WORK BOOK FOR

INTERMEDIATE

SECOND YEAR

MATHEMATICS PAPER –II(B)

[COORDINATE GEOMETRY AND CALCULUS]

BY

Sri.V.Ramakrishna I.R.S

Special commissioner &secretary

Board of intermediate Education

Andhra Pradesh

MATHEMATICS WORK BOOK(IIB) COMMITTEE

COURSE – CO-ORDINATORS

Dr.K.Chandrasekhar Rao, P.hD.

Regional joint Dirctor (Retd) Zone IV –Y.S.R. Kadapa.

Course writers

M.sreelakshmi,

Junior Lecturer in Mathematics,

K.S.R Govt.Junior college,

Anantapuram.

P.Harinatha Achari,

Junior Lecturer in Mathematics,

Dr. M.S.M.N Govt Junior

college(G), Nagari, Chittoor.

M. Srihari Rao

Junior Lecturer in Mathematics,

J.B.A.Junior college,

Kavali,Nelloore.

G.Srinivasa Reddy

Junior Lecturer in Mathematics,

M.T Govt.Junior college,

Nuzvid.

L.Sanyasi Naidu

Junior Lecturer in Mathematics,

DR.V.S.K.R Govt.Junior college

Visakapatnam.

B.K.R. Chowdary

Junior Lecturer in Mathematics,

Govt.Junior college,Mandapet , EGD

PREFACE

I hear and I forget; I see and I remember;
I do and I understand; I Think and I learn.

The Board of Intermediate Education, Andhra Pradesh, Vijayawada made an attempt to provide work books for the first time to the Intermediate students with relevant and authentic material with an aim to engage them in academic activity and to motivate them for self learning and self assessment. These work books are tailored based on the concepts of *"learning by doing"* and *"activity oriented approach"* to sharpen the students in four core skills of learning – *Understanding, Interpretation, Analysis and Application.*

The endeavor is to provide ample scope to the students to understand the underlying concepts in each topic. The workbooks enable the students to practice more and acquire the skills to apply the learned concept in any related context with critical and creative thinking. The inner motive is that the students should shift from the existing rote learning mechanism to the conceptual learning mechanism of the core concepts.

I am sure that these compendia are perfect tools in the hands of the students to face not only the Intermediate Public Examinations but also the other competitive Examinations.

My due appreciation to all the course writers who put in all their efforts in bringing out these work books in the desired modus.

V. RAMAKRISHNA, I.R.S.
SECRETARY
B.I.E., A.P., VIJAYAWADA.

CONTENT

BIBOARD OF INTERMEDIATE EDUCATION: A.P. VIJAYAWDA

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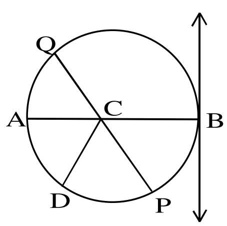
MATHEMATICS-IIB WORK BOOK

Concepts:

1. Path traced by a person in a moving Giant wheel is a circle.
2. Path traced by a pot on a potter's wheel in motion is a circle.
3. Path traced by a point moving with a fixed distance from a fixed point is a circle.
4. All the points on a circle are at same distance from centre of that circle (yes/no)
5. Distance between centre and a point on the circle is radius.
6. Line segment joining any two points on a circle is called a chord.
7. A chord passing through centre of a circle is called a diameter.
8. Midpoint of a diameter = centre.
9. Length of diameter of a circle = $2 \times$ radius
10. A diameter divide a circle into two equal parts. Each part is called a semi link.
11. Line passing through any two points on a circle is called secant line.
12. Line touching the circle at only one point is called tangent and the common point is called point of contact.
13. From the figure;



a) Name centre : C

b) Name all chords: RS, AB, PQ

c) Name all diameters : PQ, AB

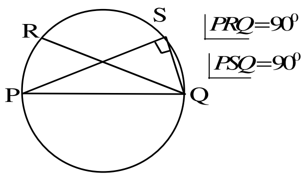
d) Name all tangents : BL

e) Name all radii : CA, CD, CP,CB,CQ

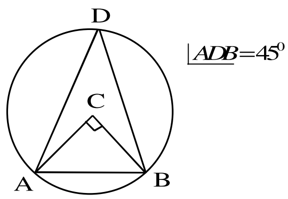
f) Name point of tangents : B

14. Match the following :

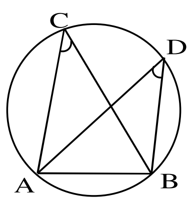
<table>
<tr><th></th><th>Reason</th></tr>
<tr><td>1.



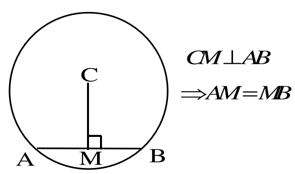
</td><td>a) In a circle, angles in the same segment and made by same chord are equal.</td></tr>
<tr><td>2.



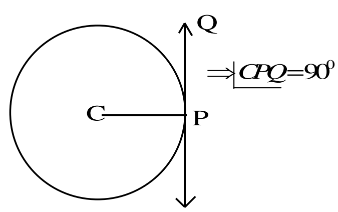
</td><td>b) Angle in a semi circle = 90^0</td></tr>
<tr><td>3.



$\Rightarrow \angle ACB = \angle ADB$</td><td>c) Perpendicular from centre of a circle to any on i0t's chords divides the chord into two equal parts.</td></tr>
<tr><td>4.



</td><td>d) radius is perpendicular to tangent at the point of contact.</td></tr>
<tr><td>5.



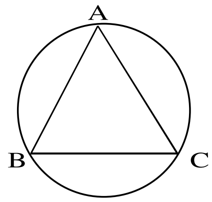
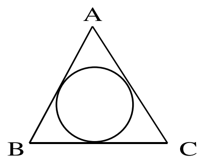
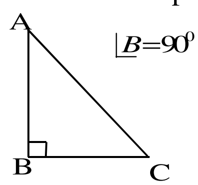
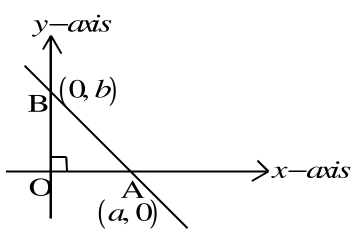
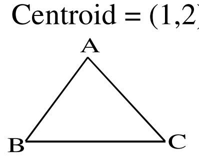
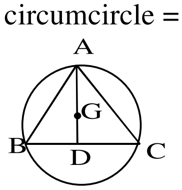
</td><td>e) Angle made by a chord at centre is twice the angle made at circumtence of major sector.</td></tr>
</table>

15. Circle passing through vertices of a triangle is called <u>circumcircle</u> of that triangle.

16. Circle touching all the three sides of a triangle internally is called <u>Incircle</u> of that trinalge.

17. If "G" is centroid of ΔABC then AG : GD =2 : 1 where AD is a median. True/False.

18. Match the following:

<table>
<tr><th></th><th></th></tr>
<tr><td>1. Name the circle

</td><td>a) In a circle of ΔABC</td></tr>
<tr><td>2. Name the circle

</td><td>b) Circumcircle of ΔABC</td></tr>
<tr><td>3. Circumcentre = midpoint of AC

</td><td>c) In a right angled triangle hypotenuse is a diameter of it's circumcircle and hence circumcentre = midpoint of hypotense</td></tr>
<tr><td>4. Circumcentre of ΔOAB is mid point of $AB = \left(\frac{a}{2}, \frac{b}{2}\right)$

</td><td>d) In a equilateral triangle circumcentre = Incentre = centroid = orthocentre</td></tr>
<tr><td>5. ΔABC is an equilater triangle
It's incentre =(1,2)
Then circumcentre = (1,2)
Centroid = (1,2)

</td><td>e) Because in an equilated incentre = circumcentre = centroid = G
$\therefore$ radius = AG = $\frac{2}{3}(3) = 2$
as $AG : GD = 2 : 1$</td></tr>
<tr><td>6. ΔABC is an equilateral triangle AD is a height of length 3 units, radius of circumcircle = 2

</td><td></td></tr>
</table>

19. Circles with same centre but with different radii are called <u>concentric circles</u>.

20. If $P(x, y)$ is a point on a circle with radius r and centre = (h, k) which of the following are true.

    a) $cp = r$ b) $(x-h)^2+(y-k)^2=r^2$ c) both (1) & (2)

21. $A(x_1, y_1), B(x_2, y_2)$ are end points of a diameter of a circle. $P(x, y)$ is a point on the circle. Then which of the following are true.

    i) slope of APX slope of B = –1

    ii) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$

22. degree of eqn of a circle is <u>2</u>

23. Standard eqn of a circle is $x^2+y^2+2gx+2fy+c=0$

24. Match the following:

List – I	**List – II**
1. Eqn $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a circle iff	a) centre = $(-g, -f)$ radius = $\sqrt{g^2+f^2-c}$
2. If $g^2+f^2-c \geq 0$ then represents a circle then it's centre = ——— Radius = ———	b) $(x-h)^2+(y-k)^2=r^2$
3. eqn of a circle with centre = (h, k), radius = r is	c) $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$
4. eqn of a circle with centre = (0, 0), radius = r is	d) $a=b\neq 0,\ h\geq 0,\ g^2+f^2-ac\geq 0$
5. If $(x_1, y_1),(x_2, y_2)$ are end points of a diameter, then eqn of the circle is	e) $x^2+y^2=r^2$

25. General coordinates of any point on a circle in terms of single variable Q are called parametric coordinates of P.

    Match the following:

Cartesian eqn of a circle	Parametric eqns & parametric coordinates of any point on the circle
1. $(x-h)^2+(y-k)^2=r^2$ 2. $x^2+y^2=r^2$	a) $x=r\cos\theta, x=r\sin\theta(r\cos\theta,\ r\sin\theta)$ b) $x=h+r\cos\theta, y=k+r\sin\theta$ $P(\theta)=(h+r\cos\theta,\ k+r\sin\theta)$

3. $S = x^2 + y^2 + 2gx + 2fy + c = 0$	c) $x = 2\cos\theta, y = k + r\sin\theta$ $P(\theta) = (2\cos\theta,\ 2\sin\theta)$
4. $x^2 + y^2 = 4$	d) $x = -g + r\cos\theta,\ y = 2\sin\theta$ $P(\theta) = (-g + r\cos\theta,\ -f + r\sin\theta)$

26. P is an point on the circle $(x-1)^2 + (y-2)^2 = 9$ then $P(\theta) = \underline{(1+3\sin\theta,\ 2+3\sin\theta)}$

27. Radius of a point circle is 0

28. Radius of unit circle is 1

29. Radius of a circle passing through the (0, 0) $(\alpha,\ 0)$ $(0,\ \beta)$ is $\underline{\dfrac{\sqrt{\alpha^2+\beta^2}}{2}}$

30. Radius of the circle $3x^2 + 3y^2 - 6x + 4y - 4 = 0$ is $\dfrac{7}{3}$ and it's centre = $\underline{(1,\ -2/3)}$

31. Every point in a plane is a point circle (yes/no)

32. If P = (1, 2) then eqn of the point circle at P is $(x-1)^2 + (y-2)^2 = 0$ (yes/no)

33. A point circle has only centre and zero radius (yes/no)

34. If $x^2 + y^2 + 2gx + 2fy + c = 0$ is a point circle, then which of the followed is correct.

    1) $g^2 + f^2 - c = 0$ 2) $g^2 + f^2 - c = 1$ 3) $g^2 + f^2 - c = 2$

35. Circles with same centre but with different radii are called concentric circles.

36. Eqns of concentric circles differ only in constants say yes/no

37. Eqn of circle concentric with $x^2 + y^2 + 4x + 6y - 1 = 0$ is of the form $x^2 + y^2 - 2y = 0$ (yes/no)

38. Eqn of circle concentric with $2x + y - 3 = 0$ and radius 2 is $2x + y - 1 = 0$

39. Eqn of circle concentric with $x^2 + y^2 - 6x - 4y - 12 = 0$ and passing through (1, 2) is $\underline{x^2 + y^2 - 6x - 4y - 21 = 0}$

40. Circle passing through two given points has minimum radius if the points are end points of a diameter (yes/no)

41. Let $L_1 = 0,\ L_2 = 0, L_3 = 0$ are three non-concurrent lines and no two of them are paralled to each other. Then

    No of circles touching all the three lines = 4

42. $L_1 = 0,\ L_2 = 0, L_3 = 0$ are three non-concurrents lines Two of then are parallel. Then

    No of circles touching all the three lines = 2

43. $L_1 = 0,\ L_2 = 0, L_3 = 0$ are three parallel lines then No of circles touching all three lines $=$ 0

44. No of circles passing through three non-collinear points in a plane = 1

45. Eqn of circle passing through (1, 2), (3, 4) and has radius as small as possible is $(x-1)(x-3)+(y-2)(y-4)=0$

46. Four points lying on a circle are called concyclic points

47. $A=(a_1,\ 0)\ B(a_2,0)\ C(b_1,\ 0)\ D(c_2,\ 0)$ are on coordinate axes. Then A, B, C, D are concyclic points iff

    1) $OA \cdot OB = OC \cdot OD$ (yes/no)

    2) $a_1a_2 = b_1b_2$ (yes/no)

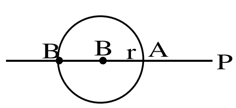
48. Two straight lines $a_1x+b_1y+c_1=0$, $a_2x+b_2y+c_2=0$ intersect coordinate axes at 4 different points. Then the four points are concyclic iff $a_1a_2 = b_1b_2$

49. $L_1 = 0, L_2 = 0, L_3 = 0$ are three given straight lines which are non-concurrent and no two of them are parallel lines consider $\lambda_1(L_1L_2)+\lambda_2(L_2L_3)+\lambda_3(L_3L_1)=0$

    1) It is a curve passing through all the intersecting points of given lines

    2) This is the circumcircle of the triangle formed by the given lines if $x^2 coeff = y^2 coeff$ & $xy\, coeff = 0$

50.



    i) minimum distance of P dran the circle = PA = $|(p-r)|$

    ii) longest distance of P from the circle = PB = $|CP+r|$

51. Intercept made by the circle S=0 on x–axis is $2\sqrt{g^2-c}$

52. Intercept made by the circle S =0 on y-axis is $2\sqrt{f^2-c}$

53. If a circle touches x–axis then x–intercept made by the circle S = 0 and $g^2 = c$

54. If a circle touches y–axis then y–intercept made by the circle S = 0 and $f^2 = c$

55. If a circle $S = x^2+y^2+2gx+2fy+c=0$ touches both the coordinate axes, then

    i) $2\sqrt{g^2-c}=0, 2\sqrt{f^2-c}=0$ (yes/no)

ii) $g^2 = f^2 = c$ (yes/no)

iii) g = 0 (yes/no)

iv) c= 0 (yes/no)

56. If a circle with centre (h, k) touches

i) x–axis then radius $r = |k|$ (yes/no)

ii) y-axis, then radius $r = |h|$ (yes/no)

iii) touches both the axes, then $|h| = |k| = r$ (yes/no)

iv) coordinates of centre are +ve or –ve depends on the quadrant in which centre say true or false (yes/no)

57. Say true or false

If a circle with radius r touches coordinate axes

i) in first quadrant then centre = (r, r)

ii) in second quadrant then centre = (–r, r)

iii) in third quadrant then centre = (–r, –r)

iv) in fourth quadrant then centre = (r, –r)

58. When a circle touches both the coordinate axes then centre of the circle lines on angular bisectors of coordinate axes i.e. on y = x or y = –x (yes/no)

59. When a circle passing through (0, 0) and making intercepts α, β on x, y axes respectively. Then eqn of the circle is $x^2 + y^2 \pm \alpha \pm \beta y = 0$

60. If a circle touches x–axis at $(\alpha, 0)$ and cuts off intercept on y–axis of length 2l, then radius = $r = \sqrt{\alpha^2 + l^2}$ and centre = $(\alpha, \pm r)$

61. when a circle touches y–axis at $(0, \beta)$ and cuts off intercept on y–axis of length 2k, then radius = $r = \sqrt{\beta^2 + k^2}$ and centre = $(\pm r, \beta)$

62. A pont $P(x_1, y_1)$ is an interior point or on the circumference of the circle or an exterior point of the circle S =0 iff $S_{11} \underset{>}{\overset{<}{=}} 0$ (true/false)

63. Power of a point $P(x_1, y_1)$ w.r.to a circle S=0 is

i) $Cp^2 - r^2$ ii) S_{11}

iii) $PA \cdot PB$ if a line through P intersects the circle of A & B

iv) All the above

64. If $P=(-1,2)$, $S=x^2+y^2+2x+2y+1=0$. A line through P cuts the circle of A & B then $PA\cdot PB=$

i) 8 ii) 2 iii) 0 iv) 6

65.

66.

67. Power $P(\alpha,\beta)$ w.r.to circle $x^2+y^2=r^2$ is

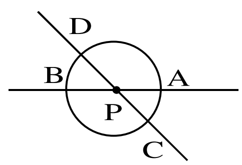
i) $\alpha^2+\beta^2-r^2$ ii) $\alpha^2+\beta^2\pm r^2$ iii) $\frac{\alpha^2+\beta^2}{r^2}$ iv) None

68. Power of a point P w.r.to a circle S=0 is –ve then P lies

i) inside the circle ii) on the circle

iii) outside the circle

69.



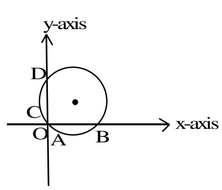
from the figure, power of P w.r.t. the circle is

i) $PA\cdot PB$ ii) $PC\cdot PD$ iii) $PA\cdot PB=PC\cdot PI=S_{11}$ iv) All the above

70. If power of P w.r.to a circle is, +ve, then P lies

i) inside the circle ii) outside the circle iii) on the circle

71. from the figure $OA\cdot OB=\underline{OC\cdot OD}$



72. Power of a point on a circle w.r.to the same circle = 0

73. length of tangent drawn from an external point to a circle is $\underline{\sqrt{S_{11}}}$

74. No of tangents drawn at a point on the circle = 2

75. No of tangents drawn at a point on the circle = 1

76. No of tangents drawn from an internal point to the same circle = 0

77. No of tangents possible to a point circle drawn from it's centre = Infinitely many

78. length of tangent drawn from (1, 3) into circle $x^2+y^2-2x=0$ is $2\sqrt{2}$

79. length of tangent drawn from (1, –3) to the circle $x^2+y^2+4x+6y-12=0$ is

i) 4 ii) 5 iii) not possible

80. If the length of tangent from (5, 4) to the circle $x^2+y^2+2ky=0$ is 1, then k = <u>–5</u>

81. Let 'C' is centre, r is radius of a circle, d is perpendicular distance from centre to a given line L = 0 then which of the following are correct?

a) the line intersects the circle at two distinct points if d < r True

b) The line touches the circle at only are point if d = r True

c) The line does not meet the circle if d > r

82. 1) The line y = mx+c touches the circle $x^2+y^2=r^2$ if $c^2=r^2\left(1+m^2\right)$ (yes/no)

2) $y=mx+c$ meets the circle at two distinct points if $c^2<r^2\left(1+m^2\right)$ (yes/no)

3) $y=mx+c$ does not meet the circle $x^2+y^2=r^2$ if $c^2>r^2\left(1+m^2\right)$ (yes/no)

83. Condition for a line $lx+my+n=0$ to touch the circle $S=0$ is

$\left(l^2+m^2\right)\left(g^2+f^2-c\right)=\left(\text{lg}+mf-n\right)^2$ (yes/no)

84. When a line touches a circle, then point of contact =

1) foot of the perpendicular from centre on to the tangent

2) point of intersection of the circle & the tangent

3) both (1) & (2)

85. Eqn of tangent at $\left(x_1,\ y_1\right)$ on the circle S = 0 is $\underline{x^2+y^2-2x=0}$

86. Eqn of tangent at $2\sqrt{2}$ to the circle $x^2+y^2+4x+6y-12=0$ is $\underline{x^2+y^2+2ky=0}$

87. Eqn of tangent with slope m to circle $x^2+y^2=r^2$ is $y=mx\pm r\sqrt{l+m^2}$

88. Eqn of tangent with slope m to circle $S=x^2+y^2+2gx+2fy+c=0$ is

$y+f=m\left(x+g\right)\pm r\sqrt{l+m^2}$

89. If P is a point on a circle S = 0 then normal at P

i) passes through P

ii) perpendicular to tangent at P

iii) passes through centre of P

iv) All the above

90. Let 'C' is centre of a circle, then normal through 'P' is the line $\overleftrightarrow{CP}$ (true/false)

91. length of the chord intercepted on a normal by the circle = <u>2r</u>

92. Tangents drawn at the end points of a chord meet at point P, then the chord is called chord of contact of P w.r.to the given circle.

93. Tangents drawn at the end points of chords passing through a fixed point P intersect on a straight line called polar of p w.r.to the given circle. The point is called pole of that line

94. If $P(x_1, y_1)$ is inside the circle, then the chord of contact of p = Tangent at P (true/false)

95. If $P(x_1, y_1)$ is inside the circle, then the chord of contact of p does not exist (true/false)

96. Every point on the polar of P w.r.to a circle is point of intersection of two tangents to the same circle (True/false)

97. If P is outside the circle, then polar of P is subset of locus of chord of contact of P (true/false)

98. No point on a polar w.r.to a circle can be an interior point to that circle. (yes/no)

99. If p is inside the circle then polar of p lies outside the circle (yes/no)

100. If p is on the circle then polar of p = Tangent at P. (yes/no)

101. Pole of diameter does not exist (yes/no)

102. Polar of centre of a circle w.r.to the same circle does not exist (yes/no)

103. Pole of $lx+my+n=0$ w.r.to circle $x^2+y^2=r^2$ is $\left(\frac{-r^2 l}{n}, \frac{-r^2 m}{n}\right)$ (yes/no)

104. Pole of the line $lx+my+n=0$ w.r.to the circle S = 0 is

$$\left[-g+\frac{lr^2}{\lg+mf-n}, -f+\frac{mr^2}{\lg+mf-n}\right]$$

105. If two points $P(x_1, y_1)$ $Q(x_2, y_2)$ are conjugate points w.r.to a circle S = 0, then

i) polar of one point passes through the second point

ii) $S_{12}=0$

iii) both (1) & (2)

106. If two lines $l_1x+m_1y+n_1=0$, $l_2x+m_2y+n_2=0$ are conjugate lines with respect to a circle $x^2+y^2=r^2$, then

i) pole of one line lies on the second line ii) $r^2(l_1l_2+m_1m_2)=n_1n_2$

iii) both (1) & (2)

107. Two points P, Q are inverse points to each other w.r.to a circle then

1) C, P, Q are collinear 2) P, Q lie on the same side of centre C 3) $CP \cdot CQ = r^2$

4) Inverse point of P is point of intersection of the line $\overleftrightarrow{CP}$ and polar of P

5) All the above

108. If P is on the circle, then inverse point of p = <u>itself</u> (true/false)

109. Inverse point of centre w.r.to same circle does not exist. (true/false)

110. Eqn to pair of tangents drawn from an external point $P(x_1, y_1)$ to a circle S = 0 then $SS_{11} = S_1^2$

111. If θ is the angle between the tangents drawn from an external point $P(x_1, y_1)$ to a circle $S = 0$ then $\tan\theta/2 = \underline{r/\sqrt{S_{11}}}$

112. Locus of point of intersection of perpendicular tangents to a circle is called <u>director circle.</u>

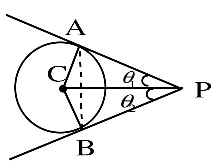
112. radius of a director ciercle of a circle is $\sqrt{2}$ (radius of given circle)

112. Eqn to director circle of $x^2 + y^2 = r^2$ is $x^2 + y^2 = 2r^2$

113. Eqn to director circle of $(x-h)^2 + (y-k)^2 = r^2$ is $(x-h)^2 + (y-k)^2 = 2r^2$

114. Director circle of a circle is concentric with given circle (true/false)

115.



