I.E.S./ISS EXAM- 2023

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T.B.C. : ASRT-B-STT **Test Booklet Series**

Serial 1005805

A

TEST BOOKLET

STATISTICS

Paper II

Time Allowed : Two Hours ***Maximum Marks : 200***

INSTRUCTIONS

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET ***DOES NOT*** HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. **Please note that it is the candidate's responsibility to encode and fill in the Roll Number and Test Booklet Series Code A, B, C or D carefully and without any omission or discrepancy at the appropriate places in the OMR Answer Sheet. Any omission/discrepancy will render the Answer Sheet liable for rejection.**
3. You have to enter your Roll Number on the Test Booklet in the Box provided alongside. ***DO NOT*** write ***anything else*** on the Test Booklet.
4. This Test Booklet contains **80** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose ***ONLY ONE*** response for each item.
5. You have to mark all your responses ***ONLY*** on the separate Answer Sheet provided. See directions in the Answer Sheet.
6. All items carry equal marks.
7. Before you proceed to mark in the Answer Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions sent to you with your Admission Certificate.
8. After you have completed filling in all your responses on the Answer Sheet and the examination has concluded, you should hand over to the Invigilator ***only the Answer Sheet***. You are permitted to take away with you the Test Booklet.
9. Sheets for rough work are appended in the Test Booklet at the end.
10. **Penalty for wrong answers :**

    THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.

    (i) There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.

    (ii) If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.

    (iii) If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.

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1. The radius of a circle is measured with an error of measurement, which is $N(0, \sigma^2)$. Let $x_1, x_2, \dots x_n$ be n measurements of the radius. Let $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ be the sample mean and $S^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$ be the sample variance. What is the unbiased estimate of the area of the circle ?

(a) $\pi(\bar{x})^2$

(b) $\pi \frac{\sum_{i=1}^{n} x_i^2}{n}$

(c) $\pi\left(\frac{\sum_{i=1}^{n} x_i^2}{n} - S^2\right)$

(d) $\pi[(\bar{x})^2 - S^2]$

2. Let 2·5, – 2·0, 1·5, 3·5, 0·5 be the observations of a random sample of size 5 from the continuous distributions with pdf $f(x) = \frac{1}{8} e^{-|x-2|} + \frac{3}{4\sqrt{2\pi}} e^{-\frac{1}{2}(x-\theta)^2}$; $x, \theta \in R$ and θ is unknown. Then the method of moment estimators of θ belongs to the interval :

(a) (0·60, 0·70)

(b) (0·70, 0·80)

(c) (0·80, 0·90)

(d) (0·90, 0·99)

3. Consider the following statements :

Statement-I :

The method of moments provides consistent estimators of population moments.

Statement-II :

From the Weak Law of Large Numbers, it follows that $\frac{1}{n}\sum_{i=1}^{n} x_i^j \xrightarrow{P} E(X_j)$ provided $E(X_j)$ exists.

Which one of the following is correct in respect of the above Statements ?

(a) Statement-I and Statement-II are individually correct and Statement-II is the correct explanation of Statement-I.

(b) Statement-I and Statement-II are individually correct but Statement-II is ***not*** the correct explanation of Statement-I.

(c) Statement-I is correct but Statement-II is incorrect.

(d) Statement-I is incorrect but Statement-II is correct.

4. Let X be a random variable which assumes only two values 1 and 0, respectively representing occurrence of success or failure in an experiment with only two outcomes. The probability of getting success in the experiment is $p(\theta)$ defined as :

$$p(\theta) = \begin{cases} \theta, & \text{if } \theta \text{ is rational} \\ 1-\theta, & \text{if } \theta \text{ is algebraic irrational} \end{cases}$$

Define an estimator, $T = \frac{\Sigma_{i=1}^{n} x_i}{n}$ = Sample proportion of success.

Which one of the following is correct ?

(a) T is MLE as well as consistent for θ.

(b) T is unbiased and consistent for θ.

(c) T is MLE but not consistent for θ.

(d) T is consistent but not MLE tending to θ.

5. Let {6, 11, 4, 13, 5} be a random sample from the exponential distribution $f(x, \theta) = e^{-(x-\theta)}$, $x \geq \theta$, $\theta \in (-\infty, 3]$ is unknown. Then the Maximum Likelihood Estimate of e^{θ} is :

(a) e^3

(b) e^4

(c) $e^{7 \cdot 8}$

(d) e^6

6. Let {1, 0, 0, 0, 1, 1} be a random sample from a binomial distribution $b(1, \theta)$, $0 < \theta < 1$. Then UMVUE of $\theta(1 + \theta)$ is :

(a) 0·7

(b) 0·6

(c) 0·5

(d) 0·3

7. Consider the following statements regarding unbiased estimators of parameter θ :

1. Let T_0 be an UMVUE of θ and T_1 is any other unbiased estimator of θ with efficiency 0·64, then the correlation coefficient between T_0 and T_1 is 0·8.

