

PHYSICS

Paper – I

Time Allowed : **Three Hours**

Maximum Marks : **200**

**Question Paper Specific Instructions**

**Please read each of the following instructions carefully before attempting questions :**

There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.

Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Neat sketches may be drawn, wherever required.

Answers must be written in **ENGLISH** only.

**Useful Constants :**

Electron charge (e)	= $1.602 \times 10^{-19}$ C
Electron rest mass ( $m_e$ )	= $9.109 \times 10^{-31}$ kg
Proton mass ( $m_p$ )	= $1.672 \times 10^{-27}$ kg
Vacuum permittivity ( $\epsilon_0$ )	= $8.854 \times 10^{-12}$ farad/m
Vacuum permeability ( $\mu_0$ )	= $1.257 \times 10^{-6}$ henry/m
Velocity of light in free space (c)	= $3 \times 10^8$ m/s
Boltzmann constant (k)	= $1.380 \times 10^{-23}$ J/K
Electron volt (eV)	= $1.602 \times 10^{-19}$ J
Planck constant (h)	= $6.626 \times 10^{-34}$ J s
Stefan constant ( $\sigma$ )	= $5.67 \times 10^{-8}$ W m <sup>-2</sup> K <sup>-4</sup>
Avogadro number (N)	= $6.022 \times 10^{26}$ kmol <sup>-1</sup>
Gas constant (R)	= $8.31 \times 10^3$ J kmol <sup>-1</sup> K <sup>-1</sup>
exp (1)	= 2.718

## SECTION A

**Q1.** (a) (i) Define the angular velocity  $\omega$  of a rigid body rotating about some axis.

(ii) Then derive the relation

$$\vec{v} = \vec{\omega} \times \vec{r}$$

for the velocity of a point in the body with position vector  $\vec{r}$  relative to an origin on the axis. Draw a diagram explaining all the vectors.

8

(b) A comet of mass  $m$  moves towards the Sun with initial velocity  $v_0$ . The mass of the sun is  $M$  and its radius is  $R$ . Find the total cross-section  $\sigma$  for striking the Sun. Take the Sun to be at rest and ignore all other bodies.

8

(c) Obtain the Lorentz transformations for the components of momentum-energy four vector.

8

(d) A body executes Simple Harmonic Motion such that its velocity at the mean position is  $2.0$  m/s and acceleration at one of the extremities is  $1.57$  m/s<sup>2</sup>. Calculate the time period of vibration.

8

(e) Consider that a continuous laser beam which is assumed to be perfectly monochromatic with a wavelength of  $6500 \text{ \AA}$  is chopped into  $2$  ns pulses using a shutter. Calculate the number of photons in each pulse, if the power of each pulse is  $5$  mW.

8

**Q2.** (a) During take-off, an aircraft accelerates horizontally in a straight line at a rate  $A$ . A small bob of mass  $m$  is suspended on a string attached to the roof of the cabin, and a hydrogen balloon (total mass  $m$ ) is tethered to the floor by a string. For each, determine the tension in the string and the equilibrium angle  $\theta$  between string and vertical. Draw a neat diagram to explain your answer.

10

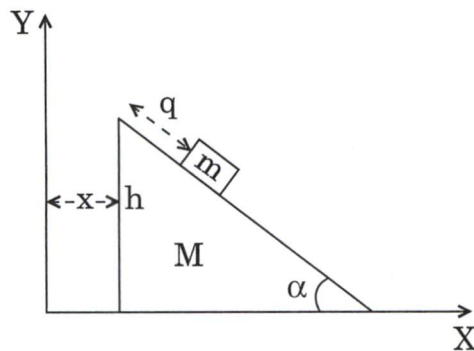
(b) A rigid body is rotating under the influence of an external torque ( $N$ ) acting on it. If  $T$  is the kinetic energy and  $\omega$  is the angular velocity,

$$\text{show that } \frac{dT}{dt} = N \cdot \omega$$

in the principal axes system.

15

- (c) A block of mass  $m$  is sliding on a wedge of mass  $M$  as shown in the figure below. The wedge can slide on the horizontal table. Find the equation of motion. 15



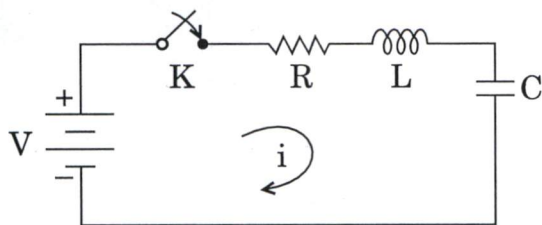
- Q3.** (a) What are beats ?  
 A tuning fork gives 6 beats per second with 25.4 cm length of stretched sonometer wire. The same 6 beats are heard per second when the length of the same wire is reduced to 24.6 cm. Calculate the frequency of the tuning fork. 10
- (b) (i) What is achromatisation ? Obtain the condition for achromatism of two thin lenses separated by a distance of  $x$  metres. 10
- (ii) Two glasses have dispersive powers in the ratio 2 : 3. These glasses are used in the manufacture of an achromatic contact objective of focal length 50 cm. What are the focal lengths of the lenses ? 5
- (c) What is the Resolving power of a grating ? Obtain an expression for the Resolving power of a grating, based on Rayleigh's criterion of resolution. 15
- Q4.** (a) On a space-time diagram after the Lorentz transformation with given  $v$  we get new tilted axis  $ct'$  and  $x'$ . For a point on the  $ct'$  axis which is at the (geometrical) distance  $\delta$  on the diagram measured from the origin, find the values of  $t'$  and  $t$ . 10
- (b) (i) A transparent film of glass of refractive index 1.5 is introduced normally in the path of one of the interfering beams of Michelson interferometer which is illuminated with light of wavelength 5893 Å. This causes 500 dark fringes to shift across the field. Determine the thickness of the film. 8
- (ii) What is holography ? Briefly discuss the applications of holography. 7
- (c) What is the principle of propagation in optical fibers ? Explain with the help of neat diagram, the ray propagation in Step index and Parabolic index optical fibers. How can pulse dispersion be minimized using Parabolic index optical fibers ? 15

