d - g

2) a - f; b - e; c- g; d- h

3) a - f; b - e; c - h; d - g

4) a - e; b - g; c - f; d - h

KEY:3

97. Match layer of ionosphere and height of the layer from surface of earth for atmosphere

a) D	e) 250 - 400 km
b) E	f) 100 km
c) F	g) 65 - 75 km
d) F_2	h) 170 - 190 km

1) a-g; b-f; c-h; d-e 2) a-h; b-g; c-e; d-f
3) a-g; b-h; c-f; d-e 4) a-g; b-h; c-e; d-f

KEY:1

98. The frequency band used for radar relay systems & T.V is
1) UHF 2) VLF 3) VHF 4) EHF

KEY:1

99. Match layer of ionosphere and frequencies most affected

a) D	e) helps surface waves and reflects HF
b) E	f) efficiently reflects HF waves
c) F	g) partially absorbs HF
d) F_2	h) reflects LF and absorbs MF

1) a-g; b-f; c-h; d-e 2) a-h; b-e; c-g; d-f
3) a-h; b-e; c-f;d -g 4) a-e; b-h; c-g; d-f

KEY:2

100. Match the frequency band with the type of use service frequency band

a) AM broadcast	e) 88 - 108 MHz
b) satellite communication	f) 896 - 935 MHz
c) FM broadcast	g) 540 - 1600 kHz
d) Cellular mobile	h) 3.7 - 6.5 GHz

1) a-g; b-h; c-f; d-e 2) a-h; b-g; c-e; d-f
3) a-f; b-h; c-e; d-g 4) a-g; b-h; c-e; d-f

KEY:4

101. Match List-1 with List-2

List-1 Communication mode	List-2 Example
a) Point to point communication	e) RADAR
b) broadcast communication	f) AM Radio
c) Line of sight communication	g) FM Radio
d) Satellite communication	h) Traditional telephony
	i) Mobile telephony
	j) TV

1) a-h; b-f,g,j; c-e;d-i,j 2) a-e; b-i,j; c-h;d-g,f
3) a-f ; b-h,i,j; c-g;d-e 4) a-g; b-f,g,j; c-e;d-i,j

KEY:1

102. Match List-1 with List-2

List-1	List-2
Name	Frequencies most affected
a) Part of stratosphere(D)	e) VHF(upto several GHz)
b) Part of mesophere (F_1)	f) reflects LF
c) Part of thermosphere(F_2)	g) Partially absorbs HF
d) Troposphere	h) Efficiently reflects HF

1) a-e;b-f,g,h;c-i;d-h   2) a-f;b-f,g,h;c-f;d-e
3) a-f,g,h;b-g;c-i;d-e   4) a-g;b-f,g,h;c-f;d-e

KEY:3

103. Match List-1 with List-2

List-1	List-2
Type of propagation	Frequencies
a) sky waves	e) 1.5 MHz
b) space wave	f) 20 MHz
c) ground wave	g) 30MHz
d) micro wave	h) 50MHz
	i) 3 GHz

1) a-e,i;b-e;c-h;d-i   2) a-f,g;b-h;c-e;d-i
3) a-i,g;b-e;c-h;d-f   4) a-g;f-h;c-i,b;d-e

KEY:2

104. In AM, the centpercent modulation is achieved when
1) Carrier amplitude = signal amplitude
2) Carrier amplitude = signal amplitude
3) Carrier frequency = signal frequency
4) Carrier frequency = signal frequency

KEY:1

105. A signal emitted by an antenna from a certain point can be received at another point of the surface in the form of
1) sky wave   2) ground wave
3) sea wave   4) both 1 and 2

KEY:4

106. The process of superimposing signal frequency (i.e., audio wave) on the carrier wave is known as
1) Transmission   2) Reception
3) Modulation   4) Detection

KEY:3

107. The difference between phase and frequency modulation
1) practically they are same but theoretically they differ
2) lies in the poorer audio response of phase modulation
3) lies in the poorer audio response of frequency modulation
4) lies in the definitions of modulation and their modulation index

KEY:1

108. The better propagation mode to propagate television frequency and radar signals is

1) satellite communication
2) ground propagation
3) polarized communication
4) skywave communication

KEY:1

109. The need for doing modulation is

1) to increase the intensity of audio signal
2) to decrease the intensity of audio signal
3) to transmit audio signal to large distances
4) to increase the frequency of the audio signal

KEY:3

110. Amplitude modulation is used for broad casting because

1) it is more noise immune
2) it requires less transmitting power
3) it has simple circuit
4) it has high fidelity (faithful reproduction)

KEY:3

111. A: It is necessary for transmitting antenna must be at same height as that of receiving antenna for line of sight communication.

