# STATISTICS

## Paper – II

*Time Allowed : **Three** Hours*

*Maximum Marks : **200***

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### Question Paper Specific Instructions

***Please read each of the following instructions carefully before attempting questions :***

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are **compulsory**. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections A and B.*

*Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off.*

*All questions carry equal marks. The number of marks carried by a question/part is indicated against it.*

*Unless otherwise mentioned, symbols and notations have their usual standard meanings.*

*Assume suitable data, if necessary and indicate the same clearly.*

*Answers must be written in **ENGLISH** only.*

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## SECTION A

**Q1.** (a) Consider a Weibull distribution with scale parameter $\alpha$, location parameter $\mu = 0$, and shape parameter $\gamma$. Show that its hazard rate is constant when $\gamma = 1$ and increases with time when $\gamma = 2$. What happens when $\gamma > 2$ ? *8*

(b) The following are the number of defectives found on items produced in 15 days :

2, 3, 1, 2, 2, 1, 3, 2, 2, 1, 2, 2, 1, 0 and 0

Construct a control chart for this process and comment on whether the process is in control. *8*
(Use graph sheet provided)

(c) A forest department conducted a two-year testing of a new brand of an insecticide on ten plants. The survival times in months are recorded and are given below. The + symbol next to an observation signifies that the observation is censored (either removed or dropped from study).

24+, 16+, 8, 19, 10, 8+, 5, 17, 20, 10

Obtain an estimate of S(t) by Kaplan-Meier method and plot it against t. What is the estimated probability of 15 month survival ? *8*
(Use graph sheet provided).

(d) Solve the two-person zero-sum game, whose pay-off matrix is given below, by graphical method. *8*

$$\begin{pmatrix} 3 & 5 \\ -1 & 6 \\ 4 & 1 \\ 2 & 2 \\ 1 & -3 \end{pmatrix}$$

(Use graph sheet provided)

(e) Let 0·35, 0·69, 0·05, 0·87, 0·43 be five simulated values of uniform (0, 1) random variable. Using these, simulate values of a random variable X whose probability mass function is

| x : | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| p(x) : | 0·25 | 0·35 | 0·30 | 0·10 |

*8*

**Q2.** (a) Seven forest departments gave their reports in a format that runs into 1500 pages. The central office, on cross-checking, found the following number of errors in their reports :

| Department Name : | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| Number of Errors : | 4 | 6 | 6 | 2 | 15 | 4 | 4 |

Construct a control chart for the (above sequence of) data. What are upper and lower control limits ? Is the process under control ? *15*
(Use graph sheet provided)

(b) (i) Develop a formula to estimate the percentage defectives from process capability ratio, by assuming normal distribution for the process characteristic. *8*

(ii) The specification limits (in inches) for the inside diameter of tubes are (0·995, 1·005). The standard deviation of inside diameters of tubes generated by a lathe machine is estimated to be 0·002 inch. Compute Process Capability Ratio and estimate percentage defectives from it. Assume normal distribution for inside diameters and its mean as 1. *7*

(Use normal distribution table provided at the end of the booklet)

(c) If the hazard rate is $\frac{k}{x}$, for $x > x_0$, where k and $x_0$ are some constants, then find the distribution of life time. *10*

**Q3.** (a) Solve the following linear programming problem by simplex method :

Maximize $z = -3x_1 + x_2 + x_3$

subject to

$$x_1 - x_2 + x_3 \leq 11$$

$$-4x_1 + x_2 + 2x_3 \geq 3$$

$$2x_1 - x_3 = -1$$

$$x_1, x_2, x_3 \geq 0$$

*15*

(b) (i) In a single period, single item inventory model, the stock is ordered at the beginning of the period. The unsold items at the end of the period are salvaged. Let c be the purchase cost, h be the holding cost per unit, and p be the penalty cost per shortage item. Derive the formula for optimal order that minimizes the total expected cost. *7*