PA & PB are tangents drawn from point to a circle with ‘C’ as centre.

Which of the following are true.

a) PC is diameter of circle passing through points P, A, B

b) Area of quadrilateral PACB = $r\sqrt{S_{11}}$

c) Area of $\Delta PAB = \dfrac{r(S_{11})^{3/2}}{S_{11} + r^2}$

d) If $\theta = 90^0$, then $r = \sqrt{S_{11}}$

e) All the above are true

116. Eqn to direction circle of $x^2 + y^2 = 16$ is $\underline{x^2 + y^2 = 32}$

117. If two circles lie on the same side of their common tangent then the common tangent is called <u>direct common tangent</u>

117. If two circles lie on either side of their common tangent then the common tangent is called Transverse common tangent

118. Point of intersection of direct common tangents is <u>External centre of similitude</u>.

119. Point of intersection of Transverse common tangents is <u>Internal centre of similitude</u>.

120. C_1, C_2 are centres, r_1, r_2 are radii of two circles, then internal centre of similaitude devides $\overline{C_1C_2}$ in the ratio $r_1 : r_2$ internal (ture/false)

121. External centre of similitude devides $\overline{C_1C_2}$ in the ratio $r_1 : r_2$ externally. (true/false)

122. Length of direct common tangent of two circles with radii r_1, r_2 and 'd' distance between their centres = $\underline{\sqrt{d^2 - (r_1 - r_2)^2}}$

123. Length of transverse common tangent of two circles = $\sqrt{d^2 - (r_1 + r_2)^2}$

124. C_1, C_2 are centres, r_1, r_2 are radii of two circles.

Math the following:

Condition	Relative position of the two circles
1. $C_1C_2 > r_1 + r_2$	a. touch each other externaly
2. $C_1C_2 = r_1 + r_2$	b. touch each other internally
3. $C_1C_2 < \|r_1 - r_2\|$	c. one is inside other No common points
4. $\|r_1 - r_2\| < C_1C_2 < r_1 + r_2$	d. one is outside the other and no common points
5. $C_1C_2 = \|r_1 - r_2\|$	e. intersect each other at two distinct points

125. Match the following:

Condition	No of common Tangents
1. $C_1C_2 > r_1 + r_2$	a. 2
2. $C_1C_2 = r_1 + r_2$	b. 1
3. $\|r_1 - r_2\| < C_1C_2 < r_1 + r_2$	c. 3
4. $C_1C_2 = \|r_1 - r_2\|$	d. 0
5. $C_1C_2 < \|r_1 - r_2\|$	e. 4

LEVLE – 1

1. Centre, radius of circle $x^2+y^2+4x+8y-5=0$ is $\underline{(-2,-4), r=5}$

2. centre, radius of $3x^2+3y^2-6x+4y-4=0$ is $\underline{(2,-4/3), r=\sqrt{88}/3}$

3. centre of the circle passing through the points (0, 0), (1,0),(0,1)is $\underline{\left(\frac{1}{2},\frac{1}{2}\right)}$

4. If the line $x+2by+7=0$ is a diameter of the circle $x^2+y^2-6x+2y=0$, then b = $\underline{5/2}$

5. If the radius of the circle $x^2+y^2-18x+12y-k=0$ be 11, then k = $\underline{4}$

6. The point diametrically opposite to the point (1, 0) on the circle $x^2+y^2-4x+3y+3=0$ is $\underline{(3,-3)}$

7. If (1, 4) (3, x) are end points of a diameter of a circle with centre (y, 2) then values of x & y are $\underline{x=0, y=2}$

8. If the eqn $kx^2+(4+l)xy+3y^2-6x+12y-4=0$ represents a circle, then $k+l=$ $\underline{3-4=-1}$

9. Eqn of the circle passing through (3, 4) and having centre at (1, 1) is $\underline{(x-1)^2+(y-1)^2=13}$

10. Eqn of the circle with centre (1, 2) and passing through centre of the circle $x^2+y^2+2x-8=0$ is $\underline{(x-1)^2+(y-2)^2=8}$

11. Eqn of the circle passing through (0, 0) (a, 0) (0, b) is $\underline{(x-a)x+y(y-b)=0}$

12. Which of the following is a diameter of circle $x^2+y^2-4x-2y+4=0$

    a) $x+y-3=0$ b) $x+y+2=0$ c) $3x+y=0$

13. The circle concentric with $x^2+y^2+2x-8y+1=0$ and having radius 3 is $\underline{x^2+y^2+2x-8y+8=0}$

14. Eqn of the circle passing through centre of circle $x^2+y^2-4x-6y=0$ and concentric with the circle $x^2+y^2+8x-4y+4=0$ is $\underline{x^2+y^2+8x-4y-17=0}$

15. The eqn $3x^2+3y^2+4x+8y+9=0$ represents a circle (true/false)

16. Area of the circle $x^2+y^2+4kx=0$ is $\underline{4\pi k^2}$

17. Eqn of the circle whose end points of a diameter are (1, 2) (3, 4) is

    $(x-1)(x-3)+(y-2)(y-4)=0$

18. Eqn of the circle passing through (2,0) (0,2) and has it's radius as small as possible is

    $\underline{(x-2)x+(y-2)y=0}$

19. If the points (0, 0) (0, 1) (–1, 0) and (k, 1) are concyclic then k = <u>0, –1</u>

20. If (1, 0) (0, 1) (4, 0) (0, k) are concyclic then <u>k = 4</u>

21. The minimum & maximum distances from a point (1, –4) to the circle

    $x^2+y^2+2x-3=0$ are <u>2, 6</u>

22. The eqn of the circle of radius 2 and touching coordinate axes in 3rd quadrant is

    $(x+2)^2+(y+2)^2=4$

23. The eqn of a circle with centre (3, –2) and touching x–axis is $(x-3)^2+(y+2)^2=4$

24. x–intercept made by the circle $x^2+y^2-2x+4y-9=0$ is $\underline{2\sqrt{10}}$

25. Intercept made by the circle with centre (3, 2) and radius 4 on x–axis is $\underline{4\sqrt{3}}$

26. The point (1, –2) lies in <u>the interior</u> of the circle $x^2+y^2-4x+6y+6=0$

27. Power of p(1, 5) w.r.to the circle $x^2+y^2-2x+2y-23=0$ is <u>11</u>

28. A line through p cuts a circle at A & B. Also $PA \cdot PB$ is negative. Then what can you say about position <u>Inside the circle</u>.

29. The circle $x^2+y^2+2x-2y+1=0$ touches

    1) x–axis at (–1, 0) only

    2) y–axis at (0, 1) only

    3) Both the axes x–axis at (–1, 0), y–axis at (0, 1)

    4) Neither of the axes

30. The circle $x^2+y^2+2x+4y+4=0$ touches

    a) x–axis b) y–axis c) both the axes d) Neither of the axes

31. The circle with centre (2, 3) touches the line $3x+4y+7=0$ the radius = <u>5</u>

32. Centre of the circle $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

33. centre of the circle $(x-2)(x-3)+(y-1)(y-2)=0$ is $\underline{\left(\frac{5}{2}, \frac{3}{2}\right)}$

34. Eqn of the circle passing through three non-collinear points A, B, C can be of the form (Eqn of the circle with AB as diameter) + λ (eqn of the line AB) = 0 (true/false)

35. $x^2+y^2=0$ is

1) A point 2) A circle of radius 3) x–axis 4) y–axis

36. The eqn $x^2+y^2+2x+4y+5=0$ represents

1) a point 2) A circle of radius 3) x–axis 4) y–axis

37. Locus of point of intersection of perpendicular tangents of the circle $x^2+y^2=16$ is

i) a circle passing through (0, 0)

ii) A circle of radius 4

iii) A concentric circle of radius $\sqrt{2}\times 4$

iv) None of these

38. centre of the circle $x=2+5\cos\theta,\ y=-1+5\cos\theta$ is (2, –1)

39. eqn of chord of the circle $x^2+y^2-6x=0$ whose midpoint (2,0) is

a) x–2=0 b) x+2=0 c) y–2=0 d) y+2=0

40. Midpoint of chord of the circle $x^2+y^2=4$ interecepted on the line x+2y=4 is

a) (1,2) b) (–1, 2) c) $\left(\frac{4}{5},\frac{8}{5}\right)$ d) (0, 0)

41. A line $lx+my+n=0$ meet the circle $x^2+y^2=25$ at A & B The tangent at A & B meet at C. Then

1) $lx+my+n=0$ is chord of contact of c w.r.to the circle.

2) pole of $lx+my+n=0$ w.r.to the circle=c & $c=\left[\frac{-25l}{n},\frac{-25m}{n}\right]$

3) both (1) & (2)

4) None of the above

42. A circle is passing through (0, 0). Lengths of chords intercepted by it an coordinate axes are 6, 4 respectively, then radius of the circle is

1) $\sqrt{10}$ 2) $\sqrt{13}$ 3) 13 4) 10

43. Equation to chord of contact of p(3, 4) to the circle $(x-1)^2+(y-2)^2=4$ is

1) $x+y-2=0$ 2) $x+y-5=0$ 3) $x+y+3=0$ 4) $x+y-1=0$

44. $x\cos\alpha+y\sin\alpha=p$ is a tangent to the circle $x^2+y^2-2ax\cos\alpha-2ay\sin\alpha=0$ is if p =

1) 0 or 2a 2) –2a 3) a 4) 1

45. The equation of tangent to the circle $x^2+y^2=a^2$ at (a, b) is $ax+by-\lambda=0$, then $\lambda=$

1) b^2 2) –1 3) a^2 4) a

46. Equation of tangent to the circle $x^2+y^2=10$ at the points where the line y + 3 = 0 meets it are

1) $x-3y-10=0,\ x+3y+10=0$

2) $x-y+10=0,\ x+y+10=0$

3) $2x+y+1=0,\ 2x-y+1=0$

47. The straight line $x+y-7=0$ touches the circle $x^2+y^2-2x-4y-3=0$ at the point whose coordinate are

1) (3, 4) 2) (1, 4) 3) (1, –2) 4) (–3, 4)

48. Eqn of normal to the circle $x^2+y^2+4x+6y-39=0$ through (–3, –4) is

1) $y+3=1(x+2)$ 2) $y-3=1(x+2)$ 3) $y+3=1(x+4)$ 4) $y+4=1(x-2)$

49. The line $lx+my+n=0$ will be a normal to the circle $x^2+y^2+2ax+2by+c=0$ iff

1) $x=la+mb$ 2) $n=la+mb$ 3) $x-la+mb=0$ 4) $y-la-mb=0$

50. The point at which normal to the circle $x^2+y^2-4x=0$ at (1, 2) will meet the circle again at _____

1) (3, –2) 2) (3, 2) 3) (–3, 2) 4) (2, 3)

51. no of tangents that can be drawn from (1, 2) to the circle $x^2+y^2+6x-8y+1=0$ is

1) 0 2) 2 3) 1 4) 3

52. length of the tangent drawn from (–2, 3) on to the circle $x^2+y^2-2x-4y-1=0$ is

1) 2 2) 4 3) $\sqrt{9}$ 4) 5

53. The angle between the tangents drawn from (0, 0) to the circle $(x-4)^2+(y-2)^2=16$ is

1) $2Tan^{-1}2$ 2) $Tan^{-1}2$ 3) $Tan^{-1}\frac{1}{2}$ 4) None of these

54. Eqns of tangents drawn from (0, 1) to the circle $x^2+y^2-2x+4y=0$ are

1) $(2x-y+1)(x+2y-2)=0$ 2) $(2x+y-1)(x-2y+2)=0$

3) $(x+y-1)(x-y+1)=0$

55. Eqn of common tangent of touching circles $x^2+y^2-6x-2y+1=0,\ x^2+y^2+2x-8y+13=0$ at the point of contact is

1) $8x-6y+12=0$ 2) $4x-3y+1=0$

3) $3x-4y+2=0$ 4) $x+y+7=0$

56. Tangents drawn from (0, 0) will be perpendicular to each other if

1) $g^2+f^2=2c$ 2) $g^2+f^2=c$ 3) $g^2+f^2+c=0$ 4) $g^2=0$

57. Length of the tangents drawn from points on a circle $x^2+y^2+2gx+2fy+c_1=0$ to the circle $x^2+y^2+2gx+2fy+c_2=0$ is

1) $\sqrt{c_2-c_1}$ 1) $\sqrt{c_1-c_2}$ 3) $\sqrt{c_1+c_2}$

LEVLE – 2

1. If $2x-3y=5,\ 3x-4y=7$ are two diameters of a circle of radius 7, then eqn of the circle is

1) $(x-1)^2+(y+1)^2=49$ 2) $(x+1)^2+(y-1)^2=49$

3) $x^2+y^2=49$ 4) $x^2+y^2+2x+1=0$

2. Eqn of circle with radius 2 and whose centre is image of (1, 2) in the line $y+x=1$

1) $(x+1)^2+y^2=4$ 2) $(x-1)^2+y^2=4$

3) $x^2+y^2=4$

3. $2x+y-7=0,\ x+3y-11=0$ are diameters of a circle then eqn of the circle which also pass through (5, 7) is

1) $(x-2)^2+(y-3)^2=25$ 2) $(x+2)^2+(y+3)^2=25$

3) $(x-7)^2+y^2=5$

4. Eqn of the circle concentric with the circle $x^2+y^2-6x+12y-45=0$ and of double its area is

1) $(x-3)^2+(y+6)^2=100$ 2) $(x+3)^2+(y-6)^2=10$

3) $(x-2)^2+(y+6)^2=100$

5. locus of centre of a circle of radius 3 which rolls on the circumference of the circle $x^2+y^2+6x-6y+9=0$ is

1) $(x+3)^2+(y-3)^2=6^2$ 2) $(x-3)^2+(y-3)^2=6^2$

3) $(x+3)^2+(y+3)^2=6^2$

6. Area of the circle in which a chord of length 4 units makes an angle $\frac{\pi}{2}$ at the centre is

1) 8π 2) 30π 3) 35π 4) π

7. The centre of a circle is (3, –2) and the circumference of it is 10π, then eqn of the circle is

1) $(x-3)^2+(y+2)^2=5^2$ 2) $(x+3)^2+(y-2)^2=25$

3) $(x-3)^2+(y+2)^2=5$

8. $2x-3y+4=0,\ x+y-3=0$ are diameters of a circle of area 5π, then eqn of the circle is

1) $(x-1)^2+(y-2)^2=5$ 2) $(x-1)^2+(y-2)^2=5^2$

3) $(x+1)^2+(y-2)^2=5$

9. The eqn of the circumcircle of the triangle formed by the line ax+by+c=0 and the coordinate axes is

1) $\left(x+\frac{c}{a}\right)x+\left(y+\frac{c}{a}\right)y=0$ 2) $\left(x+\frac{c}{a}\right)x+\left(y+\frac{c}{b}\right)y=0$

3) $ax^2+ay^2+ax+by=0$

10. Eqn of circumcircle of the triangle formed by the line $2x+3y-4=0$ and coordinate axes

1) $(x-2)x+\left(y-\frac{4}{3}\right)y=0$ 2) $(x-2)x+(y-4)y=0$

3) $x^2+y^2+2x+4y=0$

11. The eqn of a circle with (0, 0) as centre and passing through vertices of an equilateral triangle whose median is of length 3a is

1) $x^2+y^2=(2a)^2$ 2) $x^2+y^2=(3a)^2$ 3) $x^2+y^2=a^2$

12. Radius of a circle inscribed in a square of length 10 is __________

1) 5 2) 10 3) $\frac{10}{3}$ 4) $\frac{5}{2}$

13. ABCD is a square of length 4 units. A = (0, 0) AB and AD are along +ve x, y axes respectively Then eqn of the circle passing through A, B, C, D is

1) $(x-4)x+(y-4)y=0$ 2) $x^2+y^2=16$

3) $x^2+y^2-2x-2y=16$

14. centre of the circle inscribed in a rectangle formed by the lines $x^2-8x+12=0,\ y^2-14y+45=0$ is

1) (4, 7) 2) (4, 8) 3) (7, 4)

15. Eqn of mirror image of the circle $x^2+y^2+4x+8y-16=0$ in the line $x+y+2=0$ is

1) $(x-2)^2+y^2=36$ 2) $x^2+(y-2)^2=36$

3) $x^2+y^2=36$

16. If the lines $2x+3y+1=0,\ kx+2y+3=0$ intersect coordinate axes at concyclic points, then k = ————

1) 3 2) 6 3) 4 4) 1

17. centre of the circle touching x–axis at (3, 0) and making intercept of 6 untis on +ve y–axis is

1) $(3,\ 3\sqrt{2})$ 2) $(3\sqrt{2},\ 3)$ 3) $(\sqrt{2},\ 3)$

18. The lines $2x-y-3=0,\ x-3y-4=0$ are diameters of a circle of area 88 square units, then eqn of the circle is

1) $(x-1)^2+(y+1)^2=28$ 2) $(x-1)^2+(y+1)^2=38$

3) $(x-1)^2+(y-1)^2=28$

19. Eqns of tangents to the circle $x^2+y^2-6x-8y=0$ which are inclined at 45^0 with x–axis

1) $y=x+5\sqrt{2}$ 2) $x-y+1+5\sqrt{2}=0$ 3) $x+y+1+5\sqrt{2}=0$

20. If the lines $3x-4y+4=0,\ 6x-8y-7=0$ are tangents to a circle, then radius = ———

1) $\frac{3}{4}$ 2) $\frac{3}{2}$ 3) 4 4) 2

21. Eqn of circle passing through (0, 0) and making intercepts of 6 &8 on positive coordinate axes is ———

1) $(x-6)x+(y-8)y=0$ 2) $(x+6)x+(y+8)y=0$

3) $(x-6)x+y^2=0$

22. For the line $3x+4y-5=0$ and the circle $x^2+y^2-4x-12y+4=0$ which of the following statements is true.

1) line touches the circle 2) line intersects the circle

3) line is a diameter of circle

23. Eqn of the circle touching x = 0, y = 0 and x = 2 is

1) $(x-1)^2+(y-1)^2=1^2$ 2) $(x-1)^2+(y+1)^2=1^2$

3) $(x+1)^2+(y-1)^2=1^2$ 4) both (1) & (2)

24. Locus of centre of the circle touching both the coordinate axes is

1) $x^2-y^2=0$ 2) $x^2+y^2=0$ 3) $x+y=0$

25. Eqn of circle concentric with $x^2+y^2-2x-6y+1=0$ and touching y–axs is

1) $(x-1)^2+(y-3)^2=1^2$ 2) $(x-1)^2+(y-3)^2=3^2$

3) $x^2+y^2-6y=0$

26. Eqn of the circle passing through (–2, 1) and touching the line 3x–2y–6=0 at (4, 3) is

1) $7x^2+7y^2+4x-82y+55=0$ 2) $x^2+y^2+4x+82y+30=0$

3) $x^2+y^2+4x+7y+2=0$

27. Locus of midpoints of chords of the circle $(x-1)^2+y^2=1$ which passes through the (0, 0) is

1) $x^2+y^2+x=0$ 2) $x^2+y^2-x=0$

3) $x^2+y^2+x+y=0$

28. Locus of midpoints of chords of the circle $x^2+y^2=9$ which subtend a right angle at (0, 0) is

1) $2(x^2+y^2)=9$ 2) $x^2+y^2=9$

3) $x^2+y^2=8$

29. Locus of midpoints of chords of the circle $x^2+y^2=16$ that subtend an angle of $\frac{2\pi}{3}$ at (0, 0) is

1) $x^2+y^2=6$ 2) $x^2+y^2=4$ 3) $x^2+y^2=8$

30. Tangents AB & AC are drawn from the point A(0, 1) to the circle $x^2+y^2-4x-2y+1=0$ which of the following are true about the circle passing through A, B, C

1) It is a circle with AP as diameter. Where P is centre of the given circle

2) eqn of that circle is $x(x-2)+(y-1)(y-1)=0$

3) both (1) & (2)

31. circle $x^2+y^2+4x+8y-5=0$ intersect the line $4x+3y=m$ in two distinct points, then range of m is

1) $-45<m<5$ 2) $-45<m<-5$ 3) $5<m<45$

32. If the line $y=2\sqrt{2}+k$ touches the circle $x^2+y^2=25$ then k = ———

1) ±15 2) 5 3) 10 4) 4

33. Eqns of tangents to the circle $x^2+y^2-4x+6y-12=0$ which are parallel to x+y-8=0

1) $x+y+1\pm5\sqrt{2}=0$ 2) $x+y+1\pm\sqrt{2}=0$

3) $x+y+1\pm6\sqrt{2}=0$

34. Equations of tangents to the circle $x^2+y^2+2x-2y-14=0$ that are perpendicular to $3x+4y+1=0$

1) $4x-3y+26=0, 4x-3y-13=0$ 2) $4x-3y+27=0,\ 4x-3y-13=0$

3) $4x-3y+1=0, 3x+4y+26=0$

35. Eqn of normal to the circle $x^2+y^2-2y=0$ that is parallel to $2x+y-3=0$ is $\underline{2x+y-1=0}$

36. Tangents drawn from P(a, b) to the circle $x^2+y^2=r^2$ are PA & PB, then the circumcentre of ΔPAB is $\left(\frac{a}{2},\frac{b}{2}\right)$

37. length of tangents drawn from any point on the circle $(x-2)^2+(y-3)^2=5r^2$ to the circle $(x-2)^2+(y-3)^2=r^2$ is 16 units, then area between the two circles is $\underline{256\pi}$

1) 256π 2) 64π 3) 8π

38. An infinite no of tangents can be drawn to the circle $x^2+y^2-4x-2y+k=0$ from (2, 1) then k = ———

1) 5 2) 0 3) 4 4) 8

39. For an equilateral triangle centre is (0, 0) and length of altitude is a. Then eqn of the circumcircle is

1) $9x^2+9y^2=4a^2$ 2) $x^2+y^2=a^2$

3) $3x^2+3y^2=a^2$

40. From any point on the circle $x^2+y^2=a^2\sin^2\alpha$, then angle between them is

1) $\theta=\alpha$ 2) $\theta=2\alpha$ 3) $\theta=\frac{\alpha}{2}$

41. circles $x^2+y^2+2gx+2fy=0,\ x^2+y^2 2g'x+2f'y=0$ touch externally if

1) $f'g=fg'$ 2) $f'g=\sqrt{fg'}$ 3) $f^2g=f'^2g'$

42. Relative positions of the two circles $x^2+y^2-2x-3=0$ & $x^2+y^2-4x-6y-3=0$

1) The intersect each other at two distinct points

2) They touch each other externally

3) Touch internally

43. No of circles touching all the three lines x = 0, x = 2, y+1=0

1) 2 2) 4 3) 0

44. A square is inscribed in circle $x^2+y^2-2x+4y-93=0$ with sides parallel to axes, then which of the following can be axes then which of the following can be one of vertices of the square

1) $(-6,-9)$ 2) $(2,4)$ 3) $(-1,2)$ 4) $(8,1)$

45. If the length of the tangent from $(f,\ g)$ to the circle $x^2+y^2=6$ be twice the length of tangent from same point to the circle $x^2+y^2+3x+3y=0$ then

$f^2+g^2+4f+4g+2=$ ——

1) 0 2) 2 3) 4 4) 8

46. Find area of the circle $(x+1)(x+2)+(y-1)(y+3)=0$

1) $\frac{17\pi}{4}$ 2) $\frac{18\pi}{4}$ 3) 4π

47. If the circle $x^2+y^2+6x-2y+k=0$ bisects the circumference of the circle $x^2+y^2+2x-6y-15=0$ then k = ———

1) –23 2) 23 3) 0 4) 24

48. length of tangent from (6, 8) to circle $x^2+y^2=4$ is

1) $4\sqrt{6}$ 2) 96 3) $2\sqrt{6}$

49. Find eqn of circle with radius 5 units and touching the curve $x^2+y^2-2x-4y-20=0$ at (5, 5)

1) $x^2+y^2-18x-16y+120=0$ 2) $x^2+y^2-8x-8y+6=0$

3) $x^2+y^2+6x+8y=0$

50. Eqn of circle toucing x–axis and whose centre is (1, 2) is

1) $(x-1)^2+(y-2)^2=4$ 2) $(x-1)^2+(y-2)^2=1^2$

3) $(x-1)^2+(y-2)^2=5$

51. If a line drawn from a fixed point $M(a,\, b)$ cuts the circle $x^2+b^2=k^2$ at C & D then $MC \times MD =$ ———

1) $a^2+b^2-k^2$ 2) $a^2+b^2+k^2$ 3) a^2+b^2

52. Angle between pair of tangents from (1, 3) to the circle $x^2+y^2-2x+4y-11=0$ is—

1) $\sin^{-1}\left(\frac{24}{25}\right)$ 2) $Tan^{-1}\left(\frac{4}{3}\right)$ 3) $\frac{\pi}{2}$

53. Length of tangent drawn from midpoint of line joining (0, 0) (4, –4) to the circle $2x^2+2y^2-y=0$ is

1) 3 2) 4 3) 8 4) 0

54. Area quadrilateral formed by the tangents drawn from (4, 5) to the circle $x^2+y^2-4x-2y-11=0$ with a pair of radii joining the points of contact of these tangents is

1) 8 2) 6 3) 4 4) 9

55. If the chord of contact of tangents from a point A to a given circle passes through θ, then the circle with AB as diameter will

1) touch the given circle externally.

