Indian Forest Service
(Main) Exam, 2021

ZCVB-B-STSC

STATISTICS

Paper – II

Time Allowed : **Three** Hours

Maximum Marks : **200**

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

There are ***EIGHT*** *questions in all, out of which* ***FIVE*** *are to be attempted.*

Questions no. ***1*** *and* ***5*** *are* ***compulsory****. Out of the remaining* ***SIX*** *questions,* ***THREE*** *are to be attempted selecting at least* ***ONE*** *question from each of the two Sections A and B.*

Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicate the same clearly.

Answers must be written in ***ENGLISH*** *only.*

SECTION A

Q1. (a) Distinguish between defects and defectives in SQC. Give some examples of defects for which the c-chart is applicable. How do you calculate control limits for a c-chart ? 8

(b) If T is a random variable representing the hours of failure for a device with following pdf :

$$f(t) = t\, e^{-t^2/2} \; ; \; t \geq 0$$

find the reliability function and hazard function. If 50 devices are placed in operation and 27 are still in operation 1 hour later, find approximately the expected number of failures in the time interval from 1 to 1·1 hours using hazard function. 8

(c) Solve the following linear programming problem graphically : 8

Minimize $Z = 20x_1 + 20x_2$

subject to

$$x_1 + 2x_2 \geq 1$$

$$2x_1 + x_2 \geq 2$$

$$2x_1 + 3x_2 \geq 3$$

$$3x_1 + 2x_2 \geq 4$$

$$x_1, x_2 \geq 0$$

(d) A company has a team of four salesmen and there are four districts where the company wants to start its business. The profit per day (in ₹) for each salesman in each district is estimated as given below :

		Districts			
		1	2	3	4
Salesman	A	16	10	14	11
	B	14	11	15	15
	C	15	15	13	12
	D	13	12	14	15

Find the assignment of salesmen to various districts which will yield maximum profit. 8

(e) Using the Rule of Dominance, solve the game with the following pay-off matrix : 8

Player B's strategies

Player A's strategies		1	2	3	4	5
	1	2	5	10	7	2
	2	3	3	6	6	4
	3	4	4	8	12	1

Q2. (a) Show the probability that at least one of the two points $\bar{X}$ and R goes outside the control limits

$$1-\left[S(\sqrt{n}\,T-3\rho)-S(\sqrt{n}\,T+3\rho)\right].\left[P\left(\frac{R}{\sigma}\leq D_2\,\rho\right)-P\left(\frac{R}{\sigma}\leq D_1\,\rho\right)\right]$$

where $\rho = \dfrac{\sigma'}{\sigma}$ $\quad T = \dfrac{\mu' - \mu}{\sigma}$

$$S(x) = \int_x^{\infty} e^{-\frac{1}{2}t^2}\,dt$$

assuming that the control charts are based on μ' population mean and σ' as population standard deviation, where the actual values of these parameters are μ and σ respectively. 15

(b) The time to failure in operating hours of a critical solid state power unit has the hazard rate function given by

$$\lambda(t) = (0{\cdot}003)\left(\frac{t}{500}\right)^{1/2} \text{ ; for } t \geq 0$$

(i) What is the reliability if the power unit must operate continuously for 50 hours ?

(ii) Determine the design life if a reliability of 0·90 is desired.

(iii) Compute the MTTF.

(iv) Given that the unit has operated for 50 hours, what is the probability that it will survive a second 50 hours of operation ? 10

(c) Use Simplex method to find an optimal solution to the following LPP :

Maximize $Z = 3x_1 + 2x_2$

subject to

$$-x_1 + 2x_2 \leq 4$$

$$3x_1 + 2x_2 \leq 14$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

Does it indicate the existence of an alternative optimal solution ? If yes, obtain it. *15*

Q3. (a) Define renewal function and renewal density function. Derive the following forward renewal equation :

$$U(t) = f_1(t) + \int_0^t U(t - \tau)\, f(\tau)\, d\tau$$ *15*

(b) The Optimal Simplex table of the following LPP :

Maximize $Z = 10x_1 + 15x_2 + 20x_3$

subject to

$$2x_1 + 4x_2 + 6x_3 \leq 24$$

$$3x_1 + 9x_2 + 6x_3 \leq 30$$

$$x_1, x_2, x_3 \geq 0$$

is given below :

C_B	C_j / X_B	10 x_1	15 x_2	20 x_3	0 x_4	0 x_5	b
20	x_3	0	-1	1	1/2	$-1/3$	2
10	x_1	1	5	0	-1	1	6
	$Z_j - C_j$	0	15	0	0	10/3	Z=100

Perform the sensivity analysis to check whether optimality of the table is affected if the profit coefficients are changed from (10, 15, 20) to (7, 14, 15). *10*

(c) Show that the Markov Chain with transition probability matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{pmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

is irreducible, periodic and all its states are persistent. *15*

Q4. (a) The number of customer complaints received daily by an organisation is given below :

Day	Complaint
1	2
2	3
3	0
4	1
5	9
6	2
7	0
8	0
9	4
10	2
11	0
12	7
13	0
14	2
15	4

Does it mean that the number of complaints is under statistical control ? Establish a control scheme for the future. Draw control chart. *15*