2. Let T_0 be an MVUE of $g(\theta)$ and T_1 is any other unbiased estimator with efficiency less than 1, then any unbiased linear combination of T_0 and T_1 will also be MVUE of $g(\theta)$.

Which of the above statements is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

8. Let $x_1, x_2, \ldots x_n$ be a random sample from $U(\theta - 0 \cdot 5, \theta + 0 \cdot 5)$, $\theta \in R$. Which of the following statements is/are correct ?

1. $(X_{(1)}, X_{(n)})$ is sufficient as well as complete.

2. $\frac{X_{(1)} + X_{(n)}}{2}$ is MLE for θ.

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

9. Let {4·5, 3·8, 2·5, 5·2} be a random sample from N(μ, 1) distribution. If the parametric space for parameter μ is $\Omega = \{-2, -1, 1, 2\}$, then the MLE of μ is :

(a) 5·2

(b) 4

(c) – 2

(d) 2

10. Let {2·5, 4·0, 7·1, 6·3, 8·9, 5·1} be a random sample from the distribution with pdf $f(x, \theta) = \theta 2^{\theta} x^{-(\theta+1)}$, $x > 2$, $\theta > 2$. The Cramer-Rao Lower Bound for the variance of an unbiased estimate of $ln\ \theta$ is :

(a) 6

(b) $\frac{\theta^2}{6}$

(c) $\frac{6}{\theta^2}$

(d) $\frac{1}{6}$

11. Let $X \sim U(7, 7 + \theta)$, $\theta > 0$; U being Uniform distribution. Define $T = a(X - b)^2$. Then the statistic T becomes unbiased for θ^2 if :

(a) a = 3, b = 0

(b) a = 7, b = 0

(c) a = 3, b = 7

(d) a = 7, b = 7

12. Let $X_1, X_2, \ldots X_n$ be iid exponential variates with mean $\lambda > 0$. Define $T = \Sigma_{i=1}^{n} x_i$, a sufficient statistic for λ. Then pivotal statistic to construct confidence interval for λ is :

(a) $2\lambda + T$

(b) $\frac{2T}{\lambda}$

(c) $\frac{2\lambda}{T^2}$

(d) $\frac{\lambda}{2T^2}$

13. If we have a sample of size n = 5, then the number of distinct bootstrap samples with replacement of size 5 is :

(a) 625

(b) 306

(c) 252

(d) 126

14. Consider the following distributions :

1. Bernoulli distribution $b(1, \theta)$
2. Poisson distribution $P(\theta)$
3. Cauchy distribution $C(\theta, 1)$
4. Logistic distribution with location parameter θ

How many of the above distributions possess Monotone Likelihood Ratio property ?

(a) Only one

(b) Only two

(c) Only three

(d) All four

15. Let X follow normal distribution $N(\mu, \sigma^2)$. Consider the hypotheses for the test as $H_0 : \mu = 2$ versus $H_1 : \mu = 3$. Which one of the following statements is correct ?

(a) H_0 is simple but H_1 is composite.

(b) Both H_0 and H_1 are simple.

(c) H_0 is composite but H_1 is simple.

(d) Both H_0 and H_1 are composite.

16. In which one of the following cases, MLE for a parameter θ need not be unique based on a random sample of size n ?

(a) $U(0, \theta)$

(b) $U(\theta - 0{\cdot}5, \theta + 0{\cdot}5)$

(c) $f(x, \theta) = e^{-(x-\theta)}, x \geq \theta$

(d) $f(x, \theta) = (1 + \theta) x^{\theta}, 0 < x < 1$

17. Let X be a random variable with pdf $f(x, \theta) = 2\theta x + 1 - \theta, 0 < x < 1, -1 \leq \theta \leq 1$. Based on a sample size one, the Uniformly Most Powerful (UMP) critical region for testing $H_0 : \theta = 0$ versus $H_1 : \theta > 0$ at level $\alpha = 0{\cdot}05$ is given by :

(a) $x \geq \frac{1}{20}$

(b) $x \leq \frac{1}{20}$

(c) $x \leq \frac{19}{20}$

(d) $x \geq \frac{19}{20}$

18. Consider the following statements :

Statement-I :

If T is MLE of θ, then log T is MLE of log θ.