2) intersect the given circle at the distinct points

3) lie inside the given circle

56. In ΔABC $\angle A = 90^0$, B=(2, –4) c = (1, 5) eqn of circum circle of ΔABC is

1) $(x-1)(x-2)+(y-5)(y+4)=0$ 2) $(x-1)(x+2)+(y-5)(y-4)=0$

3) $(x-1)(x-2)+(y-5)(y-4)=0$

57. Eqn of circumcircle of the triangle formed by the lines $x+y=6,\ 2x+y=4,\ x+2y=5$ is

1) $x^2+y^2-17x-19y+50=0$ 2) $x^2+y^2-17x-17y+50=0$

3) $x^2+y^2-7x-9y+50=0$

58. If (1, 4) lies inside the circle $x^2+y^2-6x-10y+p=0$ & the circle does not touch or intersect coordinate axes, then

1) $25<p<29$ 2) $p<29$ only 3) $p>25$ only 4) $0<p<2$

59. length of diameter of circle $x^2+y^2-6x-8y=0$ is <u>10</u>

60. Polar of (1, –2) w.r.to $x^2+y^2-10x-10y+25=0$

1) $4x+7y-30=0$ 2) $4x+7y-31=0$

3) $2x+7y+4=0$

61. If two circles $(x-1)^2+(y-3)^2=r^2$, $x^2+y^2-8x+2y+8=0$ intersect in two different points, then what can we conclude about r?

1) 2 < r < 8 2) r > 10 3) 0 < r < 2

62. $3x+y+k=0$ is a tangent to the circle $x^2+y^2=10$ the k = ——

1) ±10 2) ±4 3) 8 4) 2

63. centre = (2, –3) circumference of the circle = 10π then eqn of the circle is

1) $(x-2)^2+(y+3)^2=5^2$ 2) $(x-2)^2+(y+3)^2=16$

3) $(x-2)^2+(y-3)^2=10$

64. Find eqn of circle having normal $(x-1)(y-2)=0$ and a tangent $3x+4y=6$ is

1) $(x-1)^2+(y-2)^2=1$ 2) $(x-1)^2+(y-2)^2=2$

3) $(x-1)^2+(y-2)^2=4$

65. eqn of circle with centre = (5, 4) and touch y–axis is ––

1) $(x-5)^2+(y-4)^2=5^2$ 2) $(x-5)^2+(y-4)^2=4^2$

3) $(x-5)^2+(y-4)^2=(\sqrt{41})^2$

66. If circle $x^2+y^2+6x+2ky+25=0$ touches y–axis is ––

1) ±5 2) 1 3) ±4 4) ±2

67. The point on the circle $x^2+y^2=4$ whose distance from $4x+3y-12=0$ is $\frac{4}{5}$ units is

1) (2, 0) 2) (2, 1) 3) (8, 0) 4) (4, 2)

68. eqn of circle passing through (0, 0) which makes intercepts a & b an axes is

1) $x^2+y^2\pm ax\pm by=0$ 2) $x^2+y^2\pm ax\pm ay=0$

3) $x^2+y^2\pm bx\pm by=0$

69. Find the value of $m+n$ if the circumference of $x^2+y^2+8x+8y-m=0$ is bisected by the circle $x^2+y^2-2x+4y+n=0$ is –56

70. eqn of normal to the circle $x^2+y^2=16$ at $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$ is $x-y=0$

71. Area of an equilateral triangle inscribed in circle $x^2+y^2-6x+2y-28=0$ is $\underline{\frac{57\sqrt{3}}{2}}$

72. PQ, RS be tangents at extremities of a diameter PR of a circle of radius r such that PS & RQ intersect at X on the circumference of the circle. Then 2r equals to $\underline{\sqrt{PQ \cdot RS}}$

73. A circle is drawn touching x–axis with its centre at the point of reflexian of (m, n) on the line y –x = 0 then eqn of the circle is $\underline{(x-n)^2+(y-m)^2=m^2}$

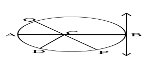
74. length of the chord intercepted by the circle $x^2+y^2-6x+8y-5=0$ on the line $2x-y=5$ is $\underline{10}$

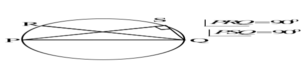
75. The incentre of an equilateral triangle is (1, 1) and eqn of one side is $3x+4y+3=0$. Then eqn of circumcircle of the triangle is $\underline{(x-1)^2+(y-1)^2=4^2}$

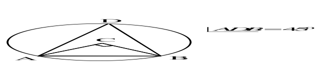
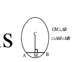
76. Eqn of the circle with centre (2, 3) and touch in the line $3x-4y+1=0$ is $\underline{x^2+y^2-4x-6y+12=0}$

LEVLE – 3

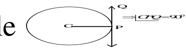
1. A square is inscribed in the circle $x^2+y^2-2x+6y-6=0$ whose diagonals are parallel to axes and a vertex in 3rd quadrant is A, then OA = $\underline{3\sqrt{2}}$

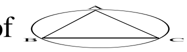
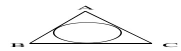
2. Area of the triangle formed by tangent, Normal drawn at $\left(1, \sqrt{3}\right)$ to the circle  and x–axis (positive) is $\underline{2}$

3. If one of the diameters of the circle  is a chord of circle S, whose centre is at (3, 2), then radius of circle 'S' is $\underline{7}$

4. Two perpendicular tangents can be drawn from (0, 0) to the circle  then the value of  is 

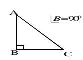
5. Read the following statements carefully.
Mark the correct option out of the options given below.
a) both statements (1), (2) are true : statement (2) is correct explanation of statement (1)
b) (1) is true, (2) is true but (2) is not correct explanation of statement (1)
c) statement (1) is true, (2) is false
d) statement (1) is false, (2) is true.

5. Statement I : Tangents drawn from the point p(13, 9) to the circle  are perpendicular to each other.

Statement II: eqn to director circle of  is 

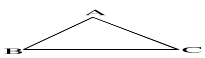
Answer : a

Comprehension:

In the diagram shown, a circle is drawn with centre (1, 1) and radius l and a line L. The line L is a tangent to the circle at Q. Further L meets y–axis at R, and x–axis at p in such a way that the angle  equals θ, where $0 < \theta < \frac{\pi}{2}$

6. The coordinate of Q are

a)  b) $(\sin\theta, \cos\theta)$

c)  d) $(1+\sin\theta, 1+\cos\theta)$

7. Equation of the line PR is

a) 

b) $x\sin\theta + y\cos\theta = \cos\theta + \sin\theta + 2$

c) $x\sin\theta + y\cos\theta = \cos\theta + \sin\theta + 1$

d) $x\tan\theta + y = 1 + \cot\theta/2$

8. Area of ΔOPR when $\theta = \frac{\pi}{4}$ is

a) $3-2\sqrt{2}$ b) $3+2\sqrt{2}$ c) $6+4\sqrt{2}$ d) none of these

Passage II

If θ-chord of a circle be that chord which subtends an angle θ at the centre of the circle

9. $x+y=2$ is θ-chord of $x^2+y^2=4$ then $\theta =$ ——

a) $\frac{\pi}{2}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{4}$ d) $\frac{3\pi}{2}$

10. If slope of a $\frac{\pi}{3}$–chord of chord of $x^2+y^2=1$ is 1, then it's equation is

a) $2x-2y\pm\sqrt{3}=0$ b) $x-y\pm\sqrt{3}=0$ c) $x-y-2\sqrt{3}=0$ d) $x-y+1=0$

11. Distance $\frac{\pi}{2}$–chord of $x^2+y^2-2x-6y+1=0$ from centre is

1) $\frac{3}{\sqrt{2}}$ 2) 1 3) $\frac{1}{\sqrt{2}}$ 4) $3\sqrt{2}$

MATHEMATICS WORK BOOK

PARABOLA

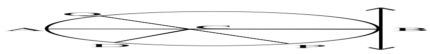
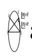
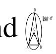
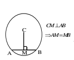
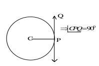
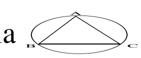
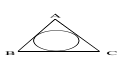
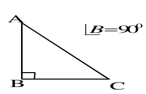
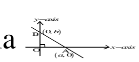
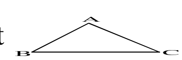
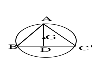
LEVEL -1(I P E)

1. My Dear student can you find any geometrical figure in the above picture? If yes, what is that
2. The standard form of the parabola is $y^2 = 4ax$
3. The conic with eccentricity unity is called parabola
4. Please try to draw the graph of parabola $x^2 = 4ay$?
5. the general equation of the tangent to the parabola $y^2 = 4ax$ at $P(x_1, y_1)$ is $yy_1 = 2a(x + x_1)$
6. The equation of the normal at to the parabola $y^2 = 4ax$ at $P(x_1, y_1)$ is $y - y_1 = \frac{2a}{y_1}(x - x_1)$
7. The parametric coordinates of the parabola $y^2 = 4ax$ at t = (......, ...)
8. The equation of the tangent at t is = ...
9. In the figure-1, If the line y=mx+c intersect the parabola $y^2 = 4ax$ in two distinct points then the condition is?
10. In then figure-2, If the line y = mx+c touches the parabola $y^2 = 4ax$ then the condition is $c = \frac{a}{m}$ [yes/no]
11. In then figure-3, If the line y=mx+c lies exterior of the parabola $y^2 = 4ax$ then the condition is $c > \frac{a}{m}$ [yes/no]
12. If the points $P(t_1), Q(t_2)$ and $R(t_3)$ on the parabola $y^2 = 4ax$ then area of the ΔPQR is? (from figure-4)
13. If the points $P(x_1, y_1), Q(x_2, y_2)$ and $R(x_3, y_3)$ are on the parabola $y^2 = 4ax$ then area of the ΔLMN formed by the tangents at P, Q and R is (see Figure-5)

    $$\frac{1}{16a}\left|(y_1 - y_2)(y_2 - y_3)(y_3 - y_1)\right| \text{ sq.units.} \qquad \text{[yes/no]}$$

14. The path of a projectile is a parabola and the "sixer" hit by a batsman in the cricket game trace of a parabola (yes/no)

    Eccentricity of the parabola is 1 (yes/no)

15. Standard form of the parabola $y^2 = 4ax$ (a>0). The parabola lies in the II[nd] and III[rd] quadrants (yes/no)
16. The parabola meets the axes at only one point (0,0) (yes/no)
17. The locus of point of intersection of orthogonal tangents to the parabola $y^2 = 4ax$ is its directrix x+ a=0 (yes/no)
18. Locus of a point moving in the plane such that its distance from a fixed point is equal to its distance from a fixed straight line is called parabola (yes/no)
19. To the parabola $y^2 = 4ax$ for any value of x we obtain two values of y but opposite signs. This shows that the curve is symmetric about..................
20. A chord through a point P on the parabola and perpendicular to the axis is ....of the parabola.
21. A chord of the parabola passing through focus of the parabola is called .... of the parabola.
22. The quadratic equation $y = ax^2 + bx + c\ (\Delta \neq 0)$ traces the graph of ....
23. The condition for the general equation of second degree  to represent (i) .... (ii) ....
24. If the chord joining the points  and  on the parabola  is focal chord then ............
25. If  is one end of the focal chord of a parabola  then other end is ....
26. If  is a parabola and PQ is a focal chord passing through focus S then  ....
27. A comet moves in a parabola orbit with sun as focus when the comet is 2×10^7 K.M. from the sun to it makes an angle $\pi/2$ with axis of the orbit. Then the nearest distance between sun and comet is ....
28. From an external point p, tangents are drawn to the parabola  and there tangents makes and angles θ_1, θ_2 with its axis such that  is conatant b then p lies on the straight line is ....
29. If the tangent at P to the parabola $y^2 = 4ax$ meets the tangent at the vertex in Q then SQ is perpendicular to tangent P then ...... where S is focus, A is vertex

30. Length of Latusrectum of the parabola $y^2 = 4ax$ is (a>0) [ ]

a) 2a b) a c) 3a d) 4a

31. Equation of the directrix of the parabola $y^2 = 4ax$ is (a>0) [ ]

a) x+a=0 b) x–a = 0 c) y+a= 0 d) y – a = 0

32. If $P(x_1, y_1)$ is any point on the parabola $y^2 = 4ax$ (a>0) the focal distance is [ ]

a) $x_1 + a$ b) $y_1 + a$ c) $x_1 - a$ d) $y_1 - a$

33. If the line $y = mx + c(m \neq 0)$ touches the parabola then condition is [ ]

a) $c = \frac{a}{m}$ b) $m = ac$ c) $c = -am^2$ d) $c = am^2$

34. Equation of the normal in the slope form of the parabola $y^2 = 4ax$ if m is slope of the normal [ ]

a) $y = mx - 2am - am^3$ b) $y = mx + 2am + am^3$

c) $y = mx + \frac{a}{m}$ d) $y = mx - \frac{a}{m}$

LEVEL – 2 (EAMCET)

35. If θ is angle between the tangent is drawn from (x_1, y_1) to the parabola $y^2 = 4ax$ then $\tan\theta =$ [ ]

a) $\left|\frac{\sqrt{S_{11}}}{x_1 + a}\right|$ b) $\left|\frac{\sqrt{S_{11}}}{y_1 + a}\right|$ c) $\left|\frac{2\sqrt{S_{11}}}{x_1 + a}\right|$ d) $\left|\frac{2\sqrt{S_{11}}}{y_1 + a}\right|$

36. The coordinate of the focus of the parabola $x^2 = -4y$ is [ ]

a) (1, 0) b) (–1, 0) c) (0, 1) d) (0, –1)

37. The equation of the parabola whose vertex is (3, –2) and focus is (3, 1) is [ ]

a) $(x-3)^2 = 12(y+2)$ b) $(x-3)^2 = -12(y+2)$

c) $(x+3)^2 = 12(y-3)$ d) $(x-2)^2 = 12(y+3)$

38. If the coordinates of the ends of a focal chord of the parabola $y^2 = 4ax$ are (x_1, y_1) and (x_2, y_2) then $\frac{y_1 y_2}{x_1 x_2} =$ [ ]

a) 1 b) –2 c) 4 d) –4

39. The equation of Normal to the parabola $y^2 = 4ax$ which is parallel to $y - 2x + 5 = 0$ is [ ]

a) $2x - y - 12 = 0$ b) $y - 2x + 1 = 0$

c) $y - 2x + 7 = 0$ d) $y - 2x + 4 = 0$

40. The normal at a point t_1 on the parabola $y^2 = 4ax$ meets the parabola again t_2 then t_2 [ ]

a) $2 + t_1 t_2$ b) $t_1 + \frac{2}{t_2}$ c) $-t_1 - \frac{2}{t_2}$ d) $-t_2 - \frac{2}{t_1}$

41. For the parabola $y^2 + 6y - 2x + 5 = 0$

I : The vertex is (–2, 3)

II : The directrix is y+3 = 0

Which of the following is correct?

a) Both I and II are true b) I is true II is false

c) I is false and II is true d) Both I and II are false

LEVEL-3 (JEE)

42. Observe the following lists

List – I	**List –II**
A) The directrix of the parabola $y^2 - 2y + 8x - 23 = 0$	1) x + 2 = 0
B) The equation of the Latusrectum of the parabola $x^2 - 2x - 4y - 3 = 0$	2) y –5 = 0
C) The axis of the parabola $8x + x^2 + 12y + 4 = 0$	3) x + 4 = 0
D) Equation of the tangent at the Vertex of $y^2 - 6y - 12y - 15 = 0$	4) x – 5= 0
	5) y = 0

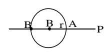
Correct match for list – I from list – II is

	A	B	C	D
1)	4	3	5	1
2)	4	5	3	1
3)	4	2	5	1
4)	4	5	3	2

43. If t_1, t_2 are two point on the parabola $y^2 = 4ax$ then observe the following lists

List – I	**List – II**
A) Equation of the tangent at t_1 is	i) $yt_1 = x + at_1^2$
B) Equation of the Normal at t_1 is	ii) $y + xt_1 = 2at_1 + 2at_1 + at_1^3$
C) The point of intersection tangents at t_1 and t_2	iii) $\left(at_1t_2,\ a\left(t_1 + t_2\right)\right)$
D) The point of intersection normal at t_1 and t_2	iv) $\left[2a + a\left(t_1^2 + t_1t_2 + t_2^2\right), -at_1t_2\left(t_1 + t_2\right)\right]$

44. Assertion (A) : If the line x = 3y + k touches the parabola $3y^2 = 4x$ then k = 5

Reason (R) : Equation to the tangent 

inclined at an angle 30^0 to the axis is

$x - \sqrt{3}y + 6 = 0$

Then one of the following is the correct answer.

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true R is false
4) A is false R is true

45. Assertion (A) : PQ is a double ordinate of a parabola $\overline{QR}$ is a focal chord then $\overline{PR}$ is perpendicular to the directrix

Reason (R) : The chord joining the points t_1 and t_2 of a parabola is a focal chord iff $t_1t_2 = -1$

Then one of the following is the correct answer.