## SECTION B

- Q5.** (a) Write the Laplace's equation in Cartesian, spherical polar and cylindrical coordinates in one-dimensional space. Using a suitable Laplace's equation out of these three, find the capacitance of parallel plate capacitor. 8
- (b) Justify the importance of method of images. Using this method, evaluate induced charge density on the surface of a grounded conducting sphere when a point charge is placed near it. 8
- (c) How many nitrogen molecules must strike a  $1 \text{ cm}^2$  surface each second to exert a pressure of 1 atmosphere ? (Assume the molecules are all moving at same speed, corresponding to a temperature of 300 K, and at an angle of  $45^\circ$  to the wall). 8  
[Molecular mass of  $\text{N}_2$  is 28 u]
- (d) A gas of  $N$  spinless Bose particles of mass  $m$  is enclosed in a volume  $V$  at a temperature  $T$ .
- (i) Find an expression for the density of single-particle state  $D(\epsilon)$  as a function of the single-particle energy  $\epsilon$ . Sketch the result.
- (ii) Write down an integral expression which implicitly determines  $\mu(T)$ . Referring to your sketch in (i), determine in which direction  $\mu(T)$  moves as  $T$  is lowered. 8
- (e) Write a brief note on Chandrasekhar Limit. 8
- Q6.** (a) Write Maxwell's equations in differential and algebraic forms. Give the physical significance of each. Show that the equation of continuity is contained in Maxwell's equations. 10
- (b) Define gauge transformation. Under what condition can the gauge transformation be regarded as restricted gauge transformation ? Further, generate the symmetrical forms of Maxwell's equations in terms of scalar and vector potentials under Lorentz gauge. 10

- (c) (i) Show that in an A.C. circuit with inductor only, current lags behind emf in phase by  $\frac{\pi}{2}$ . If the current through 0.5 Henry inductor varies sinusoidally with an amplitude of 10 amp and a frequency 50 Hz, calculate the potential difference across the terminals of the inductor. 10

- (ii) In the circuit as shown in figure :

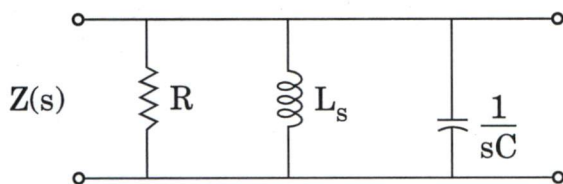


where  $V = 10 \text{ V}$ ,  $R = 10 \Omega$ ,  $L = 1 \text{ H}$ ,  $C = 10 \mu\text{F}$ , and  $v_c(0) = 0$ ,

find  $i(0+)$ ,  $\frac{di}{dt}(0+)$  and  $\frac{d^2i}{dt^2}(0+)$ . 10

- Q7.** (a) The impedance function of a parallel R, L and C circuit has poles located at  $-3 \pm 4j$  radians/second. If the value of  $L = 1 \text{ Henry}$ , determine the following :

- (i) The value of R and C.  
 (ii) What is the quality factor of this circuit? Derive an expression for series and parallel circuit. 15



- (b) (i) Consider one gm of ice at a temperature  $T_1 \text{ K}$ . Show that when this ice changes into steam at a temperature  $T_2 \text{ K}$ , the total gain in entropy is

$$\Delta S = \frac{L_i}{T_1} + C \log_e \left( \frac{T_2}{T_1} \right) + \frac{L_s}{T_2}$$

where  $L_i$  is latent heat of ice,  $C$  is specific heat of water,  $L_s$  is latent heat of steam.  $T_1 = 273 \text{ K}$ . 10

- (ii) Calculate the work done in expanding one mole of an ideal gas at  $127^\circ\text{C}$  to double its initial volume. 5

- (c) Find the pressure at which water would boil at  $150^{\circ}\text{C}$  if the change in specific volume when one gm of water is converted into steam is 1676 c.c. Given  $J = 4.2 \times 10^7$  ergs/cal, one atmosphere =  $10^6$  dyne/cm<sup>2</sup> and latent heat of vapourization of steam = 540 cal/gm. 10

**Q8.** (a) Define a black body and evaluate Planck's radiation law. Show that at high temperature, this law resembles Rayleigh – Jeans law. 15

(b) What are geomagnetic storms and why are they potentially disruptive to power grids? 10

(c) A hypothetical engine, with an ideal gas as the working substance, operates in the cycle shown below. Show that the efficiency of the engine is 15

$$\eta = 1 - \frac{1}{\gamma} \left( \frac{1 - \frac{P_3}{P_1}}{1 - \frac{V_1}{V_3}} \right).$$

