

STATISTICS

Paper – I

Time Allowed : **Three Hours**

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. 1 and 5 are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

*Answers must be written in **ENGLISH** only.*

SECTION A

Q1. (a) (i) Let $P(A_n) = 0$ for each $n \geq 1$. With proper justification show that

$$P\left(\bigcup_{n=1}^{\infty} A_n\right) = 0.$$

(ii) Let $P(A_n) = 1$ for each $n \geq 1$. With proper justification show that

$$P\left(\bigcap_{n=1}^{\infty} A_n\right) = 1. \quad 6+2=8$$

(b) Do the following functions define cumulative distribution functions ?

$$(i) \quad F(x) = \begin{cases} 0 & \text{if } x < 0, \\ x & \text{if } 0 \leq x < \frac{1}{2}, \\ 1 & \text{if } x \geq \frac{1}{2}. \end{cases}$$

$$(ii) \quad F(x) = \begin{cases} 0 & \text{if } x \leq 1, \\ 1 - \frac{1}{x} & \text{if } x > 1. \end{cases} \quad 4+4=8$$

(c) Construct sequential probability ratio test for testing $H_0 : p = p_0$ against $H_1 : p = p_1$, when X is a Bernoulli random variable defined with probability function given by

$$f(x; p) = p^x (1 - p)^{1-x}, \quad x : 0, 1; \quad 0 < p < 1. \quad 8$$

(d) A company wishes to purchase one of five different machines : A, B, C, D or E. In an experiment designed to determine whether there is a performance difference among the machines, five experienced operators were assigned to work on the machines for equal times. Data given below represents the number of units produced by each machine :

A	68	72	77	42	53
B	72	53	63	53	48
C	60	82	64	75	72
D	48	61	57	64	50
E	64	65	70	68	53

Test the hypothesis that there is no difference between the machines at 0.05 significance level.

[Note : Refer the chi-square table for the theoretical value of χ^2].

Chi-square distribution table is provided at the end of the booklet.

- (e) Let X_1 and X_2 be independent and identically distributed random variables with common probability mass function

$$P[X = \pm 1] = \frac{1}{2}.$$

Write $X_3 = X_1X_2$. Show that X_1, X_2, X_3 are pairwise independent but not independent. 8

- Q2.** (a) (i) A lot of five identical batteries is life tested. The probability assignment is assumed to be

$$P(A) = \int_A \frac{1}{\lambda} e^{-x/\lambda} dx$$

for any event $A \subseteq [0, \infty)$, where $\lambda > 0$ is a known constant. Thus the probability that a battery fails after time t is given by

$$P(t, \infty) = \int_t^{\infty} \frac{1}{\lambda} e^{-x/\lambda} dx, \quad t \geq 0.$$

If the times to failure of the batteries are independent, what is the probability that at least two batteries will be operating after t_0 hours? 7

- (ii) Let (X_1, X_2, \dots, X_n) be a random sample of size n drawn from a population whose probability density function is given by the form

$$f(x; \theta) = (1 + \theta) x^\theta, \quad 0 < x < 1; \quad \theta > 0.$$

Find the maximum likelihood estimator for θ and examine whether such an estimator is sufficient for θ . 5+3=8

- (b) Consider a Markov chain assuming values in the state space $S = \{0, 1, 2, 3, 4\}$ and having the following transition probability matrix :

$$P = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right] \end{matrix}$$

- (i) Determine the period of each of the states.
 (ii) Identify the states that are transient and the ones that are recurrent. 10

- (c) (i) A practitioner is interested in testing whether the number of accidents occurring daily in a certain industrial plant is Poisson distributed with some unknown mean λ . Assume that the outcome of the number of accidents on a given day is in Region 1 if there are 0 accidents, Region 2 if there is 1 accident, Region 3 if there are 2 or 3 accidents, Region 4 if there are 4 or 5 accidents, and Region 5, if there are more than 5 accidents. Under these given conditions, if the weekly number of accidents over a 30-week period is as given below, test the hypothesis that the number of accidents in a week has a Poisson distribution.