Statement-II :

If T is MLE of θ, then distribution of T will be normal in case of large sample size.

The properties of MLE associated with Statement-I and Statement-II are respectively :

(a) Asymptotic property and consistency property

(b) Invariance property and asymptotic property

(c) Consistency property and efficiency property

(d) Invariance property and consistency property

19. In a test of randomness of a given set of observations {7, 19, 3, 1, 8, 5, 10, 4}, when the null hypothesis of randomness of the observations is true, then the expected number of runs U and its variance are respectively :

(a) $4, \frac{12}{7}$

(b) $5, \frac{12}{7}$

(c) $4, \frac{20}{7}$

(d) $5, \frac{20}{7}$

20. Let $x_1, x_2, ...: x_n$ be a random sample from a population with probability density function

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}; & x > \theta \\ 0, & \text{otherwise} \end{cases}$$

For parameter θ, consider the following statements :

1. Min $(x_1, x_2, ... x_n)$ is a sufficient statistic of θ.

2. $\overline{x} = \frac{1}{n}\Sigma_{i=1}^{n} x_i$ is a minimum variance bound estimator of θ.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

21. Let $x_1, x_2, \dots x_n$ be a random sample from Normal distribution $N(\mu, \sigma^2)$. Assume that μ is known and σ is unknown. Define an estimator T as $T = k\sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$. Then the constant k for T to be unbiased estimator of σ is :

(a) $\sqrt{2}\,\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)}$

(b) $\sqrt{\frac{n}{2}}\,\frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}$

(c) 1

(d) $\sqrt{\frac{n}{2}}\,\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n+1}{2}\right)}$

22. Let $x_1, x_2, \dots x_n$ be a random sample from a Normal distribution $N(\mu, \sigma^2)$. Suppose both μ and σ are unknown. Which of the following are jointly sufficient statistic for (μ, σ) ?

1. $\left\{\sum_{i=1}^{n} x_i,\ \sum_{i=1}^{n} x_i^2\right\}$
2. $\left\{\sum_{i=1}^{n} x_i,\ \sum_{i=1}^{n} (x_i - \bar{x})^2\right\}$
3. $\left\{\sum_{i=1}^{n} x_i^2,\ \sum_{i=1}^{n} (x_i - \bar{x})^2\right\}$

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

23. Let $x_1, x_2, \dots x_n$ be a random sample from a Cauchy distribution having pdf $f(x, \theta) = \frac{1}{\pi[1 + (x - \theta)^2]}$; $-\infty < x, \theta < \infty$:

Consider the following statements :

1. $\overline{X}$ is unbiased for θ but not consistent estimator for θ.
2. Sample median is consistent estimator for θ.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

24. Let $x_1, x_2, \dots x_n$ be a random sample from a Poisson distribution with mean $\theta > 0$. Then minimum variance unbiased estimator for $P(x = k)$ is :

(a) $T_1 = \begin{cases} 1; & \text{if } x = k \\ 0; & \text{otherwise} \end{cases}$

(b) $T_2 = \begin{cases} k; & \text{if } x = 0 \\ 0; & \text{otherwise} \end{cases}$

(c) $T_3 = \binom{\sum_{i=1}^{n} x_i}{k} \frac{(n-1)^{\left(\sum_{i=1}^{n} x_i - k\right)}}{n^{\left(\sum_{i=1}^{n} x_i\right)}}$

(d) $T_4 = \binom{\sum_{i=1}^{n} x_i}{k} \left(1 - \frac{1}{n}\right)^{\left(\sum_{i=1}^{n} x_i - k\right)}$

25. Let $x_1, x_2, \ldots x_n$ be a random sample with density function $f(x, \theta) = \theta e^{-\theta x}$; $x > 0$. The best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (> \theta_0)$ is :

(a) $x : \Sigma_{i=1}^{n} x_i \leq \frac{1}{2\theta_0} \chi^2_{1-\alpha, 2n}$

(b) $x : \Sigma_{i=1}^{n} x_i \geq \frac{1}{2\theta_0} \chi^2_{1-\alpha, 2n}$

(c) $x : \Sigma_{i=1}^{n} x_i \geq \frac{1}{2\theta_0} \chi^2_{\alpha, 2n}$

(d) $x : \Sigma_{i=1}^{n} x_i \leq \frac{1}{2\theta_0} \chi^2_{\alpha, 2n}$

26. Let X be a random variable with $N(\theta, 1 + a\theta^2)$, $a > 0$. The locally Most Powerful Test for testing $H_0 : \theta = 0$ against $H_1 : \theta > 0$ is :