1) Both A and R are true and R is the correct explanation of A
2) Both A and R are true and R is not the correct explanation of A
3) A is true R is false
4) A is false R is true

Comprehension passage

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$ they intersect at P and Q in the first and fourth quadrants respectively. Tangents to the circle at P and Q intersect the x – axis at R and tangents to the parabola at P and Q intersect to x–axis at s

46. The ratio of the Areas of the triangles PQS and PQR is

a) $\frac{1}{\sqrt{2}}$ b) 1 : 2 c) 1 : 4 d) 1 : 8

47. The radius of the circum circle of the triangle PRS is

 a) 5 b) $3\sqrt{3}$ c) $3\sqrt{2}$ d) $2\sqrt{3}$

48. The radius of the in circle of the triangle PQR is

 a) 4 b) 3 c) 8/3 d) 2

PARABOLA WORK BOOK KEY

1. Yes, Parabola
2. Yes
3. Yes
4. For a >0
5. Yes
6. No
7. $\left(at^2, 2at\right)$
8. $yt = x + at^2$
9. $c < \frac{a}{m}$
10. Yes
11. Yes
12. $\frac{1}{8a}\left|(y_1 - y_2)\right|(y_2 - y_3)(y_3 - y_1)$
13. Yes
14. Yes
15. Yes
16. Yes
17. Yes
18. Yes
19. X–axis
20. Double ordinate
21. Focal chord
22. Parabola
23. Parabola is (i) $h^2 = ab$ (ii) $\Delta \equiv abc + 2fgh - af^2 - bg^2 - ch^2 \neq 0$
24. $t_1t_2 = -1$

25. $\left(\frac{a}{x_1}, \frac{-4a^2}{y_1}\right)$

26. $\frac{1}{a}$

27. $10^7 KM$

28. $y = bx$

29. SP.SA

30. d

31. x+a=0

32. $x_1 + a$

33. a

34. a

35.a

36.b

37. b

38. d

39.a

40.c

41.c

42.2

43.1

44.4

45.4

46.c

47.b

48.d

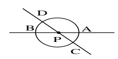
WORK BOOK FOR INTERMEDIATE STUDENTS
ELLIPSE MATHS IIB
LEVEL – I

Choose the correct answer for the following:

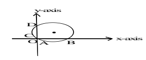
1. Eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is [ ]

   a) $\sqrt{\frac{a^2-b^2}{a^2}}$ b) $\sqrt{\frac{a^2+b^2}{a^2}}$ c) $\sqrt{\frac{a^2-b^2}{b^2}}$ d) $\sqrt{\frac{a^2+b^2}{b^2}}$

2. Focii of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a<b)$ is [ ]

   a) $(\pm ae, 0)$ b) $(0, \pm ae)$ c)  d) $(\pm be, 0)$

3. Focii of an ellipse $\frac{(x-b)^2}{a^2}+\frac{(y-k)^2}{b^2}=1(a>b)$ is [ ]

   a) $(h\pm ae, k)$ b) $(h, k\pm ae)$ c) $(h\pm be, k)$ d) 

4. Equations of directrices of an ellipse $\frac{(x-b)^2}{a^2}+\frac{(y-k)^2}{b^2}=1(a<b)$ is [ ]

   a) $x=h\pm a/e$ b) $y=k\pm b/e$ c) $x=k\pm a/e$ d) $y=h\pm b/e$

5. Equations of directrices of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ is [ ]

   a) $x=\pm a/e$ b) $x=\pm b/e$ c) $y=\pm a/e$ d) $y=\pm b/e$

6. Length of latusrectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a<b)$ is [ ]

   a) $\frac{2b^2}{a}$ b) $\frac{2a^2}{b}$ c) $\frac{2b^2}{a^2}$ d) $\frac{2a^2}{b^2}$

7. If S, S' are foci of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1(a>b)$ and p is any point on ellipse then $SP+S'P$ is .... [ ]

   a) length of minor axis b) length of major axis

   c) length of latus rectum d) length of focal distance

8. Condition for the line $x\cos\alpha+y\sin\alpha=p$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is .... [ ]

   a) $p^2=a^2\cos^2\alpha+b^2\sin^2\alpha$ b) $p^2=a^2\cos^2\alpha-b^2\sin^2\alpha$

c) $a^2 = p^2\cos^2\alpha + b^2\sin^2\alpha$ d) $b^2 = a^2\cos^2\alpha + p^2\sin^2\alpha$

9. Locus of the foot of the perpendicular drawn from either of the foci to any tangent to the ellipse is .... [ ]

a) auxiliary circle b) director circle

c) semi circle d) none

10. If the normal at the end of latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ passes through one end of the minor axis then $e^4+e^2=$ [ ]

a) 1 b) –1 c) 0 d) –2

11. If the line $lx+my+n=0$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then —— [ ]

a) $n^2 = a^2l^2 + b^2m^2$ b) $n^2 = a^2m^2 + b^2l^2$

c) $n^2 = a^2m^2 - b^2l^2$ d) $n^2 = a^2b^2 - b^2m^2$

12. If $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then the point of contact is..[ ]

a) $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$ b) $\left(\frac{a^2m}{c}, \frac{-b^2}{c}\right)$ c) $\left(\frac{a^2m}{c}, \frac{b^2}{c}\right)$ d) $\left(\frac{-a^2m}{c}, \frac{b^2}{c}\right)$

13. Condition for the line $lx+my+n=0$ is normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is.... [ ]

a) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)}{m^2}$ b) $\frac{a^2}{l^2}-\frac{b^2}{m^2}=\frac{\left(a^2+b^2\right)}{n^2}$

c) $\frac{a^2}{l^2}+\frac{b^2}{m^2}=\frac{\left(a^2-b^2\right)}{n^2}$ d) none

14. If θ_1, θ_2 are the eccentric angles of the extremities of a focal chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\left(a>b\right)$ then $\frac{e+1}{e-1}=$——— [ ]

a) $\cot\frac{\theta_1}{2}\cot\frac{\theta_2}{2}$ b) $\tan\frac{\theta_1}{2}\tan\frac{\theta_2}{2}$

c) $\cot\frac{\theta_1}{2}\tan\frac{\theta_2}{2}$ d) $\tan\frac{\theta_1}{2}\cot\frac{\theta_2}{2}$

15. Equation of normal at $p(\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is —— [ ]

a) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$ b) $\frac{ax}{\cos\theta}+\frac{by}{\sin\theta}=a^2-b^2$

c) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2+b^2$ d) $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

Fill the following blanks with suitable answers:

16. length of latusrectum of an ellipse $9x^2+16y^2=144$ is ———

17. length of major and minor axis of an ellipse $4x^2+y^2-8x+2y+1=0$ ————

18. Equation of ellipse whose distance between foci is 2 and length of latur rectum is $15/2$ is ———

19. Eccentricity of an ellipse $16x^2+25y^2=400$ is —————

20. If the length of the latus rectum is equal to half of its major axis of an ellipse then its eccentricity is —————

21. Equation of ellipse whose distance between foci is 8 and distance between directrices is 2 is —————

22. Radius of a circle passing through the foci of an ellipse $9x^2+16y^2=144$ and having least radius is ——————

23. Equation of tangent of the ellipse $9x^2+16y^2=144$ at the end of latusrectum in the first quadrant is —————

24. Equation of normal at (2, –1) to the ellipse $x^2+2y^2-4x+12y+14=0$ is —————

25. If $4x+y+k=0$ is tangent to the ellipse $x^2+3y^2=3$ then k = ————

26. Equation of normal at $\theta=\frac{\pi}{4}$ to the ellipse $4x^2+9y^2=36$ is —————

27. Equation of tangent to the ellipse $2x^2+y^2=8$ which makes an angle 45^0 with x–axis is ———

Write either true or False of the following statements:

28. A conic with eccentricity (e) is lessthan one is on ellipse [ ]

29. A chord of the ellipse passing through either of the foci of the ellipse is called double ordianate

30. A focal chord of on ellipse perpendicular to the major axis of an ellipse is called latusrectum.

31. The circle whose diameter is major axis of an ellipse is called auxiliary circle

32. $x=a\sec\theta,\ y=b\tan\theta$ are parametric equations of ellipse

33. Equation of tangent at $p(\theta)$ on the ellipse S = 0 is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

34. Almost four normals can be drawn from any point to ellipse

35. If S, S' are foci of an ellipse $S = 0(a > b)$ then $S'P$ is constant

36. Equation of normal at $\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ of ellipse is not defined

37. The locus of point of intersection of perpendicular tangent an ellipse S = 0 is called director circle.

Match the following:

38.

List – I	**List – II**
A) The equation of the major axis	1) 3x –34
B) The equation of the directrices	2) y –2 = 0
C) The equation of the latus rectum	3) x = 6
D) The equation of minor axis	4) 32/5
E) length of latus rectum	5) x – 3 = 0

39.

Equation of ellipse	**Length of latus rectum**
A) $\frac{(x-2)^2}{36}+\frac{(y-3)^2}{11}=1$	1) 2
B) $\frac{(x+1)^2}{16}+\frac{(y+1)^2}{25}=1$	2) 18/5
C) $\frac{(x-2y+1)^2}{49}+\frac{(2x-y+3)^2}{7}=1$	3) 11/3
D) $x=3\cos\theta,\ y=5\sin\theta$	4) 9/2
E) $9x^2+16y^2=144$	5) 32/5

40.

List – I	**List – II**
A) Tangent at $\frac{\pi}{6}$ to $\frac{x^2}{4}+\frac{y^2}{3}=1$	1) $\sqrt{2}x-y=1$
B) Tangent at (1, 1) to $x^2+2y^2=3$	2) $x^2+y^2=18$
C) Normal at $\frac{\pi}{4}$ to $x^2+2y^2=4$	3) $x+2y-3=0$
D) Normal at (2, 3) to $x^2+4y^2=40$	4) $3\sqrt{3}x+2\sqrt{3}y=12$
E) Auxillary circle of $2x^2+y^2=36$	5) $6x-y-9=0$

LEVEL – II

1. The centre of the ellipse $\frac{(2x-3y-1)^2}{16}+\frac{(3x+2y-8)^2}{9}=1$ is

   a) (1, 1) b) (1, –2) c) (2, 1) d) (2, –1)

2. The eccentricity of an ellipse is $\frac{1}{2}$ and one of the directrices is x = 4 then equation of ellipse is __________

   a) $3x^2+4y^2=1$ b) $4x^2+3y^2=1$ c) $4x^2+3y^2=12$ d) $3x^2+4y^2=12$

3. The distance between the foci of the ellipse $x=3\cos\theta,\ y=4\sin\theta$ is _________

   a) $2\sqrt{7}$ b) $7\sqrt{2}$ c) $\sqrt{7}$ d) $3\sqrt{7}$

4. The eccentricity of the ellipse $x^2+4y^2+2x+16y+13=0$ is _________

   a) $\frac{\sqrt{3}}{2}$ b) $\frac{1}{2}$ c) $\frac{1}{\sqrt{3}}$ d) $\frac{1}{\sqrt{2}}$

5. In an ellipse the distance between the foci is 6 and its minor axis is 8 then its eccentricity is

   a) $\frac{4}{5}$ b) $\frac{1}{\sqrt{52}}$ c) $\frac{3}{5}$ d) $\frac{1}{2}$

6. The equation of the latus rectum of the ellipse $9x^2+25y^2-36x+50y-164=0$ are___

   a) $x-6=0,\ x+2=0$ b) $x-6=0,\ x+2=0$

   c) $x+6=0,\ x-2=0$ d) $x-4=0,\ x+5=0$

7. If P is a point on the ellipse $9x^2+36y^2=324$ whose foci are S, S' then $PS+PS'=$_________

   a) 9 b) 12 c) 27 d) 36

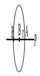
8. The equation $\frac{x^2}{2-r}+\frac{y^2}{r-5}+1=0$ represents an ellipse then ____

   a) r > 2 b) r > 5 c) 2 < r < 5 d) r < 2 or r > 5

9. The value of m so that the line y = 4x + m touches the ellipse $x^2+4y^2=4$ is ____

   a) $\pm\sqrt{45}$ b) $\pm\sqrt{60}$ c) $\pm\sqrt{65}$ d) $\pm\sqrt{72}$

10. If tangents are drawn from any point on the circle $x^2 + y^2 = 25$ to the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ then angle between the tangents is ____________

a) $\frac{\pi}{4}$ b)  c) $\frac{\pi}{2}$ d) $\frac{2\pi}{3}$

11. The product of the perpendiculars from the foci on any tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is ————

a) a^2 b) a^2-b^2 c) b^2 d) $\sqrt{a^2+b^2}$

12. If $\frac{x}{a}+\frac{y}{b}=\sqrt{2}$ touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then its ellntie angle θ is equal to ____

a) 0^0 b) 90^0 c) 45^0 d) 60^0

13. The pole of the line $y-x+2$ with respect to the ellipse $x^2+4y^2-2x-16y-10=0$ is

a) $\left(\frac{9}{4},\frac{-5}{24}\right)$ b) $\left(\frac{6}{7},\frac{-17}{4}\right)$ c) $\left(-26,\frac{35}{4}\right)$ d) None

14. If $2x-y+3=0,\ 4x+ky+3=0$ are conjugate with respect to the ellipse $5x^2+6y^2=15$ then k = ————

a) 1 b) 2 c) 6 d) 6

15. If (1, 2), (k, –1) are conjugate with respect to the ellipse $2x^2+3y^2=6$ is

a) 2 b) 2 c) 6 d) 8

16. If the chord of contact of the point (1, –2) with respect to the ellipse $4x^2+5y^2=20$ is $ax+by+c=0$ then the ascending order of a, b, c is ———

a) a, b, c b) c, b, a c) c, a, b d) b, a, c

17. If the line $2x+5y=12$ intersects the ellipse $4x^2+5y^2=20$ in two distinct points A and B then the midpoint of AB is ——————

a) (0, 1) b) (1, 2) c) (1, 0) d) (2, 1)

18. A : Equation of the ellipse whose latus rectum 8 and eccentricity $\frac{1}{\sqrt{2}}$ is $\frac{x^2}{64}+\frac{y^2}{32}=1$

B : Equation of ellipse whose minor axis 6 and eccentricity $\frac{1}{2}$ is $\frac{x^2}{12}+\frac{y^2}{9}=1$

a) only A is true b) only B is true

c) Both A and B are true d) neither A nor B are true

19. A : The vertices of the ellipse $3x^2+4y^2+6x-8y-5=0$ are (1, 1), (–3, 1)

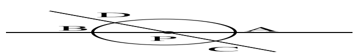
B : The vertices of the ellipse $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1(a>b)$ are $(\alpha \pm a, \beta)$

a) Both A and R are true and R is the correct explanation of A

b) Both A and R are true and R is not correct explanation of A

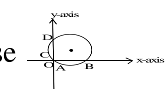
c) A is true but R is false   d) A is false but R is true

20.Match the following:

Ellipse	**Eccentricity**
A) 	1) $\sqrt{5}/3$
B) $9x^2+25y^2-18x-100y-116=0$	2) $\sqrt{3}/2$
C) $36x^2+144y^2-36x-96y-119=0$	3) $4/5$

a) a, b, c   b) b, c, a   c) c, a, b   d) a, c, b

LEVEL – III

1. The equation of the circle passing through the foci of the ellipse  and having centre at (0, 3) is ______

a) $x^2+y^2-6y-5=0$ b) $x^2+y^2-6y+5=0$

c) $x^2+y^2-6y-7=0$ d) none

2. The area of the quadrilateral formed by the tangents at the end points of the latusrecta to the ellipse $\frac{x^2}{9}+\frac{y^2}{5}=1$ is ______

a) $\frac{27}{4}$ b) 18   c) $\frac{27}{2}$ d) 27

3. If ‘P’ is a point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with foci S and S' then the max value of the area of $\Delta PSS'$ is _________

a) ab   b) 2ab   c) abc   d) abc^2

4. A : The focii of the ellipse $\frac{(x-1)^2}{5}+\frac{(y-5)^2}{9}=1$ are (1, 7), (1, 3)

R : The focii of the ellipse $\frac{(x-\alpha)^2}{a^2}+\frac{(y-\beta)^2}{b^2}=1(a<b)$ are $(\alpha, \beta \pm be)$

Hyperbola – Maths IIB

LEVEL – I

Choose the correct answer for the following:

1. Eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$ is ____

   a) $\sqrt{\frac{a^2+b^2}{a^2}}$ b) $\sqrt{\frac{a^2+b^2}{b^2}}$ c) $\frac{\sqrt{a^2+b^2}}{a}$ d) $\frac{\sqrt{a^2+b^2}}{b}$

2. coordinates of foci of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$ are _____

   a) $(0, \pm ae)$ b) $(\pm ae, 0)$ c) $(0, \pm be)$ d) $(\pm be, 0)$

3. Equations of charectrices of hyperboal $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}-1$ are ____

   a) $x=h\pm a/e$ b) $x=k\pm a/e$ c) $y-k\pm b/e$ d) $y-h\pm b/e$

4. Length of latusrectum of hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}-1=0$ is ________

   a) $\frac{2b^2}{a}$ b) $\frac{2a^2}{b}$ c) $\frac{2b^2}{a^2}$ d) $\frac{2a^2}{b^2}$

5. Equation of the director circle of $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is _____________

   a) $x^2+y^2=a^2+b^2$ b) $x^2+y^2=b^2$

   c) $x^2+y^2=a^2-b^2$ d) $x^2+y^2=a^2$

6. Equation of normal at $P(\theta)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is ____

   a) $\frac{ax}{\sec\theta}-\frac{by}{\tan\theta}=a^2-b^2$ b) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2-b^2$

   c) $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2$ d) none

7. Hyperbola is $S-\frac{x^2}{a^2}-\frac{y^2}{b^2}-1=0$ and its conjugate hyperbola is $S=\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$

   and pair of asymptotes is $A=\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$ then $S+S'=$___

   a) 2S b) $2S'$ c) $2A$ d) SS'

8. Eccentricities of hyperbola and its conjugate are e, e' respectively then $\frac{1}{e^2}+\frac{1}{e'^2}==$ __

a) 0 b) –1 c) 1 d) 2

9. Product of length of the perpendiculars from any point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is ____

a) $\frac{16}{25}$ b) $\frac{12}{25}$ c) $\frac{144}{25}$ d) $\frac{256}{25}$

10. The equation of rectangular hyperbola is ______

a) $x^2-y^2=a^2+b^2$ b) $x^2-y^2=a^2$

c) $x^2-y^2=b^2-a^2$ d) $x^2-y^2=a^2-b^2$

Fill the following blanks with suitable answer:

11. The hyperbola whose transverse and conjugate axes are respectively the conjugate and transverse axis of a given hyperbola is called ______ of the given hyperbola.

12. If $lx+my+n=0$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ then ______

13. If $3x-4y+k=0$ is tangent to $x^2-4y^2=5$ then k = ____

14. If the eccentricity of the hyperbola is $\frac{5}{4}$ then eccentricity of its conjugate hyperbola is _____

15. Equation of normal at $\theta=\frac{\pi}{3}$ to the hyperbola $3x^2-4y^2-12$ is _____

16. Equations of directrices of hyperbola $16y^2-9x^2=144$ is ______

17. centre of hyperbola $5x^2-4y^2+20x+8y+4=0$ is ________

18. Product of the perpendicular distance from any point on a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ to its asymptotes is ______

19. Equation of hyperbola whose foci are $(\pm5,\ 0)$ and the transverse axis is of length 8 is _______

20. Equation of hyperbola whose asymptotes are $3x=\pm5y$ and the vertices are $(\pm5,\ 0)$ is ________

Write either true or false of the following statements:

21. A conic with eccentricity (e) is greater than one is a hyperbola.

22. If S, S' are foci of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and P is any point on hyperbola then $S'P-SP=26$

23. Eccentricity of rectangular hyperbola is $\sqrt{2}$

24. Equations of pair of asymptotes is $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

25. condition for the line $y=mx+c$ is tangent to hyperbola is $c^2=a^2m^2+b^2$

26. A line segment along x–axis of length 2a is transverse axis of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

27. A line segment along x-axis of length 2a is conjugate axis of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$

28. A hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ doesnot intersect the x–axis

29. A circle whose diameter is transverse axis of hyperbola is auxiliary circle

30. Angle between asymptotes of hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\sec^{-1}(e)$

Match the following:

31.

Hyperbola	**Foci**
A) $\frac{(x-1)^2}{16}-\frac{(y-2)^2}{9}=1$	1) (1, –1), (–9, –1)
B) $\frac{(x+2)^2}{9}-\frac{(y-3)^2}{27}=1$	2) (6, 2), (–4, 2)
C) $\frac{(x+1)^2}{25}-\frac{(y+2)^2}{16}=1$	3) (4, 3), (–8,3)
D) $9x^2-16y^2+72x-32y-16=0$	4) $\left(-1\pm\sqrt{41},-2\right)$

32.

Hyperbola	**contre**
A) $4(x+3)^2-9(y-2)^2=36$	1) (2, 1)
B) $3(x-3)^2-4(y-1)^2=12$	2) (–3, 2)
C) $9(x-2)^2-5(y-1)^2=45$	3) (2, 1)
D) $x^2-4x-y^2-2y-8=0$	4) (3, 1)

LEVEL – II

Choose correct answer of the following:

1. The distance between the foci of the hyperbola $x^2-3y^2-9x-6y-11=0$ is ______

 a) 4 b) 6 c) 8 d) 10

2. The locus of the point $\left(\frac{e^t+e^{-t}}{2}, \frac{e^t-t^{-t}}{2}\right)$ is a hyperbola of eccentricity is _____

 a) $\sqrt{3}$ b) 3 c) $\sqrt{2}$ d) 2

3. If the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and the hyperbola $\frac{x^2}{4}-\frac{y^2}{b^2}=1$ coinside then $b^2=$___

 a) 4 b)5 c) 8 d) 7

4. The values of m for which the line $y=mx+2$ become a tangent to the hyperbola $4x^2-9y^2=36$ is _____

 a) $\pm 2/3$ b) $\pm\frac{2\sqrt{2}}{3}$ c) $\pm\frac{8}{9}$ d) $\pm\frac{4\sqrt{2}}{3}$

5. Equation of one of the tangents passes through (2, 8) to the hyperbola $5x^2-y^2=5$ is

 a) $3x+y-14=0$ b) $3x-y+2=0$

 c) $x+y+3=0$ d) $x-y+6=0$

6. The product of the perpendicular distance from the foci on any tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is _____

 a) b^2 b) a^2 c) $2a^2$ d) $2b^2$

7. The equation of the common tangent to the curve $y^2=8x$ and $xy=-1$ is

 a) $y=2x+1$ b) $2y=x+6$ c) $y=x+2$ d) $3y=8x+2$

8. The angle between the symptotes of the hyperbola $x^2-3y^2=3$ is ____

 a) $\frac{\pi}{3}$ b) $\frac{\pi}{5}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{7}$

9. Equation of hyperbola passes through (2, 3) and has the asymptotes $4x+3y-7=0$ and $x-2y-1=0$ is ____

 a) $4x^2+5xy-6y^2-11x+11y+50=0$ b) $4x^2+5xy-6y^2-11x+11y-43=0$

 c) $4x^2-5xy-6y^2-11x+11y+50=0$ d) $x^2-5xy-y^2-11x+11y-43=0$

10. The curve represented by $x = a(\cosh\theta + \sin\theta),\ y = b(\cosh\theta - \sinh\theta)$ is ___

a) a hyperbola b) an ellipse c) a parabola d) a circle

LEVEL – III

1. A circle and rectangular hyperbola xy =1 cuts at $(x_3,\ y_3), r = 1,2,3,4$ then

$x_1x_2x_3x_4$ =____

a) –1 b) 0 c) 1 d) none

2. The midpoint of the chord $4x - 3y = 5$ of the hyperbola $2y^2 - 3y^2 = 12$ is____

a) $\left(0, \frac{-5}{3}\right)$ b) $(2, 1)$ c) $\left(\frac{5}{4}, 0\right)$ d) $\left(\frac{5}{4}, 2\right)$

3. If x = 9 is a chord of contact of the hyperbola $x^2 - y^2 = 9$ then the equation of the tangent at one of the points of contact is ______

a) $x + \sqrt{3}y + 2 = 0$ b) $3x + 2\sqrt{2}y - 3 = 0$

c) $3x - \sqrt{2}y + 6 = 0$ d) $x - \sqrt{3}y + 2 = 0$

4. If the circle $x^2 + y^2 = a^2$ intersects the hyperbola $xy = c^2$ in four points (x_i, y_i), $i = 1,2,3,4$ then $y_1 + y_2 + y_3 + y_4 =$____

a) 0 b) c c) a d) c^4

5. A : The equation of the normal to the hyperbola $x^2 - 4y^2 = 5$ at (3, –1) is $4x - 3y = 15$

R : The equation of the normal to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at $(x_1,\ y_1)$ is

$$\frac{a^2x}{x_1} + \frac{b^2y}{y_1} = a^2 + b^2$$

a) Both A and R are true and R is correct explanation of A

b) Both A and R are true and R is not correct explanation of A

c) A is true but R is false d) A is false but R is true

6. A hyperbola passes through a focus of the ellipse $\frac{x^2}{169} + \frac{y^2}{25} = 1$ Its transverse and conjugate axes coincide respectively with the major and minor axes of the ellipse. The product of the eccentricity is 1. Then equation of hyperbola is _______

a) $g^2 + f^2 - c = 2$ b) $x^2 + y^2 + 4x + 6y - 1 = 0$

c) $x^2 + y^2 + 4x + 6y + c = 0$ d) $x^2 + y^2 - 6x - 4y - 12 = 0$

Key for Level – I (Hyperbola)