8

8	0	0	1	3	4	0	2	12	5
1	8	0	2	0	1	9	3	4	5
3	3	4	7	4	0	1	2	1	2

[Note : Refer the chi-square table for the theoretical value of χ^2]

Chi-square distribution table is provided at the end of the booklet.

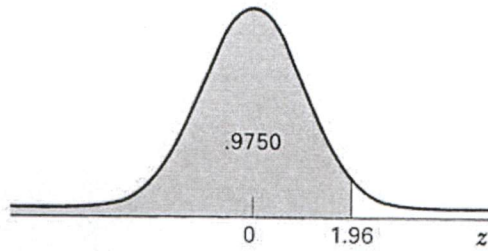
- (ii) In an experiment of testing the nature of 30 items drawn from a lot, the following sequence of defectives (D) and non-defectives (N) is obtained :

N D D N D N N N D N N D D
 N D N D N N D N D D N D
 N N D N D

- (1) Determine the number of runs.
- (2) Examine whether the sequence is random at 5% level of significance. 2+5=7

[Note : Refer the standard normal table for the theoretical value]

Normal Distribution Table



z	-0.09	-0.08	-0.07	-0.06	-0.05	-0.04	-0.03	-0.02	-0.01	0.00	z
-3.80	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	-3.80
-3.70	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	-3.70
-3.60	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	-3.60
-3.50	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	-3.50
-3.40	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	-3.40
-3.30	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	-3.30
-3.20	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	-3.20
-3.10	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	-3.10
-3.00	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	-3.00
-2.90	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	-2.90
-2.80	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	-2.80
-2.70	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	-2.70
-2.60	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	-2.60
-2.50	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	-2.50
-2.40	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	-2.40
-2.30	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	-2.30
-2.20	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	-2.20
-2.10	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	-2.10
-2.00	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	-2.00
-1.90	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	-1.90
-1.80	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	-1.80
-1.70	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	-1.70
-1.60	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	-1.60
-1.50	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	-1.50
-1.40	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	-1.40
-1.30	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	-1.30
-1.20	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	-1.20
-1.10	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	-1.10
-1.00	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	-1.00
-0.90	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	-0.90
-0.80	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	-0.80
-0.70	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420	-0.70
-0.60	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	-0.60
-0.50	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	-0.50
-0.40	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	-0.40
-0.30	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	-0.30
-0.20	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	-0.20
-0.10	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	-0.10
0.00	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	0.00

Normal Distribution Table

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	<i>z</i>
0.00	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	0.00
0.10	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	0.10
0.20	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	0.20
0.30	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	0.30
0.40	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	0.40
0.50	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	0.50
0.60	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	0.60
0.70	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	0.70
0.80	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	0.80
0.90	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	0.90
1.00	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	1.00
1.10	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	1.10
1.20	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	1.20
1.30	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	1.30
1.40	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1.40
1.50	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1.50
1.60	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1.60
1.70	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1.70
1.80	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1.80
1.90	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1.90
2.00	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	2.00
2.10	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	2.10
2.20	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	2.20
2.30	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	2.30
2.40	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	2.40
2.50	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	2.50
2.60	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	2.60
2.70	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	2.70
2.80	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	2.80
2.90	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	2.90
3.00	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	3.00
3.10	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	3.10
3.20	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	3.20
3.30	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	3.30
3.40	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	3.40
3.50	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	3.50
3.60	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.60
3.70	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.70
3.80	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	3.80

- Q3.** (a) (i) Let $\{X_n\}_{n \geq 1}$ be a strictly decreasing sequence of positive random variables and suppose that $X_n \xrightarrow{P} 0$. Show that $X_n \xrightarrow{\text{a.s.}} 0$. 7
- (ii) Let X be a random variable defined with the probability function given by

$$f(x; \theta) = \frac{1}{\theta} e^{-x/\theta}, 0 < x < \infty; 0 < \theta < \infty.$$