B: EM waves of frequency beyond 40 MHz, propagate as space waves.

1) both A and B are correct
2) both A and B are wrong
3) only A is correct
4) only B is correct

KEY:4

112. AM is used for broad casting because,

1) it is more noise immune than other modulating systems
2) it requires less transmitting power compared with other systems
3) its use avoids receiver complexity
4) no other modulation system can provide the necessary bandwidth, faithful transmission.

KEY:3

COMMUNICATION SYSTEMS

PREVIOUS JEE MAINS QUESTIONS

1. An amplitude modulated wave is represented by the expression $v_m = 5(1 + 0.6\cos 6280t)\sin(211 \times 10^4 t)$ volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively: [Sep. 02, 2020 (I)]

(a) $\frac{3}{2}\mathrm{V}, 5\mathrm{V}$ (b) $\frac{5}{2}\mathrm{V}, 8\mathrm{V}$ (c) 5 V, 8 V (d) 3 V, 5 V

SOLUTION : . (b)

From the given expression,

$$V_m = 5(1 + 0.6\cos 6280t)\sin(211 \times 10^4 t)$$

Modulation index, $\mu = 0.6$

$$A_m = \mu A_c$$

$$\frac{A_{\max} + A_{\min}}{2} = A_c = 5 \text{ (i)}$$

$$\frac{A_{\max} - A_{\min}}{2} = A_m = 3 \text{ (ii)}$$

From equation $(\mathrm{i}) + (\mathrm{ii})$,

Maximum amplitude, $A_{\max} = 8.$

From equation $(\mathrm{i}) - (\mathrm{ii})$,

Minimum amplitude $A_{\mathrm{mm}} = 2.$

2. In an amplitude modulator circuit, the carrier wave is given by, $\mathrm{C}(t) = 4\sin(20000\pi t)$ while modulating signal is given by, $m(t) = 2\sin(2000)$. The values ofmodulation index and lower side band frequency are: [12 April 2019 II]

(a) 0.5 and 10 fflz (b) 0.4 and 10 fflz (c) 0.3 and 9 kHz (d) 0.5 and 9 kHz

SOLUTION : (d)

$$\text{Modulation index, } \mu = \frac{A_m}{A_c} = \frac{2}{4} = 0.5 \text{ Given, fe} = \frac{20000\pi}{2\pi} = 10000 \text{ Hz.}$$

$$\text{and fm} = \frac{2000\pi}{2\pi} = 1000 \text{ Hz.}$$

$$\text{LSB} = f_e - f_m = 10000 - 1000 = 9000 \text{ Hz.}$$

3. A message signal of frequency 100 MHz and peak voltage100 V is used to execute amplitude modulation on a carrier wave offrequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band fiequencies are: [10 April 2019 II]

(a) 4; 1×10^8 Hz (b) 4; 2×10^8 Hz (c) $0.25; 2 \times 10^8$Hz (d) $0.25; 1 \times 10^8$T

SOLUTION : (c)

$$\text{Range of fiequency} = (f_c - f_m) \text{ to } (f_c + f_m)$$

$$\text{Band width} = 2f_m = 2 \times 100 \times 10^6 \text{ Hz}$$

$$= 2 \times 10^8 \text{ Hz}$$

$$\text{and Modulation index} = \frac{A_m}{A_c} = \frac{100}{400} = 0.25$$

4. A signal Acosoot is transmitted using v_0 sin ooOt as carrier wave. The correct amplitude modulated (AM) signal is: [9 April 2019 I]

(a) $v_0$sino)$t + \frac{A}{2}$ sin $(0)0 - c0)t + \frac{A}{2}$ sin $(c0_0 + 0))t$ (b) v_0 sin $[0)_0(1 + 0.01$Asino)t)t]

(c) $v_0$sino)t + Acosoot (d) $(v_0 + A)$ cosoot sinooOt

SOLUTION : (a)

5. The physical sizes ofthe transmitter and receiver antenna in a communication system are:

[9 April 2019 II]

(a) independent ofboth carrier and modulation fiiequency

(b) inverselyproportional to carrier frequency

(c) inversely proportional to modulation frequency

(d) proportional to carrier fiiequency

SOLUTION : . (b)

Size ofantenna, $l=\frac{\lambda}{4}$. As $\lambda=\frac{C}{f}\, l \propto \frac{1}{f}$

6. The wavelength ofthe carrier waves in a modern optical fiber communication network is close to: [8 April 2019 I]

(a) 24(X)nm (b) 15(K)nm (c) 600nm (d) 900nm

SOLUTION : (b)

Carrier waves of wavelength 1500 nm is used in modern optical fiber communication.

