SDFG–U–ELE

ELECTRICAL ENGINEERING

PAPER—I

Time Allowed : Three Hours

Maximum Marks : 300

QUESTION PAPER SPECIFIC INSTRUCTIONS

Please read each of the following instructions carefully before attempting questions

There are **EIGHT** questions divided in **TWO** Sections.

Candidate has to attempt **FIVE** questions in all.

Question Nos. **1** and **5** are compulsory and out of the remaining, **THREE** are to be attempted choosing at least **ONE** question from each Section.

The number of marks carried by a question/part is indicated against it.

Wherever any assumptions are made for answering a question, they must be clearly indicated.

Diagrams/Figures, wherever required, shall be drawn in the space provided for answering the question itself.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly.

Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

Answers must be written in **ENGLISH** only.

SECTION—A

1. *(a)* Suppose A is a 3×3 diagonalizable matrix. Then—

    (i) show that each eigenvalue of A is 0 or 1, if $A^2 = A$;

    (ii) find the trace of the matrix $B = A + A^3 + A^{-1}$, if the eigenvalues of A are 2, 3, −2.   12

(b) Evaluate the linear density in atoms per mm in the following directions in BCC iron, which has lattice constant of 2·89 Å :

    (i) [1 0 0]
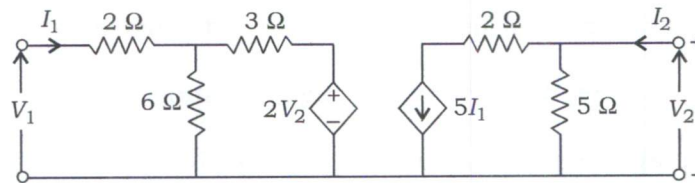
    (ii) [1 1 0]

    (iii) [1 1 1]   12

(c) Derive an expression for capacitance (C) of concentric spheres having radii a and b $(a < b)$ respectively with single dielectric.   12

(d) Find the hybrid parameters of the following circuit :

   12

(e) Construct full—

    (i) conjunctive normal form for the statement $P \rightarrow Q$;

    (ii) disjunctive normal form for the statement $(P \rightarrow (Q \vee R)) \wedge (P \vee Q)$.   6+6=12

2. *(a)* *(i)* Obtain the half-range cosine series for the function $f(x) = \sin x$ in $0 \le x \le \pi$ and hence, find the value of $\sum\limits_{n=1}^{\infty} \dfrac{1}{4n^2 - 1}$.   10

    (ii) Evaluate the integral $\iint\limits_{R} (x - y)^4 \cos^2(x + y)\, dx\, dy$, where R is the rhombus with successive vertices at $(\pi, 0)$, $(2\pi, \pi)$, $(\pi, 2\pi)$, $(0, \pi)$.   10

(b) *(i)* Determine the volume of an HCP unit cell in terms of its a and c lattice parameters. 10

(ii) Copper has an atomic radius of 0.13 nm, an FCC crystal structure and an atomic weight of 63.5 g/mol. Evaluate its theoretical density and compare the answer with its measured density. (Take Avogadro number, $N_A = 6.022 \times 10^{23}$ atoms/mol) 10

(c) *(i)* Two inductive coils having same self-inductance when connected in series carrying a current of I amperes store W joules of magnetic energy in their fields. When the connections of one of the coils are interchanged and the current is reduced to $\left(\dfrac{I}{3}\right)$ amperes, the stored energy remains the same. Calculate the ratio of mutual to self-inductance. 10

(ii) Determine the transfer function $\dfrac{V_o(s)}{V_g(s)}$ for the circuit shown below for $R_1 = 500\ \Omega$, $R_2 = 50\ \Omega$, $L = 10$ mH and $C = 2\ \mu F$:
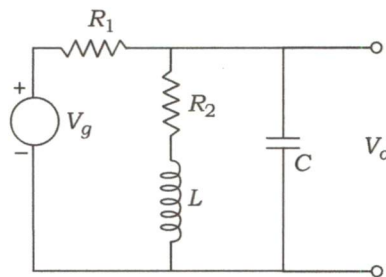
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3. *(a)* *(i)* Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} - 6\frac{\partial^2 z}{\partial y^2} = e^{x+2y} + \sin(4x+3y) + y\cos x$$

10

(ii) Compute the following integral by residue theorem :

$$\int_0^{2\pi} \frac{\sin\theta}{3 - 2\sin\theta}\,d\theta$$

10

(b) What is the electric field intensity $\vec{E}_1$ V/m due to an infinite sheet of uniform charge density σ C/m²?

(i) Derive the electric field intensity $\vec{E}_2$ V/m at P contributed by the circular portion of this infinite sheet charge of radius b metre on the perpendicular axis at a metre from the sheet as shown in the figure below.