(a) $\varphi(x) = \begin{cases} 1; & \Sigma_{i=1}^{n} x_i > \frac{Z_\alpha}{\sqrt{n}} \\ 0; & \text{otherwise} \end{cases}$

(b) $\varphi(x) = \begin{cases} 1; & \overline{X} > \frac{Z_{1-\alpha}}{n} \\ 0; & \text{otherwise} \end{cases}$

(c) $\varphi(x) = \begin{cases} 1; & \overline{X} > \frac{Z_\alpha}{n} \\ 0; & \text{otherwise} \end{cases}$

(d) $\varphi(x) = \begin{cases} 1; & \overline{X} > \frac{Z_\alpha}{\sqrt{n}} \\ 0; & \text{otherwise} \end{cases}$

where Z_α is the upper α% value of N(0, 1)

27. Let $x_1, x_2, \ldots x_n$ be a random sample from Normal distribution $N(\mu, \sigma^2)$. The Mean Squared Error (MSE) of the estimator $T = \Sigma_{i=1}^{n} (x_i - \overline{x})^2$ where $\overline{x} = \frac{\Sigma_{i=1}^{n} x_i}{n}$ for estimating σ^2 is :

(a) $(n^2 - 1)\sigma^4$

(b) $(n^2 + 1)\sigma^4$

(c) $((n - 1)^2 + 1)\sigma^4$

(d) $((n + 1)^2 + 1)\sigma^4$

28. Let $x_1, x_2, \ldots x_n$ be a random sample from Normal distribution $N(\theta, \theta^2)$. Then sufficient statistic for θ is :

(a) $\Sigma_{i=1}^{n} x_i$ only

(b) $\Sigma_{i=1}^{n} x_i^2$ only

(c) $\Sigma_{i=1}^{n} x_i + \Sigma_{i=1}^{n} x_i^2$

(d) $\left(\Sigma_{i=1}^{n} x_i , \Sigma_{i=1}^{n} x_i^2\right)$

29. Let $x_1, x_2, \ldots x_n$ be a random sample from a Poisson distribution with parameter $\lambda > 0$. Which of the following statements are correct in respect of Likelihood Ratio Test (LRT) ?

1. The LRT for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda > \lambda_0$ produces UMP test identical to the test obtained by NP theory of testing.
2. The LRT for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda < \lambda_0$ produces UMP test identical to the test obtained by NP theory of testing.
3. The LRT for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda \neq \lambda_0$ produces UMPU test identical to the test obtained by NP theory of testing.

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

30. In SPRT, to test $H_0 = X \sim f(x, \theta_0)$ versus $H_1 = X \sim f(x, \theta_1)$, Z is defined as $Z = \log\left\{\frac{f(x, \theta_1)}{f(x, \theta_0)}\right\}$ and $h(\theta)$ is non-zero solution of the equation $E(e^{zh(\theta)}) = 1$. Under H_0, the OC function $L(\theta)$ becomes :

(a) α

(b) β

(c) $1 - \alpha$

(d) $1 - \beta$

Consider the following for the next two (02) items that follow :

Let X be a random variable with pmf

$$P(x) = \begin{cases} \frac{3 + 2\theta + \mu}{15}, & x = 1 \\ \frac{5 + \theta - 2\mu}{15}, & x = 2 \\ \frac{7 - 3\theta + \mu}{15}, & x = 3 \end{cases}$$

where $\theta \geq 0$, $\mu \geq 0$ are unknown constants and $\theta + \mu \leq 2$.

For testing $H_0 : \theta + \mu = 1$ versus $H_1 : \theta = \mu = 0$, the null hypothesis is rejected if x = 2 or 3.

31. The size of the test based on a single observation is :

(a) $\frac{2}{5}$

(b) $\frac{11}{15}$

(c) $\frac{4}{5}$

(d) $\frac{13}{15}$

32. The test is

(a) biased with power $\frac{3}{5}$

(b) biased with power $\frac{2}{5}$

(c) unbiased with power $\frac{4}{5}$

(d) unbiased with power $\frac{13}{15}$

Consider the following for the next two (02) items that follow :

Let x_1, x_2, ... x_n be a random sample from the distribution having pdf $f(x, \theta) = 3\theta^3(x + \theta)^{-4}$; $0 < x < \infty, \theta > 0$.