7. In a line ofsight ratio communication, a distance ofabout 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is $70\mathrm{m}$, then the minimum height of the transmitting antenna should be: [8 April 2019 II]

(Radius ofthe Earth $=6.4\times 10^{6}\mathrm{m}$).

(a) $20\mathrm{m}$ (b) $51\mathrm{m}$ (c) $32\mathrm{m}$ (d) $40\mathrm{m}$

SOLUTION : . (c)

$$\text{LOS}=\sqrt{2h_{T}R}+\sqrt{2h_{R}R}$$

$$\text{or } 50\times 103=\sqrt{2h_{T}\times 64\times 10^{6}}+\sqrt{2\times 70\times 64\times 40^{6}}$$

On solving, $\mathrm{h_T}=32\mathrm{m}$

8. A $100\mathrm{V}$ carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index? [12 Jan. 2019 I]

(a) 0.3 (b) 0.5 (c) 0.6 (d) 0.4

SOLUTION : (c)

$$\text{Maximum amplitude} = E_m + E_c = 160$$

$$E_m + 100 = 160$$

$$E_m = 160 - 100 = 60$$

$$\text{Modulation index, } \mu = \frac{E_m}{E_c} = \frac{60}{100}$$

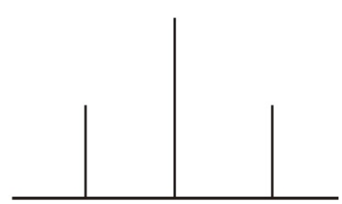
$$\mu = 0.6$$

9. To double the covering range of a TV transmittion tower, its height should be multiplied by: [12 Jan 2019 II]

(a) $\frac{1}{\sqrt{2}}$ (b) 2 (c) 4 (d) $\sqrt{2}$

SOLUTION : (c)

$$\text{As we know, Range} = \sqrt{2hR}$$



therefore to double the range height h' should be 4 times.

10. An amplitude modulated signal is given y $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$. Here t is in seconds. The sideband fiequencies (in fflz) are, [Given $\pi = 22/7$]

[11 Jan 2019 II]

(a) 1785 and1715 (b) 178.5 and 171.5

(c) 89.25 and 85.75 (d) 892.5 and 857.5

SOLUTION : . (c)

Equation given $V(t) = 10[1 + 0.3\cos(2.2 \times 10^4)]\sin\left(5.5 \times 10^5 t\right)$

$$= 10 + 1.5[\sin(57.2 \times 10^4 t) + \sin(52.8 \times 10^4 t)]$$

$$0)_{cw} + 0) = 57.2 \times 10^4 = 2\pi f_1$$

$$f_1 = \frac{57.2 \times 10^4}{2 \times \left(\frac{22}{7}\right)} = 9.1 \times 10^4 = 91\text{KHz}$$

$$0)_c - 0)_w = 52.8 \times 10^4$$

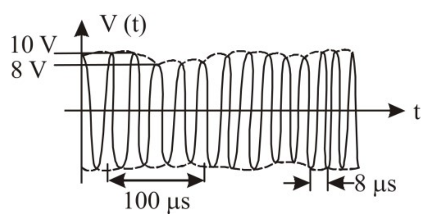
$$f_2 = \frac{52.8 \times 10^4}{2 \times \left(\frac{22}{7}\right)} = 84 \text{ KHz}$$

$$f_c - f_w \; f_c \; f_c + f_w$$

Upper side band fiiequency (f_1) is $f_1 = f_c - f_w = \frac{52.8 \times 10^4}{2\pi} \approx 85.00\text{kHz}$

Lower side band frequency (f_2) is $f_2 = f_c + f_w = \frac{57.2 \times 10^4}{2\pi} \approx 90.00\text{kHz}$

11. An amplitude modulated signal is plotted below:



Which one of the following best describes the above signal? **[11 Jan. 2019 II]**

(a) $\left(9 + \sin\left(2.5\pi \times 10^5 t\right)\right) \sin(2\pi \times 10^4 t)\text{V}$

(b) $\left(1 + 9\sin(2\pi \times 10^4 t)\right) \sin\left(2.5\pi \times 10^5 t\right)\text{V}$

(c) $\left(9 + \sin(2\pi \times 10^4 t)\right) \sin\left(2.5\pi \times 10^5 t\right)\text{V}$

(d) $\left(9+\sin\left(4\pi\times10^{4}t\right)\right)\sin\left(5\pi\times10^{5}t\right)V$

SOLUTION : (c)

After analysing the graph we may conclude that

(i) Amplitude varies as $8-10V$ or 9 ± 1

(ii) Two time period as 100 μs (signal wave) & $8\mu s$ (carrier wave)