(ii) A news magazine costs ₹ 30 per copy. The unsold items are salvaged to get a revenue of ₹ 5 per copy. If the demand per day has normal distribution with mean 300 copies and variance of 400 copies$^2$, find how many copies are to be purchased so as to have lower costs. *3*

(Use, if needed, the table of normal probabilities provided at the end of the booklet)

(c) Three plants, namely P, Q and R have the production capacity 50, 80 and 120 units respectively. Four markets A, B, C and D have the requirements of 90, 70, 40 and 50 units respectively. The shipping costs per unit from plant P to the markets A, B, C and D are ₹ 8, ₹ 9, ₹ 11 and ₹ 16 respectively. Similar shipping costs from plant Q to the above markets are ₹ 12, ₹ 7, ₹ 5 and ₹ 8 respectively, and from plant R are ₹ 14, ₹ 10, ₹ 6 and ₹ 7 respectively.

(i) Determine the optimal shipping pattern that utilizes all the supply and meets all the requirements at minimum transportation cost.

(ii) Which plant's supply is not split but fully sent to only one market ? *15*

**Q4.** (a) Prove that the state j of a Markov chain is persistent (recurrent), if and only if,

$$\sum_{n=0}^{\infty} p_{jj}^{(n)} = \infty.$$ *15*

(b) (i) Let $X_n$ be the largest number up to n tosses of a die.
Is $\{X_n, n = 1, 2, ...\}$ a Markov chain ?

(ii) Let $\{X_n\}$, $\{Y_n\}$ be two Markov chains. Is $\{Z_n\}$, where $Z_n = X_n + Y_n$, a Markov chain ?

(iii) Does a sequence of outcomes of independent traits form a time-homogeneous Markov chain ?

(iv) Give an example of a Markov chain for which the limiting distribution does not exist but the stationary distribution exists.

(v) Is the Markov chain with the following transition probability matrix an irreducible one ? *3×5=15*

$$\begin{pmatrix} 0 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(c) The times of arrivals of customers at a bank counter follow a Poisson process with a mean number of 45 arrivals per hour. The service time at the counter follows an exponential distribution with mean number of customers served per minute being one.

(i) What are the probabilities of having zero and having five customers at the counter ?

(ii) Compute $L_s$, $L_q$, $W_s$ and $W_q$. *2+8*

## SECTION B

**Q5.** (a) What is Heteroscedasticity ? In what circumstances is it observed in an econometric model ? Also discuss the Goldfeld-Quandt test to establish heteroscedasticity in an econometric model. *8*

(b) What is autocorrelation ? If $Y_t = \rho Y_{t-1} + v_t$ with $| \rho | < 1$ and $v_t$ is the random variable satisfying all the basic assumptions of ordinary least square (OLS), then propose a test for testing $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$. *8*

(c) Distinguish between an ARMA (p, q) process and ARIMA (p, d, q) process. How does Box-Jenkins method facilitate the identification of ARMA or ARIMA models ? *8*

(d) Write a brief note on the present official statistical system in India. Also discuss various official agencies responsible for data collection and their main functions. *8*

(e) 4 items are to be constructed so that they are equispaced on different scale. If the easiest item is passed by 85% of the group and the most difficult by 25%, find the percentage of individuals in the group passing the other two items. *8*

(Standard normal probability table is provided at the end of the booklet)

**Q6.** (a) Consider a general linear regression model in matrix notation as

$$Y = X\beta + u$$

Show that $\hat{\beta} = (X' X)^{-1} X'Y$. Obtain the variance-covariance matrix of $\hat{\beta}$. Also show that the estimator satisfies the Best Linear Unbiased Estimator (BLUE) property. *10*