33. What is $E\left(\frac{\partial^2 \ln f}{\partial \theta^2}\right)$ equal to ?

(a) $-\frac{3}{5\theta^2}$

(b) $-\frac{4}{5\theta^2}$

(c) $\frac{3}{5\theta^2}$

(d) $\frac{4}{5\theta^2}$

34. The C-R bound for unbiased estimator of θ^3 will be :

(a) $\frac{3n}{5\theta^2}$

(b) $\frac{5\theta^5}{3n}$

(c) $\frac{5\theta^4}{n}$

(d) $\frac{5n}{\theta^4}$

Consider the following for the next two (02) items that follow :

Let $x_1 = \frac{17}{3}$, $x_2 = \frac{10}{3}$, $x_3 = \frac{13}{3}$, $x_4 = 4$ be the observed values of a random sample of size 4 from the distribution having pdf

$$f(x, \theta) = \frac{1}{3}\left[\frac{1}{2\theta} e^{-\frac{x}{2\theta}} + \frac{1}{\theta^2} e^{-\frac{x}{\theta^2}} + e^{-x}\right]; x > 0, \theta > 0$$

35. What is E(X) equal to ?

(a) $\frac{\theta^2 + \theta + 1}{3}$

(b) $\frac{\theta^2 + 2\theta + 1}{3}$

(c) $\frac{\theta^2 + 2\theta + 2}{3}$

(d) $\frac{\theta^2 + 3\theta + 1}{3}$

36. The method of moment estimate of θ is :

(a) $\sqrt{13} - 1$

(b) $\sqrt{13} + 1$

(c) 1

(d) 3

Consider the following for the next two (02) items that follow :

Let x_1, x_2, x_3, x_4 be a random sample from Poisson distribution with mean $\lambda > 0$. To test $H_0 : \lambda = 1$ versus $H_1 : \lambda = 1{\cdot}5$, the following randomized test is considered :

$$\text{Let } \varphi(x) = \begin{cases} 1 & \text{if } \Sigma_{i=1}^{4} x_i > 2 \\ 0{\cdot}1 & \text{if } \Sigma_{i=1}^{4} x_i = 2 \\ 0 & \text{if } \Sigma_{i=1}^{4} x_i < 2 \end{cases}$$

(Given $e^{-4} \approx 0{\cdot}0183$ and $e^{-6} \approx 0{\cdot}0025$)

37. The size of the randomized test is within the interval :

(a) (0·05, 0·20)

(b) (0·30, 0·50)

(c) (0·60, 0·80)

(d) (0·80, 0·90)

38. The power of the randomized test is within the interval :

(a) (0·50, 0·70)

(b) (0·60, 0·75)

(c) (0·70, 0·85)

(d) (0·80, 0·99)

Consider the following for the next two (02) items that follow :

Let X be a random variable whose pmf under H_0 and H_1 is given as under :

x :	0	1	2	3	4
$P(x \mid H_0)$:	0·1	0·2	0·3	0·3	0·1
$P(x \mid H_1)$:	0·2	0·3	0·1	0·2	0·2

Test is conducted based on a single observed value of X to test $H_0 : X \sim P(x \mid h_0)$ versus $H_1 : X \sim P(x \mid H_1)$. The null hypothesis is rejected if X = 0 is observed. Further, null hypothesis is rejected when head 'H' is observed on tossing of an unbiased coin when X = 4 is observed.

39. The size of the test is :

(a) 0·15

(b) 0·20

(c) 0·30

(d) 0·60

40. The power of the test is :

(a) 0·15

(b) 0·30

(c) 0·40

(d) 0·70

Consider the following for the next two (02) items that follow :

Let $x_1, x_2, \dots x_n$ be a random sample from Poisson distribution with mean $\lambda > 0$.

41. If $T_1 = \Sigma_{i=1}^{n} x_i$ and $T_2 = (x_1, \Sigma_{i=2}^{n} x_i)$ are the statistics, then which of these statistics is/are sufficient for λ ?

(a) T_1 only

(b) T_2 only

(c) Both T_1 and T_2

(d) Neither T_1 nor T_2

42. If $T_3 = (x_1, x_2 + x_3, \Sigma_{i=4}^{n} x_i)$ and $T_4 = (x_1 + x_2, x_3 + x_4, \Sigma_{i=5}^{n} x_i)$ are the statistics, then which of these statistics is/are sufficient for λ ?

(a) T_3 only

(b) T_4 only

(c) Both T_3 and T_4

(d) Neither T_3 nor T_4

Consider the following for the next two (02) items that follow :

Let $x_1, x_2, \dots x_n$ be a random sample from Normal distribution $N(0, \theta), \theta > 0$.

43. Consider the following statements in respect of x_1 :

1. x_1 is unbiased for θ.
2. x_1 is complete statistic for θ.
3. x_1 is sufficient for θ.

How many of the above statements are correct ?