So, equation ofAM signal is $\left[9\pm1\sin\left(\left(\ \ \right)\right)^{\sin}\left(\left(\ \ \right)\right)\right]$

$$=[9\pm\sin\left(2\pi\times10^{4}t\right)]\sin\left(2.5\pi\times10^{5}t\right)V]$$

12. A TV transmission tower has a height of140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted $fi_{i}om$ this tower in LOS (Line of Sight) mode? (Given: radius ofearth $=6.4\times10^{6}m$). [10 Jan. 2019 I]

(a) 65km (b) 48km (c) 80hn (d) 40km

SOLUTION : . (a)

Maximum distance upto which signal can be broadcasted $d_{max}=\sqrt{2Rh_{T}}+\sqrt{2Rh_{R}}$

where h_T and h_R are heights of transmission tower and receiving antenna respectively.

Putting values R, hT and hR $d_{max}=\sqrt{2\times64\times106}\left[\sqrt{140}+\sqrt{40}\right]$

or, $d_{max}=65$ km

13. The modulation fiiequency of an AM radio station is 250 kHz, which is 10% ofthe carrier wave. Ifanother AM station approaches you for license what broadcast frequency will you allot? [10 Jan. 2019 I]

(a) $2750kHz$ (b) $2900kHz$ (c) $2250kHz$ (d) $2000kHz$

SOLUTION : (d)

According to question, modulation frequency, 250 Hz is 10% ofcarrier wave

$$f_{camer}=\frac{250}{0.1}=2500\text{ KHZ}$$

Range ofsignal $2500 \pm 250\text{KHz} = 2250\text{E}$ to 2750 Hz For 2000KHZ

$$f_{mod} = 200\text{Hz}$$

Range $= 1800$ KHZ to 2200 KHZ

14. In a communication system operating at wavelength 800 nm, only one percent ofsource frequency is available as signal bandwidth. The number ofchannels accomodated for transmitting TV signals ofband width 6 MHz are(Take velocity oflight $c = 3 \times 10^8 \text{m/s}$, $h = 6.6 \times 10$ 34J - s) [9 Jan. 2019 II]

(a) 3.75×10^6 (b) 3.86×10^6 (c) 6.25×10^5 (d) 4.87×10^5

SOLUTION : . (c)

$$\text{Frequency, } f = \frac{V}{\lambda} = \frac{3\times10^8}{8\times10^{-7}} = \frac{30}{8} \times 10^{14} \text{ Hz} = 3.75 \times 10^{14} \text{ Hz}$$

$$1\% \text{ off} = 0.0375 \times 10^{14} \text{ Hz}$$

$$= 3.75 \times 10^{12}\text{Hz} = 3.75 \times 10^6\text{MHz}$$

As we know, number of channels accomodated for transmission $=$

$$\frac{\text{tota1bandwidthofChanne1}}{\text{bandwidthneededperchanne1}} = \frac{3.75\times10^6}{6} = 6.25 \times 10^5$$

15. Atelephonic communication service is working at carrier frequency of 10 GHz. Only10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz? [2018]

(a) 2×10^3 (b) 2×10^4 (c) 2×10^5 (d) 2×10^6

SOLUTION : (c)

Ifn $=$ no. ofchannels

10% of 10 GHz $= n \times 5$ KHz or,

$$\Rightarrow n = 2 \times 10^5$$

16. A carrier wave ofpeak voltage 14V is used for transmitting a message signal. The peak voltage of modulating signal given to achieve a modulation index of80% will be: [2018]

(a) $11.2V$ (b) $7V$ (c) $22.4V$ (d) $28V$

SOLUTION : (a)

Given: modulation index m=80% $= 0.8$

$$E_c = 14V, E_m = ?$$

$$\text{using, } m = \frac{E_m}{E_c} \Rightarrow E_m = m \times E_c = 0.8 \times 14 = 11.2V$$

17. The number of amplitude modulated broadcast stations that can be accommodated in a 300 kHz band width for the highest modulating fiiequency 15fflz will be: [Online Apri115, 2018]

(a) 20 (b) 10 (c) 8 (d) 15

SOLUTION : . (b)

Given, modulating $\text{fi}_{\text{i}}\text{equency} f_m = 15$ KHz

Bandwidth ofone channel $= 2f_m = 30\text{kHz}$

No ofchannels accommodate $= \frac{300\text{kHz}}{30\text{kHz}} = 10$

18. The carrier fiiequency of a transmitter is provided by a tank circuit ofa coil ofinductance $49\mu\text{H}$ and a capactiance of$2.5\text{n}\Gamma$. It is modulated by an audio signal of $\text{l}2\text{kHz}$. The frequency range occupied by the side bands is: [Online Apri115, 2018]