1.b	2.c	3.a	4.b	5.c
6. a	7. c	8. c	9. c	10. b
11.conjugate hyperbola	12. $x^2+y^2+4x+6y+$	13. $x^2+y^2-6x-4y-$	14. $x^2+y^2-6x-4y-$	15. $L_1=0,\ L_2=0, L$
16. 5y+9=0	17. (–2, 1)	18. $L_1=0,\ L_2=0, L_3=$	19. $L_1=0,\ L_2=0, L_3=$	20. $9x^2-25y^2=22$
21. T	22. F	23. T	24. T	25. F
26. T	27. T	28. F	29. T	30. T
31. b,c,d,a	32. b, d, c, a			

Key for Level – II

1.c	2.c	3.b	4.b	5.b
6.a	7.c	8.c	9. c	10. a

Key for Level – III

1.c	2.b	3.b	4.a	5.a	6.b

MATHEMATIC WORKBOOK

INDEFINITE INTEGRATION

Level I (IPE)

1. If $\frac{d}{dx}\left(F(x)\right)=2x$ then what is $F(x)$

a) x^3 b) x^4-3 c) x^2+c d) All of the above

2. If $\int 2x\,dx = x^2+c$ then what is $\int x\,dx$

3. $\int \cos(x)\,dx=$

4. $\int \sec^2(x)\,dx=$

5. If $\int \frac{1}{x}dx=$

a) $\log(x)$ b) $\log(x)+c$ c) $\log(|x|)$ d) $\log(|x|)+c$

6. $\int e^{\log\left(1+\cot^2(x)\right)}dx=$

7. $\int (ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{(n+1)a}$ then what is $\int (3x+7)^{10}dx$

8. $\int \cos ec^2(5x+9)\,dx=$

9. If $\int \frac{f'(x)}{f(x)}dx=\log(|f(x)|)$ then what is $\int \tan(x)\,dx$

a) $\log\left(\sec^2(x)\right)+c$ b) $\log\left(\cot^2(x)\right)+c$

c) $\log\left(\sec(x)\right)+c$ d) $\log\left(\cot(x)\right)+c$

10. $\int \frac{\cos\left(\log(x)\right)}{x}dx,\ x>0=$

11. $\int \frac{1}{x\log(x)}dx,(x>1)=$

a) $\log\left(|\log(x)|\right)+c$ b) $\frac{1}{\log(|x|)}+c$

c) $\log(|x|)+c$ d) $\left(\log(x)\right)^2+c$

12. $\int \frac{(1+x)e^x}{\cos^2\left(xe^x\right)}dx =$

a) $\cos^2\left(xe^x\right)+c$ b) $\sec^2\left(xe^x\right)+c+$

c) $\tan\left(xe^x\right)+c$ d) $\sin\left(xe^x\right)+c$

13. $\int \frac{\sin^4(x)}{\cos^6(x)}dx =$

a) $\frac{\cos^6(x)}{7}+c$ b) $\frac{\tan^5(x)}{5}+c$

c) $\frac{\sin^4(x)}{7}+c$ d) $\frac{\sec^4(x)}{5}+c$

14. $\int \frac{x^8}{1+x^{18}}dx =$

15. $\int \sec(x)\log\left(\sec(x)+\tan(x)\right)dx =$

16. $\int \frac{1}{\sqrt{a^2-x^2}}dx =$

17. $\int \frac{1}{\sqrt{1-4x^2}}dx =$

18. $\int \frac{3}{\sqrt{9x^2-1}}dx =$

19. $\int \frac{1}{(x+1)(x+2)}dx =$

20. $\int \frac{1}{4\sin^2(x)+9\cos^2(x)}dx =$

21. $\int e^x\left(\tan^{-1}+\frac{1}{1+x^2}\right)dx =$

a) $e^x\tan^{-1}(x)+c$ b) $e^x\frac{1}{1+x^2}+c$

c) $e^x\cot^{-1}(x)+c$ d) $e^x\left(1+x^2\right)+c$

22. $\int \frac{xe^x}{(x+1)^2}dx =$

a) $\log(x+1)+c$ b) $\frac{e^x}{1+x}+c$ c) xe^x+c d) $\frac{e^x}{1+x^2}+c$

23. $\int \cos(\log(x))\,dx =$

24. $\int xe^{x}dx =$

25. $\int \log(x)\,dx =$

a) $\frac{1}{x}+c$ b) $x\log(x)-x+c$ c) $\log(x)+c$ d) $\frac{(\log(x))^2}{2}+c$

26. $\int \sin^{-1}(x)\,dx =$

27. If $\int \sin^{n}(x)\,dx = I_n$ and if $I_n = \frac{-\sin^{n-1}(x)\cos(x)}{n}+\frac{n-1}{n}I_{n-2}$ what is $\int \sin^{4}(x)\,dx$

28. If $I_n = \int \tan^{n}(x)\,dx$ and $I_n = \frac{\tan^{n-1}(x)}{n-1}-I_{n-2}$ what is $\int \tan^{6}(x)\,dx$

29. $\int \frac{1}{1+\cos^{2}(x)}dx =$

30. $\int \frac{1}{1+\sin(2x)}dx =$

Level 2 (EAMCET)

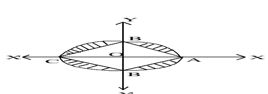
31. If $\int \frac{x^{49}\tan^{-1}(x^{50})}{1+x^{100}}dx = k\left(\tan^{-1}(x^{50})\right)^2+c$ then k =

a) $\frac{1}{50}$ b) $\frac{-1}{50}$ c) $\frac{1}{100}$ d) $\frac{-1}{100}$

32. If $\int \frac{\sin(x)}{\sin(x-\alpha)}dx = Ax+B\log\sin(x-\alpha)+c$ then the value of (A, B) is

a) $(\sin(\alpha),\cos(\alpha))$ b) $(-\cos(\alpha),\ \sin(\alpha))$

c) $(\cos(\alpha),\sin(\alpha))$ d) $(-\sin(\alpha),\ \cos(\alpha))$

33. If  then $f(x)=$

a) $2\log(e^{x}+1)$ b) $\log(e^{2x}-1)$

c) $2\log(e^{x}+1)-x$ d) $\log(e^{2x}+1)$

34. $\int \frac{3^{x}}{\sqrt{9^{x}-1}}dx =$

a) $\frac{1}{\log 3}\log\left(\left|3^{x}+\sqrt{9^{x}-1}\right|\right)+c$ b) $\frac{1}{\log 3}\log\left(\left|3^{x}-\sqrt{9^{x}-1}\right|\right)+c$

c) $\frac{1}{\log 9}\log\left(\left|3^x+\sqrt{9^x-1}\right|\right)+c$ d) $\frac{1}{\log 9}\log\left(\left|9^x+\sqrt{9^x-1}\right|\right)+c$

35. If $\int \frac{7x^8+8x^7}{\left(1+x+x^8\right)^2}dx = f(x)+c$ then what is $f(x)$

a) $\frac{x^8}{1+x+x^8}$ b) $28\log\left(1+x+x^8\right)$ c) $\frac{1}{1+x+x^8}$ d) $\frac{-1}{1+x+x^8}$

36. $\int\left(\sqrt{\frac{a+x}{a-x}}+\sqrt{\frac{a-x}{a+x}}\right)dx =$

a) $2\sin^{-1}\left(\frac{x}{a}\right)+c$ b) $2a\sin^{-1}\left(\frac{x}{a}\right)+c$

c) $2\cos^{-1}\left(\frac{x}{a}\right)+c$ d) $2a\cos^{-1}\left(\frac{x}{a}\right)+c$

37. $\int\frac{dx}{\sqrt{x}(x+a)} =$

a) $\frac{2}{3}\tan^{-1}\left(\sqrt{x}\right)+c$ b) $\frac{2}{3}\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)+c$

c) $\tan^{-1}\left(\sqrt{x}\right)+c$ d) $\tan^{-1}\left(\frac{\sqrt{x}}{3}\right)+c$

38. $\int\frac{dx}{a^2\sin^2(x)+b^2\cos^2(x)} =$

a) $\frac{1}{ab}\tan^{-1}\left(\frac{a\tan(x)}{b}\right)+c$ b) $\tan^{-1}\left(\frac{a\tan(x)}{b}\right)+c$

c) $\frac{1}{ab}\tan^{-1}\left(\frac{b\tan(x)}{a}\right)+c$ d) $\tan^{-1}\left(\frac{b\tan(x)}{a}\right)+c$

39. If $\int\frac{x-\sin(x)}{1+\cos(x)}dx = x\tan\left(\frac{x}{2}\right)+p\log\left(\left|\sec\left(\frac{x}{2}\right)\right|\right)+c$ then p =

a) –4 b) 4 c) 2 d) –2

40. If $\int\sin^{-1}\left(\frac{2x}{1+x^2}\right)dx = f(x)-\log\left(1+x^2\right)+c$ then $f(x)=$

a) $2x\tan^{-1}(x)$ b) $-2x\tan^{-1}(x)$ c) $x\tan^{-1}(x)$ d) $-x\tan^{-1}(x)$

41. $\int e^{\sqrt{x}}dx =$

a) $2x\tan^{-1}(x)$ b) $-2x\tan^{-1}(x)$ c) $x\tan^{-1}(x)$ d) $-x\tan^{-1}(x)$

42. $\int\left(\frac{\log(x)-1}{1+(\log(x))^2}\right)dx =$

a) $\frac{\log(x)}{(\log(x))^2+1}+c$ b) $S'P-SP=26$

c) $\sqrt{2}$ d) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

Level 3 (IIT)

43. $y=mx+c$

a) $c^2=a^2m^2+b^2$ b) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

c) $\frac{x^2}{a^2}-\frac{y^2}{b^2}+1=0$ d) None of the above

44. If $\int\frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}dx = Ax+B\log(9e^{2x}-4)+c$ then A =

a) $\frac{-3}{2}$ b) $\frac{3}{2}$ c) $\frac{-2}{3}$ d) $\frac{2}{3}$

45. If $\int\frac{dx}{(x^2+a^2)^2}=\frac{1}{ka^2}\left(\frac{x}{x^2+a^2}+\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right)+c$ then then value of k =

a) 1 b) 2 c) 3 d) 4

46. The value of $(x-1)^2+(y-2)^2=4$ is

a) $(x-5)^2+(y-4)^2=5^2$ b) $(x-5)^2+(y-4)^2=4^2$

c) $x-\log\left(\left|\sin\left(x-\frac{\pi}{4}\right)\right|\right)+c$ d) $x-\log\left(\left|\cos\left(x-\frac{\pi}{4}\right)\right|\right)+c$

47. **Match the following:**

Column I	**Column – II**
A) If $I=\int\frac{\sin(x)-\cos(x)}{\lvert\sin(x)-\cos(x)\rvert}dx$ where $\frac{\pi}{4}<x<\frac{3\pi}{8}$ then I is equal to	p) $\sin(x)$
B) If $\int\frac{x^2}{(x^3+1)(x^3+2)}dx=\frac{1}{3}f\left(\frac{x^3+1}{x^2+1}\right)+c$ then $f(x)$ is equal to	q) $x+c$

C) If $\int \sin^{-1}(x)\cos^{-1}(x)\,dx =$ r) $\log_e(|x|)$

$$f^{-1}(x)\left(\frac{\pi}{2}x - xf'(x) - 2\sqrt{1-x^2}\right) + 2x + c$$

then $f(x)$ is equal to

D) If $\int \frac{dx}{xf(x)} = f(f(x)) + c$ 4) $\sin^{-1}(x)$

then $f(x)$ is equal to

Passage Type

Let $f: R \to R$ be a function and $f(x) = (x-1)(x+2)(x-3)(x-6) - 100$. If $g(x)$ is a polynomial of degree ≤ 3 such that $\int \frac{g(x)}{f(x)}dx$ does not contain any logarithmic function and $g(-2) = 10$, then

48. The equation $f(x) = 0$ has

a) all four distinct roots b) three distinct real roots

c) two real and two imaginary d) all four imaginary roots

49. The minimum value of $f(x)$ is

a) –136 b) –100 c) –84 d) –68

50. $\int \frac{g(x)}{f(x)}dx$ equals to

a) $\tan^{-1}\left(\frac{x-2}{2}\right) + c$ b) $\tan^{-1}\left(\frac{x-1}{1}\right) + c$

c) $\tan^{-1}(x) + c$ d) d) None of the above

More than one correct option

51. $\int e^{\sin^2(x)}\left(\cos(x) + \cos^2(x)\right)\sin(x)\,dx =$

a) $\frac{1}{2}e^{\sin^2(x)}\left(3 - \sin^2(x)\right) + c$

b) $e^{\sin^2(x)}\left(1 + \frac{1}{2}\cos^2(x)\right) + c$

c) $e^{\sin^2(x)}\left(3\cos^2(x) + 2\sin^2(x)\right) + c$

d) $e^{\sin^2(x)}\left(2\cos^2(x) + 3\sin^2(x)\right) + c$

52. If $I=\int\left(\sqrt{\tan(x)}+\sqrt{\cot(x)}\right)dx=f(x)+c$ then $f(x)=$

a) $\sqrt{2}\sin^{-1}\left(\sin(x)-\cos(x)\right)$

b) $\frac{\pi}{\sqrt{2}}-\sqrt{2}\cos^{-1}\left(\sin(x)-\cos(x)\right)$

c) $\sqrt{2}\tan^{-1}\left(\frac{\tan(x)-1}{\sqrt{2\tan(x)}}\right)$

d) None of the above

MATHEMATICS WORKBOOK

KEY

INDEFINITE INTERGRATION

Level 1(IPE)

1. (3)

2. $\frac{x^2}{2}+c$

3. $\sin(x)+c$

4. $\tan(x)+c$

5. (4)

6. $-\cot(x)+c$

7. $\frac{(3x+7)^{11}}{33}+c$

8. $\frac{-\cot(5x+9)}{5}+c$

9. (3)

10. $\sin(\log(x))+c$

11. (1)

12. (3)

13. (2)

14. $\frac{1}{9}\tan^{-1}\left(x^3\right)+c$

15. $\frac{1}{2}\left(\log\left(\sec(x)+\tan(x)\right)\right)^2+c$

16. $\sin^{-1}\left(\frac{x}{a}\right)+c$

17. $\frac{1}{2}\sin^{-1}(2x)+c$

18. $\cosh^{-1}(3x)+c$

19. $\log\left(\left|\frac{x+1}{x+2}\right|\right)+c$

20. $\frac{1}{6}\tan^{-1}\left(\frac{2}{3}\tan(x)\right)+c$

21. (1)

22. (2)

23. $\frac{x}{2}\left(\cos(\log(x))+\sin(\log(x))\right)+c$

24. xe^x-e^x+c

25. (2)

26. $x\sin^{-1}(x)+\sqrt{1-x^2}+c$

27. $-\frac{\sin^3(x)\cos(x)}{4}-\frac{3}{8}\sin(x)\cos(x)+\frac{3}{8}x+c$

28. $\frac{\tan^5(x)}{5}-\frac{\tan^3(x)}{3}+\tan(x)-x+c$

29. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\left(\frac{1}{\sqrt{2}}\right)\tan(x)\right)+c$

30. $-\frac{1}{1+\tan(x)}+c$

Level 2 (EAMCET)

31.(3)

32. (3)

33. (3)

34. (1)

35. (1)

36. (2)

37. (2)

38. (1)

39. (1)

40. (1)

41. (2)

42. (4)

Level 3 (IIT)

43. (1)

44. (1)

45. (2)

46. (2)

47. A–q, B–r, C–p, D–r

Passage Type

48. (3)

49. (3)

50. (1)

More than one correct option

51. (1) and (2)

52. (1), (2) and (3)

INTERMEDIATE MATHS WORKBOOK

MATHS –IIB

DEFINITE INTEGRALS

I. To write the following statements are True or Fals

1. If f is integrable ofn [a, b] and if there is a differentiable function F on [a, b] suchthat $F' = f$ Then $\int_a^b f(x)dx = F(b) - F(a)$

2. $\int_0^a f(x)dx = 0$

3. $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

4. If $f(x)$ is even that $\int_{-a}^{a} f(a-x)dx$

5. If $f(x)$ is odd then $\int_{-a}^{a} f(x) = 2\int_0^a f(x)dx$

6. If $f(2a-x) = f(x)$ then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$

7. If $f(2a-x) = -f(x)$ then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$

8. $\int_0^{\pi/2} \cos^n x dx = \int_0^{\pi/2} \sin^n x dx$

9. $\underset{n\to\infty}{Lt} \frac{1}{n}\sum_{i=1}^{n} f\left(\frac{i}{n}\right) = \int_0^1 f(x)dx$

10. $\int_0^P f(x)dx = \underset{n\to\infty}{Lt} \frac{1}{n}\sum_{i=1}^{np} f\left(\frac{i}{n}\right)$

Answers:

1. T 2. T 3. T 4. F 5. T 6. T 7. F
8. T 9. T 10. T

II.

1. $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+f(\cos x)}dx = \int_0^{\pi/2} \frac{f(\cos x)}{f(\sin x)+f(\cos x)}dx$

2. $\int_0^{\pi/2} \sin^4 dx \neq \int_0^{\pi/2} \cos^4 dx$

3. $\int_0^{\pi/2} \frac{1}{1+Tanx}dx = \int_0^{\pi/2} \frac{1}{1+\cos x}dx$

4. $\int_{-1}^{1} |x| dx \neq -\int_{-1}^{0} xdx + \int_0^1 xdx$

5. $\int_0^4 |2-x| dx = \int_0^2 |2-x| dx + \int_2^4 |2-x| dx$

KEY

1. T 2. F 3. T 4. F 5. T

MATCHING I

	Group A		Group B
1.	If $f(x)$ is integrable on[0, a] then $\int_0^a \frac{f(x)}{f(x)+f(a-x)}dx$	[ ]	a) $\frac{1}{2}$
2.	$\int_0^{2a} \frac{f(x)}{f(x)+f(2a-x)}dx$	[ ]	b) a
3.	$\int_0^b \frac{f(x)}{f(x)+f(a+b-x)}dx$	[ ]	c) $\frac{b-a}{2}$
4.	$\int_3^6 \frac{\sqrt{x}}{\sqrt{a-x}+\sqrt{x}}dx$	[ ]	d) $\frac{3}{2}$
5.	$\int_0^1 \frac{f(x)}{f(x)+f(1-x)}dx$	[ ]	e) $\frac{a}{2}$

KEY

1. e 2. b 3. c 4. d 5. a

MATCHING II

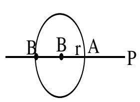
	Group A		Group B
1.	$\underset{n\to\infty}{Lt} \sum_{i=1}^{n} \frac{1}{n}\left[\frac{n-i}{n+i}\right]$	[ ]	a) $\frac{7}{5}$

2.	$\underset{n\to\infty}{Lt}\sum_{i=1}^{n}\frac{1}{n}\left[\frac{i-n}{i+n}\right]$	[ ]	b) $\frac{1}{6}$
3.	$\underset{n\to\infty}{Lt}\sum_{i=1}^{n}\frac{1}{n}\left[\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{6^n}\right]$	[ ]	c) $\log 5$
4.	$\underset{n\to\infty}{Lt}\sum_{i=1}^{n}\frac{1}{n}\left[\frac{1}{n+1}+\frac{1}{n+2}+.....+\frac{1}{5^n}\right]$	[ ]	d) $\log 6$
5.	$\underset{n\to\infty}{Lt}\left[\frac{1+2^4+3^4+....+n^4}{n^5}\right]$	[ ]	e) $-1+2\log 2$
			f) $1-2\log 2$
			g) $2\log 2+1$

KEY

1. e 2. f 3. d 4. c 5. a

MATCHING III

	Group A		Group B
1.	$\int_0^{\pi/2}\sin^{10}x\,dx$	[ ]	a) $\frac{16}{35}$
2.	$\int_0^{\pi/2}\cos^{8}x\,dx$	[ ]	b) $\frac{3}{16}\pi$
3.		[ ]	c) $\frac{256}{693}$
4.	$\int_0^{\pi/2}\sin^{4}x\,dx$	[ ]	d) $\frac{256}{693}\pi$
5.	$\int_0^{\pi/2}\cos^{11}x\,dx$	[ ]	e) $\frac{63}{512}\pi$
			f) $\frac{35}{256}\pi$
			g) $\frac{35}{256}$

KEY

1. e 2. f 3. a 4. b 5. c

MULTIPLE CHOICE

1. $\int_2^3\frac{2x}{1+x^2}dx$

a) 1 b) 0 c) 2 d) $\log 2$

2. $\int_{-1}^{1} e^{-x}\,dx$

a) $e-\frac{1}{e}$ b) $\frac{1}{e}-e$ c) $e+\frac{1}{e}$ d) 0

3. $\int_{0}^{2} |1-x|\,dx$

a) 0 b) 1 c) 2 d) –1

4. $\int_{1}^{5} \frac{dx}{\sqrt{2x-1}}$

a) 1 b) 5 c) 2 d) 6

5. $\int_{0}^{1} \frac{x^2}{x^2+1}\,dx$

a) $1-\frac{\pi}{4}$ b) $\frac{\pi}{4}-1$ c) $\frac{\pi}{4}+1$ d) $-\frac{\pi}{4}-1$

6. $\int_{0}^{\pi/2} \frac{e^{\cos x}}{e^{\cos x}+e^{\sin x}}\,dx$

a) 2π b) π c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

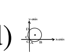
7. $\int_{0}^{\pi/2} \frac{2000\sin x+200\cos x}{\cos x+\sin x}\,dx$

a) 2200π b) 1100π c) 550π d) 1800π

8. $\int_{0}^{\pi/2} \frac{dx}{1+\tan^n x}$

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c)  d) 2π

9. $\int_{0}^{\pi/2} \frac{1}{1+\cot x}\,dx$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) π d) 

10. $\int_{0}^{\pi/2} \frac{a\sec x+b\,\mathrm{cosec}\,x}{\sec x+\mathrm{cosec}\,x}\,dx$

a) $(a+b)\frac{\pi}{2}$ b) $(a+b)\frac{\pi}{4}$ c) $(a+b)\pi$ d) $(a-b)\frac{\pi}{4}$

11. $\underset{n\to\infty}{Lt}\left[\frac{1}{n}\left[\sqrt{1+\frac{1}{n}}+\sqrt{2+\frac{2}{n}}+......+\sqrt{1+\frac{n}{4}}\right]\right]$

a) $\frac{2}{3}\left(2^{2/3}-1\right)$ b) $\frac{2}{3}\left(2^{2/3}-1\right)$ c) $\frac{3}{2}\left(2^{3/2}-1\right)$ d)

12. $\int_0^1 \sqrt{1+x}dx$

a) $\frac{3}{2}\left(2^{3/2}-1\right)$ b) $\frac{2}{3}\left(2^{3/2}-1\right)$ c) $\frac{2}{3}\left(2^{2/3}-1\right)$ d) $\frac{2}{3}\left(1-2^{2/3}\right)$