Given a random sample (X_1, X_2, \dots, X_n) of size n drawn from the population having the above probability density function, show

that the sample mean $\bar{X} = \frac{1}{n} \sum_{i=1}^n X_i$ is the minimum variance

bound estimator for θ and has variance $\frac{\theta^2}{n}$. 8

- (b) Let X_1, X_2, \dots, X_n be independent and identically distributed $U(0, \theta)$ random variables, where $\theta > 0$ is an unknown parameter. Show that

$$\max\{X_1, X_2, \dots, X_n\} \xrightarrow{P} \theta. \quad 15$$

- (c) Define Monotone Likelihood Ratio (MLR) property. State its significance. Examine whether MLR property holds for the family of distributions when a random sample of n observations is drawn on X which follows a binomial distribution. 10

- Q4.** (a) (i) Let X_1, X_2, \dots be a sequence of independent and identically distributed random variables with common mean μ and variance

$$\sigma^2. \text{ Also, let } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i \text{ and } S^2 = \frac{1}{(n-1)} \sum_{i=1}^n (X_i - \bar{X})^2. \text{ Show}$$

that $\sqrt{n}(\bar{X} - \mu)/S \xrightarrow{L} Z$, where $Z \sim N(0, 1)$. 7

- (ii) Let p be the probability of getting a success in a Bernoulli trial. Assume that five independent Bernoulli trials are conducted for testing the hypothesis $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{3}{4}$. Find the size and power of the statistical test under the condition that H_0 is rejected if more than three successes occur. 4+4=8

- (b) (i) Let (Ω, \mathcal{F}, P) be a probability space. Let $A, B, C \in \Omega$ with $P(B)$ and $P(C) > 0$. If B and C are independent, show that

$$P(A|B) = P(A|B \cap C)P(C) + P(A|B \cap C^c)P(C^c). \quad 7$$

- (ii) Let X and Y be random variables defined on a probability space (Ω, \mathcal{F}, P) .

(1) If $P(X \geq 0) = 1$, then show that $E(X|Y) \geq 0$ with probability 1.

(2) If X_1 and X_2 are two random variables defined on the same probability space and if $P(X_1 \geq X_2) = 1$, then show that $E(X_1|Y) \geq E(X_2|Y)$ with probability 1. 8

- (c) Define sufficiency of an estimator. Let X_1, X_2 and X_3 be three independent observations on a random variable X which follows a Bernoulli distribution with parameter θ . Examine the sufficiency of the statistic $T = X_1 + 2X_2 + 3X_3$ for θ . 10

SECTION B

- Q5.** (a) What do you mean by canonical correlations ? If $\mathbf{x} : r \times 1$ and $\mathbf{y} : s \times 1$ be two vectors such that $r \leq s$, $\text{Cov}(\mathbf{x}) = \Sigma_{11}$, $\text{Cov}(\mathbf{y}) = \Sigma_{22}$, $\text{Cov}(\mathbf{x}, \mathbf{y}) = \Sigma_{12}$ and $\Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}$ has full rank. How many canonical correlations exist ? If λ_i is an eigenvalue of $\Sigma_{11}^{-1/2} \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21} \Sigma_{11}^{-1/2}$ with associated eigenvector \mathbf{e}_i , then show that λ_i is also an eigenvalue of $\Sigma_{11}^{-1} \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}$ with associated eigenvector $\Sigma_{11}^{-1/2} \mathbf{e}_i$. 8
- (b) Suppose that ρ is a correlation matrix of order five with all off-diagonal elements equal to 0.80. Calculate the proportion of the total population variance explained by the first principal component. 8
- (c) From a list of 2400 minor household expenditures, a simple random sample of 180 items was drawn in order to estimate the total amount spent for operation of the household. Certain types of expenditures (on clothing and car upkeep) were not considered relevant. Of the 180 sample items, 150 were relevant. The sum and uncorrelated sum of squares of the relevant amounts (in ₹) were as follows :
- $$\sum y_i = 433.53, \quad \sum y_i^2 = 1941.83.$$
- Estimate the total expenditure for household operation and find the standard error of the estimate. Note that y_i represents household expenditure for i^{th} relevant sample item. 8
- (d) What is meant by a uniformity trial ? How would you study the relative efficiency of RBD (Randomized Block Design) over CRD (Completely Randomized Design) using uniformity trial ? Obtain the expressions for estimated relative efficiency of RBD over CRD. 8