(a) $18\text{Rz} - 30\text{kHZ}$ (b) $63\text{Rz} - 75\text{kHZ}$

(c) $442\text{kHz} - 466\text{kHz}$ (d) $13482\text{kHz} - 13494\text{kHz}$

SOLUTION : (c)

Given : Inductance, $L = 49\mu\text{H} = 49 \times 10^{-6}\text{H}$, capacitance $C = 2.5\text{n}\Gamma = 2.5 \times 10^{-9}\Gamma$

$$\text{Using } (j) = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{49\times10^{-6}\times\frac{25}{10}\times10^{-9}}} = \frac{1}{7\times5\times10^{-8}} = \frac{10^8}{7\times5}$$

$$\text{or, } \frac{10^8}{7\times5} = 2\pi \times f = 2 \times \frac{22}{7} \times f\ ((j)^{=2\pi f)}$$

$$\text{or, } f=\frac{10^{7}}{22}=\frac{10^{4}}{22}\text{kHz}=454.54\text{kHz}$$

Therefore frequency range $454.54 \pm 12\text{kHz}$ i. e., 442 $\mathbb{HE}$ $-$ 466 fflz

19. In amplitude modulation, sinusoidal carrier fiequencyused is denoted by $(i\text{J}_{\text{c}}$ and the signal frequency is denoted by $(i\text{J}_{m}$. The bandwidth $(\Delta(i\text{J}_{m})$ ofthe signal is such that $\Delta(i\text{J}_{m} < (i\text{J}_{\text{c}}$. Which ofthe following frequencies is not contained in the modulated wave? [2017]

(a) $(i\text{J}_{m} + (i\text{J}_{\text{c}}$ (b) $0)_{\text{c}} - 0)_{m}$ (c) $\text{t}0_{m}$ (d) $(i\text{J}_{\text{c}}$

SOLUTION : (c)

Modulated carrier wave contains frequency $\omega_{cand}\ \text{t}i\text{J}_{c} \pm Ci\text{J}_{m}$

20. A signal is to be transmitted through a wave of wavelength λ, using a linear antenna. The length 1 ofthe antenna and effective power radiated P_{eff} will be given respectively as :

(K is a constant ofproportionality) [Online April 9, 2017]

(a) $\lambda, \text{P}_{\text{eff}}=\text{K}\left(\frac{1}{\lambda}\right)^{2}$ (b) $\frac{\lambda}{8}, \text{P}_{\text{eff}}=\text{K}\left(\frac{1}{\lambda}\right)$

(c) $\frac{\lambda}{16}, \text{P}_{\text{eff}}=\text{K}\left(\frac{1}{\lambda}\right)^{3}$ (d) $\frac{\lambda}{5}, \text{P}_{\text{eff}}=\text{K}\left(\frac{1}{\lambda}\right)^{\frac{1}{2}}$

SOLUTION : (a)

Length ofantenna $=$ comparable to λ

Power radiated by linear antenna inversely depends on the square ofwavelength and directly on the length ofthe antenna.

$$\text{Hence, Power P}=\mu\left(\frac{1}{\lambda}\right)^{2}$$

here $\mu=\text{K}$

21. Asignal offiiequency20 kHz and peak voltage of5 Volt is used to modulate a carrier wave offrequency 1.2 MHz and peak voltage 25 Volts. Choose the correct statement.

[Online April 8, 2017]

(a) Modulation index $=5$, side frequency bands are at 1400 fflz and 1000 fflz

(b) Modulation index $= 5$, side frequency bands are at 21.2 ℍE and 18.8 ℍE

(c) Modulation index $= 0.8$, side frequency bands are at 1180 fflz and 1220 fflz

(d) Modulation index $= 0.2$, side frequency bands are at 1220 fflz and 1180 fflz

SOLUTION : (d)

$$\text{Modulation index } (m) = \frac{V_m}{V_0} = \frac{5}{25} = 0.2$$

Given, fiiequency of carrier wave $(f_c) = 1.2 \times 10^6$ Hz$= 1200\text{kHz}$.

Frequency of signal $(f_0) = 20\text{kHz}$.

Side frequency bands $= f_c \pm f_0$

$f_1 = 1200 - 20 = 1180$ fflz

$f_2 = 1200 + 20 = 1220$fflz

22. Choose the correct statement: [2016]

(a) In fiiequency modulation the amplitude of the high fiiequency carrier wave is made to vary in proportion to the amplitude ofthe audio signal.

(b) In fiiequency modulation the amplitude of the high fiiequency carrier wave is made to vary in proportion to the fiiequency of the audio signal.

