MATHEMATICS
Paper – II

Time Allowed : ***Three*** *Hours*

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are ***EIGHT*** *questions in all, out of which* ***FIVE*** *are to be attempted.*

Questions no. ***1*** *and* ***5*** *are* ***compulsory****. Out of the remaining* ***SIX*** *questions,* ***THREE*** *are to be attempted selecting at least* ***ONE*** *question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary, and indicate the same clearly.

Answers must be written in ***ENGLISH*** *only.*

SECTION A

Q1. (a) Let G be a finite commutative group. Let $n \in \mathbf{Z}$ be such that n and order of G are relatively prime. Show that the function $\phi : G \to G$ defined by $\phi(a) = a^n$, for all $a \in G$, is an isomorphism of G onto G. *8*

(b) Apply Cauchy's Principle of Convergence to prove that the sequence $<f_n>$ defined by

$$f_n = 1 + \frac{1}{4} + \frac{1}{7} + \ldots + \frac{1}{3n-2}$$

is not convergent. *8*

(c) Find $\frac{dy}{dx}$, when

$$f(x, y) = \log(x^2 + y^2) + \tan^{-1}\left(\frac{y}{x}\right) = 0,$$

on using derivatives of Implicit Functions. *8*

(d) An automobile dealer wishes to put four repairmen R_1, R_2, R_3 and R_4 to four different jobs J_1, J_2, J_3 and J_4. But R_3 cannot do the job J_2. The dealer has estimated the number of man-hours that would be required for each job-man on one-one basis as given in the following table :

	R_1	R_2	R_3	R_4
J_1	6	2	3	4
J_2	9	7	–	5
J_3	6	4	7	5
J_4	6	8	8	9

Formulate the above as a Linear Programming Problem. *8*

(e) If $f(z) = u + iv$ is any analytic function of the complex variable z and $u - v = e^x(\cos y - \sin y)$, find f(z) in terms of z. *8*

Q2. (a) Prove that every group is isomorphic to a permutation group. *10*

(b) Examine the convergence of $\int_0^\infty \frac{dx}{(1+x)\sqrt{x}}$ and find its value, if possible. *15*

(c) Find the Taylor's series expansion of a function of the complex variable

$$f(z) = \frac{1}{(z-1)(z-3)}$$

about the point z = 4. Find its radius of convergence. *15*

Q3. (a) Examine the existence of maxima and minima of the function, $u(x, y) = xy + \frac{8}{x} + \frac{8}{y}$. *10*

(b) (i) Let R be a non-zero commutative ring with unity. If every ideal of R is prime, prove that R is a field.

(ii) Let R be a commutative ring with unity such that $a^2 = a, \forall a \in R$. If I be any prime ideal of R, find all the elements of $\frac{R}{I}$. *8+7*

(c) Consider the following Linear Programming Problem as primal :

Minimize $z = 30x_1 + 20x_2$

s/t,

$3x_1 + 5x_2 \geq 100$

$2x_1 + x_2 \geq 120$

$5x_1 + 3x_2 \geq 90$

$x_1, x_2 \geq 0$

Then using the principle of duality, find the optimal solution of the primal. *15*

Q4. (a) Show that

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta}\, d\theta = \frac{\pi}{12}.$$ 15

(b) Show that an element x in a Euclidean domain is a unit if and only if $d(x) = d(1)$, where the notations have their usual meanings. 10

(c) Starting with Least Cost Method, find all the solutions to the following transportation problem : 15

Warehouses

		I	II	III	IV		
	A	8	6	5	3	18	
Plants	B	6	7	6	8	20	Supply
	C	10	8	4	5	18	
		15	16	12	13		

Demand

SECTION B

Q5. (a) Find the complete primitive of

$$4r - 4s + t = 16 \log_e (x + 2y),$$

r, s, t bear their usual meanings. *8*

(b) From the following table, estimate the number of students who obtained marks between 40 and 46 : *8*

Marks :	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students :	32	43	55	40	30

(c) Consider the following integers and their 8-bits binary representations :

13 = 00001101, 20 = 00010100

Perform the following bitwise operations and express the results in decimal system : *2+2+2+2*

(i) 13 & 20 (Bitwise AND)

(ii) 13 | 20 (Bitwise OR)

(iii) 13 ^ 20 (Bitwise XOR)

(iv) ~ 20 (Bitwise Compliment)

(d) Examine the motion of a particle sliding on a parabolic wire given by $x^2 = 2y$. *8*

(e) Find the orthogonal trajectory of the following family of curves :

$$x^2 - y^2 = a^2$$

Then sketch the two families to demonstrate whether they cut orthogonally. *8*

Q6. (a) Solve the following by Charpit's method : *10*

$$pxy + pq + qy = yz,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$$

(b) Using Regula-Falsi method, find the fourth root of 28 correct to three decimal places. *15*

(c) Verify whether the motion given by

$$\vec{q} = (3x\,\hat{i} - 2y\,\hat{j})\, xy^2$$

is a possible fluid motion. If so, is it of the potential kind ? Accordingly find out the streamlines and the velocity potential or the angular velocity if the fluid was replaced by a rigid solid. *15*

Q7. (a) Write down the algorithm and flowchart of Runge-Kutta method of fourth order to find the numerical solution at $x = 0{\cdot}8$ for

$$\frac{dy}{dx} = \sqrt{2(x+y)},\ y(0{\cdot}4) = 0{\cdot}82.$$ *7+8*

Assume the step length $h = 0{\cdot}2$.

(b) Discuss the flow given by the complex potential

$$w = \log_e \left(z - \frac{a^2}{z} \right).$$

Draw sketches of the streamlines and explain the flow directions along the streamlines. *15*

(c) Solve the following differential equation : *10*

$$(y^2 + z^2 - x^2)\, p - 2xyq + 2xz = 0,\ p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$$

Q8. (a) Derive the Lagrange's equation for a spherical problem. *15*

(b) Solve the following system of equations by Gauss-Seidel method : *10*

$$20x + y - 3z = 16$$

$$2x + 20y - z = -19$$

$$3x - 2y + 20z = 25$$

starting with the initial solution $x_0 = y_0 = z_0 = 0$.

(c) Find the singular solution of $yp^2 - 2xp + y = 0$. Also trace the graph. *15*