(a) Only one

(b) Only two

(c) All three

(d) None

44. Consider the following statements in respect of x_1^2 :

1. x_1^2 is complete for θ.
2. x_1^2 is sufficient for θ.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

45. Let $X_1, X_2, \ldots X_n$ be iid random variables with $N(\theta, 1)$. The UMPU test of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$ is given by

$$\varphi(x) = \begin{cases} 1; & \Sigma_{i=1}^{n} x_i < c_1 \text{ or } \Sigma_{i=1}^{n} x_i > c_2 \\ 0; & \text{otherwise} \end{cases}$$

What are the values c_1 and c_2 ?

(a) $\left(n\theta_0 - \sqrt{n}Z_{\left(\frac{\alpha}{2}\right)}, n\theta_0 + \sqrt{n}Z_{\left(\frac{\alpha}{2}\right)}\right)$

(b) $\left(n\theta_0 - \sqrt{n}Z_{\alpha}, n\theta_0 + \sqrt{n}Z_{1-\alpha}\right)$

(c) $\left(n\theta_0 - \sqrt{n}Z_{1-\alpha}, n\theta_0 + \sqrt{n}Z_{\alpha}\right)$

(d) $\left(n\theta_0 - \sqrt{n}Z_{1-\left(\frac{\alpha}{2}\right)}, n\theta_0 + \sqrt{n}Z_{\left(\frac{\alpha}{2}\right)}\right)$

46. Let $x_1, x_2, \ldots x_9$ be a random sample from Normal distribution $N(\mu, \sigma^2)$, μ and σ are unknown parameters, $\mu \in R$, $\sigma > 0$. The observed values of $\bar{x} = \frac{\Sigma_{i=1}^{9} x_i}{9} = 9{\cdot}8$ and $\frac{1}{8}\Sigma_{i=1}^{9} (x_i - \bar{x})^2 = 1{\cdot}69$. If LRT is used to test $H_0 : \mu = 8{\cdot}8$ versus $H_1 : \mu > 8{\cdot}8$, then what is the calculated value of the statistic ?

(a) 1·86

(b) 2·31

(c) 2·90

(d) 3·36

47. Let $x_1, x_2, \ldots x_n$ be a random sample from $e^{-(x-\theta)}$, $x \geq \theta$, $\theta > 0$. What is the confidence coefficient of the interval $\left(x_{(1)} - \frac{ln\, 4}{n}, \; x_{(1)} + \frac{ln\, 2}{n}\right)$ for θ, where $x_{(1)} = \min(x_1, x_2, \ldots x_n)$?

(a) 0·5

(b) 0·75

(c) 0·95

(d) 0·99

48. Consider the following statements :

1. If an MVB estimator T exists for θ, then Likelihood equation will have a solution equal to the estimator T.

2. MVB estimator for location parameter θ in the Cauchy distribution does not exist for θ.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

49. Consider the following statements :

1. If a sufficient estimator exists, it is a function of the maximum likelihood estimator.

2. If a sufficient estimator exists, it is always unique.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

50. Consider the following statements about Likelihood Ratio Test (LRT) as :

1. LRT is generally Uniformly Most Powerful (UMP), if a UMP test exists.

2. Under certain conditions, LRT is consistent.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

51. If **Y** is $N_3(\mu, \Sigma)$ where $\mu' = (3, -2, 1)$ and $\Sigma = \text{diag}\ (2, 4, 3)$, then the distribution of $\mathbf{Y}' \Sigma^{-1} \mathbf{Y}$ is :

(a) $\chi^2(3, 2\cdot9167)$

(b) $\chi^2(3, 5\cdot8333)$

(c) $\chi^2(3, 1\cdot45835)$

(d) $\chi^2(3, 1\cdot9445)$

52. Three different methods of analysis are used to determine the amount of a certain constituent in a sample. Five different analysts generate one observation each under each of these methods. Various sum of squares are obtained as

TSS = 97·6,

SSA (Analysts) = 4·27,

SSB (Methods) = 79·6.

What is the value of MSE, the Mean Sum of Squares due to error ?

(a) 2·89

(b) 1·96

(c) 1·72

(d) 1·53

53. Let $\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim N_3(\mathbf{0}, \mathbf{I}_3)$ and

$$\mathbf{A} = \frac{1}{3}\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix}.$$

What is the value of Var(**Y′AY**) ?

