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II

Total No. of Questions : 24

Total No. of Printed Pages : 3

Part - III

MATHEMATICS - PAPER - I(A)

(English Version)

Time : 3 Hours **Max. Marks : 75**

Note : *This question paper consists of three Sections-A, B and C.*

SECTION - A

1. Very Short Answer Type Questions : **10x2=20**

(i) Answer *all* the questions.

(ii) Each question carries *two marks.*

1. If $f(x)=2x-1, g(x)=\frac{x+1}{2}$ for all $x\in R$, then find $(gof)(x)$.

2. Find the domain of the real valued function $f(x)=\sqrt{x^2-25}$.

3. Define symmetric matrix and give an example.

4. If $A=\begin{bmatrix}1 & 0 & 0\\ 2 & 3 & 4\\ 5 & -6 & x\end{bmatrix}$ and $\det A=45$ then find x.

5. If $\overrightarrow{OA}=\bar{i}+\bar{j}+\bar{k}, \overrightarrow{AB}=3\bar{i}-2\bar{j}+\bar{k}, \overrightarrow{BC}=\bar{i}+2\bar{j}-2\bar{k}$ and $\overrightarrow{CD}=2\bar{i}+\bar{j}+3\bar{k}$ then find the vector $\overrightarrow{OD}$.

6. Find the vector equation of the plane passing through the points $\bar{i}-2\bar{j}+5\bar{k}, -5j-\bar{k}$ and $-3\bar{i}+5\bar{j}$.

7. Let $\bar{a}$ and $\bar{b}$ be non-zero, non-collinear vectors. If $|\bar{a}+\bar{b}|=|\bar{a}-\bar{b}|$, then find the angle between $\bar{a}$ and $\bar{b}$.

8. Find the value of $\sin330^\circ \cdot \cos120^\circ + \cos210^\circ \cdot \sin300^\circ$

9. If $A - B = \frac{3\pi}{4}$, then show that $(1 - \tan A)(1 + \tan B) = 2$.

10. Show that $\tanh^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}\log_e^3$.

SECTION - B

5×4=20

II. Short Answer Type Questions :
(i) Answer *any five* questions.
(ii) Each question carries *four marks*.

11. If $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$ is a non-singular matrix, then prove that A is invertible and

$A^{-1} = \frac{AdjA}{detA}$.

12. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}$, $2\bar{i} + 3\bar{j} - 4\bar{k}$, $-\bar{i} + \bar{j} + 2\bar{k}$ and $4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = \frac{-146}{17}$.

13. If $\bar{a} = 2\bar{i} + \bar{j} - \bar{k}$, $\bar{b} = -\bar{i} +$ $\cdot(\bar{b} \times \bar{c})$.

14. For $A \in R$, prove that $\cos A \cos\left(\frac{\pi}{3} + A\right)\cos\left(\frac{\pi}{3} - A\right) = \frac{1}{4}\cos3A$ and hence deduce that

$\cos\frac{\pi}{9} \cdot \cos\frac{2\pi}{9} \cdot \cos\frac{3\pi}{9} \cdot \cos\frac{4\pi}{9} = \frac{1}{16}$.

15. Solve $\cot^2 x - (\sqrt{3}+1)\cot x + \sqrt{3} = 0; 0 < x < \frac{\pi}{2}$.

16. Prove that $\text{Tan}^{-1}\frac{1}{2} + \text{Tan}^{-1}\frac{1}{5} + \text{Tan}^{-1}\frac{1}{8} = \frac{\pi}{4}$.

17. In a ΔABC, show that $\frac{b^2 - c^2}{a^2} = \frac{\sin(B-C)}{\sin(B+C)}$.

SECTION C

III. Long Answer Type Questions : **5x7=35**

(i) Answer *any five* questions.

(ii) Each question carries *Seven marks*.

18. Let $f: A \to B$ be a bijection. Then prove that $fof^{-1} = I_B$ and $f^{-1}of = I_A$.

19. By using Mathematical Induction, to prove the statement :

$$\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\ldots\ldots\ldots+\frac{1}{(2n-1)(2n+1)}=\frac{n}{2n+1},\ \forall n\in N.$$

20. Show that $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}^2 = \begin{vmatrix} 2bc-a^2 & c^2 & b^2 \\ c^2 & 2ac-b^2 & a^2 \\ b^2 & a^2 & 2ab-c^2 \end{vmatrix} = (a^3+b^3+c^3-3abc)^2$

21. Solve $2x-y+3z=8,\ -x+2y+z=4,\ 3x+y-4z=0$ by using matrix inversion method.

22. Find the shortest distance between the Skew lines.

$\bar{r}=(6\bar{i}+2\bar{j}+2\bar{k})+t(\bar{i}-2\bar{j}+2\bar{k})$ and $\bar{r}=(-4\bar{i}-\bar{k})+s(3\bar{i}-2\bar{j}-2\bar{k})$.

23. If $A+B+C=\frac{\pi}{2}$, then prove that $\cos2A+\cos2B+\cos2C=1+4\sin A\sin B\sin C$.

24. In ΔABC, prove that $r+r_1+r_2-r_3=4R\cos C$.