(e) Find \mathbf{b} and A so that the joint density function

$$f(x, y) = \frac{1}{2.4 \pi} \exp \left\{ -\frac{1}{0.72} \left(\frac{x^2}{4} - 0.8xy + y^2 \right) \right\}, -\infty < x, y < \infty$$

can be written in the form ($\mathbf{x}' = (x, y)$)

$$\frac{|A|^{1/2}}{(2\pi)^{p/2}} \exp \left\{ -\frac{1}{2} (\mathbf{x} - \mathbf{b})' A (\mathbf{x} - \mathbf{b}) \right\}.$$

Also find $\mu_x, \mu_y, \sigma_x, \sigma_y$ and ρ_{xy} .

8

Q6. (a) Let $\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3$ and \mathbf{X}_4 be independent and identically distributed 3×1 random vectors with

$$\boldsymbol{\mu} = \begin{pmatrix} -3 \\ 1 \\ 4 \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 2 \end{pmatrix}.$$

If $\mathbf{Y}_1 = \frac{1}{2} \mathbf{X}_1 + \frac{1}{2} \mathbf{X}_2 + \frac{1}{2} \mathbf{X}_3 + \frac{1}{2} \mathbf{X}_4$, and

$$\mathbf{Y}_2 = \frac{1}{2\sqrt{3}} \mathbf{X}_1 + \frac{1}{2\sqrt{3}} \mathbf{X}_2 + \frac{1}{2\sqrt{3}} \mathbf{X}_3 - \frac{\sqrt{3}}{2} \mathbf{X}_4$$

be linear combination of vectors, then obtain (i) $E(\mathbf{Y}_1)$, (ii) $E(\mathbf{Y}_2)$, (iii) $\text{Var}(\mathbf{Y}_1)$, (iv) $\text{Var}(\mathbf{Y}_2)$, and (v) $\text{Cov}(\mathbf{Y}_1, \mathbf{Y}_2)$. If \mathbf{X}_i ($i = 1, 2, 3, 4$) has trivariate normal distribution, then what is the joint distribution of \mathbf{Y}_1 and \mathbf{Y}_2 ?

Further if $\mathbf{X} \sim N_3(\boldsymbol{\mu}, \Sigma)$, then write the joint distribution of $(\mathbf{X}_1 - \mathbf{X}_2 + \mathbf{X}_3)$

and $(\mathbf{X}_1 + \mathbf{X}_2 - \mathbf{X}_3)$. Here $\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 \\ \mathbf{X}_2 \\ \mathbf{X}_3 \end{pmatrix}$, $\boldsymbol{\mu}$ and Σ are as given above.

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- (b) Nelson's goal was to estimate the lifetime (in hours) of an encapsulating resin for gold-aluminium bonds in integrated circuits operating at 120°C. Thirty-seven units were assigned at random to one of the five different temperature stresses, ranging from 175° to 250°. Following table gives the \log_{10} (lifetimes) in hours for the test units.

log ₁₀ times till failure of a resin under stress									
Temperature (°C)									
175°		194°		213°		231°		250°	
2.04	1.85	1.66	1.66	1.53	1.35	1.15	1.21	1.26	1.02
1.91	1.96	1.71	1.61	1.54	1.27	1.22	1.28	0.83	1.09
2.00	1.88	1.42	1.55	1.38	1.26	1.17	1.17	1.08	1.06
1.92	1.90	1.76	1.66	1.31	1.38	1.16			

Compute the overall mean and treatment effects. Compute the Analysis of Variance table. Write your conclusions.

(F-distribution table is provided at the end of the booklet).

15

- (c) Three independent measurements on each of the angles A, B and C of a triangle are as follows :

A	B	C
39.5	60.3	80.1
39.3	62.2	80.3
39.6	60.1	80.4

Obtain the best estimates of the three angles taking into account the relation that sum of angles is equal to 180°.

