Part - III
MATHEMATICS - PAPER - I(B)
(English Version)

Time : 3 Hours   Max. Marks : 75

Note : *This question paper consists of three Sections A, B and C.*

SECTION - A

I.   Very short answer type questions.   10x2=20

   (i)   Attempt *all* questions.

   (ii)   Each question carries *two marks*.

1.   Transform the equation $3x+4y+12=0$ into
      (i)   slope-intercept form
      (ii)   intercept form

2.   Find the value of p, if the straight lines $x+p=0$, $y+2=0$ and $3x+2y+5=0$ are concurrent.

3.   If $(3, 2, -1)$, $(4, 1, 1)$ and $(6, 2, 5)$ are three vertices and $(4, 2, 2)$ is the centroid of a tetrahedron, find the fourth vertex.

4.   Find the equation of the plane whose intercepts on X, Y, Z-axes are 1, 2, 4 respectively.

5.   Compute $\lim\limits_{x \to 0} \dfrac{e^x - 1}{\sqrt{1+x} - 1}$.

6.   Compute $\lim\limits_{x \to \infty} \dfrac{8|x| + 3x}{3|x| - 2x}$.

7.   If $f(x) = \sin(\log x)$, $(x > 0)$ find $f'(x)$.

8.   Find the derivative of $\cos(\log x + e^x)$.

9.   Find the slope of the normal to the curve $x = a\cos^3\theta$, $y = a\sin^3\theta$ at $\theta = \dfrac{\pi}{4}$.

10.   Find the intervals on which $f(x) = x^2 - 3x + 8$ is increasing or decreasing.

SECTION - B

II. Short answer type questions. 5x4=20

   (i)   Attempt *any five* questions.

   (ii)   Each question carries *four marks*.

11. If the distance from P to the points (2, 3) and (2, −3) are in the ratio 2 : 3, then find the equation of the locus of P.

12. A(5, 3) and B(3, −2) are two fixed points. Find the equation of the locus of P, so that the area of triangle PAB is 9.

13. A straight line with slope 1 passes through Q(−3, 5) and meets the straight line $x + y − 6 = 0$ at P. Find the distance PQ.

14. If f, given by $f(x) = \begin{cases} k^2x - k & \text{if } x \geq 1 \\ 2 & \text{if } x < 1 \end{cases}$ is a continuous function on R, then find the values of k.

15. Find $\dfrac{dy}{dx}$ for the function $x = \dfrac{a(1 - t^2)}{1 + t^2}, y = \dfrac{2bt}{1 + t^2}$.

16. Find the angle between the curve $2y = e^{\frac{-x}{2}}$ and Y-axis.

17. Show that $\dfrac{x}{1 + x} < \ln(1 + x) < x, \forall x > 0$.

SECTION - C

III. Long answer type questions. 5x7=35

   (i)   .Attempt *any five* questions.

   (ii)   Each question carries *seven marks*.

18. If p and q are the lengths of the perpendiculars from the origin to the straight lines $x\sec\alpha + y\csc\alpha = a$ and $x\cos\alpha - y\sin\alpha = a\cos2\alpha$, prove that $4p^2 + q^2 = a^2$.

19. Show that the pairs of straight lines $6x^2 - 5xy - 6y^2 = 0$ and $6x^2 - 5xy - 6y^2 + x + 5y - 1 = 0$ form a square.

20. Find the values of k if the lines joining the origin to the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.

21. Find the direction cosines of two lines which are connected by the relations $l + m + n = 0$ and $mn - 2nl - 2lm = 0$.

22. If $x^y = y^x$ then $\dfrac{dy}{dx} = \dfrac{y\,(x\,\log y - y)}{x\,(y\,\log x - x)}$.

23. Show that the tangent at $P(x_1, y_1)$ on the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ is $y\,y_1^{\frac{-1}{2}} + x\,x_1^{\frac{-1}{2}} = a^{\frac{1}{2}}$.

24. The profit function $P(x)$ of a company, selling x items per day is given by $P(x) = (150 - x)x - 1600$. Find the number of items that the company should sell for maximum profit. Also find the maximum profit.

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