(c) In amplitude modulation the amplitude of the high fiiequency carrier wave is made to vary in proportion to the amplitude ofthe audio signal.

(d) In amplitude modulation the fiiequency of the high fiiequency carrier wave is made to vary in proportion to the amplitude ofthe audio signal.

SOLUTION : (c)

In amplitude modulation, the amplitude ofthe high $afi_{iequenc}mp1itu(t^{rcarrierwavemade}_{eofaudi_{os}igna1}$ to vary in proportional to the $\sim_{L}-\backslash_{--}$ Audio signal

q( )$_{J}^{\prime}$11(1t$^{(}$t · t1\1$l$tt^{1}{] Camer wave

$\lambda_{\sim}Reject^{((}r_{l;}$}J↓V\1 T$^{\backslash 1\backslash}$/L†1^{n}L^{Reject}(

23. Amodulated signal $C_m(t)$ has the form $C_m(t) = 30 \sin 300\pi t + 10(\cos 200\pi t - \cos 400\pi t)$. The carrier frequency f_c, the modulating fiiequency (message fiiequency) f_{c0} and the modulation indix μ are respectively given by: [Online Apri110, 2016]

(a) $f_c = 200$ Hz; $f_v = 50$ Hz; $\mu = \frac{1}{2}$

(b) $f_c = 150$ Hz; $b = 50$ Hz; $\mu = \frac{2}{3}$

(c) $f_c = 150$ Hz; $b = 30$ Hz; $\mu = \frac{1}{3}$

(d) $f_c = 200$ Hz; $b = 30$ Hz; $\mu = \frac{1}{2}$

SOLUTION : (b)

Comparing the given equation with standard modulated signal wave equation, $m =$

$$A_c \sin o)_c t + \frac{\mu A_c}{2}$$

$$\cos (0)_c - 0)_s)t - \frac{\mu A_c}{2} \cos (0)_c + 0)_s)t$$

$$\mu \frac{A_c}{2} = 10 \Rightarrow \mu = \frac{2}{3} \text{(modulation index)}$$

$$A_c = 30$$

$$0)_c - 0)_s = 200\pi$$

$$0)_{cs} + 0) = 400\pi$$

$$\Rightarrow f_c = 150, f_s = 50 \text{ Hz.}$$

24. An audio signal consists of two distinct sounds: one a human speech signal in the fiiequency band of200 Hz to 2700 Hz, while the other is a high frequency music signal in the fiiequencyband of10200 Hz to 15200 Hz. The ratio of the AM signal bandwidth required to send both the signals together to the AM signal bandwidth requried to send just the human speech is:

[Online April 9, 2016]

(a) 2

(b) 5

(c) 6

(d) 3

SOLUTION : . (c)

$$\text{Ratio ofAM signal Bandwidths} = \frac{15200-200}{2700-200} = \frac{15000}{2500} = 6.$$

25. A signal of 5 kHz fiiequency is amplitude modulated on a carrier wave offrequency 2 MHz. The fiiequencies ofthe resultant signal is/are: [2015]

(a) 2005 ℍE, 2000 fflz and 1995 fflz

(b) 2000 fflz and 1995 fflz

(c) 2 MHz only

(d) 2005 fflz and 1995 fflz

SOLUTION : (a)

Amplitude modulated wave consists of three frequencies are $0)_{cmcm}+0), 0), 0)-0)$

i.e. 2005 ℍE, 2000kHz, 1995 ℍb

26. Long range radio transmission is possible when the radio waves are reflected from the ionosphere. For this to happen the frequency ofthe radio waves must be in the range:

[Online Apri119, 2014]

(a) 80 - 150 MHz

(b) 8 - 25 MHz

(c) 1 - 3 MHz

(d) 150 - 1500kHz

SOLUTION : . (b)

Frequency ofradio waves for sky wave propagation is2 MHZ to 30 MHZ.

27. For sky wave propagation, the radio waves must have a frequency range in between:

[Online Apri112, 2014]

(a) 1 MHz to 2 MHz

(b) 5 MHz to 25 MHz

(c) 35 MHz to 40 MHz

(d) 45 MHz to 50 MHz

SOLUTION : . (b)

Sky wave propagation is suitable for fiequencyrange 5 MHz to 25 MHz.