13. $\underset{n\to\infty}{Lt}\sum\frac{i^3}{i^4+n^4}=k\log 2$ where k =

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) 1

14. $\underset{n\to\infty}{Lt}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right).....\left(1+\frac{n^2}{n^2}\right)\right]^{1/n}=k,\ k=$

a) $e^{\pi-4}$ b) $e^{\frac{\pi-4}{2}}$ c) $2e^{\frac{\pi-4}{2}}$ d) $2e^{\frac{4-\pi}{2}}$

15. $\underset{n\to\infty}{Lt}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}=k,\ k=$

a) e b) $\frac{1}{e}$ c) 1 d) –1

16. $\underset{n\to\infty}{Lt}\frac{2^k+4^k+6^k+......+(2n)^k}{n^{k+1}}=l,\quad l=$

a) $\frac{2}{k+1}$ b) $\frac{2^k}{k}$ c) $\frac{2^{k+1}}{k+1}$ d) $\frac{2^k}{k+1}$

17. $\underset{n\to\infty}{Lt}\frac{3^k+6^k+9^k+.....+(3n)^k}{n^k+1}=l$

a) $\frac{3}{k+1}$ b) $\frac{3^k}{k}$ c) $\frac{3^k+1}{k+1}$ d) $\frac{3^k}{k+1}$

18. $\underset{n\to\infty}{Lt}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2^2}}+\frac{1}{\sqrt{n^2+2^2}}+....+\frac{1}{\sqrt{n^2+n^2}}\right]$

a) $\log\left(1+\sqrt{2}\right)$ b) $\log\left(1-\sqrt{2}\right)$ c) $\log\left(\sqrt{2}-1\right)$ d) 0

19. $\int_0^1 \sqrt{\frac{1+x}{1-x}}dx$

a) $\frac{\pi}{2}$ b) 2π c) $\frac{\pi}{2}-1$ d) $\frac{\pi}{2}+1$

20. $\underset{n\to\infty}{Lt} \frac{1}{n}\sqrt{\frac{n+i}{n-i}}$

a) $\frac{\pi}{2}$ b) 2π c) $\frac{\pi}{2}-1$ d) $\frac{\pi}{2}-1$

21. In $[0, \pi]$ Area between the curve $f(x)=\sin x$ and x–axos

a) 2 b) 4 c) 1 d) 3

22. The area enclosed within the curves $|x|+|y|=1$

a) 1 b) 2 c) 1 d) 3

23. The area of the region enclosed by $y=e^x, y=x, x=0, x=1$

a) e –1 b) e –2 c) $e-\frac{1}{2}$ d) $e-\frac{3}{2}$

24. Area bounded between the curves $y^2=4ax, x^2=4bx$

a) $\frac{16}{3}ab$ b) $16ab$ c) $4ab$ d) $8ab$

25. Area bounded between the curves $y=x^2, y=\sqrt{x}, x\ge 0$

a) $\frac{16}{3}$ b) $\frac{4}{3}$ c) $\frac{1}{3}$ d) 8

26. Area bounded between the curves $y^2=4x,\ x^2=4y$

a) 16 b) $\frac{16}{3}$ c) $\frac{8}{3}$ d) 8

27. Area of the region bounded by $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

a) π b) πab c) $\frac{(\pi-2)}{4}ab$ d) $(\pi-c)ab$

28. Area of the region bounded by $9x^2+16y^2=144$

a) 144 b) 144π c) 12π d) 12

29. Area bounded by the curve $y=\ln x$, x–axis and the straight line x = e

a) 0 b) 1 c) e d) $\frac{1}{e}$

30. $\int_{-\pi/2}^{\pi/2} \sin|x|dx$

a) 0 b) 1 c) 2 d) π

31. $\underset{n\to\infty}{Lt}\left[\frac{n}{n^2+1}+\frac{n}{n^2+2^2}+.....+\frac{n}{2n^2}\right]=$

a) $\frac{\pi}{2}$ b) π c) $\frac{\pi}{4}$ d) 2π

32. $\underset{n\to\infty}{Lt}\left[\frac{1}{na}+\frac{1}{na+1}+\frac{1}{na+2}+......+\frac{1}{nb}\right]=$

a) $\log\frac{a}{b}$ b) $\log\frac{b}{a}$ c) $\log a$ d) $\log b$

33. $\underset{n\to\infty}{Lt}\left[\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)........\left(1+\frac{n^2}{n^2}\right)\right]^{1/4}=k$, k =

a) $e^{\pi-4}$ b) $e^{\frac{\pi-4}{2}}$ c) $2e^{\frac{\pi-4}{2}}$ d) e

34. $\underset{n\to\infty}{Lt}\frac{1}{n}\sum_{i=1}^{n}Tan\frac{\pi}{4}\left(\frac{i}{4}\right)$

a) $2\log 2$ b) $\log 2$ c) $2\pi\log 2$ d) $\frac{2}{\pi}\log 2$

35. If $\int_1^4 |x-3|\,dx = 2A+B$ then

a) $A=\frac{3}{2}, B=4$ b) $A=\frac{1}{2}, B=1$

c) $A=2, B=\frac{-3}{2}$ d) $A=1, B=1$

36. If $f(t)=\int_{-t}^{t}\frac{e^{-|x|}}{2}dx$ then $\underset{x\to\infty}{Lt} f(t)$

a) 1 b) $\frac{1}{2}$ c) 0 d) –1

37. $\int_0^{\pi/2}\left(\sin^{2020}x-\cos^{2020}x\right)dx=$

a) $\frac{1}{2020}$ b) $\frac{2020!}{(2020)^{2020}}$ c) $\frac{\pi}{2020}$ d) 0

38. $\int_0^{2\pi} |\cos x|\, dx =$

a) 3 b) 4 c) 5 d) 6

39. If $a < 0 < b$ then $\int_a^b \frac{|x|}{x} dx =$

a) $a-b3$ b) $b-a$ c) $a+b$ d) 0

40. $\int_0^b \sqrt{(x-a)(b-x)}dx$

a) $\frac{(a-b)^2}{4}\pi$ b) $\frac{(a-b)^2}{4}$ c) $\frac{(a-b)^2}{2}\pi$ d) $(a-b)^2 \pi$

41. $\int_a^b \sqrt{\frac{x-0}{b-x}}dx$

a) $\frac{\pi}{2}(b-a)$ b) π c) $\frac{\pi}{8}(b-a)^2$ d) 2π

42. $\int_a^b \frac{1}{\sqrt{(x-a)(b-x)}}dx$

a) $\frac{\pi}{2}(b-a)$ b) π c) $\frac{\pi}{8}(b-a)^2$ d) 2π

43. $\int_a^b \sqrt{(x-a)(b-x)}dx$

a) $\frac{\pi}{2}(b-a)$ b) π c) $\frac{\pi}{8}(b-a)^2$ d) 2π

44. $\int_4^8 \sqrt{(8-x)(x-4)}dx$

a) π b) 2π c) 3π d) 4π

45. $\int_3^7 \sqrt{\frac{7-x}{x-3}}dx$

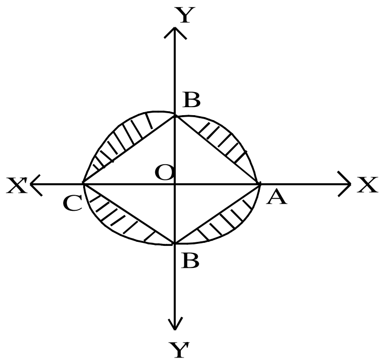
a) 4π b) 3π c) 2π d) π

46. $\int_4^9 \frac{dx}{\sqrt{(9-x)(x-4)}}$

a) 4π b) 3π c) 2π d) π

47. Observe the given figure and give the answers of $OA = a, OB = b,$ and

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ a>b$$



(i) In a 1st quadrant area of AOB

a) $\frac{\pi ab}{a}$ b) $ab(\pi-2)$ c) $\frac{ab(\pi-2)}{2}$ d) $\frac{1}{2}ab$

(ii)Area of the Δ le AOB

a) πab b) $ab(\pi-2)$ c) $\frac{ab(\pi-2)}{2}$ d) $\frac{1}{2}ab$

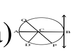
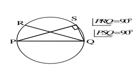
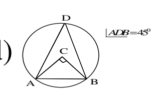
(iii)Shades area in Q_1

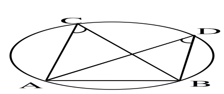
a) πab b) $ab(\pi-2)$ c) $ab\frac{(\pi-2)}{2}$ d) $\frac{ab}{4}(\pi-2)$

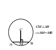
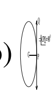
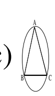
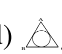
(iv)Total shaded area

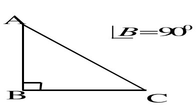
a) πab b)2ab   c) $ab(\pi-2)$ d) $\frac{ab}{4}(\pi-2)$

v)Total unshaded area

a)    b)2ab   c)    d) 

48. The value of 

a)    b)    c)    d) 

49. If 

then $I_2+I_4, I_3+I_5, I_4+I_6, ...$ are in

a) AP   b) GP   c) HP   d) none of these

50. If $I_n = \int_0^{\pi/4} \cot^n x dx$, $I_2 + I_4, I_3 + I_5, I_4 + I_6, ...$

a) AP b) GP c) HP d) none of these

51. If $I_n = \int_{-1}^{1} \frac{\cosh x}{1+e^{2x}} dx$

a)0 b) 1 c) $\frac{e^2-1}{2e}$ d) $\frac{e^2+2}{2e}$

52. $\int_0^{2\pi} x \sin^6 x \cos^5 x \, dx =$

a) 2π b) $\frac{\pi}{2}$ c) 0 d) –4

53. $I = \int_0^1 x(1-x)^n \, dx$ value

a) $\frac{1}{n+2}$ b) $\frac{1}{n+1} - \frac{1}{n+2}$ c) $\frac{1}{n+1} + \frac{1}{n+2}$ d) $\frac{1}{n+1}$

54. $\int_{-\pi/2}^{\pi/2} \sin^4 x \cos^6 x \, dx =$

a) $\frac{3\pi}{128}$ b) $\frac{\pi}{512}$ c) $\frac{3\pi}{512}$ d) none of these

55. $\int_0^{3\pi/2} |\cos x| dx =$

a) 3 b) 4 c) 5 d) 6

Answers:

1.d	2.a	3.b	4.c	5.a	6.d	7.c	8.b	9.a	10.b
11.a	12.b	13.c	14.c	15.b	16.d	17.d	18.a	19.c	20.c
21.a	22.b	23.d	24.a	25.c	26.b	27.b	28.c	29.b	30.c
31.c	32.b	33.c	34.d	35.c	36.c	37.d	38.b	39.b	40.a
41.a	42.b	43.c	44.b	45.c	46.s	47.(i) a, (ii) d, (iii) d (iv) c, (v) b			
48. b	49. c	50.a	51.c	52.c	53.b	54. b	55.a		

SYNOPSIS

I. Differential equation of different families of curves

Equation of a curve a,b,c are parameters	Differential equation $D^n y = yn = \frac{d^n y}{dx^n}$
1. $y = a e^{m_1 x}$	$(D - m_1) y = 0$
2. $y = ae^{m_1 x} + be^{m_2 x}$	$(D - m_1)(D - m_2) y = 0$
3. $y = ae^{m_1 x} + be^{m_2 x} + ce^{m_3 x}$	$(D - m_1)(D - m_2)(D - m_3) y = 0$
4. $y = (a + bx) e^{m_1 x}$	$(D - m_1)^2 y = 0$
5. $y = (a + bx + cx^2) e^{m_1 x}$	$(D - m_1)^3 y = 0$
6. $y = e^{\alpha x} (a \cos \beta x + b \sin \beta x)$	$\left[(D - \alpha)^2 + \beta^2\right] y = 0$
7. $y = a \cos \beta x + b \sin \beta x$	$(D^2 + \beta^2) y = 0$
8. $y = ax^m + bx^n$	$\left(x^2 D^2 - (m + n - 1) xD + mn\right) y = 0$
9. $y = cx + f(c)$ clairault's equation	$y = px + f(P)$ where $P = \frac{dy}{dx}$

II. Solution by Inspection.

1. $d(xy)$	$xdy + ydx$
2. $d\left(\frac{x}{y}\right)$	$\frac{ydx - xdy}{y^2}$
3. $d\left(\frac{y}{x}\right)$	$\frac{ydx - xdy}{x^2}$
4. $d(\log(xy))$	$\frac{xdy + ydx}{xy}$
5. $d\left(\log\left(\frac{x}{y}\right)\right)$	$\frac{ydx - xdy}{xy}$
6. $d\left(\log\left(\frac{y}{x}\right)\right)$	$\frac{xdy - ydx}{xy}$
7. $d\left(\tan^{-1}\left(\frac{y}{x}\right)\right)$	$\frac{xdy - ydx}{x^2 + y^2}$
8. $d\left(\tan^{-1}\left(\frac{x}{y}\right)\right)$	$\frac{ydx - xdy}{x^2 + y^2}$
9. $d\left(\frac{1}{2}(x^2 + y^2)\right)$	$xdx + ydy$
10. $d(\log(x + y))$	$\frac{dx + dy}{x + y}$

III. Linear Differential Equation

1. $\frac{dy}{dx}+p(x)y=Q(x)$	Integration Factor (I.F) = $e^{\int pdy}$ Solution: y(I.F) = $\int Q.(IF)dx+c$
2. $\frac{dx}{dy}+p(y)x=Q(y)$	$I.F=e^{\int Pdy}$

IV. Bernoulli's Differential Equation.

$$\frac{dy}{dx}+P(x)y=Q(x)y^n \text{ ———(1)}$$

$$\Rightarrow y^{-n}\frac{dy}{dx}+P(x)y^{1-n}=Q(x) \text{ ———(2)}$$

Put $t=y^{1-n}\Rightarrow\frac{dt}{dx}=(1-n)y^{-n}\frac{dy}{dx}$

$$\therefore\frac{dt}{dx}+(1-n)p(x)\cdot t=(1-n)Q(x) \text{ ———(3)}$$

Now Eq(3) is a Linear Differential Equation.

V. Geometrical Application of Differential Equation.

Form a differential equation from a given geometrical problem of ten following formulae are useful to remember.

(i)Length of tangent (L_T)	$\left\lvert y\sqrt{1+\frac{1}{m^2}}\right\rvert$
(ii)Length of Normal (L_N)	$\left\lvert y\sqrt{1+m^2}\right\rvert$
(iii)Length of Sub-tangent (L_{ST})	$\left\lvert\frac{y}{m}\right\rvert$
(iv)Length of Sub-normal (L_{SN})	$\lvert ym\rvert$

Where y is the ordinate of the point; m is the slope of the tangent $\left(\frac{dy}{dx}\right)_P$

VI. Homogeneous Differential Equation.

Its general form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}=\phi\left(\frac{y}{x}\right)$

Method–I: put $y=\upsilon x\Rightarrow\frac{dy}{dx}=\upsilon+x\frac{d\upsilon}{dx}$

Substituting eq (2) in (1), we can solve (1)

Method–II:

Given equation (1) trans form into $Mdx + Ndy = 0$

If $\frac{dM}{dy} = \frac{dN}{dx}$ Then (1) is a Exact differential Equation. In this case solution of (1) is

$\int Mdx + \int (terms\ of\ \ N\ not\ containing\ x)\,dy\ =\ $ C

(y–constant)

If (2) is not an Exact differential Equation

Then Equation (2) multiplying with I.F = $\frac{1}{Mx + Ndy}$ and find the solution by above formula.

V. LEVEL –I; QUESTION BANK

1. The order of the differential equation $\left(\frac{dy}{dz}\right)^3 + \left(\frac{dy}{dx}\right)^2 + y^4 = 0$ is

a) 1 b) 2 3) 3 d) 4

2. The order and degree of the differential equation $\left(1 + 3\frac{dy}{dx}\right)^{\frac{2}{3}} = 4\frac{d^3y}{dx^3}$ are

a) $\left(1, \frac{2}{3}\right)$ b) $(3, 1)$ c) $(3, 3)$ d) $(1, 2)$

3. The order and the degree of the differential equation : $y = px + \sqrt{a^2p^2 + b^2}$ where $p = \frac{dy}{dx}$ are respectively:

a) $(2, 1)$ b) $(1, 2)$ c) $(1, 1)$ d) $(2, 2)$

4. The degree of $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is

a) 1 b) 2 c) 3 d) can not be defined

5. The order of the D.E whose general solution is given by :

$y = (c_1 + c_2)\cos(x + c_3) - c_4e^x + c_5$

a) 5 b) 4 c) 3 d) 2

6. **Statement (I) :** The elimination of arbitrary constants from α, β and γ from $y = (\alpha + \beta + \gamma)x$ results in a D.E of order 3

Statement (II): The elimination of arbitrary constants α, β and γ from :

$y = \alpha x + \beta \sin x + \gamma e^x$ results in a D.E of order 3

a) I is true and II is false

b) I is false and II is false

c) I is true and II is true

d) I is false and II is true

7. Match the following family of curves with their differential equations.

(i) $y = Ae^{5x} + Be^{4x} (A, B)$	a) $y'' - 2py' + p^2 y = 0$
(ii) $y = e^x (A\cos x + B\sin x)(A, B)$	b) $y''' + y'' - 6y' = 0$
(iii) $y = a + be^{2x} + ce^{-3x} (a, b, c)$	c) $y'' - 9y' + 20 = 0$
(iv) $x^2 + y^2 = r^2$	d) $(x-h)^2 + (y-k)^2 = r^2$

8. The D.E of the family: $x^2 + y^2 = r^2$ of curves where a; b; c are arbitrary constants is:

a) $S = x^2 + y^2 + 2gx + 2fy + c = 0$ b) $y''' + 3y'' - 3y' - y = 0$

c) $y''' - 3y'' - 3y' + y = 0$ d) $y''' - 3y'' + 3y' - y = 0$

9. $y = Ae^x + Be^{2x} + Ce^{3x}$ satisfies the D.E

a) $y''' - 6y'' + 11y' - 6y = 0$ b) $y''' + 6y'' + 11y' + 6y = 0$

c) $y''' + 6y'' - 11y' + 6y = 0$ d) $y''' - 6y'' - 11y' + 6y = 0$

10. The D.E of the simple harmonic motion given by : $y - la - mb = 0$ is

a) $x^2 + y^2 - 4x = 0$ b) $\frac{d^2x}{dt^2} + n^2 x = 0$

c) $\frac{dx}{dt} - \frac{d^2x}{dt^2} = 0$ d) $\frac{d^2x}{dt^2} - \frac{dx}{dt} + nx = 0$

11. If $y = A\cos nx + B\sin nx;$ then $y_2 + n^2 y$ is equal to

a) 0 b) 1 c) y d) –1

12. The D.E which represents the family of curves $y = c_1 e^{c_2 x}$ where c_1 and c_2 are arbitrary constants:

a) $y'' = y'y$ b) $yy'' = y'$ c) $yy'' = (y')^2$ d) $y' = y^2$

13. Order of the D.E of the family of all concentric circles centred at (h, k) is:

a) 1 b) 2 c) 3 d) 4

12. Match the following family of curves with their differential Equations.

(i) All non-vertical lines in a plane a) $\frac{d^3y}{dx^3}=0$

(ii) All non-horizontal lines in a plane b) $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

(iiii) All parabolas whose axes are parallel to y–axis is c) $\frac{d^2x}{dy^2}=0$

(iv) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ d) $\frac{d^2y}{dx^2}=0$

13. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ then $\frac{d^2y}{dx^2}=$

a) $\frac{-b^4}{a^2y^3}$ b) $\frac{b^2}{ay^2}$ c) $\frac{-b^2}{a^2y^3}$ d) $\frac{b^3}{a^2y^2}$

14. The D.E corresponding to the family of circles having centres on X–axis and passing through the origin is:

(the D.E corresponding to the family of circles in the plane touching the y–axis at the origin is

a) $y^2+x^2+\frac{dy}{dx}=0$ b) $y^2-x^2+\frac{dy}{dx}=0$

c) $y^2+x^2+2xy\frac{dy}{dx}=0$ d) $y^2-x^2-2xy\frac{dy}{dx}=0$

15. The D.E for the family of circles $x^2+y^2=2ay=0$ where 'a' is an arbitrary constant is

a) $\left(x^2+y^2\right)y'=2xy$ b) $2\left(x^2+y^2\right)y'=xy$

c) $\left(x^2-y^2\right)y'=2xy$ d) $2\left(x^2-y^2\right)y'=xy$

16. The D.E of the family of parabolas with focus as the origin and the axis as x–axis is

a) $yy_1^2+4xy_1=4y$ b) $-yy_1^2=2xy_1-y$

c) $yy_1^2+y=2xyy_1$ d) $yy_1^2+2xyy_1+y=0$

17. The D.E whose solution is $y=ax^3+bx^2$

a) $x^2y_2-4xy_1+6y=0$ b) $x^2y_2-4y_1+6y=0$

c) $x^2y_2+4y_1+6y=0$ d) $xy_2=2xy$

18. If $y = a + bx^2$ where a, b are arbitrary constants then

a) $y_2 = 2xy$ b) $xy_2 = y_1$ c) $xy_2 + y_1 + y = 0$ d) $xy_2 = 2xy$

19. The D.E of the family of curves $x^2 = 4b(y+b); b \in R$ is:

a) $xy'' = y'$ b) $x(y')^2 = x + 2yy'$ c) $x(y')^2 = x - 2yy'$ d) $x(y')^2 = 2yy' - x$

20. The order of the D.E corresponding to the family of parabolas whose axes are along the x–axis and whose foci are at the origin is:

a) 4 b) 3 c) 2 d) 1

21. If c is a parameter, then the D.E. of the family of curves: $x^2 = c(y+c)^2$ is

a) $xy_1^3 + yy_1^2 - 1 = 0$ b) $xy_1^3 - yy_1^2 + 1 = 0$

c) $xy_1^3 + yy_1^2 + 1 = 0$ d) $xy_1^3 - yy_1 - 1 = 0$

22. The degree and order respectively of the D.E of the family of the curves represented by $y = \sqrt{c\left(x + \sqrt{c}\right)}$ are (c is a parameter)

a) 1, 3 b) 2, 3 c) 3, 1 d) 2, 2

23. The degree and order of the D.E of the family of parabolas whose axis is x–axis are respectively:

a) 2, 3 b) 2, 1 c) 3, 1 d) 2, 2

24. The D.E representing the family of circles of constant radius r is:

a) $r^2 y'' = \left(1 + (y')^2\right)^2$ b) $r^2 (y')^2 = \left(1 + (y')^2\right)^2$

c) $r^2 (y'')^2 = \left(1 + (y')^2\right)^3$ d) $(y'')^2 = r^2\left(1 + (y')^2\right)^2$