(ii) Find b, if $a = 0 \cdot 5$ m and $\vec{E}_2 = \dfrac{\vec{E}_1}{2}$.

Infinite sheet charge
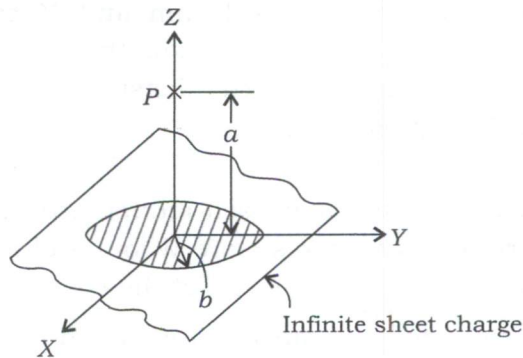
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(c) *(i)* In low-voltage Schering bridge designed for measurement of permittivity, the branch ab consists of two electrodes between which the specimen under test may be inserted; arm bc is a non-reactive resistor R_3 in parallel with a standard capacitor C_3 and cd is a non-reactive resistor R_4 in parallel with a standard capacitor C_4; arm da is a standard air capacitor of capacitance C_2. Without the specimen between the electrodes, balance is obtained with the following values :

$$C_3 = 150 \text{ pF}$$

$$C_4 = 200 \text{ pF}$$

$$C_2 = 250 \text{ pF}$$

$$R_3 = 5 \times 10^3 \ \Omega$$

$$R_4 = 10 \times 10^3 \ \Omega$$

With specimen inserted, these values become

$$C_3 = 200 \text{ pF}$$

$$C_4 = 1200 \text{ pF}$$

$$C_2 = 1000 \text{ pF}$$

and R_3 and R_4 remain as previous. In each case, the frequency is $\omega = 10 \times 10^3$ rad / s. Determine the relative permittivity of the specimen.

12

(ii) Draw the connections and phasor diagram of Anderson's bridge along with its advantages and disadvantages.

8

4. *(a)* *(i)* For the network shown in the following figure, compute $i_L(t)$ and $i_1(t)$, if the initial current through the inductor is 0 ampere :
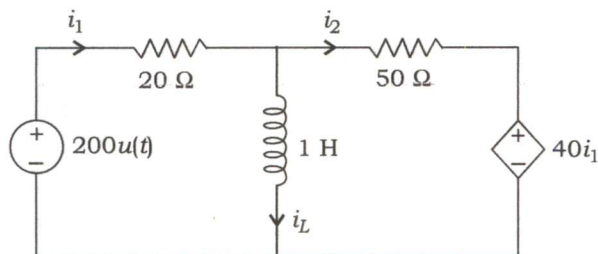
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(ii) Determine the current at $t > 0$, if AC voltage V is applied, when switch S is moved from position 2 to position 1 at $t = 0$, for the network shown in the following figure. Assume steady-state current of 1 ampere in the network, when the switch is at position 1 :
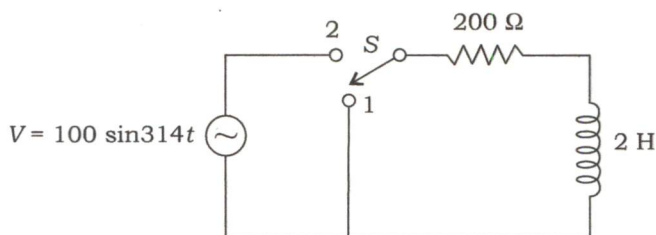
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(b) *(i)* Give the variation of resistivity of purified mercury with temperature. Also, represent the resistivity of normal metal as a function of temperature (T) along with pure and impure superconductors. 12

(ii) Compute the drift mobility and mean scattering time of conduction electrons in copper at room temperature, given that the density of copper is $8 \cdot 98$ g/cm^3, the conductivity of copper is $5 \cdot 95 \times 10^5$ Ω^{-1} cm^{-1} and the atomic mass of copper is $63 \cdot 5$ g/mol. Take Avogadro number, $N_A = 6 \cdot 02 \times 10^{23}$ and charge on electron (e) = $1 \cdot 6 \times 10^{-19}$ coulomb, mass of electron (m_e) = $9 \cdot 1 \times 10^{-31}$ kg. 8

(c) *(i)* A moving-coil instrument has a resistance of 5 Ω between terminals and full-scale deflection is obtained with a current of 15 mA. This instrument

is to be used with a manganin shunt to measure 100 A at full scale. Calculate the error caused by a 10 °C rise in temperature—

(1) when the internal resistance of 5 Ω is due to copper only;

(2) when a 3 Ω manganin swamping resistance is used in series with a copper coil of 2 Ω resistance.