28. A transmitting antenna at the top ofa tower has height 32 m and height ofthe receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in line of sight (LOS) mode? [Online April 9, 2014]

(a) 55.4km (b) 45.5km (c) 54.5km (d) 455km

SOLUTION : (b)

Given: $h_R = 32m$

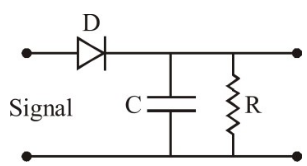
$h_T = 50m$

Maximum distance, $d_M = ?$

Applying, $d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$= \sqrt{2 \times 64 \times 10^6 \times 50} + \sqrt{2 \times 64 \times 10^6 \times 32} = 45.5 \text{ km}$$

29. Adiode detector is used to detect an amplitudemodulated wave of60% modulation byusing a condenser ofcapacity 250 picofarad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it. [2013]



(a) 10.62 MHz (b) 10.62 KE (c) 5.31 MHz (d) 5.31 ℍb

SOLUTION : . (b)

Given : Resistance $R = 100$ kilo ohm$= 100 \times 10^3 \Omega$

Capacitance $C = 250$ picofarad$= 250 \times 10^{-12} \Gamma$

$$\tau = RC = 100 \times 10^3 \times 250 \times 10^{-12} \text{ sec}$$

$$= 2.5 \times 10^7 \times 10^{-12} \text{ sec} = 2.5 \times 10^{-5} \text{ sec}$$

The higher frequency whcih can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC} = \frac{1}{2\pi \times 0.6 \times 2.5 \times 10^{-5}} HZ$$

$$= \frac{100\times10^4}{25\times1.2\pi} HZ = \frac{4}{1.2\pi}\times10^4 Hz = 10.61 \text{ KHz}$$

This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate ofdecay modulated singnal voltage for proper detection ofmdoulated signal.

30. Which of the following modulated signal has the best noise – tolerance?

[Online April 25, 2013]

(a) Long – wave **(b) Short – wave**

(c) Medium – wave **(d) Amplitude – modulated**

SOLUTION : . (b)

Short – wave has the best noise tolerance.

31. Which ofthe following statement is NOT correct? [Online April 23, 2013]

(a) Ground wave signals are more stable than the sky wave signals.

(b) The critical fiiequency of an ionospheric layer is the highest fiiequency that will be reflected back by the layer when it is vertically incident.

(c) Electromagnetic waves of fieqencies higher than about 30 MHz cannot penetrate the ionosphere.

(d) Sky wave signals in the broadcast frequency range are stronger at night than in the daytime.

SOLUTION : (c)

Above critical frequency (f_c) , an electromagnetic wave penetrates the ionosphere and is not reflected by it

32. This question has Statement – l and Statement – 2. Of the four choices given after the Statements, choose the one that best describes the two Statements.

Statement – l: Short wave transmission is achieved due to the total internal reflection of the e – m wave from an appropriate height in the ionosphere.

Statement - 2: Refiactive index ofa plasma is independentofthe frequency of e - m waves.

[Online April 22, 2013]

(a) Statement - 1 is true, Statement - 2 is false.

(b) Statement - 1 is false, Statement - 2 is true.

(c) Statement - l is true, Statement - 2 is true but Statement - 2 is not the correct explanation of statement - l.

(d) Statement - l is true, Statement - 2 is true and Statement - 2 is the correct explanation of Statement - l.

SOLUTION : (a)

Effective refractive index ofthe ionosphere

$$n_{eff} = n_0 \left[1 - \frac{80.5N}{f^2}\right]^{1/2}$$

where f is the frequency ofem waves

33. Ifa carrier wave $c(t) = A \sin \omega_c t$ is amplitude modulated by a modulator signal

$m(t) = A \sin o)_m t$ then the equation of modulated signal $[C_m(t)]$ and its modulation index are Respectively [Online April 9, 2013]

(a) $C_m(t) = A(1 + \sin tiJ_m t) \sin \omega_c t$ and 2

(b) $C_m(t) = A(1 + \sin tiJ_m t) \sin tiJ_m t$ and 1

(c) $C_m(t) = A(1 + \sin tiJ_m t) \sin \omega_c t$ and 1

(d) $C_m(t) = A(1 + \sin \omega_c t) \sin tiJ_m t$ and 2

SOLUTION : . (c)

Modulation index$m_a = \frac{E_m}{E_c} = \frac{A}{A} = 1$

Equation ofmodulated signal $[C_m(t)] = E_{(C)} + m_a E_{(C)} \sin o)_m t$

$$= A(1 + \sin w_C t) \sin o)_m t$$

(As $\mathrm{E}_{(c)} = \mathrm{A}\sin\omega_c \mathrm{t}$)

34. Aradar has apower oflkW and is operating at a fiiequency of10 GHz. It is located on a mountain top ofheight 500 m. The maximum distance upto which it can detect object located on the surface ofthe earth (Radius ofearth $= 6.4 \times 10^6 \mathrm{m}$) is : [2012]