10

- Q7.** (a) Explain Invariance property of Hotelling's T^2 statistic. Verify this property for testing $H_0 : \mu' = (9, 5)$ vs $H_1 : \mu' \neq (9, 5)$ using data matrix for a random sample of size $n = 3$ from a bivariate normal population

$$X = \begin{pmatrix} 6 & 9 \\ 10 & 6 \\ 8 & 3 \end{pmatrix}$$

and matrix $C = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Test H_0 at $\alpha = 0.05$ level of significance. Write your conclusion.

(Table of F-distribution percentage points ($\alpha = 0.05$) is attached at the end of the booklet).

15

- (b) Explicitly state the main objective of randomized complete block design. State the corresponding additive model and the associated assumptions. Describe the procedure for testing the treatment effects when such a design is used.

15

- (c) How does curvilinear regression differ from linear regression ? Given

$$f(x, y) = \begin{cases} 1, & \text{if } |y| < x, \quad 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

show that the regression of Y on X is linear but regression of X on Y is curvilinear.

10

- Q8.** (a) Following table gives the data for 10 systematic samples with $n = 4$ and $N = 40$. Each column represents a systematic sample and the rows are the strata.

Strata	Systematic sample numbers									
	1	2	3	4	5	6	7	8	9	10
I	0	1	1	2	5	4	7	7	8	6
II	6	8	9	10	13	12	15	16	16	17
III	18	19	20	20	24	23	25	28	29	27
IV	26	30	31	31	33	32	35	37	38	38

Compare the precision of systematic sampling with simple random sampling.

15

- (b) To construct a procedure for detecting potential hemophilia A carriers, blood samples were assayed for two groups of women and measurements on two variables, $X_1 = \log_{10}$ (AHF activity) and $X_2 = \log_{10}$ (AHF-like antigen) recorded. After investigation, we have the following information. Here $n_1 = 30$, $n_2 = 22$.

$$\bar{\mathbf{x}}_1 = \begin{pmatrix} -0.0065 \\ -0.0390 \end{pmatrix}, \quad \bar{\mathbf{x}}_2 = \begin{pmatrix} -0.2483 \\ 0.0262 \end{pmatrix},$$

$$S_{\text{Pooled}}^{-1} = \begin{pmatrix} 131.158 & -90.423 \\ -90.423 & 108.147 \end{pmatrix}.$$

Obtain Fisher's linear discriminant function. Classify the new observation $(-0.210, -0.044)$ in one of the two groups when $P_1 = P_2$ and $C(1|2) = C(2|1)$. If $C(1|2) =$ cost incurred when a π_2 observation is incorrectly classified as $\pi_1 = 10$ and $C(2|1) =$ cost incurred when a π_1 observation is incorrectly classified as $\pi_2 = 5$, then classify the new observation $(-0.210, -0.044)$ in one of the two groups π_1 or π_2 . ($P_1 =$ prior probability for group I, $P_2 =$ prior probability for group II and $C(1|2)$ and $C(2|1)$ are cost of misclassification).

15

- (c) A population consists of five numbers : 2, 3, 6, 8, 11. Consider all possible samples of size 2 that can be drawn without replacement from this population. Find the following :

10

- (i) The mean of the population
- (ii) The standard deviation of the population
- (iii) The mean of the sampling distribution of means
- (iv) The standard error of means

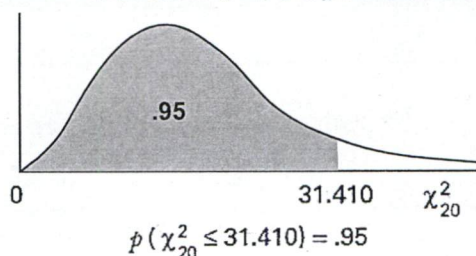
F-distribution Table

		$F_{0.95}$								
Denominator Degrees of Freedom	Numerator Degrees of Freedom									
	1	2	3	4	5	6	7	8	9	
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	