(a) 1

(b) 2

(c) 4

(d) 6

54. Let $\mathbf{Y} = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} \sim N_3(\mathbf{0}, \mathbf{I}_3)$ and

$$\mathbf{A} = \begin{pmatrix} \frac{1}{3} & 0 & -\frac{\sqrt{2}}{3} \\ 0 & 0 & 0 \\ -\frac{\sqrt{2}}{3} & 0 & \frac{2}{3} \end{pmatrix}.$$

What is the distribution of $(\mathbf{Y'AY})$?

(a) Exp(1)

(b) N(0, 3)

(c) $\chi^2_{(1)}$

(d) $\chi^2_{(3)}$

55. Let $Y_1, Y_2 \ldots Y_n$ be an iid N(0, 1) variates and

$\mathbf{Y} = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ Y_n \end{bmatrix}$. If quadratic forms $\mathbf{Y'A_1Y}$ and $\mathbf{Y'A_2Y}$ are independently Chi-square distributed, then :

(a) $\mathbf{A_1A_2 = I}$

(b) $\mathbf{A_1A_2 = 0}$

(c) $\mathbf{A_1A_2} = n\mathbf{I}$

(d) $\mathbf{A_1A_2} = n^{-1}\mathbf{I}$

Consider the following for the next two (02) items that follow :

Let X_i, Y_i, Z_i where i = 1, 2, 3 be nine independent observations with common variance σ^2, and $E(X_i) = \theta_1$, $E(Y_i) = \theta_2$ and $E(Z_i) = \theta_1 - \theta_2$; i = 1, 2, 3. Let $X = \Sigma_{i=1}^{3} X_i$, $Y = \Sigma_{i=1}^{3} Y_i$ and $Z = \Sigma_{i=1}^{3} Z_i$.

56. What is the BLUE of θ_1 equal to ?

(a) $\frac{1}{9}[2X + 3Y - Z]$

(b) $\frac{1}{9}[3X + Y - 2Z]$

(c) $\frac{1}{9}[2X + Y + Z]$

(d) $\frac{1}{9}[2X + Y - Z]$

57. What is the BLUE of θ_2 equal to ?

(a) $\frac{1}{9}[X + 3Y - 2Z]$

(b) $\frac{1}{9}[3X + Y - 2Z]$

(c) $\frac{1}{9}[2X + Y - Z]$

(d) $\frac{1}{9}[X + 2Y - Z]$

Consider the following for the next three (03) items that follow :

In a Simple Linear Regression Model $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$; i = 1, 2, ... n where $E(\varepsilon_i) = 0$, $Var(\varepsilon_i) = \sigma^2$ and $Cov(\varepsilon_i, \varepsilon_j) = 0, i \neq j$.

58. What is $Var(\hat{\beta}_0)$ equal to ?

(a) $\frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(b) $\frac{n\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(c) $n\sigma^2 + \frac{\sigma^2(\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(d) $\frac{\sigma^2}{n} + \frac{\sigma^2(\bar{x})^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

59. What is $Cov(\hat{\beta}_0, \hat{\beta}_1)$ equal to ?

(a) $\frac{\bar{x}\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(b) $-\frac{\bar{x}\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(c) $\sigma^2 \sum_{i=1}^{n}(x_i - \bar{x})^2$

(d) 0

60. What is $Var(\hat{\beta}_1)$ equal to ?

(a) $\frac{\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(b) $\frac{n\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(c) $\frac{(n-1)\sigma^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$

(d) $\sigma^2 \sum_{i=1}^{n}(x_i - \bar{x})^2$

61. For a two-way classification with one observation per cell, if p and q are the levels of the two factors A and B, then the degrees of freedom for error are :

(a) pq – 1

(b) p(q – 1)

(c) q(p – 1)

(d) (p – 1) (q – 1)

62. Let **A** be a matrix of order n × p. Then any generalized inverse of **A** is of order :

(a) p × n

(b) n × n

(c) n × p

(d) p × p

63. Consider the following statements in respect of a symmetric matrix **A** :

1. A generalized inverse of a symmetric matrix is not necessarily symmetric.
2. A symmetric generalized inverse of **A** can always be found.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

64. Let **A** be an n × p matrix of rank r. Let $\mathbf{A}^-$ be any generalized inverse of **A**. Which of the statements given below is/are correct ?

1. rank $(\mathbf{A}\mathbf{A}^-)$ = rank $(\mathbf{A}^-\mathbf{A})$ = rank$(\mathbf{A})$ = r
2. $(\mathbf{A}^-)'$ is a generalized inverse of $\mathbf{A}'$

Select the correct answer using the code given below :

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

65. Consider the following ANOVA table :

Sources of Variation	Degrees of freedom	Sum of squares	Mean sum of squares	Test statistic
Treatments	m	p	r	5·22
Error	12	q	25	
Total	14	561		

What are the values of p, q and r respectively ?