The resistance temperature coefficients of copper and manganin are 0·004/°C and 0·000015/°C respectively.　　10

(ii) Draw the block diagram of ramp-type digital voltmeter and explain its functioning.　　10

SECTION—B

5. (a) For a series *R-L-C* circuit excited from an AC source, find the resonant frequency, bandwidth and quality factor, if $R = 100$ Ω, $L = 0·5$ H and $C = 0·4$ μF.　　12

(b) Define line defects in materials. Explain different types of line defects and compare them. Also, explain their cause of creation.　　12

(c) Design a circuit that accepts a 3-bit number and gives an output 0, if input represents even decimal number and gives an output 1, if input represents an odd decimal number.　　12

(d) A current transformer having a bar primary is rated at 500/5 A, 50 Hz with an output of 20 VA. At rated load with non-inductive burden, the inphase and quadrature components (referred to the flux) of the exciting mmf are 8 A and 10 A respectively. The number of turns in the secondary winding is 98 and the impedance of the secondary winding is $(0·4 + j0·3)$ Ω. Calculate the ratio and phase angle errors.　　12

(e) (i) The reverse recovery time t_{rr} of a diode is 2 μs. In the conducting mode to reverse blocking mode operation, the diode needs the rate of fall of forward current of 50 amperes/μs. Determine the storage charge and the peak reverse current.　　6

(ii) A diode with 500 mW power dissipation at 25 °C has 5 mW/°C derating factor. If the forward voltage drop remains constant at 0·7 V, calculate the maximum forward current at 50 °C.　　6

6. *(a)* *(i)* Calculate the DC voltages V_o and V_i, and the bias currents for the feedback pair of transistors shown in the figure below, given that $\beta_1 = 100$, $\beta_2 = 150$, $V_{CC} = 15$ volts, $R_C = 200\ \Omega$ and $R_B = 1\ \text{M}\Omega$:
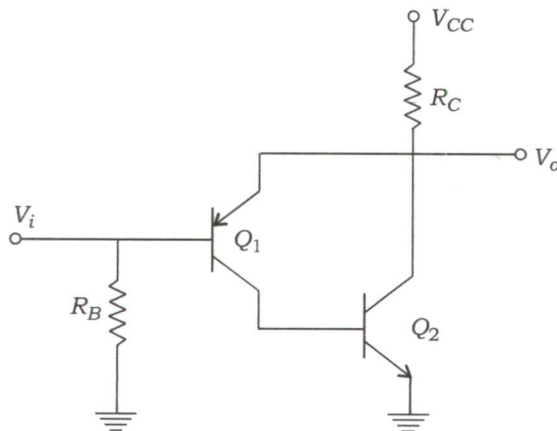
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(ii) In the current mirror circuit shown in the figure below, the current is mirrored in two transistors. All the three transistors are identical. Calculate the load current I assuming $\beta = 100$:
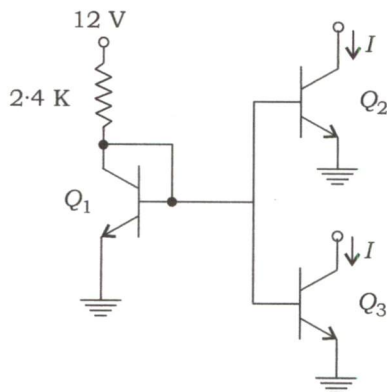

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(b) State Gauss divergence theorem. Let R be the region bounded by the closed cylinder $x^2 + y^2 = 4$, $z = 0$ and $z = 2$. Verify this theorem, if $\vec{F} = 3x^2\hat{i} + y^2\hat{j} + z\hat{k}$. 20

(c) *(i)* What is Random Access Memory? Explain Static Random Access Memory and Dynamic Random Access Memory. 10

(ii) Write a program in C to find whether the given number is even or odd and if it is odd, find whether it is prime or not. 10

7. *(a)* For the JFET amplifier circuit shown in the figure below, $g_m = 2$ mS, $r_d = 200$ kΩ, $C_{gs} = 10$ pF, $C_{gd} = 2$ pF, $R_S = 1$ kΩ, $R_1 = 10$ MΩ, $R_2 = 100$ kΩ, $R_D = 5$ kΩ, $C_{C_1} = C_{C_2} = 0 \cdot 1$ μF. Assume output capacitor $C_o = 10$ pF, C_S and R_L to be very large. Find—

(i) mid-frequency gain;

(ii) lower cut-off frequency;

(iii) higher cut-off frequency;