F-distribution Table

		$F_{0.99}$									
Denominator Degrees of Freedom	Numerator Degrees of Freedom										
	10	12	15	20	24	30	40	60	120	∞	
1	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3	
2	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50	
3	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53	
4	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63	
5	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36	
6	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67	
7	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23	
8	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93	
9	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71	
10	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54	
11	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40	
12	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30	
13	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21	
14	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13	
15	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07	
16	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01	
17	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96	
18	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92	
19	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88	
20	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84	
21	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81	
22	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78	
23	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76	
24	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73	
25	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71	
26	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69	
27	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67	
28	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65	
29	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64	
30	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62	
40	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51	
60	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39	
120	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.25	
∞	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00	

Percentiles of the Chi-Square Distribution



d.f.	$\chi^2_{.005}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$
1	.0000393	.000982	.00393	2.706	3.841	5.024	6.635	7.879
2	.0100	.0506	.103	4.605	5.991	7.378	9.210	10.597
3	.0717	.216	.352	6.251	7.815	9.348	11.345	12.838
4	.207	.484	.711	7.779	9.488	11.143	13.277	14.860
5	.412	.831	1.145	9.236	11.070	12.832	15.086	16.750
6	.676	1.237	1.635	10.645	12.592	14.449	16.812	18.548
7	.989	1.690	2.167	12.017	14.067	16.013	18.475	20.278
8	1.344	2.180	2.733	13.362	15.507	17.535	20.090	21.955
9	1.735	2.700	3.325	14.684	16.919	19.023	21.666	23.589
10	2.156	3.247	3.940	15.987	18.307	20.483	23.209	25.188
11	2.603	3.816	4.575	17.275	19.675	21.920	24.725	26.757
12	3.074	4.404	5.226	18.549	21.026	23.336	26.217	28.300
13	3.565	5.009	5.892	19.812	22.362	24.736	27.688	29.819
14	4.075	5.629	6.571	21.064	23.685	26.119	29.141	31.319
15	4.601	6.262	7.261	22.307	24.996	27.488	30.578	32.801
16	5.142	6.908	7.962	23.542	26.296	28.845	32.000	34.267
17	5.697	7.564	8.672	24.769	27.587	30.191	33.409	35.718
18	6.265	8.231	9.390	25.989	28.869	31.526	34.805	37.156
19	6.844	8.907	10.117	27.204	30.144	32.852	36.191	38.582
20	7.434	9.591	10.851	28.412	31.410	34.170	37.566	39.997
21	8.034	10.283	11.591	29.615	32.671	35.479	38.932	41.401
22	8.643	10.982	12.338	30.813	33.924	36.781	40.289	42.796
23	9.260	11.688	13.091	32.007	35.172	38.076	41.638	44.181
24	9.886	12.401	13.848	33.196	36.415	39.364	42.980	45.558
25	10.520	13.120	14.611	34.382	37.652	40.646	44.314	46.928
26	11.160	13.844	15.379	35.563	38.885	41.923	45.642	48.290
27	11.808	14.573	16.151	36.741	40.113	43.194	46.963	49.645
28	12.461	15.308	16.928	37.916	41.337	44.461	48.278	50.993
29	13.121	16.047	17.708	39.087	42.557	45.722	49.588	52.336
30	13.787	16.791	18.493	40.256	43.773	46.979	50.892	53.672
35	17.192	20.569	22.465	46.059	49.802	53.203	57.342	60.275
40	20.707	24.433	26.509	51.805	55.758	59.342	63.691	66.766
45	24.311	28.366	30.612	57.505	61.656	65.410	69.957	73.166
50	27.991	32.357	34.764	63.167	67.505	71.420	76.154	79.490
60	35.535	40.482	43.188	74.397	79.082	83.298	88.379	91.952
70	43.275	48.758	51.739	85.527	90.531	95.023	100.425	104.215
80	51.172	57.153	60.391	96.578	101.879	106.629	112.329	116.321
90	59.196	65.647	69.126	107.565	113.145	118.136	124.116	128.299
100	67.328	74.222	77.929	118.498	124.342	129.561	135.807	140.169