(b) At a certain petrol pump, customers arrive in a Poisson process with an average time of 5 minutes between arrivals. The time intervals between services at the petrol pump follow exponential and as such the mean time taken to service a unit is 2 minutes. Then find the average number of customers in the queuing system and how long, on an average, a customer has to wait in the queue ? *10*

(c) Write the applications of duality. Use Dual Simplex method to find the solution of the following LPP : *15*

Minimize $Z = x_1 + 4x_2 + 3x_4$

subject to

$$x_1 + 2x_2 - x_3 + x_4 \geq 3$$

$$-2x_1 - x_2 + 4x_3 + x_4 \geq 2$$

$$x_1, x_2, x_3, x_4 \geq 0$$

SECTION B

Q5. (a) Explain seasonal component of a time series. Explain Ratio to Moving Average method for measurement of seasonal variation in a time series data. *8*

(b) For the General Linear Model given by

$$\mathbf{Y} = X\beta + \mathbf{U}$$

with assumptions $E(\mathbf{U}) = 0$, $E(\mathbf{UU}') = \sigma^2 I_n$, X is a set of fixed numbers with $\rho(X) = k < n$, $\mathbf{B} = (\beta_1, \beta_2, \ldots \beta_k)'$ and $\mathbf{U} = (U_1, U_2, \ldots U_n)'$.

Obtain ordinary least square estimator $\hat{\beta}$ of β and show that it is unbiased estimator of β. Hence find $Var(\hat{\beta})$. *8*

(c) Define Age-specific Fertility Rate and the Total Fertility Rate (TFR). Also define Gross Reproduction Rate (GRR). *8*

(d) Describe the logistic growth model in population projection. State the situation when time series data would follow logistic law. Also state the limitation. *8*

(e) What is measurement scale in statistics ? Also describe the types of measurement scales with example in each type. *8*

Q6. (a) Define Autocorrelation of order k and Correlogram. For an infinite series generated by the moving average of a random component with equal weights, the correlogram is given by

$$r_k = \begin{cases} 1 - \dfrac{k}{m} & \text{for} \quad k \le m \\ 0 & \text{for} \quad k > m \end{cases}.$$ *15*

(b) State the need of standardization of Crude Death Rate (CDR). Also compute and compare the standardized CDR of the populations A and B in the following table : *15*

Age group (in years)	Death rate per 1000 population		Standard Population
	Population A	Population B	
0 – 4	3·0	10·0	1000
5 – 14	2·8	6·0	1500
15 – 24	1·0	2·0	1100
25 – 34	0·8	2·0	900
35 – 44	2·0	3·0	800
45 – 59	4·0	6·5	500
60 – 74	10·0	15·0	400
≥ 75	25·0	30·0	300

(c) (i) Describe standardization of scale by Z-score and Min-Max scaling in psychometric measurements.

(ii) Describe percentile and find the percentile from the following data :

The scores obtained by 10 students are 38, 47, 49, 58, 60, 65, 70, 79, 80, 92. Using the percentile formula, calculate the percentile for score 70. *10*

Q7. (a) Calculate Fisher's Ideal Index from the following data and verify that it satisfies time and factor reversal tests : *10*

Commodity	2015 – 16		2016 – 17	
	Price	Value	Price	Value
A	10	100	12	96
B	8	96	8	104
C	12	144	15	120
D	20	300	25	250
E	5	40	8	64
F	2	20	4	24

(b) Describe various columns of a life table. Also, fill in the blanks in the portion of the table given below with question mark (?). *15*

Age (X) in years	l_X	d_X	p_X	q_X	L_X	T_X	e_X^0
4	100000	500	?	?	?	5110000	?
5	?	400	?	?	?	?	?

(c) Describe the present statistical system in India and the role of National Statistical Office. *15*

Q8. (a) Prove the following relationship between the force of mortality (μ_X) at age X and the expectation of life (e_X^0) at age X.

$$\mu_X = \frac{1}{e_X^0}\left[1 + \frac{d}{dX} e_X^0\right].$$ *15*

(b) What are the consequences of multicollinearity and the consequences of auto correlated disturbances ? For the General linear model given by

$$\mathbf{Y} = X\boldsymbol{\beta} + \mathbf{U}, \text{ where } \boldsymbol{\beta} = (\beta_1, \beta_2, \dots \beta_k)', \ \mathbf{U} = (U_1, U_2, \dots U_n)'$$

with the assumption $E(\mathbf{U}) = 0$ and $E(\mathbf{UU}') = \sigma^2\Omega$ where σ^2 is unknown but Ω is known symmetric positive definite matrix of order n.

Obtain the generalized least squares estimator $\hat{\mathbf{b}}$ of $\boldsymbol{\beta}$ with variance of $\hat{\mathbf{b}}$. Also obtain unbiased estimator of σ^2. *15*

(c) What is the significance of cost of living index number ? The following table gives the per capita income and the cost of living index of a community. Calculate the real income taking into account the rise in the cost of living. *10*

Year	Cost of living index (Base 2001)	Per capita income (₹ in "000") per year
2001	100	360
2002	104	400
2003	115	480
2004	160	520
2005	210	550
2006	260	590
2007	300	610
2008	320	650