(b) Consider the following structural equation model, assuming Y's as endogeneous and X's as predetermined variables :

$$Y_1 = \alpha_{10} + \alpha_{12} Y_2 + \alpha_{13} Y_3 + \beta_{11} X_1 + U_1$$

$$Y_2 = \alpha_{20} + \alpha_{23} Y_3 + \beta_{21} X_1 + \beta_{22} X_2 + U_2$$

$$Y_3 = \alpha_{30} + \alpha_{31} Y_1 + \beta_{31} X_1 + \beta_{32} X_2 + U_3$$

$$Y_4 = \alpha_{40} + \alpha_{41} Y_1 + \alpha_{42} Y_2 + \beta_{43} X_3 + U_4$$

where $U_1$, $U_2$, $U_3$ and $U_4$ are stochastic terms. Use Rank and Order conditions to identify the equations. *15*

(c) A test is administered on 400 pupils. It gives mean 60 and standard deviation 12. Complete the following table of equivalent raw scores. *15*

| Raw score : | 84 | 78 | 72 | 66 | 60 | 54 | 48 | 42 | 36 |
|---|---|---|---|---|---|---|---|---|---|
| σ-score (Z) | – | – | 1 | – | 0 | – | – | – | – |
| Standard score (X') : | – | – | 0 | – | – | 45 | – | – | – |

**Q7.** (a) Estimate the standardised death rates for the two countries from the following table : *15*

| Age group (in years) | Death rate per 1000 | | Standard Population (in Lakhs) |
|---|---|---|---|
| | Country A | Country B | |
| 0 – 4 | 20·00 | 5·00 | 100 |
| 5 – 14 | 1·00 | 0·50 | 200 |
| 15 – 24 | 1·40 | 1·00 | 190 |
| 25 – 34 | 2·00 | 1·00 | 180 |
| 35 – 44 | 3·30 | 2·00 | 120 |
| 45 – 54 | 7·00 | 5·00 | 100 |
| 55 – 64 | 15·00 | 12·00 | 70 |
| 65 – 74 | 40·00 | 35·00 | 30 |
| 75 and above | 12·00 | 110·00 | 10 |

(b) Define national income statistics and write the method of income estimation. What are the drawbacks of expenditure method of national income estimation ? *10*

(c) Fit a logistic curve to the following census data by Rhode's method : *15*

| Year | Population (in million) |
|---|---|
| 1900 | 10·616 |
| 1910 | 14·480 |
| 1920 | 19·276 |
| 1930 | 25·732 |
| 1940 | 34·138 |
| 1950 | 46·384 |
| 1960 | 62·886 |
| 1970 | 77·116 |
| 1980 | 100·312 |
| 1990 | 125·896 |
| 2000 | 151·990 |
| 2010 | 182·944 |
| 2020 | 217·029 |

**Q8.** (a) In the usual notations, show that

(i) $P_x = P_x \cdot P_{x+1} \cdots P_{x+n-1}$

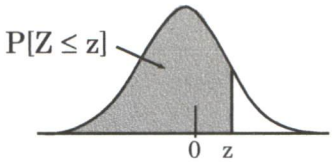
(ii) $e_x = \left( \sum_{n=1}^{\infty} l_{x+n} \right) \Big/ l_x$ *15*

(b) Prove that Fisher's ideal index number lies between Laspeyres' and Paasche's index numbers. *10*

(c) Construct the wholesale price index number for 2004 and 2005 using 2003 as the base year, from the data given below : *15*

| Commodity | Wholesale price (in '00 ₹) per quintal | | |
|---|---|---|---|
| | 2003 | 2004 | 2005 |
| A | 140 | 160 | 190 |
| B | 120 | 130 | 140 |
| C | 100 | 105 | 108 |
| D | 75 | 80 | 90 |
| E | 250 | 270 | 300 |
| F | 400 | 420 | 450 |

## Standard Normal Probabilities




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| 0.1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| 0.2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| 0.3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| 0.4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| 0.5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| 0.6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| 0.7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| 0.8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| 0.9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |
| 3.5 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 | .9998 |