(a) 261, 300, 130·5

(b) 300, 261, 130·5

(c) 250, 300, 128·5

(d) 261, 300, 128·5

66. Which one of the following is ***not*** a fundamental principle of official statistics ?

(a) Professional standards and ethics

(b) Accountability and transparency

(c) Confidentiality and coordination

(d) Pricing and advertisement

67. With regard to the conduct of the Census, which of the following, as per the Census Act 1948, is correct ?

(a) The Census must be conducted every ten years.

(b) The Central Government may notify whenever it may consider it necessary or desirable to take a Census.

(c) The Central Government may conduct after amending the periodicity clause.

(d) The Registrar General of India, who is the Census Commissioner, may decide and his decision will be final.

68. Consider the following pairs :

1. Marine Hospital – Union List
2. Public Health – State List
3. Sanitation – State List
4. Population Control – Concurrent List

How many pairs given above are correctly matched ?

(a) Only one

(b) Only two

(c) Only three

(d) All four

69. Which of the following Agencies/ Organizations generate labour force data ?

1. National Sample Survey Office
2. Labour Bureau and the Directorate General of Employment and Training
3. Office of Registrar General of India

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

70. Which of the following Index Numbers are prepared and published by the Government of India ?

1. Index numbers of agricultural production
2. Index numbers of foreign trade
3. Annual indices of industrial production

Select the correct answer using the code given below :

(a) 1 and 2 only
(b) 2 and 3 only
(c) 1 and 3 only
(d) 1, 2 and 3

71. Consider the following statements in respect of GDP :

1. GDP is composed of goods and services produced for sale in the market.
2. GDP includes some non-market production as well (such as defence services and education services).
3. Not all productive work is included in GDP.
4. Voluntary work is not included in GDP.

How many statements given above are correct ?

(a) Only one
(b) Only two
(c) Only three
(d) All four

72. Marshall and Edgeworth price index number formula utilizes the weights as :

(a) Quantities of the base year only
(b) Quantities of the given year only
(c) Combined quantities of the base year and given year
(d) Neither quantities of base year nor quantities of given year

73. Consider the following in respect of an NSSO survey :

1. Collecting data from the respondents	–	Field Operations Division
2. Formulating sample design for a survey	–	Survey, Design and Research Division
3. Disseminating the data to public	–	Field Operations Division
4. Preparing survey reports	–	Survey Coordination Division

How many of the pairs given above are correctly matched ?

(a) Only one pair

(b) Only two pairs

(c) Only three pairs

(d) All four pairs

74. Periodic Labour Force Survey is conducted by :

(a) Labour Bureau, Ministry of Labour and Employment

(b) Office of Economic Adviser, Ministry of Commerce and Industry

(c) National Statistical Office

(d) None of the above

75. Which of the following Census tables are covered in the Census of India ?

1. Social and cultural tables
2. Migration tables
3. Fertility tables

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

76. Which of the following are International Statistical Organizations ?

1. Economic and Social Commission for Asia and the Pacific
2. North Atlantic Treaty Organization
3. The Asian Development Bank
4. The International Labour Organization

Select the correct answer using the code given below :

(a) 1, 2 and 3

(b) 1, 2 and 4

(c) 1, 3 and 4

(d) 2, 3 and 4

77. Consider the following statements regarding Global Hunger Index (GHI) :

1. GHI is a 100 point scale, where 0 (zero) is the best score and 100 is the worst score.
2. GHI consists of four indicators that together capture the multidimensional nature of hunger.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

78. Consider the following statements:

1. The population census is a Union subject and is listed at serial number 69 of the Eighth Schedule of the Constitution of India.
2. The responsibility of conducting the census rests with the office of the Registrar General and Census Commissioner of India, Ministry of Home Affairs, Government of India.

Which of the statements given above is/are correct ?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

79. Which of the following are monthly High Frequency Indicators (HFIs) ?

1. Consumer Price Index
2. Rail Freight Traffic
3. E-way Bills
4. Population

Select the correct answer using the code given below :

(a) 1, 2 and 3

(b) 1, 2 and 4

(c) 1, 3 and 4

(d) 2, 3 and 4

80. Which of the following Goods and Services are omitted from GDP ?

1. Household production such as preparing meals, gardening, etc.
2. Leisure time activities
3. Helping children with homework and similar activities

Select the correct answer using the code given below :

(a) 1 and 2 only

(b) 2 and 3 only

(c) 1 and 3 only

(d) 1, 2 and 3

SPACE FOR ROUGH WORK

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