(iv) gain-bandwidth product.
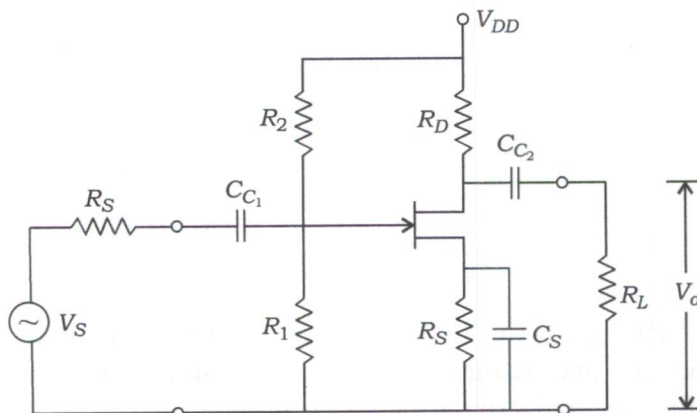
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(b) A balanced 240 V, 3-phase voltage is applied to an unbalanced delta-connected load having the following phase impedances :

$$Z_{AB} = 25\angle 90° \text{ Ω}, \quad Z_{BC} = 15\angle 30° \text{ Ω}, \quad Z_{CA} = 20\angle 0° \text{ Ω}$$

(i) Calculate the line currents.

(ii) Obtain the readings of the two wattmeters whose current coils are connected in the lines A and B, and the voltage coils are connected across the line C.

Consider ABC system for supply voltage and V_{BC} as reference. 20

(c) (i) Suppose that X and Y are independent random variables having the common density function

$$f(x) = \begin{cases} e^{-x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find the density function of the random variable $X|Y$. 8

(ii) A root of the equation $xe^x - 1 = 0$ lies in the interval $(0 \cdot 5, 1 \cdot 0)$. Determine this root correct to three decimal places using regula-falsi method. First find how many least decimal digits are required for three decimal places accuracy. 12

8. *(a)* *(i)* The power in a single-phase circuit is measured by an electrodynamo-meter wattmeter. The voltage across the load is 100 V and the load current is 10 A at a power factor of 0·2 lagging. The wattmeter circuit has a resistance of 3500 Ω and an inductance of 30 mH. Estimate the percentage error in the wattmeter reading when the pressure coil is connected (1) on the supply side and (2) on the load side. The current coil has a resistance of 0·1 Ω and negligible inductance. The supply frequency is 50 Hz. 15

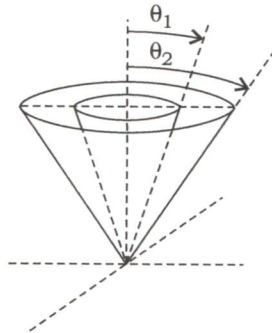*(ii)* The limiting errors for a four-dial resistance box are :

Units	: ± 0·2%	Hundreds	: ± 0·05%
Tens	: ± 0·1%	Thousands	: ± 0·02%

If the resistance value is set at 3525 Ω, calculate the limiting error in the resistance value. 5

(b) In the region between the two coaxial cones with insulated vertices as shown in the figure below, the voltage at $\theta_1 = 30°$ is 0 volt and at $\theta_2 = 45°$ is 125·5 volts :
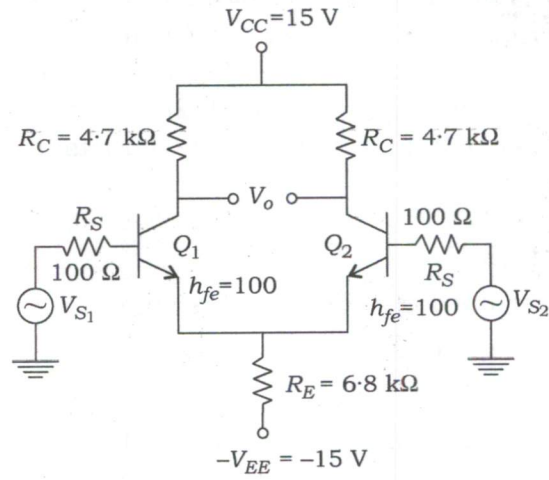

(i) Calculate the angle θ at which the voltage is 75 volts. Assume air as the dielectric in the region between the two coaxial cones.

(ii) Find the charge distribution on the conducting plane at $\theta_2 = 90°$. 20

(c) A dual input, balanced output differential amplifier is configured using silicon transistors which are identical having $h_{ie} = 2·8$ kΩ as shown in the figure below.

(i) Calculate the differential gain, common mode gain and CMRR.

(ii) What is the peak-to-peak output voltage V_o, if V_{S_1} is 50 mV peak-to-peak at 2 kHz and V_{S_2} is 30 mV peak-to-peak at 2 kHz?

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