25. The D.E obtained by elimination the arbitrary constants a and b from : $xy = ae^x + be^{-x}$ is

a) $xy'' + 2y' - xy = 0$ b) $y'' + 2yy' - xy = 0$

c) $xy'' + 2y' + xy = 0$ d) $y'' + y' - xy = 0$

26. The D.E whose solution is $Ax^2 + By^2 = 1$ where A and B are arbitrary constants is of

a) second order and second degree

b) first order and second degree

c) first order and first degree

d) second order and first degree

LEVEL – II

27. If K and l respectively are the order and degree of the d.e whose general solution represents the family of circles of constant radius, then $k^2+l^2=$

a) 1 b) 2 c) 3 d) 4

28. If m and n are the order and degree of the D.E of the family of parabolas with focus at the origin and X–axis as its axis; then mn – m +n =

a) 1 b) 2 c) 3 d) 4

29. Consider the following D.E's

$D_1 : y = 4\frac{dy}{dx} + 3x\frac{dx}{dy}$

$D_2 : \frac{d^2y}{dx^2} = \left(3+\left(\frac{dy}{dx}\right)^2\right)^{\frac{4}{3}}$

$D_3 : \left(1+\left(\frac{dy}{dx}\right)\right)^2 = \left(\frac{dy}{dx}\right)^2$ The ratio of the sum of the orders of $D_1 : D_2$ and D_3 to the sum of their degree is:

a) 1 : 2 b) 1:1 c) 2 : 3 d) 4 : 7

30. The D.E of the family of parabolas with vertex at (0, –1) and having axis along the y–axis is

a) $yy'+2xy+1=0$ b) $xy'+y+1=0$ b

c) $xy'-2y-2=0$ d) $xy'-y-1=0$

31. If the order of a D.E $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+\sin\left(\frac{dy}{dx}\right)+y=0$ is 'l' and the degree of the D.E $\left(1+\frac{d^2y}{dx^2}\right)^{\frac{2}{3}} = \left(2-\left(\frac{dy}{dx}\right)^3\right)^{\frac{3}{2}}$ is m then the D.E corresponding to the family of curves: $y = Ax^l + Be^m$ where A and B are constants is

a) $\left(4x^2-2x\right)y''+\left(16x^2-2\right)y'+\left(32x-8\right)y=0$

b) $\left(2x^2-x\right)y''+\left(8x^2-2\right)y'+\left(16x-4\right)y=0$

c) $\left(2x^2-4x\right)y''-\left(8x^2-1\right)y'+\left(16x-4\right)y=0$

d) $\left(4x^2-2x\right)y''+\left(8x^2-1\right)y'+\left(16x-4\right)y=0$

32. If l and m are the degree and order respectively of the family of the D.E of the family of all circles in the XY–plane with radius 5 units then $2l+3m=$

a) 5 b) 10 c) 15 d) 7

33. The D.E corresponding to all the circles lying in the first quadrant and touching the coordinate axes is:

a) $(x-y)^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=\left(x+y\frac{dy}{dx}\right)^2$

b) $(x-y)^2\left[1+\frac{dy}{dx}\right]^2=\left(x+y\frac{dy}{dx}\right)^2$

c) $(x-y)^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=x+y\left(\frac{dy}{dx}\right)^2$

d) $(x-y)^2\left(1+\frac{dy}{dx}\right)=\left(x+y\frac{dy}{dx}\right)^{\frac{1}{2}}$

34. If c is a parameter; then the D.E whose solution is $y=c^2+\frac{c}{x}$ is:

a) $y=y_1^2-y_2$ b) $y=x^4y_1^2-xy_1$ c) $y=y_1^2-xy_1$ d) $y=xy_1-2x^2y_2$

35. A solution of the D.E $\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}+y=0$ is

a) y = 2 b) y = 2x c) y=2x–4 d) $y=2x^2-4$

36. If $x^2+y^2=1$ then

a) $yy''-2(y')^2+1=0$ b) $yy''+(y')^2+1=0$

c) $yy''+(y')^2-1=0$ d) $yy''+2(y')^2+1=0$

37. The number of arbitrary constants in the general solution of an nth order D.E is

a) 1 b) n –1 c) n d) n + 1

38. The differential equation of hyperbolas with co-ordinate axes as asymptotes is:

a) $xy_1-y=0$ b) $xy_1+y=0$ c) $y_1=xy$ d) $xy_1=y^2$

39. The D.E of the system of curves given by $\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1$ (λ parameter) is:

a) $x^2-xyy_1=a^2$ b) $x^2-\frac{xy}{y_1}=a^2$

c) $x^2+xy=a^2y_1$ d) $x^2-xy=a^2y_1$

LEVEL – I, QUESTION BANK

40. The solution of D.E $xdx + ydy = 0$ is:

a) $x^2 + y^2 = c$ b) $x^2 + xy = c$ c) $xy + y^2 = c$ d) $xy = c$

41. The solution of D.E $xdx + ydy = 0$ is:

a) $x^2 + y^2 = c$ b) $x^2 + xy = c$ c) $xy + y^2 = c$ d) $xy = c$

42. The solution of D.E $\frac{dy}{dx} = e^{x-y}$ is:

a) $e^x + e^y = c$ b) $e^x - e^y = c$ c) $e^{x-y} = c$ d) $e^{x+y} = c$

43. The general solution of $x\sqrt{1+y^2}dx + y\sqrt{1+x^2}dy = 0$ is

a) $\sqrt{1+x^2} + \sqrt{1+y^2} = c$ b) $\sqrt{1+x^2} - \sqrt{1+y^2} = c$

c) $\sinh^{-1} x + \sinh^{-1} y = c$ d) $\left(1+x^2\right)\left(1+y^2\right) = c$

44. The solution of $e^{x-y}dx + e^{y-x}dy = 0$ is

a) $e^{2x} - e^{2y} = c$ b) $e^{2x} + e^{2y} = c$ c) $e^x + e^y = c$ d) $e^{x-y} = c$

45. The solution of D.E $Ln\left(\frac{dy}{dx}\right) = ax + by$ is:

a) $be^{ax} + ae^{-by} = c$ b) $be^{ax} - ae^{-by} = c$

c) $be^{-ax} + ae^{-by} = c$ d) $be^{ax} - ae^{by} = c$

46. The solution of the D.E $\frac{dy}{dx} = \frac{xy+y}{xy+x}$ is:

a) $x + y = \log\left(\frac{cy}{x}\right)$ b) $x + y = \log(cxy)$

c) $x - y = \log\left(\frac{cx}{y}\right)$ d) $y - x = \log\left(\frac{cx}{y}\right)$

47. The general solution of D.E $\frac{dy}{dx} = 1 + x + y + xy$ is

a) $\log(1+x) = y + \frac{x^2}{2} + k$ b) $y = x + \frac{x^2}{2} + k$

c) $\log(1+y) = \frac{x^3}{3} + k$ d) $y = ke^{x+\frac{x^2}{2}} - 1$

48. Solution of $\frac{dy}{dx}=\frac{x\log x^2+x}{\sin y+y\cos y}$ is

a) $y\sin y=x^2\log x+c$ b) $y\sin y=x^2+c$

c) $y\sin y=x^2+\log x$ d) $y\sin y=x\log x+c$

49. The solution of $\frac{dy}{dx}=\left(\frac{x}{y}\right)^{-\frac{1}{3}}$ is

a) $x^{\frac{2}{3}}+y^{\frac{2}{3}}=c$ b) $y^{\frac{2}{3}}-x^{\frac{2}{3}}=c$ c) $x^{\frac{1}{3}}+y^{\frac{1}{3}}=c$ d) $y^{\frac{1}{3}}-x^{\frac{1}{3}}=c$

50. The solution of $xdx+ydy=x^2dx-xy^2dx$ is

a) $x^2-1=(1+y)^2c$ b) $x^2+1=c\left(1-y^2\right)$

c) $x^2-1=c\left(1-y^2\right)$ d) $x^2+1=c(1-y)$

51. The solution of $x^2+y^2\frac{dy}{dx}=4$ is

a) $x^2+y^2=12x+c$ b) $x^2+y^2=3x+c$

c) $x^2+y^2=8x+c$ d) $x^3+y^3=12x+c$

52. The general solution of the D.E $(y\sin x+y)\frac{dy}{dx}-\cos^2 x=0$ is:

a) $y^2=x-\cos x+c$ b) $y=1+\sin x+c$

c) $y^2=2x-2\sin x+c$ d) $y^2=2x+2\cos x+c$

53. Match the following differential equations with their solution:

(i) $2xy\frac{dy}{dx}=1+y^2$ a) $y=A\left(1+x^2\right)$

(ii) $\frac{dy}{dx}-\frac{2xy}{1+x^2}=0$ b) $x^y=c$

(iii) $\frac{dy}{dx}=\left(1+y^2\right)\left(1+x^2\right)^{-1}$ c) $y-x=c(1+xy)$

(iv) $yx^{y-1}dx+x^y\log xdy=0$ d) $1+y^2=Ax$

54. Which of the following statements is correct?

Statement(I): If $dy+2xydx=2e^{-x^2}dx$ then $ye^{x^2}=2x+c$

Statement(II): If $ye^{x^2}-2x=c$ then $dx=\left(2e^{-x^2}-2xy\right)dy$

a) Both I and II are true b) Neither I nor II is true

c) I is true, II is false d) I is false, II is true

55. If $dx+dy=(x+y)(dx-dy)$ then $\log(x+y)=$

a) $x+y+c$ b) $x+2y+c$ c) $x-y+c$ d) $2x+y+c$

56. Solution of the D.E : $ydx+(x+x^2y)dy=0$ is

a) $\log y=cx$ b) $\frac{-1}{xy}+\log y=c$ c) $\frac{1}{xy}+\log y=c$ d) $-\frac{1}{xy}=c$

57. The solution of the D.E $y^2(xdx+ydy)=\left(\sqrt{x^2+y^2}\right)(ydx-xdy)$ is: ____

58. The general solution of the D.E : $\cos(x+y)dy=dx$ is ____

a) $y=\sec(x+y)+c$ b) $y-\tan\left(\frac{x+y}{2}\right)=x+c$

c) $y=\tan\left(\frac{x+y}{2}\right)+c$ d) $y=\frac{1}{2}\tan(x+y)+c$

59. The solution of the D.E $\frac{dy}{dx}=1-\cos(y-x)\cot(y-x)$ is:

a) $x\tan(y-x)=c$ b) $x=\tan(y-x)+c$

c) $x=\sec(y-x)+c$ d) $x+\sec(y-x)=c$

60. If $\frac{dy}{dx}+2x\tan(x-y)=1$ then $\sin(x-y)=$

a) Ae^{-x^2} b) Ae^{2x} c) Ae^{x^2} d) Ae^{-2x}

61. The solution of $\tan y\frac{dy}{dx}=\sin(x+y)+\sin(x-y)$ is

a) $\sec y=2\cos x+c$ b) $\sec y=-2\cos x+c$

c) $\tan y=-2\cos x+c$ d) $\sec^2 y=-2\cos x+c$

62. The solution of the D.E : $\frac{dy}{dx}=\sin(x+y)\tan(x-y)$ is

a) $\cos ec(x+y)+\tan(x+y)=x+c$

b) $x+\cos ec(x+y)=c$

c) $x+\tan(x+y)=c$ d) $x+\sin(x+y)=c$

63. The solution of $\frac{dy}{dx}+1=e^{x+y}$ is:

a) $e^{-(x+y)}+x+c=0$ b) $e^{-(x+y)}-x+c=0$

c) $e^{x+y}+x+c=0$ d) $e^{x+y}-x+c=0$

64. The solution of $\frac{dy}{dx}=\tan^2(x+y)$ is:

a) $\tan(x+y)=x+c$ b) $2(x+y)+\sin(2x+2y)=4x+c$

c) $2(x+y)-\sin(2x+2y)=4x+c$ d) $\sec(x+y)=c$

65. If $\frac{dy}{dx}=y+3>0$ and $y(0)=2$, then $y(Ln2)=$

a) 5 b)13 c) –2 d) 7

66. If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$ then $y\left(\frac{\pi}{2}\right)=$

a) $\frac{4}{3}$ b) $\frac{1}{3}$ c) $-\frac{2}{3}$ d) $-\frac{1}{3}$

67. The solution of the D.E: $\frac{dy}{dx}=(4x+y+1)^2$ where $y(0)=1$ is

a) $y=2x^2-1-\frac{\pi}{8}$ b) $y=4x-\left(1+\frac{\pi}{8}\right)$

c) $y=2\tan\left(2x+\frac{\pi}{4}\right)-4x-1$ d) $y=2\tan\left(x+\frac{\pi}{8}\right)+4x-1$

68. The solution of $xdx+ydy+\frac{xdx-ydy}{x^2+y^2}=0$

a) $x^2+y^2+2\tan^{-1}\left(\frac{y}{x}\right)=c$ b) $x^2+y^2+2\tan^{-1}\left(\frac{x}{y}\right)=c$

c) $x^2+y^2-2\tan^{-1}\left(\frac{y}{x}\right)=c$ d) $x^2+y^2+\log(xy)=c$

69. Then solution of $\frac{xdy}{x^2+y^2}=\left(\frac{y}{x^2+y^2}-1\right)dx$ is

a) $y^2=x^3\tan(c-x)$ b) $y=x\cot(c-x)$

c) $y=x\tan(c-x)$ d) $y=x\cos(c-x)$

70. The equation of the curve through (1, 1) which satisfies the equation $(y+xy)dx+(x-xydy)=0$ is

a) $\log(xy)+x+y=0$ b) $\log(xy)+x+y+1=0$

c) $\log(xy)+x-y-1=0$ d) $\log(xy)+x-y=0$

71. The equation of the curve through $\left(0, \frac{\pi}{4}\right)$ satisfying the D.E

$e^x \tan y dx + \left(1+e^x\right)\sec^2 y dy = 0$ is given by

a) $\left(1+e^x\right)\tan y = 2$ b) $\left(1+e^x\right) = 2\tan y$

c) $1+e^x = 2\sec y$ d) $\left(1+e^x\right)\tan y = k$

72. The equation of the curve passing through $\left(\frac{\pi^2}{4}, 1\right)$ which has a solution of the

equation as $y^2 \cos\sqrt{x}dx - 2\sqrt{x}e^y dy = 0$ is

a) $\sin\sqrt{x} + e^{\frac{1}{y}} = e$ b) $\sin\sqrt{x} - e^{\frac{1}{y}} = 1-e$

c) $\sin\sqrt{x} + e^{\frac{1}{y}} = 1-e$ d) $\sin\sqrt{x} + e^{\frac{1}{y}} = e$

73. For the primitive integral equation : $\frac{d^2y}{dx^2} = e^{-2x}$ is:

a) $\frac{e^{-2x}}{4}$ b) $\frac{e^{-2x}}{4} + cx + d$ c) $\frac{1}{4}e^{-2x} + cx^2 + d$ d) $\frac{e^{-2x}}{4} + cx + d$

74. Let a solution $y = g\left(x\right)$ of the differential equation $x\sqrt{x^2-1}dy - y\sqrt{y^2-1}dx = 0$

satisfy $y\left(2\right) = \frac{2}{\sqrt{3}}$

Statement(I): $y\left(x\right) = \sec\left(\sec^{-1} x - \frac{\pi}{6}\right)$ and

Statement(II): $y\left(x\right)$ is given by $\frac{1}{y} = \frac{2\sqrt{3}}{x} - \sqrt{1-\frac{1}{x^2}}$

a) statement –I is true, statement – II is true

b) statement – I is true; statement – II is false

c) statement – I is false; statement – II is false

d) statement – I is false; statement – II is true

75. The equation of the curve passing through the origin and satisfying the

$\Delta E = \frac{dy}{dx} = \left(x-y\right)^2$ is:

a) $e^{2x}\left(1-x+y\right) = 1+x-y$ b) $e^{2x}\left(1+x-y\right) = 1-x+y$

c) $e^{2x}\left(1-x+y\right) = -\left(1+x+y\right)$ d) $e^{2x}\left(1+x+y\right) = 1-x+y$

76. If $y = g(x)$ satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right)dy = \left(\sqrt{4+\sqrt{9+x}}\right)^{-1} dx; x>0 \text{ on } g(0)=\sqrt{7}; \text{ then } y(256)=$$

a) 3 b) 9 c) 16 d) 80

77. The solution of the D.E: $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$ satisfying y = 1 when x = 1 is

a) $y\left(e^{x^3} - (1+2e)\right) - x = 0$ b) $y\left(e^{x^3} + (1-e)\right) + x = 0$

c) $y\left(e^{x^3} + (1+e)\right) - x = 0$ d) $y\left(e^{x^3} - (1+e)\right) + x = 0$

78. The solution of the D.E: $\frac{dx}{dy} + 2yx = 2y$ which passes through the point (2, 0) is:

a) $(x-1) = 2e^{y^2}$ b) $(x-1) = 2e^{-y^2}$

c) $(x-1) = e^{y^2}$ d) $(x-1) = e^{-y^2}$

79. Let $y = f(x)$ be a solution of the D.E $\sqrt{1-x^2}\frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1$.

If $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ then $y\left(-\frac{1}{2}\right) =$

a) $-\frac{1}{\sqrt{2}}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{\sqrt{2}}$ d) $\frac{\sqrt{3}}{2}$

80. The general solution of $y' + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is

a) $\log\left|\tan\left(\frac{y}{2}\right)\right| = c + \sin\left(\frac{x}{2}\right)$ b) $\log\left|\tan\left(\frac{y}{2}\right)\right| = c - 2\sin x$

c) $\log\left|\tan\left(\frac{y}{4}\right)\right| = c - 2\sin\left(\frac{x}{2}\right)$ d) $\log\left|\tan\left(\frac{y}{2}\right)\right| = c - 4\sin\left(\frac{x}{2}\right)$

81. If $f(x); f'(x); f''(x)$ are positive functions and $f(0)=1, f'(0)=2$ then the solution of the D.E $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$ is

a) e^{2x} b) $2\sin x + 1$ c) $\sin^2 x + 2x + c$ d) e^{4x}

82. The general solution of $\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}$ is:

a) $Tan^{-1}\left(\frac{y}{x}\right) + c = \log x$ b) $2Tan^{-1}\left(\frac{y}{x}\right) + c = \log x$

c) $Tan^{-1}\left(\frac{y}{x}\right)=\log cx$ d) $2Tan^{-1}\left(\frac{y}{x}\right)+c=c+x$

83. The general solution of $\frac{dy}{dx}=\frac{y^3+3x^2y}{2x^3}$ is:

a) $y^3=x$ b) $y^2=x\left(x^2+y^2\right)$ c) $y^3=cy\left(x^2+y^2\right)$ d) $y^2=cx\left(x^2+y^2\right)$

84. The general solution of $y^2+x^2\frac{dy}{dx}=xy\frac{dy}{dx}$ is:

a) $e^{\frac{x}{y}}=cx$ b) $e^{-\frac{y}{x}}=cy$ c) $e^{\frac{y}{x}}=cy$ d) $e^{-\frac{y}{x}}=cy$

85. The solution of $\left(x^2+y^2\right)dx=2xydy$ is:

a) $c\left(x^2-y^2\right)=x$ b) $c\left(x^2+y^2\right)=x$

c) $c\left(x^2-y^2\right)=y$ d) $c\left(x^2+y^2\right)=y$

86. The solution of $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is:

a) $e^{\frac{y}{x}}=kx$ b) $e^{\frac{y}{x}}=ky$ c) $e^{\frac{y}{x}}=kx$ d) $e^{-\frac{y}{x}}=k$

87. The general solution of $y^2dx+\left(x^2-xy+y^2\right)dy=0$ is

a) $Tan^{-1}\left(\frac{y}{x}\right)=\log y+c$ b) $2Tan^{-1}\left(\frac{x}{y}\right)+\log x+c=0$

c) $\log\left(y+\sqrt{x^2+y^2}\right)+\log y+c=0$ d) $\sinh^{-1}\left(\frac{x}{y}\right)+\log y+c=0$

88. The general solution of $xdy-ydx=\left(\sqrt{x^2+y^2}\right)dx$ is:

a) $x+\sqrt{x^2+y^2}=cy^2$ b) $y+\sqrt{x^2+y^2}=cx^2$

c) $y+\sqrt{x^2+y^2}=cx$ d) $x+\sqrt{x^2+y^2}=cy$

89. If $\frac{dy}{dx}=\frac{y+x\left(\tan\left(\frac{y}{x}\right)\right)}{x}$ then $\sin\left(\frac{y}{x}\right)$ is equal to

a) cx^2 b) cx c) cx^3 d) cx^4

90. The solution of the D.E: $\frac{dy}{dx}=\frac{y}{x}+\frac{\phi\left(\frac{y}{x}\right)}{\phi'\left(\frac{y}{x}\right)}$ is:

a) $x\phi\left(\frac{y}{x}\right)=k$ b) $\phi\left(\frac{y}{x}\right)=kx$ c) $y\phi\left(\frac{y}{x}\right)=k$ d) $\phi\left(\frac{y}{x}\right)=ky$

91. The general solution of the $D.E$ $yy'=x\left(\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right)$ where ϕ is an arbitrary function is:

a) $x\phi\left(\frac{y^2}{x^2}\right)=cy$ b) $x^2\phi\left(\frac{y^2}{x^2}\right)=c$

c) $x^2\phi\left(\frac{y^2}{x^2}\right)=cy^2$ d) $\phi\left(\frac{y^2}{x^2}\right)=cx^2$

92. The general solution of the D.E: $x^2 ydx=-\left(x^3+y^3\right)dy=0$ is

a) $y^3=3x^3\log cx$ b) $c\left(x^3-y^3\right)=x^2$

c) $\log|y|-\frac{x^3}{3y^3}=c$ d) $c^2\left(x^2-y^2\right)=y^2-x^2$

93. The general solution of the D.E$\left(x^3-3xy^2\right)dx=\left(y^3-3x^2y\right)dy$ is:

a) $c^2\left(x^2+y^2\right)=y^2-x^2$ b) $c^2\left(x^2+y^2\right)=\left(y^2-x^2\right)^2$

c) $c^2\left(x^2+y^2\right)^2=y^2-x^2$ d) $c^2\left(y^2-x^2\right)$

94. The solution of $\frac{dy}{dx}=\frac{x+y}{x-y}$ is

a) $Tan^{-1}\left(\frac{y}{x}\right)=\log\left(\sqrt{x^2+y^2}\right)+c$ b) $Tan^{-1}\left(\frac{y}{x}\right)=\log\left(\sqrt{x^2+y^2}\right)+c$

c) $\sin^{-1}\left(\frac{y}{x}\right)=\log\left(\sqrt{x^2+y^2}\right)+c$ d) $\cot^{-1}\left(\frac{y}{x}\right)=\log\left(\sqrt{x^2-y^2}\right)+c$

95. If $x\frac{dy}{dx}=y\left(\log y-\log x+1\right)$ then the solution of the D.E is:

a) $y\log\left(\frac{x}{y}\right)+cx$ b) $x\log\left(\frac{y}{x}\right)=cy$

c) $\log\left(\frac{y}{x}\right)=cx$ d) $\log\left(\frac{x}{y}\right)=cy$

96. The solution of the D.E $\frac{dy}{dx}=\frac{x+y}{x}$ satisfying the condition $y(1)=1$ os

a) $y=Lnx+x$ b) $y=xLnx+x^2$

c) $y=xe^{x-1}$ d) $y=xLn(x)+x$

97. A family of curves has the D.E $xy\frac{dy}{dx}=2y^2-x^2$ Then the family of curves is:

a) $y^2=cx^2+x^3$ b) $y^2=cx^4+x^3$

c) $y^2=x+cx^4$ d) $e^{-\frac{y}{x}}+\log x=1$

98. The general solution of $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$ is:

a) $ye^{\frac{y}{x}}+x=c$ b) $ye^{\frac{x}{y}}-x=c$ c) $ye^{\frac{x}{y}}+y=c$ d) $ye^{\frac{x}{y}}+x=c$