(a) 80hn **(b) 16hn** **(c) 40km** **(d) 64hn**

SOLUTION :

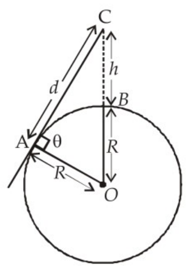
(a) Let d is the maximum distance, upto which it can detect the objects

From ΔAOC

$OC^2 = AC^2 + AO^2$

$(h+R)^2 = d^2 + R^2$

$\Rightarrow \quad d^2 = (h+R)^2 - R^2$



$$d = \sqrt{(h+R)^2 - R^2}; d = \sqrt{h^2 + 2hR}$$

$$d = \sqrt{500^2 + 2 \times 64 \times 10^6} = 80\mathrm{km}$$

35. A radio transmitter transmits at 830 kHz. At a certain distance from the transmitter magnetic field has amplitude $4.82 \times 10^{-11}\mathrm{T}$. The electric field and the wavelength are Respectively [Online May 26, 2012]

(a) $0.014\mathrm{N/C}, 36\mathrm{m}$ **(b) $0.14\mathrm{N/C}, 36\mathrm{m}$**

(c) $0.14\mathrm{N/C}, 360\mathrm{m}$ **(d) $0.014\mathrm{N/C}, 360\mathrm{m}$**

SOLUTION : **(d)**

Frequency of EM wave $\mathrm{u} = 830$ KHz$= 830 \times 10^3$ Hz.

Magnetic field, $B = 4.82 \times 10^{-11}\mathrm{T}$

As we know, frequency, $0 = \frac{c}{\lambda}$

or $\lambda = \frac{c}{v} = \frac{3\times10^8}{830\times10^3}$

$$\lambda = 360\text{m}$$

And, $E = BC = 4.82 \times 10^{-11} \times 3 \times 10^8 = 0.014\text{N/C}$

36. Given the electric field ofa complete amplitude modulated wave as $\vec{E} = \hat{\imath}E_c(^1\left(+\frac{E_m}{E_c}\cos ti\text{J}_{m}{}^{t})_c\right)_{\cos(j)t}$.Where the subscript c stands for the carrier wave and m for the modulating signal. The fiequencies present in the modulatedwave are

[Online May 19, 2012]

(a) $(j)_c$ and $\sqrt{0)_{cm}2_{+0)}2}$

(b) $i\text{J}_c$, $ti\text{J}_c + ci\text{J}_m$ and $(i\text{J}_c - ti\text{J}_m$

(c) $(j)_c$ and $(j)_m$

(d) $(j)_c$ and $\sqrt{\text{oo}_c\text{oo}_m}$

SOLUTION : . (b)

The frequencies present in amplitude modulated wave are :

Carrier frequency $= \omega_c$

Upper side band frequency $= ti\text{J}_c + ti\text{J}_m$

Lower side band frequency $= ti\text{J}_c - ti\text{J}_m$.

37. A 10kW transmitter emits radio waves ofwavelength 500 m. The number of photons emitted per second by the transmitter is ofthe order of [Online May 12, 2012]

(a) 10^{37}

(b) 10^{31}

(c) 10^{25}

(d) 10^{43}

SOLUTION : . (b)

$$\text{Power} = \frac{nhc}{\lambda}$$

(where, $n = \text{no.}$ ofphotons per second)

$$\Rightarrow n = \frac{10\times10^3\times500}{6.6\times10^{-34}\times3\times10^8} = 10^{31}$$

38. This question has Statement – I and Statement – 2. Ofthe four choices given after the statements, choose the one that best describes the two statements. [2011]

Statement– 1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement– 2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(a) Statement – I is true, Statement – 2 is true, Statement – 2 is the correct explanation of Statement – I.

(b) Statement – I is true, Statement – 2 is true, Statement – 2 is not the correct explanation of Statement – I.

(c) Statement – I is false, Statement – 2 is true.

(d) Statement – I is true, Statement – 2 is false.

SOLUTION : (b)

For long distance communication, sky wave signals are used.

Also, the state ofionosphere varies every time.

So, both statements are correct.

39. Which of the following four alternatives is not correct? We need modulation : [2011 RS]

(a) to reduce the time lag between transmission and reception ofthe information signal

(b) to reduce the size ofantenna

(c) to reduce the fiiactional band width, that is the ratio of the signal band width to the centre frequency

(d) to increase the selectivity

SOLUTION : (a)

Low frequencies cannot be transmitted to long distances. Therefore, they are super imposed on a high fiiequency carrier signal by a process known as modulation. Speed of electro - magnetic waves will not change due to modulation. So there will be time lag between transmission and reception ofthe information signal.