99. The solution of $x\frac{dy}{dx}=y+xe^{\frac{y}{x}}$ with $y(1)=0$ is

a) $e^{\frac{y}{x}}+\log x=1$ b) $e^{-\frac{y}{x}}=\log x$ c) $e^{-\frac{y}{x}}+2\log x=1$ d) $e^{-\frac{y}{x}}+\log x=1$

100. The solution of the D.E $3xy'-3y+\left(x^2-y^2\right)^{\frac{1}{2}}=0$ satisfying the condition $y(1)=1$ is

a) $3\cos^{-1}\left(\frac{y}{x}\right)=Ln|x|$ b) $3\cos\left(\frac{y}{x}\right)=Ln|x|$

c) $3\cos^{-1}\left(\frac{y}{x}\right)=2Ln|x|$ d) $3\sin^{-1}\left(\frac{y}{x}\right)=Ln|x|$

101. A curve passes through the point $\left(1,\frac{\pi}{6}\right)$. Let the slope of the curve at each point (x y) be $\frac{y}{x}+\sec\left(\frac{-y}{x}\right); x>0$ Then the equation of the curve is:

a) $\sin\left(\frac{y}{x}\right)=Ln(x)+\frac{1}{2}$ b) $\cos ec\left(\frac{y}{x}\right)=Ln(x)+2$

c) $\sec\left(\frac{2y}{x}\right)=Lnx+2$ d) $\cos\left(\frac{2y}{x}\right)=Ln+\frac{1}{2}$

102. If $\frac{dy}{dx}=\frac{xy}{x^2+y^2}; y(1)=1$; then a value of x satisfying $y(x)=e$ is:

a) $\sqrt{3}e$ b) $\frac{1}{2}\sqrt{3}e$ c) $\sqrt{2}e$ d) $\frac{e}{\sqrt{2}}$

103. The solution of the D.E $\left(2x^3 - xy^2\right)dx + \left(2y^3 - x^2y\right)dy = 0$ is:

a) $x^4 - x^2y^2 + y^4 = c$ b) $x^2 + x^2y^2 + y^2 = c$

c) $x^4 + x^2y^2 + y^4 = c$ d) $x^2 - x^2y^2 + y^2 = c$

104. The solution of D.E: $\frac{dy}{dx} = \frac{x-2y+1}{2x-4y}$ is:

a) $\left(x-2y\right)^2 + 2x = c$ b) $\left(x-2y\right)^2 + x = c$

c) $x - 2y + 2x^2 = c$ d) $x - 2y + x^2 = c$

105. The solution of the DE: $\left(2x-3y+5\right)dx + \left(9y-6x+7\right)dy = 0$ is:

a) $3x-3y+8\log\left|6x-9y-1\right| = c$ b) $3x-9y+8\log\left|6x-9y-1\right| = c$

c) $3x-9y+8\log\left|2x-3y-1\right| = c$ d) $3x-9y+4\log\left|2x-3y-1\right| = c$

106. The solution of the DE: $\left(2x+4y+3\right)\frac{dy}{dx} + \left(x-2y+1\right) = 0$ is

a) $\log\left(\left(2x-4y\right)+3\right) = x-2y+c$

b) $\log\left(2\left(2x-4y\right)+3\right) = 2\left(x-2y\right)+c$

c) $\log\left(2\left(x-2y\right)+5\right) = 2\left(x+y\right)+c$

d) $\log\left(4\left(x-2y\right)+5\right) = 4\left(x+2y\right)+c$

107. The solution of the DE $\left(\frac{x+y-1}{x+y-2}\right)\frac{dy}{dx} = \frac{x+y+1}{x+y+2}$ given that y = 1 when x = 1 is

a) $2\left(y-x\right)+\log\left(\frac{\left(x+y\right)^2-2}{2}\right) = 0$ b) $\log\left(\frac{\left(x+y\right)^2-2}{2}\right) = \left(x-y\right)^2$

c) $\log\left(\frac{\left(x-y\right)^2+2}{2}\right)+2\left(y-x\right) = 0$ d) $\left(x-y\right)+\log\left|\frac{\left(x+y\right)^2-2}{2}\right| = 0$

108. If the solution of the DE $\frac{dy}{dx} = \frac{3y-7x-3}{3x-7y+7}$ is $\left(y-x-\lambda\right)^m\left(y+x-\mu\right)^n = c$

(where λ, μ, m, n are constants), then match the following:

i) λ	a) 1
ii) μ	b) 2
iii) m	c) 7
iv) n	d)
v) c	e)

LEVEL – I; QUESTION BANK

109. If Integration factor of $\frac{dy}{dx}-\frac{2}{x^2}y=\frac{1}{x^3}$ is:

a) $-\frac{1}{x}$ b) x^2 c) $e^{\frac{2}{x^2}}$ d) $e^{\frac{2}{x}}$

110. If of $x\cos x\frac{dy}{dx}+\left(x\sin x+\cos x\right)y=1$ is

a) $x\cos x$ b) $x\sin x$ c) $x\sec x$ d) $x\cos ecx$

111. If of $x\cos x\frac{dy}{dx}+y\log x=e^x x^{\frac{1}{2}\log x}$ is

a) $e^{\frac{\log x}{2}}$ b) $x^{2\log x}$ c) $\frac{1}{2}(\log x)^2$ d) $\frac{1}{2}\log x$

112. If of $(x+y+1)\frac{dy}{dx}=1$ is:

a) e^{-x} b) e^x c) e^{-y} d) e^y

113. An integrating factor of $\left(1+y+x^2y\right)dx+\left(x+x^3\right)dy=0$ is:

a) e^x b) x^2 c) $\frac{1}{x}$ d) x

114. An I.F of $\left(1-x^2\right)\frac{dy}{dx}+xy=\frac{x^4}{\left(1+x^5\right)}\left(\sqrt{1-x^2}\right)^3$ is:

a) $\sqrt{1-x^2}$ b) $\frac{x}{\sqrt{1-x^2}}$ c) $\frac{x^2}{\sqrt{1-x^2}}$ d) $\frac{1}{\sqrt{1-x^2}}$

115. If of $\left(x+2y^3\right)\frac{dy}{dx}=y^2$ is:

a) $e^{\frac{1}{y}}$ b) $e^{-\frac{1}{y}}$ c) y d) $-\frac{1}{y}$

116. Assertion (A) : If of $\frac{dy}{dx}+y=x^2$ is e^x

Reason (R): If of $\frac{dy}{dx}+p(x)y=Q(x)$ is, $e^{\int pdx}$

a) Both (A) and (R) are true and R is the correct explanation of (A)

b) Both (A) and (R) are true but (R) is not the correct explanation of (A)

c) A is true; R is false

d) A is false; R is true

117. The solution of $\frac{dy}{dx}+\frac{1}{3}y=1$ is:

a) $y=3+ce^{\frac{x}{3}}$ b) $y=3+ce^{-\frac{x}{3}}$ c) $3y=c+e^{\frac{x}{3}}$ d) none

118. The general solution of $\frac{dy}{dx}+y\tan x=2x+x^2\tan x$ is

a) $y-x^2=c\sec x$ b) $y\cos x=x^2\sec x+c$

c) $y\sec x=x^2+\cos x$ d) $y=x^2+c\cos x$

119. $y+x^2=\frac{dy}{dx}$ has the solution

a) $y+x^2+2x+2=ce^x$ b) $y+x+2x^2+2=ce^x$

c) $y+x+x^2+2=ce^{2x}$ d) $y+x+x^2+2=ce^{2x}$

120. The solution of $\frac{dy}{dx}+y=e^x$ is

a) $2y=e^{2x}+c$ b) $2ye^{2x}=e^x+c$

c) $2ye^x=e^{2x}+c$ d) $2ye^x=2e^x+c$

121. The solution of $\left(1+x^2\right)\frac{dy}{dx}+2xy-4x^2=0$ is

a) $3x\left(1+y^2\right)=4y^3+c$ b) $3y\left(1+x^2\right)=4x^3+c$

c) $y=e^x\sin x+c$ d) $y\sin x=e^x+c$

123. If $-\frac{\pi}{4}<x<\frac{\pi}{4}$; then the general solution of the D.E $\cos^2 x\frac{dy}{dx}-\left(tam2x\right)y=\cos^4 x$ is

a) $y=\frac{1}{2}\left(\frac{\tan 2x+c}{1-\tan^2 x}\right)$ b) $y=\frac{1}{2}\left(\frac{\cos 2x+c}{1-\tan^2 x}\right)$

c) $y=\frac{1}{2}\left(\frac{\sin 2x+c}{1-\tan^2 x}\right)$ d) $y=\frac{1}{2}\left(\frac{\sin x+c}{1-\tan^2 x}\right)$

124. The D.E $\frac{dy}{dx}=\frac{1}{ax+by+c}$ where a; b; c are all non-zero real numbers is:

a) Linear in y b) Linear in x

c) Linear in both x and y d) Homogeneous equation

125. The general solution of the D.E $\frac{dy}{dx}=\frac{1}{x+y+1}$ is (k, c are arbitrary constant)

a) $y=\log_e\left(\frac{x+y+2}{k}\right)$ b) $x=\log_e\left(\frac{x+y+2}{k}\right)$

c) $x = ce^{y} + y + 2$ d) $y = ce^{x} + x + 2$

126. The general solution of $\sin y \frac{dy}{dx} = \cos y\left(1 - x\cos y\right)$ is

a) $\sec y = x - 1 - ce^{x}$ b) $\sec y = x + 1 + ce^{x}$

c) $\sec y = x + e^{x} + c$ d) $\sec y = x - e^{x} + c$

127. The solution of the DE $\sqrt{1-y^2}dx + xdy - \sin^{-1} ydy = 0$ is

a) $x = \sin^{-1} y - 1 + ce^{-\sin^{-1} y}$ b) $y = x\sqrt{1-y^2 + \sin^{-1} ydy + c}$

c) $x = 1 + \sin^{-1} y + ce^{\sin^{-1} y}$ d) $y = \sin^{-1} y - 1 + \sqrt{1-y^2}x + c$

128. The solution of the DE: $\left(1+y^2\right) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$ is

a) $xe^{2\tan^{-1} y} - e^{\tan^{-1} y} = c$ b) $\left(x-2\right)e^{-\tan^{-1} y} = c$

c) $2xe^{\tan^{-1} y} - e^{2\tan^{-1} y} = c$ d) $xe^{\tan^{-1} y} + e^{2\tan^{-1} y} = c$

129. The general solution of the D.E $\frac{dx}{dy} + \frac{x}{y} = x^2$ is

a) $\frac{1}{y} = cx - y\log x$ b) $\frac{1}{x} = cy + x\log x$

c) $\frac{1}{x} = cy - y\log y$ d) $\frac{1}{y} = \left(x + y\log x\right)$

130. The solution of the D.E: $\left(x + 2y^3\right)\frac{dy}{dx} = y$ is

a) $x = y^3 + c$ b) $x = y^3 + cy$ c) $y = x^3 + c$ d) $y = x^3 + cx + d$

131. The solution of the D.E $\left(x - 4y^3\right)\frac{dy}{dx} - y = 0\left(y > 0\right)$ is

a) $x = y^3 + cy$ b) $x + 2y^3 = cy$ c) $y = x^3 + cx$ d) $y + 2x^3 = cx$

132. The solution of $\cos y + \left(x\sin y - 1\right)\frac{dy}{dx} = 0$ is

a) $x\sec y = \tan y + c$ b) $\tan y - \sec y = cx$

c) $\tan y + \sec y = cx$ d) $x\sec y + \tan y = c$

133. The solution of the D.E $y' = \frac{1}{e^{-y} - x}$ si

a) $x = e^{-y}\left(y + c\right)$ b) $y + e^{-y} = x + c$

c) $x = e^{y}\left(y + c\right)$ d) $x + y = e^{-y} + c$

134. The solution of $\frac{dy}{dx}+\frac{1}{x}=\frac{e^y}{x^2}$ is

a) $2x=\left(1+cx^2\right)e^y$ b) $x=\left(1+cx^2\right)e^y$

c) $2x^2=\left(1+cx^2\right)e^{-y}$ d) $x^2=\left(1+cx^2\right)e^{-y}$

135. If $x^2y-x^3\frac{dy}{dx}=y^4\cos x$ then x^3y^{-3} is equal to

a) sinx   b)2sinx+c   c) 3sinx+c   d) 3cosx+c

136. If $y=A(x)e^{-\int Pdx}$ is a solution of $\frac{dy}{dx}+p(x)y=Q(x)$ then $A'(x)=$

a) $e^{\int pdx}$ b) $Q(x)e^{-\int pdx}$ c) $\int Q(x)e^{\int pdx}+c$ d) $Q(x)e^{\int pdx}$

137. The solution of the D.E: $\frac{dy}{dx}+2y\tan x=\sin x$ satisfying y = 0 when $x=\frac{\pi}{3}$ is

a) $y=2\sin^2 x+\cos x-2$ b) $y=2\sin^2 x-\cos x-2$

c) $y=2\cos^2 x-\sin x+2$ d) $y=2\cos x-\sin^2 x-1$

138. If $x\log x\frac{dy}{dx}+y=\log x^2$ and $y(e)=0$ then $y\left(e^2\right)=$

a) 0   b) 1   c) $\frac{1}{2}$ d) $\frac{3}{2}$

139. If of $\frac{dy}{dx}=\frac{x+y+1}{x+1}$ is

a) $\frac{1}{y+1}$ b) $\frac{1}{x+1}$ c) $\log(x+1)$ d) $\log(y+1)$

LEVEL – II; QUESTION BANK

140. Let $y(x)$ be the solution of the D.E $x\log x\frac{dy}{dx}+y=2x\log x\,(x\geq 1)$; then $y(e)=$

a) 2   b) 2e   c) e   d) 0

141. If $y=y(x)$ is the solution of the D.E : $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1)=1$; then

$y\left(\frac{1}{2}\right)=$

a) $\frac{7}{64}$ b) $\frac{1}{4}$ c) $\frac{49}{16}$ d) $\frac{13}{16}$

142. The solution of the D.E $x\frac{dy}{dx}+2y=x^2\left(x\neq 0\right)$ with $y\left(1\right)=1$ is

a) $y=\frac{4}{5}x^3+\frac{1}{5x^2}$ b) $y=\frac{x^3}{5}+\frac{1}{5x^2}$

c) $y=\frac{x^2}{4}+\frac{3}{4x^2}$ d) $y=\frac{3x^2}{4}+\frac{1}{4x^2}$

143. Solution of the D.E $\cos x dy=y\left(\sin x-y\right)dx; 0<x<\frac{\pi}{2}$ is:

a) $y\sec x=\tan x+c$ b) $\sec x=\left(\tan x+c\right)y$

c) $\tan x=\left(\sec x+c\right)y$ d) $\sec x=\left(\tan x+c\right)y$

144. Let $y=y\left(x\right)$be the solution curve of the D.E $\sin x\frac{dy}{dx}+y\cos x=4x; x\in\left(0,\pi\right)$.

If $y\left(\frac{\pi}{2}\right)=0$ then $y\left(\frac{\pi}{6}\right)=$

a) $\frac{-8\pi^2}{9\sqrt{3}}$ c) $\frac{-8\pi^2}{9}$ c) $\frac{-4\pi^2}{9}$ d) $\frac{4\pi^2}{9\sqrt{3}}$

145. Let $y=y\left(x\right)$ be the solution curve of the D.E $\left(y^2-x\right)\frac{dy}{dx}=1$ satisfying $y\left(0\right)=1$; this curve intersects the x–axis at a point whose obscissa is

a) $2+e$ b) 2 c) $2-e$ d) –e

146. If for $x\geq 0; y=y\left(x\right)$ is the solution of the D.E

$\left(1+x\right)dy=\left[\left(1+x\right)^2+\left(y-3\right)\right]dx; y\left(2\right)=0$ then $y\left(3\right)$ is equal to _________

147. Let I be the purchase value of an equipment and $v'\left(t\right)$ be the value after it has been used for t years the value $v\left(t\right)$ depreciates at a rate given by D.E $\frac{dv\left(t\right)}{dt}=-K\left(T-E\right)$, where k > 0 is a constant and T is the total life in years of the equipment.

Then the scrap value V(T) of the equipment is:

a) $I-\frac{KT^2}{2}$ b) $I-\frac{K\left(T-E\right)^2}{2}$ c) e^{-KT} d) $T^2-\frac{1}{K}$

148. The population $p\left(t\right)$ at time t of a certain mouse species satisfies the D.E

$\frac{dp\left(t\right)}{dt}=0.5p\left(t\right)-450$. If p(0) = 850, then the time at which the population becomes zero is:

a) $2l\,n(18)$ b) $l\,n(9)$ c) $\frac{1}{2}l\,n(18)$ d) $l\,n(18)$

149. At present; a firm is manufacturing 2000 items. It is estimated that the rate of change of production p w.r.t additional numbe of workers x is given by $\frac{dp}{dx}=100-12\sqrt{x}$ If the firm exployes 20 more workers then the new level of production of items is

a) 250 b) 300 c) 3500 d) 4500

150. Let the population of rabbits surviving at time t be governed by the D.E

$\frac{dp(t)}{dt}=\frac{1}{2}p(t)-200$

a) $600-500e^{\frac{t}{2}}$ b) $400-300e^{-\frac{t}{2}}$ c) $400-300e^{\frac{t}{2}}$ d) $300-200e^{-\frac{t}{2}}$

151. If a curve $y=f(x)$ passes through the point (1, –1) and satisfies the D.E: $y(1+xy)=xdy$ then $f\left(-\frac{1}{2}\right)$ equal to

a) $\frac{2}{5}$ b) $\frac{4}{3}$ c) $-\frac{2}{5}$ d) $-\frac{4}{5}$

152. **Integer value correct type**

(1) Let $y'(x)+y(x)g'x=g(x)g'(x); y(0)=0; x\in R$ where $f'(x)$ denotes $\frac{d\,f(x)}{dx}$ and $g(x)$ is a given non-constant differentiable function on R with $g(0)=g(2)=0$. Then the value of $y(2)$ is_____

(2) Let $f:R\to R$ be differentiable function with $f(0)=0$. If $y=f(x)$ satisfies the differential equation $\frac{dy}{dx}=(2+5y)(5y-2)$; then the value of $\lim_{x\to-\infty} f(x)$ is ____

(3) Let $f:R\to R$ be a differentiable function with $f(0)=1$ and satisfying the equation $f(x+y)=f(x)f'(y)+f'(x)f(y)$ for all $x, y\in R$, Then the value of $\log_e(f(4))$ is____

MISCELLANEOUS PROBLEMS:

153. The family of curves represented by the general solution of $y'=\frac{y}{2x}$ contains

a) circles b) ellipses c) hyperbolas d) parabolas

154. If at any point on the curve $y = f(x)$ the length of the subnormal is a constant then the curve will be a lan:

a) circle b) ellipse c) parabola d) straight line

155. The family of curves in which sub-tangent at any point to any curve is double the abscissa is given by

a) $x = cy^2$ b) $y = cx^2$ c) $x^2 = cy^2$ d) $y^2 = cx^2$

156. The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with

a) variable radii and a fixed centre at (0, 1)

b) variable radii and a fixed centre at (0, –1)

c) fixed radius i and variable centres along the x–axis

d) fixed radius i and variable centres along the y–axis

157. At any point on a curve, the slope of the tangent is equal to the sum of abscissa and the product of ordinate and abscissa of that point. It the curve passes through (0, 1) then the equation of the curve is

a) $y = 2e^{\frac{x^2}{2}} - 1$ b) $y = 2e^{x^2}$ c) $y = e^{-x^2}$ d) $y = 2e^{-x^2} - 1$

158. The equation of family of curves whose sub-tangents are of constant length k is:

a) $y = ce^{\frac{x}{k}}$ b) $\frac{ce^x}{k} = y$ c) $y^k = cx$ d) $y = c$

159. Equation of the curve for which sub-normal at a pt is twice the square of the ordinate is:

a) $\log y = x^2 + c$ b) $y = ce^{2x}$ c) $y = ce^{-2x}$ d) $2y = ce^x$

160. If $y(x)$ is a solution of the D.E: $\frac{dy}{dx} + 3y = 2$, then $\lim_{x \to \infty} y(x)$ is equal to

a) 1 b) 0 c) $\frac{3}{2}$ d) $\frac{2}{3}$

161. Consider the D.E, $ydx - (x + y^2)dy = 0$. If for y= 1, x takes value 1, then the value of x when $y = 4$ is

a) 9 b) 16 c) 36 d) 64

162. Consider the D.E $y^2dx+\left(x-\frac{1}{y}\right)dy=0$. If $y(1)=1$ then x is given by:

a) $4-\frac{2}{y}-\frac{e^{\frac{1}{y}}}{e}$ b) $3-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$ c) $1+\frac{1}{y}-\frac{e^{\frac{1}{y}}}{e}$ d) $1-\frac{1}{y}+\frac{e^{\frac{1}{y}}}{e}$

163. If the general solution of the DE $y'=\frac{y}{x}+\phi\left(\frac{x}{y}\right)$; for some function ϕ is given by $y\log|cx|=x$ where c is an arbitrary constant, then $\phi(2)$ is equal to

a) 4 b) $\frac{1}{4}$ c) –4 d) $\frac{1}{4}$

164. If $\frac{dy}{dx}+y\tan x=sn2x$ and $y(0)=1$; then $y(r)$ is equal to

a) 1 b) –1 c) –5 d) 5

165. The general solution of the DE $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$ is

a) $y\sqrt{\tan x}=x+c$ b) $y\sqrt{\cot x}=x+c$

c) $y\sqrt{\tan x}=\cot x+c$ d) $y\sqrt{\cot x}=x+c$

166. A normal is drawn at a point $p(x, y)$ of a curve. If meets the x–axis at Q. If PQ is of constant length k; then S.T the differential equation describing such curve is $y\frac{dy}{dx}=\pm\sqrt{k^2-y^2}$. Find the equation of such a curve passing through (0, k)

167. A curve $y=f(x)$ passes through (1, 1) and at p(x, y), tangent cuts the x–axis and y–axis at A and B respectively such that $B_P : A_P = 3:1$, then

a) equation of curve is $xy'-3y=0$

b) normal at (1, 1) is $x+3y=4$

c) curves passes through $\left(2, \frac{1}{8}\right)$

d) equation of curve is $xy'+3y=0$

168. If $y(x)$ satisfies the D.E : $y'-y\tan x=2x\sec x$ and $y(0)=0$, then

a) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ b) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

c) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ d) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$